MA322 – Scientific Computing Laboratory

Lab – 14

Dipanshu Goyal 210123083

Ques – 1

By using the five-point stencil method, the given BVPs are estimated and the linear algebraic equations formed in the process are solved directly by forming a matrix and solving Ax = b i.e. $x = A^{-1}b$.

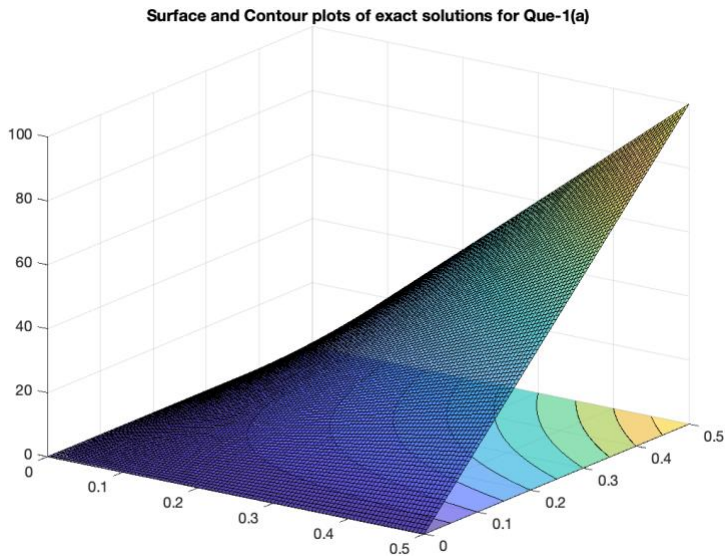
Then the following 4 plots are plotted accordingly: -

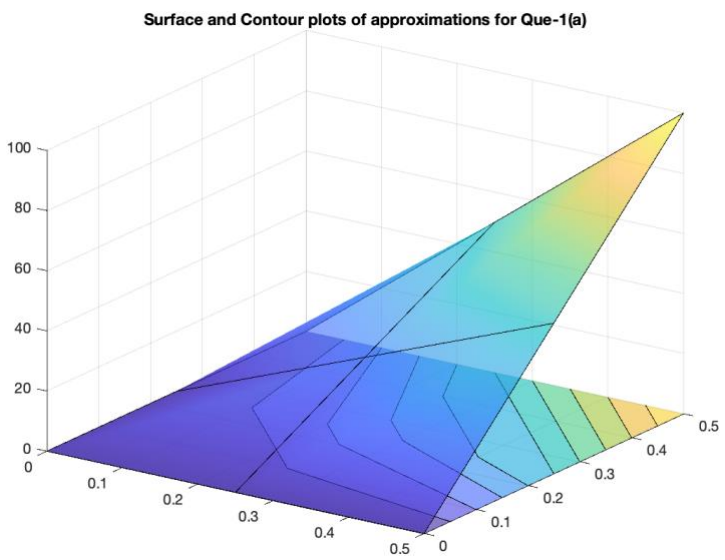
- ➔ Surface and Contour plots of approximate solutions.
- ➔ Surface and Contour plots of Exact Solutions.
- ➔ Surface plot of Errors.
- ➔ delx vs Max Error plot on loglog scale.

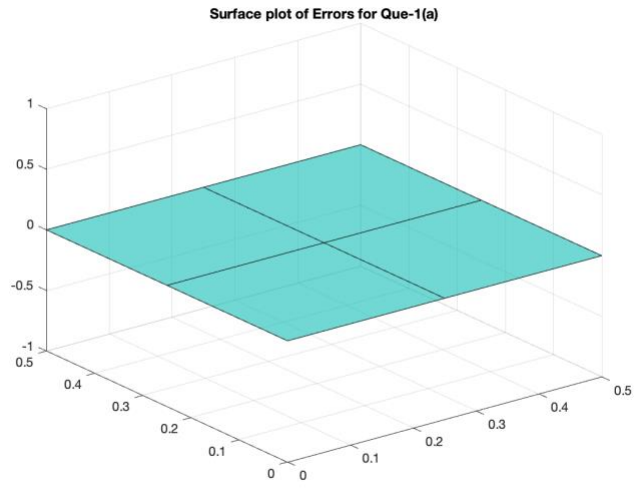
(a) For h = 0.25

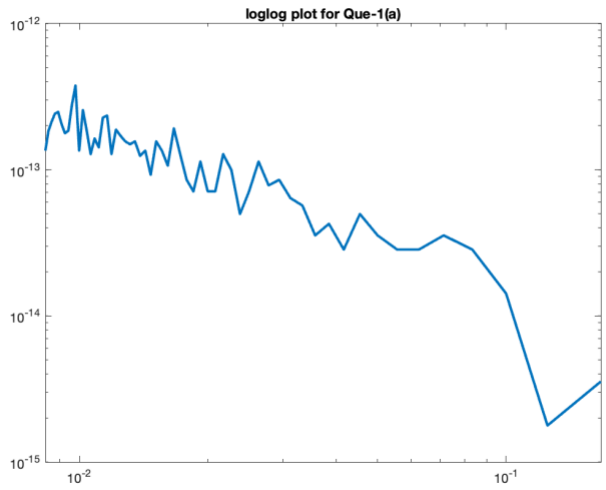
```
Que-1(a)

A =

     0    50   100
     0    25    50
     0     0     0


Err =

     0     0     0
     0     0     0
     0     0     0
```

Here in matrix A, x is increasing from left to right and y is increasing from bottom to top.





