Indian Institute of Information Technology-Vadodara
MA 102: Introduction to Discrete Mathematics
Tutorial 4

1. Show that negation of $\forall x(P(x) \Rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.

2. Express the statement "If a person is female and is a parent, then this person is someone's mother" as a logical expression involving predicates, quantifiers with a domain consisting of all people, and logical connectives.

3. Express the negation of the statement $\forall x \exists y(xy = 1)$ so that no negation precedes a quantifier.

4. Use rules of inference to show that if $\forall x(P(x) \Rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true then $\forall x(R(x) \wedge S(x))$ is true.

5. For following set of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises
i) Every student has an Internet account.
Homer does not have an Internet account.
Maggie has an Internet account.
ii) All foods that are healthy to eat do not taste good.
Tofu is healthy to eat.
You only eat what tastes good.
You do not eat tofu.
Cheeseburgers are not healthy to eat.

6. Use resolution to show the hypotheses "Allen is a bad boy or Hillary is a good girl" and "Allen is a good boy or David is happy" imply the conclusion "Hillary is a good girl or David is happy."

7. Use resolution to show that the hypotheses "It is not raining or Geeta has her umbrella," "Geeta does not have her umbrella or she does not get wet," and "It is raining or Geeta does not get wet" imply that "Geeta does not get wet."

8. What is wrong with this argument? Let $H(x)$ be "x is happy." Given the premise $\exists x H(x)$, we conclude that $H(Lola)$. Therefore, Lola is happy.

9. Find DNF, CNF, PDNF, PCNF of the following formulae
a) $Q \wedge (P \vee \neg Q)$
b) $P \Rightarrow (P \wedge (Q \Rightarrow P))$
c) $(Q \Rightarrow P) \wedge (\neg P \wedge Q)$

10. A statement is in prenex normal form (PNF) if and only if it is of the form

$$Q_1 x_1 Q_2 x_2 \cdots Q_k x_k P(x_1, x_2, \ldots, x_k),$$

where each $Q_i, i = 1, 2, ..., k$, is either the existential quantifier or the universal quantifier, and $P(x_1, ..., x_k)$ is a predicate involving no quantifiers. For example, $\exists x \forall y(P(x, y) \wedge Q(y))$ is in prenex normal form, whereas $\exists x P(x) \wedge \forall x Q(x))$ is not (because the quantifiers do not all occur first). Find PNF of the following: a) $\forall x P(x) \wedge \exists x Q(x)$; b) $\forall x P(x) \vee \exists x Q(x)$