1.3 Exercises

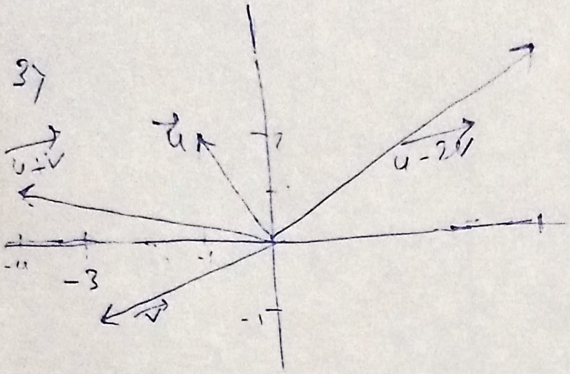
1) $u = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \; v = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$u+v = \begin{bmatrix} -4 \\ 1 \end{bmatrix} \quad u-2v = \begin{bmatrix} -1+6 \\ 2+2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

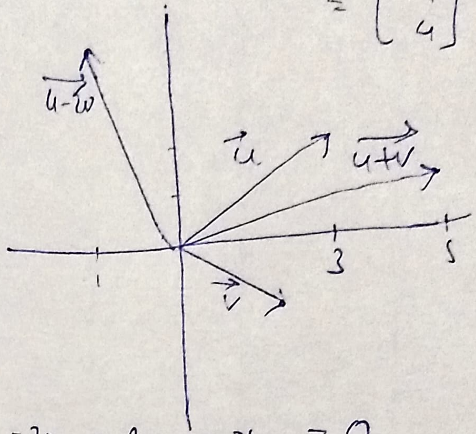
2) $u = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \; v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$u+v = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \quad u-2v = \begin{bmatrix} 3-4 \\ 2+2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

3)



4)



5) $6x_1 - 3x_2 = 1$

$-x_1 + 4x_2 = -7$

$5x_1 = -5$

6) $-2x_1 + 8x_2 + x_3 = 0$

$3x_1 + 5x_2 + (-6)x_3 = 0$

7) $x_1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1/2 \end{bmatrix} = a$

$b = \begin{bmatrix} -3 \\ 3 \end{bmatrix} = b_1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + b_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$c = \begin{bmatrix} 7 \\ 1 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$d = \begin{bmatrix} -7 \\ 4 \end{bmatrix} = d_1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

8) $w = \begin{bmatrix} 4 \\ 1 \end{bmatrix} = w_1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + w_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$y = \begin{bmatrix} 3 \\ -3 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

to reach y walk $-2u$ then $4v$

$\Updownarrow$

7) a, to reach a from origin, 1 unit in $\vec{u}$ direction & $-2\ \vec{v}$ direction,

$\boxed{a = u - 2v}$

b, to reach b from origin, 2 unit in $\vec{u}$ & $-2\ \vec{v}$

$\boxed{b = 2u - 2v}$,

c, to reach c from origin, 2 unit in $\vec{u}$ $-3.5\ \vec{v}$

$\boxed{c = 2u - 3.5v}$

d, to reach d from origin 3 units in $\vec{u}$ & -4 unit in $\vec{v}$

$\therefore \boxed{d = 3u - 4v}$

8) similarly
$w = -u + 2v$, $x = -2u + 2v$

$y = -2u + 4v$, $z = -3u + 4v$

9) $x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

10) $x_1 \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -7 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 15 \end{bmatrix}$

11) so, prove $b = x_1 a_1 + x_2 a_2 + x_3 a_3$

$\therefore x_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\Rightarrow$ this linear system has solution,

$\begin{cases} x_2 + 4x_3 = 3 \\ x_1 + 5x_3 = 2 \end{cases}$

x_3 free variable

12) $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 5 & 0 \\ 2 & 5 & 8 \end{bmatrix}$, $b = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{bmatrix}$

$\Rightarrow$

$\Rightarrow \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & -4 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

$\Rightarrow$ <u>no soln</u>,

$\cancel{5x_2 - 4x_3 = 1}$

$\cancel{5x_2 = 1 + 4x_3}$

$\cancel{x_1 = -5 - 2x_3}$

13> $\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

no, b is not linear combination of vectors.

14> $\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 1 & 7/3 & -5/3 \\ 0 & 0 & 11 & -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 1 & 7/3 & -5/3 \\ 0 & 0 & 5/3 & 14/3 \end{bmatrix}$, yes, sol^n exists,

$\dfrac{11 - \frac{20}{3}}{3}$

so, b is linear combinat of $\dfrac{-2 + \frac{20}{3}}{3}$ column vector.

