MA323 – Monte Carlo Simulation

Lab - 1

Question - 1

The outcomes on the basis of given values are as follows:

1. a = 6, b = 0, m = 11, x_0 = 0
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   distinct values = 1

2. a = 6, b = 0, m = 11, x_0 = 1
   1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5
   distinct values = 10

3. a = 6, b = 0, m = 11, x_0 = 2
   2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10
   distinct values = 10

4. a = 6, b = 0, m = 11, x_0 = 3
   3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4
   distinct values = 10

5. a = 6, b = 0, m = 11, x_0 = 4
   4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9
   distinct values = 10

6. a = 6, b = 0, m = 11, x_0 = 5
   5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3
   distinct values = 10

7. a = 6, b = 0, m = 11, x_0 = 6
   6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8
   distinct values = 10

8. a = 6, b = 0, m = 11, x_0 = 7
   7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2
   distinct values = 10

9. $a = 6, b = 0, m = 11, x_0 = 8$
   8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7
   distinct values = 10

10. $a = 6, b = 0, m = 11, x_0 = 9$
    9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1
    distinct values = 10

11. $a = 6, b = 0, m = 11, x_0 = 10$
    10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6 3 7 9 10 5 8 4 2 1 6
    distinct values = 10

- When seed $x_0 = 0$, there is one repeating distinct value 0. For $x_0 = 1$ to 10, there are 10 distinct values from 1 to 10 appearing for each x_0 before repetition starts, so period length = 10.

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1. $a = 3, b = 0, m = 11, x_0 = 0$
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   distinct values = 1

2. $a = 3, b = 0, m = 11, x_0 = 1$
   1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3
   distinct values = 5

3. $a = 3, b = 0, m = 11, x_0 = 2$
   2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6
   distinct values = 5

4. $a = 3, b = 0, m = 11, x_0 = 3$
   3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9
   distinct values = 5

5. $a = 3, b = 0, m = 11, x_0 = 4$
   4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1
   distinct values = 5

6. $a = 3, b = 0, m = 11, x_0 = 5$
   5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4
   distinct values = 5

7. $a = 3, b = 0, m = 11, x_0 = 6$
   6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7
   distinct values = 5

8. $a = 3, b = 0, m = 11, x_0 = 7$
   7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10
   distinct values = 5

9. $a = 3, b = 0, m = 11, x_0 = 8$
   8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2
   distinct values = 5

10. $a = 3, b = 0, m = 11, x_0 = 9$
    9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5 4 1 3 9 5
    distinct values = 5

11. $a = 3, b = 0, m = 11, x_0 = 10$
    10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8 2 6 7 10 8
    distinct values = 5

- When seed $x_0 = 0$, there is one repeating distinct value 0. For $x_0 = 1$ to 10, there are 5 distinct values from 1 to 10 appearing for each x_0 before repetition starts, so period length = 5.

- Inference :- The input with maximum period length will be better. The largest possible period length of linear congruence generator is $m – 1$. This value is achieved when $a = 6$ (full period), while period length for $a = 3$ is only 5, so $a = 6$ is preferred over $a = 3$. This is because more numbers in sequence → more randomness. Also, x_0 should not be 0.

