

Linear search

Unsorted / sorted

first occurrence / last occurrence / all occurrences

Best case  $\rightarrow \mathcal{O}(1)$  first iteration

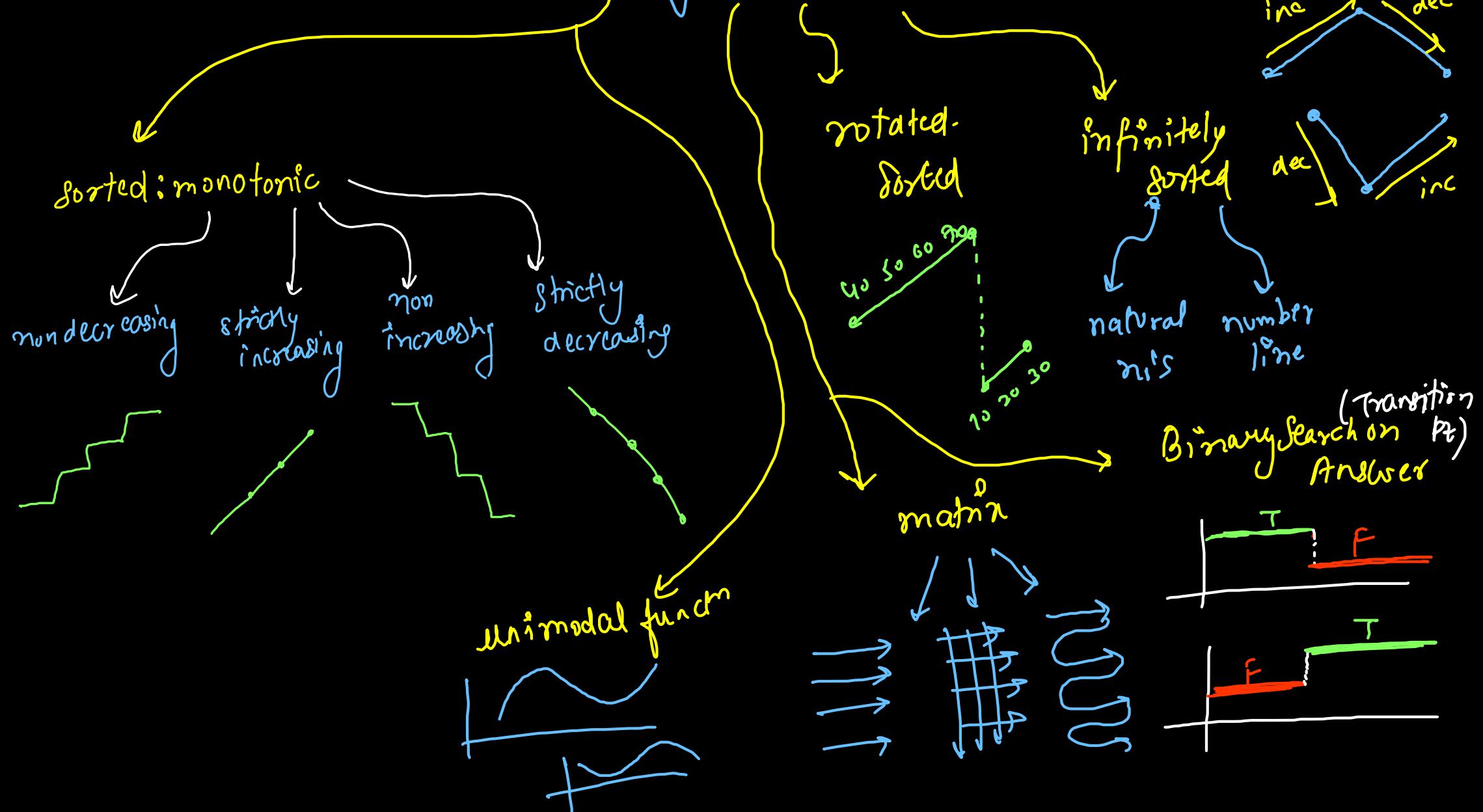
Avg case  $\rightarrow \Theta(n)$

Worst case  $\rightarrow \mathcal{O}(n)$  last iteration

Search space after each iteration

$N \rightarrow N-1$

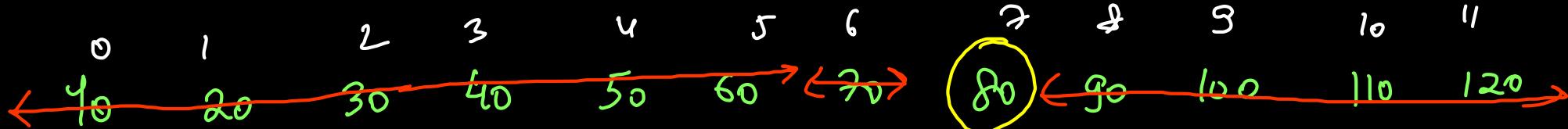
# Binary Search



# Binary Search

$\downarrow$

Divide & Conquer



$n \rightarrow n/2 \rightarrow n/4 \rightarrow n/8 \rightarrow \dots \rightarrow 1$   
 $\Rightarrow \alpha(\log_2 n)$

target = 80

- ①  $60 < 80$
- ②  $90 > 80$
- ③  $70 < 80$

left right  
mid

```

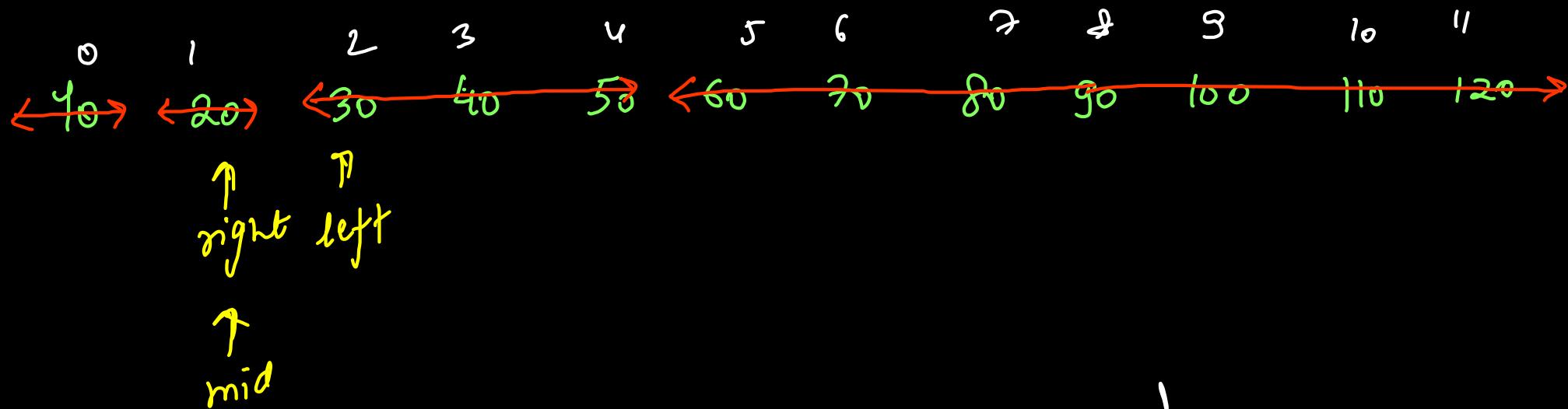
int left = 0, right = n-1;
while(left <= right) {
    int mid = (left + right)/2;
}

```

```

if (arr[mid] == target)
    return mid;
else if (arr[mid] < target)
    left = mid+1;
} else
    right = mid-1;
return -1;

```



target = 25

$60 > 25$   
 $30 > 25$   
 $10 < 25$   
 $20 < 25$

un successful  
 search  
 $\Theta(\log_2 N)$

# Leetcode 704

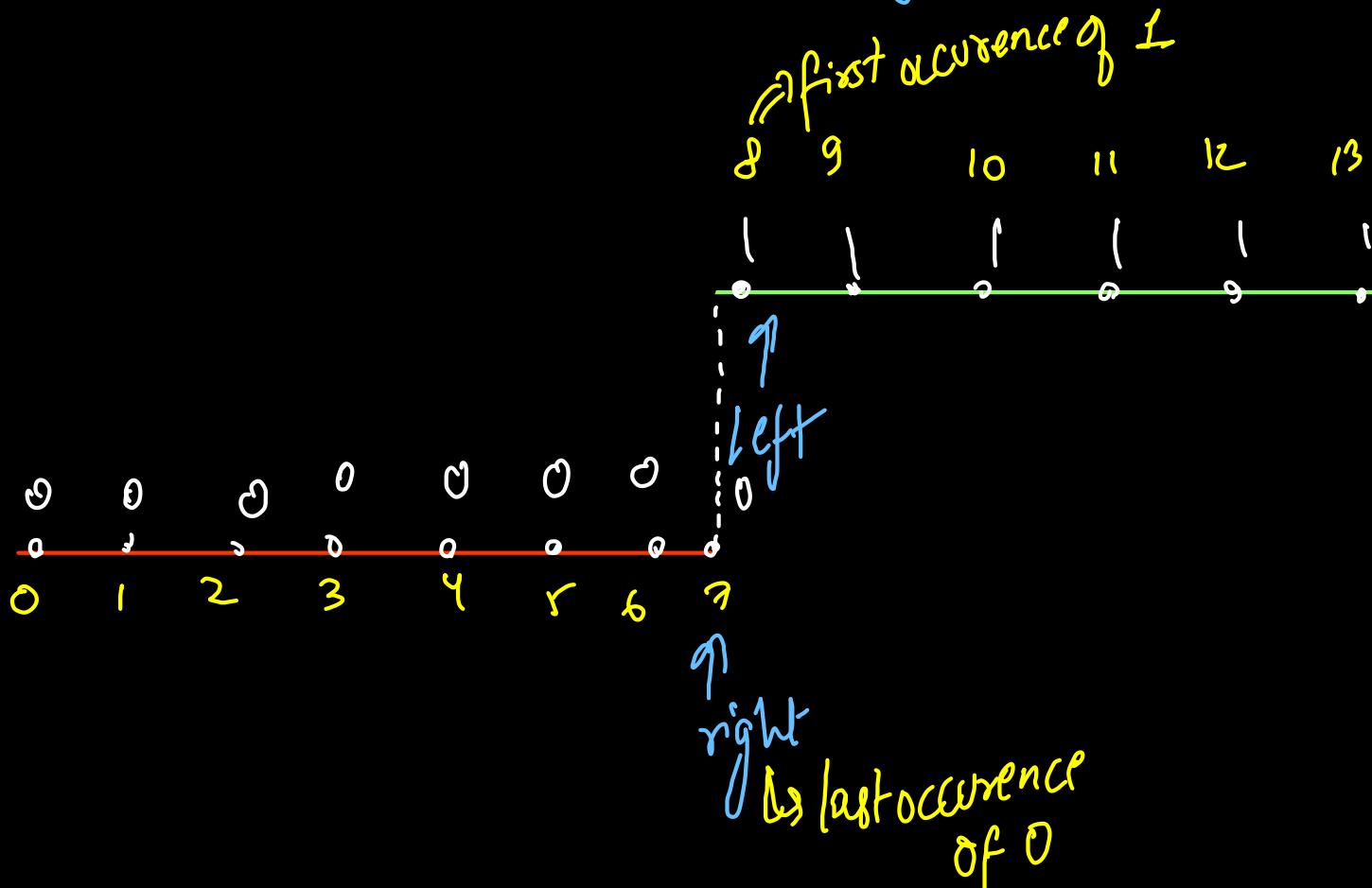
```
public int search(int[] nums, int target) {  
    int left = 0, right = nums.length - 1;  
  
    while(left <= right){  
        int mid = (left + right) / 2;  
  
        if(nums[mid] == target) return mid; // successful search  
        else if(nums[mid] < target) left = mid + 1;  
        else right = mid - 1;  
    }  
  
    return -1; // unsuccessful search  
}
```

Time :  $\Omega(1)$   
Bestcase :  $\Theta(\log n)$   
Avgcase :  $\Theta(\log n)$   
Worstcase :  $O(\log n)$

Space :  $O(1)$   
inplace

## Transition Point

Sorted Binary Array



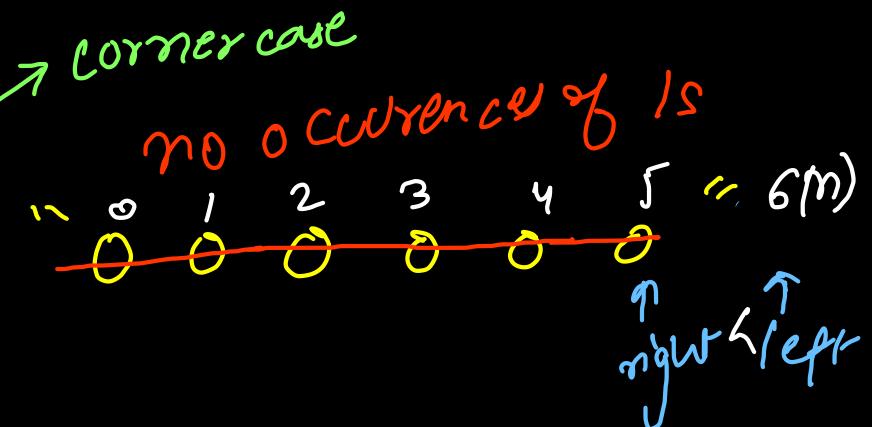
```

int transitionPoint(int arr[], int n) {
    if(n == 0 || arr[n - 1] == 0) return -1;

    int left = 0, right = n - 1;
    while(left <= right){
        int mid = (left + right) / 2;

        if(arr[mid] == 0) left = mid + 1;
        else right = mid - 1;
    }
    return left;
}

```



Aditya's  
Doubt  
left = 1      right = 1

$$(left + right)/2 = (1+1)/2 = 2/2 = 1$$

$$left/2 + right/2 = 1/2 + 1/2 = 0 + 0 = 0 \times$$

# First Bad Version {Google}

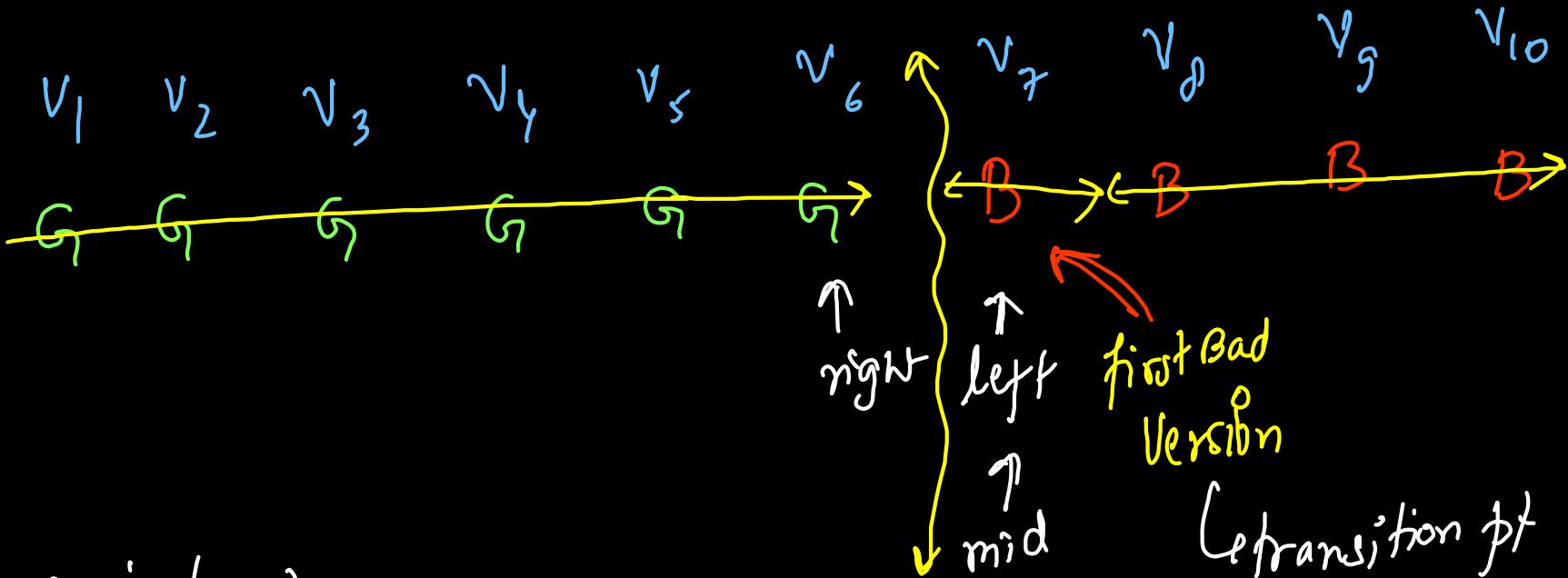
You are a product manager and currently leading a team to develop a new product. Unfortunately, the latest version of your product fails the quality check. Since each version is developed based on the previous version, all the versions after a bad version are also bad.

Suppose you have  $n$  versions  $[1, 2, \dots, n]$  and you want to find out the first bad one, which causes all the following ones to be bad.

You are given an API `bool isBadVersion(version)` which returns whether `version` is bad. Implement a function to find the first bad version. You should minimize the number of calls to the API.



eg  $n = 10$



`isBadVersion(rnd)`

True  
Bad

False  
Good

Binary Search

```

public class Solution extends VersionControl {
    public int firstBadVersion(int n) {
        int left = 1, right = n; ↳ natural no's
        while(left <= right){
            int mid = (left + right) / 2; TLE ↳ wrong
            if(isBadVersion(mid) == false) left = mid + 1;
            else right = mid - 1;
        }
        return left;
    }
}

```

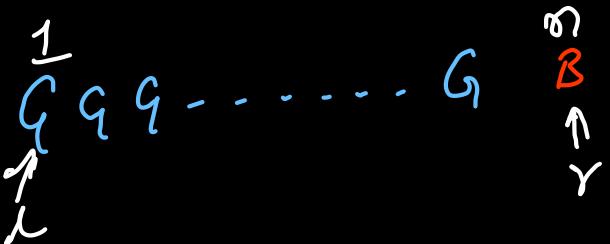
$\cancel{(left + right)} / 2$

$$= \cancel{(2^{31} - 1 + 1)} / 2 \quad \text{overflow}$$

$$= \cancel{(-garbage)} / 2 = \cancel{-ve}$$

Acc to Constraint

$$n = 2^{31} - 1 = \text{Integer, MAX-VALUE}$$



$$l = 1 = n$$

$$r = 2^{31} - 1 = n$$

$$\cancel{l + (right - left) / 2} \quad \text{left + } \frac{\cancel{right - left}}{2}$$

$$l + \cancel{\frac{2^{31} - 1 - 1}{2}} \quad \text{not overflowing}$$

```
public int firstBadVersion(int n) {  
    int left = 1, right = n;  
  
    while(left <= right){  
        int mid = left + (right - left) / 2;  
  
        if(isBadVersion(mid) == false) left = mid + 1;  
        else right = mid - 1;  
    }  
  
    return left;  
}
```

Accepted

Time  $\rightarrow O(\log n)$   
Space  $\rightarrow O(1)$   
Google

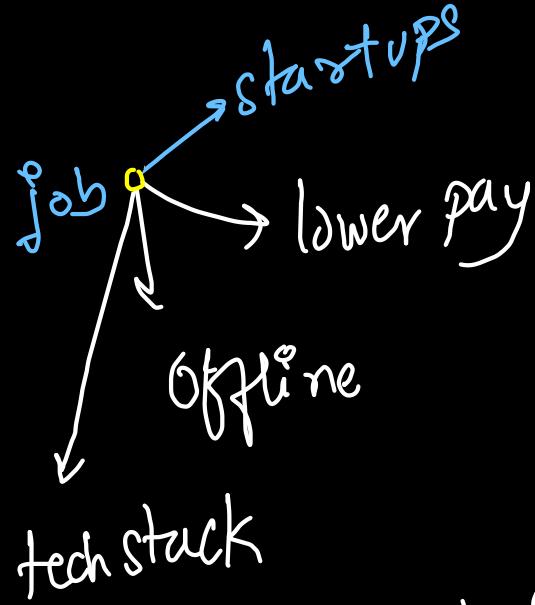
recession {us companies}

hiring slowdown

or

hiring freeze

} freshers



OR

layoffs } job switch

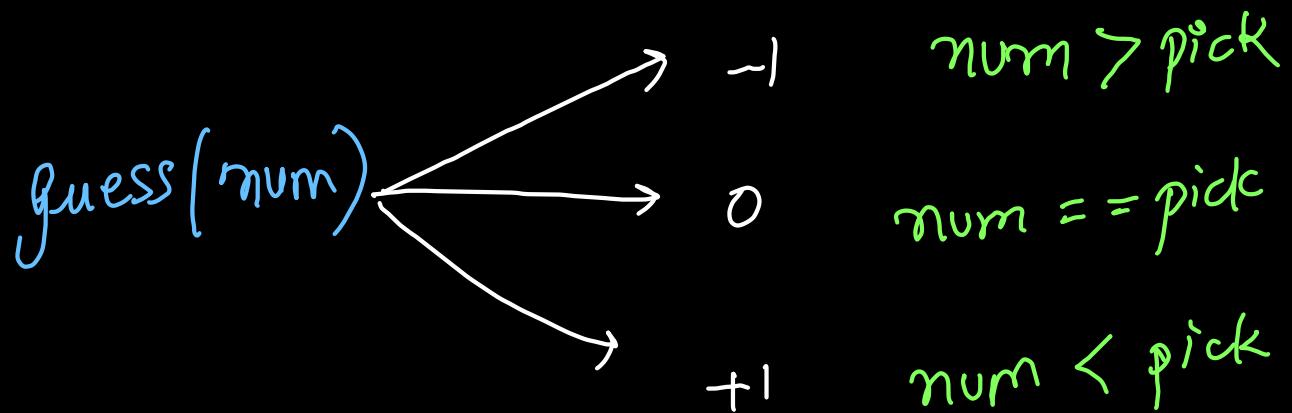
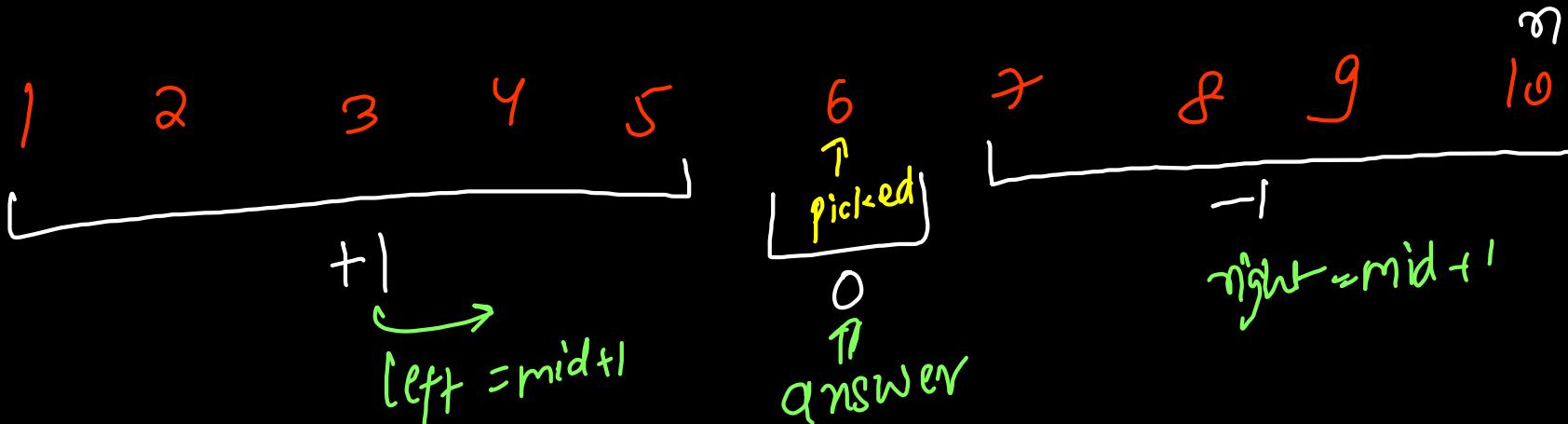
} backup

offer letter

↓

interviews

# Guess No Higher or Lower



```
public class Solution extends GuessGame {  
    public int guessNumber(int n) {  
        int left = 1, right = n;  
  
        while(left <= right){  
            int mid = left + (right - left) / 2;  
  
            int choice = guess(mid);  
            if(choice == 0) return mid;  
            else if(choice == 1) left = mid + 1;  
            else right = mid - 1;  
        }  
  
        return -1;  
    }  
}
```

