Indian Institute of Information Technology Vadodara

MA201 Probability and Statistics

Autumn 2024-25

Mid-sem Exam | 30 marks | Two hours

Que. 1 **03 marks** In an institute, 45% of the students are girls. If 20% of the girl students are be-spectacled and 40% of the be-spectacled students are girls. Find the probability that a randomly chosen student will be a boy without spectacle.

Que. 2 **02 marks** Let X is score of an examination that follows the Normal law with mean value of 100 and variance being 225. What score falls at 95^{th} percentile? Given, $\Phi(1.66) = 0.95$.

Que. 3 **04 marks** Roll a four-sided die twice. The set of all possible outcomes is $S = \{(d_1, d_2) : d_1 = 1, 2, 3, 4;\ d_2 = 1, 2, 3, 4\}$. Let X equal the larger of the two outcomes if they are different and the common value if they are the same. We assume that each of the outcome is equally likely. (a) Find PMF of X, (b) find $E(X)$ and $Var(X)$.

Que. 4 **03 marks** Which of the following(s) is/are valid covariance matrix(ces)? why or why not? comment on the dependence between the random variables referring to the identified valid covariance matrix(ces). (i) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, (ii) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, (iii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and (iv) $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

02 mark Construct an example covariance matrix for which $Var(X_1 + X_2) = Var(X_1)$, where X_1 and X_2 are random variables. You need to write the entries like given in above.

Que. 5 **03 marks** Let a random variable $X \sim \mathcal{N}(\mu, \Sigma)$, where $\mu = [1\ 2\ 2]^T$ and $\Sigma = \begin{bmatrix} 1 & 0 & 0 \\ & 5 & 2 \\ & & 5 \end{bmatrix}$.

Find the value of probability density function (PDF) of X at $x_0 = [0.5\ 0\ 1]^T$.

Que. 6 **05 marks** Let $Y = X_1 + X_2 + \ldots\ldots X_{10}$ where X_i are independent and identically distributed (iid) Bernoulli random variables with $p = P\{X = 1\} = 0.5$, $\forall i$

(a) Find the true value of $P\{Y \geq 9\}$ (b) Find the approximate value of $P\{Y \geq 9\}$ using Markov's Inequality. (c) Find the approximate value of $P\{Y \geq 9\}$ using Chebyshev's inequality.

Que. 7 **05 marks** Two continuous random variables X and Y have the joint density

$$f_{X,Y}(x, y) = C(x^2 + y) \text{ for } -1 \leq x \leq 1, 0 \leq y \leq 1$$

(a) Compute the constant C.

(b) Find the marginal densities of X and Y. Are these two variables independent?

(c) Compute probabilities $P\{Y < 0.6\}$ and $P\{Y < 0.6 | X < 0.5\}$.

Que. 8 **03 marks** A new test for a rare disease is 95% accurate (true positive rate) and 90% accurate for healthy individuals (true negative rate). If 1% of the population has the disease, what is the probability that a person who tests positive actually has the disease?

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