(Please write your Exam Roll No.) **Exam Roll No. ....**

END TERM EXAMINATION

THIRD SEMESTER [B.TECH.] FEBRUARY 2023

Paper Code: ES-201 **Subject: Computational Methods**

Time: 3 Hours **Maximum Marks: 75**

Note: Attempt any five questions including Q.No.1 which is compulsory. Select one question from each unit. Assume missing data, if any. Scientific calculator is allowed.

Q1 (a) In the subtraction of 37.593621-37.584216, how many significance will be lost. **(1.5)**

(b) How many steps of the bisection algorithm are needed to compute a root of f(x) to full machine single precision on a 32-bit word length computer if a=16 and b=17. **(1.5)**

(c) Using Simpson's 1/3 rule compute the integral $\int_0^{\pi/2} \sqrt{\sin(x)}\,dx$ for n=4 with an accuracy to five decimal places. **(1.5)**

(d) Using Lagrange interpolation find the unique polynomial of degree 2, such that f(0)=1, f(1)=3, f(3)=55. **(1.5)**

(e) Define eigen Values of a matrix and eigen values of the matrix **(1.5)**

(f) Define Cubic Spline function and write the conditions required for cubic spline interpolation **(1.5)**

(g) Solve IVP $\frac{dy}{dx} = x + y^2, y(0) = 1$ using Picard's method upto 2nd iterations. **(3)**

(h) Classify whether the equations are hyperbolic, elliptical or parabolic:

$(i) e^x u_{xx} + \cos y\, u_{xy} - u_{yy} = 0$

$(ii) u_{xx} + u_{yy} + u_x + \sin x\, u_y - u = x^2 + y^2$ **(3)**

UNIT – I

Q2 a) If the secant method is used on $f(x) = x^5 + x^3 + 3$ *with* $x_0 = -1$ *and* $x_1 = 1$, *what will be* x_8? **(8)**

b) If f(x) = 3x-cos(x)-1 and x_0= 0.6, what are x_1 and x_2 in the Newton iteration? **(7)**

Q3 a) Minimize the function f(x) = $4x^3 + x^2 - 7x + 14$ within the interval [0,1] using golden section method. **(7.5)**

b) Find minimum value of $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using steepest decent method taking starting point x_0(0, 0). **(7.5)**

UNIT-II

Q4 a) Prove that $e^x = \left(\frac{\Delta^2}{E}\right) e^x . \frac{Ee^x}{\Delta^2 e^x}$ the interval of differentiation being unity. **(6.5)**

b) Construct a divided difference table for the function f(x) given in the following table and find the Newton divided difference interpolating polynomial of f(x)

x	1	3/2	0	2
y	3	13/4	3	5/3

Also find f(0.5). **(8.5)**

P.T.O.

Q5 (a) If the integral $\int_0^{\pi} e^{\cos x} dx$ is to be computed with absolute error less than $\frac{1}{2} \times 10^{-3}$, and if we are going to use composite trapezoidal rule with a uniform partition, how many subintervals are needed? **(6)**

(b) Evaluate the definite integral $\int_0^1 e^{-x^2} dx$ using the basic Simpson's 1/3 rule and the basic Simpson's 3/8 rule carries five significant digits. Also, find the length of correctness in the significant decimal places (rounded) in both the rules. **(9)**

UNIT-III

Q6 (a) Show that the system of equations:

$$\begin{cases} x_1 + 4x_2 + \alpha x_3 = 6 \\ 2x_1 - x_2 + 2\alpha x_3 = 3 \\ \alpha x_1 + 3x_2 + x_3 = 5 \end{cases}$$

Possesses a unique solution when α=0, no solution when α≠ -1 and infinitely many solution when α =1. **(6)**

(b) Solve the system of linear equations:

$$\begin{cases} 0.0001x + y = 1 \\ x + y = 2 \end{cases}$$

Using Gauss elimination method in order to apply no pivoting and partial pivoting. Also analyse, how roundoff error effect the computation if the calculation carry at most five significant digits of precision (rounding). **(9)**

Q7 (a) Determine the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using power series method taking starting eigen vector$X^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. **(6)**

(b) Determine the Cholesky factorization of the matrix A, where A is:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}$$ **(9)**

UNIT – IV

Q8 (a) Using Taylor's series method, compute y(0.2) to three decimal precision from $\frac{dy}{dx} = 1 - 2xy$ *given that* $y(0) = 0$. Compare the result with the exact result for the value of y at 0.2. **(7.5)**

(b) Consider the initial value problem $\begin{cases} \frac{dx}{dt} = 2 + (x - t - 1)^2 \\ x(1) = 2 \end{cases}$. Using Runge-Kutta method of 4th order find x(1.5) taking h=0.5 **(7.5)**

Q9 (a) Solve the initial value problem $\frac{dy}{dx} = 1 + xy^2, y(0) = 1$ and find y(0.4) using Milne's method, when it is given that: **(6)**

x	0.1	0.2	0.3
y	1.105	1.223	1.355

(b) Solve the partial differential equation $u_{xx} + u_{yy} = -81xy,\ 0 < x < 1,\ 0 < y < 1$ given that u(x,0)=0, u(0,y)=0, u(1,y)=100, u(x,1)=100 and h=1/3. **(9)**

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