1) $2x_1 - 5x_2 + 8x_3 = 0$
$-2x_1 - 7x_2 + x_3 = 0$
$4x_1 + 2x_2 + 7x_3 = 0$

$$\Rightarrow \begin{bmatrix} 2 & -5 & 8 & 0 \\ -2 & -7 & 1 & 0 \\ 4 & 2 & 7 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \\ 0 & 12 & -9 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \underline{\text{yes}}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 = (8x_3 + 5x_2)/2 \\ x_2 = \cancel{\dots} \tfrac{9x_3}{12} = \tfrac{3}{4}x_3 \\ x_3 \text{ free} \end{bmatrix}$$

2) $$\begin{bmatrix} 1 & -3 & 7 & 0 \\ -2 & 1 & -4 & 0 \\ 1 & 2 & 9 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 5 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 0 & 12 & 0 \end{bmatrix}$$

~~x_3 is free, so, yes~~
~~has non-trivial sol^n.~~
No free variable,
the system has only
trivial sol^n.

3) $$\begin{bmatrix} -3 & 5 & -7 & 0 \\ -6 & 7 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 5 & -7 & 0 \\ 0 & -3 & 15 & 0 \end{bmatrix}$$

yes, x_3 ~~...~~ free

4) $$\begin{bmatrix} -5 & 7 & 9 & 0 \\ 1 & -2 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 6 & 0 \\ -5 & 7 & 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 6 & 0 \\ 0 & -3 & 39 & 0 \end{bmatrix}$$

yes, x_3 ~~&~~ free

5) $$\begin{bmatrix} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

x_3 is free

$x_2 = -2x_3, \quad x_1 = 5x_3$

$x_1 + 3x_2 - 5x_3 = 0$

$x_1 + 4x_2 - 8x_3 = 0$

$-3x_1 - 7x_2 + 9x_3 = 0$

$$\begin{bmatrix} 1 & 3 & -5 & 0 \\ 1 & 4 & -8 & 0 \\ -3 & -7 & 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_3 free, $x_2 = 3x_3$, $x_1 = -4x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

7) $$\begin{bmatrix} 1 & 3 & -3 & 7 & 0 \\ 0 & 1 & -4 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 9 & -8 & 0 \\ 0 & 1 & -4 & 5 & 0 \end{bmatrix}$$

$x_2 - 4x_3 + 5x_4 = 0$

$x_1 + 9x_3 - 8x_4 = 0$

$x_1 = 8x_4 - 9x_3$

$x_2 = -5x_4 + 4x_3$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -9x_3 + 8x_4 \\ 4x_3 - 5x_4 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} -9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 8 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

8) $$\begin{bmatrix} 1 & -2 & -9 & 5 \\ 0 & 1 & 2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -5 & -7 & 0 \\ 0 & 1 & 2 & -6 & 0 \end{bmatrix}$$

$x_2 + 2x_3 - 6x_4 = 0$

$x_1 - 5x_3 - 7x_4 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5x_3 + 7x_4 \\ -2x_3 - 6x_4 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ -6 \\ 0 \\ 1 \end{bmatrix}$$

9) $$\begin{bmatrix} 3 & -9 & 6 \\ -1 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 2 & 0 \\ 3 & -9 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 3x_2 + 2x_3 = 0$

$x_1 = 3x_2 - 2x_3$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

10)
$$\begin{bmatrix} 1 & 3 & 0 & -4 & 0 \\ 2 & 6 & 0 & -8 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 3x_2 - 4x_4 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3x_2 + 4x_4 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

11
$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & -2 & 0 & 0 & 7 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_5 - 4x_6 = 0 \Rightarrow x_5 = 4x_6$$

$$x_3 - x_6 = 0 \Rightarrow x_3 = x_6$$

$$x_1 - 4x_2 + 5x_6 = 0$$

$$\Rightarrow x_1 = 4x_2 - 5x_6$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4x_2 - 5x_6 \\ x_2 \\ x_6 \\ x_4 \\ 4x_6 \\ x_6 \end{bmatrix} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

12)
$$\begin{bmatrix} 1 & 5 & 2 & -6 & 9 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 0 & 8 & 1 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_6 = 0$$

$$x_3 - 7x_4 + 4x_5 = 0$$

$$x_4 = \frac{4x_5}{7}$$

$$x_1 + 5x_2 + 8x_4 + x_5 = 0$$

$$x_1 = -5x_2 - 8x_4 - x_5$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -5x_2 - 8x_4 - x_5 \\ x_2 \\ x_3 \\ 4/7\, x_5 \\ x_5 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 4/7 \\ 1 \\ 0 \end{bmatrix}$$

