# END-SEMESTER EXAMINATION, JULY-2023
# CALCULUS - B (MTH-2101)

**Programme: B. Tech** | **Semester: 2nd**
**Full Marks: 60** | **Time: 3 Hours**

| Subject/Course Learning Outcome | *Taxonomy Level | Ques. Nos. | Marks |
|---|---|---|---|
| Determine the distances, volume, equations of spheres, lines, and planes using vector operations in space. | L3 | 1. a, b, c, 2. a, b, c, 3. a | 2,2,2<br>2,2,2<br>2 |
| Compute the length of a curve, curvature, tangent, and normal vectors. | L3 | 3. b, c, 4. a, b, c | 2,2<br>2,2,2 |
| Determine limit, derivatives, directional derivatives, maxima/minima, and Jacobian of a function of two or several variables. | L3 | 5. a, b, c. | 2,2,2 |
| Apply the computational and conceptual principles of calculus to the solution of double and triple integrals. | L3 | 5.a,b<br>6.b,c<br>7.c | 2,2<br>2,2<br>2 |
| Use Green's theorem to compute line integrals in terms of double integrals. | L3 | 5.c<br>6.a<br>7.a,b<br>9.c | 2<br>2<br>2,2<br>2 |
| Apply the most important theorems of vector calculus, such as the curl, divergence, and Stoke's theorems to simplify integration problems. | L3 | 8.a,b,c<br>9.a,b | 2,2,2<br>2,2 |

*Bloom's taxonomy levels: Remembering (L1), Understanding (L2), Applying (L3), Analysing (L4), Evaluating (L5), Creating (L6)

**Answer all questions. Each question carries equal mark.**

1 (a) Determine an equation of the largest sphere with centre (5,4,9) that is contained in the first octant. [2]

(b) Determine the centre and radius of the sphere $2x^2+2y^2+2z^2=8x-24z+1$. [2]

(c) For the scalar field $u=\frac{x^3}{4}+\frac{y^3}{5}$, calculate magnitude of the gradient at the point (1,3). [2]

2 (a) Calculate the directional derivative of $f(x,y,z)=xy^2+yz^2+zx^2$ at the point $P(2,-1,1)$ in the direction of the vector $\langle 1,2,2\rangle$. [2]

(b) Determine the area of the triangle with vertices $K(1,2,3), L(1,3,6), M(3,8,6)$. [2]

(c) Compute the distance from the point (-6,3,5) to the plane $x-2y-4z=8$. [2]

3 (a) Determine the point where the curve $\vec{r}(t)=\langle t,0,2t-t^2\rangle$ intersects the paraboloid $z=x^2+y^2$. [2]

(b) Determine $\vec{r}(t)$ if $\vec{r}\,'(t)=\langle t,e^t,te^t\rangle$ and $\vec{r}(0)=\langle 1,1,1\rangle$. [2]

(c) A ball is thrown at an angle of $45^\circ$ to the ground. If the ball lands $90m$ away, then determine the initial speed of the ball. [2]

4 (a) Determine the tangential component of the acceleration vector where $\vec{r}(t)=\langle 3t-t^3,3t^2\rangle$. [2]

(b) Compute $\lim\limits_{(x,y)\to(0,0)}\frac{xy}{\sqrt{x^2+y^2}}$, if it exists, or show that the limit does not exist. [2]

(c) Determine $\frac{\partial^3 z}{\partial u\partial v\partial w}$ of the function $z=u\sqrt{v-w}$. [2]

5 (a) If $z=x^2-xy+3y^2$ and $(x,y)$ changes from $(3,-1)$ to $(2.96,-0.95)$, then compare between $\Delta z$ and $dz$. [2]

(b) Determine the Curl and divergent of the vector function $\langle -y,x,0\rangle$. [2]

(c) Determine $\vec{\nabla}.(\vec{\nabla}\times\vec{V})$, where $\vec{V}=3yz\hat{i}+4xz\hat{j}+2xy\hat{k}$. [2]

6 (a) Determine the equation of the the normal line to the surface $x+y+z=e^{xyz}$ at the point $(0,0,1)$. [2]

(b) Compute the local maximum and minimum values and saddle point of the function $f(x,y)=xy+\frac{1}{x}+\frac{1}{y}$. [2]

(c) Compute the three positive numbers whose sum is 100 and whose product is a maximum. [2]

7 (a) Calculate the iterated integral $\int_0^1\int_0^1\frac{x-y}{(x+y)^3}dxdy$. [2]

(b) Calculate the double integral $\iint_R\frac{xy^2}{x^2+1}dA$, where $R=\{(x,y)\mid 0\le x\le 1,-3\le y\le 3\}$. [2]

(c) Compute the volume of the solid enclosed by the surface $z=1+e^x\sin y$ and the planes $x=\pm 1, y=0, y=\pi$, and $z=0$. [2]

8 (a) Determine the double integral $\iint_D x\,dA,\ D=\{(x,y)\mid 0\le x\le\pi, 0\le y\le\sin x\}$. [2]

(b) Sketch the region whose area is given by the integral $\int_{\pi/4}^{\pi/2}\int_1^3 r\,dr\,d\theta$ and determine the integral. [2]

(c) Compute $\iint_D e^{-x^2-y^2}dA$, where $D$ is the region bounded by the semicircle $x=\sqrt{4-y^2}$ and the $y-$axis. [2]

9 (a) Use triple integral to determine the volume of the solid enclosed by the paraboloids $y=x^2+z^2$ and [2]

$y=8-x^2-z^2$.

**(b)** Compute the line integral [2]

$\int_C y^3 ds,\ C: x=t^3, y=t,\ 0\le t\le 3$.

**(c)** Determine the line integral $\int_C \langle xy, 3y^2\rangle \bullet d\vec{r}$, where. [2]

$C: \vec{r}(t)=10t^4 i+3t^3 j,\ 0\le t\le 1$.

**10** **(a)** Determine a function $f$ such that $\vec{F}=\vec{\nabla}f=\langle x^2, y^2\rangle$ [2]

and use it to compute $\int_C \left(x^2\hat{i}+y^2\hat{j}\right).d\vec{r}$, where $C$ is

the parabola $y=2x^2$ from $(1,2)$ to $(2,8)$.

**(b)** Use Green's Theorem to determine the line integral [2]

$\oint_C xy^2 dx + x^2 y\, dy$ where $C$ is the closed curve defined

by the circle $x^2+y^2=1$ oriented anti-clockwise.

**(c)** Use Stoke's theorem to determine [2]

$\iint_S curl\left(2y\cos z\hat{i}+e^x\sin z\hat{j}+xe^y\hat{k}\right).\vec{ds}$, where $S$

is the hemisphere $x^2+y^2+z^2=9,\ z\ge 0$ oriented

upward.

***