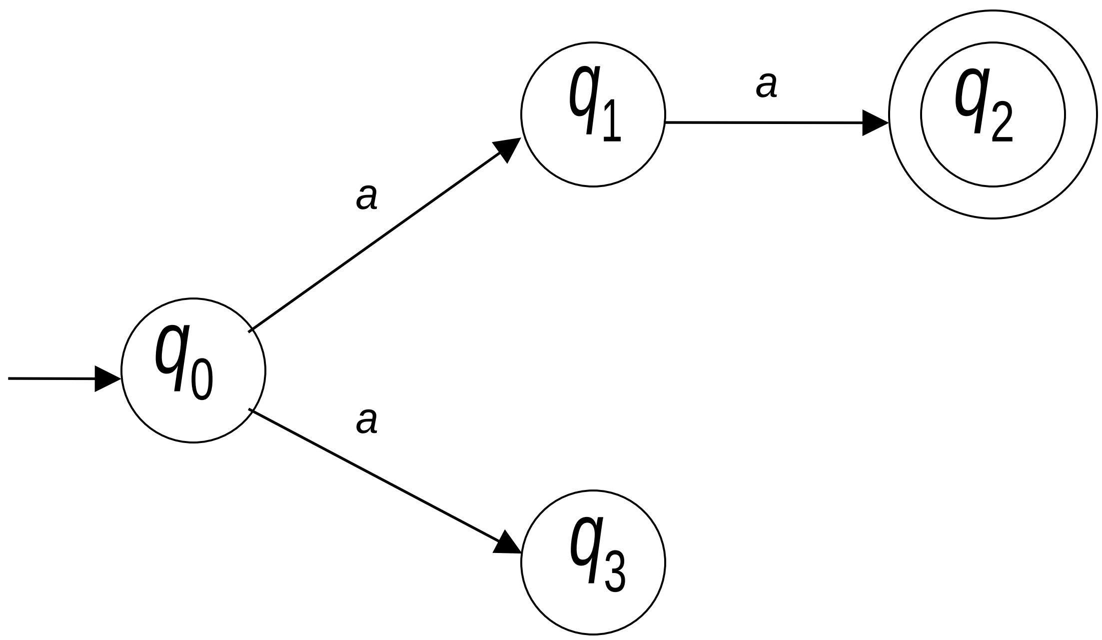
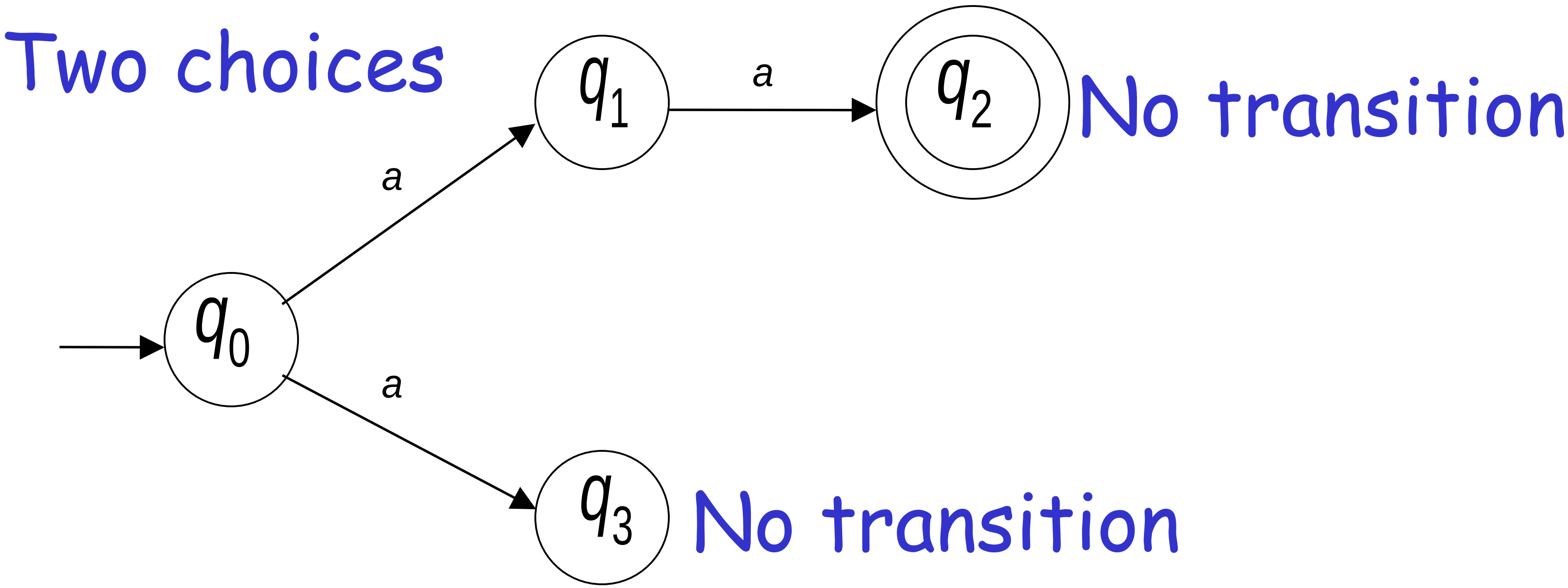
Non-Deterministic Finite Automata

Nondeterministic Finite Automaton (NFA)

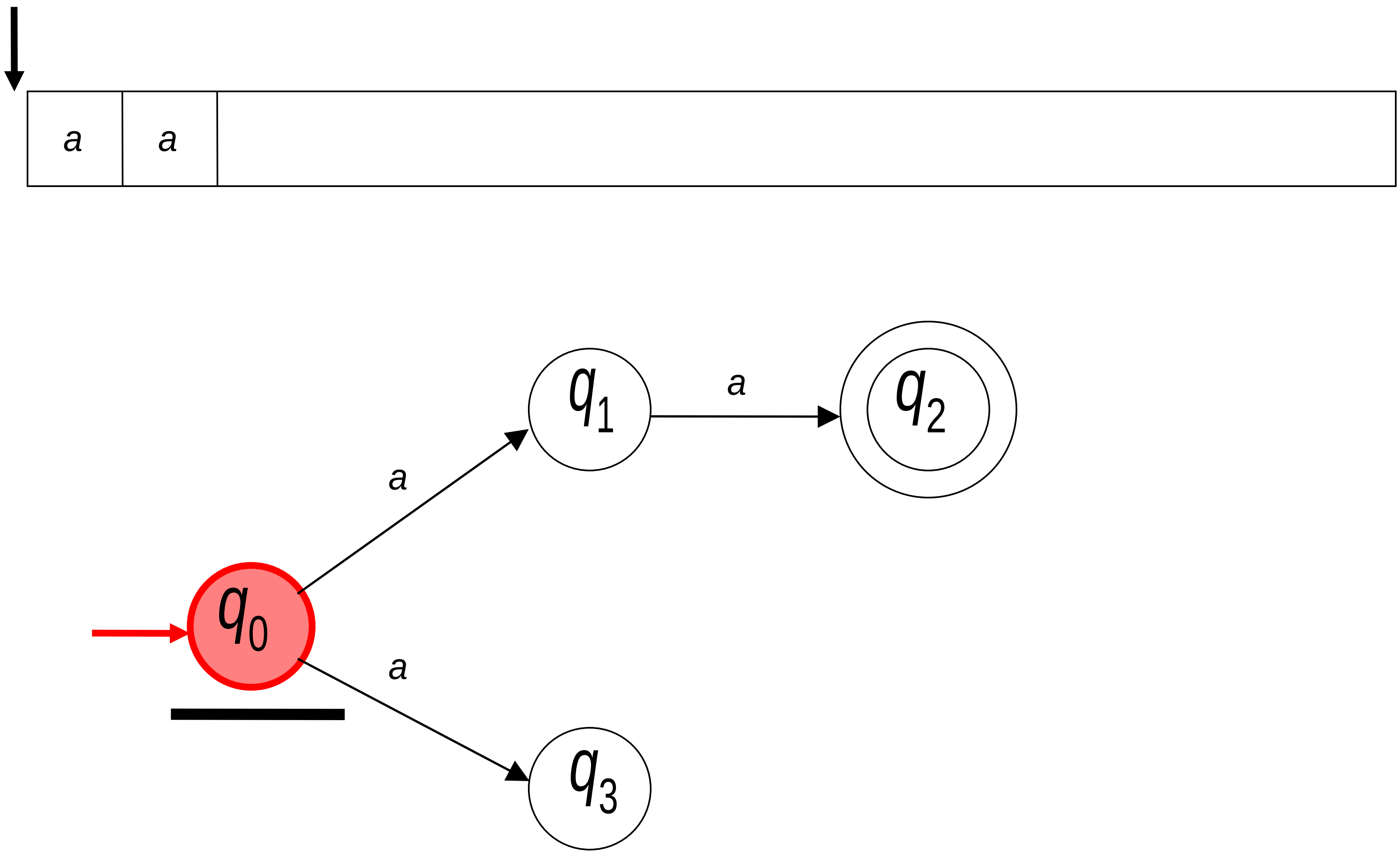
Alphabet = $\{a\}$



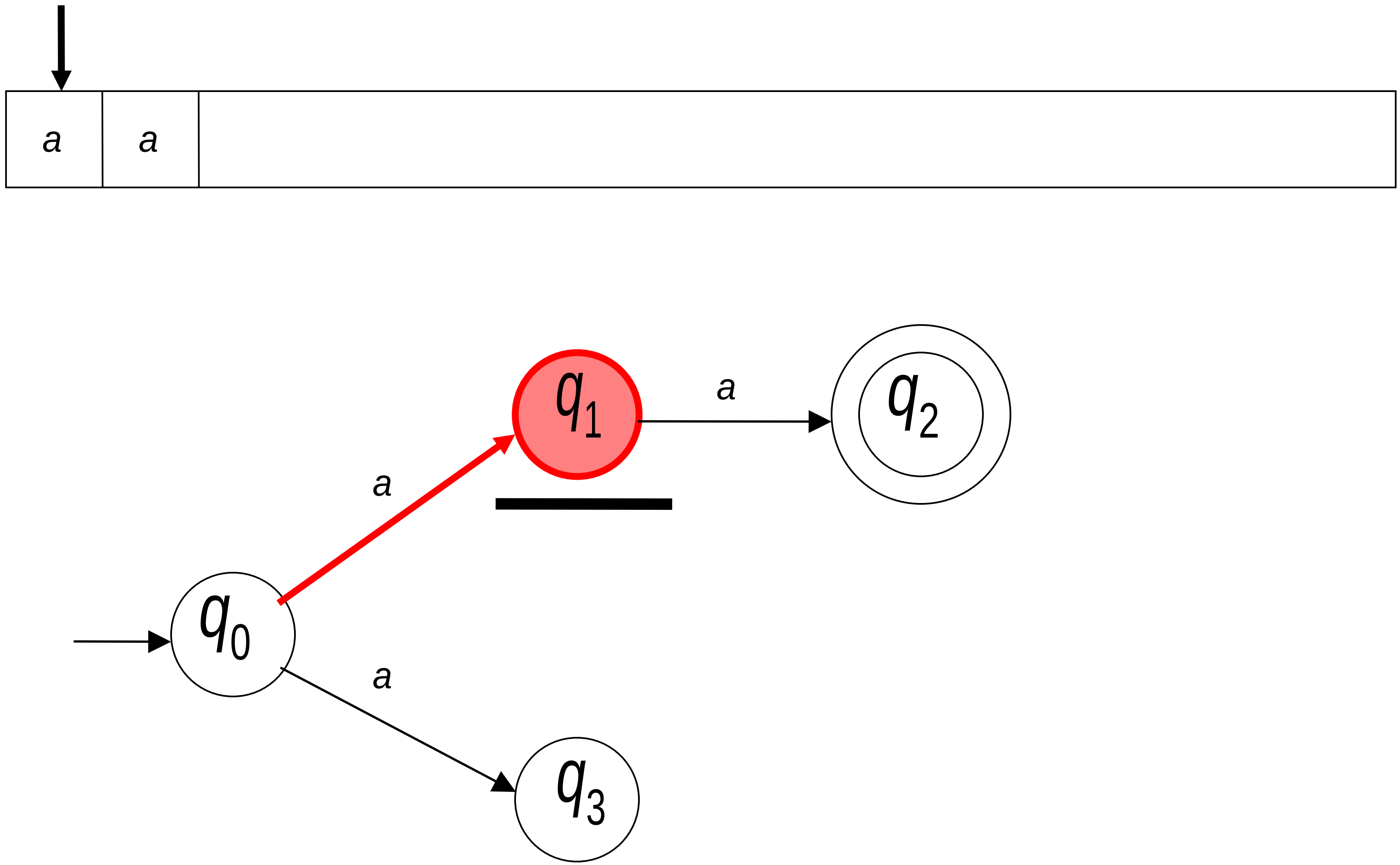
Alphabet = $\{a\}$



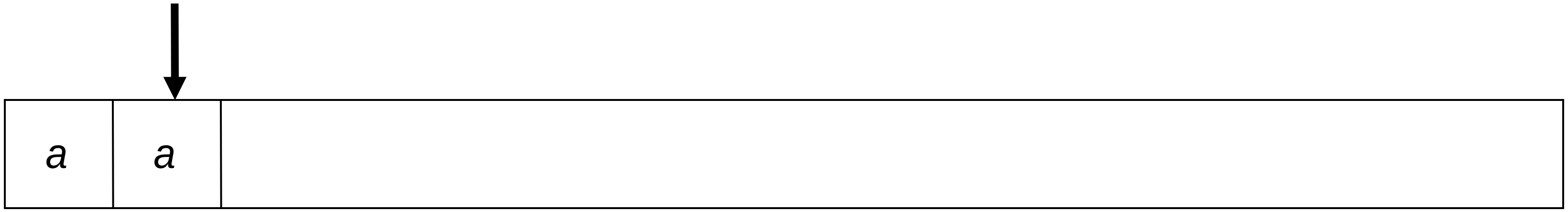
First Choice



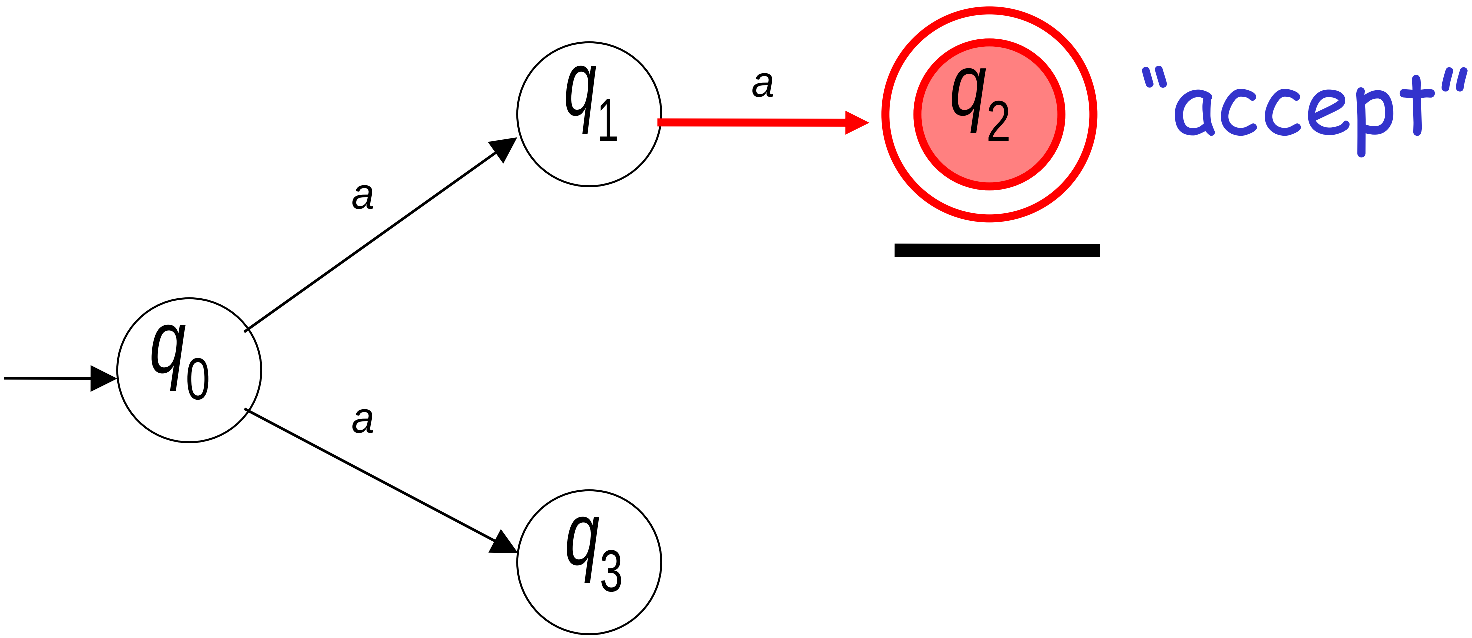
First Choice



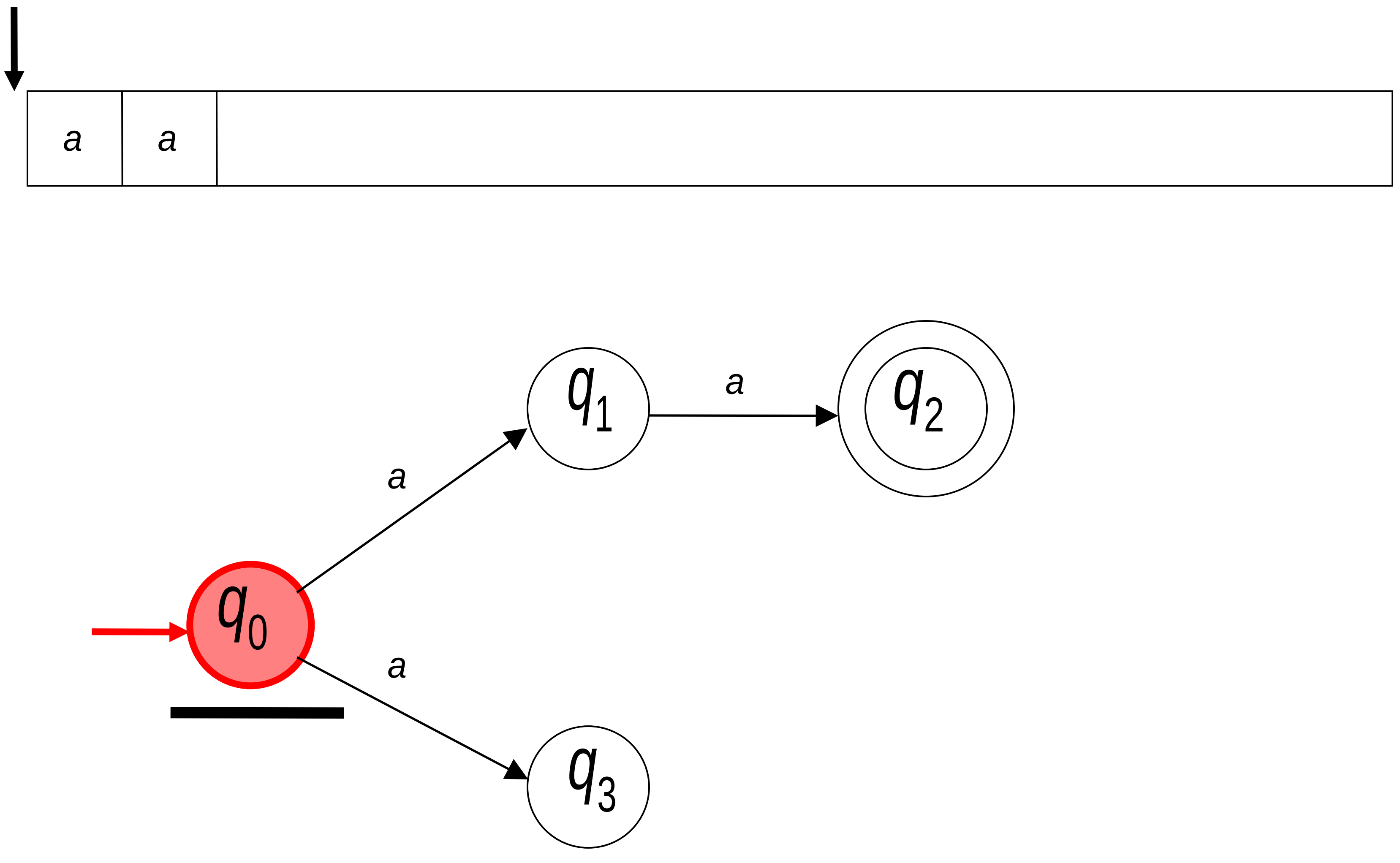
First Choice



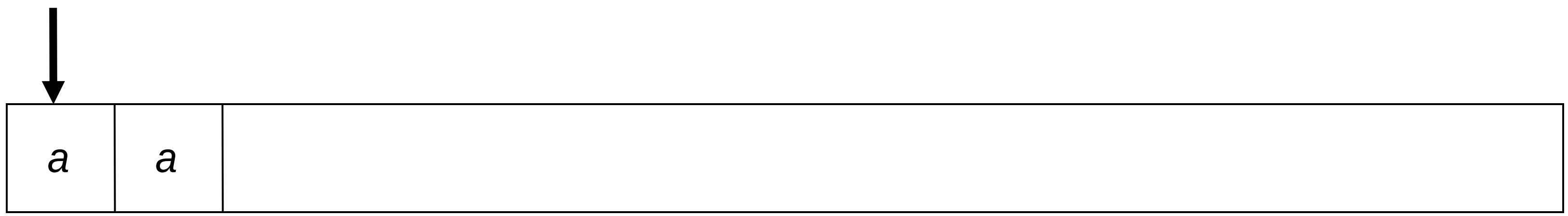
All input is consumed



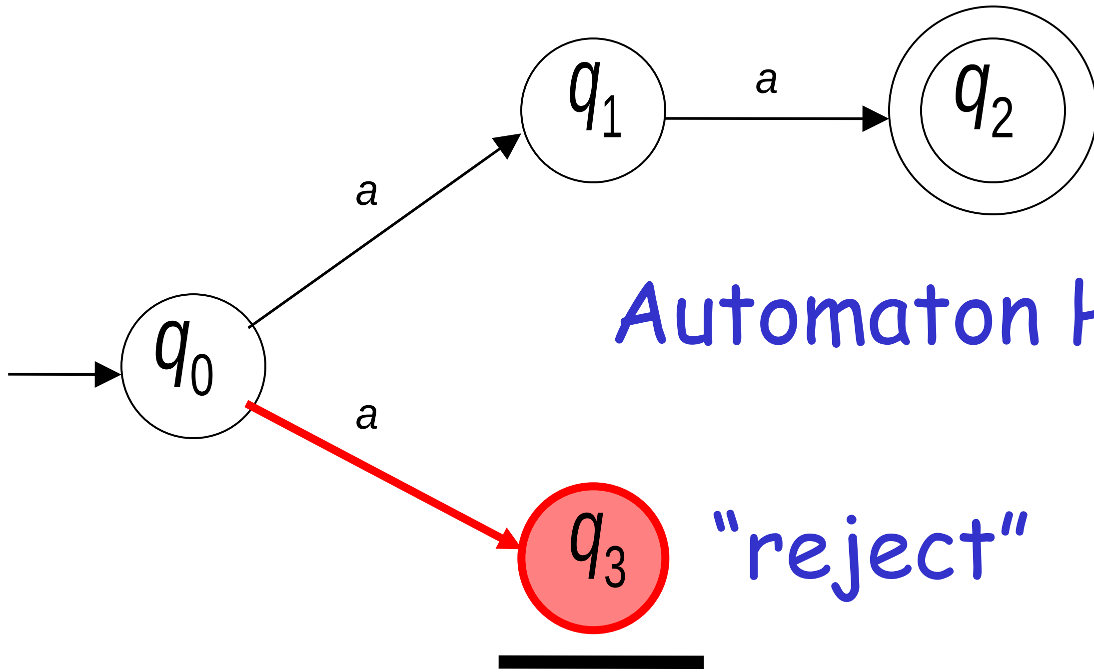
Second Choice



Second Choice



Input cannot be consumed



Automaton Halts

"reject"

