END-SEMESTER EXAMINATION, June-2024

CALCULUS B (MTH2101)

Programme: B.Tech(All Branches) **Semester: 2nd**

Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Use the knowledge of three dimensions and vectors to describe the region, lines, planes and surfaces.	L3	1a, b, c, 2a, 2b	2,2,2, 2,2
Compute the length of the curve, curvature, tangent normal vector, tangent plane.	L3	2c, 3a	2,2
Apply the concept of function of several variables to find the limit, derivative, directional derivative, linearization and maxima minima.	L3	3b, 3c, 4a, b, c, 5a, b,	2,2,2, 2,2,2, 2
Apply the concept of double and triple integration to evaluate the integral, to find the surface area.	L3	5c, 6a, b, c, 7a, b, 7c, 8a	2,2,2, 2,2,2, 2,2
Apply the concept of line integral to evaluate it, in conservative vector field and in Greens theorem.	L4	8b, c, 9a, b,	2,2,2, 2
Apply the concept of curl, Divergence, surface integrals and volume integrals	L4	9c, 10a, b, c	2,2,2, 2

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1.	(a)	If A (0, 4, 3), B (0,0,0) and C (3, 0, 4) are three points defined in x, y, z co-ordinate system, then find the vector perpendicular to both vectors $\left(\overrightarrow{AB}\right)$ and $\left(\overrightarrow{BC}\right)$	2
	(b)	Compute the area of parallelogram with vertices A (-2,1), B (0,4), C (4,2) and D (2, -1).	2
	(c)	Find the position vector of the particle that has given acceleration and the specified initial velocity and position: $a(t)=t\hat{i}+e^{t}\hat{j}+e^{-t}\hat{k}, v(0)=\hat{k}, r(0)=\hat{j}+\hat{k}$	2

2.	(a)	Find the parametric equation for the tangent line to the curve $x = t, y = e^{-t}, z = 2t - t^2$ at $(0,1,0)$.	2
	(b)	At what point on the curve $x = t^3, y = 3t, z = t^4$ is the normal plane parallel to the plane 6x + 6y – 8z = 1?	2
	(c)	If **a** = <3,0,1>, find a vector **b** such that Comp$_{\mathbf{a}}$**b** = 2.	2
3.	(a)	Find the curvature of $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at the point(1,1, 1).	2
	(b)	Calculate: $\lim\limits_{t \to 0} \left(e^{-3t} i + \dfrac{t^2}{\sin^2 t} j + \cos 2t k \right)$	2
	(c)	Find the domain of the vector function, r(t) = $\left\langle \sqrt{4-t^2}, e^{-3t}, \ln(t+1) \right\rangle$	2
4.	(a)	Explain if the function is differentiable and find the linearization L (x, y) of the function $f(x, y) = \sqrt{x + e^{4y}}$ at the point $(3,0)$.	2
	(b)	Use the Chain Rule to find $\partial z / \partial s$ and $\partial z / \partial t$ for $z = e^{x+2y}, x = s/t, y = t/s$.	2
	(c)	Find the directional derivative of the function $f(x,y) = e^x \sin y$ at $(0, \pi/3)$ in the direction of the vector $v = \langle -6, 8 \rangle$.	2
5.	(a)	Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = xy(1 - x - y)$.	2
	(b)	Find out points at which the direction of fastest change of the function $f(x,y,) = x^2 + y^2 + -2x - 4y$ is **i** + **j**.	2
	(c)	If R = [0,4] x [-1,2], Use a Riemann sum with m=2, n=3 to estimate the value of $\iint_R (1 - xy^2)\, dA$. Take the sample points to be the upper left corners of the rectangle.	2
6.	(a)	Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin y$ and the planes $x = \pm 1, y = 0, y = \pi, and\ z = 0$.	2

	(b)	Evaluate the double integral: $\iint_R (x + xy^{-2})\, dA$, where $R = \{(x, y) \mid 0 \le x \le 1, -3 \le y \le 3\}$	2
	(c)	Evaluate the integral by reversing the order of integration $\int_0^1 \int_x^1 e^{x/y}\, dy dx$	2
7.	(a)	Evaluate the iterated integral by converting to polar coordinate $\int_0^1 \int_y^{\sqrt{2-y^2}} (x + y)\, dx dy$	2
	(b)	Find the surface area of the part of the plane 3x + 2y+z =6 that lies in the first octant.	2
	(c)	Evaluate the iterated integral $\int_0^{\pi/2} \int_0^y \int_0^x \cos(x + y + z)\, dz\, dx\, dy$	2
8.	(a)	Find the Jacobian of the transformation $x = uv,\ y = u/v$.	2
	(b)	Evaluate the line integral $\int_C \vec{F} \cdot dr$, if $\vec{F}(x,y,z) = (x+y)\hat{i} + (y-z)\hat{j} + z^2\hat{k}$, where $r(t) = t^3\hat{i} - t^2\hat{j} + t\hat{k}$, $0 \le t \le 1$	2
	(c)	Determine whether or not $\vec{F}$ is a conservative vector field. If it is, find a function f such that $\vec{F} = \nabla f$. If $\vec{F}(x, y) = (2xy + y^{-2})\hat{i} + (x^2 - 2xy^{-3})\hat{j},\ y > 0$.	2
9.	(a)	Use Green's Theorem to evaluate the line integral $\int_C \cos y\, dx + x^2 \sin y\, dy$, C is the rectangle with vertices (0,0), (5,0), (5, 2), and (0,2).	2
	(b)	Find the curl and the divergence of the vector field $\vec{F}(x, y, z) = xye^2\hat{i} + yze^x\hat{k}$	2

	(c)	Find the equation of the tangent plane to the given parametric surface $x = u + v, y = 3u^2, z = u - v$ at the point (2,3,0).	2
10	(a)	Evaluate the surface integral $\iint_S y\,ds$, S is the helicoid with vector equation $r(u,v)=\langle u\cos v, u\sin v, v\rangle$, $0\le u\le 1, 0\le v\le \pi$.	2
	(b)	Use Stokes' Theorem to evaluate $\iint_S curl\ \vec{F}\cdot dS$. $\vec{F}(x,y,z)=\tan^{-1}(x^2yz^2)i+x^2yj+x^2z^2k$. S is the cone $x=\sqrt{y^2+z^2}, 0\le x\le 2$, oriented in the direction of the positive x-axis.	2
	(c)	Use the Divergence theorem to calculate the surface integral $\iint_S \vec{F}.ds$, i.e; calculate the flux of $\vec{\mathbf{F}}$ across **S**, if $\vec{\mathbf{F}}(x,y,z)=xye^z + xy^2z^3\,\mathbf{j} - ye^z\,\mathbf{k}$, S is the surface of the box bounded by the coordinate planes and x=1, y=2 and z=3.	2
		End of Questions	