Subject Name: Soft Computing **Subject Code: IT 701**

Fuzzy Set: Introduction, Basic Definition and Terminology, Properties and Set-theoretic Operations, Fuzzy Relations, Membership Functions and their assignment, Fuzzy rules and fuzzy Reasoning, Fuzzy if-then Rules, Fuzzy Inference Systems. Application of Fuzzy logic in solving engineering problems.

Course Objectives

1) The objective of this course is to understand terminology and operations in Fuzzy set.
2) To understand application of fuzzy logic to solve engineering problem also fuzzy rule and reasoning.

Fuzzy Set: Introduction

The word "fuzzy" means "vagueness". Fuzziness occurs when the boundary of a piece of information is not clear-cut. Fuzzy sets have been introduced by Lotfi A. Zadeh (1965) as an extension of the classical notion of set.

Human thinking and reasoning frequently involve fuzzy information originating from inherently inexact human concepts. Human can give satisfactory answers, which are probably true. However, our systems are unable to answer many questions. The reason is most systems are designed based upon classical set theory and two-valued logic which is unable to cope with unreliable and incomplete information and give expert opinions.

Fuzzy set theory is an extension of classical set theory where elements have degree of membership.

Basic Definition

A set is any well-defined collection of objects. An object in a set is called an element or member of that set.

Sets are defined by a simple statement describing whether a particular element having a certain property belongs to that particular set.

Classical set theory enumerates all its elements using

A = {a1, a2, a3, a4, …….an}

If the elements ai (i=1,2,3,4,…n) of a set A are subset of universal set X, then set A can be represented for all elements x ∈ X by its characteristics function.

Terminology

A set A is well defined by a function called characteristics function. This function defined on the universal space X, assumes:

A value of 1 for those elements x that belongs to set A and

A value of 0 for those elements x that do not belong to set A

The notation used to express these mathematically are

A: X -> [0,1]

A(x) = 1, x is a member of A

A(x) = 0, x is not a member of A

Properties

Properties on sets play an important role for obtaining the solution. Following are the different properties of classical sets –

Commutative Property

Having two sets A and B, this property states –

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associative Property

Having three sets A, B and C, this property states –

$$A\cup(B\cup C)=(A\cup B)\cup C$$
$$A\cap(B\cap C)=(A\cap B)\cap C$$

Distributive Property

Having three sets A, B and C, this property states –

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$$
$$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$$

Idempotency Property

For any set A, this property states –

$$A\cup A=A$$
$$A\cap A=A$$

Identity Property

For set A and universal set X, this property states –

$$A\cup\phi=A$$
$$A\cap X=A$$
$$A\cap\phi=\phi$$
$$A\cup X=X$$

Transitive Property

Having three sets A, B and C, the property states –

If $A\subseteq B\subseteq C$, then $A\subseteq C$

Involution Property

For any set A, this property states –

$$\overline{\overline{A}} = A$$

De Morgan's Law

It is a very important law and supports in proving tautologies and contradiction. This law states –

$$\overline{A\cap B} = \overline{A}\cup\overline{B}$$

$$\overline{A\cup B} = \overline{A}\cap\overline{B}$$

Set-theoretic Operations

Set Operations include Set Union, Set Intersection, Set Difference, Complement of Set, and Cartesian product.

Union

The union of sets A and B (denoted by A ∪ BA ∪ B) is the set of elements which are in A, in B, or in both A and B. Hence, $A \cup B = \{x|x\in A \text{ OR } x \in B\}$.

Example − If A = {10,11,12,13} and B = {13,14,15}, then A ∪ B = {10,11,12,13,14,15} – The common element occurs only once.

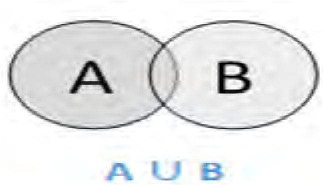

Figure 4.1: Union

Intersection

The intersection of sets A and B (denoted by A ∩ B) is the set of elements which are in both A and B. Hence, A ∩ B = {x|x∈ A AND x ∈ B}.

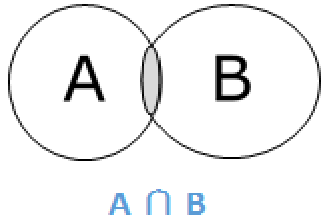

Figure 4.2: Intersection

Difference

The set difference of sets A and B (denoted by A–B) is the set of elements which are only in A but not in B. Hence, A − B = {x|x∈ A AND x ∉ B}.

