1.6) 1)

	A (output)	S (output)
A (input)	.2	.7
S (input)	.8	.3

Let P_A & P_S be the sectors annual output respectively,

$\therefore P_A = .2P_A + .7P_S$ — (1)

(income) (expense)

Similarly, $P_S = .8P_A + .3P_S$ — (2)

(income) (expense)

Moving all variables to left of the eqns

$$.8P_A - .7P_S = 0$$

$$-.8P_A + .7P_S = 0$$

Row reduce the augmented matrix

$$\Rightarrow \begin{bmatrix} .8 & -.7 & 0 \\ -.8 & +.7 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} .8 & -.7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -.875 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $P_A = .875\,P_S$. equilibrium price

2) The ratios of the prices remain same, no matter what currency is used.

3) a)

	Chemicals & Metals	Fuels & Power	Machinery
	outputs		
	.2	.8	.4
	.3	.1	.4
	.5	.1	.2

input
Chemicals & Metals
Fuels & Power
Machinery

b) So, $P_C = .2P_C + .8P_F + .4P_M$ (Income) (expenses)

$P_F = .3P_C + .1P_F + .4P_M$

$P_M = .5P_C + .1P_F + .2P_M$

~~$-.2P_C + 0.9P_F - 0.4P_M = 0$~~

~~$-$~~ $.8P_C - 0.8P_F - 0.4P_M = 0$

$-.3P_C + 0.9P_F - 0.4P_M = 0$

$-.5P_C - 0.1P_F + 0.8P_M = 0$

$$\begin{bmatrix} .8 & -0.8 & -0.4 & 0 \\ -.3 & 0.9 & -0.4 & 0 \\ -.5 & -0.1 & 0.8 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -8 & -4 & 0 \\ -3 & 9 & -4 & 0 \\ -5 & -1 & 8 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1/2 & 0 \\ 0 & 6 & -5.5 & 0 \\ 0 & -6 & 5.5 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & -0.5 & 0 \\ 0 & 1 & -.917 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1.417 & 0 \\ 0 & 1 & -.917 & 0 \\ 0 & 0 & 0 & \end{bmatrix}$$

$P_F = .917P_M$

$P_C = 1.417P_M$

$\therefore$ $P_C = 1.417$
$P_F = .917$
P_M : free

4) a) Agriculture Energy Manufacturing Transportation

Inputs \ outputs				
Agriculture	.65	.3	.3	.2
Energy	.1	.1	.15	.1
Manufacturing	.25	.35	.15	.3
Transp	0	.25	.4	.4

b) Income = Expenses

$$P_A = .65P_A + .3P_E + .3P_M + .2P_T$$
$$P_E = .1P_A + .1P_E + .15P_M + .1P_T$$
$$P_M = .25P_A + .35P_E + .15P_M + .3P_T$$
$$P_T = 0P_A + .25P_E + .4P_M + .4P_T$$

$$.35P_A - 0.3P_E - 0.3P_M - 0.2P_T = 0$$
$$-0.1P_A + 0.9P_E - 0.15P_M - 0.1P_T = 0$$
$$-0.25P_A - 0.35P_E + .85P_M - 0.3P_T = 0$$
$$-0.25P_E - 0.4P_M + 0.6P_T = 0$$

$$\begin{bmatrix} 35 & -30 & -30 & -20 \\ -10 & 90 & -15 & -10 \\ -25 & -35 & 85 & -30 \\ 0 & -25 & -40 & -60 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -6/7 & -6/7 & -4/7 \\ 0 & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot \end{bmatrix}$$

$90 - \frac{60}{7} \cdot 85$

$= \frac{-6}{7}$

So, $P_A = 2.03P_T$ $P_E = .53P_T$ $P_M = 1.17P_T$, P_T = free

$P_T = 100$, $P_A = 200$ $P_E = 53$, $P_M = 120$

5) $B_2S_3 \begin{bmatrix}2\\3\\0\\6\end{bmatrix}$, $H_2O \begin{bmatrix}0\\0\\2\\1\end{bmatrix}$, $H_3BO_3 \begin{bmatrix}1\\0\\3\\3\end{bmatrix}$, $H_2S \begin{bmatrix}0\\1\\2\\0\end{bmatrix}$ $\begin{matrix}B\\S\\H\\O\end{matrix}$

So, $x_1\begin{bmatrix}2\\3\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\\1\end{bmatrix} = x_3\begin{bmatrix}1\\0\\3\\3\end{bmatrix} + x_4\begin{bmatrix}0\\1\\2\\0\end{bmatrix}$

So, $\begin{bmatrix}2 & 0 & -1 & 0 & 0\\ 3 & 0 & 0 & -1 & 0\\ 0 & 2 & -3 & -2 & 0\\ 0 & 1 & -3 & 0 & 0\end{bmatrix}$

