

Lab – 13

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Ques – 1

To implement the Linear Shooting method, we assumed $v = u'$ and thus got a system of 2 ODEs, which we solved using the fourth order Runge-Kutta method for system of equations. Using the Linear Shooting method for given BVP, we get the following results: -

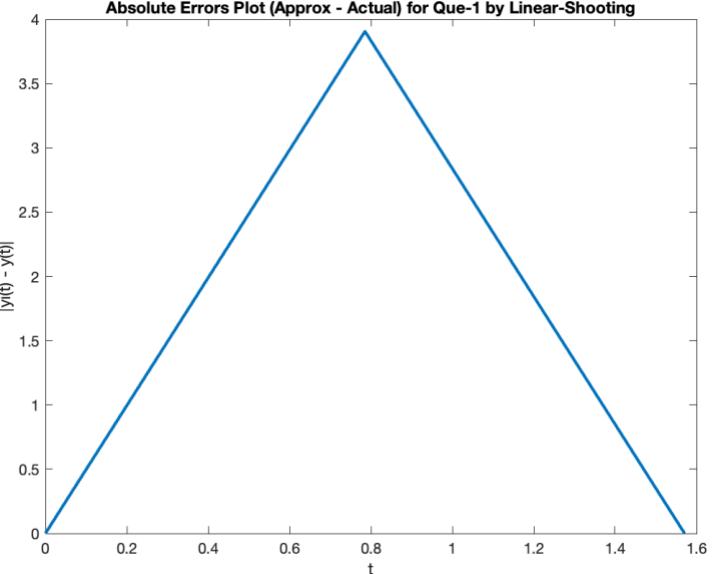
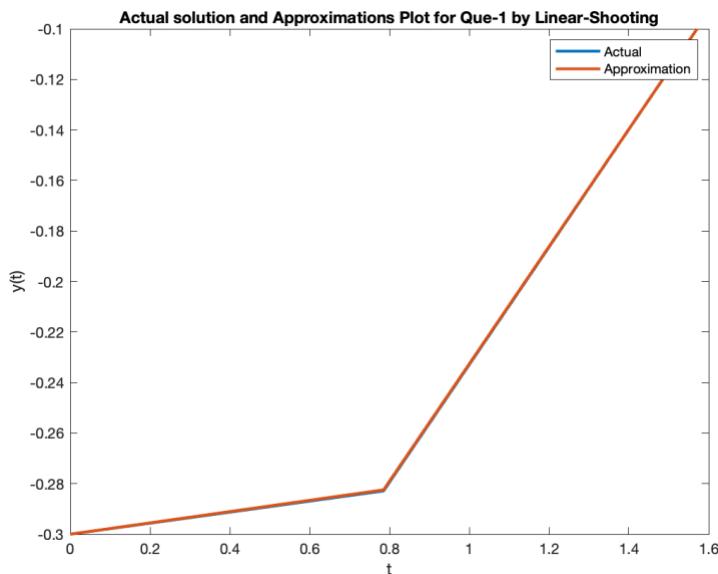
(a) For $h = \pi/4$

Que-1

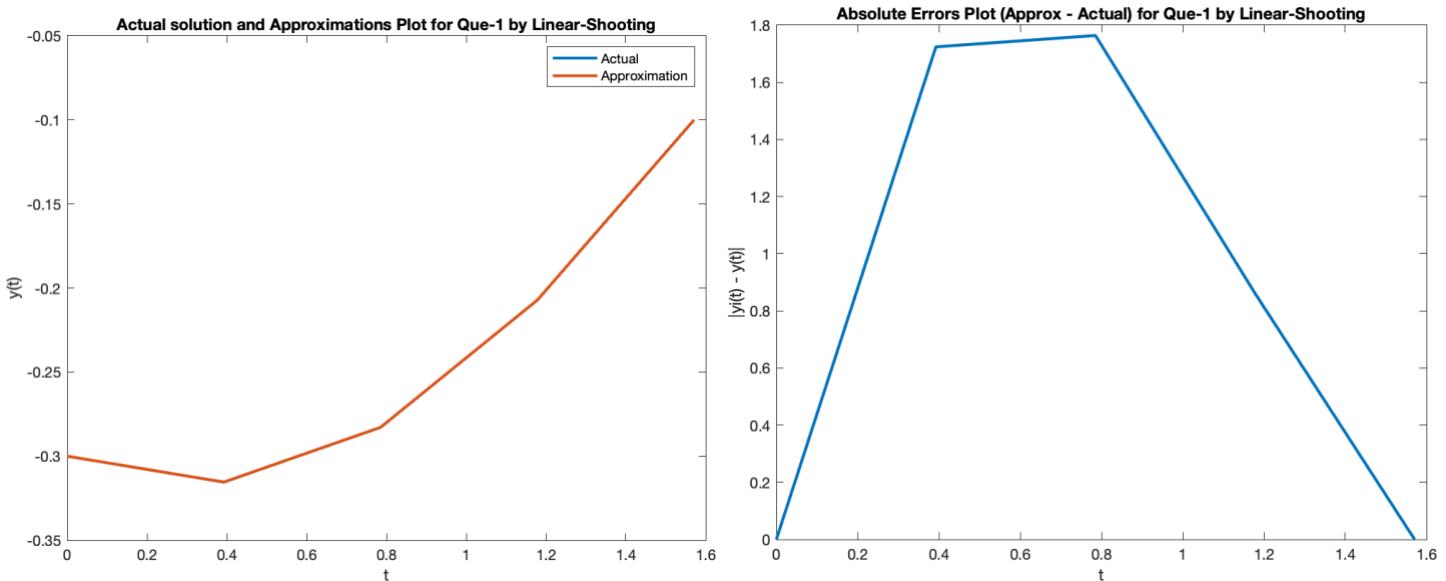
Que-1 by Linear-Shooting

For $h = 7.853982e-01$

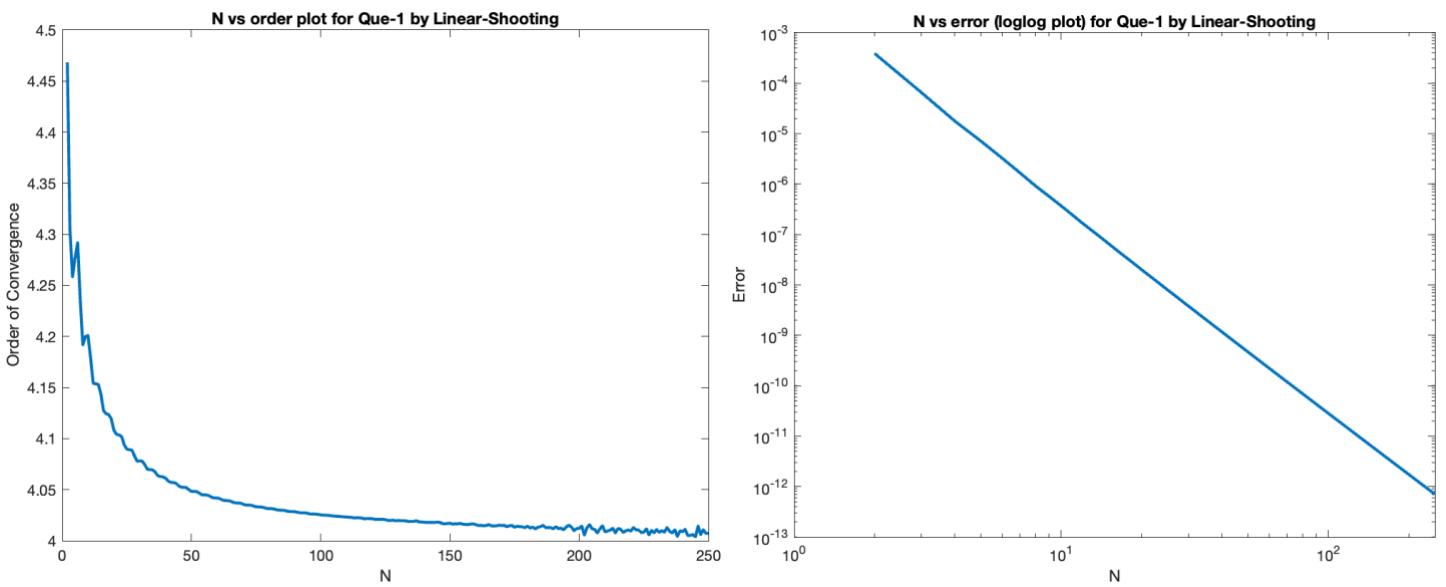
t	Approximation	Exact	Error(Exact - Approx.)
0.000000	-0.3000000	-0.3000000	0.0000000
0.785398	-0.2824522	-0.2828427	0.0003905
1.570796	-0.1000000	-0.1000000	0.0000000

(b) For $h = \pi/8$ For $h = 3.926991e-01$

t	Approximation	Exact	Error(Exact - Approx.)
0.000000	-0.3000000	-0.3000000	0.0000000
0.392699	-0.3154150	-0.3154322	0.0000172
0.785398	-0.2828251	-0.2828427	0.0000176
1.178097	-0.2071844	-0.2071930	0.0000086
1.570796	-0.1000000	-0.1000000	0.0000000



Since the Runge-Kutta method is used, the order of convergence should go to 4 which can be noticed in the below mentioned plot: -



We can notice that since the errors are very low, the approximation and exact plots seem to coincide, however if those plots are zoomed in, the error can be noticed.

Ques – 2

For this question, the six parts have total 3 types of boundary conditions, which are: -

- Dirichlet Conditions. (parts (a) and (b))
- Neumann Conditions. (parts (c) and (d))
- Mixed Conditions. (parts (e) and (f))

Now, for every type of boundary condition, three types of schemes are implemented where the second derivative is always replaced by the central difference, while the first derivative is replaced by Forward difference, Central Difference and Backward Difference for schemes 1, 2 and 3 respectively.

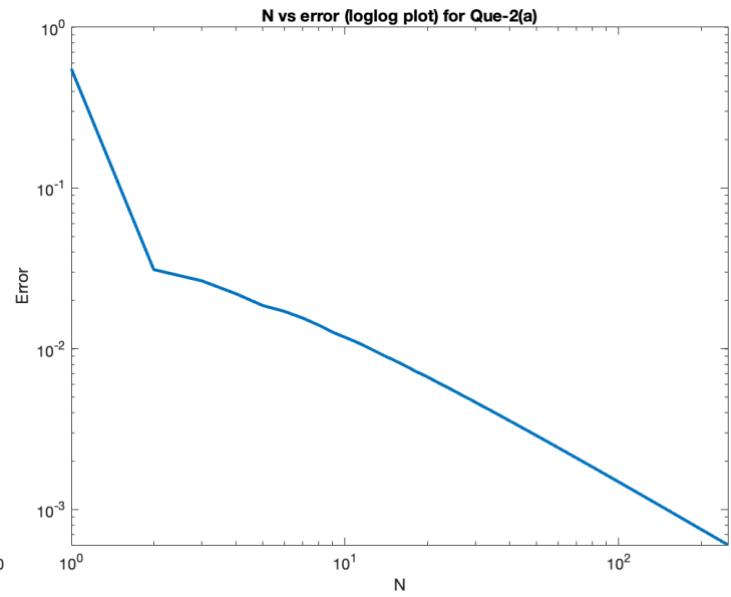
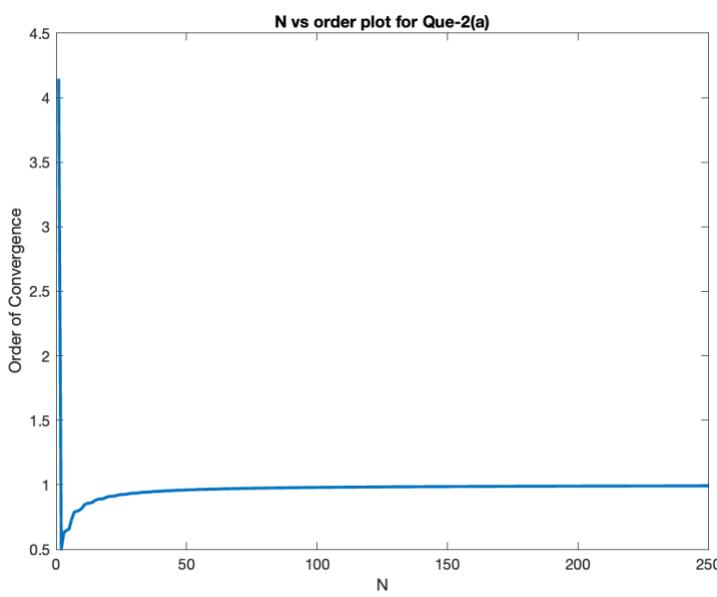
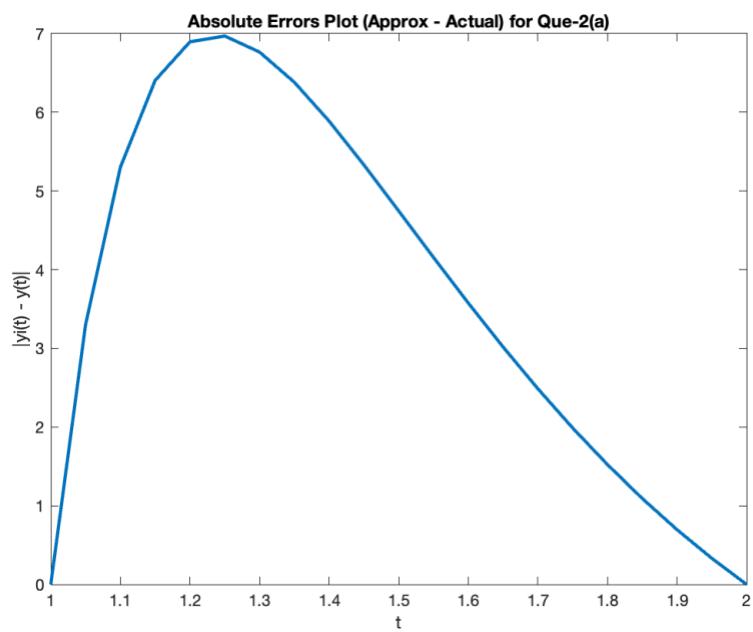
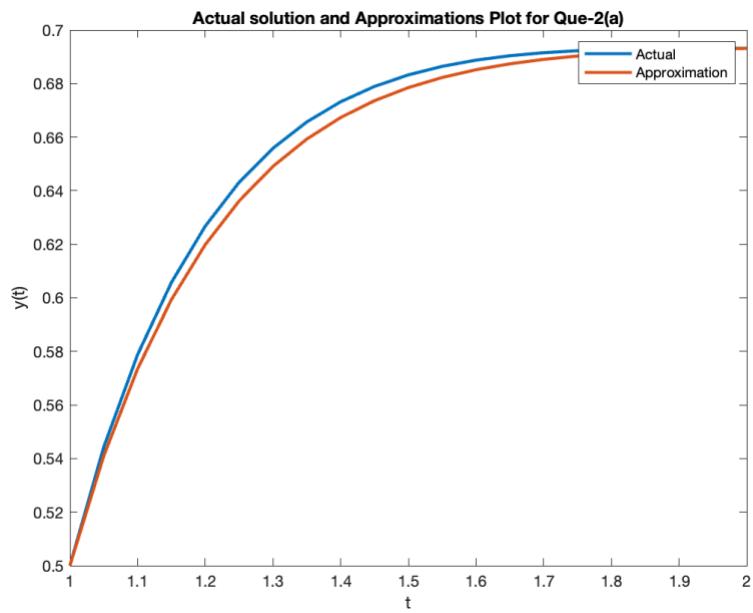
By replacing the derivatives by finite differences, we can manipulate the linear equations to get a $N \times N$ diagonally dominant sparse matrix A such that $Aw = b$, and we eliminate the w_0 and w_{N+1} by using the given boundary conditions and making sufficient changes in first and last elements of b and the matrix entries $A(1, 1)$ and $A(N, N)$. Thus, by solving the system of linear equations $Aw = b$, we get $w = [w_1, w_2, \dots, w_N]$. Now w_0 and w_{N+1} can be calculated using boundary conditions and appended in this array.