Example − If A = {10,11,12,13} and B = {13,14,15}, then (A − B) = {10,11,12} and (B − A) = {14,15}. Here, we can see (A − B) ≠ (B − A)

Figure 4.3: Difference

Complement of a Set

The complement of a set A (denoted by A′) is the set of elements which are not in set A. Hence, A′ = {x|x∉ A}. More specifically, A′ = (U–A) where U is a universal set which contains all objects.

Example − If A = {x|x belongs to set of add integers} then A′ = {y|y does not belong to set of odd integers}

Cartesian Product / Cross Product

The Cartesian product of n number of sets A1,A2,…An denoted as A1 × A2...× An can be defined as all possible ordered pairs (x1,x2,…xn) where x1 ∈ A1,x2 ∈ A2,…xn∈ An

Example − If we take two sets A = {a,b} and B = {1,2},

The Cartesian product of A and B is written as − A × B = {(a,1),(a,2),(b,1),(b,2)}

And, the Cartesian product of B and A is written as − B × A = {(1,a),(1,b),(2,a),(2,b)}

Fuzzy Relations

A fuzzy relation R is a 2D MF:

$$R = \{((x, y), \mu_R(x, y)) | (x, y) \in X \times Y\}$$

Examples:

- x is close to y (x and y are numbers)
- x depends on y (x and y are events)
- x and y look alike (x, and y are persons or objects)
- if x is large, then y is small (x is an observed reading and Y is a corresponding action)

Max-Min Composition

The max-min composition of two fuzzy relations R1 (defined on X and Y) and R2 (defined on Y and Z) is

$$\mu_{R_1 \circ R_2}(x, z) = \vee_{y} [\mu_{R_1}(x, y) \wedge \mu_{R_2}(y, z)]$$

Properties:

Associativity:

$$R \circ (S \circ T) = (R \circ S) \circ T$$

Distributivity over union

$$R \circ (S \bigcup T) = (R \circ S) \bigcup (R \circ T)$$

Week distributivity over intersection

$$R \circ (S \bigcap T) \subseteq (R \circ S) \bigcap (R \circ T)$$

Monotonicity

$$S \subseteq T \Rightarrow (R \circ S) \subseteq (R \circ T)$$

Membership functions and assignment

Definition:A membership function for a fuzzy set A on the universe of discourse X is defined as μ_A: X → [0, 1], where each element of X is mapped to a value between 0 and 1. This value, called membership value or degree of membership, quantifies the grade of membership of the element in X to the fuzzy set A.

Membership functions allow us to graphically represent a fuzzy set. The *x* axis represents the universe of discourse, whereas the *y* axis represents the degrees of membership in the [0, 1] interval.

Simple functions are used to build membership functions. Because we are defining fuzzy concepts, using more complex functions does not add more precision.

Some of the Fuzzy member functions are as follows:

1. Triangular MFs

A triangular MF is specified by three parameters {a, b, c} as follows:

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{m-a}, & a < x \le m \\ \frac{b-x}{b-m}, & m < x < b \\ 0, & x \ge b \end{cases}$$

The parameters {a, b, c} (with a < b < c) determine the x coordinates of the three corners of the underlying triangular MF.

2. Trapezoidal MFs

A trapezoidal MF is specified by four parameters {a, b, c, d} as follows:

$$\text{trapezoid}(x; a, b, c, d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right).$$

The parameters {a, b, c, d} (with a < b <= c < d) determine the x coordinates of the four corners of the underlying trapezoidal MF.

3. Gaussian MFs

A Gaussian MF is specified by tow parameters:

$$\text{gaussian}(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}.$$

A Gaussian MF is determined complete by c and σ; c represents the MFs center and σ determines the MFs width.

Fuzzy Rules & Fuzzy Reasoning

Extension Principle

A is a fuzzy set on X

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \cdots + \mu_A(x_n)/x_n$$

The image of A under f() is a fuzzy set B:

$$B = \mu_B(x_1)/y_1 + \mu_B(x_2)/y_2 + \cdots + \mu_B(x_n)/y_n$$

Where $Y_i = f(x_i)$, i=1 to n

If f() is a many to one mapping then

$$\mu_B(y) = \max_{x=f^{-1}(y)} \mu_A(x)$$

Fuzzy Reasoning

Following are the different modes of approximate reasoning −

Categorical Reasoning

In this mode of approximate reasoning, the antecedents, containing no fuzzy quantifiers and fuzzy probabilities, are assumed to be in canonical form.

Qualitative Reasoning

In this mode of approximate reasoning, the antecedents and consequents have fuzzy linguistic variables; the input-output relationship of a system is expressed as a collection of fuzzy IF-THEN rules. This reasoning is mainly used in control system analysis.

Syllogistic Reasoning

In this mode of approximation reasoning, antecedents with fuzzy quantifiers are related to inference rules.

