MA323 – Monte Carlo Simulation

Lab - 4

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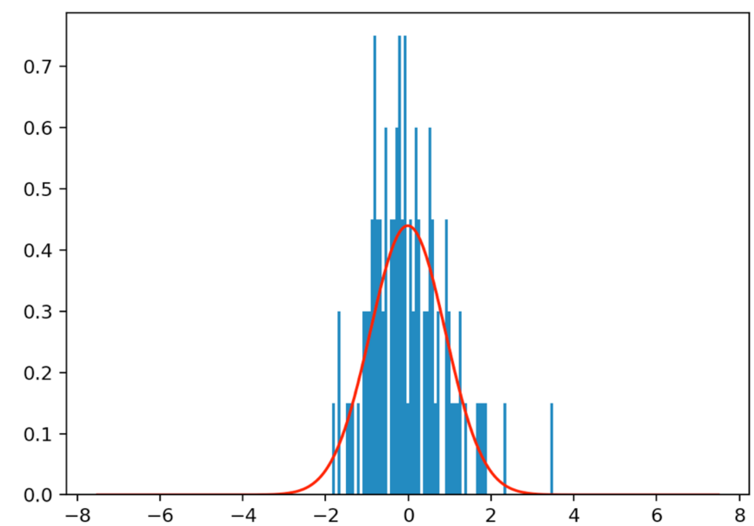
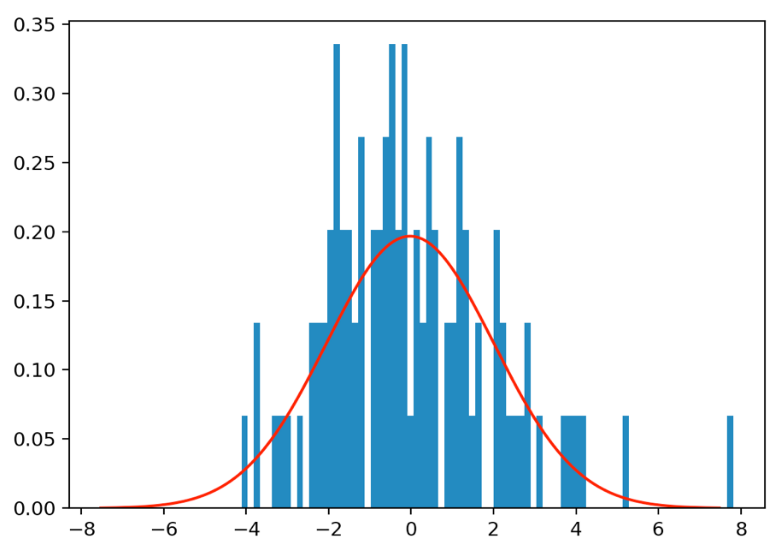
1. Box Muller Method

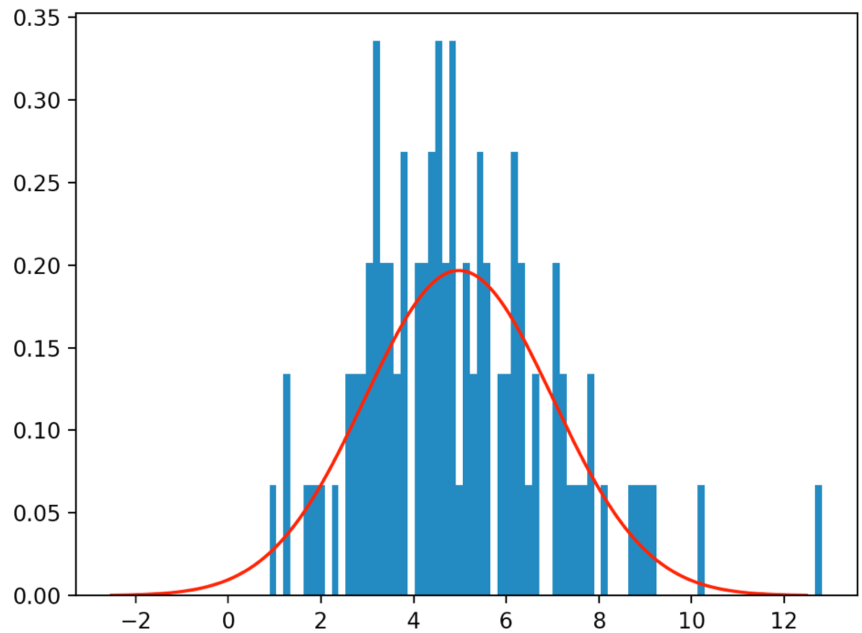
(a) 100 Samples from N (0,1) distribution:



