MA-323 MONTE CARLO SIMULATION LAB-2

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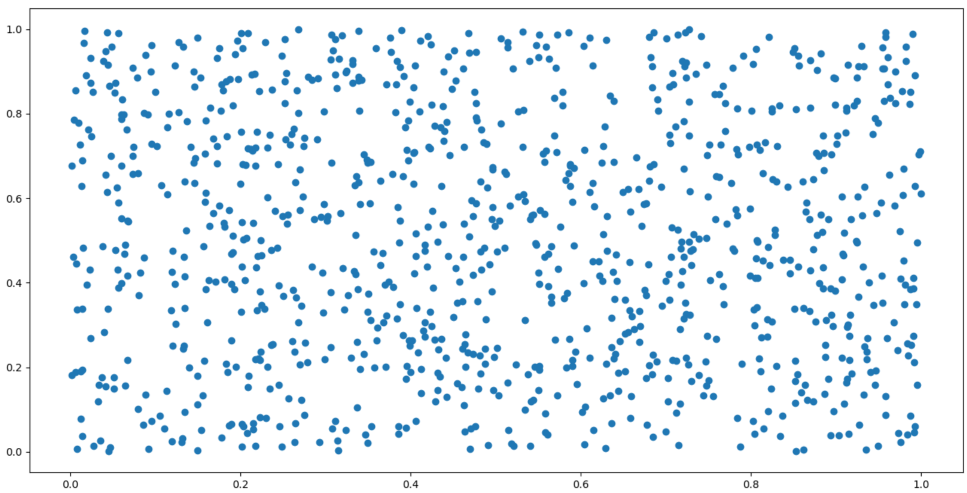
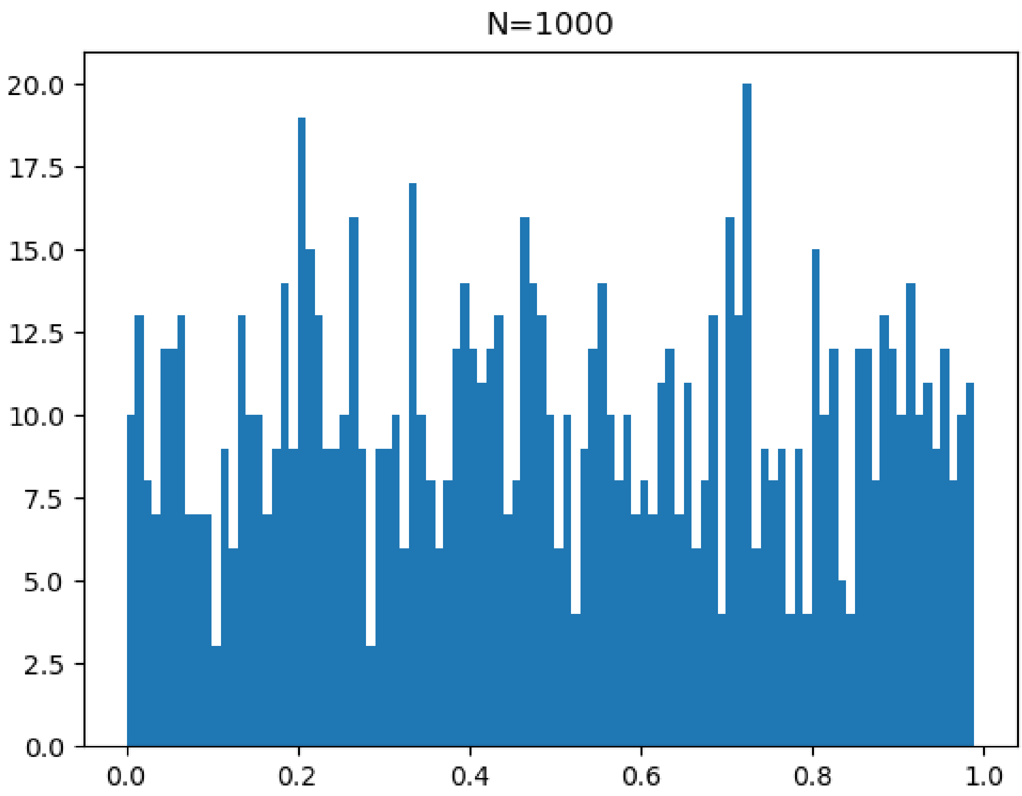
Question 1

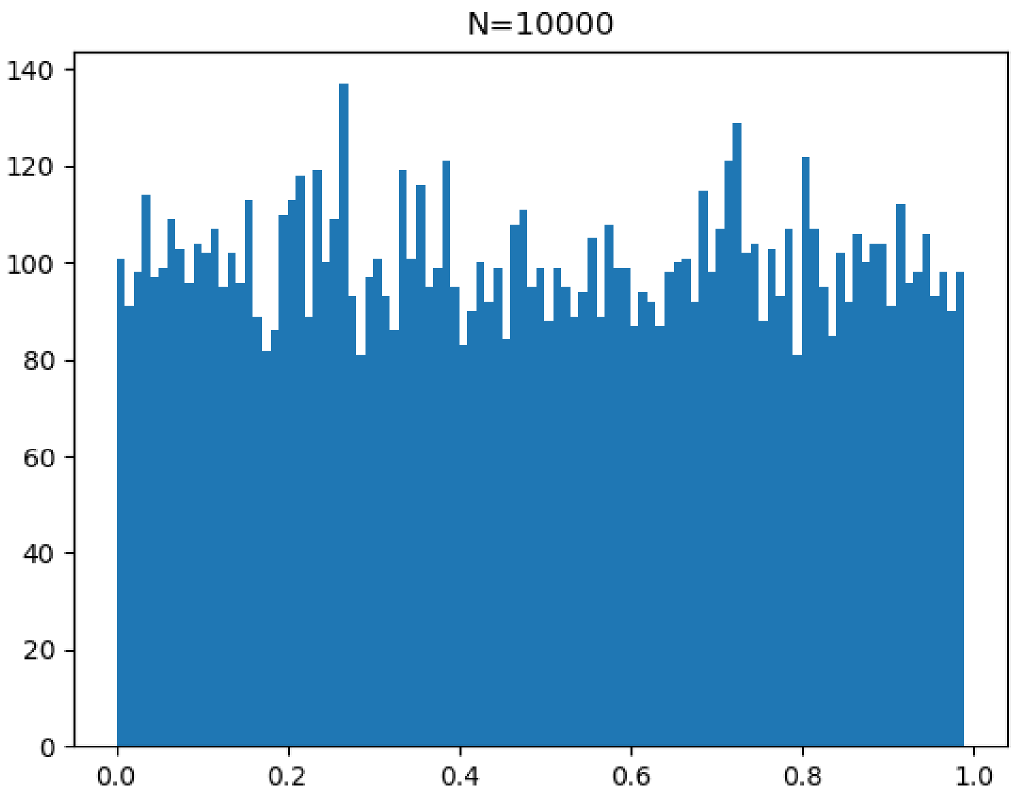
Linear Congruence Generator
x0 = 1 , a(multiplier) = 1229 , m(modulus) = 2048 and b = 1

