MA374 – Financial Engineering Laboratory

Lab - 03

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Ques - 1

The initial option prices for American Call Option and American Put Option are: -

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American Call Option Price = 15.736778626185817
American Put Option Price = 8.923113287677742
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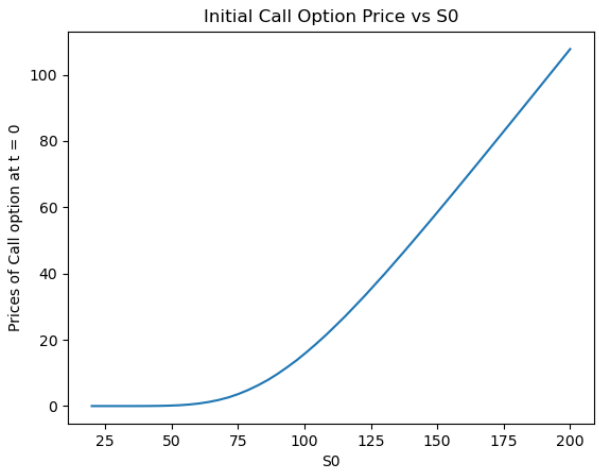
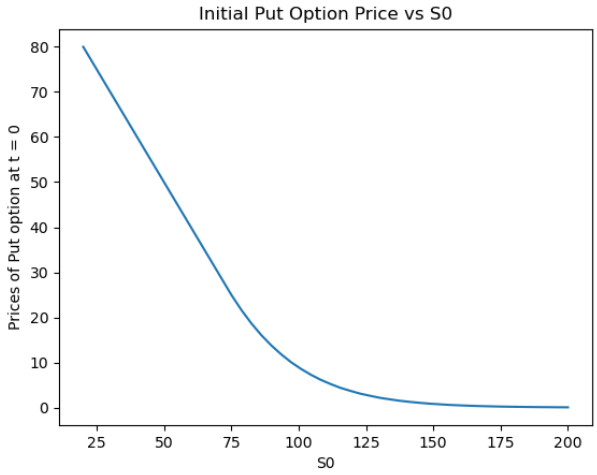
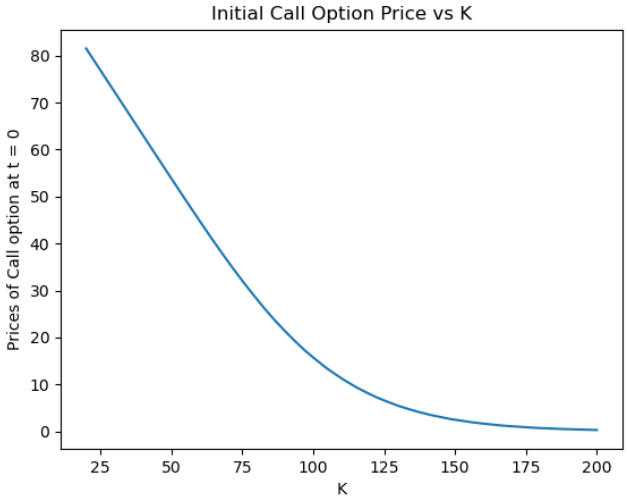
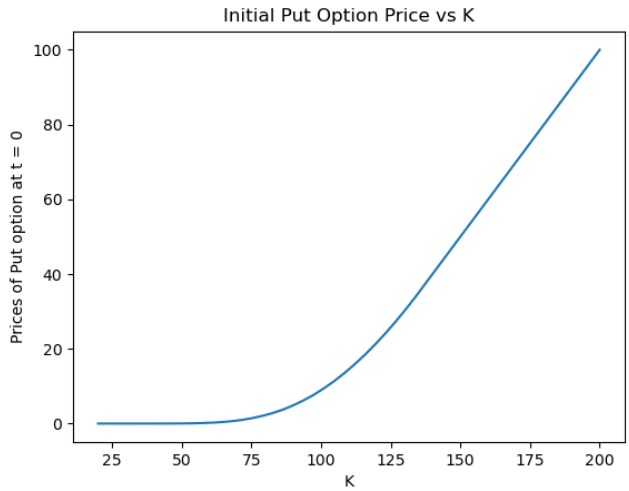
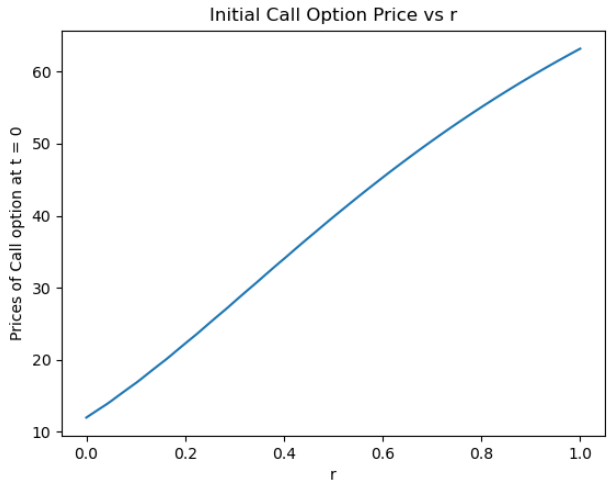
Sensitivity Analysis

