Indian Institute of Information Technology Vadodara
MA 101: Linear Algebra and Matrices
Tutorial 5

1. Find the rank of following matrices. What can you say about propoerties (one to one, onto and bijective) of its corresponding linear transformation.

$$\begin{bmatrix} -5 & 10 & -5 & 4 \\ 8 & 3 & -4 & 7 \\ 4 & -9 & 5 & -3 \\ -3 & -2 & 5 & 4 \end{bmatrix}$$

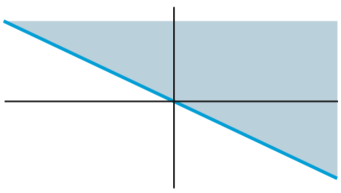
$$\begin{bmatrix} 4 & -7 & 3 & 7 & 5 \\ 6 & -8 & 5 & 12 & -8 \\ -7 & 10 & -8 & -9 & 14 \\ 3 & -5 & 4 & 2 & -6 \\ -5 & 6 & -6 & -7 & 3 \end{bmatrix}$$

2. Is following subset of $\mathbb{R}^n$ a subspace? If yes then prove it else give reason.

a)

$$V = \{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} | x_1 \geq 0 \}$$

b)



3. Determine if w is in the subspace generated by v_1, v_2, where

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ -5 \\ 8 \end{bmatrix}, \text{ and } \mathbf{w} = \begin{bmatrix} 8 \\ 2 \\ -9 \end{bmatrix}.$$

4. Find a basis for Col(A) and a basis for Null(A). Here matrix and its REF is given.

$$A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. What can you say about Nul(B) when B is a 5×4 matrix with linearly independent columns?

6. Let W be the subset of $\mathbb{R}^3$ defined by

$$W = \{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 | 5x_1 - 2x_2 + x_3 = 0 \}.$$

   Find a basis of W.

7. Let U and V be subspaces of the n-dimensional vector space $\mathbb{R}^n$. Prove that the intersection $U \cap V$ is also a subspace of $\mathbb{R}^n$. What can you say about union of two subspaces of $\mathbb{R}^n$?

8. Construct a nonzero 3×3 matrix A and a vector b such that b is not in Col(A).

9. Let T be a linear transformation from $\mathbb{R}^5$ to $\mathbb{R}^5$. If T is one to one then show that T is onto. Is converse true?

10. Give a basis of $\mathbb{R}^3$ containing a vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.