267968

Course Name: Matrices, Differential Equations, and Laplace transform

Course Outcome

CO1- Know the rank of a matrix and its applications in solving systems of linear equations

CO2- Find the Eigen values and Eigen vectors of a square matrix

CO3- Solve linear ordinary and partial differential equations of higher orders

CO4- Classify the linear partial differential equations as elliptic, parabolic and hyperbolic

CO5- Apply Laplace transform to Engineering problems using properties

CO6- Apply Inverse Laplace transform to Engineering problems

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End Term Examination, Even Semester 2023-24

B.Tech. Year-I, Semester-II

Sub. Code & Name- BMAS 1105; Matrices, Differential Equations and Laplace transform

Time: 3 Hours **Maximum Marks: 50**

Instructions to student:

1. Attempt ALL sections.
2. The terms have their usual meanings.
3. Assume $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$

Section – A

Attempt All Questions. 4 X 5 = 20 Marks

No.	Detail of Question	Marks	CO	BL	KL
1	A matrix $$A = \begin{bmatrix} a & b & b^2 \\ 1 & 2 & -1 \\ 0 & 2 & 2 \end{bmatrix}$$ is such that the rank of A is 2 and sum of its eigen values are $\mathbf{0}$. Find the value of $\boldsymbol{a+b}$, when $\boldsymbol{b > 0}$.	4	1	C	M
2	Solve the following differential equation: $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\cos x.$$	4	3	U	F
3	Find the general solution of (a) $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}(1 + y^2)x\,dy = 0$ (b) $\frac{d^4y}{dx^4} - a^4y = 0$, where 'a' is constant.	4	3	E	P

4	Consider two thin, rectangular composite plates lying in the xy-plane, These plates are made up of different materials in different regions. The temperature distribution, $T(x, y)$, in both the plates are governed by the following differential equations: (a) $T_{xx} + (1 + y^2)^2 T_{yy} - 2y(1 + y^2) T_y = 0,$ (b) $x^2 T_{xx} - 2xyT_{xy} + y^2 T_{yy} = 0.$ Determine whether the governing equations for the temperature distribution are hyperbolic, parabolic or elliptic.	4	4	U	F
5	In an analysis of electric circuits, the engineers often encounter complex voltage or current waveforms which can be described by some mathematical functions. One such waveform is encountered by an engineer trying to solve for the current through a critical component of a high precision equipment. The equipment performance is described by a function that represents a challenging integral, the solution of which is pivotal for adjusting the circuit to achieve optimal efficiency. The engineer models the behavior of the current $I(t)$ through this component for $t > 0$ using an unusual yet perfectly valid mathematical expression that combines exponential and trigonometric functions as follows: $I(t) = \int_0^\infty e^{-t} \sin at \cos bt \, dt,$ where, a and b are the constants representing specific frequency components of the waveform. Your task is to assist in solving this integral, providing a clear pathway to the solution by employing the Laplace transform. **OR,** (i) Find: $L^{-1}\left\{\frac{p-2}{p(p-1)(p+2)}\right\}$ (ii) Solve: $(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y)$	4	5, 3	C	M

Section – B

Attempt All Questions. 3 X 5 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
6	(a) Solve the partial differential equation: $$(D^2 - DD' + D' - 1)z = 0$$ (b) Use second shifting property of Laplace transform to find: $$L[F(t)]$$ where, $$F(t) = \begin{cases} (t-2)^3, & t > 2 \\ 0, & t < 2 \end{cases}$$ **OR,** Evaluate: $L[F(t)]$ where, $$F(t) = t\, e^{-t} \cosh 2t$$ Hence find: $L[\int_0^t F(t)\, dt]$	3	3, 5	A	P
7	Solve the partial differential equation: $$(4D^2 - 4DD' + D'^2)z = 16 \log(x + 2y)$$	3	3	E	P
8	Find: $$L^{-1}\left\{\log\left(\frac{p+1}{p+2}\right)\right\}$$	3	6	A	C
9	If $$L\{f(t)\} = \frac{p-1}{p^2+1}$$ then show that: $$L\{f(2t)\} = \frac{p-2}{p^2+4}$$	3	5	An	C

10	A student was performing some practical in an electronics lab during his course of study in B.Tech. programme. While performing a practical, he saw a waveform in C.R.O. (Cathode Ray Oscilloscope). A cathode ray oscilloscope is an electrical test device used to produce waveforms in response to several input signals. The excited student hurriedly noted down the functional form of the waveform but could not identify the period of that periodic function. His teacher told him that the waveform satisfies the relation $f(t+4) = f(4)$. Now, his assignment was to find the Laplace transform of the function: $$f(t) = \begin{cases} 1; & 0 \le t < 2 \\ -1; & 2 \le t < 4 \end{cases}$$ Determine his result.	3	5	C	M

Section – C

Attempt All Questions. 5 X 3 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
11	Solve the following partial differential equation: $$(D^2 + DD' - 6D'^2)z = x^2y + e^{3x+y}.$$ **OR,** Show that: if $L\,[\,F\,(t)] = f\,(p)$ then, $$L\left\{\frac{\cos at - \cos bt}{t}\right\} = \frac{1}{2}\log\left(\frac{p^2+b^2}{p^2+a^2}\right)$$	5	3, 5	A	P
12	Solve the following partial differential equations: (a) $(D^2 - DD' - 2D'^2)z = e^x\,(y-1)$ (b) $s + ap + bq + abz = e^{mx+ny}; m \neq -a, n \neq -b$	5	3	E	P
13	Consider the motion of a particle governed by $$y'' + 9y = 6\cos 3t;$$ Given that: $y(0) = 2,\ y'(0) = 0.$ Determine the position of that particle at any time t using Laplace transform technique. **OR,** Apply convolution theorem to evaluate: $$L^{-1}\left[\frac{p}{(p^2+4)^2}\right]$$	5	6, 5	C	M