Today’s class

- Standard Forms:

  Sum-of-Products and Product-of-Sums implementation

- Gate-level minimization – part 1

Sum-of-Products (SOP) implementation

- SOP is a Boolean expression with AND terms, called as *products* with one or more literals.
- The *sum* denotes ORing of these product terms.

- Example: $F = y' + xy + x'yz'$

- Realize this expression in two-level implementation.

Product-of-Sums (POS) implementation

- POS is a Boolean expression with OR terms, called as *sums* with one or more literals.
- The *product* denotes ANDing of these sum terms.

- Example: $F = x\,(y' + z)\,(x' + y + z')$

- Realize this expression in two-level implementation.

Gate-level Minimization

- Designing an optimal gate-level implementation of the Boolean functions describing a digital circuit.

- This manual designing helps us to develop the skills for designing and/or developing modern digital design tools.

The MAP method

- Recall: Truth-table => Unique representation
  Boolean simplification => Not unique

- Boolean simplifications lack in specific rules to predict each step in the process.

- Map method is simple, straightforward procedure for minimizing Boolean functions.

- This method is kind of pictorial form of the Truth-table and known as the *Karnaugh map* -or- K-map.

K-map

- K-map is a diagram made of squares, with each square representing one minterm of the function.

- By recognizing various patterns, one can derive a simplest expression for the function.

- The simplified functions are in either of the standard forms of Boolean expression,

  i.e., SOP or POS.

K-map: A simple example
2-input OR gate

Truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

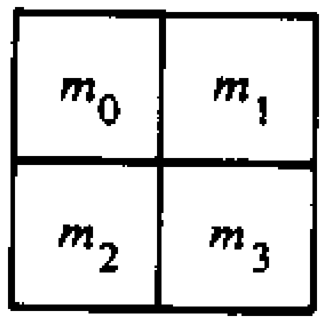
	$\overline{B}$	B
$\overline{A}$		1
A	1	1

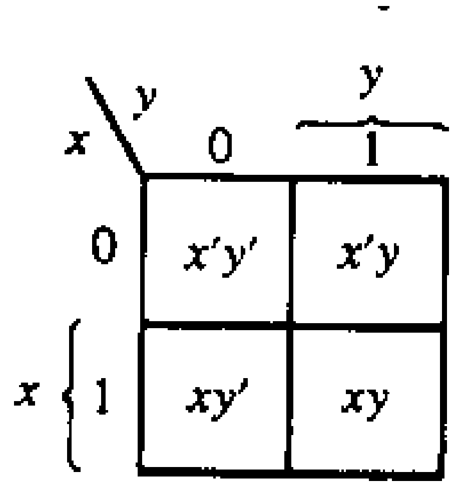
Sum-of-products K-map

	$\overline{B}$	B
$\overline{A}$		
A		1

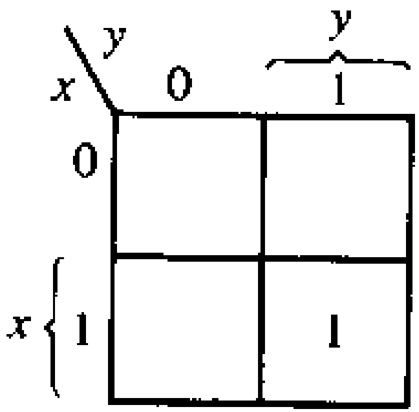
Product-of-sums K-map

Two-variable K-map

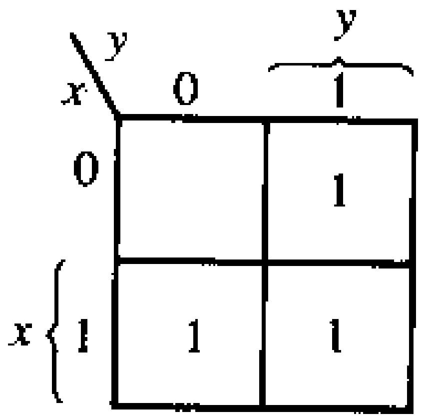




Examples: Two-variable K-maps



(a) xy



(b) $x + y$

Three-variable K-map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

$x \backslash yz$	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
x 1	$xy'z'$	$xy'z$	xyz	xyz'

