MA322 – Scientific Computing Laboratory

Lab – 11

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For each part of both the questions, 4 plots are formed, namely: -

- Y(t) vs t plot for actual and approximated solutions.
- Absolute error vs t plot.
- N vs error. (Loglog plot)
- N vs order of convergence.

Where, N is number of intervals, i.e., N = (b - a)/h.

To get the last two plots, N is varied from 1 to 100, and maximum errors are taken for N and 2N. The order is calculated by $\log_2(E_N/E_{2N})$, E_N and E_{2N} being maximum errors while computing the mentioned method for that particular value of N.

Ques – 1

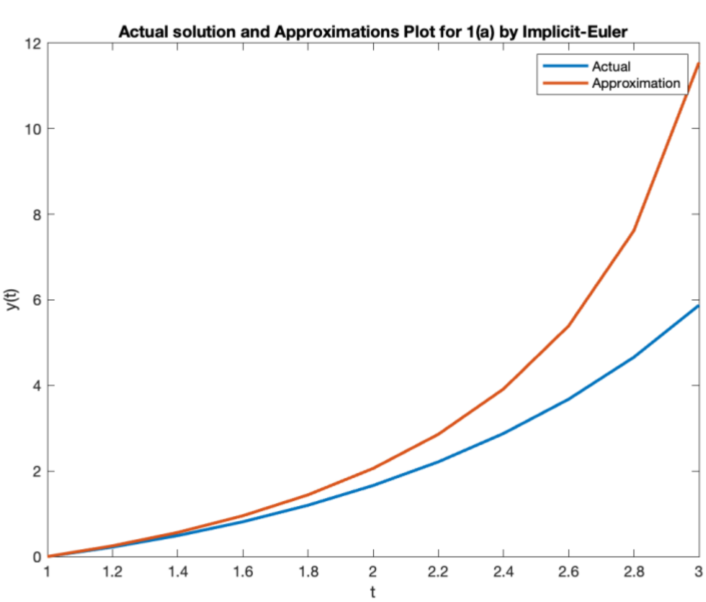
Here, the following methods are implemented for given two parts: -

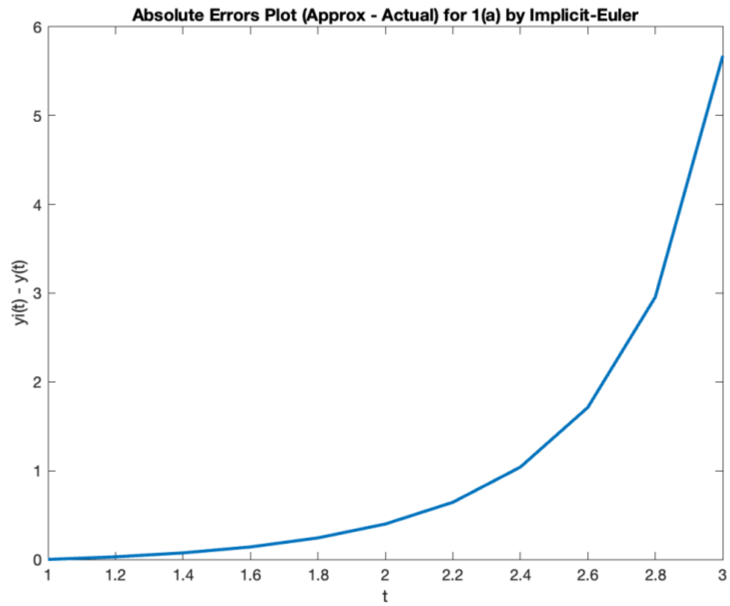
- Implicit-Euler's Method.
- 2nd Order Runge-Kutta Method for c2 = ½, 2/3 and 1.
- 4th Order Runge-Kutta Method (classical and Kutta Method).

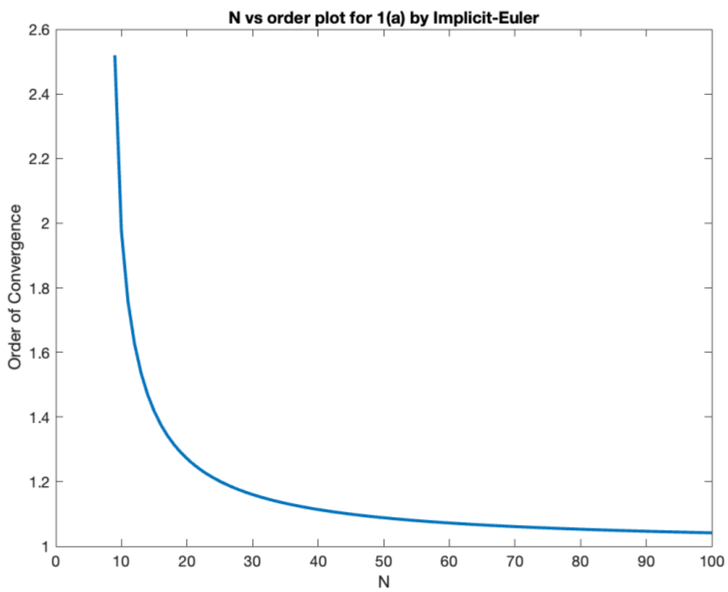
Following are the results thus obtained: -

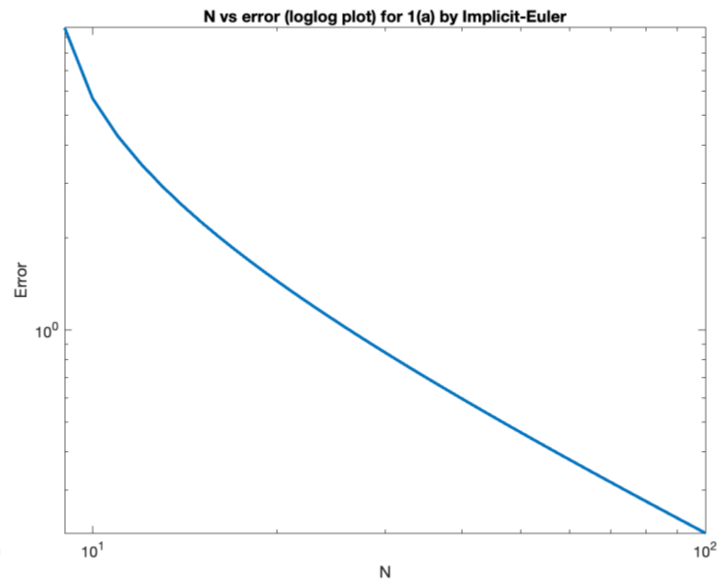
1(a) by Implicit-Euler

t	Approximation	Exact	Error(|Exact - Approx.|)
1.000000	0.000000	0.000000	0.000000
1.200000	0.250455	0.221243	0.029212
1.400000	0.563306	0.489682	0.073624
1.600000	0.953529	0.812753	0.140777
1.800000	1.442151	1.199439	0.242712
2.000000	2.060476	1.661282	0.399195
2.200000	2.857735	2.213502	0.644233
2.400000	3.916831	2.876551	1.040279
2.600000	5.391615	3.678475	1.713139
2.800000	7.614693	4.658665	2.956028
3.000000	11.548079	5.874100	5.673979



