- Space, time and energy are limited, use them wisely. If you answer all the questions sequentially you will earn a bonus of 5 points. Points are mentioned against each question and maximum cumulative points that can be earned is 90.
- This is an open book/ notes exam. Students may carry one book along with their own hand written notes during the examination. Notes should be properly clipped together with student ID written on each page. Notes with loose pages are not permitted.
- Use of scientific calculator is permitted during the examination.
- It is mandatory to self assess your answers and enter the expected points for attempted questions in the assessment table on the first page of the answer script. You are required to create an additional column on the first page. The questions will not be evaluated for which the entries are missing.

(Student Name) Dipean Dasgupta (Student ID) 20215188

Question 1

A drilling tool is attached to a two-link robotic arm anchored at the origin. The lengths of both the links are one unit each. You can control the movement of the drilling tool in the workspace ($(x_1, x_2) \in \mathbb{R}^2$) by controlling the motors, i.e. joint angles (ranging from 0 to 360 degrees in steps of 1 degree resolution). However, there are regions where drilling tool can not enter (but the links can), such regions are referred to as obstacles. One such workspace configuration is shown in Figure 1.

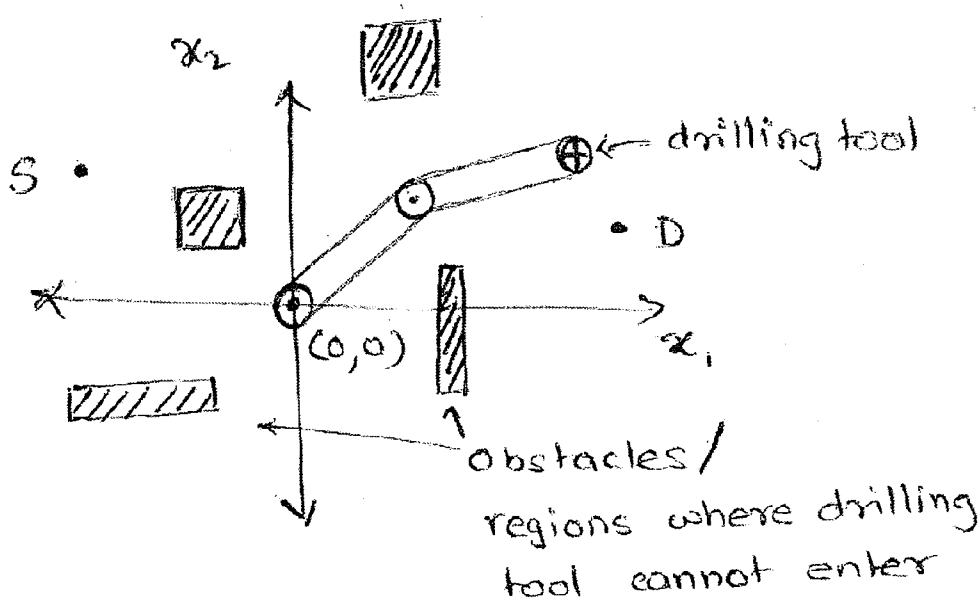

Figure 1: A two link robotic arm

(a) Develop an algorithm for the robotic arm which plans for the collision free movement of the drilling tools from a source S to a destination D in the workspace. The plan corresponds to an appropriate sequence of pairs of angles for both the actuators (motors) controlling the motion of the drilling tool. (10)

(b) Assuming that there are no obstacles in the workspace, what will be the plan for moving the drilling tool from $(2, 0)$ to $(-1, -1)$ in the workspace? (5)

Question 2

Let W denote the weight matrix of a Hopfield network of n units and let θ be the n-dimensional row vector of units' thresholds. The energy $E(x)$ of a state x (a row vector) of the network is given by

$$E(x) = -\frac{1}{2} x W x^T + \theta x^T.$$

The energy function can also be written in the form $E(x) = -\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} w_{ij}x^i x^j + \sum_{i=1}^{n}\theta_i x^i$, where x^i is the i^{th} component of state vector x.