Question – 2

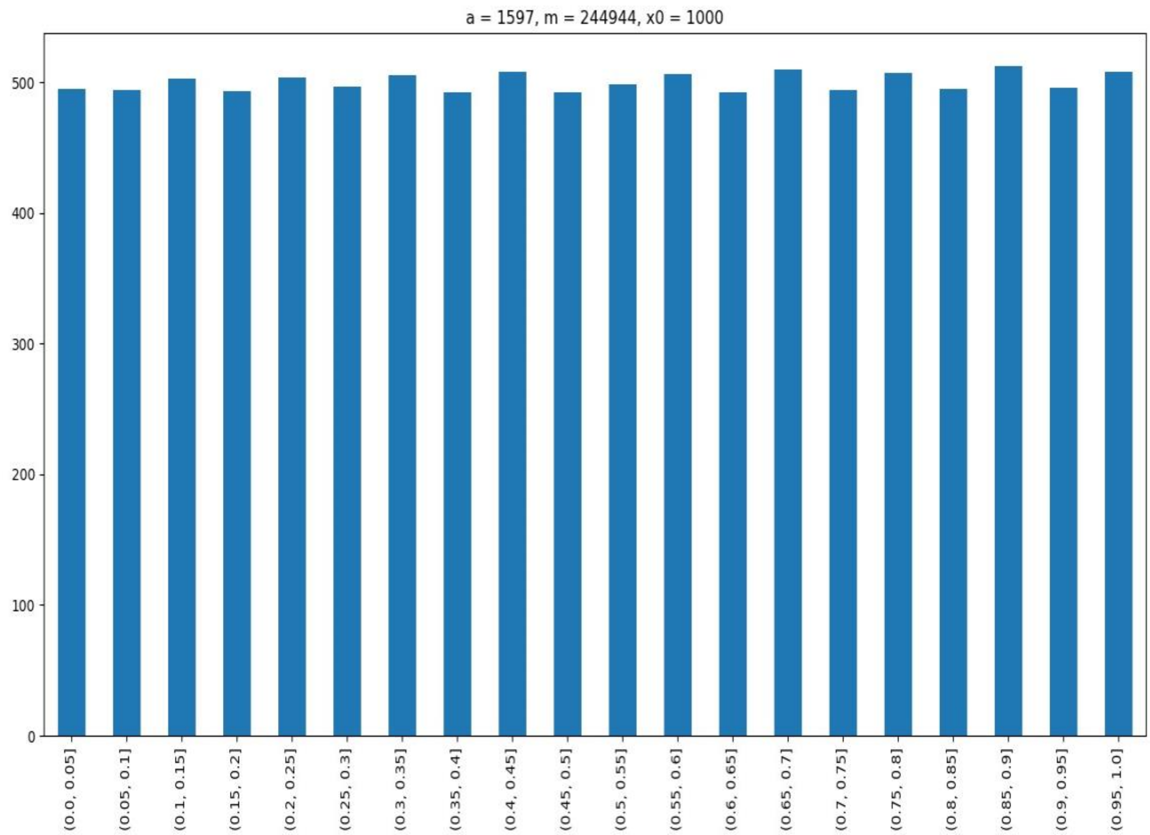
Tabulated Data :-

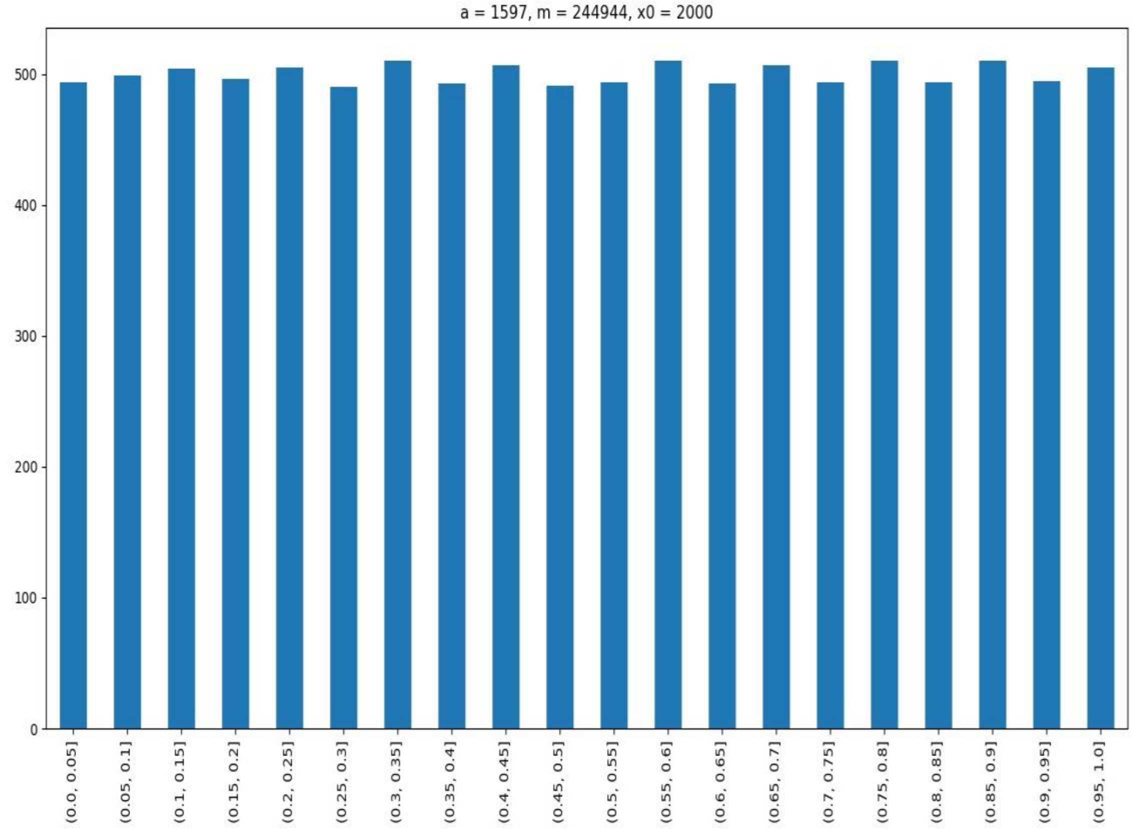
a = 1597, m = 244944, no. of samples = 10000					
	Seed				
	1000	2000	3000	4000	5000
(0.0, 0.05]	495	494	492	495	493
(0.05, 0.1]	494	499	495	493	506
(0.1, 0.15]	503	504	494	513	492
(0.15, 0.2]	493	496	492	496	496
(0.2, 0.25]	504	505	495	505	507
(0.25, 0.3]	497	490	495	492	495
(0.3, 0.35]	505	510	534	510	510
(0.35, 0.4]	492	493	493	493	496
(0.4, 0.45]	508	507	492	510	506
(0.45, 0.5]	492	491	493	489	498
(0.5, 0.55]	498	494	493	508	508
(0.55, 0.6]	506	510	497	492	491
(0.6, 0.65]	492	493	495	497	494
(0.65, 0.7]	510	507	535	507	512
(0.7, 0.75]	494	494	494	493	491
(0.75, 0.8]	507	510	494	510	509
(0.8, 0.85]	495	494	495	492	490
(0.85, 0.9]	512	510	494	510	506
(0.9, 0.95]	496	495	492	492	495
(0.95, 1.0]	508	505	537	504	506

