Indian Institute of Information Technology Vadodara
MA 101: Linear Algebra and Matrices
Tutorial 8

1. Construct the general solution of $x' = Ax$ involving complex eigenfunctions and then obtain the general real solution, where $A = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$

2. Let $x(t)$ be the position of a particle at time t. Let A be a 2×2 matrix with eigenvalues -3 and -1 corresponding to eigenvectors $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Solve the initial value problem $x' = Ax$ with $x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

3. Find the minimal polynomial of following matrix and answer diagonalizabilty and triangulability of the matrix by analysing minimal polynomial.
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

4. Find a 3×3 matrix whose minimal polynomial is λ^2.

5. Show that every matrix $A \in M_n(\mathbb{R})$ with $A^2 = A$ is diagonalisable over $\mathbb{R}$.

6. Give an example of a matrix which is not diagonalisable over $\mathbb{C}$. Show that every square matrix with complex entries is triangulable over $\mathbb{C}$.

7. Let $P(t) = a_0 + a_1 t + \cdots + a_{n-1} t^{n-1} + t^n$. Consider $A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & & & & \vdots \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}$.
Show that characteristics polynomial of A is $P(t)$. What can you say about minimal polynomial of A? Construct a 4×4 matrix which is triangulable over $\mathbb{R}$ but not diagonalizable over $\mathbb{R}$.