Indian Institute of Information Technology Vadodara
MA 101: Linear Algebra and Matrices
Tutorial 6

1. Let V be the set of all 3×3 anti-symmetric matrices with entries from $\mathbb{R}$. Is V a vector space? If yes then find its basis and dimension. What can you say if 3 is replaced by n?

2. Let V be the set of all vectors of the form $\begin{bmatrix} a-2b+5c \\ 2a+5b-8c \\ -a-4c+7c \\ a+b+4c \end{bmatrix}$. Is V a vector space? If yes then find its basis.

3. Which of the following sets are linearly independent set in $\mathbb{R}[X]$?
   (a) $\{1, 2X+1, 3X^2, X\}$
   (b) $\{2X+1, 3X+2\}$
   (c) $\{1, X+1, X^2+X+1, X^3+X^2+X+1\}$

4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by the formula $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} x_1+3x_2-2x_3 \\ 2x_1+3x_2 \\ x_2-x_3 \end{bmatrix}$.
   Determine whether T is an isomorphism and if so find the formula for the inverse linear transformation T^{-1}.

5. Let S be a finite minimal spanning set of a vector space V. That is, S has the property that if a vector is removed from S, then the new set will no longer span V . Prove that S must be a basis of V.

6. Show that if A is $n \times n$ and B is $n \times p$, then rank $(AB) \leq$ rank(A). [Hint: Explain why every vector in the column space of AB is in the column space of A.

7. Show that if A is $n \times n$ and B is $n \times p$, then rank$(AB) \leq$ rank(B). [Hint: Use previous exercise.]

8. Let V be a vecor space over $\mathbb{R}$ of dimension n. Let V' be the set of all linear transformations from V to V. Show that V' is also a vector space over $\mathbb{R}$ and find its basis.

9. Suppose that U and W are finite dimensional subspaces of a vector space V. Then show that $U + W$ is also finite dimensional subspace and $dim(U+W) = dim(U) + dim(W) - dim(U \cap W)$, where $U+W = \{x | x = y + z \text{ for some } y \in U, z \in W\}$.
(Note: $U + W$ is a smallest subspace containing $U \cup W$.)