MA322 – Scientific Computing Laboratory

Lab – 09

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- For the error estimate, I have calculated the actual integral using the inbuilt int (f, a, b) function in MATLAB and took its difference with the approximated integral.
- In Ques – 3, we're asked to perform Gaussian Quadrature, so I applied both Gauss-Legendre and Gauss-Lobatto Quadrature.

Ques – 1

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Que - 1
(a)
The approximated integral by Newton-cotes formula for n = 2 is 0.228074
The actual integral = 0.192259
The error in approimation is 3.581477e-02

The approximated integral by Gauss-Lobatto quadrature for n = 2 is 0.228074
The actual integral = 0.192259
The error in approimation is 3.581477e-02

(b)
The approximated integral by Newton-cotes formula for n = 2 is -0.177764
The actual integral = -0.176820
The error in approimation is 9.443255e-04

The approximated integral by Gauss-Lobatto quadrature for n = 2 is -0.177764
The actual integral = -0.176820
The error in approimation is 9.443255e-04
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Ques – 2

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Que - 2
(a)
The approximated integral by Newton-cotes formula for n = 2 is 4.143260
The actual integral = 2.588629
The error in approimation is 1.554631e+00

The approximated integral by Gauss-Lobatto quadrature for n = 2 is 4.143260
The actual integral = 2.588629
The error in approimation is 1.554631e+00
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The approximated integral by Newton-cotes formula for n = 3 is 2.583696
The actual integral = 2.588629
The error in approimation is 4.932229e-03

The approximated integral by Gauss-Lobatto quadrature for n = 3 is 2.583696
The actual integral = 2.588629
The error in approimation is 4.932229e-03

The approximated integral by Newton-cotes formula for n = 4 is 2.585789
The actual integral = 2.588629
The error in approimation is 2.839581e-03

The approximated integral by Gauss-Lobatto quadrature for n = 4 is 2.587786
The actual integral = 2.588629
The error in approimation is 8.425024e-04

The approximated integral by Newton-cotes formula for n = 5 is 2.587968
The actual integral = 2.588629
The error in approimation is 6.601757e-04

The approximated integral by Gauss-Lobatto quadrature for n = 5 is 2.588623
The actual integral = 2.588629
The error in approimation is 5.134422e-06


(b)
The approximated integral by Newton-cotes formula for n = 2 is -0.866667
The actual integral = -0.733969
The error in approimation is 1.326975e-01

The approximated integral by Gauss-Lobatto quadrature for n = 2 is -0.866667
The actual integral = -0.733969
The error in approimation is 1.326975e-01

The approximated integral by Newton-cotes formula for n = 3 is -0.739105
The actual integral = -0.733969
The error in approimation is 5.136164e-03

The approximated integral by Gauss-Lobatto quadrature for n = 3 is -0.739105
The actual integral = -0.733969
The error in approimation is 5.136164e-03

The approximated integral by Newton-cotes formula for n = 4 is -0.736428
The actual integral = -0.733969
The error in approimation is 2.458521e-03

The approximated integral by Gauss-Lobatto quadrature for n = 4 is -0.734204
The actual integral = -0.733969
The error in approimation is 2.346887e-04
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The approximated integral by Newton-cotes formula for n = 5 is -0.734157
The actual integral = -0.733969
The error in approimation is 1.874933e-04

The approximated integral by Gauss-Lobatto quadrature for n = 5 is -0.733980
The actual integral = -0.733969
The error in approimation is 1.125727e-05
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We observe that the approximations by Newton-Cotes and Gauss-Lobatto Quadrature are same for n = 2,3 and after that, Gauss-Lobatto Quadrature is giving a better approximation.

Ques – 3

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Que - 3
(a)
The approximated integral by Gauss-Legendre quadrature for n = 2 is 0.665844
The actual integral = 0.663494
The error in approimation is 2.350027e-03

The approximated integral by Gauss-Lobatto quadrature for n = 2 is 1.977795
The actual integral = 0.663494
The error in approimation is 1.314302e+00

The approximated integral by Gauss-Legendre quadrature for n = 4 is 0.663493
The actual integral = 0.663494
The error in approimation is 2.273214e-07

The approximated integral by Gauss-Lobatto quadrature for n = 4 is 0.662818
The actual integral = 0.663494
The error in approimation is 6.759082e-04


(b)
The approximated integral by Gauss-Legendre quadrature for n = 2 is 1.962973
The actual integral = 1.933421
The error in approimation is 2.955126e-02

The approximated integral by Gauss-Lobatto quadrature for n = 2 is 1.667460
The actual integral = 1.933421
The error in approimation is 2.659614e-01

The approximated integral by Gauss-Legendre quadrature for n = 4 is 1.933417
The actual integral = 1.933421
The error in approimation is 4.602032e-06

The approximated integral by Gauss-Lobatto quadrature for n = 4 is 1.933467
The actual integral = 1.933421
The error in approimation is 4.512592e-05
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Here, we observe that Gauss-Legendre Quadrature is giving better approximations than Gauss-Lobatto Quadrature.

Ques – 4

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Que - 4
(a)
The approximated integral by Gauss-Legendre quadrature for n = 2 is 0.746595
The actual integral = 0.746824
The error in approimation is 2.294445e-04

The approximated integral by Gauss-Legendre quadrature for n = 4 is 0.746824
The actual integral = 0.746824
The error in approimation is 3.353198e-07

The approximated integral by Gauss-Legendre quadrature for n = 6 is 0.746824
The actual integral = 0.746824
The error in approimation is 7.772834e-11

(b)
The approximated integral by Gauss-Legendre quadrature for n = 2 is 1.263158
The actual integral = 2.651635
The error in approimation is 1.388477e+00

The approximated integral by Gauss-Legendre quadrature for n = 4 is 2.047285
The actual integral = 2.651635
The error in approimation is 6.043503e-01

The approximated integral by Gauss-Legendre quadrature for n = 6 is 2.411689
The actual integral = 2.651635
The error in approimation is 2.399464e-01
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Here is a reference to the Gaussian Quadrature Rules: -
https://www.dam.brown.edu/people/alcyew/handouts/GLquad.pdf