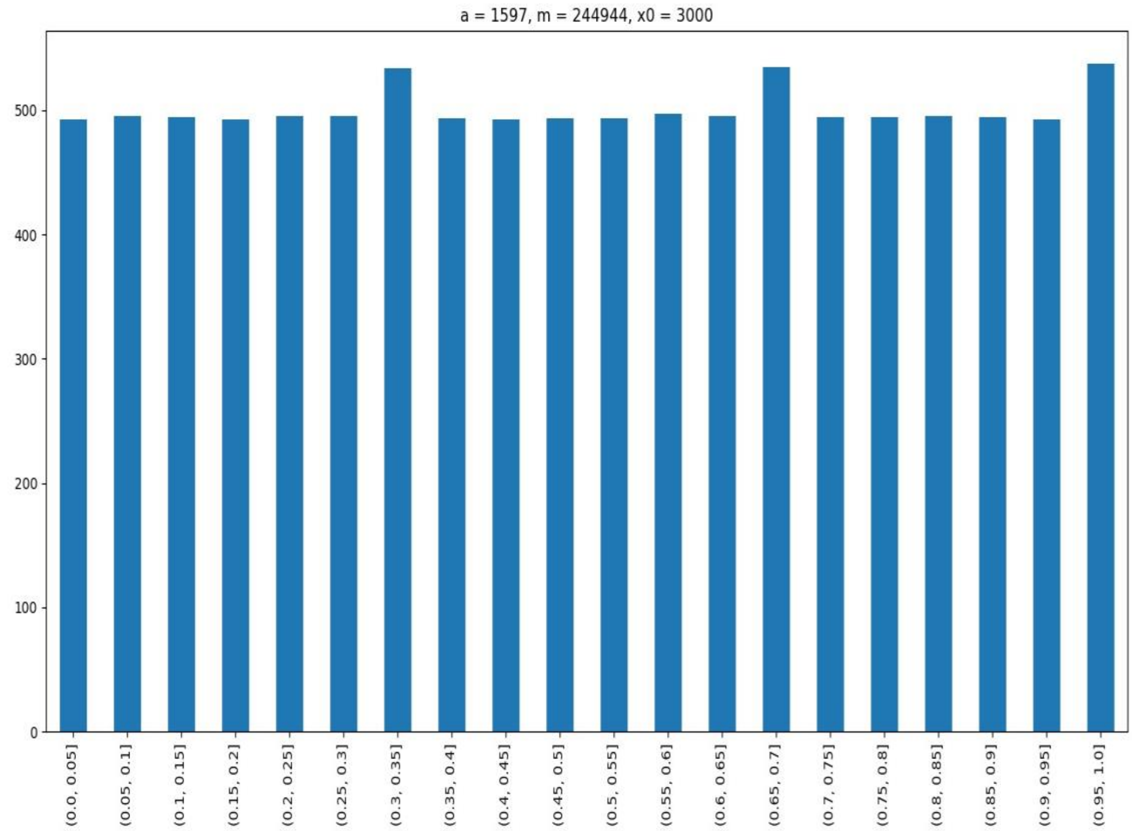
a = 51749, m = 244944, no. of samples = 10000					
	Seed				
	1000	2000	3000	4000	5000
(0.0, 0.05]	494	500	495	498	498
(0.05, 0.1]	493	489	554	499	499
(0.1, 0.15]	509	492	430	491	491
(0.15, 0.2]	493	492	555	489	490
(0.2, 0.25]	490	513	495	515	514
(0.25, 0.3]	493	512	432	496	495
(0.3, 0.35]	520	491	557	495	494
(0.35, 0.4]	514	511	495	493	493
(0.4, 0.45]	497	494	494	515	514
(0.45, 0.5]	495	492	496	512	512
(0.5, 0.55]	495	497	493	515	516
(0.55, 0.6]	491	491	493	515	517
(0.6, 0.65]	516	519	492	493	492
(0.65, 0.7]	509	496	553	493	495
(0.7, 0.75]	496	514	432	494	493
(0.75, 0.8]	495	518	495	512	512
(0.8, 0.85]	495	495	557	497	497
(0.85, 0.9]	518	495	431	495	495
(0.9, 0.95]	493	498	557	490	490
(0.95, 1.0]	495	492	495	494	494

