Mechanics: Physical Quantities

1 Dimensions:

The dimension of a physical quantity is defined as the power to which the base quantities are raised to express the physical quantity.
Representing the base quantities mass, length and time as [M], [L] and [T] respectively, we can write,

$$\text{velocity} = \frac{\text{displacement}}{\text{time taken}}$$
$$\text{or, } [v] = \frac{[L]}{[T]} = [LT^{-1}] = [M^0LT^{-1}]$$

The dimensional formula is defined as the expression of the physical quantity interms of its basic unit with proper dimensions. For example,
dimensional formula of force is $[MLT^{-2}]$
An equation containing physical quantities with dimensional formula is known as dimensional equation. For example.
dimensional equation of $v = u + at$ is

$$[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T]$$
$$\text{or, } [LT^{-1}] = [LT^{-1}] + [LT^{-1}]$$
$$\text{or, } [M^0LT^{-1}] = [M^0LT^{-1}]$$

Dimensional formula of some physical quantities.

$$Acceleration = \frac{Velocity}{Time} = \frac{[M^0LT^{-1}]}{[T]} = [M^0LT^{-2}]$$

$$Pressure = \frac{Force}{Area} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

$$Work = Force \times Distance = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$$

$$Power = \frac{Work}{Time} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

Mechanics

S. N.	Quantity	Unit	Dimension
(1)	Velocity or speed (v)	m/s	$[M^0L^1T^{-1}]$
(2)	Acceleration (a)	m/s^2	$[M^0LT^{-2}]$
(3)	Momentum (P)	$kg\text{-}m/s$	$[M^1L^1T^{-1}]$
(4)	Impulse (I)	$Newton\text{-}sec$ or $kg\text{-}m/s$	$[M^1L^1T^{-1}]$
(5)	Force (F)	$Newton$	$[M^1L^1T^{-2}]$
(6)	Pressure (P)	$Pascal$	$[M^1L^{-1}T^{-2}]$
(7)	Kinetic energy (E_K)	$Joule$	$[M^1L^2T^{-2}]$
(8)	Power (P)	$Watt$ or $Joule/s$	$[M^1L^2T^{-3}]$
(9)	Density (d)	kg/m^3	$[M^1L^{-3}T^0]$
(10)	Angular displacement (θ)	$Radian$ ($rad.$)	$[M^0L^0T^0]$
(11)	Angular velocity (ω)	$Radian$/sec	$[M^0L^0T^{-1}]$
(12)	Angular acceleration (α)	$Radian/sec^2$	$[M^0L^0T^{-2}]$
(13)	Moment of inertia (I)	$kg\text{-}m^2$	$[M^1L^2T^0]$
(14)	Torque (τ)	$Newton\text{-}meter$	$[M^1L^2T^{-2}]$

Heat

S. N.	Quantity	Unit	Dimension
(1)	Temperature (T)	*Kelvin*	$[M^0L^0T^0\theta^1]$
(2)	Heat (Q)	*Joule*	$[ML^2T^{-2}]$
(3)	Specific Heat (c)	*Joule / kg-K*	$[M^0L^2T^{-2}\theta^{-1}]$
(4)	Thermal capacity	*Joule / K*	$[M^1L^2T^{-2}\theta^{-1}]$
(5)	Latent heat (L)	*Joule / kg*	$[M^0L^2T^{-2}]$
(6)	Gas constant (R)	*Joule / mol-K*	$[M^1L^2T^{-2}\theta^{-1}]$
(7)	Boltzmann constant (k)	*Joule / K*	$[M^1L^2T^{-2}\theta^{-1}]$
(8)	Coefficient of thermal conductivity (K)	*Joule / m-s-K*	$[M^1L^1T^{-3}\theta^{-1}]$
(9)	Stefan's constant (σ)	*Watt / m^2-K^4*	$[M^1L^0T^{-3}\theta^{-4}]$
(10)	Wien's constant (b)	*Meter-K*	$[M^0L^1T^0\theta]$
(11)	Planck's constant (h)	*Joule-s*	$[M^1L^2T^{-1}]$
(12)	Coefficient of Linear Expansion (□)	*Kelvin*$^{-1}$	$[M^0L^0T^0\theta^{-1}]$
(13)	Mechanical eq. of Heat (J)	*Joule / Calorie*	$[M^0L^0T^0]$
(14)	Vander wall's constant (a)	*Newton-m^4*	$[ML^5T^{-2}]$
(15)	Vander wall's constant (b)	m^3	$[M^0L^3T^0]$