For every part, N is taken to be 19 and therefore, $h = (b-a)/20$. After running the implemented schemes for each part, we get the following results: -

(a)

➤ Forward-difference for first derivative: -

Que-2(a)			
t	Approximation	Exact	Error(Exact - Approx.)
1.000000	0.5000000	0.5000000	0.0000000
1.050000	0.5409536	0.5442550	0.0033015
1.100000	0.5734796	0.5787813	0.0053016
1.150000	0.5993298	0.6057355	0.0064057
1.200000	0.6198702	0.6267660	0.0068958
1.250000	0.6361740	0.6431436	0.0069695
1.300000	0.6490897	0.6558554	0.0067657
1.350000	0.6592916	0.6656739	0.0063822
1.400000	0.6673189	0.6732069	0.0058880
1.450000	0.6736043	0.6789357	0.0053314
1.500000	0.6784966	0.6832429	0.0047463
1.550000	0.6822780	0.6864339	0.0041559
1.600000	0.6851774	0.6887536	0.0035762
1.650000	0.6873811	0.6903988	0.0030177
1.700000	0.6890409	0.6915279	0.0024870
1.750000	0.6902807	0.6922688	0.0019881
1.800000	0.6912020	0.6927249	0.0015230
1.850000	0.6918874	0.6929796	0.0010922
1.900000	0.6924049	0.6931004	0.0006955
1.950000	0.6928098	0.6931417	0.0003319
2.000000	0.6931472	0.6931472	0.0000000

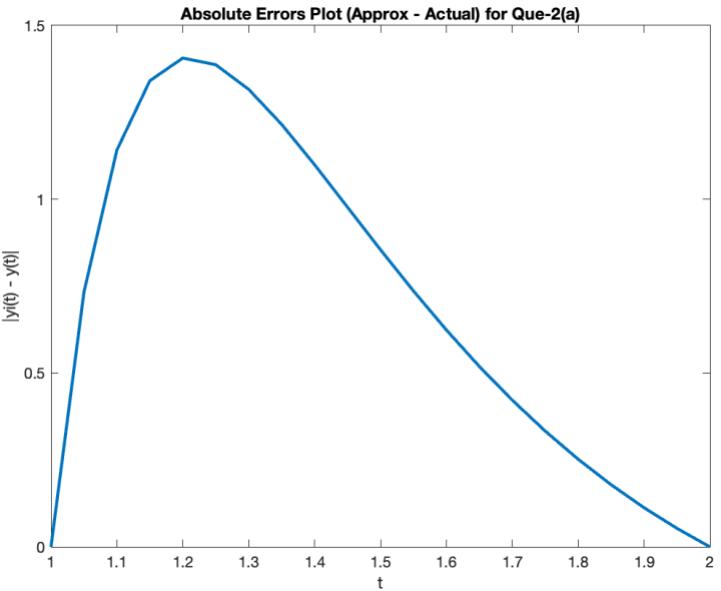
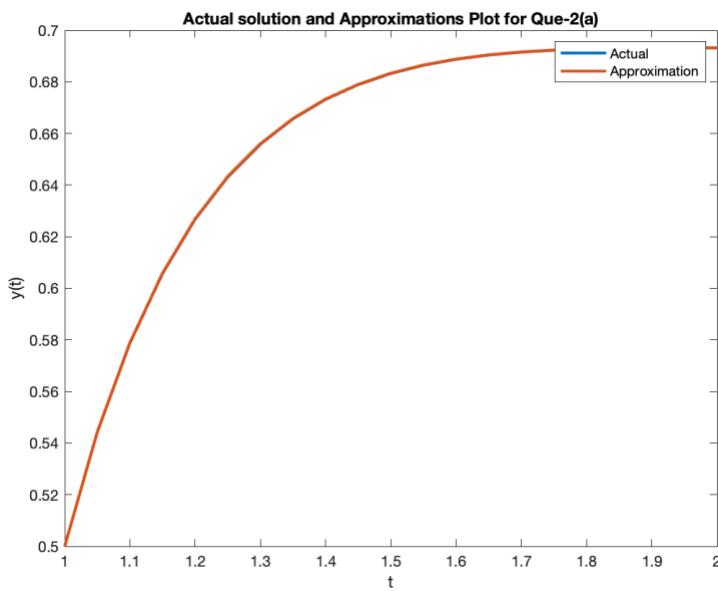


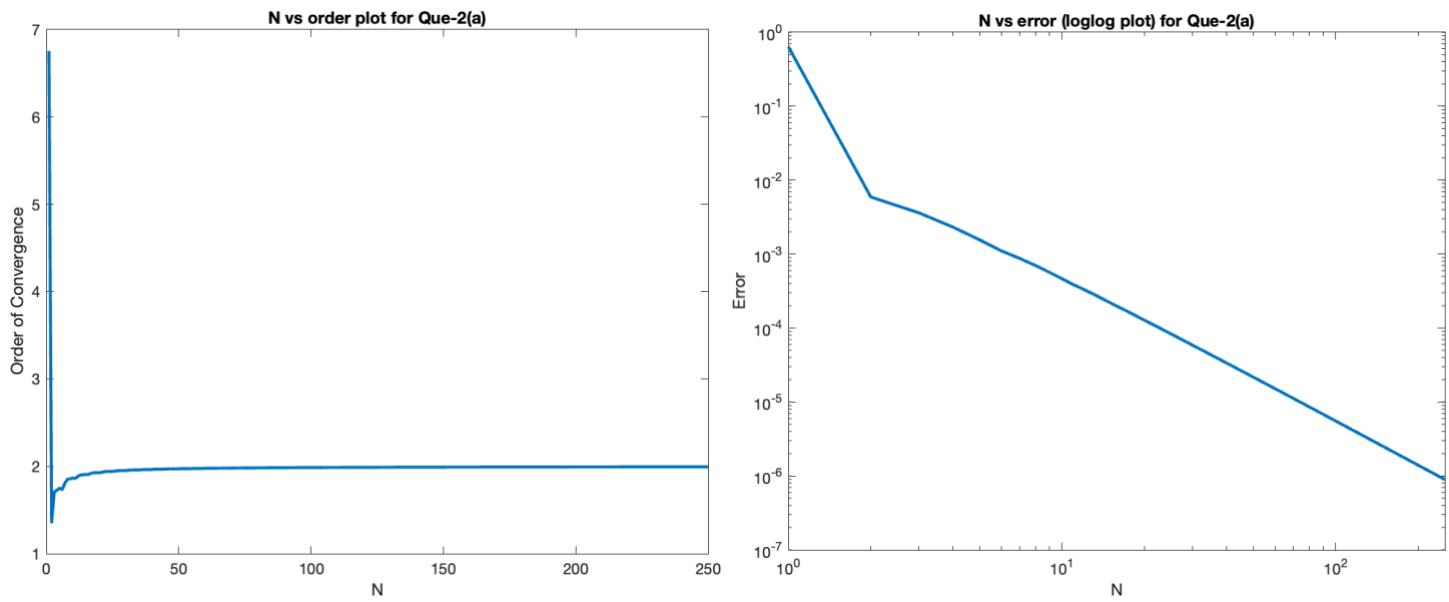
➤ Central-difference for first derivative: -

Que-2(a)

For $h = 5.000000e-02$

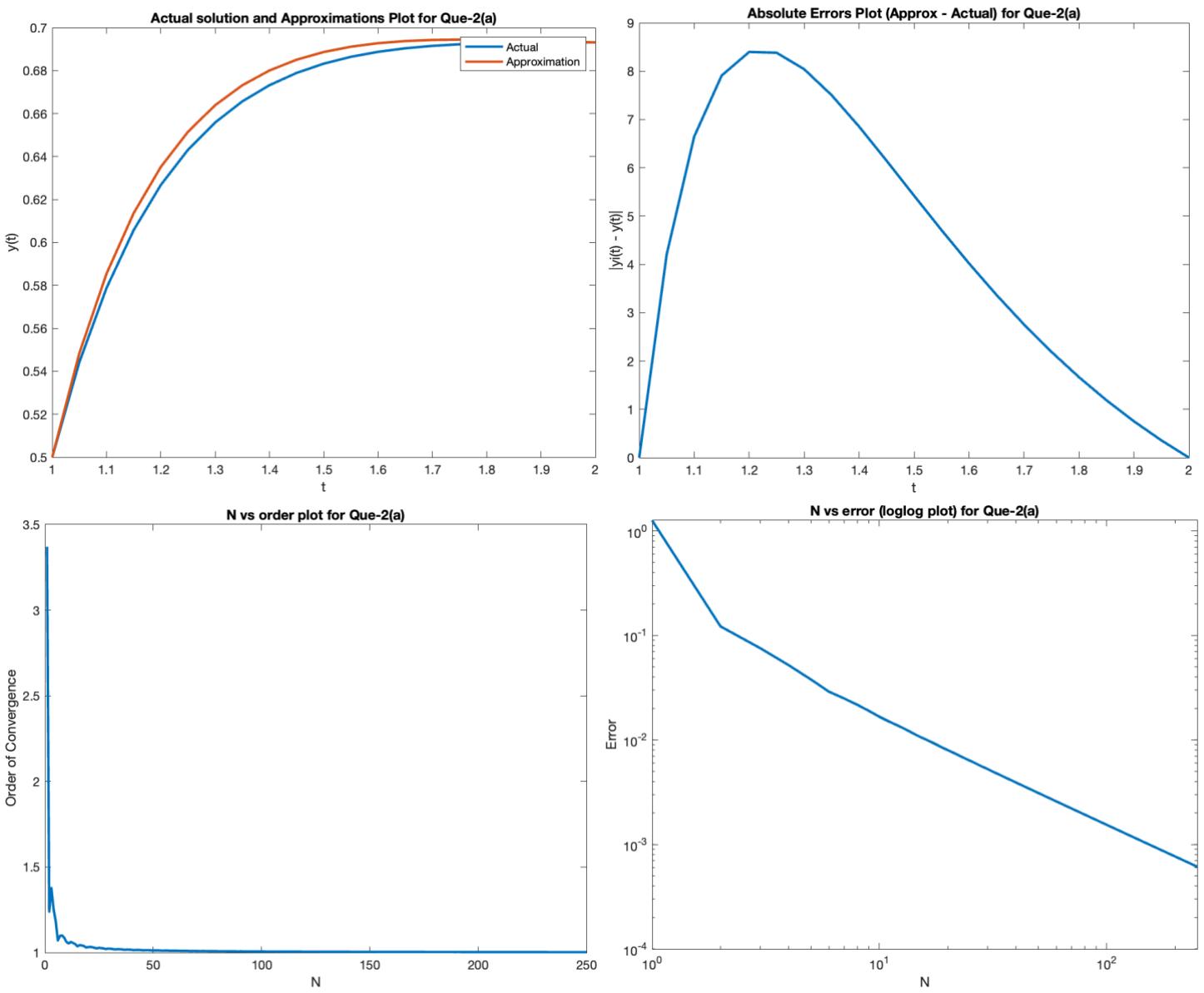
t	Approximation	Exact	Error(Exact - Approx.)
1.000000	0.5000000	0.5000000	0.0000000
1.050000	0.5443283	0.5442550	0.0000733
1.100000	0.5788954	0.5787813	0.0001142
1.150000	0.6058696	0.6057355	0.0001341
1.200000	0.6269066	0.6267660	0.0001406
1.250000	0.6432823	0.6431436	0.0001387
1.300000	0.6559870	0.6558554	0.0001316
1.350000	0.6657954	0.6656739	0.0001216
1.400000	0.6733169	0.6732069	0.0001099
1.450000	0.6790334	0.6789357	0.0000977
1.500000	0.6833284	0.6832429	0.0000855
1.550000	0.6865076	0.6864339	0.0000736
1.600000	0.6888160	0.6887536	0.0000624
1.650000	0.6904507	0.6903988	0.0000519
1.700000	0.6915701	0.6915279	0.0000422
1.750000	0.6923022	0.6922688	0.0000333
1.800000	0.6927502	0.6927249	0.0000252
1.850000	0.6929975	0.6929796	0.0000179
1.900000	0.6931117	0.6931004	0.0000113
1.950000	0.6931470	0.6931417	0.0000053
2.000000	0.6931472	0.6931472	0.0000000





➤ Backward-difference for first derivative: -

Que-2(a) by scheme-3 For h = 5.000000e-02			
t	Approximation	Exact	Error(Exact - Approx.)
1.000000	0.5000000	0.5000000	0.0000000
1.050000	0.5484617	0.5442550	0.0042066
1.100000	0.5854264	0.5787813	0.0066452
1.150000	0.6136451	0.6057355	0.0079096
1.200000	0.6351645	0.6267660	0.0083985
1.250000	0.6515250	0.6431436	0.0083815
1.300000	0.6638970	0.6558554	0.0080416
1.350000	0.6731776	0.6656739	0.0075038
1.400000	0.6800599	0.6732069	0.0068529
1.450000	0.6850824	0.6789357	0.0061467
1.500000	0.6886666	0.6832429	0.0054237
1.550000	0.6911435	0.6864339	0.0047096
1.600000	0.6927746	0.6887536	0.0040210
1.650000	0.6937667	0.6903988	0.0033679
1.700000	0.6942840	0.6915279	0.0027561
1.750000	0.6944574	0.6922688	0.0021886
1.800000	0.6943908	0.6927249	0.0016659
1.850000	0.6941671	0.6929796	0.0011875
1.900000	0.6938522	0.6931004	0.0007518
1.950000	0.6934984	0.6931417	0.0003568
2.000000	0.6931472	0.6931472	0.0000000



(b)

