Indian Institute of Information Technology Vadodara
MA 101: Linear Algebra and Matrices
Tutorial 9

1. Find the distance between $u = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}, v = \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}$.

2. Let $u = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$, and let W be the set of all vectors of $\mathbb{R}^3$ which are orthogonal to u. Show that W is a subspace of $\mathbb{R}^3$ and find an orthogonal basis of W. Give an orthogonal basis of $\mathbb{R}^3$ containing u.

3. Verify the parallelogram law for vectors u and v:
$||u+v||^2 + ||u-v||^2 = 2.||u||^2 + 2.||v||^2$

4. Compute the orthogonal projection of $u = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ on to the line through origin and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and distance of u to the line.

5. Write $v = \begin{bmatrix} 4 \\ 5 \\ -3 \\ 3 \end{bmatrix}$ as a linear combination of u_i, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}, \quad \mathbf{u}_4 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -2 \end{bmatrix}$$

6. Let $W = span\{u_1, u_2\}$ and $U = [u_1 \ \ u_2]$. Compute $UU^T, U^TU, \text{Proj}_W y, UU^Ty$. What do you observe?

$$\mathbf{y} = \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$$

7. Let $A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 2 & -2 & 9 \\ 4 & -14 & -3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find $p, u \in \mathbb{R}^3$ such that $x = p + u, p \in \text{Row}(A), u \in \text{Null}(A)$.

8. Find an orthonormal basis-$\{u_1, u_2, u_3\}$ for the column space of the matrix $A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$. Let $Q = [u_1 \ u_2 \ u_3]$ with its columns and $R = Q^T A$. Verify $A = QR$? Use this factorisation to find least square solution of $AX = b$, where $b = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$

9. Describe all least-squares solutions of the equation $Ax = b$ where $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$.

10. Show that $Null(A) = Null(A^T A)$ for any $m \times n$ matrix A.