MA322 – Scientific Computing Lab

# Lab - 02

Dipanshu Goyal 210123083

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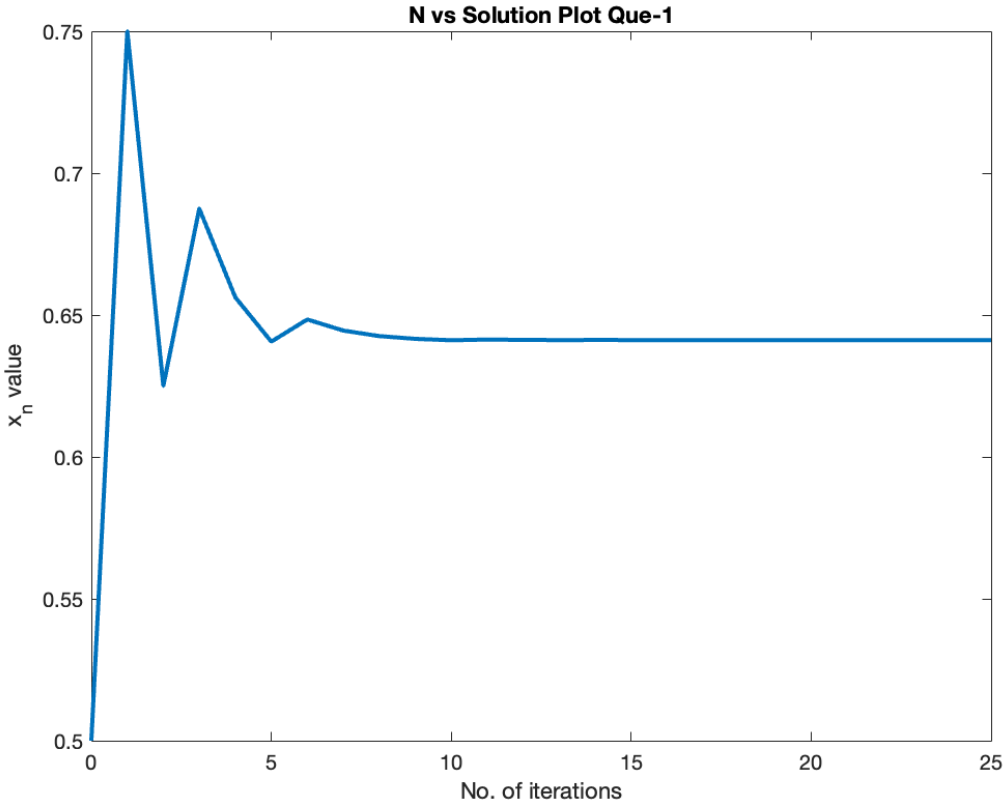
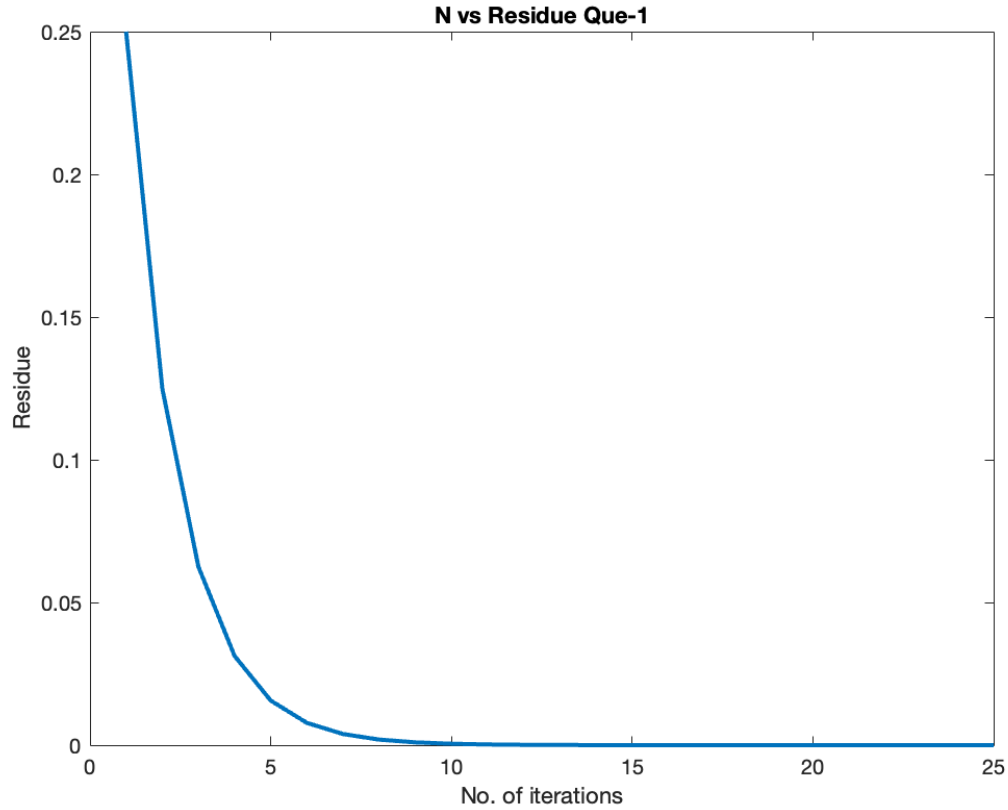
The residue or error is taken as $|x_{n+1} - x_n|$. Run the file named output_file.m.

## Ques – 1

```
Bisection Method for Q-1
No. Of Iterations       Approx. Soln.           Error
1                       0.750000000000000       2.500000e-01
2                       0.625000000000000       1.250000e-01
3                       0.687500000000000       6.250000e-02
4                       0.656250000000000       3.125000e-02
5                       0.640625000000000       1.562500e-02
6                       0.648437500000000       7.812500e-03
7                       0.644531250000000       3.906250e-03
8                       0.642578125000000       1.953125e-03
9                       0.641601562500000       9.765625e-04
10                      0.641113281250000       4.882812e-04
11                      0.641357421875000       2.441406e-04
12                      0.641235351562500       1.220703e-04
13                      0.641174316406250       6.103516e-05
14                      0.641204833984375       3.051758e-05
15                      0.641189575195312       1.525879e-05
16                      0.641181945800781       7.629395e-06
17                      0.641185760498047       3.814697e-06
18                      0.641183853149414       1.907349e-06
19                      0.641184806823730       9.536743e-07
20                      0.641185283660889       4.768372e-07
21                      0.641185522079468       2.384186e-07
22                      0.641185641288757       1.192093e-07
23                      0.641185700893402       5.960464e-08
24                      0.641185730695724       2.980232e-08
25                      0.641185745596886       1.490116e-08

The required root is: 0.641186
The number of iterations performed: 25
```


N vs Solution Plot Que-1
0.75
0.7
0.65
0.6
0.55
0.5
x_n value
0
5
10
15
20
25
No. of iterations

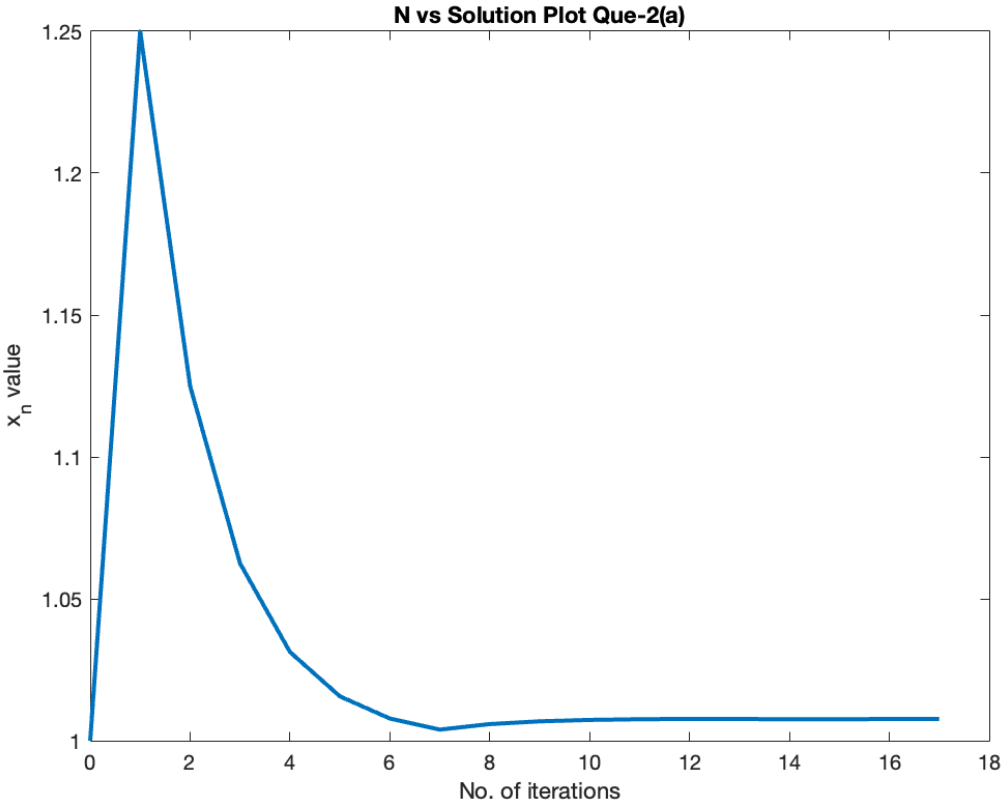

N vs Residue Que-1
0.25
0.2
0.15
0.1
0.05
0
Residue
0
5
10
15
20
25
No. of iterations