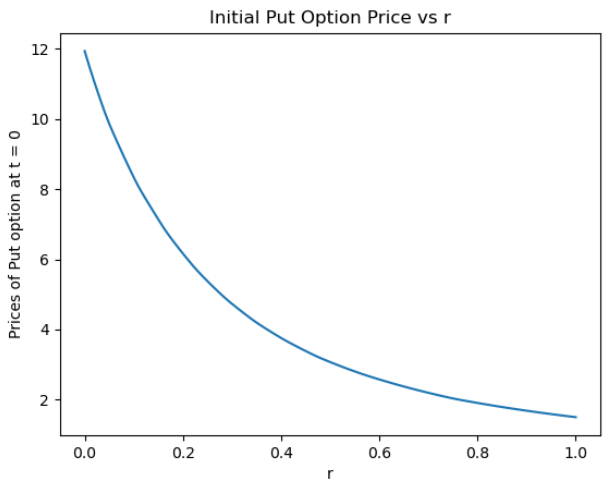


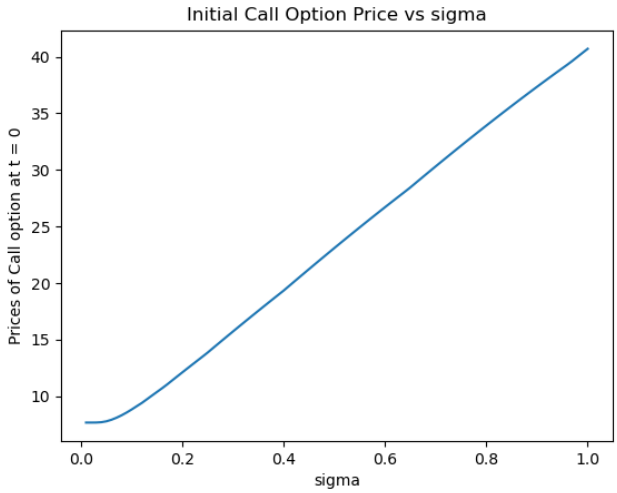




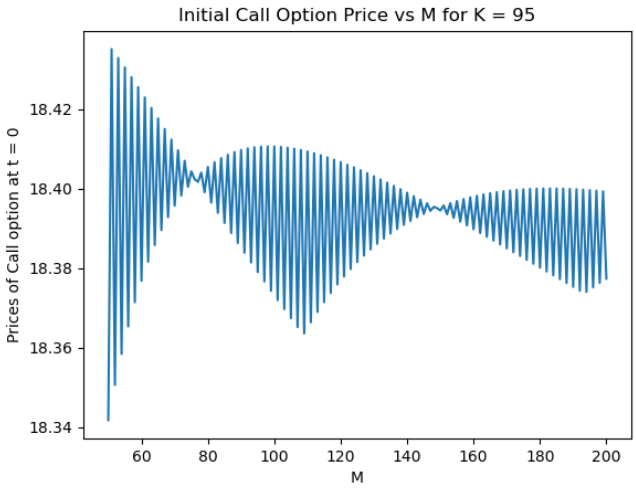


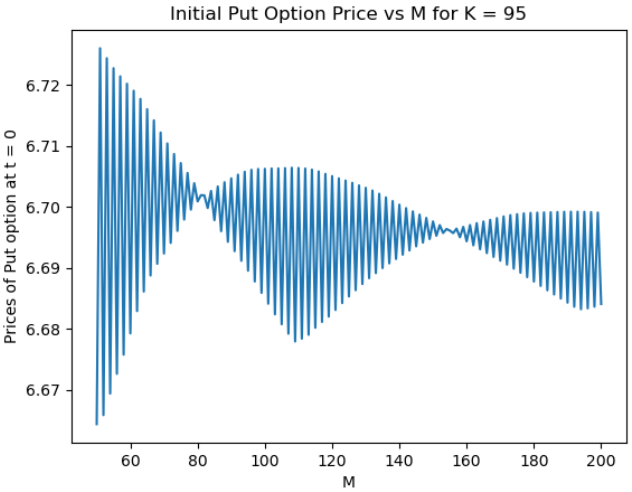

Initial Call Option Price vs r
Prices of Call option at t = 0
r

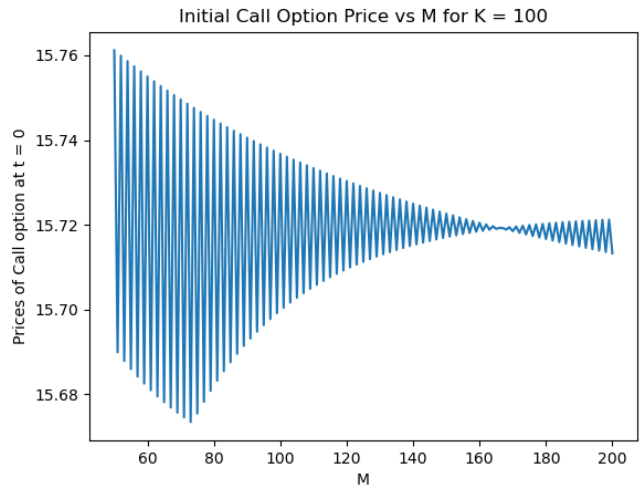

Initial Put Option Price vs r
Prices of Put option at t = 0
r

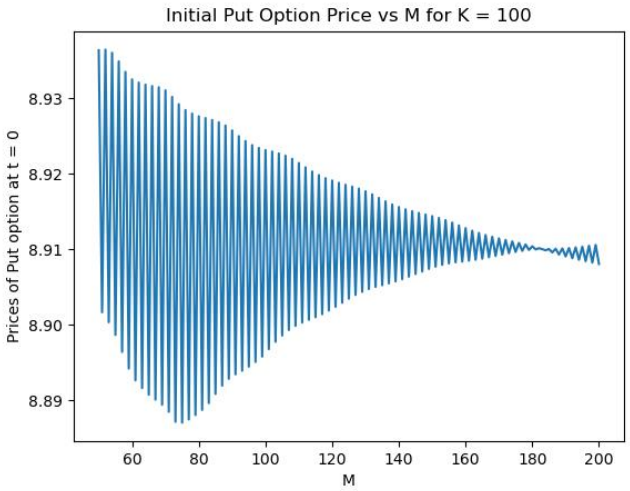

Initial Call Option Price vs sigma