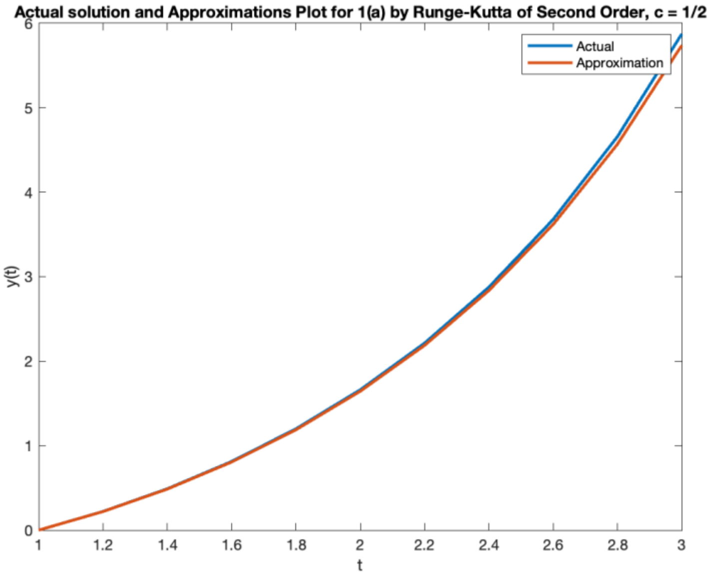


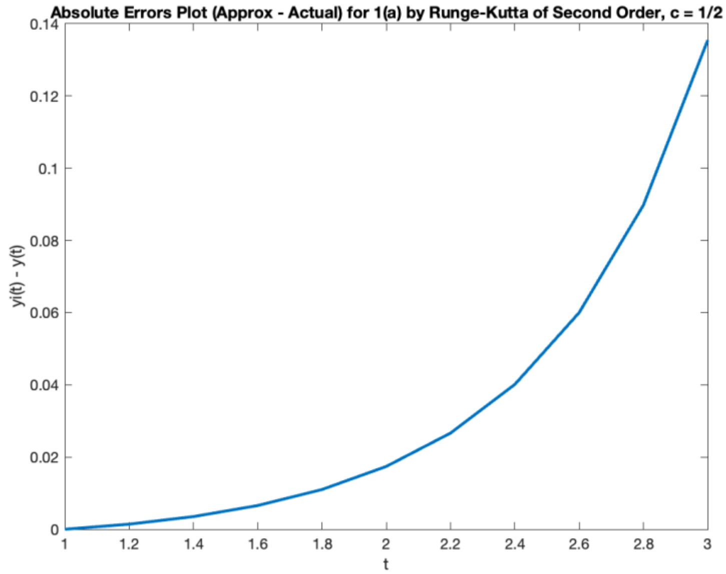


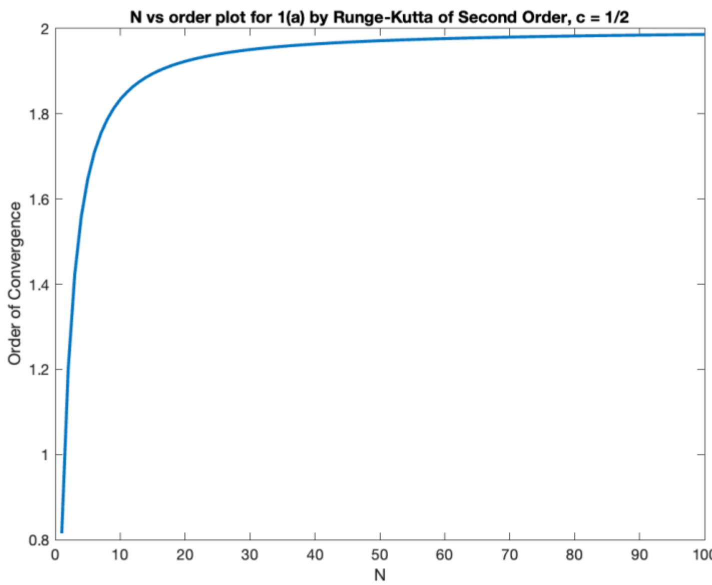


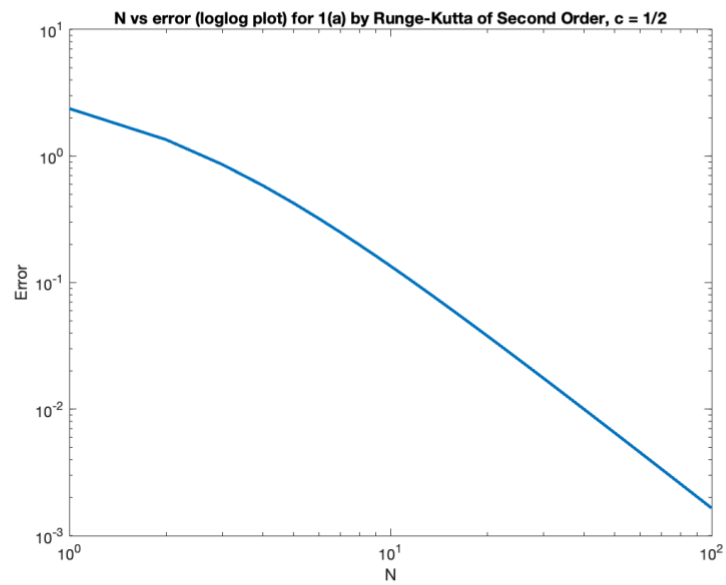
1(a) by Runge-Kutta of Second Order, c2 = 1/2

t	Approximation	Exact	Error(|Exact - Approx.|)
1.000000	0.000000	0.000000	0.000000
1.200000	0.219835	0.221243	0.001408
1.400000	0.486177	0.489682	0.003505
1.600000	0.806185	0.812753	0.006568
1.800000	1.188439	1.199439	0.010999
2.000000	1.643889	1.661282	0.017393
2.200000	2.186861	2.213502	0.026641
2.400000	2.836436	2.876551	0.040116
2.600000	3.618493	3.678475	0.059983
2.800000	4.568894	4.658665	0.089771
3.000000	5.738647	5.874100	0.135453



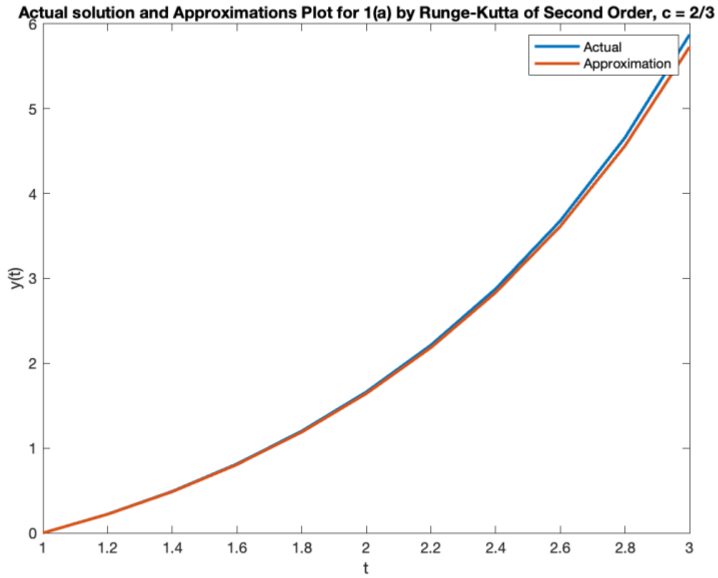


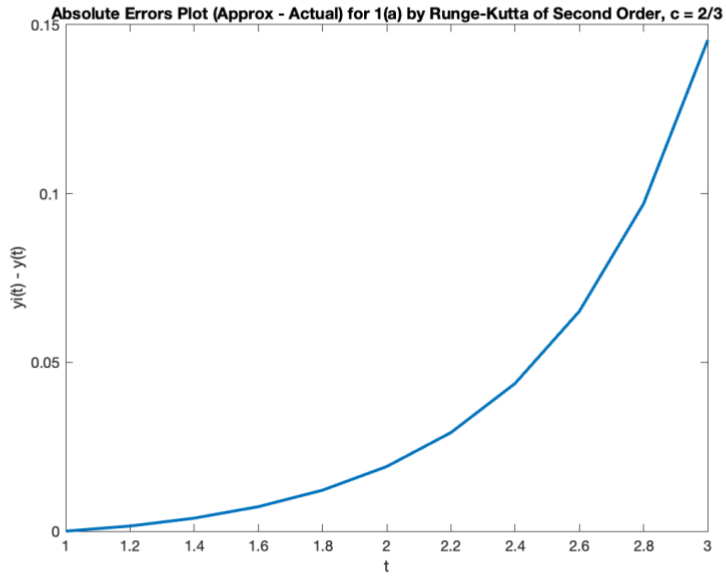


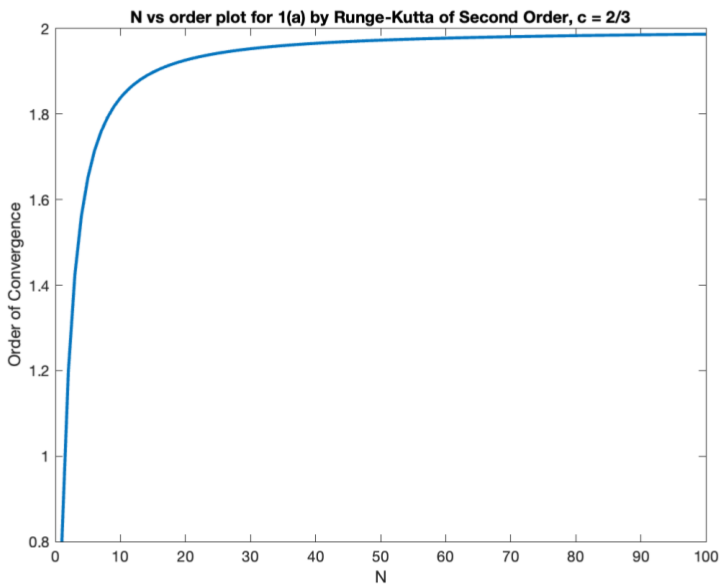


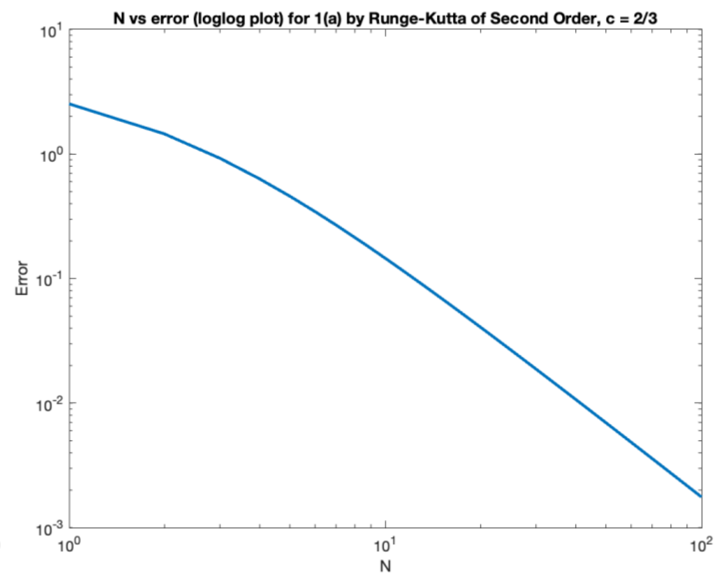
1(a) by Runge-Kutta of Second Order, c2 = 2/3

t	Approximation	Exact	Error(|Exact - Approx.|)
1.000000	0.000000	0.000000	0.000000
1.200000	0.219723	0.221243	0.001520
1.400000	0.485831	0.489682	0.003850
1.600000	0.805502	0.812753	0.007251
1.800000	1.187299	1.199439	0.012139
2.000000	1.642139	1.661282	0.019143
2.200000	2.184295	2.213502	0.029207
2.400000	2.832773	2.876551	0.043779
2.600000	3.613343	3.678475	0.065133
2.800000	4.561706	4.658665	0.096959
3.000000	5.728625	5.874100	0.145475



