Indian Institute of Information Technology Vadodara
MA 102: Introduction to Discrete Mathematics
Tutorial 7

1. State and prove divisibility test of 3, 5, 11.

2. What is the remainder obtained when $2^{70} + 3^{70}$ is divided by 13?

3. Find the multiplicative inverse of each non-zero element of $\mathbb{Z}_{11}$ to verify $\mathbb{Z}_{11}$ is a field.

4. Find all integers x such that $2x \equiv 3 \bmod 5, 3x \equiv 4 \bmod 7, x \equiv 5 \bmod 11$.

5. Use Fermat's little theorem to compute 5^{2021} mod 7, 5^{2021} mod 11, and 5^{2021}mod 13. Use Chinese remainder theorem to find 5^{2021} mod 1001.

6. Let GCD(a, 143)=1. Show that $a^{142} \equiv 1 \bmod 143$.

7. Prove that if n is a positive integer such that the sum of the divisors of n is n + 1, then n is prime.

8. Show that Goldbach's conjecture, which states that every even integer greater than 2 is the sum of two primes, is equivalent to the statement that every integer greater than 5 is the sum of three primes.

9. A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes
   a) are there in total?
   b) contain exactly two heads?
   c) contain at most three tails?
   d) contain the same number of heads and tails?

10. How many bit strings of length 10 either begin with 3 zeros or end with 3 ones?