

Graphs

- $G = (V, E)$
- V is the vertex set.
 - Vertices are also called nodes and points.
- E is the edge set.
 - Each edge connects two different vertices.
 - Edges are also called arcs and lines.
 - Directed edge has an orientation (u, v) .



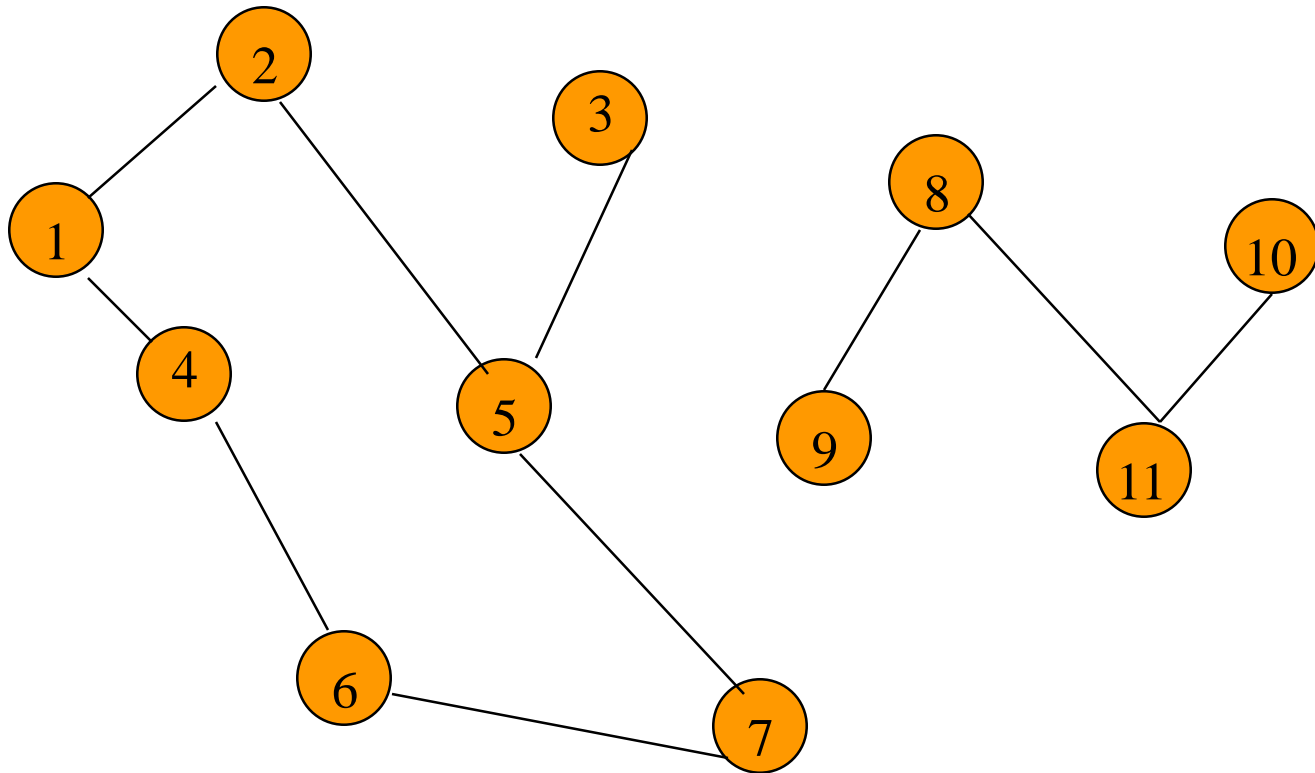
Graphs

- Undirected edge has no orientation (u,v) .

