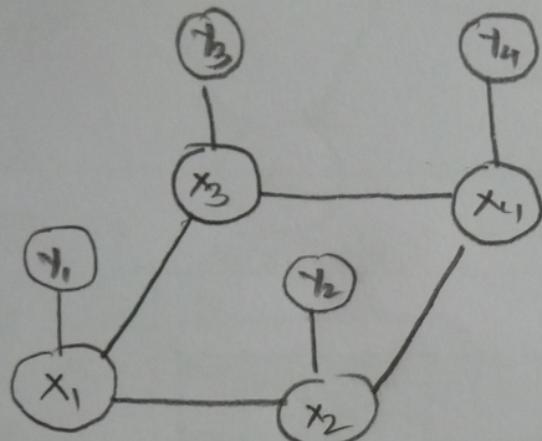
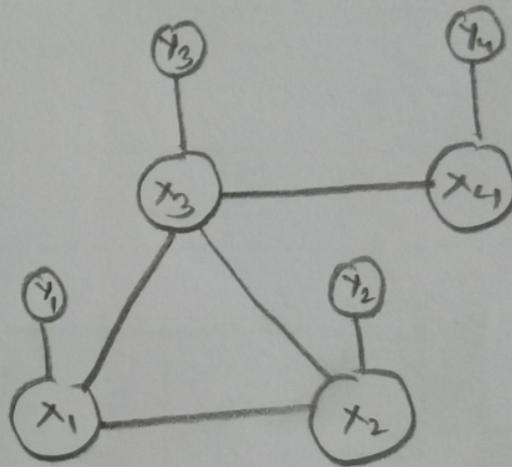


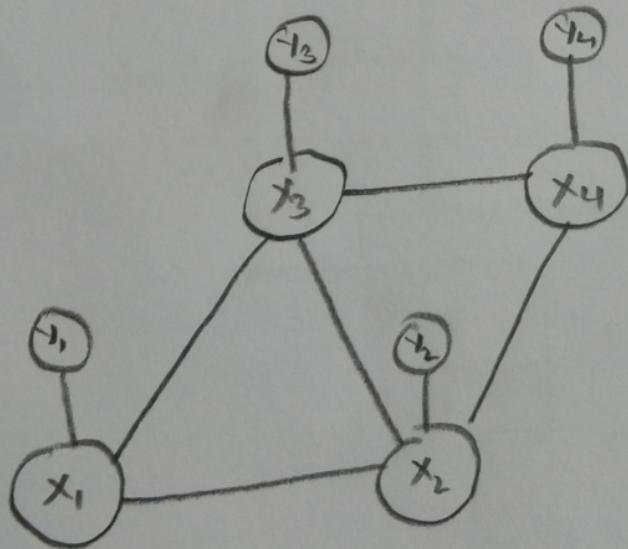
Q1 For the UGMs shown in figures below, write the factorizations of the joint probability density function  $P(x_1x_2x_3x_4y_1y_2y_3y_4)$  over the maximal cliques.



(A)



(B)



[3 points]

Q.2 A directed graphical model on three variables (binary) is shown in Figure below.

$x_1$	$P(x_1=x)$	$x_1$	$x_2$	$\frac{x_2}{x_2}$	$P(x_2=x)$
0	a			0	b
1	1-a			1	1-b

$x_1, x_2$	$P(x_3 x_1, x_2)$
00	$c_1 \quad 1-c_1$
01	$c_2 \quad 1-c_2$
10	$c_3 \quad 1-c_3$
11	$c_4 \quad 1-c_4$

Data Table

$x_1$	$x_2$	$x_3$
0	0	0
0	0	1
0	1	0
0	0	1
0	1	1
1	0	0
0	0	0
0	1	0
0	0	0
0	1	1
0	0	1
1	1	1
1	0	1
1	1	1
1	0	0

A table of data (in rows) is provided to learn the CPT parameters  $\{a, b, c_1, c_2, c_3, c_4\}$  such that it maximizes likelihood of observing data table (DT).

$$\max_{\theta} p(DT|\theta) \sim \left[ \begin{array}{c} \text{log likelihood} \\ \max_{\theta} \log P(DT|\theta) \end{array} \right]$$

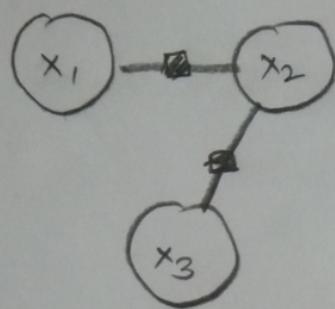
$$P(DT|\theta) = \prod_{\text{Row}} P(DT(\text{Row})|\theta) \quad [3 \text{ Points}]$$

Ex.  $P(DT(\text{highlighted row})|\theta) = P(x_1=1, x_2=0, x_3=0 | a, b, c_1, c_2, c_3, c_4)$

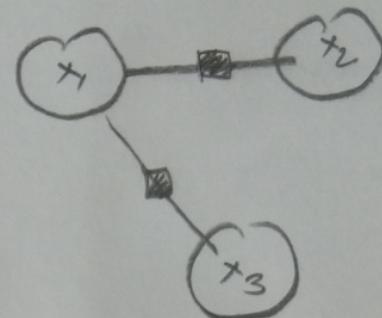
$$= \underbrace{P(x_1=1|a)}_{\downarrow} \underbrace{P(x_2=0|b)}_{\downarrow} \underbrace{P(x_3=0|x_1=1, x_2=0, a, b, c_1, c_2, c_3, c_4)}_{\downarrow}$$

$$= (1-a) \times (b) \times (c_3).$$

Q.3 For the data table of Q2. UGMs are to be trained with following factor graphs.



(a)



(b)

Write down the factorizations for each of the above UGM along with the factor functions (factor tables) over maximal cliques.

[1 point]

Q.4 Answer the following queries with learnt models of Q2 and Q3.

(I)  $P(x_1=0, x_2=0, x_3=0)$

(II)  $P(x_1=0, x_2=0 | x_3=0)$

Write down your observations in each case.

[3 points]