1.4) 1) $\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$

$3 \times 2 \quad (3,)$

So, we have 2 vectors $\in \mathbb{R}^3$, but 3 weights, so, product is not possible

2) $\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$, similarly, 1 vector & 2 weights, so product not possible

3) $\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 2\begin{bmatrix} 6 \\ -4 \\ 7 \end{bmatrix} + (-3)\begin{bmatrix} 5 \\ -3 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 12 \\ -8 \\ 14 \end{bmatrix} + \begin{bmatrix} -15 \\ 9 \\ -18 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$

4) $\begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -4 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

5) $\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$

$= 5\begin{bmatrix} 5 \\ -2 \end{bmatrix} - \begin{bmatrix} 1 \\ -7 \end{bmatrix} + 3\begin{bmatrix} -8 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$

6) $\begin{bmatrix} 7 & -3 \\ 2 & 1 \\ 9 & -6 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$

$= -2\begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix} - 5\begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$

7) $\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

8. $z_1\begin{bmatrix}4\\-2\end{bmatrix}+z_2\begin{bmatrix}-4\\5\end{bmatrix}+z_3\begin{bmatrix}-5\\4\end{bmatrix}+z_4\begin{bmatrix}3\\0\end{bmatrix}=\begin{bmatrix}4\\13\end{bmatrix}$

$\Rightarrow \begin{bmatrix}4 & -4 & -5 & 3\\-2 & 5 & 4 & 0\end{bmatrix}\begin{bmatrix}z_1\\z_2\\z_3\\z_4\end{bmatrix}=\begin{bmatrix}4\\13\end{bmatrix}$

9) $3x_1 + x_2 - 5x_3 = 9$

$x_2 + 4x_3 = 0$

$\Rightarrow x_1\begin{bmatrix}3\\0\end{bmatrix}+x_2\begin{bmatrix}1\\1\end{bmatrix}+x_3\begin{bmatrix}-5\\4\end{bmatrix}=\begin{bmatrix}9\\0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}3 & 1 & -5\\0 & 1 & 4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}9\\0\end{bmatrix}$

10) $8x_1 - x_2 = 4$

$5x_1 + 4x_2 = 1$

$x_1 - 3x_2 = 2$

$\Rightarrow x_1\begin{bmatrix}8\\5\\0\end{bmatrix}+x_2\begin{bmatrix}-1\\4\\-3\end{bmatrix}=\begin{bmatrix}4\\1\\2\end{bmatrix}$

$\Rightarrow \begin{bmatrix}8 & -1\\5 & 4\\0 & -3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}4\\1\\2\end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix}, \; b = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ soln.

12> $A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & -2 & -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3/5 \\ -4/5 \\ 1 \end{bmatrix}$

(5) $u = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$

so, u in plan (?) is span(A),

so, need to check if u is linear combination of columns of A,

i.e., check if soln. exist for augmented matrix $[A \; u]$

$$\Rightarrow \begin{bmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ -2 & 6 & 4 \\ 3 & -5 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 0 & -8 & -12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 0 & 0 & 0 \end{bmatrix}$$, so soln exists, so $\vec{u}$ in span(A)

14) Just as 13, find the soln of augmented matrix $[A \ \vec{u}]$

$$\Rightarrow \begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 5 & 8 & 7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & -7 & 7 & -8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -29 \end{bmatrix}$$ — no soln, inconsistent

so, $\vec{u}$ not in Span(A).

15) lets, try find the soln for $[A \ b]$

$$\Rightarrow \begin{bmatrix} 2 & -1 & b_1 \\ -6 & 3 & b_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -1 & b_1 \\ 0 & 0 & b_2+3b_1 \end{bmatrix}$$ no soln,

for soln to exist; $b_2 + 3b_1 = 0$

$b_2 = -3b_1$

16) $A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$ $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -4 & b_1 \\ -3 & 2 & 6 & b_2 \\ 5 & -1 & -8 & b_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & b_2+3b_1 \\ 0 & 14 & 12 & b_3-5b_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & b_2+3b_1 \\ 0 & 0 & 0 & b_3-5b_1+2b_2+6b_1 \end{bmatrix}$$

