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MA322: Scientific Computing Lab – 7 (2024)

1. By using *Hermite* interpolation, construct an approximating polynomial for the following data which is generated using the function $f(x) = \ln(\exp(x) + 2)$:

x	$f(x)$	$f'(x)$
-1	0.86199480	0.15536240
-0.5	0.95802009	0.23269654
0	1.0986123	0.33333333
0.5	1.2943767	0.45186776

   Approximate $f(0.25)$ and calculate the absolute error.

2. A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

   (a) Use a *Hermite* polynomial to predict the position of the car and its speed when $t = 10$ seconds.

   (b) Use the derivative of the *Hermite* polynomial to determine whether the car ever exceeds a 55 mi/h speed limit on the road. If so, what is the first time the car exceeds this speed?

   (c) What is the predicted maximum speed for the car?

3. Use the cubic splines for the value of $x = 0$ and $x = 0.5$ to approximate $f(x)$ and $f'(x)$, and calculate the actual error.

   (a) $f(x) = e^{2x}$; approximate $f(0.43)$ and $f(0.43)$.

   (b) $f(x) = \ln(e^x + 2)$; approximate $f(0.25)$ and $f(0.25)$.

4. Construct the clamped cubic spline using the data of Exercise 1 and the fact that

   (a) $f'(-1) = 0.15536240$ and $f'(0.5) = 0.45186276$.

5. A car traveling along a straight road is clocked at a number of points. The data from the observations is given in the following table where the time is in seconds, and the distance in feet:

Time	0	3	5	8	13
Distance	0	225	383	623	993

(a) Use *the natural cubic spline* interpolation to predict the position of the car and its speed when $t = 10$ seconds.

(b) Use *the clamped cubic spline* interpolation to predict the position of the car and its speed when $t = 10\,sec$. In this case use the fact that the speed at the beginning and the end of the race is 75 and 72 feet per second, respectively.