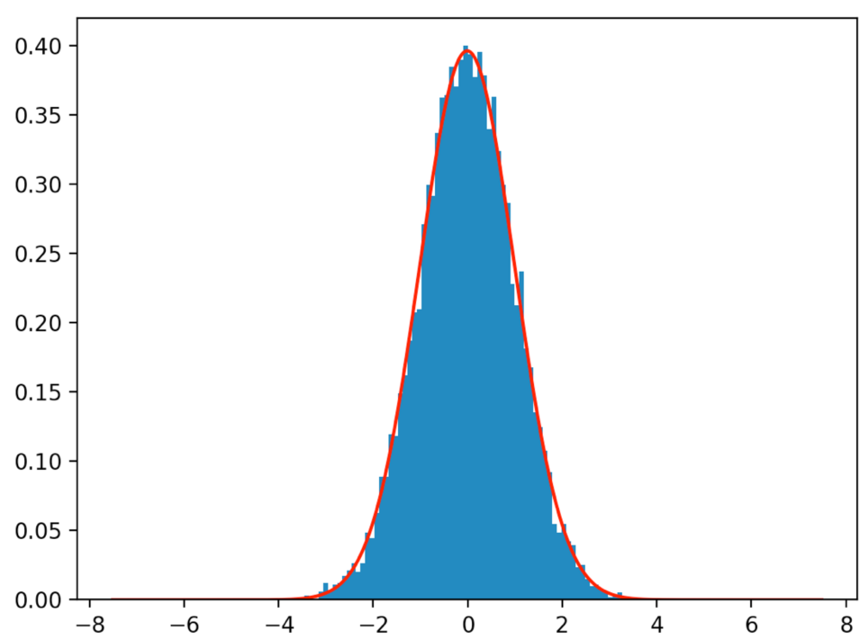
(b) 100 Samples from N (0,5) distribution:



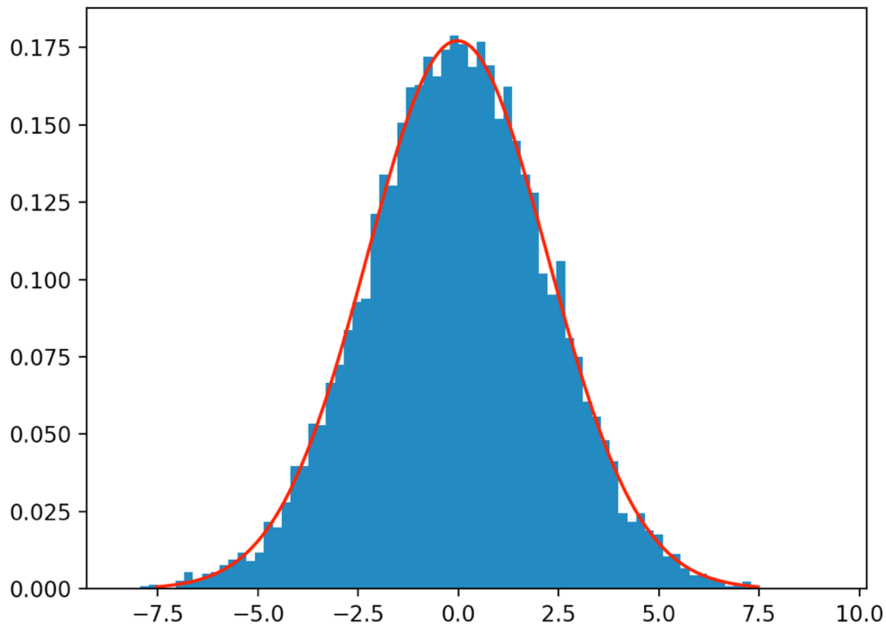
(c) 100 Samples from N (5,5) distribution:



