Indian Institute of Information Technology Vadodara
MA 101: Linear Algebra and Matrices
Tutorial 4

1. Find determinant of following matrices:

$$A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{bmatrix}, D = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}$$

2. Show that det(A)=0, where $A = [a_{ij}]_{5\times 5}$, where $a_{ij} = i + j$

3. Suppose $CD = -DC$. Find the mistake in following sentence:
   Taking determinant gives det (C) det $(D) = -$ det (D) det (C), so either det $(C) = 0$, or det $(D) = 0$.

4. Find the determinant of $n \times n$ identity matrix when its i^{th} column is replaced by $[x_1, x_2, \cdots, x_i, \cdots, x_n]^T$.

5. Find the area of the parallelogram whose corner points are $(2, 2), (5, 2), (3, 6), (6, 6)$.

6. Find a formula for the area of the triangle (in terms of determinant) whose vertices are $0, v_1, v_2$ in $\mathbb{R}^2$.

7. Find a formula for $det(rA)$ when A is an $n \times n$ matrix and r is a real no.

8. Use the concept of area of a parallelogram to write a statement about a 2×2 matrix A that is true if and only if A is invertible.

9. For any square matrix A, show that $A.adj(A) = det(A).I$.