MA322 – Scientific Computing Laboratory

Lab – 06

Dipanshu Goyal 210123083

- Run output_file.m to run the code.

Ques – 1

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Forward and Backward differences for Que - 1
The Forward Difference Table will be:-
2.718e+00       1.763e+00       1.137e+00       7.636e-01
4.482e+00       2.900e+00       1.900e+00       0.000e+00
7.382e+00       4.801e+00       0.000e+00       0.000e+00
1.218e+01       0.000e+00       0.000e+00       0.000e+00

The approximate value of f(2.25) by forward difference is: 9.49692500

The Backward Difference Table will be:-
2.718e+00       0.000e+00       0.000e+00       0.000e+00
4.482e+00       1.763e+00       0.000e+00       0.000e+00
7.382e+00       2.900e+00       1.137e+00       0.000e+00
1.218e+01       4.801e+00       1.900e+00       7.636e-01

The approximate value of f(2.25) by backward difference is: 9.49692500

The actual value of f(2.25) is: 9.48773584
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The approximate value of f (2.25) is same for both forward and backward difference method because both the methods will result in same interpolating polynomial p(x), both are just different approaches to achieve it.

Also, we observe that, the obtained value is very close to the actual value.

Ques – 2

For forming interpolating polynomials of degree one, two and three respectively, I took the first two points for linear polynomial, then took next point and updated the difference table for quadratic polynomial and repeated same for last point to get cubic polynomial.

(a)

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Forward difference for different degrees for Que - 2
The Forward Difference Table will be:-
1.000e+00       6.487e-01
1.649e+00       0.000e+00

The approximate value of f(0.43) by forward difference of degree 1 is: 2.11579840

The Updated Divided Difference Table can be given as:-
1.000e+00       6.487e-01       4.208e-01
1.649e+00       1.070e+00       0.000e+00
2.718e+00       0.000e+00       0.000e+00

The approximate value of f(0.43) by forward difference of degree 2 is: 2.37638253

The Updated Divided Difference Table can be given as:-
1.000e+00       6.487e-01       4.208e-01       2.730e-01
1.649e+00       1.070e+00       6.939e-01       0.000e+00
2.718e+00       1.763e+00       0.000e+00       0.000e+00
4.482e+00       0.000e+00       0.000e+00       0.000e+00