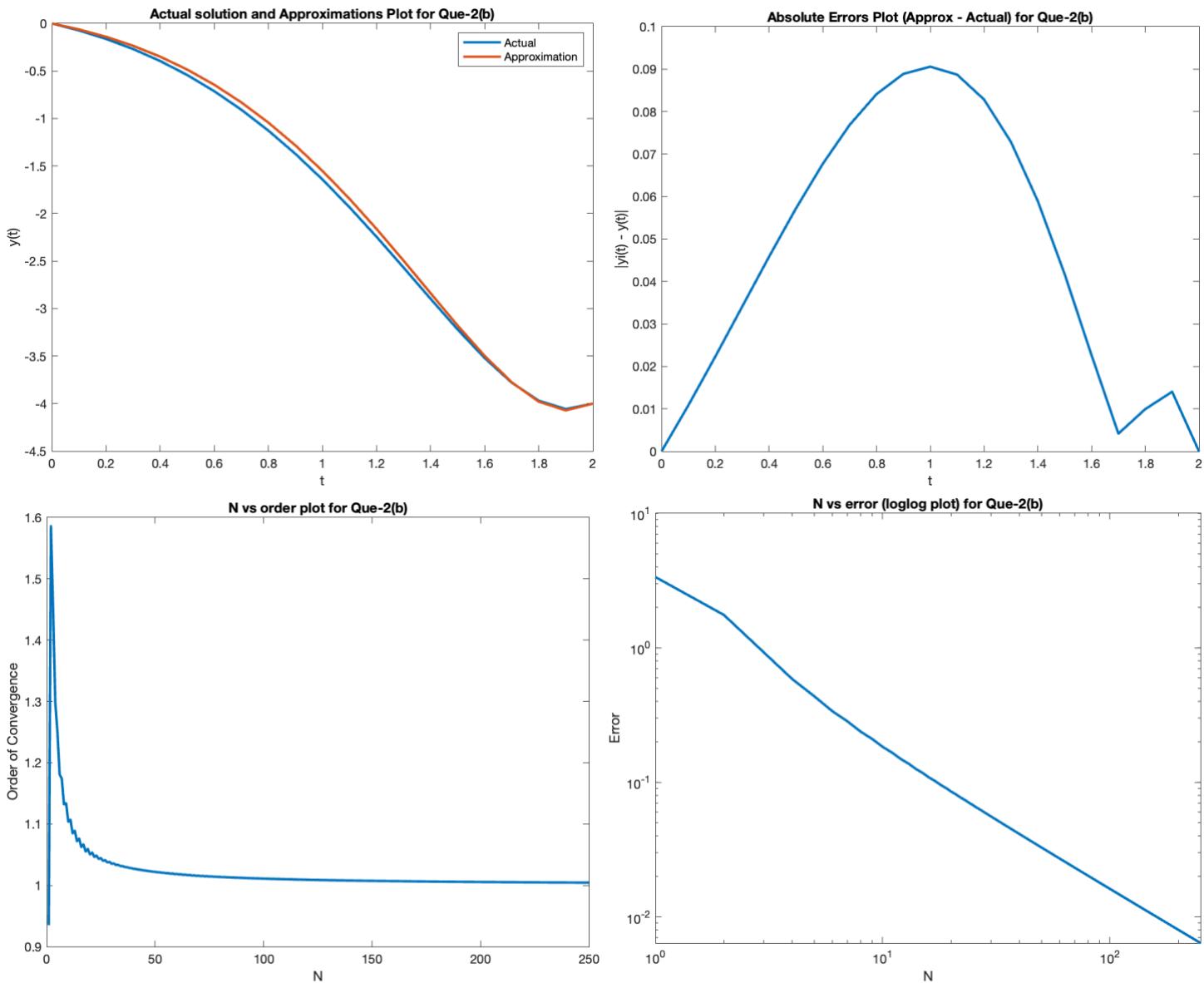
➤ Forward-difference for first derivative: -

Que-2(b)

For $h = 1.000000e-01$

t	Approximation	Exact	Error($ Exact - Approx. $)
0.000000	0.000000	0.000000	0.0000000
0.100000	-0.0628025	-0.0736691	0.0108666
0.200000	-0.1403892	-0.1627002	0.0223110
0.300000	-0.2350642	-0.2691374	0.0340733
0.400000	-0.3491576	-0.3949876	0.0458300
0.500000	-0.4849508	-0.5421435	0.0571927
0.600000	-0.6445759	-0.7122849	0.0677090
0.700000	-0.8298842	-0.9067532	0.0768690
0.800000	-1.0422757	-1.1263932	0.0841175
0.900000	-1.2824812	-1.3713567	0.0888755
1.000000	-1.5502865	-1.6408591	0.0905726
1.100000	-1.8441861	-1.9328815	0.0886954
1.200000	-2.1609509	-2.2438063	0.0828554
1.300000	-2.4950933	-2.5679753	0.0728820
1.400000	-2.8382065	-2.8971552	0.0589487

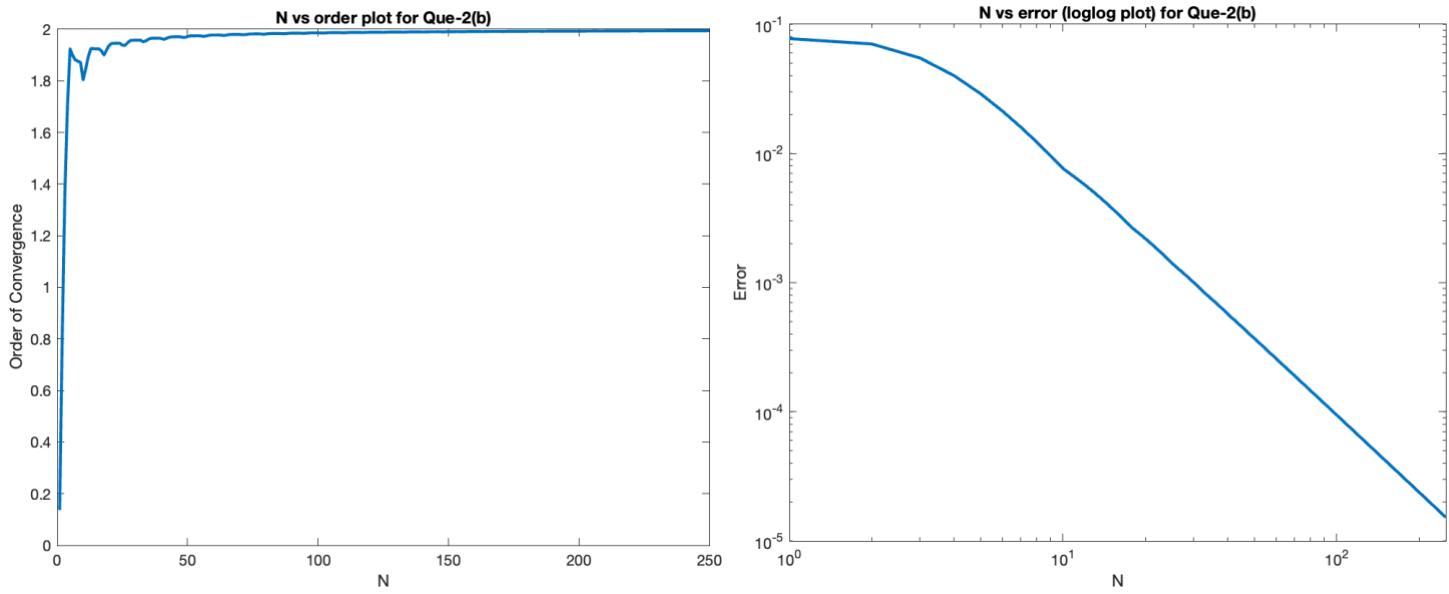
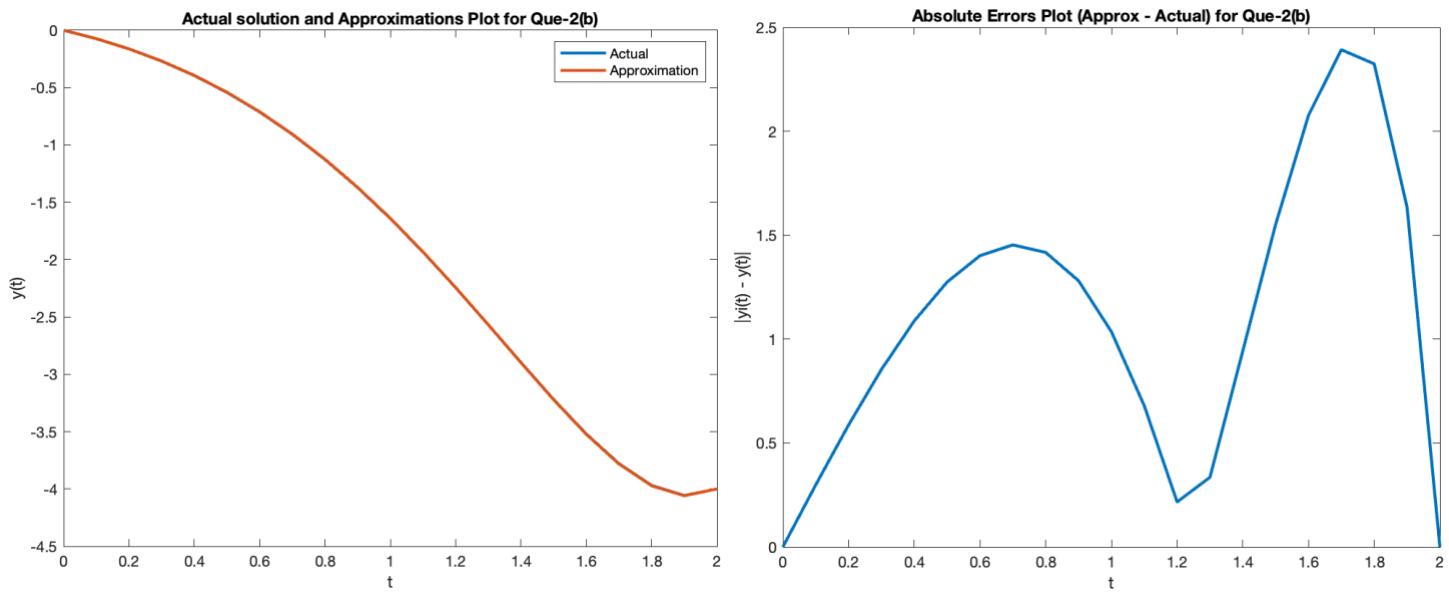
1.400000	-2.8382065	-2.8971552	0.0589487
1.500000	-3.1781545	-3.2198944	0.0417399
1.600000	-3.4980809	-3.5207515	0.0226706
1.700000	-3.7752022	-3.7793722	0.0041700
1.800000	-3.9793425	-3.9693901	0.0099523
1.900000	-4.0711589	-4.0571185	0.0140404
2.000000	-4.0000000	-4.0000000	0.0000000



➤ Central-difference for first derivative: -

Que-2(b)			
For $h = 1.000000e-01$			
t	Approximation	Exact	Error ($ Exact - Approx. $)
0.000000	0.0000000	0.0000000	0.0000000
0.100000	-0.0733704	-0.0736691	0.0002987
0.200000	-0.1621132	-0.1627002	0.0005870
0.300000	-0.2682834	-0.2691374	0.0008540
0.400000	-0.3938999	-0.3949876	0.0010877
0.500000	-0.5408686	-0.5421435	0.0012749
0.600000	-0.7108833	-0.7122849	0.0014016
0.700000	-0.9052996	-0.9067532	0.0014537
0.800000	-1.1249758	-1.1263932	0.0014174

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0.900000	-1.3700756	-1.3713567	0.0012811
1.000000	-1.6398229	-1.6408591	0.0010362
1.100000	-1.9322018	-1.9328815	0.0006797
1.200000	-2.2435895	-2.2438063	0.0002169
1.300000	-2.5683108	-2.5679753	0.0003355
1.400000	-2.8980991	-2.8971552	0.0009438
1.500000	-3.2214472	-3.2198944	0.0015528
1.600000	-3.5228285	-3.5207515	0.0020770
1.700000	-3.7817647	-3.7793722	0.0023924
1.800000	-3.9717148	-3.9693901	0.0023246
1.900000	-4.0587529	-4.0571185	0.0016344
2.000000	-4.0000000	-4.0000000	0.0000000

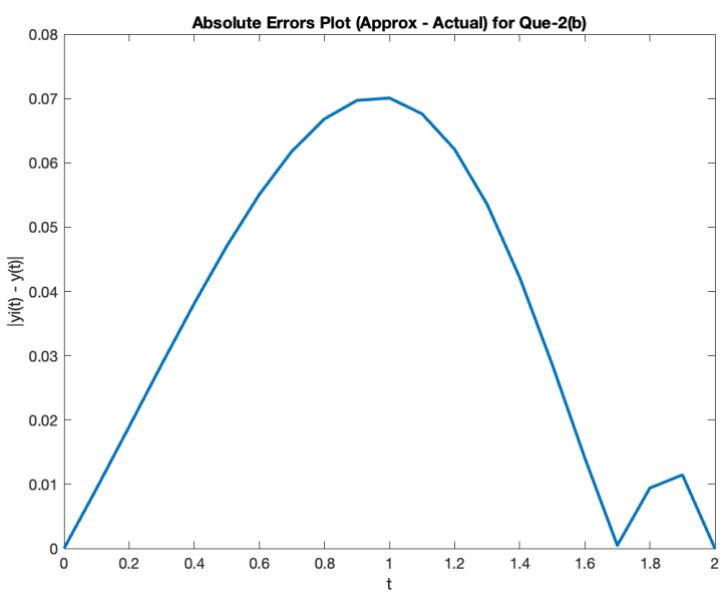
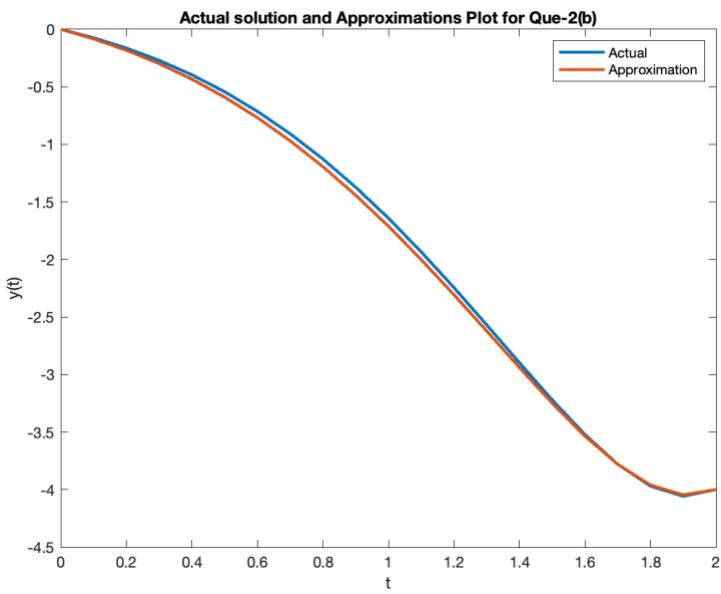