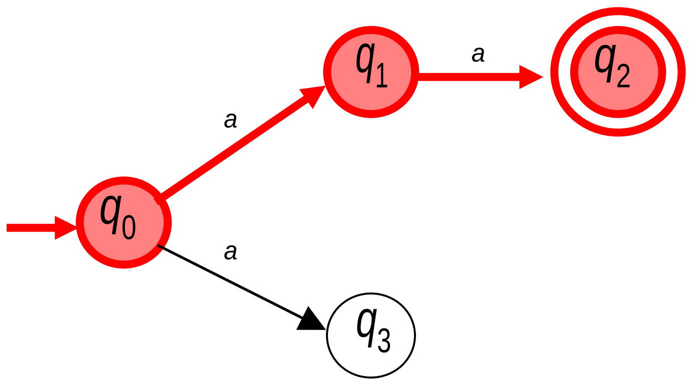
An NFA accepts a string:

if there is a computation of the NFA that accepts the string

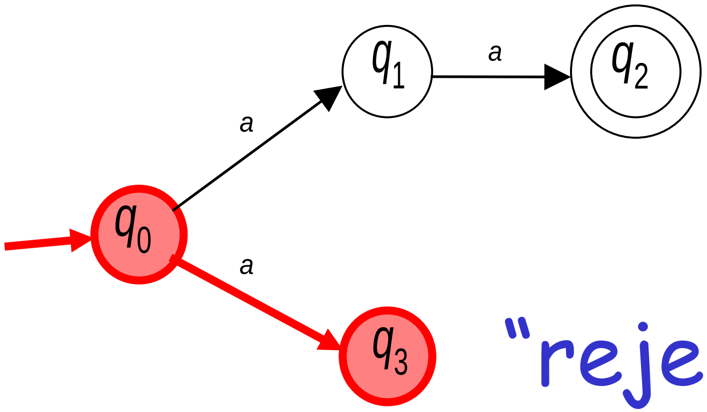
i.e., all the input string is processed and the automaton is in an accepting state

aa is accepted by the NFA:

"accept"



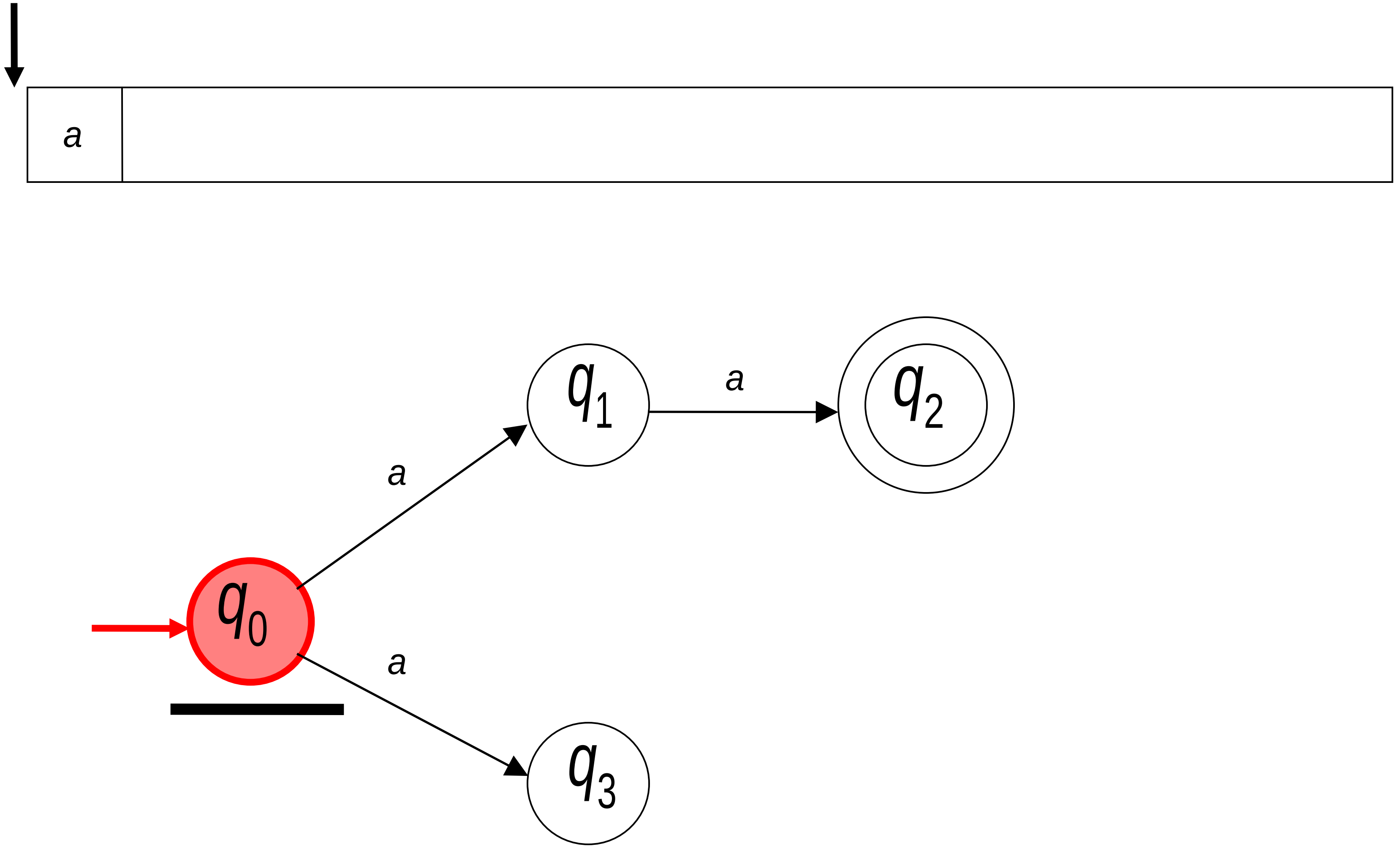
because this computation accepts aa



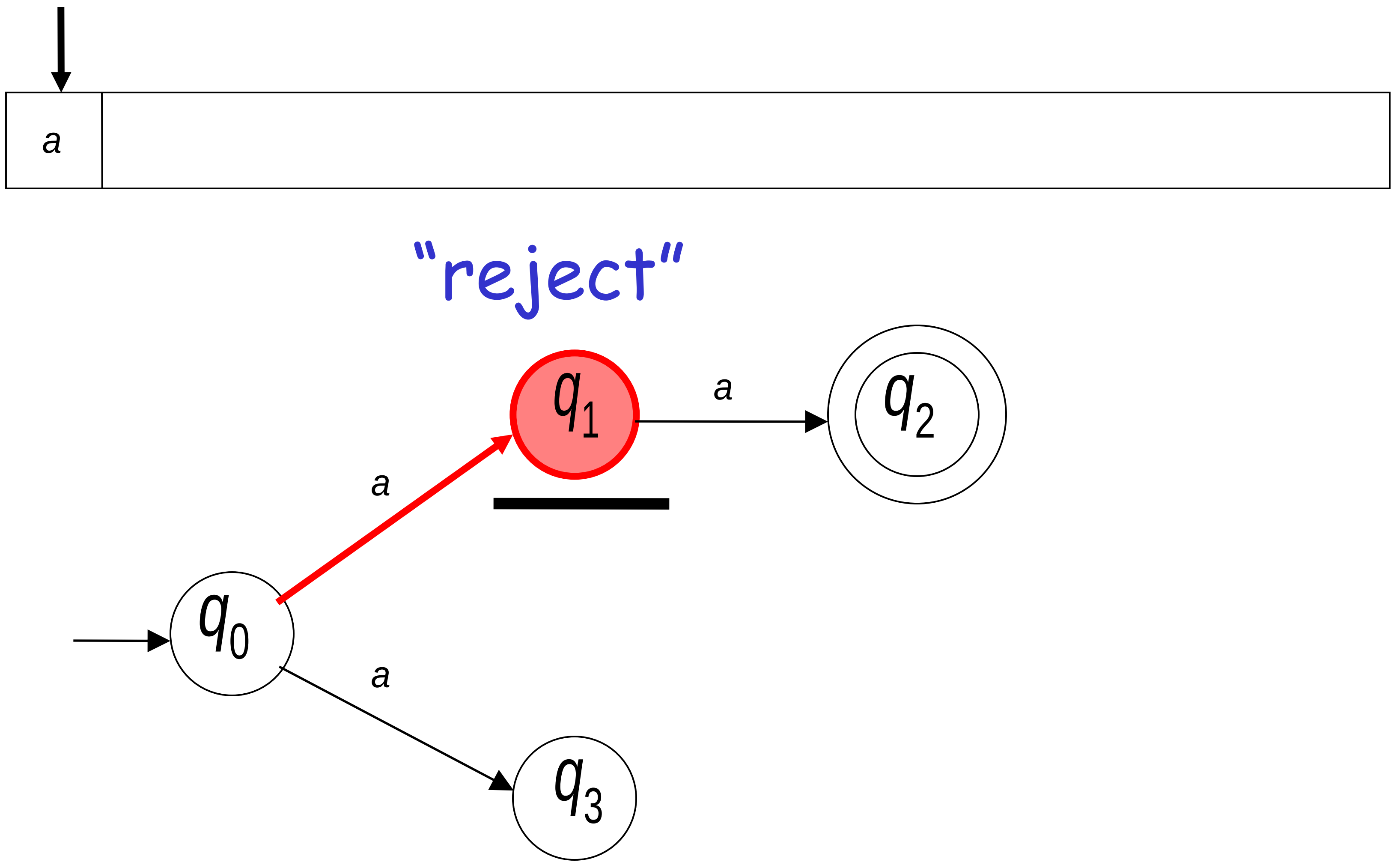
"reject"