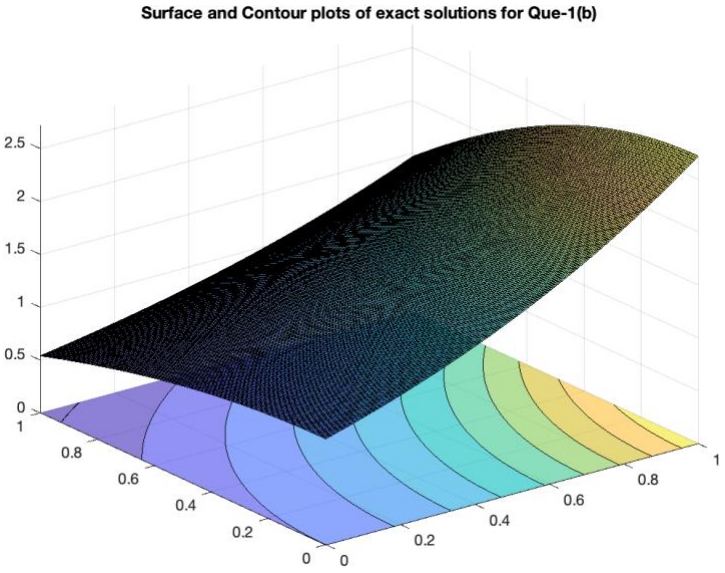


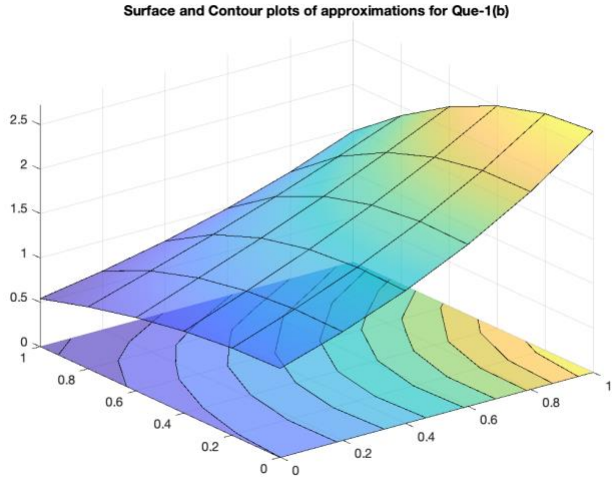


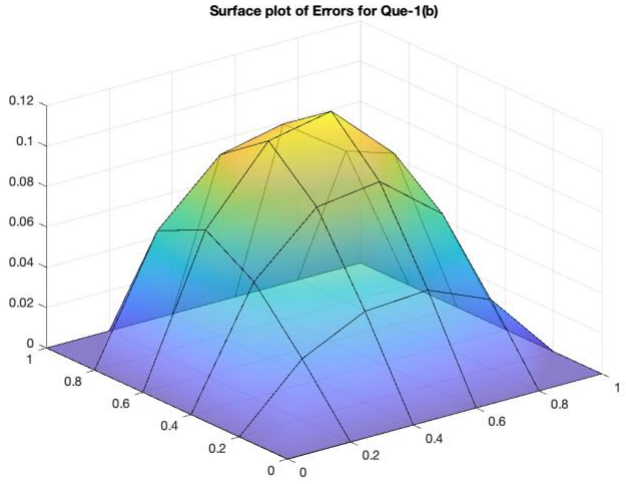
(b) For h = 0.2

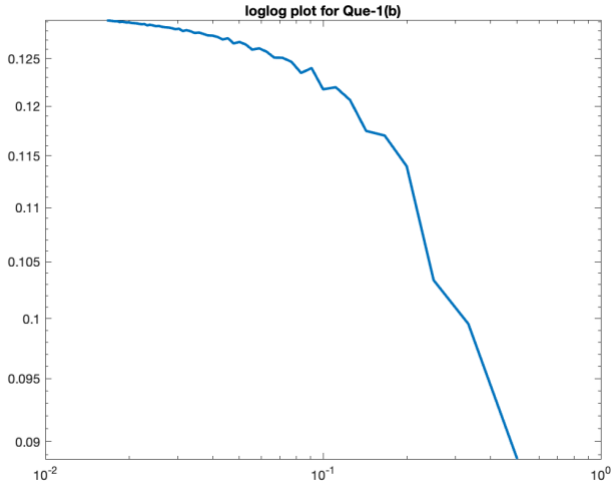
```
Que-1(b)

A =

    0.5403    0.6599    0.8060    0.9845    1.2025    1.4687
    0.6967    0.9117    1.1292    1.3661    1.6254    1.8938
    0.8253    1.0802    1.3390    1.6178    1.9214    2.2435
    0.9211    1.1823    1.4600    1.7684    2.1155    2.5037
    0.9801    1.2273    1.5074    1.8331    2.2155    2.6641
    1.0000    1.2214    1.4918    1.8221    2.2255    2.7183


Err =

         0         0         0         0         0         0
         0    0.0607    0.0899    0.0966    0.0748         0
         0    0.0721    0.1077    0.1140    0.0846         0
         0    0.0573    0.0859    0.0901    0.0656         0
         0    0.0302    0.0453    0.0473    0.0343         0
         0         0         0         0         0         0
```

Here also, for matrix A, the x and y increase in same direction as (a) part.







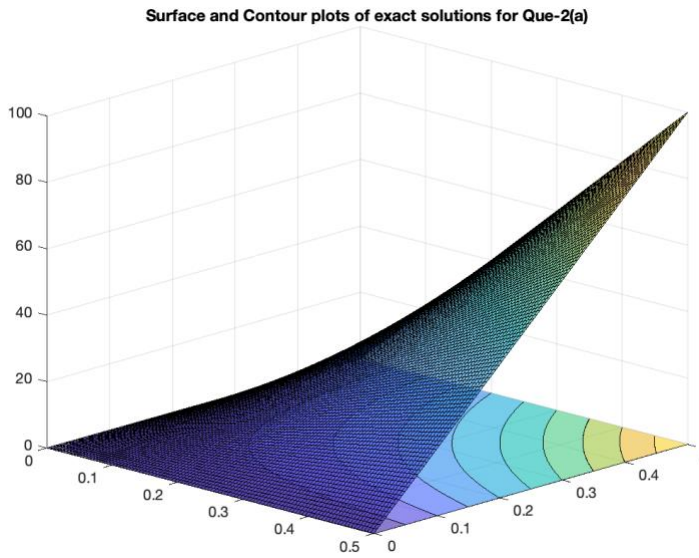


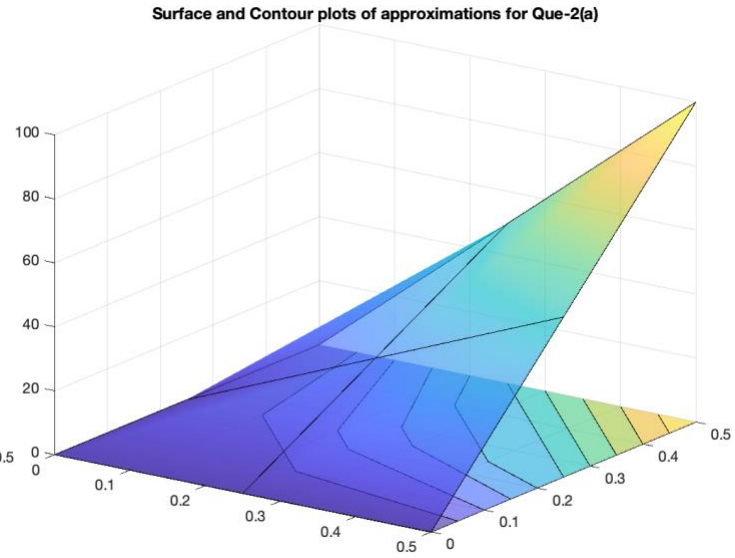
Ques – 2

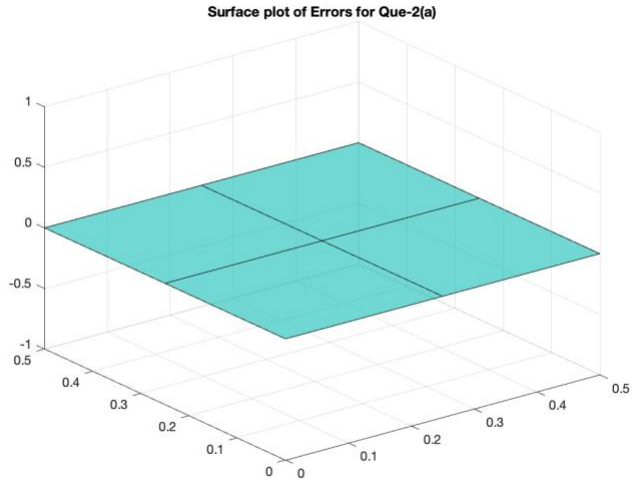
The equations obtained in Question-1 are solved by Gauss-Seidel method here for both parts taking a maximum number of iterations as 1000.

