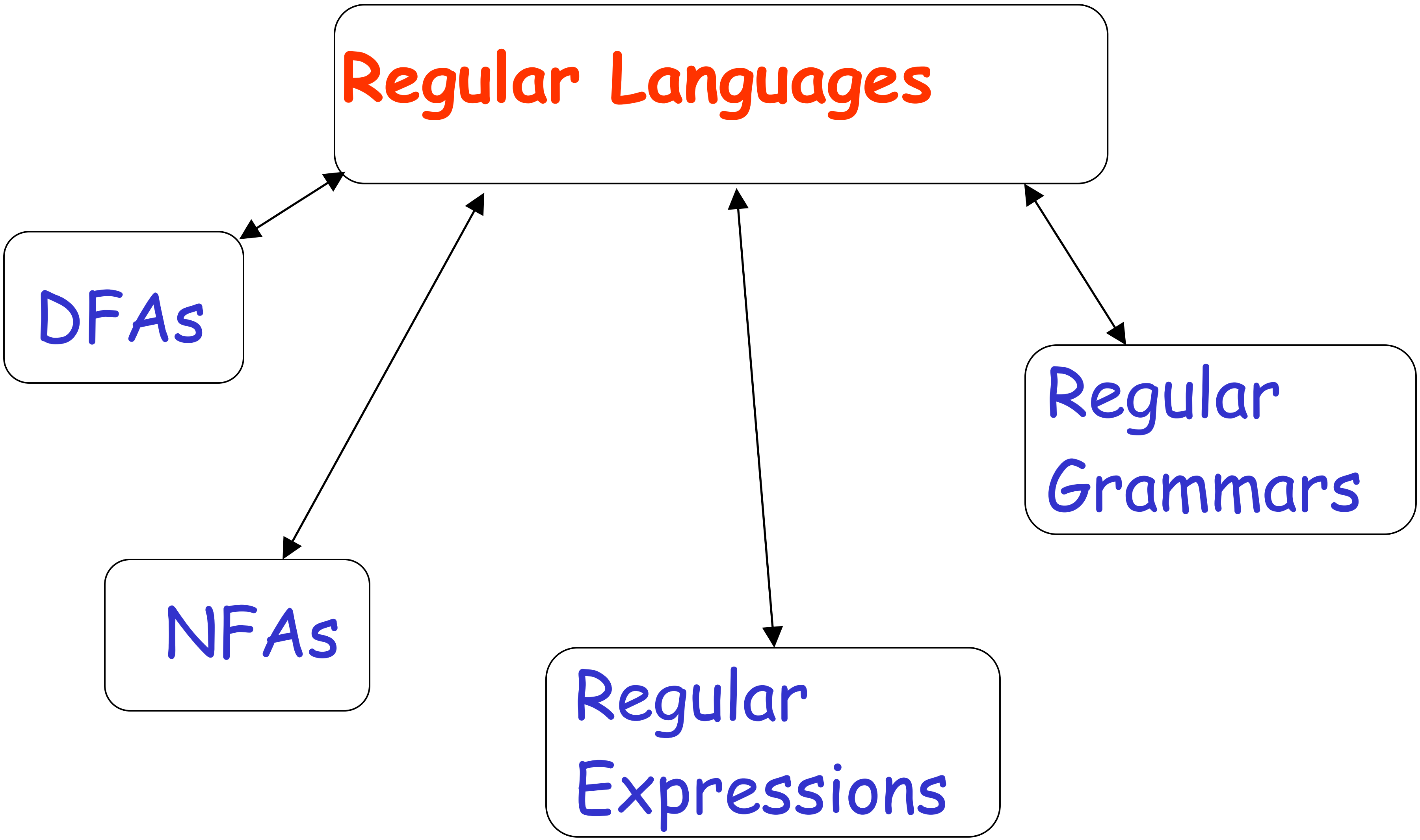
Standard Representations of Regular Languages



When we say: We are given a Regular Language L

We mean: Language L is in a standard representation

Elementary Questions

about

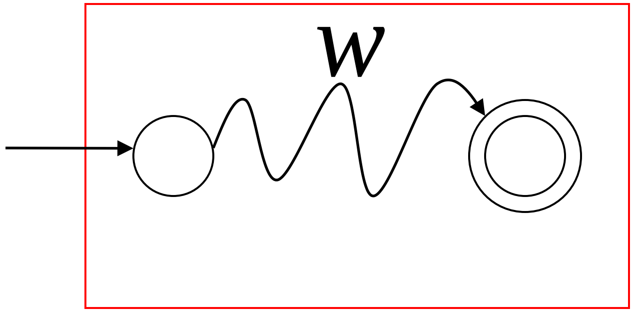
Regular Languages

Membership Question

Question: Given regular language L
and string w
how can we check if $w \in L$?