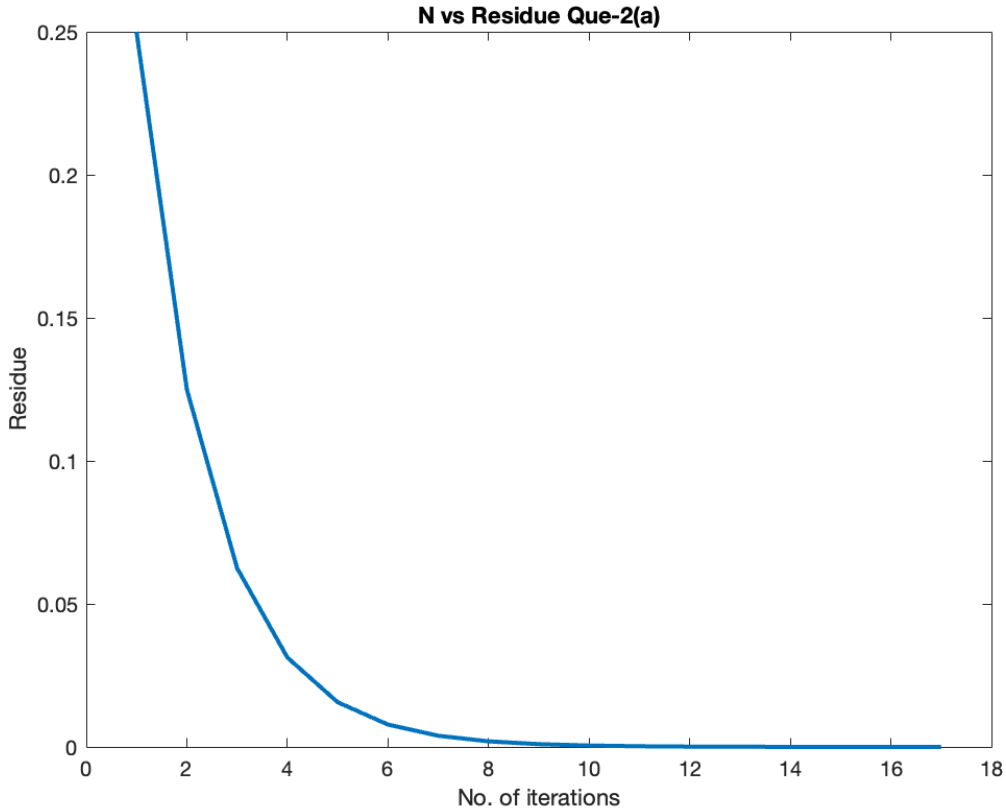
# Ques – 2

(a)

```
Bisection Method for Q-2(a)
No. Of Iterations       Approx. Soln.          Error
1                       1.250000000000000      2.500000e-01
2                       1.125000000000000      1.250000e-01
3                       1.062500000000000      6.250000e-02
4                       1.031250000000000      3.125000e-02
5                       1.015625000000000      1.562500e-02
6                       1.007812500000000      7.812500e-03
7                       1.003906250000000      3.906250e-03
8                       1.005859375000000      1.953125e-03
9                       1.006835937500000      9.765625e-04
10                      1.007324218750000      4.882812e-04
11                      1.007568359375000      2.441406e-04
12                      1.007690429687500      1.220703e-04
13                      1.007629394531250      6.103516e-05
14                      1.007598876953125      3.051758e-05
15                      1.007614135742188      1.525879e-05
16                      1.007621765136719      7.629395e-06
17                      1.007625579833984      3.814697e-06

The required root is: 1.007626
The number of iterations performed: 17
```


N vs Solution Plot Que-2(a)
1.25
1.2
1.15
1.1
1.05
1
$x_n$ value
0
2
4
6
8
10
12
14
16
18
No. of iterations


N vs Residue Que-2(a)
0.25
0.2
0.15
0.1
0.05
0
Residue
0
2
4
6
8
10
12
14
16
18
No. of iterations

(b)

Bisection Method for Q-2(b)

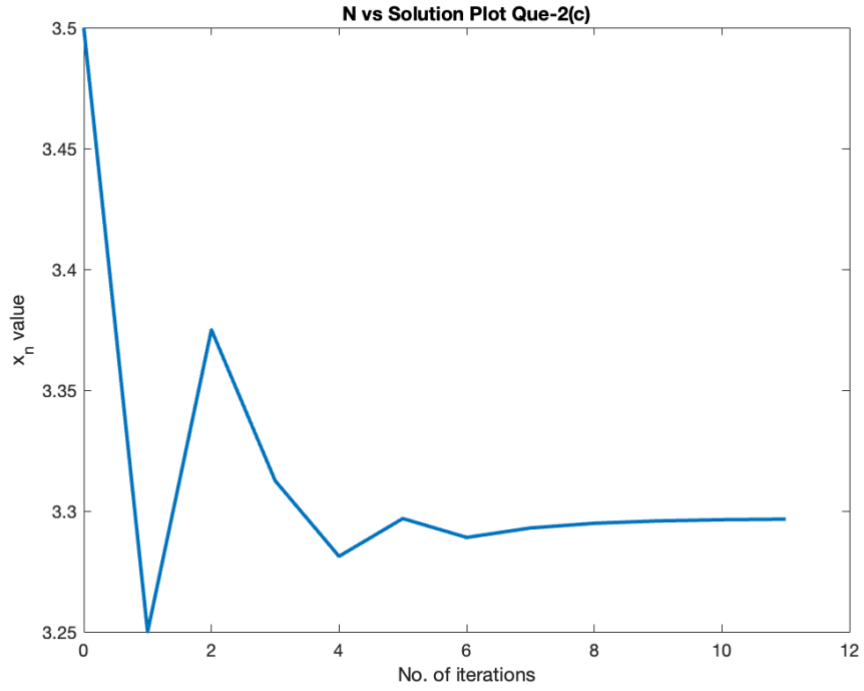
| No. Of Iterations | Approx. Soln. | Error |
|---|---|---|
| 0 | 0.000000 | 0 |

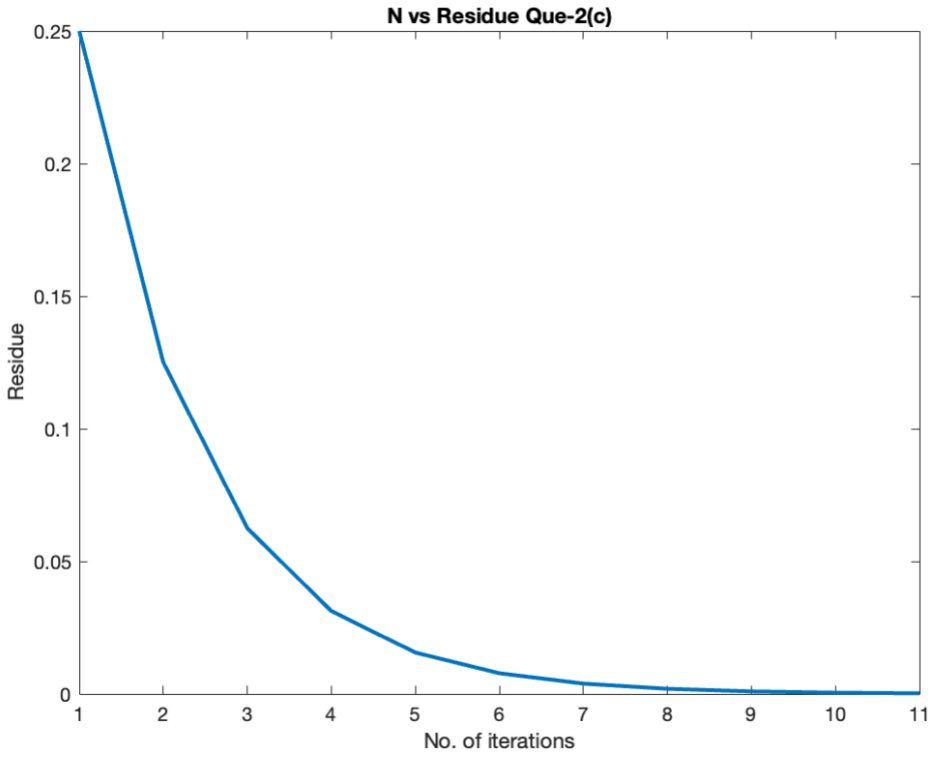
(c)

Bisection Method for Q-2(c)

| No. Of Iterations | Approx. Soln. | Error |
|---|---|---|
| 1 | 3.250000000000000 | 2.500000e-01 |
| 2 | 3.375000000000000 | 1.250000e-01 |
| 3 | 3.312500000000000 | 6.250000e-02 |
| 4 | 3.281250000000000 | 3.125000e-02 |
| 5 | 3.296875000000000 | 1.562500e-02 |
| 6 | 3.289062500000000 | 7.812500e-03 |
| 7 | 3.292968750000000 | 3.906250e-03 |
| 8 | 3.294921875000000 | 1.953125e-03 |
| 9 | 3.295898437500000 | 9.765625e-04 |
| 10 | 3.296386718750000 | 4.882812e-04 |
| 11 | 3.296630859375000 | 2.441406e-04 |

