



Indian Institute of Information Technology, Vadodra

Subject: Probability and Statistics
Branch: CSE/IT
Timing: 10:00 AM to 12:00 PM
Session: Autumn 2022-23

Subject Code: MA201
Semester: 3
Date: 17th October 2022
Total Marks: 30

Mid Semester Examination

General Guidelines :

1. All questions are compulsory.
2. Scientific calculator is allowed. ✓

✓ 1. Two number x and y are selected at **random** between zero and one. Let the event A, B, C be defined as follows. $A = \{x > 0.5\}$, $B = \{y > 0.5\}$ and $C = \{x > y\}$. (b)

- (a) Find $P[AB]$, $P[BC]$ and $P[CA]$. (2)
- (b) Find $P[A|B]$, $P[B|C]$ and $P[C|A]$. (2)
- (c) Discuss dependency of all events with each other based on calculations. (1)

✓ 2. A service provider who has leased 30 satellite channels and expects that 10% of channels would be ideal signs a contract with 33 users. He can access additional satellite channels at 50% extra cost at short notice if more than 30 users demand channels simultaneously.

- (a) What is the probability that exactly 30 users request satellite channels? (1)
- (b) What is the probability that all users making a request is served? (2)
- (c) Justify that the service provider make profit/loss from the arrangement. (2)

✓ 3. A coin toss two times. Probability of head is $\frac{1}{3}$. X is random variable that shows number of heads. Y is random variable that shows number of tails.

- (a) Calculate and Draw PMF of X and Y . (2)
- (b) Calculate and Draw PMF of $Z = X + Y$ using Convolution. (3)

✓ 4. A urn consist of 2 black and 2 white balls. A person pick 2 balls randomly. X_1 is number of black balls. X_2 is number of ~~black~~ ^{white} balls. X_3 is numbers of colours presents in selected balls. (2)

- (a) Calculate PMF of X_1, X_2 and X_3 . (2)
- (b) Derive covariance matrix for X_1, X_2 and X_3 . (2)
- (c) Derive correlation matrix for X_1, X_2 and X_3 . (1)

✓ 5. X and Y are random variables with joint density given by (5)

$$f_{X,Y}(x,y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find Covariance $COV(X, Y)$.

✓ 6. For any two independent random variables X and Y , $Z = \min(X, Y)$. (5)

- (a) Find $f_Z(z)$. (3)
- (b) If X and Y are uniform random variable between 0 and 1. find $f_Z(z)$. (2)

Best wishes