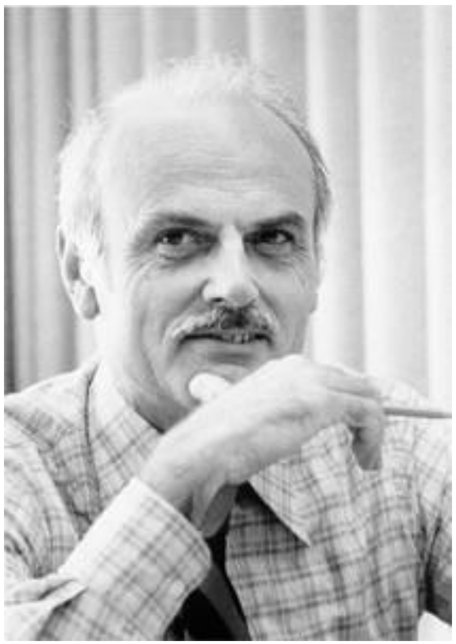
Intro to the Relational Model

Edgar F. Codd (1923-2003)



- Pilot in the Royal Air Force in WW2
- Inventor of the relational model and algebra while at IBM
- Turing Award, 1981

http://en.wikipedia.org/wiki/File:Edgar_F_Codd.jpg

Outline

- Part 1: Relational data model

- Part 2: Relational algebra

Example for Relational data model

User

uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	name
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	gid
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

relations (or tables)

Example for Relational data model

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

attributes (or columns)

Example for Relational data model

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

string int float

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

domain (or type)

Example for Relational data model

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

tuples (or rows)

Duplicates are not allowed

Ordering of rows doesn't matter (even though output is always in some order)

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

Example for Relational data model

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

User: {⟨142, Bart, 10, 0.9⟩, ⟨123, Milhouse, 10, 0.2⟩, … }
Group: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, … }
Member: {⟨142, dps⟩, ⟨123, gov⟩, … }

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a set of attributes (or columns)
- Each attribute has a name and a domain (or type)
  - The domains are required to be atomic
- Each relation contains a set of tuples (or rows)
  - Each tuple has a value for each attribute of the relation
  - Duplicate tuples are not allowed
    - Two tuples are duplicates if they agree on all attributes

Schema vs. instance

- Schema (metadata)
  - Specifies the logical structure of data
  - Is defined at setup time, rarely changes

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

- Instance
  - Represents the data content
  - Changes rapidly, but always conforms to the schema

User: {⟨142, Bart, 10, 0.9⟩, ⟨123, Milhouse, 10, 0.2⟩, … }
Group: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, … }
Member: {⟨142, dps⟩, ⟨123, gov⟩, … }

Integrity constraints

- A set of rules that database instances should follow
- Example:
  - *age* cannot be negative
  - *uid* should be unique in the *User* relation
  - *uid* in *Member* must refer to a row in *User*

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

User: {⟨142, Bart, 10, 0.9⟩, ⟨123, Milhouse, 10, 0.2⟩, … }
Group: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, … }
Member: {⟨142, dps⟩, ⟨123, gov⟩, … }

Integrity constraints

- An instance is only valid if it satisfies all the integrity constraints.

- Reasons to use constraints:
  - Ensure data entry/modification respects to database design
  - Protect data from bugs in applications

Types of integrity constraints

- Tuple-level
  - Domain restrictions, attribute comparisons, etc.
    - E.g. *age* cannot be negative
- Relation-level
  - Key constraints (focus in this lecture)
    - E.g. *uid* should be unique in the *User* relation
  - Functional dependencies (discussed later)
- Database-level
  - Referential integrity – foreign key (focus in this lecture)
    - *uid* in *Member* must refer to a row in *User* with the same *uid*

Key (Candidate Key)

Def: A set of attributes K for a relation R if

- Condition 1: In no instance of R will two different tuples agree on all attributes of K
  - That is, K can serve as a "tuple identifier"
- Condition 2: No proper subset of K satisfies the above condition
  - That is, K is minimal

- Example: *User* (*uid*, *name*, *age*, *pop*)
  - *uid* is a key of *User*
  - *age* is not a key (not an identifier)
  - {*uid*, *name*} is not a key (not minimal), but a superkey

Satisfies only Condition 1

Schema vs. instance

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3

- Is *name* a key of *User*?
  - Yes? Seems reasonable for this instance
  - No! User names are not unique in general

Isn't it confusing?