Prices of Call option at t = 0
sigma

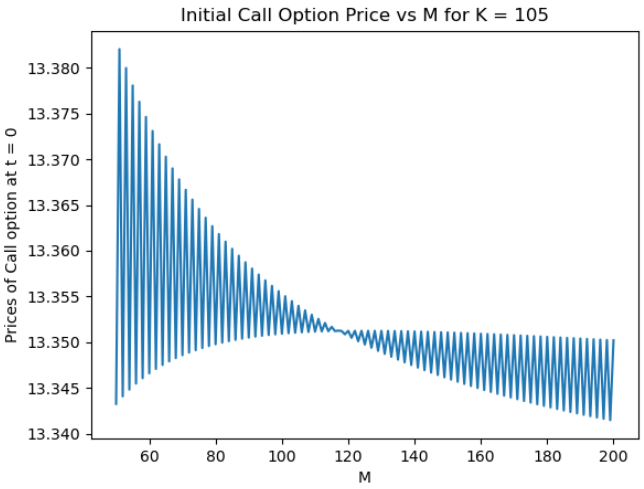

Initial Put Option Price vs sigma
Prices of Put option at t = 0
sigma

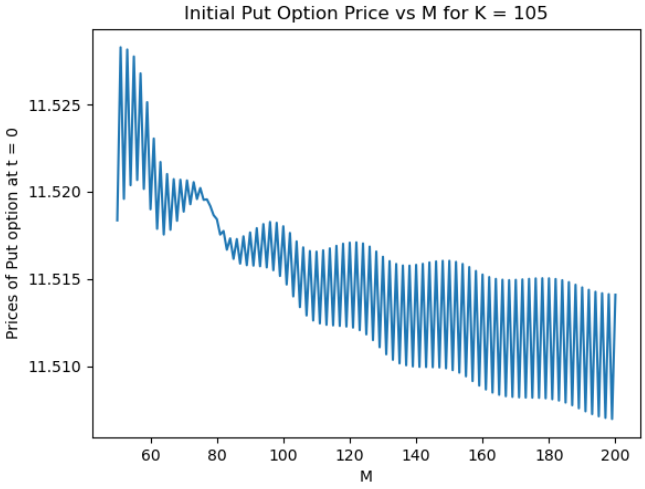

Initial Call Option Price vs M for K = 95
Prices of Call option at t = 0
M


Initial Put Option Price vs M for K = 95
Prices of Put option at t = 0
M









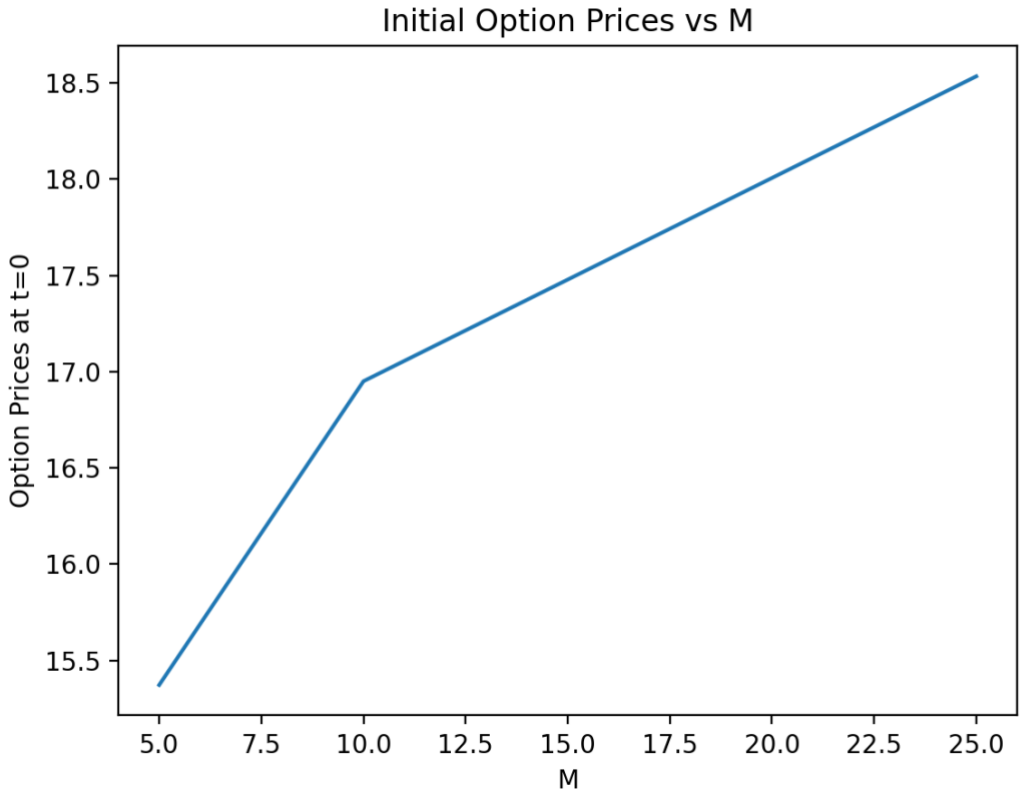
Ques – 2

