

END TERM EXAMINATION**SECOND SEMESTER [B.TECH] JULY 2023****Paper Code: BS-112****Subject: Applied Mathematics-II****Time: 3 Hours****Maximum Marks: 75**

Note: Attempt five questions in all including Q.No.1 which is compulsory. Select one question from each unit.

Q1. (a) Find all the point at which the following mapping is not Conformal $w = \frac{z+1}{4z^2+2}$. (2.5)

(b) Split the real and imaginary part of i^i . (2.5)

(c). Find the Laplace transform of $t^2 e^{-2t}$. (2.5)

(d) Using half range sine series of function $f(x) = 1$ for $0 < x < \pi$,

prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$ (2.5)

(e) Taylor series expansion of $\frac{1}{z-2} \ln |z| < 1$ is..... (2.5)

(f) Classify the type of PDE: $\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + 5 \frac{\partial^2 u}{\partial y^2} = 0$, whether it is parabolic, elliptic or hyperbolic? (2.5)

UNIT-I

Q 2. (a) Find all the values of $(-1 + \sqrt{3} i)^{\frac{3}{2}}$. (7)

(b) Verify that the function $u(x, y) = x^3 y - xy^3$ is harmonic and find the harmonic conjugate of $u(x, y)$ to express the function $f(z) = u + iv$ as an analytic function. (8)

Q 3. (a) Evaluate the integral $\oint_C \frac{e^{2z}}{(z-1)(z-2)} dz$ along the curve C , where C is a circle $|z| = 3$. (7)

(b) Integrate the function $f(z) = \bar{z}$ along the curve C , where C is the square with vertices $z = 0, 2, 2i, 2 + 2i$ (8)

UNIT-II

Q 4.(a) Find the bilinear transformation or Mobious transformation which maps $1, i, 1$ of the z - plane onto $1, i, -1$ of the w - plane respectively. Also find the fixed points or invariant points. (8)

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- (b) Find the singular points, type of singularities and the corresponding residues of the function $f(z) = \frac{1}{(z^2-1)^2}$. (7)

Q 5.(a) Sketch and graph the given region: $|z| \leq \frac{1}{2}, -\frac{\pi}{8} < \text{Arg}(z) < \frac{\pi}{8}$,

and its image under the given mapping: $w = z^2$. (7.5)

(b) Prove that the integral $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx = \frac{\pi}{3}$. (7.5)

UNIT-III

Q 6. (a) Using Laplace transform solve the ordinary differential equation

$$y'''(t) + 2y''(t) - y'(t) - 2y = 0, \text{ with conditions } y(0) = y'(0) = 0, \text{ and } y''(0) = 6. \quad (8)$$

(b) Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$. (7)

Q 7.(a) Find the Fourier series to represent the function $f(x) = x^2$ in the interval $(-\pi, \pi)$. (7.5)

(b) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$

Hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$. (7.5)

UNIT-IV

Q 8. (a) Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature zero, assuming that the initial temperature is

$$f(x) = \begin{cases} x & \text{for } 0 < x < \frac{L}{2} \\ L-x & \text{for } \frac{L}{2} < x < L \end{cases} \quad (8)$$

(b) A tightly stretched string with fixed end points $x = 0$ and $x = l$, is initially in a position given by $y = \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$. <https://www.ggsipuonline.com> (7)

Q 9. (a) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions

$$u(0, y) = u(l, y) = u(x, 0) = 0 \text{ and } u(x, a) = \sin \frac{n\pi x}{l}. \quad (8)$$

(b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, with boundary condition $u(x, 0) = 3\sin(n\pi x)$,

$$u(0, t) = 0 \text{ and } u(1, t) = 0, \text{ where } 0 < x < 1, t > 0. \quad (7)$$