Time  $\rightarrow O(\log_2 n)$

Space  $\rightarrow O(1)$

Lower Bound  
vs  
Upper Bound

Flour  
vs  
Celi

First Occurrence  
vs  
Last Occurrence

Binary Search with Duplicates, Sorted

10 10 20 20 20 20 30 50 50 50 60 60 70 80 80 80  
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

first occurrence (10) = 0

Last occurrence (10) = 1

First<sup>leftmost</sup>occ(20) = 2

Lastocc(20) = 5

rightmost

firstocc(30) = 6

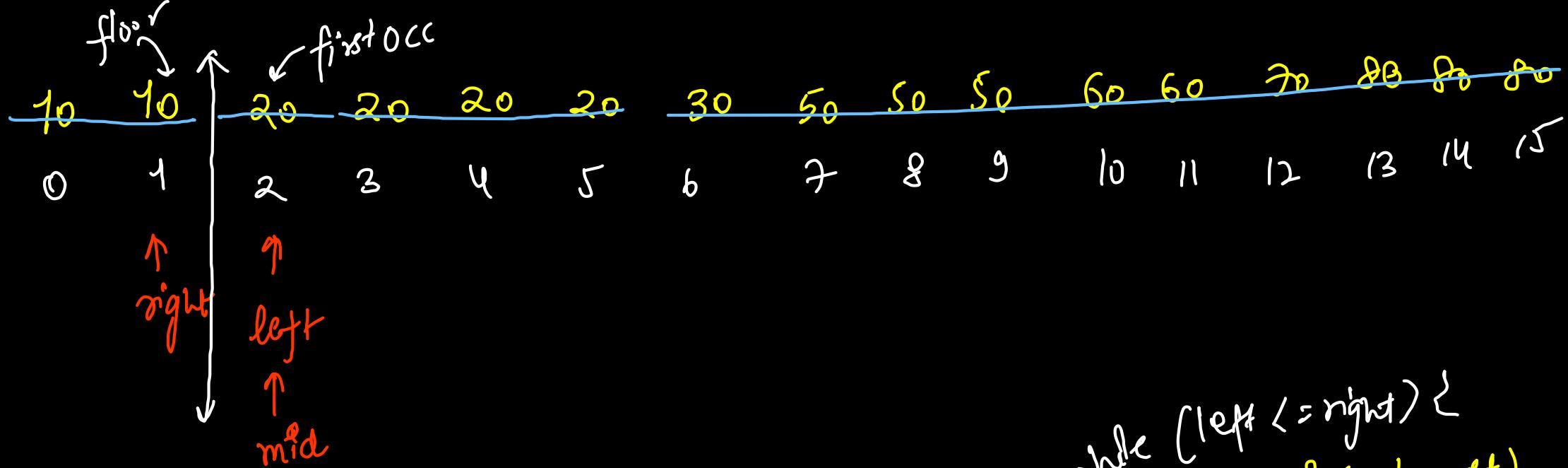
Lastocc(30) = 6

single occ

firstocc(40) = -1

Lastocc(40) = -1

target since



$\text{target} = 20$

$\text{firstOcc}(20) = ②$

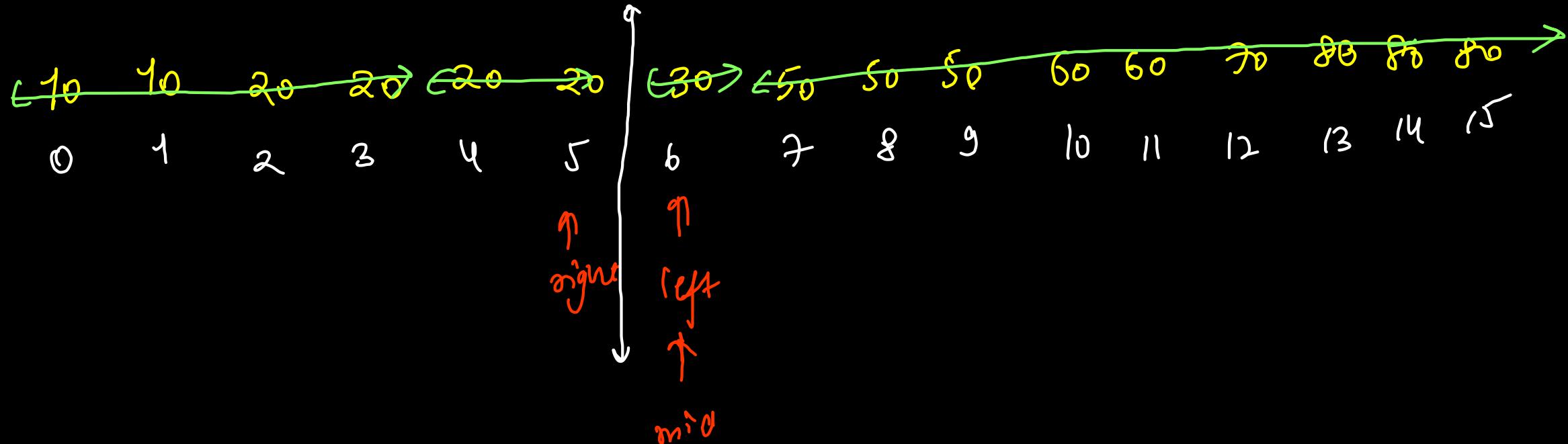
```

if (arr[mid] == target)
    right = mid - 1;
else if (arr[mid] < target)
    left = mid + 1;
else
    right = mid - 1;
  
```

$\text{return left};$

```

while (left <= right) {
    if (arr[mid] <= target)
        right = mid - 1;
    else
        left = mid + 1;
}
return left;
  
```



$$\text{lastOcc}(20) = \textcircled{5}$$

```

int lastOccurrence(int[] nums, int target){
    int left = 0, right = nums.length - 1;

    while(left <= right){
        int mid = left + (right - left) / 2;

        if(arr[mid] > target) right = mid - 1;
        else left = mid + 1;
    }

    return right;
}

```

```

int firstOccurrence(int[] nums, int target){
    int left = 0, right = nums.length - 1;

    while(left <= right){
        int mid = left + (right - left) / 2;

        if(arr[mid] >= target) right = mid - 1;
        else left = mid + 1;
    }

    return left;
}

```

 $\mathcal{O}(\log_2 n)$ 

```

int lastOccurrence(int[] nums, int target){
    int left = 0, right = nums.length - 1;

    while(left <= right){
        int mid = left + (right - left) / 2;

        if(arr[mid] > target) right = mid - 1;
        else left = mid + 1;
    }

    return right;
}

```

 $\mathcal{O}(\log_2 n)$ 

```

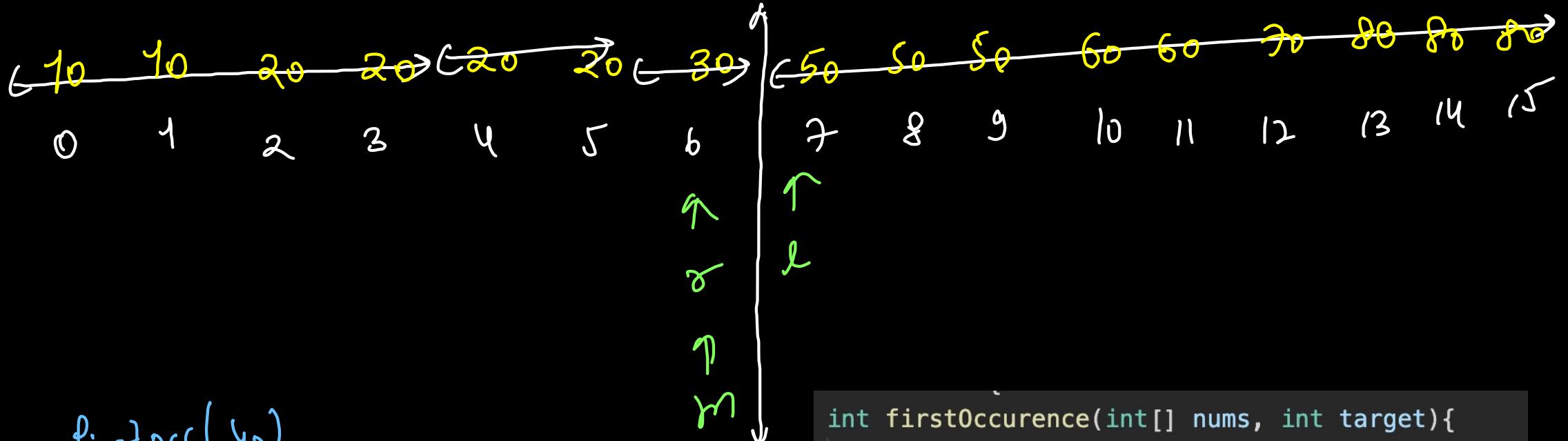
public int[] searchRange(int[] nums, int target) {
    int fi = firstOccurrence(nums, target);
    int li = lastOccurrence(nums, target);

    if(fi > li) return new int[]{-1, -1};
    return new int[]{fi, li};
}

```

target  
 nor  
 found

 $\mathcal{O}(2 \log_2 n)$ 
 $\approx \mathcal{O}(\log_2 n)$



firstOcc(40)

lastOcc(40)

```

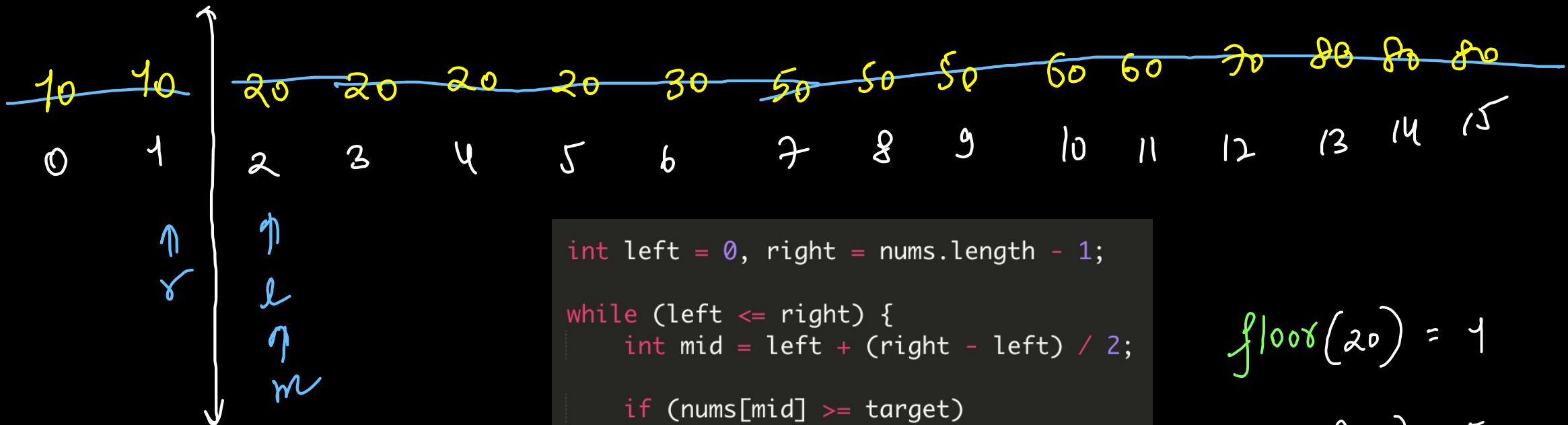
int firstOccurrence(int[] nums, int target){
    int left = 0, right = nums.length - 1;

    while(left <= right){
        int mid = left + (right - left) / 2;

        if(arr[mid] >= target) right = mid - 1;
        else left = mid + 1;
    }

    return left;
}

```



*if (arr[mid] == target)  
right = mid - 1*

*else if (arr[mid] < target)  
left = mid + 1*

*else  
right = mid - 1*

```
int left = 0, right = nums.length - 1;

while (left <= right) {
    int mid = left + (right - left) / 2;

    if (nums[mid] >= target)
        right = mid - 1;
    else
        left = mid + 1;
}

return right;
```

$$\text{floor}(20) = 1$$

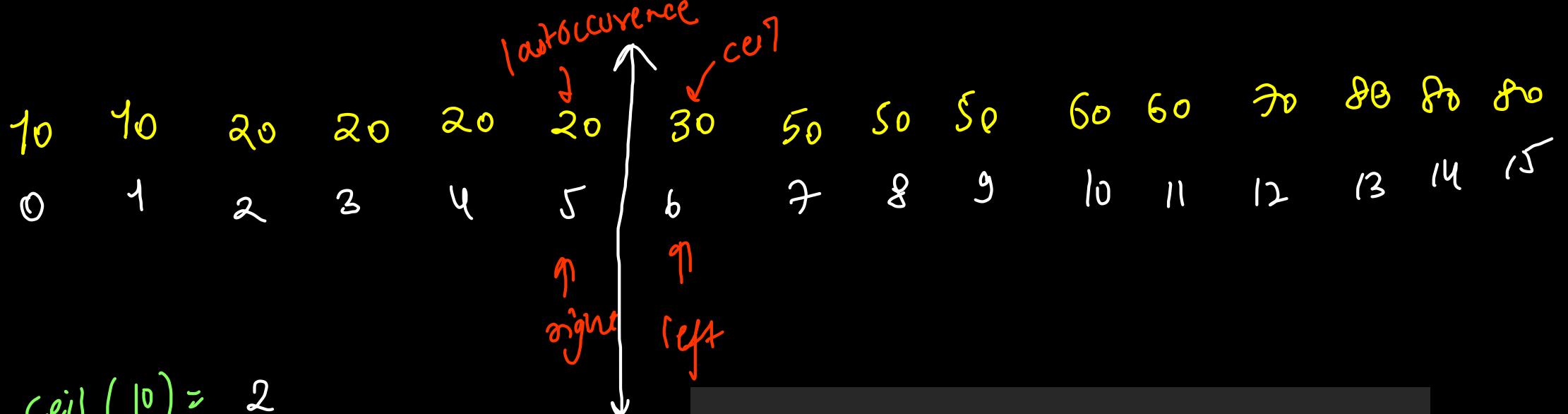
$$\text{floor}(25) = 5$$

$$\text{floor}(50) = 6$$

$$\text{floor}(100) = 15$$

$$\text{floor}(10) = 1$$

$$\text{floor}(0) = -1$$



$$\text{ceil}(10) = 2$$

$$\text{ceil}(20) = 6$$

$$\text{ceil}(25) = 6$$

$$\text{ceil}(80) = 16 \text{ (arr.length)}$$

$$\text{ceil}(100) = 16 \text{ (arr.length)}$$

```

int lastOccurrence(int[] nums, int target){
    int left = 0, right = nums.length - 1;

    while(left <= right){
        int mid = left + (right - left) / 2;

        if(arr[mid] > target) right = mid - 1;
        else left = mid + 1;
    }

    return return return left;
}

```

## floor (on GFG)

```
static int findFloor(long nums[], int n, long target)
{
    int left = 0, right = n - 1;

    while (left <= right) {
        int mid = left + (right - left) / 2;

        if(nums[mid] == target)
            return mid;
        if (nums[mid] > target)
            right = mid - 1;
        else
            left = mid + 1;
    }

    return right;
}
```

## ceil (on Codestudio)

```
int left = 0, right = nums.length - 1;

while (left <= right) {
    int mid = left + (right - left) / 2;

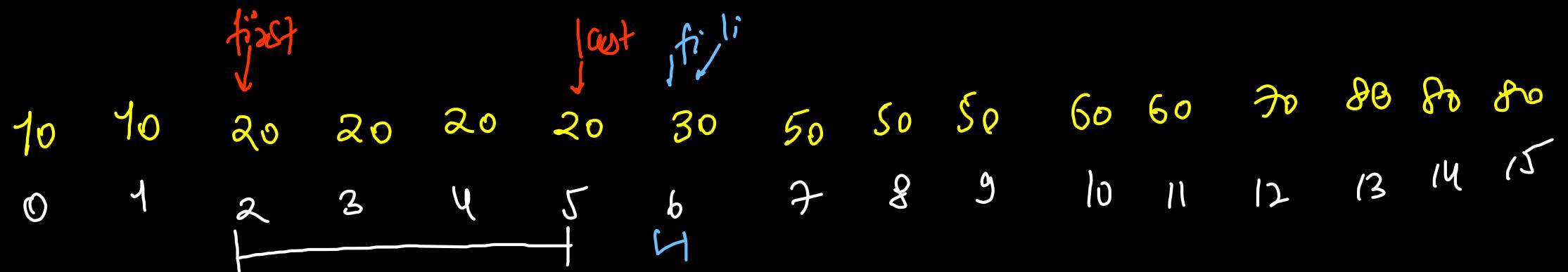
    if(nums[mid] == target) return nums[mid];
    if (nums[mid] > target)
        right = mid - 1;
    else
        left = mid + 1;
}

if(left == nums.length) return -1;
return nums[left];
```

Acc to GFG  $\text{floor}[x] \leq x$

Acc to Codestudio

$\text{ceil}[x] \geq x$



$$\text{Count occurrence} (\text{ans}) = \text{li} - \text{fi} + 1;$$

```

class Solution {
    int firstOccurrence(int[] nums, int target) { }  $\xrightarrow{\mathcal{O}(\log n)}$ 
    int lastOccurrence(int[] nums, int target) { }  $\xrightarrow{\mathcal{O}(\log n)}$ 
    int count(int[] nums, int n, int target) {
        int fi = firstOccurrence(nums, target);
        int li = lastOccurrence(nums, target);
    }  $\xrightarrow{\mathcal{O}(1)}$ 
    return li - fi + 1;
}

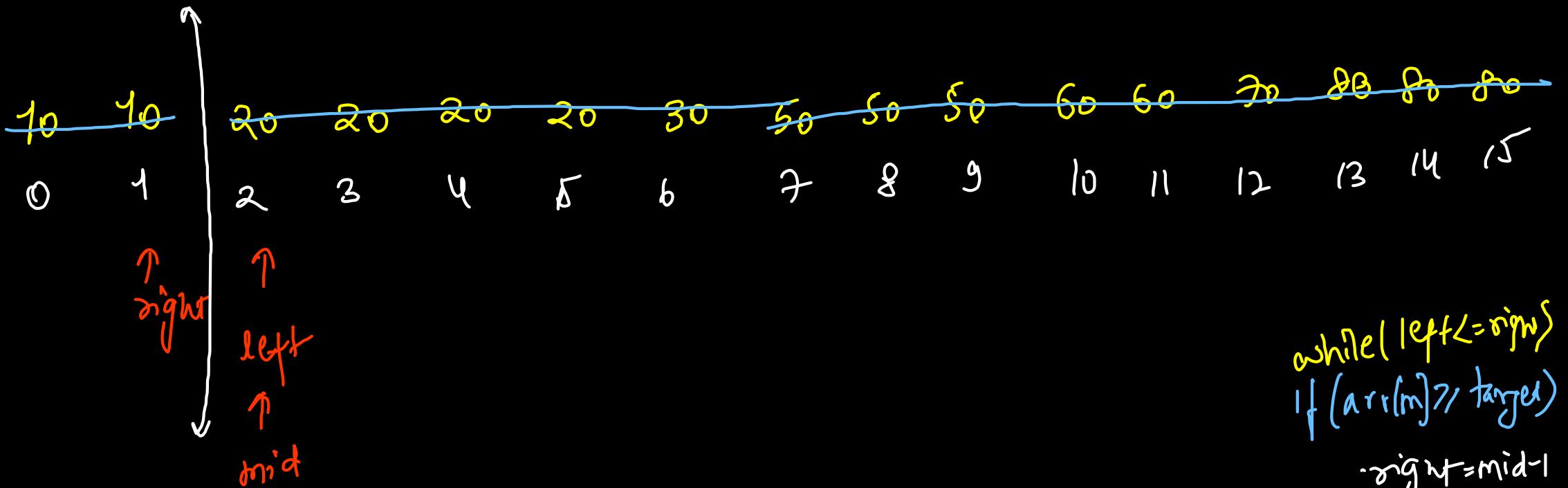
```