y: columns 11, 10; z: columns 01, 11

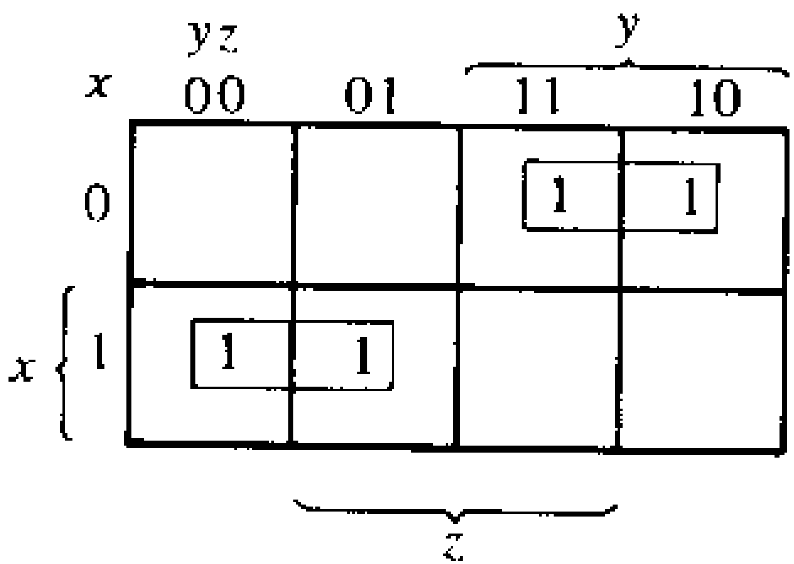
Example: Three-variable K-map

- $F = xy'z + xyz$
- Draw the K-map and find the simplified expression.

- Solution: $F = xz$

Example 1

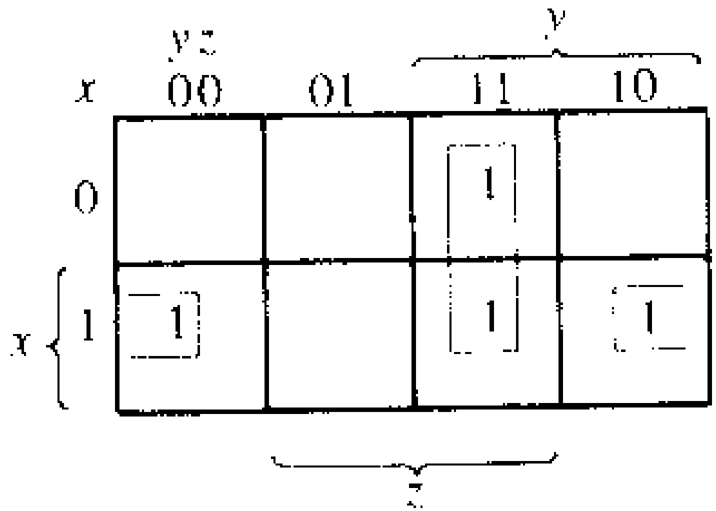
Simplify Boolean function F (x,y,z) = ∑ (2,3,4,5)



Simplification: F (x,y,z) = ∑ (2,3,4,5) = x'y + xy'

