MA322 – Scientific Computing Laboratory

Lab – 12

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Ques – 1

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Que-1(a)
 1(a) by Adams-Bashforth
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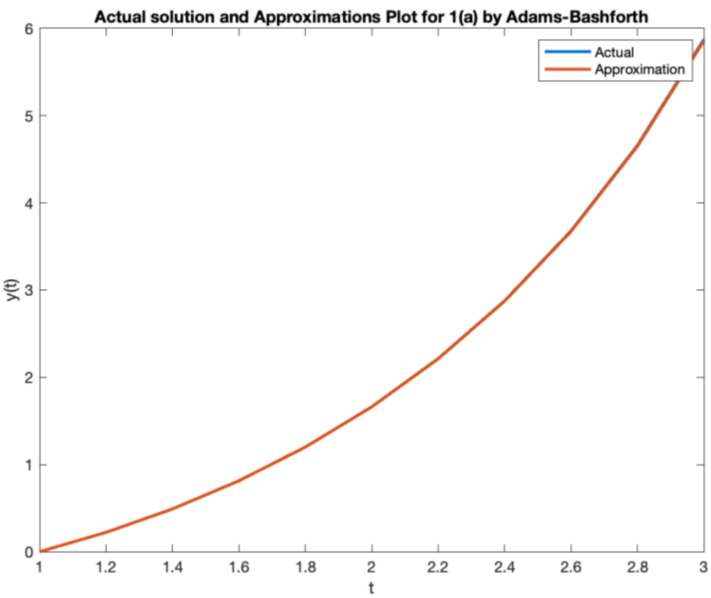
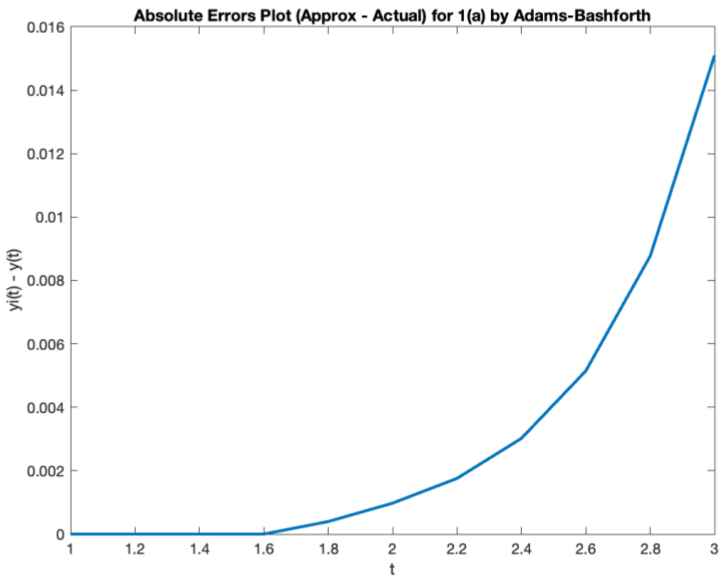
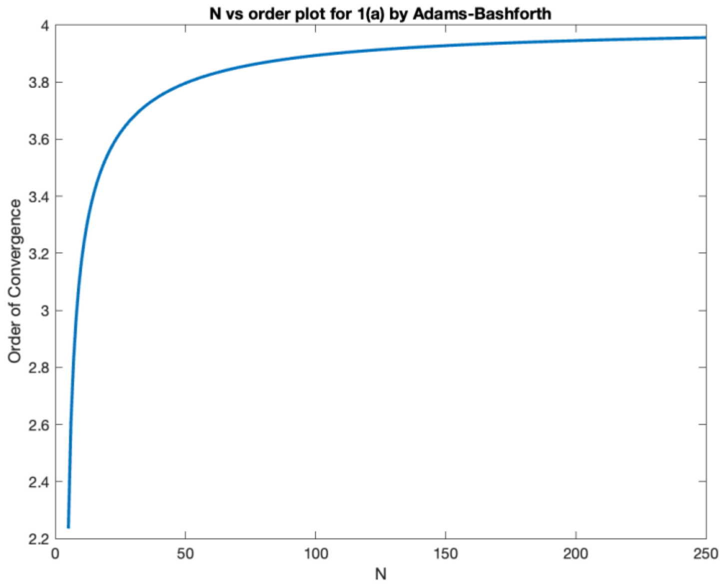
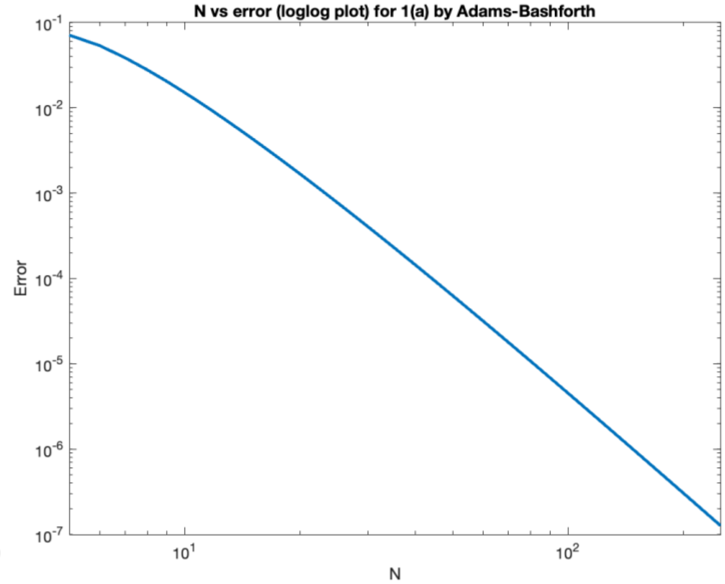
t	Approximation	Exact	Error(|Exact - Approx.|)
1.000000	0.0000000	0.0000000	0.0000000
1.200000	0.2212428	0.2212428	0.0000000
1.400000	0.4896817	0.4896817	0.0000000
1.600000	0.8127527	0.8127527	0.0000000
1.800000	1.1990436	1.1994386	0.0003951
2.000000	1.6603071	1.6612818	0.0009746
2.200000	2.2117462	2.2135018	0.0017556
2.400000	2.8735340	2.8765514	0.0030174
2.600000	3.6733294	3.6784753	0.0051459
2.800000	4.6498974	4.6586651	0.0087677
3.000000	5.8589994	5.8741000	0.0151005



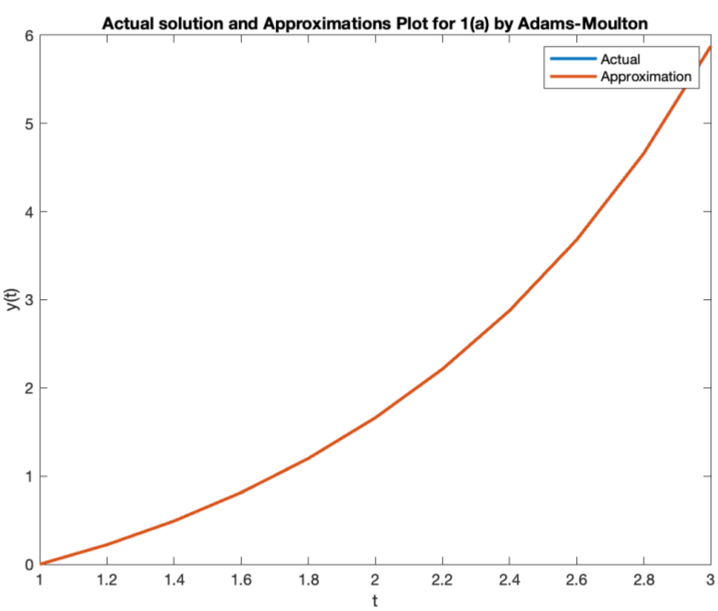


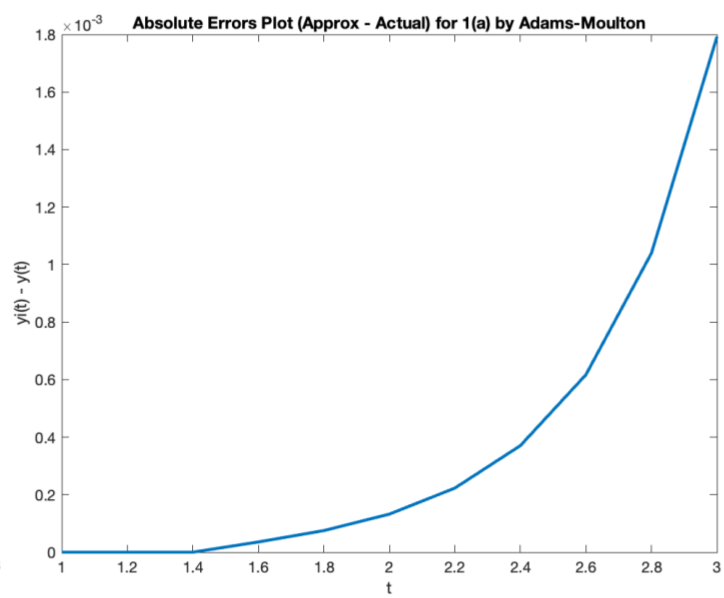


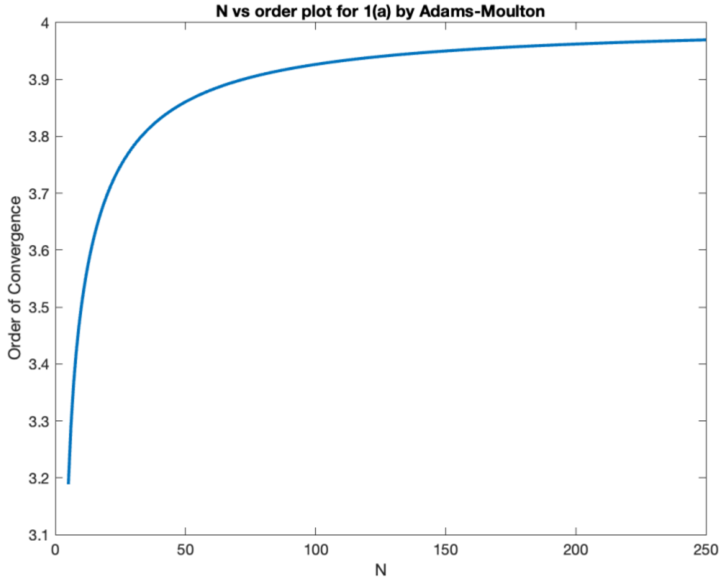


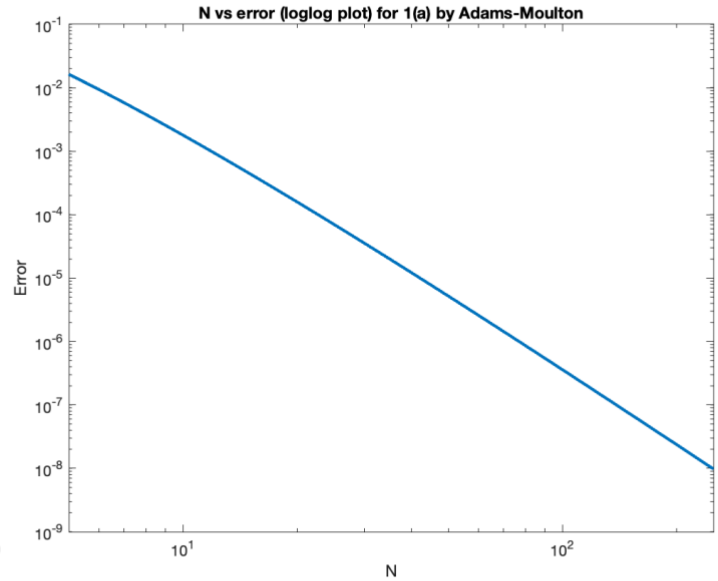
1(a) by Adams–Moulton

t	Approximation	Exact	Error(|Exact – Approx.|)
1.000000	0.0000000	0.0000000	0.0000000
1.200000	0.2212428	0.2212428	0.0000000
1.400000	0.4896817	0.4896817	0.0000000
1.600000	0.8127883	0.8127527	0.0000356
1.800000	1.1995138	1.1994386	0.0000751
2.000000	1.6614141	1.6612818	0.0001323
2.200000	2.2137247	2.2135018	0.0002229
2.400000	2.8769220	2.8765514	0.0003706
2.600000	3.6790924	3.6784753	0.0006170
2.800000	4.6597048	4.6586651	0.0010398
3.000000	5.8758921	5.8741000	0.0017921







