END SEMESTER EXAMINATION, JULY-2022
INTRODUCTORY GRAPH THEORY (CSE 1004)

Programme:B.Tech(CSE & CSIT)
Full Marks: 60

Semester:2nd
Time: 3 Hours

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Able to understand the fundamental concepts of graphs and apply them to study graph isomorphisms, Eulerian graphs, graphic sequences and digraphs.	L2,L3, L3,L2, L3,L3	1(a),1(b), 1(c),2(a), 2(b),2(c)	2,2, 2,2, 2,2
Able to understand the concepts of trees, spanning trees and study its various concepts and apply the Kruskal's algorithm to find the minimum spanning tree and Dijkstra's algorithm to find the shortest path of connected weighted graphs	L3,L3, L3	3(a),3(b), 3(c)	2,2, 2
Able to understand matchings and factorization of graphs and its various applications.	L3,L3, L3	4(a),4(b), 4(c)	2,2, 2
Able to understand and analyze coloring of graphs, it's enumerative aspects and its applications.	L3,L3, L3,L3, L3,L3	5(a),5(b), 5(c),6(a), 6(b),6(c)	2,2, 2,2, 2,2
Able to understand and analyze planar graphs and its various applications.	L3,L3, L3,L3, L3,L3	7(a),7(b), 7(c),8(a), 8(b),8(c)	2,2, 2,2, 2,2
Able to understand the concepts of line graphs, edge-coloring and the various aspects of Hamiltonian cycles.	L3,L3, L3,L2, L3,L3	9(a),9(b), 9(c),10(a), 10(b),10(c)	2,2, 2,2, 2,2

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Define a k-regular graph and give an example of a 3-regular graph. 2

   (b) Draw all simple nonisomorphic graphs of order 3. 2

(c) Prove that every graph has an even number of vertices of odd degree. 2

2. (a) Define an Eulerian graph and give an example. 2

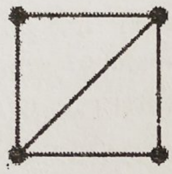
(b) Prove or disprove: If u and v are the only vertices of odd degree in a graph G, then G contains a u,v-path. 2

(c) Determine, whether the following is a graphic sequence. If, yes construct the graph. 2

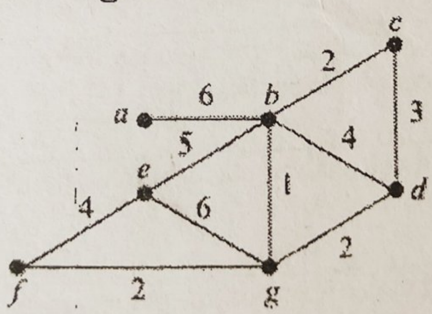
$$d: 5, 5, 4, 4, 2, 2, 1, 1$$

3. (a) Prove that if G is a simple graph, then $\operatorname{diam} G \geq 3 \Rightarrow \operatorname{diam} \overline{G} \leq 3$. 2

(b) Determine the number of spanning trees of the given graph by using Matrix Tree computation. 2

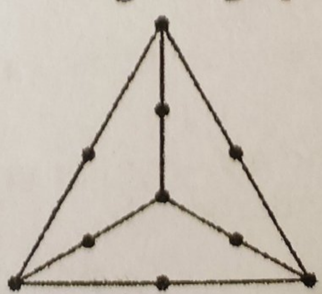


(c) Find the minimum spanning tree of the given graph by using Kruskal's algorithm. 2



4. (a) How many perfect matchings does the graph K_4 have? Draw all the perfect matchings of K_4. 2

(b) Determine whether the given graph has a 1-factor or not. 2



(c) Prove that if G is a bipartite graph with no isolated vertices, then $\alpha(G) = \beta'(G)$. 2

5. (a) Compute the chromatic number, the clique number and the independence number of the graph given below. 2

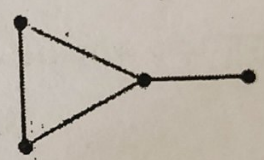
(b) Exhibit a graph G with a vertex v so that $\chi(G-v) < \chi(G)$ and $\chi(\overline{G}-v) < \chi(\overline{G})$. 2

(c) Prove or disprove: For every graph G, 2

$$\chi(G) \leq n(G) - \alpha(G) + 1.$$

6. (a) Draw $P_4 \vee K_3$ and compute $\chi(P_4 \vee K_3)$. 2

(b) Compute the chromatic polynomial of the given graph. 2



(c) Prove that if T is a tree with n vertices, then $\chi(T\,;k) = k(k-1)^{n-1}$. 2

7. (a) Show that $K_{3,3}$ is nonplanar. 2

(b) If $l(F_i)$ denotes the length of face F_i in a plane graph G, then show that $2e(G) = \sum l(F_i)$. 2

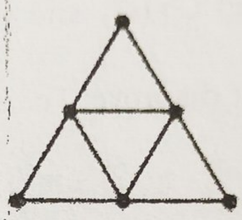
(c) Prove or disprove: Every subgraph of a nonplanar graph is nonplanar. 2

8. (a) Prove that if G is a plane graph and every face of G has even length, then the dual graph G^* of G is Eulerian. 2

(b) Prove that every simple planar graph has a vertex of degree atmost 5. 2

(c) If G is a simple planar graph with atleast 3 vertices then prove that $e(G) \le 3n(G) - 6$. 2

9. (a) For the given graph G, draw the line graph $L(G)$. 2



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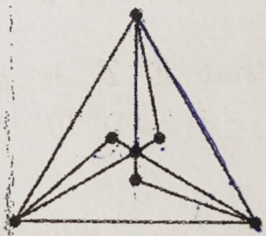
(b) Define edge-chromatic number $\chi'(G)$ of a graph G and determine $\chi'(K_4)$. 2

(c) Determine whether the given graph is Hamiltonian or not. 2



10 (a) Define a 1-factorable graph and give an example of a graph that is 1-factorable. 2

(b) Prove that if a graph G has a Hamiltonian cycle, then for each nonempty set $S \subseteq V(G)$, the graph $G - S$ has atmost $|S|$ components. 2

(c) For $n > 1$, prove that $K_{n,n}$ has $(n-1)!n!/2$ Hamiltonian cycles. 2