Indian Institute of Information Technology Vadodara
MA 101: Linear Algebra and Matrices
Tutorial 1

1. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}$. Show that A is row equivalent to identity matrix by finding the elementary matrices $E_1, E_2, \ldots, E_k$ such that $E_k E_{k-1} \cdots E_1 A = I$.

2. Let A, B, C be square matrices of same dimension. If $AB = BC = I$, then show that $A = C$. This will show that if a matrix has left inverse as well as right inverse then it is unique.

3. In a region around Gandhinagar, about 6% of a city's population moves to the surrounding suburbs each year, and about 4% of the suburban population moves into the city. In 2015, there were 10,000,000 residents in the city and 800,000 in the suburbs. Set up a system of equations that describes this situation, where x_0 is the initial population in 2015. Then estimate the populations in the city and in the suburbs two years later, in 2017.

4. Let A be a 2×2 matrix which commutes with all 2×2 matrices (i.e., $AB = BA$ for every 2×2 matrix B). What can you say about A?

5. Find the inverse of following matrix using block matrix inversion discussed in the class:
$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 3 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

6. If A and B are square matrices and $I - AB$ is invertible then show that $I - BA$ is also invertible. Hint: Use $B(I - AB) = (I - BA)B$.

7. Choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) infinitely many solutions. Give separate answers for each part.
$x_1 + hx_2 = 2$
$4x_1 + 8x_2 = k$

8. Find the interpolating polynomial $P(t) = a_0 + a_1 t + a_2 t^2$ whose graph will pass through (1, 12), (2,15), (3, 16). Does there exists a cubic polynomial which will pass through these points? What about degree n polynomial?