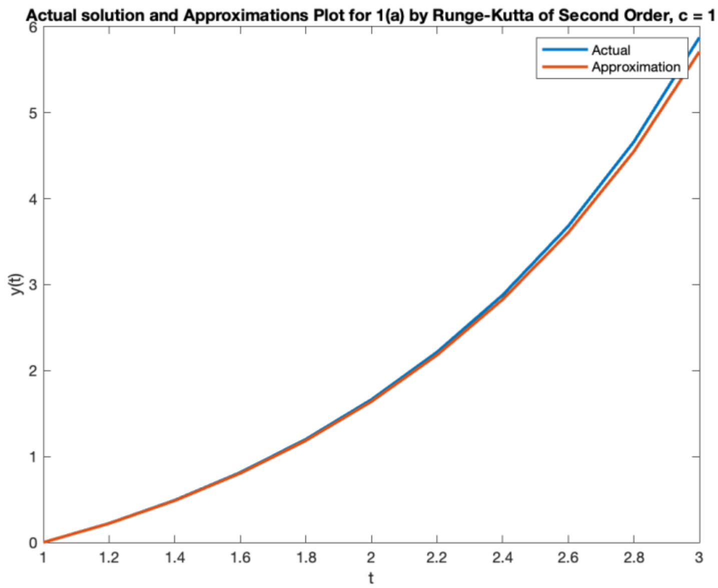


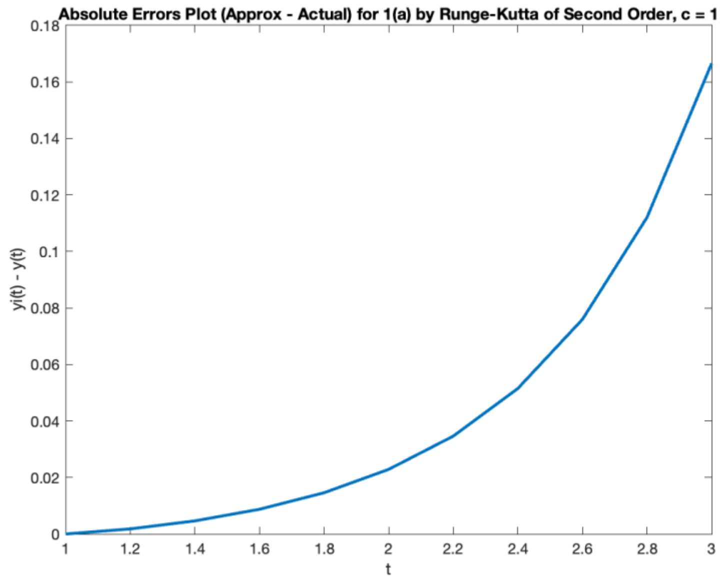


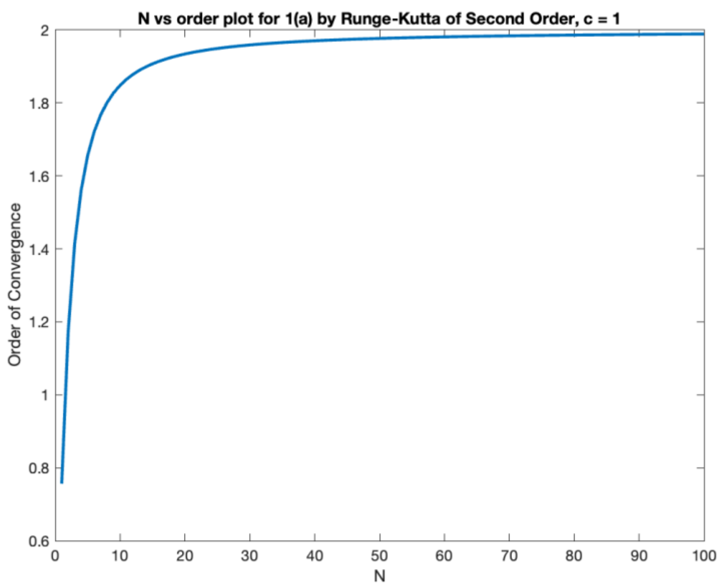


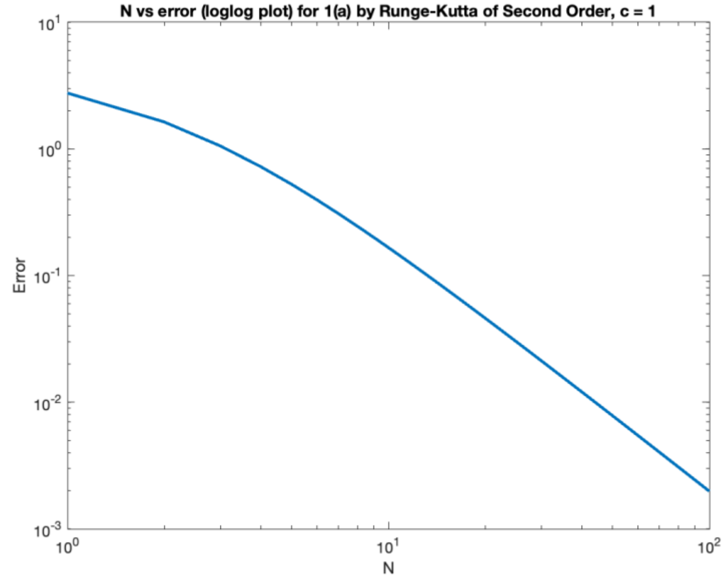
1(a) by Runge-Kutta of Second Order, c2 = 1

t	Approximation	Exact	Error(|Exact - Approx.|)
1.000000	0.000000	0.000000	0.000000
1.200000	0.219444	0.221243	0.001798
1.400000	0.485049	0.489682	0.004632
1.600000	0.804012	0.812753	0.008741
1.800000	1.184856	1.199439	0.014583
2.000000	1.638423	1.661282	0.022859
2.200000	2.178877	2.213502	0.034625
2.400000	2.825065	2.876551	0.051486
2.600000	3.602525	3.678475	0.075951
2.800000	4.546614	4.658665	0.112052
3.000000	5.707570	5.874100	0.166530



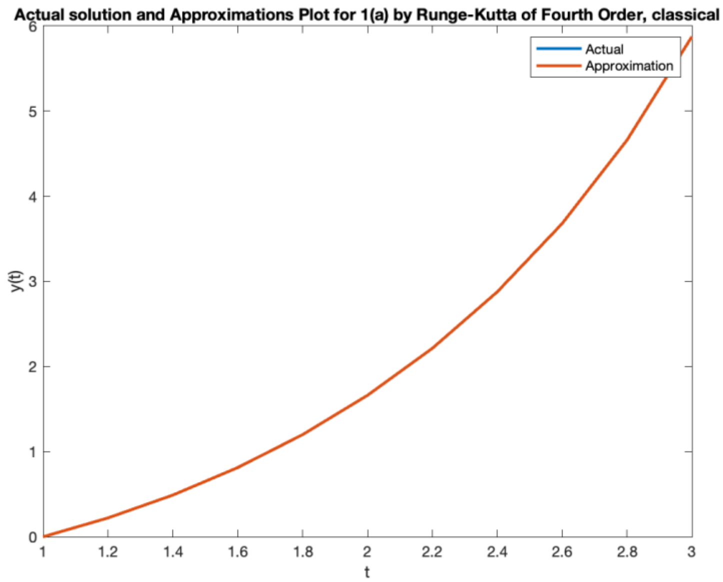


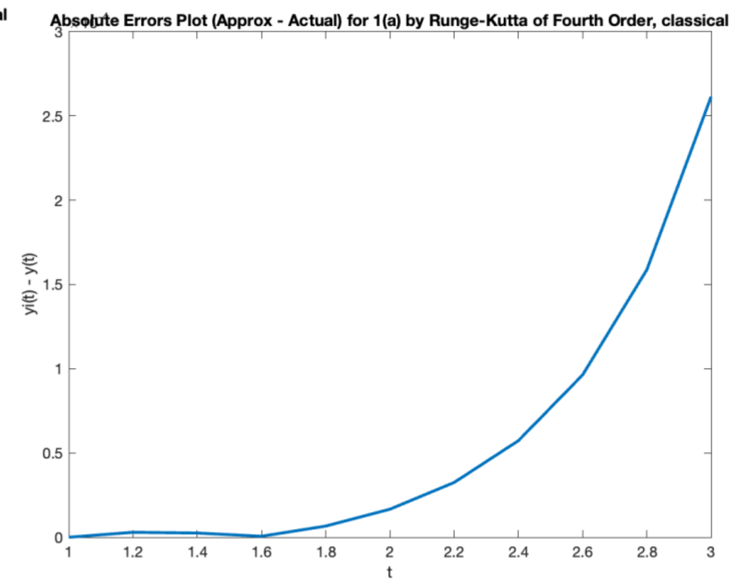


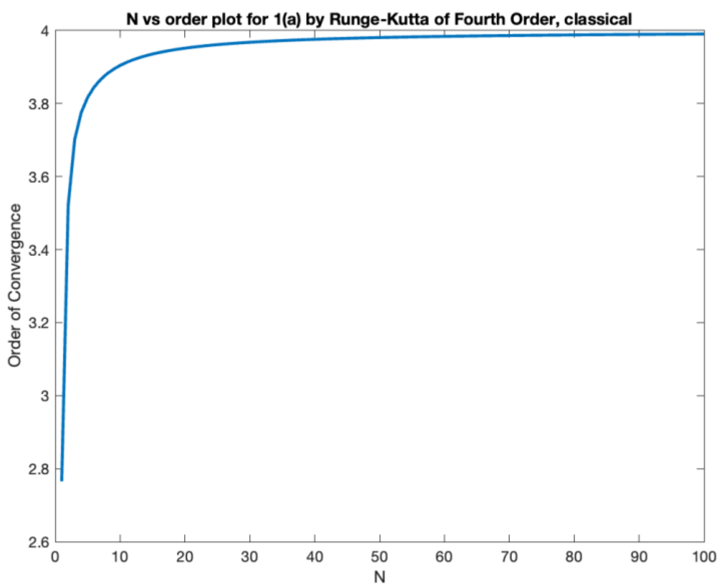


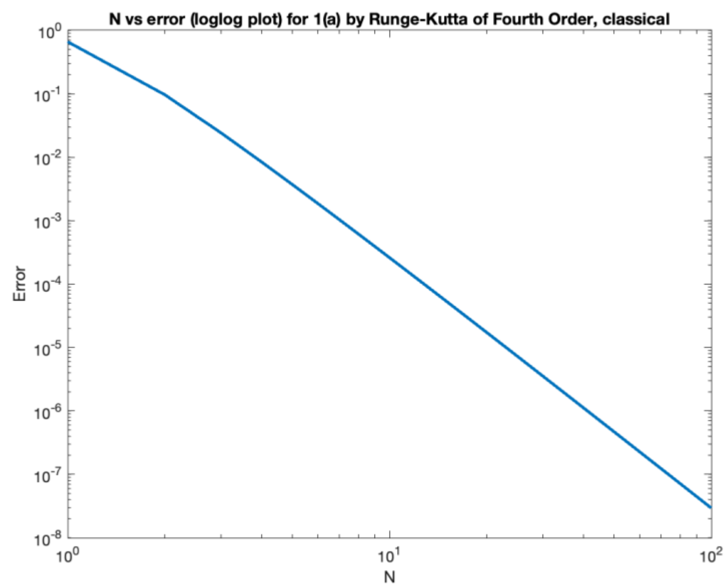
1(a) by Runge-Kutta of Fourth Order, classical

t	Approximation	Exact	Error(|Exact - Approx.|)
1.000000	0.000000	0.000000	0.000000
1.200000	0.221246	0.221243	0.000003
1.400000	0.489684	0.489682	0.000003
1.600000	0.812752	0.812753	0.000001
1.800000	1.199432	1.199439	0.000007
2.000000	1.661265	1.661282	0.000017
2.200000	2.213469	2.213502	0.000032
2.400000	2.876494	2.876551	0.000057
2.600000	3.678379	3.678475	0.000096
2.800000	4.658506	4.658665	0.000159
3.000000	5.873839	5.874100	0.000261