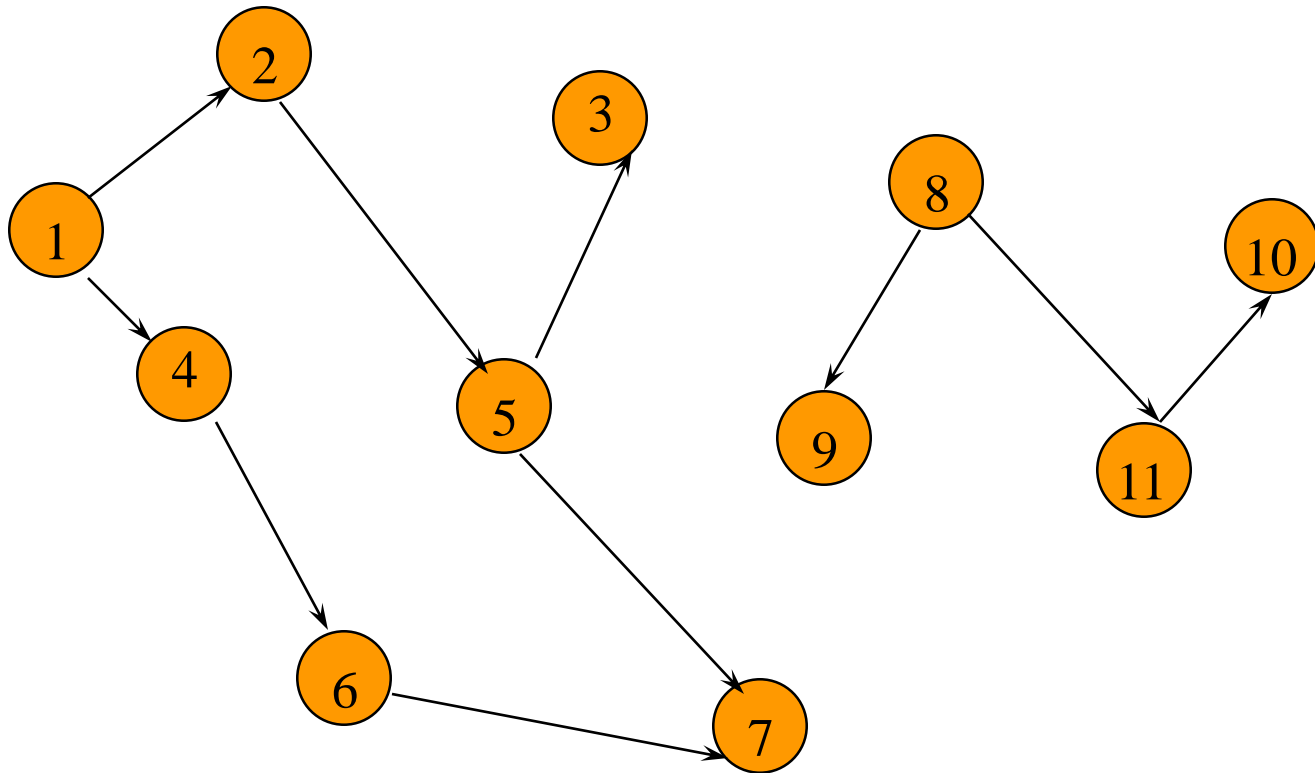
$u \text{ --- } v$

- Undirected graph \Rightarrow no oriented edge.
- Directed graph \Rightarrow every edge has an orientation

