DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 322: Scientific Computing Lab - VI

1. The following data represents the function $f(x) = \exp(x)$.

x	1.0	1.5	2.0	2.5
$f(x)$	2.7183	4.4817	7.3819	12.1825

   Estimate the value of $f(2.25)$ using the (i) Newton's forward difference interpolation and (ii) Newton's backward difference interpolation. Compare with the exact value.

2. Use Newton forward-diiference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials

   a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

   b. $f\left(\frac{-1}{3}\right)$ if $f(-0.75) = -0.07181250, f(-0.5) = -0.02475000, f(-0.25) = 0.33493750, f(0) = 1.10100000$

   Also plot the obtained interpolating polynomials.

3. Let $f(x) = \dfrac{1}{1+x^2}$ for $-5 \le x \le 5$. For each $n = 1, 2, \ldots, 10$, let $h = 10/n$ and $y_n = P_n(1+\sqrt{10})$, where $P_n(x)$ is the interpolating polynomial for $f(x)$ at the nodes $x_0^{(n)}, x_1^{(n)}, \ldots, x_n^{(n)}$ and $x_j^{(n)} = -5 + jh$, for each $j = 0, 1, \ldots, n$. Does the sequence $\{y_n\}$ appear to converge to $f(1+\sqrt{10})$? Explain your observations with reasons.

   Take P_n as Lagrange interpolant, Newton-forward and Newton-backward.