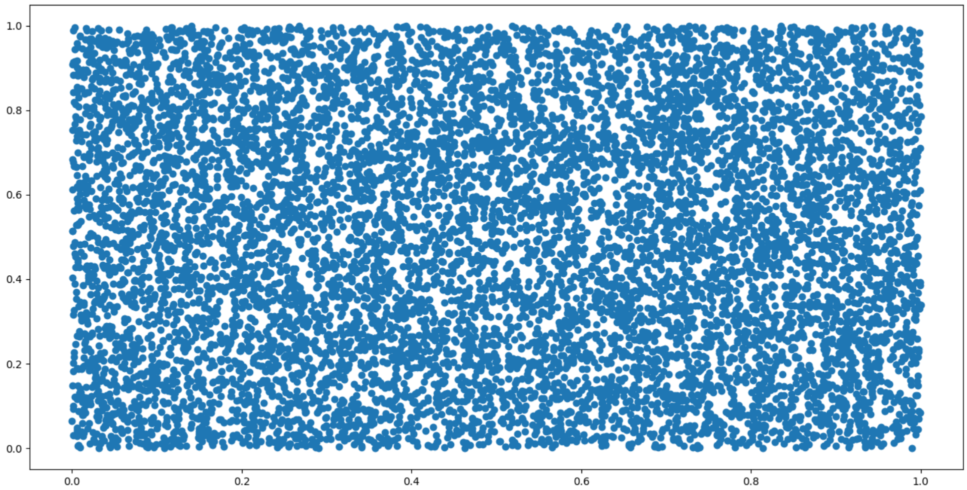
- **Plot for n=1000**



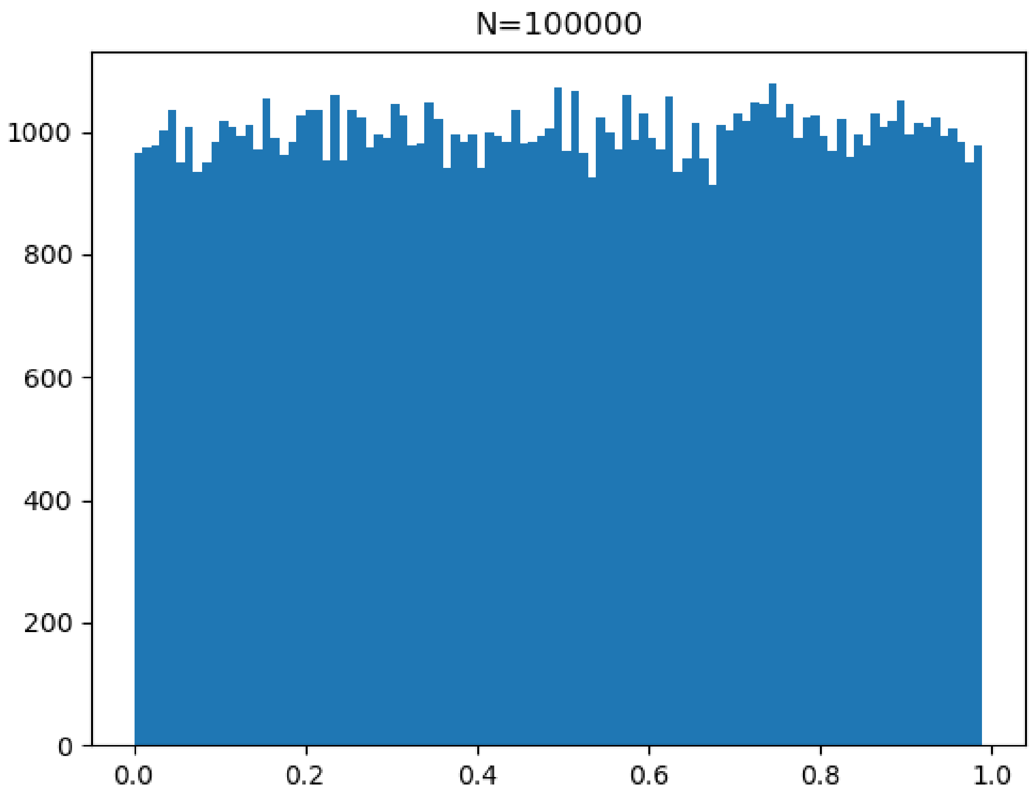


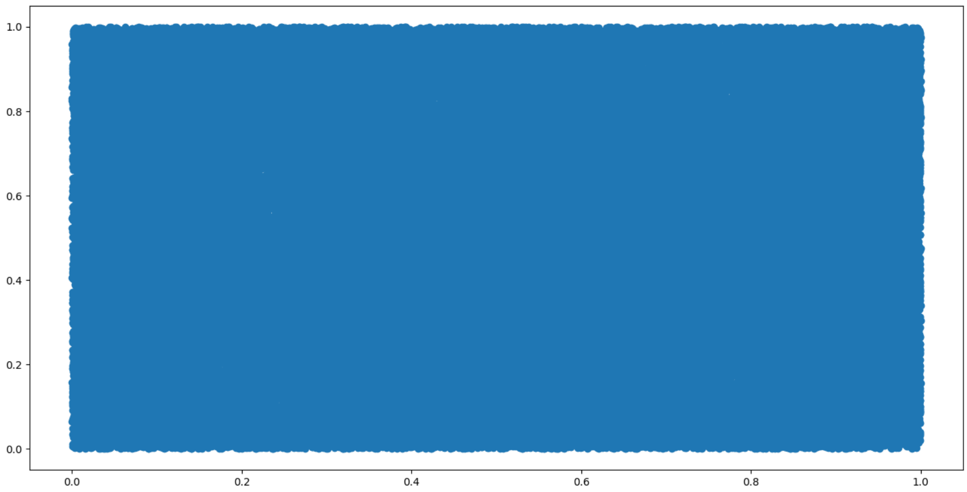
- **Plot for n= 10000**





- **Plot for n= 100000**


N=100000
1000
800
600
400
200
0
0.0
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

CONCLUSIONS

1) This scatter plot seems random. There does not seem to be a definite relationship between Ui and Ui+1. Ui 's do not follow a particular pattern and thus are random.

2) The histogram and Kde plot suggest that the lagged Fibonacci generator closely follows what an ideal generator should follow i.e. all the values sandwiched between 0 and 1 and
All Ui to be mutually independent.

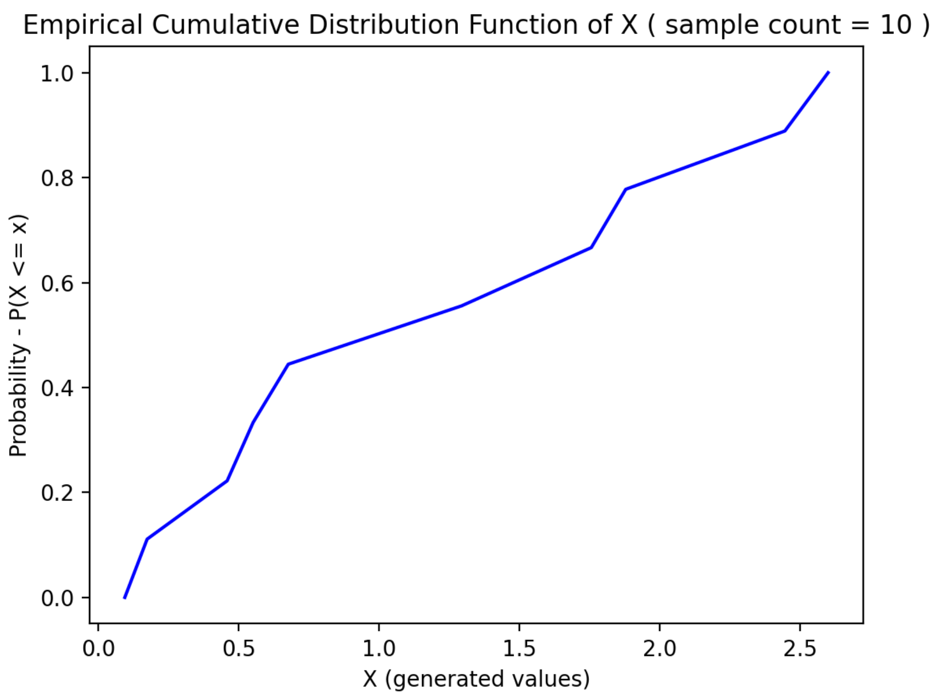
3) Also as we increase the value of n(or size you can say) we can see all histogram bars getting at the same level(One can also see the KDE plot line getting straighter as n increases).
When n=1000 we can histogram bars height differing considerably but as n increases say to 100000 height differences reduces. It pertains to having uniform distribution.

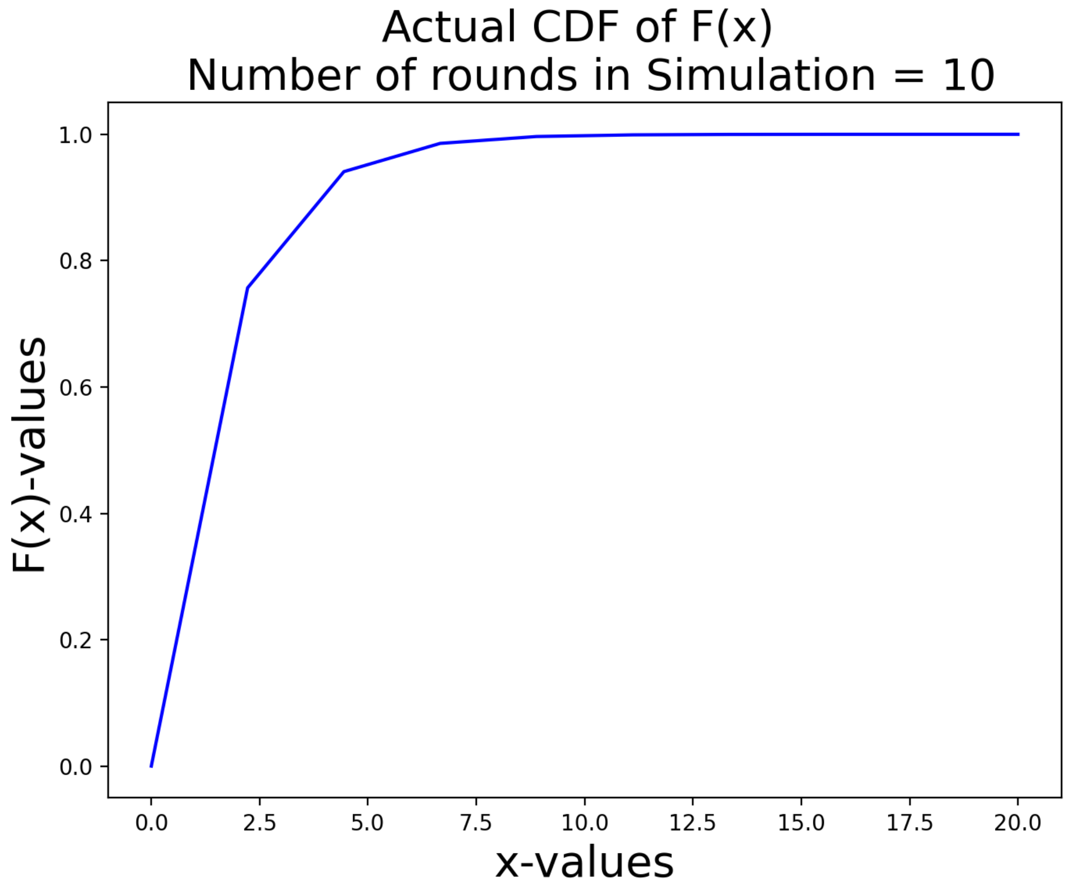
Question 2

Assumed mean = pi/2(1.57)