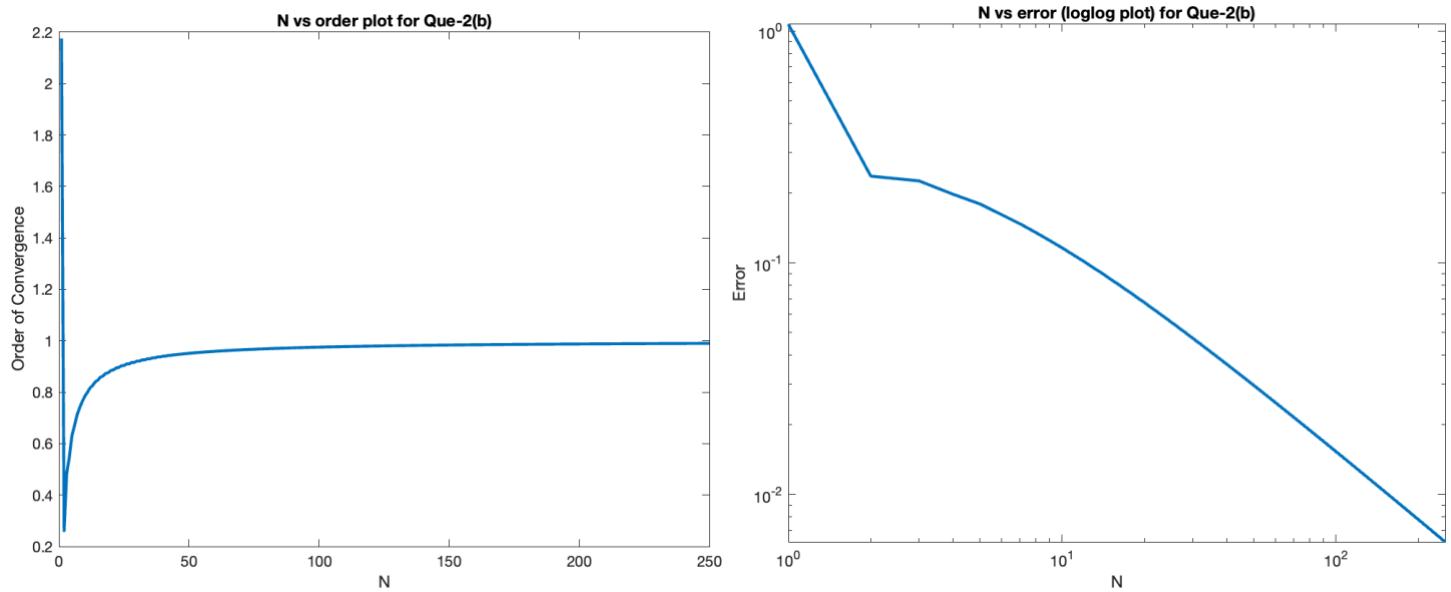
➤ Backward-difference for first derivative: -

Que-2(b)

For $h = 1.000000e-01$

t	Approximation	Exact	Error(Exact - Approx.)
0.000000	0.000000	0.000000	0.000000
0.100000	-0.0829901	-0.0736691	0.0093210
0.200000	-0.1816433	-0.1627002	0.0189431
0.300000	-0.2977677	-0.2691374	0.0286303
0.400000	-0.4330899	-0.3949876	0.0381023
0.500000	-0.5891782	-0.5421435	0.0470348
0.600000	-0.7673489	-0.7122849	0.0550640
0.700000	-0.9685475	-0.9067532	0.0617943
0.800000	-1.1932040	-1.1263932	0.0668108
0.900000	-1.4410555	-1.3713567	0.0696988
1.000000	-1.7109303	-1.6408591	0.0700712
1.100000	-2.0004880	-1.9328815	0.0676065
1.200000	-2.3059064	-2.2438063	0.0621001
1.300000	-2.6215081	-2.5679753	0.0535328
1.400000	-2.9393142	-2.8971552	0.0421590
1.500000	-3.2485156	-3.2198944	0.0286212
1.600000	-3.5348467	-3.5207515	0.0140953
1.700000	-3.7798471	-3.7793722	0.0004749
1.800000	-3.9599920	-3.9693901	0.0093981
1.900000	-4.0456723	-4.0571185	0.0114462
2.000000	-4.0000000	-4.0000000	0.0000000





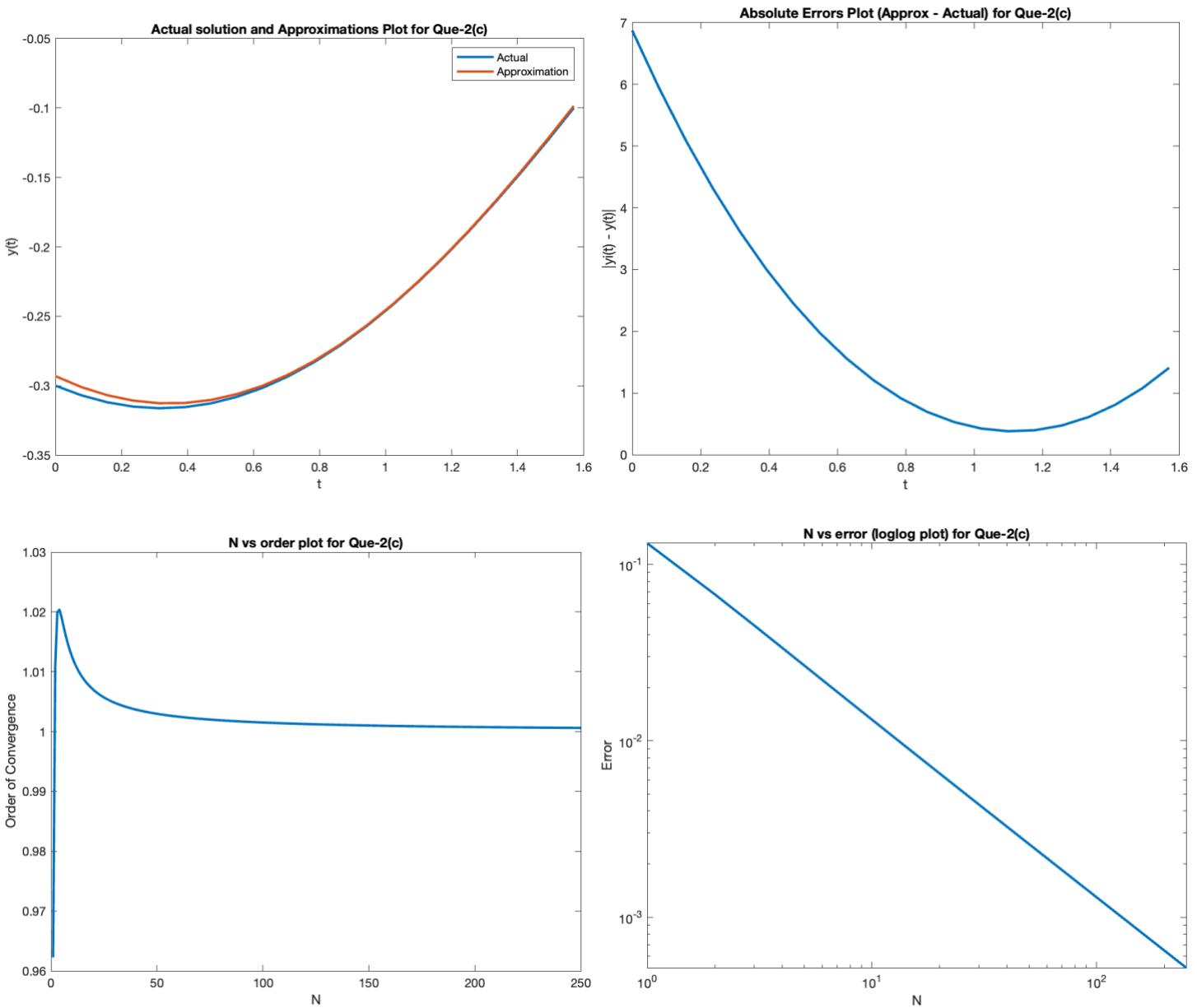
(c)

➤ Forward-difference for first derivative: -

Que-2(c)

For $h = 7.853982e-02$

t	Approximation	Exact	Error($ Exact - Approx. $)
0.000000	-0.2931252	-0.3000000	0.0068748
0.078540	-0.3009792	-0.3069211	0.0059420
0.157080	-0.3068586	-0.3119499	0.0050913
0.235619	-0.3107357	-0.3150555	0.0043198
0.314159	-0.3125943	-0.3162187	0.0036244
0.392699	-0.3124298	-0.3154322	0.0030024
0.471239	-0.3102496	-0.3127010	0.0024514
0.549779	-0.3060727	-0.3080419	0.0019692
0.628319	-0.2999299	-0.3014836	0.0015537
0.706858	-0.2918633	-0.2930666	0.0012033
0.785398	-0.2819265	-0.2828427	0.0009162
0.863938	-0.2701837	-0.2708750	0.0006913
0.942478	-0.2567098	-0.2572373	0.0005275
1.021018	-0.2415896	-0.2420136	0.0004240
1.099557	-0.2249175	-0.2252978	0.0003803
1.178097	-0.2067965	-0.2071930	0.0003964
1.256637	-0.1873380	-0.1878107	0.0004728
1.335177	-0.1666604	-0.1672706	0.0006102
1.413717	-0.1448890	-0.1456992	0.0008102
1.492257	-0.1221545	-0.1232295	0.0010749
1.570796	-0.0985926	-0.1000000	0.0014074



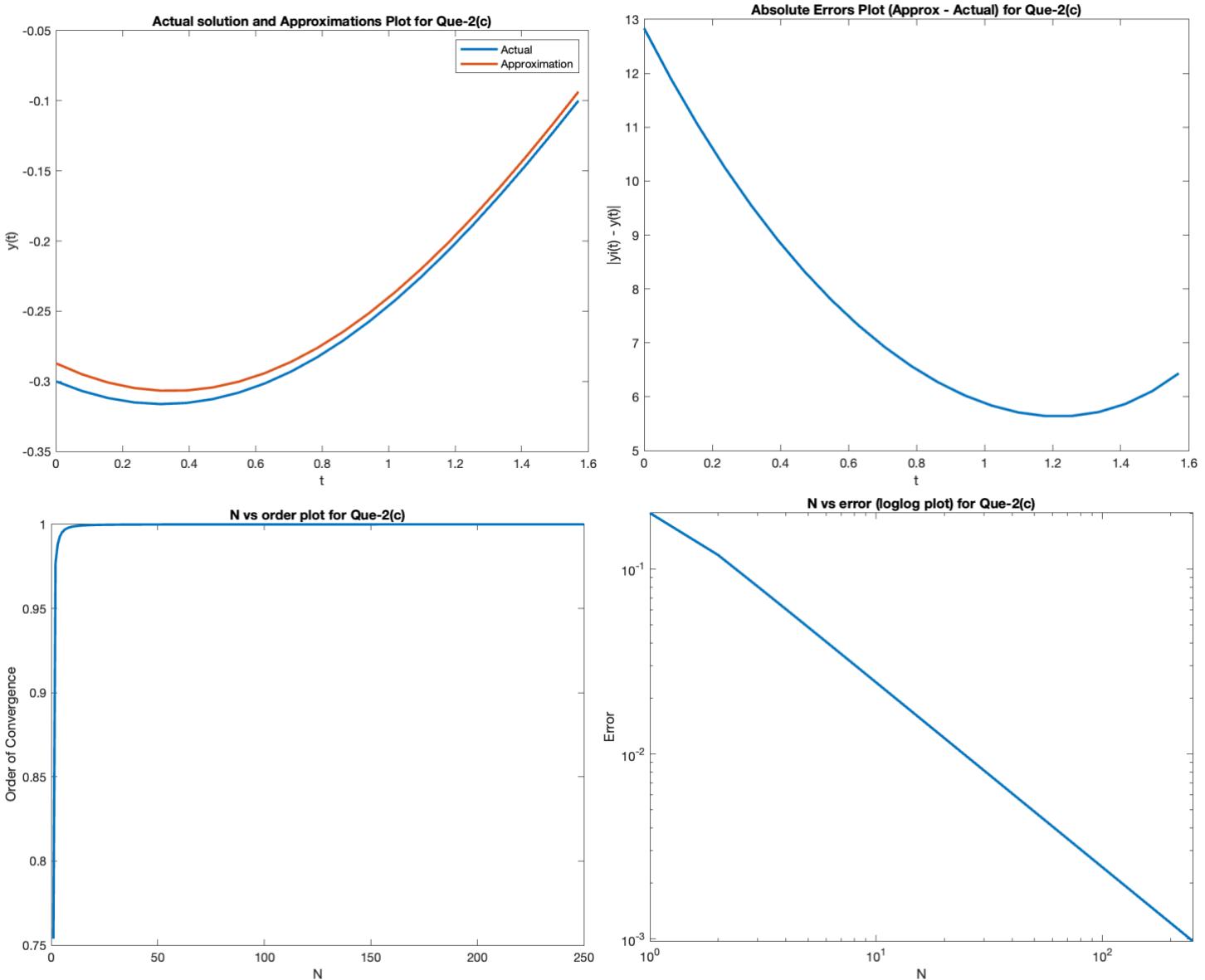
➤ Central-difference for first derivative: -

Que-2(c)

For $h = 7.853982e-02$

t	Approximation	Exact	Error ($ Exact - Approx. $)
0.000000	-0.2871653	-0.3000000	0.0128347
0.078540	-0.2950193	-0.3069211	0.0119018
0.157080	-0.3009029	-0.3119499	0.0110470
0.235619	-0.3047899	-0.3150555	0.0102656
0.314159	-0.3066654	-0.3162187	0.0095533
0.392699	-0.3065257	-0.3154322	0.0089065
0.471239	-0.3043789	-0.3127010	0.0083221
0.549779	-0.3002443	-0.3080419	0.0077976
0.628319	-0.2941528	-0.3014836	0.0073308
0.706858	-0.2861462	-0.2930666	0.0069204
0.785398	-0.2762772	-0.2828427	0.0065655
0.863938	-0.2646092	-0.2708750	0.0062658
0.942478	-0.2512153	-0.2572373	0.0060220

0.942478	-0.2512153	-0.2572373	0.0060220
1.021018	-0.2361784	-0.2420136	0.0058352
1.099557	-0.2195904	-0.2252978	0.0057075
1.178097	-0.2015511	-0.2071930	0.0056419
1.256637	-0.1821683	-0.1878107	0.0056425
1.335177	-0.1615560	-0.1672706	0.0057146
1.413717	-0.1398345	-0.1456992	0.0058647
1.492257	-0.1171284	-0.1232295	0.0061010
1.570796	-0.0935665	-0.1000000	0.0064335



