DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 322: Scientific Computing Lab - 13

1. The boundary-value problem

$$y'' = y' + 2y + \cos x, \quad 0 \le x \le \frac{\pi}{2}, \quad y(0) = -0.3, \quad y(\frac{\pi}{2}) = -0.1$$

has the solution $y(x) = -\frac{1}{10}(\sin x + 3\cos x)$. Use the Linear Shooting method to approximate the solution, and compare the results to the actual solution.
(a). With $h = \frac{\pi}{4}$; (b). With $h = \frac{\pi}{8}$.

2. Solve the following two-point boundary–value problems by using forward, backward and central difference for the first–order derivative and central difference for the second–order derivative.

$$\begin{cases} y'' + \frac{4}{x}y' + \frac{2}{x^2}y = \frac{2\ln x}{x^2}, \quad 1 < x < 2. \\ \text{Boundary conditions:} \\ y(1) = \frac{1}{2}, \quad y(2) = \ln 2. \\ \text{Exact solution:} \\ y(x) = \frac{4}{x} - \frac{2}{x^2} + \ln x - \frac{3}{2}. \end{cases}$$

$$\begin{cases} y'' - 2y' + y = xe^x - x, \quad 0 < x < 2. \\ \text{Boundary conditions:} \\ y(0) = 0, \quad y(2) = -4. \\ \text{Exact solution:} \\ y(x) = \frac{1}{6}x^3e^x - \frac{5}{3}xe^x + 2e^x - x - 2. \end{cases}$$

$$\begin{cases} y'' - y' - 2y = \cos x, \quad 0 < x < \frac{\pi}{2}. \\ \text{Boundary conditions:} \\ y'(0) = -\frac{1}{10}, \quad y'\left(\frac{\pi}{2}\right) = \frac{3}{10}. \\ \text{Exact solution:} \\ y(x) = -\frac{1}{10}(\sin x + 3\cos x). \end{cases}$$

$$\begin{cases} y'' + xy' - 2y = 2 + (2 + x^2)e^x, \quad -1 < x < 0. \\ \text{Boundary conditions:} \\ y'(-1) = -2, \quad y'(0) = 1. \\ \text{Exact solution:} \\ y(x) = x^2 + xe^x. \end{cases}$$

$$\begin{cases} y'' + 2y' + y = x, \quad 0 < x < 1. \\ \text{Boundary conditions:} \\ y(0) + y'(0) = e - 3, \\ y(1) + y'(1) = 1 - \frac{2}{e}. \\ \text{Exact solution:} \\ y(x) = 2e^{-x} + (e - 2)xe^{-x} + x - 2. \end{cases}$$

$$\begin{cases} y'' + \cos(x)y' + y = (\sqrt{2} - 1)\cos^2 x - \frac{\sin 2x}{2}, \; 0 < x < \frac{\pi}{4}. \\ \text{Boundary conditions:} \\ y(0) + y'(0) = \sqrt{2}, \\ y\left(\frac{\pi}{4}\right) + y'\left(\frac{\pi}{4}\right) = 2 - \sqrt{2}. \\ \text{Exact solution:} \\ y(x) = \cos x + (\sqrt{2} - 1)\sin x. \end{cases}$$