CS203 Design and Analysis of Algorithm

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Mid Semester Examination - **Set A**
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1. **[3+3 = 6 Marks]** Consider a recurrence relation $T(n) = \sqrt{n}T(\sqrt{n}) + n$. Prove that $\Theta(nlogn)$ and $\Theta(n)$ are **NOT** suitable bounds as per definition of Θ bound [Hint: You can use substitution method].

2. **[3+3 = 6 Marks]** Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 4T(\frac{n}{2}+2) + n$. Use the substitution method to verify your answer.

3. **[6 Marks]** Write an algorithm for 3-way Merge Sort algorithm as discussed in the class. Also, specify complexity of your approach.

4. **[6 Marks]** Attempt any one from the two choices.

   (a) Write an optimal algorithm (using any technique) for maximum product subarray problem where you are given any array having positive and negative numbers and find the maximum product subarray. Justify your solution.

   OR

   (b) Given n pairs of parentheses, write an efficient algorithm (using recursion technique) to generate all combinations of well-formed parentheses.

   Input: n = 3
   Output: ["((()))","(()())","(())()","()(())","()()()"]
   Input: n = 1
   Output: ["()"]

5. **[6 Marks]** Attempt any one from the two choices.

   (a) What is the best way to multiply a chain of matrices with dimension that are $10 \times 5, 5 \times 2, 2 \times 20, 20 \times 12, 12 \times 4$, and 4×60. Show your work. No need to write algorithm.

   OR

   (b) Let $S = \{a, b, c, d, e, f, g\}$ be a collection of objects with benefit-weight values as follows: a:(12,4), b:(10,6), c:(8,5), d:(11,7), e:(14,3), f:(7,1), and g:(9,6). What is the optimal solution to the fractional knapsack for S assuming we have sack (bag) that can hold objects with total weight 18? Show your work.

6. **[5+5=10 Marks]** Attempt any one from the two choices.

   (a) You are given with n files having records and you are asked to merge those files in optimal way. The good news is all records are sorted in the file and when you merge them then don't worry about sorting part. However, you have to create always a new file to merge two files at a time. Write an algorithm to find an optimal merging way to merge n files. Use your optimal binary merge pattern for ten files whose length are 28, 32, 12, 5, 84, 53,91, 35, 3

and 11.

OR

(b) You arrive at Disney Land, where you can participate in various activities and games. Each activity or game has different costs. You can choose only one activity. You've been given a set C of n coins in various values $(c_1, c_2, c_3, \ldots, c_n)$. Write an algorithm to find the fewest number of coins to pay for your activity assuming that you have infinite number of coins for every element in set C. Justify your algorithm by taking a suitable example with minimum 10 coins.