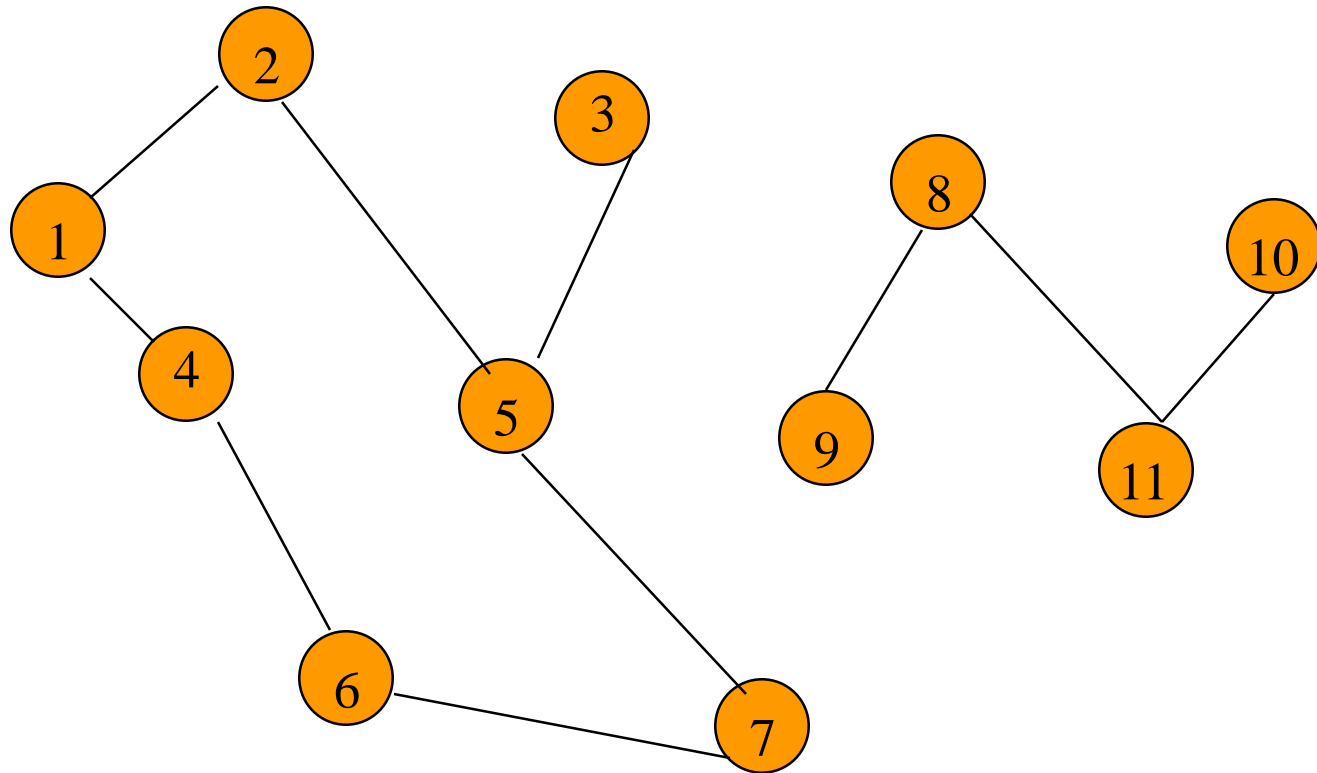
Undirected Graph



Directed Graph (Digraph)