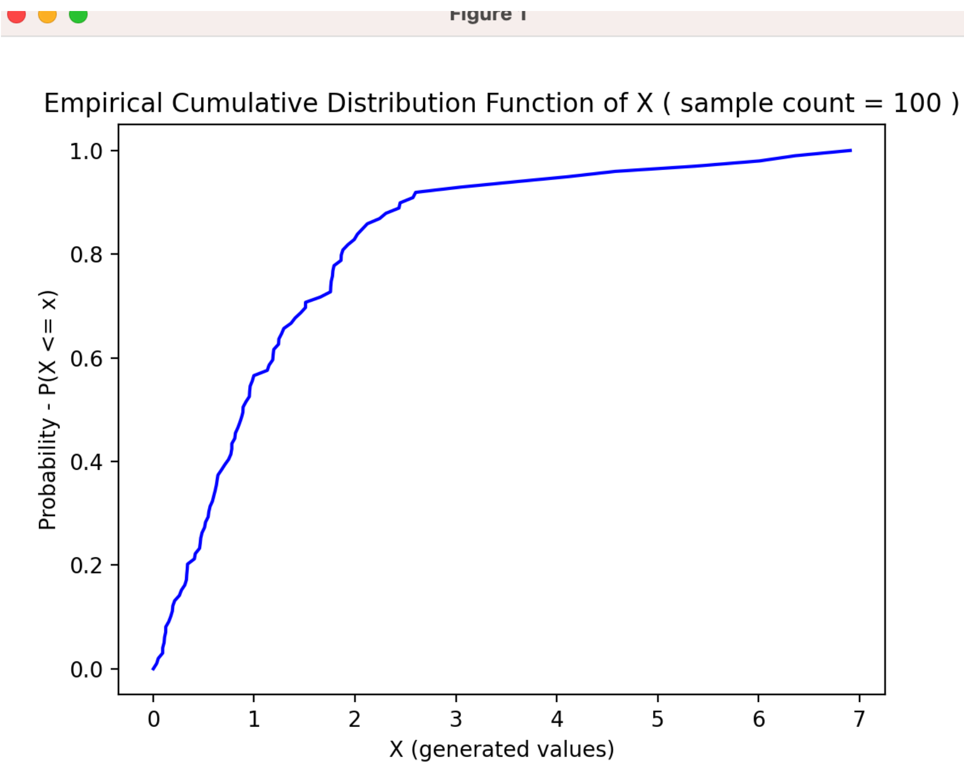
Below are graphs for different values of n :-