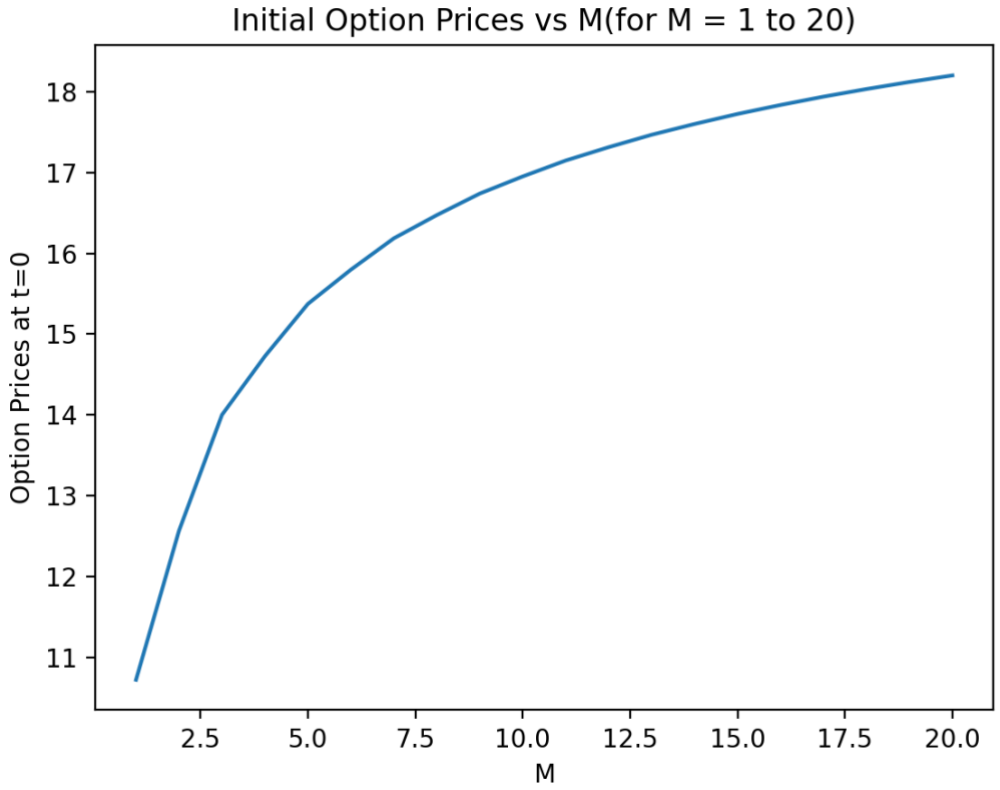
(a) The initial option prices for Lookback Option are: -

M	Initial Option Price	Execution Time
5	15.372952215663778	0.0005900859832763672 sec
10	16.95034049177767	0.006894111633300781 sec
25	18.533781500094165	217.85930705070496 sec
50	Not Feasible	Not Feasible

For M = 50, the algorithm is unable to calculate the option price since it has exponential time complexity. So, computation for M = 50 is not feasible with this algorithm for finding out the loopback option.

(b) The plots comparing the above values are as follows:





As the value of M increases, the initial option price increases, and it seems that the prices tend to converge as M is increased further.

(c) The values of the options at all intermediate time points for M = 5:
(<u>Note:</u> 't' shows time intervals wrt M)