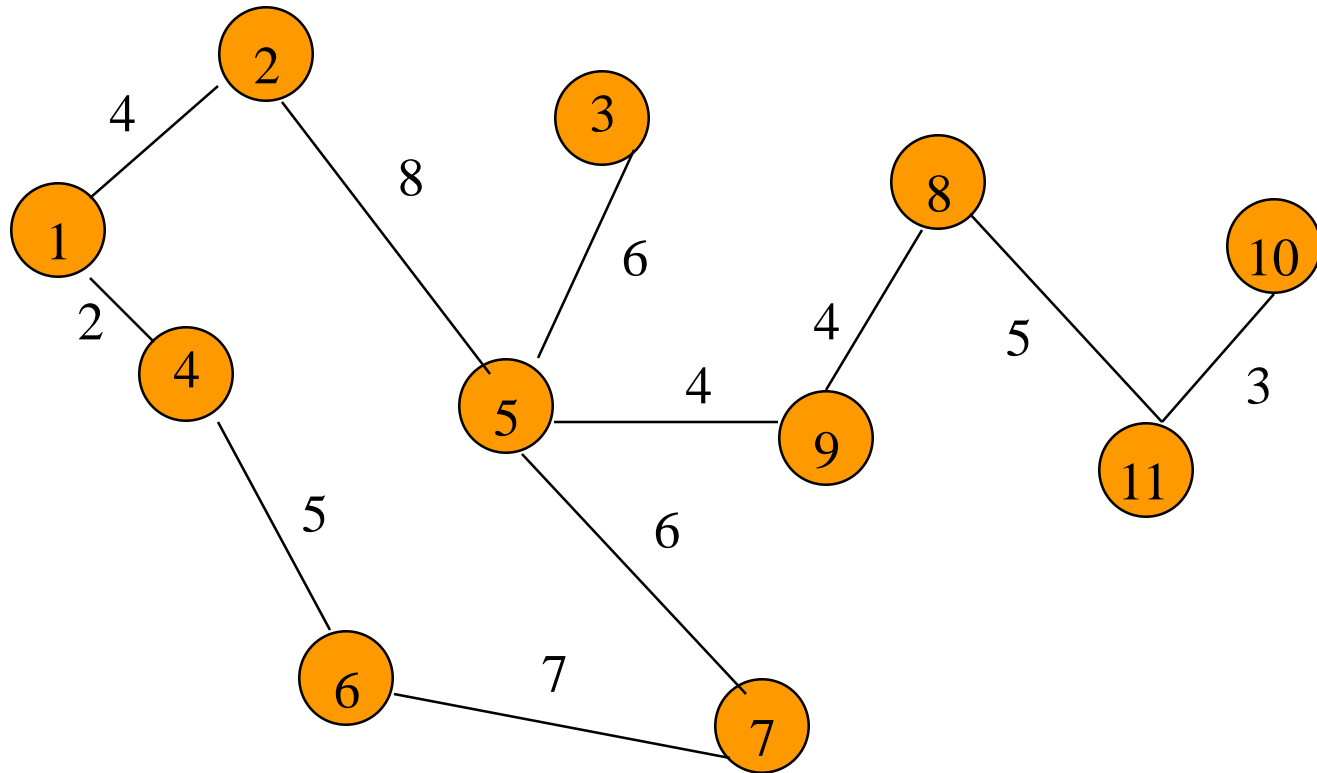


Applications—Communication Network



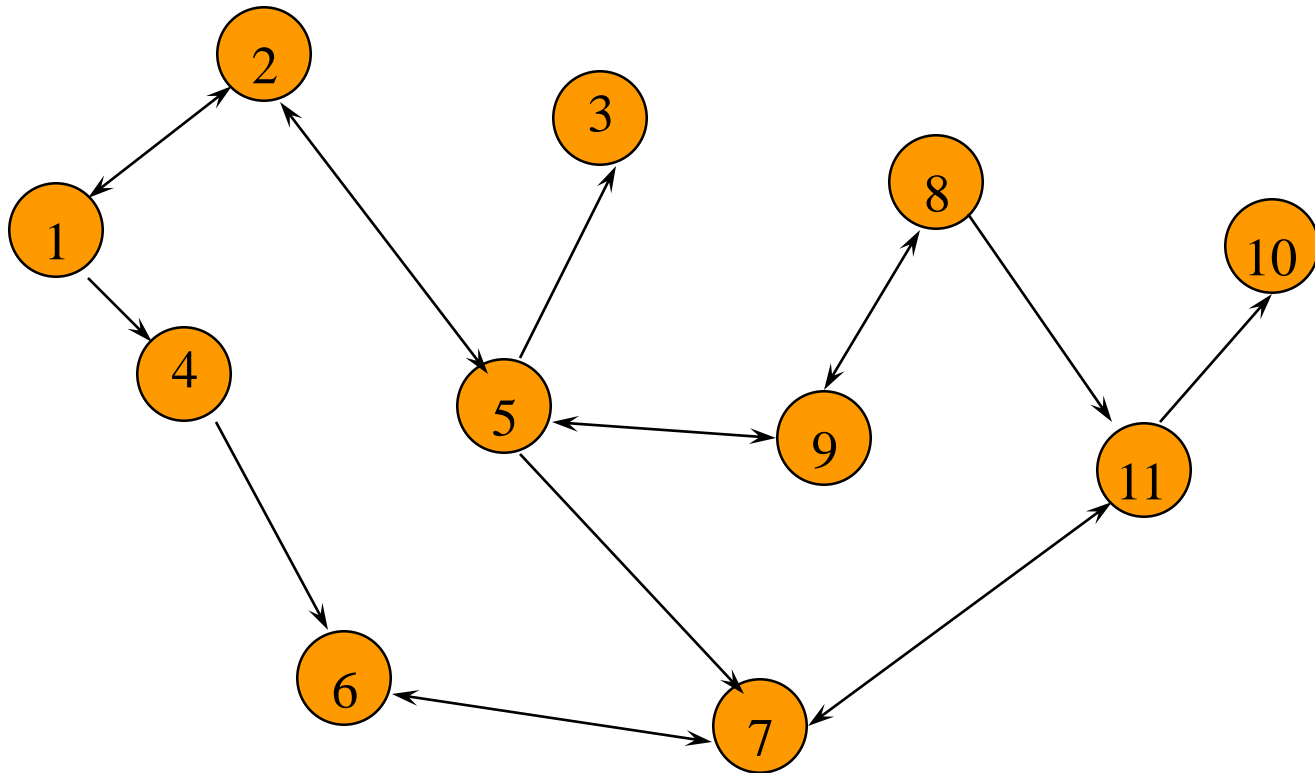
- Vertex = city, edge = communication link.

Driving Distance/Time Map



- Vertex = city, edge weight = driving distance/time

Street Map



- Some streets are one way.

