

Indian Institute of Information Technology Vadodara
MA 101: Linear Algebra and Matrices
Tutorial 2

1. Find an LU factorization of the matrices, if possible.

$$\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$$

2. Solve the equation $Ax = b$ by using the LU factorization given for A .

$$A = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

3. Use the definition of Ax to write the matrix equation as a vector equation.

$$\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$$

4. Determine if $Ax = b$ has a solution by checking whether b is a linear combination of a_1, a_2 , and a_3 , where i^{th} column of A is equal to a_i .

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, a_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

5. Give a geometric description of $\text{Span}\{v_1, v_2\}$ for the vectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

6. A steam plant burns two types of coal: anthracite (A) and bituminous (B). For each ton of A burned, the plant produces 27.6 million Btu of heat, 3100 grams (g) of sulfur dioxide, and 250 g of particulate matter (solid-particle pollutants). For each ton of B burned, the plant produces 30.2 million Btu, 6400 g of sulfur dioxide, and 360 g of particulate matter. Over a certain time period, the steam plant produced 162 million Btu of heat, 23,610 g of sulfur dioxide, and 1623 g of particulate matter. Determine how many tons of each type of coal the steam plant must have burned.
7. Describe all solutions of $Ax = 0$ in parametric vector form, where A is

$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

8. Construct a 3×3 nonzero matrix A such that the following vector v is a solution to $AX = 0$.

$$v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

9. Given $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, find one nontrivial solution of $Ax = 0$ by inspection.