END-SEMESTER EXAMINATION, JULY-2022
CALCULUS-II (MTH 2001)

Programme: B.Tech **Semester: 2nd**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Use the knowledge of three dimensions and vectors to describe the region, lines, planes and surfaces.	L1,L1,L1 L1,L1,L3	1(a),(b),(c) 2(a),(b), (c)	2 × 6
Compute the length of the curve, curvature, tangent normal vector, tangent plane.	L1,L1,L1 L1,L1,L1	3(a),(b), (c) 4(a) ,(b),(c)	2 × 6
Apply the concept of function of several variables to find the limit, derivative, directional derivative, linearization and maxima minima.	L1,L1,L3 L1,L1,L3 L1,L3	5(a),(b), (c) 6(a),b), (c) 7(a),(b)	2 × 8
Apply the concept of double and triple integration to evaluate the integral, to find moment of inertia of lamina and surface area.	L3 L1,L5,L5 L1	7(c) 8(a,(b), (c) 9(a)	2 × 5
Apply the concept of line integral to evaluate it, in conservative vector field and in Green's theorem.	L5,L5 L1	9(b),(c) 10(a)	2 ×3
Apply the concept of curl, divergence in Stokes theorem and the divergence theorem.	L1,L3	10(b),(c)	2 x 2

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Find the volume of the parallelepiped with adjacent edges PQ, PR, PS ,where $P(3,0,1), Q(-1,2,5), R(5,1-1), S(0,4,2)$. 2

 (b) Find the center and radius of sphere $x^2+y^2+z^2-2x-4y+8z=15$. 2

 (c) Find the angle between the vector $\langle 4,0,2\rangle$ and $\langle 2,-1,0\rangle$. 2

2. (a) Find the equation of plane passing through the point (6, 3, 2) and 2

perpendicular to the vector $\langle -2,1,5\rangle$.

(b) Find whether the planes $x+y+z=1, x-y+z=1$ are parallel, perpendicular or neither. If neither, then find the angle between them. 2

(c) Find 2

$$\lim_{t\to 1}\left\langle \frac{t^2-t}{t-1}, \sqrt{t+8}, \frac{\sin \pi t}{\ln t}\right\rangle$$

3. (a) Find a vector function that represents the curve of intersection of the two surfaces, the cone $z=\sqrt{x^2+y^2}$ and the plane $z=1+y$. 2

(b) Find the curvature of the curve $\vec{r}(t)=\left\langle \sqrt{2}\,t, e^t, e^{-t}\right\rangle$. 2

(c) A gun is fired with angle of elevation 30^0. What is the muzzle speed if the maximum height of the shell is 500m? 2

4. (a) Find the equation of the tangent line of the curve $x=1+2\sqrt{t}, y=t^3-t, z=t^3+t$ at the point $(3,0,2)$. 2

(b) Find the tangent and normal vector of the vector $\vec{r}(t)=\langle \cos t, \sin t, \ln \cos t\rangle$ at $(1,0,0)$. 2

(c) Find the tangential and normal components of acceleration vector for $\vec{r}(t)=\langle t, \cos^2 t, \sin^2 t\rangle$. 2

5. (a) Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2}$ if it exists, or show that the limit does not exist. 2

(b) Find and sketch the domain of the function $f(x,y)=\ln(9-x^2-9y^2)$. 2

(c) Find $\dfrac{\partial^3 u}{\partial r^2 \partial\theta}$ of the function $u=e^{r\theta}\sin\theta$ 2

6. (a) Determine whether $u = x^2 - y^2$ is a solution of Laplace's equation $u_{xx} + u_{yy} = 0$ 2

(b) Find the directional derivative of the function $f(x,y) = ye^{-x}$ in the direction of $\theta = \frac{2\pi}{3}$ at the point $(0,4)$. 2

(c) Use the chain rule to find $\frac{\partial z}{\partial s}$ if $z = e^r \cos\theta, r = st$ and $\theta = \sqrt{s^2 + t^2}$. 2

7. (a) Find the maximum rate of change of $f = (x+y)/z \ at (1,1,-1)$ and the direction in which it occurs. 2

(b) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y) = x^2 + y^2$ subject to the given constraint $xy = 1$. 2

(c) Find the local maximum and minimum values and saddle points of the function $f(x,y) = \sin x, \sin y, -\pi < x < \pi, -\pi < y < \pi$. 2

8. (a) Evaluate the double integral 2

$\iint x^3 dA,\ D = \{(x,y)|\ 1 \le x \le e, 0 \le y \le \ln x\}$

(b) Use polar coordinates to find the volume of the solid that lies below the paraboloid $z = 18 - 2x^2 - 2y^2$ and above the xy – plane. 2

(c) Evaluate the iterated integral $\int_0^1 \int_0^1 \int_0^{\sqrt{1-z^2}} \frac{z}{y+1} dxdzdy$. 2

9. (a) Find the volume of the tetrahedron enclosed by the coordinate planes and the plane 2x+y+z = 4. 2

$\ln\frac{(1+x)}{x} = 1$

$\frac{\pi \cos \pi t}{\frac{1}{t}}$

-1

(b) Find spherical coordinates of the point with rectangular coordinates $\left(\sqrt{3},-1,2\sqrt{3}\right)$. 2

(c) Find the area of the surface, that is a part of plane $3x + 2y + z = 6$ which lies in the first octant. 2

10. (a) Assuming the appropriate partial derivatives exist and are continuous, show that 2

$$div(\vec{F}\times\vec{G})=\vec{G}.curl\vec{F}-\vec{F}.curl\vec{G}$$

(b) Use Green's theorem to evaluate the line integral 2

$\int_C y^3dx - x^3dy$ along the positively oriented curve C, a circle $x^2 + y^2 = 4$.

(c) Use the Divergence theorem to calculate the flux of 2

$\vec{F}=\left\langle x^3+y^3, y^3+z^3, z^3+x^3\right\rangle$, across the surface S, a sphere with center the origin and radius 2.

End of Questions