MA323 – Monte-Carlo Simulation

Lab - 08

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Q1)

$$f(x) = k\lambda x^{k-1} e^{-\lambda x^k} \quad \text{for } x > 0.$$

Q) Can you think of the justification for clubbing S ≥ 6 into one strata?
Clubbing S ≥ 6 into one strata is justified because, in the given model, having a large number of storms in the coming month (S ≥ 6) is a rare occurrence due to the relatively low value of λ (λ = 2.9). This rarity results in a low probability of such events. Combining them into a single strata simplifies the stratification process, increases the sample size within the strata, and improves the overall efficiency of the calculation, as well as making the analysis more practical and computationally efficient.

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For n = 100
Simple Monte Carlo
Probability            = 0.74
Confidence Interval    = [0.6400037085095548, 0.8399962914904452]
Variance               = 0.1507070707070708
Interval Length        = 0.1999925829808904

Stratification Method
Probability            = 0.8169523203608925
Confidence Interval    = [0.7878459710399872, 0.8460586696817978]
Variance               = 0.1507070707070708
Interval Length        = 0.058212698641810556
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For n = 10000
Simple Monte Carlo
Probability            = 0.7703
Confidence Interval    = [0.7594248384516114, 0.7811751615483886]
Variance               = 0.1782531753175283
Interval Length        = 0.02175032309677727

Stratification Method
Probability            = 0.8011054299833138
Confidence Interval    = [0.797837453367397, 0.8043734065992306]
Variance               = 0.1782531753175283
Interval Length        = 0.006535953231833647
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1. Observations:
2. Stratification improved precision by grouping rare events, resulting in more stable probability estimates.
3. The stratification method typically led to narrower 99% confidence intervals, enhancing result reliability.

4. Stratification optimized sample utilization and computational efficiency when dealing with rare events.
5. The analysis has practical applications in water resource management, aiding proactive planning based on probability estimates.
6. The model's assumptions about rainfall patterns are recognized, and their potential impact on accuracy is acknowledged. Question2: The code effectively calculates the probability μ

Q2)

$$f(x_1, \ldots, x_{38}) = \begin{cases} \frac{\Gamma(\sum_{j=1}^{38} \alpha_j)}{\prod_{j=1}^{38} \Gamma(\alpha_j)} \prod_{j=1}^{38} x_j^{\alpha_j - 1} & \text{if } x_j > 0 \text{ and } \sum_{i=1}^{38} x_i = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Compute the $\mu = P(X_{19} = \max_i X_i)$ using conditional Monte Carlo technique, where the values of the parameters are given in the following table.

j	1	2	3	4	5	6	7	8	9	10	11	12	13
α_j	2082	1999	2008	2047	2199	2153	1999	2136	2053	2121	1974	2110	2110
j	14	15	16	17	18	19	20	21	22	23	24	25	26
α_j	2168	2035	2019	2044	2191	2284	1912	2196	2099	2041	2192	2188	1984
j	27	28	29	30	31	32	33	34	35	36	37	38	
α_j	2158	2019	2032	2051	2192	2133	2142	2113	2150	2221	2046	2127	

Using Conditional Monte Carlo Technique, ANSWER: 0.02636:

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Observations:

1. The problem involves generating random variables from a Dirichlet distribution with specific alpha parameters, which is essential for probability estimation.

2. The code utilizes the gamma distribution to generate random variables with shape parameters corresponding to the given alpha values, facilitating the calculation of the Dirichlet random variable.

3. The code conditions on the value of Y19 (associated with alpha[18]) to determine if it is the largest Yj, simplifying the probability calculation.
4. The approach is computationally efficient and avoids the need to calculate the Dirichlet density directly.

Q3)

$$f(\boldsymbol{x}) = \max\left\{0, \frac{1}{5}\sum_{i=1}^{5} x_i\right\}$$

Xi – Independent log-normal random variables

Chosen parameters: (μ_i, σ_i^2),

(mu1, s1), (mu2, s2), (mu3, s3), (m4, s4), (m5, s5)

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The mu_values are [1.0, 1.2, 0.8, 1.5, 1.3]

The sigma2_values are [0.1, 0.2, 0.15, 0.3, 0.25]

Total number of iteration to estimate actual expectation is 100000

Estimated mean using covariate method comes out to be 3.6596101850643707
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Observations:

1. The code successfully implements the covariate technique to estimate μ = E(f(X)), considering specific log-normal random variables.
2. It allows users to specify parameters (μ and σ^2) for each log-normal distribution, offering flexibility for different scenarios.
3. A function to sample log-normal variables is defined, making it reusable and aligned with the chosen parameters.
4. Samples are generated for each log-normal variable, with a substantial sample size (n = 10,000) for accurate estimation.
5. ➢ The code features a dedicated function for estimating μ using the covariate technique, considering the maximum of the specific function f(X).