END SEMESTER EXAMINATION, JULY-2023
INTRODUCTORY GRAPH THEORY (CSE 1004)

Programme: B.Tech | **Semester:2nd**
Full Marks: 60 | **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Able to understand the fundamental concepts of graphs and apply them to study graph isomorphisms, Eulerian graphs, graphic sequences and digraphs.	L3, L3, L3, L3, L3, L3	1(a),1(b), 1(c),2(a), 2(b),2(c)	2,2, 2,2, 2,2
Able to understand the concepts of trees, spanning trees and study its various concepts and apply the Kruskal's algorithm to find the minimum spanning tree and Dijkstra's algorithm to find the shortest path of connected weighted graphs.	L3, L3, L3, L3, L3, L3	3(a),3(b), 3(c),4(a), 4(b), 4(c)	2,2, 2,2, 2,2
Able to understand matchings and factorization of graphs and its various applications.	L3, L3, L3	5(a),5(b), 5(c)	2,2, 2
Able to understand and analyze coloring of graphs, it's enumerative aspects and its applications.	L3, L3, L3, L3, L3, L3	6(a),6(b), 6(c),7(a), 7(b),7(c)	2,2, 2,2, 2,2
Able to understand and analyze planar graphs and its various applications.	L3, L3, L3, L3, L3, L3	8(a),8(b), 8(c),9(a), 9(b),9(c)	2,2, 2,2, 2,2
Able to understand the concepts of line graphs, edge-coloring and the various aspects of Hamiltonian cycles.	L3, L3, L3	10(a),10(b), 10(c)	2,2, 2

***Bloom's taxonomy levels:** *Remembering (L1), Understanding (L2), Applying (L3), Analysing (L4), Evaluating (L5), Creating (L6)*

Answer all questions. Each question carries equal mark.

1. (a) Prove that every u,v-walk contains a u,v-path. 2

   (b) Draw four simple nonisomorphic graphs of order 4. 2

   (c) Prove or disprove: If u and v are the only vertices of odd degree in a graph G, then G contains a u,v-path. 2

2. (a) Prove that if every vertex of a graph G has degree atleast 2, then G contains a cycle. 2

(b) Prove or disprove: If D is an orientation of a simple graph with 10 vertices, then the vertices of D cannot have distinct outdegrees. 2

(c) Determine, whether the following is a graphic sequence. If yes, construct the graph. 2

$$d: 5, 5, 4, 4, 2, 2, 1, 1$$

3. (a) Prove that every tree with at least two vertices has at least two leaves. 2

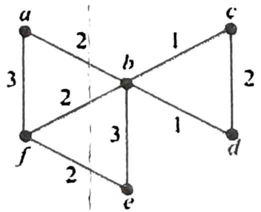
(b) Determine the diameter and radius of the cycle graph C_5 by calculating the eccentricity of each vertex of the graph. 2

(c) Prove that G is a tree if and only if adding any edge with endpoints in $V(G)$ creates exactly one cycle. 2

4. (a) Let T be an n-vertex tree having one vertex of each degree i with $2 \le i \le k$; the rest $n-k+1$ vertices are leaves. Determine n in terms of k. 2

(b) Let G be a complete undirected graph with 4 vertices and 6 edges having weights 1, 2, 3, 4, 5 and 6. Determine the maximum possible weight that a minimum weight spanning tree of G can have. 2

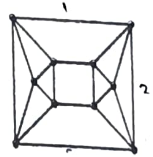
(c) Find the minimum spanning tree of the given graph by using Kruskal's algorithm. 2



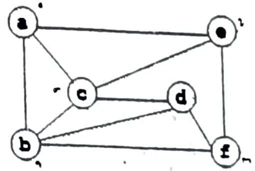
5. (a) How many perfect matchings does the graph $K_{2,2}$ have? Draw all the perfect matchings of $K_{2,2}$. 2

(b) Let T be a tree with n vertices, and let k be the maximum size of an independent set in T. Determine $\alpha'(T)$ in terms of n and k. 2

(c) In the graph given below exhibit a k-factor for $k=1$ and $k=2$. 2



6. (a) Determine the chromatic number of the given graph. 2

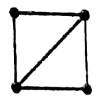


(b) Prove that the chromatic number of a graph equals the maximum of the chromatic numbers of its components. 2

(c) Prove or disprove: If $G = F \cup H$, then $\chi(G) \le \chi(F) + \chi(H)$. 2

7. (a) Draw $P_3 \vee K_3$ and compute $\chi(P_3 \vee K_3)$. 2

(b) Compute the chromatic polynomial of the given graph. 2



(c) Determine $\chi(C_n \vee K_1; k)$ and hence find $\chi(C_3 \vee K_1; 4)$. 2

8. (a) Show that K_5 is nonplanar. 2

(b) Prove or disprove: Every subgraph of a planar graph is planar. 2

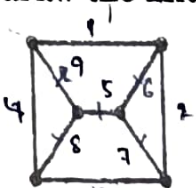
(c) Draw the dual graph of K_4. 2

9. (a) If G is a triangle-free simple planar graph with atleast 3 vertices then prove that $e(G) \le 2n(G) - 4$. 2

(b) If in an undirected connected planar graph G, there are eight vertices and five faces, then determine the number of edges in G. 2

(c) Let G be a simple undirected planar graph with 10 vertices and 15 edges. If G is connected, then determine the number of bounded faces in any embedding of G on the plane. 2

10 (a) For the given graph G, compute the edge-chromatic number $\chi'(G)$ and draw the line graph $L(G)$. 2

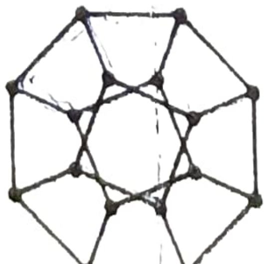


G

(b) Determine the maximum edge multiplicity $\mu(G)$ of the graph G given below. 2

G

(c) Determine whether the given graph is Hamiltonian or not. If yes, show the Hamiltonian cycle by suitably naming the vertices. 2



End of Questions