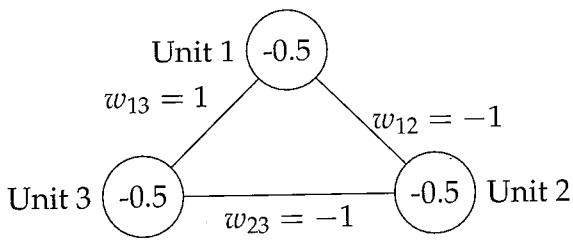

Figure 2: Three unit Hopfield network

(a) Consider a Hopfield network with three units as shown in Figure 2. Each one of them can assume the state 1 or -1. The threshold values for each unit are mentioned in the node representing the unit. If the total excitation of the unit selected for a state update is greater than threshold set for that unit, the unit attains state 1, otherwise it attains state -1. Being asynchronous, at each step only one of the state is selected for update. The edge weights between i^{th} unit and j^{th} unit, w_{ij} is marked on the edge. Sketch state-energy diagram indicating all the transitions. (8)

(b) In a Hopfield network, Hebbian learning is implemented by loading m selected n-dimensional stable states represented in the form of row vectors $\{x_1, x_2, \ldots, x_m\}$ on the network and by updating the network's weights (initially set to zero) iterating on all the m vectors according to the following rule (5)

$$w_{ij} = w_{ij} + x_k^i x_k^j, \; for\; i, j = 1, \ldots, n \; and i \neq j.$$

The symbols x_k^i and x_k^j denote the i^{th} and j^{th} component respectively of the row vector x_k. In other words, for the first vector x_1 the weight matrix is given by the expression $W_1 = x_1 x_1^T - I$, where I denotes the $n \times n$ identity matrix.

Show that the minimum of the energy function of a Hopfield network with the weight matrix W_1 is located at x_1. What is the condition under which Hebbian rule can be used to store all the m patterns?

(c) Justify: A Hopfield network with n units and asynchronous dynamics, which starts from any given network state, eventually reaches a stable state at a local minimum of the energy function. (2)

Question 3

Two players are involved in a relationship whereby, if both players devote more effort to the relationship, they are both better off. For any given effort of player j, the return on individual i's effort starts by increasing, and then decreases. We represent an effort level by a non-negative number. For every effort level s_i, the utility function for any player i is defined as $u_i(s_i, s_j) = s_i(c + s_j - s_i)$, where s_j is the effort level of player j and $c > 0$ is a constant.

(a) Formulate a two player game with the above information between two players where each strategy set $S_i := [0, 3c]$ is the interval of non-negative real numbers and the payoffs are given by u_i, $i = 1, 2$. (5)

(b) Find the best response functions for both the players. (5)

(c) Find Nash equilibrium if it exists. (5)

Question 4

You are working as a project manager and technical training lead with FUN-AI. The task assigned to you is to groom a new batch of trainees. You decided to conduct daily quizzes as part of the assessment process. You are required to compile a report with comments on each trainee and submit the same to HR department for further actions.

Based on your past experience you have developed a graphical model[1] as shown in Figure 3.

A quick description of the variables:

- X_t : Random variable indicating the preparedness of the trainee - WellPrepared (WP), NotPrepared (NP)
- Y_t : Random variable modeling the exam day behaviour of the trainee and other exceptional events like trainee falling sick, trainee adopting unfair means, etc. - Normal (N), Exceptional (E)
- O_t : Random variable indicating the score/ performance of the trainee in a quiz - AboveAverage (AA), BelowAverage (BA)

List of conditional probabilities for λ:

- $P(X_0 = WP) = 0.6$
- $P(Y_t = N|X_t = WP) = 0.999$, $P(Y_t = N|X_t = NP) = 0.6$
- $P(O_t = AA|X_t = WP, Y_t = N) = 0.99$, $P(O_t = AA|X_t = WP, Y_t = E) = 0.5$, $P(O_t = AA|X_t = NP, Y_t = N) = 0.2$, $P(O_t = AA|X_t = NP, Y_t = E) = 0.1$

[1]Similar to Hidden Markov Model.