Complete Undirected Graph

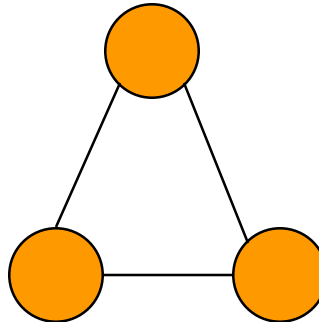
Has all possible edges.



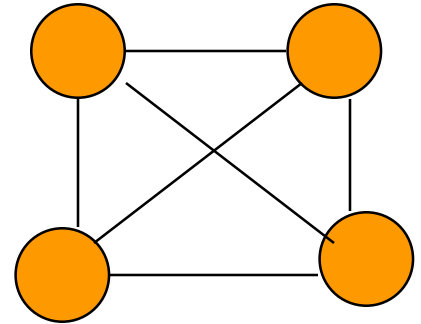
$n = 1$



$n = 2$



$n = 3$



$n = 4$

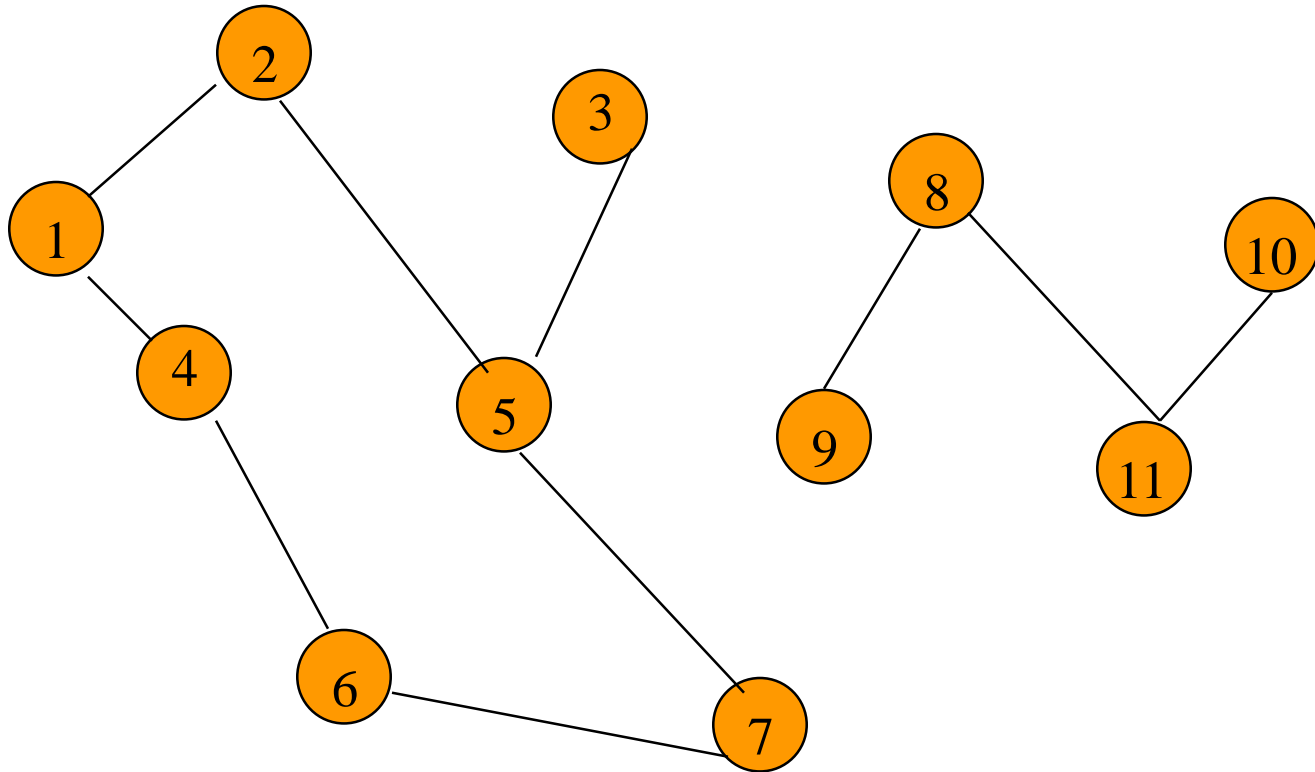
Number Of Edges—Undirected Graph

- Each edge is of the form (u, v) , $u \neq v$.
- Number of such pairs in an n vertex graph is $n(n-1)$.
- Since edge (u, v) is the same as edge (v, u) , the number of edges in a complete undirected graph is $n(n-1)/2$.
- Number of edges in an undirected graph is $\leq n(n-1)/2$.

