(Please write your Exam Roll No.) *Exam Roll No. .....*

END TERM EXAMINATION

THIRD SEMESTER [B.TECH] DECEMBER-2024

Paper Code: ES-201 ***Subject: Computational Methods***

Time: 3 Hours ***Maximum Marks: 60***

Note: Attempt five questions in all including Q. no.1 which is compulsory. Select one question from each unit. Scientific calculator is allowed.

Q1 (a) Determine the decimal number that correspond to the machine word $[45DE4000]_{16}$ **(2)**

(b) Using secant method find the root of $x\sin(x) - 3\cos(x) = 0$ between (0.0, 1.8) with accuracy of 2 digits after decimal point. **(3)**

(c) For following given set of data for x & y, formulate the Newton divided differences table. **(3)**

x	0.00	2.00	4.00	6.0	8.0
y	0.00	8.00	64.00	216.00	512.00

(d) Compute $\int_0^1 \frac{\sin(x)}{x} dx$ by using the composite Trapezoid rule with six uniform points. Assign the value $\left(\frac{\sin x}{x}\right) = 1$ *at* $x = 0$. **(3)**

(e) Define Decomposition of any matrix. Explain the required condition for Doolittle and Crout decomposition method. **(2)**

(f) Find the value of λ for which the system of equations $x+y+4z=1$, $x+2y-2z=1$ & $\lambda x+y+z=1$, will have unique solution. **(3)**

(g) Using Euler's method find the value of y at x=0.10 of the ODE $\frac{dy}{dx} = x + y + xy$ Where initial conditions are $x_0 = 0.00$ & $y_0 = 1.00$ and using step size 0.10. **(3)**

(h) Give an example and conditions of Parabolic, Hyperbolic and Elliptic partial differential equations. **(1)**

UNIT – I

Q2 (a) Find the value of function $f(x) = 1 - \cos(x)$ *at* $x = 0.1$. Modify the expression so that loss of significant digits can be avoided and calculate the value again. Compare two values with the true value 0.4996×10^{-2}. **(5)**

(b) Define rate of convergence and stability of iterative method. Prove that the rate of convergence of Newton-Raphson method is 2. **(5)**

Q3 (a) Define Multivariate unconstraint minimization problem with an example. Using Newton Method to minimize multivariate function, minimize $f(x^k) = 4x_1^2 + x_2^2 - 2x_1x_2$ *starting at* $x_0^k = [1,1]^T$, *where* $x^k = [x_1, x_2]^T$ **(5)**

(b) Determine the minimum point of the function $f(x) = x^2 - 7x + 12$ by Fibonacci search method, if the first uncertainty interval is [2,4] . **(5)**

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UNIT-II

Q4 (a) Explain error estimation in Newton-Gregory Forward interpolation. Also prove that the maximum error in Newton's Forward interpolation is 1 when $|x-x_0| < h$, where h is step size of the given data. **(5)**

(b) For the given set of data for X and Y construct the table of forward differences. Find the interpolated value of Y at X=4.60 using Newton forward interpolation formula. Also find the estimated error in interpolated value. **(5)**

X	1.0	2.0	3.0	4.0	5.0	6.0
Y	7.0	13.0	21.0	32.0	48.0	70.0

Q5 (a) Find $I = \int_0^1 x dx$, by Gaussian Quadrature formula for n=4. Where the values of 'abscissae and Weights' corresponding to n=4 are given.
Abscissae = ($\pm$0.33998 & $\pm$ 0.86114)*and corresponding Weights* = (0.65214 & 0.34785) **(4)**

(b) Evaluate the definite integral $\int_0^1 \frac{1}{1+x} dx$correct to three decimal point using the basic trapezoidal rule with h = 0.5, 0.25 and 0.125 then obtain a better estimate using Romberg's method. Compare the results with the true value. **(6)**

UNIT-III

Q6 (a) Test the consistency of the system of following given equations:

$$\begin{cases} 5x_1 + 3x_2 + 7x_3 = 4 \\ 3x_1 + 26x_2 - 2x_3 = 9 \\ 7x_1 + 2x_2 + 10x_3 = 3 \end{cases}$$

If the system is consistent, is the system have finite solution or infinite solution? **(4)**

(b) Explain the concept of partial pivoting and hence solve the following given system of linear equation by Guass-Jordan method. The system of linear. **(6)**

$$\text{equations is: } \begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 4x_1 + 3x_2 - x_3 &= 6 \\ 3x_1 + 5x_2 + 3x_3 &= 4 \end{aligned}$$

Q7 (a) Obtain the linear Spline for the function f(x) define by the data given below

x	1	2	4	8
f(x)	3	7	21	73

Evaluate the function at x=3. **(4)**

(b) Solve the following given system of linear equations using Cholesky factorization method: **(6)**

$$\begin{aligned} 25x + 15y - 5z &= 35 \\ 15x + 18y &= 33 \\ -5x + 11z &= 6 \end{aligned}$$

P.T.O.

UNIT-IV

Q8 (a) Using Picard method, compute y(0.2) to three decimal precision from $\frac{dy}{dx} = x + y$ *given that* $y(0) = 1$. Compare the result with the exact result for the value of y at 0.2. **(4)**

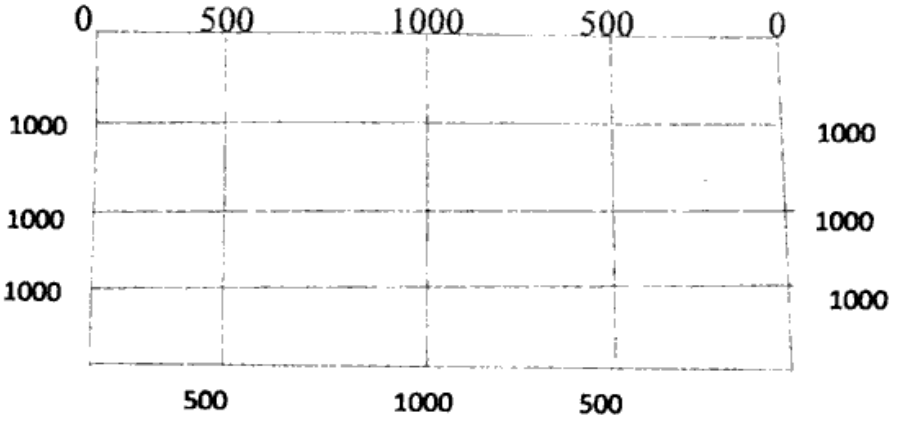
(b) Consider the initial value problem $\begin{cases} \frac{dx}{dt} = t + x^2 \\ x(0) = 1 \end{cases}$

Using Runge-Kutta method of 4th order find x(0.2) taking h=0.1. **(6)**

Q9 (a) Solve the initial value problem

$\frac{dy}{dx} = x - y^2$ *in the range* $0 \le x \le 1$, *for the initial condition* $y(0) = 0$. **(4)**

(b) Solve the elliptical equation $u_{xx} + u_{yy} = 0$ for the following square mesh with oundary values given as **(6)**



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