The approximate value of f(0.43) by forward difference of degree 3 is: 2.36060473
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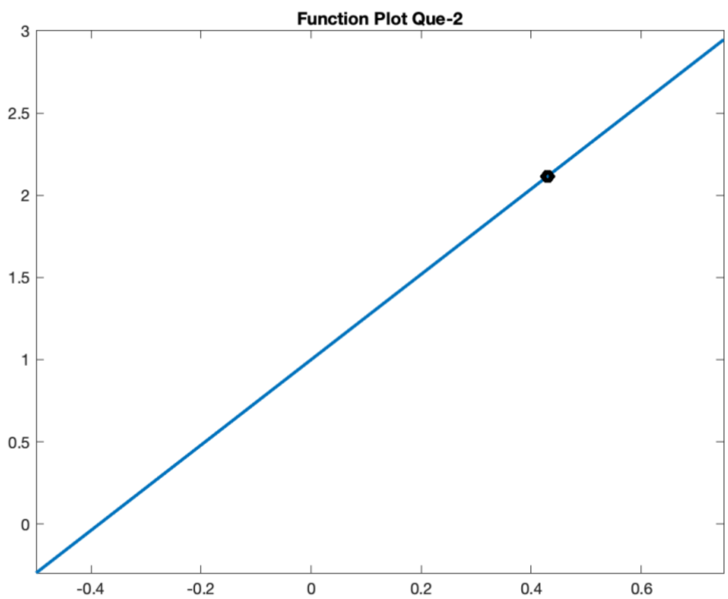
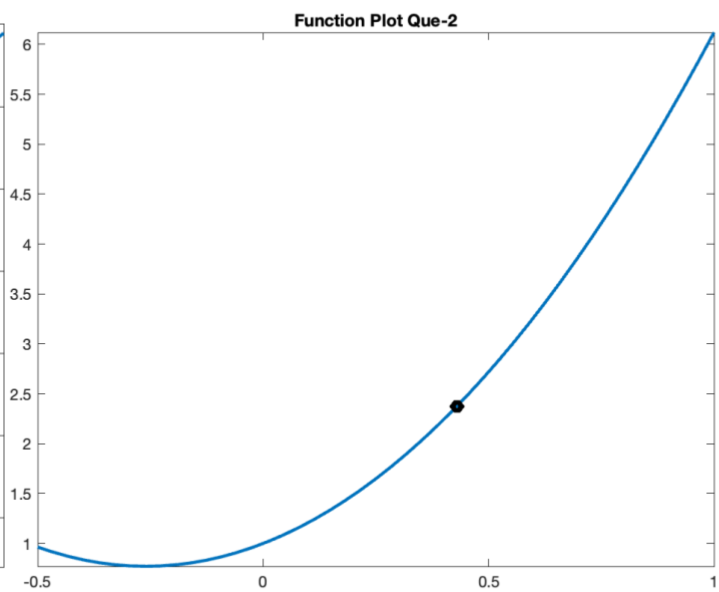
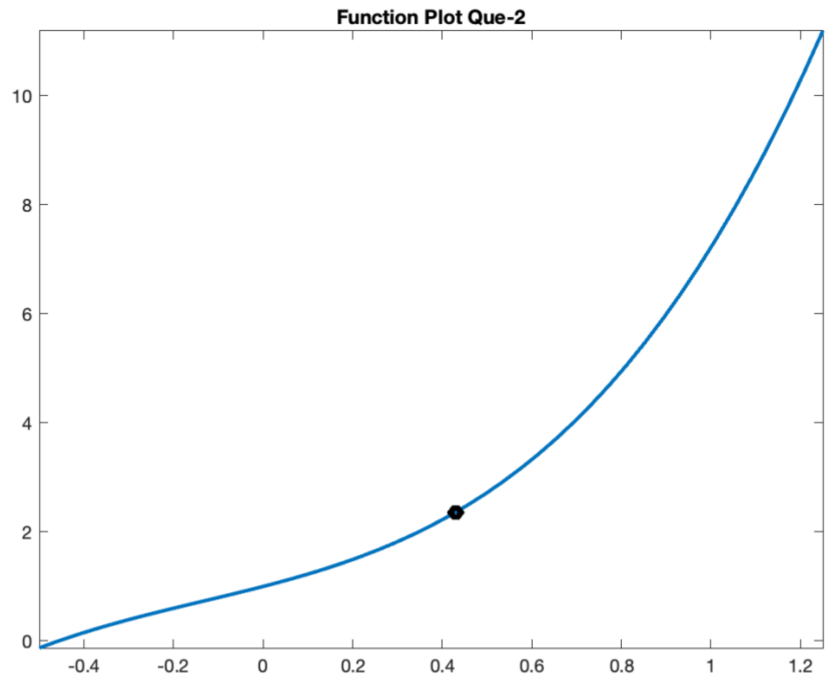
The interpolating polynomials of degree one, two and three can be plotted as follows: -







(b)

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The Forward Difference Table will be:-
-7.181e-02      4.706e-02
-2.475e-02      0.000e+00

The approximate value of f(-1/3) by forward difference of degree 1 is: 0.00662500

The Updated Divided Difference Table can be given as:-
-7.181e-02      4.706e-02       3.126e-01
-2.475e-02      3.597e-01       0.000e+00
3.349e-01       0.000e+00       0.000e+00

The approximate value of f(-1/3) by forward difference of degree 2 is: 0.18030556

The Updated Divided Difference Table can be given as:-
-7.181e-02      4.706e-02       3.126e-01       9.375e-02
-2.475e-02      3.597e-01       4.064e-01       0.000e+00
3.349e-01       7.661e-01       0.000e+00       0.000e+00
1.101e+00       0.000e+00       0.000e+00       0.000e+00