Number Of Edges—Directed Graph

- Since edge (u,v) is not the same as edge (v,u) , the number of edges in a complete directed graph is $n(n-1)$.
- Number of edges in a directed graph is $\leq n(n-1)$.

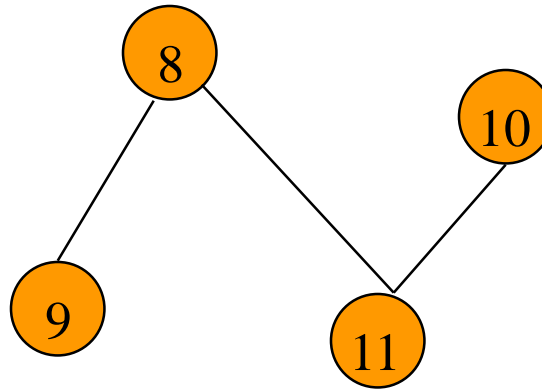
Vertex Degree



Number of edges incident to vertex.

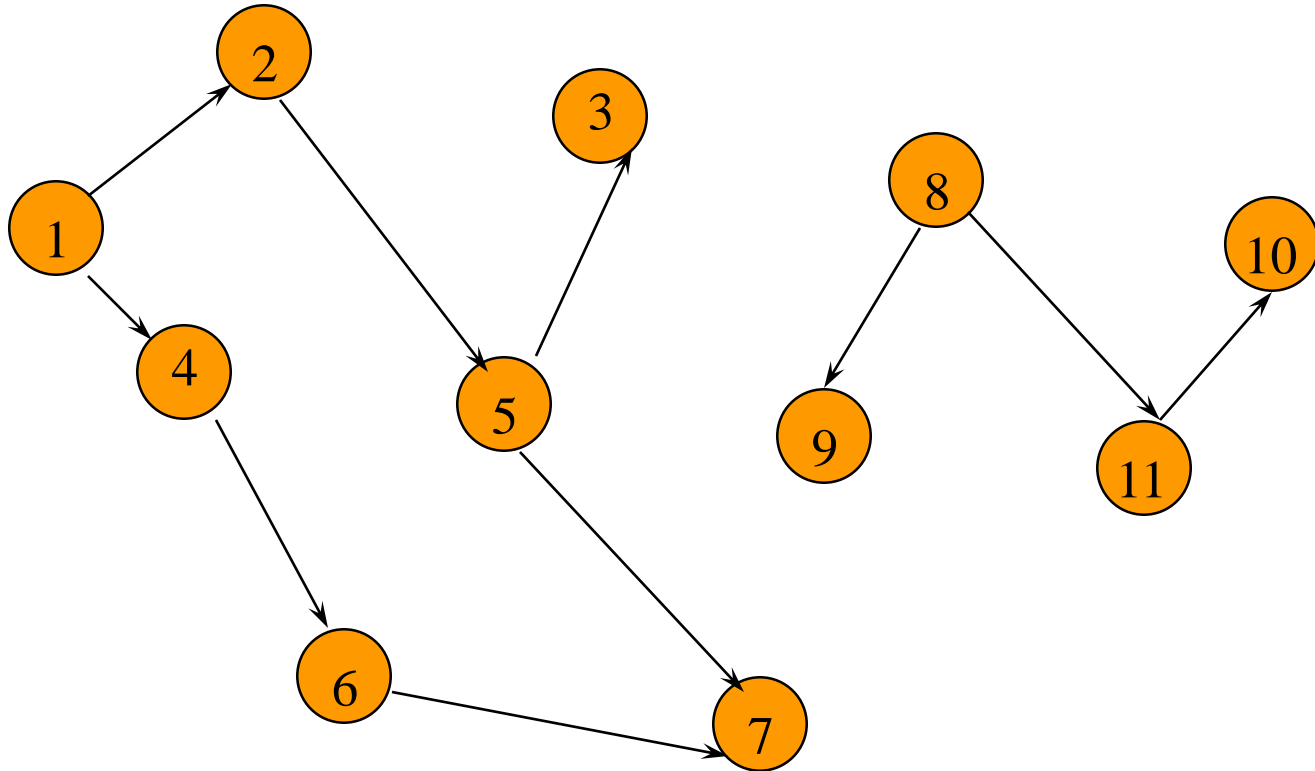
$\text{degree}(2) = 2$, $\text{degree}(5) = 3$, $\text{degree}(3) = 1$