Example 2

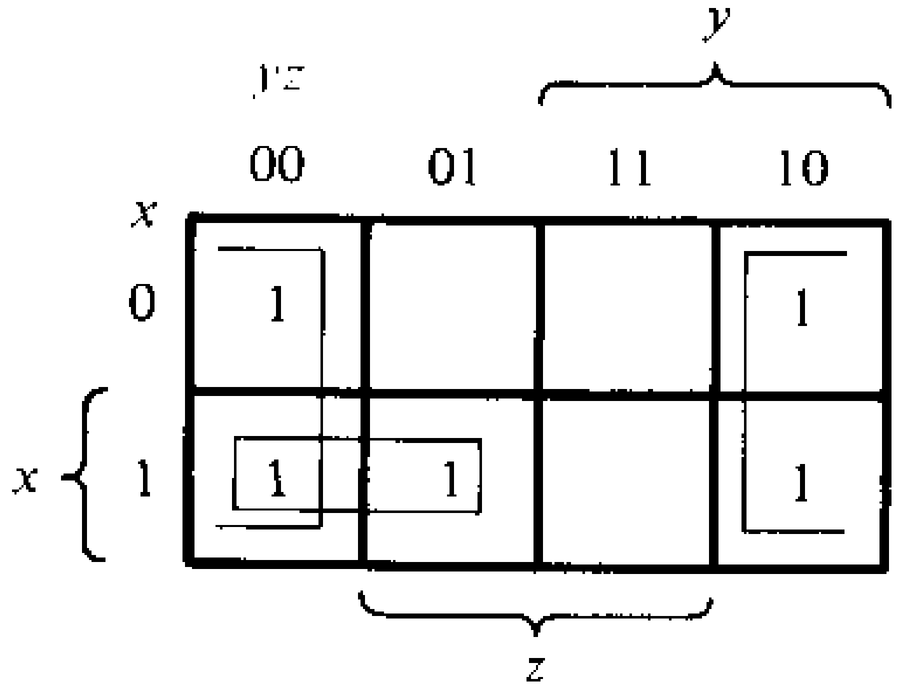
Simplify Boolean function F (x,y,z) = ∑ (3,4,6,7)



Simplification: F (x,y,z) = ∑ (3,4,6,7) = yz + xz’

Example 3

Simplify Boolean function F (x,y,z) = ∑ (0,2,4,5,6)



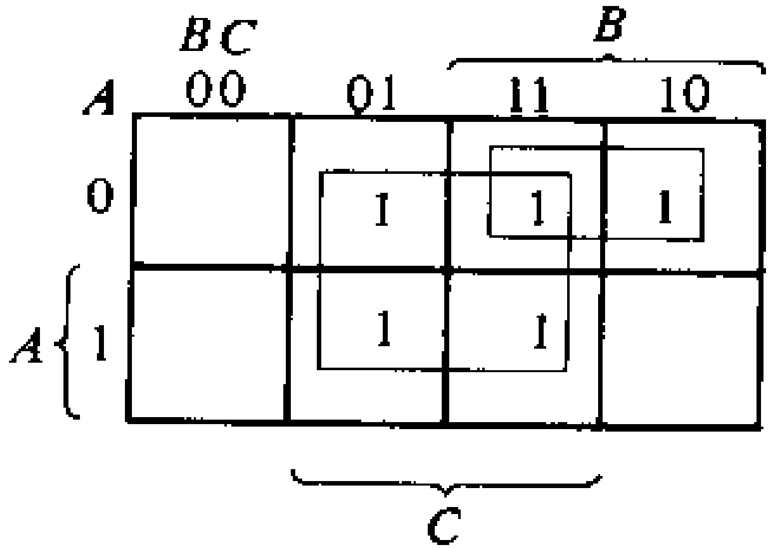
Simplification: F (x,y,z) = ∑ (0,2,4,5,6) = z’ + xy’

Example 4

Given: $F = A'C + A'B + AB'C + BC$

1) Express as a sum of minterms .

2) Find minimal sum-of-products expression.



1) Expression: $F(A,B,C) = \sum(1,2,3,5,7)$

2) Sum-of-Products: $F = C + A'B$

Example 5

Simplify: $S\ (x,y,z) = \sum\ (1,2,4,7)$

Draw the K-map

$S(x,y,z) = x'y'z + x'yz' + xy'z' + xyz$