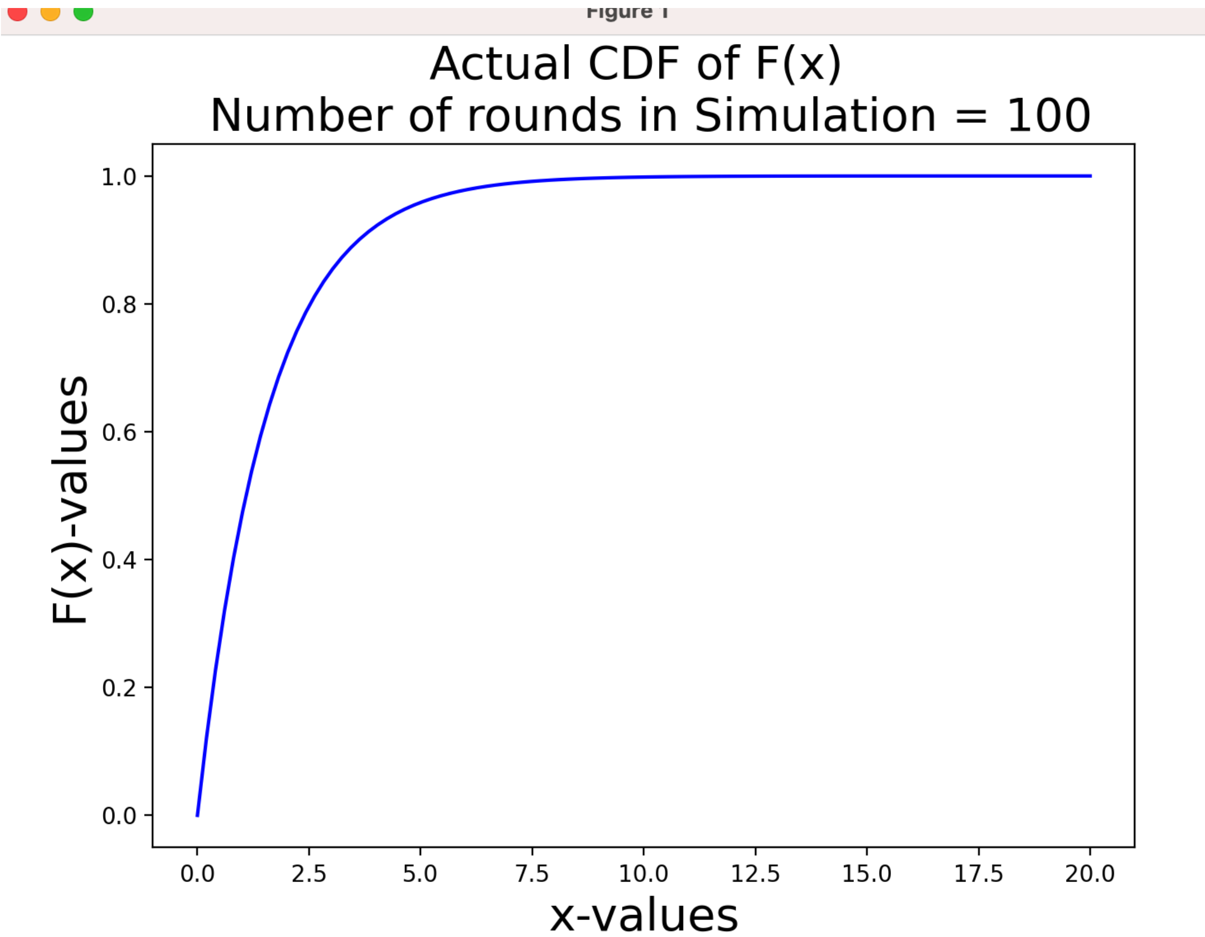
Plot for n=10



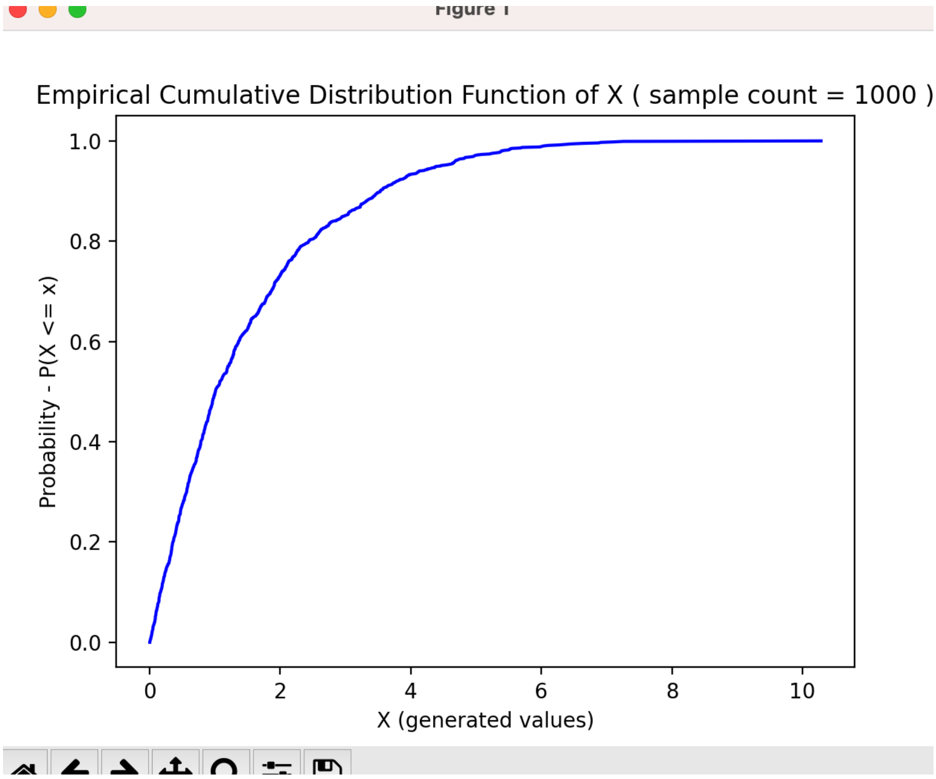


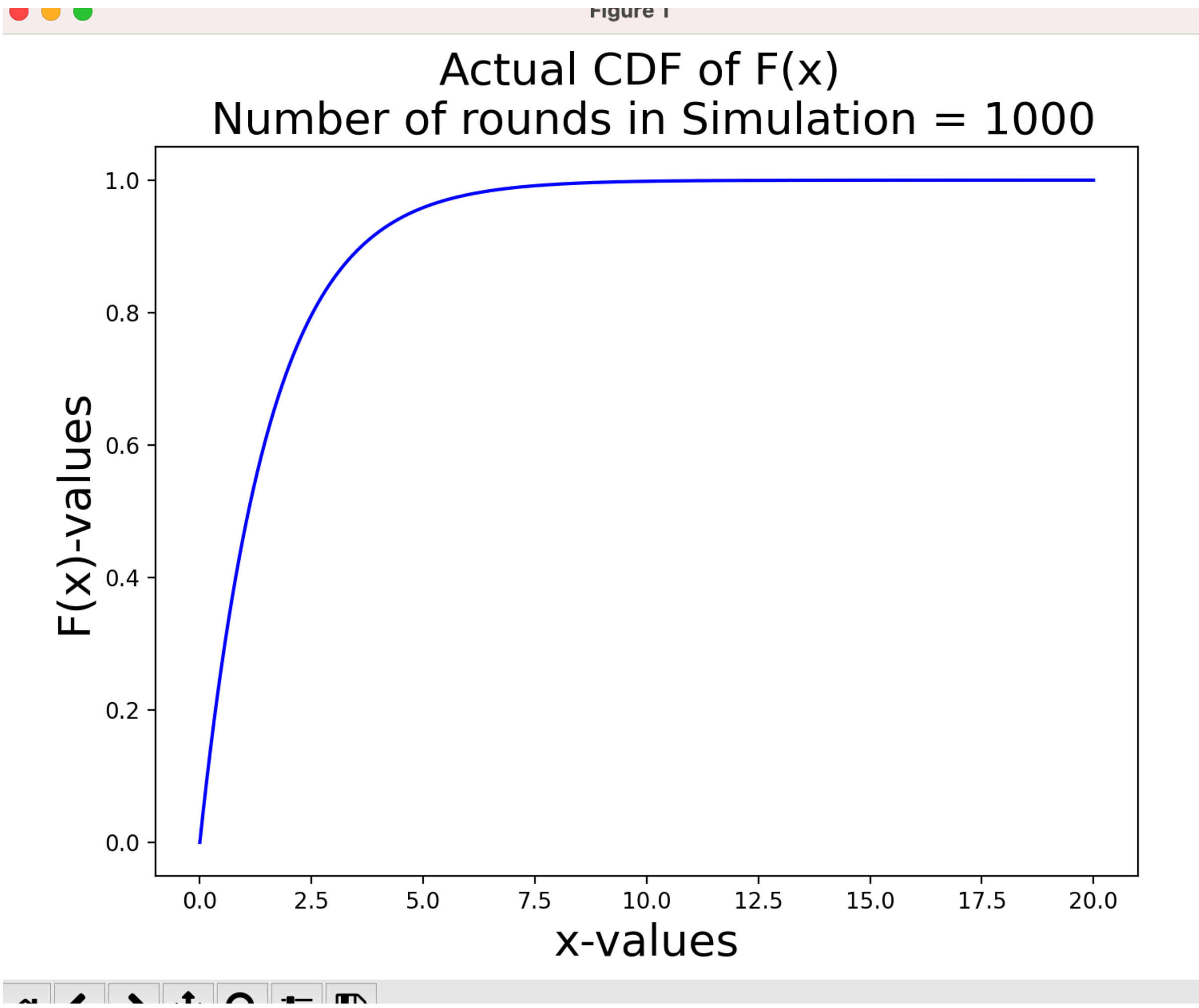
Plot for n=100





Plot for n=1000





Plot for n=10000

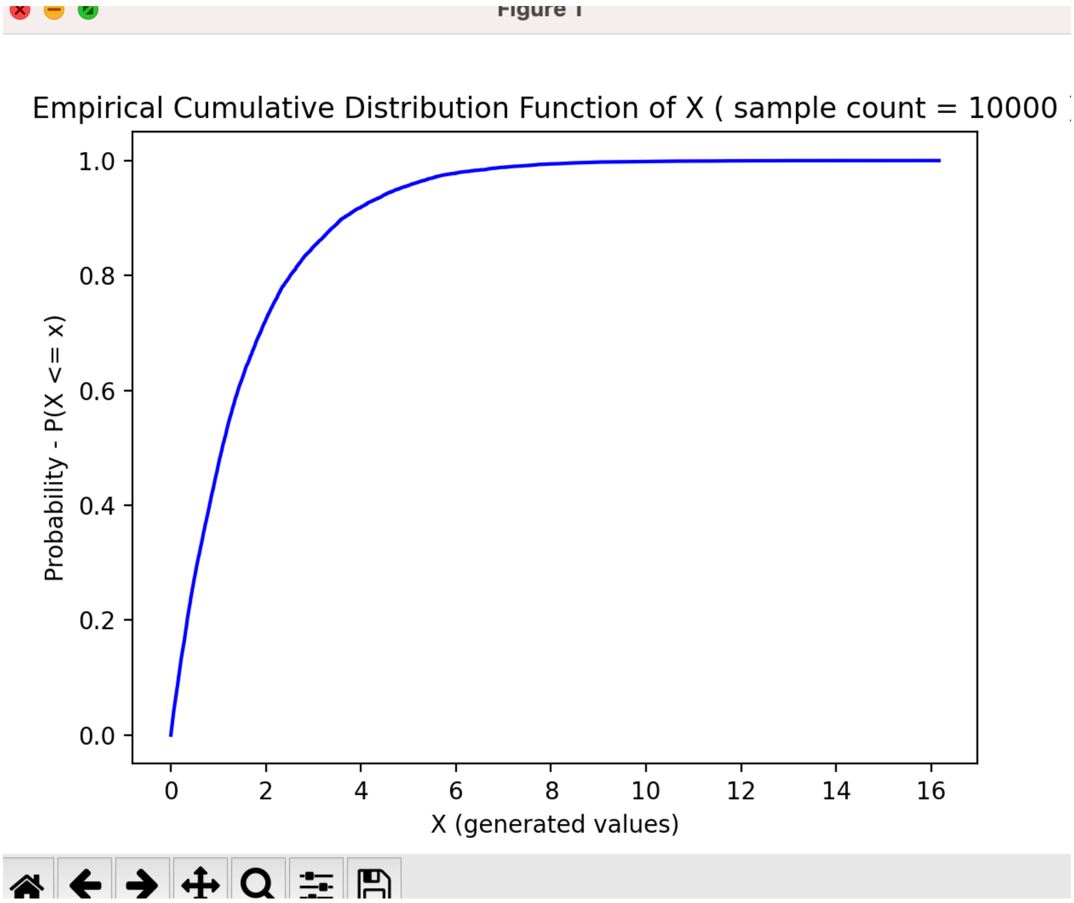


Figure 1