- Key declarations are part of the schema

More examples of keys

- *Member* (*uid*, *gid*)
  - {*uid*, *gid*}
  - ☞ **A key can contain multiple attributes**
- *Address* (*street_address*, *city*, *state*, *zip*)
  - Key 1: {*street_address*, *city*, *state*}
  - Key 2: {*street_address*, *zip*}
  - ☞ **A relation can have multiple keys!**

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

- Primary key: a designated candidate key in the schema declaration
  - Underline all its attributes, e.g., *Address* (*street_address*, *city*, *state*, *zip*)

Use of keys

- More constraints on data, fewer mistakes
- Look up a row by its key value
  - Many selection conditions are "key = value"
- "Pointers" to other rows (often across tables)

"Pointers" to other rows

- Foreign key: primary key of one relation appearing as attribute of another relation

User

uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

"Pointers" to other rows

- Referential integrity: A tuple with a non-null value for a foreign key that does not match the primary key value of a tuple in the referenced relation is not allowed.

Member

uid	*gid*
142	dps
123	gov
857	Xi
857	gov
456	abc
456	gov
…	…

Group

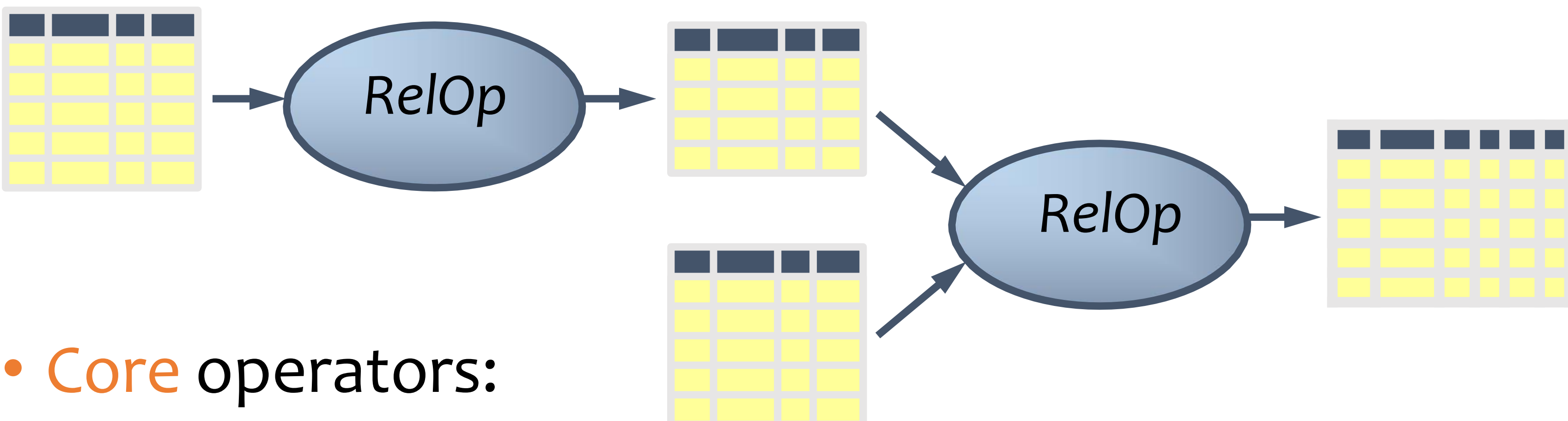
gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society

Outline

- Part 1: Relational data model
  - Data model
  - Database schema
  - Integrity constraints (keys)
  - Languages
    - Relational algebra (focus in this lecture)
    - SQL (next lecture)

Relational algebra

A language for querying relational data based on "operators"



- Core operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

Core operator 1: Selection

- Example: Users with popularity higher than 0.5

$$\sigma_{pop>0.5} User$$

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

$\sigma_{pop>0.5}$

uid	*name*	*age*	*pop*
142	Bart	10	0.9
[redacted]	[redacted]	[redacted]	[redacted]
857	Lisa	8	0.7
[redacted]	[redacted]	[redacted]	[redacted]
…	…	…	…

Core operator 1: Selection

- Input: a table R
- Notation: $\sigma_p R$
  - p is called a selection condition (or predicate)
- Purpose: filter rows according to some criteria
- Output: same columns as R, but only rows of R that satisfy p

More on selection

- Selection condition can include any column of R, constants, comparison ($=$, $\leq$, etc.) and Boolean connectives ($\wedge$: and, $\vee$: or, $\neg$: not)
  - Example: users with popularity at least 0.9 and age under 10 or above 12

$$\sigma_{pop \geq 0.9 \wedge (age < 10 \vee age > 12)}\ User$$

- You must be able to evaluate the condition over each single row of the input table!
  - Example: the most popular user

$$\sigma_{pop \geq every\ pop\ in\ User}\ User$$

WRONG!

