Regular Expressions

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RE

Def^n: i) Σ, λ, ϕ are RE

ii) R_1 & R_2 then union of R_1 & R_2 is a R.E.

$$R_1 + R_2$$

iii) Concatenation of R_1 & R_2 i.e. $R_1.R_2$

iv) iteration on R i.e R^*

v) If R is a RE then (R) is also RE.

vi) RE w.r.t. Σ which can be obtained from above rules is also RE.

Ex. 1) L_1 = set of all string on $\Sigma = \{0,1\}$ ending in 00.

$$R_1 = (0+1)^* 00$$

2) L_2 = $\Sigma = \{0,1\}$ ~~being~~ starting with 0 & ending with 1.

$$R_2 = 0(0+1)^* 1$$

3) $L_3 = \{\lambda, 11, 1111, 111111, \ldots\}$

$$R_3 = (11)^*$$

Identities of REs

1) $\phi + R = R$ 2) $\phi . R = R . \phi = \phi$

3) $\lambda R = R \lambda = R$ 4) $\lambda^* = \lambda$ & $\phi^* = \lambda$

5) $R + R = R$ 6) $R^* R^* = R^*$

7) $R R^* = R^* R$ 8) $(R^*)^* = R^*$

9) $\lambda + R R^* = R^* = \lambda + R^* R$

10) $(RQ)^* R = R(QR)^*$

11) $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

12) $(P+Q) R = PR + QR$ &

$$R(P+Q) = RP + RQ$$

Arden's Theorem

Let P & Q be RE over Σ. If P does not contains λ then the following eq^n in R viz. $R = Q + RP$ has unique sol^n given by $R = QP^*$

Ex. Prove that

$$(1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1)$$
$$= 0^*1(0+10^*1)^*$$

$(1+00^*1)+(1+00^*1)(0+10^*1)^*(\ldots)$

$= 0^* 1 (0+10^*1)^*$

LHS

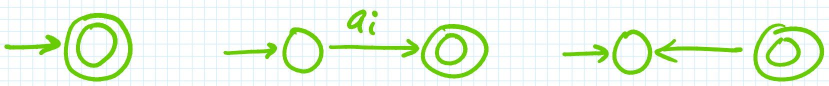
$= \underbrace{(1+00^*1)}_{R} [\lambda + \underbrace{(0+10^*1)^*}_{P^*} \underbrace{(0+10^*1)}_{P}]$

$\equiv (\lambda+00^*) 1 \; P^*$

$\equiv 0^* 1 (\quad\quad) \equiv$ RHS

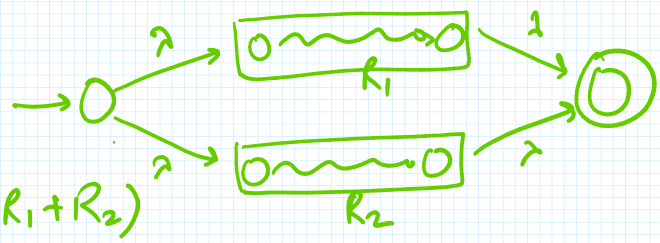
FA & RE

Theorem: Every RE R can be recognised by a transition system and $\exists$ a path from initial state to final state with path value w.



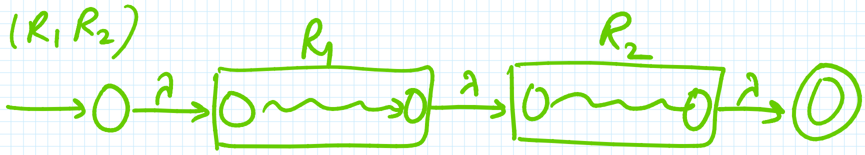
$R = \lambda$ $\qquad$ $a_i \in \Sigma$, $R = a_i$ $\qquad$ $R = \phi$

$R_1 \quad R_2 \qquad R_1 + R_2$

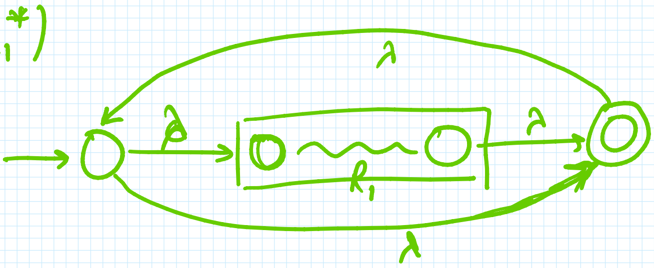


$L(R_1+R_2)$

$L(R_1 R_2)$

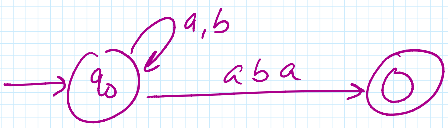


$L(R_1^*)$

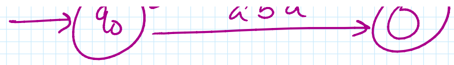


Theorem: Any set L accepted by a FA M is represented by a RE.