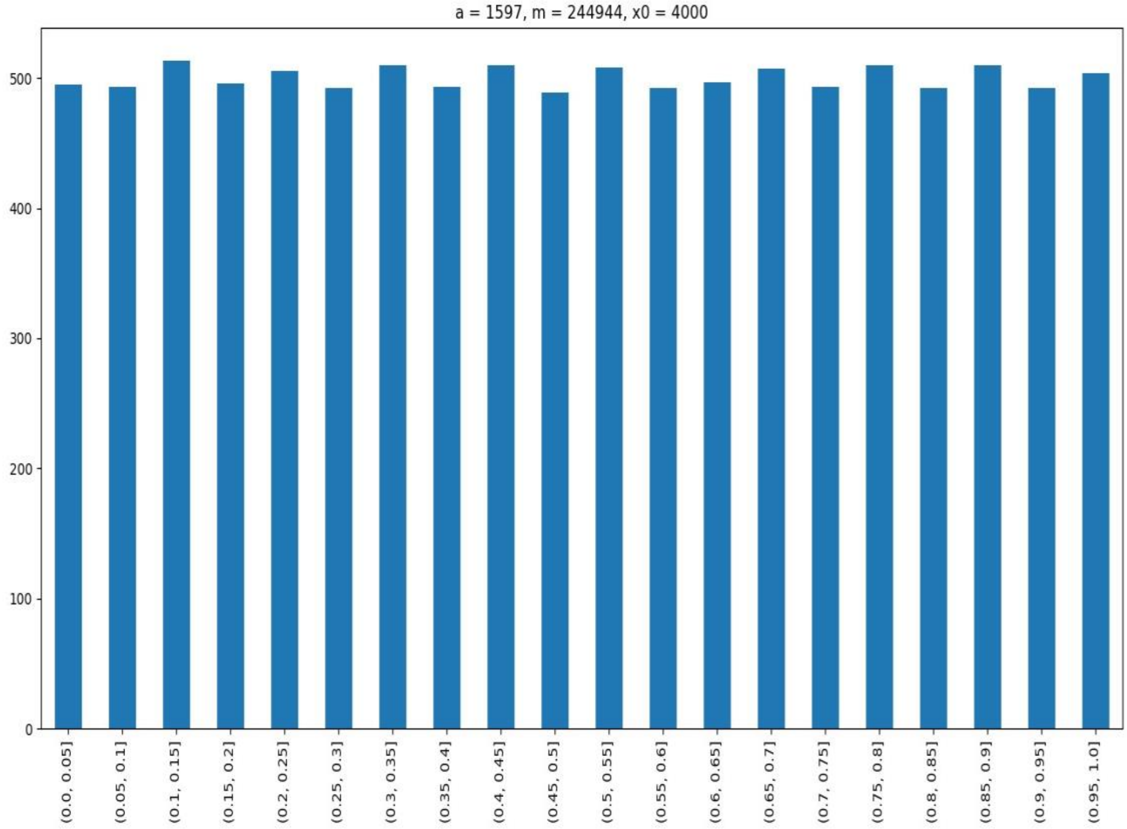
Graphs :-

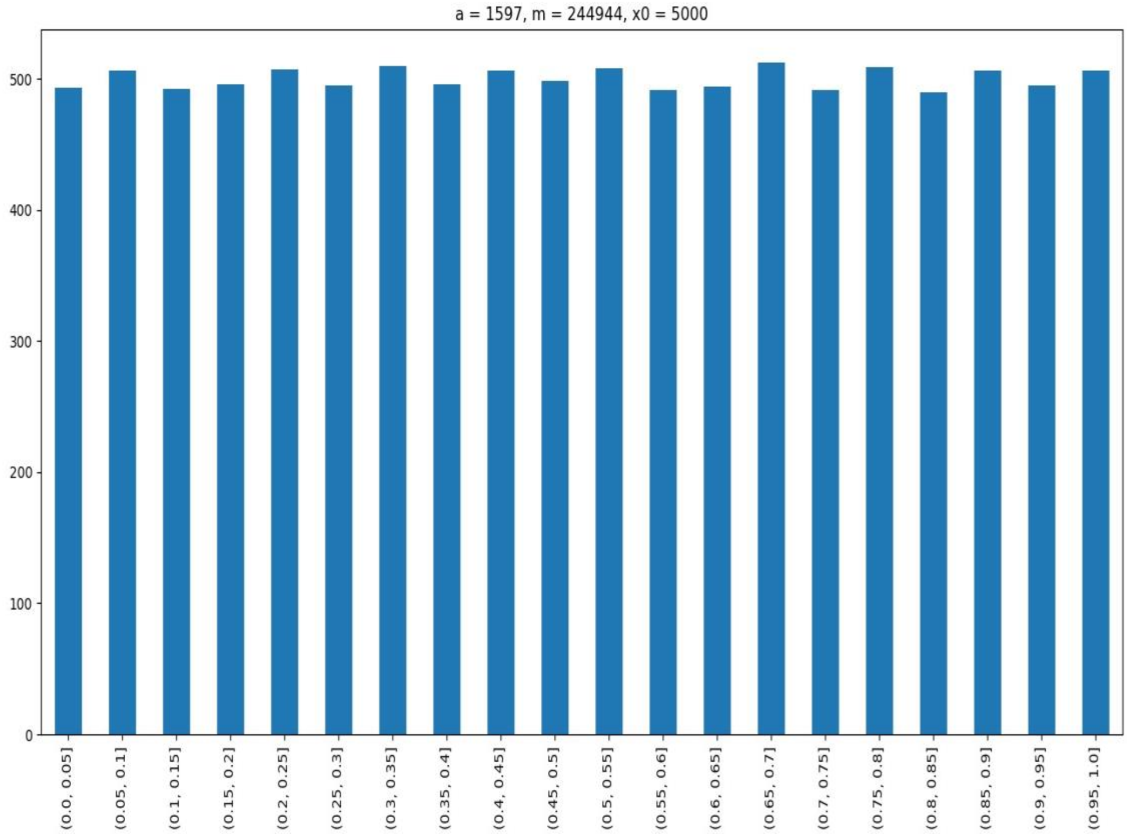
1. a = 1597, m = 244944



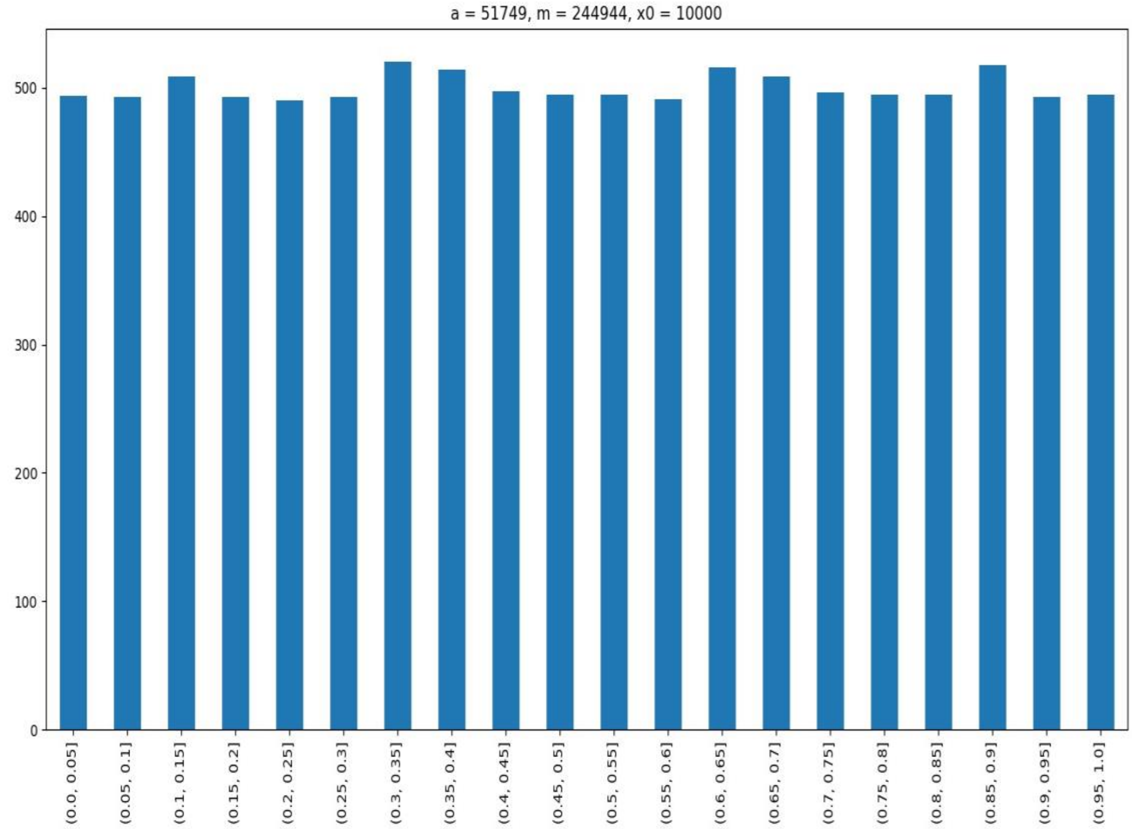

a = 1597, m = 244944, x0 = 2000
0
100
200
300
400
500
(0.0, 0.05]
(0.05, 0.1]
(0.1, 0.15]
(0.15, 0.2]
(0.2, 0.25]
(0.25, 0.3]
(0.3, 0.35]
(0.35, 0.4]
(0.4, 0.45]
(0.45, 0.5]
(0.5, 0.55]
(0.55, 0.6]
(0.6, 0.65]
(0.65, 0.7]
(0.7, 0.75]
