Midterm   **Marks: 30**

Course Instructor: Dr. Dibyendu Roy   Time Limit: 120 minutes
Instructions: Clearly write your name and roll number. Solutions must be argued properly for getting credits. Scientific calculator is allowed.

(Q1)   [5 marks]

Prove that $T(n) = 5n \log_2 n + 500n$ is $\Omega(n)$ and $O(n^{1+\epsilon})$ where $\epsilon > 0$ is an arbitrary small positive constant.

(Q2)   [5 marks]

An integer array A is k-even-mixed if there are exactly k even integers in A, and the odd integers in A appear in sorted order. Given a k-even-mixed array A containing n distinct integers for $k = \lceil \frac{n}{\log n} \rceil$. describe an $O(n)$-time algorithm to sort A.

(Q3)   [5 marks]

A Pythagorean Quad consists of four integers (a, b, c, d) such that $d = \sqrt{a^2 + b^2 + c^2}$. Given an array A containing n distinct positive integers, describe an $O(n^2)$-time algorithm to determine whether four integers from A form a Pythagorean Quad, where integers from A may appear more than once in the Quad. State whether your running time is worst-case or average-case.

(Q4)   [5 marks]

Suppose you have a sorted array A containing n integers. each of which fits into a single machine word. Now suppose someone performs some $\log \log n$ swaps between pairs of adjacent items in A so that A is no longer sorted. Describe an algorithm to best re-sort the integers in A.

(Q5)   [5 marks]

Describe Heap Sort algorithm and prove the followings: (a) correctness (b) complexity.

(Q6)   [5 marks]

Describe a dynamic programming algorithm to compute the minimum number of scalar multiplications required to compute $A = A_1 \times A_2 \times \cdots \times A_m$ where A_i is a matrix of dimension $p_{i-1} \times p_i$. Using your algorithm find the minimum number of multiplications required to compute $A_1 \times A_2 \times \cdots \times A_6$ where the dimensions of the matrices are:

matrix	A_1	A_2	A_3	A_4	A_5	A_6
dimension	30×35	35×15	15×5	5×10	10×20	20×25