The required root is: 3.296631
The number of iterations performed: 11



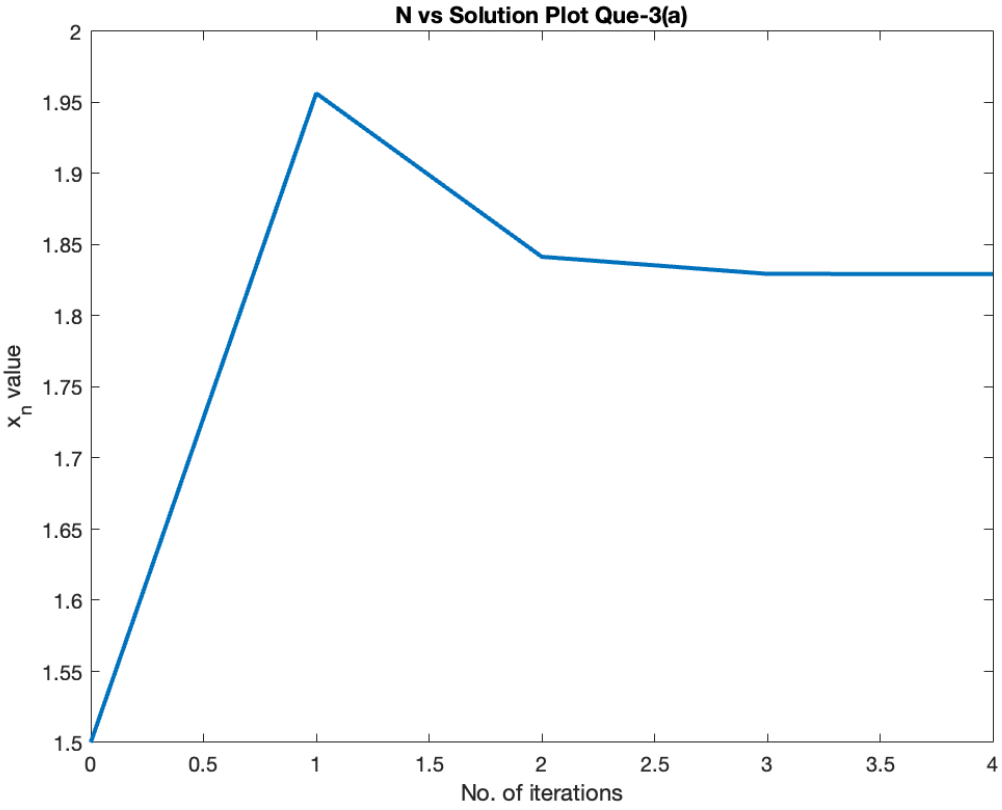


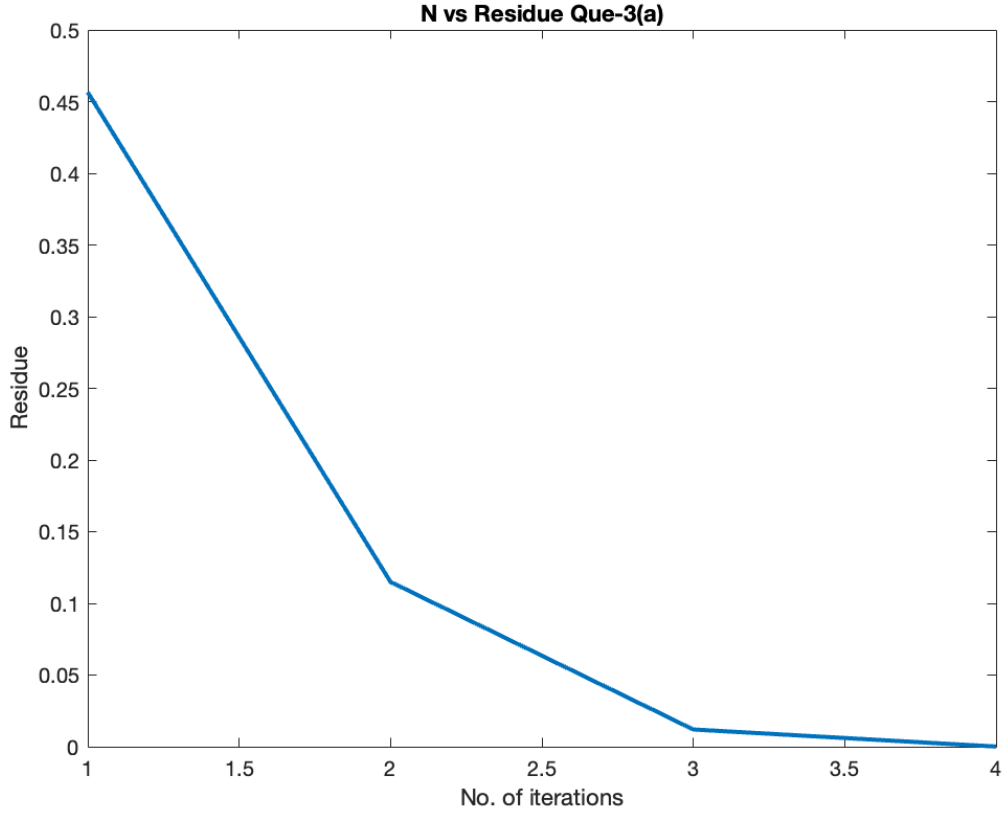

# Ques – 3

(a)

```
Newton Method for Q-3(a)
No. Of Iterations       Approx. Soln.          Error
1                       1.956489721124210      4.564897e-01
2                       1.841533061042061      1.149567e-01
3                       1.829506013203651      1.202705e-02
4                       1.829383614494166      1.223987e-04

The required root is: 1.829384
The number of iterations performed: 4
```

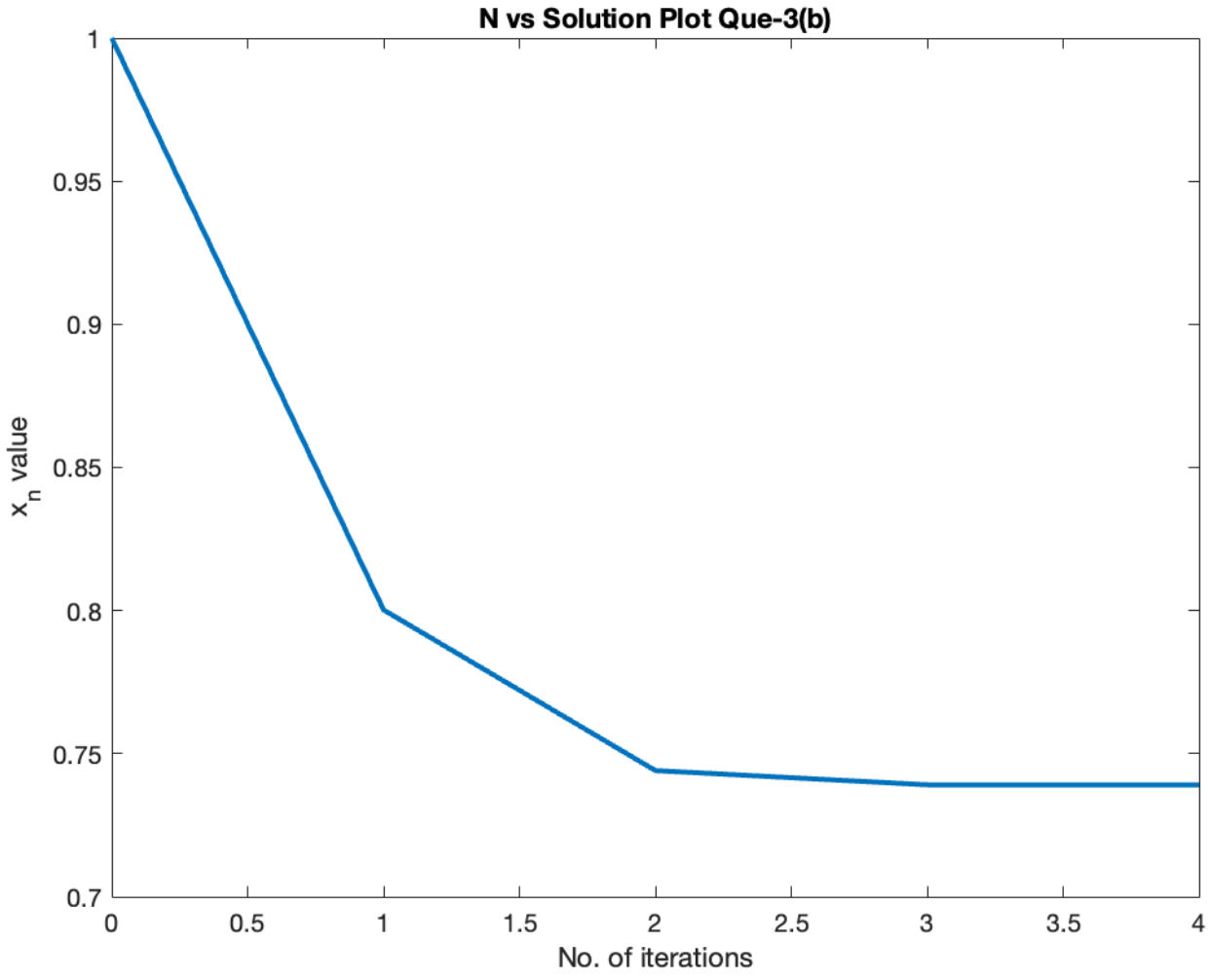

N vs Solution Plot Que-3(a)
x_n value
No. of iterations
2
1.95
1.9
1.85
1.8
1.75
1.7
1.65
1.6
1.55
1.5
0
0.5
1
1.5
2
2.5
3
3.5
4

