

- (b) What will be the resulting makespan of running this greedy algorithm on a given sequence of $n=m(m-1)+1$ jobs with processing time $t_j=1$ for $1 \leq j \leq n-1$ and $t_n=m$, for $m=4$ identical machines? 2
- (c) Compare the resulting assignment in Q5(b) with the optimal assignment which can be obtained by assigning the largest job to one of the machines and evenly spreading the remaining jobs over the other $m-1$ machines. 2

*** End of Questions ***

MID SEMESTER EXAMINATION, APRIL-2024 ALGORITHM DESIGN-2 (CSE 4131)

Programme: B.Tech.(CSE/CSIT/CDS/CIOT/CAIML/CCS)

Semester: 4th Full Marks: 30

Time: 2 Hours

Course Outcome	Taxonomy Level	Ques. Nos.	Marks
CO1: understand the network flow problem and apply it to real-world problems.	L3, L4, L5	1(a), 1(b), 1(c),	2+2+2
CO2: distinguish between computationally tractable and intractable problems. - define and relate class-P, class-NP and class NP-complete, PSPACE, PSPACE-complete. - given a problem in NP, define an appropriate certificate and the verification algorithm.	L3, L4, L5,	2(a), 2(b), 2(c), 3(a), 3(b), 3(c), 4(a), 4(b), 4(c),	2+2+2+2+2+2+2
CO3: understand approximation algorithms and apply this concept to solve problems.	L4, L5, L6	5(a), 5(b), 5(c)	2+2+2
CO4: understand local search techniques and apply this concept to solve problems.			
CO5: understand randomization and apply this concept to solve problems.			
CO6: identify and apply an appropriate algorithmic approach to solve a problem and explain the challenges to solve it.			

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. Consider a flow-network $G(V, E)$, where $V = \{s, a, b, c, d, e, f, g, h, i, t\}$ and capacities of the edges in E are $c(s,a)=2, c(s,b)=3, c(s,c)=2, c(a,d)=2, c(b,d)=1, c(b,e)=1, c(c,e)=2, c(c,f)=1, c(d,g)=3, c(e,h)=2, c(f,i)=1, c(g,t)=2, c(h,t)=3, c(i,t)=2$ where s and t are the source and sink node respectively.
- (a) Compute the maximum flow from s to t of the above graph using Ford-Fulkerson algorithm. 2
- (b) Show the minimum-cut and find the capacity of minimum cuts. 2

- (c) A bipartite graph $G = (V, E)$ has the sets $V = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ of vertices and $E = \{(0, 9), (1, 5), (1, 7), (2, 6), (2, 8), (3, 5), (3, 6), (3, 9), (4, 6), (4, 7)\}$ of edges. Given a maximal matching $M_0 = \{(0, 9), (1, 7), (2, 8), (3, 6)\}$ of the four pairs of vertices, find a maximum matching of all five pairs of vertices as follows: "Select two possible augmenting paths P_1 and P_2 for M_0 and justify your selection."

2.

- (a) Give a simple reduction algorithm to reduce Sorting problem to Convex-Hull problem. 2
- (b) Define the class NP-Hard. How does it differ from NP-Complete? Provide examples of NP-Hard problems that are not necessarily in NP. Explain why they are classified as NP-Hard. 2
- (c) Given clauses : $C_1 = x_1 \vee 0 \vee 0$, $C_2 = x_2 \vee 0 \vee 0$, $C_3 = x'_1 \vee x'_2 \vee 0$ 2
 (i) Is $C_2 \wedge C_3$ satisfiable ?
 (ii) Is $C_1 \wedge C_2 \wedge C_3$ satisfiable ?

3. (a) Give the formal problem statement for both decision and optimization version of "Set-Cover" problem. 2
- (b) Give a reduction algorithm for Vertex-Cover \leq_p Set-Cover. 2
- (c) Given a graph $G(V, E)$ where V is the set of vertices and set of all edges $E = \{(v1, v2), (v1, v9), (v2, v3), (v2, v8), (v3, v5), (v4, v5), (v5, v6), (v6, v7), (v7, v8), (v7, v9)\}$, can a Vertex-Cover of size 4 possible for the given graph? Justify your answer. 2

4. (a) Give a proper example scenario where we can solve a NP-Complete problem by sacrificing one of its 2

features, i.e. - "solve arbitrary instances of the given problem".

- (b) Give a formal problem statement for decision-quantified-satisfiable problem. How it is different from the 3-satisfiable problem? 2
- (c) Consider the problem of "Given a 3×3 board with 8 tiles (every tile has one number from 1 to 8) and one empty space. The objective is to place the numbers on tiles to match the final configuration using the empty space. The primary challenge of the 8-puzzle problem lies in starting from a given initial state and finding a sequence of moves that leads to the goal state". Model this problem as a planning problem. (Hint: set of conditions, initial state, goal state, set of operators etc.) 2

5.

Greedy-Load-Balance ($n, m, t[1 \dots n]$) {
 start with no jobs assigned
 set $T_i = 0$ and $A(i) = \emptyset$ for all machines M_i
 for $j = 1, \dots, n$ do {
 let M_i be a machine with the minimum $\min_k T_k$
 assign job j to machine M_i
 set $A(i) \leftarrow A(i) \cup \{j\}$
 set $T_i \leftarrow T_i + t_j$
 }
 return $A[1], A[2], \dots, A[m]$
 }

- (a) For the above given approximation algorithm, one possible lower bound on the optimal assignment is - "one of the m machines must do atleast the amount of work same as the maximum processing time of any job". Provide a critical view on this lower bound. 2