Indian Institute of Information Technology-Vadodara
MA 102: Introduction to Discrete Mathematics
Tutorial 5

1. Prove that if x is irrational then $1/x$ also irrational. Identify the method of your proof.

2. Prove that if n is an integer and $3n + 2$ is even, then n is even using
   a) a proof by contraposition.
   b) a proof by contradiction.

3. Prove or disprove that the product of two irrational numbers is irrational.

4. Let $P(n)$ be the proposition "If a and b are positive real numbers, then $(a + b)^n \geq a^n + b^n$." Prove that $P(1)$ is true. What kind of proof did you use?

5. Prove that if x and y are real numbers, then $\max(x, y) + \min(x, y) = x + y$. Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x < y$, respectively.

6. Give a constructive proof to show that there is a positive integer that equals the sum of the positive integers not exceeding it.

7. Prove that either $2 * 10^{500} + 15$ or $2 * 10^{500} + 16$ is not a perfect square.

8. Prove or disprove that there is a rational number x and an irrational number y such that x^y is irrational.

9. Prove that there is no positive integer n such that $n^2 + n^3 = 100$. Which method did you use?

10. Suppose that $A = \{1, 2, 3, 4\}$ and R be the relation on A defined as $(a, b) \in R$ iff $a < b$. Find the matrix, graph representation of R with respect to the natural ordering.

11. Suppose that the relation R on a set is represented by the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

    Is R reflexive, symmetric, antisymmetric?

12. Determine whether the relation R on the set of real numbers- $\mathbb{R}$ is reflexive, symmetric, antisymmetric, transitive, where $(x, y) \in R$ if and only if
    (i) $x + y = 0$
    (ii) $xy \geq 0$
    (iii) $x = 1$ or $y = 1$

13. Let A be the relation "to be wife of" and B "to be father of" on the set of all humans. What does the relation $A \circ B$ mean in this case?