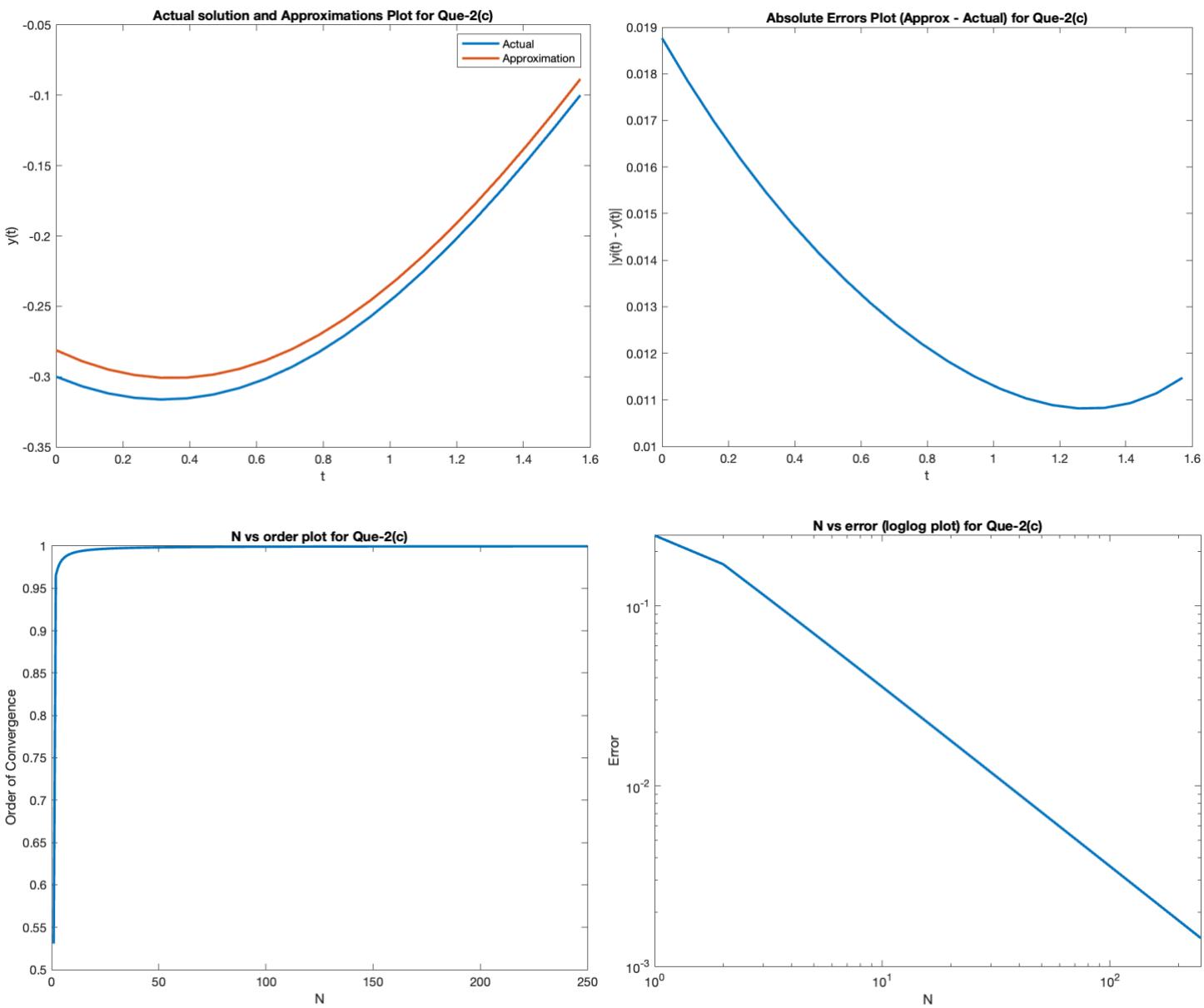
➤ Backward-difference for first derivative: -

Que-2(c)

For $h = 7.853982e-02$

t	Approximation	Exact	Error($ Exact - Approx. $)
0.000000	-0.2812352	-0.3000000	0.0187648
0.078540	-0.2890892	-0.3069211	0.0178319
0.157080	-0.2949771	-0.3119499	0.0169729
0.235619	-0.2988739	-0.3150555	0.0161816
0.314159	-0.3007659	-0.3162187	0.0154527
0.392699	-0.3006506	-0.3154322	0.0147817

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0.471239	-0.2985363	-0.3127010	0.0141647
0.549779	-0.2944428	-0.3080419	0.0135991
0.628319	-0.2884008	-0.3014836	0.0130828
0.706858	-0.2804519	-0.2930666	0.0126147
0.785398	-0.2706481	-0.2828427	0.0121946
0.863938	-0.2590515	-0.2708750	0.0118235
0.942478	-0.2457338	-0.2572373	0.0115034
1.021018	-0.2307761	-0.2420136	0.0112375
1.099557	-0.2142677	-0.2252978	0.0110301
1.178097	-0.1963056	-0.2071930	0.0108874
1.256637	-0.1769941	-0.1878107	0.0108167
1.335177	-0.1564432	-0.1672706	0.0108274
1.413717	-0.1347683	-0.1456992	0.0109309
1.492257	-0.1120888	-0.1232295	0.0111407
1.570796	-0.0885268	-0.1000000	0.0114732



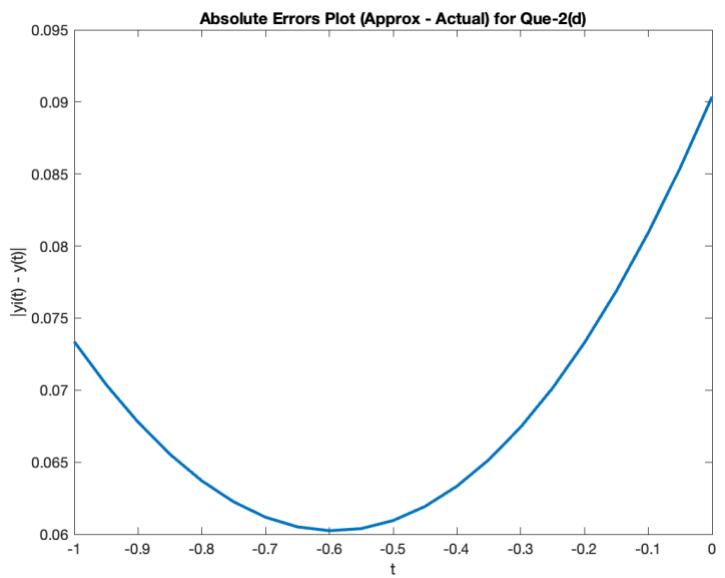
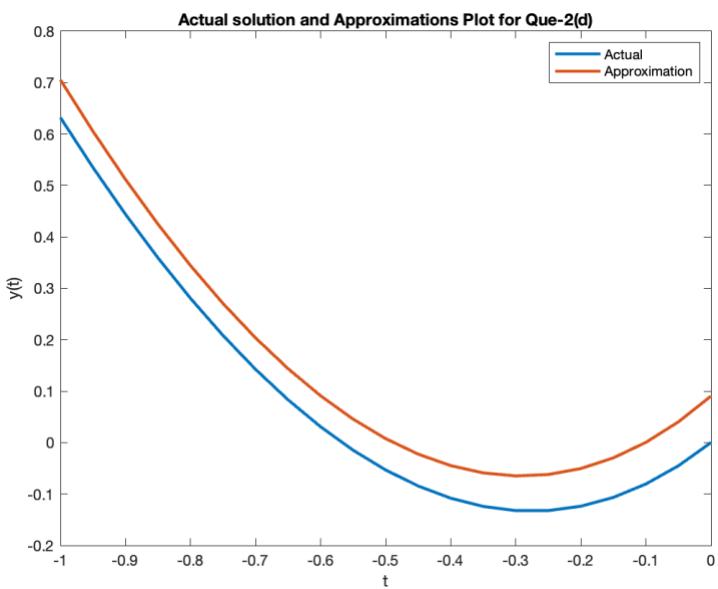
(d)

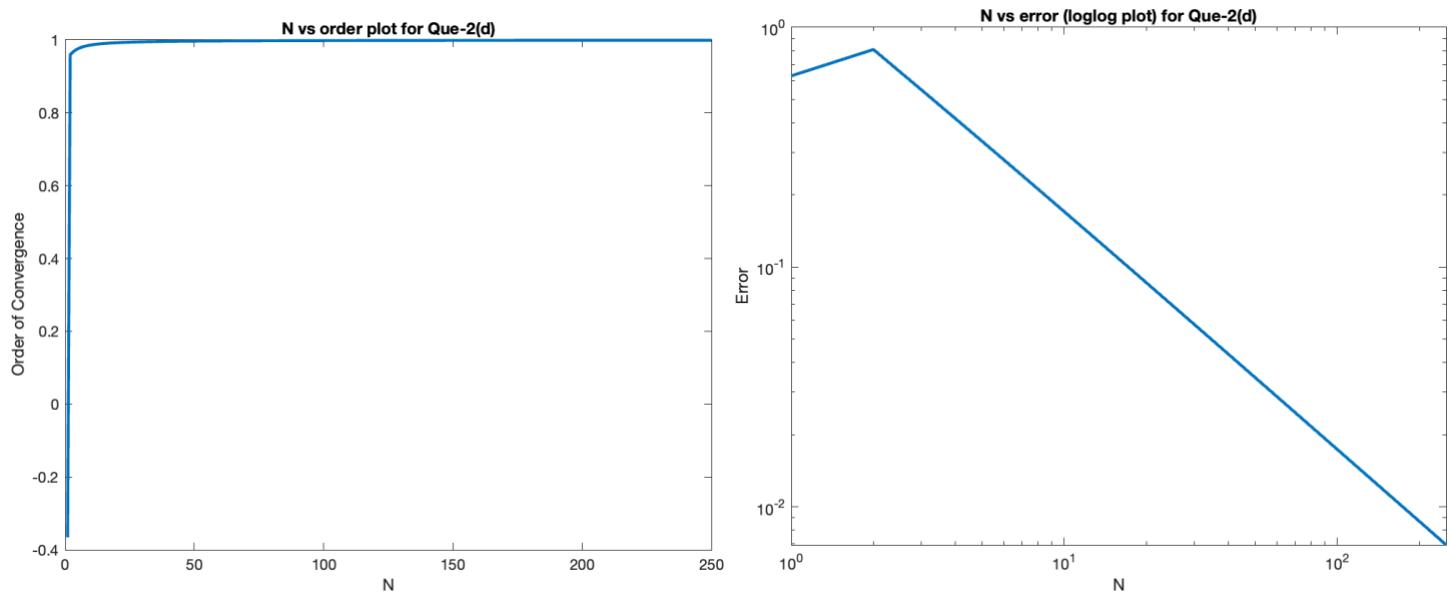
➤ Forward-difference for first derivative: -

Que-2(d)

For $h = 5.000000e-02$

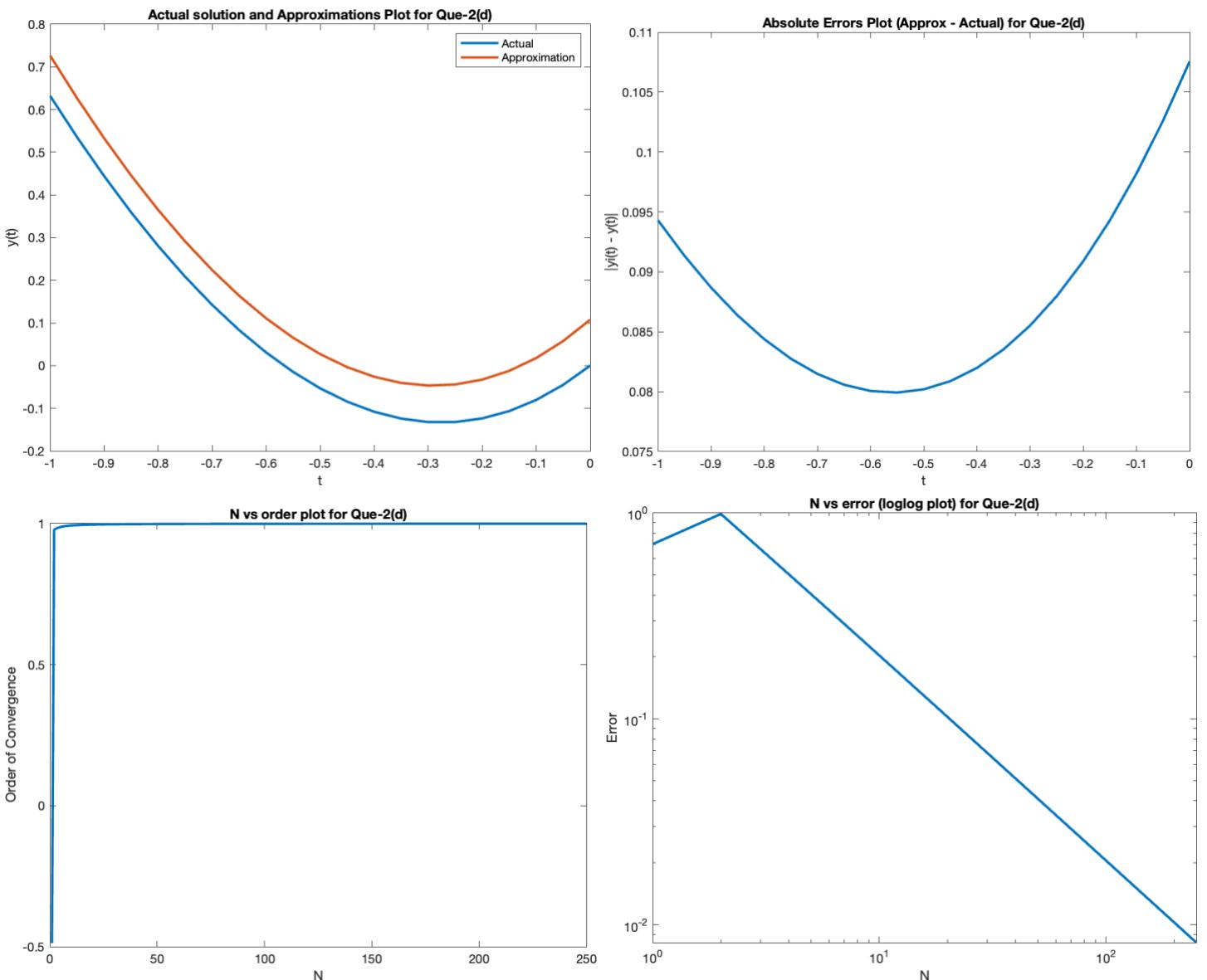
t	Approximation	Exact	Error(Exact - Approx.)
-1.000000	0.7054778	0.6321206	0.0733573
-0.950000	0.6054778	0.5350960	0.0703818
-0.900000	0.5118649	0.4440873	0.0677776
-0.850000	0.4247471	0.3591973	0.0655498
-0.800000	0.3442407	0.2805368	0.0637039
-0.750000	0.2704702	0.2082251	0.0622451
-0.700000	0.2035694	0.1423903	0.0611791
-0.650000	0.1436815	0.0831702	0.0605113
-0.600000	0.0909603	0.0307130	0.0602473
-0.550000	0.0455702	-0.0148224	0.0603926
-0.500000	0.0076872	-0.0532653	0.0609526
-0.450000	-0.0225003	-0.0844327	0.0619324
-0.400000	-0.0447909	-0.1081280	0.0633372
-0.350000	-0.0589693	-0.1241408	0.0651716
-0.300000	-0.0648054	-0.1322455	0.0674401
-0.250000	-0.0620535	-0.1322002	0.0701467
-0.200000	-0.0504511	-0.1237462	0.0732950
-0.150000	-0.0297181	-0.1066062	0.0768881
-0.100000	0.0004444	-0.0804837	0.0809281
-0.050000	0.0403555	-0.0450615	0.0854170
0.000000	0.0903555	0.0000000	0.0903555