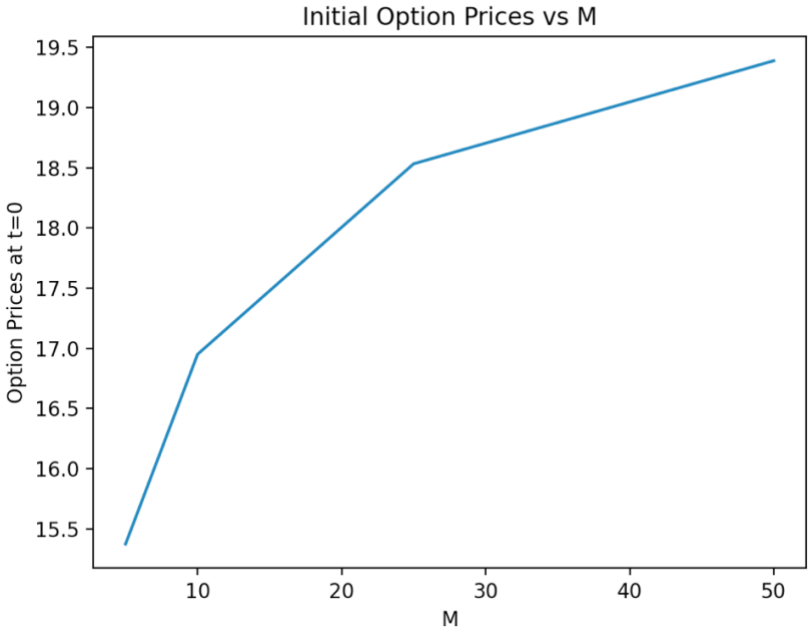
	t=0	t=1	t=2	t=3	t=4	t=5
1.	15.3729522	15.5321315	15.1997501	13.3861693	10.3324806	0.0
2.	x	15.7096998	16.3657735	17.5044647	16.8729784	21.0024917
3.	x	x	11.6225925	10.2358255	7.9008017	0.0
4.	x	x	20.3053101	23.0262154	27.6767603	34.2971452
5.	x	x	x	10.2358255	7.9008017	0.0
6.	x	x	x	13.3849082	12.9020379	16.0596983
7.	x	x	x	12.7023232	12.1032854	14.1899412
8.	x	x	x	28.5664894	34.6964628	42.0619748
9.	x	x	x	x	7.9008017	0.0
10.	x	x	x	x	12.9020379	16.0596983
11.	x	x	x	x	6.0414018	0.0
12.	x	x	x	x	21.1632233	26.2255457
13.	x	x	x	x	6.0414018	0.0
14.	x	x	x	x	19.7757554	24.6019481
15.	x	x	x	x	19.7757554	24.6019481
16.	x	x	x	x	38.2824322	45.9144885
17.	x	x	x	x	x	0.0
18.	x	x	x	x	x	16.0596983
19.	x	x	x	x	x	0.0
20.	x	x	x	x	x	26.2255457
21.	x	x	x	x	x	0.0
22.	x	x	x	x	x	12.2801577
23.	x	x	x	x	x	10.8504352
24.	x	x	x	x	x	32.1629756
25.	x	x	x	x	x	0.0
26.	x	x	x	x	x	12.2801577
27.	x	x	x	x	x	9.4405893
28.	x	x	x	x	x	30.7531297
29.	x	x	x	x	x	9.4405893
30.	x	x	x	x	x	30.7531297
31.	x	x	x	x	x	30.7531297
32.	x	x	x	x	x	47.0499089

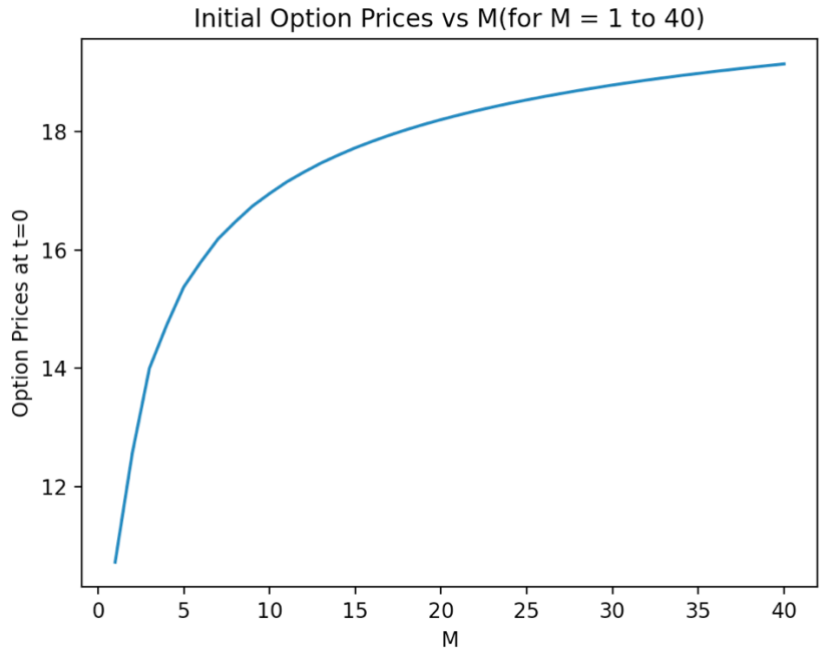
Ques - 3

Using Markov property, the efficient algorithm gives option prices for lookback option as follows: -

