- **Boolean Algebra:**
- **When no. of variables are less i.e. 1,2,3.**
- **It is preferred when output is 0 or 1.**

- **K-Map:**
- **When no. of variables are less i.e. 2,3,4 (upto 5 variables).**
- **It is preferred when output is 0, 1 or X.**
- **Tabulation Method:**
- **It is preferred when no. of variables are greater than 5.**

- **Operator order in Boolean expression:**
- **Parentheses**
- **NOT**
- **AND**
- **OR**

- **Example:**

- $x + (y + z) = (x + y) + z$
- $x . (y . z) = (x . y) . Z$

- **Remember the operators order.**

- **Basic theorem based on Boolean postulates:**
- **NOT:**
- **(X')'= X**
- **AND:**
- **X.X = X**
- **X.1 = X**
- **X.0 = 0**
- **X.X' = 0**

- **OR:**
- **X+X = X**
- **X+1 = X**
- **X+0 = X**
- **X+X' = 1**

- $\mathbf{X + (X \,.\, Y) = X}$
- $\mathbf{X \,.\, (X + Y) = X}$

- **Problem:**
- **XY+XY'**

- **Solution:**
- **XY+XY'**
  **X(Y+Y')= X, [Since: Y+Y'= 1]**

- **Theorem: Identity**
- 0(+)
- 1(.)
- **Theorem: Commutative**
- X+Y = Y+X
- X.Y = Y.X

- **Theorem: Distributive**
- X(Y+Z)= (X+Y) + (Y+Z)
- X + (Y.Z)= (X+Y) . (Y+Z)

- **Theorem: Transposition**
- (X+Y) . (X+Z) = (X+YZ)

- **Theorem: DeMorgan's**
- (X+Y)'= X'. Y'
- (X.Y)' = X' + Y'

- **Problem: Minimize the following Boolean function:**
- XY' + XYZ' + XY'Z'W

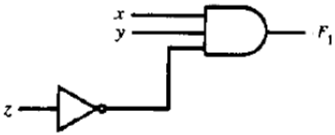
- **Solution:**
- XY' + XYZ' + XY'Z'W = XYZ' + XY' (1 + Z'W)
- XYZ' + XY', [Since: 1 + A = 1]
- X (Y + YZ')
- X (Y' + Z'), [Since: Y + YZ' = Y' + Z' ]
- XY' + XZ'

- **Advantages of minimization:**
- **No. of logic Gates: Less**
- **Speed: High**
- **Power dissipation: Lower**
- **Complexity of logic circuits: Less**
- **Fan-in: Low**
- **Cost: Low**

- **Boolean Function:**
- **An algebraic expression formed by binary variables.**
- **An operators AND and OR, unary operator NOT.**
- **Parentheses, and equal sign.**
- **Example.**
- F1 = xyz'
- variables: x, y, z
- Gates: 1 NOT gate; 1 AND Gate.
- Output: F1 can be either 0 or 1, for a given input.

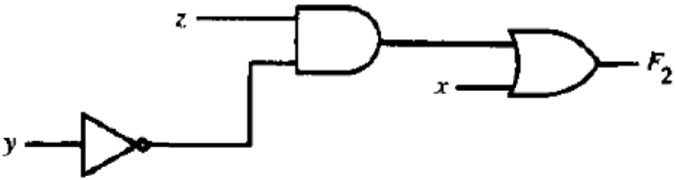
# F1 = xyz'
# when output is high / low ?

| $x$ | $y$ | $z$ | $F_1$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |



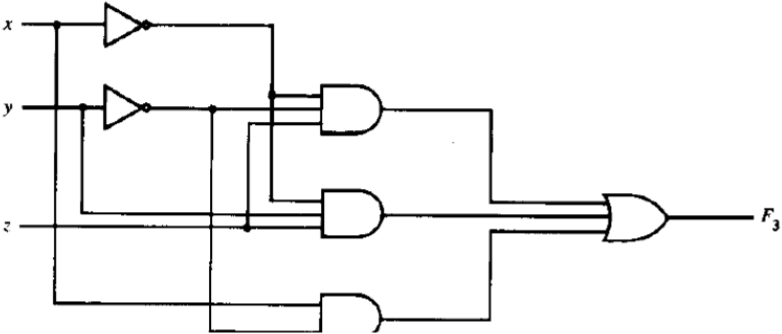
$F2 = x + y'z$

when output is high / low ?



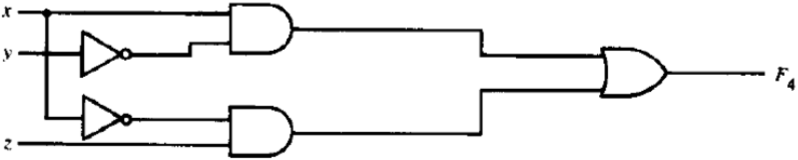
$$F3 = x'y'z + x'yz + xy'$$

when output is high / low ?



