DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 322: Scientific Computing Lab - II

1. Use the Bisection method to find the root of the equation $\frac{1}{x} = 2^x$ on $[0, 1]$.

2. Use the Bisection method to find solution accurate to within 10^{-5} for the following problems.

   a. $2 + \cos(e^x - 2) - e^x = 0 \quad \text{for} \quad 0.5 \leq x \leq 1.5$

   b. $x - \tan(x) \quad \text{for} \quad 0 \leq x \leq 4$

   c. $e^{-x}(3.2\sin(x) - 0.5\cos(x)) = 0 \quad \text{for} \quad 3 \leq x \leq 4$

3. Use Newton's method to find solution accurate to within 10^{-5} for the following problems.

   a. $e^x + 2^{-x} + 2\cos(x) - 6 = 0 \quad \text{for} \quad 1 \leq x \leq 2$

   b. $x\cos(x) - x^2 = 0 \quad \text{with initial guess} \quad x_0 = 1$

   c. $3x = \cos(x) + 1$ find its real root

   d. $\sin x - e^{-x} = 0 \quad \text{for} \quad 6 \leq x \leq 7$

4. Consider

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   Clearly 0 is the only solution of $f(x) = 0$. Choose $x_0 = 0.0001$ as your initial guess and solve it using Newton's Method. Can you go below 0.00005?

5. The fourth degree polynomial $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$, has two real zeros, one in $[-1, 0]$ and the other in $[0, 1]$, Attempt to approximate these zeros to within 10^{-6} using the

   a. Newton's method

   b. Bisection method

   Use the midpoints of each interval as the initial approximation in (a).

6. Use Newton's method to find an approximate value of $\mathcal{K}$, accurate to within 10^{-6} for the population equation

$$1,564,000 = 1,000,000e^{\mathcal{K}} + \frac{435,000}{\mathcal{K}}(e^{\mathcal{K}} - 1).$$

7. Consider the polynomial $p(x) = x^3 + 94x^2 - 389x + 294$ having zeros as $1, 3$ and -98. Start with the initial guess $x_0 = 2$, carry out the calculation using Newton's method and explain what happens.