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Que-1(b)
 1(b) by Adams-Bashforth
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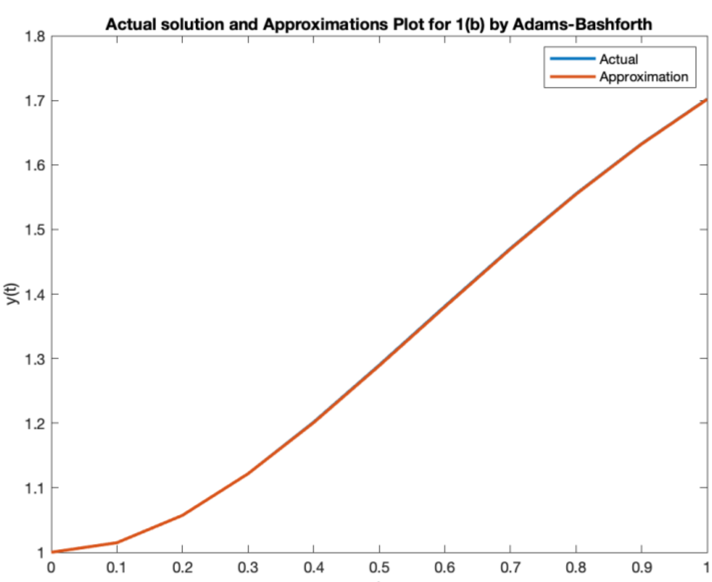
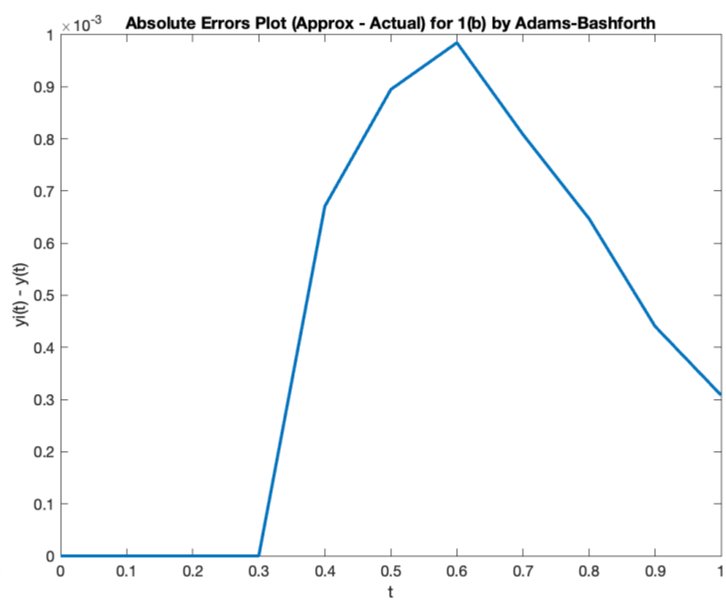
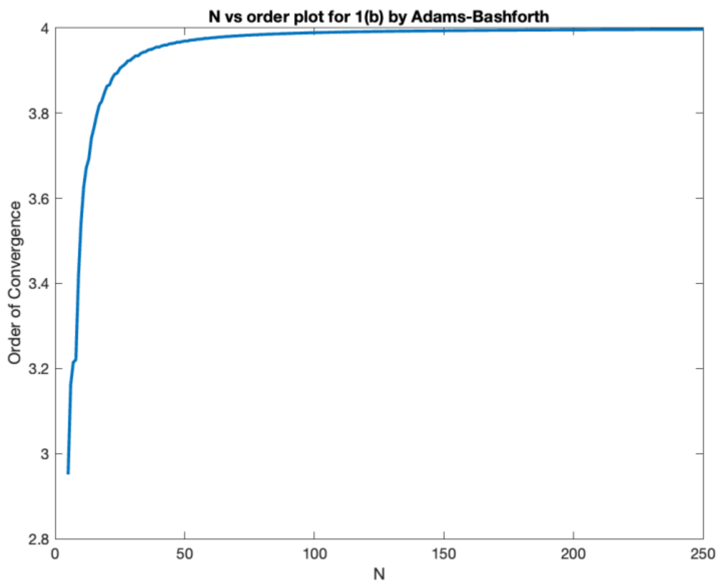
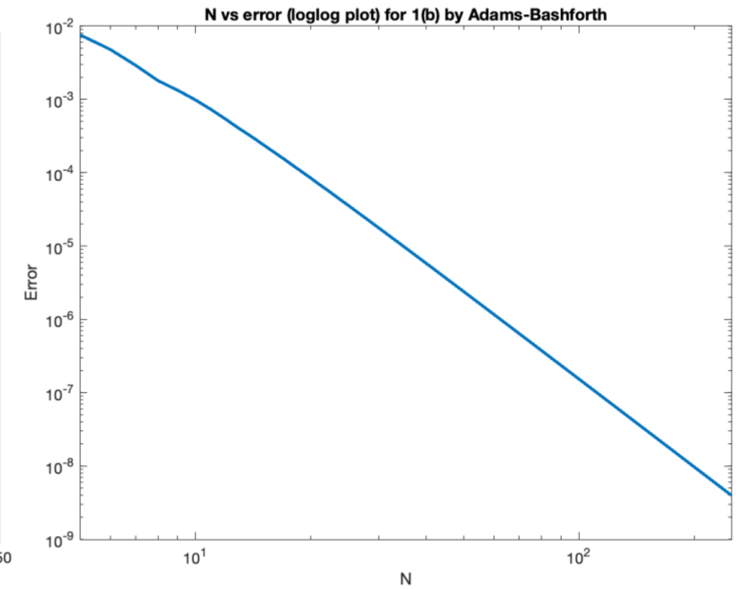
t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.0000000	1.0000000	0.0000000
0.100000	1.0148155	1.0148155	0.0000000
0.200000	1.0571810	1.0571810	0.0000000
0.300000	1.1216980	1.1216980	0.0000000
0.400000	1.2008154	1.2014860	0.0006707
0.500000	1.2889104	1.2898053	0.0008949
0.600000	1.3799467	1.3809312	0.0009845
0.700000	1.4696066	1.4704152	0.0008086
0.800000	1.5543842	1.5550314	0.0006472
0.900000	1.6321727	1.6326132	0.0004405
1.000000	1.7015617	1.7018701	0.0003083



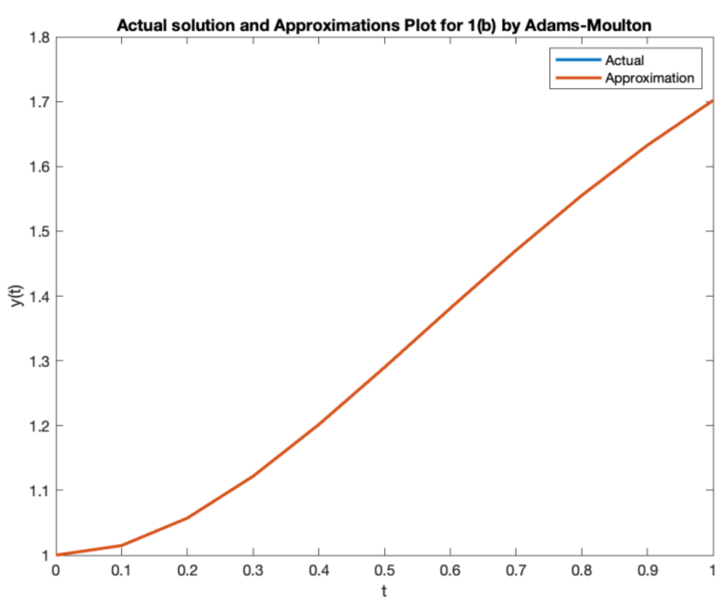


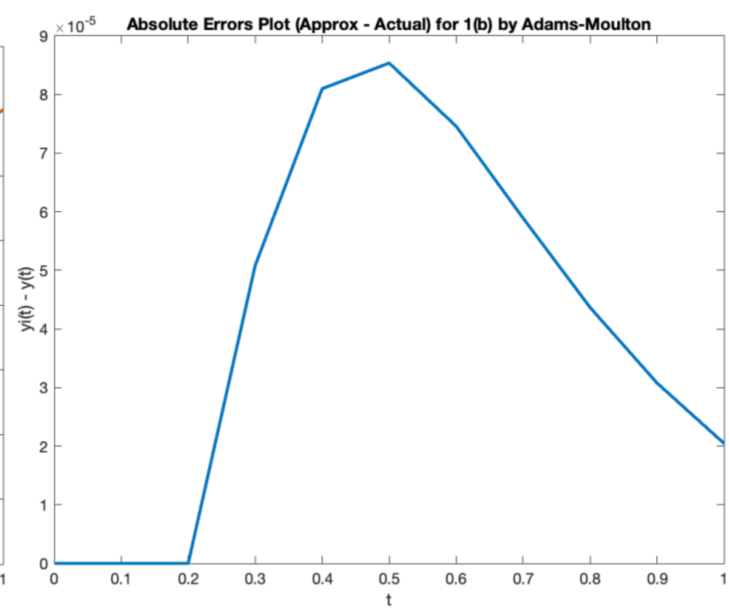


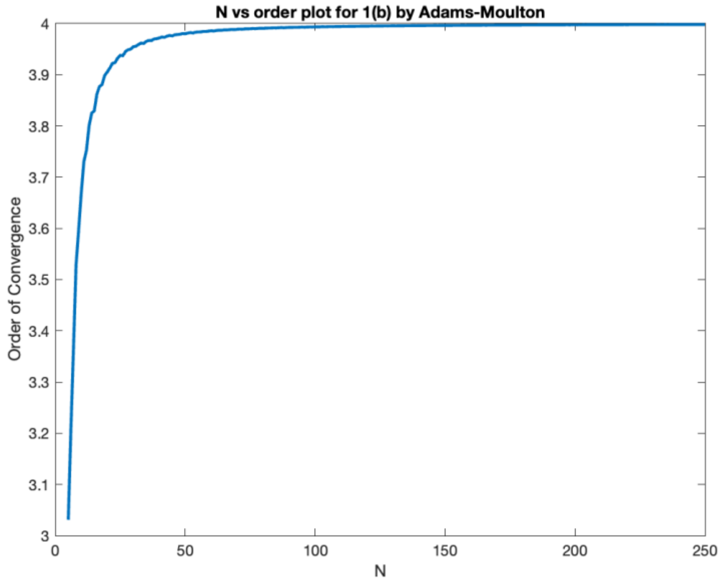


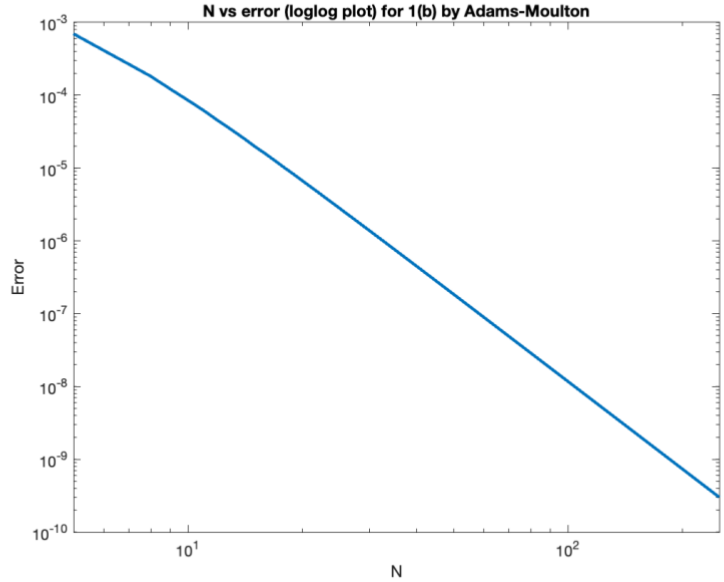
1(b) by Adams-Moulton

t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.0000000	1.0000000	0.0000000
0.100000	1.0148155	1.0148155	0.0000000
0.200000	1.0571810	1.0571810	0.0000000
0.300000	1.1217488	1.1216980	0.0000508
0.400000	1.2015670	1.2014860	0.0000810
0.500000	1.2898906	1.2898053	0.0000854
0.600000	1.3810058	1.3809312	0.0000746
0.700000	1.4704741	1.4704152	0.0000589
0.800000	1.5550751	1.5550314	0.0000437
0.900000	1.6326439	1.6326132	0.0000308
1.000000	1.7018905	1.7018701	0.0000205









