End-semester Examination (Pen and paper mode) 2023

PH110: Waves and Electromagnetics

Marks: 45

Time: 180 Minutes

- **All questions are compulsory and their marks is indicated in square bracket.**
- **All questions needs to be answered sequentially without fail. Non-compliance of instruction will invite deduction in marks.**
- **In case you feel any question/s is/are incorrect or have insufficient instruction then write in the answer book with your justification without wasting any time**

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1. (i) Write down the Maxwell's equations for free space and inside the linear medium with no free charge or free current in both differential and integral form.

(ii)

Suppose

$$\mathbf{E}(r,\theta,\phi,t) = A\frac{\sin\theta}{r}\left[\cos(kr-\omega t) - (1/kr)\sin(kr-\omega t)\right]\hat{\boldsymbol{\phi}}, \quad \text{with } \frac{\omega}{k} = c.$$

(a) Show that $\mathbf{E}$ obeys all four of Maxwell's equations, in vacuum, and find the associated magnetic field.

(b) Calculate the Poynting vector. Average $\mathbf{S}$ over a full cycle to get the intensity vector $\mathbf{I}$. (Does it point in the expected direction? Does it fall off like r^{-2}, as it should?)

(c) Integrate $\mathbf{I}\cdot d\mathbf{a}$ over a spherical surface to determine the total power radiated.

(iii)

Show that the mode TE_{00} cannot occur in a rectangular wave guide. Consider a rectangular wave guide with dimensions $2.28\ \text{cm} \times 1.01\ \text{cm}$.

What TE modes will propagate in this wave guide, if the driving frequency is 1.70×10^{10} Hz? Suppose you wanted to excite only *one* TE mode; what range of frequencies could you use? What are the corresponding wavelengths (in open space)?

[5+5+5 = 15 Marks]

2. (a)

Sea water at frequency $\nu = 4 \times 10^8$ Hz has permittivity $\epsilon = 81\epsilon_0$, permeability $\mu = \mu_0$, and resistivity $\rho = 0.23\ \Omega\cdot\text{m}$. What is the ratio of conduction current to displacement current? [Consider a parallel-plate capacitor immersed in sea water and driven by a voltage $V_0\cos(2\pi\nu t)$.]

(b)

An atomic electron (charge q) circles about the nucleus (charge Q) in an orbit of radius r; the centripetal acceleration is provided, of course, by the Coulomb attraction of opposite charges. Now a small magnetic field dB is slowly turned on, perpendicular to the plane of the orbit. Show that the increase in kinetic energy, dT, imparted by the induced electric field, is just right to sustain circular motion *at the same radius* r.

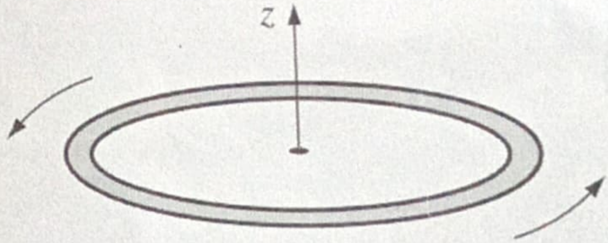
(c)

A thin uniform donut, carrying charge Q and mass M, rotates about its axis as shown in Fig.

(a) Find the ratio of its magnetic dipole moment to its angular momentum. This is called the **gyromagnetic ratio** (or **magnetomechanical ratio**).

(b) What is the gyromagnetic ratio for a uniform spinning sphere?

(c) According to quantum mechanics, the angular momentum of a spinning electron is $\frac{1}{2}\hbar$, where $\hbar$ is Planck's constant. What, then, is the electron's magnetic dipole moment, in $\mathrm{A}\cdot m^2$?



[5+5+5 = 15 Marks]

3. Write short note on: (i) Scalar/vector fields and fundamental theorem of divergence and curl, (ii) Bound surface and volume charge (current) density, (iii) Maxwell's correction in Ampere's law, (iv) Self/Mutual inductance and Faraday's law, (v) Solutions of Laplace equation and Uniqueness theorem, (vi) Conservation of charges and equation of continuity or Dirac Delta functions.

[2.5*6 = 15 Marks]

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In spherical polar coordinates:

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}.$$

Curl:

$$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}}$$
$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}.$$