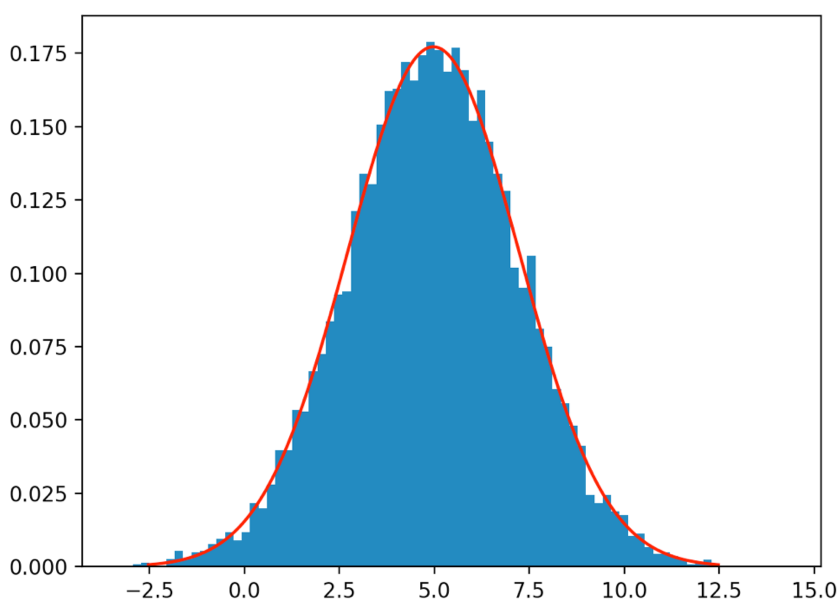
(d) 10000 Samples from N (0,1) distribution:



(e) 10000 Samples from N (0,5) distribution:

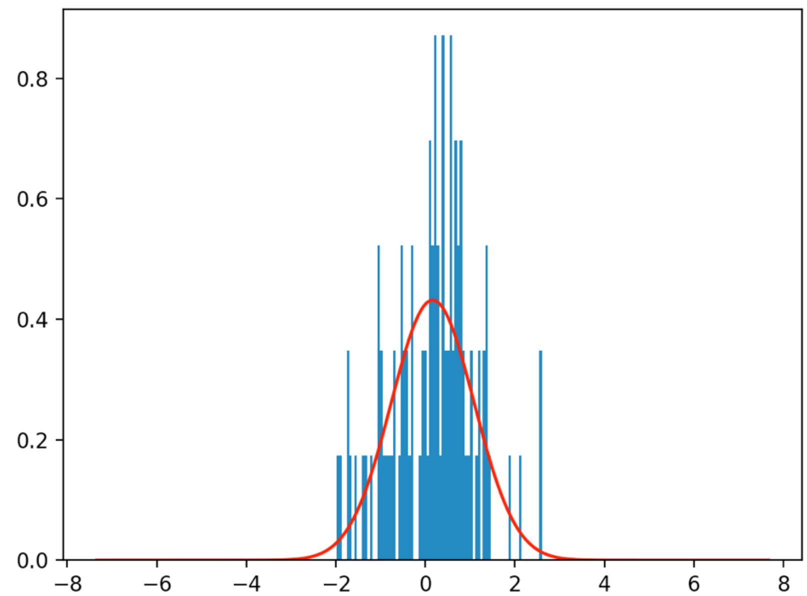


(f) 10000 Samples from N (5,5) distribution:

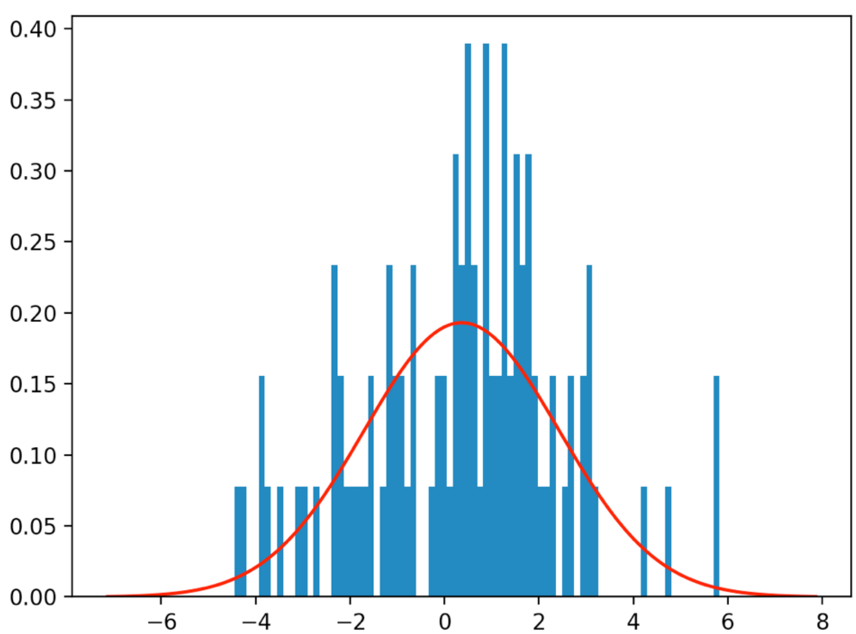


2. Marsaglia and Bray Method

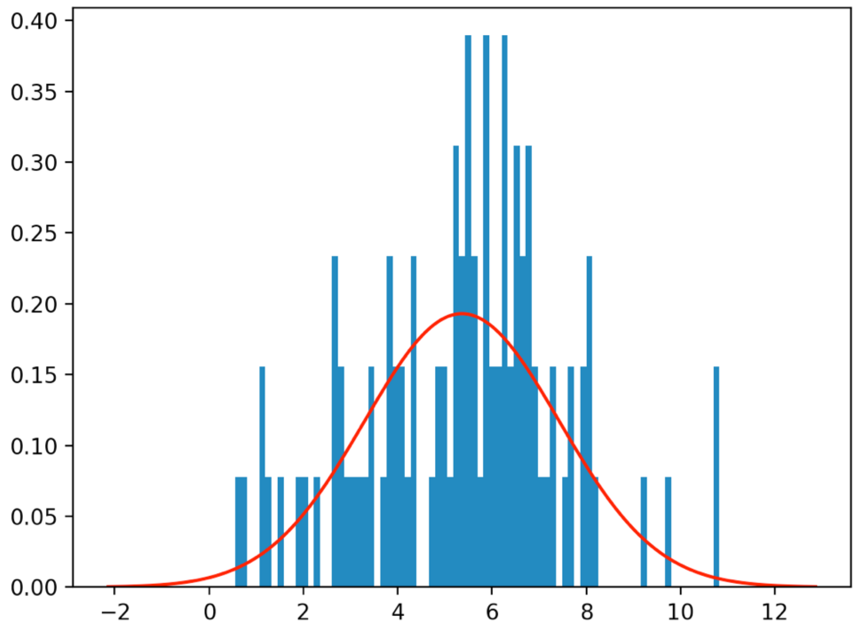
(a) 100 Samples from N (0,1) distribution:



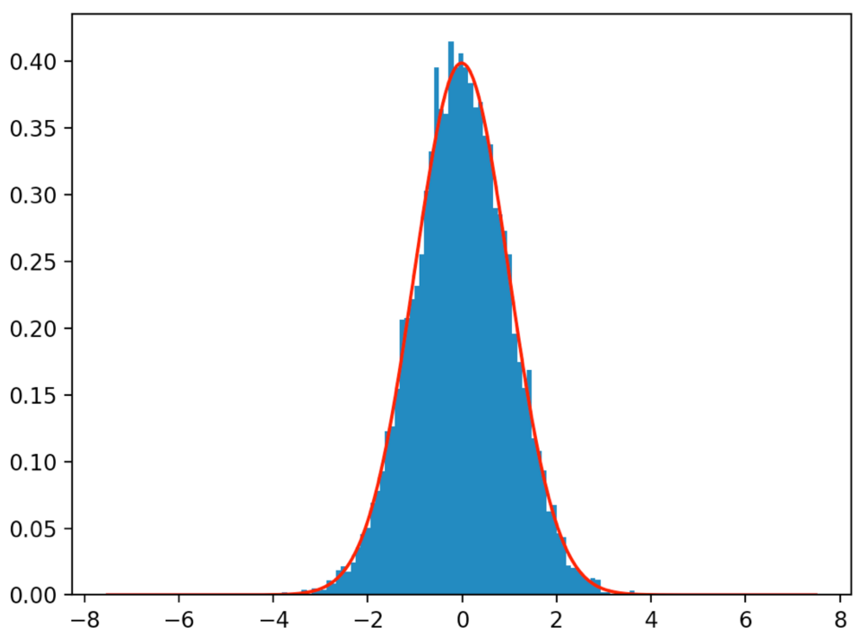
(b) 100 Samples from N (0,5) distribution:



