(Please write your Exam Roll No.) *Exam Roll No. ....*

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] JULY 2023

Paper Code: ETMA-102	**Subject: Applied Mathematics-II**
Time: 3 Hours	**Maximum Marks: 75**

Note: Attempt five questions in all including Q.No1 which is compulsory. Select one question from each unit.

Q1).

a) If $u = e^{xyz}$, show that $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$. [4]

b) Form a partial differential equation of $z = (x^2 + a^2)(y^2 + b^2)$. [4]

c) Find Laplace transform of $f(t) = te^{-4t}\sin 3t$. [4]

d) Prove that the function sinhz is analytic and find its derivative. [4]

e) Check whether the vector field $F = 2xyz^3 i + x^2z^3 j + 3x^2yz^2 k$ is irrotational. [4]

f) Evaluate $\int_0^1 \int_{e^x}^{e} \frac{1}{\log y}\, dydx$, *by changing the order of integration.* [5]

UNIT - I

Q2.

a) If $x + y + z - u = 0,\ y + z - uv = 0,\ z - uvw = 0$, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$. [6]

b) Examine the function $f(x, y) = x^3 + y^3 - 12x - 3y + 20$ for extremum. [6.5]

Q3.

a) Find the maximum and minimum distances of the point $(1,2,-1)$ from the sphere $x^2 + y^2 + z^2 = 24$. [6.5]

b) If $V = r^m$, and $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m + 1)r^{m-2}$. [6]

UNIT-II

Q4.

a) Find Laplace transform of $\int_0^t \int_0^t \int_0^t (t\sin t)\,dtdtdt$. [6]

b) State convolution theorem and using it find inverse Laplace transform of $\frac{1}{s^2(s^2+a^2)}$ [6.5]

Q5.

a) Find Laplace transform of $f(t) = \frac{\cos 2t - \cos 3t}{t}$. [6]

b) Using Laplace transform solve: $y''' + 2y'' - y' - 2y = 0$, given $y(0) = 1$, $y'(0) = 2$, $y''(0) = 2$. [6.5]

UNIT - III

Q6.

a) Find the bilinear transformation which maps the points $z = 1, -i, -1$ into the points $w = i, 0, -i$ [6]

b) Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{1 - 2a\sin\theta + a^2}$, $0 < a < 1$. [6.5]

P.T.O.

Q7.

a) Find an analytic function $f(z) = u + iv$, if $u - v = (x - y)(x^2 + y^2 + 4xy)$. [6.5]

b) Evaluate the integral $\oint \frac{3z^2+z}{z^2-1} dz$, along the boundary of the circle $|z - 1| = 1$. [6]

UNIT-IV

Q8.

a) Evaluate $\iint xy(x + y)dxdy$, over the region bounded by the curve $y = x^2$ and the line $y = x$. [6]

b) Verify Stoke's theorem for the vector function $F = xyi + yzj + z^2k$, over the cube with vertices (0, 0, 0), (a, 0, 0), (0, a, 0), (0, 0, a), (a, a, 0), (0, a, a), (a, 0, a), (a, a, a), if the face of the cube in XOY plane is missing. [6.5]

Q9.

a) If the vector $A = 2xzi - xj + y^2k$, evaluate $\iiint Adv$, over the volume V bounded by the surfaces $x = 0, y = 0, x = 2, y = 6, z = x^2, z = 4$. [6.5]

b) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q has coordinates (5, 0, 4). [6]

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