Empirical Cumulative Distribution Function of X (sample count = 10000
1.0
0.8
0.6
0.4
0.2
0.0
Probability - P(X <= x)
0
2
4
6
8
10
12
14
16
X (generated values)

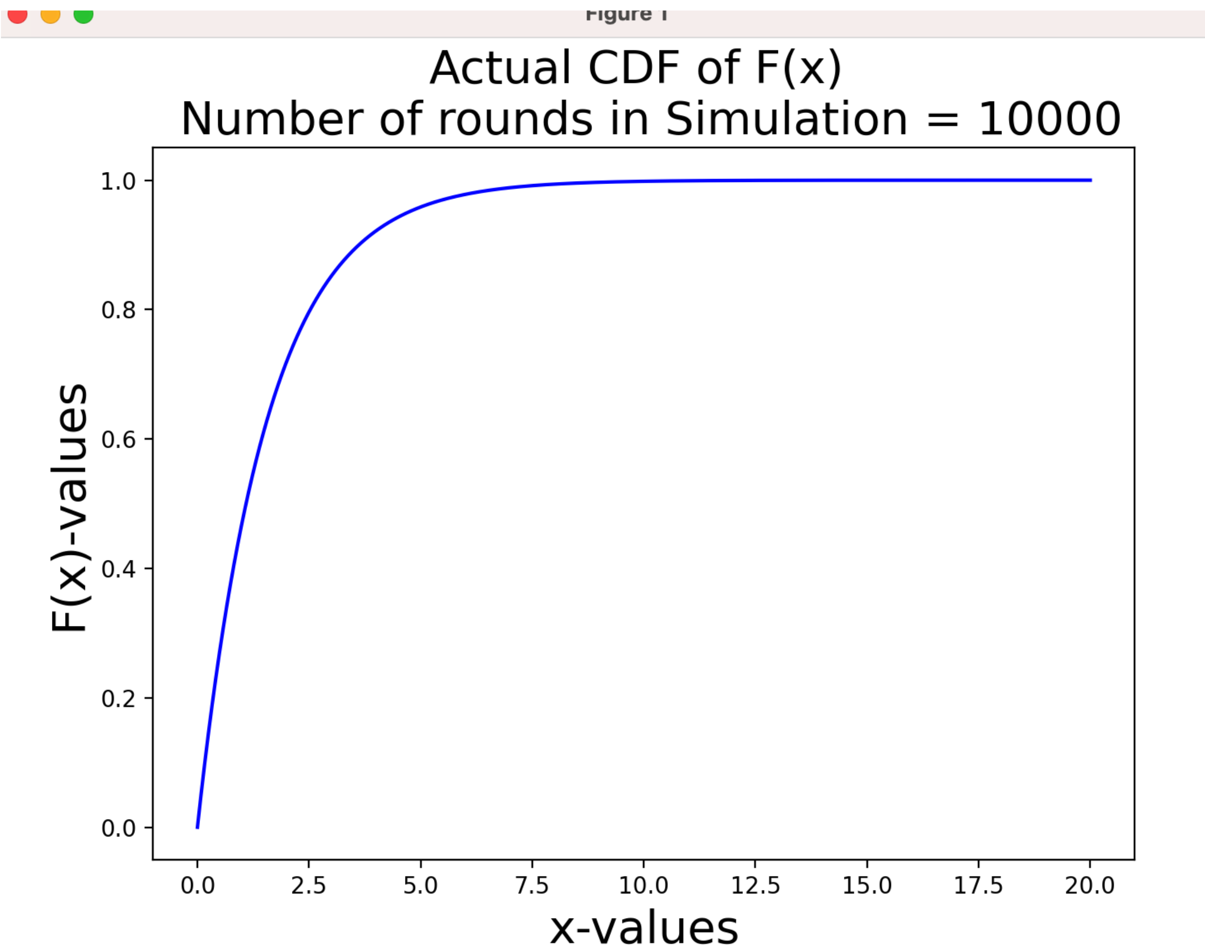
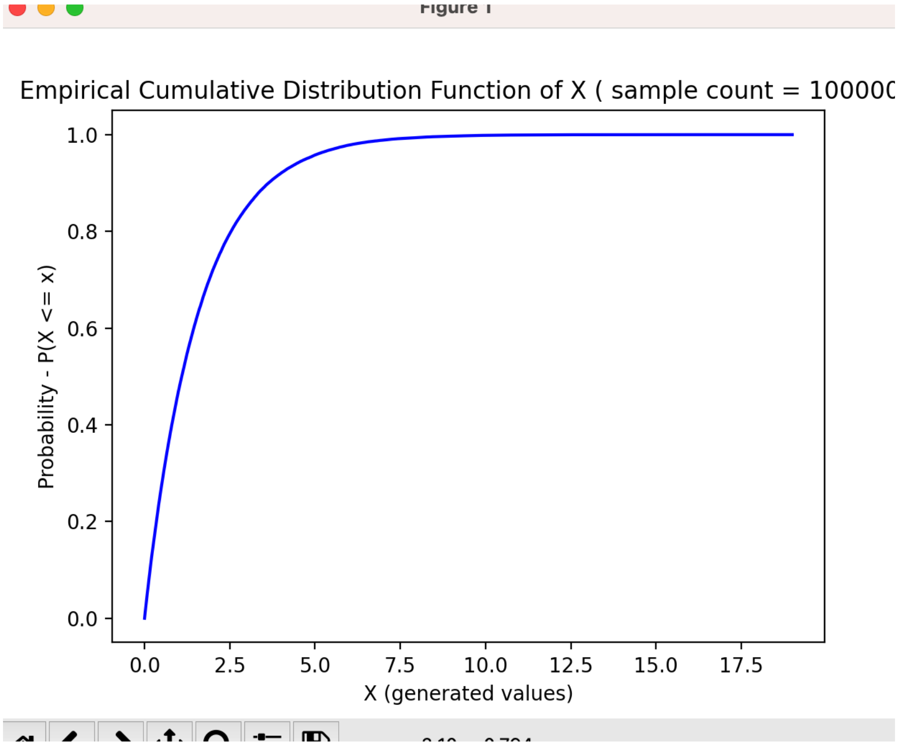
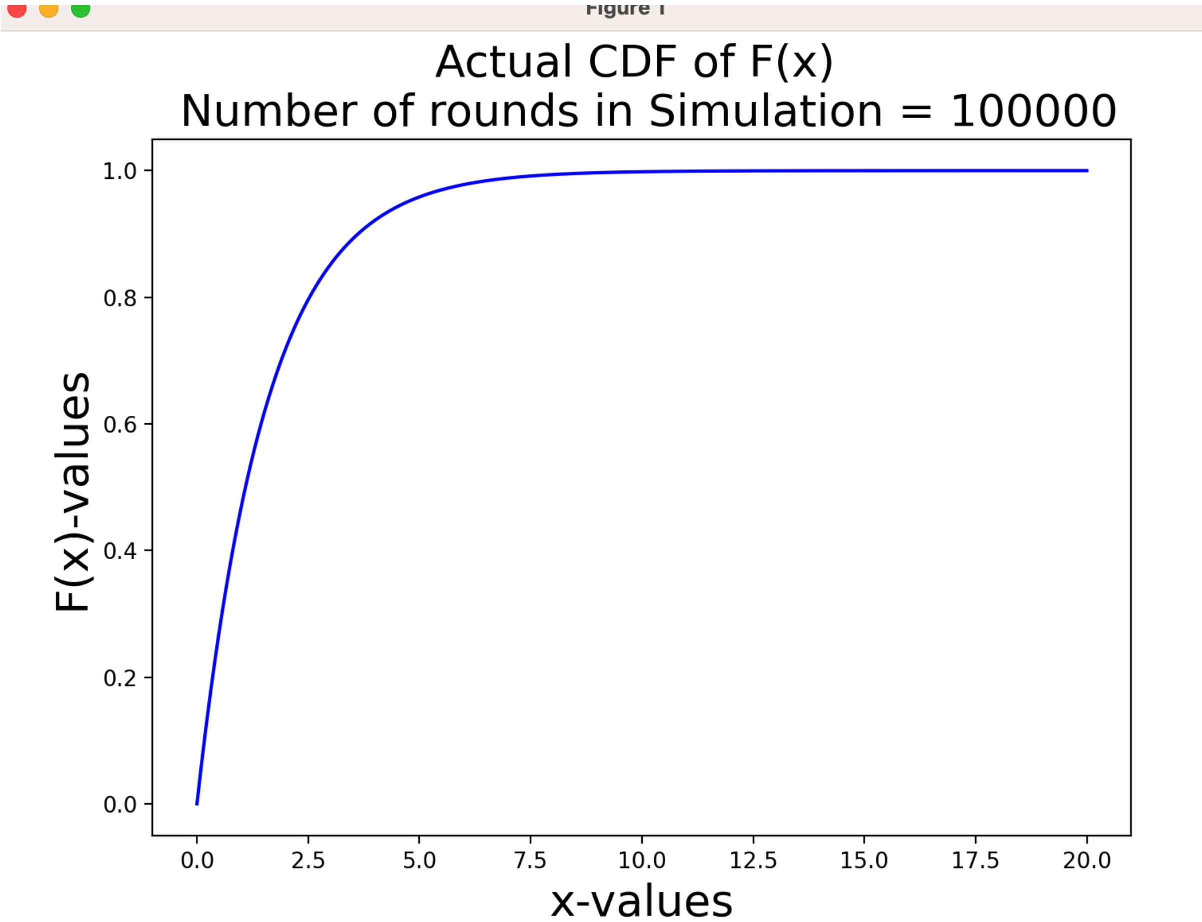