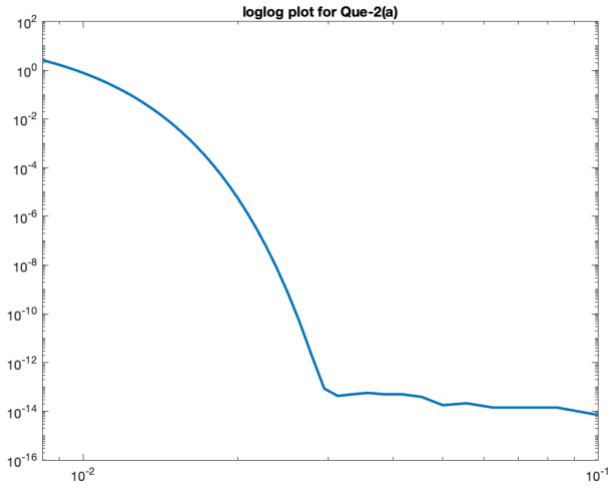
(a) For h = 0.25

```
Que-2(a)

A =

     0    50   100
     0    25    50
     0     0     0


Err =

     0     0     0
     0     0     0
     0     0     0
```





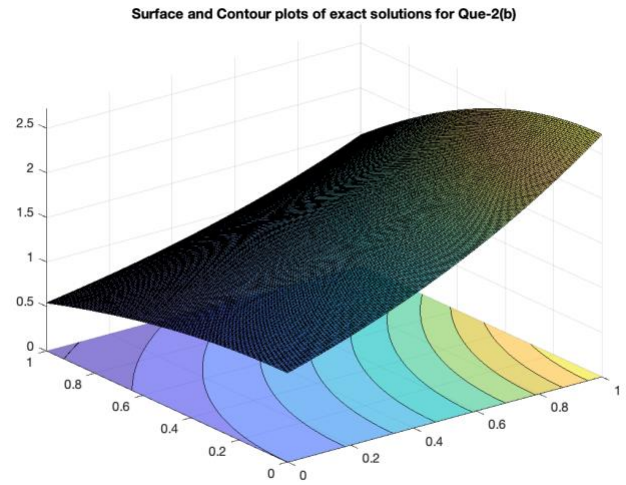


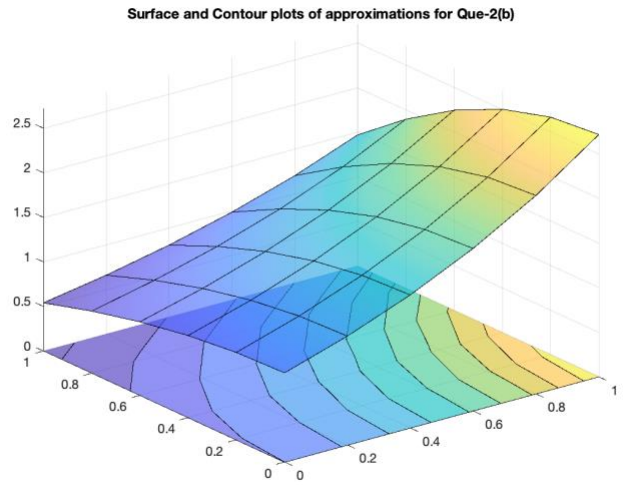


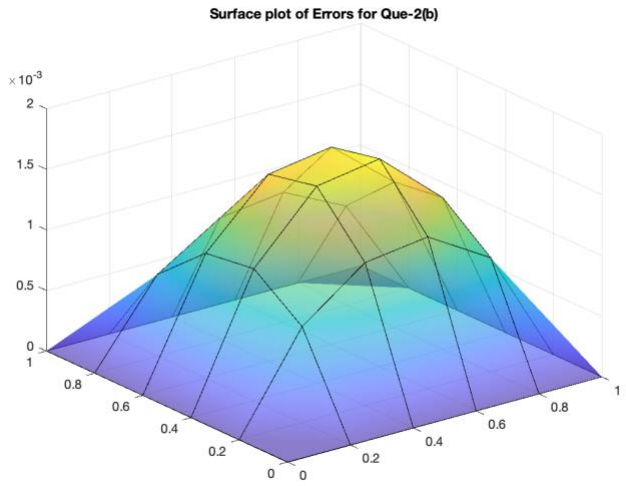
(b) For h = 0.2

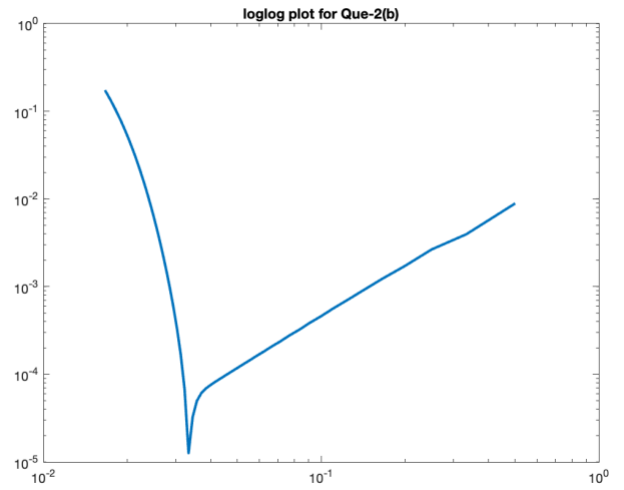
```
Que-2(b)

U =

    1.0000    1.2214    1.4918    1.8221    2.2255    2.7183
    0.9801    1.1978    1.4633    1.7871    2.1821    2.6641
    0.9211    1.1261    1.3757    1.6800    2.0511    2.5037
    0.8253    1.0091    1.2328    1.5055    1.8380    2.2435
    0.6967    0.8516    1.0404    1.2706    1.5514    1.8938
    0.5403    0.6599    0.8060    0.9845    1.2025    1.4687


Err =

         0         0         0         0         0         0
         0    0.0008    0.0012    0.0013    0.0009         0
         0    0.0011    0.0016    0.0017    0.0013         0
         0    0.0010    0.0015    0.0016    0.0012         0
         0    0.0007    0.0010    0.0011    0.0008         0
         0         0         0         0         0         0
```

Here, for part (b) y axis in matrix U is increasing from top to bottom unlike before.