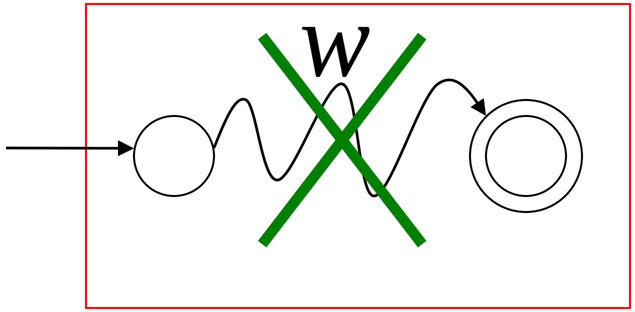
Answer: Take the DFA that accepts L
and check if w is accepted

DFA



$w \in L$

DFA



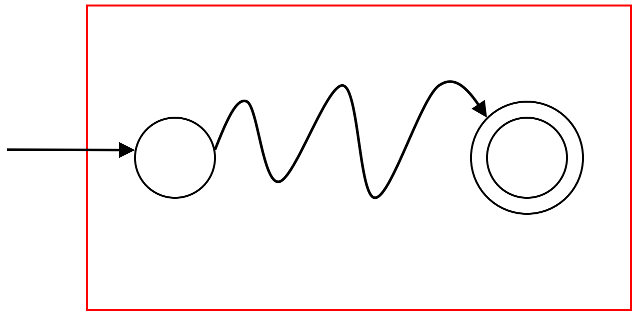
$w \notin L$

Question: Given regular language L
how can we check
if L is empty: $(L = \emptyset)$?

Answer: Take the DFA that accepts L

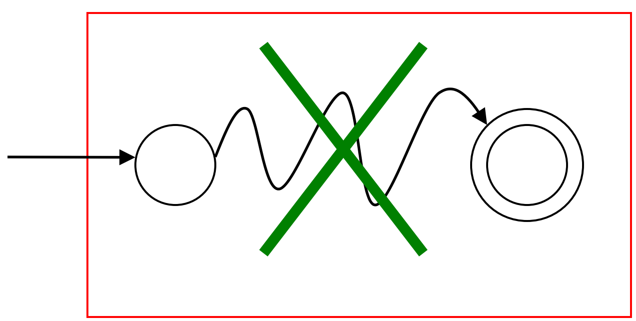
Check if there is any path from
the initial state to a final state

DFA



$$L \neq \emptyset$$

DFA



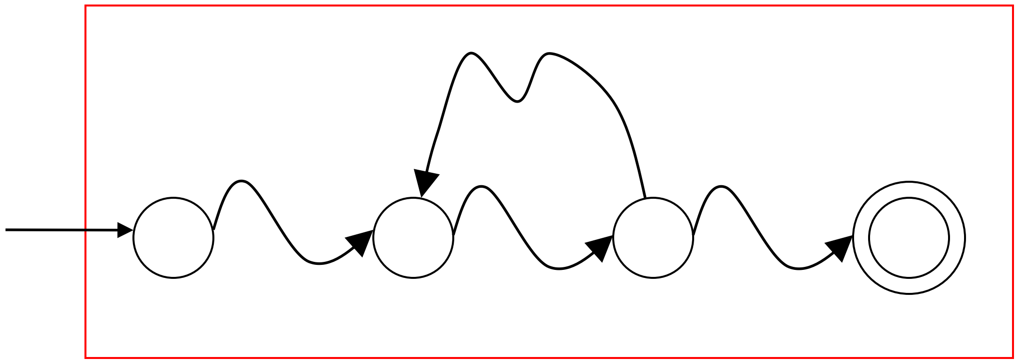
$$L = \emptyset$$

Question: Given regular language L how can we check if L is finite?

Answer: Take the DFA that accepts L

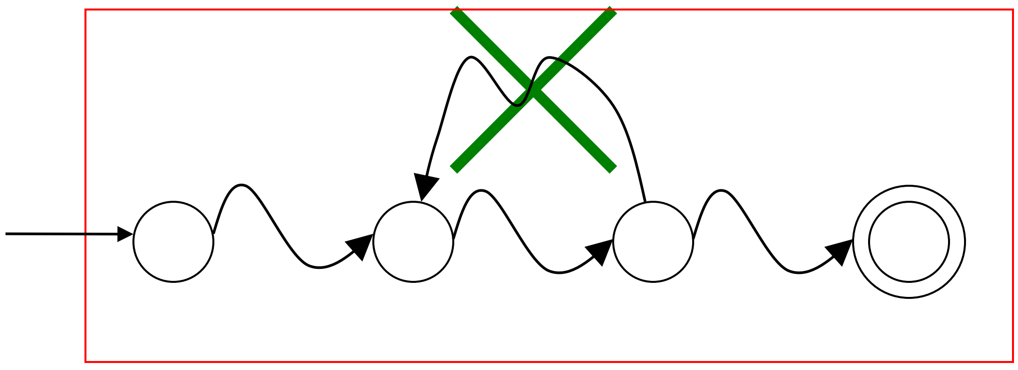
Check if there is a walk with cycle from the initial state to a final state

DFA



L is infinite

DFA



L is finite

Question:

Given regular languages L_1 and L_2
how can we check if $L_1 = L_2$?