Figure 1
Actual CDF of F(x)
Number of rounds in Simulation = 10000
1.0
0.8
0.6
0.4
0.2
0.0
F(x)-values
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
x-values

Plot for n=100000





Conclusion:

1) Actual mean is approx 1.570796
2) Actual variance is approx 2.467401

Here variance = mean*mean and mean is pi/2.

Sample count = 10
Mean = 1.1936962912966962
Variance = 0.7814990069541273

Sample count = 100
Mean = 1.2994460101344962
Variance = 1.762657457409761

Sample count = 1000
Mean = 1.4965421325216637
Variance = 1.9869188361903516

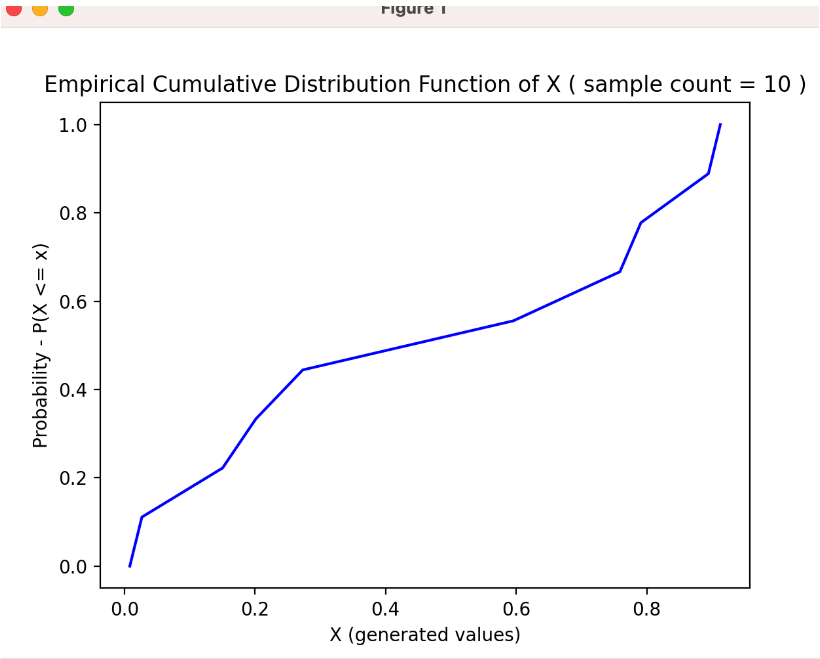
Sample count = 10000
Mean = 1.5701159836132759
Variance = 2.488296189302775

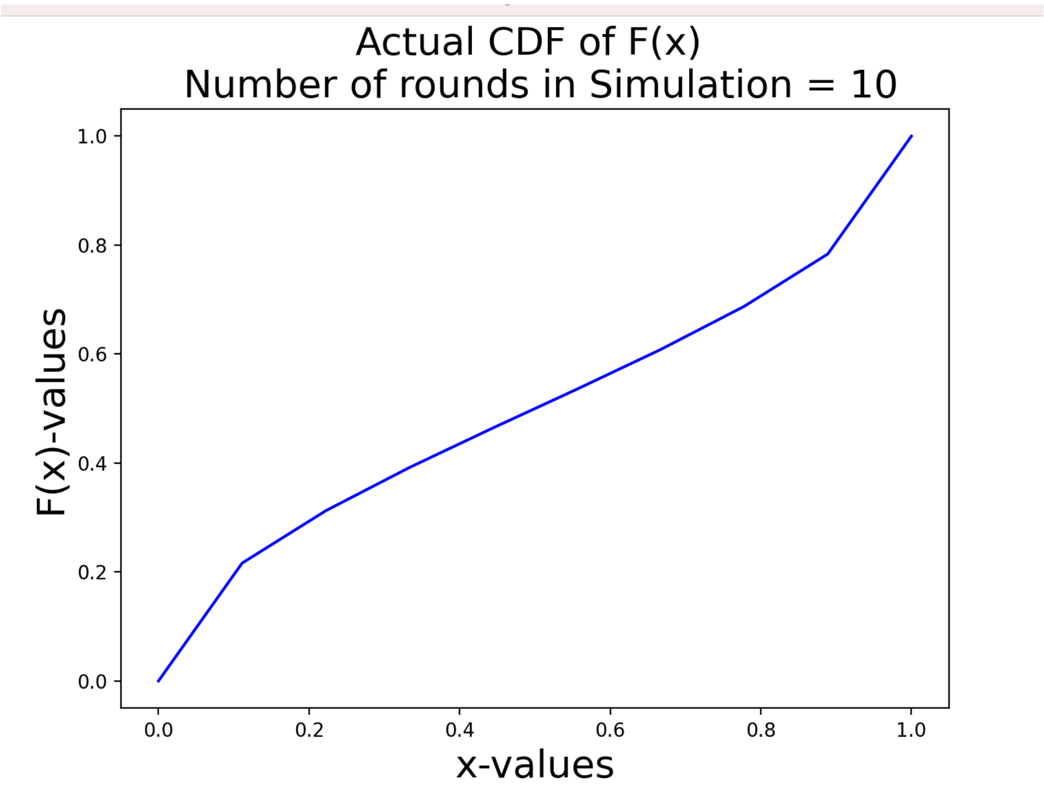
Sample count = 100000
Mean = 1.572536789079683
Variance = 2.461267751081261

As we increase the value of n we can see sample mean and sample variance coming closer to actual mean and actual variance.