(0.75, 0.8]
(0.8, 0.85]
(0.85, 0.9]
(0.9, 0.95]
(0.95, 1.0]

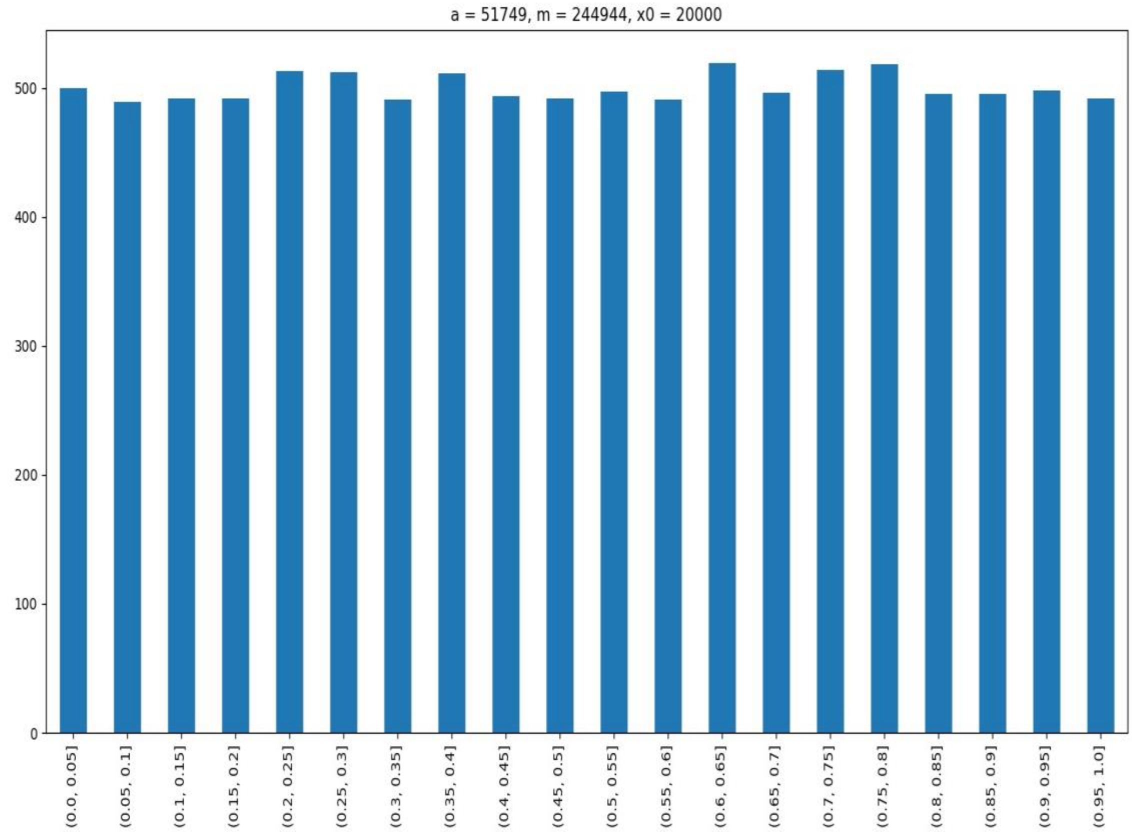

a = 1597, m = 244944, x0 = 3000
0
100
200
300
400
500
(0.0, 0.05]
(0.05, 0.1]
(0.1, 0.15]
(0.15, 0.2]
(0.2, 0.25]
(0.25, 0.3]
(0.3, 0.35]
(0.35, 0.4]
(0.4, 0.45]
(0.45, 0.5]
(0.5, 0.55]
(0.55, 0.6]
(0.6, 0.65]
(0.65, 0.7]
(0.7, 0.75]
(0.75, 0.8]
(0.8, 0.85]
(0.85, 0.9]
(0.9, 0.95]
(0.95, 1.0]