➤ Central-difference for first derivative: -

Que-2(d)			
For h = 5.000000e-02			
t	Approximation	Exact	Error(Exact - Approx.)
-1.000000	0.7264312	0.6321206	0.0943107
-0.950000	0.6264312	0.5350960	0.0913352
-0.900000	0.5327702	0.4440873	0.0886829
-0.850000	0.4455596	0.3591973	0.0863623
-0.800000	0.3649191	0.2805368	0.0843822
-0.750000	0.2909770	0.2082251	0.0827519
-0.700000	0.2238713	0.1423903	0.0814810
-0.650000	0.1637497	0.0831702	0.0805795
-0.600000	0.1107706	0.0307130	0.0800576
-0.550000	0.0651037	-0.0148224	0.0799261
-0.500000	0.0269306	-0.0532653	0.0801959
-0.450000	-0.0035544	-0.0844327	0.0808783
-0.400000	-0.0261433	-0.1081280	0.0819847
-0.350000	-0.0406138	-0.1241408	0.0835270
-0.300000	-0.0467282	-0.1322455	0.0855173
-0.250000	-0.0442326	-0.1322002	0.0879676
-0.200000	-0.0328558	-0.1237462	0.0908903
-0.150000	-0.0123081	-0.1066062	0.0942981
-0.100000	0.0177196	-0.0804837	0.0982034
-0.050000	0.0575575	-0.0450615	0.1026189
0.000000	0.1075575	0.0000000	0.1075575



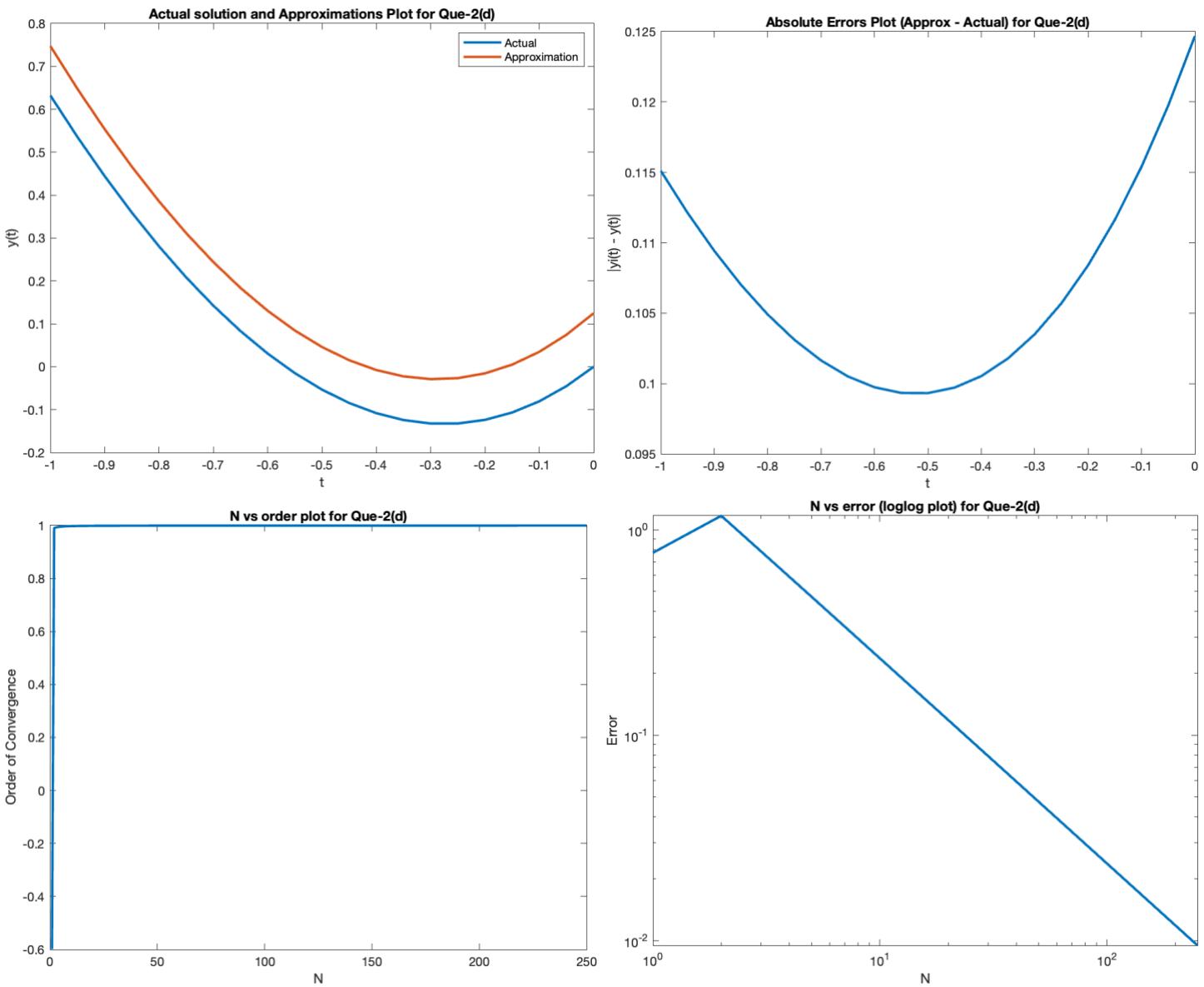
➤ Backward-difference for first derivative: -

Que-2(d)

For $h = 5.000000e-02$

t	Approximation	Exact	Error($ Exact - Approx. $)
-1.000000	0.7472250	0.6321206	0.1151044
-0.950000	0.6472250	0.5350960	0.1121289
-0.900000	0.5535174	0.4440873	0.1094301
-0.850000	0.4662167	0.3591973	0.1070194
-0.800000	0.3854459	0.2805368	0.1049091
-0.750000	0.3113371	0.2082251	0.1031120
-0.700000	0.2440320	0.1423903	0.1016417
-0.650000	0.1836826	0.0831702	0.1005123
-0.600000	0.1304519	0.0307130	0.0997388
-0.550000	0.0845145	-0.0148224	0.0993369
-0.500000	0.0460575	-0.0532653	0.0993228
-0.450000	0.0152811	-0.0844327	0.0997137
-0.400000	-0.0076005	-0.1081280	0.1005276
-0.350000	-0.0223579	-0.1241408	0.1017829
-0.300000	-0.0287461	-0.1322455	0.1034993
-0.250000	-0.0265031	-0.1322002	0.1056970

-0.250000	-0.0265031	-0.1322002	0.1056970
-0.200000	-0.0153489	-0.1237462	0.1083972
-0.150000	0.0050156	-0.1066062	0.1116218
-0.100000	0.0349099	-0.0804837	0.1153936
-0.050000	0.0746750	-0.0450615	0.1197365
0.000000	0.1246750	0.0000000	0.1246750



(e)

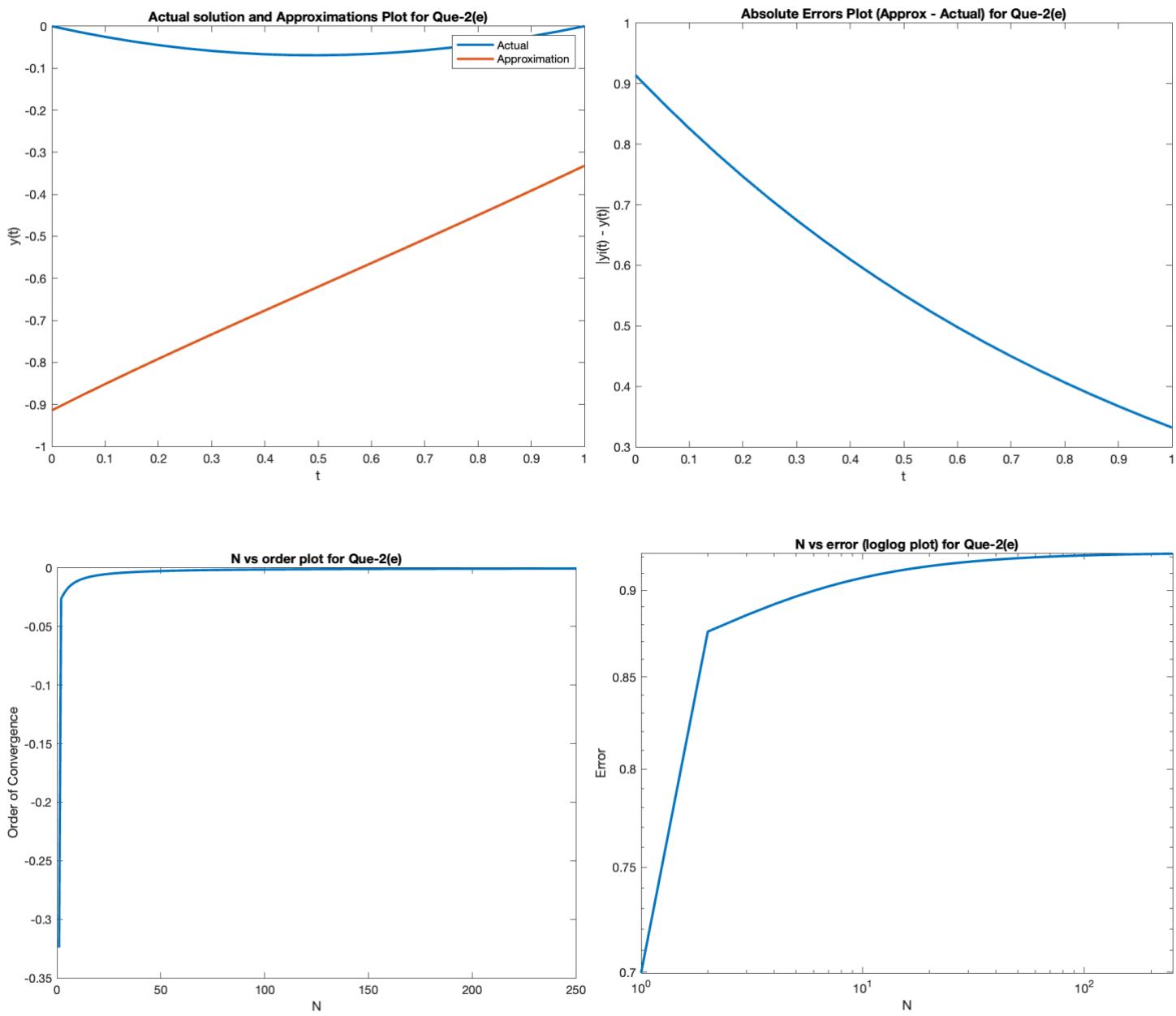
➤ Forward-difference for first derivative: -

Que-2(e)

For $h = 5.000000e-02$

t	Approximation	Exact	Error($ Exact - Approx. $)
0.000000	-0.9139505	0.0000000	0.9139505
0.050000	-0.8823389	-0.0133786	0.8689603
0.100000	-0.8514821	-0.0253323	0.8261497
0.150000	-0.8212680	-0.0358494	0.7854186
0.200000	-0.7915932	-0.0449226	0.7466706
0.250000	-0.7623625	-0.0525488	0.7098137
0.300000	-0.7334884	-0.0587287	0.6747597
0.350000	-0.7048903	-0.0634662	0.6414241

0.350000	-0.7048903	-0.0634662	0.6414241
0.400000	-0.6764945	-0.0667684	0.6097261
0.450000	-0.6482336	-0.0686452	0.5795885
0.500000	-0.6200459	-0.0691087	0.5509372
0.550000	-0.5918752	-0.0681735	0.5237017
0.600000	-0.5636703	-0.0658559	0.4978144
0.650000	-0.5353847	-0.0621740	0.4732107
0.700000	-0.5069765	-0.0571477	0.4498289
0.750000	-0.4784077	-0.0507977	0.4276101
0.800000	-0.4496443	-0.0431462	0.4064981
0.850000	-0.4206556	-0.0342164	0.3864392
0.900000	-0.3914144	-0.0240322	0.3673822
0.950000	-0.3618965	-0.0126184	0.3492781
1.000000	-0.3320804	0.0000000	0.3320804