this computation is ignored

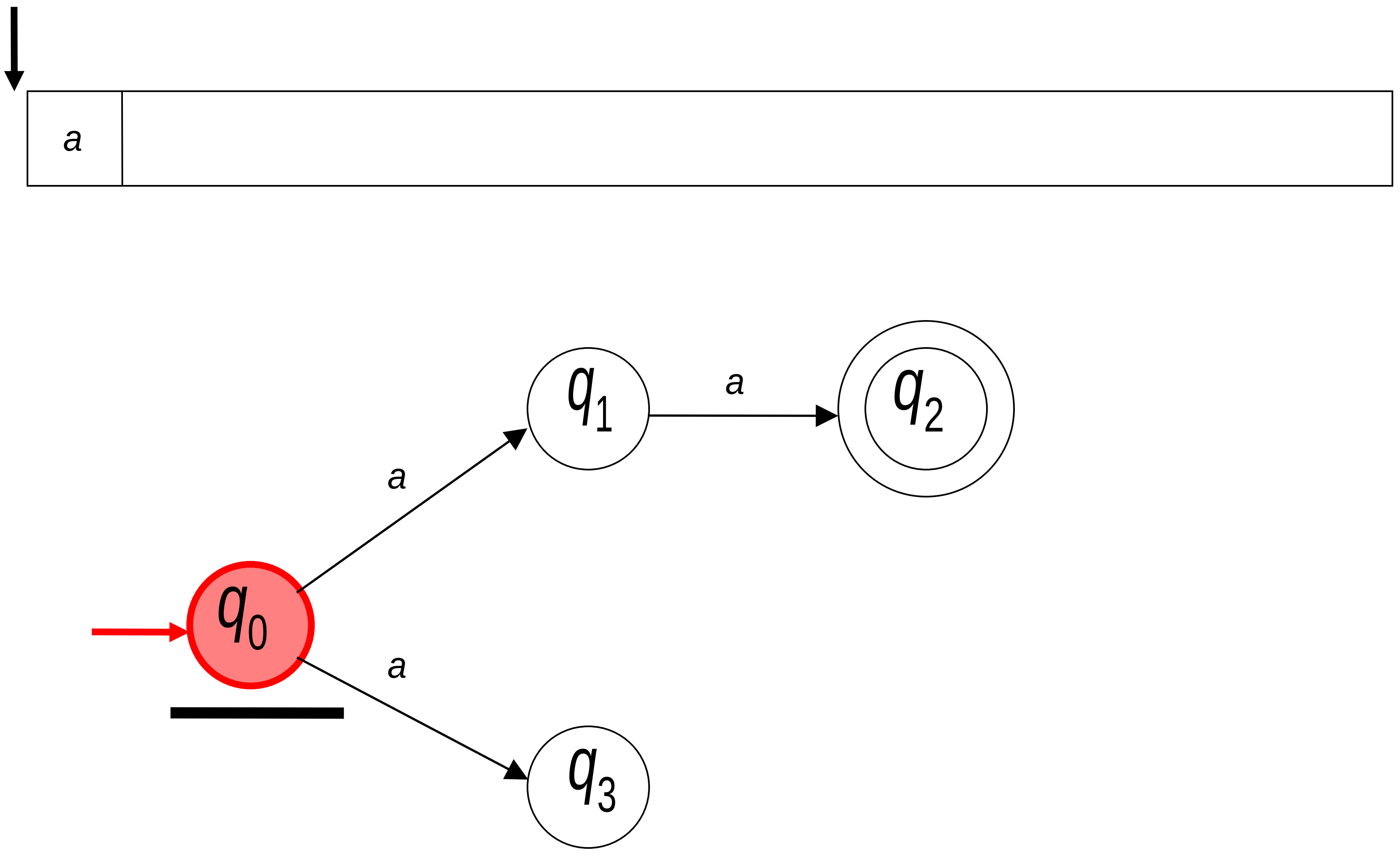
Rejection example



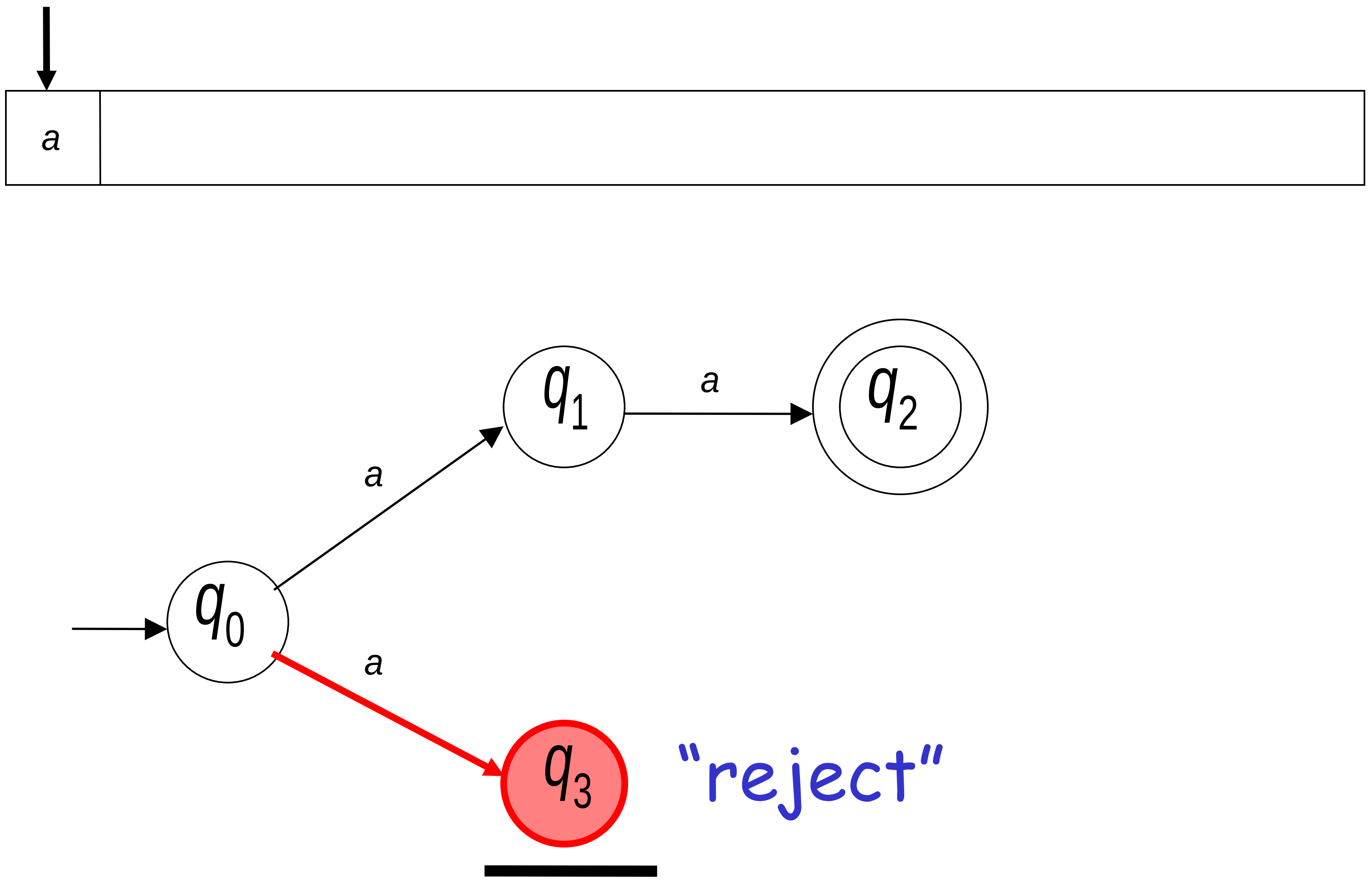
First Choice



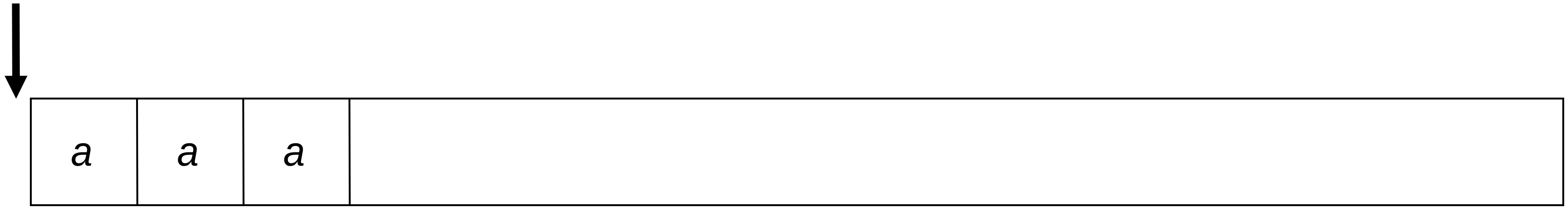
Second Choice

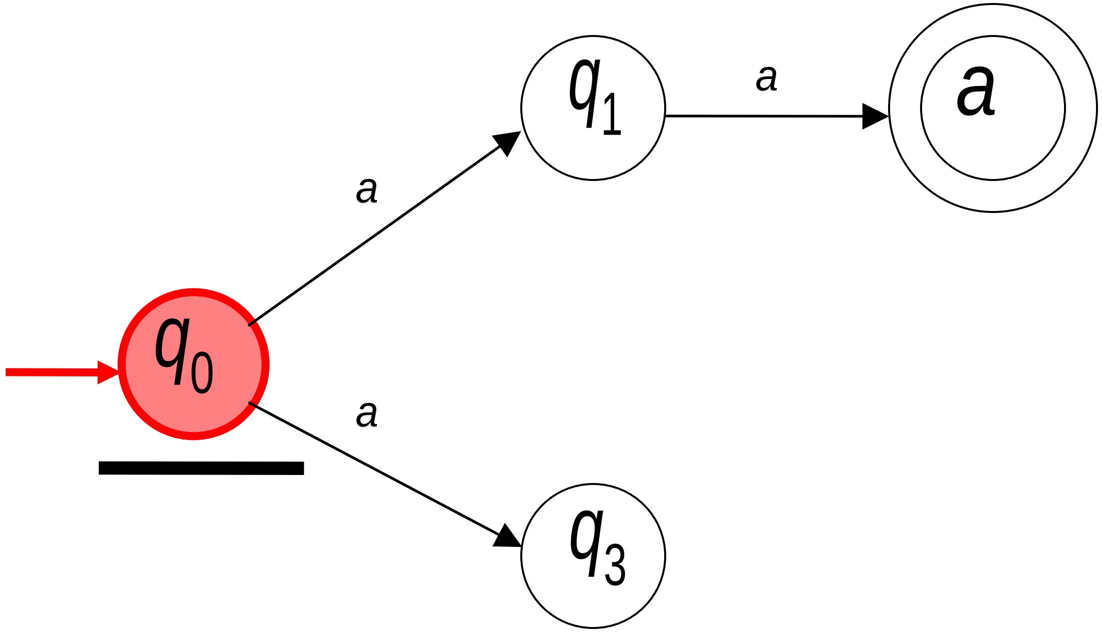


Second Choice

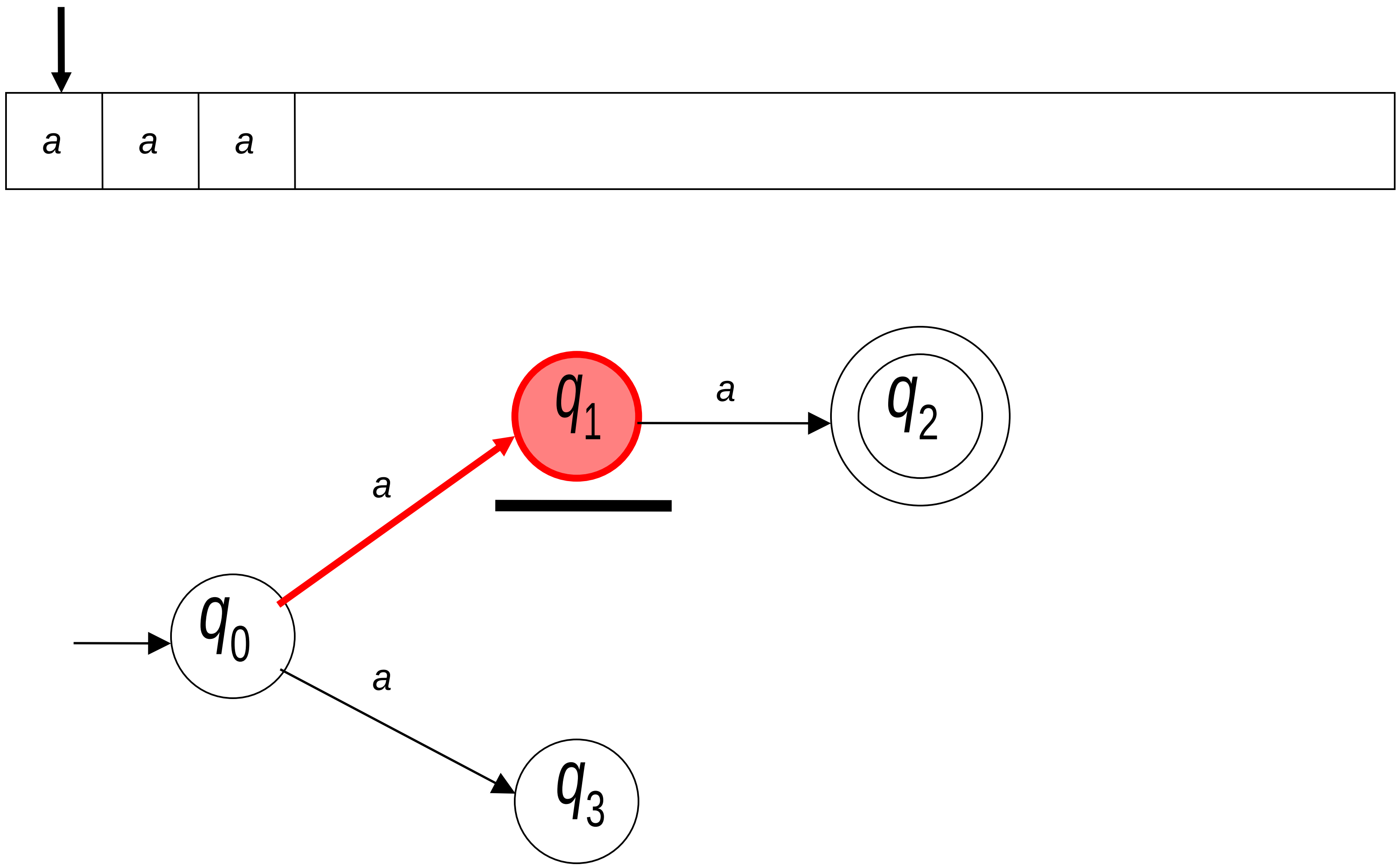


Another Rejection example

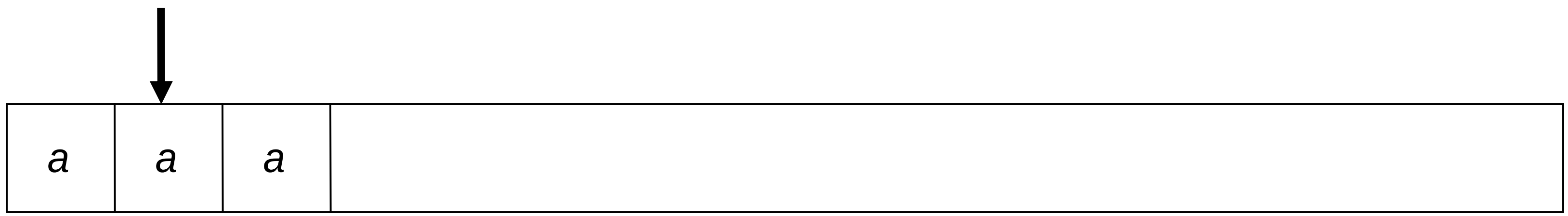




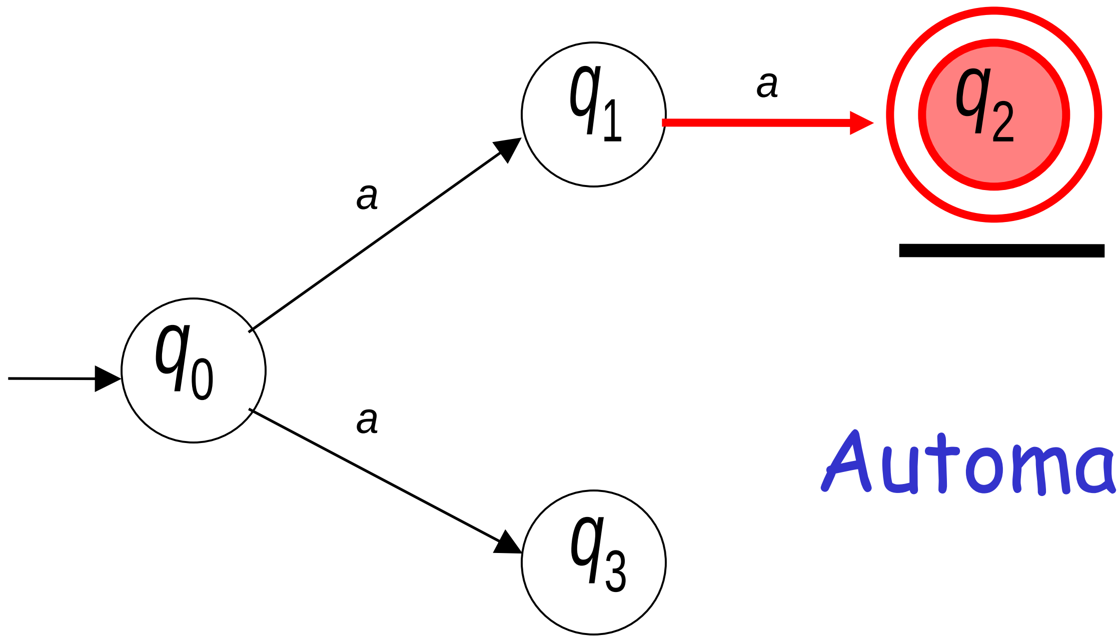
First Choice



First Choice



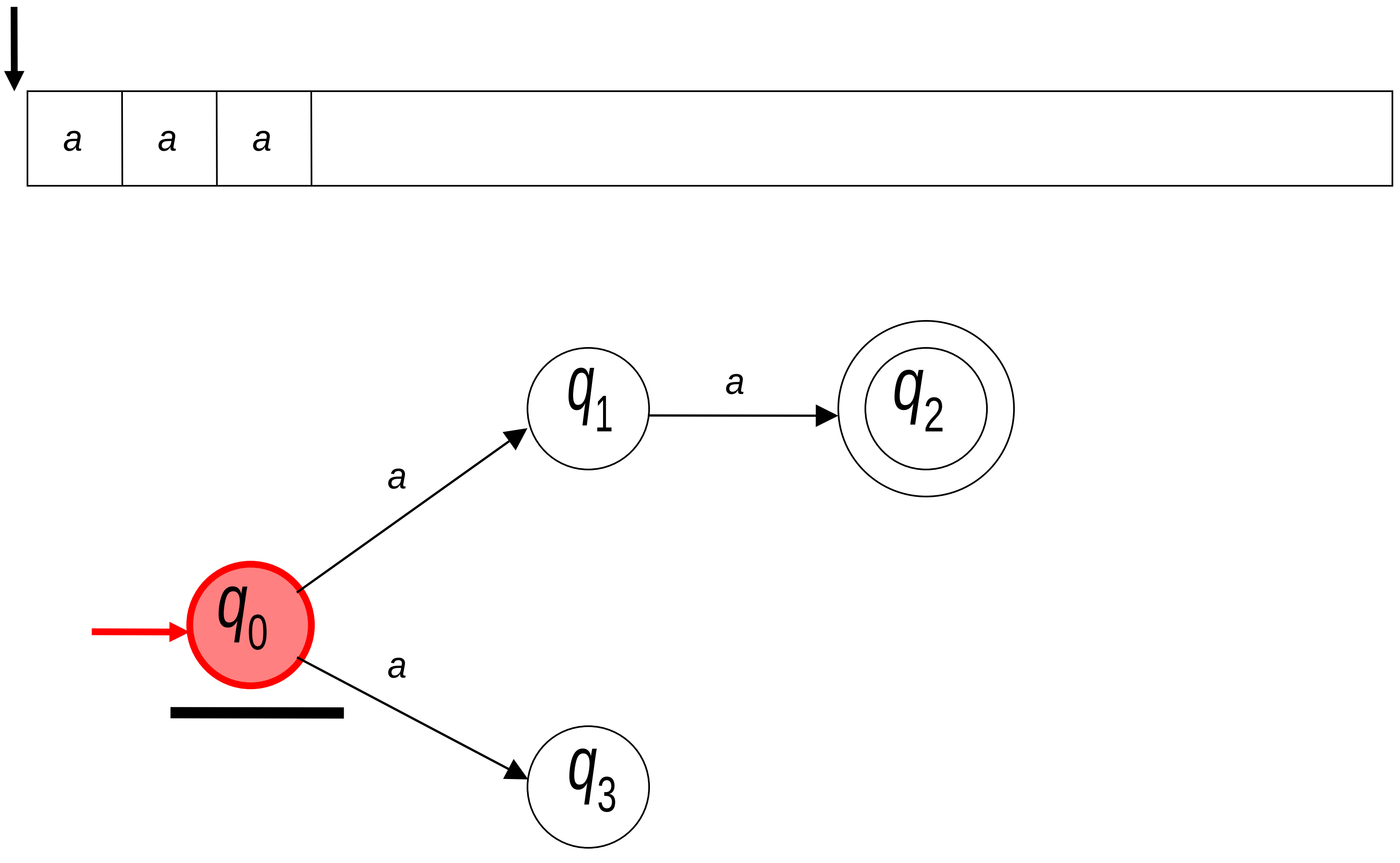
Input cannot be consumed



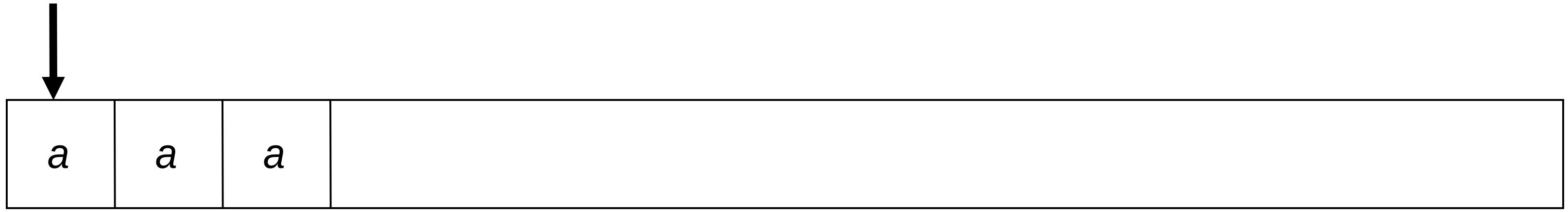
"reject"

Automaton halts

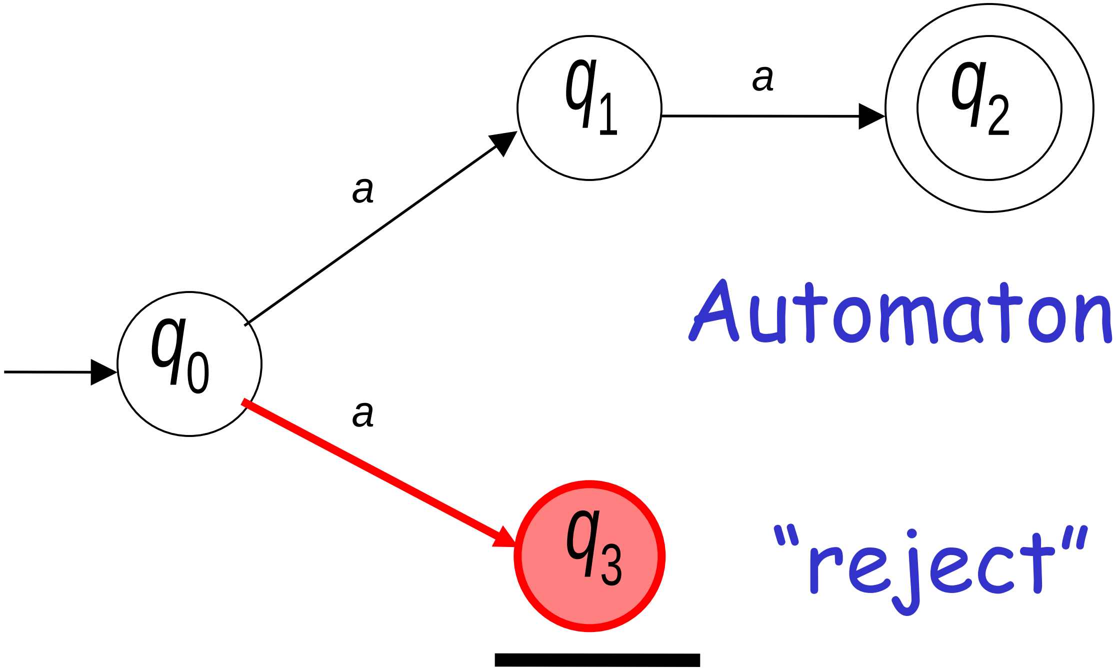
Second Choice



Second Choice



Input cannot be consumed



Automaton halts

"reject"

An NFA rejects a string:

if there is no computation of the NFA that accepts the string.

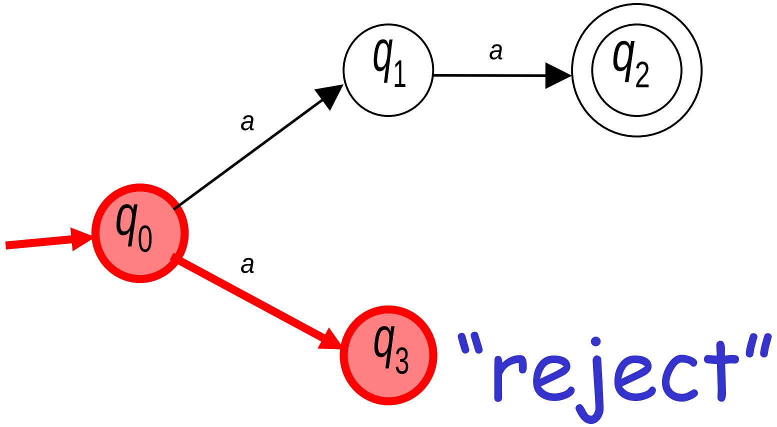
For each computation:

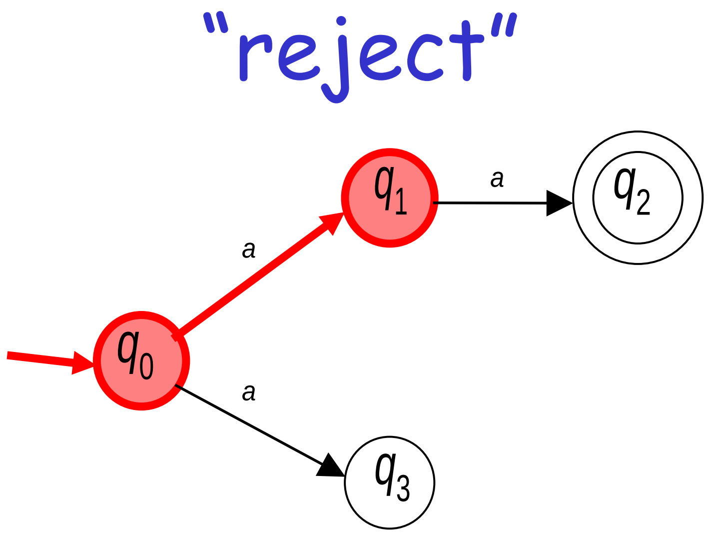
- All the input is consumed and the automaton is in a non accepting state

OR

- The input cannot be consumed

a is rejected by the NFA:

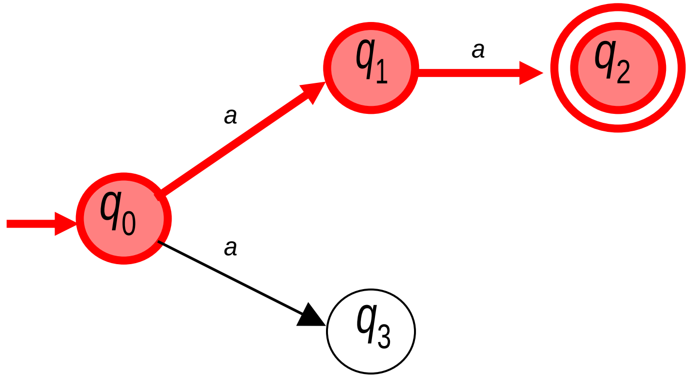


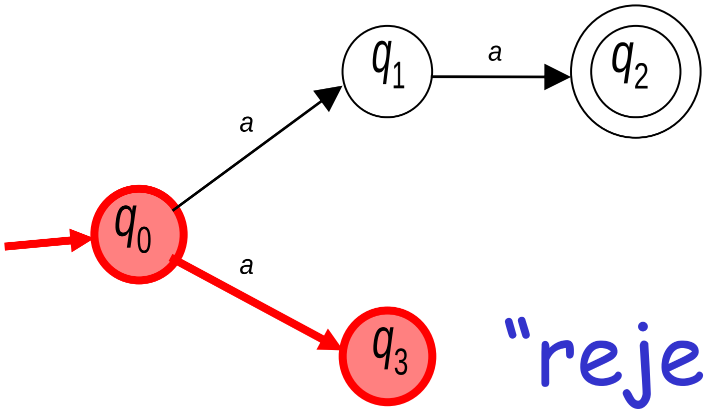


All possible computations lead to rejection

aaa is rejected by the NFA:

"reject"

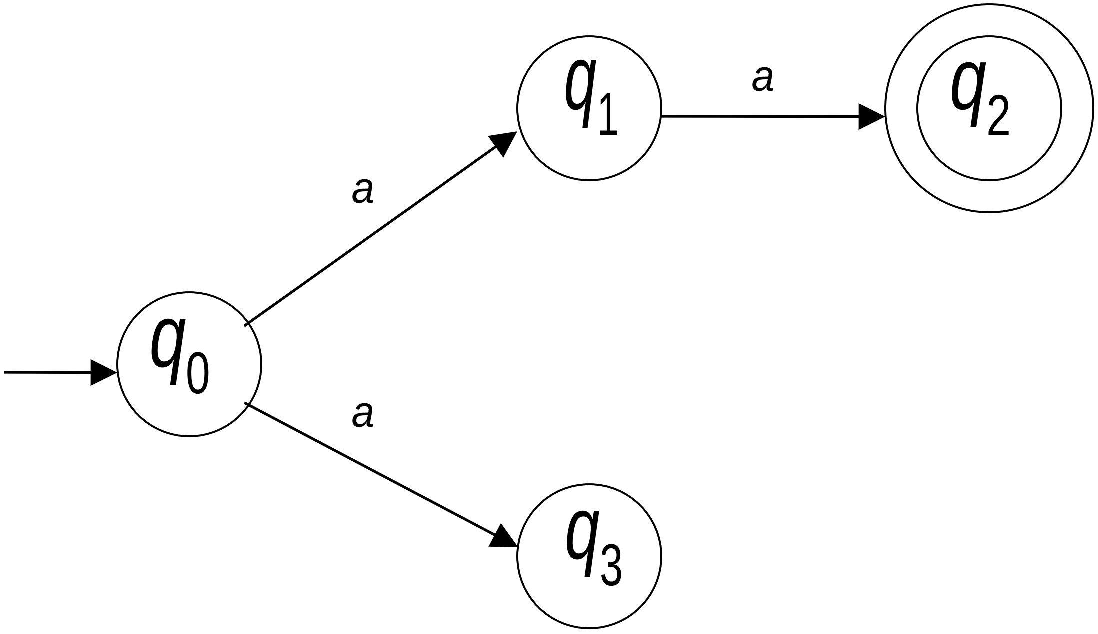




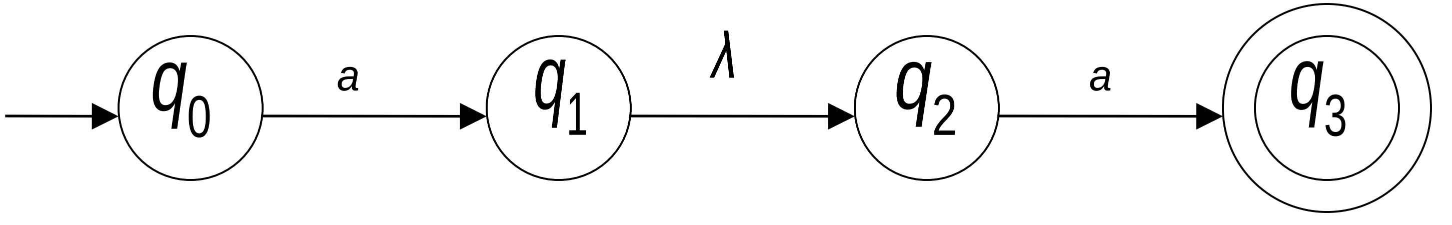
"reject"

