END SEMESTER EXAM, JANUARY -2024
PROBABILITY AND STATISTICS (MTH 2002)

Program: B.Tech. **Semester: 3rd**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Apply probability axioms to compute probability and conditional probability.	L3,L3,L4,L3	1(a,b), 2(a)	2*3
Define random variables and compute probability distributions, joint & marginal distribution.	L4,L4,L1,L5, L5	2(b,c)3(a, b,c)	2*5
Compute expectation of random variables and their functions and compute moments and moment generating functions of a random variable.	L4,L1	2 (c) 4(a),6(a)	2*3
Discuss discrete probability distribution viz: Binomial, Poisson & Hypergeometric and continuous probability distribution distributions viz: Uniform, Normal Gamma & Exponential.	L4,L5,L3, L4,L5,L3,L4, L1	4(b,c), 5(a), 6(b,c),7(b)	2*6
Estimate the population mean and variance of a normal distribution by point and interval estimation	L3, L5	5(b,c) 7(a,c)	2*4
Infer about population parameter through hypothesis testing with the help of a random sample. Analyze linear regression and co-relation	L2,L4,L4, L3,L4,L4 L3,L5,L5	8(a,b,c), 9(a,b),10 (a) 9(c),10(b ,c)	2*9

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Calculate the 10% trimmed mean of the following data: [2]

   8,2,2,12,5,7,15,14,15,3,11.

   (b) Interest centers around the nature of an oven purchased at a particular department store. It can be either a gas or an electric oven. Consider the decisions made by six distinct customers. Suppose that the probability is 0.40 that at most two of these individuals purchase an electric oven. Compute the probability that at least three purchase the electric oven. [2]

(c) For any two events A and B, prove that [2

$$P(A' \cap B') = 1 + P(A \cap B) - P(A) - P(B).$$

2. (a) In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability that incorrectly diagnosing a person without cancer as having the disease is 0.06, evaluate the probability that a person diagnosed as having cancer actually has the disease. [2

(b) Suppose that the random variable X having probability density function $f(x) = \begin{cases} kx^2, & -1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$. Then evaluate k. [2]

(c) With reference to 2(b), evaluate the cumulative distribution function of the random variable X. [2]

3. (a) Suppose that the random variable X has probability density function $f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$. Calculate the variance of X. [2]

(b) Let X and Y be two independent random variables where E(X)=4, E(Y)=5 and $\sigma_X^{\ 2} = 2, \sigma_Y^{\ 2} = 3$. Evaluate the mean and variance of the Random variable $Z = 3X - 2Y$. [2]

(c) Suppose that the joint density function of random variables X and Y is $f(x,y) = \begin{cases} e^{-(x+y)}, & x > 0,\ y > 0 \\ 0, & \text{elsewhere} \end{cases}$. [2]

Show that the random variables X and Y are independent.

4. (a) Suppose that X and Y have the following joint probability function [2]

f(x,y)		x = 0	x = 1
y	0	0.10	0.15
	1	0.20	0.35
	2	0.10	0.10

Calculate μ_x and μ_y.

(b) With reference to 4(a), evaluate $P(X=1|Y \le 1)$. [2]

(c) A random variable X has a mean $\mu = 20$ and variance $\sigma^2 = 9$. Using Chebyshev's theorem, find $P(11 < X < 29)$. [2]

(a) In a certain industrial facility, accident occurs infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. Calculate the probability that in any given period of 400 days there are at most 3 days with an accident. [2]

(b) In an NBA championship series, the team that wins three games out of five is the winner. Suppose that teams A and B face each other in the championship games and that team A has probability 0.6 of winning a game over team B. Calculate the probability that team A would win the series. [2]

(c) Suppose X follows a continuous uniform distribution from 0 to 5. Determine the conditional probability $P(X > 2.5|X \le 4)$. [2]

6. (a) A pair of fair die rolled 180 times, using the normal approximation to binomial distribution, calculate the probability that a total of 7 occurs at least 25 times. [2]

(b) Let X have probability distribution $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$. [2]

Compute the probability distribution of $Y=2X-1$.

(c) The moment generating function of certain random variable X is given by $M_X(t)=e^{2(e^t-1)}$. Evaluate the mean and variance of the random variable X. [2]

7. (a) The probabilities are 0.4, 0.2, 0.3 and 0.1 respectively, that a delegate to a certain convention arrived by air, bus, automobile, or train. Compute the probability that among 9 delegates randomly selected at this convention, 3 arrived by air, 3 arrived by bus, 1 arrived by automobile, and 2 arrived by train? [2]

(b) If the random variable X, has a Gamma distribution with $\alpha = 2$ and $\beta = 1$. Evaluate $P(1.8 < X < 2.4)$. [2]

(c) Compute the maximum likelihood estimator for 'p' of binomial distribution from the sample of observations $x_1, x_2, \ldots, x_n$. [2]

8. (a) Given that X has a normal distribution with mean $\mu = 300$ and $\sigma = 50$. Calculate the probability that X assumes a value greater than 362. [2]

(b) Suppose that a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected and removed from the box in succession without replacing the first, calculate the probability that both fuses are defective. [2]

(c) A lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of trip times to be normally distributed. Calculate the probability that a trip will take at least 30 minutes. [2]

9. (a) The length of life of light bulbs that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, estimate a 96% confidence interval for the population mean of all bulbs. [2]

(b) The heights of a random sample of 25 college students showed a mean of 174.5 cm. and a standard deviation of 6.9 cm. Construct a 95% confidence interval for the mean height of all college students. [2]

(c) A random sample of 20 students yielded a mean 72 and a variance 16 for scores on a college placement test in mathematics. Assuming the score to be normally distributed, construct a 98% confidence interval for the variance σ^2. [2]

10. (a) The grades of a class of 5 students on a midterm report (x) and on the final examination (y) are as follows: [2]

x	77	50	71	72	81
y	82	66	78	34	47

Calculate the correlation coefficient.

(b) Estimate the regression line from 10(a). [2]

(c) Estimate the final examination grade of a student who received a grade of 75 on the midterm report. [2]

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