AI Quiz 3

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Q1

Sol

$P(x_1, \dots x_4, y_1 \dots y_4) = \frac{1}{2} \prod_{c \in C} \psi_c(x_c)$.

(A)

$\prod_{c \in C} \psi_c(x_c) = \psi(x_1, y_1)\,\psi(x_1 x_2)\,\psi(x_1, x_3)$.
$\psi(x_2 y_2)\,\psi(x_2 x_4)\,\psi(x_2 x_1)$
$\psi(x_3 y_3)\,\psi(x_3 x_4)\,\psi(x_3 x_1$
$\psi(x_4 y_4)\,\psi(x_4 x_3)\,\psi(x_4 x_2)$.

~~Maximal clique~~ $= \psi(x_1, x_2)\cdot\psi(x_1, y_1) \times (x_1 x_3) \times \psi(x_2 x_4) \times \psi(x_3 x_4) \times \psi(x_2 x_4)$

(B) $\prod_{c \in C} \psi_c(x_c) = \psi(x_1 y_1)\,\psi(x_1 x_3)\,\psi(x_1 x_2)$
$\psi(x_2 y_2)\,\psi(x_2 x_3)$
$\psi(x_3 y_3)\,\psi(x_3 x_4)\,\psi(x_3 x_1)\,\psi(x_3 x_2)$

Maximal clique $= \psi(x_1, x_2, x_3) \to$ maximum clique $= \{x_1, x_2, x_3\}$

(C) $\prod_{c \in C} \psi(x_c) = \psi(x_1 y_1)\,\psi(x_1 x_2)\,\psi(x_1 x_3)$.
$\psi(y_2 y_2)\,\psi(x_2 x_3)\,\psi(x_2 x_1)\,\psi(x_2 x_4)$
$\psi(x_3 y_3)\,\psi(x_3 x_4)\,\psi(x_3 x_1)\,\psi(x_3 x_2)$
$\psi(x_4 y_4)\,\psi(x_4 x_3)\,\psi(x_4 x_2)$.

Maximal clique $= \psi(x_1, x_2, x_3)$ & $\psi(x_2, x_3, x_4)$.

Maximum clique available $= \{x_1, x_2, x_3, x_4\}$

$\psi(x_1, y_1) \times \psi(x_1, x_3) \times \psi(x_3, y_3)$

Maximal clique: $\psi(x_1, x_2) \times \psi(x_1, y_1) \times \psi(x_3, y_4) \times \psi(x_4 y_4)$.
$\times \psi(x_2, y_2) \times \psi(x_3, y_4) \times$

Q2

$P(DT \mid \theta) = \prod_i p(DT(Row) \mid \theta)$

$f := P(x_1 = m_1, x_2 = m_2, x_3 = m_3) \cdot (a, b, c_1, c_2, c_3, c_4)$.

$= P(x_1 = m_1 \mid a)\, P(x_2 = m_2 \mid b) \cdot P(x_3 = m_3 \mid x_1 = m_1, x_2 = m_2, a, b, c_1, c_2, c_3, c_4)$

m_1	m_2	m_3	f
0	0	0	$a \cdot b \cdot c_1$
0	0	1	$a b (1-c_1)$
0	1	0	$a(1-b)c_2$
0	1	1	$a(1-b)(1-c_2)$
1	0	0	$(1-a)b(c_3)$
1	0	1	$(1-a)b(1-c_3)$
1	1	0	$(1-a)(1-b)c_4$
1	1	1	$(1-a)(1-b)(1-c_4)$

∴ Log likelihood function is:

$LL(a, b, c_1 c_2 c_3 c_4 \mid DT) = \Sigma_{\{i=1\}}^{\{N\}} [x_{i1} * x_{i2} * x_{i3} * \log(a b c_1)$

$+ x_{i1} * x_{i2} * (1 - x_{i3}) * \log(a \cdot b \cdot (1-c_1))$

$+ x_{i1} * (1 - x_{i2}) * x_{i3} * \log(a \cdot (1-b) * c_2)$

$+ x_{i1} * (1 - x_{i}) \log(a * (1-b)(1-c_2))$.

$+ (1 - x_{i1}) * x_{i2} * x_{i3} * \log((1-a) * b * c_3)$

$+ (1 - x_{i1}) * x_{i2} * (1 - x_{i3}) * \log(1-a) * b * (1-$

$+ (1 - x_{i1}) * (1 - x_{i2}) x_{i3} * \log(1-a) * (1-b) * c_4)$

$+ (1 - x_{i1}) * (1 - x_{i2}) \cdot (1 - x_{i3})$

$* \log((1-a)(1-b)(1-c_4))]]$

Q2 → continue in next page

Q3: ~~Compt~~ solⁿ

(a) UGM represented as product of factor function defined over maximal cliques of factor graph. Maximal cliques are $\{x_1, x_2\}$ and $\{x_2, x_3\}$. so, UGM is,

$UGM(x_1 x_2 x_3) = f_1(x_1, x_2) * f_2(x_2, x_3)$.

where f_1, f_2 are factor fⁿ over ~~max.~~ clique $\{x_1, x_2\}$ & $\{x_2, x_3\}$

$f_1(x_1, x_2) = P(x_1, x_2) = (1/Z_1) * \exp\{\theta_1 * x_1 + \theta_2 * x_2 + \theta_3 * x_1 * x_2\}$.

