

MID-SEMESTER EXAMINATION, April-2024

Applied Linear Algebra (MTH 3003)

Programme: B.Tech.(All Except ME)
Full Marks: 30

Semester: 4th
Time: 2 Hours

Subject/Course Learning Outcome	Taxonomy Level	Ques. Nos.	Marks
Concept of column picture to understand the geometrical meaning of the solution of the system of equations, Gaussian elimination method and singular system. Also, explain the role of elementary matrices to convert a matrix into upper triangular form.	L4, L4, L3,	1. a, b, c	2, 2, 2
Explain the concepts of triangular factorization, matrix inverse using Gauss Jordan method.	L5, L4, L3, L4, L5, L4	2. a, b, c 3. a, b, c	2, 2, 2 2, 2, 2
Explain the concept vector space, subspaces, column space and null space, echelon form to find the rank.	L3, L3	4. a, b	2, 2
Explain the concepts of row reduced echelon form of matrices, linear independence and dependence of vectors, basis and dimension.	L3, L4, L3	4. c, 5. a	2, 2
Explain the concepts of four fundamental subspaces, left-inverse, right inverse of a matrix and the concept of linear transformation.	L4, L5	5. b, c	2, 2

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Explain the column picture for the equations
- $$x - 2y = 0$$
- $$x + y = 6.$$

2

- (b) Discuss the relations between b_1 and b_2 and the number of solutions of the corresponding system 2
 $3x - 2y = b_1$
 $6x - 4y = b_2$
- (c) Solve the following system using Gaussian elimination method. 2
 $u + v = 0$
 $u + 2v + w = 1$
 $2u - v - 3w = 3$
2. (a) Elaborate skew-symmetric matrix with an example of a 3×3 matrix 2
- (b) Factorize the matrix 'A' as $A = LDU$ form, where L is the lower triangular matrix, D is the diagonal matrix and U is the upper triangular matrix. 2
 $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- (c) Determine the inverse of the following matrix using Gauss-Jordan method. 2
 $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
3. (a) Explain about singular matrix. For which three values of c the following matrix is singular. 2
 $\begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$
- (b) Design three elementary matrices multiplying which the following matrix is upper triangular. 2
 $\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$
- (c) Explain permanent breakdown of elimination process with proper examples. 2

4. (a) Discuss about the column space and the null space of the following matrix. 2
 $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$
- (b) Reduce the following matrix into its echelon form and find its rank. 2
 $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$
- (c) Determine the complete solution the following system. 2
 $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$
5. (a) Examine whether the given vectors $(1,2,3)$, $(2,1,3)$ and $(3,2,1)$ form a basis for \mathbb{R}^3 ? Justify your answer. 2
- (b) Determine the dimensions of the four fundamental spaces of the given matrix. 2
 $A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 0 \end{bmatrix}$
- (c) Construct a the basis and of each of the four fundamental subspaces for the following matrix. 2
 $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

End of Questions