Core operator 2: Projection

- Example: IDs and names of all users

$$\pi_{uid,name}\ User$$

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

$\rightarrow \pi_{uid,name} \rightarrow$

uid	*name*
142	Bart
123	Milhouse
857	Lisa
456	Ralph
…	…

Core operator 2: Projection

- Input: a table R
- Notation: $\pi_L R$
  - L is a list of columns in R
- Purpose: output chosen columns
- Output: "same" rows, but only the columns in L

More on projection

- Duplicate output rows are removed (by definition)
  - Example: user ages

$$\pi_{age}\, User$$

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

$\rightarrow \pi_{age} \rightarrow$

age
10
[redacted]
8
[redacted]
…

Core operator 3: Cross product

$$User \times Member$$

User

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7

Member

uid	gid
123	gov
857	abc
857	gov

$\times$

uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov

Core operator 3: Cross product

- **Input:** two tables R and S
- **Notation:** $R \times S$
- **Purpose:** pairs rows from two tables
- **Output:** for each row r in R and each s in S, output a row rs (concatenation of r and s)

A note a column ordering

- Ordering of columns is unimportant as far as contents are concerned

uid	*name*	*age*	*pop*	*uid*	*gid*
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

=

uid	*gid*	*uid*	*name*	*age*	*pop*
123	gov	123	Milhouse	10	0.2
857	abc	123	Milhouse	10	0.2
857	gov	123	Milhouse	10	0.2
123	gov	857	Lisa	8	0.7
857	abc	857	Lisa	8	0.7
857	gov	857	Lisa	8	0.7
…	…	…	…	…	…

- So cross product is commutative, i.e., for any R and S, $R \times S = S \times R$ (up to the ordering of columns)

Derived operator 1: Join

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	gid
123	gov
857	abc
857	gov
…	…

$\bowtie_{User.uid=Member.uid}$

uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
[redacted]					
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

Prefix a column reference with table name and "." to disambiguate identically named columns from different tables

Derived operator 1: Join

- Input: two tables R and S
- Notation: $R \bowtie_p S$
  - p is called a join condition (or predicate)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row r in R and each row s in S, output a row rs if r and s satisfy p
- Shorthand for $\sigma_p(R \times S)$
- (A.k.a. “theta-join”)

Derived operator 2: Natural join

$$User \bowtie Member$$
$$= \pi_{uid,name,age,pop,gid}\left(User \bowtie_{\substack{User.uid= \\ Member.uid}} Member\right)$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	gid
123	gov
857	abc
857	gov
…	…

$\bowtie$

uid	name	age	pop	[redacted]	gid
123	Milhouse	10	0.2	[redacted]	gov
[redacted]					
857	Lisa	8	0.7	[redacted]	abc
857	Lisa	8	0.7	[redacted]	gov
…	…	…	…	[redacted]	…

Derived operator 2: Natural join

- Input: two tables R and S
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
  - Enforce equality between identically named columns
  - Eliminate one copy of identically named columns
- Shorthand for $\pi_L\left(R \bowtie_p S\right)$, where
  - p equates each pair of columns common to R and S
  - L is the union of column names from R and S (with duplicate columns removed)

Core operator 4: Union

- Input: two tables R and S
- Notation: $R \cup S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R and all rows in S (with duplicate rows removed)

R

uid	gid
123	gov
857	abc

$\cup$

S

uid	gid
123	gov
901	edf

=

$R \cup S$

uid	gid
123	gov
857	abc
901	edf

Core operator 5: Difference

- Input: two tables R and S
- Notation: $R - S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R that are not in S

R

uid	gid
123	gov
857	abc

$-$

S

uid	gid
123	gov
901	edf

$=$

R - S

uid	gid
857	abc

Derived operator 3: Intersection

- Input: two tables R and S
- Notation: $R \cap S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows that are in both R and S
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