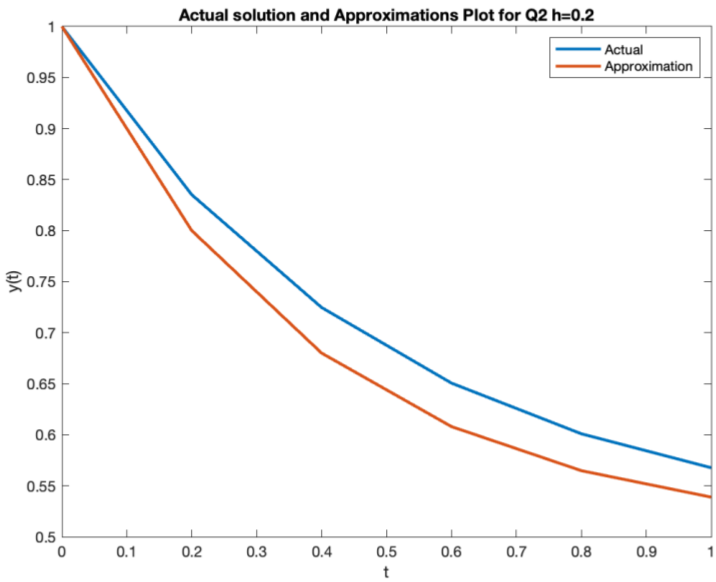
Ques – 2

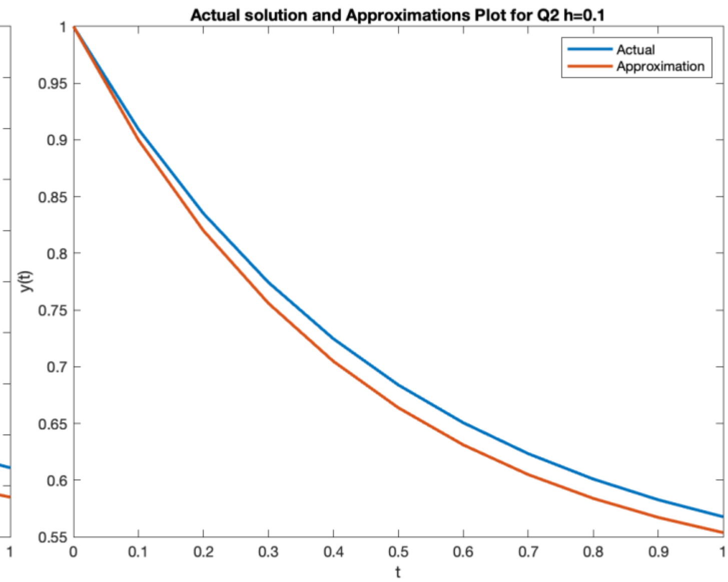
In this question, the IVP is solved by given three methods for five different values of h, the actual value, approximation, and error table is mentioned for one value of h, rest are printed in the code only.

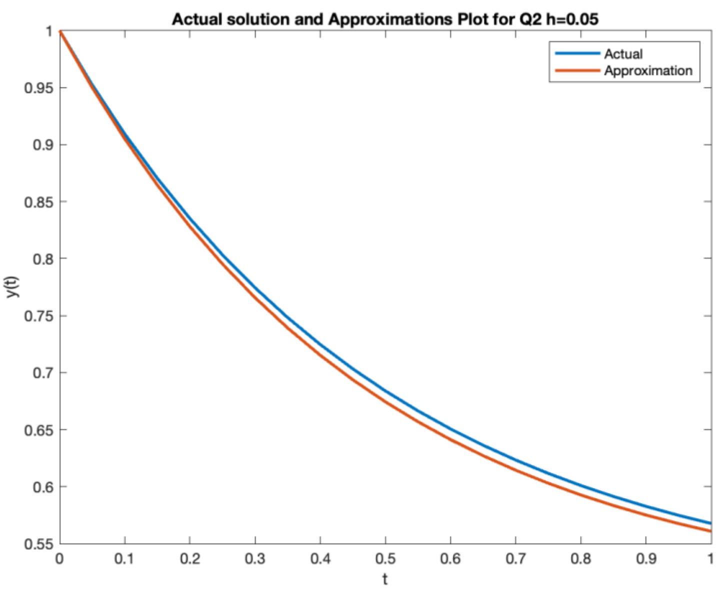
➔ Explicit-Eulers: -

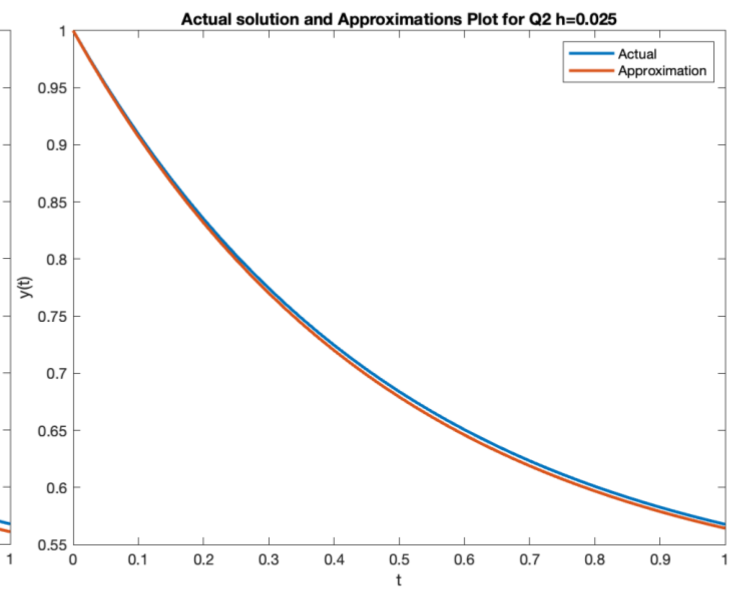
Explicit Euler for h = 0.050000

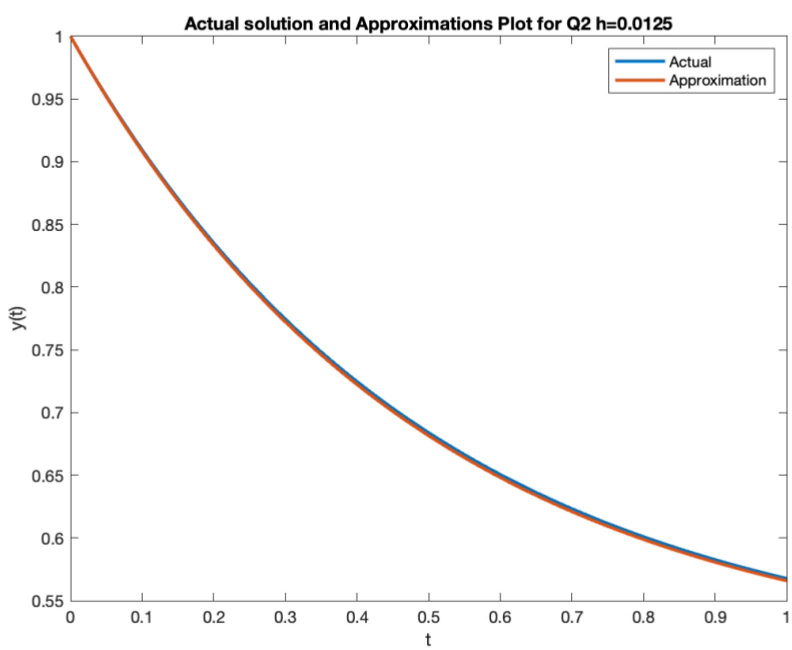
t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.0000000	1.0000000	0.0000000
0.050000	0.9500000	0.9524187	0.0024187
0.100000	0.9050000	0.9093654	0.0043654
0.150000	0.8645000	0.8704091	0.0059091
0.200000	0.8280500	0.8351600	0.0071100
0.250000	0.7952450	0.8032653	0.0080203
0.300000	0.7657205	0.7744058	0.0086853
0.350000	0.7391485	0.7482927	0.0091442
0.400000	0.7152336	0.7246645	0.0094309
0.450000	0.6937102	0.7032848	0.0095746
0.500000	0.6743392	0.6839397	0.0096005
0.550000	0.6569053	0.6664355	0.0095302
0.600000	0.6412148	0.6505971	0.0093823
0.650000	0.6270933	0.6362659	0.0091726
0.700000	0.6143840	0.6232985	0.0089145
0.750000	0.6029456	0.6115651	0.0086195
0.800000	0.5926510	0.6009483	0.0082972
0.850000	0.5833859	0.5913418	0.0079559
0.900000	0.5750473	0.5826494	0.0076021
0.950000	0.5675426	0.5747843	0.0072417
1.000000	0.5607883	0.5676676	0.0068793