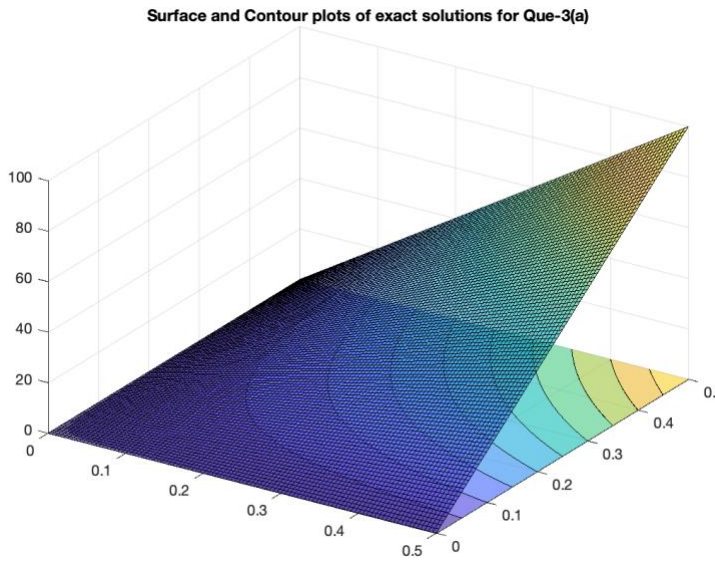


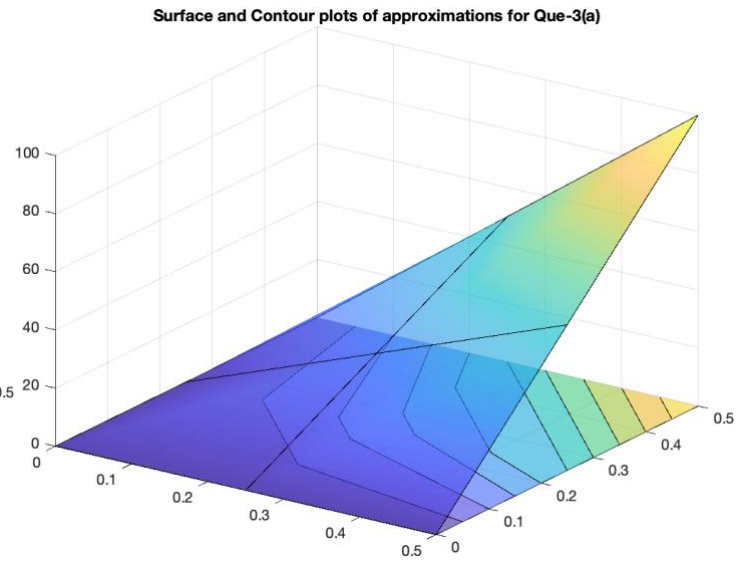
Ques – 3

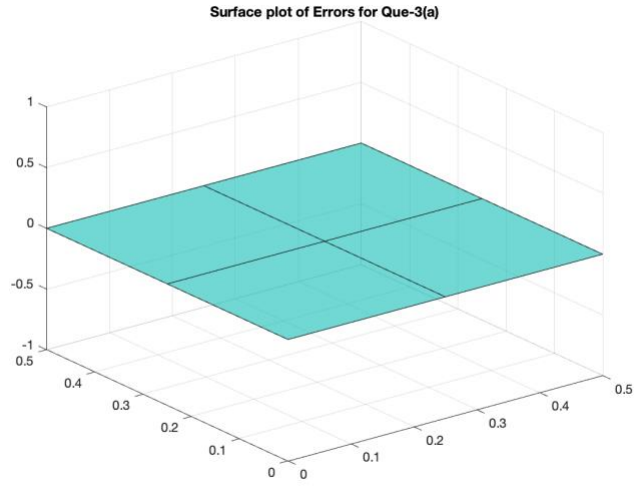
The equations obtained in Question-1 are solved by Jacobi method here for both parts taking a maximum number of iterations as 1000.

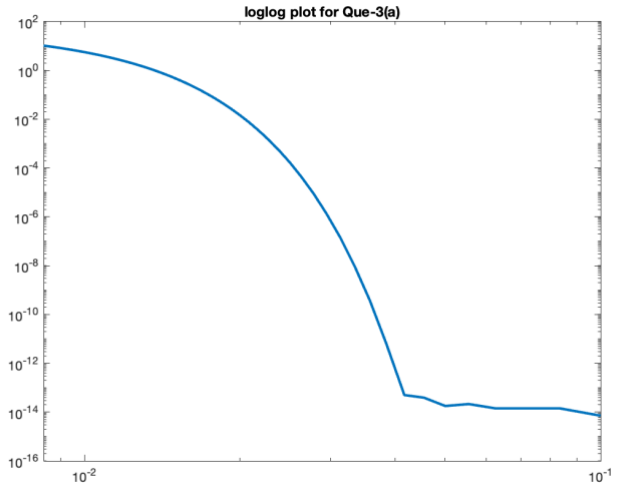
(a) For h = 0.25

```
Que-3(a)

A =

     0    50   100
     0    25    50
     0     0     0


Err =

     0     0     0
     0     0     0
     0     0     0
```





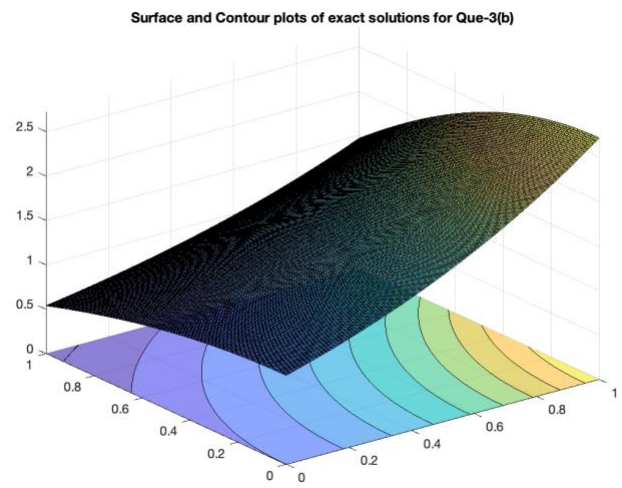


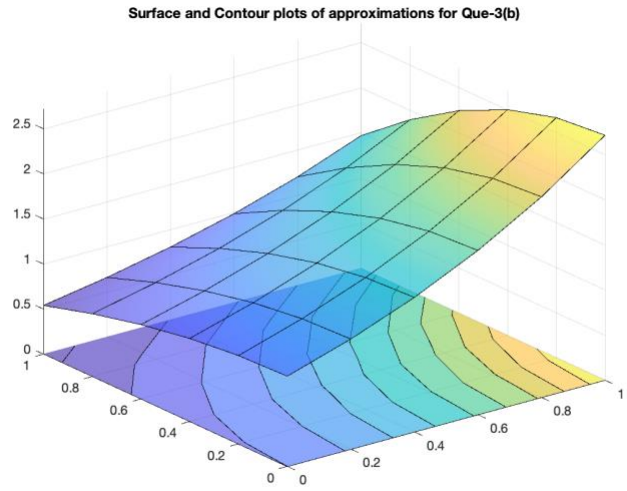


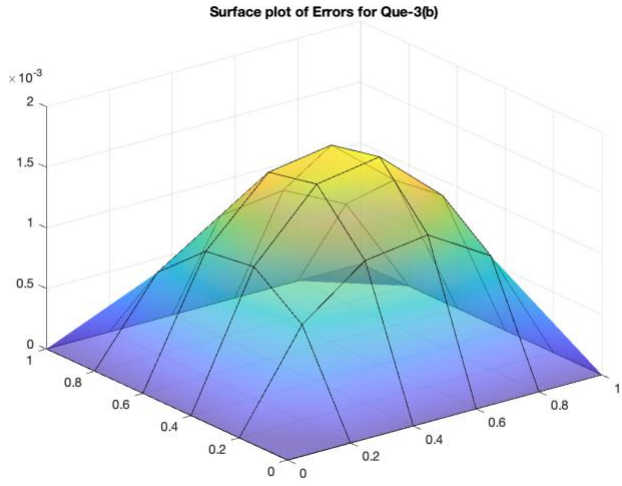
(b) For h = 0.2

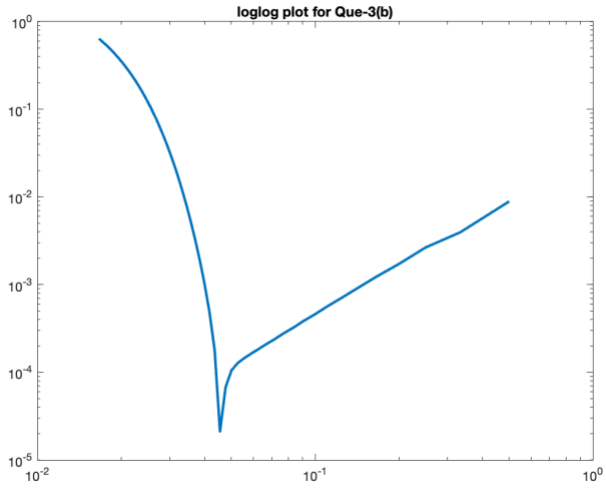
```
Que-3(b)

U =

    1.0000    1.2214    1.4918    1.8221    2.2255    2.7183
    0.9801    1.1978    1.4633    1.7871    2.1821    2.6641
    0.9211    1.1261    1.3757    1.6800    2.0511    2.5037
    0.8253    1.0091    1.2328    1.5055    1.8380    2.2435
    0.6967    0.8516    1.0404    1.2706    1.5514    1.8938
    0.5403    0.6599    0.8060    0.9845    1.2025    1.4687


Err =

         0         0         0         0         0         0
         0    0.0008    0.0012    0.0013    0.0009         0
         0    0.0011    0.0016    0.0017    0.0013         0
         0    0.0010    0.0015    0.0016    0.0012         0
         0    0.0007    0.0010    0.0011    0.0008         0
         0         0         0         0         0         0
```

Here, just like Question-2, x is increasing from left to right in all matrices while y is increasing from top to bottom in part (b) and from bottom to top in part (a).