F4 = xy' + x'z

when output is high / low ?



- **Problem:**
- **XY + XY'Z + XY'Z', find the minimum number of NAND Gates.**

- **Solution:**
- **XY + XY'Z + XY'Z'**
- XY + XY' (Z + Z')
- XY + XY'
- X (Y + Y')
- X

**Zero NAND Gate.**

- **Problem:**
- **(X+Y+Z)(X+Y'+Z)(X+Y+Z')**

- **Solution:**
- **Take A = X+Y**
- (A+Z)(X+Y'+Z)(A+Z')
- (A+ZZ')(X+Y'+Z)
- A (Y + Y' + Z)
- (X+Y)(X+Y'+Z)
- X + YZ

- **Problem:**
- $\mathbf{(X+Y)(X+Y')(X'+Y)(X'+Y')}$

- **Solution:**
- **(X+YY')(X'+YY')**
- **X.X'**
- **0**

- **Redundant term: Not needed**
- **Example:**
- XY+X'Z+YZ
- YZ is redundant term
- **Solution:**
- XY+X'Z+YZ(X+X')
- XY+X'Z+XYZ+X'YZ)
- XY(1+Z)+X'Z(1+Y)
- XY+X'Z)

- **Redundancy Theorem:**
- **Three variables.**
- **Each variable comes twice.**
- **Out of three variables one variable is complement.**
- **The term which is complement is taken.**
- **Example:**
- AB+A'C+BC
- AB+A'C

- **Complement of a Boolean function:**
- **Use of DeMorgan's theorem**
- **Use of Duality**

# Complement by DeMorgan's theorem

Example: $F1 = x'yz' + x'y'z$

Find $F1'$

$$
\begin{aligned}
F1' &= (x'yz' + x'y'z)' \\
&= (x'yz')' (x'y'z)' \\
&= (x + y' + z)(x + y + z')
\end{aligned}
$$

<u>Check:</u> truth-table

- **Problem:**
- **Find F2' for F2 = x (y'z' + yz)**

- **Solution:**
- **$F2' = (x\,(y'z' + yz))'$**
- $F2' = x' + (y'z' + yz)'$
- $F2' = x' + (y'z')'\,(yz)'$
- $F2' = x' + (y + z)\,(y' + z')$

# Complement by Duality

- Each AND => OR
- Each OR => AND

Dual of Boolean function

- Complement each literal.

# Again do F1' using Duality

$F1 = x'yz' + x'y'z$

$$F1' = (x' + y + z')(x' + y' + z) \quad \text{(dual of F1)}$$
$$= (x + y' + z)(x + y + z') \quad \text{(complementing each literal)}$$

Look back to the previous answer of F1'.

# Now let us do F2' by Duality

$F2 = x\,(y'z' + yz)$

$F2' = x + (y' + z')\,(y + z)$ (dual of F2)

$= x' + (y + z)\,(y' + z')$ (complement of literals)

- **Minterms or Standard products:**
- Two binary variables expression can have following four AND operations or product terms:
- x.y
- x'.y
- x.y'
- x'.y'

- **Maxterms or Standard sums:**
- Two binary variables expression can have following four OR operations or product terms:
- x+y
- x'+y
- x+y'
- x'+y'

| | | | Minterms | | Maxterms | |
|---|---|---|---|---|---|---|
| $x$ | $y$ | $z$ | Term | Designation | Term | Designation |
| 0 | 0 | 0 | $x'y'z'$ | $m_0$ | $x + y + z$ | $M_0$ |
| 0 | 0 | 1 | $x'y'z$ | $m_1$ | $x + y + z'$ | $M_1$ |
| 0 | 1 | 0 | $x'yz'$ | $m_2$ | $x + y' + z$ | $M_2$ |
| 0 | 1 | 1 | $x'yz$ | $m_3$ | $x + y' + z'$ | $M_3$ |
| 1 | 0 | 0 | $xy'z'$ | $m_4$ | $x' + y + z$ | $M_4$ |
| 1 | 0 | 1 | $xy'z$ | $m_5$ | $x' + y + z'$ | $M_5$ |
| 1 | 1 | 0 | $xyz'$ | $m_6$ | $x' + y' + z$ | $M_6$ |
| 1 | 1 | 1 | $xyz$ | $m_7$ | $x' + y' + z'$ | $M_7$ |

- **Expressing a Boolean function from its Truth-table:**

| $x$ | $y$ | $z$ | Function $f_1$ | Function $f_2$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

F1 = x'y'z + xy'z' + xyz = m1 + m4 + m7

F2 = x'yz + xy'z + xyz' + xyz = m3 + m5 + m6 + m7