The approximate value of f(-1/3) by forward difference of degree 3 is: 0.17451852
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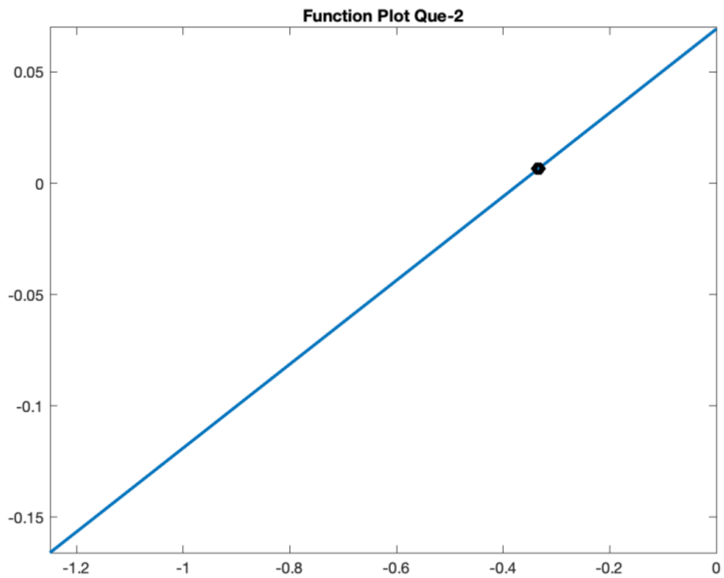
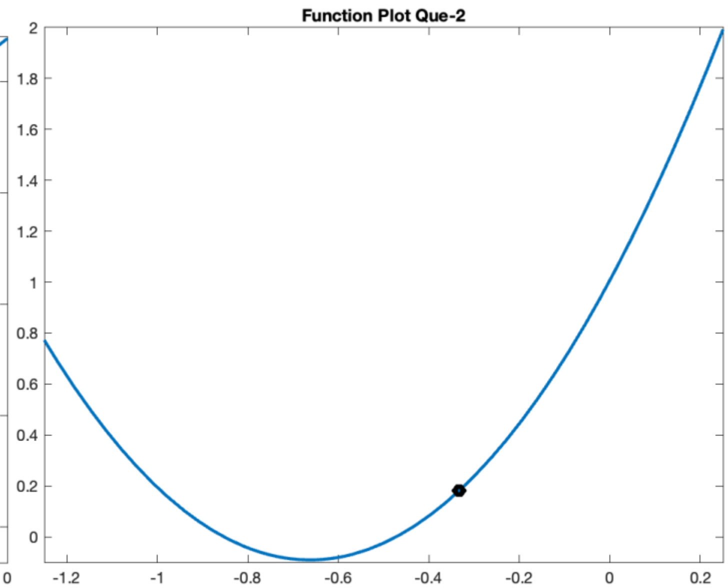
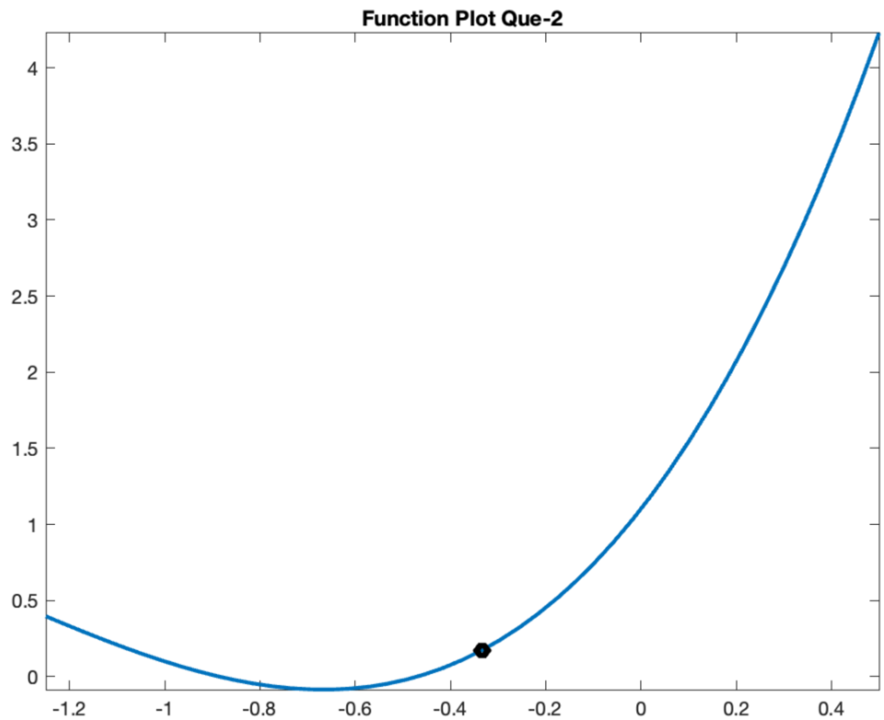
The interpolating polynomials of degree one, two and three can be plotted as follows: -







