

21 Jun

## Big O theta Omega notation: →

### II Insertion sort: →

A.length =  $n$  = input array length

Best case time complexity =  $a n + b \times 1$

Worst " " " " =  $a' n^2 + b' n + c \times 1$

Space complexity = 3 variables

$n \uparrow$  (and  $b$ )  $\uparrow$

↓  
increases

ignore the constants: →

→  $a n + b \rightarrow O(n)$

→  $n \uparrow$   $n \uparrow$   $n \uparrow$

↳ much more increases

10x	[	$n = 10,$	$n^2 = 100$	] 100x
		$n = 100$	$n^2 = 10,000$	

~~⊗~~  $a n^2 + b n + c \rightarrow O(n^2)$

Analysis of algorithm.

⊗ space complexity = 3 variable  
=  $3 \times 1$



## Notations $\rightarrow$ Big O

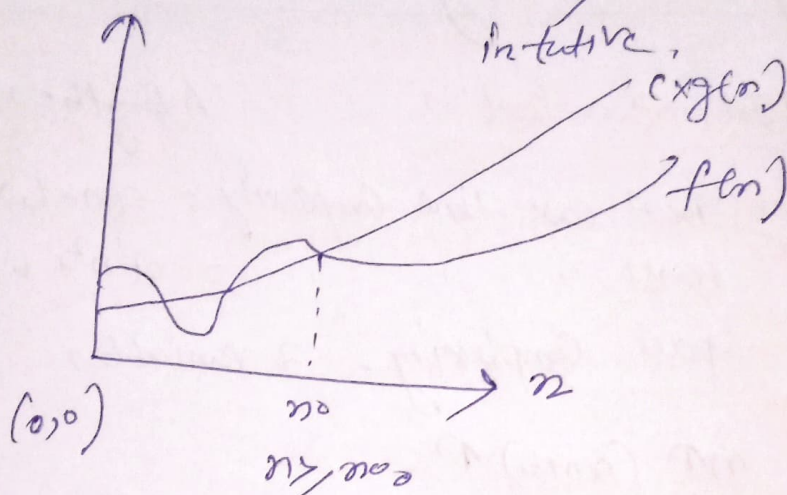
Order of  $n$

best

$$f(n) = a n + b + 1 \rightarrow O(n)$$

$$f(n) = a' n^2 + b' n + c' \rightarrow O(n^2)$$

$$f(n) = 3 = c \rightarrow O(1)$$



$$f(n) = O(g(n))$$

if and only if, there exists  $n_0$  and  $c$  such that  $0 \leq f(n) \leq c \cdot g(n)$  for all  $n \geq n_0$

$$f(n) = 2n^2 + 1 \cdot n + 3 \rightarrow O(n^2)$$

$\uparrow \quad \quad \uparrow \quad \quad \uparrow$   
 $a' \quad \quad b' \quad \quad c'$

$\uparrow$   
 $g(n)$

$$2n^2 + 1 \cdot n + 3 \leq c \cdot n^2 \text{ for all } n \geq n_0$$

let,  $c = 10$

$$2n^2 + 1 \cdot n + 3 \leq 10n^2 \text{ for all } n \geq n_0$$

$n \geq 1$

$$\Rightarrow 2n^2 + n + 3 \leq 10n^2 \text{ for all } n \geq n_0$$

$$\Rightarrow 2 + \frac{1}{n} + \frac{3}{n^2} \leq 10$$

$$\underline{n=1} \Rightarrow 2 + \frac{1}{1} + \frac{3}{1} \leq 10$$

$$\Rightarrow 6 \leq 10$$

$$\underline{n=2},$$

$$2 + \frac{1}{2} + \frac{3}{4} \leq 10 \quad \checkmark$$

$$n=3, \quad 3 + \frac{1}{3} + \frac{3}{9} \leq 10$$

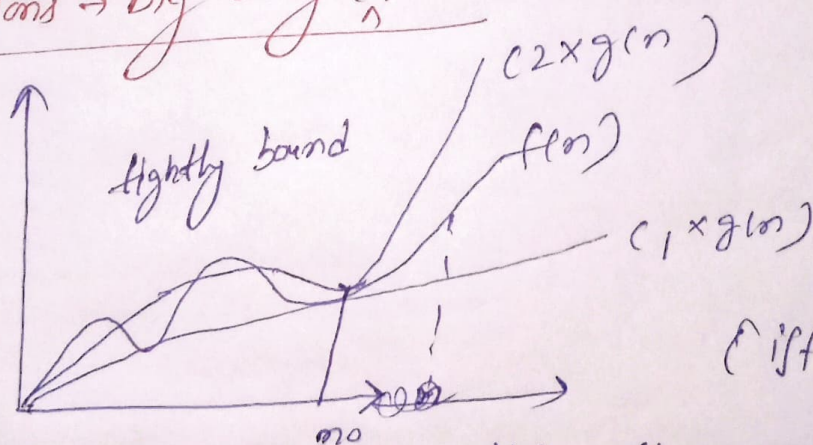
⋮

$$\text{for all } n \geq 1, \quad f(n) = 2n^2 + n + 3 \rightarrow O(n^2)$$

$$\Rightarrow f(n) \leq 10 \times n^2 \quad \text{for all } n \geq 1$$

$$\Rightarrow \boxed{f(n) = O(n^2)}$$

Notations  $\rightarrow$  Big  $\omega$ ,  $\Theta$ ,  $\Omega$

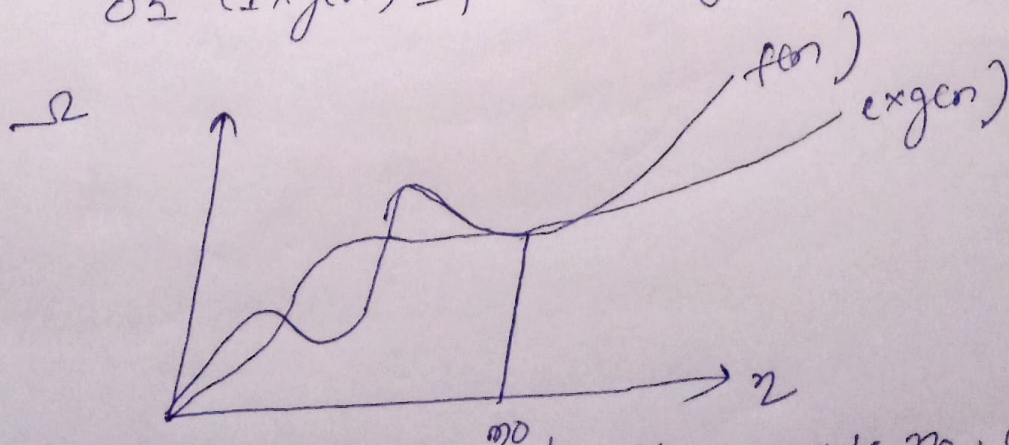


(iff  $\rightarrow$  if & only if)

$f(n) = \Theta(g(n))$  iff there exists

$n_0, c_1, c_2$  such that

$$0 \leq c_1 \times g(n) \leq f(n) \leq c_2 \times g(n) \quad \text{for all } n \geq n_0.$$



$f(n) = \Omega(g(n))$  iff there exists  $n_0, c$  so that  $0 \leq c \times g(n) \leq f(n)$  for all  $n \geq n_0$