M	Initial Option Price	Execution Time
5	15.372952215663778	0.0002429485321044922 sec
10	16.95034049177767	0.0016510486602783203 sec
25	18.533781500094165	0.11969828605651855 sec
50	19.390465235522452	7.935082197189331 sec

The plots comparing the above values are as follows: -





The values of the options at all intermediate time points for M = 5: -

	t=0	t=1	t=2	t=3	t=4	t=5
1.	15.3729522	15.5321315	15.1997501	13.3861693	10.3324806	0.0
2.	x	15.7096998	16.3657735	17.5044647	16.8729784	21.0024917
3.	x	x	11.6225925	10.2358255	7.9008017	0.0
4.	x	x	20.3053101	23.0262154	27.6767603	34.2971452
5.	x	x	x	10.2358255	7.9008017	16.0596983
6.	x	x	x	13.3849082	12.9020379	14.1899412
7.	x	x	x	12.7023232	12.1032854	42.0619748
8.	x	x	x	28.5664894	34.6964628	0.0
9.	x	x	x	x	12.9020379	16.0596983
10.	x	x	x	x	6.0414018	0.0
11.	x	x	x	x	21.1632233	26.2255457
12.	x	x	x	x	6.0414018	24.6019481
13.	x	x	x	x	19.7757554	24.6019481
14.	x	x	x	x	19.7757554	45.9144885
15.	x	x	x	x	38.2824322	26.2255457
16.	x	x	x	x	x	0.0
17.	x	x	x	x	x	12.2801577
18.	x	x	x	x	x	10.8504352
19.	x	x	x	x	x	32.1629756
20.	x	x	x	x	x	12.2801577
21.	x	x	x	x	x	9.4405893
22.	x	x	x	x	x	30.7531297
23.	x	x	x	x	x	9.4405893
24.	x	x	x	x	x	30.7531297
25.	x	x	x	x	x	47.0499089

Comparative Analysis: -

1. Comparing Execution times.

M	Basic	Markov Based
5	0.0005900859832763672 sec	0.0002429485321044922 sec
10	0.006894111633300781 sec	0.0016510486602783203 sec
25	217.85930705070496 sec	0.11969828605651855 sec
50	Not Feasible	7.935082197189331 sec

2. The highest value of M that can be handled by algorithm: -

   a. Basic: - Around 25 to 30.
   b. Markov Based: - Around 85-87.

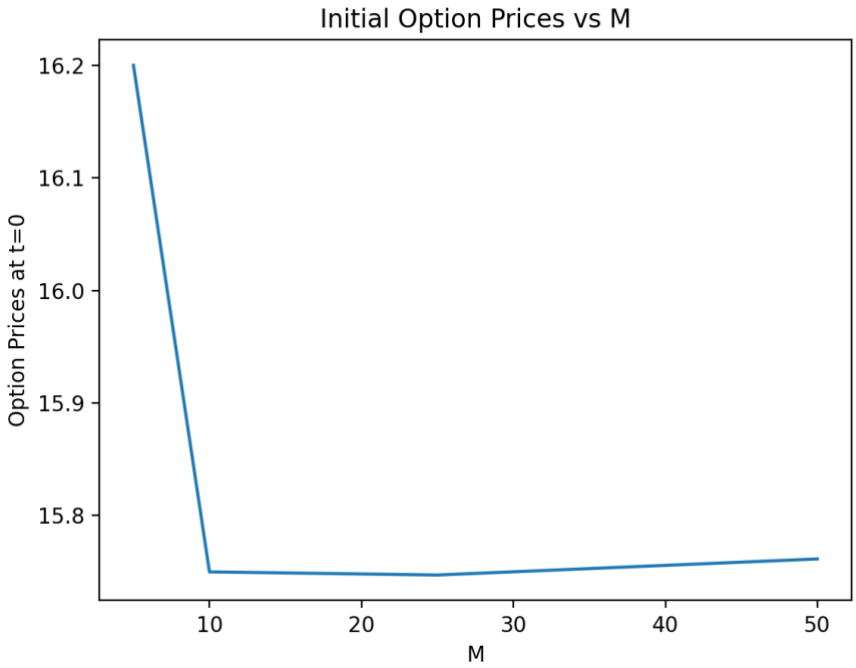
3. The basic algorithm has exponential time complexity, while Markov based doesn't have exponential time complexity as it is implemented using memorization technique in dynamic programming.

