END-SEMESTER EXAMINATION, May-2024
ALGORITHM DESIGN-2 (CSE4131)

Programme: B.Tech (CSE/CSIT/CDS/CIOT/CAIML/CCS)

Semester: 4th **Full Marks: 60** **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
CO1: understand the network flow problem and apply it to real-world problems.	L2, L4, L5	1(a),1(b),1(c)	2+2+2
CO2: distinguish between computationally tractable and intractable problems. define and relate class P, class NP and class, NPcomplete, PSPACE, PSPACE complete. given a problem in NP, define an appropriate certificate and the verification algorithm.	L1, L4, L5	2(a),2(b),2(c) 3(a),3(b),3(c) 4(a),4(b),4(c)	2+2+2 2+2+2 2+2+2
CO3: understand approximation algorithms and apply this concept to solve problems.	L3, L4, L5	5(a),5(b),5(c) 6(a),6(b),6(c)	2+2+2 2+2+2
CO4: understand local search techniques and apply this concept to solve problems.	L3, L4, L5	7(a),7(b),7(c)	2+2+2
CO5: understand randomization and apply this concept to solve problems.	L3, L4, L5	8(a),8(b),8(c) 9(a),9(b),9(c)	2+2+2 2+2+2
CO6: identify and apply an appropriate algorithmic approach to solve a problem and explain the challenges to solve it.	L5, L6	10(a),10(b),10(c)	2+2+2

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

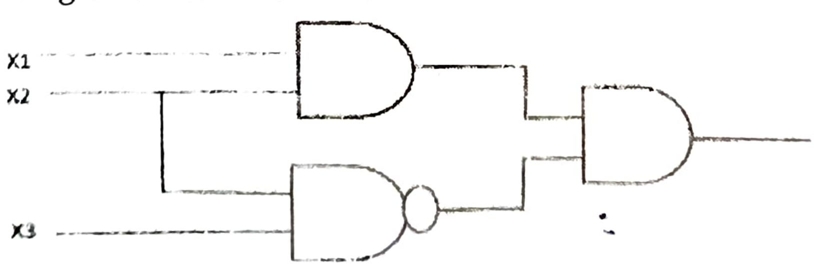
1. (a) Consider the given network flow graph $G(V, E)$, where V = {A, B, C, D, E, F, G} and capacities of the edges in E are c(A, B) = 8, c(B, C) = 10, c(C, G) = 16, c(A, D) = 11, c(D, E) = 9, c(E, G) = 12, c(B, F) = 7, c(F, C) = 13, c(D, F) = 8, c(F, E) = 9. {Note: c(u, v)=w means direction of edge is u->v with weight w}. Draw the flow graph and Identify the Source and Sink nodes with proper justification. 2

   (b) For the Flow Network in [Q(1a)], estimate the maximum flow with less number of iterations with proper justification for each of the iterations. 2

   (c) What can be the maximum number of edges in a residual graph at any arbitrary iteration for a given flow graph during the estimation of maximum flow? Justify your answer with a suitable example. 2

2. (a) Give a suitable relation between the problem classes P, NP, NPH, NPC and PSPACE? 2

   (b) Convert the given circuit to CNF. 2



(c) Define the problem statement for a Decision Set Cover problem and show that it is in NPC. 2

3. (a) Convert the given 3-SAT problem instance to an instance of an Independent Set and find the maximal Independent Set. 2
The given 3-SAT is $\varphi(x)= (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$.

(b) Suppose, the 3-SAT problem has k clauses, then it is satisfiable if and only if the corresponding graph has an independent set of size k. Now, give an example to show that 3-SAT is not satisfiable if and only if the corresponding graph does not have any independent set of size k. 2

(c) What do you mean by a Certificate of a problem? What is the significance of the polynomial time certifier? 2

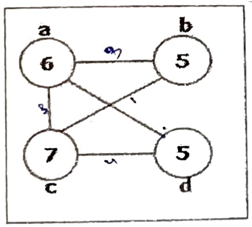
4. (a) Define PSPACE. Why Q-SAT ∈ PSPACE? 2

(b) Suppose $Y \le_P X$. If Y cannot be solved in polynomial time, then X cannot be solved in polynomial time. True or False? Justify your answer. 2

(c) Given a Complete undirected Graph G(V,E) with the total number of vertices are 8 i.e. |V|=8. Then there cannot be a vertex cover of size 3 True or False? Justify your answer. 2

5. Given an approximation algorithm for weighted-vertex-cover problem as below: 2

```
Vertex-Cover-Approx(G,w){
 Set  p_e = 0 for all e ∈ E
 While (there is an edge (i, j) such
          that neither i nor j is tight){
    Select such an edge e
    Increase p_e as much as possible
                      until i or j tight}
    S ← set of all tight nodes
    return S
    }
```



(a) Using the above algorithm based on pricing method, find the weighted-vertex-cover for the given graph with four vertices having vertex weights/costs 6, 5, 7 and 5.