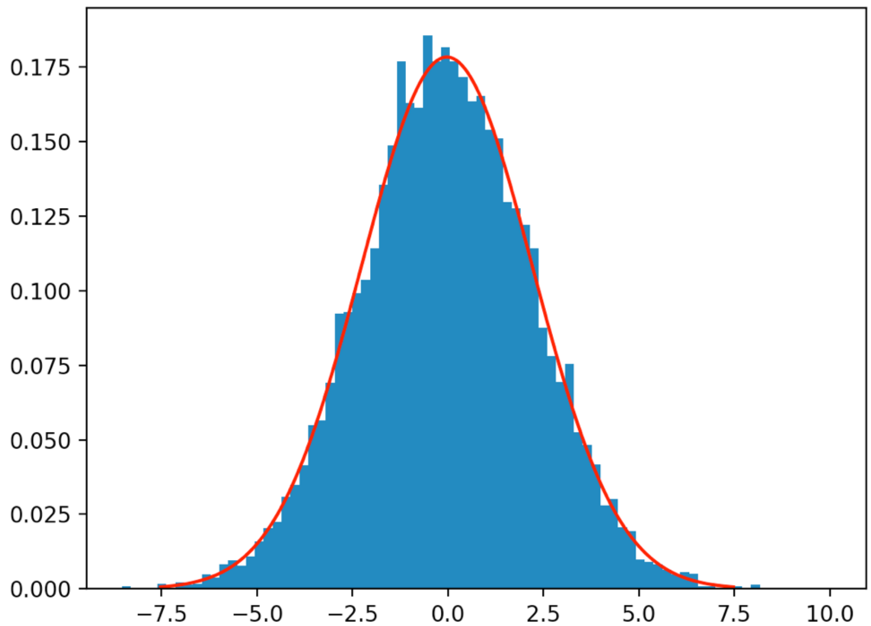
(c) 100 Samples from N (5,5) distribution:



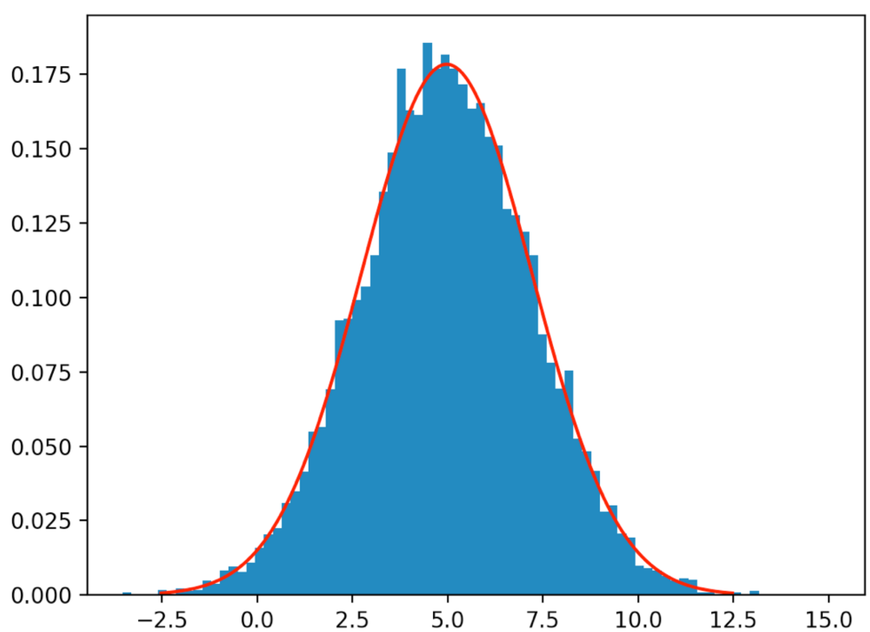
(d) 10000 Samples from N (0,1) distribution:



(e) 10000 Samples from N (0,5) distribution:



(f) 10000 Samples from N (5,5) distribution:



Following are the observations from the above data generated: -

	A	B	C
1	Box Muller Method		
2	Observations:	Mean	Variance
3	100 Samples from N(0,1) Distribution	-0.010157983	0.823030224
4	100 Samples from N(0,5) Distribution	-0.022713941	4.115151118
5	100 Samples from N(5,5) Distribution	4.977286059	4.115151118
6	10000 Samples from N(0,1) Distribution	-0.010356661	1.014419416
7	10000 Samples from N(0,5) Distribution	-0.023158199	5.072097078
8	10000 Samples from N(5,5) Distribution	4.976841801	5.072097078

E	F	G
Marsaglia and Bray Method		
Observations:	Mean	Variance
100 Samples from N(0,1) Distribution	0.162081779	0.853926542
100 Samples from N(0,5) Distribution	0.362425875	4.269632712
100 Samples from N(5,5) Distribution	5.362425875	4.269632712
10000 Samples from N(0,1) Distribution	-0.01115081	1.001307427
10000 Samples from N(0,5) Distribution	-0.024933968	5.006537133
10000 Samples from N(5,5) Distribution	4.975066032	5.006537133

Following is the comparison of the computational times of both the methods: -

Time Comparision(in sec)		
Observations:	Box Muller	Marsaglia and Bray
100 Samples from N(0,1) Distribution	0.001631021	0.00121212
10000 Samples from N(0,1) Distribution	0.074239016	0.10046196

Note: - Computational time may vary depending on which machine we're running the code; it'll not be exactly the same. The code was run 10 times and then the average of observed values was taken for the comparison.

For Marsaglia and Bray Method, the proportion of values rejected while computing the samples for N (0,1) distribution are as follows: -

Observations:	Proportions of values Rejected
100 Samples from N(0,1) Distribution	0.090909
10000 Samples from N(0,1) Distribution	0.205845

Observations:

1) From the data generated above, it can be observed that as the sample size increases, the empirical mean and variance converge to the theoretical mean and variance of the normal density from which it is sampled.

To generate sample **Y** of the form **N (mean, var)** from **N (0, 1)**
Y = mean + Z * sqrt(var) where **Z ~ N(0, 1)**.

It is observed that as the sample size increases from 100 to 10,000, the empirical density plots become more similar to the density plot generated from the formula.

It is observed that as the variance becomes more, the shape of the density plots of **N(0, 5)** and **N(5, 5)** are more distributed than **N(0, 1)** the distribution spreads out more. The shape of the density plots of **N(0, 5)** and **N(5, 5)** are same as the plot of **N(5, 5)** is obtained by shifting the plot of **N(0, 5)** by 5 units to the right.

2) The execution times for Box-Muller and Marsaglia and Bray methods were obtained by averaging over 10 simulations.

It is observed that on average, for sample size 100, the Marsaglia and Bray method runs faster than Box-Muller method, but for sample size 10,000, the Marsaglia and Bray method runs slower than Box-Muller method. This is in contrast to the theory according to which Marsaglia and Bray method is quicker as it avoids the computation of *Cosine* and *Sine* functions.

But, it is observed that this trend may not always hold as Acceptance Rejection technique involves accepting only the suitable values and rejecting the unsuitable ones. As the sample size increases, this technique leads to a significant overhead due to which it becomes slower than Box-Muller method with *Cosine* and *Sine* functions. The computational times of Box-Muller method depends on how the sine and cosine functions are implemented in the system. Due to better implementation, the observed value of computational times for Box-Muller method is lesser than Marsaglia and Bray method.

3) Theoretically, the proportion of values rejected should be equal to $1 - \pi/4$. This is the area of the discarded region from a box of unit area. Random numbers are chosen such that they lie inside a circle which is inscribed in a square of unit area. As a result, the area of the remaining square is $1 - \pi/4$, which measures the proportion of the values rejected.

It is observed that as the sample size increases from 100 to 10,000, the rejection rate converges to the theoretical rate of rejection i.e. 0.2146. This shows us that the accuracy of algorithm increases as the sample size decreases and the proportion of values rejected becomes close to theoretical value.