Lower Bound  $\Rightarrow$  If target is found  $\Rightarrow$  first occurrence

else target is not present  $\Rightarrow$  ceil

Upper Bound  $\Rightarrow$  Always the ceil value

\* If target is not found  $\Rightarrow$  Lower Bound = upper bound



$$\text{lowerBound}(20) = 2$$

$$\text{lowerBound}(30) = 6$$

$$\text{lowerBound}(80) = 13$$

$$\text{lowerBound}(5) = 0$$

$$\text{lowerBound}(18) = 2$$

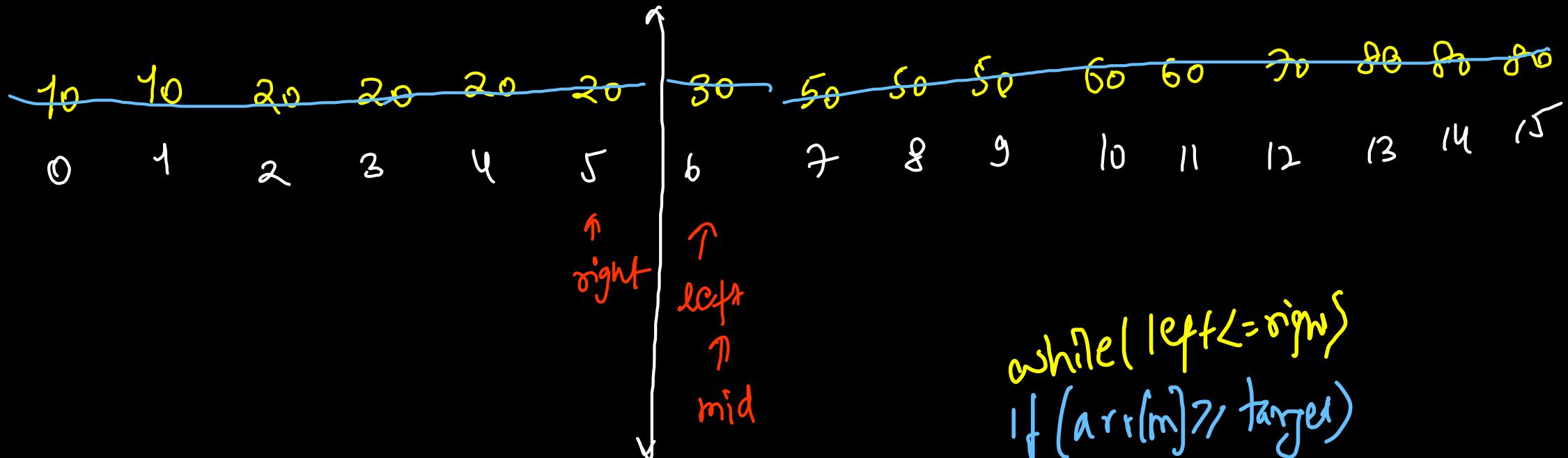
$$\text{lowerBound}(25) = 6$$

$$\text{lowerBound}(85) = \underline{\underline{\text{arr.length}}} \text{ or } -1$$

```

while(left <= right)
    if(arr[m] >= target)
        right = mid - 1
    else
        left = mid + 1
}
return left;

```

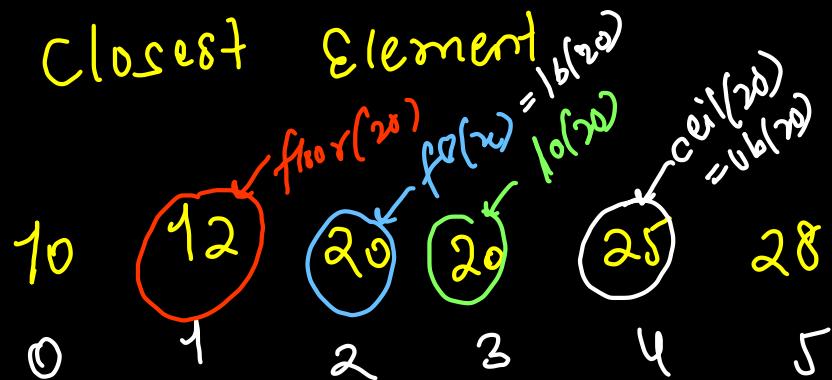


lower Bound (25) =

```

while (left <= right)
    if (arr[m] > target)
        right = mid - 1
    else
        left = mid + 1
}
return left;

```

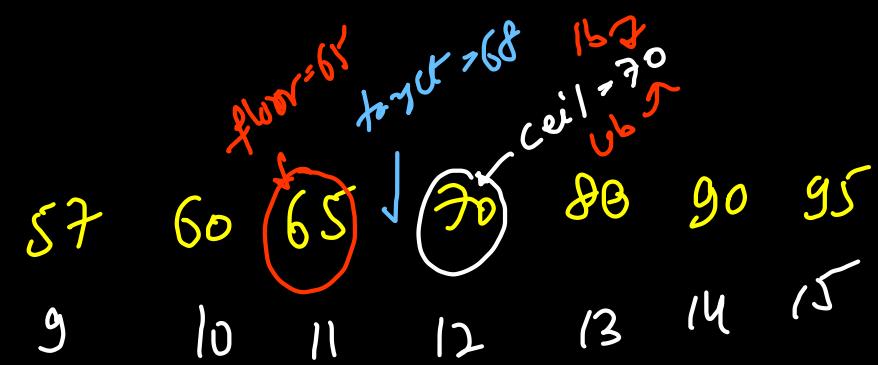


$$\text{target} = 45 \rightarrow 30 \quad |45 - 30| = 15$$

$$\text{target} = 45 \rightarrow 50 \quad |50 - 45| = 5$$

$$\text{target} = 60 \rightarrow 60 \quad |60 - 60| = 0$$

$$\text{target} = 60 \rightarrow 65 \quad |65 - 60| = 5$$



lower Bound  $\rightarrow$  target  $\vee \rightarrow$  first succ  
 vs  
 lower Bound - 1  $\Rightarrow$  floor

```

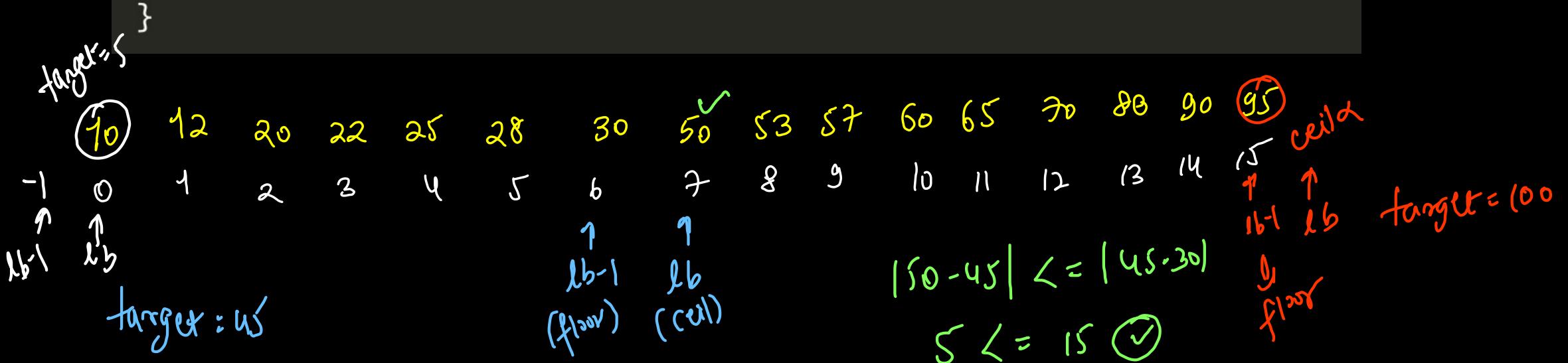
public static int lowerBound(int[] nums, int target) { ↵ }

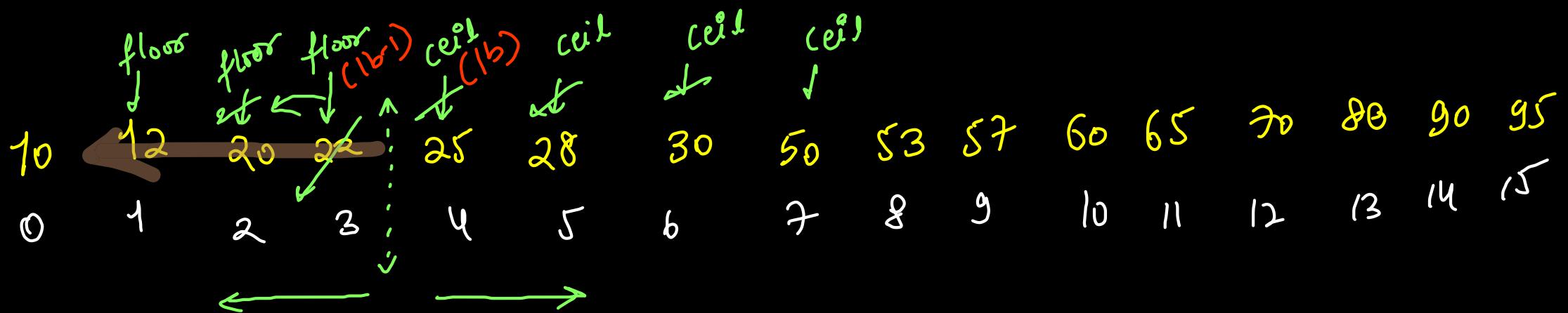
public static int findClosest(int arr[], int n, int target)
{
    int lb = lowerBound(arr, target);

    if(lb == 0) return arr[lb]; // floor does not exist
    if(lb == arr.length) return arr[lb - 1]; // ceil does not exist

    if(arr[lb] - target <= target - arr[lb - 1]) return arr[lb];
    else return arr[lb - 1];
}

```





target = 23

22, 25, 20, 28, 30

k = 5

```

public int lowerBound(int[] nums, int target) {
    // Same as First Occurrence
    int left = 0, right = nums.length - 1;

    while (left <= right) {
        int mid = left + (right - left) / 2;

        if (nums[mid] >= target)
            right = mid - 1;
        else
            left = mid + 1;
    }

    return left; // if target -> first occ, else ceil
}

```

$O(\log n)$

Time

$$O(\log n + k + \log c)$$

↑  
BS  
2 pointer

Collections.sort(res)  $\Rightarrow O(k)$

```

public List<Integer> findClosestElements(int[] nums, int k, int target) {
    int right = lowerBound(nums, target);
    int left = right - 1;

    List<Integer> res = new ArrayList<>();

    while(left >= 0 && right < nums.length && res.size() < k){
        if(target - nums[left] <= nums[right] - target){
            res.add(nums[left]);
            left--;
        } else {
            res.add(nums[right]);
            right++;
        }
    }
}

```

Two pointer

$$\Rightarrow O(k)$$

```

while(left >= 0 && res.size() < k){
    res.add(nums[left]);
    left--;
}

while(right < nums.length && res.size() < k){
    res.add(nums[right]);
    right++;
}

Collections.sort(res);  $\Rightarrow O(\log k)$ 
return res;
}

```

Square Root → Integral  $\text{LC } 69$

Square Root → Fractional CodeStudio

Valid perfect square  $\text{LC } 367$

N<sup>m</sup> Root GFG

# Square Root

$$N = 16$$

$$\sqrt{N} = 4$$

$$N = 25$$

$$\sqrt{N} = 5$$

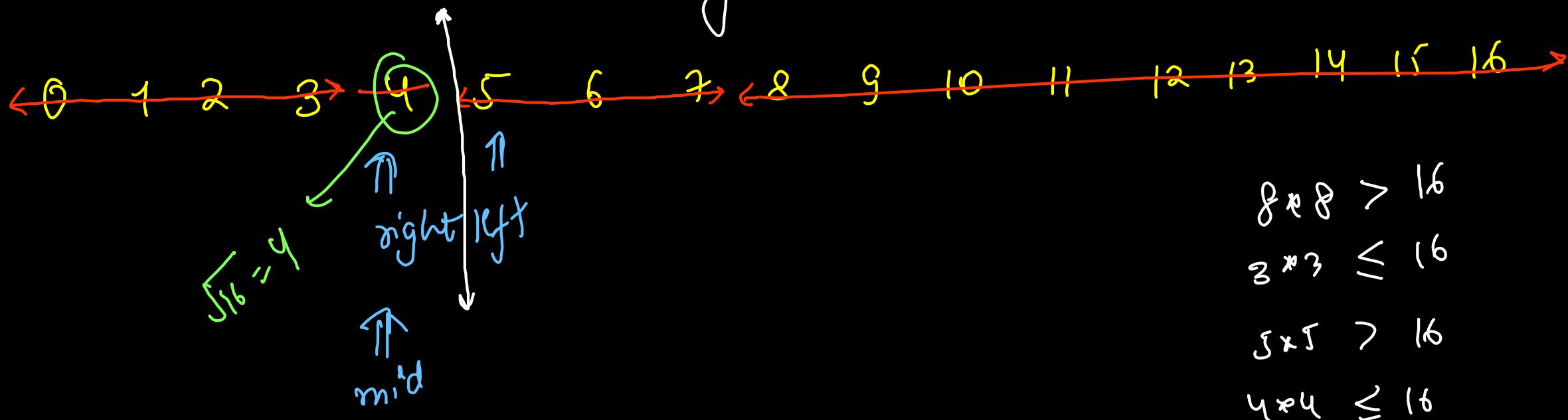
$$N = 20$$

$$\begin{aligned}\sqrt{N} &= \lfloor 4.4 \rfloor \\ &= 4\end{aligned}$$

$$N = 30$$

$$\begin{aligned}\sqrt{N} &= \lfloor 5.3 \rfloor \\ &= 5\end{aligned}$$

Binary Search    0 - N

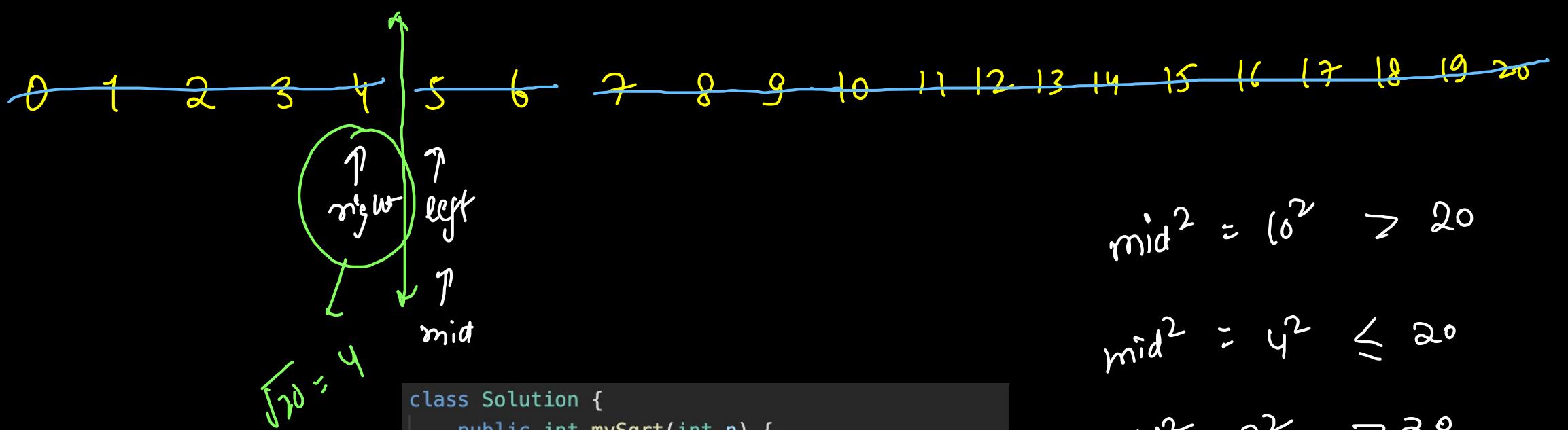


$$8 * 8 > 16$$

$$3 * 3 \leq 16$$

$$5 * 5 > 16$$

$$4 * 4 \leq 16$$



$$mid^2 = 0^2 > 20$$

$$mid^2 = 4^2 \leq 20$$

$$mid^2 = 2^2 > 20$$

$$\{^2 > 20$$

$O(\log n)$

```
class Solution {
    public int mySqrt(int n) {
        int left = 1, right = n;

        while(left <= right){
            int mid = left + (right - left) / 2;

            if(1l * mid * mid <= n){
                left = mid + 1;
            } else {
                right = mid - 1;
            }
        }

        return right;
    }
}
```

## Square Root Fractional

$$N = 16 \quad D = 3 \quad \Rightarrow \quad 4.000 \quad \text{floor}(\sqrt{N}) = 4.000$$

$$N = 20 \quad D = 2 \quad \Rightarrow \quad 4.47 \quad \text{floor}(\sqrt{N}) = 4.47$$

$$N = 30 \quad D = 5 \quad \Rightarrow \quad 5.47722 \quad \text{floor}(\sqrt{N}) = 5$$

$$x_{\max}^2 = 10^{15} \quad D_{\max} = 6$$

Floor  $\rightarrow$  Square Root + Linear Search on Every Decimal

$$\mathcal{O}(\log_2 n) + \mathcal{O}(10 * D)$$

$j=0$

$$4 + 0 \times 0^0 \downarrow$$

$$\begin{array}{r} 4.0 \\ 4.0 \\ \downarrow +0.1 \\ j=1 \quad 4.1 \quad 4+1 \times 0^0 \downarrow \quad (0)^{-1} \\ \downarrow +0.1 \quad \approx 1/10 \end{array}$$

$$\begin{array}{r} j=2 \quad 4.2 \quad 4+2 \times 0^0 \downarrow = 4+0.2 \div 4.2 \\ \downarrow +0.1 \end{array}$$

$$\begin{array}{r} j=3 \quad 4.3 \quad 4+3 \times 0^0 \downarrow = 4.3 \\ \downarrow \end{array}$$

$$j=4 \quad 4.4$$

$$\begin{array}{r} \swarrow \\ 4.5 \end{array}$$

$j=1$

$$4.00$$

$$\begin{array}{r} \downarrow +0.01 \\ 4.01 \quad 10^{-2} = 1/(100) \quad 4.001 \quad 10^{-3} = 1/1000 \\ \downarrow +0.01 \quad \downarrow +0.001 \end{array}$$

$$4.02$$

$$\begin{array}{r} \downarrow \\ \vdots \end{array}$$

$$\begin{array}{r} \downarrow +0.01 \\ 4.03 \end{array}$$

$$4.09$$

$j=2$

$$4.000$$

$$\begin{array}{r} \downarrow +0.001 \\ 4.001 \quad 10^{-3} = 1/(1000) \quad 4.0001 \quad 10^{-4} = 1/10000 \\ \downarrow +0.001 \quad \downarrow +0.0001 \end{array}$$

$$4.002$$

$$\begin{array}{r} \downarrow \\ \vdots \end{array}$$

$$\begin{array}{r} \downarrow \\ \vdots \end{array}$$

$$4.009$$

```

public static long mySqrt(long n) {
    long left = 1, right = n;

    while (left <= right) {
        long mid = left + (right - left) / 21;
        if (mid * mid <= n) {
            left = mid + 1;
        } else {
            right = mid - 1;
        }
    }

    return right;
}

```

*O(log n)*

```

public static double squareRoot(long n, int d) {
    long floor = mySqrt(n);
    double ans = floor;
    double factor = 1.0;

    for(int i = 0; i < d; i++){
        factor = factor / 10;
        for(int j = 0; j < 10; j++){
            if((ans + factor) * (ans + factor) <= n)
                ans += factor;
            else break;
        }
    }

    return ans;
}

```

*O(d \* 10<sup>d</sup>)*

$$n=20$$

$\text{factor} = 0.1$

$$\text{ans} = 4 + 0.1 \text{ (floor)}$$

$$= 4.1$$

$$+ 0.1$$

$$= 4.2$$

$$+ 0.01$$

$$= 4.21$$

$$+ 0.01$$

$$= 4.22$$

$$+ 0.01$$

$$= 4.23$$

$$+ 0.01$$

$$= 4.24$$

$$+ 0.01$$

$$= 4.25$$

$$+ 0.01$$

$$= 4.26$$

$$+ 0.01$$

$$= 4.27$$

$$+ 0.01$$

$$= 4.28$$

$$+ 0.01$$

$$= 4.29$$

$$+ 0.01$$

$$= 4.3$$

$$+ 0.01$$

$$= 4.31$$

$$+ 0.01$$

$$= 4.32$$

$$+ 0.01$$

$$= 4.33$$

$$+ 0.01$$

$$= 4.34$$

$$+ 0.01$$

$$= 4.35$$

$$+ 0.01$$

$$= 4.36$$

$$+ 0.01$$

$$= 4.37$$

$$+ 0.01$$

$$= 4.38$$

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$$= 6.0$$

$\text{factor} = 0.01$

$$4.04^2 \leq 20$$

$$4.1^2 \not\leq 20$$

$$4.11^2 \leq 20$$

$$4.12^2 \not\leq 20$$

$$4.13^2 \leq 20$$

$$4.14^2 \not\leq 20$$

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$$4.97^2 \leq 20$$

$$4.98^2 \not\leq 20$$

$$4.99^2 \leq 20$$

$$5.0^2 \not\leq 20$$

left = 0.0

right = 20.0

mid = 10.0

$$10.0^2 = 100 \not\leq 20$$

left = 0.0

right = 10.0

mid = 5.0

$$5.0^2 \not\leq 20$$

left = 0.0

right = 5.0

mid = 2.5

$$2.5^2 \leq 20$$

left = 2.5

right = 5.0

mid = 3.75

$$3.75^2 \leq 20$$

$\lambda = 3.75$   $\gamma = 5.0$

$$m = 4.375 \quad 4.375^2 \leq 20$$

$\lambda = 4.375$   $\gamma = 5.0$

$$m = 4.6875^2 > 20$$

$$\lambda = 4.375 \quad \gamma = 4.6875$$

$$m = 4.531$$

$$N = 20 \quad D = 2$$

$4.6875 \rightarrow 4.75$   
 $\lambda$

$$10^{-2} = 10^{-D}$$

while ( $right - left \geq 0.01$ ) {

double mid =  $\lambda + (\gamma - \lambda) / 2.0;$

if ( $mid^2 \leq n$ )  $left = mid + 0.01;$

else  $right = mid - 0.01;$

```
public static double squareRoot(long n, int d) {  
    double left = 1.0, right = n;  
    double epsilon = Math.pow(10, -d);  
  
    while (right - left >= epsilon) {  
        double mid = left + (right - left) / 2.0;  
  
        if (mid * mid <= n) {  
            left = mid;  
        } else {  
            right = mid;  
        }  
    }  
  
    return left;  
}
```

Approach 2

# Valid Perfect Square

$$N = 16 \quad \checkmark$$

$$\sqrt{N} = 4$$

$$\text{floor} = 4$$

$$4^2 = 16$$

$$N = 20 \quad \times$$

$$\sqrt{N} = 4.47 \dots$$

$$\text{floor} = 4$$

$$4^2 \neq 20$$

Leetcode 367

```
public int mySqrt(int n) {  
    int left = 1, right = n;  
  
    while (left <= right) {  
        int mid = left + (right - left) / 2;  
  
        if (1l * mid * mid <= n) {  
            left = mid + 1;  
        } else {  
            right = mid - 1;  
        }  
    }  
  
    return right;  
}  
  
public boolean isPerfectSquare(int num) {  
    int floor = mySqrt(num);  
    return (floor * floor == num);  
}
```