Ques – 3

I performed the 10 iterations of interpolation of $f(x) = 1/(1 + x^2)$ by $P_n(x)$ by using the Lagrange method, forward difference method and backward difference method and compared the polynomial value with exact value of function at $x_0 = 1+ \text{sqrt}(10)$ i.e. 4.162278 to form the sequence $\{y_n\}$, where $y_n = P_n(x_0)$.

```
Lagrange method, forward and backward differences for Que - 3
f(4.162278) by lagrange method is :- 0.0384615385
f(4.162278) by lagrange method is :- 0.3336709492
f(4.162278) by lagrange method is :- 0.1166052060
f(4.162278) by lagrange method is :- -0.3717596394
f(4.162278) by lagrange method is :- -0.0548918740
f(4.162278) by lagrange method is :- 0.6059346282
f(4.162278) by lagrange method is :- 0.1902492330
f(4.162278) by lagrange method is :- -0.5133526169
f(4.162278) by lagrange method is :- -0.0668173424
f(4.162278) by lagrange method is :- 0.4483348123
Exact value of f(4.162278) is :- 0.0545715835
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The approximate value of f(4.162278) by forward difference is: 0.03846154
The approximate value of f(4.162278) by forward difference is: 0.33367095
The approximate value of f(4.162278) by forward difference is: 0.11660521
The approximate value of f(4.162278) by forward difference is: -0.37175964
The approximate value of f(4.162278) by forward difference is: -0.05489187
The approximate value of f(4.162278) by forward difference is: 0.60593463
The approximate value of f(4.162278) by forward difference is: 0.19024923
The approximate value of f(4.162278) by forward difference is: -0.51335262
The approximate value of f(4.162278) by forward difference is: -0.06681734
The approximate value of f(4.162278) by forward difference is: 0.44833481
Exact value of f(4.162278) is :- 0.0545715835