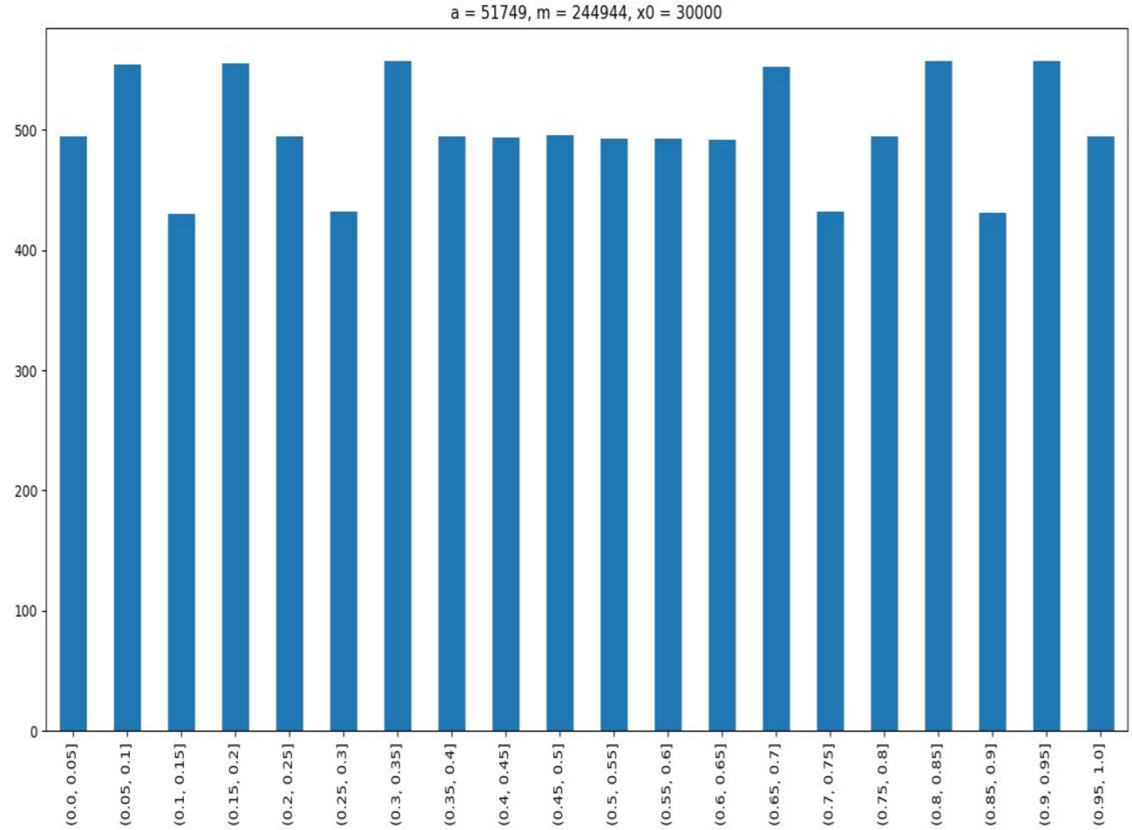

a = 1597, m = 244944, x0 = 4000
0
100
200
300
400
500
(0.0, 0.05]
(0.05, 0.1]
(0.1, 0.15]
(0.15, 0.2]
(0.2, 0.25]
(0.25, 0.3]
(0.3, 0.35]
(0.35, 0.4]
(0.4, 0.45]
(0.45, 0.5]
(0.5, 0.55]
(0.55, 0.6]
(0.6, 0.65]
(0.65, 0.7]
(0.7, 0.75]
(0.75, 0.8]
(0.8, 0.85]
(0.85, 0.9]
(0.9, 0.95]
(0.95, 1.0]