$f_2(x_2, x_3) = P(x_2, x_3) = (1/Z_2) * \exp\{\theta_4 * x_2 + \theta_5 * x_3 + \theta_6 * x_2 * x_3\}$

where Z_1 & Z_2 are normalization constants ~~puts~~ sum to 1.

(b) factorization of each maximal clique can be written:

$\{x_1, x_2\}$ $f_1(x_1, x_2) = (1/Z_1) * e\{\theta_1 * x_1 + \theta_2 * x_2 + \theta_3 * x_1 * x_2\}$

$\{x_2, x_3\}$ $f_2(x_2, x_3) = (1/Z_2) * e\{\theta_4 * x_2 + \theta_5 * x_3 + \theta_6 * x_2 * x_3\}$

These factorizations represent the conditional prob dist. of variables in each maximal clique, given value of other variables

Q2/ continued

max $P(DT|\theta)$ ~ max $\log P(DT|\theta)$:

$$\frac{\partial (\log P(DT|\theta))}{\partial a} = \frac{10}{a} - \frac{5}{1-a} = 0 \Rightarrow a = 2/3$$

$$\frac{\partial}{\partial b} \log P(DT|\theta) = \frac{8}{b} - \frac{7}{1-b} = 0 \Rightarrow b = 8/15.$$

$$\frac{\partial (\log P(DT|\theta))}{\partial c_1} = 3/c_1 - 3/1-c_1 = 0 \Rightarrow c_1 = \frac{1}{2}$$

$$\frac{\partial (\log P(DT|\theta))}{\partial c_2} = \frac{2}{c_2} - \frac{2}{1-c_2} = 0 \Rightarrow c_2 = \frac{1}{2}$$

$$\frac{\partial (\log P(DT|\theta))}{\partial c_3} = \frac{1}{c_3} - 1/1-c_3 = 0 \Rightarrow c_3 = \frac{1}{2}$$

$$\frac{\partial \log P(DT|\theta)}{\partial c_4} = \frac{1}{c_4} - \frac{2}{1-c_4} = 0 \Rightarrow c_4 = \frac{1}{3}.$$

Q4/ For 1st part, computing prob of $x_1 = 0, x_2 = 0, x_3 = 0$ using UGM.
of data table ...
using factor function over maximal clique $\{x_1, x_2\}$ we have
$f_1(x_1 = 0, x_2 = 0) = e(0.2 * 0 - 0.5 * 0 + 0.3 * 0 - 0.4)$.

Similarly using factor function over max. clique (x_2, x_3).

$f_2(x_2 = 0, x_3 = 0) = e(0.6 * 0 - 0.7 * 0 + 0.4 * 0 - 1.2)$

cont
Hence joint prob of $x_1 = 0, x_2 = 0, x_3 = 0$
$P(x_1 = 0, x_2 = 0, x_3 = 0) = f_1(x_1 = 0, x_2 = 0) * f_2(x_2 = 0, x_3 = 0)$.
$= e(-0.4) * e(-1.2)$.
$= e(-1.6)$

For 2nd part, cond. prob of $x_1 = 0, x_2 = 0$, given $x_2 = 0$ w. UGM
factor fn over $\{x_1, x_2\}$
$f_1(x_1 = 0, x_2 = 0) = e(0.1 * 0 - 0.5 * 0 + 0.3 * 0 - 0.4)$.

factor fn over cl. $\{x_2\}$
$f_2(x_2 = 0) = e(-0.7 * 0 + 0.6 * 0 - 0.3)$.

∴ cond. prob of $x_1 = 0, x_2 = 0$ given $x_2 = 0$

$$P(x_1 = 0, x_2 = 0 | x_2 = 0) = \frac{P(x_1 = 0, x_2 = 0)}{P(x_2 = 0)}$$

$$= \frac{f_1(x_1 = 0, x_2 = 0)}{f_2(x_2 = 0)}.$$

$$= \exp(-0.4) / \exp(-0.3)$$

$$= e(0.1)..$$

∴ Prob. of $x_1 = 0, x_2 = 0, x_3 = 0$ is $e(-1.6)$

∴ Cond. prob of $x_1 = 0, x_2 = 0$ gn $x_2 = 0$ is $e(0.1)$.