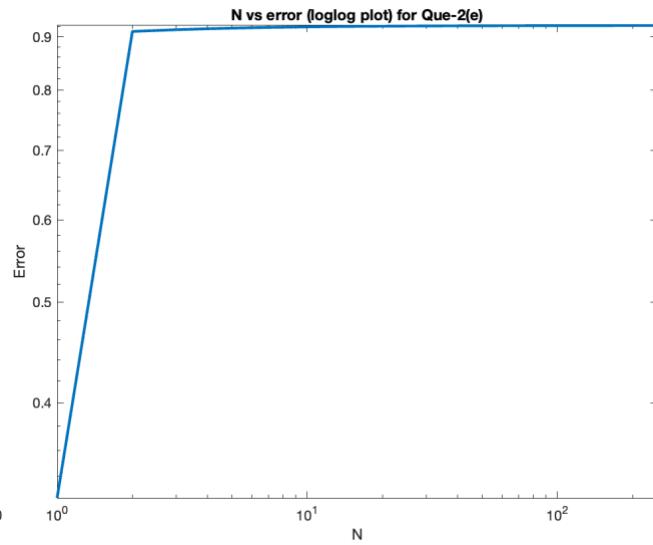
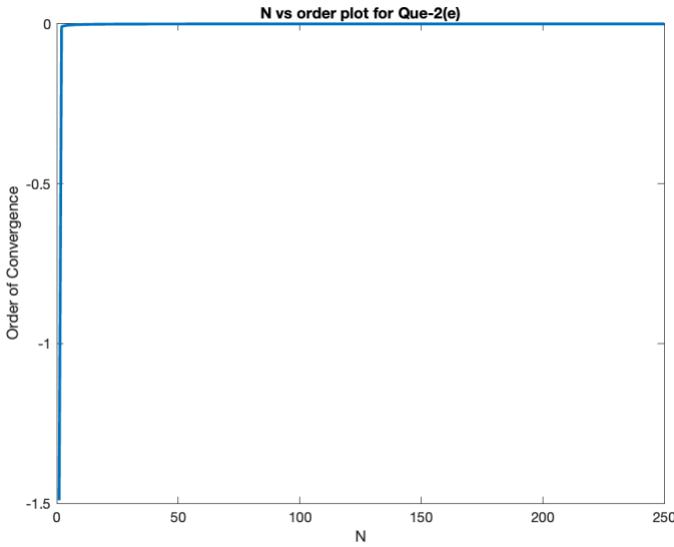
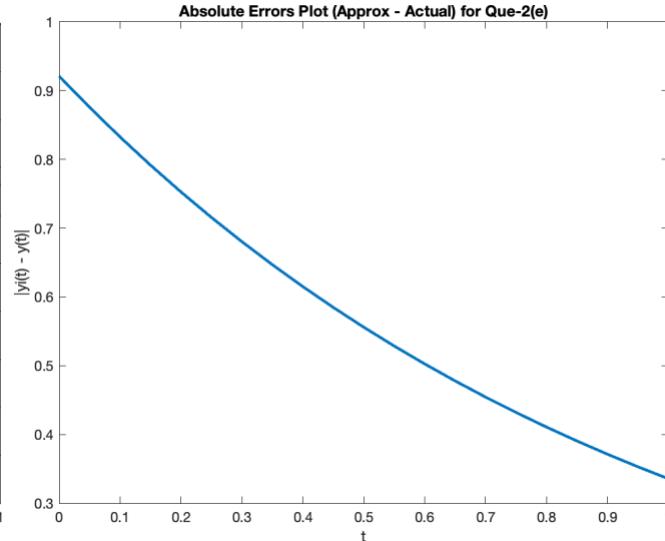
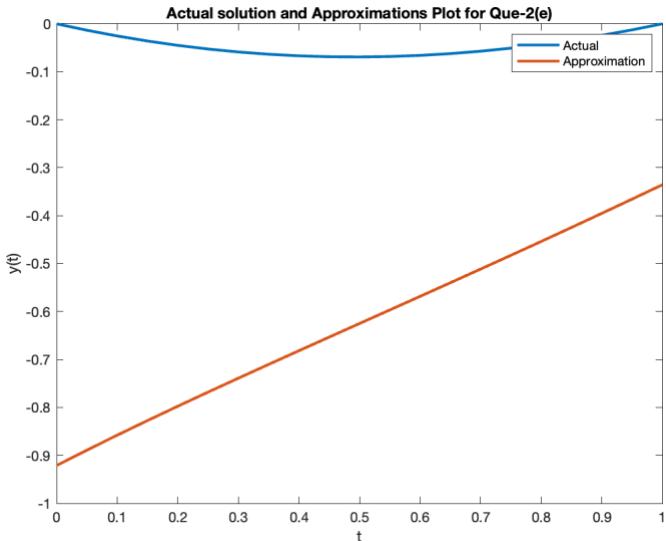


➤ Central-difference for first derivative: -

Que-2(e)

For $h = 5.000000e-02$

t	Approximation	Exact	Error(Exact - Approx.)
0.000000	-0.9210422	0.0000000	0.9210422
0.050000	-0.8890760	-0.0133786	0.8756974
0.100000	-0.8579183	-0.0253323	0.8325860
0.150000	-0.8274473	-0.0358494	0.7915979
0.200000	-0.7975510	-0.0449226	0.7526284
0.250000	-0.7681268	-0.0525488	0.7155780
0.300000	-0.7390808	-0.0587287	0.6803522
0.350000	-0.7103272	-0.0634662	0.6468610
0.400000	-0.6817873	-0.0667684	0.6150189
0.450000	-0.6533899	-0.0686452	0.5847447
0.500000	-0.6250699	-0.0691087	0.5559611
0.550000	-0.5967682	-0.0681735	0.5285948
0.600000	-0.5684316	-0.0658559	0.5025757
0.650000	-0.5400117	-0.0621740	0.4778376
0.700000	-0.5114651	-0.0571477	0.4543174
0.750000	-0.4827528	-0.0507977	0.4319551
0.800000	-0.4538399	-0.0431462	0.4106937
0.850000	-0.4246952	-0.0342164	0.3904788
0.900000	-0.3952912	-0.0240322	0.3712590
0.950000	-0.3656036	-0.0126184	0.3529853
1.000000	-0.3356110	0.0000000	0.3356110

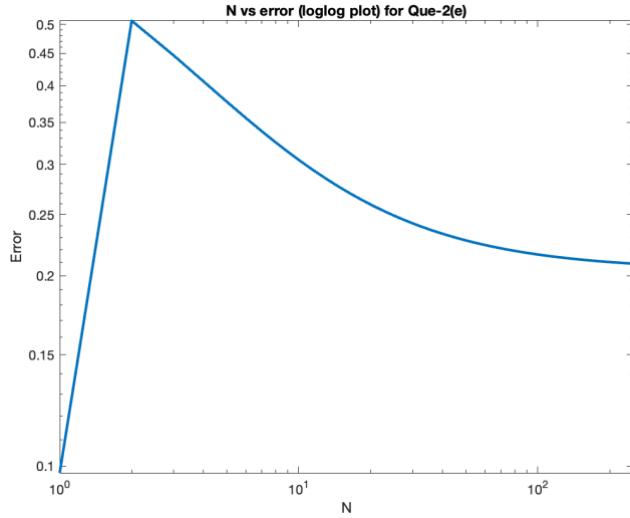
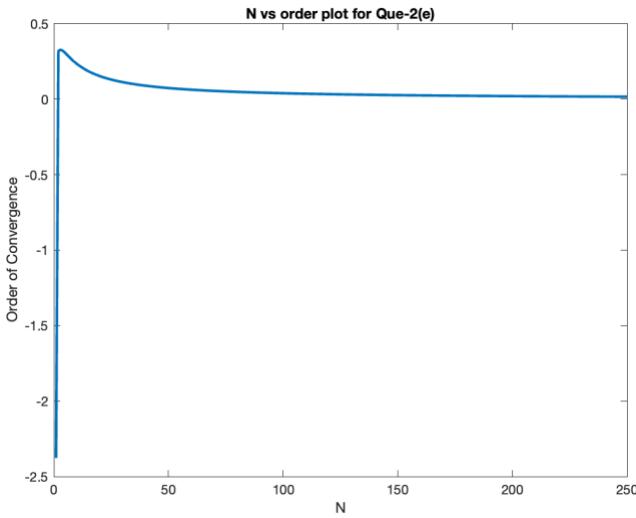
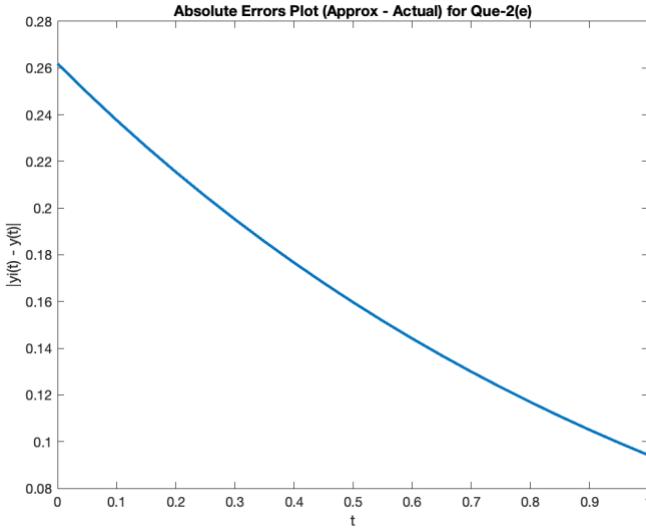
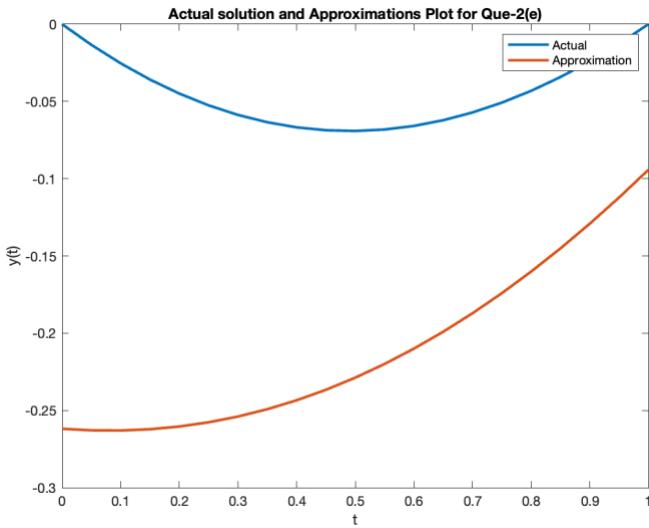


➤ Backward-difference for first derivative: -

Que-2(e)

For $h = 5.000000e-02$

t	Approximation	Exact	Error(Exact - Approx.)
0.000000	-0.2618880	0.0000000	0.2618880
0.050000	-0.2628795	-0.0133786	0.2495009
0.100000	-0.2629897	-0.0253323	0.2376574
0.150000	-0.2621814	-0.0358494	0.2263319
0.200000	-0.2604234	-0.0449226	0.2155008
0.250000	-0.2576902	-0.0525488	0.2051414
0.300000	-0.2539611	-0.0587287	0.1952324
0.350000	-0.2492200	-0.0634662	0.1857538
0.400000	-0.2434549	-0.0667684	0.1766865
0.450000	-0.2366577	-0.0686452	0.1680126
0.500000	-0.2288236	-0.0691087	0.1597149
0.550000	-0.2199509	-0.0681735	0.1517774
0.600000	-0.2100405	-0.0658559	0.1441846
0.650000	-0.1990961	-0.0621740	0.1369220
0.700000	-0.1871234	-0.0571477	0.1299757
0.750000	-0.1741301	-0.0507977	0.1233324
0.800000	-0.1601258	-0.0431462	0.1169796
0.850000	-0.1451217	-0.0342164	0.1109053
0.900000	-0.1291302	-0.0240322	0.1050979
0.950000	-0.1121650	-0.0126184	0.0995466
1.000000	-0.0942409	0.0000000	0.0942409



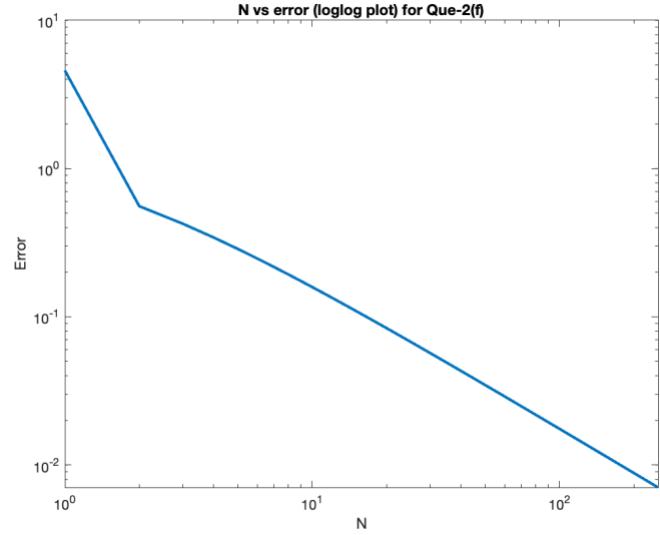
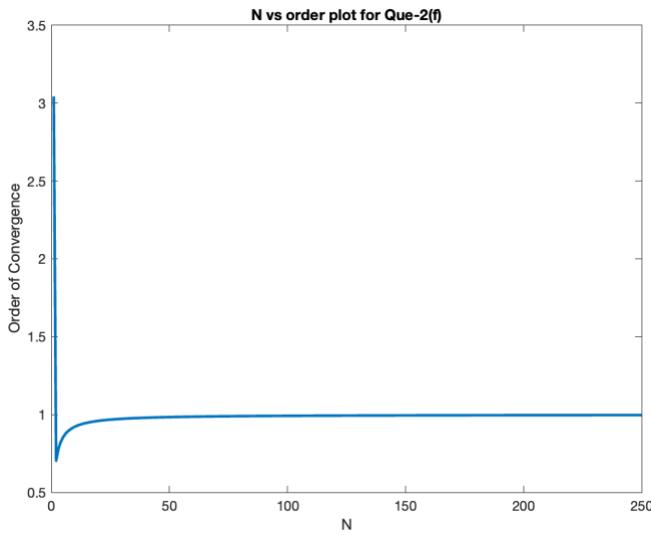
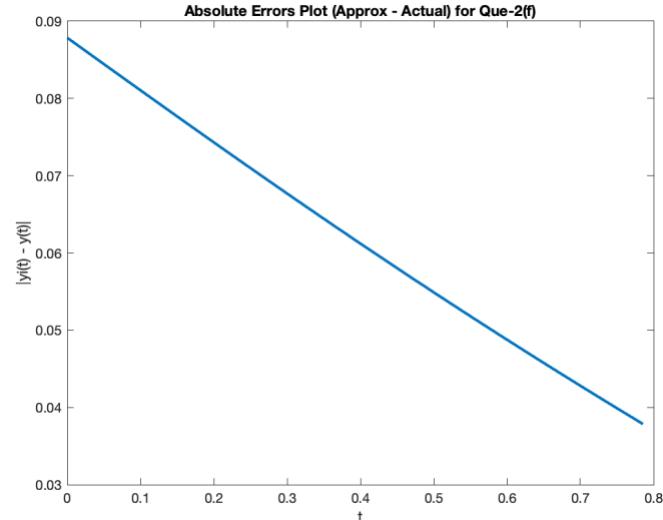
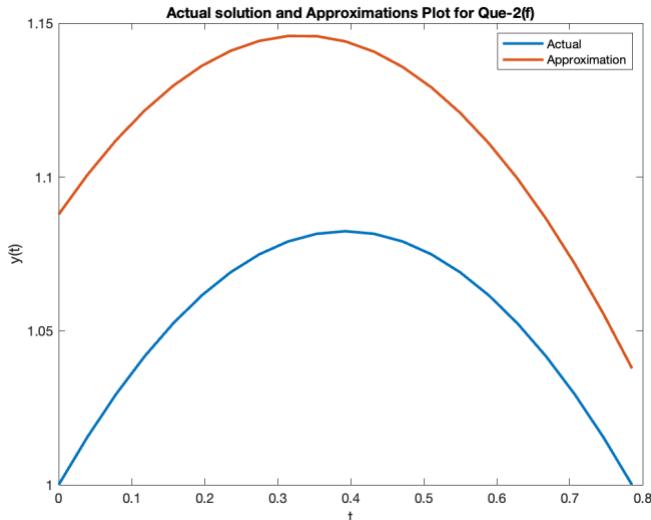
(f)