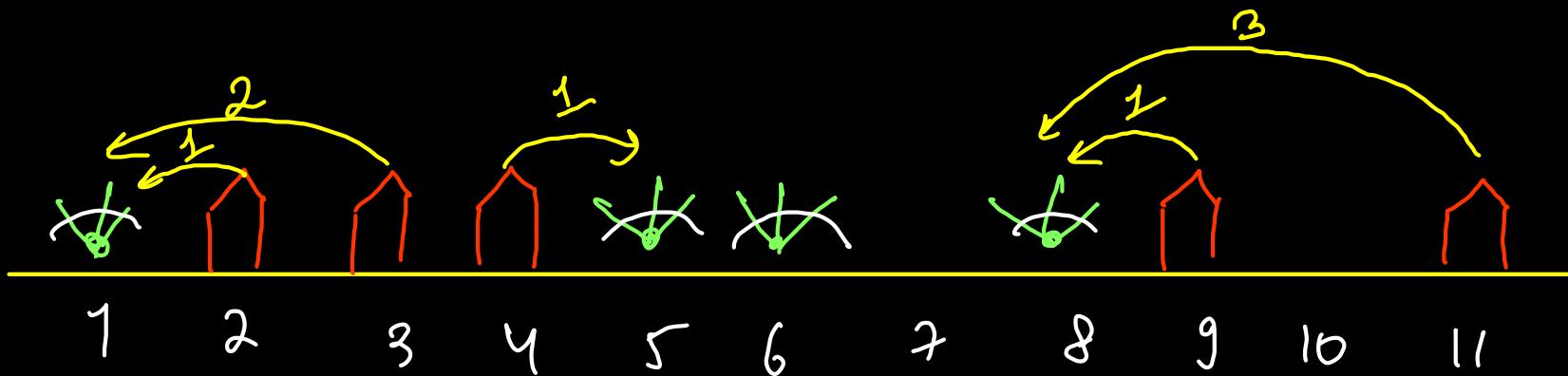
# Heaters

houses: 3, 9, 2, 4, 11

Min radius → Farthest house from any heater.  
Max radius of closest heater  
for each house

minimum radius = ③

heaters: 1, 8, 5, 6 <sup>sort</sup> → 1, 5, 6, 8



```

public int findRadius(int[] houses, int[] heaters) {
    Arrays.sort(heaters); → O( $n_2 \log n_2$ )
    int minRadius = 0;
    for(int val: houses){   O( $n_1 * \log n_2$ )
        int closestHeater = findClosest(heaters, val);
        int radius = Math.abs(closestHeater - val);
        minRadius = Math.max(radius, minRadius);
    }
    return minRadius;
}

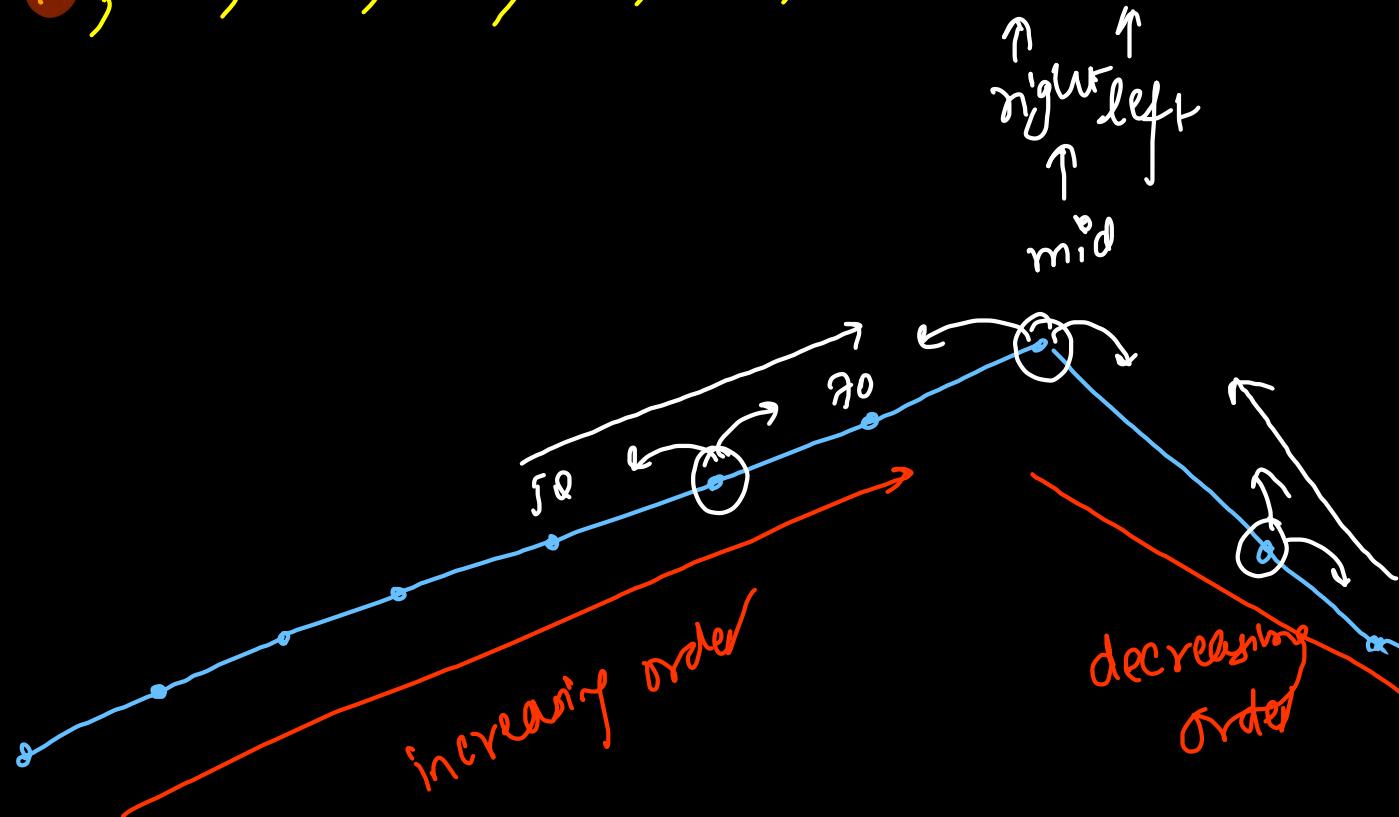
```

$n_1$   $n_2$   
 Total time  $O((n_1 + n_2) * \log(n_2))$   
 $\Rightarrow O(n_1 \log n_2)$

Mountain Array  $\Rightarrow$  Peak Element  $\Rightarrow$  Bitonic Array

Leetcode 852 :

[ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 ]  
[ 10, 20, 30, 40, 50, 60, 70, 80, 50, 30, 20, 5 ]

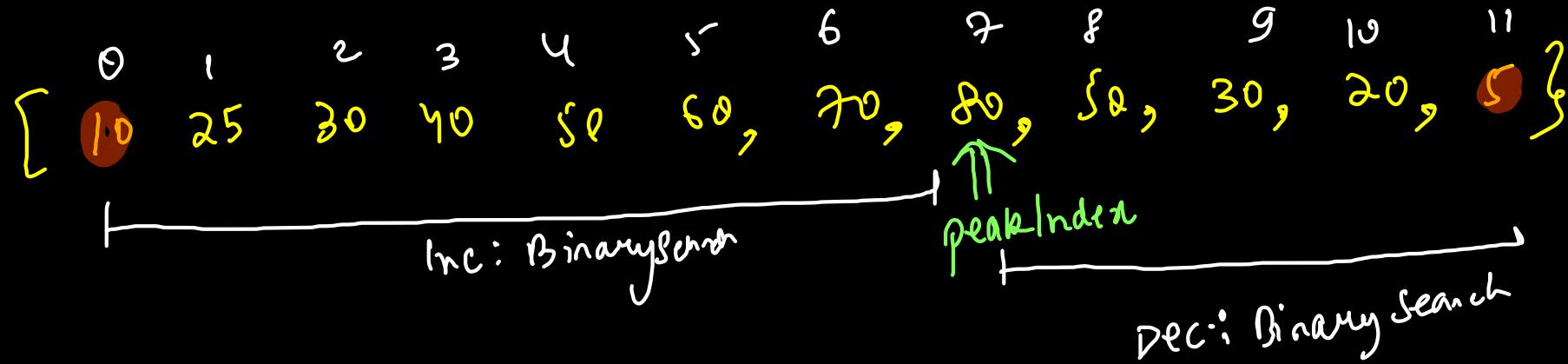


```
if (arr[m] > arr[m-1])  
    &amp; arr[m] > arr[m+1])  
    return m;  
else if (arr[m] < arr[m+1])  
    left = m+1;  
else right = m-1;
```

```
public int peakIndexInMountainArray(int[] arr) {  
    int left = 1, right = arr.length - 2;  
  
    while(left <= right){  
        int mid = left + (right - left) / 2;  
  
        if(arr[mid] > arr[mid - 1] && arr[mid] > arr[mid + 1]){  
            return mid;  
        }  
        else if(arr[mid] < arr[mid + 1]){  
            left = mid + 1;  
        } else {  
            right = mid - 1;  
        }  
    }  
  
    return -1;  
}
```

Time  $\rightarrow \Theta(\log n)$

Space  $\rightarrow \Theta(1)$



target = 20  $\Rightarrow$  10<sup>m</sup>

target = 40  $\Rightarrow$  3<sup>rd</sup>

target = 80  $\Rightarrow$  7<sup>m</sup>

target = 100, 15, 0 : -1

```

public int binarySearchInc(int[] arr, int left, int right, int target){
    while(left <= right){
        int mid = left + (right - left) / 2;

        if(arr[mid] == target) return mid;
        else if(arr[mid] < target) left = mid + 1;
        else right = mid - 1;
    }
    return -1;
}

public int binarySearchDec(int[] arr, int left, int right, int target){
    while(left <= right){
        int mid = left + (right - left) / 2;

        if(arr[mid] == target) return mid;
        else if(arr[mid] > target) left = mid + 1;
        else right = mid - 1;
    }
    return -1;
}

public int solve(int[] arr, int target) { → O(logn)
    int peakIdx = peakIndexInMountainArray(arr);

    int lans = binarySearchInc(arr, 0, peakIdx, target);
    if(lans != -1) return lans; // Inc search successful

    return binarySearchDec(arr, peakIdx + 1, arr.length - 1, target); // Target not found
}

```

Time  $\hookrightarrow O(\log n)$

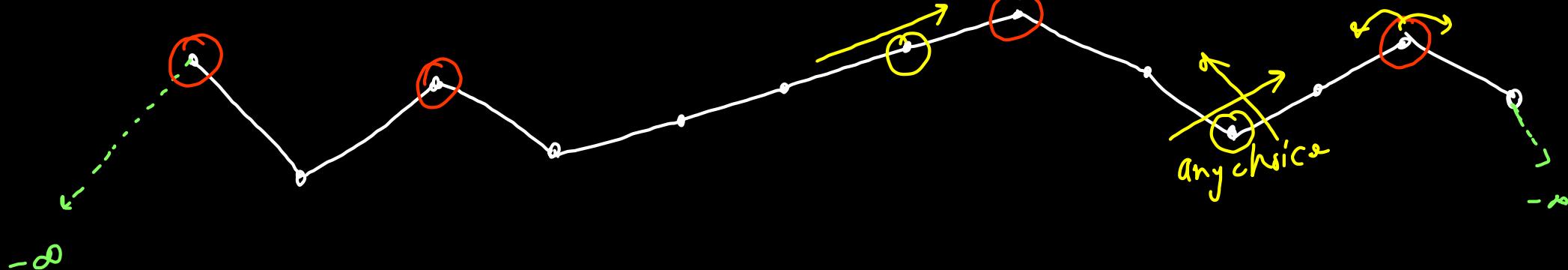
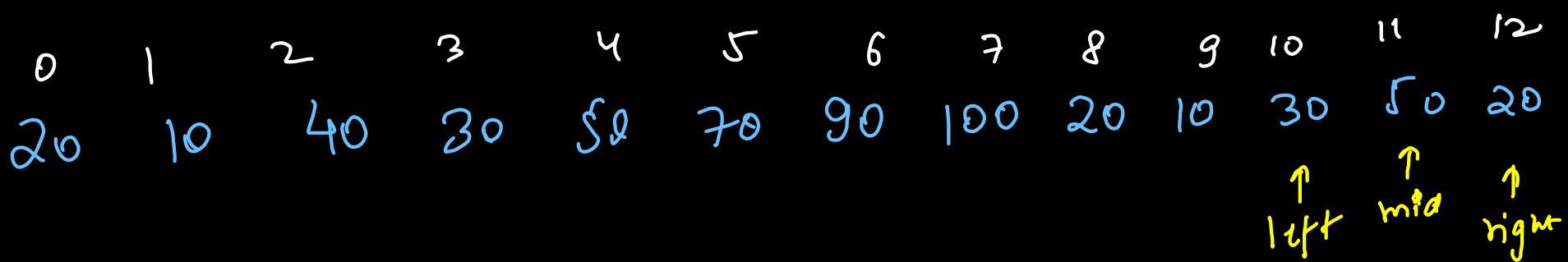
Space  $\hookrightarrow O(1)$

# Peak Element Leetcode 162

- \*
  - array: unsorted
  - multiple peaks → any peak
  - $\text{nums}[-1] = \text{nums}[n] = -\infty$

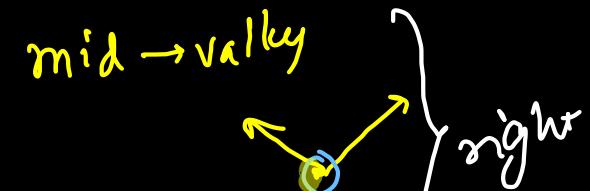
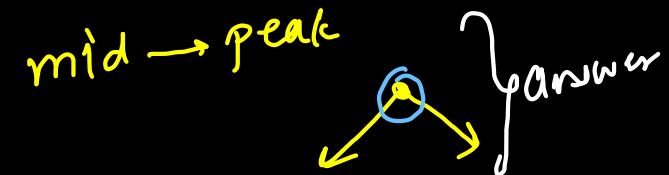
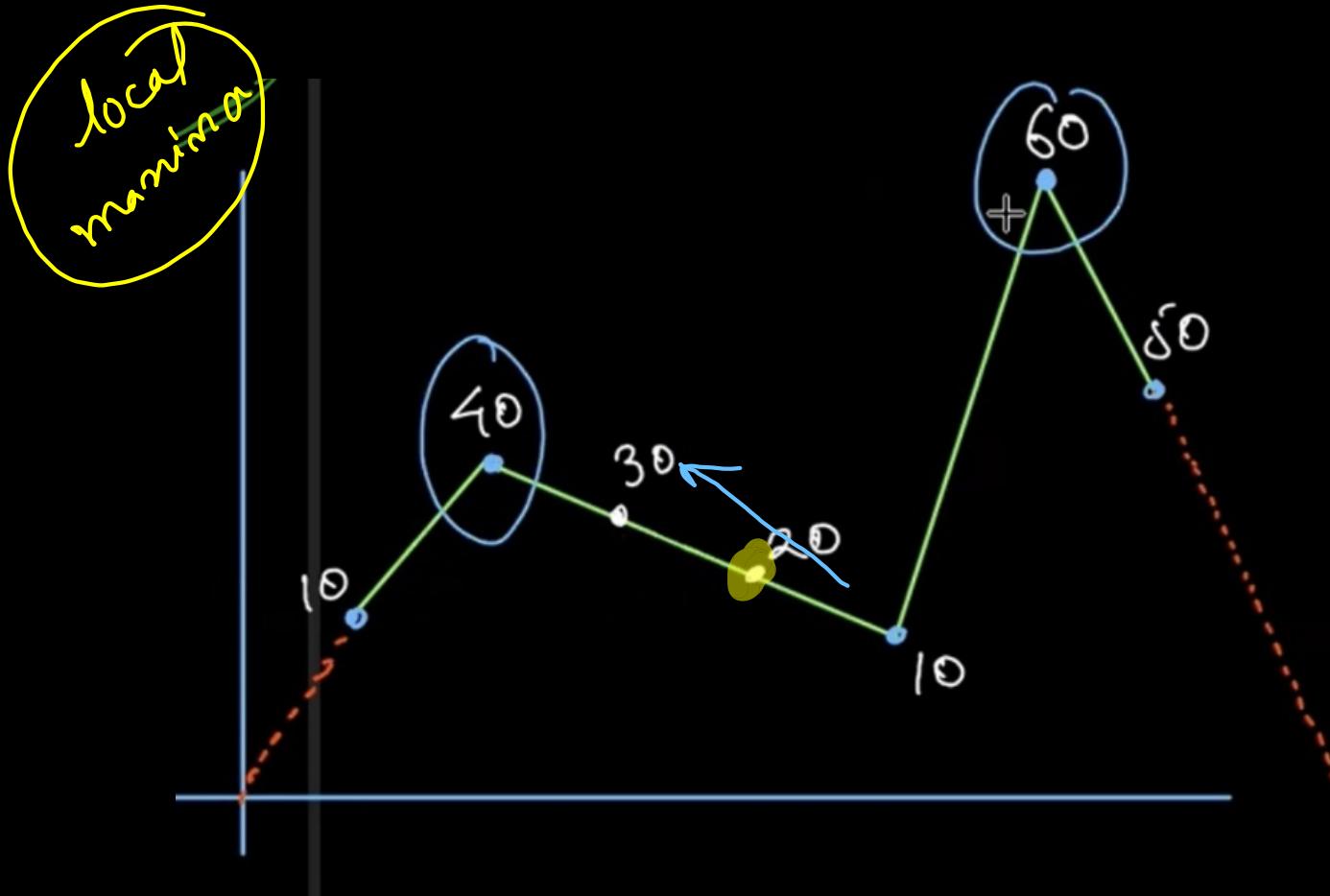
~~global peak~~  
local peak

Example 1



Example 2

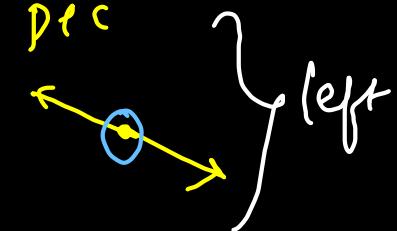
0 1 2 3 4 5 6  
10, 40, 30, 20, 10, 60, 50  
↑ ↑ ↑ ↑ ↑ ↑ ↑



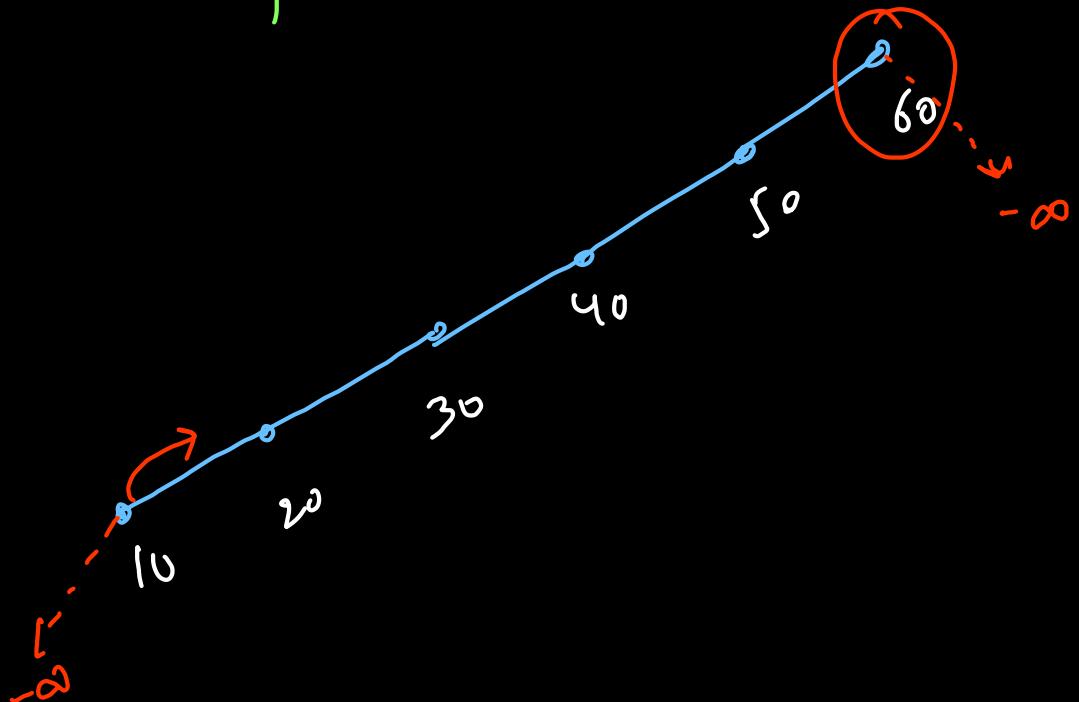
mid → inc



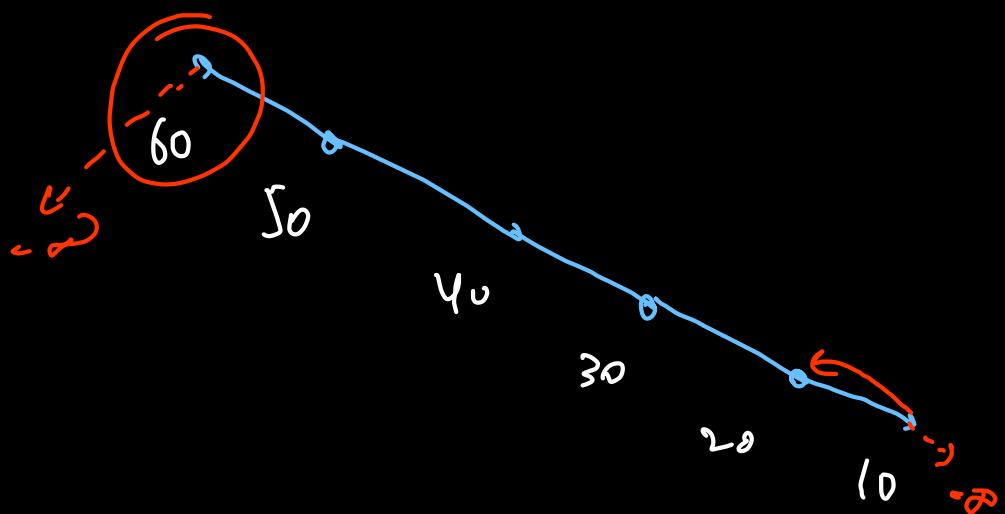
mid → dec



Example 3; cornercase (hnc)