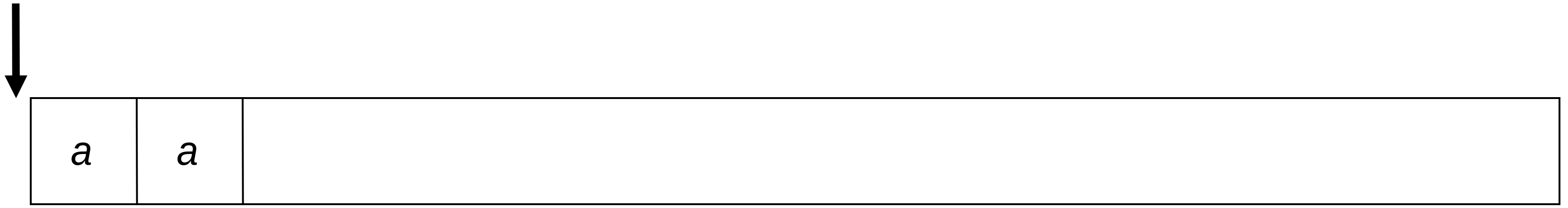
All possible computations lead to rejection

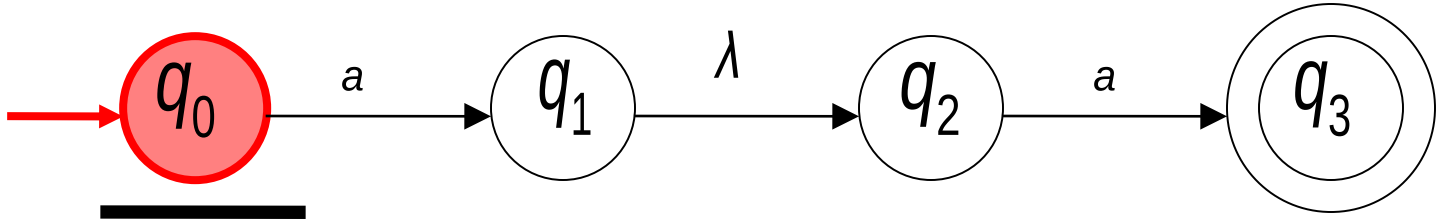
Language accepted: $L = \{aa\}$

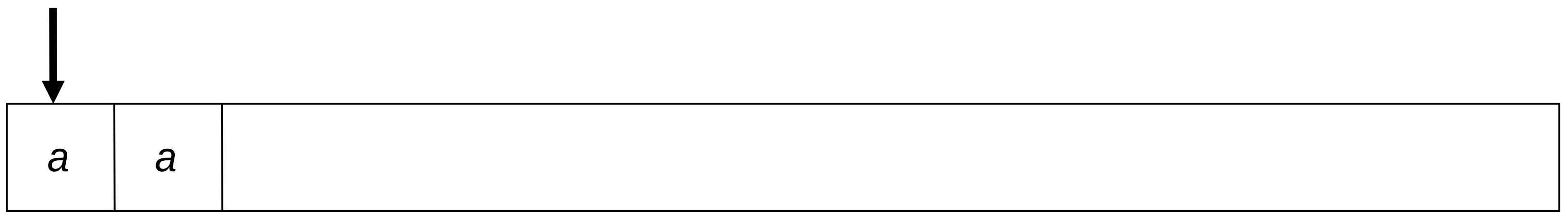


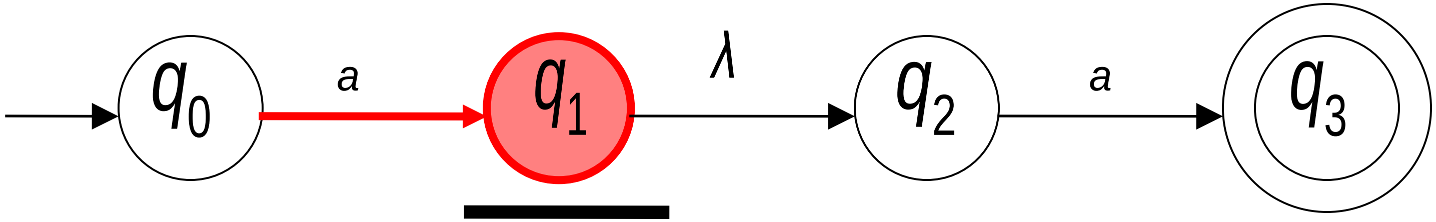
Lambda Transitions



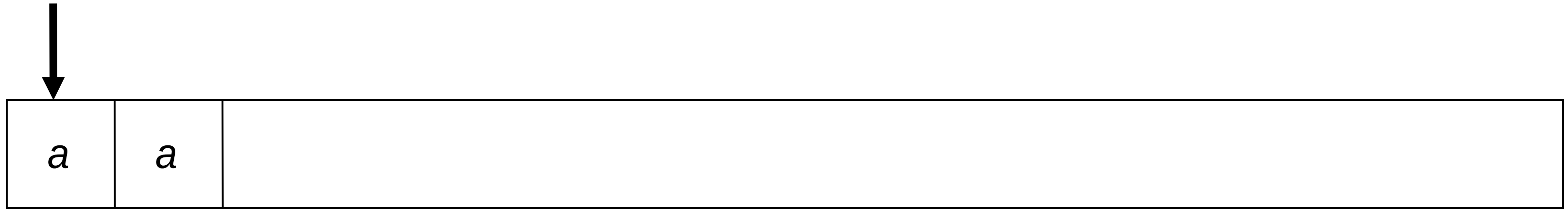

a
a

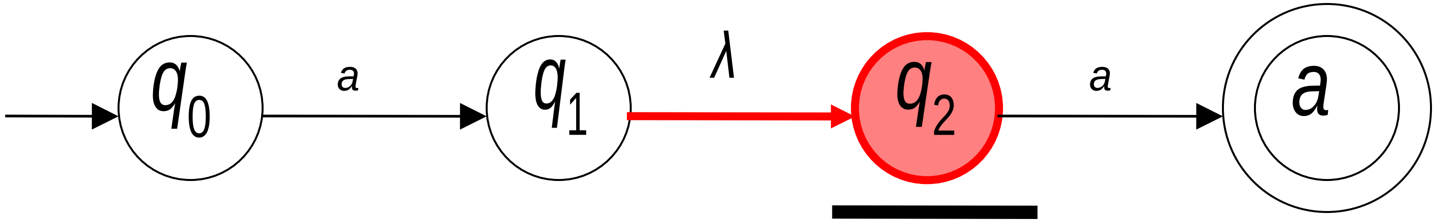

q_0
a
q_1
λ
q_2
a
q_3


a
a


q_0
a
q_1
λ
q_2
a
q_3

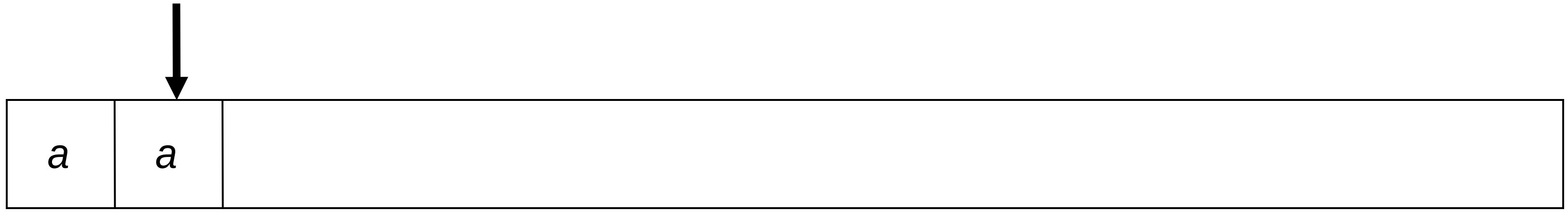
input tape head does not move



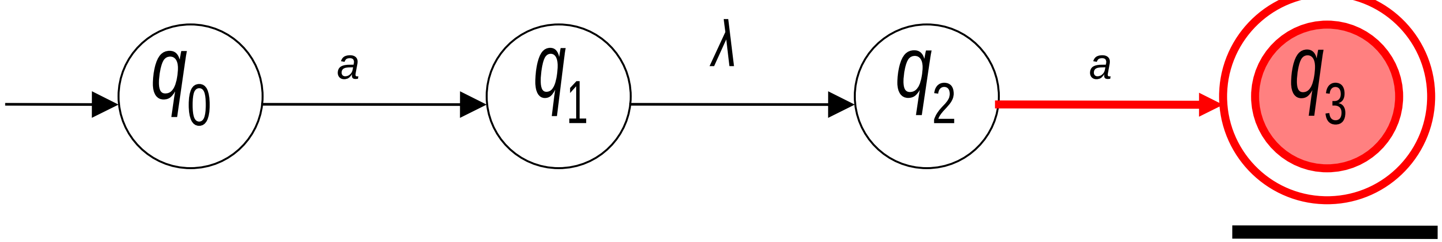


Automaton changes state

all input is consumed

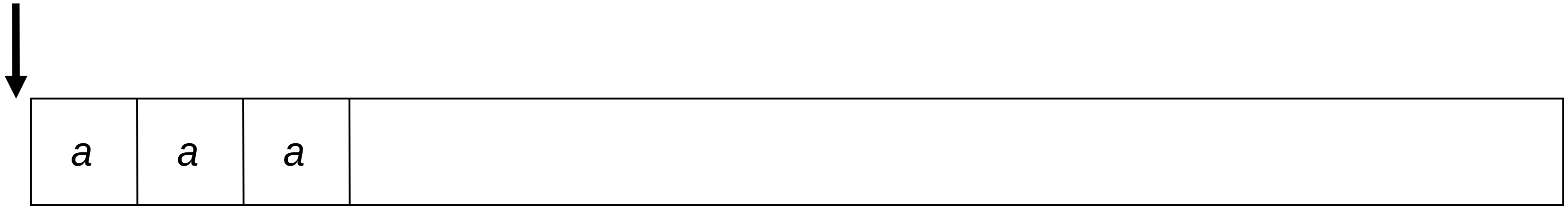


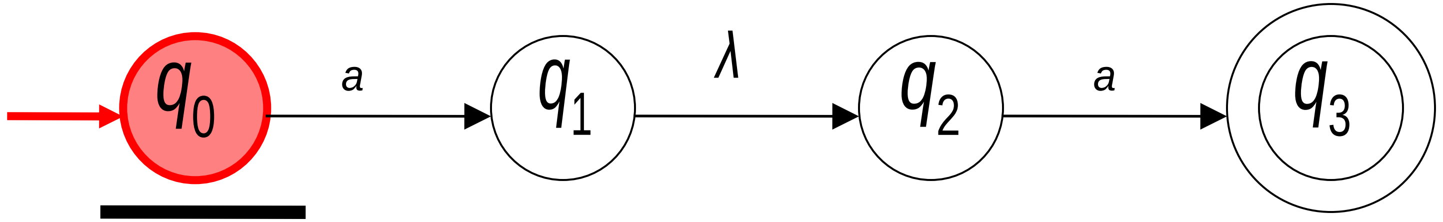
“accept”

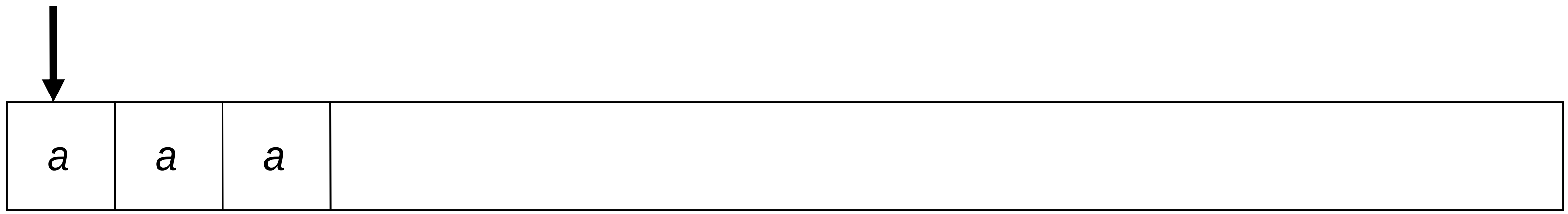


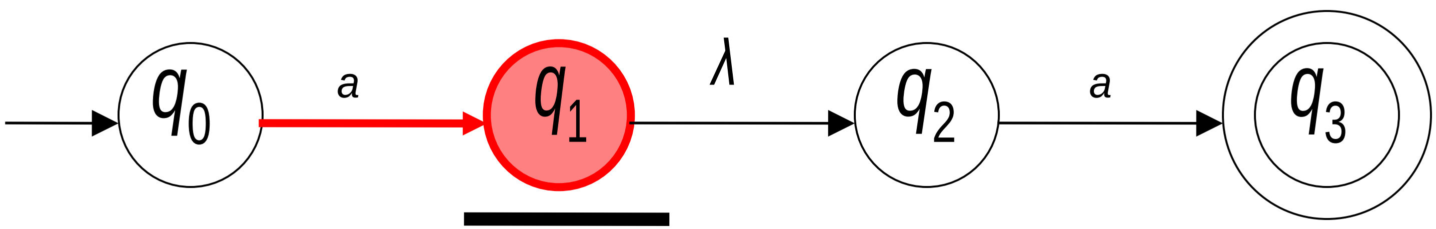
String aa is accepted

Rejection Example

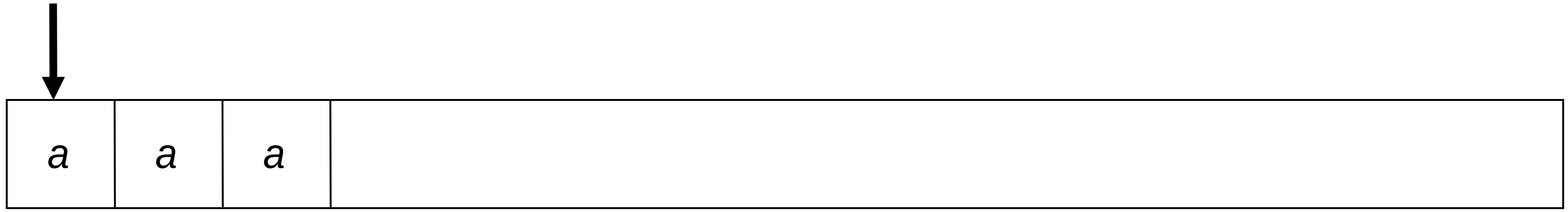


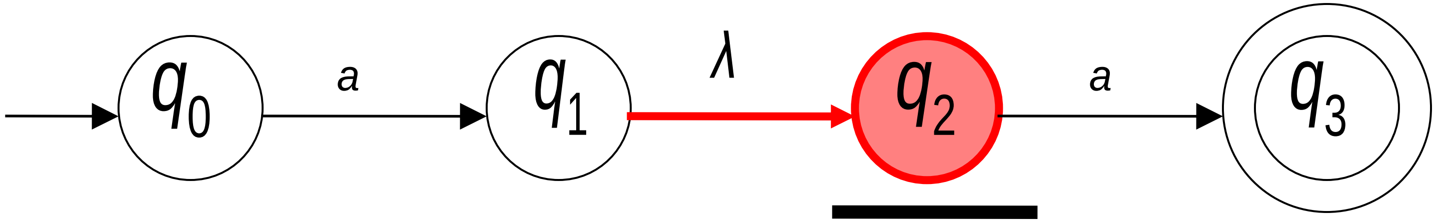



a
a
a

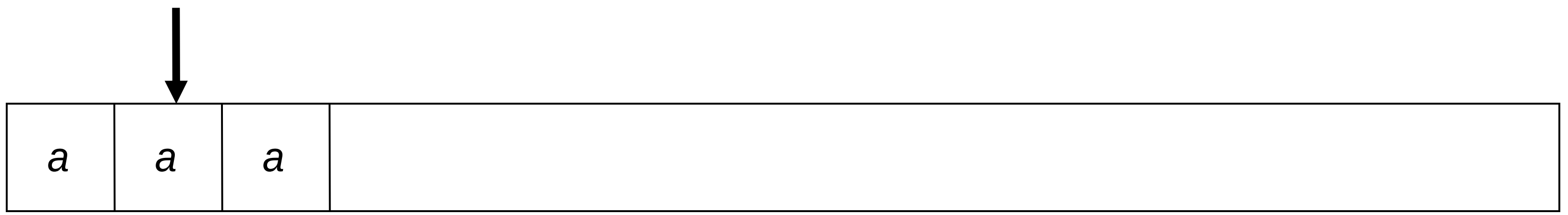

q_0
a
q_1
λ
q_2
a
q_3

(read head doesn't move)



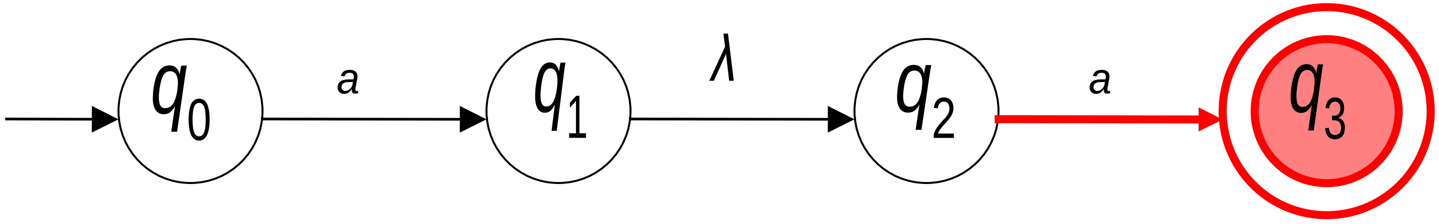


Input cannot be consumed



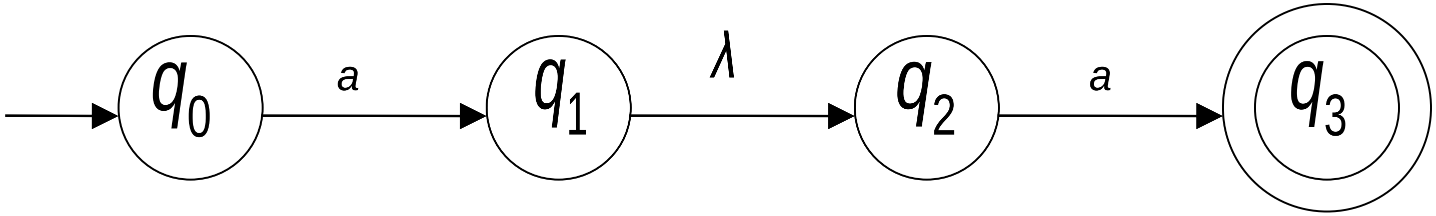
Automaton halts

"reject"