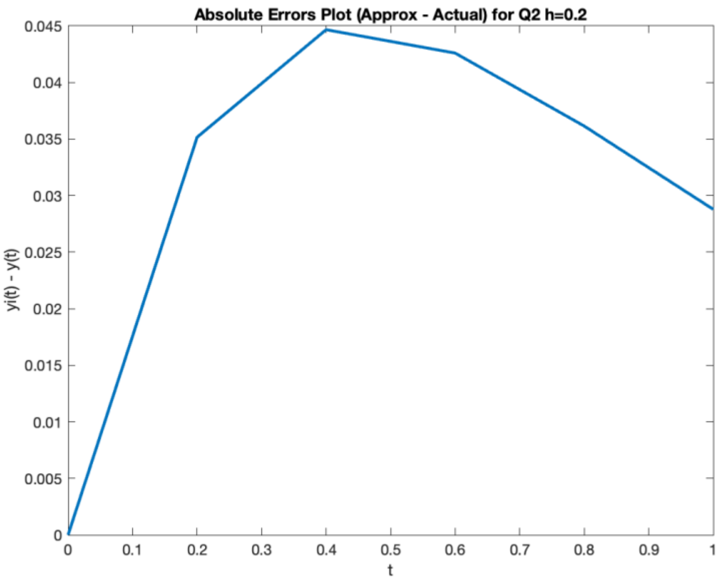

Actual solution and Approximations Plot for Q2 h=0.2
Actual
Approximation
y(t)
t

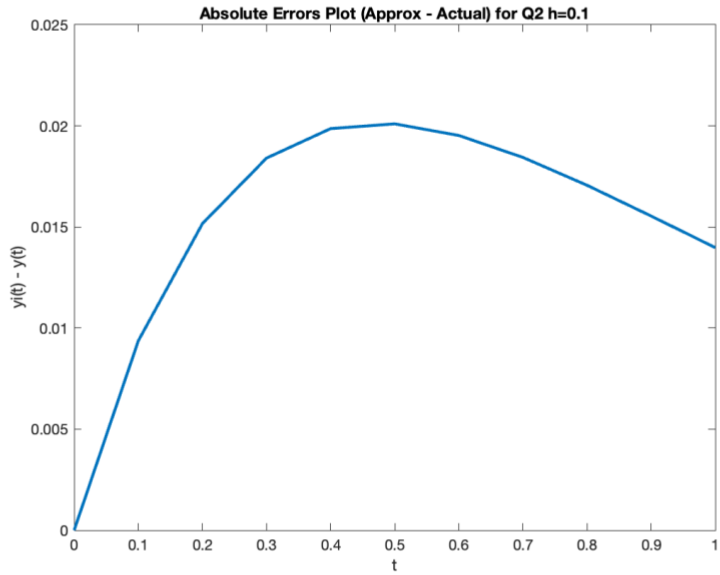

Actual solution and Approximations Plot for Q2 h=0.1
Actual
Approximation
y(t)
t

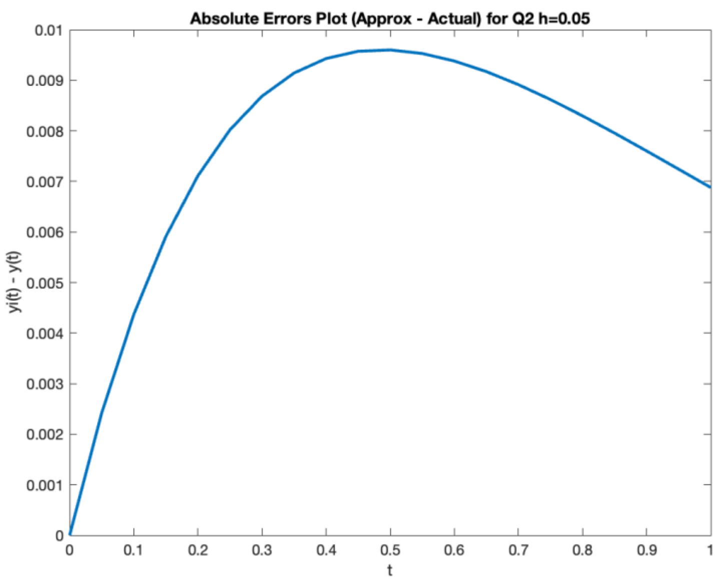

Actual solution and Approximations Plot for Q2 h=0.05
Actual
Approximation
y(t)
t

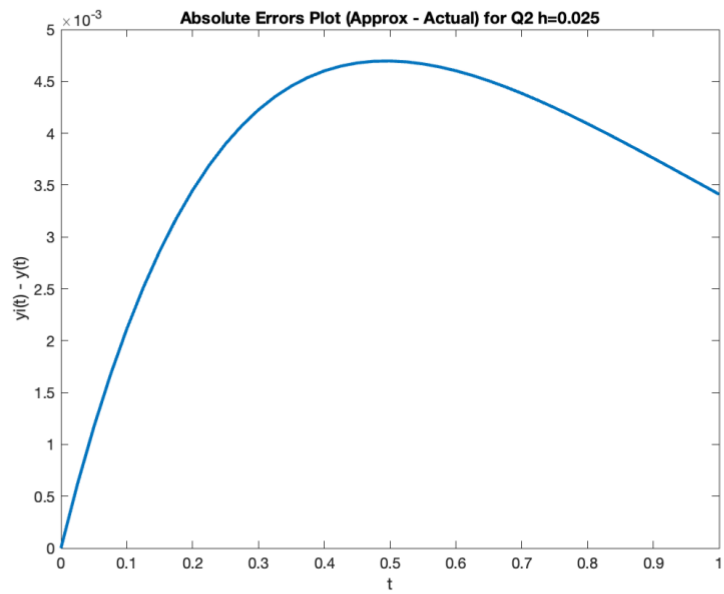

Actual solution and Approximations Plot for Q2 h=0.025
Actual
Approximation
y(t)
t

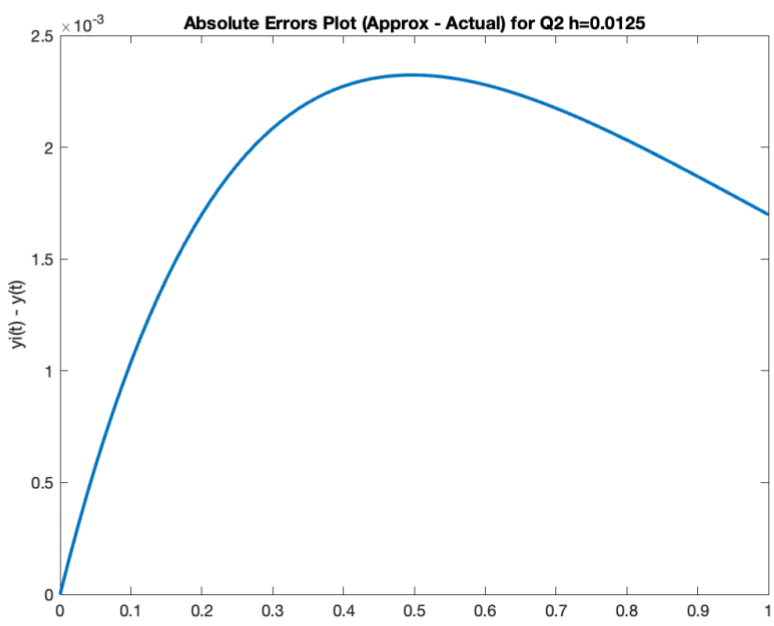

Actual solution and Approximations Plot for Q2 h=0.0125
Actual
Approximation
y(t)
t


Absolute Errors Plot (Approx - Actual) for Q2 h=0.2
yi(t) - y(t)
t
0 0.005 0.01 0.015 0.02 0.025 0.03 0.035 0.04 0.045
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Absolute Errors Plot (Approx - Actual) for Q2 h=0.1
yi(t) - y(t)
t
0 0.005 0.01 0.015 0.02 0.025
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Absolute Errors Plot (Approx - Actual) for Q2 h=0.05
yi(t) - y(t)
t
0 0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.01
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Absolute Errors Plot (Approx - Actual) for Q2 h=0.025
× 10^-3
yi(t) - y(t)
t
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

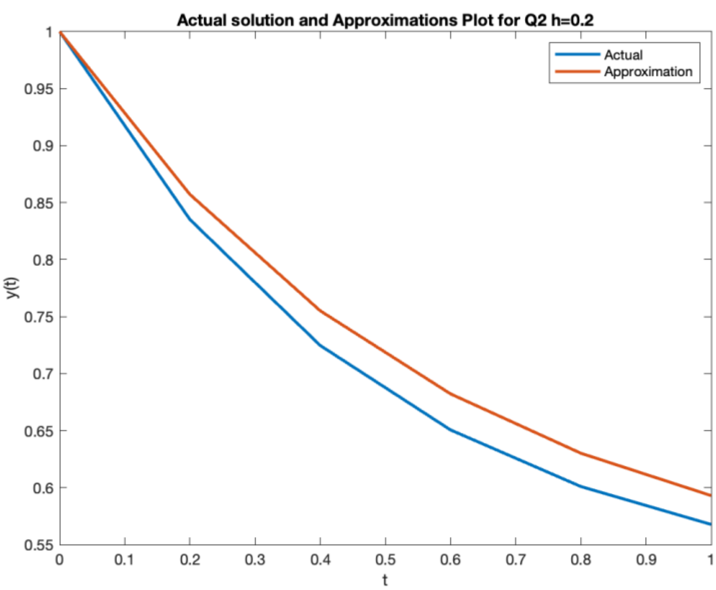

Absolute Errors Plot (Approx - Actual) for Q2 h=0.0125
× 10^-3
yi(t) - y(t)
t
0 0.5 1 1.5 2 2.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

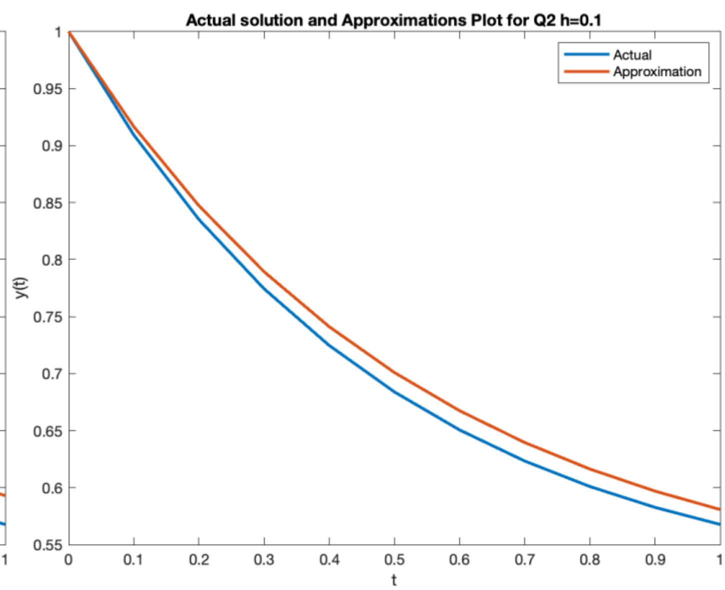
- The Absolute error vs t plot is bitonic, and is getting smoother with decrease in value of h, the approximation is getting better and error is not oscillating, this shows that **Explict-Euler method is stable here**.

