DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 322: Scientific Computing Final Lab Assignment

1. Approximate the solutions to the following elliptic PDEs by using the five–point stencil finite difference scheme:

   (a)
   $$\begin{cases} u_{xx} + u_{yy} = 0, \quad 0 < x < 0.5,\ 0 < y < 0.5; \\ u(x,0) = 0, \quad u(x,0.5) = 200x,\ 0 \le x \le 0.5; \\ u(0,y) = 0, \quad u(0.5,y) = 200y,\ 0 \le y \le 0.5. \end{cases}$$
   Use $h = k = 0.25$ and compare the results to the exact solution $u(x,y) = 400xy$.

   (b)
   $$\begin{cases} u_{xx} + u_{yy} + u_x + u_y + u = e^x(2\cos y - \sin y), \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = e^x, \quad u(x,1) = e^x\cos(1),\ 0 \le x \le 1; \\ u(0,y) = \cos(y), \quad u(1,y) = e\cos(y),\ 0 \le y \le 1. \end{cases}$$
   Use $h = k = 0.2$ and compare the results to the exact solution $u(x,y) = e^x\cos(y)$.

2. Solve the system of linear algebraic equations of the above elliptic BVPs by *Gauss-Seidel iterative method.*

3. Solve the system of linear algebraic equations of the above elliptic BVPs by *Jacobi iterative method.*

   Provide the following:

   (a) Draw the surface plot of the exact and numerical solutions.
   (b) Draw the contour plot of the exact and numerical solutions.
   (c) Draw the surf plot of the absolute error.
   (d) Plot $\triangle x (= \triangle y)$ versus $Max.Error$ in loglog scale.

4. For various $\triangle t$ and $\triangle x$, solve the following parabolic initial–boundary–value problem, numerically, by

   i. forward–time and central space (FTCS) discretization scheme,
   ii. backward–time and central space (BTCS) discretization scheme,
   iii. Crank–Nicolson scheme.

   (a)
   $$\begin{cases} \dfrac{\partial u}{\partial t} - \left(\dfrac{1}{16}\right)\dfrac{\partial^2 u}{\partial x^2} = 0, \quad (x,t) \in (0,1)\times(0,1), \\ u(x,0) = 2\sin(2\pi x) \quad x \in (0,1), \\ u(0,t) = u(1,t) = 0, \quad t \in (0,1]. \end{cases}$$
   The exact solution is given by
   $$u(x,t) = 2e^{-\pi^2 t/4}\sin(2\pi x).$$

(b)

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0, \quad (x,t) \in (0,1) \times (0,1), \\ u(x,0) = \sin\left(\dfrac{\pi\, x}{2}\right) + \dfrac{1}{2}\sin\left(2\pi x\right), \quad x \in (0,1), \\ u(0,t) = 0, \quad u(1,t) = e^{-\pi^2 t/4}, \quad t \in (0,1]. \end{cases}$$

The exact solution is given by

$$u(x,t) = e^{-\pi^2 t/4} \sin\left(\frac{\pi\, x}{2}\right) + \frac{1}{2} e^{-4\pi^2 t} \sin(2\pi x).$$

Provide the following:

1. Plot the exact and numerical solutions at the final time level in different colors with some symbols.
2. Draw the surface plot of the exact and numerical solutions.
3. Plot N versus $Max.Error$ in loglog scale.