

Today's class

- Standard Forms:
Sum-of-Products and Product-of-Sums
implementation
- Gate-level minimization – part 1

Sum-of-Products (SOP) implementation

- SOP is a Boolean expression with AND terms, called as *products* with one or more literals.
- The *sum* denotes ORing of these product terms.
- Example: $F = y' + xy + x'yz'$
- Realize this expression in two-level implementation.

Product-of-Sums (POS) implementation

- POS is a Boolean expression with OR terms, called as *sums* with one or more literals.
- The *product* denotes ANDing of these sum terms.
- Example: $F = x (y' + z) (x' + y + z')$
- Realize this expression in two-level implementation.

Gate-level Minimization

- Designing an optimal gate-level implementation of the Boolean functions describing a digital circuit.
- This manual designing helps us to develop the skills for designing and/or developing modern digital design tools.

The MAP method

- Recall: Truth-table \Rightarrow Unique representation
Boolean simplification \Rightarrow Not unique
- Boolean simplifications lack in specific rules to predict each step in the process.
- **Map method** is simple, straightforward procedure for minimizing Boolean functions.
- This method is kind of pictorial form of the Truth-table and known as the *Karnaugh map* -or- **K-map**.

K-map

- K-map is a diagram made of squares, with each square representing one minterm of the function.
- By recognizing various patterns, one can derive a simplest expression for the function.
- The simplified functions are in either of the standard forms of Boolean expression, i.e., SOP or POS.

K-map: A simple example

2-input OR gate

Truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

	\bar{B}	B
\bar{A}		1
A	1	1

Sum-of-products K-map

	\bar{B}	B
\bar{A}		
A		1

Product-of-sums K-map

Two-variable K-map

m_0	m_1
m_2	m_3

		y	
		0	$\overbrace{1}^y$
x	0	$x'y'$	$x'y$
	$\left\{ \begin{array}{l} 1 \end{array} \right.$	xy'	xy

Examples: Two-variable K-maps

		y	
		0	1
x	0		
	1		1

(a) xy

		y	
		0	1
x	0		1
	1	1	1

(b) $x + y$

Three-variable K-map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

		yz			
		x	00	01	$\overbrace{11 \quad 10}^y$
x	0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
	1	$xy'z'$	$xy'z$	xyz	xyz'
		$\underbrace{\hspace{10em}}_z$			

Example: Three-variable K-map

- $F = xy'z + xyz$
- Draw the K-map and find the simplified expression.
- Solution: $F = xz$

Example 1

Simplify Boolean function $F(x,y,z) = \sum (2,3,4,5)$

		yz		y	
x		00	01	11	10
x	0			1	1
	1	1	1		

Diagram illustrating the Karnaugh map for the Boolean function $F(x,y,z) = \sum (2,3,4,5)$. The map is a 2x4 grid with rows labeled x (0 and 1) and columns labeled yz (00, 01, 11, 10). The function is 1 for minterms 2, 3, 4, and 5, which are marked with '1' in the cells (0, 11), (0, 10), (1, 00), and (1, 01). The map shows two groups of adjacent 1s: a group of two 1s in the top row (minterms 2 and 3) and a group of two 1s in the bottom row (minterms 4 and 5). These groups are highlighted by boxes. A bracket below the bottom row indicates the variable z is the common factor for the bottom row group.