Example 4'; Dec



```

public int peakIndexInMountainArray(int[] arr) {
    int left = 1, right = arr.length - 2;

    while(left <= right){
        int mid = left + (right - left) / 2;

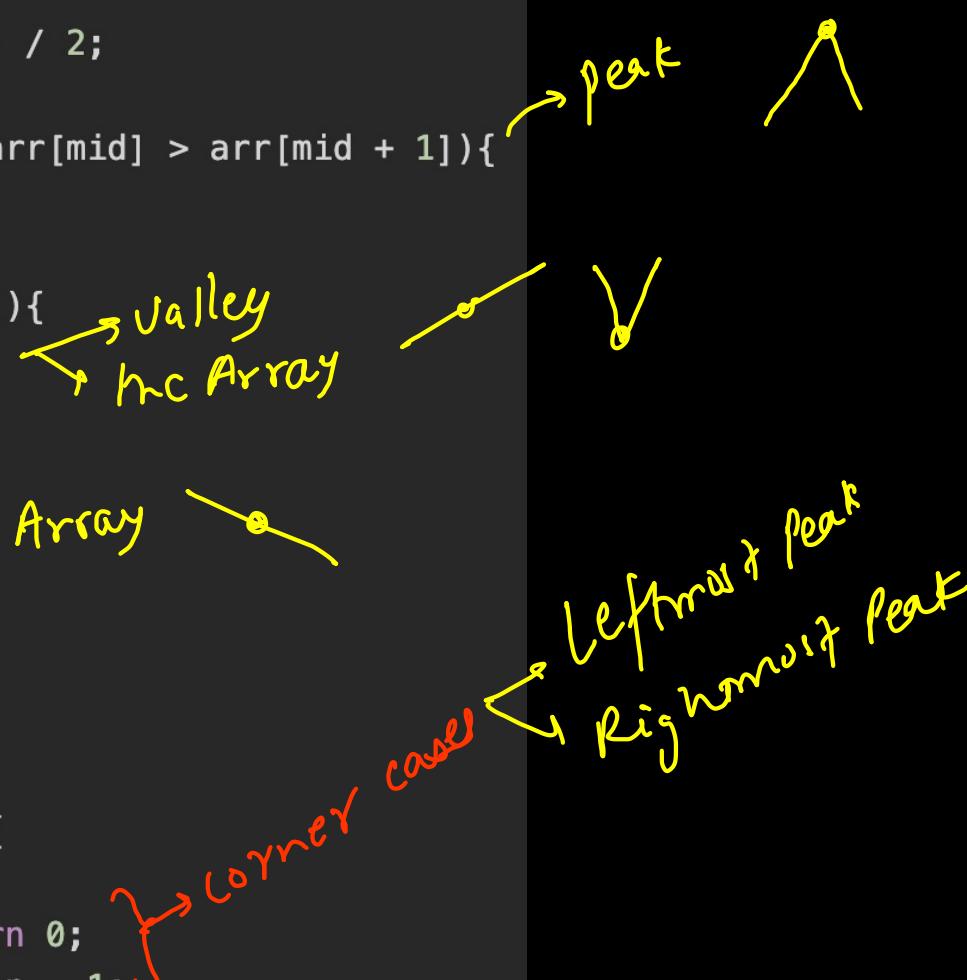
        if(arr[mid] > arr[mid - 1] && arr[mid] > arr[mid + 1]){
            return mid;
        }
        else if(arr[mid] < arr[mid + 1]){
            left = mid + 1;
        } else {
            right = mid - 1;
        }
    }
    return -1;
}

public int findPeakElement(int[] arr) {
    int n = arr.length;
    if(n == 1 || arr[0] > arr[1]) return 0;
    if(arr[n - 1] > arr[n - 2]) return n - 1;
    return peakIndexInMountainArray(arr);
}

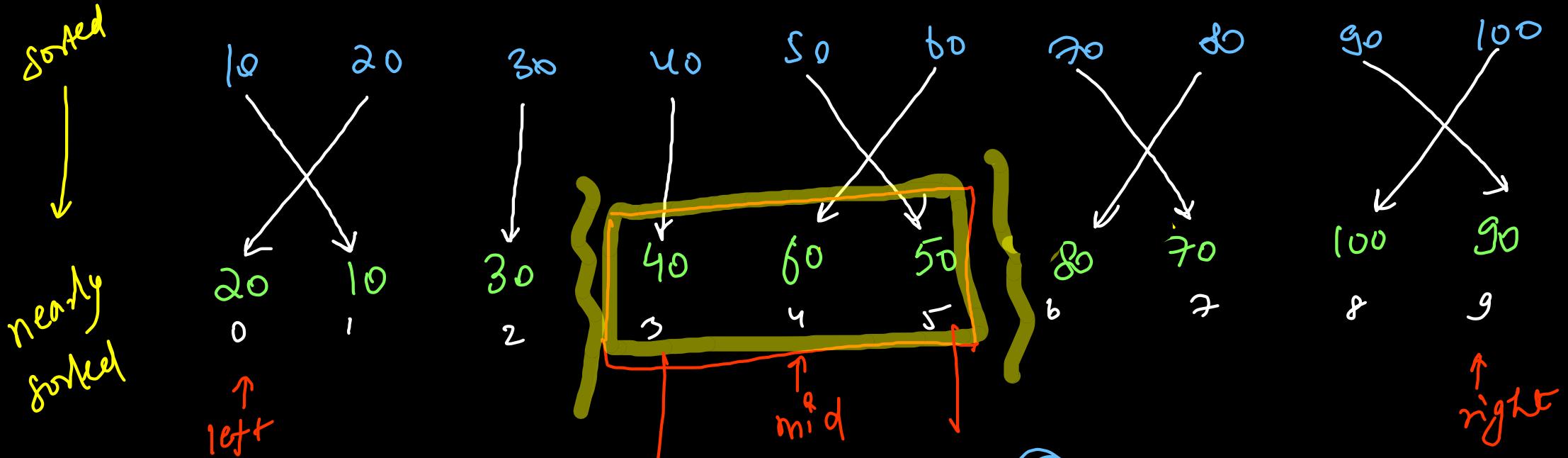
```

Time  
 $O(\log n)$

Space  
 $O(1)$



# Search in Nearly sorted Array



target  $\Rightarrow$  30 : ②

$\neq$  80 : ⑥

$\Rightarrow$  55 : ①

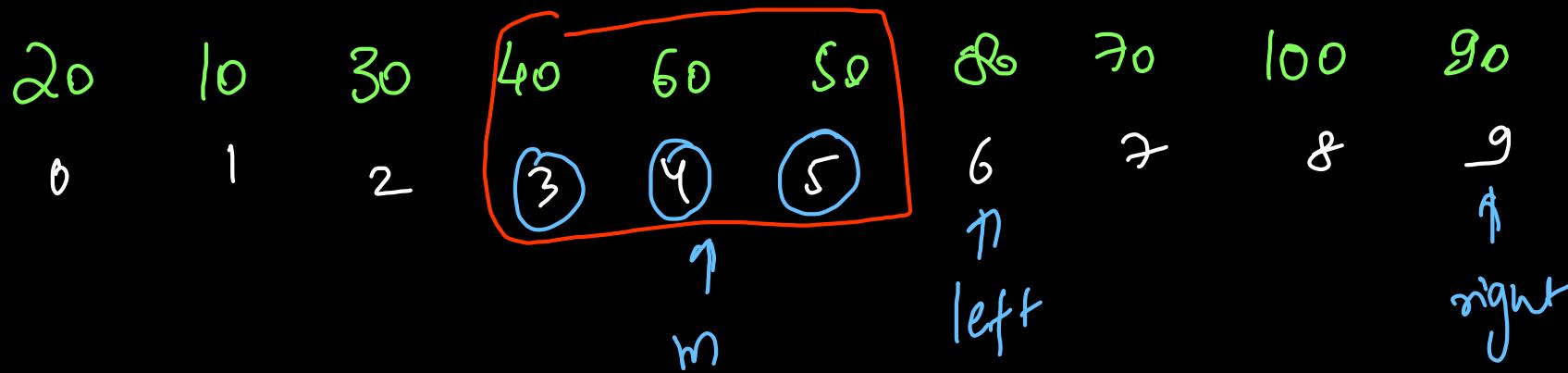
target = 30

target  
 $= \{ \}$   
return false

min = 40

mid = 60

target = 80



~~target = 30~~

~~80~~  
~~55~~

①  $\text{arr}(m)$ ,  $\text{arr}(m-1)$ ,  $\text{arr}(m+1)$   
return true;

②  $\text{target} < \text{min}$   
 $\text{right} = \text{mid} - 2$ .

③  $\text{target} > \text{max}$   
 $\text{left} = \text{mid} + 2$ ;  
④ return false

```
public static int solve(int[] arr, int target) {  
    int left = 0, right = arr.length - 1;  
    while (left <= right) {  
        int mid = left + (right - left) / 2;  
  
        int lval = (mid == 0) ? Integer.MIN_VALUE : arr[mid - 1];  
        int rval = (mid == arr.length - 1) ? Integer.MAX_VALUE : arr[mid + 1];  
  
        if (arr[mid] == target)  
            return mid;  
        if (lval == target)  
            return mid - 1;  
        if (rval == target)  
            return mid + 1;  
  
        int min = Math.min(arr[mid], Math.min(rval, lval));  
        int max = Math.max(arr[mid], Math.max(rval, lval));  
  
        if (target < min)  
            right = mid - 2;  
        else if (target > max)  
            left = mid + 2;  
        else  
            break;  
    }  
    return -1;  
}
```

Time  $\Rightarrow O(\log n)$

Space  $\Rightarrow O(1)$

## Rotated Sorted Array

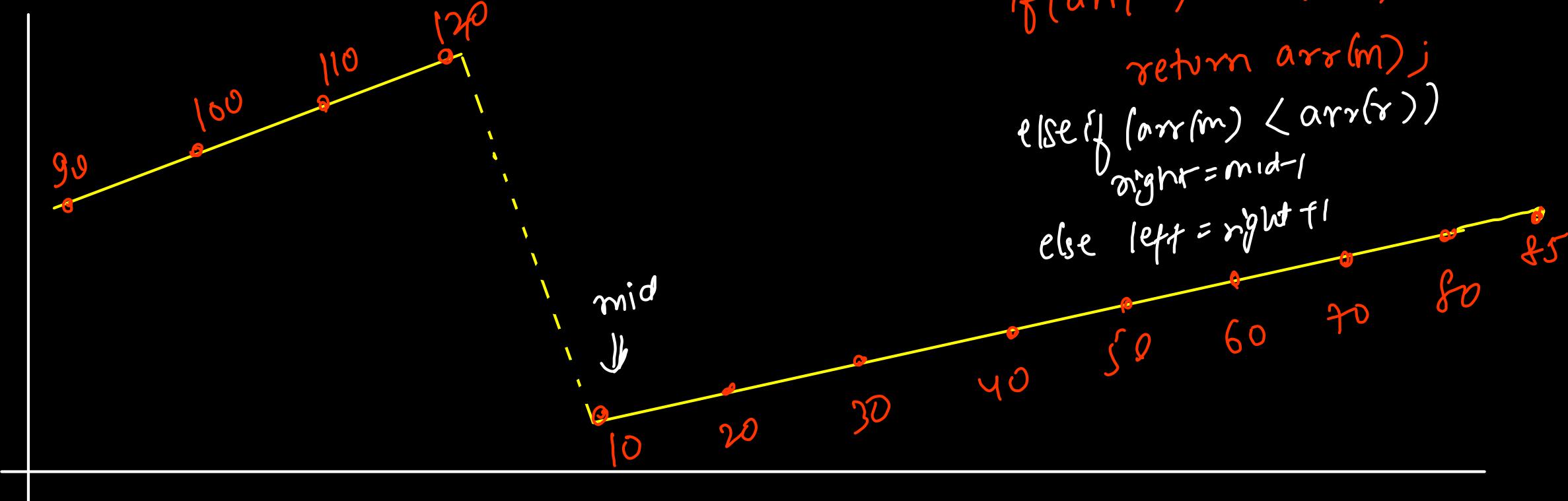
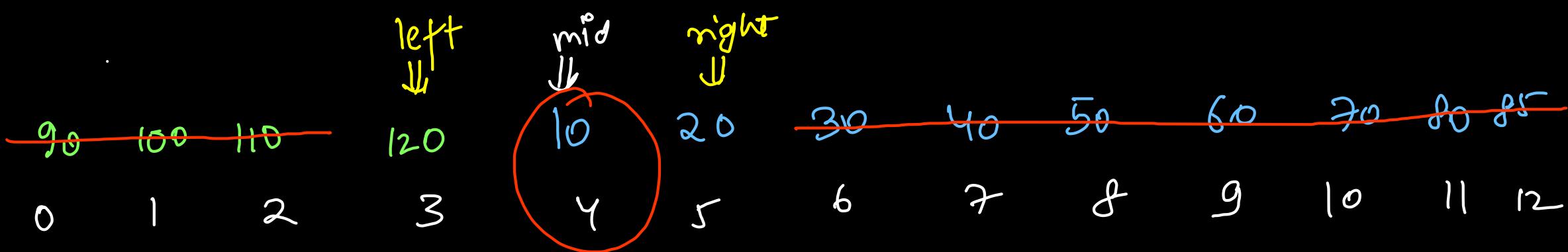
① Pivot Element / min Element -> Leetcode 153

② Rotation Count - GFG

③ Search in rotated sorted - II Leetcode 33

## Rotated Sorted Array

$k=0$	10	20	30	40	50	60	70	80	90
$k=1$	90	10	20	30	40	50	60	70	80
$k=2$	80	90	10	20	30	40	50	60	70
$k=3$	70	80	90	10	20	30	40	50	60
$k=4$	60	70	80	90	10	20	30	40	50
					!				
$k=8$	20	30	40	50	60	70	80	90	10



`if (arr(m) < arr(m-1))`

`return arr(m);`

`else if (arr(m) < arr(r))`  
`right = m-1`

`else left = right + 1`



```

public int findMin(int[] nums) {
    int n = nums.length;
    if(n == 1 || nums[0] < nums[n - 1]) return nums[0];
    // Array not rotated at all: completely sorted

    int left = 1, right = n - 1;
    while(left <= right){
        int mid = left + (right - left) / 2;
        if(nums[mid - 1] > nums[mid]) return nums[mid];
        if(nums[mid] <= nums[right])
            right = mid - 1; // unrotated part: min in left side
        else
            left = mid + 1; // rotated part: min in right side
    }

    return -1;
}

```

Time  
↳  $O(\log n)$

Space  
↳  $O(1)$

$(l-r)$  avg

Rotation Count = Pivot / min<sup>m</sup> Index = no. of rotations

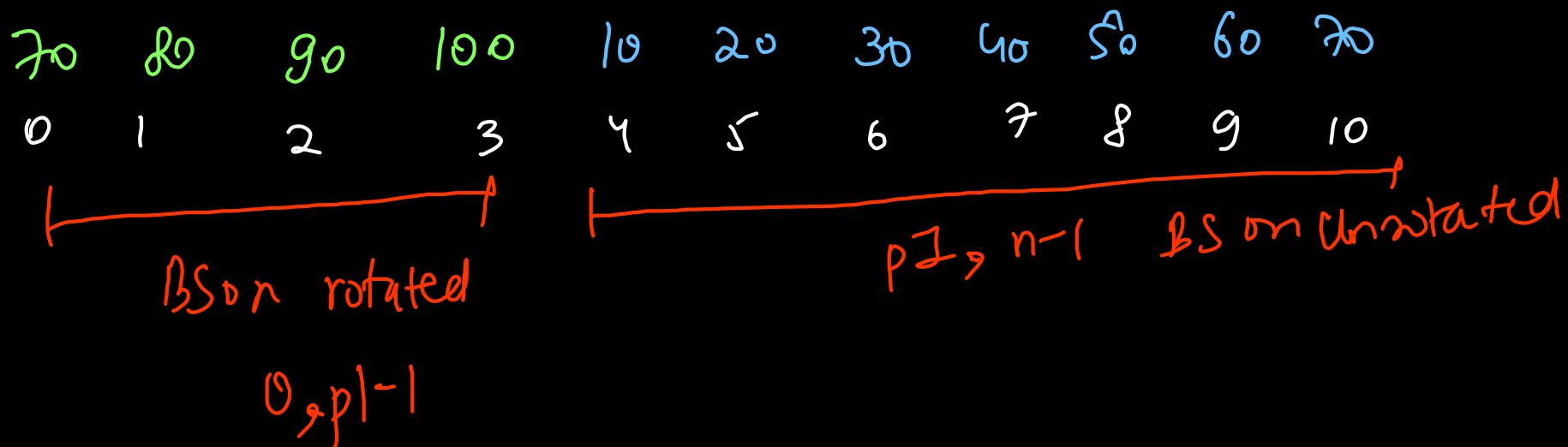
```
int findKRotation(int nums[], int n) {
    if(n == 1 || nums[0] < nums[n - 1]) return 0;

    int left = 1, right = n - 1;

    while(left <= right){
        int mid = left + (right - left) / 2;
        if(nums[mid - 1] > nums[mid]) return mid;
        if(nums[mid] <= nums[right])
            right = mid - 1; // unrotated part: min in left side
        else
            left = mid + 1; // rotated part: min in right side
    }

    return -1;
}
```

# Search in Rotated Sorted Array



target = 60

Approach: → ① Find Pivot Index

② BS on rotated

③ BS on Unrotated

```

public int binarySearch(int[] nums, int left, int right, int target){
    while(left <= right){
        int mid = left + (right - left) / 2;
        if(nums[mid] == target) return mid;
        else if(nums[mid] < target) left = mid + 1;
        else right = mid - 1;
    }
    return -1;
}

public int search(int[] nums, int target) {
    int pi = findMin(nums); → log n
    int lans = binarySearch(nums, 0, pi - 1, target); → log n_1
    if(lans != -1) return lans;

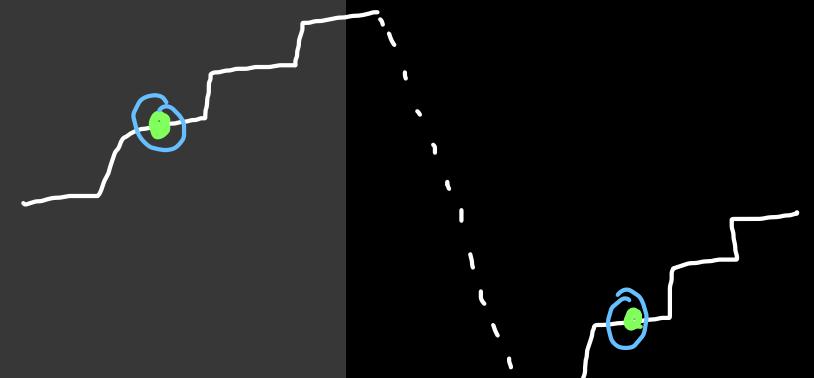
    return binarySearch(nums, pi, nums.length - 1, target); log n_2
}

```

Time ↘  
 $O(2 \log n)$   
= logarithmic

# Search Minm in Rotated sorted Array with Duplicate

```
class Solution {  
    // Time Complexity will be O(log n) in average case, but O(n) in worst case  
    // Worst case: All elements equal: We have to linearly reduce the search space  
    public int findMin(int[] nums) {  
        int low = 0, high = nums.length - 1;  
  
        while(low < high){  
            int mid = low + (high - low) / 2;  
  
            if(nums[mid] < nums[high]) {  
                high = mid;  
            } else if(nums[mid] > nums[high]) {  
                low = mid + 1;  
            } else {  
                high--; } plain region  $\Rightarrow$  linear search  
        }  
  
        return nums[low];  
    }  
}
```



{4,5,100,9,10,11,12,15,200}

Unsorted

Remove 2 min  $\geq$  Max

Sorted: 0 1 2 3 4 5 6 7 8  
4 5 9 10 11 12 15 100 200

- ①  $\min=4 \Rightarrow \max=8 \Rightarrow$  delete 9-200 : 8 deletions
- ② delete 4  $\Rightarrow \min=5 \Rightarrow \max=10 \Rightarrow$  delete 11-200  $\Rightarrow$  6 deletions
- ③ delete 4,5  $\Rightarrow \min=9 \Rightarrow \max=18 \Rightarrow$  delete 100-200  $\Rightarrow$  4 deletions

left deletion  $\Rightarrow$   $idx$   
 $\downarrow$   
right deletion  $\Rightarrow$   $n - \text{upper bound}$   
 $(2 \times arr[idx])$

Given an unsorted array **Arr** of size **N**. Find the minimum number of removals required such that twice of minimum element in the array is greater than or equal to the maximum in the array.

Dynamic Sliding Window + Binary Search

```
class Solution {    public static int upperBound(int[] nums, int target) {        }
```

```
int minRemoval(int arr[], int n) {    Arrays.sort(arr);  
    int minRemoval = n;  
  
    for(int idx = 0; idx < n; idx++){  
        int leftDel = idx;  
        int rightDel = n - upperBound(arr, 2 * arr[idx]);  
  
        minRemoval = Math.min(minRemoval, leftDel + rightDel);  
    }  
  
    return minRemoval;  
}
```

Time  $\Rightarrow n \log n$   
Space  $\Rightarrow O(1)$

## Searching in 2D Matrix

- ① unsorted : Binary Search not applicable &  
Linear search  $\rightarrow O(r * c)$
- ② only rows sorted but cols are not sorted       $r$  rows,  $c$  cols
- ~~10    20    30    40    50~~  $\rightarrow$   
~~15    25    45    55    75~~  $\rightarrow$   
~~5    15    25    35    45~~  $\rightarrow$   
~~7    27    37    47    57~~  $\rightarrow$   
~~3    13    23    33    43~~  $\rightarrow$
- target = 23
- Worstcase  $\rightarrow$  Binary Search on  
each row
- $\hookrightarrow O(r \log c)$
- $= \underline{\underline{O(n \log n)}}$

```
class Solution {
    public boolean binarySearchRow(int[][] mat, int row, int target){
        int left = 0, right = mat[0].length - 1;
        while(left <= right){
            int mid = left + (right - left) / 2;

            if(mat[row][mid] == target) return true;
            else if(mat[row][mid] < target) left = mid + 1;
            else right = mid - 1;
        }

        return false;
    }

    public boolean searchMatrix(int[][] mat, int target) {
        // Brute Force: Binary Search On Each Row
        for(int row = 0; row < mat.length; row++){
            if(binarySearchRow(mat, row, target) == true){
                return true;
            }
        }
        return false;
    }
}
```

Time  $\rightarrow n \log n$

Space  $\rightarrow O(1)$

③ rows sorted, cols sorted, Each row's last ele  $\leq$  Next row's first ele

Search matrix -1

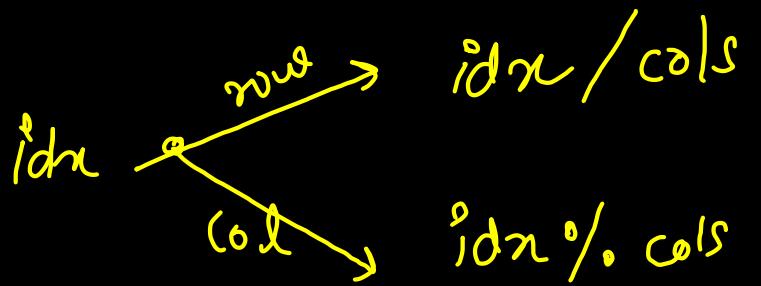
Leetcode 74

Assume BS on 1D array

- Integers in each row are sorted from left to right.
- The first integer of each row is greater than the last integer of the previous row.