$\begin{bmatrix}1 & 0 & -1/2 & 0 & 0\\ 0 & 0 & 3/2 & -1 & 0\\ 0 & 1 & -3 & 0 & 0\\ 0 & 2 & -3 & -2 & 0\end{bmatrix} = \begin{bmatrix}1 & 0 & -1/2 & 0 & 0\\ 0 & 0 & 3/2 & -1 & 0\\ 0 & 1 & -3 & 0 & 0\\ 0 & 0 & 3 & -2 & 0\end{bmatrix}$

$= \begin{bmatrix}1 & 0 & -1/2 & 0 & 0\\ 0 & 1 & -3 & 0 & 0\\ 0 & 0 & 3/2 & -1 & 0\\ 0 & 0 & 3 & -2 & 0\end{bmatrix}$

$= \begin{bmatrix}1 & 0 & -1/2 & 0 & 0\\ 0 & 1 & -3 & 0 & 0\\ 0 & 0 & 1 & -2/3 & 0\\ 0 & 0 & 3 & -2 & 0\end{bmatrix} = \begin{bmatrix}1 & 0 & -1/2 & 0 & 0\\ 0 & 1 & -3 & 0 & 0\\ 0 & 0 & 1 & -2/3 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$

$= \begin{bmatrix}1 & 0 & 0 & -1/3 & 0\\ 0 & 1 & 0 & -2 & 0\\ 0 & 0 & 1 & -2/3 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$

$x_3 = \frac{2}{3}x_4$

$x_2 = 2x_4$

$x_1 = 1/3\, x_4$

So, $x_4 = 3$,

$x_3 = 2$

$x_2 = 6$,

$x_1 = 1$ $B_2S_3 + 6H_2O \rightarrow 2H_3BO_3 + 3H_2S$

b)

$$\begin{matrix} Na \\ P \\ O \\ B \\ N \end{matrix} \quad x_1\begin{bmatrix}3\\1\\4\\0\\6\end{bmatrix} + x_2\begin{bmatrix}0\\0\\6\\1\\2\end{bmatrix} = x_3\begin{bmatrix}0\\2\\8\\3\\0\end{bmatrix} + x_4\begin{bmatrix}0\\3\\0\\1\end{bmatrix}$$

using python & numpy.rref()

$$\begin{bmatrix}3&0&0&-1&0\\1&0&-2&0&0\\4&6&-8&-3&0\\0&1&-3&0&0\\0&2&0&-1&0\end{bmatrix} = \begin{bmatrix}1&0&0&-1/3&0\\0&1&0&-1/2&0\\0&0&1&-1/6&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$$

$x_3 = \frac{1}{6}x_4$, $x_2 = \frac{1}{2}x_4$, $x_1 = \frac{1}{3}x_4$

$x_4 = 6$, $x_3 = 1$, $x_2 = 3$, $x_1 = 2$

$$2\,Na_3PO_4 + 3\,Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + 6\,NaNO_3.$$

$$x_1\,NaHCO_3 + x_2\,H_3C_6H_5O_7 \rightarrow x_3\,Na_3C_6H_5O_7 + x_4\,H_2O + x_5\,CO_2$$

$$\begin{matrix} Na \\ H \\ C \\ O \end{matrix} \quad x_1\begin{bmatrix}1\\1\\1\\3\end{bmatrix} + x_2\begin{bmatrix}0\\8\\6\\7\end{bmatrix} = x_3\begin{bmatrix}3\\5\\6\\7\end{bmatrix} + x_4\begin{bmatrix}0\\2\\0\\1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\1\\2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1&0&-3&0&0&0\\1&8&-5&-2&0&0\\1&6&-6&0&-1&0\\3&7&-7&-1&-2&0\end{bmatrix} = \begin{bmatrix}1&0&0&0&-1&0\\0&1&0&0&-1/3&0\\0&0&1&0&-1/3&0\\0&0&0&1&-1&0\end{bmatrix}$$

$x_4 = x_5$, $x_5 = 3$

$x_3 = \frac{1}{3}x_5$, $x_4 = 3$

$x_2 = \frac{1}{3}x_5$, $x_3 = 1$

$x_1 = x_5$, $x_2 = 1$

$x_1 = 3$

q) K, Mn, O, S, H

$$x_1\begin{bmatrix}1\\1\\4\\0\\0\end{bmatrix}+x_2\begin{bmatrix}0\\1\\4\\1\\6\end{bmatrix}+x_3\begin{bmatrix}0\\0\\1\\0\\2\end{bmatrix}=x_4\begin{bmatrix}0\\1\\2\\0\\6\end{bmatrix}+x_5\begin{bmatrix}2\\0\\4\\1\\0\end{bmatrix}+x_6\begin{bmatrix}0\\0\\4\\1\\2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1&0&0&0&-2&0&0\\1&1&0&-1&0&0&0\\4&4&1&-2&-4&-4&0\\0&1&0&0&-1&-1&0\\0&0&2&0&0&-2&0\end{bmatrix} = \begin{bmatrix}1&0&0&0&0&-1&0\\0&1&0&0&0&-3/2&0\\0&0&1&0&0&-1&0\\0&0&0&1&0&-5/2&0\\0&0&0&0&1&-1/2&0\end{bmatrix}$$

