Lab - 03

# MA323 – Monte-Carlo Simulation

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## Question - 1

Here, $f(x) = 20x(1-x)^3$ for all $0 < x < 1$.

I am choosing $g(x) = 1$ for all $0 < x < 1$, so the smallest value of c such that $f(x) <= cg(x)$ is: - 2.109375

a) The probability of acceptance of a generated sample is ratio of area under f(x) to the area under cg(x) which is p = 1/c. Now the expected number of iterations needed to get an accepted sample will be 1/p which is c. So, the average number of iterations required to generate a random number is c = 2.109375

b) The Expectation of PDF f(x) is E[f(x)] = 1/3 = 0.33333…

   For c = 2.109375, The sample mean = 0.33589041733507474

   For c = 6, The sample mean = 0.33092525374904125

   For c = 9, The sample mean = 0.33527137939004537

   We observe that the sample mean for c = 2.109375 and two values greater than this smallest value are approximately equal to the Expectation of PDF f(x).

c) The exact value of P(0.25 <= X <= 0.75) = 0.617188

   For c = 2.109375, the approximate value of P(0.25 <= X <= 0.75) = 0.6020748382880654

   For c = 6, the approximate value of P(0.25 <= X <= 0.75) = 0.5935764466123804

For c = 9, the approximate value of P(0.25 <= X <= 0.75) = 0.6016487373568857

On comparing these 3 values with the exact probability, we see that all 3 values are close to the exact value which shows that our generated sample is close to the PDF f for all the three values of c.

d) For a good upper bound, the average number of iterations should be around the value obtained should be close to the value obtained in (a) i.e., 2.109375.

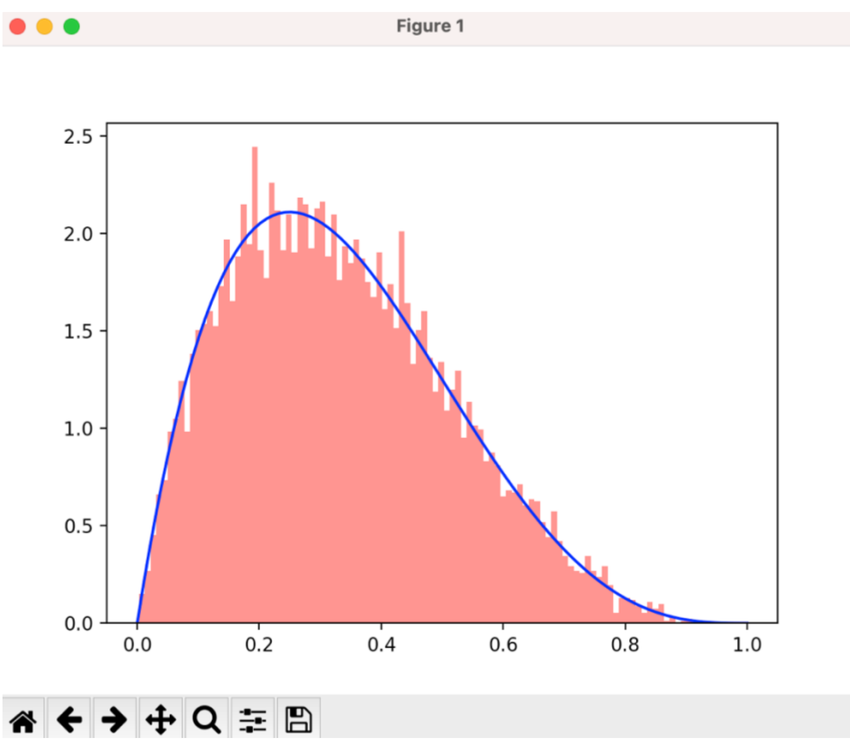
For c = 2.109375, the average number of iterations needed = 2.0986

For c = 6, the average number of iterations needed = 5.9941

For c = 9, the average number of iterations needed = 8.866

We observe that, the tighter the upper bound is, the closer is the value. Hence, the more we deviate from the smallest value of c, the more is the number of iterations we will require to generate each of the random number.

e) The histogram of the sample generated and the PDF f can be generated as: -



On comparing the histogram and the actual PDF f, we observe that the distribution of the generated sample is almost close to the actual PDF f.

# Question 2

We take f(x) as Gamma distribution with $\beta$(scale parameter) = 1 and $\alpha$ (shape parameter) = 0.5.

So, $f(x) = (x^{\alpha-1}e^{-x}) / \Gamma(\alpha)$.

Since $0 < \alpha < 1$, we take the **dominating function g(x)** = $x^{\alpha-1}/A$ **for 0 < x < 1** and **g(x)** = $e^{-x}/A$ **for x >= 1**, where $A = 1/\alpha + 1/e$. By this, we get that $f(x) <= cg(x)$ for all $x > 0$ and $c = A/\Gamma(\alpha)$.

So, the **rejection constant** = c which is stated above.

- ⇨ For alpha = 0.5, the calculated rejection constant is 1.33593291580581.

On generating 10000 random numbers from the above PDF f(x), we get the distribution of generated sample as follows:-