Indian Institute of Information Technology Vadodara
MA 102: Introduction to Discrete Mathematics
Tutorial 6

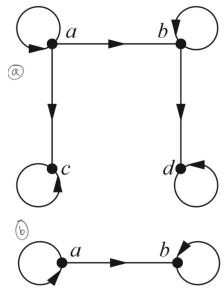
1. Let $A = \{2, 3, 4, ..., 100\}$ with partial order of divisibility.
   (a) How many maximal elements does $(A, |)$ have?
   (b) Give a subset of A that is a linear order under divisibility and is as large as possible.

2. A person's blood type is determined by the presence (T) or absence (F) of antigens A, B and Rh, as shown in the table below.

A	B	Rh	Type
F	F	F	O^-
F	F	T	O^+
F	T	F	B^-
F	T	T	B^+
T	F	F	A^-
T	F	T	A^+
T	T	F	AB^-
T	T	T	AB^+

   A person with blood type X can donate blood to a person with blood type Y, if and only if all of the antigens present in X are contained in Y. Let P be the set of the eight possible blood types, and let R be the relation on P such that XRY if and only if a person with blood type X can donate blood to a person with blood type Y. Answer the following questions.
   (a) Can a person with A^+ blood type donate to one with A^- ?
   (b) What types of blood can a person with A^+ blood type receive?
   (c) Draw a directed graph for R.
   (d) Show that R is a partial order.
   (e) Make a Hasse diagram for R.
   (f) What are the minimal (universal donor) and maximal (universal acceptor) elements of P?

3. Determine whether the relation with the directed graph shown is a partial order.



4. Display all the partial orders on a set with three elements with the help of Hasse diagram. How many of them are lattices?

5. Let R be a partial order on a finite set S. Describe how to use the matrix representation M_R to find the least and greatest element of A if they exist. **Greatest element:** $y \in (S, \preceq)$ is greatest if $x \preceq y$ for all $x \in S$.
   Least element: $z \in (S, \preceq)$ is least if $z \preceq x$ for all $x \in S$.

6. Give an example of an infinite lattice with neither a least element nor a greatest element.
7. Give an example of an infinite lattice with a least element and a greatest element.
8. What time does a 12 hour clock read 45 hours before it reads 1:00pm?
9. What is -101 mod(13)?
10. Find integers a, b, c, m which do not satisfy following statement:
    If $ac \equiv bc$ mod(m) with $m \geq 2$, then $a \equiv b$ mod(m).
11. Show that $2^{340} \equiv 1 \text{mod}(31)$.
12. Find the last digit of 333^{555}.