$x_1 = 5 + 4x_3 \quad x_2 = -2 - 7x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5+4x_3 \\ -2-7x_3 \\ x_3 \end{bmatrix} = \underset{\uparrow\, p}{\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}} + x_3 \underset{\uparrow\, q}{\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}}$$

So, the solution set is the line through $\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$ in the direction of $\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$

14) $x_1 = 3x_4, \quad x_2 = 8 + x_4 \quad x_3 = 2 - 5x_4$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_4 \\ 8+x_4 \\ 2-5x_4 \end{bmatrix} = \underset{\uparrow\, p}{\begin{bmatrix} 0 \\ 8 \\ 2 \end{bmatrix}} + x_4 \underset{\uparrow\, q}{\begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix}}$$

So, the solution set is the line through $\begin{bmatrix} 0 \\ 8 \\ 2 \end{bmatrix}$ in the direction of $\begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix}$

15) $x_1 + 3x_2 + x_3 = 1$

$-4x_1 - 9x_2 + 2x_3 = -1$

$-3x_2 - 6x_3 = -3$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & -3 & -6 & -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -5 & -2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2 + 2x_3 = 1 \qquad x_1 - 5x_3 = -2$

$x_2 = 1 - 2x_3 \qquad x_1 = -2 + 5x_3$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2+5x_3 \\ 1-2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

The soln. set is the line through $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, parallel to the line that is the solution set of the homogeneous system in exercise 5.

$x_1 + 3x_2 - 5x_3 = 4$

$x_1 + 4x_2 - 8x_3 = 7$

$-3x_1 - 7x_2 + 9x_3 = -6$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 2 & -6 & -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_2 - 3x_3 = 3 \Rightarrow x_2 = 3 + 3x_3 \\ x_1 + 3x_2 - 5x_3 = 4 \\ x_1 = 4 + 5x_3 - 3x_2 \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 + 5x_3 - 3x_2 \\ 3 + 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_2 - 3x_3 = 3 \Rightarrow x_2 = 3 + 3x_3 \\ x_1 + 4x_3 = -5 \Rightarrow x_1 = -5 - 4x_3 \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 - 4x_3 \\ 3 + 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

The sol^n. set is the line through $\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$, parallel to the line that is the sol^n. set of the homogeneous sys

$$x_1 = 4x_3 - 9x_2$$

Date: 5/10/12
Place: Asansol

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4x_3 - 9x_2 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= x_2 \underbrace{\begin{bmatrix} -9 \\ 1 \\ 0 \end{bmatrix}}_{u} + x_3 \underbrace{\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}}_{v}$$

$x_1 + 9x_2 - 4x_3 = -2$ } - non homogeneous

$$x_1 = -2 + 4x_3 - 9x_2$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 + 4x_3 - 9x_2 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} + x \begin{bmatrix} -9 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$$

So, the soln. set of the homogeneous eqn. is the plane through the origin in R3 spanned by u, v.

The soln. set of the non-homogeneous eqn is parallel to this plane & passes through point $p = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$.

$x_1 - 3x_2 + 5x_3 = 0$

$x_1 = 3x_2 - 5x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 - 5x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= x_2 \underbrace{\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}}_{u} + x_3 \underbrace{\begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}}_{v}$$

soln. set in the plane through origin in R^3 in span(u, v).

$x_1 - 3x_2 + 5x_3 = 4$

$x_1 = 4 + 3x_2 - 5x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 + 3x_2 - 5x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}}_{p} + x_2 \underbrace{\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}}_{u} + x_3 \underbrace{\begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}}_{v}$$

soln. set a plane through p, parallel to the plane for homogeneous soln.

19) $a = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$, $b = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

A line parallel to b as passing through b can be written as, $x = a + tb$, $t \in R$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ 3 \end{bmatrix} \text{ or, } \begin{cases} x_1 = -2 - 5t \\ x_2 = 3t \end{cases}$$

20) $a = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$, $b = \begin{bmatrix} -7 \\ 8 \end{bmatrix}$

similarly, $x = a + tb$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix} + t \begin{bmatrix} -7 \\ 8 \end{bmatrix} \text{ or, } \begin{cases} x_1 = 3 - 7t \\ x_2 = -4 + 8t \end{cases}$$

21) M is parallel to $(q-p)$
and passes through, p & q,
So, M can be, $p + t(q-p)$ or $q + t(q-p)$
So, $(q-p) = \begin{bmatrix} -3 & -2 \\ 1 & +5 \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$