$x_5 = \frac{1}{2}x_6$, $x_4 = \frac{5}{2}x_6$, $x_3 = x_6$ $x_2 = \frac{3}{2}x_6$, $x_1 = x_6$

$x_6 = 2$ $x_4 = 5$ $x_2 = 3$

$x_5 = 1$ $x_3 = 2$ $x_1 = 2$

q) Pb, N, Cr, Mn, O

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix}+x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix}=x_3\begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix}+x_4\begin{bmatrix}0\\0\\2\\0\\3\end{bmatrix}+x_5\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix}+x_6\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}1&0&3&0&0&0&0\\0&0&0&0&0&1&0\\6&1&0&2&0&0&0\\0&2&0&0&1&0&0\\0&8&4&3&2&1&0\end{bmatrix} = \begin{cases} x_5 = 44/45\, x_6 \\ x_4 = 11/45\, x_6 \\ x_3 = 1/18\, x_6 \\ x_2 = -22/45\, x_6 \\ x_1 = -1/6\, x_6 \end{cases}$$

Solve similarly for [illegible]

11)

	In flow	out flow
A	$x_1 + x_3$	20
B	x_2	$x_3 + x_4$
C	80	$x_1 + x_2$
Total flow	80	$x_4 + 20$

So, $x_1 + x_3 = 20$

$x_2 - x_3 - x_4 = 0$

$+ x_1 + x_2 = 80$

$x_4 = 60$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$x_4 = 60$, $x_2 = 60 + x_3$, $x_1 = 20 - x_3$

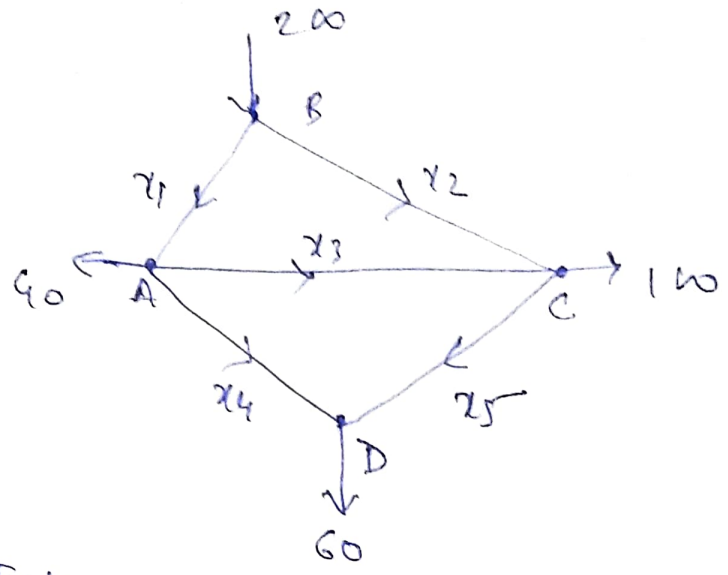
x_1 $20 - x_3$ $x_3 \leq 20$

x_2 $60 + x_3$ $x_3 \leq 20$

x_3 free

$x_4 = 60$

12)



Intersection	Flow in		out
A	x_1	=	$40 + x_4 + x_3$
B	200	=	$x_1 + x_2$
C	$x_2 + x_3$	=	$x_5 + 100$
D	$x_4 + x_5$	=	60
Total flow	200	=	200