15> $v_1 = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix}$ $v_2 = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$

let, scalar multiple be (0,0) then, $0\begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix} + 0\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

" " " (1,0) then, $\begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix}$

" " " (0,1) then $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$

(0,-1) then $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix}$

(1,1) = $\begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix} + \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$

16) $v_1 \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$

for scalars coefficients, (4P) 1 & 1, we have $\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

0, 1, we have $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$

1, 0, we have, : $\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$

-1, -1, we have $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -5 \end{bmatrix}$

1, -1, we have $\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$

17) $\begin{bmatrix} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h+17 \end{bmatrix}$

$h+17 = 0$

$h = -17$

18) $\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 2 & -3+2h \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & 7+2h \end{bmatrix} \Rightarrow 7+2h = 0$

$h = -7/2$

19) $v_2 = \frac{3}{2} v_1$

so, $av_1 + bv_2 = av_1 + b\frac{3}{2} v_1$

$= \underbrace{(a + b\tfrac{3}{2})}_{\text{scaler}} v_1$

So, span(v_1, v_2) is basically the line passing 0 & v_1 self point

20) $v_1 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ $v_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$

$3x_1 = -\frac{2}{3} \frac{3}{2}$

So, $av_1 + bv_2 = \begin{bmatrix} 3a \\ 0 \\ 2a \end{bmatrix} + \begin{bmatrix} -2b \\ 0 \\ 3b \end{bmatrix}$

$= \begin{bmatrix} 3a-2b \\ 0 \\ 2a+3b \end{bmatrix}$,

So, the vectors in $\{v_1, v_2\}$ span has $y = 0$, for all vectors, so it lies in xz plane

24) $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, Let, $y = \begin{bmatrix} h \\ k \end{bmatrix}$,

for h & k in Span{u, v},

$[u \; v \; y]$ must has soln.

$\begin{bmatrix} 2 & 2 & h \\ -1 & 1 & k \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & h/2 \\ -1 & 1 & k \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & h/2 \\ 0 & 2 & k+h/2 \end{bmatrix}$,

this augmented matrix has soln for all, k & h.

22) any augmented row, will non zero in lower row & last coln.

23) a) false, $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$ = vector

$[-4 \; 3]$ = 1×2 matrix

b) ~~False~~ NO,

c) true, $\frac{1}{2}v_1 + 0v_2$

d) true

e) false, if v is just a multiple of v_1 the span is line,

24) a $\{(-1, 2, 3, 4, 5.2)\}$

c) false, $c_1 \ldots c_p \in \mathbb{R}$

b) $v + u + (-1)v = u$ (true)

d) ~~false, line only if u, v are multiple of each other~~

e) yes

d) yes, contain, $0, \vec{u}, \vec{v}$, linear combination of $\vec{u}$ & $\vec{v}$, so yes, it contains $\vec{u}$

25] $A = \begin{array}{c} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix} \\ \begin{array}{ccc} \hat{a}_1 & \hat{a}_2 & \hat{a}_3 \end{array} \end{array}$ $b = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$

& $w = \text{Span}\{a_1, a_2, a_3\}$

a) no b is not in $\{a_1, a_2, a_3\}$, 3 vectors in the set (a_1, a_2, a_3)

b) Since, $w = \text{span}\{a_1, a_2, a_3\}$,

contains all linear combination of a_1, a_2 & a_3, so, infinite

solve, $\begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$, soln exists, yes, b is in W

c) $a_1 = 1a_1 + 0a_2 + 0a_3$

$\therefore$ a_1 is in W.

26) $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$

solve augmented matrix

$$\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 & 3 \\ -1 & 8 & 5 & 3 \\ 2 & 0 & 6 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 6 & 6 & 6 \\ 2 & 0 & 6 & 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 6 & 6 & 6 \\ 0 & 4 & 4 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Yes, sol^n exist & b is in span(A)

b) $a_3 = 1a_3 + 0a_2 + 0a_1$, so a_3 in $\underline{W}$

27) a) $5v_1 \Rightarrow$ mine #1 output in 5 days

b) $x_1 \begin{bmatrix} 20 \\ 550 \end{bmatrix} + x_2 \begin{bmatrix} 30 \\ 500 \end{bmatrix} = \begin{bmatrix} 150 \\ 2825 \end{bmatrix}$

c) $\Rightarrow \simeq \begin{bmatrix} 1 & 0 & 1.5 \\ 0 & 1 & 4 \end{bmatrix}$ so, $x_1 = 1.5$

$x_2 = 4$.

28) $A = \begin{bmatrix} 27.6 \\ 3100 \\ 250 \end{bmatrix}$ $B = \begin{bmatrix} 30.2 \\ 6400 \\ 360 \end{bmatrix}$

a) $x_1\ 27.6 + x_2\ 30.2$

b) $x_1 \begin{bmatrix} 27.6 \\ 3100 \\ 250 \end{bmatrix} + x_2 \begin{bmatrix} 30.2 \\ 6400 \\ 360 \end{bmatrix}$

c) $\Longrightarrow$

29) $m = m_1 + \cdots + m_k$. so, $\bar{v} = \frac{1}{m}\left[m_1 v_1 + \cdots + m_k v_k\right]$

$m = 2 + 5 + 2 + 1$

$= \underline{10}$

$= \frac{1}{10}\left(2\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} + 5\begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix} + 2\begin{bmatrix} -4 \\ -3 \\ -1 \end{bmatrix}\right.$