$\therefore x = \begin{bmatrix} 2 \\ -5 \end{bmatrix} + t \begin{bmatrix} -5 \\ 6 \end{bmatrix}$

22) Similarly, $q - p = \begin{bmatrix} 0 & +6 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 6 \\ -7 \end{bmatrix}$

$x = \begin{bmatrix} -6 \\ 3 \end{bmatrix} + t \begin{bmatrix} 6 \\ -7 \end{bmatrix}$

23) a) True, there always a trivial soln. i.e zero vector

b) False, implicit description.

c) ~~Ax=0~~ False, $Ax=0$, will always have trivial soln.

d) False, $x = p + tv$, it describes, line through p parallel to v.

e) True, when the solution set exists, else false

24) a) False, some entries can be zero

b) True

c) True, if zero vector is soln, its a homogeneous

d) True

e) True, its soln. set is not empty.

when, ω is the sol^n of $Ax=b$

$\therefore A\omega = b$, and $v_h = \omega - p$, then, $\omega = v_h + p$

$\Rightarrow \cancel{A(v_h+p) \neq b}$

$\Rightarrow \cancel{Av_h + Ap = b}$,

let, w & p be sol^n of $Ax=b$

$Ap = b \quad Aw = b$

So, when, $v_h = \omega - p$

$Av_h = A\omega - Ap$

$= b - b = 0$

$Ax = 0$, when $\underline{x = v_h}$

26) Since, $Ax = b$ is consistent, its solution set is obtained by translating the solution set of $Ax=0$.

So, $Ax=b$ is a single vector, if & only if ~~$Ax=0$ is a~~ solution set of $Ax=0$ is a single vector i.e. the trivial sol^n.

Also, for only one sol^n. A will have ~~pivot col^n.~~ ~~for~~ only possible when ~~all~~ pivots for all col^n. which is only possible whe

$Ax = 0$, i.e. trivial sol^n.

27) Since $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\therefore$ for any x, $Ax = 0$, i.e. ~~trivial sol^n~~.

~~all~~ sol^n. set is all vectors in R^3.

28) No, if the sol^n. set contained the origin, then $x=0$, would satisfy $A0 = b$, which is not true, since $b \neq 0$.

29) A 3×3 with 3 pivot,

for $Ax = 0$, it means, $Ax = 0$, has no free variables and hence, no non-trivial solⁿ.

for $Ax = b$, it means, A has a pivot in each of its 3 rows, the eqn. $Ax = b$ has a solⁿ for every possible 'b'.

30) A is 3×3 with two pivot,

$Ax = 0$,

A has one free variable, So, $Ax = 0$ has nontrivial solⁿ.

$Ax = b$,

A cannot have a pivot in every row, the eqn $Ax = b$ cannot have a solⁿ. for every b.

31) A is 3×2 with two pivots,

$Ax = 0$, A has no free variables, So, $Ax = 0$ has no ~~variable~~ non-trivial solⁿ.

$Ax = b$, A doesn't have pivot in every row, so the eqn. $Ax = b$ cannot have a solⁿ. for every possible b.

32. A is 2×4 with two pivot positions.

$Ax = 0$ has 2 basic vectors & 2 free variables so non-trivial solⁿ.

$Ax = b$, has pivot position in every row, $Ax = b$ has a solⁿ. for every possible b ($\mathbb{R}^2$)

33 $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$ 34 $\begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

35 $\begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & -3 \\ 1 & 2 & -3 \end{bmatrix}$ 36) $\begin{bmatrix} 2 & 1 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

37. Since the soln. set of $Ax=0$ contains the point $(4,1)$, the vector $x = (4,1)$ satisfies $Ax=0$.

So, $4a_1 + a_2 = 0$

so, $a_2 = -4a_1$

So, A can be $\begin{array}{cc} \begin{bmatrix} 1 & -4 \\ 2 & -8 \end{bmatrix} \\ a_1 \quad a_2 \end{array}$.

Finally, the only way the soln. set of $Ax=b$ could not be parallel to the line through $(1,4)$ & the origin is for the soln. set of $Ax=b$ to be empty.

39) NO, If $Ax=y$ has no soln., i.e. not even a ~~trivial~~ formal soln, then, it doesn't have a pivot in each row,

So the eqn. $Ax=z$ has at most two basic variables & at least one free variable

Thus, the soln. set for $Ax=z$ is either empty or has infinitely many elements.

40) we know, $A(u+v) = Au + Av$
$= 0$ to

$A(cu+dv) = Acu + Adv$
$= cAu + cAv = 0$

?) $Ax=0 \Rightarrow Au=0$

again $cAu = A(cu) = 0$