CS203 Design and Analysis of Algorithm

Autumn Semester 2022-23
End Semester Examination
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December 21 , 2022

1. **[3+2 = 5 Marks]** Prove the following
   (a) $O(max\{f(n), g(n)\}) = O(f(n) + g(n))$
   (b) If $d(n)$ is $O(f(n))$ and $e(n)$ is $O(g(n))$, then the product $d(n).e(n)$ is $O(f(n).g(n))$

2. **[3+2 = 5 Marks]** Justify your answer
   (a) What is the running time of Quick Sort when all elements of array A have same value?
   (b) The running time of Quick Sort is $\Theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.

3. **[7+3 = 10 Marks]** Write an algorithm to find the number of connected components in the directed graph. How to modify your approach if the given graph is strongly connected graph.

4. **[10 Marks]** Suppose you are given a connected graph G, with edge costs that you may assume are all distinct. G has n vertices and m edges. A particular edge e of G is specified. Give an algorithm with running time $O(m+n)$ to decide whether e is contained in a minimum spanning tree of G.

5. **[10 Marks]** The sets A and B have m and n elements respectively from a linear order. These sets are not necessarily sorted. Also assume that $m \leq n$. Show how to compute $A \cup B$ and $A \cap B$ in $O(nlog(m))$ time.

6. **[3+7 = 10 Marks]** Define the 0/1/2 knapsack problem for n objects to be $Max \sum_{i=1}^{n} p_i x_i$ subject to the constraints
   $\sum_{i=1}^{n} w_i x_i \leq c$ and $x_i \in \{0, 1, 2\}, 1 \leq i \leq n$
   Here you can have an 0, 1 or 2 copies of any object x_i.
   (a) Define dynamic programming functional equation to solve 0/1/2 Knapsack problem.
   (b) Give an algorithm with complexity to solve the 0/1/2 Knapsack problem.

7. **[6+4 Marks]** We have studied KMP String matching algorithms. The algorithms work on the idea that whenever there will be mismatch for a character in the given pattern P (length = m) and text T (length = n), where $m \leq n$, we shift the pattern accordingly to find another match. Now, consider a Wildcard Character (represented as ? symbol) which appears at only one position in the pattern P and can be matched with any character in the text. The idea of wildcard character is to let matching process to continue and we can tolerate the mismatch of one character. Write a modified KMP algorithm to take care the wildcard character in the matching process. Also show the steps of your modified algorithm on example: T: ABABACBDCDA and P: B?C. Here P can be matched in T at index 7.

8. **[10 Marks]** Apply Rabin-Karp Algorithm on text T: ABABACBDCBAC and pattern P: BAC using $m = 21$. (Use ASCII vales for the characters like 66*100+65*10+67 for BAC)

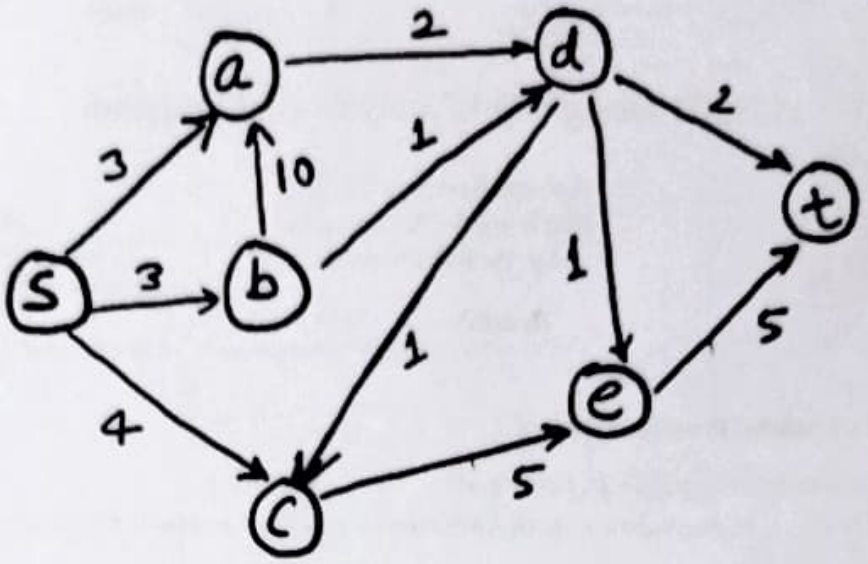

Figure 1: Graph for Ford-Fulkerson algorithm

9. **[8+2 Marks]** Apply Ford-Fulkerson algorithm on the Graph shown in Figure 1 where maximum capacity mentioned on edges. Show all your steps clearly. Also give the minimum cut of the graph.

10. **[4+6 Marks]** Write down the Floyd-Warshall algorithm and hence apply on the Graph shown in Figure 2. Also, show the predecessor matrix.

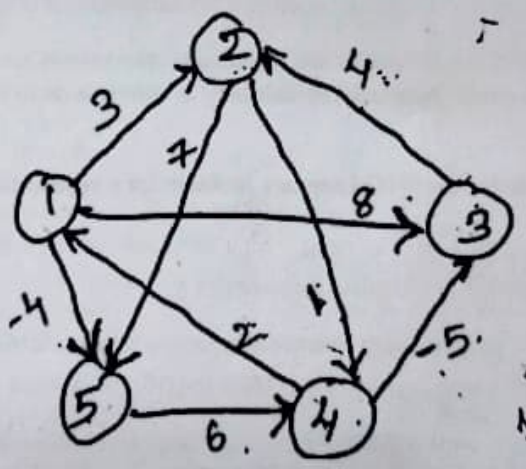

Figure 2: Graph for Flyod-Warshall algorithm

	1	2	3	4	5
1	0	3	8	∞	4
2	∞	0	∞	1	7
3	∞	4	0	∞	3
4	2	5	-5	0	-2
5	∞	11	7	6	0