$$x_1 - x_4 - x_3 = 40$$

$$x_1 + x_2 = 200$$

$$x_2 + x_3 - x_5 = 100$$

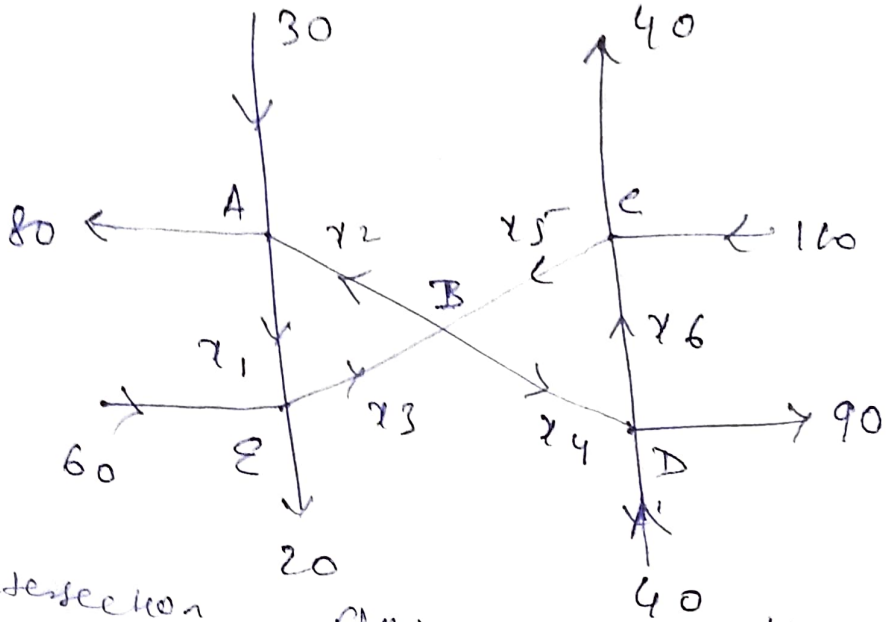
$$x_4 + x_5 = 60$$

$$\left[\begin{array}{cccccc} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right] \sim \left[\begin{array}{cccccc} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 = 100 + x_3 - x_5 \\ x_2 = 100 - x_3 + x_5 \\ x_3 \text{ free} \\ x_4 = 60 - x_5 \quad x_5 \text{ is free} \end{cases}$$

by when $x_4 = 0$, x_5 must be 60,
and
cz $x_1 = 40$, as $x_4 \geq 0$,
x_3 cannot be -ve

13)



Intersection	flow in	flow out
A	$30+x_2$	$= 80+x_1$
B	x_3+x_5	$= x_2+x_4$
C	$100+x_6$	$= x_5+40$
D	$40+x_4$	$= x_6+90$
E	x_1+60	$= 20+x_3$
Total	230	$= 230$

$$-x_1 + x_2 = 50$$

$$-x_2 + x_3 - x_4 + x_5 = 0$$

$$-x_5 + x_6 = -60$$

$$x_4 - x_6 = 50$$

$$x_1 - x_3 = -40$$

Solving,

$x_1 = x_3 - 40$ $\quad x_5 = x_6 + 60$

$x_2 = x_3 + 10$ $\quad x_6$ ~~to be~~ free

x_3 free

$x_4 = x_6 + 50$

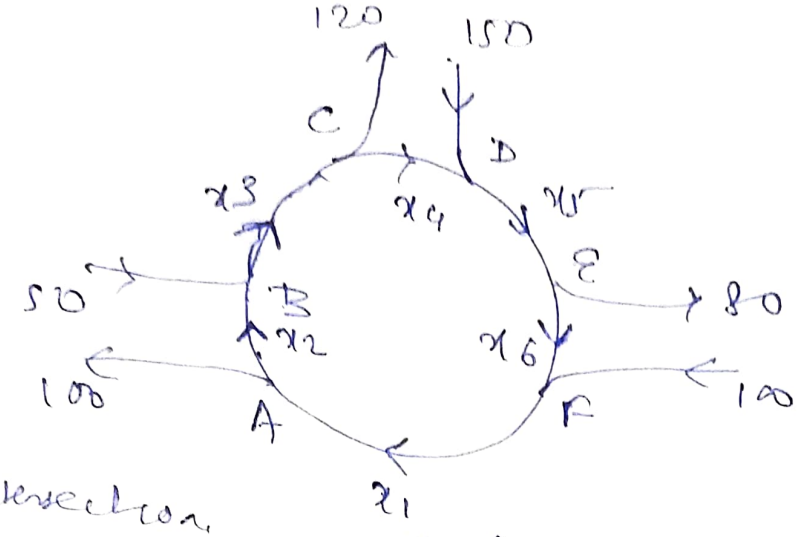
minimum flow x_1 cannot be negative

$x_3 \geq 40$, $\quad x_3 = 40$,

$x_6 \geq 0$ $\quad x_1 = 0$

$x_4 = 50$ $\quad x_2 = 50$

$x_5 \geq 60$.



Intersection	Inflow		Outflow
A	x_1	=	$100 + x_2$
B	$50 + x_2$	=	x_3
C	x_3	=	$120 + x_4$
D	$x_4 + 150$	=	x_5
E	x_5	=	$80 + x_6$
F	$x_6 + 100$	=	x_1

following

$$\begin{cases} x_1 = 100 + x_6 \\ x_2 = x_6 \\ x_3 = 50 + x_6 \\ x_4 = -70 + x_6 \\ x_5 = 80 + x_6 \\ x_6 = \text{free} \end{cases}$$