Ques – 4

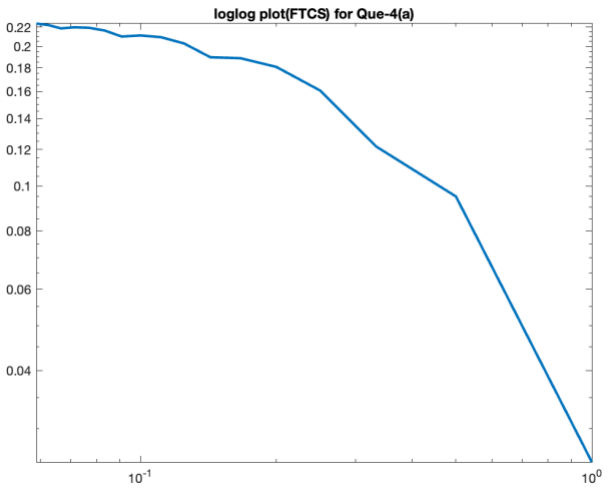
By using the FTCS method, BTCS method and the Crank-Nicolson method, the given BVPs are estimated. The **surface plot of exact solutions** is plotted once for both the parts and the following 3 plots are plotted for each method in both the parts: -

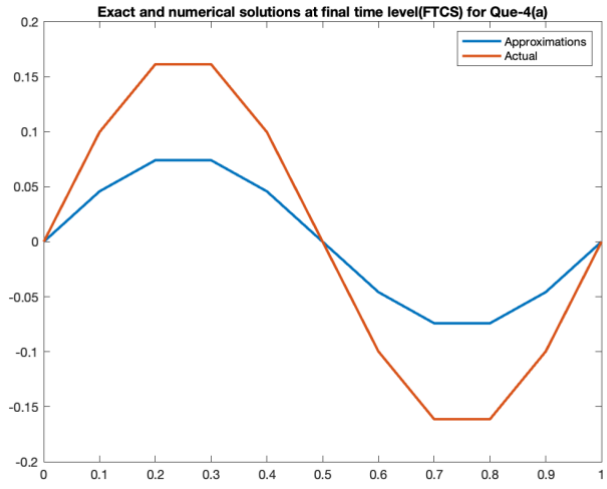
- ➔ Surface plot of approximate solutions.
- ➔ Exact and approximate solutions plot at the final time level.
- ➔ delx vs Max Error plot on loglog scale.

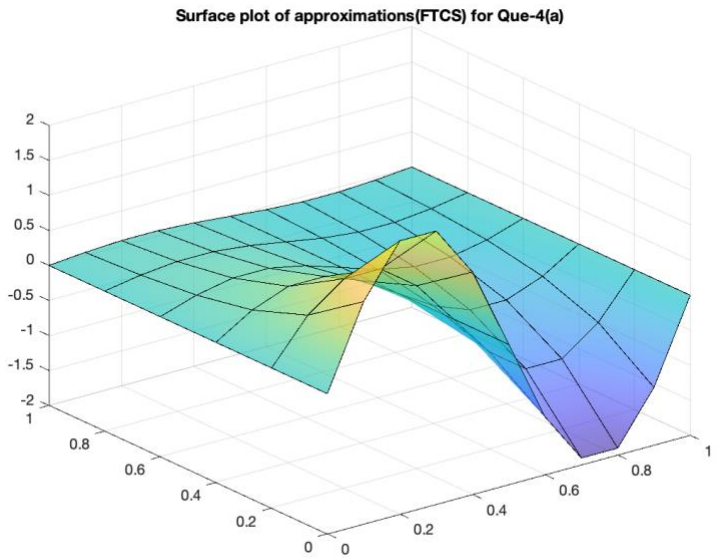
Here, for all the matrices mentioned, the x-axis is increasing from left to right and the y-axis is increasing from top to bottom.

(a) FTCS

```
Que-4(a)
FTCS Scheme:-
Estimated values:
         0    1.1756    1.9021    1.9021    1.1756    0.0000   -1.1756   -1.9021   -1.9021   -1.1756   -0.0000
         0    0.6143    0.9939    0.9939    0.6143    0.0000   -0.6143   -0.9939   -0.9939   -0.6143         0
         0    0.3210    0.5194    0.5194    0.3210   -0.0000   -0.3210   -0.5194   -0.5194   -0.3210         0
         0    0.1677    0.2714    0.2714    0.1677    0.0000   -0.1677   -0.2714   -0.2714   -0.1677         0
         0    0.0876    0.1418    0.1418    0.0876   -0.0000   -0.0876   -0.1418   -0.1418   -0.0876         0
         0    0.0458    0.0741    0.0741    0.0458    0.0000   -0.0458   -0.0741   -0.0741   -0.0458         0


Errors:
         0         0         0         0         0         0         0         0         0         0         0
         0    0.1034    0.1673    0.1673    0.1034    0.0000    0.1034    0.1673    0.1673    0.1034    0.0000
         0    0.1172    0.1896    0.1896    0.1172    0.0000    0.1172    0.1896    0.1896    0.1172    0.0000
         0    0.0998    0.1614    0.1614    0.0998    0.0000    0.0998    0.1614    0.1614    0.0998    0.0000
         0    0.0757    0.1224    0.1224    0.0757    0.0000    0.0757    0.1224    0.1224    0.0757    0.0000
         0    0.0539    0.0872    0.0872    0.0539    0.0000    0.0539    0.0872    0.0872    0.0539    0.0000
```