Core operator 6: Renaming

- Input: a table R
- Notation: $\rho_S\, R$, $\rho_{(A_1 \to A_1', \dots)} R$, or $\rho_{S(A_1 \to A_1', \dots)} R$
- Purpose: "rename" a table and/or its columns
- Output: a table with the same rows as R, but called differently

Member

uid	gid
123	gov
857	abc

$$\rho_{M1(uid \to uid_1, gid \to gid_1)}\ Member$$

M1

uid1	gid1
123	gov
857	abc

Basic operator: Renaming

- As with all other relational operators, it doesn't modify the database
  - Think of the renamed table as a copy of the original

- Used to
  - Create identical column names for natural joins
    - Example: $R(rid, \dots), S(sid, \dots.)$
    - $R \bowtie_{rid=sid} S$ can be written as $(\rho_{(\mathrm{rid}\to\mathrm{id})}R) \bowtie (\rho_{\mathrm{sid}\to\mathrm{id}}S)$
  - Avoid confusion caused by identical column names

Basic operator: Renaming

- Find IDs of users who belong to at least two groups?

$$Member \bowtie_{?} Member$$

uid	*gid*
100	gov
100	abc
200	gov

uid	*gid*
100	gov
100	abc
200	gov

$\bowtie_{?}$

uid	*gid*	*uid*	*gid*
~~100~~	~~gov~~	~~100~~	~~gov~~
100	gov	100	abc
~~100~~	~~gov~~	~~200~~	~~gov~~
100	abc	100	gov
~~100~~	~~abc~~	~~100~~	~~abc~~
~~100~~	~~abc~~	~~200~~	~~gov~~
~~200~~	~~gov~~	~~100~~	~~gov~~
~~200~~	~~gov~~	~~100~~	~~abc~~
~~200~~	~~gov~~	~~200~~	~~gov~~

Condition 1: same uid

Condition 2: different gids

Renaming example

- IDs of users who belong to at least two groups

$$Member \bowtie_{?} Member$$

$$\pi_{uid} \left(Member \bowtie_{\substack{Member.uid = Member.uid \land \\ Member.gid \neq Member.gid}} Member \right)$$

WRONG!

$$\pi_{uid_1} \left(\begin{array}{c} \rho_{(uid \to uid_1, gid \to gid_1)} Member \\ \bowtie_{uid_1 = uid_2 \land gid_1 \neq gid_2} \\ \rho_{(uid \to uid_2, gid \to gid_2)} Member \end{array} \right)$$

Expression tree notation

$$\pi_{uid_1}$$

$$\bowtie_{uid_1 = uid_2 \,\wedge\, \mathrm{gid}_1 \neq \mathrm{gid}_2}$$

$$\rho_{(uid \rightarrow uid_1, \mathrm{gid} \rightarrow \mathrm{gid}_1 \quad)} \qquad\qquad \rho_{(uid \rightarrow uid_2, \mathrm{gid} \rightarrow \mathrm{gid}_2 \quad)}$$

$$Member \qquad\qquad\qquad\qquad Member$$

Class Assignment 1

Quiz 1

- Exercise 1: IDs of groups who have at least 2 users?

- Exercise 2: IDs of users who belong to at least three groups?

Summary of operators

Core Operators

1. Selection: $\sigma_p R$
2. Projection: $\pi_L R$
3. Cross product: $R \times S$
4. Union: $R \cup S$
5. Difference: $R - S$
6. Renaming: $\rho_{\,S(A_1 \rightarrow A'_1,\ A_2 \rightarrow A'_2, \ldots)}\ R$

   Does not really add "processing" power

Note: Only use these operators for assignments & quiz

Derived Operators

1. Join: $R \bowtie_p S$
2. Natural join: $R \bowtie S$
3. Intersection: $R \cap S$

More example

User (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
Group (<u>*gid*</u> string, *name* string)
Member (<u>*uid*</u> int, <u>*gid*</u> string)