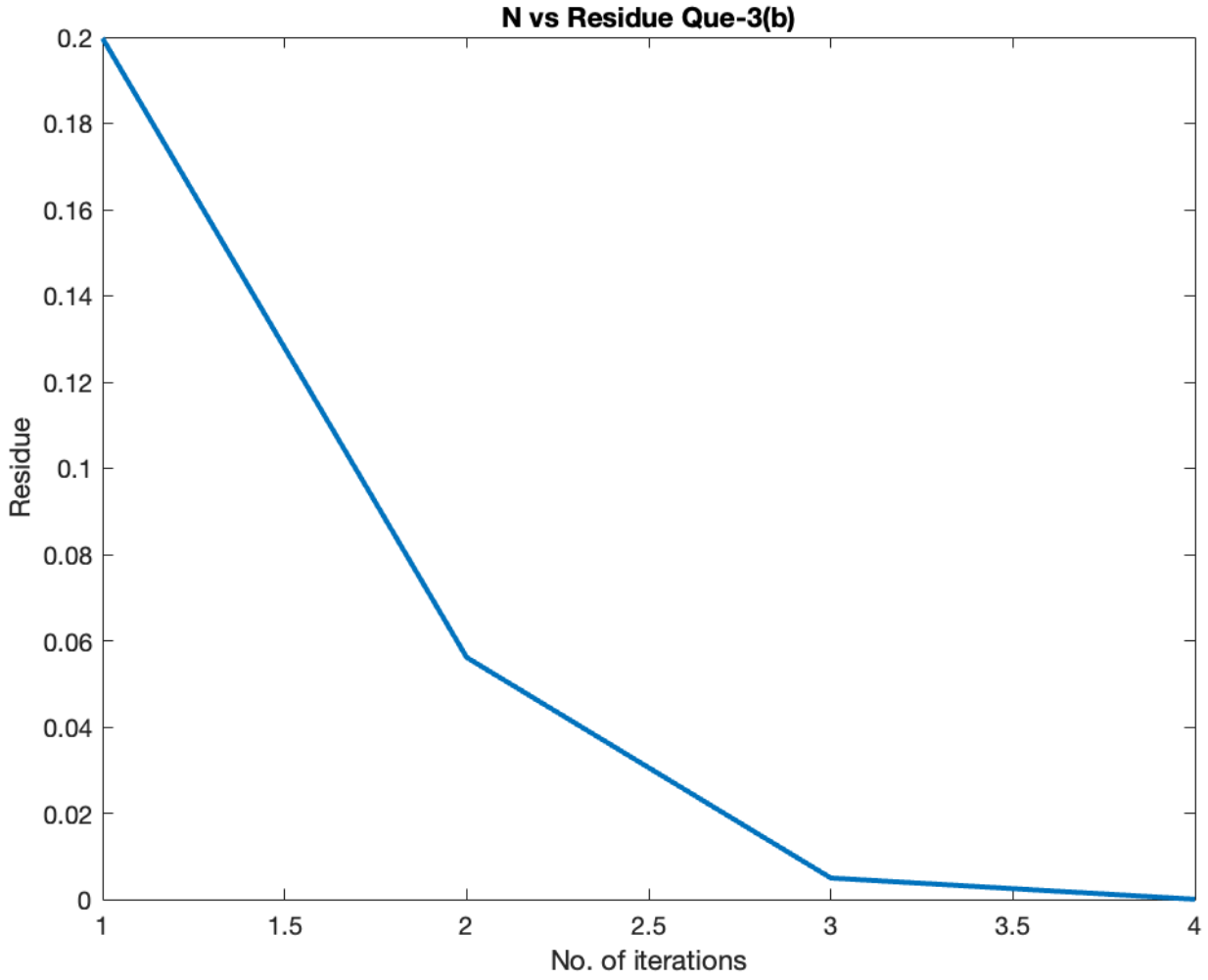

N vs Residue Que-3(a)
Residue
No. of iterations
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
1.5
2
2.5
3
3.5
4

(b)

```
Newton Method for Q-3(b)
No. Of Iterations       Approx. Soln.          Error
1                       0.800232943226195      1.997671e-01
2                       0.744094398494345      5.613854e-02
3                       0.739124068356762      4.970330e-03
4                       0.739085135600735      3.893276e-05

The required root is: 0.739085
The number of iterations performed: 4
```

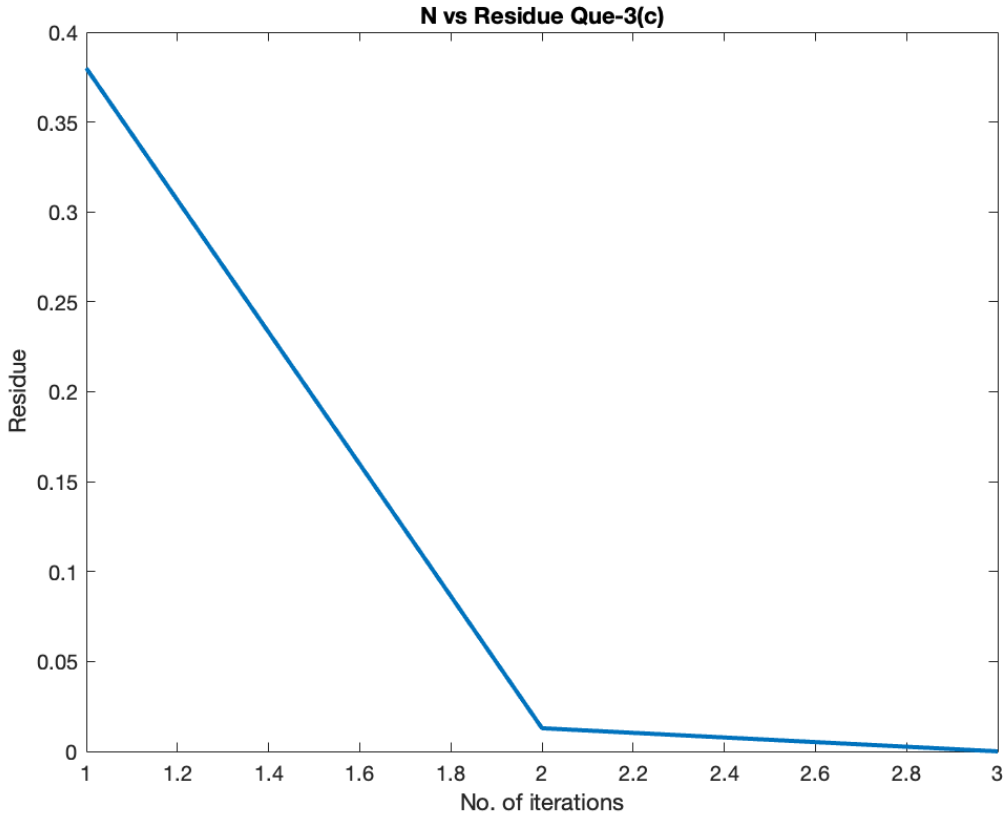


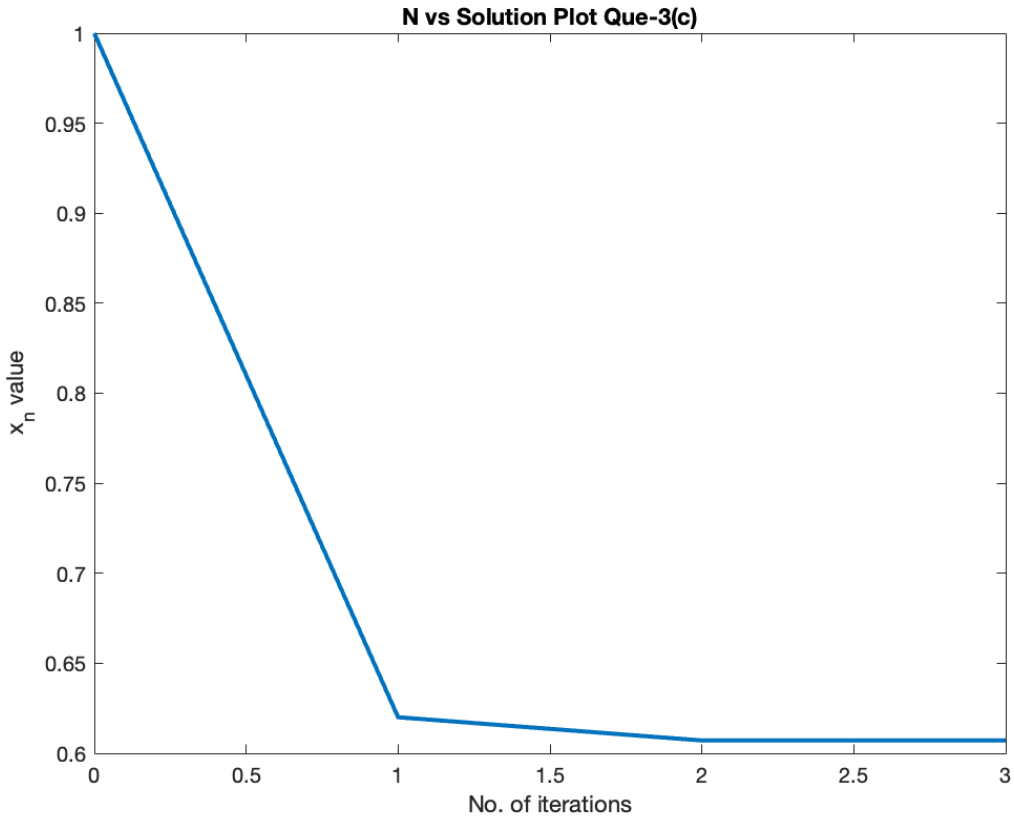



(c)

```
Newton Method for Q-3(c)
No. Of Iterations       Approx. Soln.          Error
1                       0.620015952247299      3.799840e-01
2                       0.607120658147072      1.289529e-02
3                       0.607101648144686      1.901000e-05

The required root is: 0.607102
The number of iterations performed: 3
```

