Indian Institute of Information Technology Vadodara
MA 101: Linear Algebra and Matrices
Tutorial 3

1. Suppose a 3×5 coefficient matrix, A for a system $AX = b$ has three pivot columns. Is the system consistent? Why or why not?

2. Suppose a system of linear equations $AX = b$ has a 3×5 augmented matrix $[A|b]$ whose fifth column is a pivot column. Is the system consistent? Why or why not?

3. Is the following set linearly independent and/ or span $\mathbb{R}^3$.
   a) $\{u, v, w\}$,
   b) $\{u, v\}$,
   c) $\{u, v, w, z\}$
   d) $\{u, v, w, z, 0\}$

   $\mathbf{u} = \begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -6 \\ 1 \\ 7 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$, and $\mathbf{z} = \begin{bmatrix} 3 \\ 7 \\ -5 \end{bmatrix}$

   What conclusion can you draw from above exercise?

4. Let A be the matrix given below and $v_p = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ be a particular solution to $AX = b$. Then find its all solutions.

$$\begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Determine whether following is a linear transformation or not. If yes, then find its standard matrix.
   a) $T([x_1\ x_2\ x_3\ x_4]^T) = [0\ \ x_1 + x_2\ \ x_2 + 2x_4\ \ x_2\ x_3]^T$
   b) $T([x_1\ x_2\ x_3\ x_4]^T) = [2x_1 + 4x_4]^T$

6. How many rows and columns must a matrix A have in order to define a linear transformation from $\mathbb{R}^4$ to $\mathbb{R}^6$ by the rule $T(x) = Ax$?

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(\begin{bmatrix} 1 \\ -1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Find $T(\begin{bmatrix} 5 \\ 6 \end{bmatrix})$

8. Find standard matrix representation of T given in previous exercise.

9. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects each point through the line $X + Y = 0$. Find standard matrix representation of T.

10. Given $v \neq 0$ and $p \in \mathbb{R}^n$, the line through p in the direction of v has the parametric equation $x = p + tv$. Show that a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ maps this line onto another line or onto a single point.