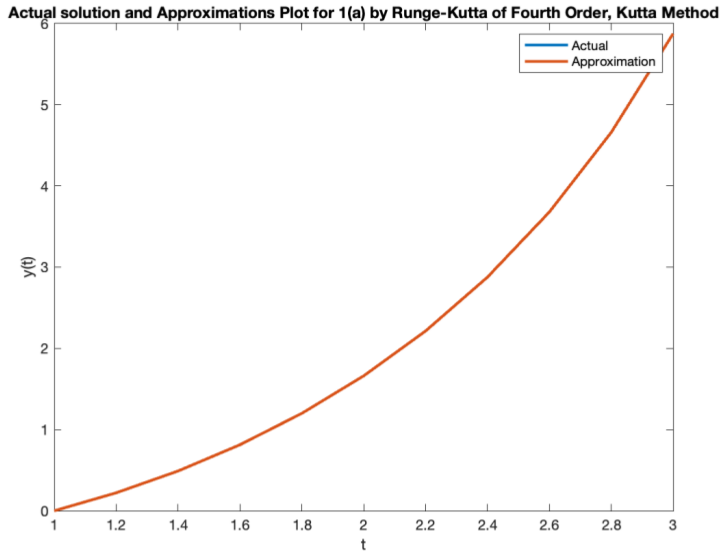


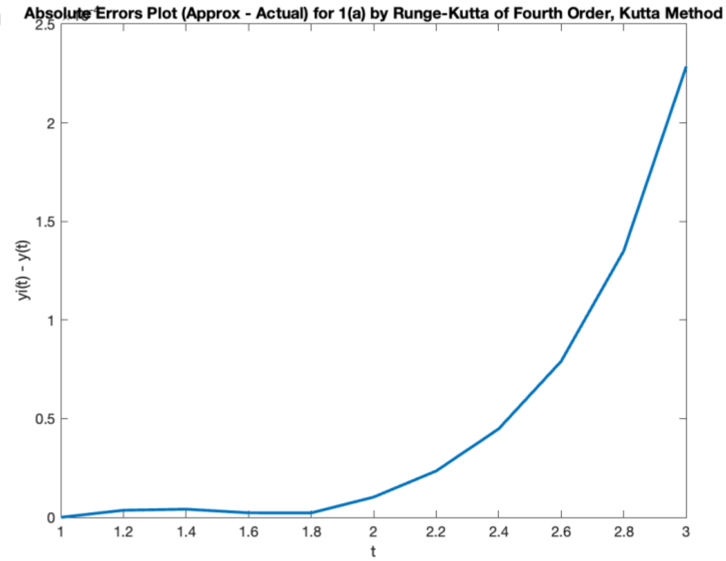


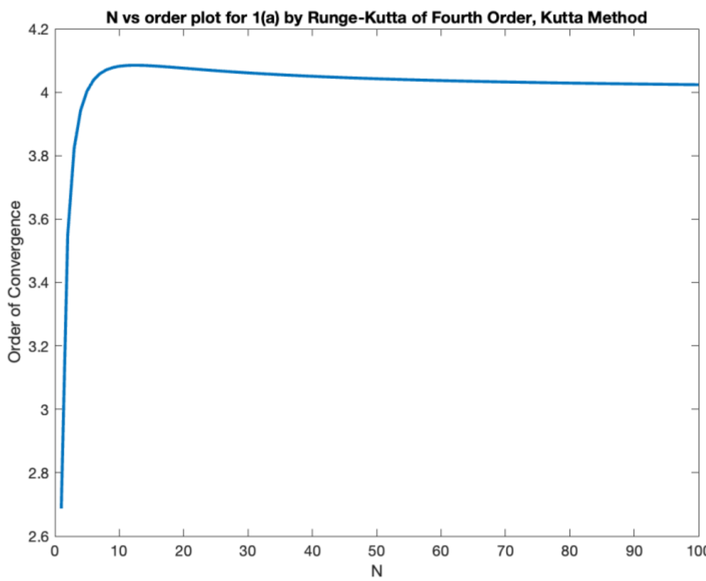


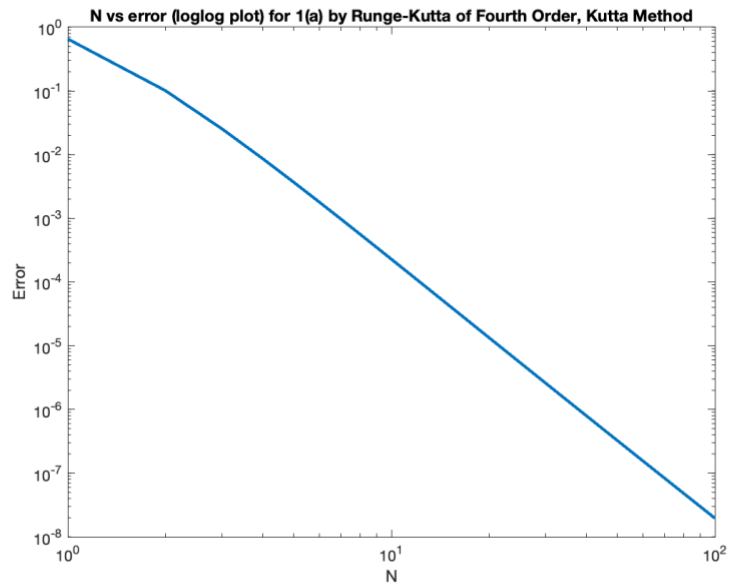
1(a) by Runge-Kutta of Fourth Order, Kutta Method

t	Approximation	Exact	Error(|Exact - Approx.|)
1.000000	0.000000	0.000000	0.000000
1.200000	0.221246	0.221243	0.000004
1.400000	0.489686	0.489682	0.000004
1.600000	0.812755	0.812753	0.000002
1.800000	1.199436	1.199439	0.000002
2.000000	1.661272	1.661282	0.000010
2.200000	2.213478	2.213502	0.000023
2.400000	2.876507	2.876551	0.000045
2.600000	3.678396	3.678475	0.000079
2.800000	4.658530	4.658665	0.000135
3.000000	5.873871	5.874100	0.000228