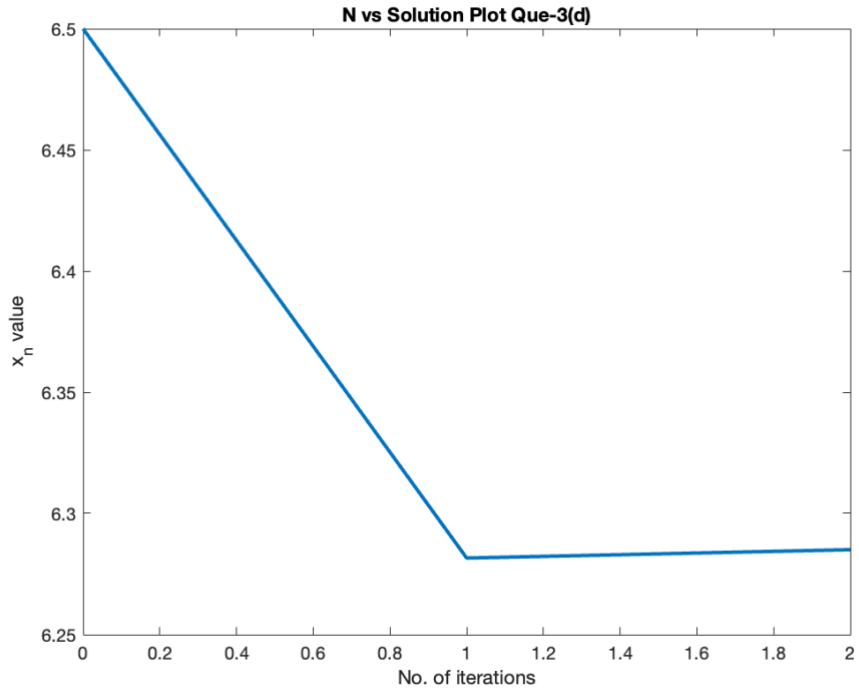

N vs Residue Que-3(c)
Residue
No. of iterations
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3

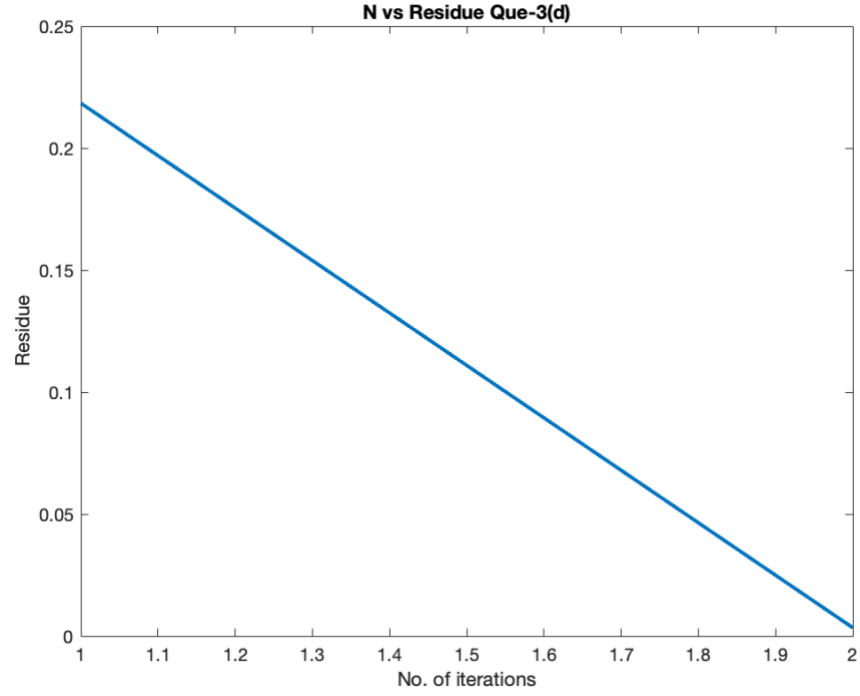

N vs Solution Plot Que-3(c)
x_n value
No. of iterations
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0
0.5
1
1.5
2
2.5
3

(d)

```
Newton Method for Q-3(d)
No. Of Iterations       Approx. Soln.          Error
1                       6.281598506973284      2.184015e-01
2                       6.285049264874215      3.450758e-03

The required root is: 6.285049
The number of iterations performed: 2
```



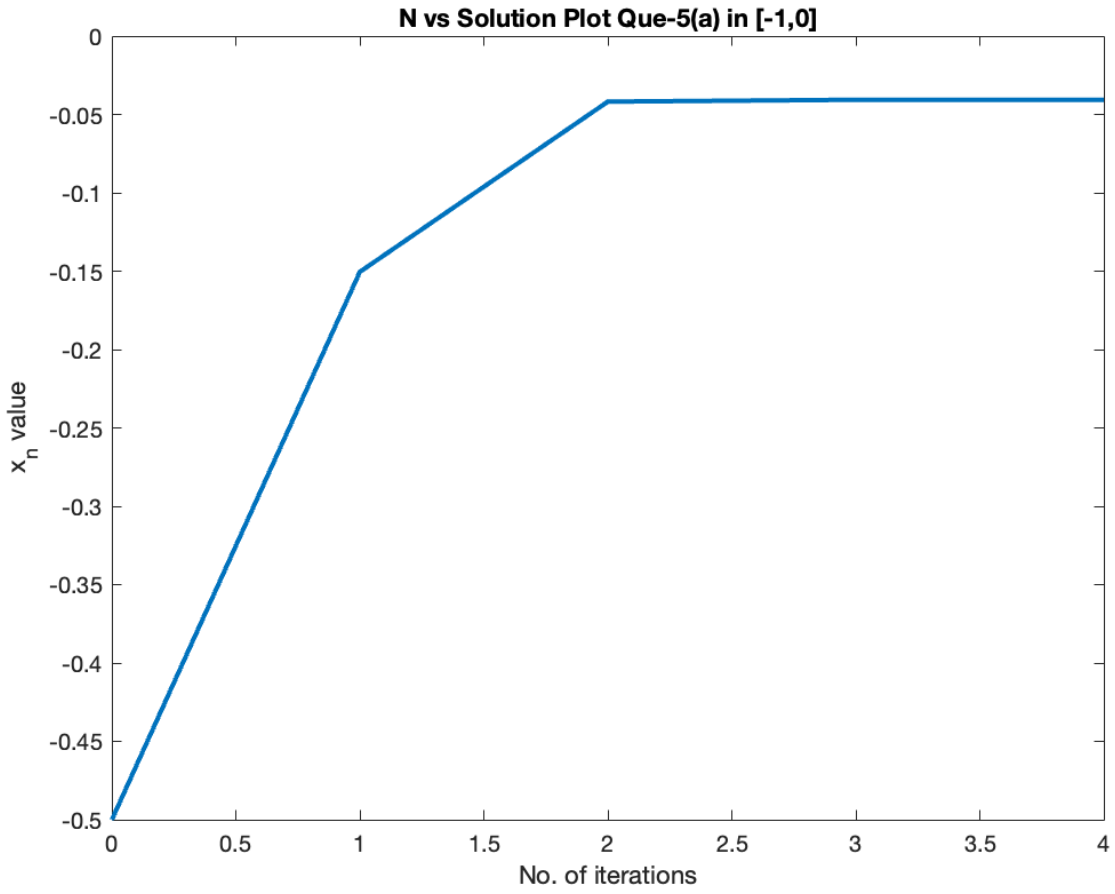



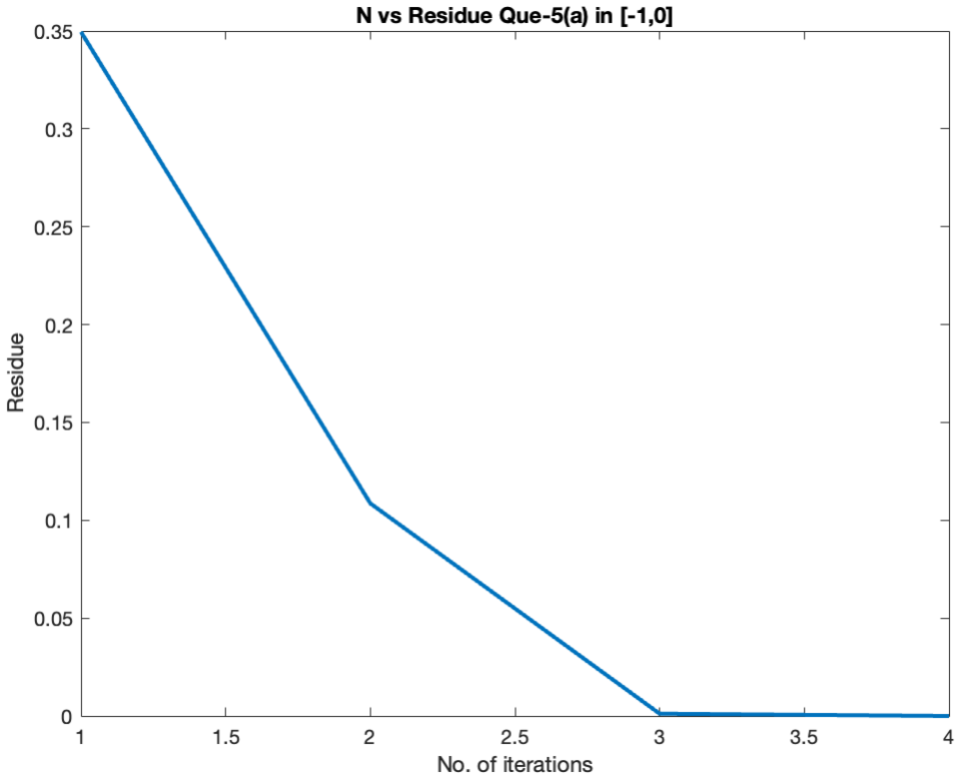
# Ques – 5

(a) [-1,0]

```
Newton Method for Q-5(a) in [-1,0]
No. Of Iterations       Approx. Soln.          Error
1                       -0.150452488687783     3.495475e-01
2                       -0.041816813948870     1.086357e-01
3                       -0.040659343497329     1.157470e-03
4                       -0.040659288315759     5.518157e-08

The required root is: -0.040659
The number of iterations performed: 4
```



