Indian Institute of Information Technology Vadodara
MA 102: Introduction to Discrete Mathematics
Tutorial 2

1. For each of the sequence of integers given below, give a simple formula or rule that gives any term of the sequence which start with the list.

    (a) $15, 8, 1, -6, -13, -20, -27, \ldots$

    (b) $3, 5, 8, 12, 17, 23, 30, \ldots$

    (c) $2, 3, 7, 25, 121, 721, 5041, 40321, \ldots$

2. Find the value of following sums

    (a) $\sum_{j=0}^{8} 3.2^j$

    (b) $\sum_{j=0}^{8} 3.(-2)^j$

    (c) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$

3. Let $f, g : A \rightarrow A$ be functions. If g is surjective and $f \circ g$ is injective then does it follow that f is injective? Give justification.

4. Let A, B, C be three subsets of a universal set U. Draw a Venn diagram and shade the area representing $A \cup (B \cap C)^c$.

5. Find a bijective map $f : [a, b] \rightarrow [c, d]$ thereby proving that $|[a, b]| = |[c, d]|$ for any $a, b, c, d \in \mathbb{R}$.

6. Find a bijective map $f : P(\mathbb{N}) \rightarrow (0, 1)$ thereby proving that $|\mathbb{R}| = |(0, 1)|$.

7. Find a bijective map $f : [a, b] \rightarrow (a, b)$ for $a < b$ thereby proving that $|[a, b]| = |(a, b)|$.

8. Determine whether each of the following sets is countable or uncountable(not countable).

    (a) $B = \{(x, y) | x \in \mathbb{N}, y \in \mathbb{Z} - \{0\}\}$.

    (b) $C = \mathbb{R} \setminus \mathbb{Q}$.

    (c) $A = $ set of all complex numbers.

9. Show that the set of all binary strings is countable, thereby proving set of all programs is countable.

10. (HILBERT'S GRAND HOTEL) We now describe a paradox that shows that something impossible with finite sets may be possible with infinite sets. The famous mathematician David Hilbert invented the notion of the Grand Hotel, which has a countably infinite number of rooms, each occupied by a guest. When a new guest arrives at a hotel with a finite number of rooms, and all rooms are occupied, this guest cannot be accommodated without evicting a current guest. However, we can always accommodate a new guest at the Grand Hotel, even when all rooms are already occupied. How can we accommodate a new guest arriving at the fully occupied Grand Hotel without removing any of the current guests?