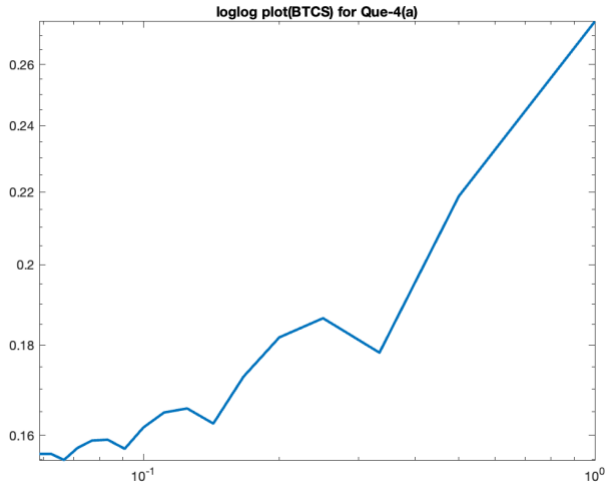


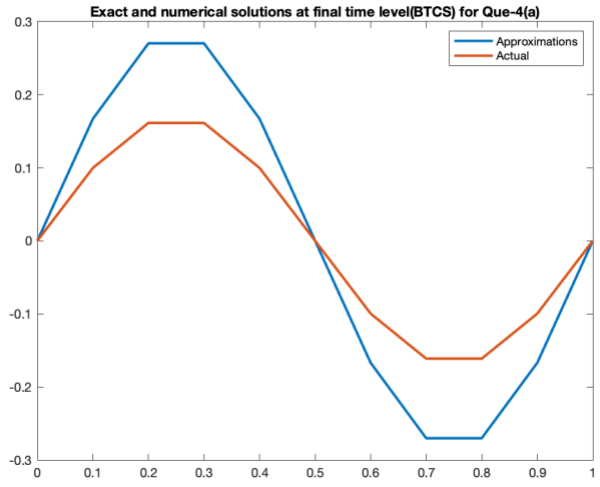


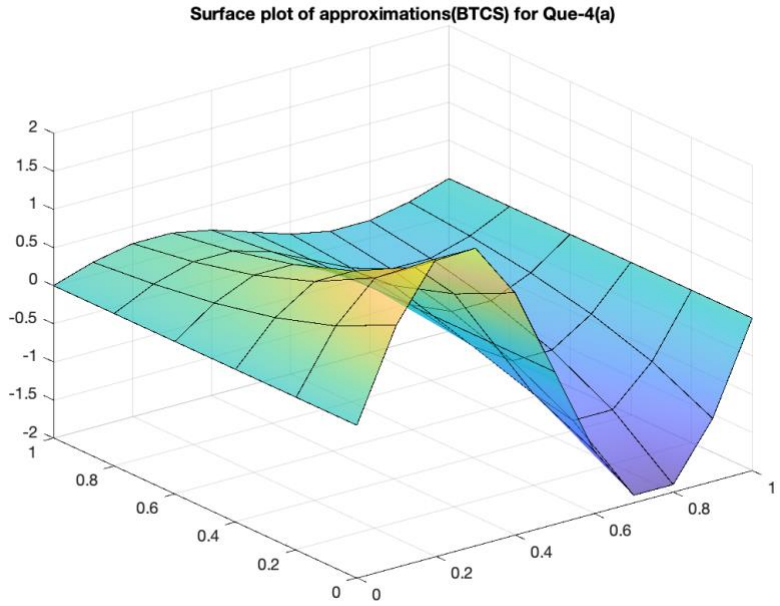


BTCS

```
BTCS Scheme:-
Estimated values:
         0    1.1756    1.9021    1.9021    1.1756    0.0000   -1.1756   -1.9021   -1.9021   -1.1756   -0.0000
         0    0.7957    1.2874    1.2874    0.7957    0.0000   -0.7957   -1.2874   -1.2874   -0.7957         0
         0    0.5385    0.8714    0.8714    0.5385    0.0000   -0.5385   -0.8714   -0.8714   -0.5385         0
         0    0.3645    0.5898    0.5898    0.3645         0   -0.3645   -0.5898   -0.5898   -0.3645         0
         0    0.2467    0.3992    0.3992    0.2467   -0.0000   -0.2467   -0.3992   -0.3992   -0.2467         0
         0    0.1670    0.2702    0.2702    0.1670         0   -0.1670   -0.2702   -0.2702   -0.1670         0


Errors:
         0         0         0         0         0         0         0         0         0         0         0
         0    0.0780    0.1262    0.1262    0.0780    0.0000    0.0780    0.1262    0.1262    0.0780    0.0000
         0    0.1004    0.1624    0.1624    0.1004    0.0000    0.1004    0.1624    0.1624    0.1004    0.0000
         0    0.0970    0.1570    0.1570    0.0970    0.0000    0.0970    0.1570    0.1570    0.0970    0.0000
         0    0.0834    0.1350    0.1350    0.0834    0.0000    0.0834    0.1350    0.1350    0.0834    0.0000
         0    0.0673    0.1089    0.1089    0.0673    0.0000    0.0673    0.1089    0.1089    0.0673    0.0000
```

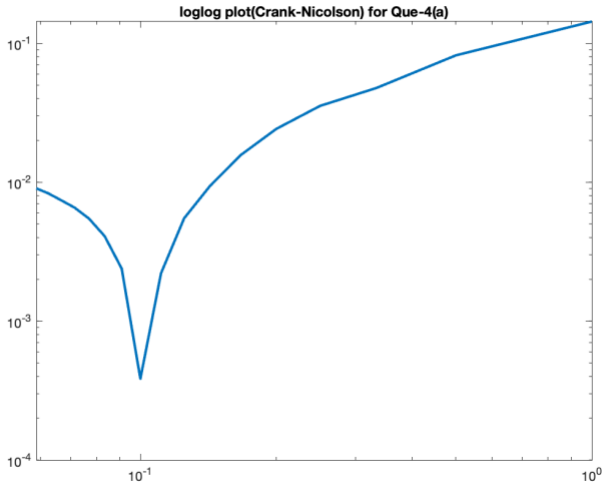


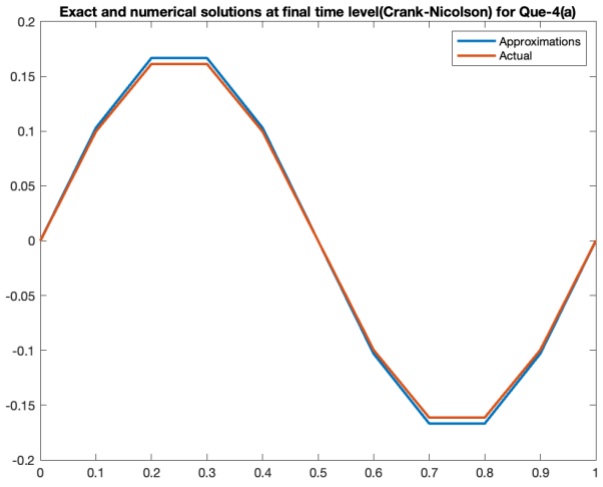


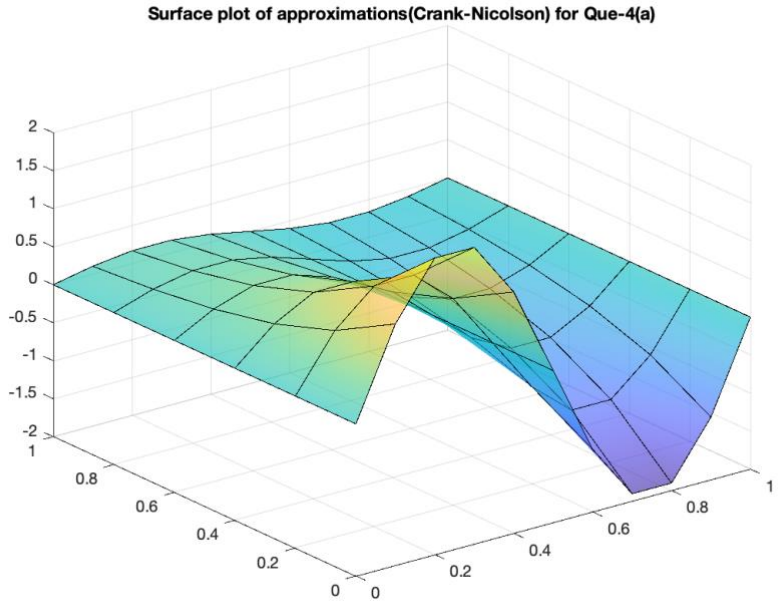


Crank-Nicolson

```
Crank-Nicolson Scheme:-
Estimated values:
         0    1.1756    1.9021    1.9021    1.1756    0.0000   -1.1756   -1.9021   -1.9021   -1.1756   -0.0000
         0    0.7225    1.1690    1.1690    0.7225    0.0000   -0.7225   -1.1690   -1.1690   -0.7225         0
         0    0.4440    0.7184    0.7184    0.4440    0.0000   -0.4440   -0.7184   -0.7184   -0.4440         0
         0    0.2729    0.4415    0.4415    0.2729    0.0000   -0.2729   -0.4415   -0.4415   -0.2729         0
         0    0.1677    0.2713    0.2713    0.1677    0.0000   -0.1677   -0.2713   -0.2713   -0.1677         0
         0    0.1031    0.1667    0.1667    0.1031    0.0000   -0.1031   -0.1667   -0.1667   -0.1031         0


Errors:
         0         0         0         0         0         0         0         0         0         0         0
         0    0.0048    0.0077    0.0077    0.0048    0.0000    0.0048    0.0077    0.0077    0.0048    0.0000
         0    0.0058    0.0095    0.0095    0.0058    0.0000    0.0058    0.0095    0.0095    0.0058    0.0000
         0    0.0054    0.0087    0.0087    0.0054    0.0000    0.0054    0.0087    0.0087    0.0054    0.0000
         0    0.0044    0.0071    0.0071    0.0044    0.0000    0.0044    0.0071    0.0071    0.0044    0.0000
         0    0.0034    0.0054    0.0054    0.0034    0.0000    0.0034    0.0054    0.0054    0.0034    0.0000
```