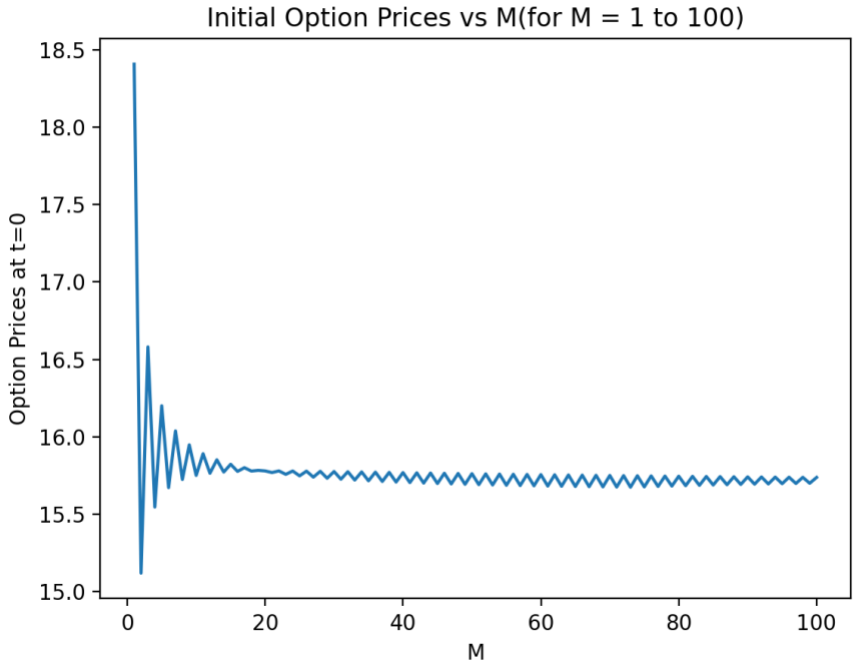
Ques - 4

(a) The initial option prices for European Call Option are: -

M	Initial Option Price
5	16.200135785709463
10	15.749706920472503
25	15.746918255600457
50	15.761196879829438

(b) The plots comparing above values are: -





The option price is converging to roughly 15.73 as M increases.

(c) The values of the option at all intermediate time points for M = 5 is as follows: -

	t=0	**t=1**	**t=2**	**t=3**	**t=4**	**t=5**
1.	16.200135785	25.375255893	38.432095157	55.877931391	7.47158700	102.55077163
2.	x	7.543996674	13.131857964	22.219195424	36.078410687	54.881827348
3.	x	x	2.1972816917	4.464542360	9.071271363	18.431444369
4.	x	x	x	0.0	0.0	0
5.	x	x	x	x	0.0	0
6.	x	x	x	x	x	0

Comparative Analysis: -

1. Comparing Execution times.

M	**Unoptimised**	**Better**	**Best**
5	0.000440120697021 sec	5.602836608886e-05 sec	5.316734313964e-05 sec
10	0.004103183746337 sec	9.393692016601e-05 sec	6.86645507812e-05 sec
25	101.93571686 sec	0.0003497600555419 sec	0.0001471042633056 sec
50	Not Feasible	0.001171112060546 sec	0.000314950942993 sec

2. The highest value of M that can be handled by algorithm: -

   a. Unoptimized: - Around 25 to 30.
   b. Better: - Around 2*10^4.
   c. Best: - Around 1000 (nCr overflows after 1000)

3. The unoptimized algorithm has exponential time and space complexity. The efficient algorithm has quadratic space and time complexity (in M), although space complexity can be decreased to linear. But since we also needed to print intermediate information, I implemented algorithm which has quadratic complexity. The most efficient algorithm has almost linear time complexity (after making use of memoization to calculate nCr), and linear space complexity.

4. The most efficient algorithm works on the same principle as that of efficient algorithm. The only difference lies in the fact that the most efficient algorithm summarises the computation of efficient algorithm to a formula. So, it is much easier to implement, and is faster than the efficient algorithm. But the upper limit of M is less due to computational issues like integer overflow and stack overflow.