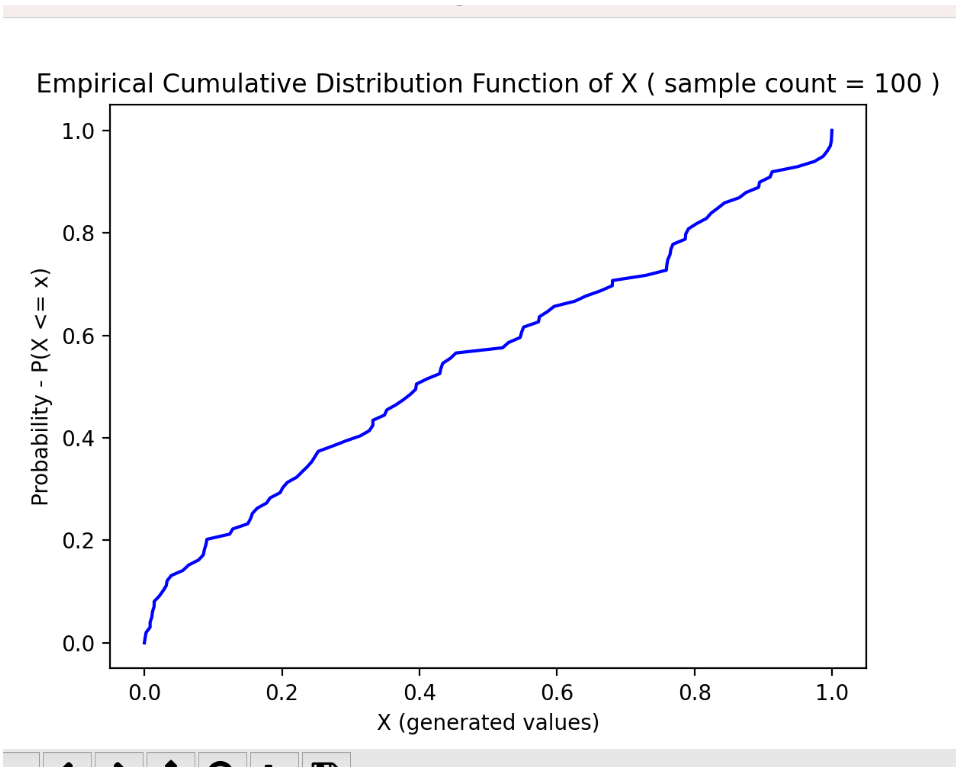
Question 3

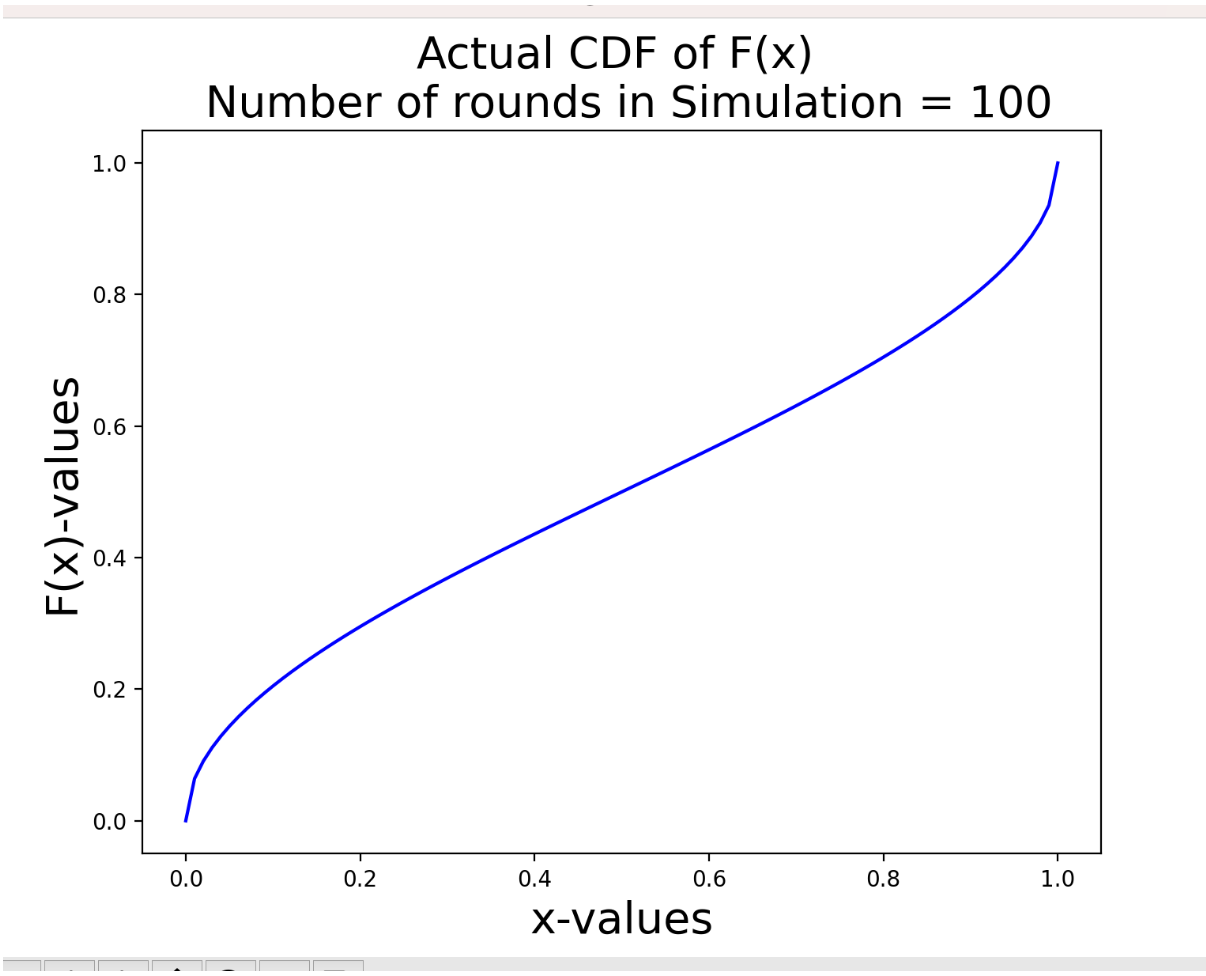
Plot for n=10



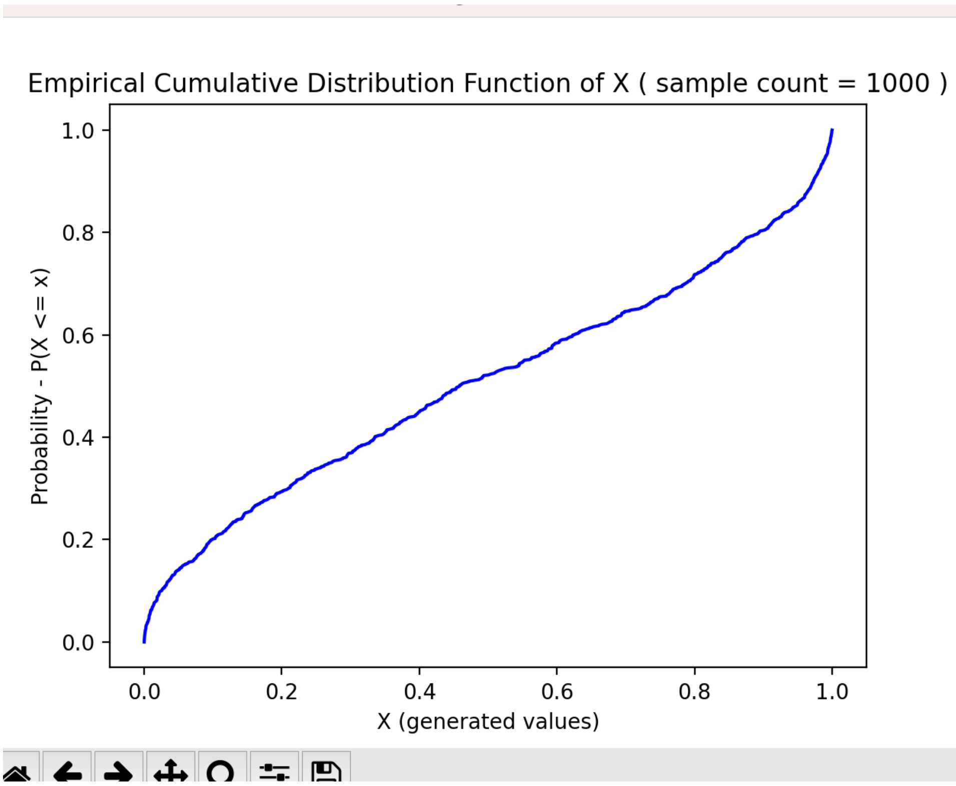


Plot for n=100

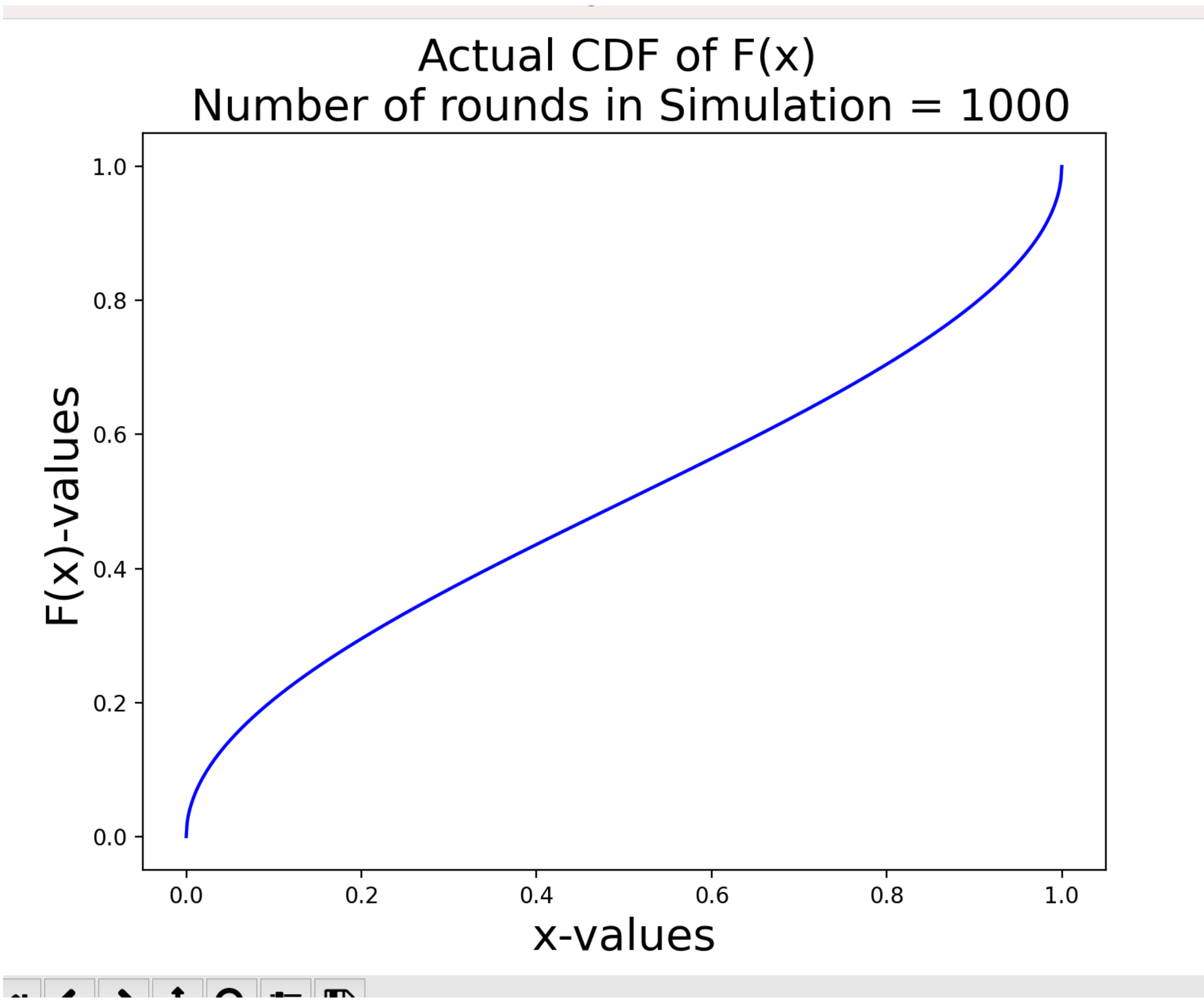




Plot for n=1000

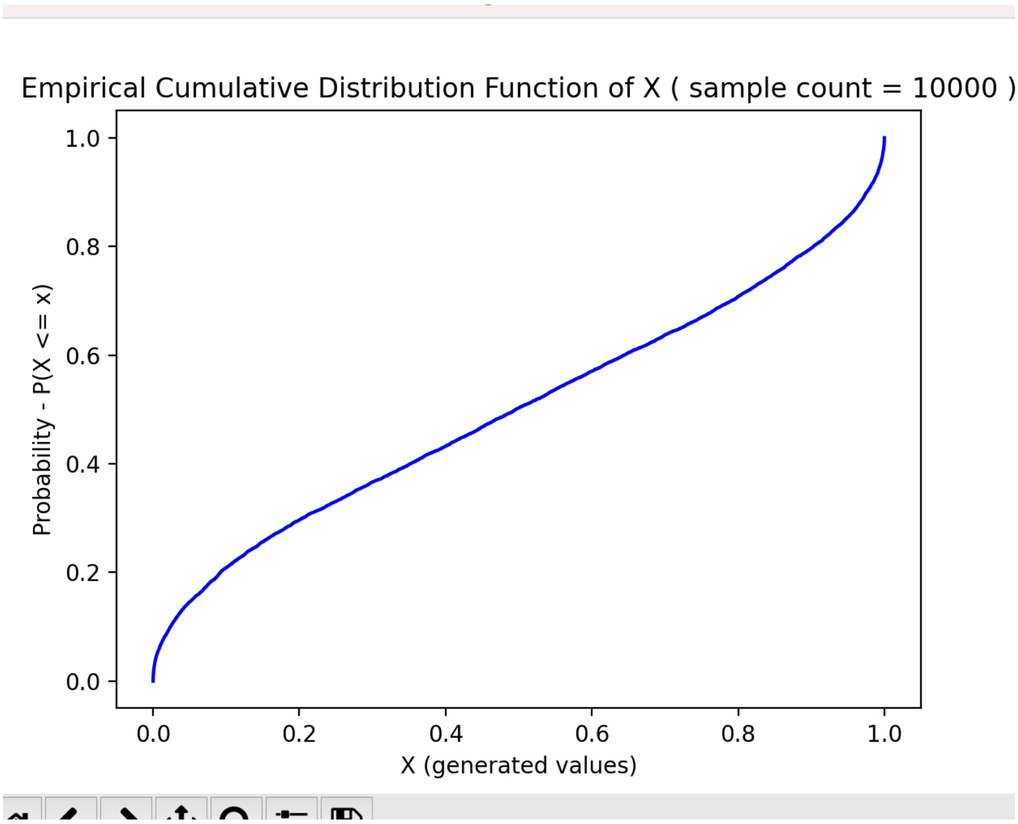


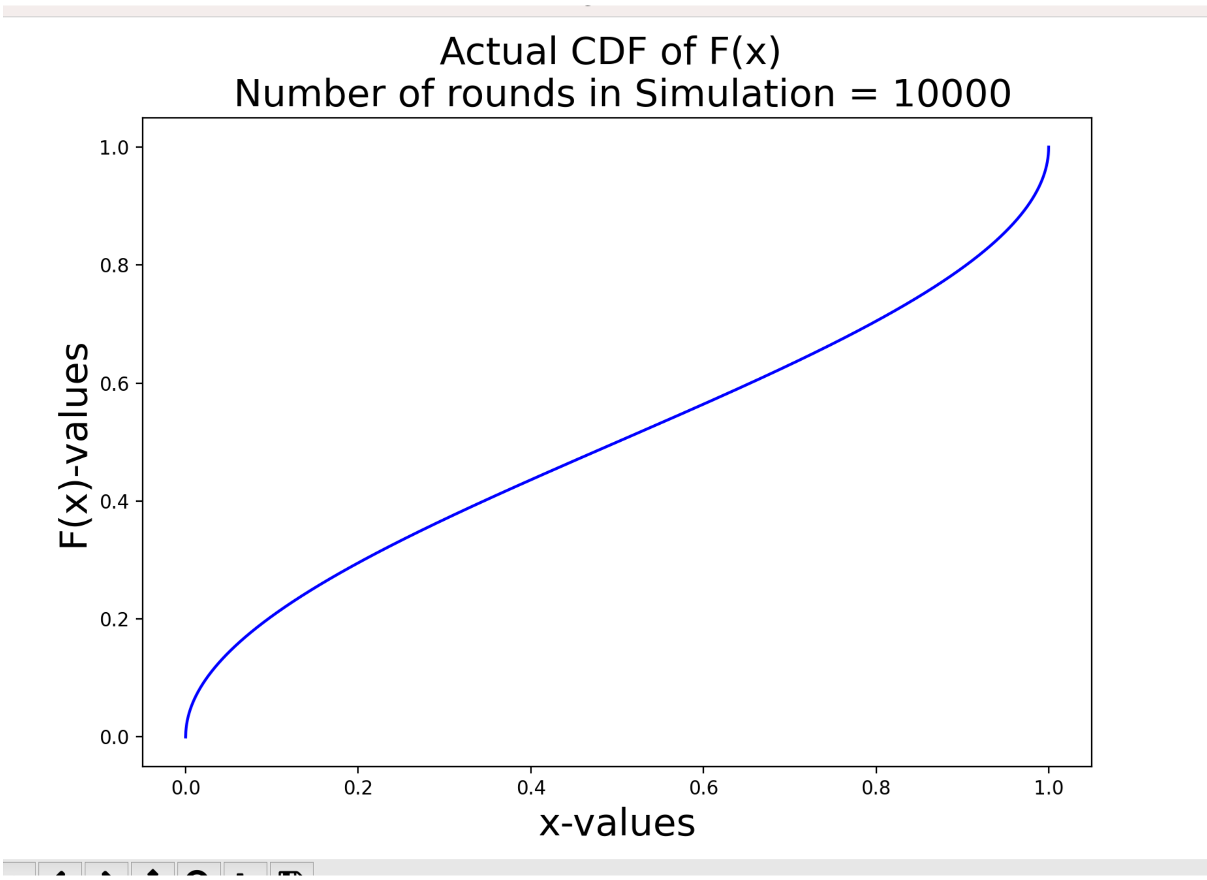
Empirical Cumulative Distribution Function of X (sample count = 1000)
Probability - P(X <= x)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
X (generated values)