0	1	2	3	4
10	20	30	40	50
(0,0)	(0,1)	(0,2)	(0,3)	(0,4)
5	6	7	8	9
60	70	80	90	100
(1,0)	(1,1)	(1,2)	(1,3)	(1,4)
10	11	12	13	14
105	110	120	140	150
(2,0)	(2,1)	(2,2)	(2,3)	(2,4)
15	16	17	18	19*
160	170	190	195	200
(3,0)	(3,1)	(3,2)	(3,3)	(3,4)
mid				

target = 190



```
class Solution {
    // Optimized Approach: Binary Search on 1D Array
    public boolean searchMatrix(int[][] mat, int target) {
        int rows = mat.length, cols = mat[0].length;
        int left = 0, right = rows * cols - 1;

        while(left <= right){
            int mid = left + (right - left) / 2;
            int row = mid / cols, col = mid % cols;

            if(mat[row][col] == target) return true;
            else if(mat[row][col] < target) left = mid + 1;
            else right = mid - 1;
        }

        return false;
    }
}
```

$O(\log(n \times \text{cols}))$

$= O(\log(n^2))$

$= O(2^{\log n})$

$\approx O(\log n)$

④ Rows sorted, cols sorted

Brute force:  $O(n \log n)$ : Binary search on each rows

1	4	7	11	15
2	5	8	12	19
3	6	9	16	22
10	13	14	17	24
18	21	23	26	30

row = 0  
row = 1  
row = 2  
row = 3  
row = 4

col = col - 1  
col = col - 1

Each row may not be smaller than next row

↓  
Binary Search  
on 1D array  
not possible  
 $O(n \log n)$

target = 21

$15 < 21$

discard row: row++

$17 < 21$

discard row

$26 > 21$

discard col

$23 > 21$   
discard col

$19 < 21$

discard row: row++

$22 > 21$

discard col: col--

$16 < 21$

discard row

You can only start the stepcase/staircase search  
from either top-right corner or bottom-left corner

1	4	7	11	15
2	5	8	12	19
3	6	9	16	22
10	13	14	17	24
18	21	23	26	30

target = 25

row = rows

1	4	7	11	15
2	5	8	12	19
3	6	9	16	22
10	13	14	17	24
18	21	23	26	30

target = -5

Search  
un-  
success-  
ful

## Stepcase Search

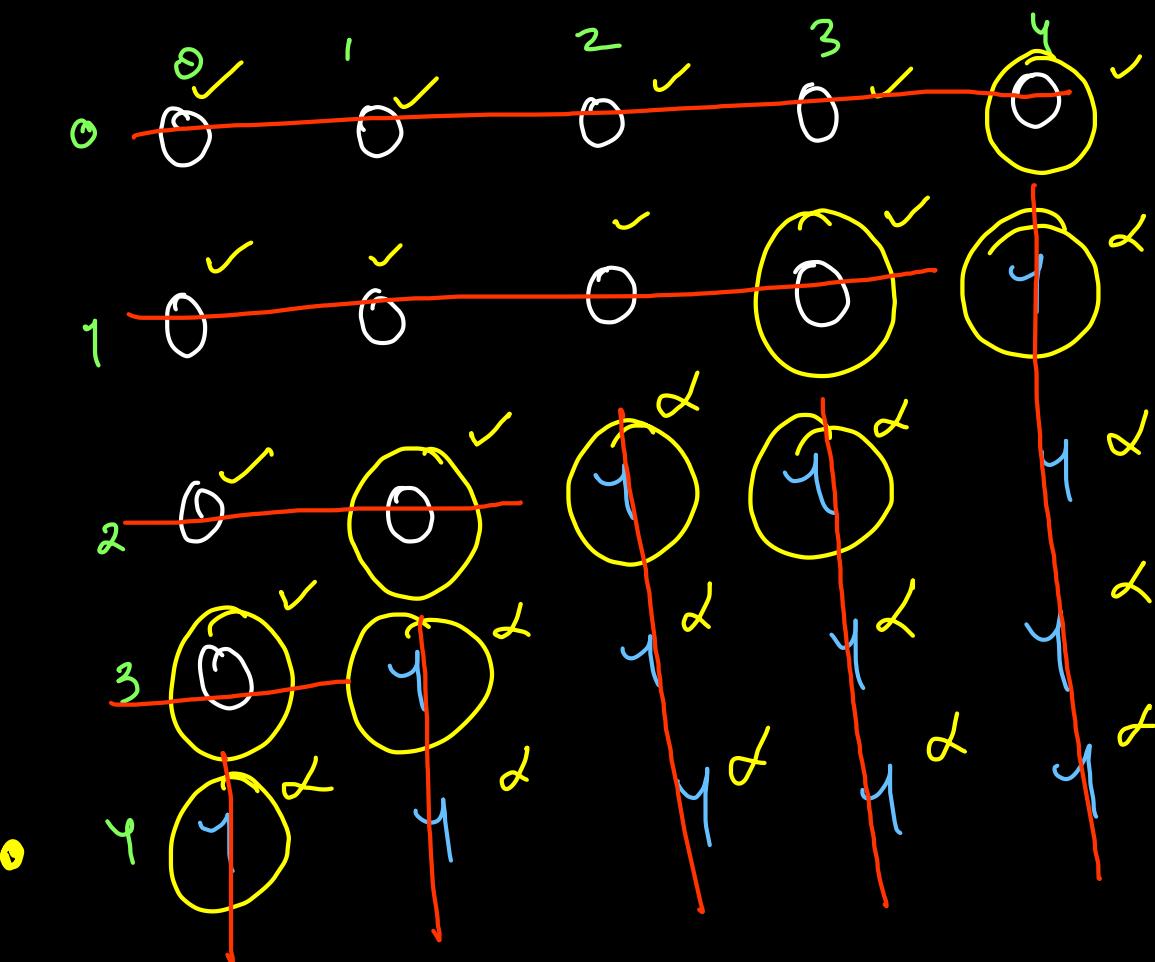
```
public boolean searchMatrix(int[][] mat, int target) {  
    int rows = mat.length, cols = mat[0].length;  
    int row = 0, col = cols - 1;  
  
    while(row < rows && col >= 0){  
        if(mat[row][col] == target) return true;  
        if(mat[row][col] < target)  
            row++; // discard the topmost row  
        else col--; // discard the rightmost column  
    }  
  
    return false;  
}
```

Time  
 $\hookrightarrow O(n)$

$O(rows + cols)$

$\uparrow \quad \uparrow$   
down + left

$$= O(n+n) = O(2n)$$



Binary matrix ↗  
↙ 1

⇒ each row is sorted

⇒ each col is sorted

Step Case Search

Count zeros

$$= 0 + 5 + 4 + 2 + 1$$

$$= 12$$

```
if ( mat[r][c] == 0 )
    zeros += (c+1);
    row++;
else
    col--;
```

```

int countZeros(int mat[][], int N) {
    int row = 0, col = N - 1, zeros = 0;

    while(row < N && col >= 0){
        if(mat[row][col] == 0){
            zeros = zeros + (col + 1); // all zeros in current row
            row++; // no more zeros: discard row
        } else {
            col--; // no zero in current column: discard column
        }
    }
    return zeros;
}

```

$\text{Time} \Rightarrow O(n)$       ↗ Stepcase Search  
 $\text{Space} \Rightarrow O(1)$       ↗

Binary matrix

	0	1	2	3	4	5
row 0	0	0	0	1	1	1
row 1	0	0	0	0	1	
row 2	0	1	1	1	1	1
row 3	0	0	1	1	1	1
row 4	0	0	0	0	0	
row 5	1	1	1	1	1	1

Row with max 1s

only rows are sorted  $\rightarrow$  cols are not sorted

Binary search on  
each row  
wrt reduced column

Ones =  $\phi$   $\beta \delta \ell$

row =  $0^{\text{th}}$   $2^{\text{nd}}$   $5^{\text{th}}$

```

int binarySearch(int arr[][], int row, int left, int right){
    while(left <= right){
        int mid = left + (right - left) / 2;

        if(arr[row][mid] == 0){
            left = mid + 1;
        } else {
            right = mid - 1;
        }
    }
    return left;
}

int rowWithMax1s(int arr[][], int rows, int cols) {
    int right = cols - 1, ans = -1;

    for(int row = 0; row < rows; row++){
        int mid = binarySearch(arr, row, 0, right);
        if(mid <= right) {
            ans = row;
            right = mid - 1; // reduced binary search on next row
        }
    }
    return ans;
}

```

}      first OLC  
      of ↓

Corner case



If entire matrix  
is 0,  
return -1;

Worst case

$\Theta(n \log n)$

Binary search on  
each row,

Binary Search on Answer

Hard concept

Very Very Important!

Minimization / maximization

Greedy

DP

Divide & conquer ✓

→ Koko Eating Bananas

→ Book Allocation Problem

→ Aggressive Cows

→ Wood cutting / EKO

# Koko Eating Bananas

## Leetcode 875

Koko loves to eat bananas. There are  $n$  piles of bananas, the  $i^{\text{th}}$  pile has  $\text{piles}[i]$  bananas. The guards have gone and will come back in  $h$  hours.

Koko can decide her bananas-per-hour eating speed of  $k$ . Each hour, she chooses some pile of bananas and eats  $k$  bananas from that pile. If the pile has less than  $k$  bananas, she eats all of them instead and will not eat any more bananas during this hour.

Koko likes to eat slowly but still wants to finish eating all the bananas before the guards return.

Return the minimum integer  $k$  such that she can eat all the bananas within  $h$  hours.

7, 3, 14, 6

hours  $h=8$

banana speed =  $b/\text{hour}$

min speed = ?

# Linear Search

max<sup>m</sup> hours h=8

- piles.length <= h <= 10<sup>9</sup>

7, 3, 11, 6

ans always valid

speed = 1

$$\text{time} = \lceil \frac{7}{1} \rceil + \lceil \frac{3}{1} \rceil + \lceil \frac{11}{1} \rceil + \lceil \frac{6}{1} \rceil = 27 \text{ hours}$$

False

speed = 2

$$\text{time} = \left\lceil \frac{7}{2} \right\rceil + \left\lceil \frac{3}{2} \right\rceil + \left\lceil \frac{11}{2} \right\rceil + \left\lceil \frac{6}{2} \right\rceil = 4+2+6+3 = 15 \text{ hours}$$

False

speed = 3

$$\text{time} = \left\lceil \frac{7}{3} \right\rceil + \left\lceil \frac{3}{3} \right\rceil + \left\lceil \frac{11}{3} \right\rceil + \left\lceil \frac{6}{3} \right\rceil = 3+1+4+2 = 10 \text{ hours}$$

False

maxSpeed  
= max<sup>m</sup> pile  
{ 11 }

speed = 4

$$\text{time} = \left\lceil \frac{7}{4} \right\rceil + \left\lceil \frac{3}{4} \right\rceil + \left\lceil \frac{11}{4} \right\rceil + \left\lceil \frac{6}{4} \right\rceil = 2+1+3+2 = 8 \text{ hours}$$

True

O(max \* n)

speed = 5

$$\text{time} = \left\lceil \frac{7}{5} \right\rceil + \left\lceil \frac{3}{5} \right\rceil + \left\lceil \frac{11}{5} \right\rceil + \left\lceil \frac{6}{5} \right\rceil = 2+1+3+2 = 8 \text{ hours}$$

True

↑  
(linear search)

Speed = 6

$$t = \left\lceil \frac{7}{6} \right\rceil + \left\lceil \frac{3}{6} \right\rceil + \left\lceil \frac{11}{6} \right\rceil + \left\lceil \frac{6}{6} \right\rceil = 2+1+2+1 = 6 \text{ hours}$$

True

```

class Solution {
    public boolean isPossible(int[] piles, long speed, long totalHours){
        long hoursRequired = 0;

        for(int bananas: piles){
            hoursRequired += bananas / speed; // floor division
            if(bananas % speed != 0) hoursRequired++; // ceil division
        }

        return (hoursRequired <= totalHours);
    }

    public long maxInArray(int[] piles){
        int max = 0;
        for(int bananas: piles)
            max = Math.max(max, bananas);
        return max;
    }

    public int minEatingSpeed(int[] piles, int totalHours) {
        long max = maxInArray(piles);
        for(long speed = 1l; speed <= max; speed++){
            if(isPossible(piles, speed, totalHours) == true)
                return (int)speed;
        }
        return (int)max;
    }
}

```

Linear search

Search space  $\Rightarrow$  max

$ispossible \Rightarrow n$

$$\begin{aligned}
 \text{Total time} &\Rightarrow O(\max * n) \\
 &\uparrow \quad \uparrow \\
 &10^9 * 10^4 \\
 &= 10^{13} \text{ TLE}
 \end{aligned}$$

`left = 1`

right = man = 11

$\text{mid} = 6$     isPossible : true  
 $\Rightarrow \text{right} = \underline{\text{mid}-1}$

left

right = 5

`mid = 3`

isPossible = false  $\Rightarrow$  left = mid + 1

left = 4

rights

$$mid = 4$$

ispossible = true  $\Rightarrow$  right = mid - 1

`left = 4 ; right = 3`

```

class Solution {
    public boolean isPossible(int[] piles, long speed, long totalHours){
        long hoursRequired = 0;

        for(int bananas: piles){
            hoursRequired += bananas / speed; // floor division
            if(bananas % speed != 0) hoursRequired++; // ceil division
        }

        return (hoursRequired <= totalHours);
    }

    public long maxInArray(int[] piles){
        int max = 0;
        for(int bananas: piles)
            max = Math.max(max, bananas);
        return max;
    }

    public int minEatingSpeed(int[] piles, int totalHours) {
        long left = 1l;
        long right = maxInArray(piles);

        while(left <= right){
            long mid = left + (right - left) / 2l;
            if(isPossible(piles, mid, totalHours) == true)
                right = mid - 1;
            else left = mid + 1;
        }

        return (int)left;
    }
}

```

- ① Array need not be sorted
- ② BS on ~~Array~~ Answer
- ③ BS  $\Rightarrow \mathcal{O}(\log N) \Rightarrow \mathcal{O}(N \log N)$
- ④ BS  $\Rightarrow$  Transition pt

$$\begin{aligned}
 & \mathcal{O}(\log(\text{Searchspace})) \\
 & = \mathcal{O}(\log \text{max}) \\
 & \quad \rightarrow \text{Total} \\
 & = \mathcal{O}(n \log \text{max}) \\
 & \quad \downarrow \log 10^9 \\
 & \quad = 9 \times 3 \\
 & \quad = 27
 \end{aligned}$$

# 1283. Find the Smallest Divisor Given a Threshold

Hint



Medium



1.8K

163



Companies

Given an array of integers `nums` and an integer `threshold`, we will choose a positive integer `divisor`, divide all the array by it, and sum the division's result. Find the **smallest** `divisor` such that the result mentioned above is less than or equal to `threshold`.

Each result of the division is rounded to the nearest integer greater than or equal to that element. (For example:  $7/3 = 3$  and  $10/2 = 5$ ).

The test cases are generated so that there will be an answer.

More  
Formal  
Def'n |

↳ Same code as `Robot Eat'N Banana`



## Book Allocation Problem

You are given **N** number of books. Every **i<sup>th</sup>** book has **A<sub>i</sub>** number of pages.

You have to allocate contiguous books to **M** number of students.  
There can be many ways or permutations to do so. In each permutation, one of the **M** students will be allocated the maximum number of pages. Out of all these permutations, the task is to find that particular permutation in which the maximum number of pages allocated to a student is the minimum of those in all the other permutations and print this minimum value.

Each book will be allocated to exactly one student. Each student has to be allocated at least one book.

### corner case

Note: Return **-1** if a valid assignment is not possible, and allotment should be in contiguous order (see the explanation for better understanding).

$$\{ 20, 10, 30, 40 \}$$

$$\text{books} = 4 \quad \text{stud} = 2$$

⇒ contiguous allocation

⇒ minimize the max<sup>m</sup> no of book pages

⇒ Each book ⇒ one student

→ Cannot skip any book

→ Cannot give same book to 2 student

$$\{ 20, 10, 30, 40 \}$$

stud = 2

invalid

→ conf1:

$$A: \{ 20 \} \quad B: \{ 10 \}$$

~~30, 40~~: not distributed

→ conf2:

$$A: \{ 20, 10 \} \quad B: \{ 30, 40 \}$$

10 → same book to  
2 students.

→ conf3:

$$A: \{ 20, \underline{40} \} \quad B: \{ 10, 30 \}$$

A: discontinuous  
allocn

→ conf4:

$$A: \{ 20, 10, 30, 40 \} \quad B: \{ \}$$

although valid; but not useful

$\{ 20, 10, 30, 40 \}$  stud = 2 $\{ 20 \}$  20 pages  
 $s_1$  $\{ 10, 30, 40 \}$  80 pages  
 $s_2$  $\{ 20, 10 \}$  30 pages  
 $s_1$  $\{ 30, 40 \}$  70 pages  
 $s_2$  $\{ 20, 10, 30 \}$  60 pages  
 $s_1$  $\{ 40 \}$  40 pages  
 $s_2$  $\therefore \text{max load} = 80 \text{ pages}$  $\therefore \text{max load} = 70 \text{ pages}$  $\boxed{\text{max load} = 60 \text{ pages}}$  $\downarrow$   
minimum max load

$$\{ 20, 10, 30, 40 \}$$
$$Stud = 3$$
$$\{ 20 \}$$
$$\} 20 \text{ load}$$
$$S_1$$
$$\{ 10 \}$$
$$\} 10 \text{ pages}$$
$$S_2$$
$$\{ 30, 40 \}$$
$$\} 70 \text{ pages}$$
$$S_3$$

Max load

(20)

$$\{ 20, 10 \}$$
$$30 \text{ load}$$
$$S_2$$
$$\{ 30 \}$$
$$30 \text{ load}$$
$$S_2$$
$$\{ 40 \}$$
$$40 \text{ pages}$$
$$S_3$$

minimized  
max load

(40)

$$\{ 20 \}$$
$$20 \text{ pages}$$
$$S_1$$
$$\{ 10, 30 \}$$
$$40 \text{ pages}$$
$$S_2$$
$$\{ 40 \}$$
$$40 \text{ pages}$$
$$S_3$$

: (40)

Linear Search : max<sup>m</sup> Load : minimize

max<sup>m</sup> possible answer

min<sup>m</sup> possible answer

= sum of array

= max<sup>m</sup> of Array

= 100

= 40

ispossible( {20, 10, 30, 40}, 40, 2 )

ref stud = 1 2 3  
currentload = 0 + 20 + 10 } false  
0 + 30  
0 + 40

ispossible( {20, 10, 30, 40}, 50, 2 )

ref stud = 1 2 3  
currentload = 0 + 20 + 10 } false  
0 + 30  
0 + 40

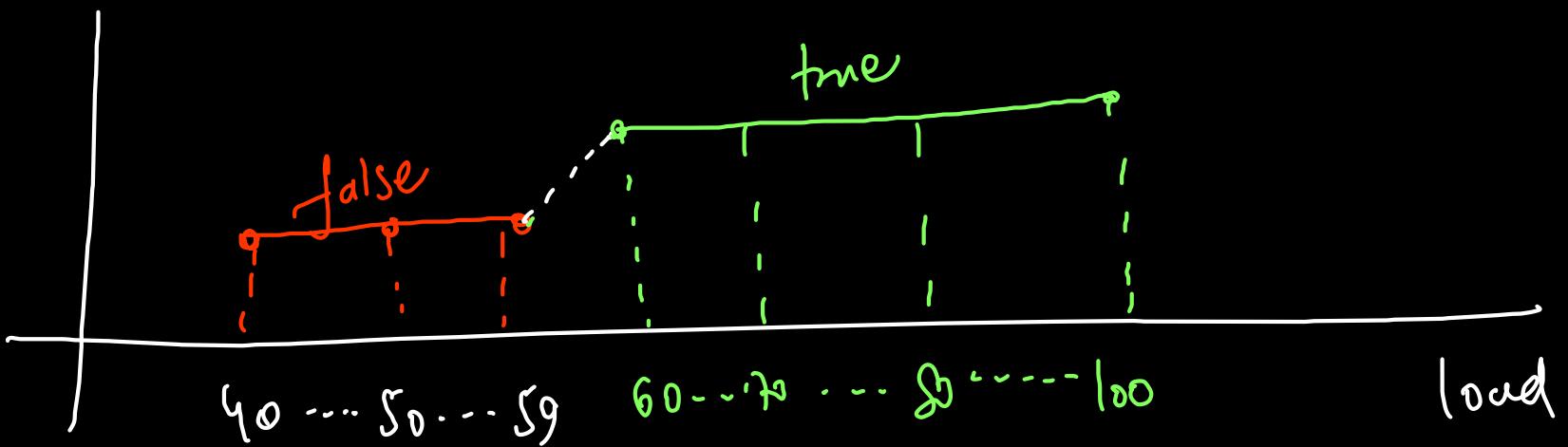
`isPossible( { 20, 10, 30, 40 }, 60, 2 )`

$\text{reqStud} = 2$

$\text{currentLoad} = 0 + 20 + 10 + 30 + \cancel{40}$

$0 + 40$

{ true }



```
// Can I Distribute all books to the totalStudents within maximum load
public static boolean isPossible(int[] pages, int maxLoad, int totalStud){
    int requiredStud = 1, currentLoad = 0;

    for(int book: pages){ → no Ob books
        if(currentLoad + book <= maxLoad) {
            currentLoad += book;
        } else {
            requiredStud++;
            currentLoad = book;
        }
    }

    return (requiredStud <= totalStud);
}

public static int maxOfArray(int[] pages){
    int max = 0;
    for(int book: pages) max = Math.max(max, book);
    return max;
}

public static int sumOfArray(int[] pages){
    int sum = 0;
    for(int book: pages) sum += book;
    return sum;
}

public static int findPages(int[] pages, int books, int stud) {
    if(books < stud) return -1; // some students will not get any book

    int left = maxOfArray(pages); // Someone will have to read the max page
    int right = sumOfArray(pages); // Someone might have to read all the books

    for(int load = left; load <= right; load++){ → SUm of pages
        if(isPossible(pages, load, stud) == true)
            return load;
    }

    return -1;
}
```

$\{ \downarrow 20, \downarrow 10, \downarrow 50, \downarrow 30, \downarrow 40 \}$  stud = 3  
 { } ↑ ↑ ↑ ↑ ↑

isPossible(50, 3)

return = 1 2 3 4

loop = 0 + 20 + 10 50 30 40 false

~~4 > 3~~

req given

is possible ( $f_0, 3$ )  
1434  
~~20 + 0~~ 50 30 40  $\frac{473}{\text{false}}$

$\text{TL} \in \mathcal{O}(\sum \sigma \text{ pass} \times \text{non book})$

$$\text{isPossible}(70, 3)$$

~~registered = 1 < 3~~

~~load = 0 + 20 + 10~~  $\frac{80}{30+40}$  ~~3 <= 3~~ true

```

public static boolean isPossible(int[] pages, int maxLoad, int totalStud){
    int requiredStud = 1, currentLoad = 0;

    for(int book: pages){  $\rightarrow O(\text{books})$ 
        if(currentLoad + book <= maxLoad) {
            currentLoad += book;
        } else {
            requiredStud++;
            currentLoad = book;
        }
    }

    return (requiredStud <= totalStud);
}

```

```

public static int maxOfArray(int[] pages){}
public static int sumOfArray(int[] pages){}

```

```

public static int findPages(int[] pages, int books, int stud) {
    if(books < stud) return -1; // some students will not get any book

    int left = maxOfArray(pages); // Someone will have to read the max pages book
    int right = sumOfArray(pages); // Someone might have to read all the books

    while(left <= right){  $\leftarrow O(\log \sum \text{of array})$ 
        int mid = left + (right - left) / 2;
        if(isPossible(pages, mid, stud) == true)
            right = mid - 1;
        else left = mid + 1;
    }

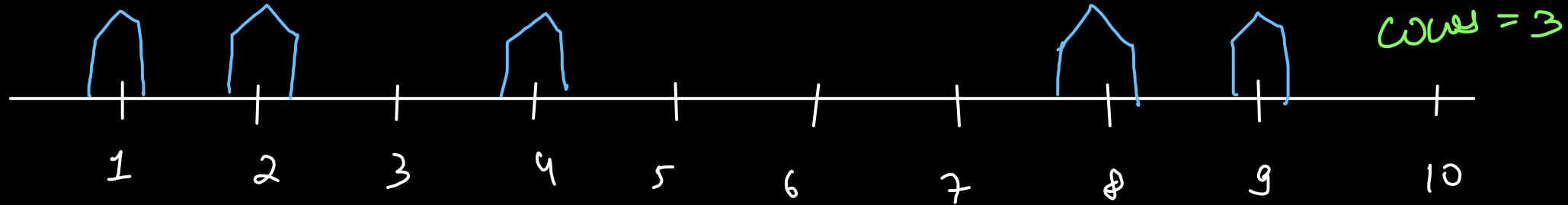
    return left;
}

```

Total time

$$O(\underbrace{\text{books}}_{10^5} * \underbrace{\log \sum}_{\log 10^6 = 20}) \approx \underline{O(n \log n)}$$

Aggressive Cows stall :  $\{4, 1, 9, 2, 8\}$  maximize the min<sup>n</sup> dist.