$\Rightarrow \frac{1}{10}\left[\begin{bmatrix} 10 \\ -8 \\ 6 \end{bmatrix} + \begin{bmatrix} 20 \\ 15 \\ -10 \end{bmatrix} + \begin{bmatrix} -8 \\ -6 \\ -2 \end{bmatrix} + \begin{bmatrix} -9 \\ 8 \\ 6 \end{bmatrix}\right.$ $\left. + \begin{bmatrix} -9 \\ 8 \\ 6 \end{bmatrix}\right)$

$\Rightarrow \frac{1}{10}\begin{bmatrix} 13 \\ 9 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1.3 \\ .9 \\ 0 \end{bmatrix}$

30) yes, as V, center of mass is linear combination of $(v_1, v_2 \dots v_k)$

31) a) $\frac{1}{3}\left(1\begin{bmatrix}0\\1\end{bmatrix}+1\begin{bmatrix}8\\1\end{bmatrix}+1\begin{bmatrix}2\\4\end{bmatrix}\right)$

$= \frac{1}{3}\left(\begin{bmatrix}10\\6\end{bmatrix}\right)$

$= \begin{bmatrix}10/3\\2\end{bmatrix}$

b) $= \frac{1}{9}\left[(w_1+1)\begin{bmatrix}0\\1\end{bmatrix} + (w_2+1)\begin{bmatrix}8\\1\end{bmatrix} + (w_3+1)\begin{bmatrix}2\\4\end{bmatrix}\right]$

$\frac{1}{9}\left[w_1\begin{bmatrix}0\\1\end{bmatrix}+w_2\begin{bmatrix}8\\1\end{bmatrix}+w_3\begin{bmatrix}2\\4\end{bmatrix}+\begin{bmatrix}10\\6\end{bmatrix}\right] = \begin{bmatrix}2\\2\end{bmatrix}$

$w_1\begin{bmatrix}0\\1\end{bmatrix}+w_2\begin{bmatrix}8\\1\end{bmatrix}+w_3\begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}18-10\\18-6\end{bmatrix}$

$\begin{bmatrix}0 & 8 & 2 & 8\\1 & 1 & 4 & 12\end{bmatrix}$

also,

$w_1+w_2+w_3=6$

$\Rightarrow \begin{bmatrix}0 & 8 & 2 & 8\\1 & 1 & 4 & 12\\1 & 1 & 1 & 6\end{bmatrix}$

$\begin{bmatrix}1 & \cancel{1} & 4 & 12\\0 & 8 & 2 & 8\end{bmatrix}$

$\begin{bmatrix}1 & & 4 & 12\\0 & 1 & 1/4 & 1\end{bmatrix}$

$w_2+\frac{1}{4}w_3=1$

$w_1+w_2+4w_3=12$

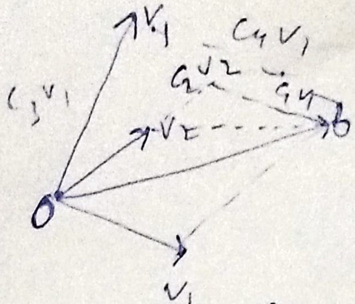
$\Rightarrow \begin{bmatrix}1 & 1 & 1 & 6\\1 & 1 & 4 & 12\\0 & 8 & 2 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 1 & 1 & 6\\0 & 0 & 3 & 6\\0 & 8 & 2 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 1 & 1 & 6\\0 & 8 & 2 & 8\\0 & 0 & 3 & 6\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 1 & 1 & 6\\0 & 1 & 1/4 & 1\\0 & 0 & 1 & 2\end{bmatrix}$

$\boxed{\begin{aligned}w_3 &= 2\\ w_2 &= 1/2\\ w_1 &= 3.5\end{aligned}}$

32)



, so, $b = c_1 v_1 + c_2 v_2$

also $b = c_3 v_3 + c_4 v_1$

so,

$x_1 v_1 + x_2 v_2 + x_3 v_3 = b$ have atleast 2 soln.

33) $u = (u_1 \dots u_n)$, $v = (v_1 \dots v_n)$, $w = (w_1 \dots w_n)$

$(u+v) + w = \cancel{(u_1 \dots u_n}$

$= ((u_1+v_1), (u_2+v_2) \dots (u_n+v_n)) + (w_1 \dots w_n)$

$= ((u_1+v_1+w_1), (u_2+v_2+w_2) \dots (u_n+v_n+w_n))$

by commucative property of R & associative

$= (u_1 + (w_1+v_1)) + (u_2 + (v_2+w_2)) \dots$

$= u + \underline{(v+w)}$

b) $c(u+v) = c((u_1+v_1), (u_2+v_2) \dots (u_n+v_n))$

by destributive propety

$= c(u_1+v_1), c(u_2+v_2) \dots$

$= (cu_1 + cv_1, (cu_2 + cv_2) \dots (cu_n + cv_n)$

$= cu + cv$

34) a) additive inverse & commucative

b) $c(du) = (cd)u$, by associative property of cd.