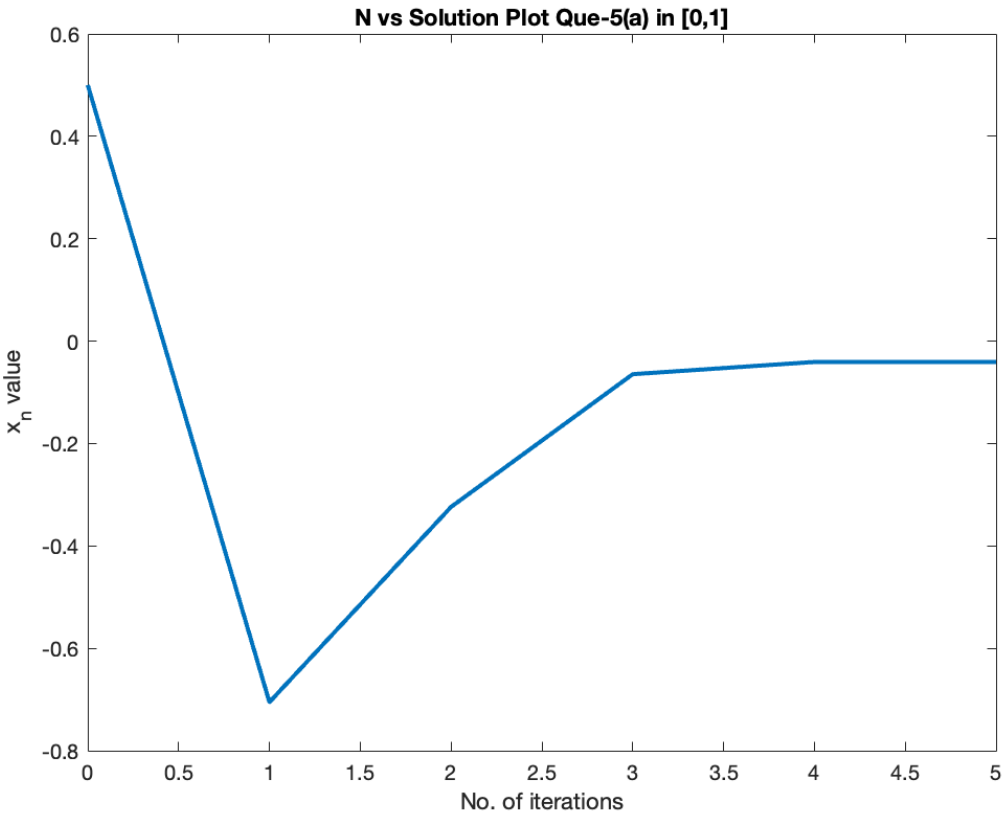


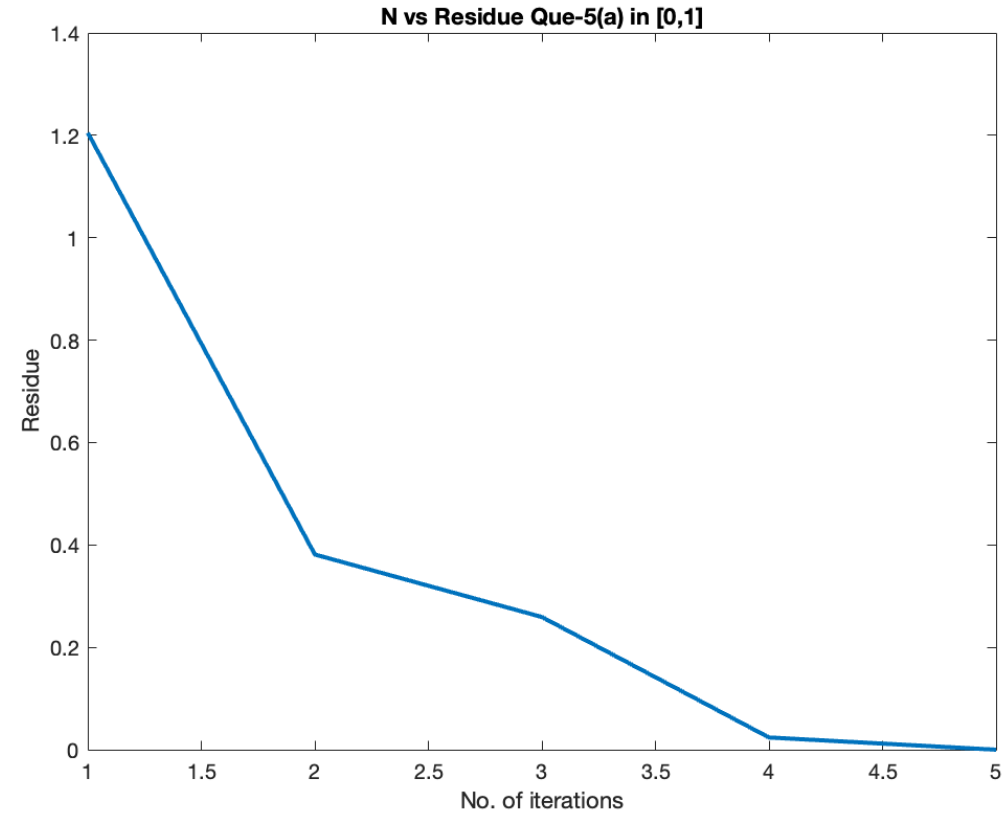

[0,1]

When calculating the root between [0,1] using Newton method by taking initial approximation as 0.5, the algorithm gives the root between [-1,0], this is because the derivative at 0.5 of the given function gives the next iterate as -0.705. Whereas using 0.6 as initial approximation, the algorithm will converge to the root between 0 and 1 which is approx. 0.962398

```
Newton Method for Q-5(a) in [0,1]
No. Of Iterations       Approx. Soln.          Error
1                       -0.705089820359281     1.205090e+00
2                       -0.323791114230475     3.812987e-01
3                       -0.064603131030575     2.591880e-01
4                       -0.040686151151956     2.391698e-02
5                       -0.040659288345335     2.686281e-05

The required root is: -0.040659
The number of iterations performed: 5
```

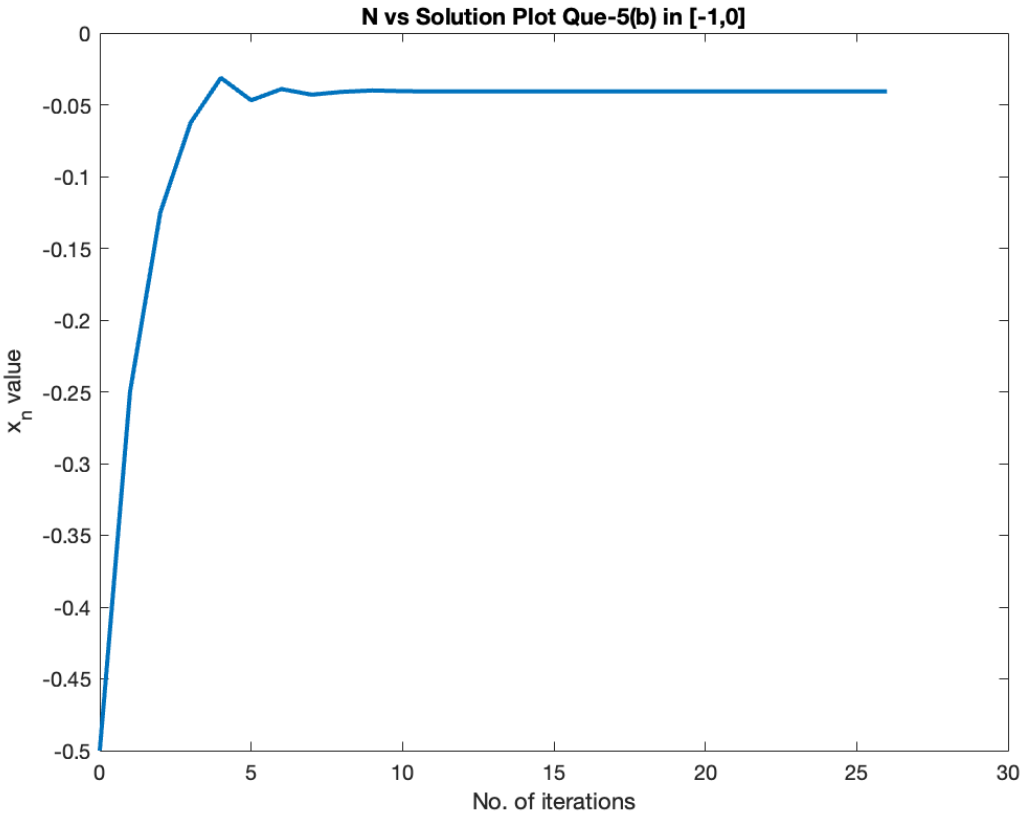

N vs Solution Plot Que-5(a) in [0,1]
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
x_n value
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
No. of iterations