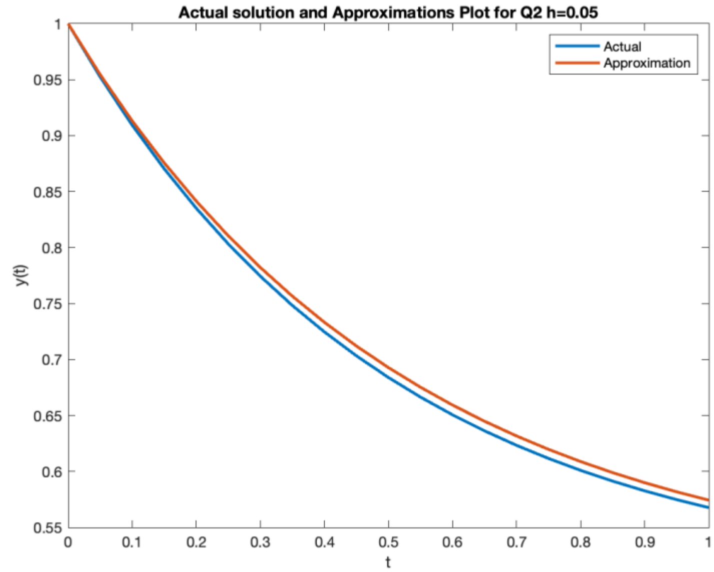
➔ Implicit-Eulers

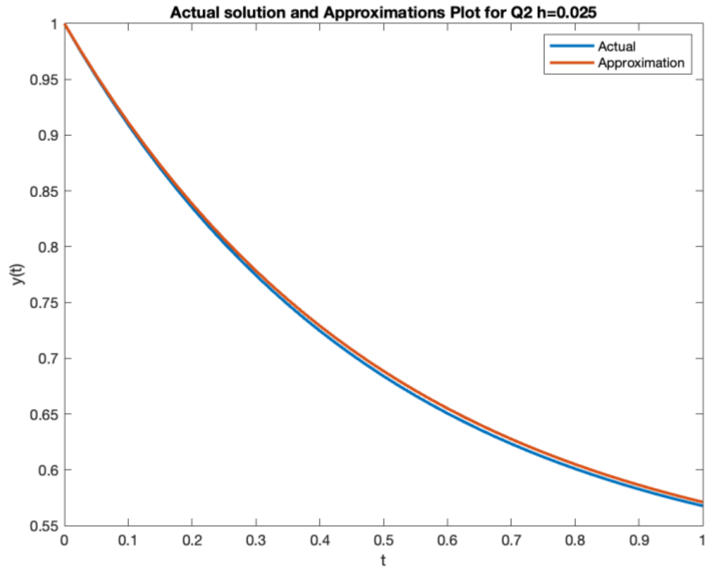
Implicit Euler for h = 0.050000

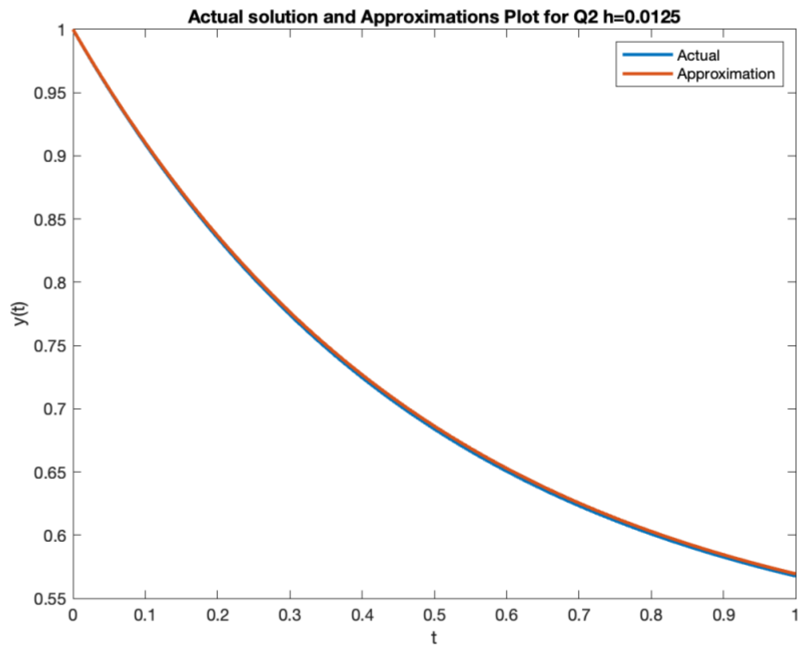
t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.0000000	1.0000000	0.0000000
0.050000	0.9545455	0.9524187	0.0021268
0.100000	0.9132232	0.9093654	0.0038578
0.150000	0.8756575	0.8704091	0.0052484
0.200000	0.8415069	0.8351600	0.0063468
0.250000	0.8104608	0.8032653	0.0071955
0.300000	0.7822371	0.7744058	0.0078313
0.350000	0.7565792	0.7482927	0.0082866
0.400000	0.7332539	0.7246645	0.0085894
0.450000	0.7120490	0.7032848	0.0087642
0.500000	0.6927718	0.6839397	0.0088321
0.550000	0.6752471	0.6664355	0.0088116
0.600000	0.6593156	0.6505971	0.0087185
0.650000	0.6448324	0.6362659	0.0085665
0.700000	0.6316658	0.6232985	0.0083673
0.750000	0.6196962	0.6115651	0.0081311
0.800000	0.6088147	0.6009483	0.0078665
0.850000	0.5989225	0.5913418	0.0075807
0.900000	0.5899295	0.5826494	0.0072800
0.950000	0.5817540	0.5747843	0.0069697
1.000000	0.5743217	0.5676676	0.0066541

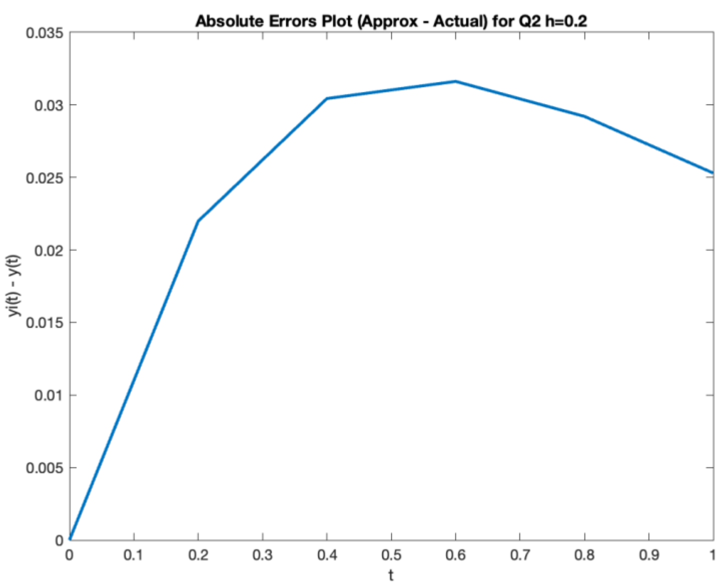


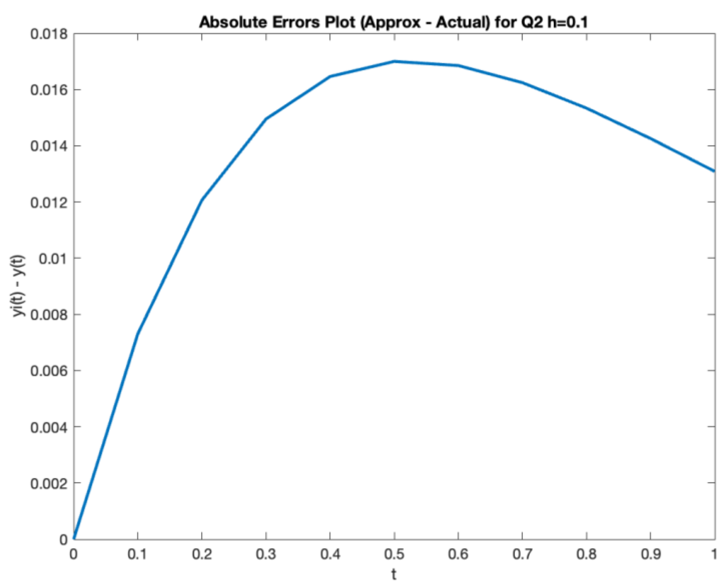


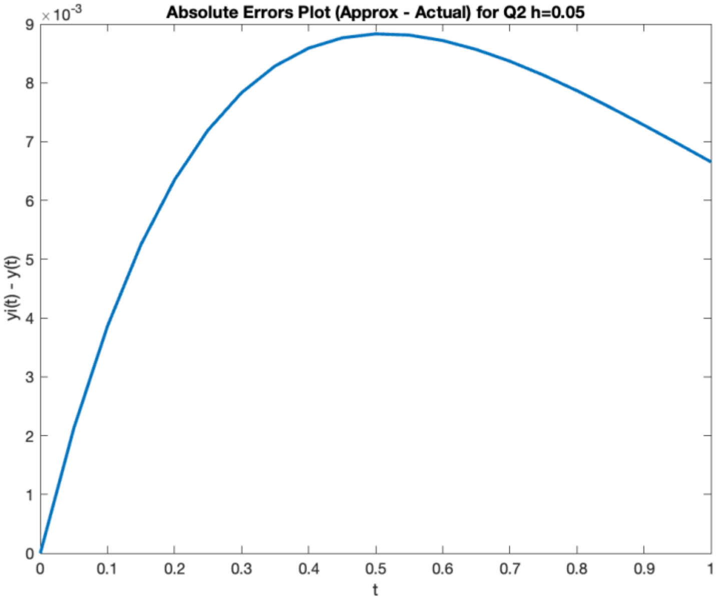

Actual solution and Approximations Plot for Q2 h=0.05
Actual
Approximation
y(t)
t

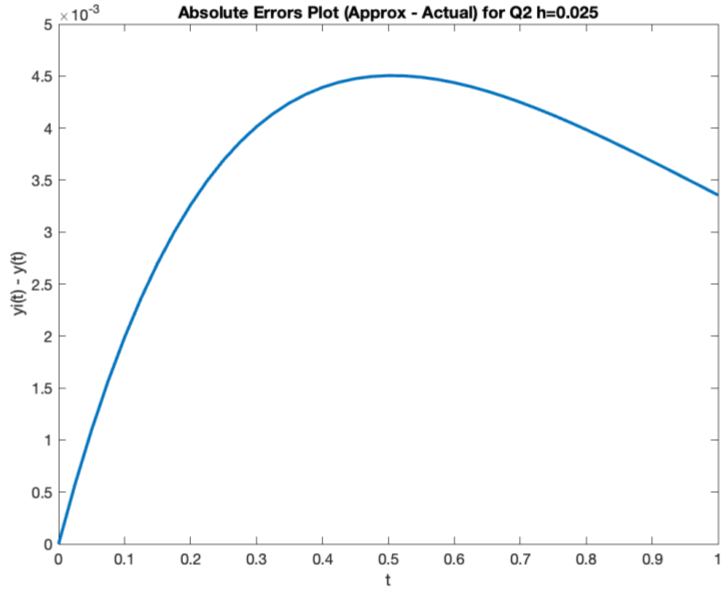

Actual solution and Approximations Plot for Q2 h=0.025
Actual
Approximation
y(t)
t

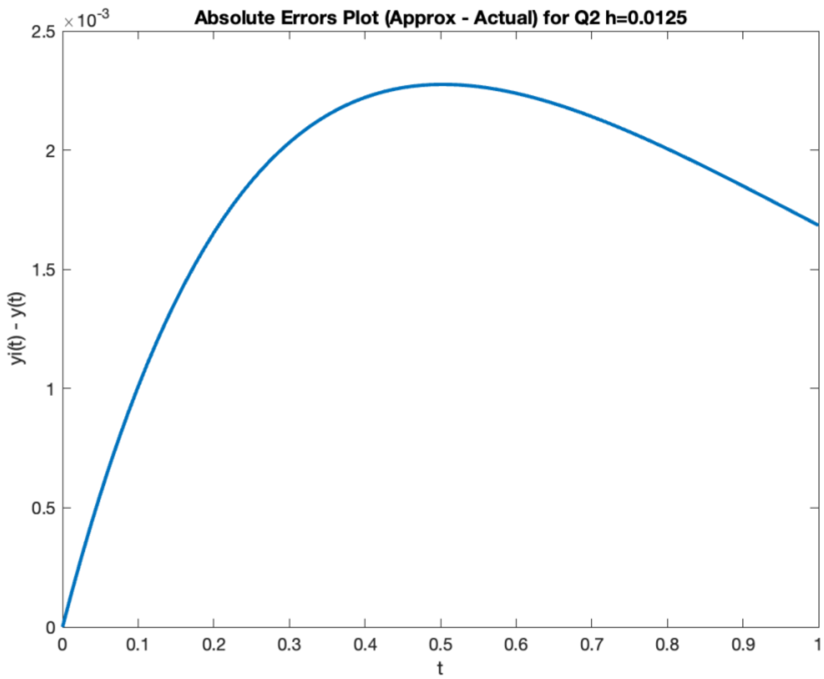

Actual solution and Approximations Plot for Q2 h=0.0125
Actual
Approximation
y(t)
t


