(Please write your Exam Roll No.) *Exam Roll No. ....*

END TERM EXAMINATION

FIRST SEMESTER [B.TECH] JANUARY 2024

Paper Code: ETMA-101	***Subject: Applied Mathematics-I***
Time: 3 Hours	***Maximum Marks: 60***

Note: Attempt five questions in all including Q. No.1 which is compulsory. Select one question from each unit. Assume missing data, if any.

Q1 Attempt all questions:- **(1.2x10=12)**

(a) Expand $log_e(1+x)$ in power of x by Maclaurin's theorem.

(b) Find the percentage error in the area of an ellipse when an error of +1 percent is made in measuring the major and minor axis.

(c) Find the asymptotes of the curve $x^2 y^2 - x^2y - xy^2 + x + y + 1 = 0$.

(d) Find the radius of curvature of the curve $\frac{1}{x^2}+\frac{1}{y^2}=1\ at\ (\frac{1}{4},\frac{1}{4})$.

(e) Define Hermitian matrix with example.

(f) Examine the system of vectors are linearly dependent or linearly independent x_1= (1,2,3) , x_2 = (2, −2 , 6).

(g) Solve $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$.

(h) Find $\int_{-1}^{1} x^3\, p_4(x)\, dx$.

(i) Write Rodrigue's Formula.

(j) Find (2.5,1.5) .

UNIT-I

Q2 (a) If y = $e^{m\cos^{-1}x}$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ And hence calculate $y_n(0)$. **(8)**

(b) Obtain the first four term in the expansion of $log \sin x$ in power of $(x-3)$ By Taylor theorem. **(4)**

Q3 (a) Find whether the series $\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\frac{x^4}{7.8}+\cdots$ x> 0. Is convergent or divergent. **(6)**

(b) Prove that the series $\frac{2}{1}+\frac{3}{4}+\frac{4}{9}+\frac{5}{16}+\cdots$ is divergent. **(6)**

UNIT-II

Q4 (a) Evaluate $\int_0^\infty \sqrt{x}\, e^{-x^{\frac{1}{3}}}dx$. **(6)**

(b) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum. **(6)**

Q5 (a) Trace the curve $r = a\sin 2\theta$. **(8)**

(b) Evaluate $\int_0^{\frac{\pi}{2}} Cos^9\,\theta\, d\theta$. **(4)**

UNIT-III

Q6 (a) Find the inverse of matrix A by Gauss Jordon method if **(6)**

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

P.T.O.

(b) Find the eigen values and eigen vector of matrix **(6)**

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Q7 (a) Verify Cayley Hamilton theorem of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find The value $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + 1$. **(8)**

(b) For which value of "a" the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ a & 13 & 10 \end{bmatrix}$ is 2 **(4)**

UNIT-IV

Q8 (a) Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \left[\left(\frac{3-x^2}{x^2}\right) \sin x - \frac{3\cos x}{x} \right]$. **(6)**

(b) Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$. **(6)**

Q9 (a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \cos x$. **(6)**

(b) Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$. **(6)**

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