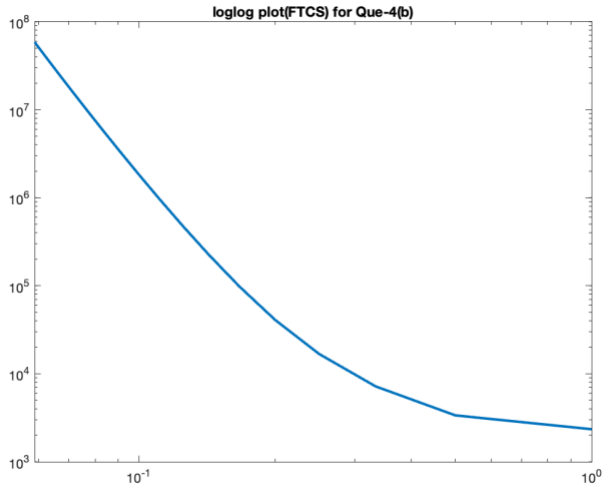


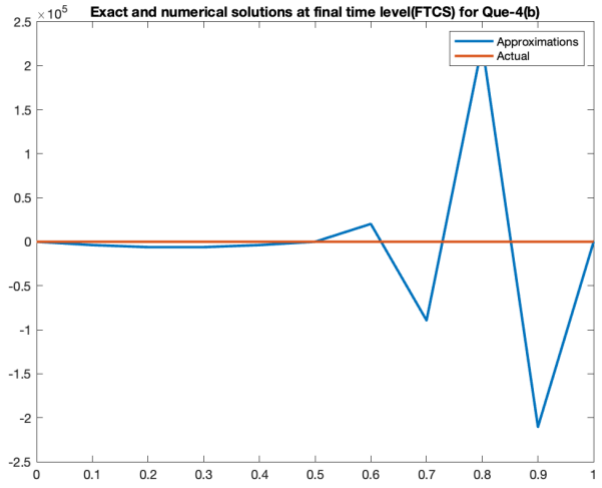


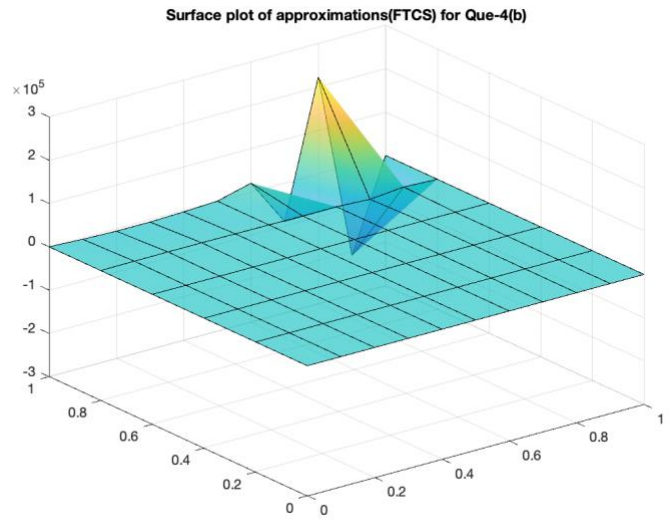


(b) FTCS

```
Que-4(b)
FTCS Scheme:-
Estimated values:
   1.0e+05 *

         0    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
         0   -0.0000   -0.0000   -0.0000   -0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
         0    0.0001    0.0002    0.0002    0.0001    0.0000   -0.0001   -0.0002   -0.0002   -0.0001    0.0000
         0   -0.0009   -0.0014   -0.0014   -0.0009    0.0000    0.0009    0.0014    0.0018    0.0001    0.0000
         0    0.0057    0.0092    0.0092    0.0057    0.0000   -0.0057   -0.0010   -0.0409    0.0330    0.0000
         0   -0.0379   -0.0613   -0.0613   -0.0379    0.0000    0.2027   -0.8932    2.2346   -2.1039    0.0000


Errors:
   1.0e+05 *

         0         0         0         0         0         0         0         0         0         0         0
         0    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000         0
         0    0.0001    0.0002    0.0002    0.0001    0.0000    0.0001    0.0002    0.0002    0.0001         0
         0    0.0009    0.0014    0.0014    0.0009    0.0000    0.0009    0.0014    0.0018    0.0001         0
         0    0.0057    0.0092    0.0092    0.0057    0.0000    0.0057    0.0010    0.0409    0.0330         0
         0    0.0379    0.0613    0.0613    0.0379    0.0000    0.2027    0.8932    2.2346    2.1039         0
```