Electricity

S. N.	Quantity	Unit	Dimension
(1)	Electric charge (q)	*Coulomb*	$[M^0L^0T^1A^1]$
(2)	Electric current (I)	*Ampere*	$[M^0L^0T^0A^1]$
(3)	Capacitance (C)	*Coulomb / volt or Farad*	$[M^{-1}L^{-2}T^4A^2]$
(4)	Electric potential (V)	*Joule / coulomb*	$M^1L^2T^{-3}A^{-1}$

Principle of Homogeneity:

This principle says that the dimensions of base quantities of all terms on both sides of a dimensional equation must be same. Consider three different quantities A, B and C such that

$$A + B = C$$

Then, according to the principle of homogeneity, dimensions of A = dimensions of B = dimensions of C.
For example,

dimensional equation of $v = u + at$ is

$$[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T]$$
$$\text{or, } [LT^{-1}] = [LT^{-1}] + [LT^{-1}]$$

Uses of Dimensional Equations:
1) To check the correctness of a physical relation.
2) To convert one system of units into another
3) To derive the relation between various physical quantities.
4) To determine the dimensions of physical constants appearing in a physical relation.

Questions:

Q. Check the correctness of the formula $t = 2\pi\sqrt{\frac{l}{g}}$ using dimensional analysis, where t is the time period of simple pendulum, l is the length of simple pendulum and g is the acceleration due to gravity.

Solution:

$$\text{Given, } t = 2\pi\sqrt{\frac{l}{g}} \qquad \ldots(1)$$

We know,
the dimensional formula of $t = [T]$
the dimensional formula of $l = [L]$
the dimensional formula of $g = [LT^{-2}]$
Since, the constant 2π has no dimension, it will not appear into the dimensional equation.
Putting the dimension of the physical quantities in equation (1), we get

$$
\begin{aligned}
[T] &= \left(\frac{[L]}{[LT^{-2}]}\right)^{1/2} \\
\text{or, } [T] &= [T^2]^{1/2} \\
\text{or, } [T] &= [T] \\
\text{or, } [M^0L^0T] &= [M^0L^0T]
\end{aligned}
$$

Hence, the given formula is dimensionally correct.

Q. Check the correctness of the formula $v = u + at^2$ using dimensional analysis, where u and v are initial and final velocity, a is acceleration and t is the time.

Q. Check the correctness of the formula $F = mv^2r$ using dimensional analysis, where F is the force, m is the mass, v is the velocity and r is the radius of orbit.

Q. Check the correctness of the formula $v = \sqrt{2gr}$ using dimensional analysis, where v is the escape velocity, g is the acceleration due to gravity and r is the radius of orbit.

Q. Convert 10 ergs in Joule

Solution:

ergs and Joules is the unit of Work in CGS system and Joule is the unit of Work in SI system.

The dimensional formula of work = $[ML^2T^{-2}]$

Now, consider n_1 be the numerical value in CGS system and n_2 be in SI units.

