END SEMESTER EXAMINATION, MARCH-2023
DISCRETE MATHEMATICS (CSE 1002)

Programme: B.Tech | **Semester: 1st**
Full Marks: 60 | **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Able to analyze and apply rules of logic to distinguish between valid and invalid arguments and use them to prove mathematical statements.	L1, L3, L3, L2, L3, L3	1(a), 1(b), 1(c), 2(a), 2(b), 2(c)	2,2, 2,2, 2,2
Able to understand sets, their various operations and use them to analyze functions and its various concepts as well as study sequences and summations.	L3, L3, L3	3(a), 3(b), 3(c)	2,2, 2
Able to analyze the searching and sorting algorithms and use the growth of functions to study the time complexity of algorithms as well as apply some of the important concepts of number theory to divisibility and modular arithmetic, integer representation of algorithms, congruences and cryptography.	L3, L3, L3, L3, L3, L3	4(a), 4(b), 4(c), 5(a), 5(b), 5(c)	2,2, 2,2, 2,2
Able to construct proofs by mathematical induction and analyze and formulate recursive definitions and develop structural induction.	L2, L3, L3	6(a), 6(b), 6(c)	2,2, 2
Able to apply different counting techniques to solve various problems.	L3, L3, L3, L3, L3, L3	7(a), 7(b), 7(c), 8(a), 8(b), 8(c)	2,2, 2,2, 2,2
Able to apply relations and their properties to analyze equivalence relations and partial orderings.	L3, L3, L3, L2, L3, L3	9(a), 9(b), 9(c), 10(a), 10(b), 10(c)	2,2, 2,2, 2,2

Bloom's taxonomy levels:** ***Remembering (L1), Understanding (L2), Applying (L3), Analysing (L4), Evaluating (L5), Creating (L6)

Answer all questions. Each question carries equal mark.

1. (a) Write the negation of the following proposition. 2
'The summer in Maine is hot and sunny."

(b) Determine whether $\neg p \to \neg q$ is logically equivalent to $p \to q$ or $q \to p$. 2

(c) Prove by the method of contraposition that if n is an integer and $n^3 + 5$ is odd, then n is even. 2

2. (a) Translate the following statement into a logical expression using predicates, quantifiers and logical connectives. 2
'Every student in this class has studied calculus.'

(b) Determine whether $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology or not. 2

(c) Show that the premises *'No man is an island.'* and *'Manhattan is an island.'* imply the conclusion *'Manhattan is not a man.'* 2

3. (a) Show that if A and B are sets, then $A - B \subseteq A$. 2

(b) Let f and g be functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Find $f \circ g$ and $g \circ f$. 2

(c) What is the missing number in the following sequence? 2
2, 12, 60, 240, 720, 1440, ------, 0

4. (a) Use the bubble sort algorithm to sort the list 3, 5, 4, 1, 2 in increasing order showing the lists obtained at each step. 2

(b) Show that $x^4 + 9x^3 + 4x + 7$ is $O(x^4)$. 2

(c) Determine whether $f(x) = \lfloor x \rfloor$ is $\Omega(x)$. 2

5. (a) Use the linear congruential generator 2

$$x_{n+1} = (7x_n + 4) \pmod 9$$

with seed $x_0 = 3$ to generate a sequence of pseudorandom numbers.

(b) What is the value of $3^{51} \bmod 5$? 2

(c) Solve the congruence $3x \equiv 4 \pmod 7$. 2

6. (a) Give recursive definition of the sequence $\{a_n\}, n = 1, 2, 3, \ldots$ if 2

(i) $a_n = 6n$ and (ii) $a_n = 5$.

(b) Use mathematical induction to prove that 3 divides $n^3 + 2n$, whenever n is a positive integer. 2

(c) Use strong induction to prove that every amount of postage of 18 cents or more can be formed using just 3-cent and 10-cent stamps. 2

7. (a) How many positive integers between 100 and 999 inclusive are divisible by 3 or 4? 2

(b) Show that among any $n+1$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers. 2

(c) If a department contains 10 men and 15 women, then how many ways are there to form a committee with six members if it must have the same number of men and women? 2

8. (a) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? 2

(b) What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$? 2

(c) How many ways are there to select five unordered elements from a set with three elements when repetition is allowed? 2

9. (a) Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric and/or transitive where $(x, y) \in R$ if and only if $x+y=0$. 2

(b) Let R be the relation $R = \{(a,b) \mid a \text{ divides } b\}$ on the set of positive integers. Find (i) R^{-1} and (ii) $\overline{R}$. 2

(c) Let R be the relation represented by the matrix 2

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Find the matrix representing R^2.

10 (a) Give a description of each of the congruence classes modulo 6. 2

(b) Draw the Hasse diagram and find the maximal and minimal elements for the poset ({2, 4, 5, 10, 12, 20, 25}, |). 2

(c) Answer the following questions for the poset ({2,4,6, 9,12,18, 27,36,48,60,72}, |). 2

(i) Find all the upper bounds of {2,9}.

(ii) Find the least upper bound of {2,9} if it exists.

End of Questions