

END TERM EXAMINATION

FOURTH SEMESTER [B.TECH] MAY-JUNE 2025

Paper Code: BS-202

Subject: Probability Statistics and
Linear Programming

Time: 3 Hours

Maximum Marks: 60

Note: Attempt five questions in all including Q.No.1 which is compulsory. Select one question from each unit.

- Q1 Attempt all questions. (4x5=20)
- (a) A factory has two plants. Records show that the plant A produces 30% of the items of the output whereas plant B produces 70% of the items. Further 5% of the items produced by plant A are defective while 1% produced by the plant B are defective. If a defective item is drawn at random, find the probability that the defective item was produced by plant B.
- (b) The joint P.D.F of random variables (X, Y) is given by
 $f(x,y) = 2; 0 < x < 1, 0 < y < x$
 $= 0; \text{elsewhere,}$
Are X and Y independent?
- (c) Prove that every regression line passes through its mean.
- (d) A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on Machine C. The profit realized by selling one chair is Rs. 300 while for a table the figure is Rs. 600. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week to maximize the profit? Formulate a mathematical model for the problem.

UNIT-I

- Q2 (a) Prove that Poisson distribution is a limiting case of Binomial distribution. If the variance of the Poisson distribution is 2. Hence find $p(x) \geq 4$. (5)
- (b) The customer accounts of certain department store have an average balance of Rs. 120 and a standard deviation of Rs. 40. Assuming that the account balances are normally distributed:
- (i) What proportion of the account is over Rs. 150?
- (ii) What proportion of account is between Rs. 100 and Rs. 15?
- (iii) What proportion of account is between Rs. 60 and Rs. 90?
- Given that tabulated value of z at 0.5 = 0.1915, at 0.75 = 0.2266 and at 1.5 is 0.4332. (5)

OR

- (a) Ten coins are tossed 1024 times and the following frequencies are observed with the expected frequencies. Compare these frequencies with the expected frequencies. (5)

P.T.O.

No. Of heads	0	1	2	3	4	5	6	7	8	9	10
Frequencies	2	10	38	106	188	257	226	128	59	7	3

- (b) In a certain city the daily consumption of electric power in millions of kilowatt-hours can be treated as a random variable having a gamma distribution with $\lambda = 3$, $\alpha = 0.5$. If the power plant of this city has a daily consumption of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day? (5)

UNIT-II

- Q3 (a) Three fair coins are tossed. Let X denote the number of heads on the first two coins and let Y denote the number of tails on the last two coins. Find the conditional distribution of Y given that $X=1$. Comment on the correlation coefficient between X and Y . (5)
- (b) Explain sampling distribution. State central limit theorem. The average weight of a water bottle is 30 kg, with a standard deviation of 1.5 kg. If a sample of 45 water bottle is selected at random from a consignment and their weights are measured, find the probability that the mean weight of the sample is less than 28 kg. (5)

OR

- (a) An urn contains four balls. Two of the balls are numbered with 1 and the other two are numbered with 2. Two balls are drawn from the urn without replacement. Let X denote the smaller of the numbers on drawn balls and Y the larger.
- Find the joint P.M.F of X AND Y .
 - Find marginal distribution of Y .
 - Find $\text{Cov}(X, Y)$.
- (b) The joint density of X and Y is given by $\{f(x, y) = 4xy e^{-(x^2+y^2)}, x \geq 0, y \geq 0\}$. Test whether X and Y are independent. Also find the conditional density of X given $Y = y$. (5)

UNIT-III

- Q4 (a) In a school the heights of six randomly chosen girls are 64, 65, 68, 69, 71 and 72 inches and those of nine randomly chosen boys 61, 62, 65, 66, 69, 70, 71, 72 and 73 inches. If the tabulated value of t for 13 D.O.F at 5% level of significance is 1.77 then test if the girls are taller than boys. (5)
- (b) Prove that the correlation coefficient ρ_{xy} is a geometric mean of two regression coefficients b_{yx} and b_{xy} . Given the bivariate data. Calculate the Karl Pearson's coefficient of correlation. (5)

x	1	5	3	2	1	1	7	3
y	6	1	0	0	1	2	1	5

OR

- (a) Apply χ^2 test to ascertain if Poisson distribution can be assumed from the following data. Given that the tabulated value of χ^2 at 5% level of significance is 9.49. (5)

No. of defectives	0	1	2	3	4	5
Frequency	6	13	13	8	4	3

- (b) Two random samples gave the following results.

Sample	Size	Sample mean	Sum of squares of deviations from mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population.
Given that $F_{(9,11)} [0.05] = 2.90$ and $t_{(20)} [0.05] = 2.086$. (5)

UNIT-IV

- Q5 (a) Use Simplex method to maximize $Z = 5x + 3y$ subject to the constraints $x + y \leq 2$, $5x + 2y \leq 10$ and $3x + 8y \leq 12$, $x, y \geq 0$. (5)
- (b) Find the optimal solution for the following transportation problem using

Source		A	B	C	D		
O1		21	16	25	13	11	Availability
O2		17	18	14	23	13	
O3		32	27	18	41	19	
Requirement		6	10	12	15	43	

V.A.M.

(5)

OR

- (a) Discuss duality. Obtain the dual problem of the following L.P.P. (5)

Maximize $Z = x_1 - 2x_2 + 3x_3$ <https://www.ggsipuonline.com>

subject to the constraints

$$-2x_1 + x_2 + 3x_3 = 2,$$

$$2x_1 + 3x_2 + 4x_3 = 1,$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

- (b) What do you mean by an assignment problem? Four jobs are to be done on four different machines. The cost in (rupees) of producing 'i' th job on the 'j' th machine is given below: (5)

		Machines			
		M1	M2	M3	M4
Jobs	J1	15	11	13	15
	J2	17	12	12	13
	J3	14	15	10	14
	J4	16	13	11	17

Assign the jobs to different machines so as to minimize the total cost.
