END SEMESTER EXAMINATION, MARCH -2023
CALCULUS-A (MTH-1101)

Programme: B.Tech | **Semester: 1st**
Full Marks: 60 | **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Use limit laws to evaluate the limit of a function and demonstrate the existence of limit and continuity of functions.	L1,L3	1.a,c	2.2
Compute slope of tangent lines and derivatives by different techniques and apply the concept of derivatives for linearizion of functions and solve various physical and Engineering problems.	L1 L1,L1,L1	1.b 2.a,b,c	2 2,2,2
Discuss the Mean Value Theorems and study maximum and minimum values of a function as well as apply L' Hospital's rule to evaluate limits of functions and sketch curves of functions	L1,L1,L1 L1,L1,L1	3.a,b,c 4.a,b,c	2,2,2 2,2,2
Compute indefinite integrals using techniques of integration and apply it to physical and Engineering problems	L1,L1 L1,L4 L4	5.a,b 6.b,c 7.c	2,2 2,2 2
Apply the concept of integration to find volume, work done, surface area and average value of an integral and study numerical integration using different methods.	L3 L1 L1, L4 L3	5.c 6.a 7.a,b 9.c	2 2 2,2 2
Apply the principles of calculus to study and calculate areas, arc lengths etc. of parametric and polar curves.	L1,L1,L2 L1,L2	8.a,b,c 9.a,b	2,2,2 2,2
Analyze infinite series and sequences and discuss their convergences using comparison test, root test and ratio test	L1,L3,L3	10.a,b,c	2,2,2

***Bloom's taxonomy levels:** *Remembering (L1), Understanding (L2), Applying (L3), Analysing (L4), Evaluating (L5), Creating (L6)*

Answer all questions. Each question carries equal mark.

1.	(a)	Find $\lim_{x\to\infty}\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$ or show that the limit does not exist.	2
	(b)	A particle moves along a straight line with equation of motion $s=f(t)=100+50t-4.9t^2$, where s is measured in meters and t in seconds. Find the velocity and speed when $t=5$.	2
	(c)	Show that the function $f(x)=\|x\|$ is differentiable for all $x<0$, as well as for all $x>0$.	2
2.	(a)	Find the points on the curve $y=1-12x+3x^2+2x^3$ where the tangent line is horizontal.	2
	(b)	If $h(2)=4$ and $h'(2)=-3$, find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)$ at $x=2$.	2
	(c)	Find the equation of the tangent line to the curve $y=\sin(\sin x)$ at the point $(\pi,0)$.	2
3.	(a)	Find $\frac{dy}{dx}$ if $y^x=x^y$.	2
	(b)	A freshly brewed cup of coffee has temperature 95^0 C in a 20^{0}C room. When its temperature is 70^{0}C, it is cooling at a rate of 1^{0}C per minute. When does this occur?	2
	(c)	Find the critical numbers of the function $h(p)=\frac{p-1}{p^2+4}$.	2
4.	(a)	Find the minimum value of the function $f(t)=t^3-3t^2-24t+100$ in the interval $[-3,3]$.	2
	(b)	Find the interval on which the function $f(x)=x^2\ln x$ is increasing or decreasing.	2
	(c)	The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?	2
5.	(a)	Use Newton's method to find a root of the equation $3\cos x=x+1$ correct to 2 decimal places.	2
	(b)	Evaluate the upper and lower sums for $f(x)=2+\sin x, 0\le x\le\pi$, with $n=4$.	2
	(c)	Find the derivative of the function $g(x)=\int_{1-2x}^{1+2x} t\sin t\,dt$.	2

6.	(a)	Evaluate the indefinite integral $\int \sqrt{x}\sin(1+x^{\frac{3}{2}})dx$.	2
	(b)	Find the volume of the solid obtained by rotating the region bounded by the curves $x=y^2, x=2y$; about y-axis.	2
	(c)	Find the average value of the function $f(x)=\sin 4x$ on the interval $[-\pi,\pi]$.	2
7.	(a)	Evaluate $\int(\tan^5 x\sec^3 x dx$.	2
	(b)	Evaluate $\int \frac{t}{t-6}dt$.	2
	(c)	Determine whether the integral $\int_e^{\infty}\frac{dx}{x(\ln x)^3}$ is convergent or divergent and if it is convergent then evaluate it.	2
8.	(a)	Use Simpson's rule with $n=10$ to estimate the arc length of the curve $y=x\sin x, 0\le x\le 2\pi$.	2
	(b)	Eliminate the parameter t from the parametric equation $x=1-t^2, y=t-2, -2\le t\le 2$ to find its Cartesian form.	2
	(c)	Describe the motion of the particle with position $(x,y)=(2\sin t, 4+\cos t)$ as t varies in $0\le t\le {}^{3\pi}\!/_{2}$.	2
9.	(a)	Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of the curve $x=e^t, y=te^{-t}$.	2
	(b)	Find the points on the curve $x=t^3-3t, y=t^3-3t^2$ where the tangent line is horizontal or vertical.	2
	(c)	Find a formula for the general term a_n of the sequence $\left\{1,-\frac{1}{3},\frac{1}{9},-\frac{1}{27},\frac{1}{81},\ldots\right\}$, assuming that the pattern of the first few terms continues.	2
10	(a)	Determine whether the geometric series $2+0.5+0.125+0.03125+\ldots$ is convergent or divergent. If it is convergent, find the sum.	2

$\frac{\pi}{2}+\frac{\pi}{4}=\frac{2\pi+\pi}{4}$ $\frac{3\pi}{4}$

	(b)	Express the number $2.\overline{516}$ as a ratio of integers.	2
	(c)	Find the Maclaurin series for $f(x) = e^{-2x}$ using the definition of the Maclaurin series.	2
		End of Questions	