This is expressed as −

x = S1A’s are B’s

y = S2C’s are D’s

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z = S3E’s are F’s

Here A,B,C,D,E,F are fuzzy predicates.

- S1 and S2 are given fuzzy quantifiers.

- S3 is the fuzzy quantifier which has to be decided.

Dispositional Reasoning
In this mode of approximation reasoning, the antecedents are dispositions that may contain the fuzzy quantifier “usually”. The quantifier usually links together the dispositional and syllogistic reasoning; hence it pays an important role.

For example, the projection rule of inference in dispositional reasoning can be given as follows –
Usually ((L, M) is R) ⇒ usually (L is [R ↓ L])
Here [R ↓ L] is the projection of fuzzy relation R on L

Fuzzy If-Then Rules
It is a known fact that a human being is always comfortable making conversations in natural language. The representation of human knowledge can be done with the help of following natural language expression –
IF antecedent THEN consequent
The expression as stated above is referred to as the Fuzzy IF-THEN rule base.

General format: if x is A then y is B
Examples:
- If pressure is high, then volume is small
- If the road is slippery, then driving is dangerous
- Is a tomato is red, then it is ripe
- If the speed is high, then apply the break a little.

Interpretations of Fuzzy IF-THEN Rules
Fuzzy IF-THEN Rules can be interpreted in the following four forms –

Assignment Statements
These kinds of statements use “=” (equal to sign) for the purpose of assignment. They are of the following form –
a = hello
climate = summer

Conditional Statements
These kinds of statements use the “IF-THEN” rule base form for the purpose of condition. They are of the following form –
IF temperature is high THEN Climate is hot
IF food is fresh THEN eat.

Unconditional Statements
They are of the following form –
GOTO 10
turn the Fan off

Fuzzy Inference System
Fuzzy Inference System is the key unit of a fuzzy logic system having decision making as its primary work. It uses the “IF…THEN” rules along with connectors “OR” or “AND” for drawing essential decision rules.

