Indian Institute of Information Technology Vadodara
Mid-semester Examination
MA 102 (Mathematics II: Linear Algebra and Matrices)

Time: 60 minutes **Max. Marks: 20**

Instructions:

1. Calculators/electronic devices are not allowed to use.
2. Write down answers neatly in sequence.
3. Each question carries 5 marks.

1. Consider the following linear system:
$x_1 - 5x_2 - 7x_3 + 6x_4 = -7$
$-x_1 + 2x_2 + 4x_3 - 3x_4 = 2$
$4x_1 - 6x_2 - 14x_3 + 10x_4 = -4$
$4x_1 - 8x_3 + 4x_4 = 6$
a) Write down the linear system in matrix form (AX=b).
b) Find all solutions of it using elementary row operations/Gaussian elimination.
c) Does the set of all solutions of it form a vector space?
d) What is the rank of A, obtained in a)?

2. Find a polynomial whose graph passes through $(1, 12), (2, 15), (3, 16), (4, 17)$. What is its degree? How many such polynomials exists?

3. Find an invertible matrix V and a diagonal matrix D such that $A = VDV^{-1}$, *where*
$$A = \begin{bmatrix} 3 & -2 & 8 \\ 0 & 5 & -2 \\ 0 & -4 & 3 \end{bmatrix}.$$
Is V orthogonal? Do the same for $A + 2I$?

4. Find the best approximate solution to $AX = b$, where
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$$
What is the projection of b onto column space of A?