Absolute Errors Plot (Approx - Actual) for Q2 h=0.2
yi(t) - y(t)
t


Absolute Errors Plot (Approx - Actual) for Q2 h=0.1
yi(t) - y(t)
t





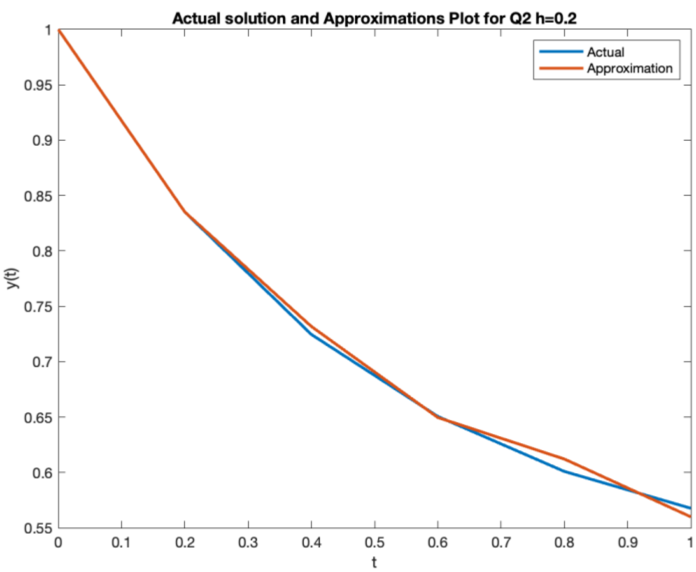


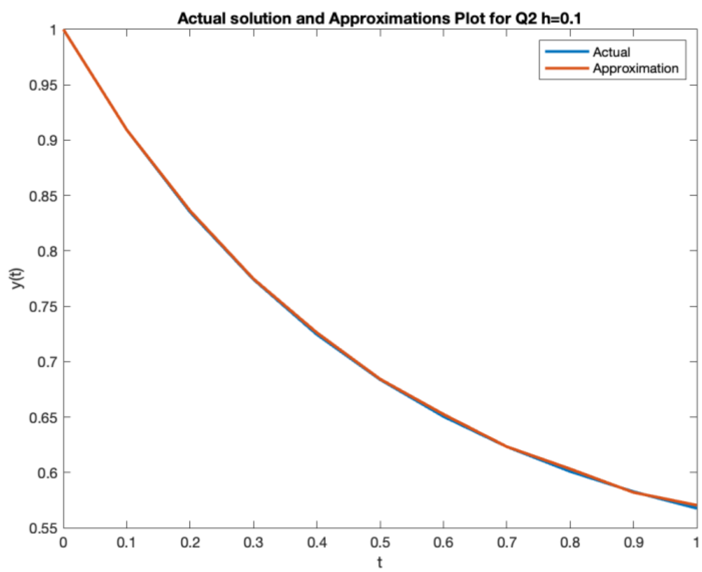
- Similar to Explicit-Eulers, the Absolute error vs t plot is bitonic, and is getting smoother with decrease in value of h, the approximation is getting better and error is not oscillating, this shows **Implicit-Euler method is also stable here**.

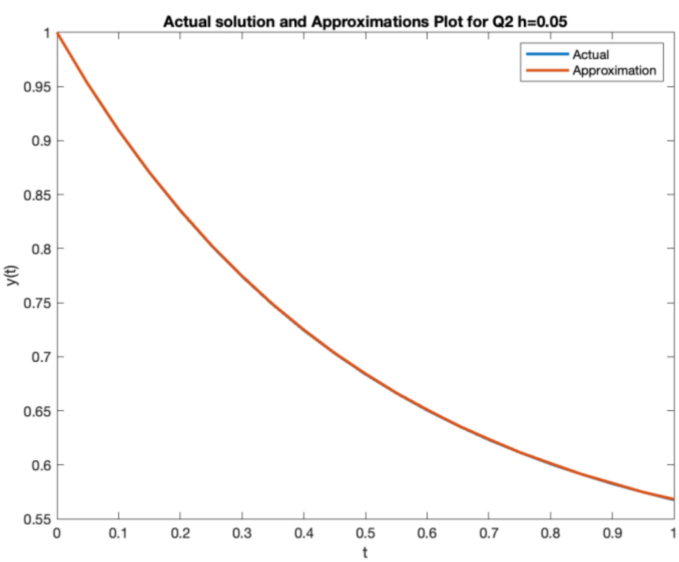
➔ Central Difference: -

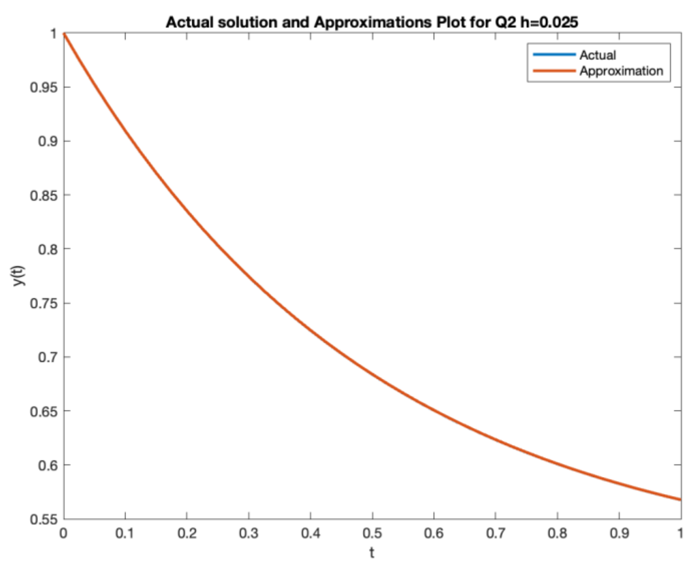
Central Difference for h = 0.050000

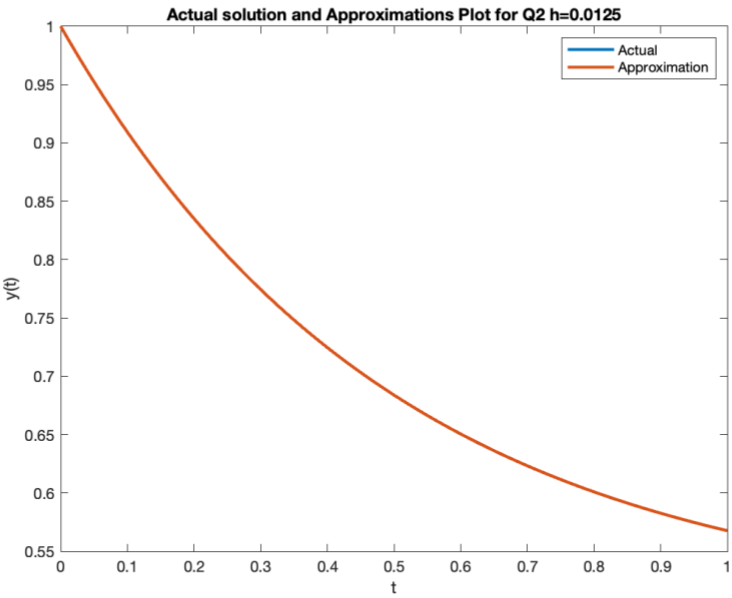
t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.050000	0.952419	0.952419	0.000000
0.100000	0.909516	0.909365	0.000151
0.150000	0.870515	0.870409	0.000106
0.200000	0.835413	0.835160	0.000253
0.250000	0.803433	0.803265	0.000167
0.300000	0.774727	0.774406	0.000321
0.350000	0.748488	0.748293	0.000195
0.400000	0.725029	0.724664	0.000365
0.450000	0.703482	0.703285	0.000197
0.500000	0.684333	0.683940	0.000393
0.550000	0.666615	0.666436	0.000180
0.600000	0.651010	0.650597	0.000413
0.650000	0.636413	0.636266	0.000147
0.700000	0.623727	0.623298	0.000429
0.750000	0.611668	0.611565	0.000103
0.800000	0.601394	0.600948	0.000445
0.850000	0.591389	0.591342	0.000047
0.900000	0.583116	0.582649	0.000466
0.950000	0.574766	0.574784	0.000018
1.000000	0.568163	0.567668	0.000495