Sum Of Vertex Degrees



Sum of degrees = $2e$ (e is number of edges)

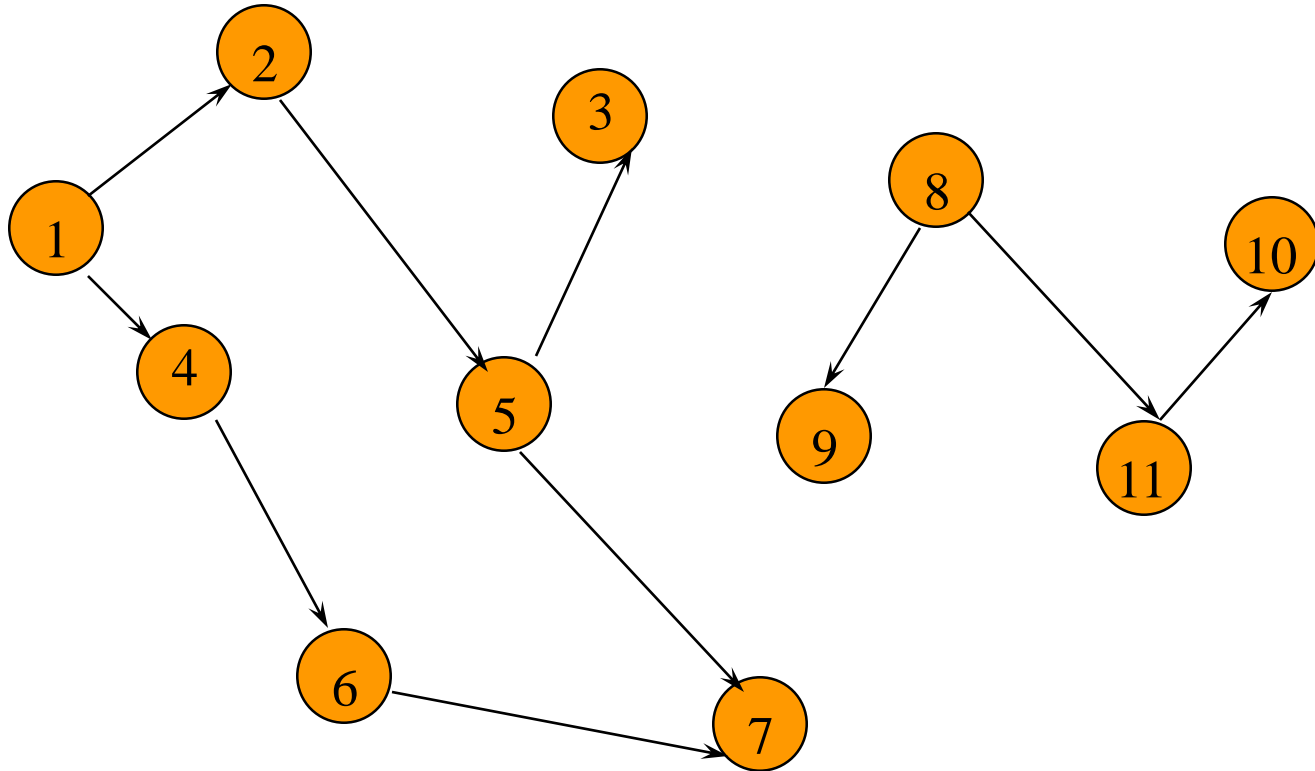
In-Degree Of A Vertex



in-degree is number of incoming edges

$\text{indegree}(2) = 1$, $\text{indegree}(8) = 0$

Out-Degree Of A Vertex



out-degree is number of outbound edges

$\text{outdegree}(2) = 1$, $\text{outdegree}(8) = 2$

Sum Of In- And Out-Degrees

each edge contributes **1** to the in-degree of some vertex and **1** to the out-degree of some other vertex

sum of in-degrees = sum of out-degrees = **e**,
where **e** is the number of edges in the digraph