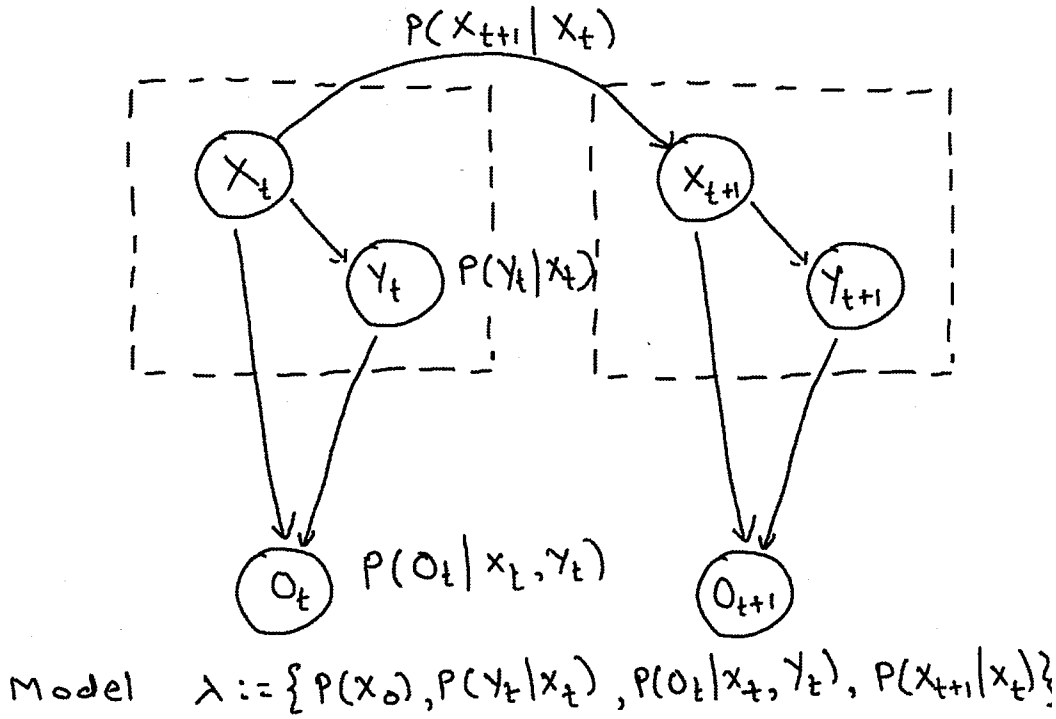

Figure 3: A Graphical Model

- $P(X_{t+1} = WP|X_t = WP) = 0.8$, $P(X_{t+1} = WP|X_t = NP) = 0.5$

For consistency, you ensure that each trainee appears for four quizzes sequentially. Accordingly, if the likelihood of an observation sequence (quiz scores) for a trainee is too low, it is a case worth analysis by HR team. You doubt one of your trainee whose observation sequence is $\mathcal{O} := \{BA,\ BA,\ AA,\ BA\}$.

(a) What is the likelihood $P(\mathcal{O}|\lambda)$? (8)

(b) Decode the preparedness sequence of the trainee. (7)

Question 5

You already know what is to be done for this question. Have fun!

(a) ? and code please. (5)

(b) ? and answer please. (10)

Question 6

A non exhaustive list of concepts covered in CS302/ CS633 is given below: Probability and Statistics; Linear Algebra; Optimization; State Space Search, Heuristic Function; Completeness, Optimality and Complexity of search algorithms; Adversarial Search, min-max agent, Alpha-Beta pruning; Graphical Models, Directed Graphical Models, Independence relation as a tuple of subsets of random variables, Markov Blanket, Factorization of joint probability density function, Naive Bayes Classifier, Hidden Markov Model, Expectation Maximization, Markov Random Field; Decision Tree, Entropy, Information Gain; Decision Conferencing, Analytical Hierarchical Process; N-arm bandit, Markov Decision Process, value func-

tion, action value function, policy evaluation, policy iteration, value iteration; Zero sum normal form games, Nash Equilibria - pure and mixed strategies, Best response, Characterization of NE with the help of best response dynamics, Dominant and Dominated Strategies; and any concepts/ subconcepts not included here but you think are important.

(a) Create AI course outline in the form of a concept dependency graph. Where the directed edge from a concept-1 to concept-2 indicates that concept-1 is required to be understood before concept-2. Is this graph a POSET? Is this graph a LATTICE? Prune this graph and represent the same with the help of a Hasse diagram and write an algorithm to find **min** and **max** elements. Highlight these elements in your graph. (8)

(b) Create AI course outline in the form of an undirected graph where the concepts are connected via an edge if they are correlated. Define strength of these correlations and assign proportionate edge weights. Write an algorithm to identify a chain of concepts (max chain) that accumulates maximum correlation. List all max chains. (7)