Simplification: $F(x,y,z) = \sum (2,3,4,5) = x'y + xy'$

Example 2

Simplify Boolean function $F(x,y,z) = \sum (3,4,6,7)$

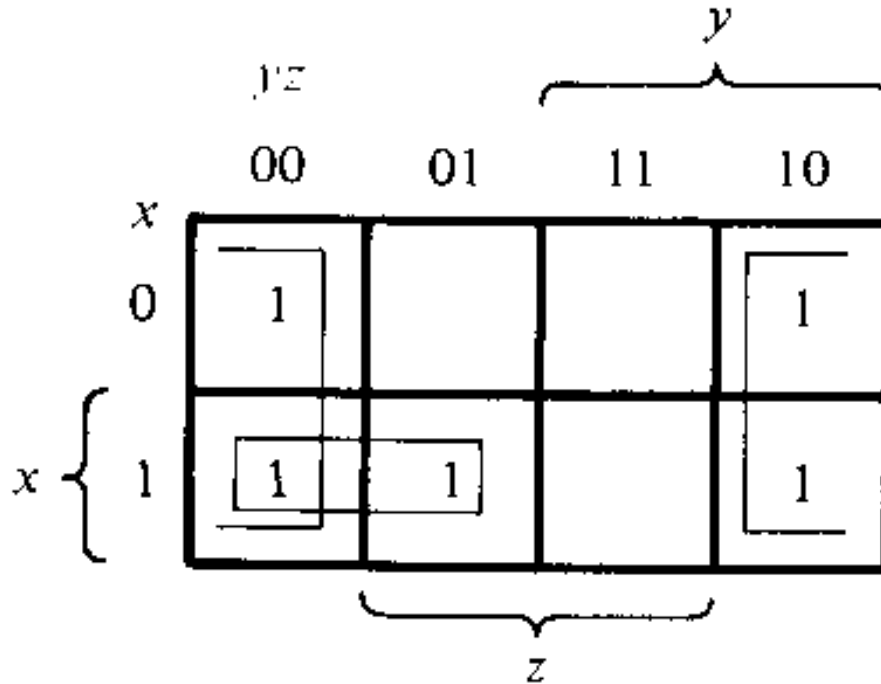
		yz		y	
x		00	01	11	10
0				1	
1		1		1	1

Diagram illustrating the Karnaugh map for the Boolean function $F(x,y,z) = \sum (3,4,6,7)$. The map is a 2x4 grid with rows labeled x (0, 1) and columns labeled yz (00, 01, 11, 10). The function value is 1 for minterms 3, 4, 6, and 7, which are circled in the map. The map shows two groups of 1s: a group of 2 (minterms 3 and 7) and a group of 3 (minterms 4, 6, and 7).

Simplification: $F(x,y,z) = \sum (3,4,6,7) = yz + xz'$

Example 3

Simplify Boolean function $F(x,y,z) = \sum (0,2,4,5,6)$

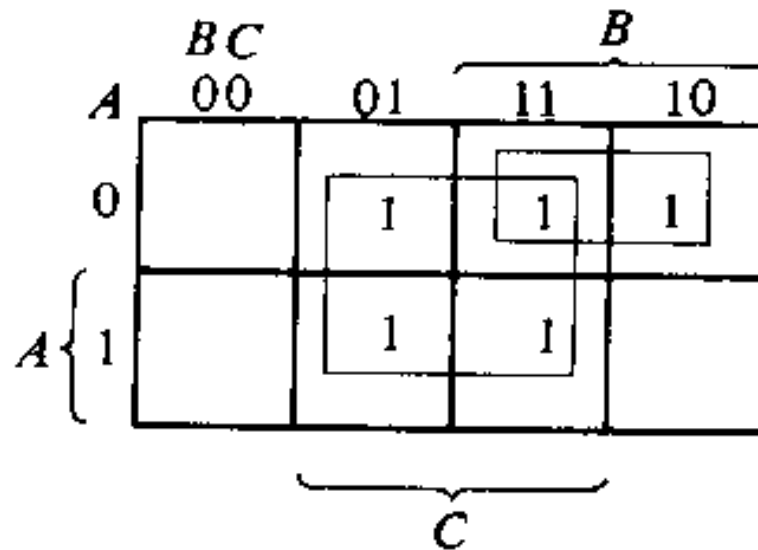


Simplification: $F(x,y,z) = \sum (0,2,4,5,6) = z' + xy'$

Example 4

Given: $F = A'C + A'B + AB'C + BC$

- 1) Express as a sum of minterms .
- 2) Find minimal sum-of-products expression.



1) Expression: $F(A,B,C) = \sum (1,2,3,5,7)$

2) Sum-of-Products: $F = C + A'B$

Example 5

Simplify: $S(x,y,z) = \sum (1,2,4,7)$

Draw the K-map

$$S(x,y,z) = x'y'z + x'yz' + xy'z' + xyz$$