END-SEMESTER EXAMINATION, May-2024
Applied Linear Algebra (MTH 3003)

Programme: B.Tech (ALL Branch except ME) **Semester: 4th**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Apply Gauss elimination principle to solve system of linear equations and elementary matrices to get LU & LDU factorization of a matrix.	L3, L3, L3 L3, L3, L3	1 (a), (b), (c), 2 (a), (b), (c)	2+2+2 2+2+2
Explain vector space, subspace, null space and column space, linear independence, basis and dimension of vector space and four fundamental subspaces, linear transformations and their applications.	L3, L3, L3	3 (a), (b), (c),	2+2+2
Explain basis and dimension of vector space and four fundamental subspaces, linear transformations and their applications.	L3, L3, L3	4 (a), (b), (c)	2+2+2
Explain orthogonality and its applications to find best fit solutions by least squares. Apply properties of determinants to solve the system of equations.	L3, L3, L3	5 (b), 7 (a), 7 (b), 8 (a)	2 2+2 2
Explain eigenvalues and eigenvectors and their application to solve system of differential equations and apply it to complex matrices, diagonalization of matrix.	L3, L3, L3 L3, L3, L3 L3, L5, L3 L3, L5, L5	5 (a), (b), (c), 6 (a), (b), (c), 7 (c) 8 (b), (c), 9 (a), (b), (c)	2+2+2 2+2+2 2+2 2 2+2+2
Examine the positive definiteness of a form and its applications to test the extreme points, Singular Value Decomposition and pseudoinverse. Identify and analyze the norm and condition number of a matrix.	L3, L3, L3	10 (a), (b), (c),	2+2+2

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1.	(a)	Determine the value of d that makes the following system singular $2x+5y+z=0,\ 4x+dy+z=2,\ y-z=3$.	2
	(b)	Solve the system of equations using column picture method for the equations $x-y=0, x+y=4$	2
	(c)	Apply Gaussian elimination to solve the following system $x+y+z=5,\ x+2y+3z=7,\ x+3y+6z=11$	2
2.	(a)	Express the following matrix as LU factorization. $A=\begin{bmatrix}2&3&3\\0&5&7\\6&9&8\end{bmatrix}$	2
	(b)	Determine two permutation matrices P_1 and P_2 so that $P_1 A P_2$ is lower triangular, where $A=\begin{bmatrix}0&0&6\\1&2&3\\0&4&5\end{bmatrix}$	2
	(c)	Use Gauass-Jordan method to determine the inverse of the following matrix $A=\begin{bmatrix}1&2&3\\0&2&6\\0&0&3\end{bmatrix}$	2
3.	(a)	Prove that the plane of vectors (b_1,b_2,b_3) that satisfies $b_3-b_2+2b_1=0$ is a subspace of $\mathbb{R}^3$.	2
	(b)	Let $P=\begin{bmatrix}1&1&-1\\1&-3&4\\3&-2&3\end{bmatrix}$ and $Q=\begin{bmatrix}-1&-2&-1\\6&12&6\\5&10&5\end{bmatrix}$ be two matrices. Determine the rank of the matrix $P - Q$.	2
	(c)	Let A and B be two n × n matrices over real numbers. Then explain which of the following statements are true ? 1. rank (AB) = rank (A) rank (B) 2. rank (A+B) ≤ rank (A) + rank (B)	2
4.	(a)	Describe the subspace of $\mathbb{R}^3$ spanned by the three vectors (0,1,1), (1,1,0) and (0,0,0).	2
	(b)	Determine the dimension of the four fundamental subspaces of the following matrix $A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$	2
	(c)	Determine the right inverse (if exists) for the matrix $A=\begin{bmatrix}1&1&0\\0&1&1\end{bmatrix}$.	2
5.	(a)	Let p and q be real numbers such that $p^2+q^2=1$. Then determine the eigenvalues of the matrix $\begin{bmatrix}p&q\\q&-p\end{bmatrix}$.	2
	(b)	Consider a 2 × 2 matrix $M=[v_1\ \ v_2]$, where, v_1 *and* v_2 are the column vectors. Suppose $M^{-1}=\begin{bmatrix}u_1^T\\u_2^T\end{bmatrix}$ where u_1^T and u_2^T are the row vectors. Determine $u_1^T v_1,\ u_1^T v_2,\ u_2^T v_1$ *and* $u_2^T v_2$.	2
	(c)	Two eigenvalues of a 3 × 3 real matrix P are $(2+\sqrt{-1})$ and 3. The determine the determinant of P.	2
6.	(a)	Solve the differential equation $\frac{du}{dt}=Au, u(0)=\begin{bmatrix}0\\6\end{bmatrix}$ where A is $A=\begin{bmatrix}1&-1\\2&4\end{bmatrix}$.	2
	(b)	Determine the 10^{th} power of A after diagonalizing it. $A=\begin{bmatrix}1&-1\\2&4\end{bmatrix}$.	2
	(c)	Prove that for a unitary matrix, the eigenvectors corresponding to different eigenvalues are orthogonal.	2
7.	(a)	Determine the projection matrix P onto A^T for $A=\begin{bmatrix}1&1&0\\0&1&1\end{bmatrix}$.	2
	(b)	Solve the following system using Cramer's rule. $2x+5y=1$ $x+4y=2$	2
	(c)	If A is a 4 by 4 matrix with eigen values 5,7,8,9 then determine $\det\left(\frac{1}{2}A\right)$ and $\det(A^{-1})$.	2
8.	(a)	Compute area of the triangle of the triangle with vertices $A(2,2), B(-1,3)$ and origin in xy plane.	2

	(b)	Determine the eigenvalues and eigenvectors of AA^T and $A^T A$ where $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.	2
	(c)	Construct the matrix A whose eigenvalues are 2 and 3, and whose eigenvectors are and respectively.	2
9.	(a)	Determine the lengths and inner product of $x = \begin{bmatrix} 1 \\ i \end{bmatrix}$ and $y = \begin{bmatrix} 2+i \\ 2-4i \end{bmatrix}$.	2
	(b)	Discuss Hermitian Matrix with proper example. Also justify that the example given satisfies the required conditions.	2
	(c)	Decide for or against the positive definiteness of the following matrix. $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$	2
10	(a)	Determine the Singular Value Decomposition and Pseudo-inverse of the matrix given in Q.8(b).	2
	(b)	Determine the norm and condition number of the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$.	2
	(c)	Compute the Gauss- Seidel matrix of the matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$.	2
		End of Questions	

2×3 $3\times$