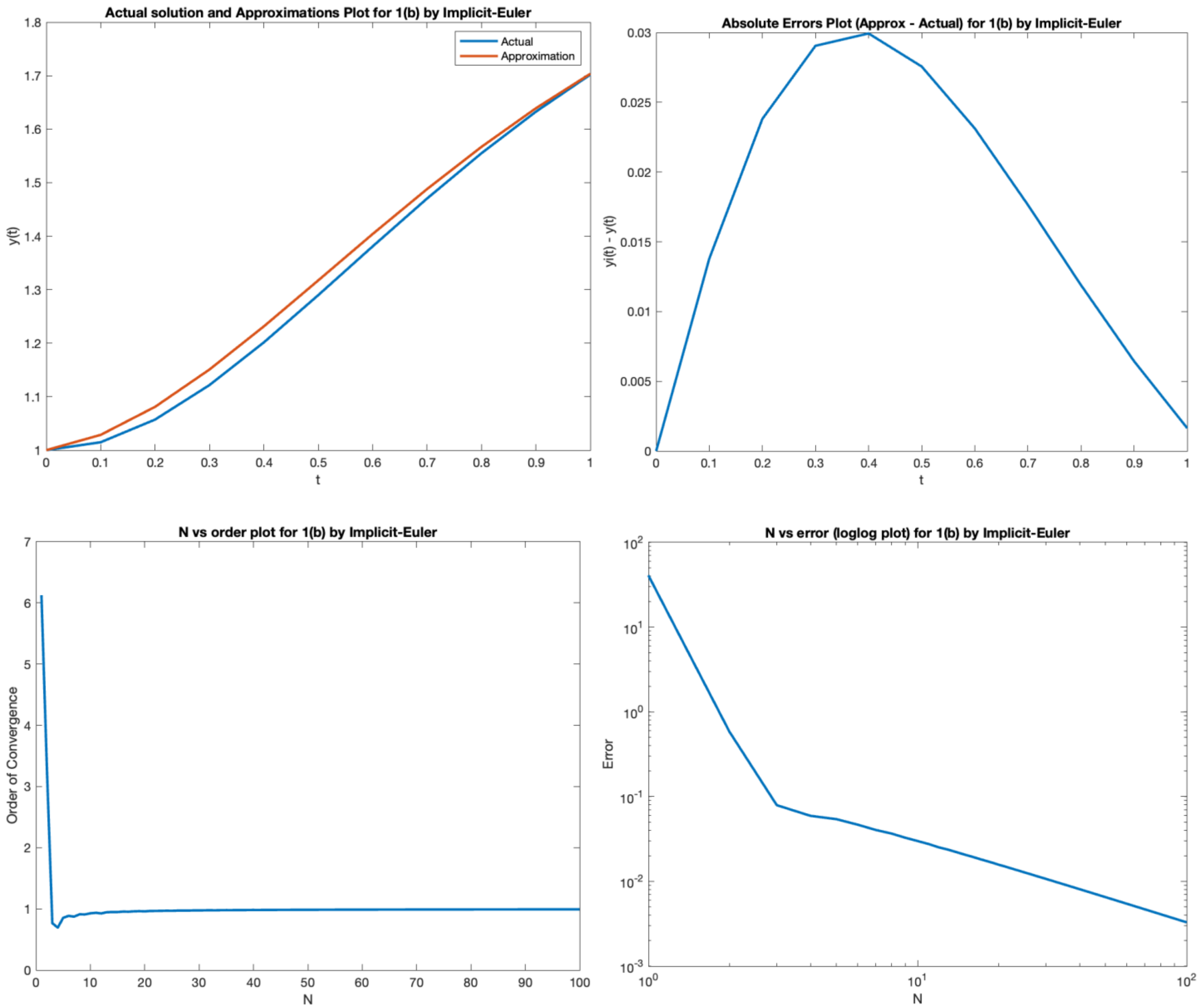






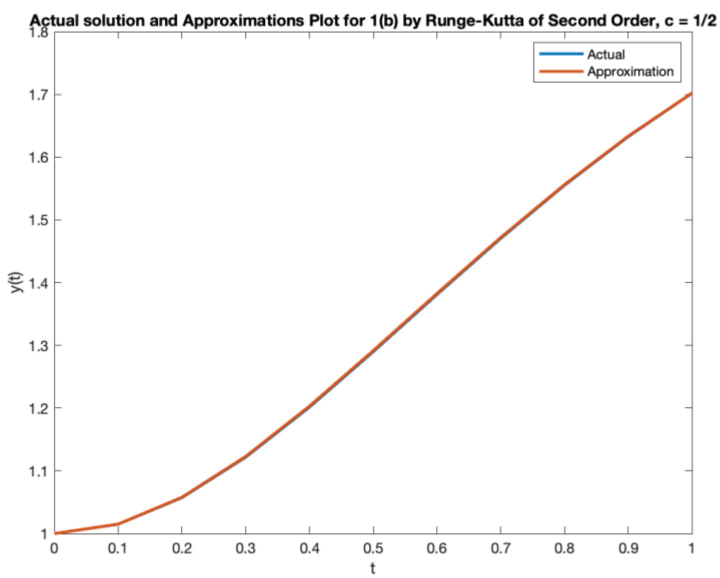
1(b) by Implicit-Euler

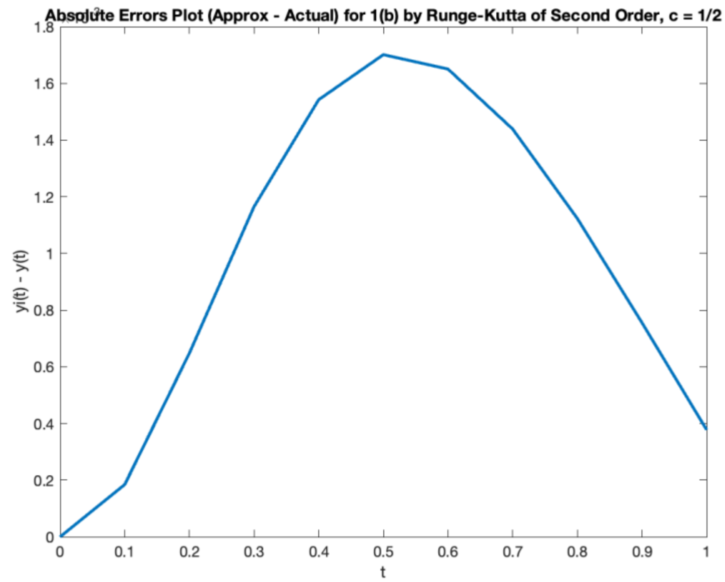
t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.100000	1.028602	1.014815	0.013786
0.200000	1.080988	1.057181	0.023807
0.300000	1.150746	1.121698	0.029048
0.400000	1.231421	1.201486	0.029935
0.500000	1.317370	1.289805	0.027564
0.600000	1.404059	1.380931	0.023128
0.700000	1.488059	1.470415	0.017644
0.800000	1.566927	1.555031	0.011895
0.900000	1.639052	1.632613	0.006438
1.000000	1.703510	1.701870	0.001640

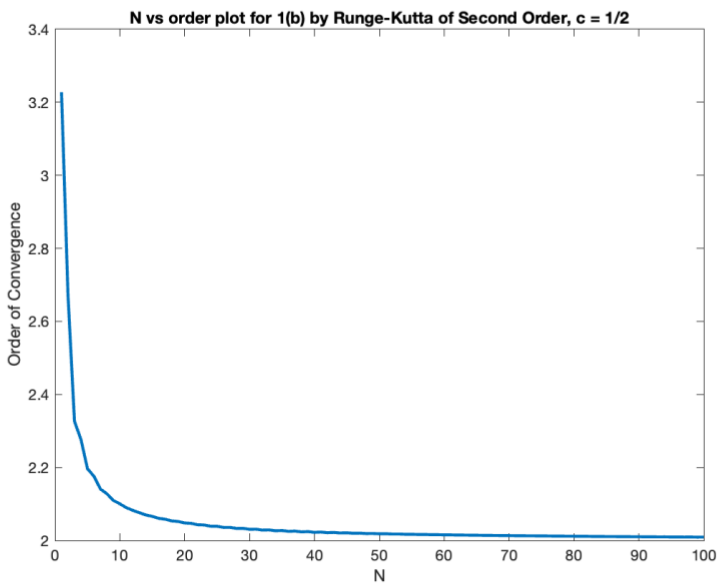


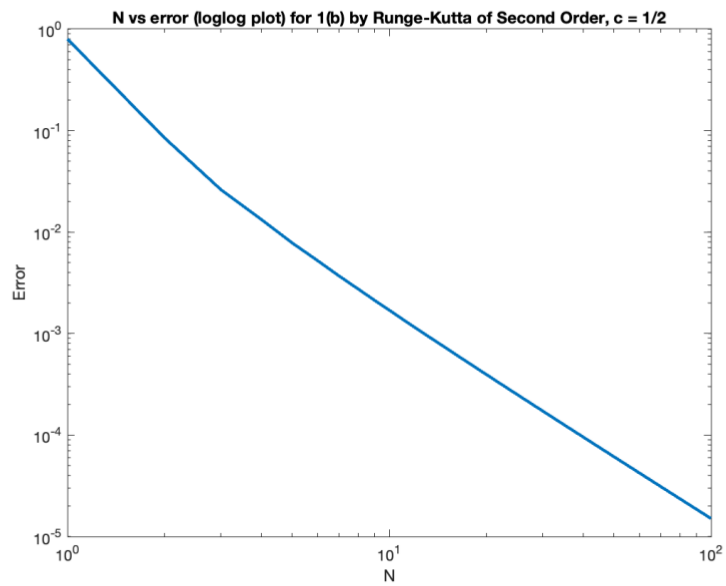
1(b) by Runge-Kutta of Second Order, c2 = 1/2

t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.100000	1.015000	1.014815	0.000185
0.200000	1.057829	1.057181	0.000648
0.300000	1.122863	1.121698	0.001165
0.400000	1.203028	1.201486	0.001542
0.500000	1.291506	1.289805	0.001701
0.600000	1.382582	1.380931	0.001650
0.700000	1.471854	1.470415	0.001439
0.800000	1.556155	1.555031	0.001123
0.900000	1.633369	1.632613	0.000756
1.000000	1.702248	1.701870	0.000378