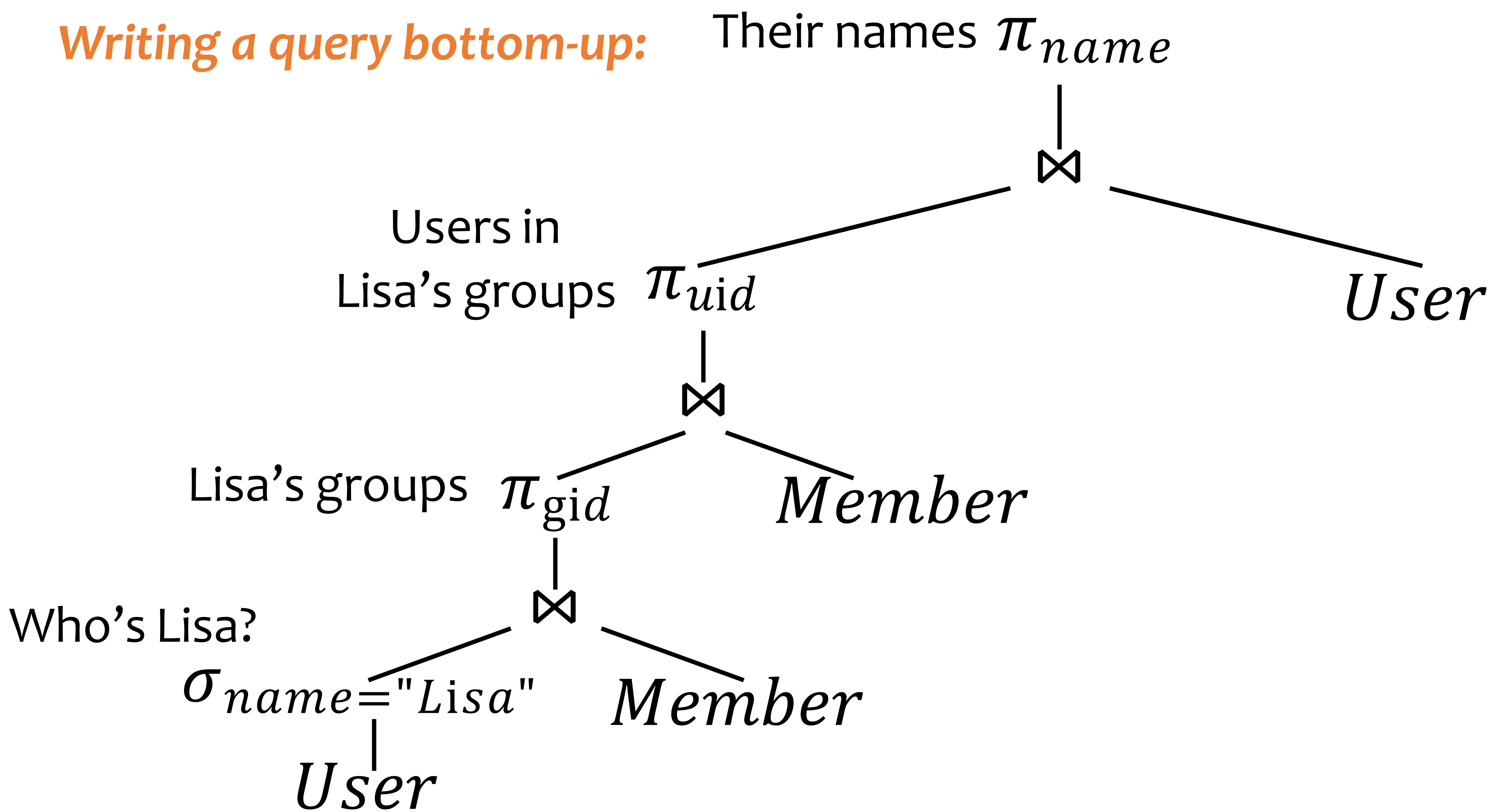
- Names of users in Lisa's groups

More example

User (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
Group (<u>*gid*</u> string, *name* string)
Member (<u>*uid*</u> int, <u>*gid*</u> string)

- Names of users in Lisa's groups

Writing a query bottom-up:



More example

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

- IDs of groups that Lisa doesn't belong to

Writing a query top-down:

More example

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

- IDs of groups that Lisa doesn't belong to

Writing a query top-down: _

More example

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

- IDs of groups that Lisa doesn't belong to

Writing a query top-down:

−

All group IDs

IDs of Lisa's groups

More example

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

- IDs of groups that Lisa doesn't belong to

Writing a query top-down: $-$

All group IDs

IDs of Lisa's groups

π_{gid}

$Group$

More example

User (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
Group (<u>*gid*</u> string, *name* string)
Member (<u>*uid*</u> int, <u>*gid*</u> string)

- IDs of groups that Lisa doesn't belong to

Writing a query top-down:

$-$

All group IDs

IDs of Lisa's groups

π_{gid}

$Group$

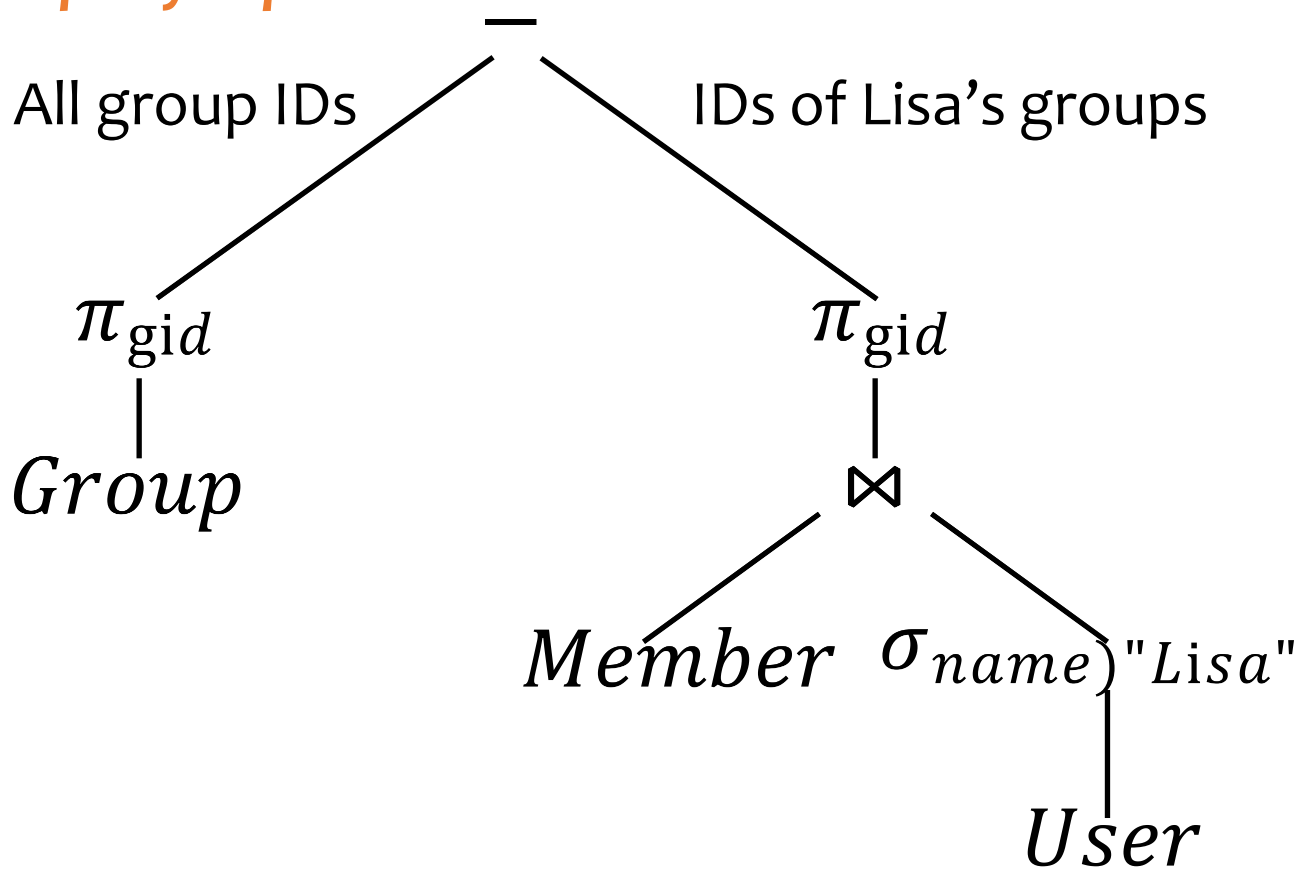
π_{gid}

More example

User (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
Group (<u>*gid*</u> string, *name* string)
Member (<u>*uid*</u> int, <u>*gid*</u> string)

- IDs of groups that Lisa doesn't belong to

Writing a query top-down:



A trickier example

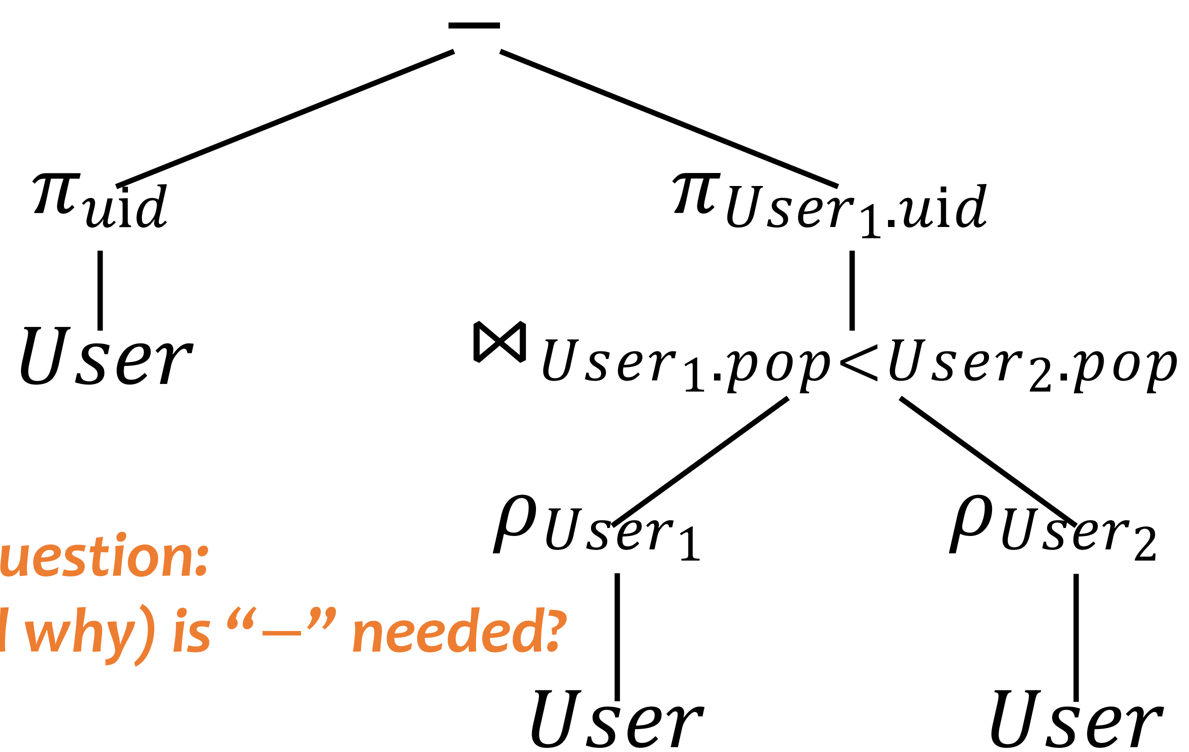
User (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
Group (<u>*gid*</u> string, *name* string)
Member (<u>*uid*</u> int, <u>*gid*</u> string)

- Who are the most popular?
  - Who do NOT have the highest *pop* rating?
  - Whose *pop* is lower than somebody else's?

A trickier example

User (*uid* int, *name* string, *age* int, *pop* float)
Group (*gid* string, *name* string)
Member (*uid* int, *gid* string)

- Who are the most popular?
  - Who do NOT have the highest *pop* rating?
  - Whose *pop* is lower than somebody else's?



A deeper question:
When (and why) is "—" needed?

Why is r.a. a good query language?

- Simple
  - A small set of core operators
  - Semantics are easy to grasp
- Declarative?
  - Yes, compared with older languages like CODASYL
  - Though operators do look somewhat "procedural"
- Complete?
  - With respect to what?

Turing machine

How does relational algebra compare with a Turing machine?

- A conceptual device that can execute any computer algorithm
- Approximates what general-purpose programming languages can do
  - E.g., Python, Java, C++, …

Alan Turing (1912-1954)

http://en.wikipedia.org/wiki/File:Alan_Turing_photo.jpg

Limits of relational algebra

- Relational algebra has no recursion
  - Example: given relation *Friend*(*uid1*, *uid2*), who can Amit reach in his social network with any number of hops?
    - Writing this query in r.a. is impossible! (cannot do aggregate operations such as transitive closure)
- Cannot perform arithmetic operations
- Cannot modify data in a DB
- Cannot sort results

- So r.a. is not as powerful as general-purpose languages

Summary

- Part 1: Relational data model
  - Data model
  - Database schema
  - Integrity constraints (keys)
  - Languages (relational algebra, relational calculus, SQL)

- Part 2: Relational algebra – basic language
  - Core operators & derived operators (how to write a query)
  - V.s. relational calculus
  - V.s. general programming language

- What's next?
  - SQL – query language used in practice (4 lectures)