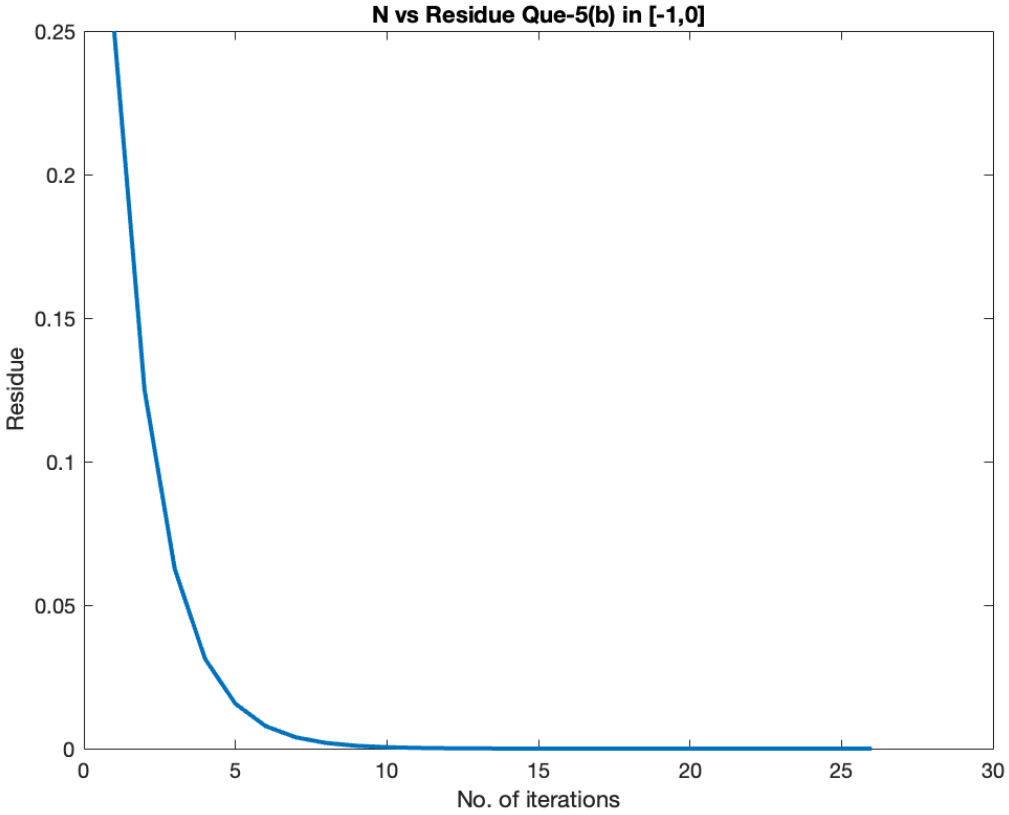

N vs Residue Que-5(a) in [0,1]
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Residue
1
1.5
2
2.5
3
3.5
4
4.5
5
No. of iterations

(b) [-1,0]

```
Bisection Method for Q-5(b) in [-1,0]
No. Of Iterations       Approx. Soln.           Error
1                       -0.250000000000000      2.500000e-01
2                       -0.125000000000000      1.250000e-01
3                       -0.062500000000000      6.250000e-02
4                       -0.031250000000000      3.125000e-02
5                       -0.046875000000000      1.562500e-02
6                       -0.039062500000000      7.812500e-03
7                       -0.042968750000000      3.906250e-03
8                       -0.041015625000000      1.953125e-03
9                       -0.040039062500000      9.765625e-04
10                      -0.040527343750000      4.882812e-04
11                      -0.040771484375000      2.441406e-04
12                      -0.040649414062500      1.220703e-04
13                      -0.040710449218750      6.103516e-05
14                      -0.040679931640625      3.051758e-05
15                      -0.040664672851562      1.525879e-05
16                      -0.040657043457031      7.629395e-06
17                      -0.040660858154297      3.814697e-06
18                      -0.040658950805664      1.907349e-06
19                      -0.040659904479980      9.536743e-07
20                      -0.040659427642822      4.768372e-07
21                      -0.040659189224243      2.384186e-07
22                      -0.040659308433533      1.192093e-07
23                      -0.040659248828888      5.960464e-08
24                      -0.040659278631210      2.980232e-08
25                      -0.040659293532372      1.490116e-08
26                      -0.040659286081791      7.450581e-09

The required root is: -0.040659
The number of iterations performed: 26
```

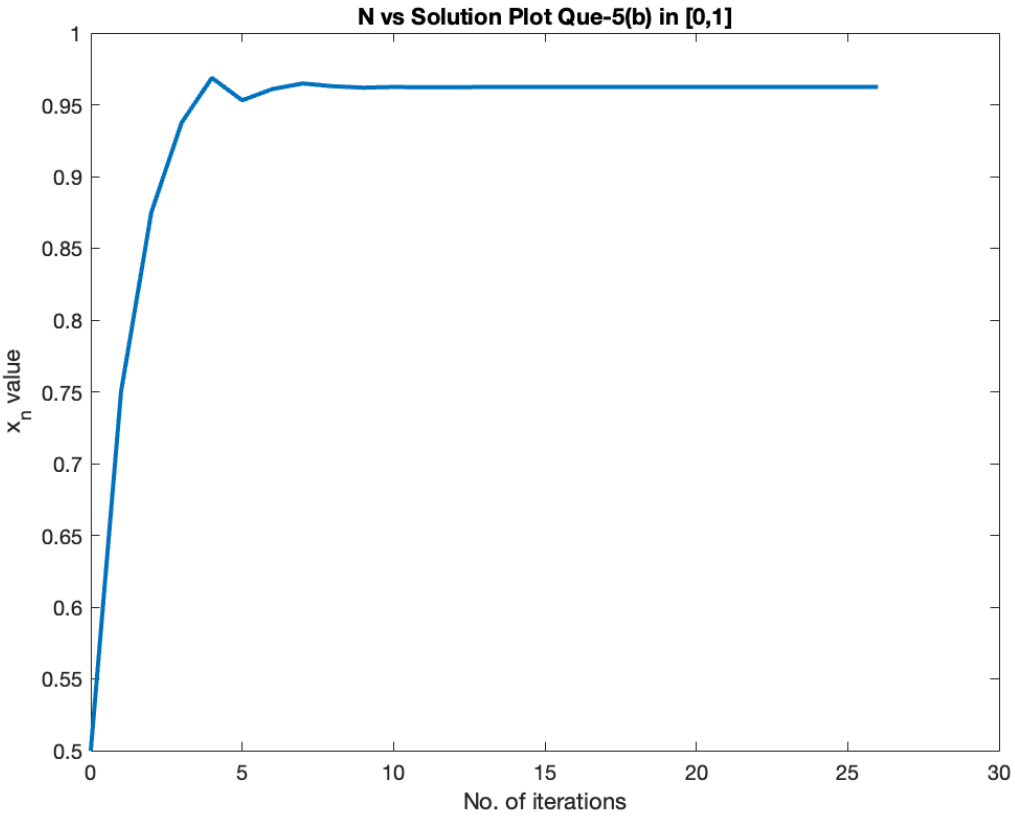

N vs Solution Plot Que-5(b) in [-1,0]
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
x_n value
0
5
10
15
20
25
30
No. of iterations

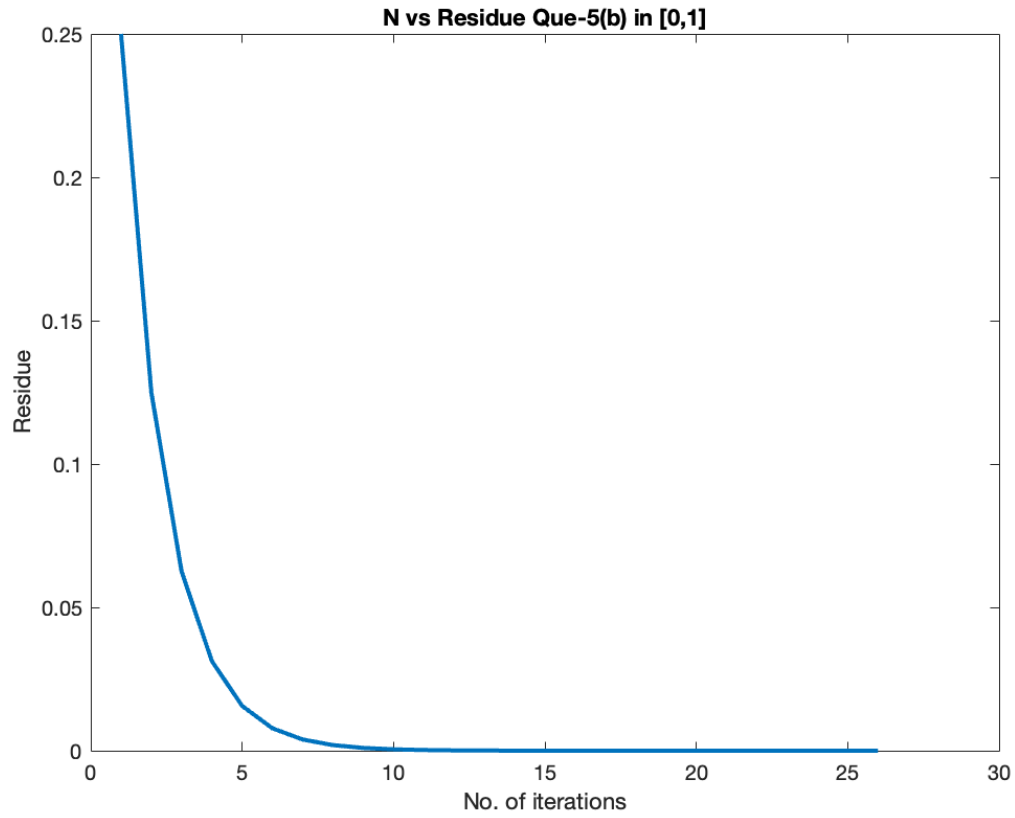

N vs Residue Que-5(b) in [-1,0]
0.25
0.2
0.15
0.1
0.05
0
Residue
0
5
10
15
20
25
30
No. of iterations