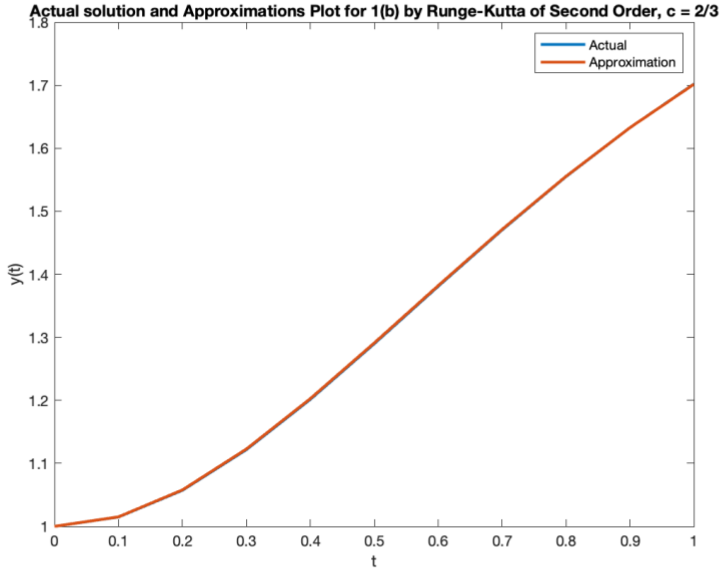


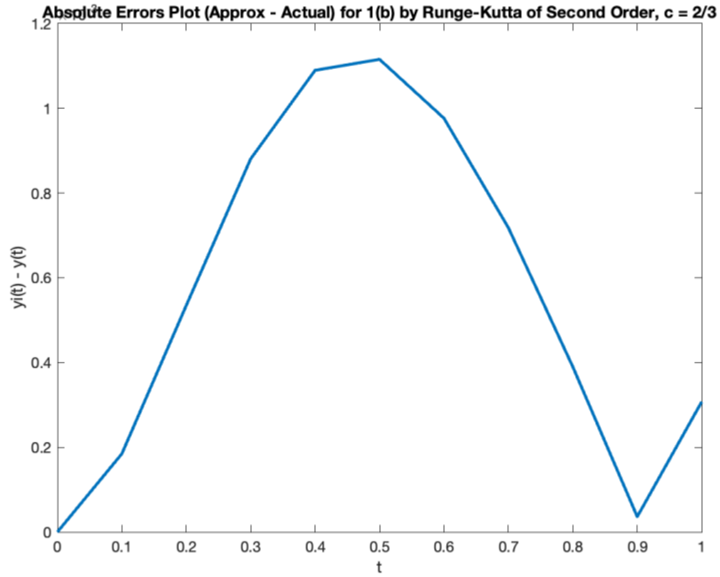


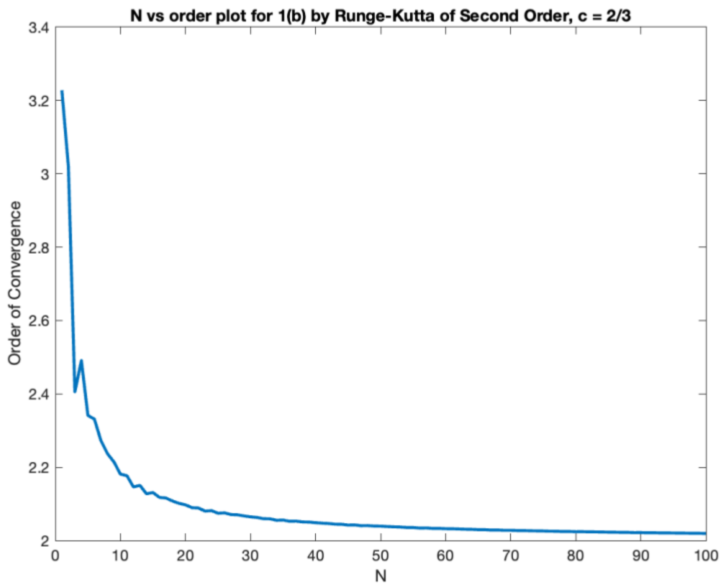


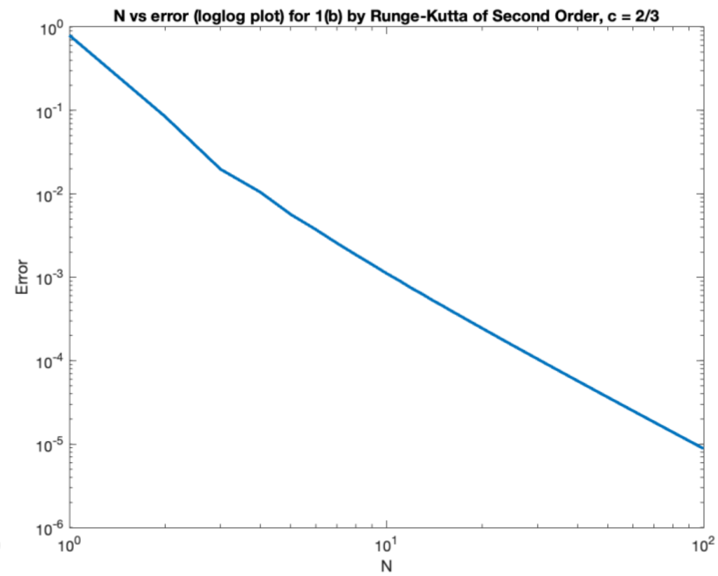
1(b) by Runge-Kutta of Second Order, c2 = 2/3

t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.100000	1.015000	1.014815	0.000185
0.200000	1.057716	1.057181	0.000535
0.300000	1.122579	1.121698	0.000881
0.400000	1.202576	1.201486	0.001090
0.500000	1.290921	1.289805	0.001116
0.600000	1.381908	1.380931	0.000977
0.700000	1.471133	1.470415	0.000718
0.800000	1.555421	1.555031	0.000390
0.900000	1.632649	1.632613	0.000036
1.000000	1.701563	1.701870	0.000307



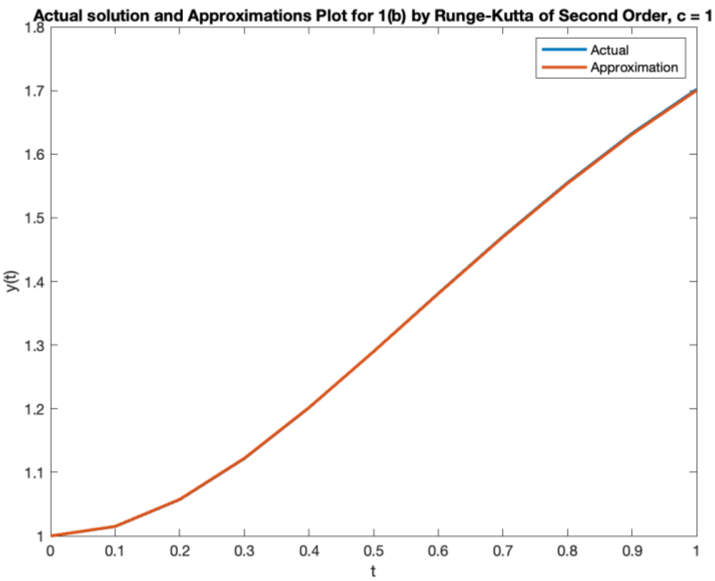


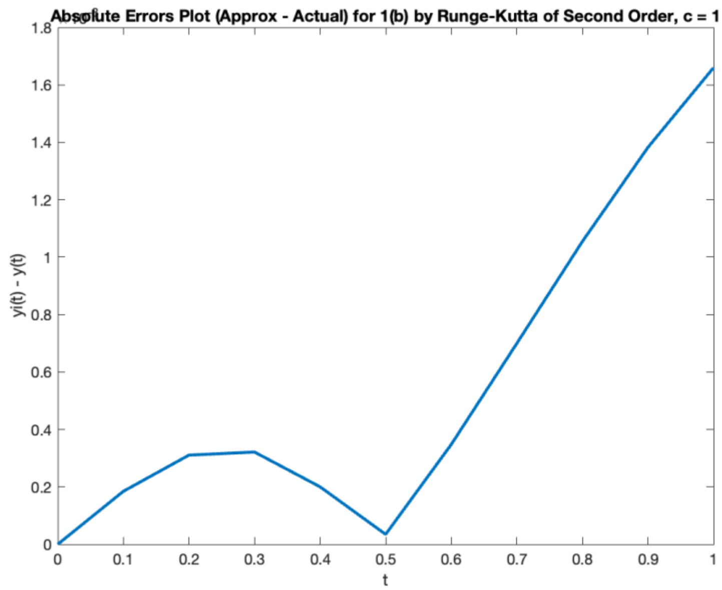


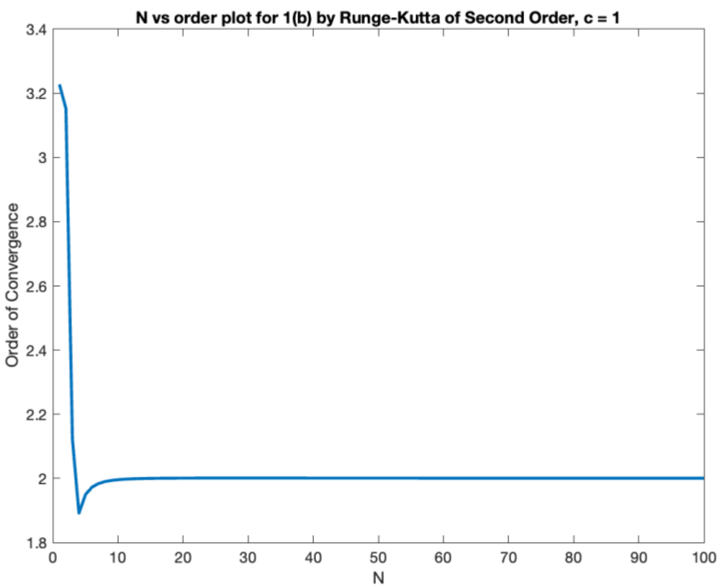


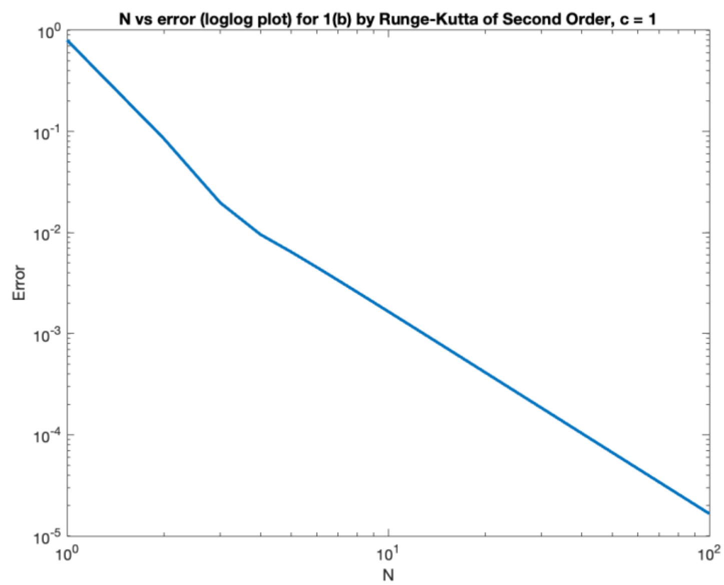
1(b) by Runge-Kutta of Second Order, c2 = 1

t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.100000	1.015000	1.014815	0.000185
0.200000	1.057492	1.057181	0.000311
0.300000	1.122019	1.121698	0.000321
0.400000	1.201686	1.201486	0.000200
0.500000	1.289771	1.289805	0.000035
0.600000	1.380584	1.380931	0.000348
0.700000	1.469716	1.470415	0.000700
0.800000	1.553977	1.555031	0.001055
0.900000	1.631231	1.632613	0.001382
1.000000	1.700210	1.701870	0.001660