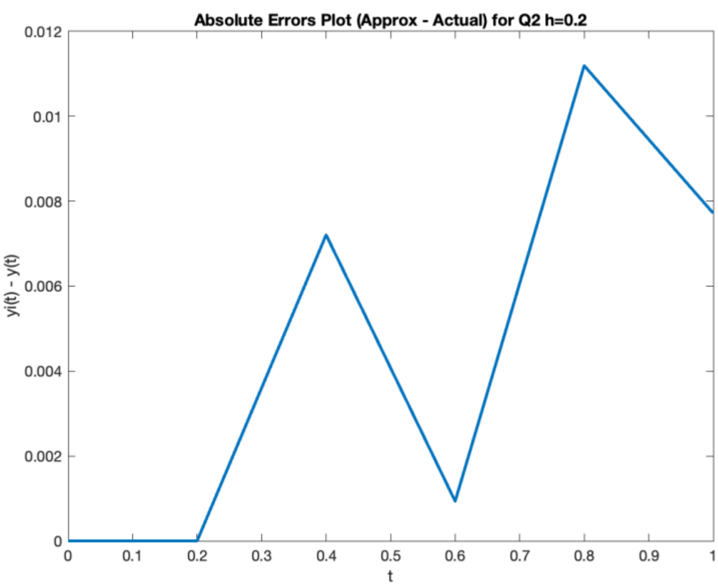


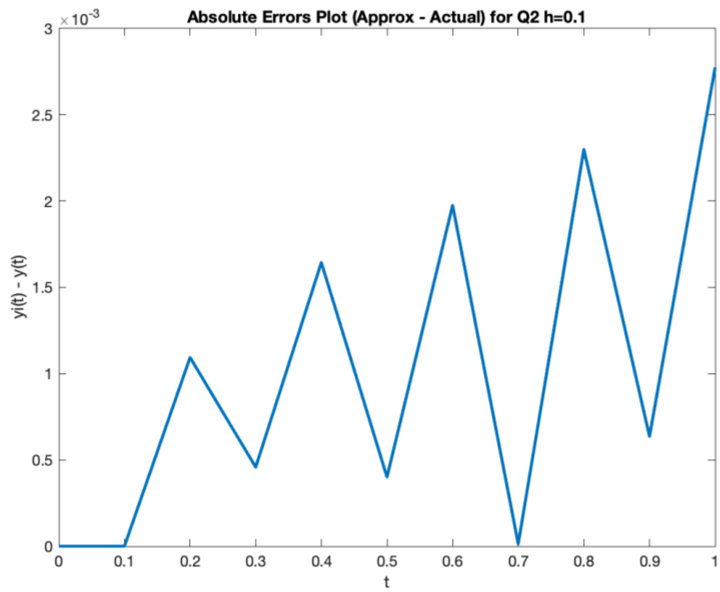


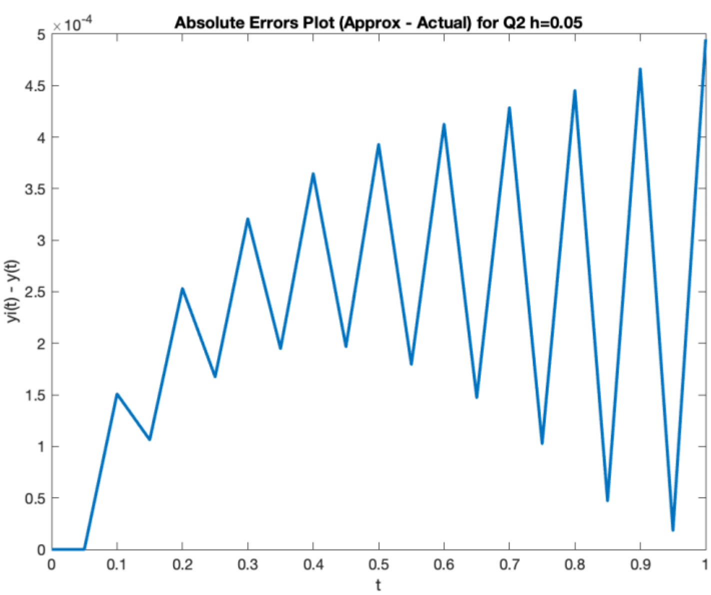


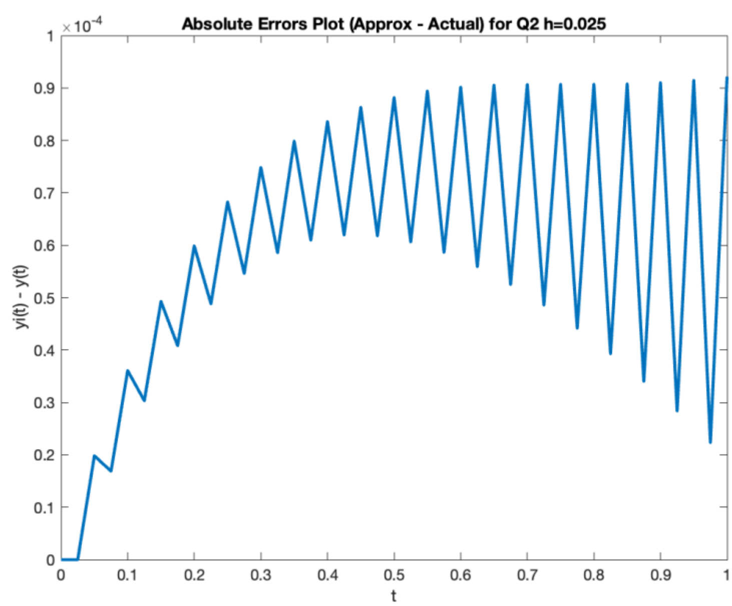


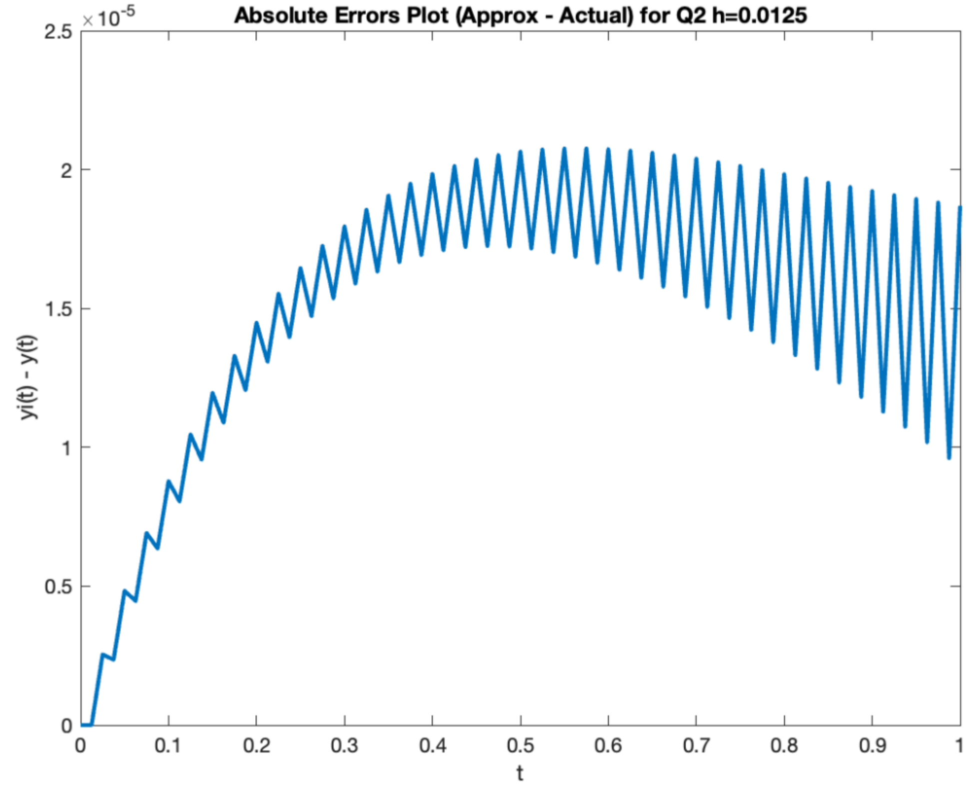

Actual solution and Approximations Plot for Q2 h=0.0125
Actual
Approximation
y(t)
t


Absolute Errors Plot (Approx - Actual) for Q2 h=0.2
yi(t) - y(t)
t


Absolute Errors Plot (Approx - Actual) for Q2 h=0.1
$\times 10^{-3}$
yi(t) - y(t)
t


Absolute Errors Plot (Approx - Actual) for Q2 h=0.05
$\times 10^{-4}$
yi(t) - y(t)
t

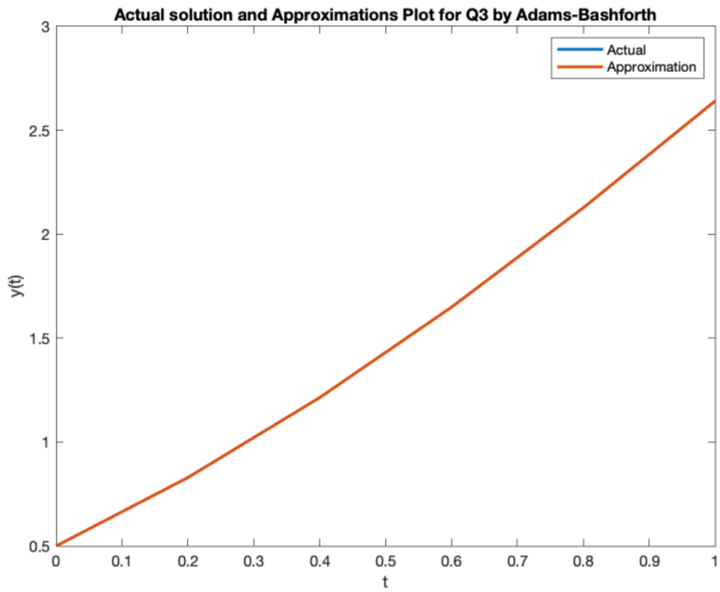

Absolute Errors Plot (Approx - Actual) for Q2 h=0.025
$\times 10^{-4}$
yi(t) - y(t)
t

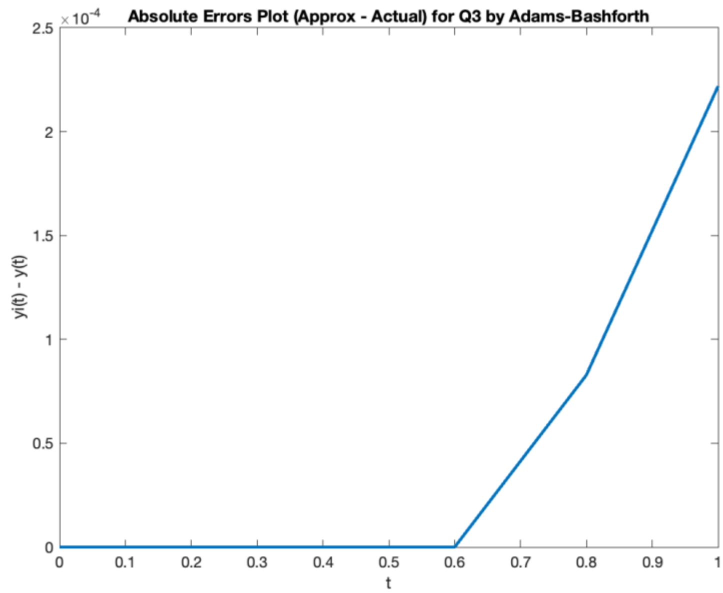


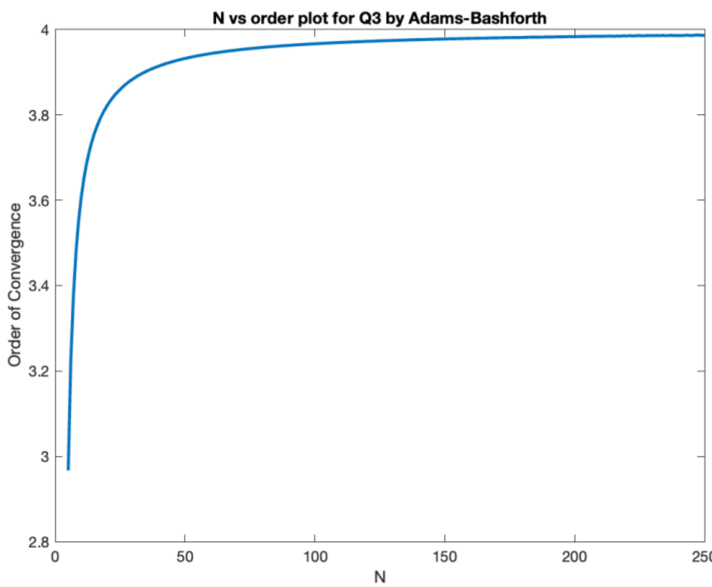
- Here, though the approximation is getting better with decreasing value of h, but the absolute error is fluctuating a lot w.r.t node points (t). This is due to the extra term we got while solving the difference equation in lecture. This fluctuation in absolute error clearly shows that **Central Difference Scheme is unstable here**.
- One reason can be that central difference is second order scheme which is being used to solve a first order IVP here, due to which it is resulting in some extra term which is making the error to oscillate and amplitude of oscillations is increasing with decreasing value of h.

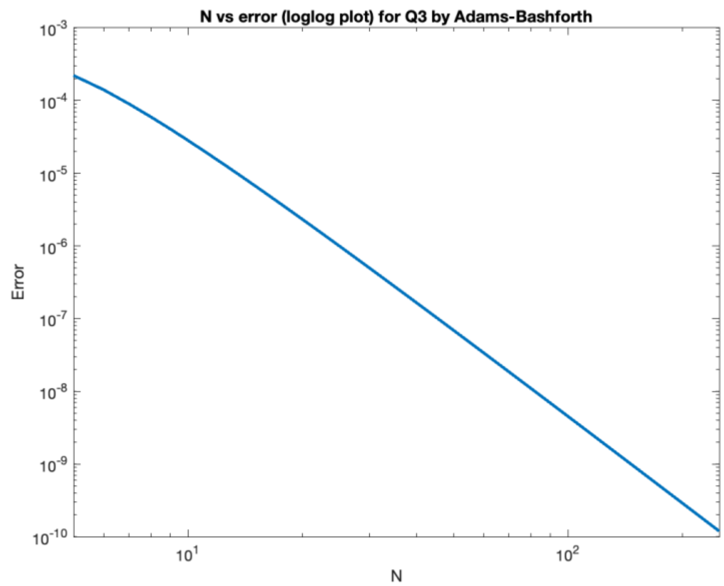
Ques – 3

```
Que-3
 Q3 by Adams-Bashforth
t                      Approximation          Exact                  Error(|Exact - Approx.|)
0.000000               0.5000000              0.5000000              0.0000000
0.200000               0.8292986              0.8292986              0.0000000
0.400000               1.2140877              1.2140877              0.0000000
0.600000               1.6489406              1.6489406              0.0000000
0.800000               2.1273124              2.1272295              0.0000828
1.000000               2.6410810              2.6408591              0.0002219
```