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The approximate value of f(4.162278) by backward difference is: 0.03846154
The approximate value of f(4.162278) by backward difference is: 0.33367095
The approximate value of f(4.162278) by backward difference is: 0.11660521
The approximate value of f(4.162278) by backward difference is: -0.37175964
The approximate value of f(4.162278) by backward difference is: -0.05489187
The approximate value of f(4.162278) by backward difference is: 0.60593463
The approximate value of f(4.162278) by backward difference is: 0.19024923
The approximate value of f(4.162278) by backward difference is: -0.51335262
The approximate value of f(4.162278) by backward difference is: -0.06681734
The approximate value of f(4.162278) by backward difference is: 0.44833481
Exact value of f(4.162278) is :- 0.0545715835
```
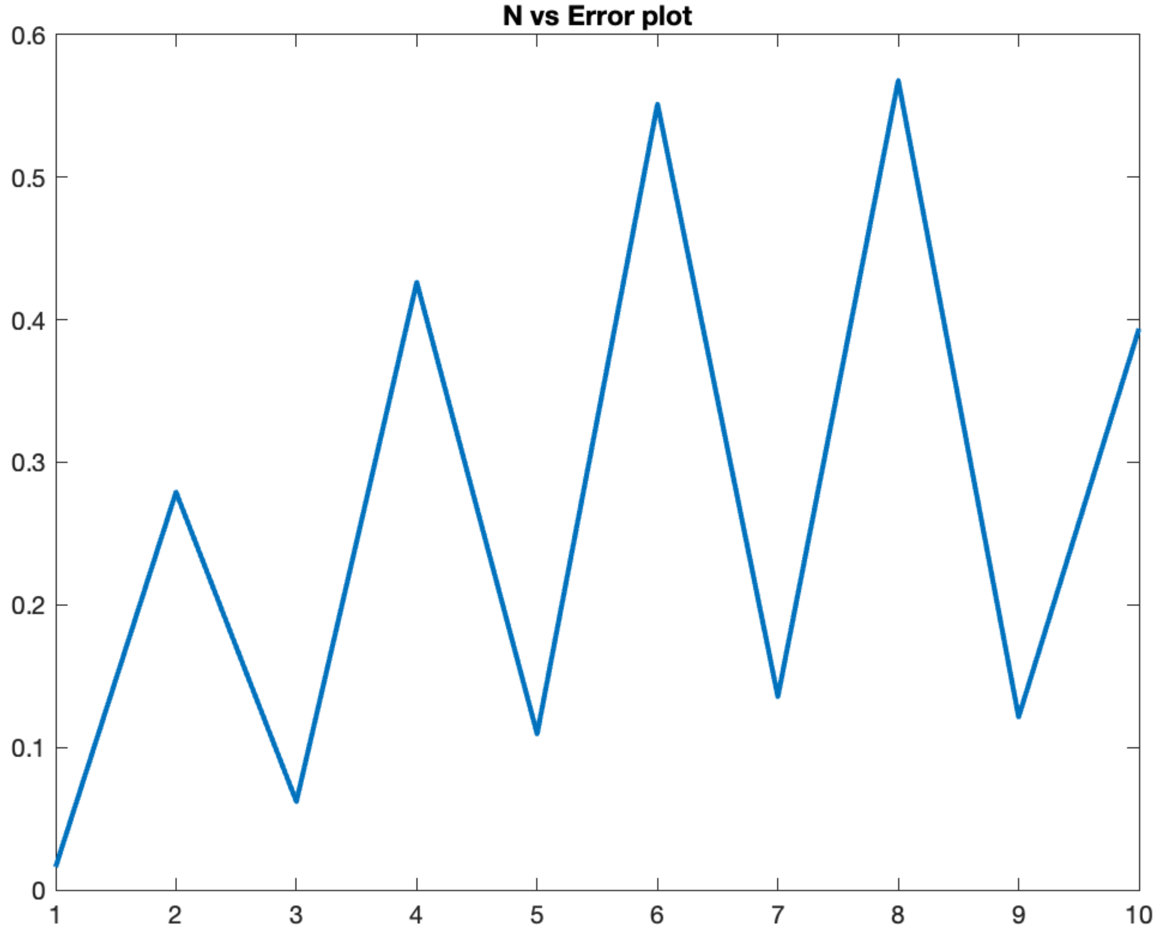

We observe that $\{y_n\}$ is not converging, instead it's continuously fluctuating over different values far from the actual value. We can visualize this with the help of a plot between number of iterations and the absolute error in the obtained value, i.e., abs. error = $| P_n(x_0) - f(x_0) |$. By the plot, it is clear that the error is fluctuating instead of converging to 0.



The reason behind this may be that the upper bound on error in interpolation of f(x) is increasing with increasing n because the magnitude of n^{th} derivative of $f(x) = 1/(1+x^2)$ increases if we increase n. This shows that increasing the number of points doesn't always guarantee convergence of interpolating polynomial to the exact function. I am adding a reference problem known as Runge's phenomenon, which explains the reasoning behind this oscillation in values of an interpolating polynomial at a point.

Runge's Phenomenon - https://en.wikipedia.org/wiki/Runge%27s_phenomenon