MA 201
Tutorial 2
(09-12/09/2024)

Note: Submit a separate book maintained only for tutorials to faculty. Loose papers will not be accepted. Enjoy the subject, carp diem. Please answer the numerical questions in the form of a decimal with three significant figures after rounding up eg.: 0.1385 to 0.139.

Q1. Coding Assignment.
This assignment aims to implement the Box-Muller Transform, a method that converts two independent uniform random variables into two independent standard normal (Gaussian) random variables. You will use MATLAB to generate Gaussian-distributed random variables from uniformly distributed random variables and verify the results by comparing them with the normal distribution function made by using the PDF formula in the range.

Tasks:

1. **Implementation: Implement the Box-Muller Transform in MATLAB to generate standard Gaussian random variables Z_1 and Z_2 from uniformly distributed random variables U_1 and U_2 (make use of the lcg function developed in tutorial 1 for generation of U_1 and U_2)**

$$Z1 = \sqrt{-2\ln(U1)}\ \cos(2\pi U2)$$

$$Z2 = \sqrt{-2\ln(U1)}\ \sin(2\pi U2)$$

2. **Validation: Plot a histogram of your generated variables and compare it with the theoretical Gaussian distribution curve. (Use histogram inbuilt function on MATLAB to plot histograms).**

Submission: Submit section-wise on the classwork in Classroom with the code with *tutorial2_<Roll No.>.m* as file name. Don't forget to add comments of your name and roll no. on the top of your code.

Template:

```
%Initialization N, X0, a, c, m %
%Construct U1 and U2%
U1 = lcg(N, X0, a, c, m)
U2 = lcg(N, X0+1, a, c, m)
```

%Construct Z1 and Z2 after applying Box-Muller transform%
%Plot of U1 and U2 before transforming:%
% Histogram of Z1 with line plot of real Gaussian plot to compare%
% Histogram of Z2 with line plot of real Gaussian plot to compare%

Q2.

Here is a process to construct a random number:

1. **Flip a biased coin that comes up heads with a probability of 3/5**
2. **If you get heads, you roll a fair die and return the result.**
3. **Otherwise, you flip a fair coin 3 times and return twice the number of heads.**

Let *N* be the number that you return. Let *F* be the indicator random variable for the first coin flip (1 if heads and 0 if tails).

a. **What is Pr [N=0]?**
b. **What is Pr [N=3]?**
c. **What is Pr [N=6 | F=0]?**
d. **What is Pr [F=0 | N=6]?**
e. **What is Pr [N+F = 5]?**

Q3.

Let *X* be a discrete random variable with the range $R_x = \{1, 2, 3, \ldots.\}$. Suppose the PMF of *X* is given by $P_x(k) = 1/2^k$ for $k = 1,2,3\ldots$.

a. **Find and plot the PDF and the CDF of *X*.**
b. **Find *P(2<X<=5)***
c. **Find *P(X>4)***

Q4.

Let X and Y be jointly continuous random variables with joint PDF as given below.

$$f_{X,Y}(x,y) = \begin{cases} cx+1 & x,y \geq 0, x+y<1 \\ \\ 0 & \text{otherwise} \end{cases}$$

1. **Show the range of (X, Y), R_{XY}, in the x-y plane.**
2. **Find the constant c.**
3. **Find the marginal PDFs $f_X(x)$ and $f_Y(y)$**
4. **Find $P(Y<2X^2)$**