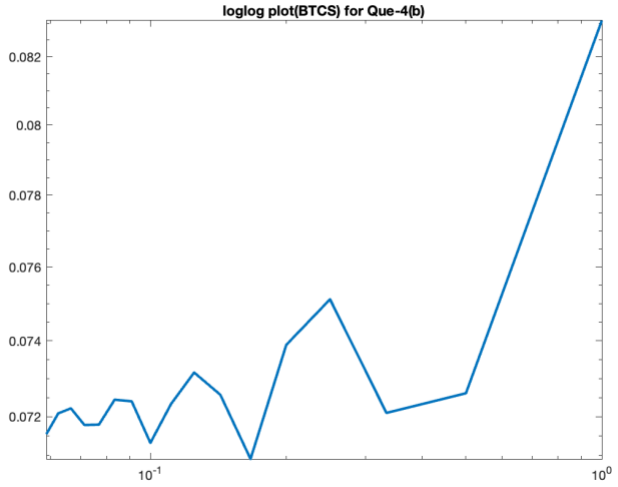


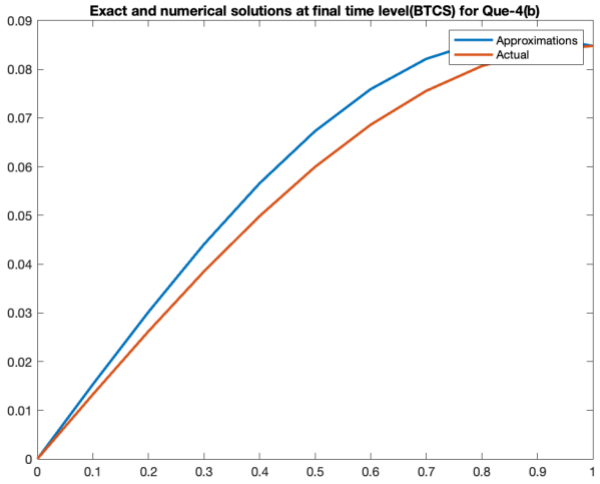


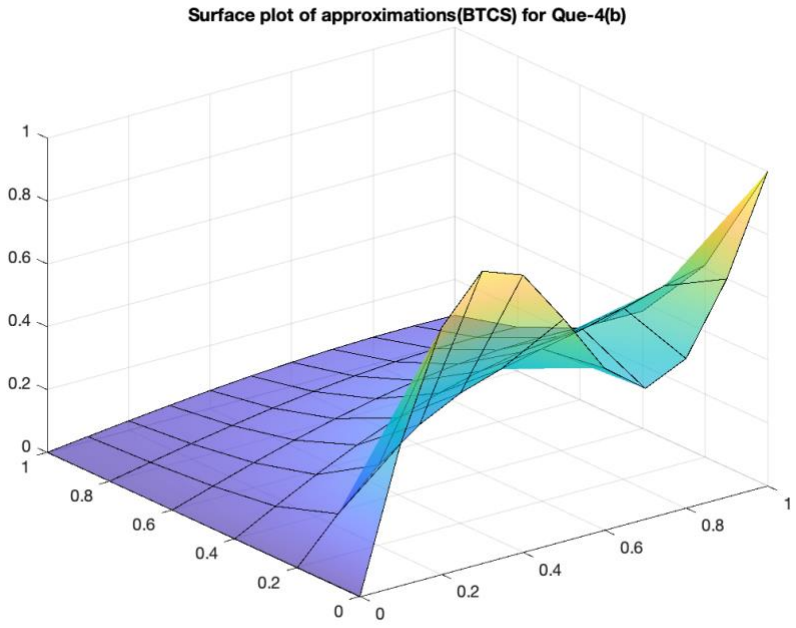


BTCS

```
BTCS Scheme:-
Estimated values:
         0    0.4503    0.7845    0.9295    0.8817    0.7071    0.5151    0.4155    0.4755    0.6938    1.0000
         0    0.1359    0.2561    0.3499    0.4147    0.4562    0.4851    0.5125    0.5447    0.5805    0.6105
         0    0.0686    0.1338    0.1930    0.2442    0.2870    0.3213    0.3474    0.3653    0.3741    0.3727
         0    0.0408    0.0802    0.1170    0.1499    0.1781    0.2009    0.2176    0.2279    0.2313    0.2275
         0    0.0250    0.0492    0.0718    0.0922    0.1097    0.1238    0.1340    0.1401    0.1417    0.1389
         0    0.0153    0.0302    0.0441    0.0566    0.0673    0.0759    0.0821    0.0858    0.0867    0.0848


Errors:
         0         0         0         0         0         0         0         0         0         0         0
         0    0.0403    0.0673    0.0726    0.0558    0.0245    0.0087    0.0313    0.0357    0.0224         0
         0    0.0103    0.0187    0.0238    0.0252    0.0235    0.0198    0.0153    0.0108    0.0060         0
         0    0.0052    0.0099    0.0137    0.0161    0.0172    0.0168    0.0149    0.0115    0.0066         0
         0    0.0033    0.0063    0.0088    0.0106    0.0115    0.0114    0.0103    0.0080    0.0045         0
         0    0.0021    0.0040    0.0056    0.0067    0.0073    0.0073    0.0066    0.0051    0.0029         0
```

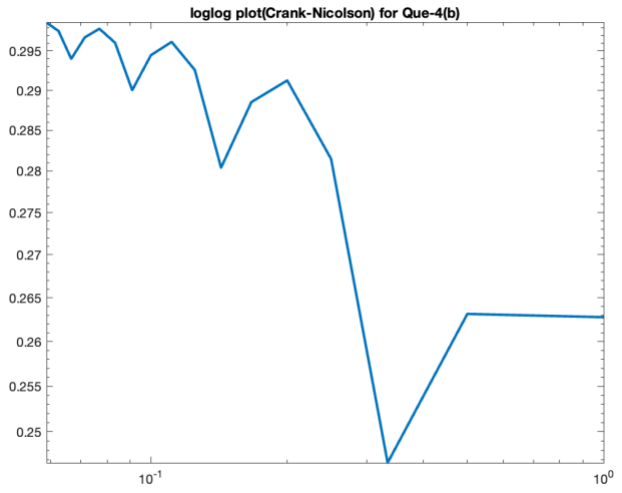


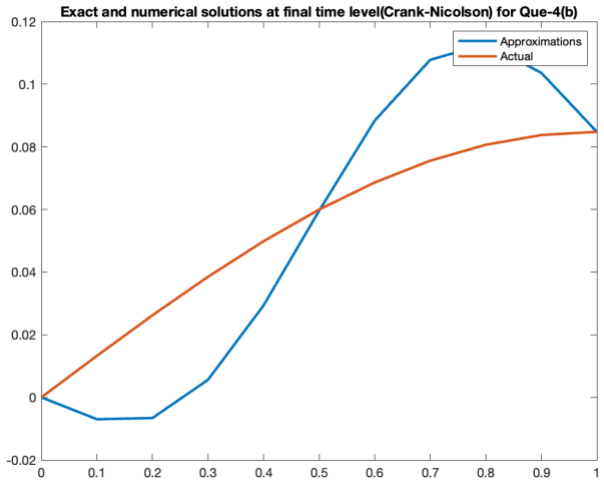


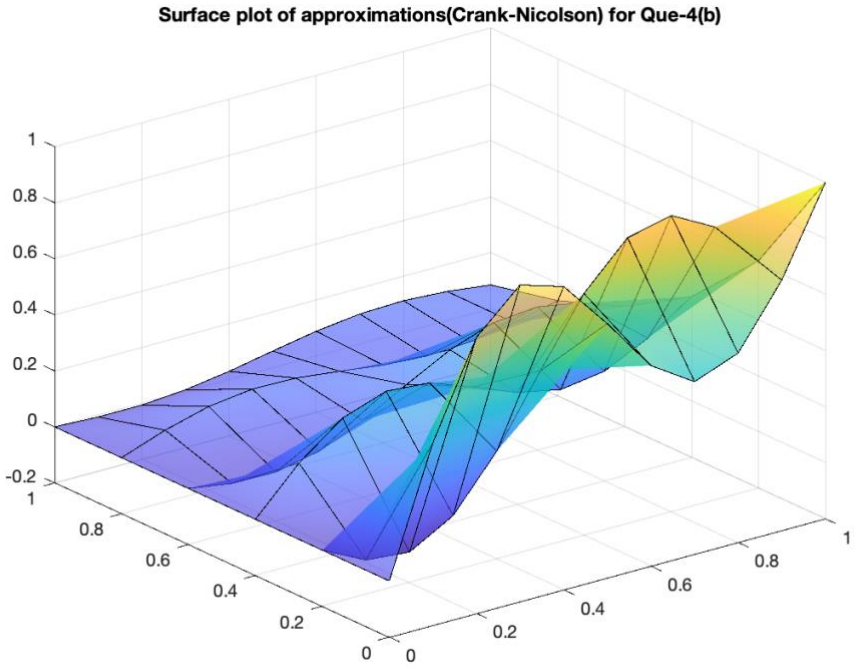


Crank-Nicolson

```
Crank_Nicolson Scheme:-
Estimated values:
         0    0.4503    0.7845    0.9295    0.8817    0.7071    0.5151    0.4155    0.4755    0.6938    1.0000
         0   -0.0772   -0.0910   -0.0031    0.1844    0.4288    0.6628    0.8193    0.8564    0.7735    0.6105
         0    0.1583    0.2769    0.3305    0.3179    0.2617    0.1991    0.1678    0.1906    0.2670    0.3727
         0   -0.0235   -0.0255    0.0073    0.0739    0.1598    0.2418    0.2969    0.3107    0.2830    0.2275
         0    0.0560    0.0982    0.1182    0.1154    0.0975    0.0773    0.0675    0.0760    0.1026    0.1389
         0   -0.0070   -0.0066    0.0056    0.0293    0.0596    0.0883    0.1078    0.1129    0.1036    0.0848


Errors:
         0         0         0         0         0         0         0         0         0         0         0
         0    0.1728    0.2798    0.2804    0.1746    0.0029    0.1690    0.2755    0.2760    0.1706         0
         0    0.1000    0.1617    0.1613    0.0988    0.0019    0.1024    0.1643    0.1638    0.1011         0
         0    0.0591    0.0958    0.0960    0.0598    0.0011    0.0577    0.0942    0.0943    0.0582         0
         0    0.0342    0.0553    0.0552    0.0338    0.0007    0.0351    0.0563    0.0561    0.0346         0
         0    0.0203    0.0328    0.0329    0.0205    0.0004    0.0197    0.0322    0.0322    0.0199         0
```







Some Observations for Question-4: -

- For part-(a), Crank-Nicolson scheme is providing the best estimate to actual solution, while for part-(b), BTCS scheme is doing that.
- In part-(b), FTCS scheme is severely failing, as we can observe that it is giving very high values (of order 10^5) than the actual solution stating that FTCS is not suitable here.
- For part-(b), Crank-Nicolson scheme is giving an oscillatory solution, this may be due to the noise from some extra unwanted term, making this scheme not suitable here.