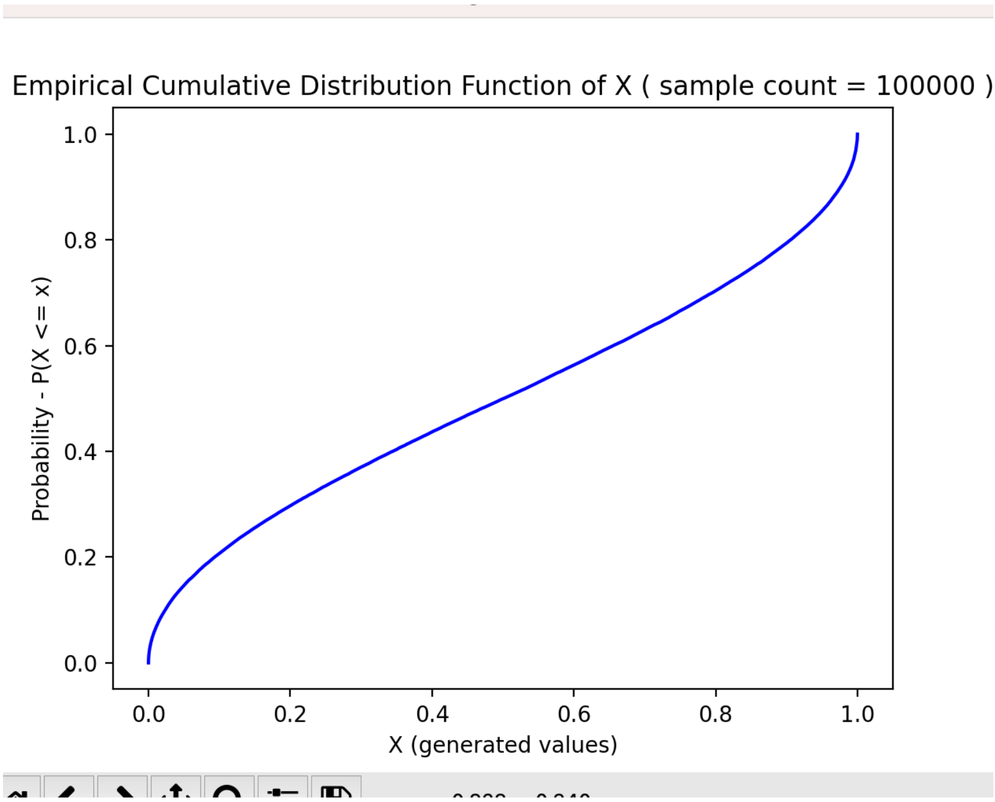
Actual CDF of F(x)
Number of rounds in Simulation = 1000
F(x)-values
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x-values

Plot for n=10000

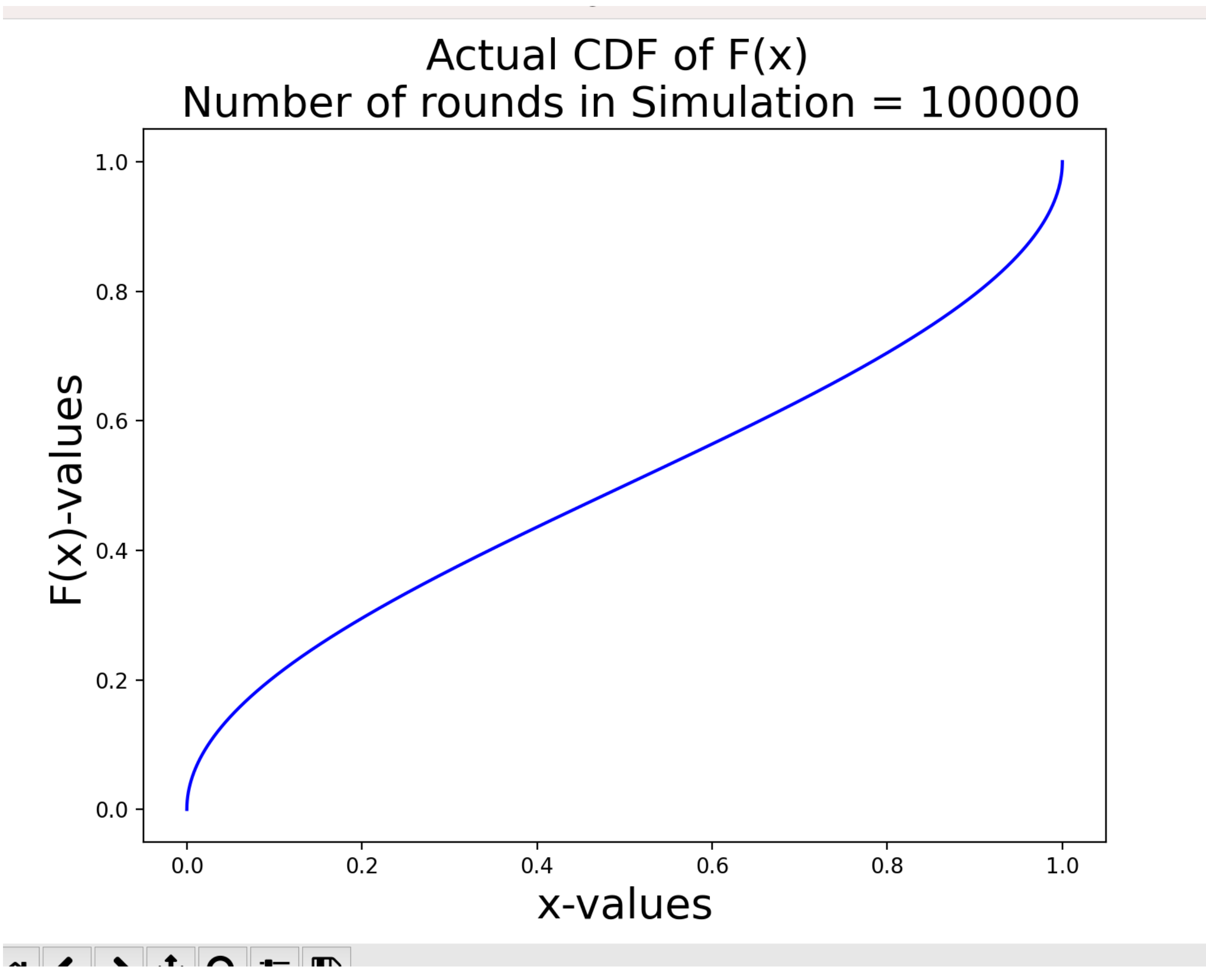




Plot for n= 100000



Empirical Cumulative Distribution Function of X (sample count = 100000)
Probability - P(X <= x)
X (generated values)
0.0
0.2
0.4
0.6
0.8
1.0



Actual CDF of F(x)
Number of rounds in Simulation = 100000
F(x)-values
x-values
0.0
0.2
0.4
0.6
0.8
1.0

CONCLUSION:

1) Actual mean of the distribution is 0.5.
2) Actual variance of distribution is 0.125.

Sample count = 10
Mean = 0.4613610544911812
Variance = 0.12002030136126798

Sample count = 100
Mean = 0.4451046366945898
Variance = 0.10441125911733588

Sample count = 1000
Mean = 0.49173183519814806
Variance = 0.12251431095826601

Sample count = 10000
Mean = 0.4985638361801597
Variance = 0.12464348755138321

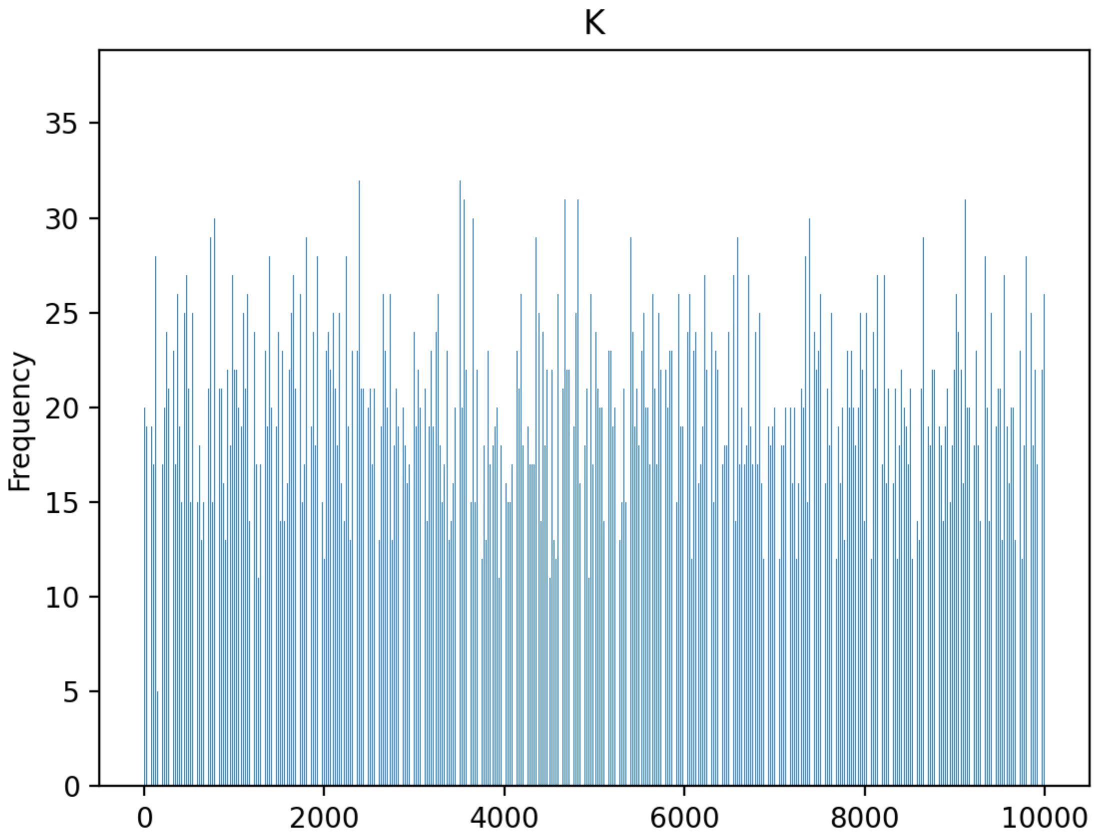
Sample count = 100000
Mean = 0.4998199454583099
Variance = 0.12574974575513317

As we increase the value of n we can see sample mean and sample variance coming closer to actual mean and actual variance.

The reason the distribution of the random variable X matches the cumulative distribution function (CDF) F(x) is because F(x) is a smooth and always increasing function. When we use a uniform distribution U on the interval [0, 1] and apply the inverse of F, denoted as $F^{-1}(U)$, the result represents a sample from the distribution of F. This relationship illustrates the concept of the Inverse Transform Method.

Question 4 :-

Discrete uniform distribution on {1, 3, 5, . . . , 9999} for sample size 100000 is as follows :-


K
Frequency
35
30
25
20
15
10
5
0
0
2000
4000
6000
8000
10000