[0,1]

```
Bisection Method for Q-5(b) in [0,1]
No. Of Iterations       Approx. Soln.           Error
1                       0.750000000000000       2.500000e-01
2                       0.875000000000000       1.250000e-01
3                       0.937500000000000       6.250000e-02
4                       0.968750000000000       3.125000e-02
5                       0.953125000000000       1.562500e-02
6                       0.960937500000000       7.812500e-03
7                       0.964843750000000       3.906250e-03
8                       0.962890625000000       1.953125e-03
9                       0.961914062500000       9.765625e-04
10                      0.962402343750000       4.882812e-04
11                      0.962158203125000       2.441406e-04
12                      0.962280273437500       1.220703e-04
13                      0.962341308593750       6.103516e-05
14                      0.962371826171875       3.051758e-05
15                      0.962387084960938       1.525879e-05
16                      0.962394714355469       7.629395e-06
17                      0.962398529052734       3.814697e-06
18                      0.962396621704102       1.907349e-06
19                      0.962397575378418       9.536743e-07
20                      0.962398052215576       4.768372e-07
21                      0.962398290634155       2.384186e-07
22                      0.962398409843445       1.192093e-07
23                      0.962398469448090       5.960464e-08
24                      0.962398439645767       2.980232e-08
25                      0.962398424744606       1.490116e-08
26                      0.962398417294025       7.450581e-09

The required root is: 0.962398
The number of iterations performed: 26
```

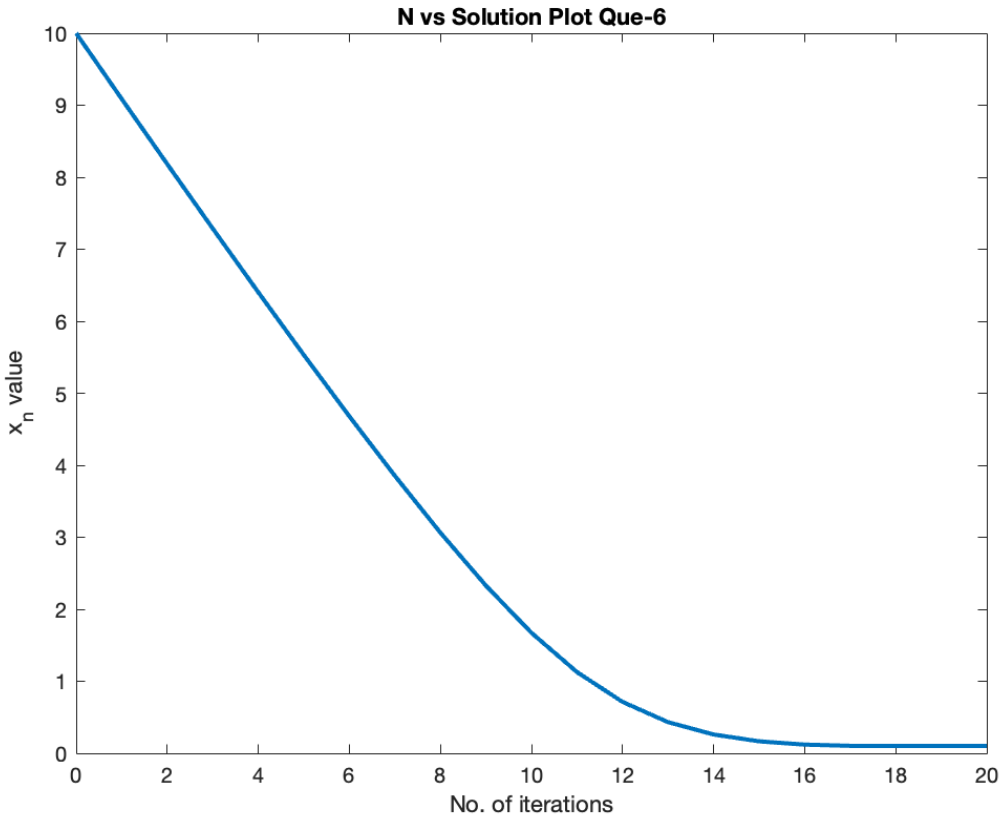

N vs Solution Plot Que-5(b) in [0,1]
x_n value
No. of iterations
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
5
10
15
20
25
30

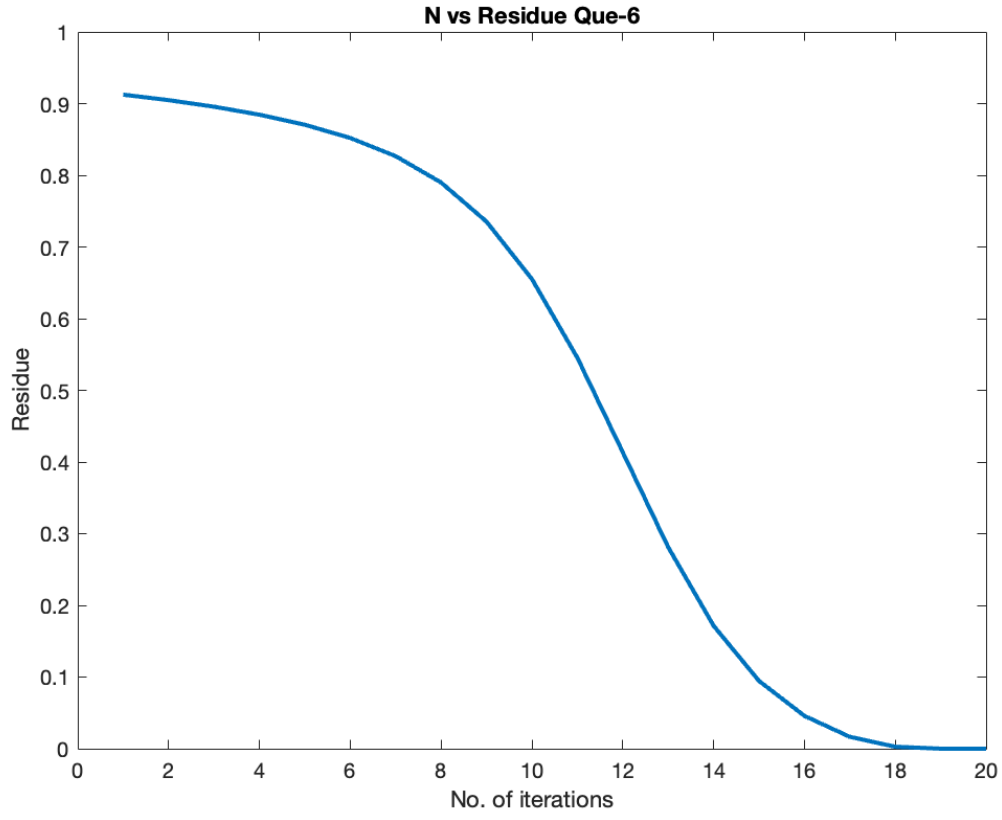

N vs Residue Que-5(b) in [0,1]
Residue
No. of iterations
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
30

# Ques – 6

```
Newton Method for Q-6
No. Of Iterations       Approx. Soln.          Error
1                       9.087508964757124      9.124910e-01
2                       8.182685525669518      9.048234e-01
3                       7.287004430692514      8.956811e-01
4                       6.402476818150976      8.845276e-01
5                       5.531990601674328      8.704862e-01
6                       4.679930050550689      8.520606e-01
7                       3.853269837544957      8.266602e-01
8                       3.063328299587984      7.899415e-01
9                       2.327977095861905      7.353512e-01
10                      1.672776460204567      6.552006e-01
11                      1.127328614132175      5.454478e-01
12                      0.713681553924737      4.136471e-01
13                      0.432433579539451      2.812480e-01
14                      0.260866148128535      1.715674e-01
15                      0.166287946536102      9.457820e-02
16                      0.120430790351644      4.585716e-02
17                      0.103829387140719      1.660140e-02
18                      0.101076673892048      2.752713e-03
19                      0.100997993903098      7.867999e-05
20                      0.100997929685793      6.421731e-08

The required root is: 0.100998
The number of iterations performed: 20
```


N vs Solution Plot Que-6
$x_n$ value
No. of iterations
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
16
18
20


N vs Residue Que-6
Residue
No. of iterations
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
2
4
6
8
10
12
14
16
18
20

# Ques – 7

```
Newton Method for Q-7
No. Of Iterations       Approx. Soln.           Error
1                       -98.000000000000000     100

The required root is: -98.000000
The number of iterations performed: 1
```

Newton's method gives solution for the function at -98. This is due to the fact that the nature of the function changes at ~2.005 from decreasing to increasing due to which, the derivative at 2 gives the next iterate or newton's method far from 2 which turns out to be -98 and the algorithm ends at it is one of the roots.

# Ques – 4

```
Newton Method for Q-4
No. Of Iterations       Approx. Soln.   Error
0                       0.000100        0

The required root is: 0.000100
The number of iterations performed: 0
```

At $x_0$ = 1e-4, the error is exp {-1 / $(0.0001)^2$} = 6.451709693e-43429449 and
at $x_1$ = 5e-5, the error is exp {-1/ $(0.00005)^2$} = 1.732603825e-173717793.

```
>> eps

ans =

   2.2204e-16
```

The machine epsilon for the computer used is: 2.2204e-16

We can clearly observe that, at $x_0$ = 0.0001 and $x_1$ = 0.00005, the actual errors are much lesser than the machine epsilon, due to which, these values are accepted as zeros of the function though they are clearly not.

So, if we try to use newton method by taking initial approximation as 0.0001 or 0.00005, we won't go any below of these, instead the algorithm will just declare the initial approximation as the root.

**Note: -** The machine epsilon is the smallest positive number that, when added to 1.0, results in a value different from 1.0 in the floating-point representation of the computer.