In the given system (CGS units)	New System (SI units)
n_1 = 10	n_2 = ?
M_1 = 1 gm	M_2 = 1 kg
L_1 = 1 cm	L_2 = 1 m
T_1 = 1 sec	T_2 = 1 sec

then, we have

$$
\begin{aligned}
n_1[M_1L_1^2T_1^{-2}] &= n_2[M_2L_2^2T_2^{-2}] \\
\text{or, } 10[M_1L_1^2T_1^{-2}] &= n_2[M_2L_2^2T_2^{-2}] \\
\text{or, } n_2 &= \left(\frac{[M_1]}{[M_2]}\right)\left(\frac{[L_1]}{[L_2]}\right)^2\left(\frac{[T_1]}{[T_2]}\right)^{-2} n_1 \\
&= \left(\frac{1gm}{1kg}\right)\left(\frac{1cm}{1m}\right)^2\left(\frac{1sec}{1sec}\right)^{-2} n_1 \\
n_2 &= \frac{1}{1000}\times\left(\frac{1}{100}\right)^2 \times 1\times 10 \\
\therefore n_2 &= 10^{-6}
\end{aligned}
$$

hence,

$$10\text{ergs} = 10^{-6}\text{Joule}$$

Q. Convert 5 Newton into dynes.

Q. Convert 19.3 gm/cm^3 into kg/m^3.

Q. The time period of a simple pendulum depends on its mass, length and acceleration due to gravity. Find the expression for the time period.

Solution:

According to question,

$$
\begin{aligned}
t &\propto m^a \\
t &\propto l^b \\
t &\propto g^c
\end{aligned}
$$

combining these equations, we get

$$
\begin{aligned}
t &\propto m^a l^b g^c \\
\text{or, } t &= k m^a l^b g^c \qquad \ldots\ldots.(1)
\end{aligned}
$$

where k is the dimensionless constant of proportionality, and a, b, c are indices to be determined.
Now, substituting the dimensions in both sides of equation (1),

$$\begin{aligned}[M^0L^0T] &= [M]^a[L]^b[LT^{-2}]^c \\ &= [M^aL^{b+c}T^{-2c}]\end{aligned}$$

Comparing, we get

$$\begin{aligned} a &= 0 \\ b+c &= 0 \\ -2c &= 1 \end{aligned}$$

solving, we get, $a = 0, b = \frac{1}{2}$ and $c = \frac{-1}{2}$
substituting these values in equation (1), we have

$$\begin{aligned} t &= km^0l^{1/2}g^{-1/2} \\ \text{or, } t &= k\sqrt{\frac{l}{g}} \qquad .....(2) \end{aligned}$$

The value of $k = 2\pi$ is found by experiment.
Thus,

$$t = 2\pi\sqrt{\frac{l}{g}} \qquad .....(3)$$

This is the expression for time period of simple pendulum.

Q. Obtain the dimensions of Gravitational Constant.
Solution:
Newton's law of Gravitation gives force between two bodies

as

$$F = \frac{Gm_1m_2}{d^2} \qquad .....(1)$$

We have, the dimensional formula of $F = [MLT^{-2}]$
the dimensional formula of $d = [L]$
the dimensional formula of $m = [M]$
then,
dimensional formula of $G = \frac{[MLT^{-2}][L]^2}{[M][M]} = [M^{-1}L^3T^{-2}]$

Hence, the dimensional formula of G are -1 in mass, 3 in length and -2 in time.

Q. Obtain the dimensions of Specific Heat Constant.

2 Limitations of Dimensional Analysis

1. It doesnot give informaton of dimensionless constant.
2. It doesnot distinguish between scalar and vector.
3. It cannot derive relation if the physical quantity depends upon more than three terms.
4. It is used only in case of power function.
5. It cannot derive physical relation involving trigonometric, exponential or logrithmic function.

3 Precision and Accuracy

Precision is the degree to which the observed values are least scattered. It means how close the different measurements are. An instrument with less tolerance can give precise data.

Similarly, during measurement the quality of data depends upon its accuracy. Accuracy is the degree to which the observed value approaches the true value. A high quality measurement should be both precise and accurate. Precision is not same as accuracy. A clock without seconds hand can be accurate but not precise.
There are four factors that limits the precision are:

1. constancy of the quality being measured
2. limitation of human observations
3. limitations of the instrument being used
4. error of calibration of the instrument.