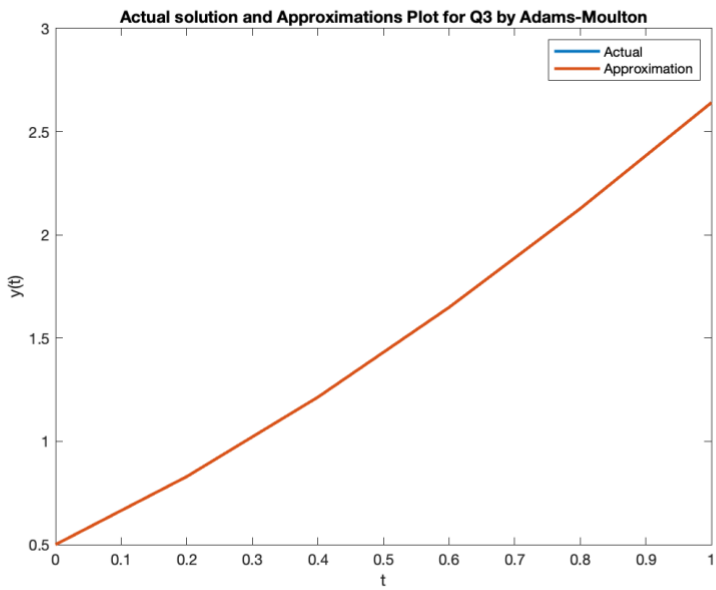


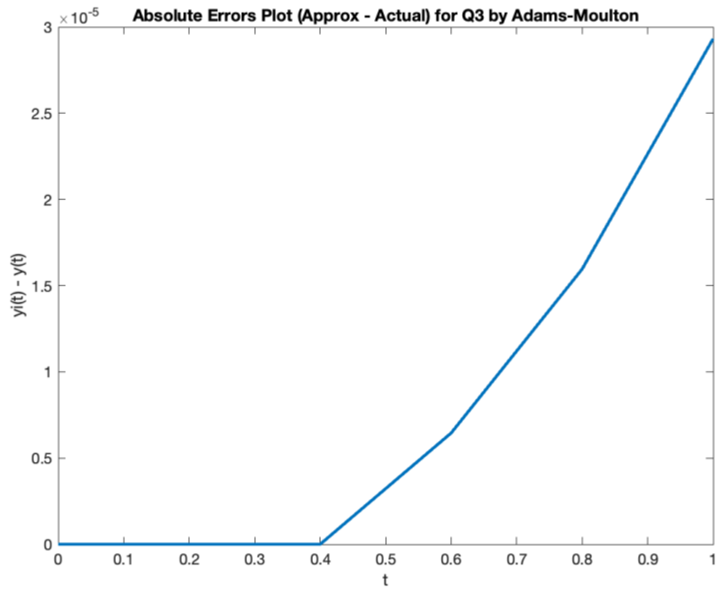


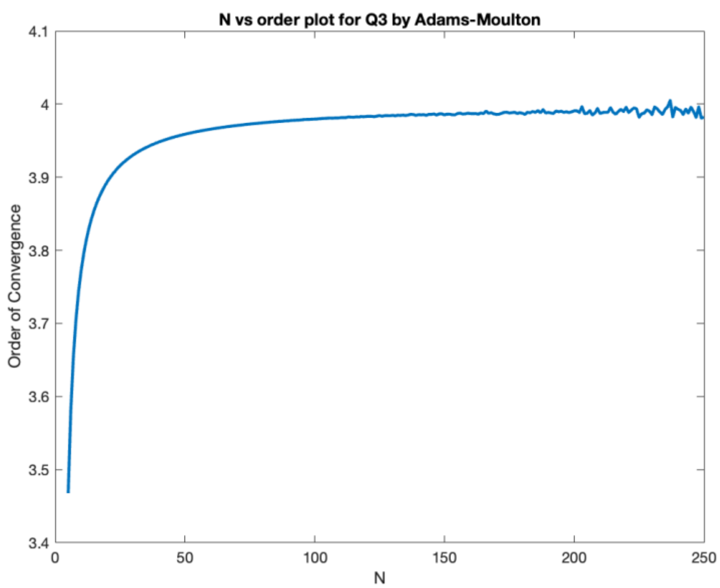


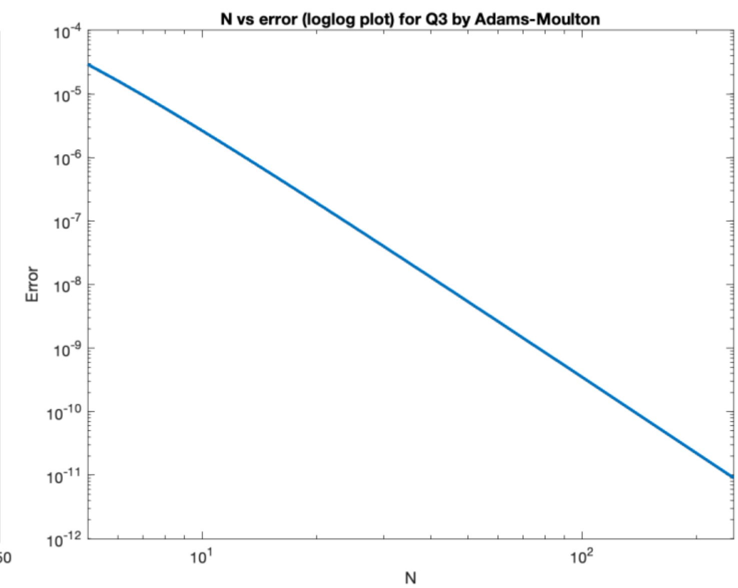
Q3 by Adams–Moulton

t	Approximation	Exact	Error(|Exact – Approx.|)
0.000000	0.5000000	0.5000000	0.0000000
0.200000	0.8292986	0.8292986	0.0000000
0.400000	1.2140877	1.2140877	0.0000000
0.600000	1.6489341	1.6489406	0.0000065
0.800000	2.1272136	2.1272295	0.0000160
1.000000	2.6408298	2.6408591	0.0000293





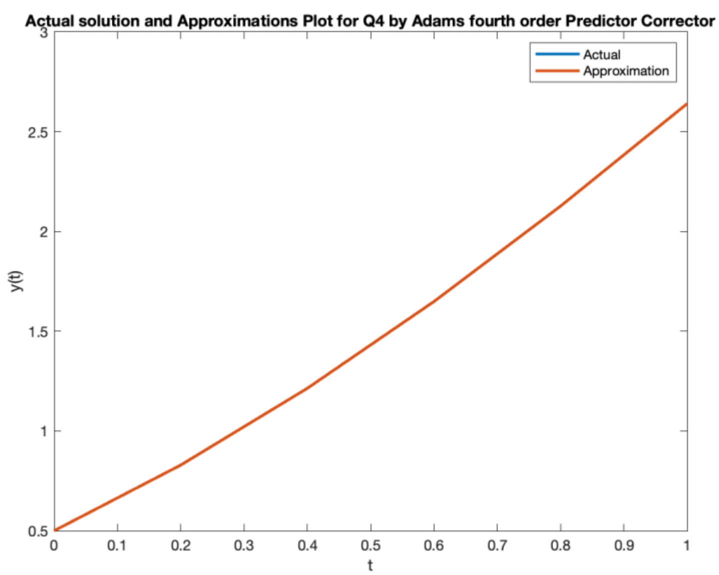


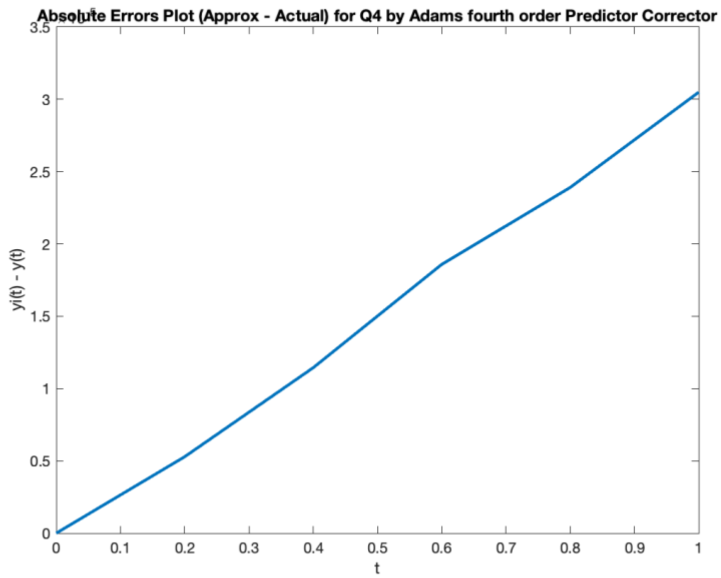


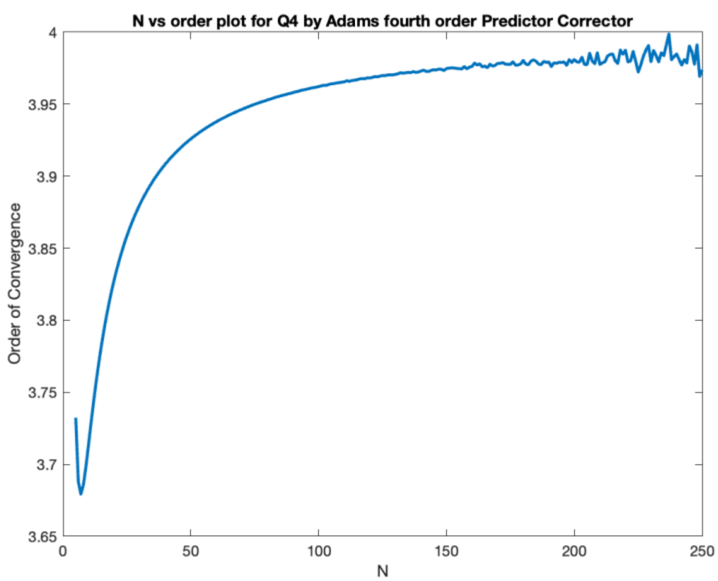
Ques – 4

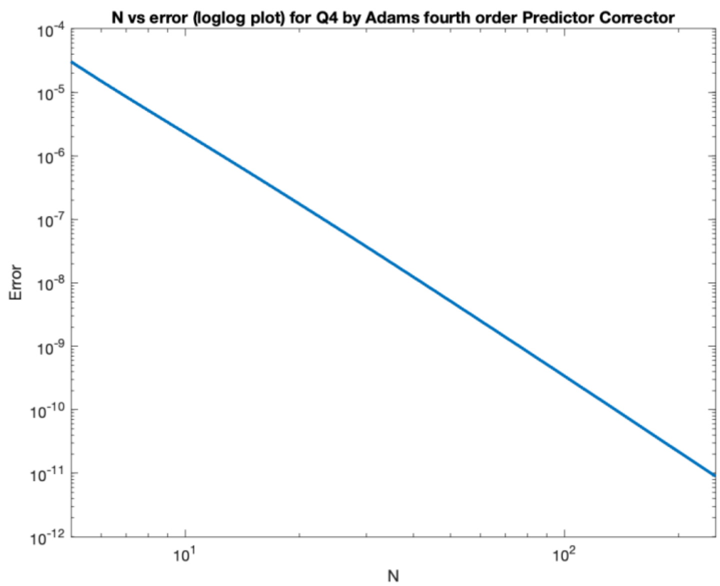
In Adams fourth order predictor-corrector method, we use the Four-step Adams-Bashforth scheme to predict Y_{i+1} and then the Three-step Adams-Moulton scheme to correct it. This gives better approximations that individual Adams-Bashforth or Adams-Moulton.

```
Que-4
 Q4 by Adams fourth order Predictor Corrector
t                       Approximation           Exact                   Error(|Exact - Approx.|)
0.000000                0.5000000               0.5000000               0.0000000
0.200000                0.8292933               0.8292986               0.0000053
0.400000                1.2140762               1.2140877               0.0000114
0.600000                1.6489220               1.6489406               0.0000186
0.800000                2.1272056               2.1272295               0.0000239
1.000000                2.6408286               2.6408591               0.0000305
```









Some Observations:-

- While implementing the Adams-Moulton scheme, the tolerance is taken to be 10^{-15} because the order of convergence plot for bigger value of tolerance (say 10^{-6}) is behaving randomly and not converging to any value.
- Since the Adams-Moulton method with low tolerance is providing a better approximation than the Adams-Bashforth method, we use Adams-Bashforth as predictor and Adams-Moulton as corrector in fourth order Adams Predictor-Corrector scheme.