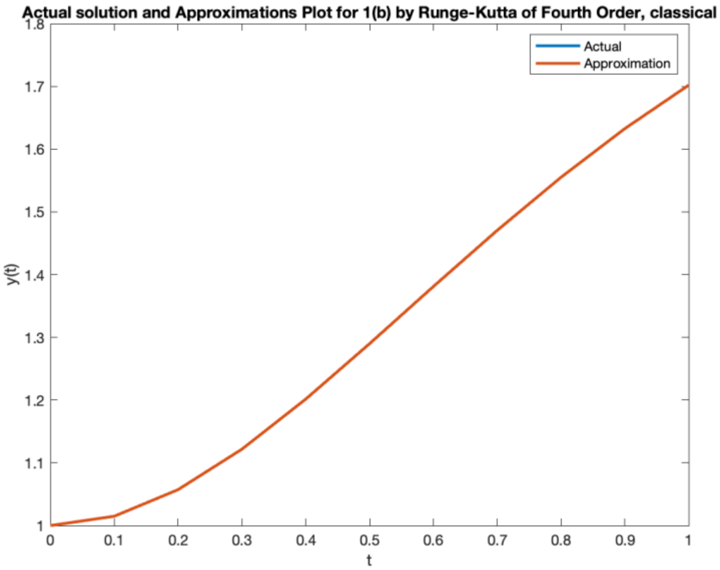


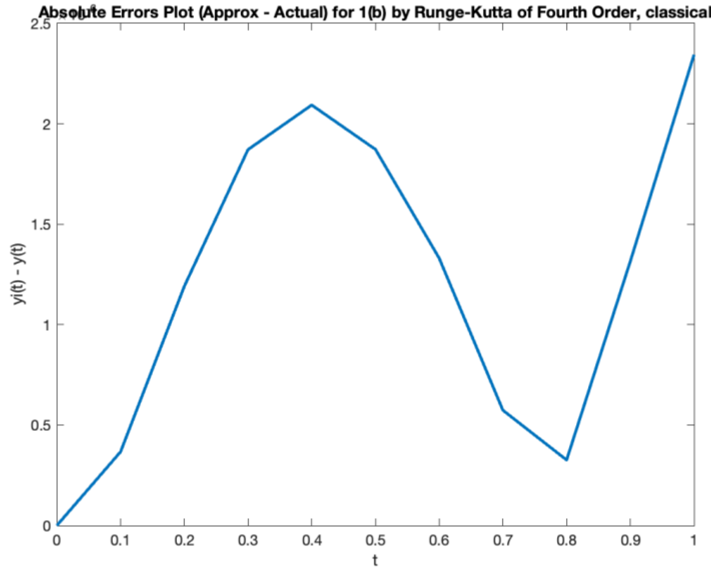


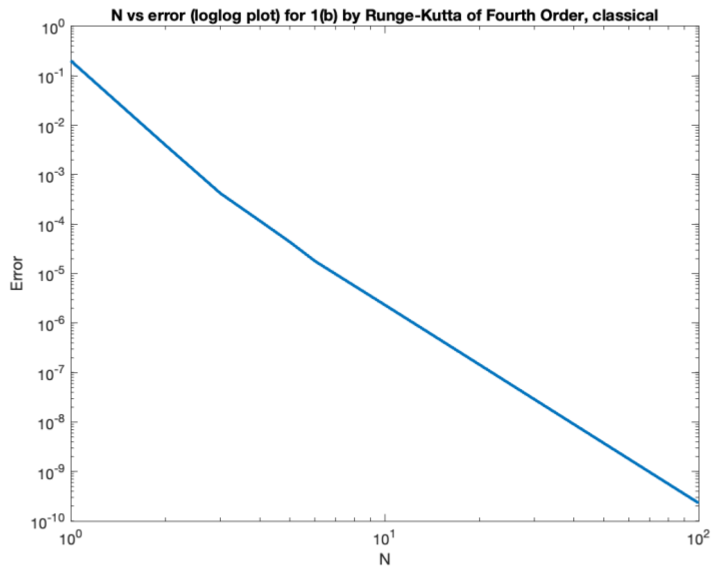


1(b) by Runge-Kutta of Fourth Order, classical

t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.100000	1.014816	1.014815	0.000000
0.200000	1.057182	1.057181	0.000001
0.300000	1.121700	1.121698	0.000002
0.400000	1.201488	1.201486	0.000002
0.500000	1.289807	1.289805	0.000002
0.600000	1.380933	1.380931	0.000001
0.700000	1.470416	1.470415	0.000001
0.800000	1.555031	1.555031	0.000000
0.900000	1.632612	1.632613	0.000001
1.000000	1.701868	1.701870	0.000002

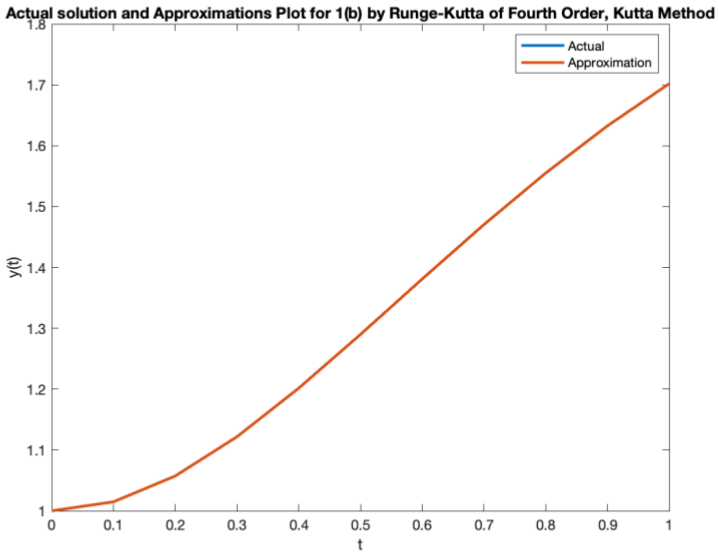


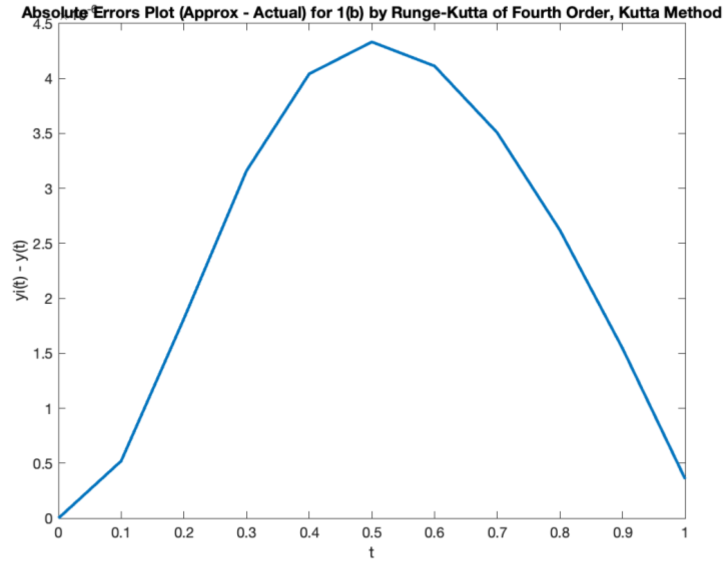


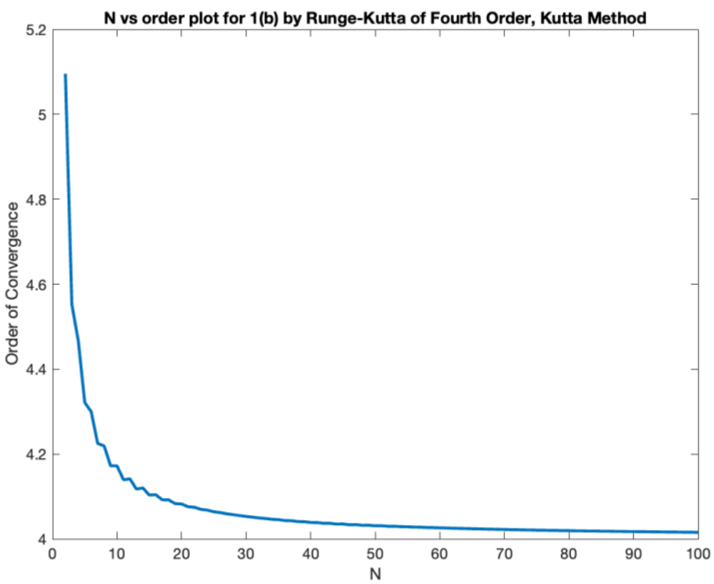


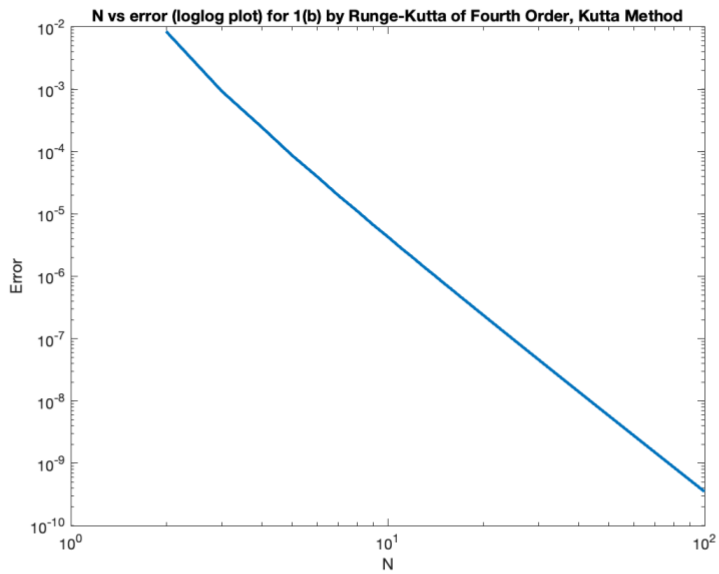
1(b) by Runge-Kutta of Fourth Order, Kutta Method

t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.100000	1.014816	1.014815	0.000001
0.200000	1.057183	1.057181	0.000002
0.300000	1.121701	1.121698	0.000003
0.400000	1.201490	1.201486	0.000004
0.500000	1.289810	1.289805	0.000004
0.600000	1.380935	1.380931	0.000004
0.700000	1.470419	1.470415	0.000004
0.800000	1.555034	1.555031	0.000003
0.900000	1.632615	1.632613	0.000002
1.000000	1.701870	1.701870	0.000000









It is observed that the Implicit-Euler is performing the worst in these methods and Fourth Order Runge Kutta is giving a very good approximation of the given IVP. The graph for actual and approximated solution is almost coinciding in case me Fourth Order Runge-Kutta methods.

Second Order Runge-Kutta methods are performing better than the Implicit-Euler's method while worse than Fourth Order Runge-Kutta method, which is as expected.

