(b) Determine area of the triangle whose vertices are (2,2), (5,2), (3,4). 2

(c) Calculate the eigenvalues and eigenvectors of AA^T and A^TA where $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. 2

9. (a) Calculate the lengths and inner product of $x = \begin{bmatrix} 1 \\ i \end{bmatrix}$ and $y = \begin{bmatrix} 2+i \\ 2-4i \end{bmatrix}$. 2

(b) Prove that eigenvalues of a Hermitian matrix are real. 2

(c) Decide for or against the positive definiteness of the following matrix. 2

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

10. (a) Determine the Singular Value Decomposition and Pseudo-inverse of the matrix given in Q.8(c). 2

(b) Calculate the norm and condition number of the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$. 2

(c) Compute the Jacobi matrix of the matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. 2

End of Questions

END SEM. EXAMINATION, JULY-2023

APPLIED LINEAR ALGEBRA (MTH - 3003)

Programme: B.Tech Semester 4th

Full Marks: 60 Time: 3 Hours

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Apply Gauss elimination principle to solve system of linear equations and elementary matrices to get LU & LDU factorization of a matrix.	L3 L3L3L3L3L3	1 a,b,c 2 a,b,c	6+6
Explain vector space, subspace, null space and column space, linear independence, rank of a matrix	L4 L4 L3	3 a,b,c	6
Explain basis and dimension of vector space and four fundamental subspaces, linear transformations and their applications	L4 L3 L3 L4,L3	4 a,b,c 5 a,b	6+4
Explain orthogonality and its applications to find best fit solutions by least squares. Apply properties of determinants to solve the system of equations	L3 L3,L3,L3 L3,L3,L3 L3	5c, 6a,b,c 7a,b,c 8b	2+6+6 +2
Explain eigenvalues and eigenvectors and their application to solve system of differential equations.	L3 L3L3L3	8a,c,9a,b	4+4
Diagonalizations and complex matrices Hermitian, skew Hermitian, Unitary matrices	L3 L3L3	9c 10 a,b,c	2+6

Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Applying (L3), Analyzing (L4), Evaluating (L5), Creating (L6)

Answer all questions. Each question carries equal mark.

1. (a) Determine the values of b that lead to missing pivot in the following system. 2

$$x + by = 0,\ \ x - 2y - z = 0,\ \ y + z = 0$$

(b) Solve by sketching the row picture for the equations 2

$x + y = 2,\ 2x - 2y = 4.$

(c) Apply Gaussian elimination to solve the system $2x+3y=0,\ 4x+5y+z=3,\ 2x-y-3z=5$. 2

2. (a) Find the LDU factorization of the following matrix, where L is lower triangular matrix, D is diagonal matrix and U is upper triangular matrix. 2

$$A=\begin{bmatrix}1&1&0\\1&2&1\\0&1&2\end{bmatrix}.$$

(b) Evaluate the value(s) of d for which the system of equations has a unique solutions. x+y+2z = 1,x+2y+3z=2, x+4y+2dz=1 2

(c) Use the Gauss-Jordan method to find inverse of the matrix 2

$$A=\begin{bmatrix}1&0&0\\1&1&1\\0&0&1\end{bmatrix}.$$

3. (a) Examine whether the following subset of R^3 is a subspace or not. $V=\{The\ plane\ of\ vectors(b_1,b_2,b_3)\ with\ b_1=0\}$. 2

(b) Describe the column space and null space of the matrix 2

$$A=\begin{bmatrix}0&2\\5&0\end{bmatrix}.$$

(c) 2

Let $P=\begin{bmatrix}1&1&-1\\2&-3&4\\3&-2&3\end{bmatrix}$ and $Q=\begin{bmatrix}-1&-2&-1\\6&12&6\\5&10&5\end{bmatrix}$ be two matrices.

Then the rank of $P+Q$ is ______

4. (a) Examine whether the following set a basis of R^3 or not. 2
$\{(1,2,2),(-1,2,1),(0,8,6)\}$

(b) Find a basis for the plane $x+y+z=0$ in R^3. Then find a basis for the intersection of that plane with $yz-$plane. 2

(c) Write the dimension of the four fundamental subspaces of the matrix 2

$$A=\begin{bmatrix}1&2&1\\2&4&3\\3&6&4\end{bmatrix}.$$

5. (a) Let $c_1, c_2 \dots, c_n$ be scalars not all zero and $a_i \epsilon R^n$ be column vectors of a matrix A such that $\sum_{i-1}^{n} c_i a_i = 0$ & $b=\sum_{i-1}^{n} a_i$. Then $Ax=b$ has how many solutions? Justify your answer. 2

(b) Calculate the left inverse (if exists) for the matrix

$$A=\begin{bmatrix}1&0\\1&1\\0&1\end{bmatrix}.$$

(c) Determine the relation between determinant of A and determinant of B. Where

$$A=\begin{bmatrix}1&2&3&4\\4&1&2&3\\3&4&1&2\\2&3&4&1\end{bmatrix} \text{ and } B=\begin{bmatrix}3&4&1&2\\4&1&2&3\\1&2&3&4\\2&3&4&1\end{bmatrix}$$

6. (a) Determine a vector x orthogonal to the row space of A and a vector y orthogonal to the column space of A, where

$$A=\begin{bmatrix}1&2&1\\2&4&3\\3&6&4\end{bmatrix}$$

(b) Determine the projection matrix that projects any vector onto the line passing through the vector perpendicular to

$$a=\begin{bmatrix}1\\3\end{bmatrix}.$$

(c) Compute the projection of $b=(1,1)$ on to the line passing through $a=(1,-1)$.

7. (a) Determine the projection of b onto the column space of A, where $A=\begin{bmatrix}1&0\\0&1\\1&1\end{bmatrix}$, $b=\begin{bmatrix}1\\1\\0\end{bmatrix}$

(b) Determine the largest eigenvalue of A

$A=uv^T$ where $u=\begin{pmatrix}1\\2\end{pmatrix}, v=\begin{pmatrix}1\\1\end{pmatrix}$

(c) If A is a 4 by 4 matrix with det. $A=\frac{1}{3}$, then calculate det. $(3A)$ and det. (A^{-1}).

8. (a) Solve the following system using Cramer's rule.
$2x+5y=1,\ x+4y=2$