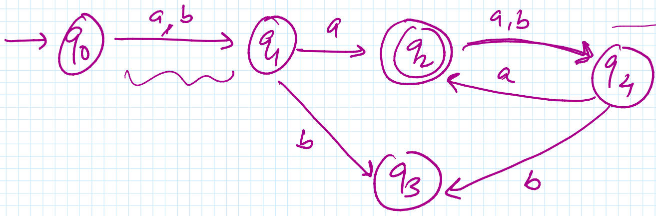
Eg.

1)  $\qquad (a+b)^* . aba$

$(a+b) \cdot ((a+b)a)^*$

1) 

2) $(a+b)\,a\,((a+b)\,a)^*$



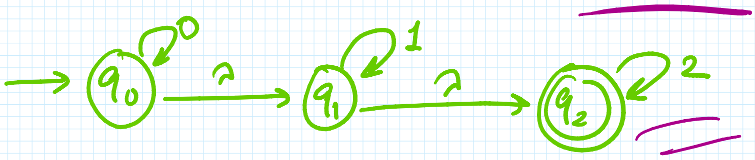
3) $baa + ab + abb$

5) $(a+b)^+$

4) $(a+b)^*\,aa + (a+b)^*$

Transition system having null moves

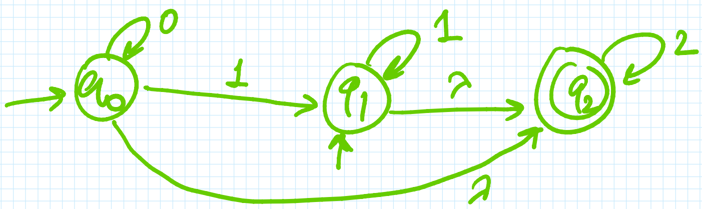
λ - move from v_1 to v_2 $v_1 \xrightarrow{\lambda} v_2$



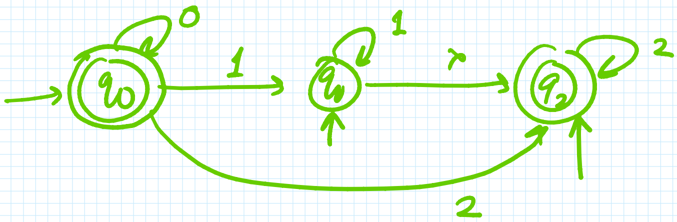
Steps

1. Find all edges starting from v_2
2. Duplicate all these edges starting from v_1 without changing the edge labels.
3. If v_1 is an initial state, make v_2 also as initial state
4. If v_2 is final state, then make v_1 as final state.

(i) λ - move $q_0 \xrightarrow{\lambda} q_1$

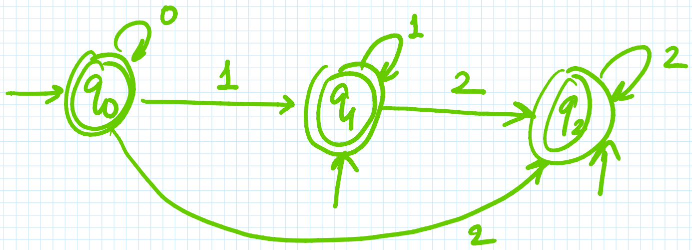


(ii) λ - move $\underset{v_1}{q_0} \xrightarrow{\lambda} \underset{v_2}{q_2}$



(iii)

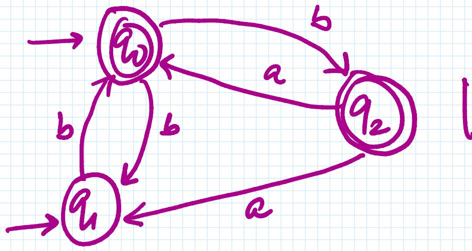
(ii) λ move $q_1 \xrightarrow{\lambda} q_2$



Conversion of non-deterministic system to deterministic system

eg:

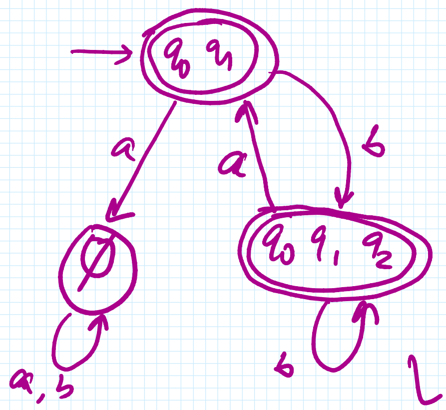
	a	b
$\rightarrow q_0$		q_1, q_2
$\rightarrow q_1$		q_0
q_2	q_0, q_1	



$L_1 = ?$

$L_1 = L_2$??

	a	b
$\rightarrow q_0, q_1$	ϕ	q_0, q_1, q_2
q_0, q_1, q_2	q_0, q_1	q_0, q_1, q_2
ϕ	ϕ	ϕ



$L_2 = ?$

bba