Ques – 2

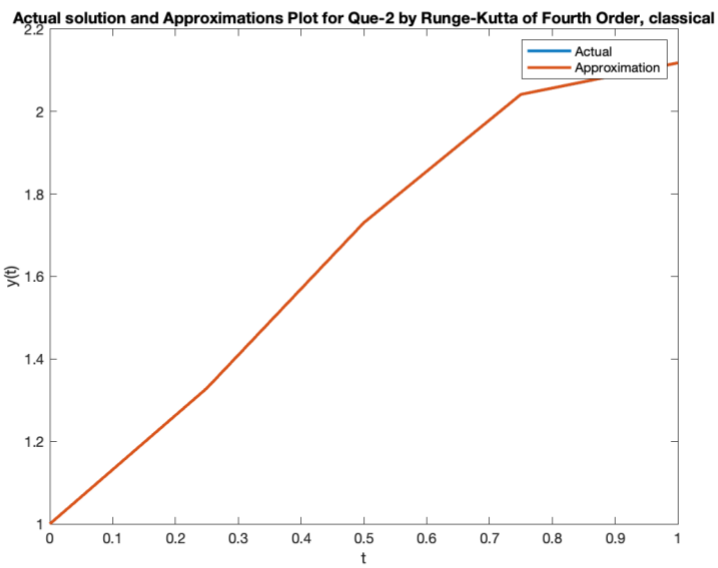
Here the Fourth Order Runge-Kutta methods are implemented for the given IVP and those 4 plots same is question-1 are formed. The results obtained are as follows: -

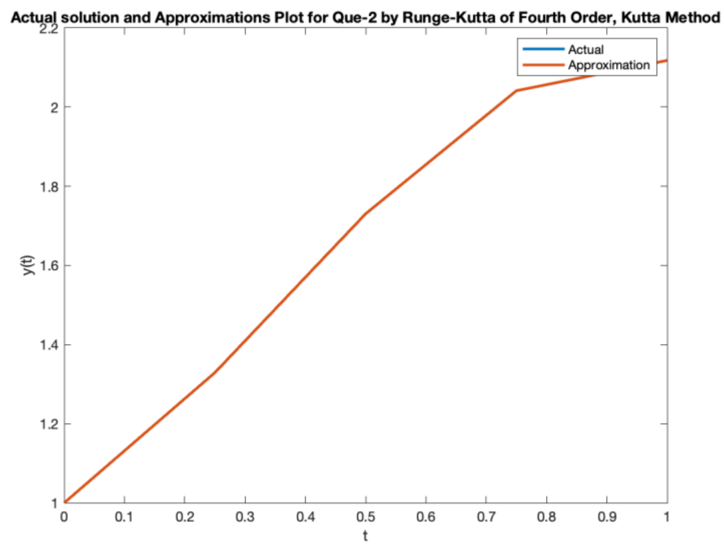
Que-2 by Runge-Kutta of Fourth Order, Kutta Method

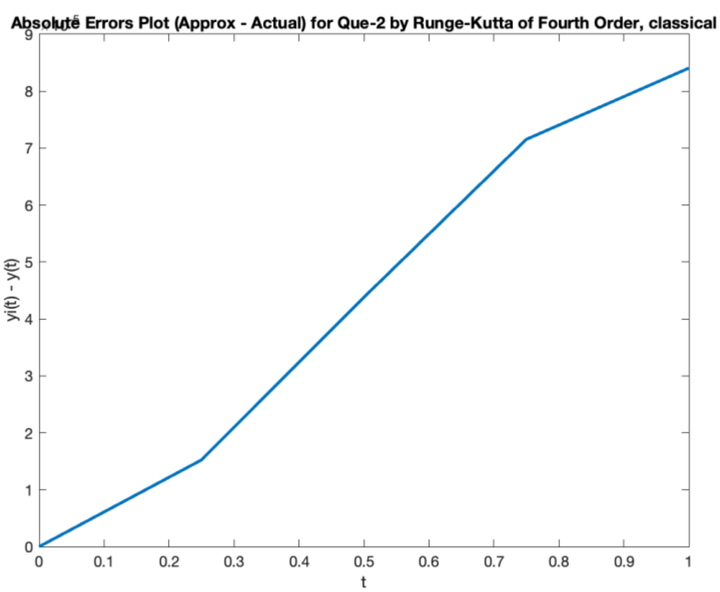
t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.250000	1.329157	1.329150	0.000007
0.500000	1.730509	1.730490	0.000019
0.750000	2.041504	2.041472	0.000032
1.000000	2.118017	2.117980	0.000037

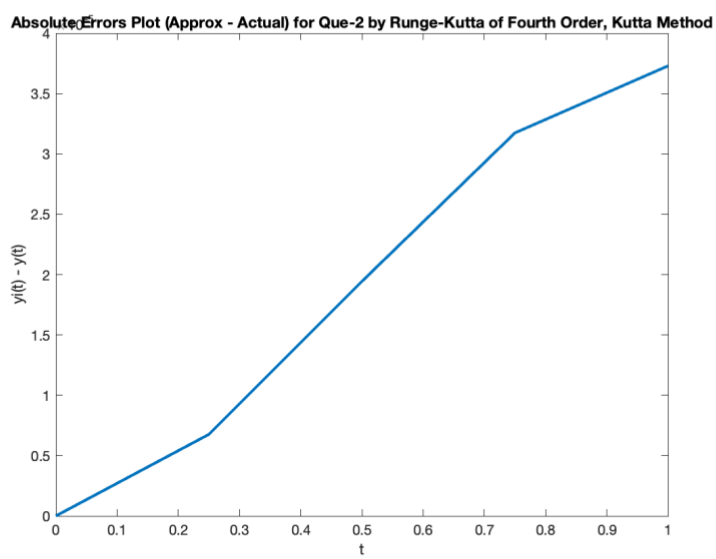
Que-2 by Runge-Kutta of Fourth Order, classical

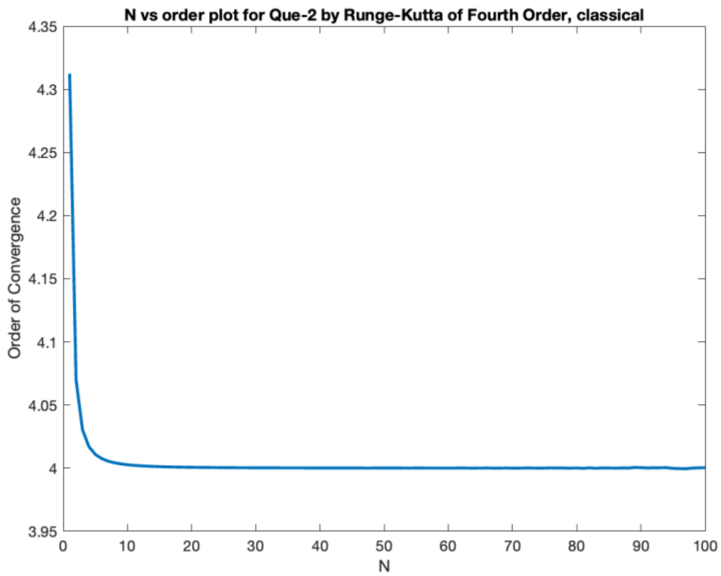
t	Approximation	Exact	Error(|Exact - Approx.|)
0.000000	1.000000	1.000000	0.000000
0.250000	1.329165	1.329150	0.000015
0.500000	1.730534	1.730490	0.000044
0.750000	2.041544	2.041472	0.000072
1.000000	2.118064	2.117980	0.000084

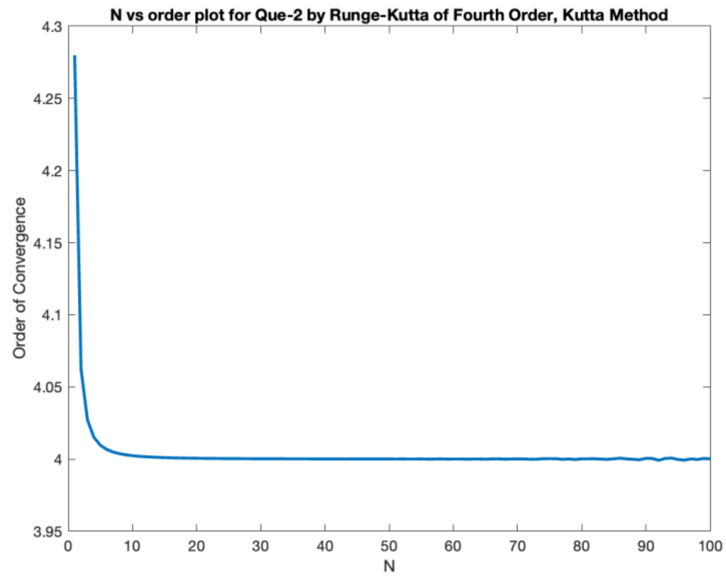


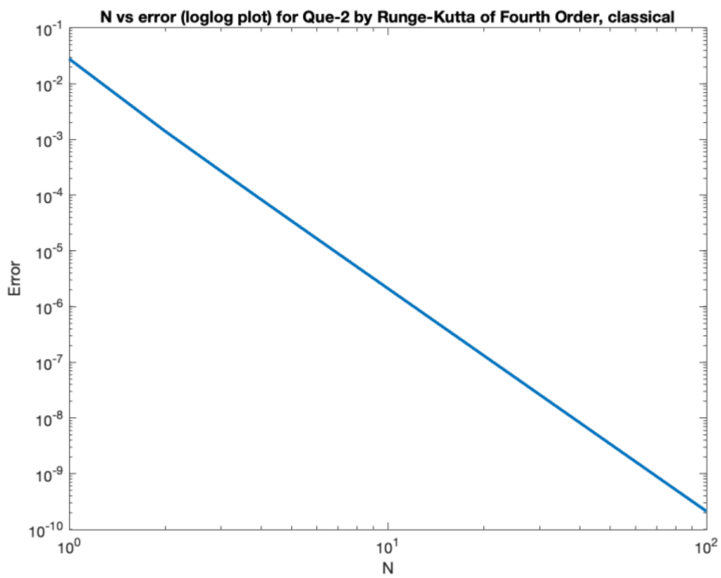


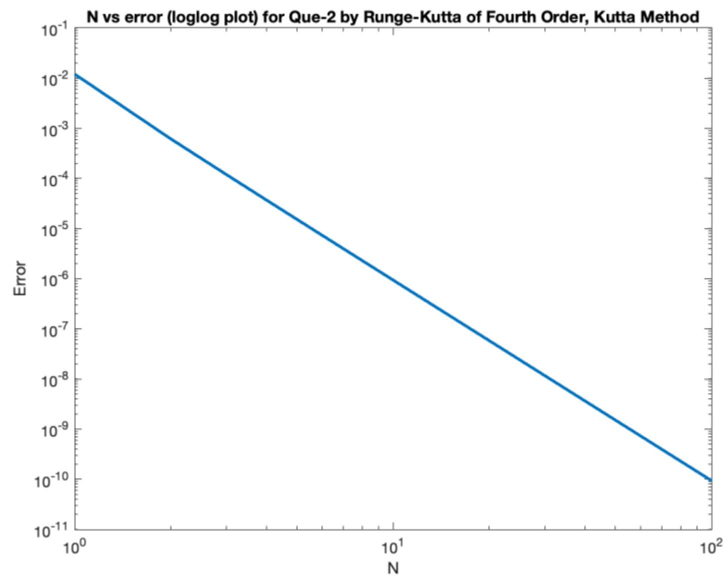












Both classical and Kutta method are performing nearly the same as each other. Kutta method is giving better final estimate in this case.