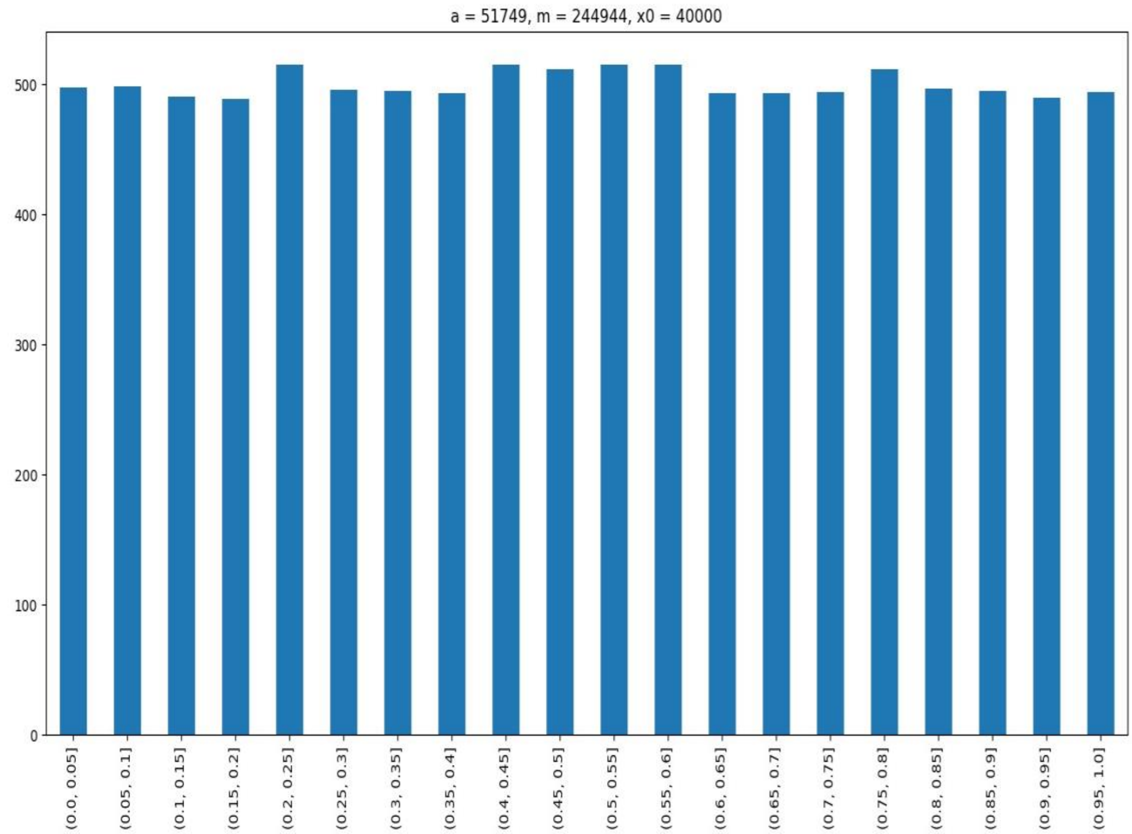

a = 1597, m = 244944, x0 = 5000
0
100
200
300
400
500
(0.0, 0.05]
(0.05, 0.1]
(0.1, 0.15]
(0.15, 0.2]
(0.2, 0.25]
(0.25, 0.3]
(0.3, 0.35]
(0.35, 0.4]
(0.4, 0.45]
(0.45, 0.5]
(0.5, 0.55]
(0.55, 0.6]
(0.6, 0.65]
(0.65, 0.7]
(0.7, 0.75]
(0.75, 0.8]
(0.8, 0.85]
(0.85, 0.9]
(0.9, 0.95]
(0.95, 1.0]

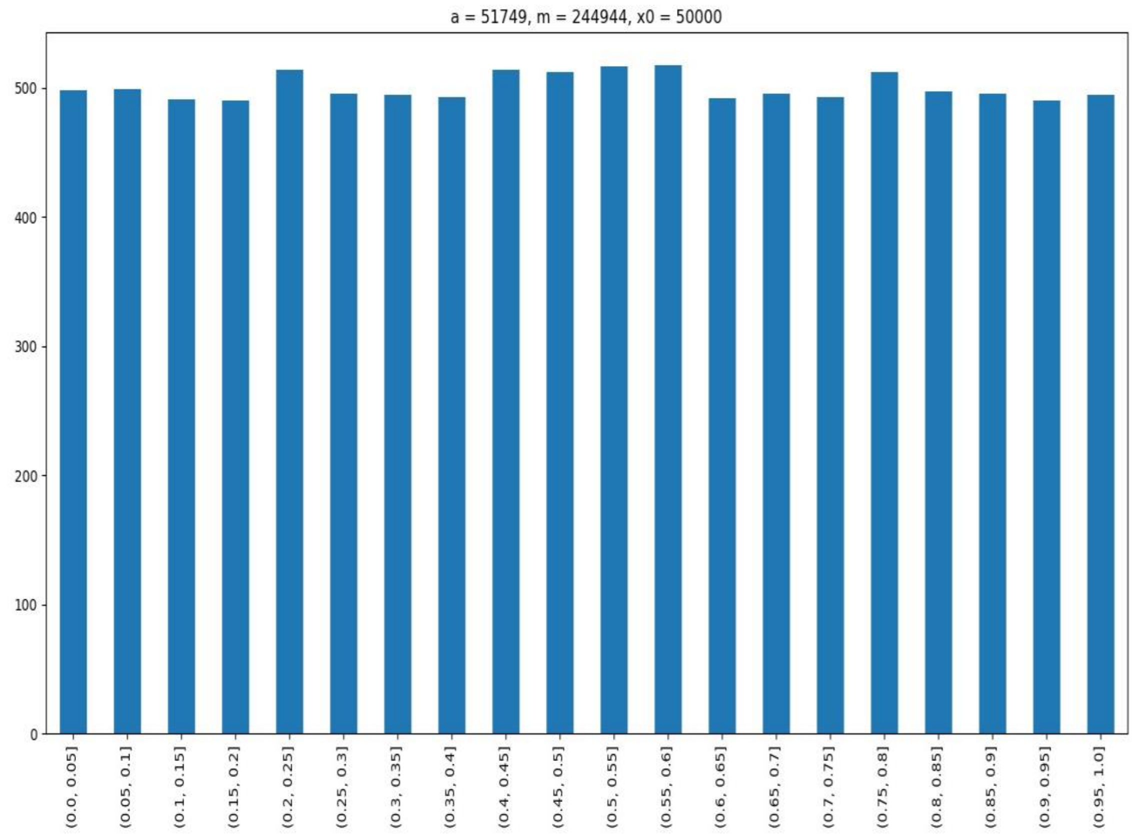
2. a = 51749, m = 244944






a = 51749, m = 244944, x0 = 30000
0
100
200
300
400
500
(0.0, 0.05]
(0.05, 0.1]
(0.1, 0.15]
(0.15, 0.2]
(0.2, 0.25]
(0.25, 0.3]
(0.3, 0.35]
(0.35, 0.4]
(0.4, 0.45]
(0.45, 0.5]
(0.5, 0.55]
(0.55, 0.6]
(0.6, 0.65]
(0.65, 0.7]
(0.7, 0.75]
(0.75, 0.8]
(0.8, 0.85]
(0.85, 0.9]
(0.9, 0.95]
(0.95, 1.0]