(b) Compare your solution with the optimal solution for this example. 2

(c) Show that "the set S returned by the above algorithm is a vertex cover, and its cost is at most twice the minimum cost of any vertex cover." 2

6. Given an approximation algorithm for weighted-set-cover problem as below:

```
Greedy-Set-Cover() {
        Start with R = U and no sets selected
        while (R ≠ Φ ){
              Select set Si that minimizes w_i/|S_i ∩ R|
                 S = S ∪ {S_i }
              Delete set S_i from R.     // R = R − {S_i}
        }
        Return the selected sets S
    }
```


Given Set-Cover instance with U={1,2,3,4,5,6,7,8}, and list of subsets such as S1={1,3,5,7}, S2={2,4,6,8}, S3={1}, S4={2}, S5={3.4}, S6={5,6,7,8} and weight array w[]= {1+ε, 1+ε, 1,1,1,1}, where ε is a very small value between 0 and 1.

(a) Using the above algorithm, find the solution S for the given set-cover problem instance. 2

(b) Compare your obtained solution with the optimal solution S*. 2

(c) Give a suitable match between the set A and B where A={ NP-Complete, P, PSPACE, NP-hard } and B={Vertex Cover Optimization Problem, Planning Problem, Flow Network, TSP Decision Problem} 2

7. (a) Given an undirected weighted graph defines the configuration S, consists of a set of nodes V={a, b, c, d, e, f}, set of edges E={(a, b), (a, e), (a, f), (b, f), (b, c), (c, d), (e, d), (f, d)}. The associated edge weight set W={-1, 4, 6, -5, 1, -2, -3, 9}. Consider the node assignment is $S_e = S_f = S_c = -1$ and for all other nodes it is +1. Draw the configuration S and find out all the good edges and bad edges of the given configuration. 2

(b) Make the configuration S of question [Q(7a)] into stable configuration using state-flipping algorithm. 2

(c) Apply gradient descent local search algorithm to find the vertex cover from the given complete bipartite graph G=(V, E), where V=L∪R, L={a, b, c, d, e}, and R={m, n, o, p, q, r}. Can gradient descent always guarantee finding the global optimum from a solution space? If not, why? 2

8.

```
Select(S, k)
{Choose a splitter a_i ∈ S.
for each element a_j of S {
      if aj < ai then put aj in S−
      if aj > ai then put aj in S+
}
if |S−| = k − 1 then
     return ai                 // the splitter ai was in fact the desired answer
else if |S−| ≥ k then
       Select(S-,k)         // the kth largest element lies in S−
     else                   // suppose |S−| = t < k −1
       Select(S+, k-t-1) // the kth largest element lies in S+
```

(a) Given a set of elements {7, 4, 2, 10, 9, 3, 6, 13, 5, 11, 8, 1, 12}. Find the median of the array using the above algorithm. Show each steps of the computation, considering the leftmost element as the splitter. 2

(b) If the splitter is an "off center" then what may be the recurrence relation of the above algorithm. Analyze the run time. 2

(c) When we choose an element as a splitter of the set under consideration that at least quarter of the elements are smaller than it and at least a quarter the elements are larger than it. Then with this process give an upper bound of the run time of the algorithm. 2

9 (a) Demonstrate what happens when we insert the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots and the hash function is $h(x) = x \bmod 9$.

(b) Let U be a universal set. Each element $x \in U$ is type of r-vector i.e $x = (x_1, x_2, \dots, x_r)$, where $0 \le x_i < p$. Let A= {set of all r-digits, p-base integers}. For $a = (a_1, a_2, \dots, a_r) \in A$, where $0 \le a_i < p$, define a hash function $h_a(x): U \to \{0, 1, \dots, p-1\}$ such that $h_a(x) = (\sum_{i=1}^{r} a_i x_i) \bmod p$. Then, a class **H** of hash functions is called universal if $Pr_{h \in \mathbf{H}}[h(x) = h(y)] \le 1/p$. 2

Using the above hash function [Q(9b)], compute the hash value of the elements $x = (2,3)$, $y = (4,5)$, $z = (2,1)$ with p=7. Where $A = \{a, b\}$ $r = 2$ and $a = (3,5)$, $b = (1,3)$. 2

(c) Find whether the $\mathbf{H} = \{h_1, h_2\}$ in [Q(9b)] is a universal family of hash functions or not ? 2

10 In a reserve forest, the forest department wants to keep an eye on animals and their movements. The department has a map of roads on which animal movements are observed. The department wants to deploy video cameras to capture live videos to be streamed. The department is concerned about deploying video cameras so efficiently that it can cover all the roads with the number of video cameras as little as possible.

(a) Formulate the problem mathematically with an appropriate data structure. 2

(b) What is your view on designing an algorithm to solve the problem? 2

(c) Which algorithm design paradigm did you prefer and What are the challenges of the algorithm? 2

End of Questions