$c1 \xleftarrow{1} c2 \xleftarrow{2} c3$

$c1 \xleftarrow{1} c2 \xleftarrow{6} c3$

$c1 \xleftarrow{1} c2 \xleftarrow{7} c3$

$c1 \xleftarrow{3} c2 \xrightarrow{4} c3$

$c1 \xleftarrow{3} c2 \xrightarrow{5} c3$

$c1 \xleftarrow{7} c2 \xrightarrow{1} c3$

These will never give you optional answers

$c1 \xrightarrow{2} c2$

$c1 \xleftarrow{2} c2$

$c1 \xleftarrow{4} c2$

$\minDist = 1$

$\minDist = 1$

$\minDist = 1$

$\minDist = 3$

$\minDist = 3$

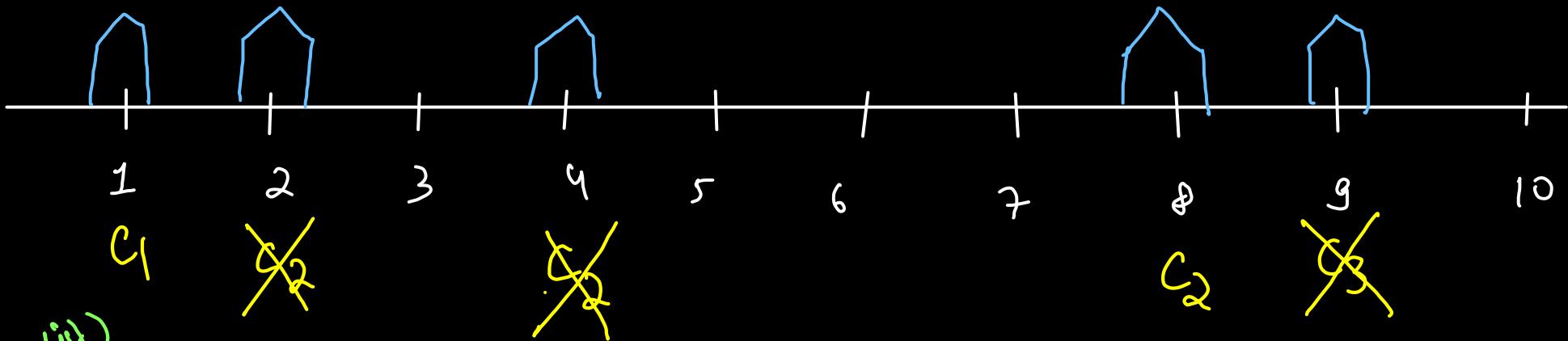
$\minDist = 1$

$\minDist = 2$

$\minDist = 2$

$\minDist = 1$

Sorted stalls :  $\{1, 2, 4, 8, 9\}$  Cows = 3



BS on  
arr[0:n-1](dist)

left = 1

$$\text{mid} = (1+8)/2 = 4$$

$$\begin{aligned}\text{right} &= \frac{\text{arr}(n-1) - \text{arr}(0)}{} \\ &= 9-1 = 8\end{aligned}$$

isPossible:  $\rightarrow$  Can I place all the cows(3)  
such that  $\min^m$  dist b/w any 2 cows  $\geq 4$

placed cows = 1      last cow =  $\text{arr}(0) = 1$   
2       $\text{arr}[3] = 8$

2 < 3  
all cows not placed  
: false :  $\text{right} = \text{mid}-1$



left = 1

$$\text{mid} = ((\text{left} + \text{right}) / 2) = 2$$

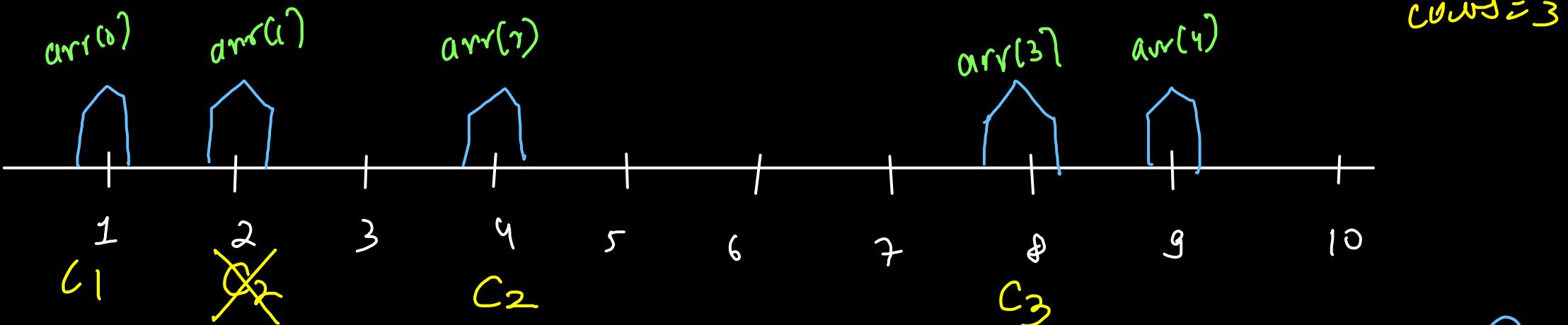
Can I place all cows such that  
min dist  $\geq 2$

$$\begin{aligned}\text{right} &= \text{mid} - 1 \\ &= 4 - 1 = 3\end{aligned}$$

placedCows = ~~1~~

~~2~~  
~~3~~  
placed all  
cows? true

lastCow = arr[0] = 1  
arr[2] = 4  
arr[3] = 8  
= left = mid + 1



$$\text{left} = \text{mid} - 1 = 2 + 1 - 1 = 3$$

$$\text{mid} = 3$$

$$\text{right} = 3$$

$\text{placed}(\text{cows} = 1)$

$\text{placed}$   
all cows

$$\begin{aligned} \text{true} \\ \text{if } (\text{left} = \text{mid} - 1) \\ = 3 + 1 = 4 \end{aligned}$$

$\text{lastRow} =$

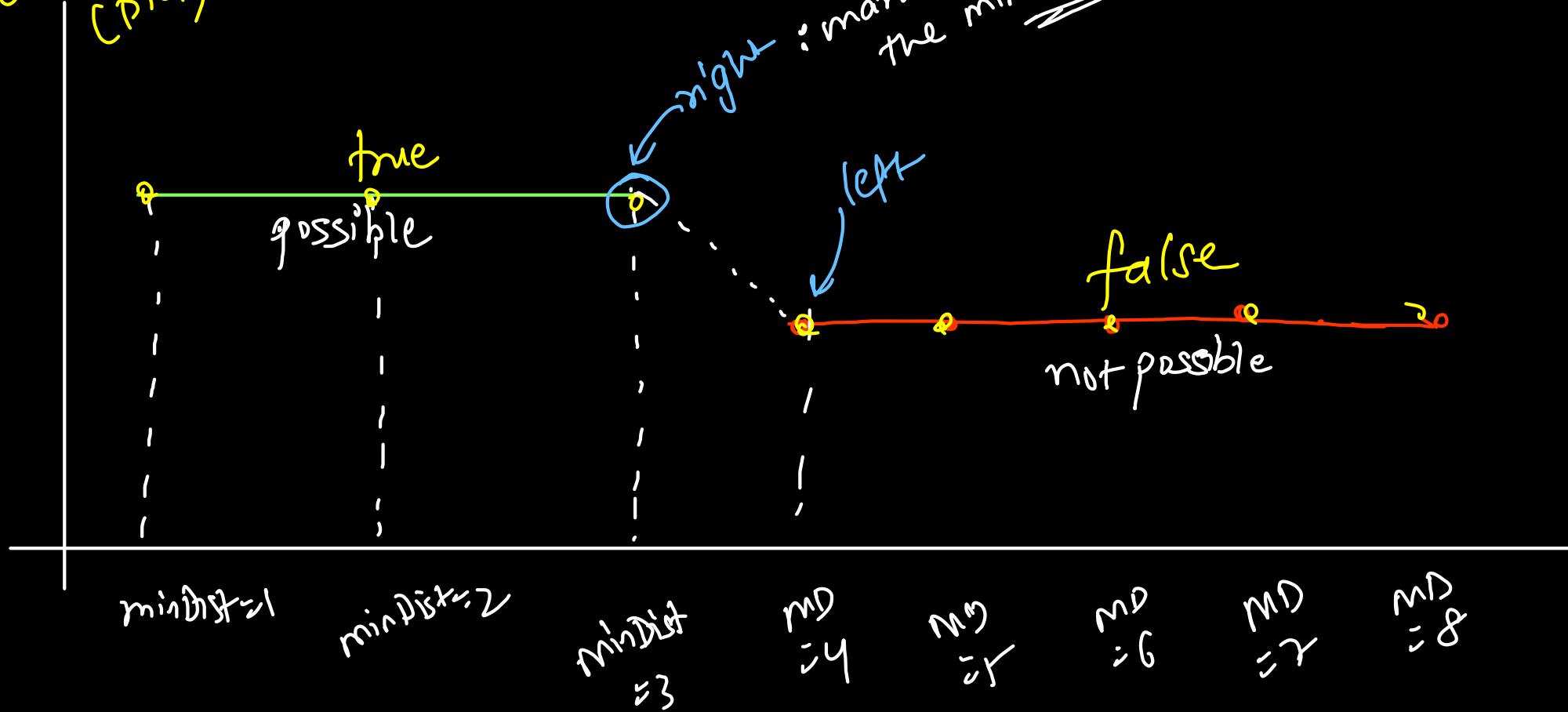
$$\cancel{\text{arr}(0)} = 1$$

$$\cancel{\text{arr}(2)} = 4$$

$$\text{arr}(3) = 8$$

answer  
after binary  
search

BS on Answer  
(Dist)



```

public static boolean isPossible(ArrayList<Integer> stalls, int minDist, int totalCows){
    int placedCows = 1, lastIdx = 0;
    → O(n)
    for(int currentIdx = 1; currentIdx < stalls.size(); currentIdx++){
        int dist = stalls.get(currentIdx) - stalls.get(lastIdx);

        if(dist >= minDist) {
            placedCows++;
            lastIdx = currentIdx;
        }
    }

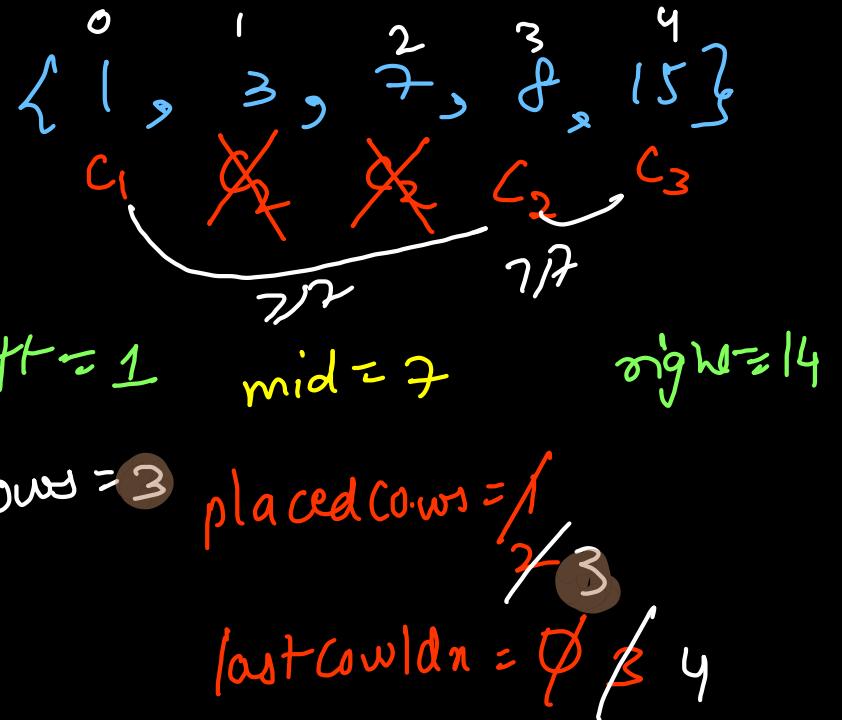
    return (placedCows >= totalCows);
}

public static int aggressiveCows(ArrayList<Integer> stalls, int cows)  {
    Collections.sort(stalls);
    // To Find Adjacent Distance, Not to apply Binary Search
    int left = 1; // Minimum Possible Adjacent Distance
    int right = stalls.get(stalls.size() - 1) - stalls.get(0);
    → O(log range)
    while(left <= right){
        int mid = left + (right - left) / 2;

        if(isPossible(stalls, mid, cows) == true){
            left = mid + 1; // maximize the minimum dist
        } else {
            right = mid - 1;
        }
    }

    return right;
}

```



Total time  $\Rightarrow O(n \log n)$   
 $\downarrow$   
 $10^5 \times \log 10^9$   
 $10^5 \times 9 \times 3$   
 $\underline{10^5 \times 27}$

## Wood cutting - Elzo

There is given an integer array **A** of size **N** denoting the heights of **N** trees.

Lumberjack Ojas needs to chop down **B** metres of wood. It is an easy job for him since he has a nifty new woodcutting machine that can take down forests like wildfire. However, Ojas is only allowed to cut a single row of trees.

Ojas's machine works as follows: Ojas sets a height parameter **H** (in metres), and the machine raises a giant sawblade to that height and cuts off all tree parts higher than **H** (of course, trees not higher than **H** meters remain intact). Ojas then takes the parts that were cut off. For example, if the tree row contains trees with heights of 20, 15, 10, and 17 metres, and Ojas raises his sawblade to 15 metres, the remaining tree heights after cutting will be 15, 15, 10, and 15 metres, respectively, while Ojas will take 5 metres off the first tree and 2 metres off the fourth tree (7 metres of wood in total).

Ojas is **ecologically minded**, so he doesn't want to cut off more wood than necessary. That's why he wants to set his sawblade as high as possible. Help Ojas find the **maximum integer height** of the sawblade that still allows him to cut off **at least B** metres of wood.

### NOTE:

- The sum of all heights will exceed **B**, thus Ojas will always be able to obtain the required amount of wood.

### Example Input

Input 1:

$$A = [20, 15, 10, 17]$$

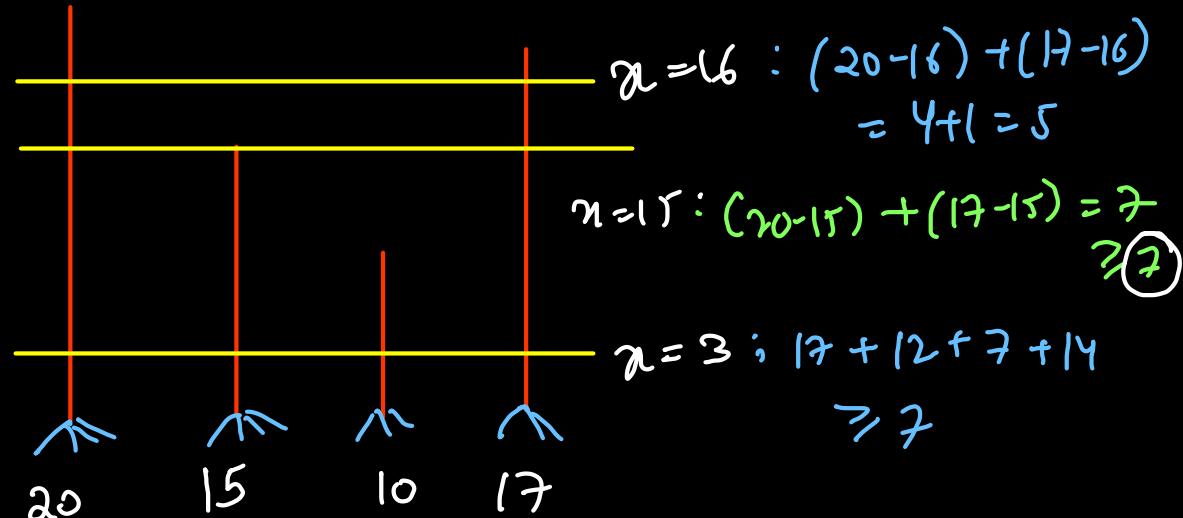
$$B = 7$$

Input 2:

$$A = [4, 42, 40, 26, 46]$$

$$B = 20$$

*Answer ≈ 15*



max<sup>m</sup> Height cut  $\Rightarrow$  requirement is fulfilled

low = 0  
(root cut)  
 $\Rightarrow$  max<sup>m</sup> wood

right = 20  
(tree top cut  $\Rightarrow$  least wood)

$$\begin{aligned} mid &= 10 \text{ (cut)} \\ \text{is possible} &= (20-10) + (15-10) \\ &\quad + (10-10) + (17-10) \\ &= 10\pi\pi + 0 + 7 = 22 \end{aligned}$$

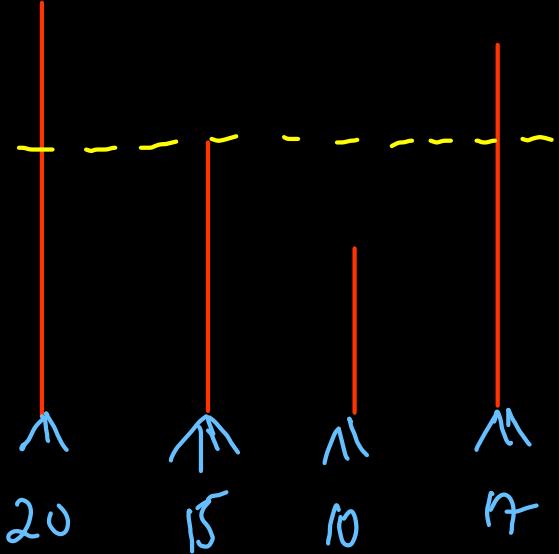
$low = 0$   
 (root cut)  
 $\Rightarrow$  ~~remain~~ wood

$right = 20$   
 (tree top cut  $\rightarrow$  east wood)

Binary search on height

$mid = 10$  (cut)

$$\begin{aligned} \text{is possible} &= (20-10) + (15-10) \\ &\quad + (10-10) + (12-10) \\ &= 10+5+0+2 = \underline{\underline{22}} \\ &\text{true} \end{aligned}$$



requirement = 7

$low = 10 + 1 = 11$  ,  $right = 20$  ,  $mid = 15$

$$\begin{aligned} \text{is possible} &= (20-15) + (15-15) + (10-\cancel{5}) \\ &\quad + (17-15) = 5+2 = \underline{\underline{7}} \\ &\text{true} \end{aligned}$$

$low = 16$  ,  $right = 20$  ,  $mid = 18$

$$\begin{aligned} \text{is possible} &= (20-18) + (\cancel{15}-\cancel{18}) + (\cancel{10}-\cancel{18}) + (\cancel{17}-\cancel{18}) \\ &= \underline{\underline{2}} \text{ false} \end{aligned}$$

$low = 16$  ,  $right = 17$

```

class Solution {
    public boolean isPossible(int[] trees, int cut, int requiredWood) {
        int wood = 0;
        for (int treeHeight : trees) {
            if (cut < treeHeight)
                wood += (treeHeight - cut);
        }
        return (wood >= requiredWood);
    }
}

```

$\mathcal{O}(n)$

```

public int solve(int[] trees, int requiredWood) {
    int left = 0, right = 1000000;

    while (left <= right)  $\rightarrow \mathcal{O}(\log n)$ 
        int mid = left + (right - left) / 2;

        if (isPossible(trees, mid, requiredWood) == true) {
            left = mid + 1; // minimize cut height  $\Rightarrow$  decrease wood
        } else {
            right = mid - 1; // wood less than required
                                $\Rightarrow$  increase wood
        }
    }
    return right;
}

```

Total time  
 $\Rightarrow \mathcal{O}(n * \log n)$

## Problem Statement

[Suggest Edit](#)

In Ninja Land, there is a famous restaurant named 'CookingNinjas'. There are 'N' cooks in 'CookingNinjas' numbered from 0 to N-1. Each cook has rank 'R' ( $1 \leq R \leq 10$ ).