a = 51749, m = 244944, x0 = 40000
0
100
200
300
400
500
(0.0, 0.05]
(0.05, 0.1]
(0.1, 0.15]
(0.15, 0.2]
(0.2, 0.25]
(0.25, 0.3]
(0.3, 0.35]
(0.35, 0.4]
(0.4, 0.45]
(0.45, 0.5]
(0.5, 0.55]
(0.55, 0.6]
(0.6, 0.65]
(0.65, 0.7]
(0.7, 0.75]
(0.75, 0.8]
(0.8, 0.85]
(0.85, 0.9]
(0.9, 0.95]
(0.95, 1.0]

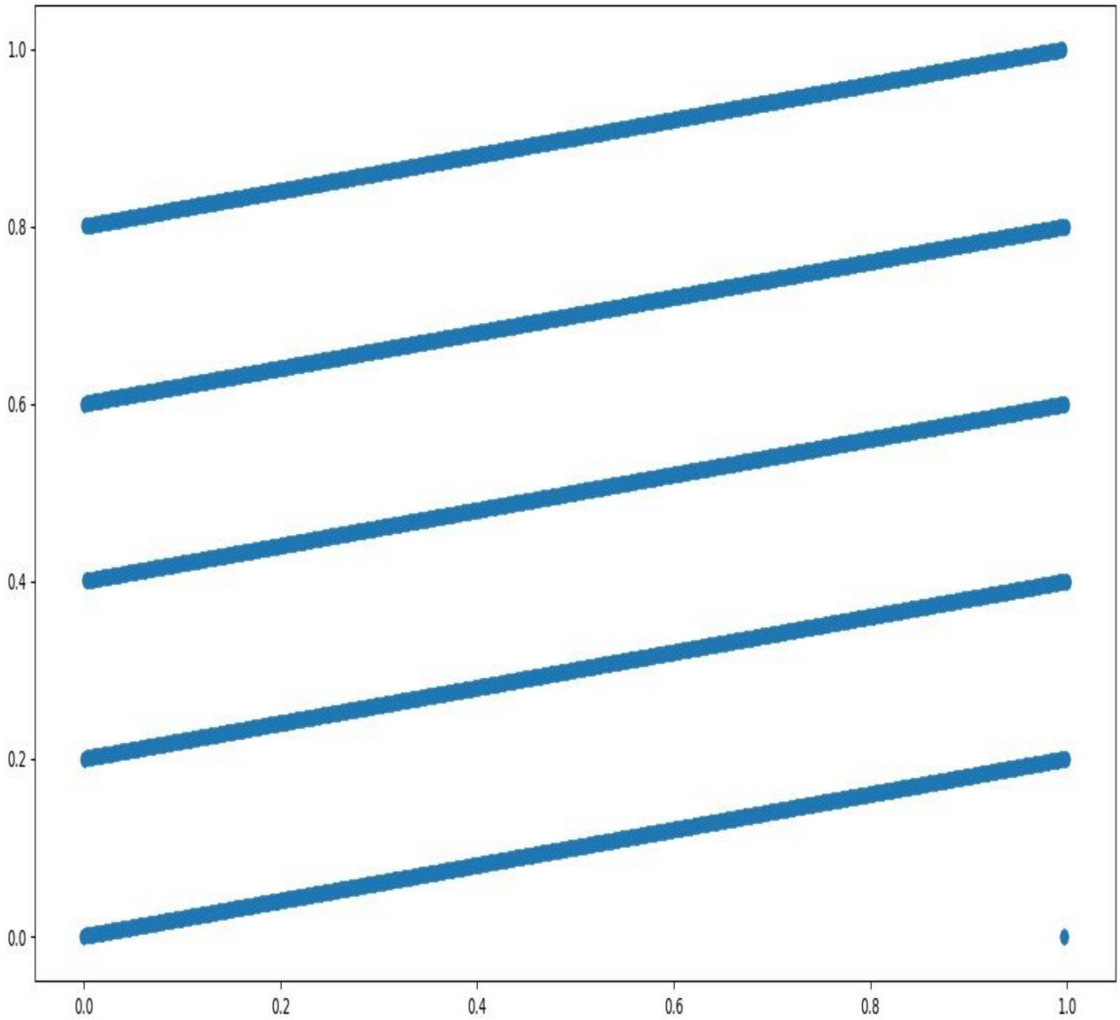


- <u>Observation</u> :-

We observe that the distribution is almost the same for every range which indicates that we have formed a uniform distribution in (0,1) i.e. we have generated random numbers X ~ U(0,1).

Question – 3

The graph of points (u_1, u_2), (u_2, u_3), …… (u_{i-1}, u_i) for a = 1229, b = 1, m = 2048 can be drawn as follows :-



- <u>Observation</u> :-

  The scatter plot contains 5 almost parallel lines originating at different y – coordinates and there is an outlier at approx. x = 1.0.