MA323 – Monte Carlo Simulation

Lab - 11

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Question 1

Generate 5000 random numbers from the following Gaussian mixture PDF:

$$f(x) = \sum_{k=1}^{3} \pi_i \frac{1}{\sigma_i} \phi\left(\frac{x-\mu_i}{\sigma_i}\right), \quad x \in \mathbb{R},$$

where ϕ is the PDF of the standard normal distribution, $(\pi_1, \pi_2, \pi_3) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$, $(\mu_1, \mu_2, \mu_3) = (-1, 0, 1)$, and $(\sigma_1, \sigma_2, \sigma_3) = \left(\frac{1}{4}, 1, \frac{1}{2}\right)$. Find the average of the generated random numbers.

The mean of the generated distribution is -0.35007415641124845.

The variance of the generated distribution is 0.9469358928593865.

Question 2

Generate 10 sample paths for the standard Brownian Motion in the time interval $[0, 5]$ using the recursion

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i}\, Z_{i+1},$$

with 5000 generated values for each of the paths. Plot all the sample paths in a single figure. Also estimate $E[W(2)]$ and $E[W(5)]$ from the 10 paths that you have generated.

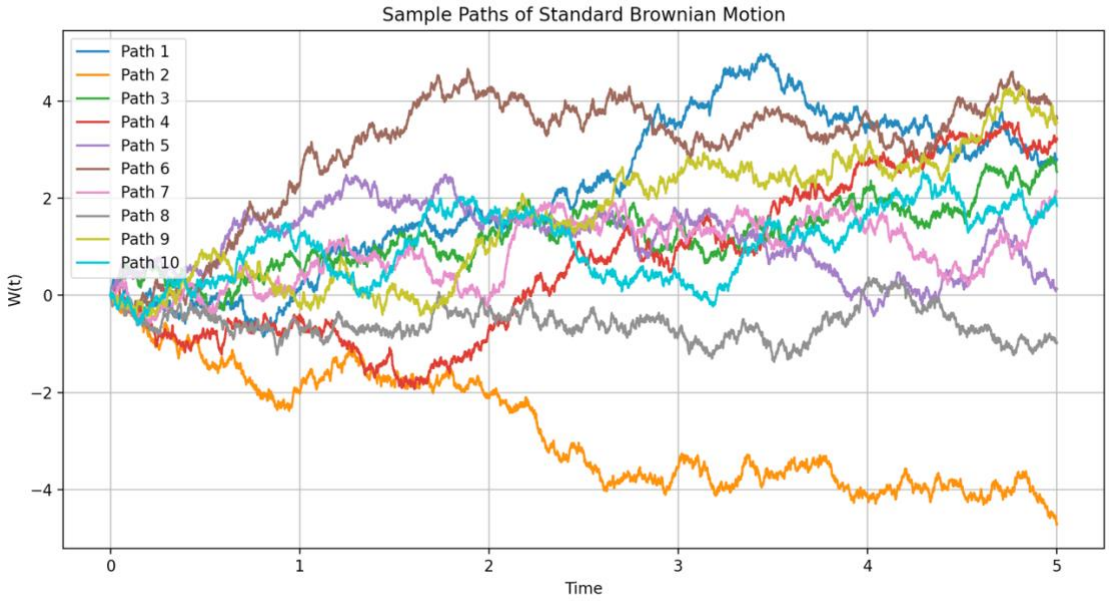
Estimated E[W(2)] = 0.8064362016488988

Estimated E[W(5)] = 1.4205472675561253

Estimated Var(W(2)) = 2.615509037500491

Estimated Var(W(5)) = 6.157998670465844

The plot we get by generating the sample paths is as follows: -



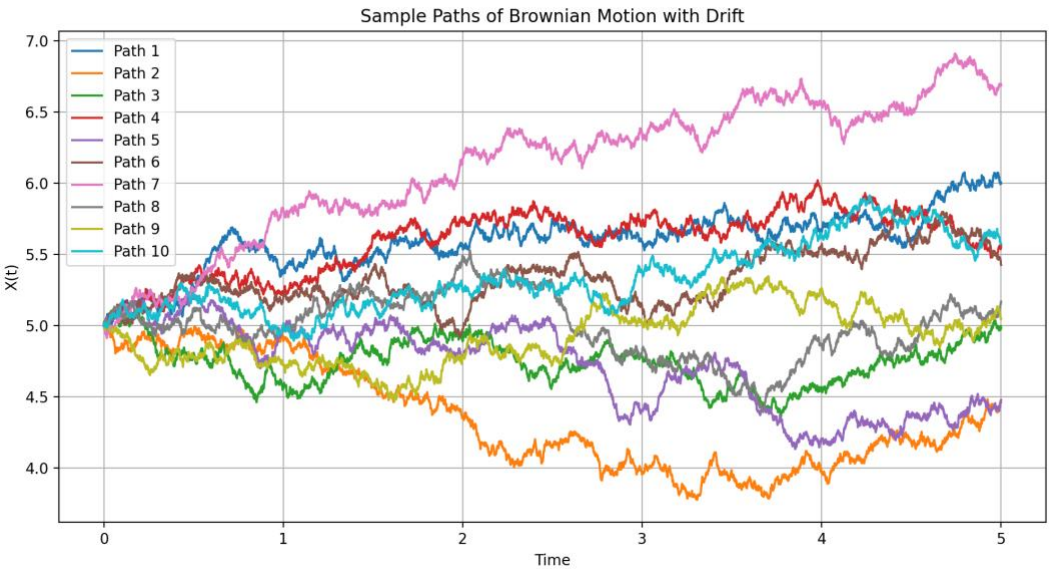
Question 3

Repeat the above exercise with the following Brownian motion ($BM(\mu, \sigma^2)$) discretization

$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}\, Z_{i+1}.$$

Take $X(0) = 5$, $\mu = 0.06$ and $\sigma = 0.3$.

The plot we get by generating the sample paths with drift is as follows: -



Estimated E[X(2)] = 5.196266468365356

Estimated E[X(5)] = 5.338483120590275