A cook with a rank 'R' can prepare 1 dish in the first 'R' minutes, 1 more dish in the next '2R' minutes, 1 more dish in next '3R' minutes, and so on (A cook can only make complete dishes) For example if a cook is ranked 2. He will prepare one dish in 2 minutes, one more dish in the next 4 mins and one more in the next 6 minutes hence in a total of 12 minutes he can make 3 dishes, Note, In 13 minutes also he can make only 3 dishes as he does not have enough time for the 4th dish).

One day 'CookingNinjas' receive an order of 'M' dishes that they need to complete as early as possible. You are given an integer array 'rank' of size 'N' in which 'rank[i]' gives 'rank' of ith cook and an integer 'M', You need to find out the minimum times required to complete this order of 'M' dishes.

Example

1 4 2 3 cooks

10 dishes

Rank = 1

4, 2, 3, 4, 5  
1<sup>st</sup> 3<sup>rd</sup> 6<sup>th</sup> 10<sup>th</sup> 15<sup>th</sup> minute

Rank = 2

2, 4, 6, 8, 10  
2<sup>nd</sup> 6<sup>th</sup> 12<sup>th</sup> 20<sup>th</sup> 30<sup>th</sup>

1	2	3	4	.
cooks				

left = 1

left = 1

left = 1

right  $\Rightarrow k$  dishes

$$R + 2R + 3R + 4R + \dots + KR$$

$$\Rightarrow R(1+2+3+\dots+k)$$

$$\Rightarrow \frac{R(k)(k+1)}{2} \text{ time}$$

10 dishes

$$1+2+3+\dots+10 = \underbrace{\frac{10 \times 11}{2}}_{55} = 55$$

$$\text{mid} = 7.5$$

is possible: true

$$\text{mid} = 55$$

is possible: true

$$\text{mid} = 27 \text{ is possible: true}$$

rank  $\cancel{2}, \cancel{3}, \cancel{0}, \cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}, \cancel{9}$

Count:  $\cancel{2}, \cancel{4}, \cancel{6}, \cancel{8}, \cancel{10}, \cancel{0}, \cancel{2}, \cancel{4}, \cancel{8}, \cancel{10}$

(10)

0 + 3

6 dishes by  $R_1$

3 dishes by  $R_2$

1 dish by  $R_3$

all dishes  
prepared by  
slowest cook

$$\text{right} = \frac{4 \times 10 \times 11}{2} \\ = 220$$

```

public static boolean isPossible(ArrayList<Integer> cooks, int timeAllowed, int requiredDishes){
    for(int rank: cooks){ → O(n)
        int mult = 1;
        for(int timeTaken = rank; timeTaken <= timeAllowed; timeTaken += rank * mult){ → O(range)
            requiredDishes--;
            if(requiredDishes == 0) return true;
        }
    }
    return false;
}

```

```

public static int minCookTime(ArrayList<Integer> cooks, int dishes) {
    Collections.sort(cooks); // Fastest Cook is assigned first
    int left = 1, right = (cooks.get(0) * dishes * (dishes + 1)) / 2;

    while(left <= right){ → O(log range)
        int mid = left + (right - left) / 2;
        if(isPossible(cooks, mid, dishes) == true){
            right = mid - 1; // minimize the time to prepare all dishes
        } else {
            left = mid + 1; // not possible in so much less time ⇒ increase time
        }
    }

    return left;
}

```

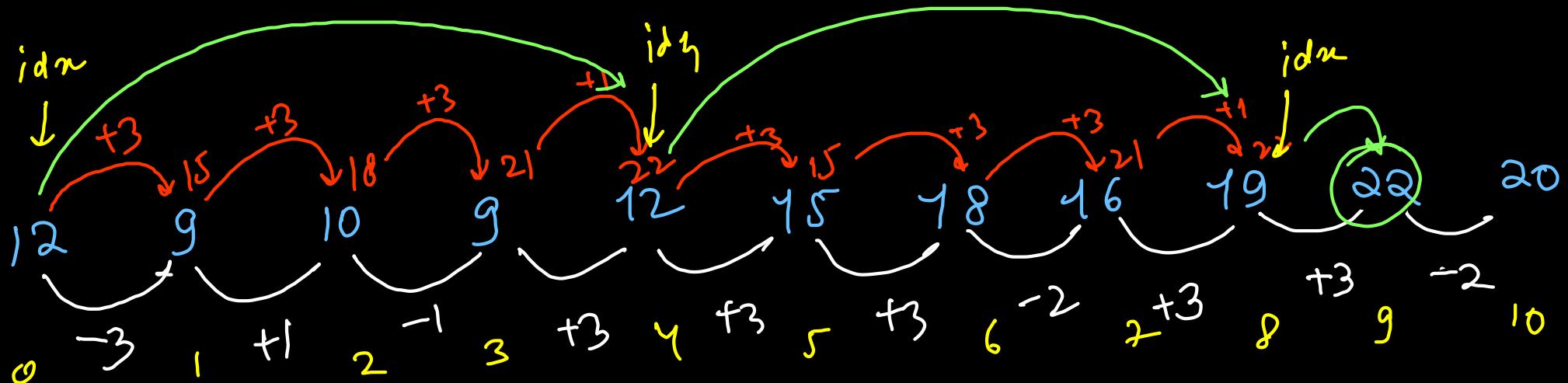
$$\begin{aligned}
 R &\mapsto R^{\frac{1}{3}} \quad k = T \\
 T &= R(F)(k+1) \\
 k &= \sqrt[3]{T}
 \end{aligned}$$

Total time

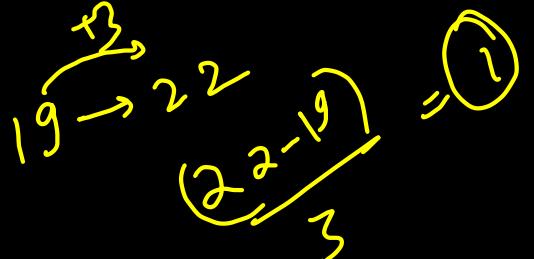
$O(n\sqrt{R} \log R)$

$R = 3$

8 step Array Search  
 JumpSearch  $\rightarrow O(\sqrt{N})$   
 Better than linear search  
 Worse than binary search  
 array sorted



target = 22

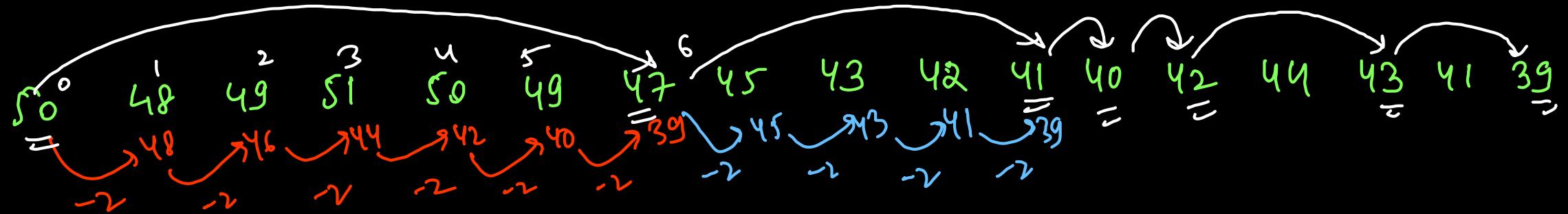


$$\text{seg jump} = (22 - 12) = 10$$

$$\text{each jump} = +3 \text{ (k)}$$

$$\min\_steps = \lceil 10/3 \rceil = 4$$

$k=2$



$$\text{req jump} \Rightarrow 50 - 39 = 11$$

$$\text{target} = 39$$

$$\text{each jump} = k = 2$$

$$\text{min jumps} = \lceil 11/2 \rceil = 6$$

$$\text{req jumps} \Rightarrow 47 - 39 = 8$$

$$\text{each jump} \Rightarrow k = 2$$

$$\text{min jumps} = \lceil 8/2 \rceil = 4$$

$$\text{req jump} = (41 - 39) = 2$$

$$\text{each jump} = 2$$

$$\text{min jumps} = 2/2 = 1$$

$$(40 - 39) = 1$$

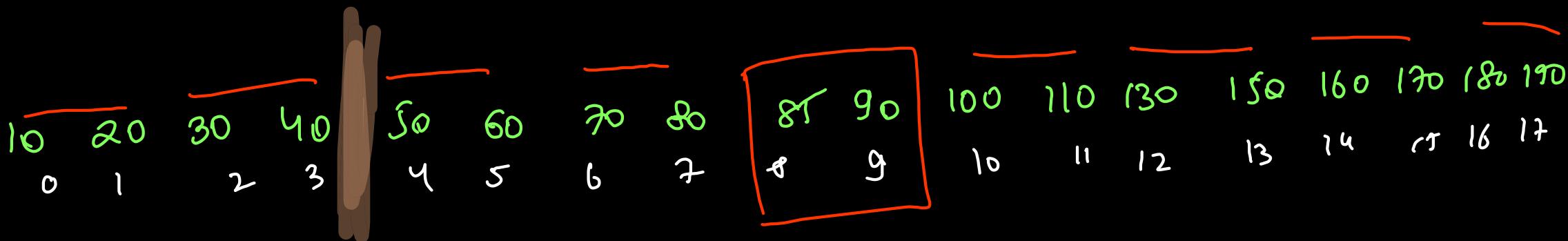
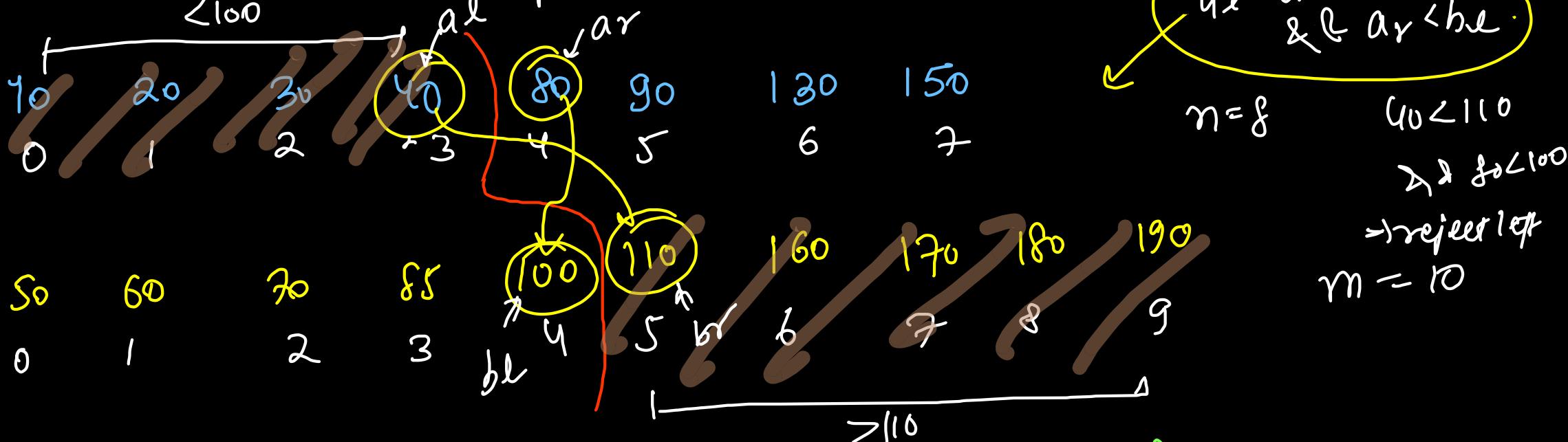
$$\lceil 1/2 \rceil = 1$$

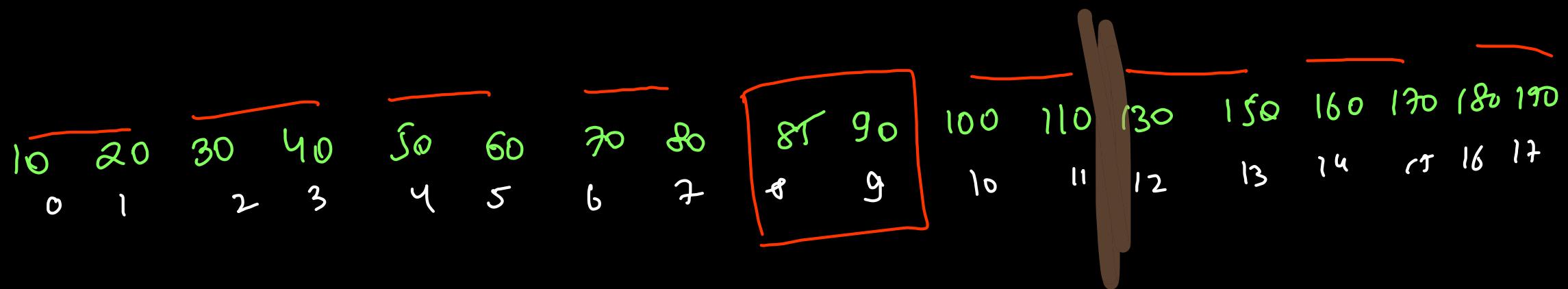
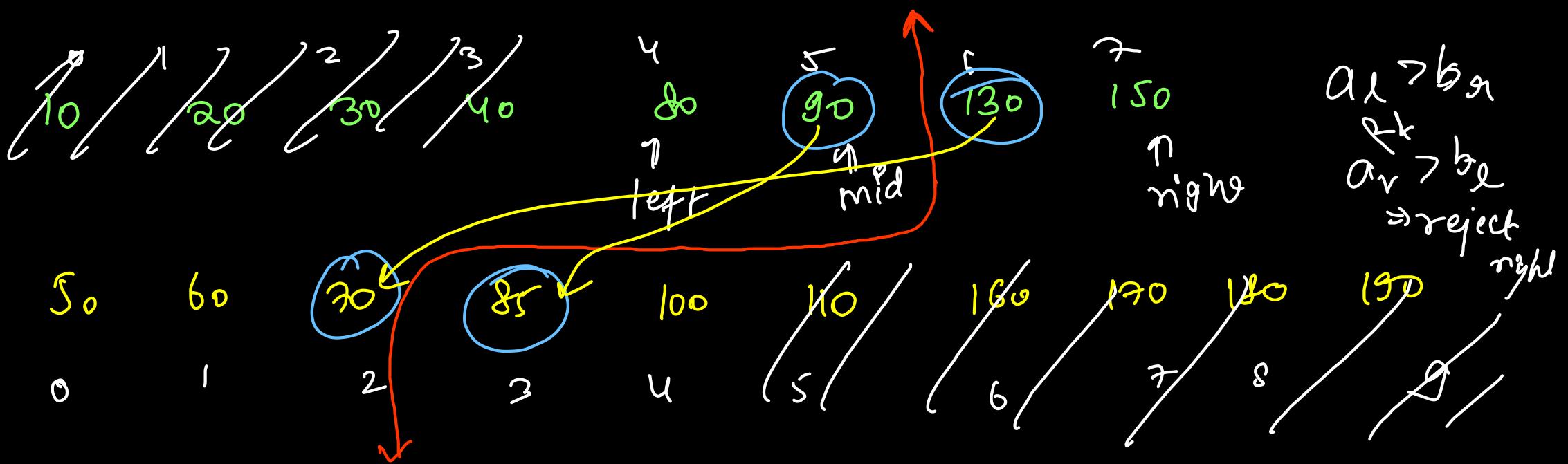
```
public static int search (int arr[], int n, int target, int jump) {  
    int idx = 0;  
  
    while(idx < n){  
        if(arr[idx] == target) return idx;  
  
        int requiredJump = Math.abs(arr[idx] - target);  
        int minJumps = requiredJump / jump; // floor division  
        if(requiredJump % jump != 0) minJumps++; // ceil division  
  
        idx = idx + minJumps;  
    }  
  
    return -1;  
}
```

Worst case  
 ↳  $O(n)$   
 linear search

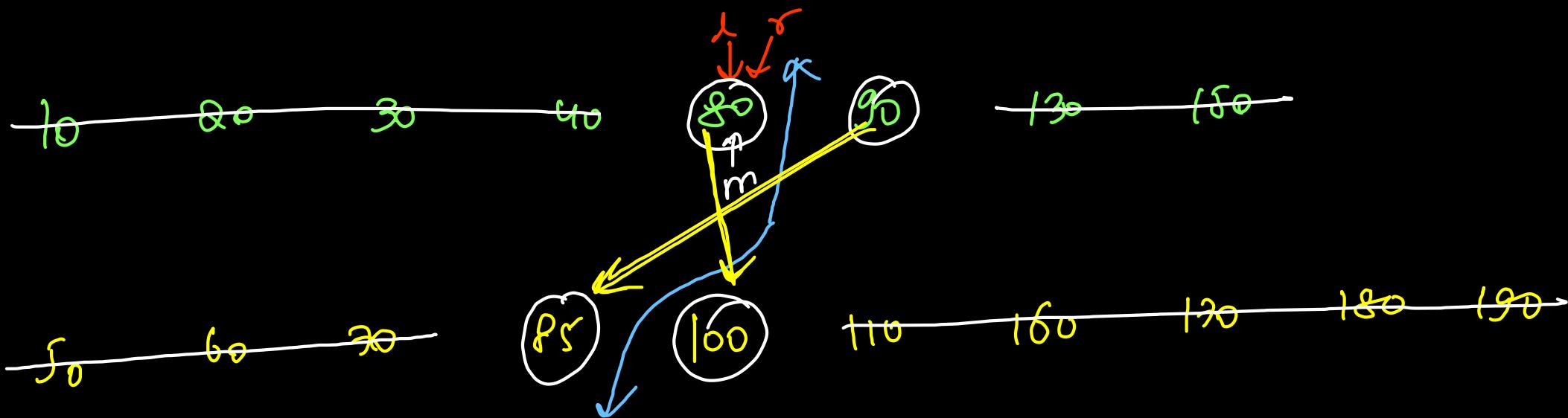
Avg case  
 ↳  $O(n/k)$

# Median of 2 sorted Arrays.





right = mid + 1



$$a_l = 80 \quad a_r = 90$$

$$b_l = 85 \quad b_r = 100$$

$$a_l < b_r \quad \& \quad a_r > b_l$$

80	90		
85	100		
0	1	2	3

~~Approach 1~~

① merge 2 Sorted Arrays  $\rightarrow O(m+n)$

② Find the median  $\xrightarrow{\text{odd}} \xrightarrow{\text{even}} O(1)$

10 20 } 30 } 40 50

10 20 } 30 40 } 50 60

~~Approach 2~~

① Binary Search on 2 arrays.

```
public double findMedianSortedArrays(int[] nums1, int[] nums2) {  
    if(nums1.length > nums2.length){  
        return findMedianSortedArrays(nums2, nums1);  
    }  
}
```

$n_1 > n_2$  : swap arrays  
corner case: 1

```
int low = 0, high = nums1.length;  
int N = nums1.length + nums2.length;  
int Nby2 = (N + 1) / 2;  
    ↙ ceil division
```

```
while(low <= high){  
    int mid = low + (high - low) / 2;  
  
    int aleft = (mid - 1 >= 0)  
        ? nums1[mid - 1] : Integer.MIN_VALUE;  
  
    int aright = (mid < nums1.length)  
        ? nums1[mid] : Integer.MAX_VALUE;  
  
    int bleft = (Nby2 - mid - 1 >= 0)  
        ? nums2[Nby2 - mid - 1] : Integer.MIN_VALUE;  
  
    int bright = (Nby2 - mid < nums2.length)  
        ? nums2[Nby2 - mid] : Integer.MAX_VALUE;  
  
    if(aleft <= bright && bleft <= aright){  
        // return median  
        if(N % 2 == 1){  
            // odd elements  
            return Math.max(aleft, bleft);  
        } else{  
            int[] arr = {aleft, bleft, aright, bright};  
            Arrays.sort(arr);  
            return (arr[1] + arr[2]) / 2.0;  
        }  
    }  
}
```

```
else if(aleft > bright){  
    high = mid - 1;  
} else {  
    // bleft > aright  
    low = mid + 1;  
}  
}  
return 0.0;
```

0 1 2 3

$a_l > b_r$  &  $a_r > b_e$

0 1 2 3

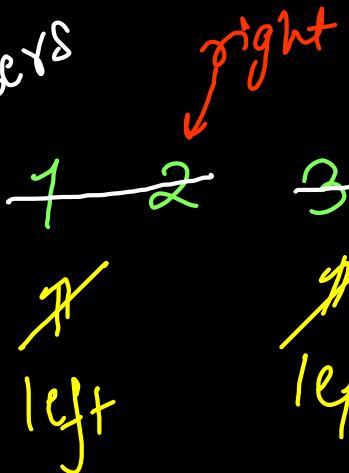
$a_l < b_r$

$a_l < b_r$   
&  $a_r < b_e$

$a_l < b_r$

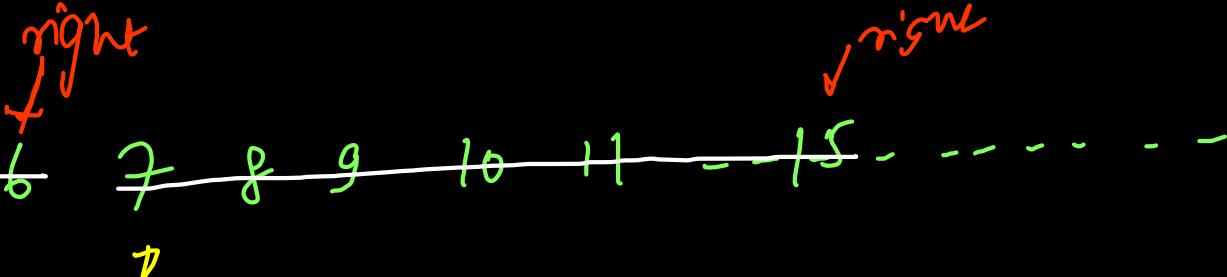
$b_l > b_e$

Natural  
Numbers



target = very large

## Unbounded BS / BS on Infinite Array



```
class UnboundedBinarySearch {  
    public int search(int[] nums, int target, int left, int right) {  
        while (left <= right) {  
            int mid = (left + right) / 2;  
  
            if (nums[mid] == target)  
                return mid; // successful search  
            else if (nums[mid] < target)  
                left = mid + 1;  
            else  
                right = mid - 1;  
        }  
  
        return -1; // unsuccessful search  
    }  
  
    public int infiniteSearch(int[] nums, int left, int right, int target) {  
        if (nums[left] >= target && target <= nums[right]) {  
            return search(nums, target, left, right);  
        } else {  
            return infiniteSearch(nums, right + 1, 2 * right, target);  
        }  
    }  
}
```

$\pi$

real  
infinity

not  $\text{IntMax}$