END-SEMESTER EXAMINATION, June-2024
INTRODUCTORY GRAPH THEORY (CSE 1004)

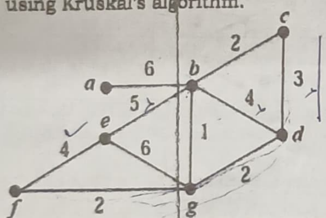
Programme: B.Tech(CSE & CSIT) **Semester: 2nd**
Full Marks: 60 **Time: 3 Hours**

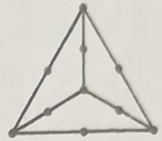
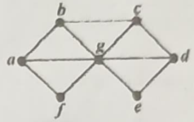
Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Able to understand the fundamental concepts of graphs and apply them to study graph isomorphisms, Eulerian graphs, graphic sequences and digraphs.	L2, L3, L3, L3, L3, L3	1(a),1(b), 1(c),2(a), 2(b),2(c)	2,2, 2,2, 2,2
Able to understand the concepts of trees, spanning trees and study its various concepts and apply the Kruskal's algorithm to find the minimum spanning tree and Dijkstra's algorithm to find the shortest path of connected weighted graphs.	L3, L3, L3, L3, L3, L3	3(a),3(b), 3(c),4(a), 4(b), 4(c)	2,2, 2,2, 2,2
Able to understand matchings and factorization of graphs and its various applications.	L3, L3, L3	5(a),5(b), 5(c)	2,2, 2
Able to understand and analyze coloring of graphs, it's enumerative aspects and its applications.	L3, L3, L3, L3, L3, L3	6(a),6(b), 6(c),7(a), 7(b),7(c)	2,2, 2,2, 2,2
Able to understand and analyze planar graphs and its various applications.	L3, L3, L3, L3, L3, L3	8(a),8(b), 8(c),9(a), 9(b),9(c)	2,2, 2,2, 2,2
Able to understand the concepts of line graphs, edge-coloring and the various aspects of Hamiltonian cycles.	L3, L3, L4	10(a),10(b), 10(c)	2,2, 2

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1.	(a)	Define decomposition of graphs and give an example of a graph which decomposes into copies of P_3.	2
	(b)	Draw all nonisomorphic graphs of order 3.	2
	(c)	Show that a k-regular graph with n vertices has $nk/2$ edges.	2
2.	(a)	Show that if $k>0$, then a k-regular bipartite graph has the same number of vertices in each partite set.	2

	(b)	Prove or disprove: The complement of a simple disconnected graph must be connected.	2
	(c)	Prove that in a digraph G $$\sum_{v\in V(G)} d^{+}(v)=e(G)=\sum_{v\in V(G)} d^{-}(v)$$	2
3.	(a)	Prove that every edge of a tree is a cut-edge.	2
	(b)	Let G be a simple graph with diameter at least 4. Prove that $\overline{G}$ has diameter at most 2.	2
	(c)	Prove that if G is an n-vertex connected graph having no cycles, then for every $u, v \in V(G)$, G has exactly one $u-v$ path.	2
4.	(a)	Determine the number of spanning trees of the given graph by the matrix tree computation method. 	2
	(b)	Find the minimum spanning tree of the given weighted graph by using Kruskal's algorithm. 	2
	(c)	Determine the number of spanning trees in a complete graph of 5 vertices.	2
5.	(a)	Let T be a tree with n vertices, and let k be the maximum size of an independent set in T. Determine $\alpha'(T)$ in terms of n and k.	2
	(b)	Draw the simple graphs for which the values of $\alpha(G)=1$ and $\alpha'(G)=1$.	2

	(c)	Determine whether the given graph has a 1-factor or not.  NO	2
6.	(a)	Define chromatic number of a graph and find the chromatic number of the given graph. 4 	2
	(b)	Prove or disprove: If G is a connected graph, then $\chi(G)\le 1+a(G)$, where $a(G)$ is the average of the vertex degrees in G.	2
	(c)	Draw $C_3 \vee P_3$ and compute $\chi(C_3 \vee P_3)$. C_3 is a cycle w/ 3 vertices. P_3 is path w/ 3 vertices. The join $G \vee H$ means connecting each vertex of G to H.	
7.	(a)	Prove that if T is a tree with n vertices, then $\chi(T;k)=k(k-1)^{n-1}$.	2
	(b)	Compute the chromatic polynomial of the given graph. 	2
	(c)	Prove that $k^4-4k^3+3k^2$ is not a chromatic polynomial.	2
8.	(a)	If G is a simple planar graph with atleast 3 vertices then prove that $e(G)\le 3n(G)-6$.	2
	(b)	Prove that every simple planar graph has a vertex of degree atmost 5.	2
	(c)	Construct a 3-regular planar graph of diameter 3 with 12 vertices.	2
9.	(a)	Show that $K_{3,3}$ is nonplanar.	2

	(b)	Prove that if G is a plane graph and every face of G has even length, then the dual graph G^* of G is Eulerian.	2
	(c)	Let G be a connected planar graph with 10 vertices. If the number of edges on each face of G is three, then determine the number of edges in G.	2
10	(a)	Draw $C_3 \square K_2$ and determine $\chi'(C_3 \square K_2)$.	2
	(b)	Determine, whether $\overline{P_5}$ is a line graph. If so, find H such that $L(H) = \overline{P_5}$.	2
	(c)	For $n > 1$, prove that $K_{n,n}$ has $(n-1)!n!/2$ Hamiltonian cycles and illustrate it for $K_{2,2}$.	2
		End of Questions	