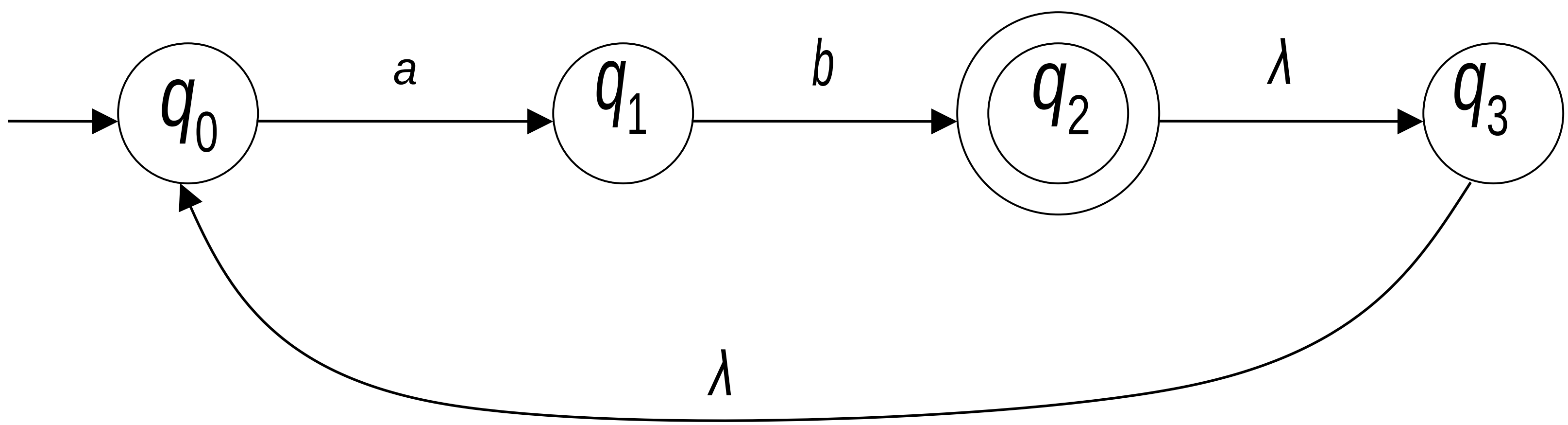


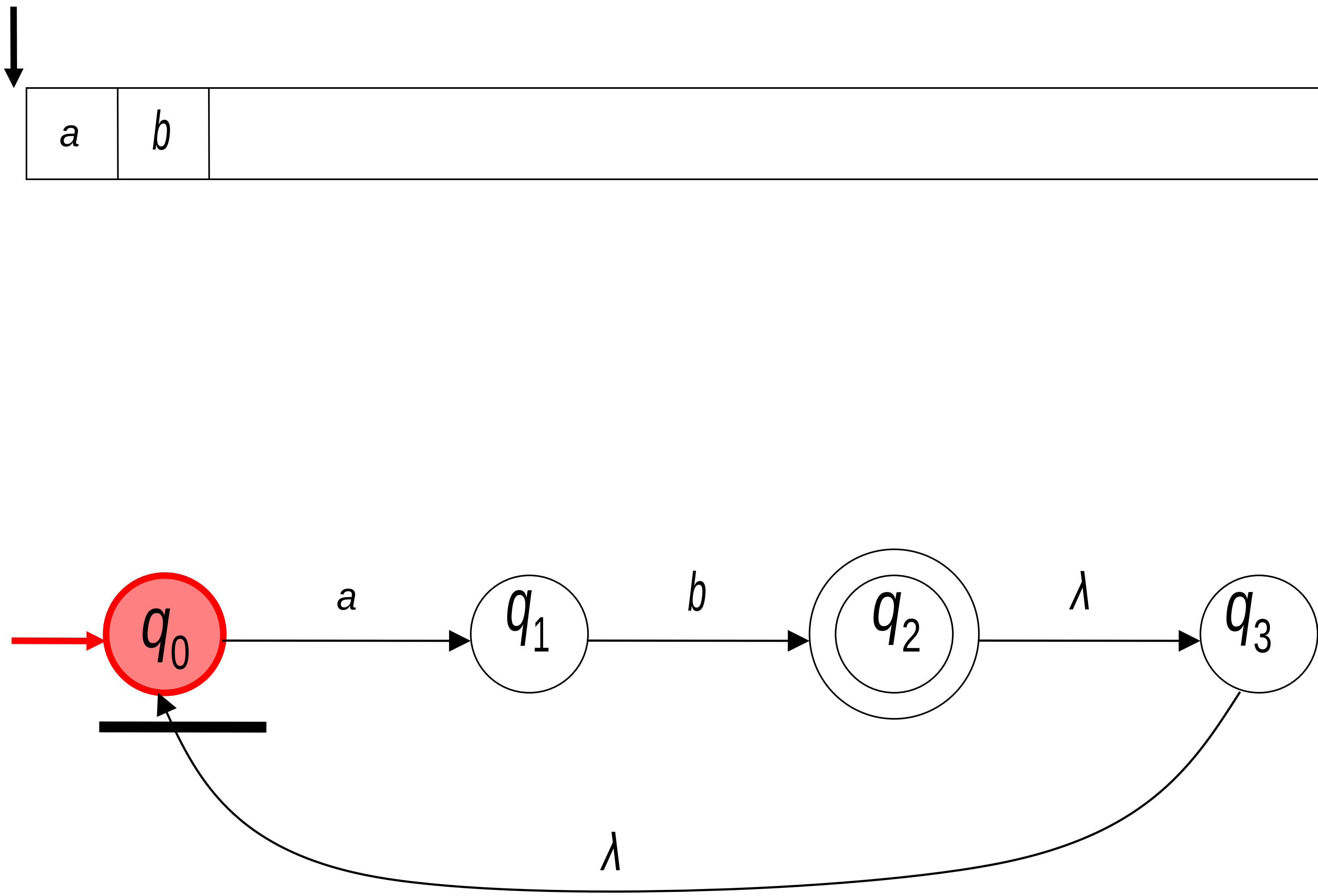
String aaa is rejected

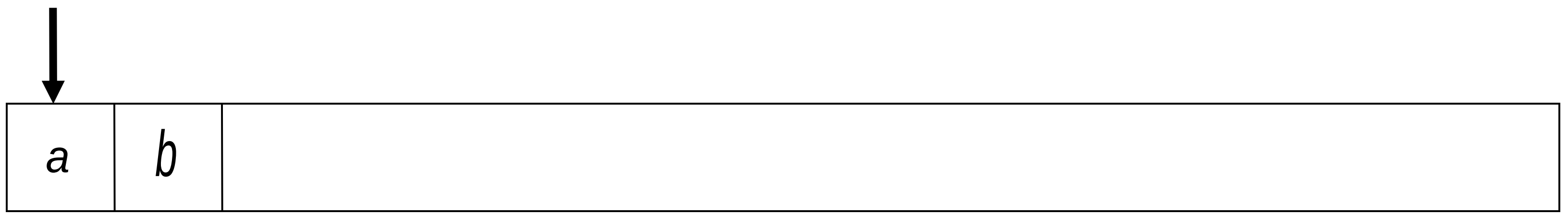
Language accepted: $L = \{aa\}$

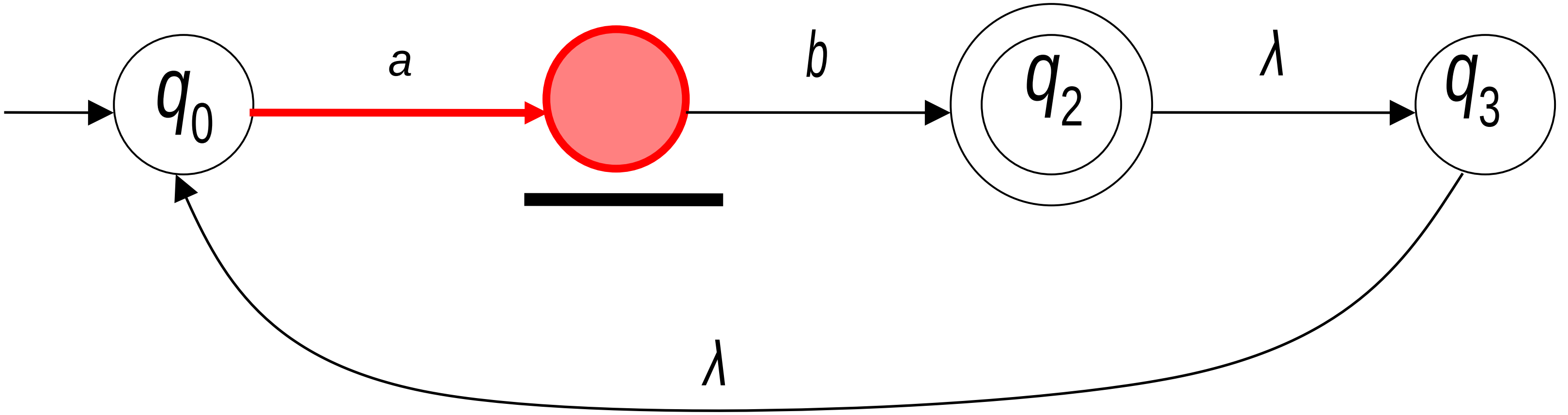


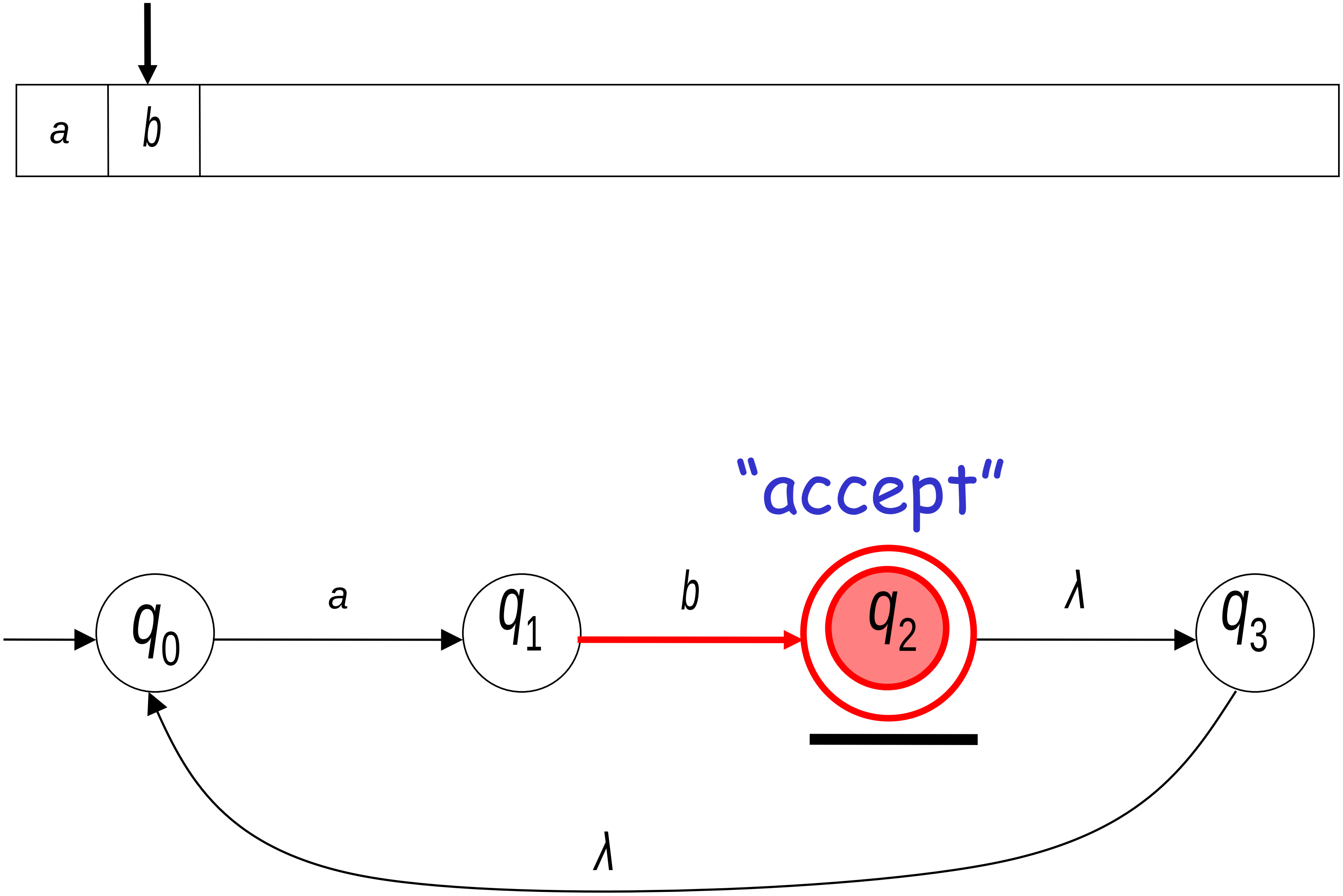
Another NFA Example



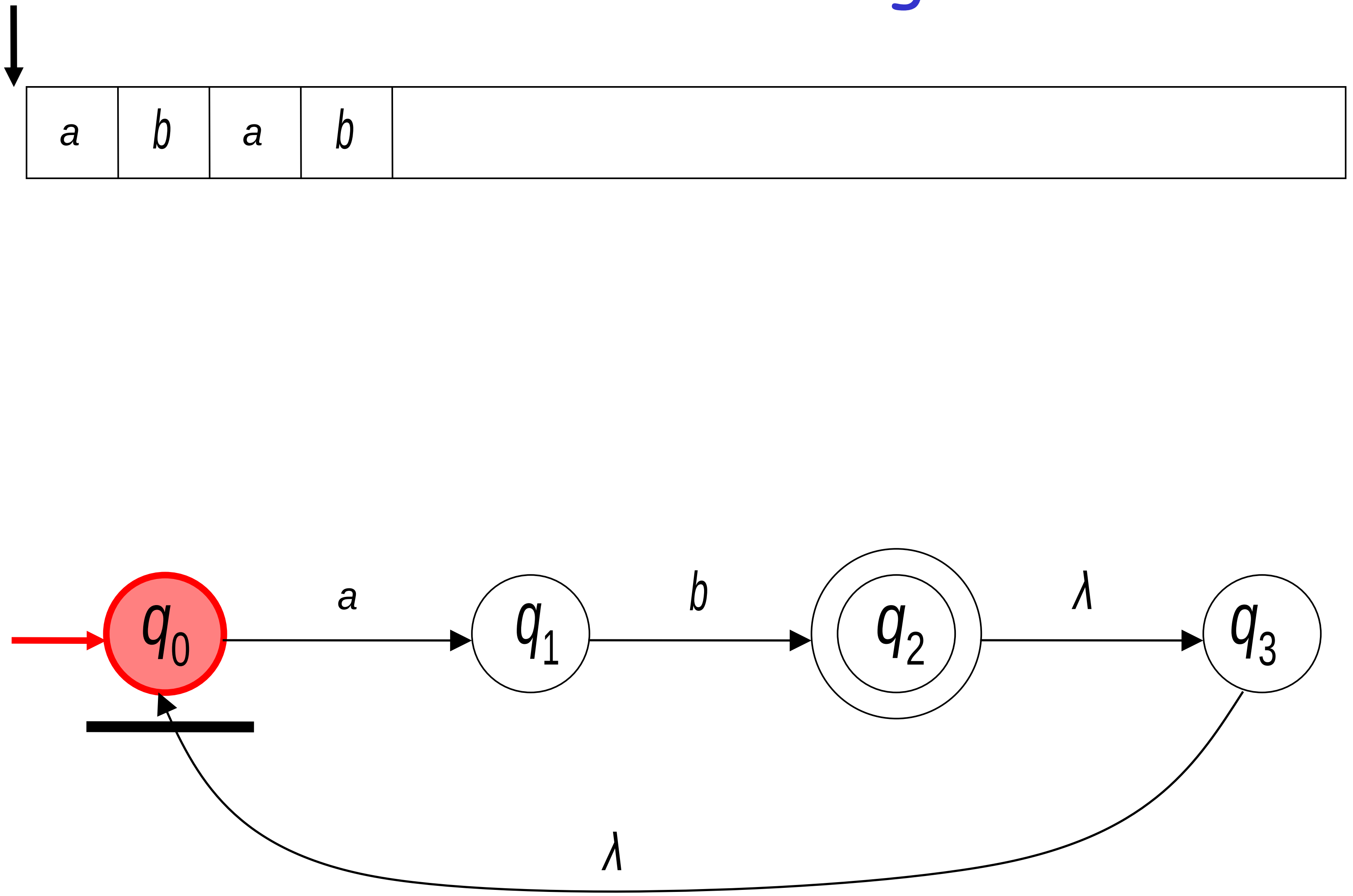

a
b
a
b
λ
λ
q_0
q_1
q_2
q_3

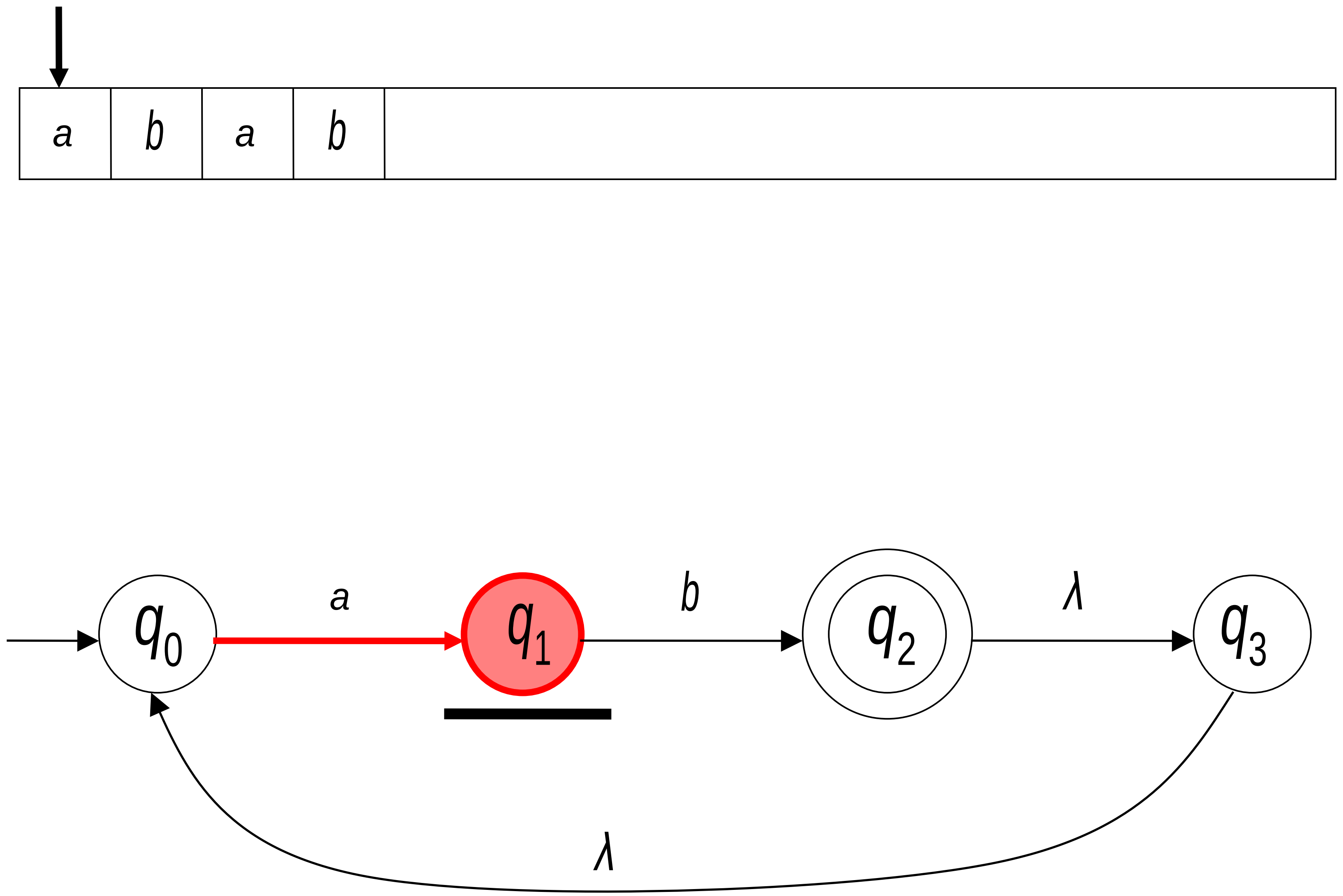

a
b

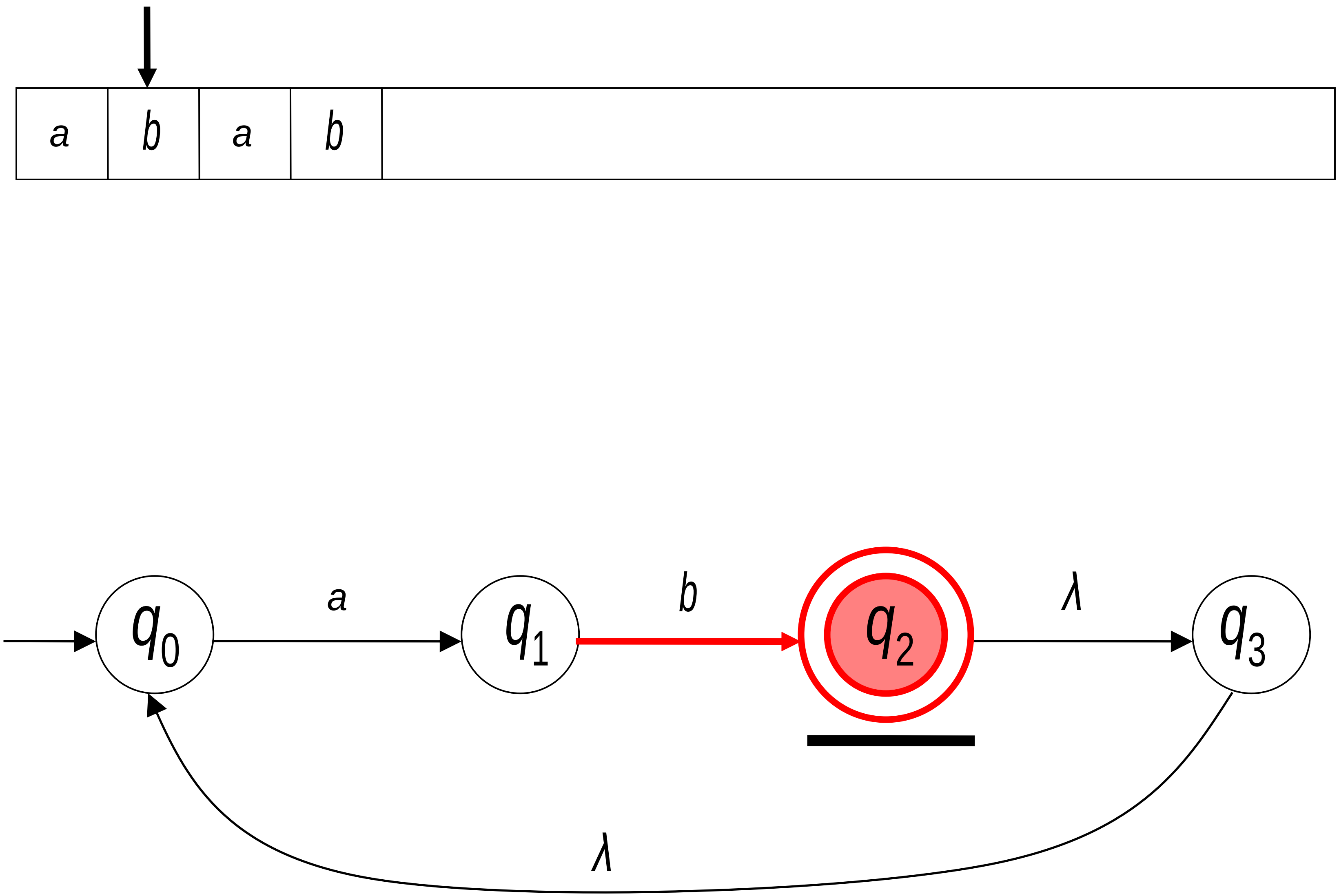

q0
a
b
q2
λ
q3
λ

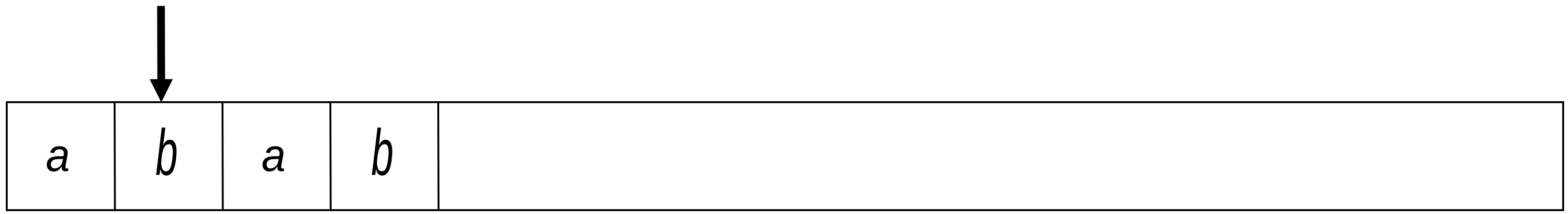

a
b
"accept"
q0
a
q1
b
q2
λ
q3
λ

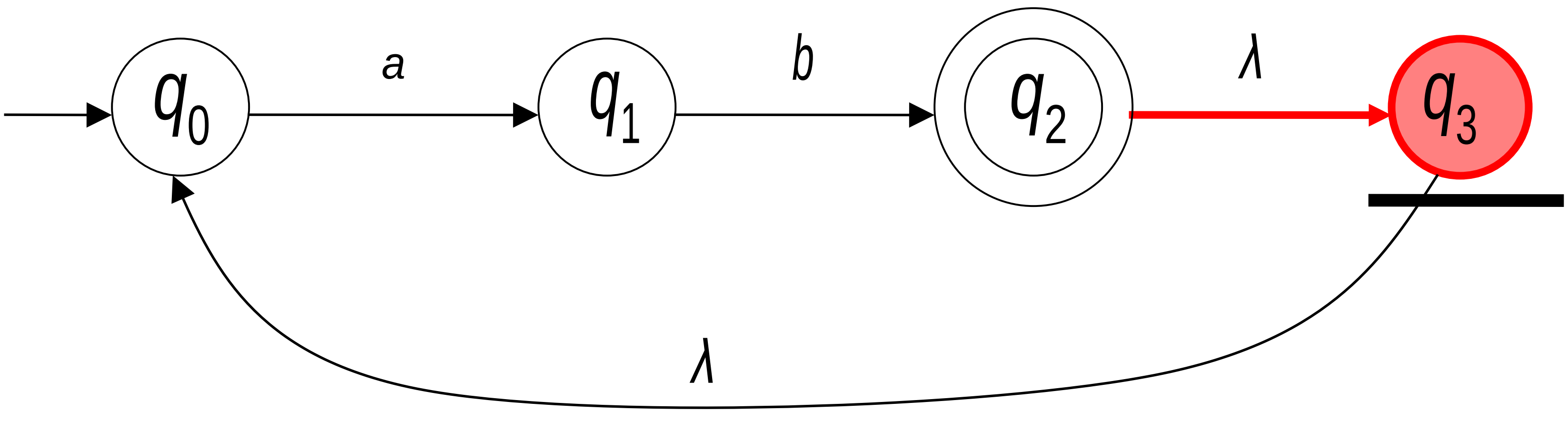
Another String

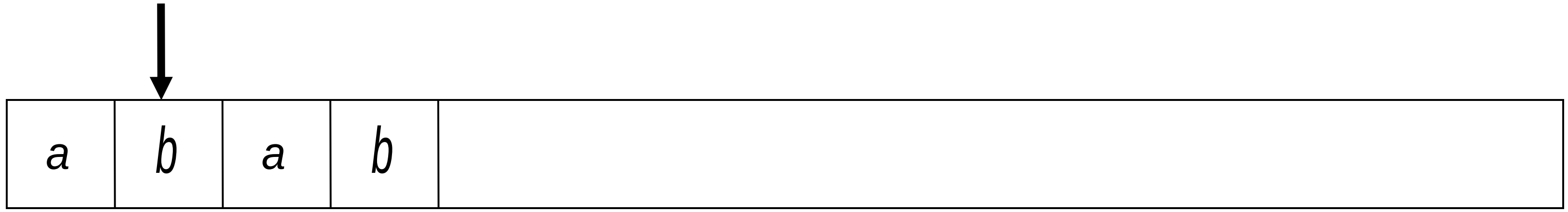



a
b
a
b
a
q_0
q_1
b
q_2
λ
q_3
λ

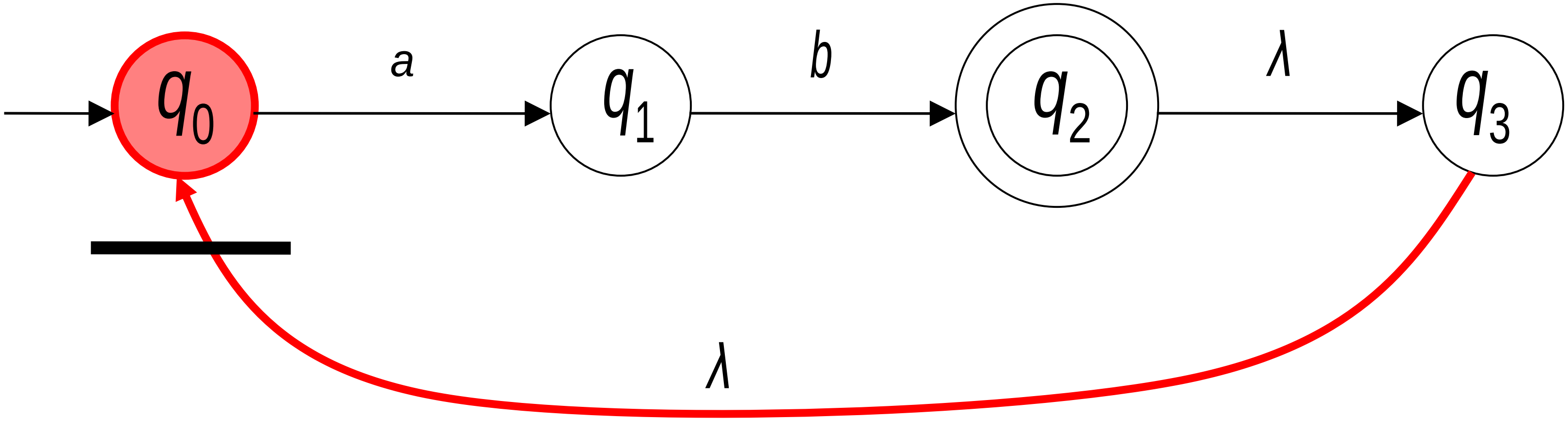

a
b
a
b
q_0
a
q_1
