MID-SEMESTER EXAMINATION, MAY-2023
CALCULUS - B (MTH-2101)

Programme: B. Tech
Full Marks: 30

Semester : 2nd
Time: 2 Hours

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Determine the distances, volume, equations of spheres, lines, and planes using vector operations in space.	L3	1. a, b, c, 2. a, b, c, 3. a	2,2,2 2,2,2 2
Compute the length of a curve, curvature, tangent, and normal vectors.	L3	3. b, c, 4. a, b, c	2,2 2,2,2
Determine limit, derivatives, directional derivatives, maxima/minima, and Jacobian of a function of two or several variables.	L3	5. a, b, c.	2,2,2

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1.	(a)	Determine the distance from $(3,7,-5)$ to the xy- plane.	2
	(b)	Determine the equation of the sphere with center $(2,-6,4)$ and radius 5 and also examine the plane obtained by the intersection of the sphere and zx -plane.	2
	(c)	If $A(0,4,3)$ $B(0,0,0)$, and $C(3,0,4)$ are three points defined in x, y, z coordinate system, then compute the vector perpendicular to both $\overline{AB}$ and $\overline{AC}$.	2

2.	(a)	Determine whether the two vectors $\langle 4,6 \rangle$ and $\langle -3,2 \rangle$ are orthogonal, parallel, or neither.	2
	(b)	Determine the parametric and symmetric equations for the line through the origin and the point (1,2,3).	2
	(c)	Calculate the volume of the parallelopiped formed by the vectors $\vec{a} = \langle 6,3,-1 \rangle, \vec{b} = \langle 0,1,2 \rangle$ and $\vec{c} = \langle 4,-2,5 \rangle$.	2
3.	(a)	Construct a vector equation and parametric equations for the line segment that joins $P(1,2,3)$, and $Q(1,-3,1)$.	2
	(b)	Determine the parametric equations for the tangent line to the curve $x = e^t, y = te^t, z = te^{t^2}$ at the point $(1,0,0)$.	2
	(c)	Compute the length of the curve $$\vec{r}(t) = \left\langle 12t, 8t^{\frac{3}{2}}, 3t^2 \right\rangle, 0 \le t \le 1.$$	2
4.	(a)	Determine the equation of the normal plane of the curve $x = 2\sin 3t, y = t, z = 2\cos 3t$ at the point $P(0,\pi,-2)$.	2
	(b)	Determine the velocity and position vectors of a particle that has the acceleration $\vec{a}(t) = \langle 1,2,0 \rangle$ and has the initial velocity and position vectors $\vec{v}(0) = \langle 0,0,1 \rangle$ and $\vec{r}(0) = \langle 1,0,0 \rangle$ respectively.	2

	(c)	A gun has muzzle speed $150m/s$. Compute the two angles of elevation that can be used to hit a target $800m$ away.	2
5.	(a)	Determine $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation $x^2 - y^2 + z^2 - 2z = 4$.	2
	(b)	Let $f(x,y) = y^x$. Determine $\dfrac{\partial^2 f}{\partial x \partial y}$ at $x = 2, y = 1$.	2
	(c)	If $u = \sqrt{r^2 + s^2}$, $r = y + x\cos t, s = x + y\sin t$; then determine the partial derivatives $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial u}{\partial t}$ when $x = 1, y = 2, t = 0$.	2
		End of Questions	

$2^x \ln 2$