➤ Forward-difference for first derivative: -

Que-2(f)

For $h = 3.926991e-02$

t	Approximation	Exact	Error($ Exact - Approx. $)
0.000000	1.0878547	1.0000000	0.0878547
0.039270	1.1006708	1.0154910	0.0851798
0.078540	1.1119252	1.0294162	0.0825090
0.117810	1.1216003	1.0417540	0.0798462
0.157080	1.1296806	1.0524856	0.0771949
0.196350	1.1361526	1.0615943	0.0745582
0.235619	1.1410050	1.0690662	0.0719388
0.274889	1.1442286	1.0748896	0.0693391
0.314159	1.1458166	1.0790555	0.0667610
0.353429	1.1457640	1.0815577	0.0642063
0.392699	1.1440685	1.0823922	0.0616763
0.431969	1.1407298	1.0815577	0.0591721
0.471239	1.1357500	1.0790555	0.0566945
0.510509	1.1291336	1.0748896	0.0542440
0.549779	1.1208873	1.0690662	0.0518211
0.589049	1.1110202	1.0615943	0.0494258
0.628319	1.0995437	1.0524856	0.0470581
0.667588	1.0864716	1.0417540	0.0447176
0.706858	1.0718201	1.0294162	0.0424039
0.746128	1.0556076	1.0154910	0.0401166
0.785398	1.0378549	1.0000000	0.0378549

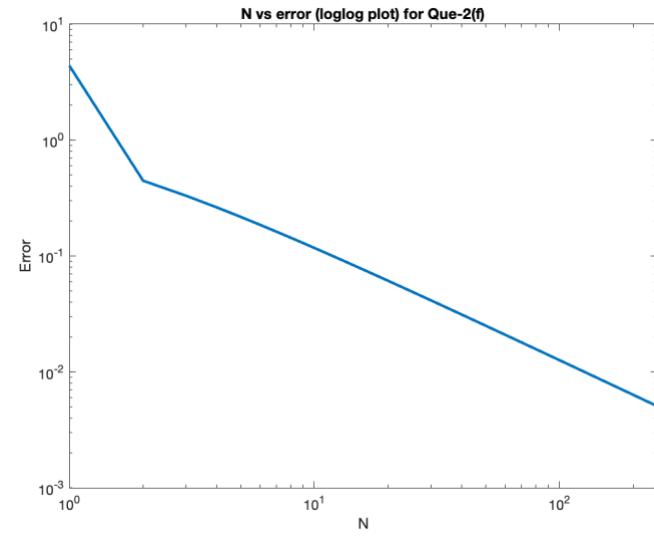
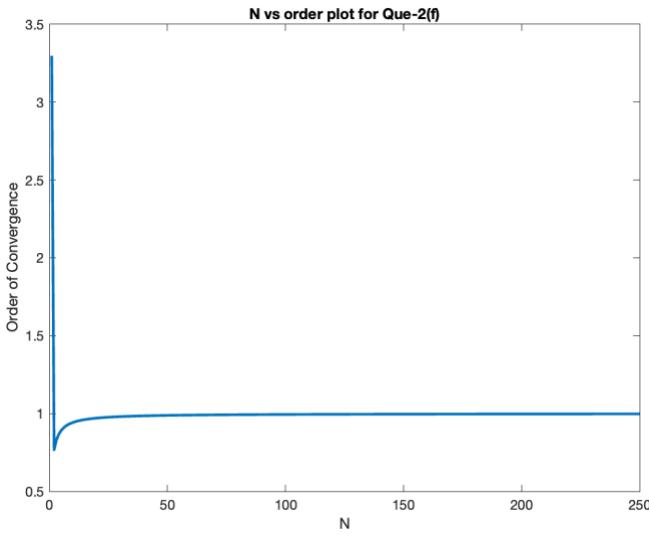
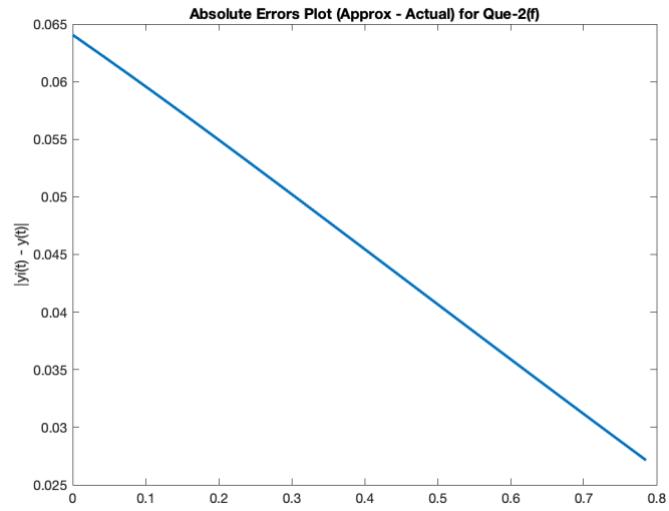
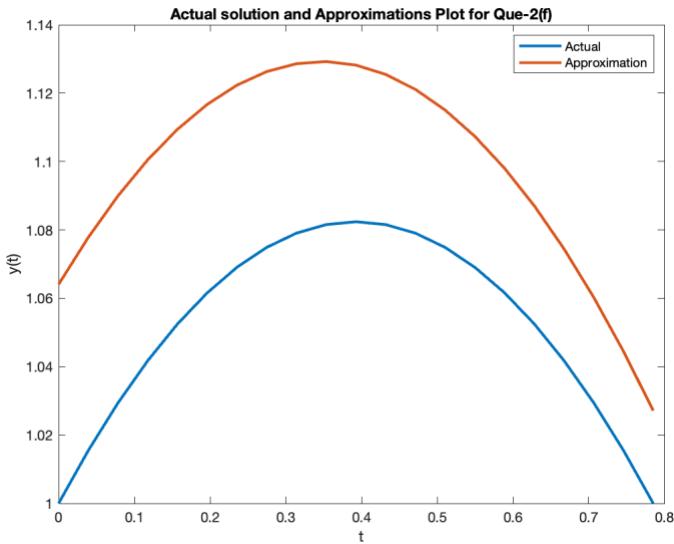


➤ Central-difference for first derivative: -

Que-2(f)

For $h = 3.926991e-02$

t	Approximation	Exact	Error(Exact - Approx.)
0.000000	1.0640664	1.0000000	0.0640664
0.039270	1.0778166	1.0154910	0.0623257
0.078540	1.0899737	1.0294162	0.0605576
0.117810	1.1005198	1.0417540	0.0587658
0.157080	1.1094391	1.0524856	0.0569535
0.196350	1.1167182	1.0615943	0.0551239
0.235619	1.1223461	1.0690662	0.0532799
0.274889	1.1263138	1.0748896	0.0514243
0.314159	1.1286150	1.0790555	0.0495594
0.353429	1.1292455	1.0815577	0.0476878
0.392699	1.1282036	1.0823922	0.0458114
0.431969	1.1254901	1.0815577	0.0439324
0.471239	1.1211079	1.0790555	0.0420523
0.510509	1.1150625	1.0748896	0.0401730
0.549779	1.1073619	1.0690662	0.0382958
0.589049	1.0980164	1.0615943	0.0364220
0.628319	1.0870386	1.0524856	0.0345529
0.667588	1.0744436	1.0417540	0.0326896
0.706858	1.0602490	1.0294162	0.0308329
0.746128	1.0444746	1.0154910	0.0289836
0.785398	1.0271426	1.0000000	0.0271426

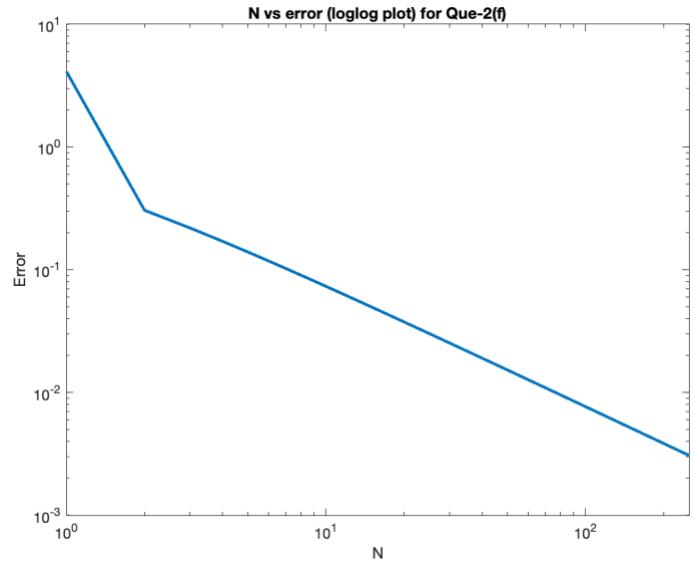
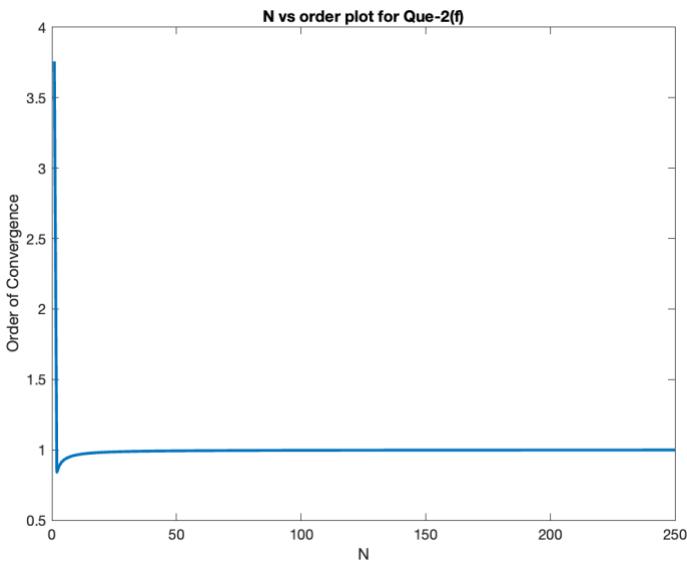
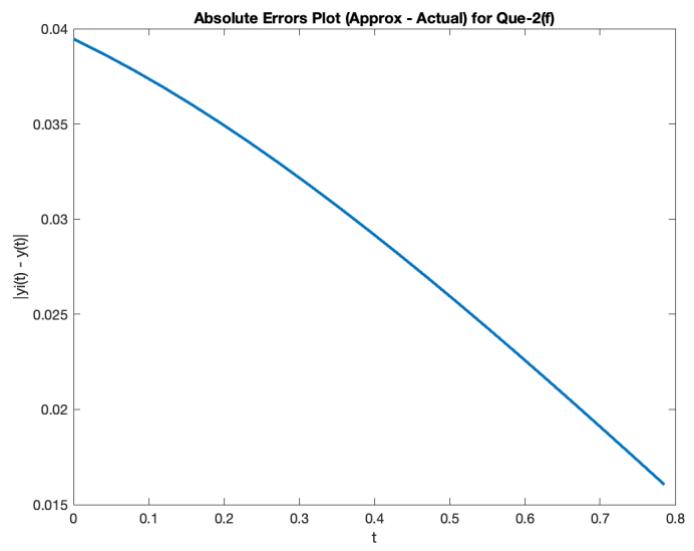
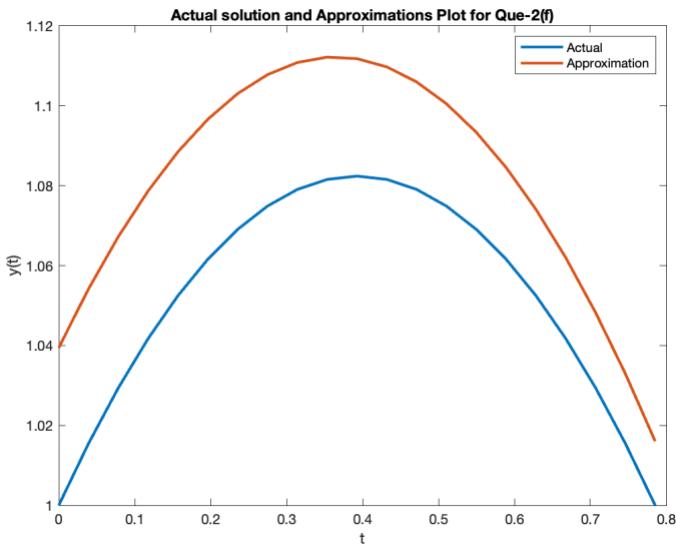


➤ Backward-difference for first derivative: -

Que-2(f)

For $h = 3.926991e-02$

t	Approximation	Exact	Error(Exact - Approx.)
0.000000	1.0394604	1.0000000	0.0394604
0.039270	1.0541769	1.0154910	0.0386859
0.078540	1.0672676	1.0294162	0.0378514
0.117810	1.0787141	1.0417540	0.0369601
0.157080	1.0885007	1.0524856	0.0360151
0.196350	1.0966140	1.0615943	0.0350196
0.235619	1.1030430	1.0690662	0.0339768
0.274889	1.1077794	1.0748896	0.0328899
0.314159	1.1108173	1.0790555	0.0317618
0.353429	1.1121533	1.0815577	0.0305956
0.392699	1.1117864	1.0823922	0.0293942
0.431969	1.1097184	1.0815577	0.0281606
0.471239	1.1059532	1.0790555	0.0268977
0.510509	1.1004976	1.0748896	0.0256081
0.549779	1.0933606	1.0690662	0.0242945
0.589049	1.0845539	1.0615943	0.0229596
0.628319	1.0740914	1.0524856	0.0216058
0.667588	1.0619898	1.0417540	0.0202357
0.706858	1.0482677	1.0294162	0.0188516
0.746128	1.0329467	1.0154910	0.0174557
0.785398	1.0160503	1.0000000	0.0160503



Some Observations: -

- The order of convergence of schemes 1 and 3(forward and backward differences respectively for first derivative) is 1 for each boundary condition.
- The order of convergence of scheme 2(central difference for first derivative) is 2 for Dirichlet's conditions, while 1 for Neumann and mixed conditions. This is because first-order finite differences are used to estimate the boundary conditions in both those cases, which dominates the overall order of convergence over all other central differences.
- For Dirichlet's conditions, scheme 2 is giving the best results, for Neumann conditions, scheme 1 is giving the best results, while for mixed conditions, scheme 3 is giving the best results.
- In the (e) part, the given BVP is not converging for any of the 3 schemes, which can be seen in the N vs order plot, where the order of convergence is going to 0 as N increases and the loglog plot is also being flat instead of showing a decreasing nature like other parts where the BVP converges with some order based on the scheme.