b
q_2
λ
q_3
λ


a
b
a
b

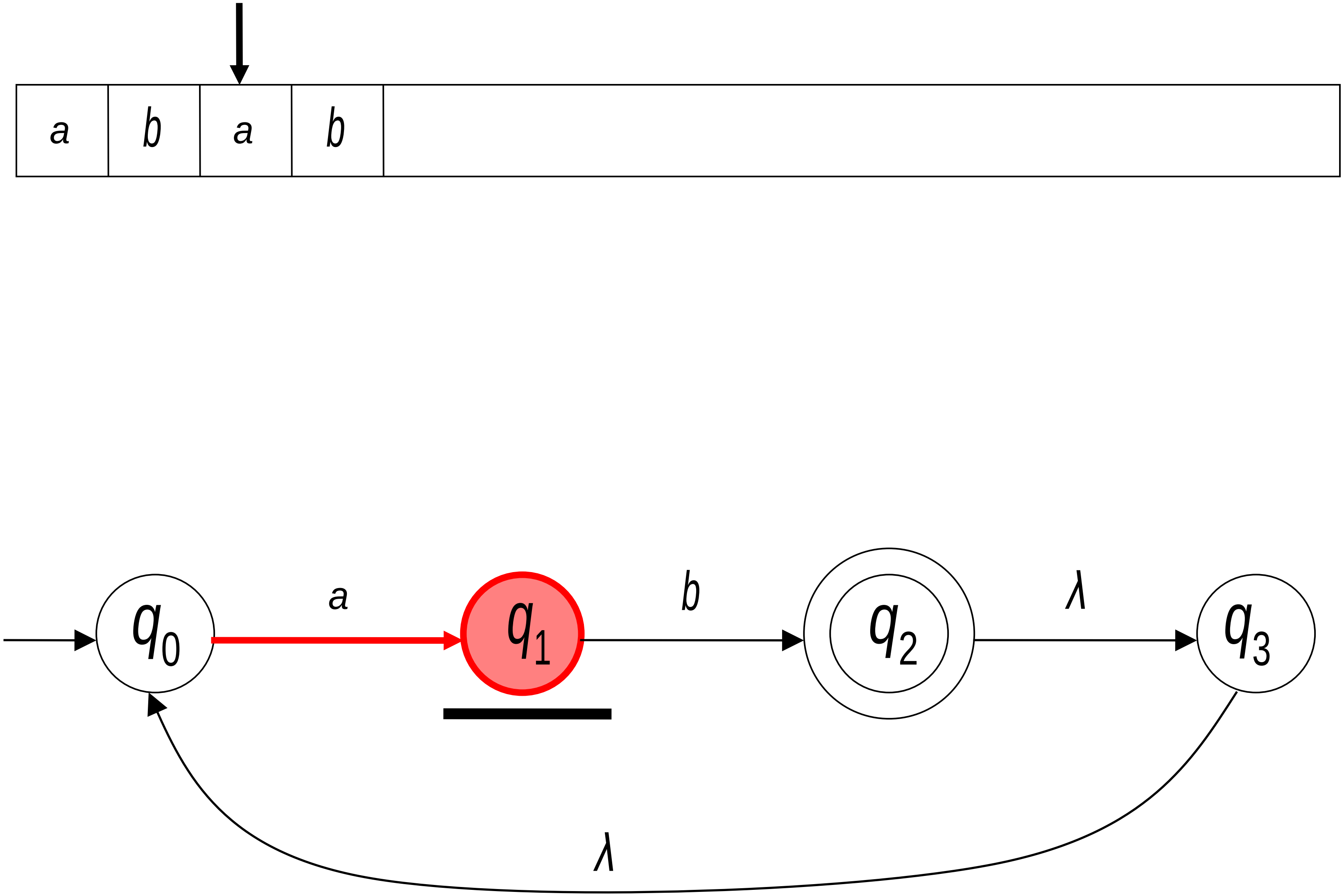

q_0
a
q_1
b
q_2
λ
q_3
λ

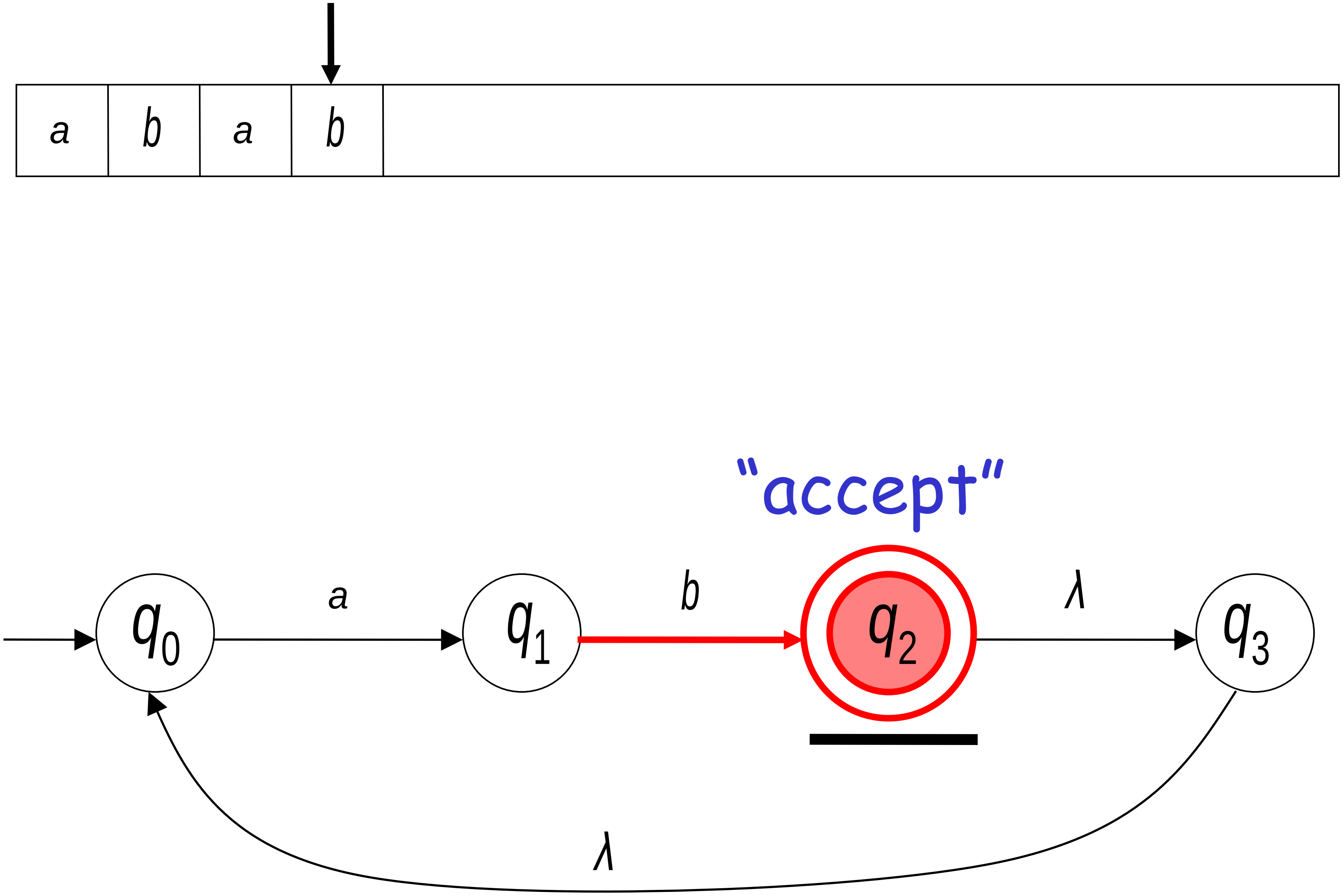


a
b
a
b



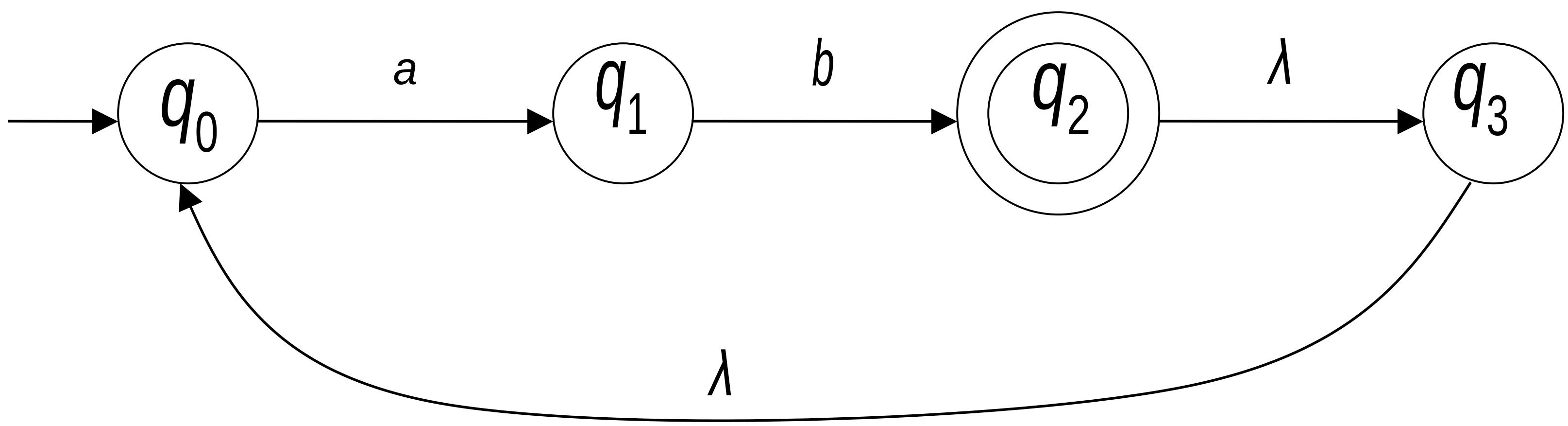
q_0
a
q_1
b
q_2
λ
q_3
λ


a
b
a
b
q_0
a
q_1
b
q_2
λ
q_3
λ

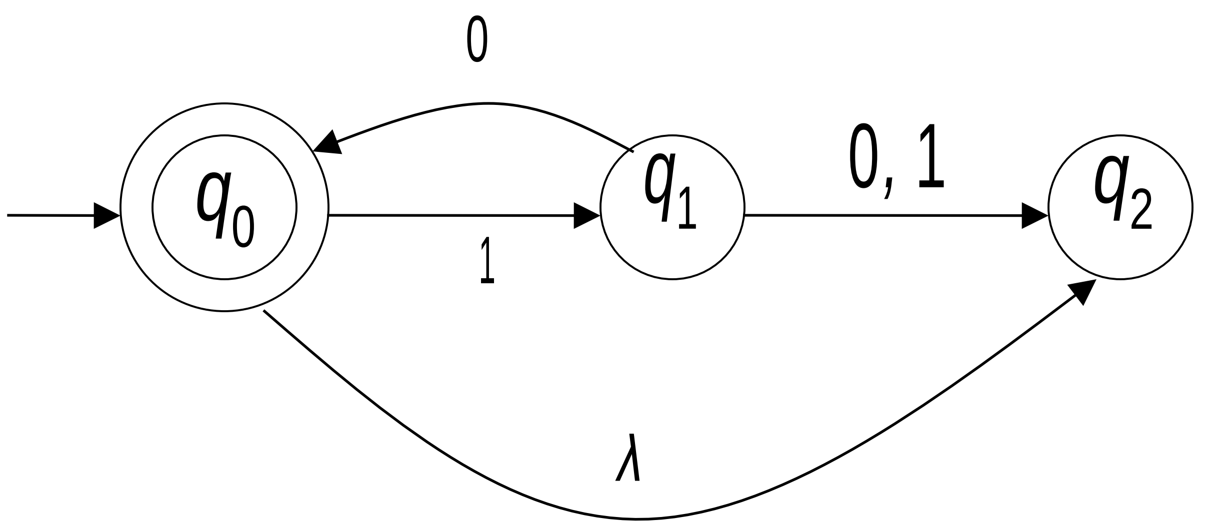

a
b
a
b
"accept"
a
b
λ
λ
q_0
q_1
q_2
q_3

Language accepted

$$L = \{ab,\ abab,\ ababab,\ \ldots\}$$
$$= \{ab\}^+$$

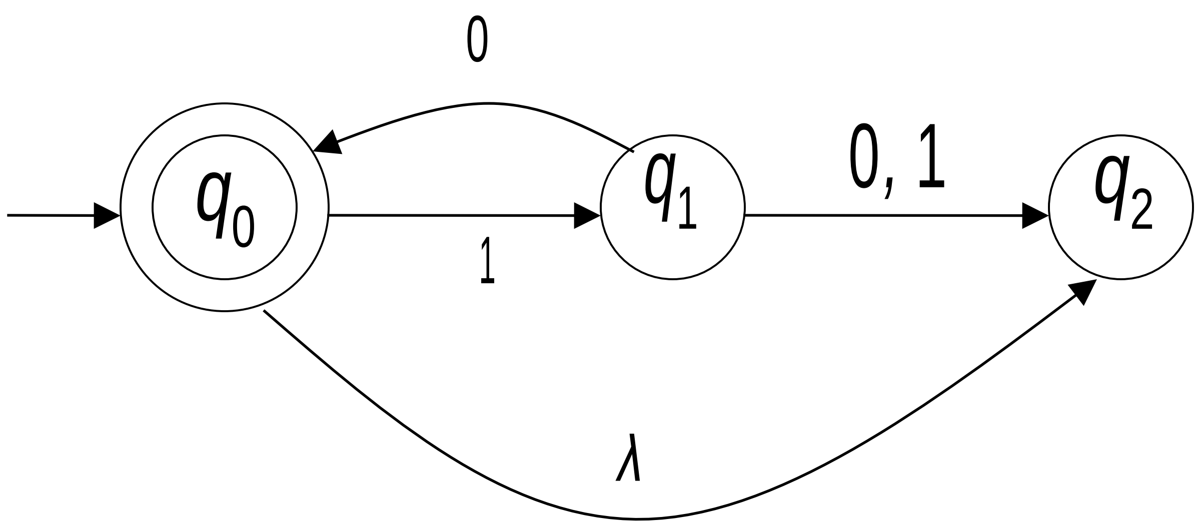


Another NFA Example



Language accepted

$$L(M) = \{\lambda, 10, 1010, 101010, \ldots\}$$
$$= \{10\}^*$$

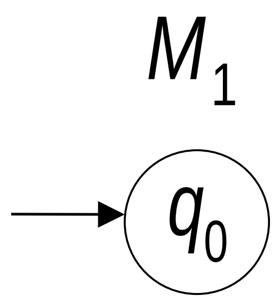


(redundant state)

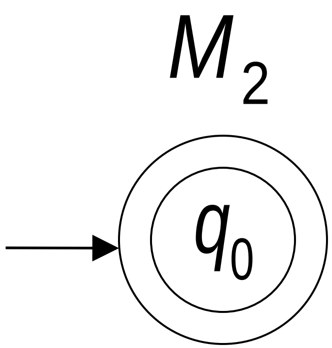
Remarks:

- The λ symbol never appears on the input tape

- Simple automata:



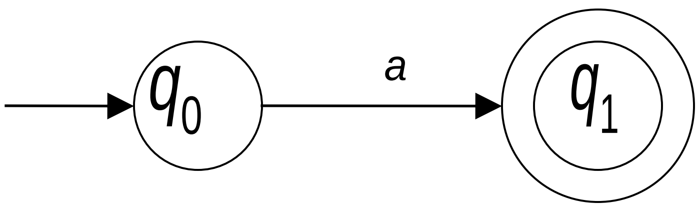
$L(M_1) = \{\}$



$L(M_2) = \{\lambda\}$

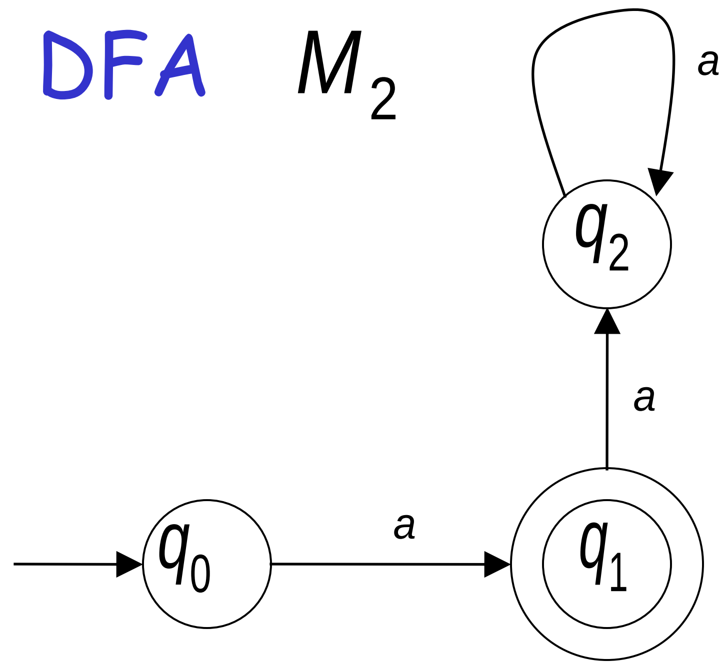
- NFAs are interesting because we can express languages easier than DFAs

NFA M_1



$$L(M_1) = \{a\}$$

DFA M_2



$$L(M_2) = \{a\}$$

Formal Definition of NFAs

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q: Set of states, i.e. $\{q_0, q_1, q_2\}$

Σ: Input aplhabet, i.e. $\{a, b\}$ $\lambda \notin \Sigma$

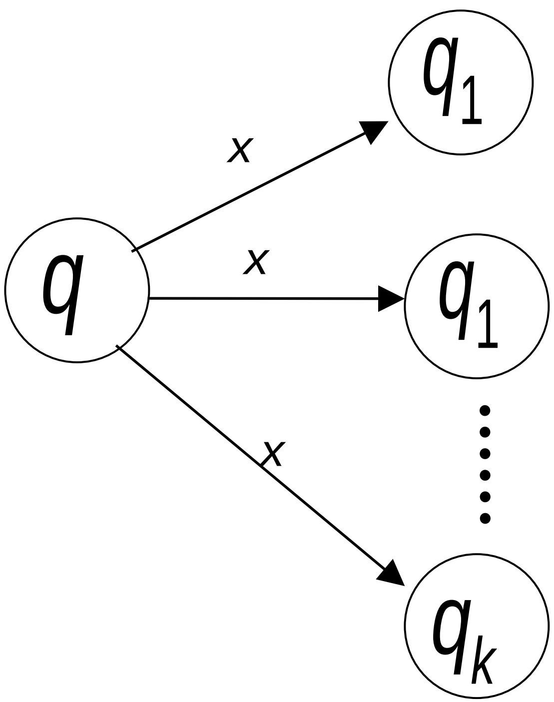
δ: Transition function

q_0: Initial state

F: Accepting states

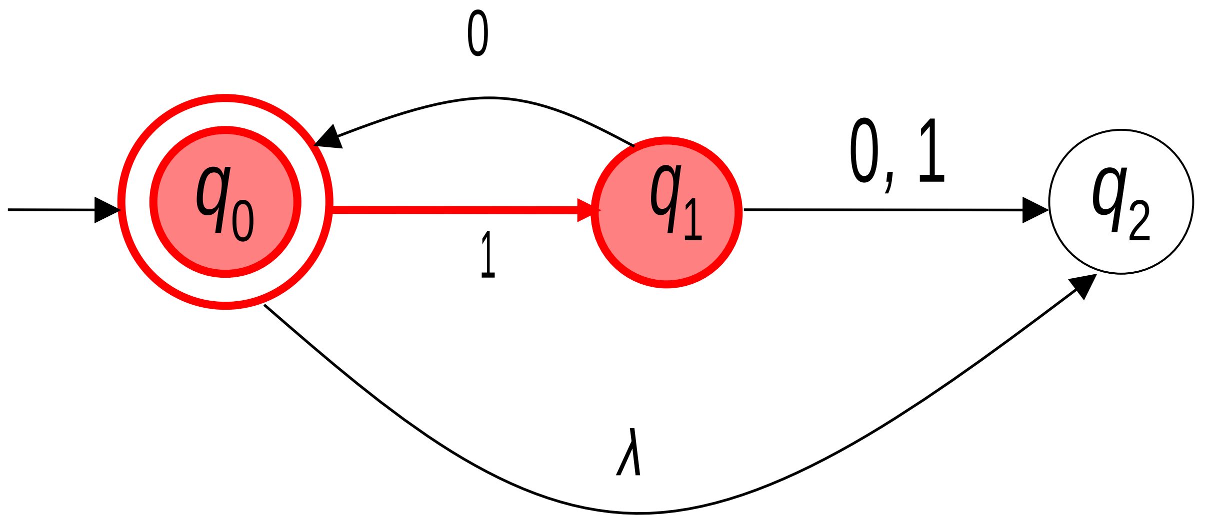
Transition Function δ

$$\delta(q, x) = \{q_1, q_2, \ldots, q_k\}$$

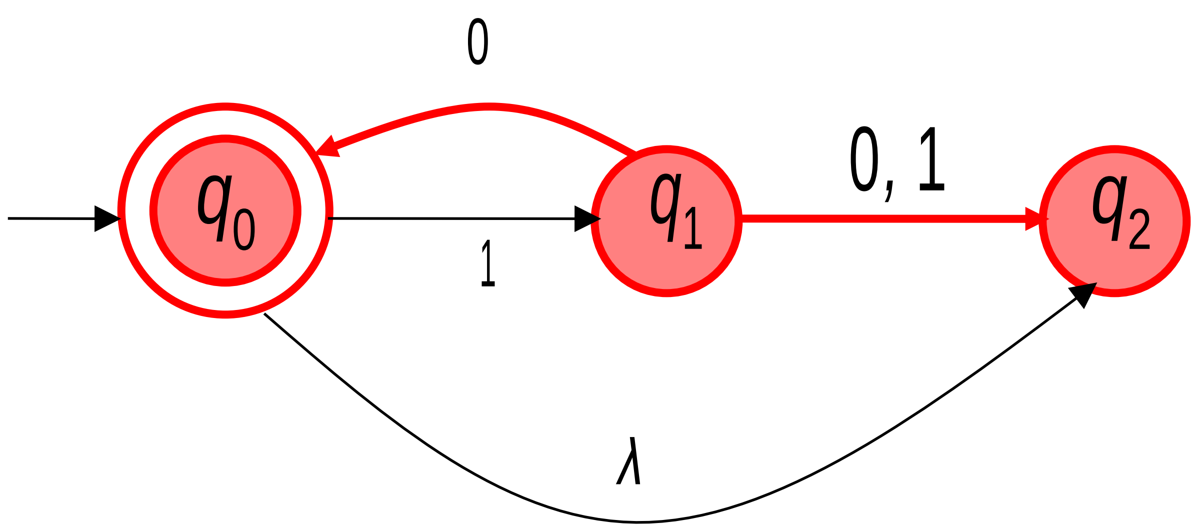


resulting states with following one transition with symbol x

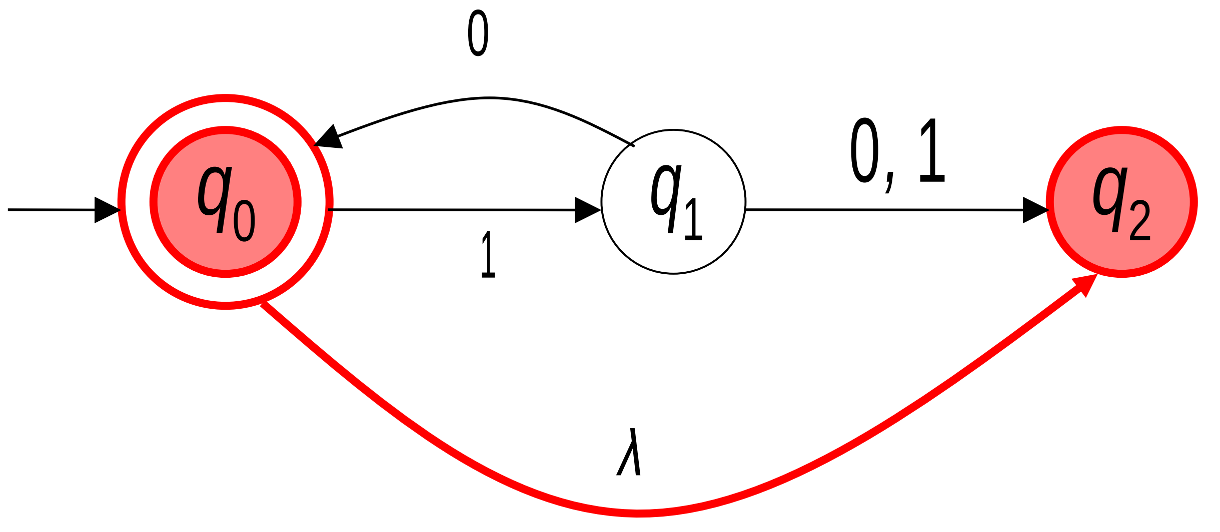
$$\delta(q_0, 1) = \{q_1\}$$



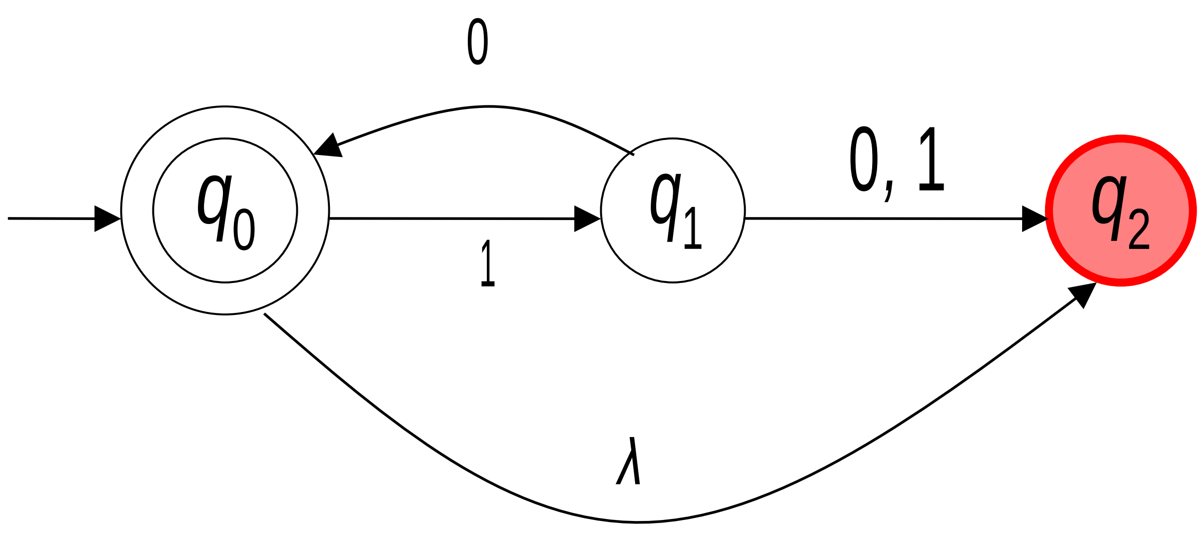
$$\delta(q_1, 0) = \{q_0, q_2\}$$



$$\delta(q_0, \lambda) = \{q_2\}$$



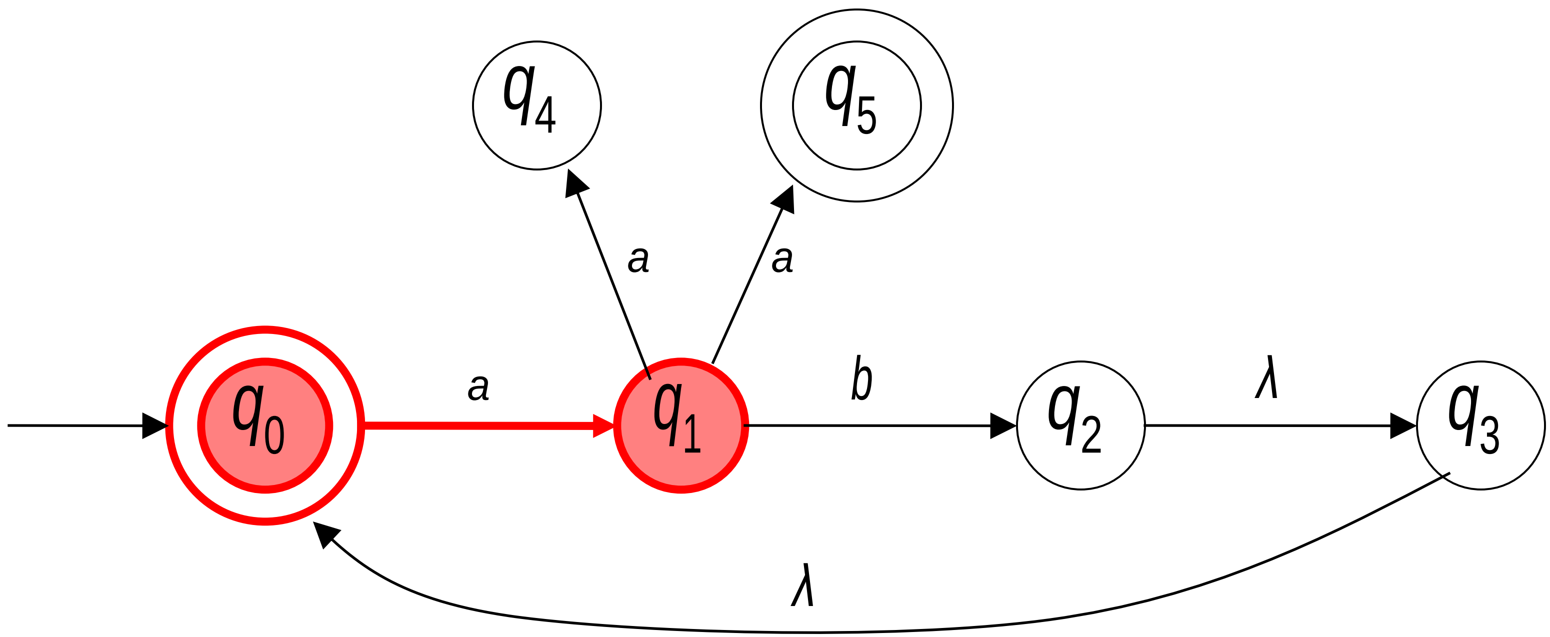
$$\delta(q_2, 1) = \emptyset$$



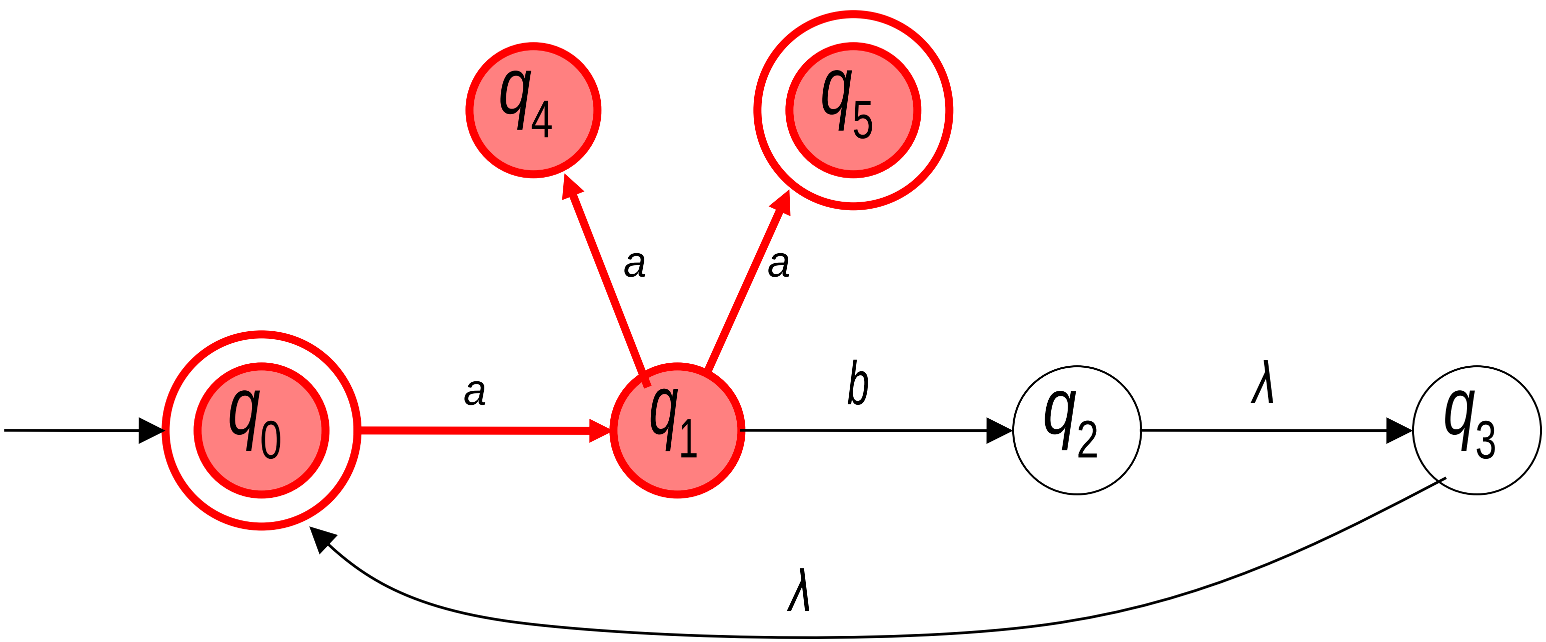
Extended Transition Function δ^*

Same with δ but applied on strings

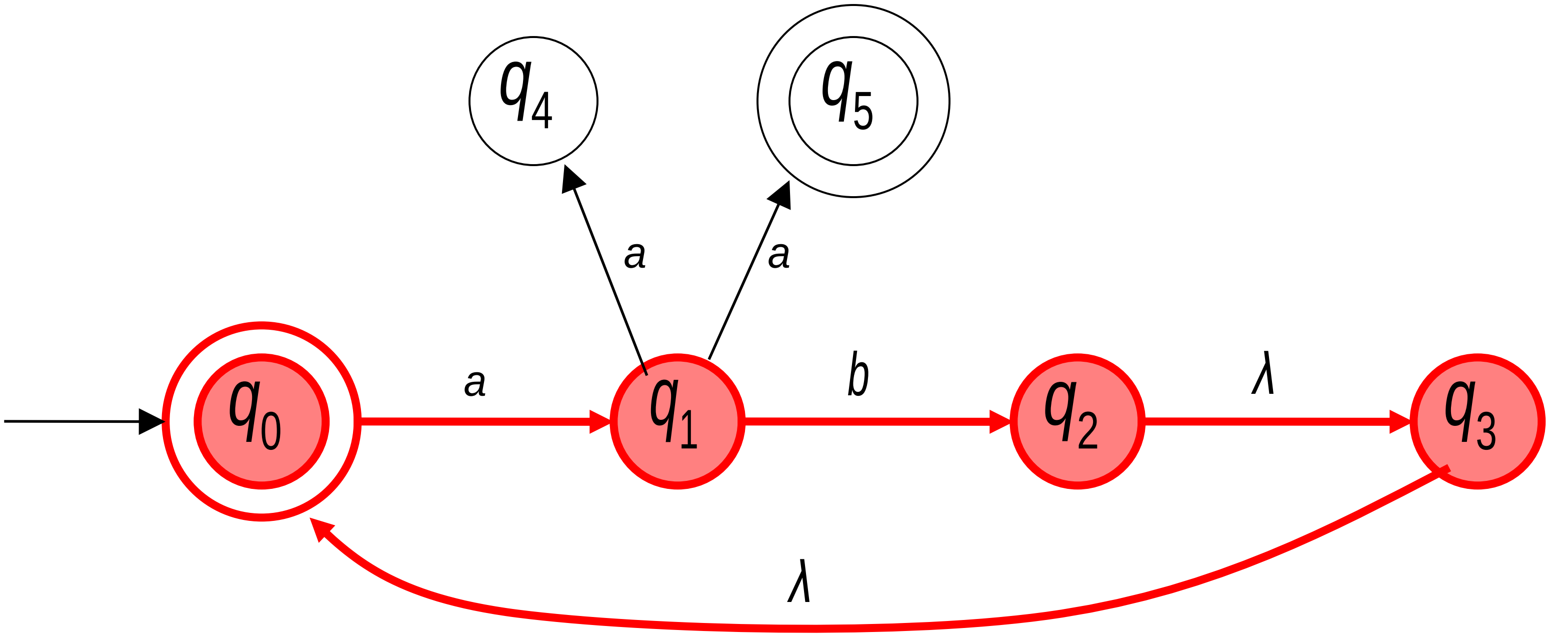
$$\delta(q_0, a) = \{q_1\}$$



$$\delta(q_0, aa) = \{q_4, q_5\}$$



$$\delta(q_0, ab) = \{q_2, q_3, q_0\}$$



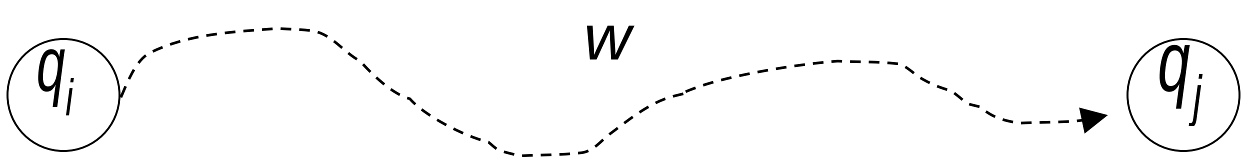
Special case:

for any state q

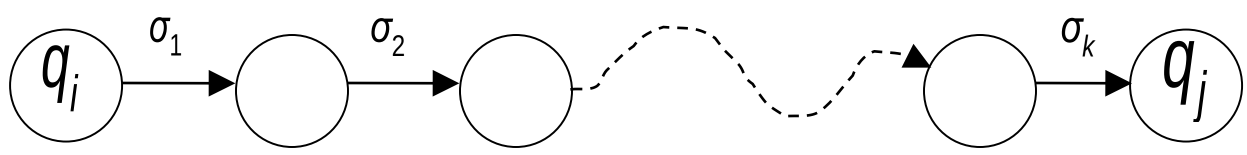
$$q \in \delta(q, \lambda)$$

In general

$q_j \in \delta(q_i, w)$: there is a walk from q_i to q_j with label w



$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



The Language of an NFA M

The language accepted by M is:

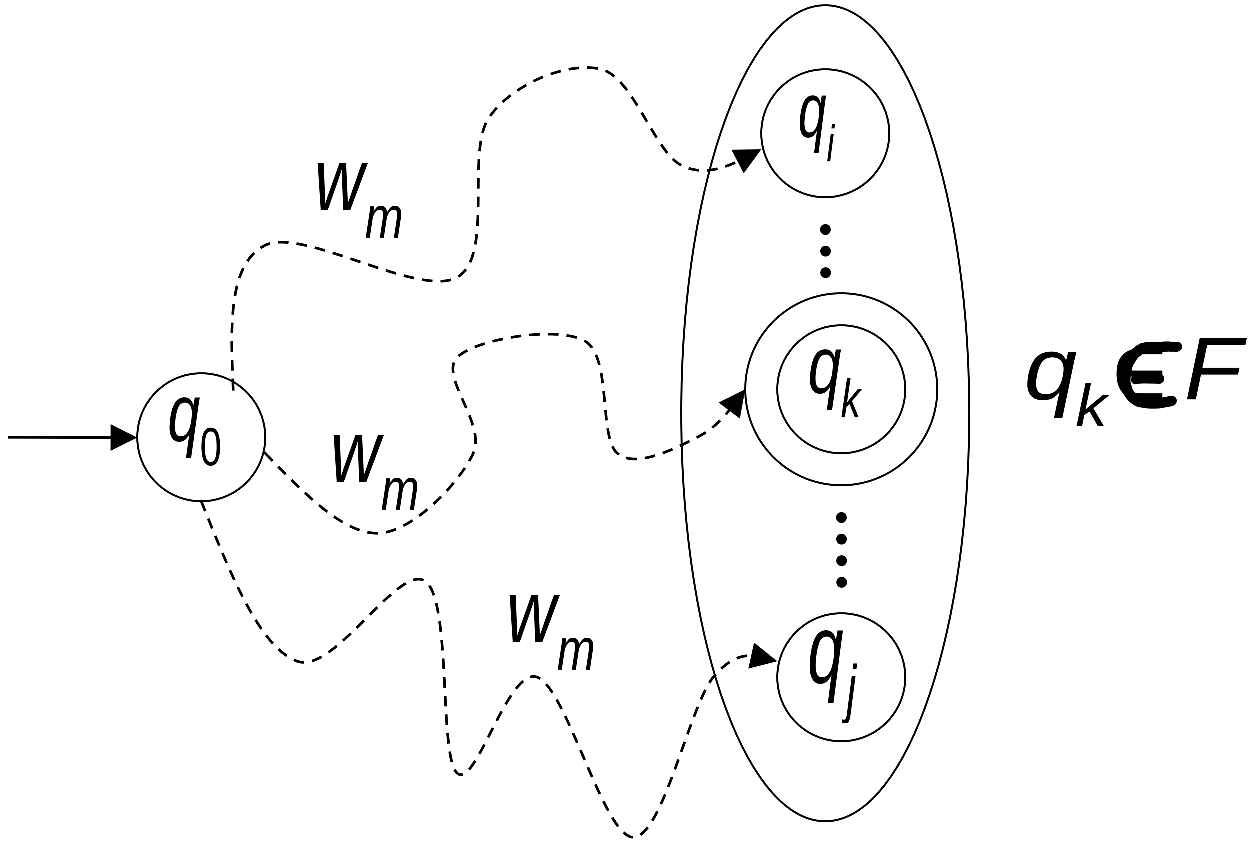
$$L(M) = \{w_1, w_2, \ldots w_n\}$$

where $\delta(q_0, w_m) = \{q_i, \ldots, q_k, \ldots, q_j\}$

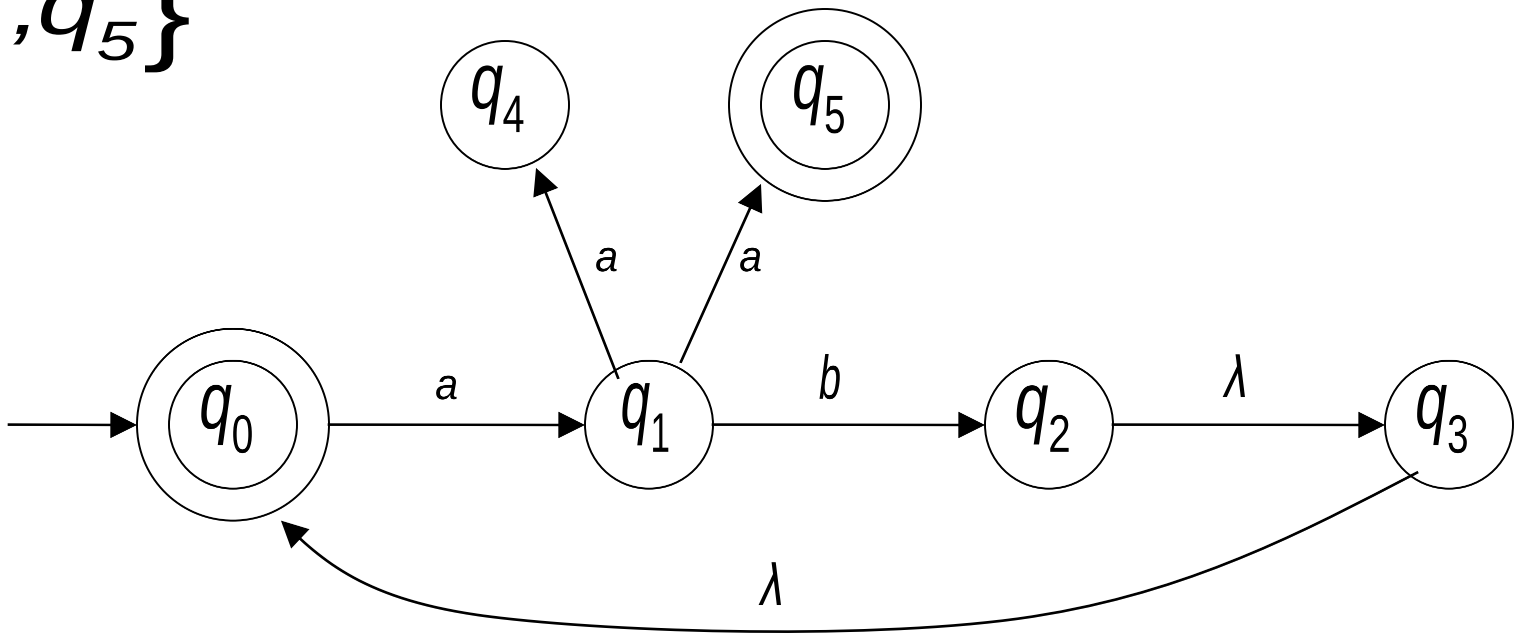
and there is some $q_k \in F$ (accepting state)

$w_m \in L(M)$

$\delta(q_0, w_m)$

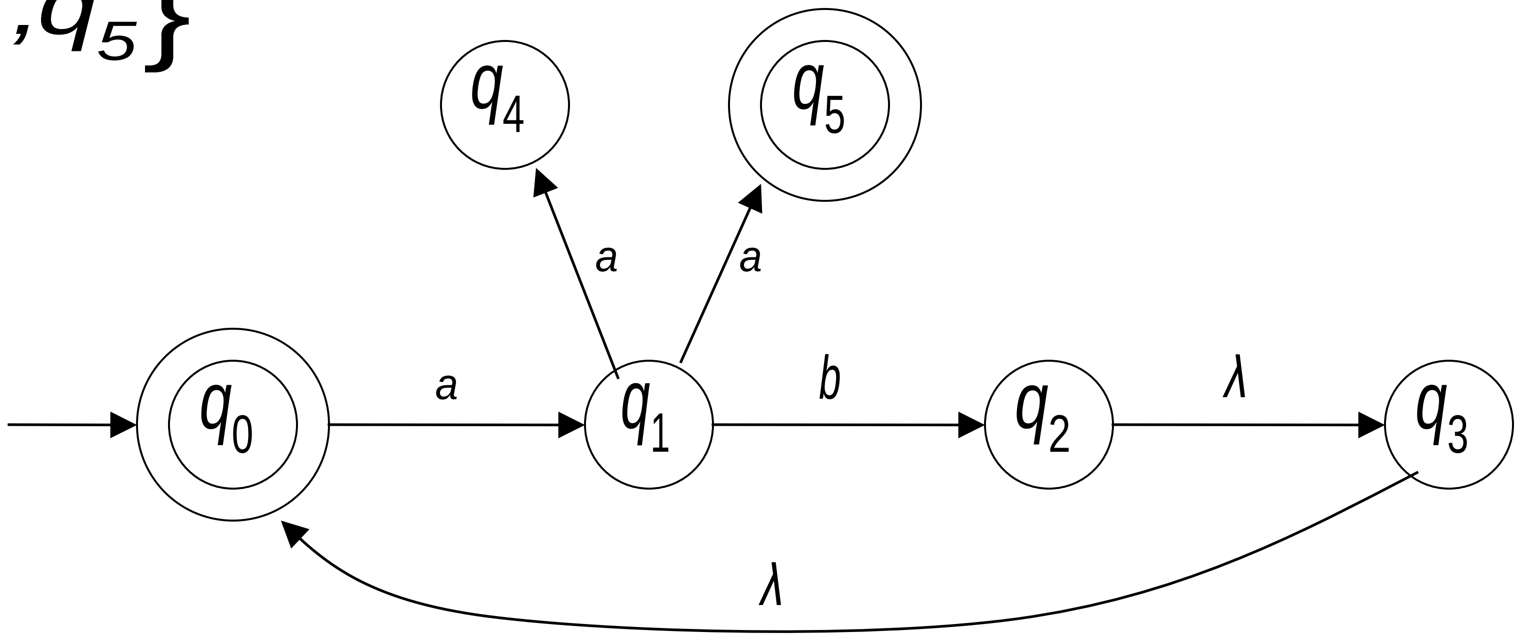


$$F = \{q_0, q_5\}$$



$$\delta(q_0, aa) = \{q_4, \underbrace{q_5}_{F}\} \Longrightarrow aa \in L(M)$$

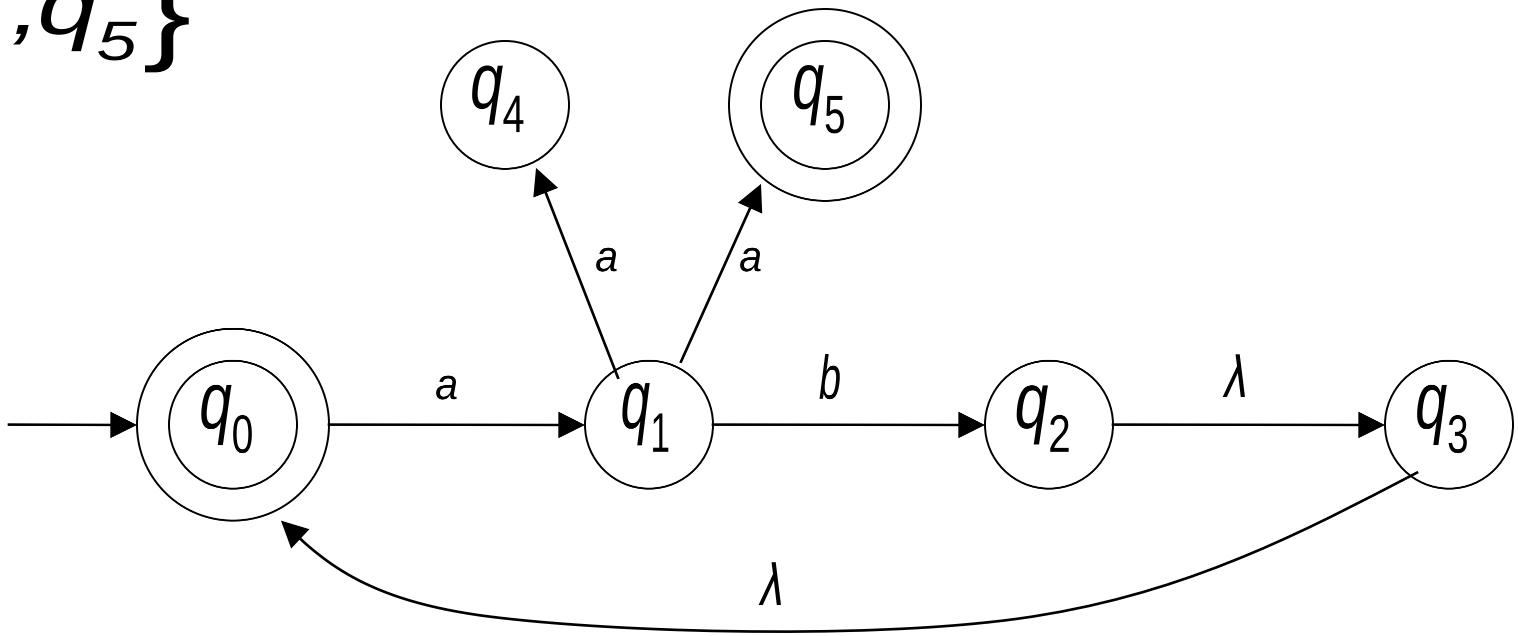
$$F = \{q_0, q_5\}$$



$$\delta(q_0, ab) = \{q_2, q_3, \underline{q_0}\} \Longrightarrow ab \in L(M)$$

F

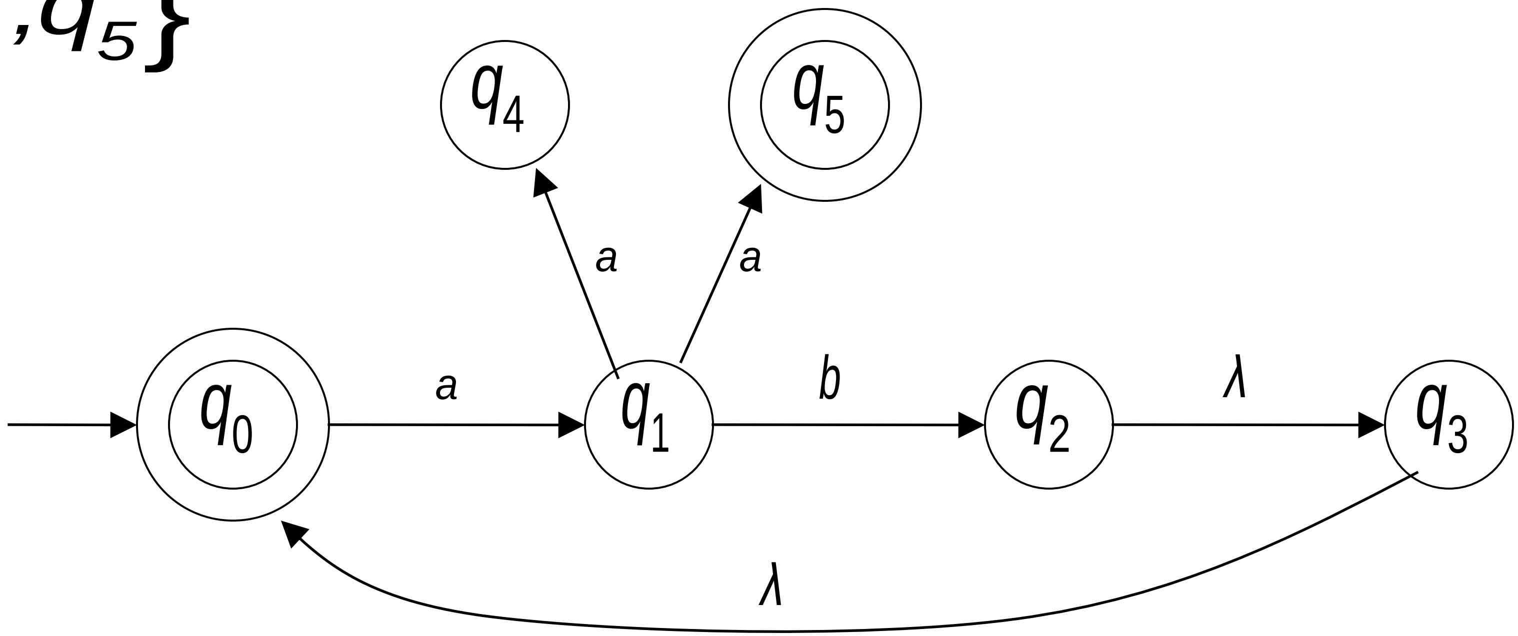
$$F = \{q_0, q_5\}$$



$$\delta(q_0, abaa) = \{q_4, \underline{q_5}\} \Longrightarrow abaa \in L(M)$$

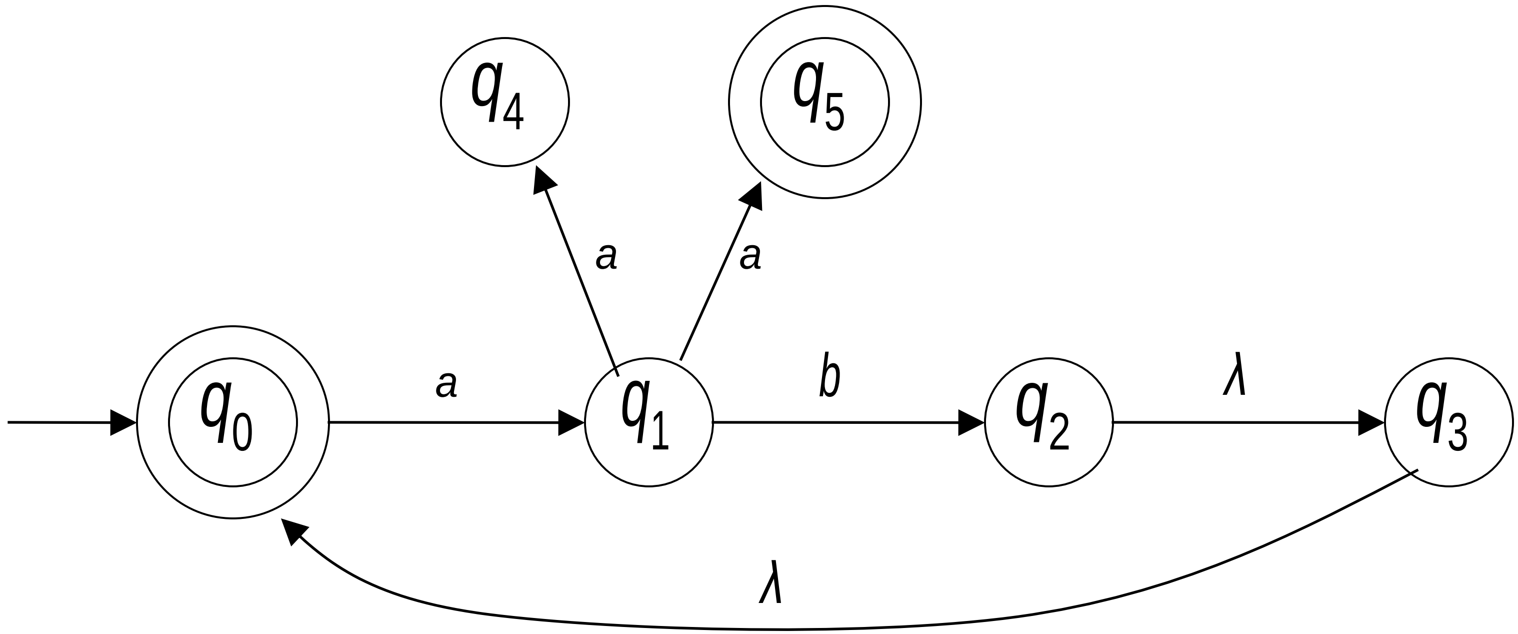
F

$$F = \{q_0, q_5\}$$



$$\delta(q_0, aba) = \{q_1\} \qquad \Longrightarrow \qquad aba \notin L(M)$$

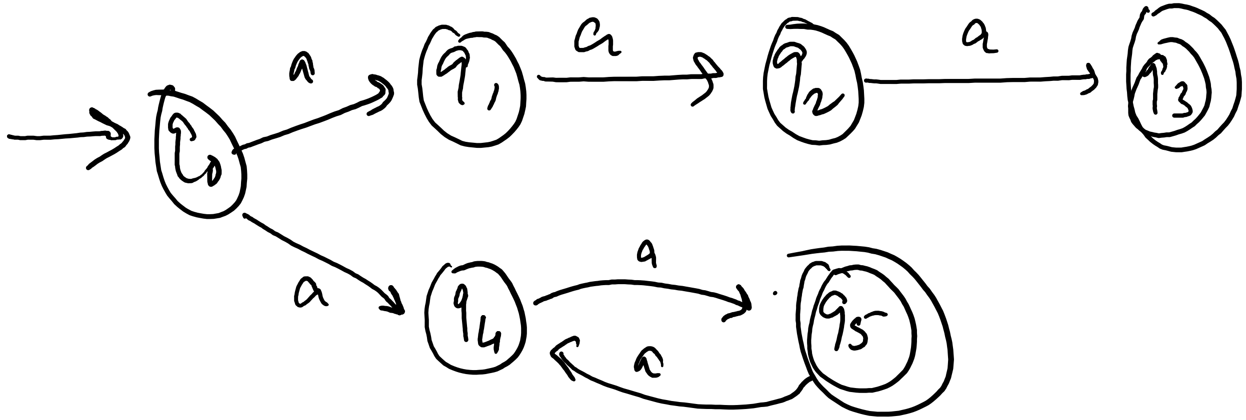
$\not\in F$



What is the language accepted by NFA?

$L(M) = \{ab\}^* \cup \{ab\}^*aa$

1



2 $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$

NFA with 4 states

3 NFA with 5 states for

$L = \{abab^n \mid n \geq 0\} \cup \{aba^n \mid n \geq 0\}$