$f(q_1) = q_1'$

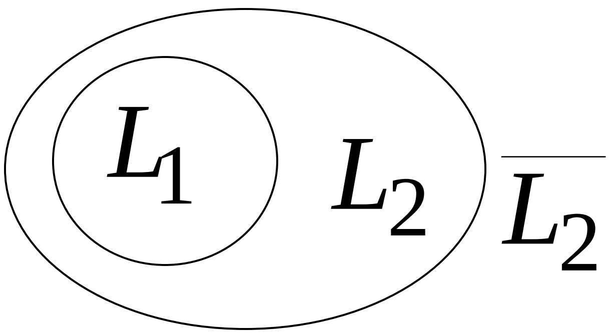
Answer: Find if $(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) = \varnothing$

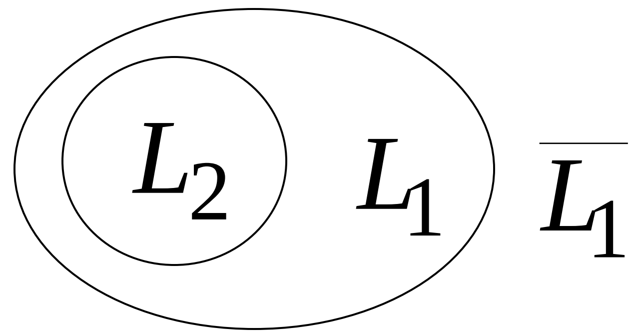
$M_1 \quad M_2$

$$(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) = \varnothing$$

$$\Downarrow$$

$L_1 \cap \overline{L_2} = \varnothing$ and $\overline{L_1} \cap L_2 = \varnothing$





$L_1 \subseteq L_2$ $\qquad$ $L_2 \subseteq L_1$

$$\Downarrow$$

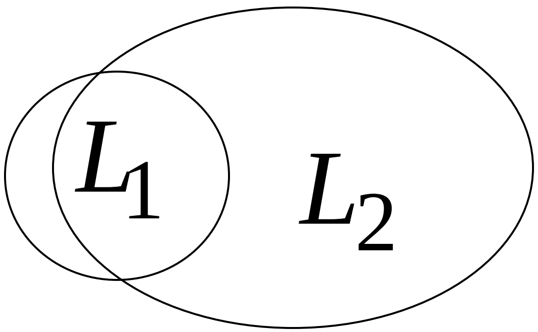
$$L_1 = L_2$$

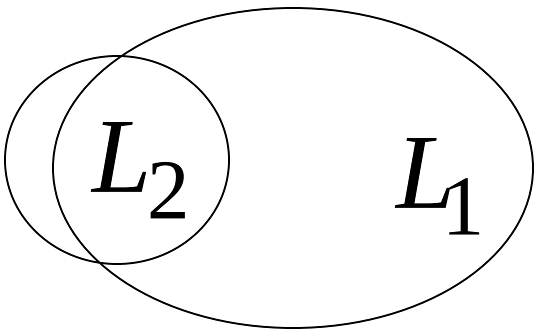
$$(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) \neq \varnothing$$

$$L_1 \cap \overline{L_2} \neq \varnothing$$

or

$$\overline{L_1} \cap L_2 \neq \varnothing$$





$$L_1 \not\subset L_2$$

$$L_2 \not\subset L_1$$

$$L_1 \neq L_2$$

Closure Properties of Regular languages

Closure Properties of Regular languages

If L1 and L2 are regular languages, then so $L1 \cup L2$; $L1 \cap L2$; $L1L2$, $L1^c$ and $L1^*$.

We say that the family of regular languages is closed under union, intersection, concatenation, complementation, and Star-closure.

Closure Properties of Regular languages

If L1 and L2 are regular languages, there exist regular expressions r1 and r2 such that L1 = L(r1) and L2 = L(r2).

By def^n, r1+r2, r1r2 and r1* are regular expressions denoting the languages L1 U L2; L1L2 and L1* respectively.

For complement of L1, we can design complemented automata (as discussed in previous classes).

For L1 ∩ L2, we can design product automata for given L1 and L2 (as discussed in previous classes).

Closure Properties of Regular languages

$L_1 \cap L_2^c$

Claim 1: If L1 and L2 are regular, then L1 - L2 (set difference) is necessarily regular also ???

Claim 2: The family of regular languages is closed under reversal (L^R) ???

Claim 3: If L is a regular language, prove that the language $\{uv : u \in L,\ v \in L^R\}$ is also regular ???

Claim 4: Show that the family of regular languages is closed under symmetric difference ???

$A \oplus B = (A - B) \cup (B - A)$

Claim 5: Show that the family of languages is closed under NOR operation ???