so, no soln if $b_1+2b_2+b_3$ is non zero,

for soln to exist

$b_1+2b_2+b_3 = 0$

17) $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -4 & 2 & -8 \\ 0 & -6 & 3 & -7 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ \boxed{0 \;\; 0 \;\; 0 \;\; 0} \\ 0 & 0 & 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2+b_1 \\ b_3+2b_2+2b_1 \\ b_4-2b_1+\cancel{3b_3} \\ +3b_2+3 \end{bmatrix}$$

or,

since 3rd row is $[0 \;\; 0 \;\; 0 \;\; 0]$ in augmented matrix, $[0 \;\; 0 \;\; 0 \;\; 0 \;\; b]$ for some b.

$b_4 - 2b_1 + 3b_2 + 3b_1$

no soln $b_1 + 3b_2 + b_4 \neq 0$

there exist some $b \in \mathbb{R}^3$ for what it is true

18)
$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & 2 & -2 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ \boxed{0 \quad 0 \quad 0 \quad 0} \\ 0 & 0 & -4 & 13 \end{bmatrix}$$

$\Rightarrow$ in augmented form,

$\begin{bmatrix} 0 & 0 & 0 & 0 & y_j \end{bmatrix}$ is possible for $y_j \neq 0$,

so, $Bx = y$ don't have a solution for each y in R^4

19) No, not all vectors in R^4 can be written as a linear combination of columns of A.

colons of A <u>do not</u> span R^4

20) No, B is in R^4, so <u>no</u>, the colums of B do not span R^4?

21) $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

so, $[v_1 \; v_2 \; v_3] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ \boxed{0 \quad 0 \quad 0} \end{bmatrix}$$, not all rows, have pivot,

so, $\{v_1, v_2, v_3\}$ span R^4?

22) $v_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}$ $v_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$

$[v_1 \ v_2 \ v_3] = \begin{bmatrix} 0 & 0 & 4 \\ 0 & -3 & -1 \\ -2 & 8 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -2 & 8 & 5 \\ 0 & -3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$, $\{v_1, v_2, v_3\}$ span $\underline{R^3}$

23) a, No, It is called Matrix eqn.

b, Yes, as soln. exist, b is a linear ~~form~~ combination of columns.

c, No, for [A b] if it has pivot in each row, then soln is inconsistent.

d, Yes, it is the sum of product of the 1st element in each vector

$[x_1 a_1 + x_2 a_2 + x_n .. a_n]$

e, Yes. f, Yes.

24) a, Yes, b, Yes c, Yes, d, Yes, e, ~~Yes~~ May be

f, Yes.

25) $c_1 = -3$, $c_2 = -1$, $c_3 = 2$ 27) $[q_1 \ q_2 \ q_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$

26) $x_1 = 3$, $x_2 = -5$ 28) $-3\begin{bmatrix} -3 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 8 \end{bmatrix} + 4\begin{bmatrix} -4 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 9 \\ -2 \end{bmatrix}$

$+ 2\begin{bmatrix} 7 \\ -4 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$.

29) $\begin{bmatrix} 2 & 1 & 8 \\ 5 & 0 & 3 \\ 2 & 7 & 4 \end{bmatrix}$ Start with a echelon matrix $\Rightarrow$ & perform row operation

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xRightarrow{\text{row op}} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 2 & 4 \end{bmatrix}$ = req. Matrix

30) Do not span R^3

$$A = \begin{bmatrix} 1 & 5 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$, do not span $\underline{R^3}$.

row operation

$$= \begin{bmatrix} 1 & 5 & 2 \\ 1 & 5 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \text{reqd. matrix}$$

31) 3×2

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

Since, 3 rows, 2 columns,

it can't have pivot in all the rows,

so, we would have, a row [0 0], so, it wi not be consistent for all values of b.

32) again for matrix $n \times m$, if $n > m$, then it is not consistent for all $\vec{b}$.

~~& if $n < m$, it will~~ i.e. the number of vector in R^m, cannot span R^m when n is less than m.

33) If eqn $Ax = b$ has a unique soln; then the associated system of eqn does not have any free variables, If every variable is a basic variable then each column of A is a pivot column. so,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$