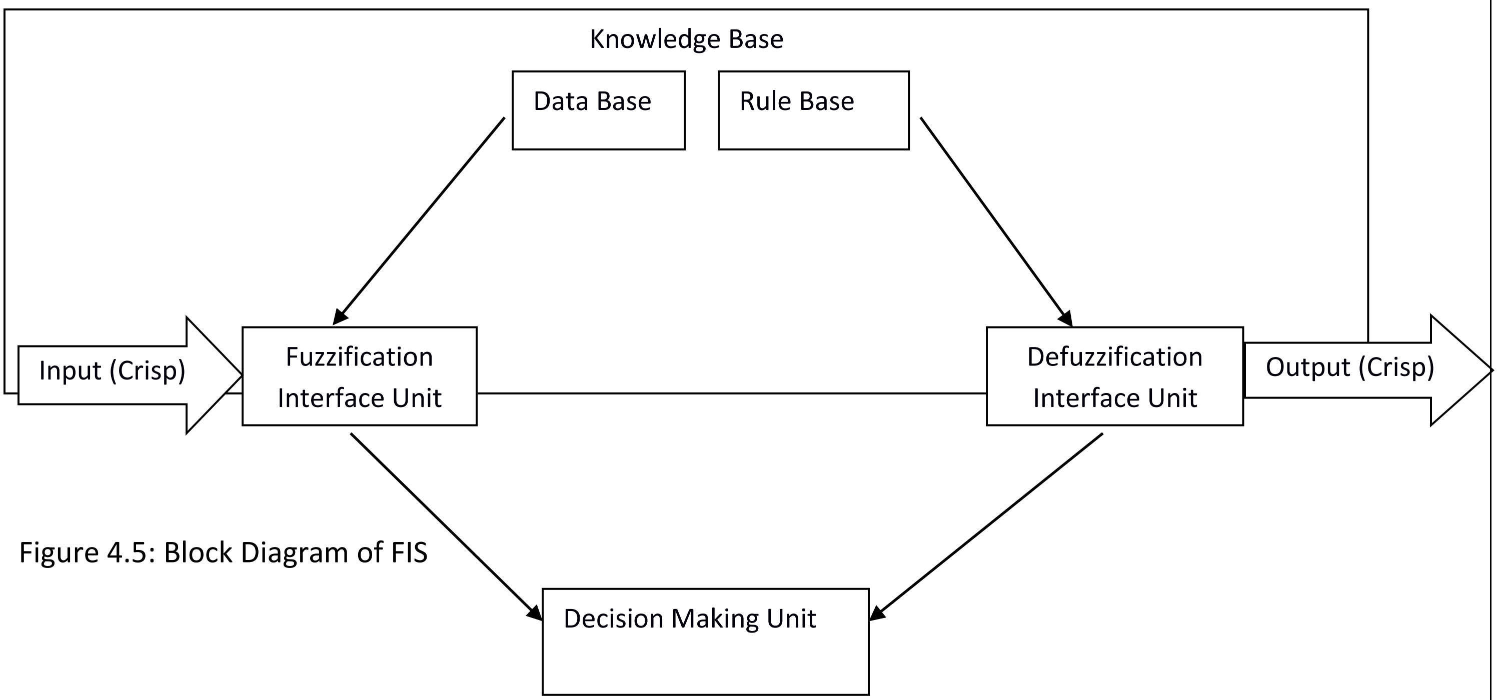

Figure 4.5: Block Diagram of FIS

Characteristics of Fuzzy Inference System

- The output from FIS is always a fuzzy set irrespective of its input which can be fuzzy or crisp.
- It is necessary to have fuzzy output when it is used as a controller.
- A defuzzification unit would be there with FIS to convert fuzzy variables into crisp variables

Functional Blocks of FIS

The following five functional blocks will help you understand the construction of FIS −
Rule Base − It contains fuzzy IF-THEN rules.
Database − It defines the membership functions of fuzzy sets used in fuzzy rules.
Decision-making Unit − It performs operation on rules.
Fuzzification Interface Unit − It converts the crisp quantities into fuzzy quantities.
Defuzzification Interface Unit − It converts the fuzzy quantities into crisp quantities. Following is a block diagram of fuzzy interference system.

Working of FIS

The working of the FIS consists of the following steps −

- A fuzzification unit supports the application of numerous fuzzification methods and converts the crisp input into fuzzy input.
- A knowledge base - collection of rule base and database are formed upon the conversion of crisp input into fuzzy input.
- The defuzzification unit fuzzy input is finally converted into crisp output.

Applications of Fuzzy Logic

Following are the applications of Fuzzy Logic in various engineering domains:

Aerospace

In aerospace, fuzzy logic is used in the following areas −

- Altitude control of spacecraft
- Satellite altitude control

- Flow and mixture regulation in aircraft deicing vehicles

Automotive

In automotive, fuzzy logic is used in the following areas –

- Trainable fuzzy systems for idle speed control
- Shift scheduling method for automatic transmission
- Intelligent highway systems
- Traffic control
- Improving efficiency of automatic transmissions

Business

In business, fuzzy logic is used in the following areas –

- Decision-making support systems
- Personnel evaluation in a large company

Defense

In defense, fuzzy logic is used in the following areas –

- Underwater target recognition
- Automatic target recognition of thermal infrared images
- Naval decision support aids
- Control of a hypervelocity interceptor
- Fuzzy set modeling of NATO decision making

Electronics

In electronics, fuzzy logic is used in the following areas –

- Control of automatic exposure in video cameras
- Humidity in a clean room
- Air conditioning systems
- Washing machine timing
- Microwave ovens
- Vacuum cleaners

Finance

In the finance field, fuzzy logic is used in the following areas –

- Banknote transfer control
- Fund management
- Stock market predictions

Industrial Sector

In industrial, fuzzy logic is used in following areas –

- Cement kiln controls heat exchanger control
- Activated sludge wastewater treatment process control
- Water purification plant control
- Quantitative pattern analysis for industrial quality assurance
- Control of constraint satisfaction problems in structural design
- Control of water purification plants

Manufacturing

In the manufacturing industry, fuzzy logic is used in following areas −

- Optimization of cheese production
- Optimization of milk production

Marine

In the marine field, fuzzy logic is used in the following areas −

- Autopilot for ships
- Optimal route selection
- Control of autonomous underwater vehicles
- Ship steering

Medical

In the medical field, fuzzy logic is used in the following areas −

- Medical diagnostic support system
- Control of arterial pressure during anesthesia
- Multivariable control of anesthesia
- Modeling of neuropathological findings in Alzheimer's patients
- Radiology diagnoses
- Fuzzy inference diagnosis of diabetes and prostate cancer

Securities

In securities, fuzzy logic is used in following areas −

- Decision systems for securities trading
- Various security appliances

Transportation

In transportation, fuzzy logic is used in the following areas −

- Automatic underground train operation
- Train schedule control
- Railway acceleration
- Braking and stopping
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Pattern Recognition and Classification

In Pattern Recognition and Classification, fuzzy logic is used in the following areas −

- Fuzzy logic-based speech recognition
- Fuzzy logic based
- Handwriting recognition
- Fuzzy logic based facial characteristic analysis
- Command analysis
- Fuzzy image search

Psychology

In Psychology, fuzzy logic is used in following areas −

- Fuzzy logic-based analysis of human behavior
- Criminal investigation and prevention based on fuzzy logic reasoning