SUNWAY
INT'L BUSINESS SCHOOL

Programme Name: __________**BCS HONS**____

Course Code: __**MATH 1023**________

Course Name: _________**Additional Mathematics**____________________________

Mathematics Open Book Examination

Date of Submission: _______**10/4/2020**_____________

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MATH1023_Dipesh Tha Shrestha

1. A. Prove that: cotθ + tanθ = secθ cscθ

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Q No 1.

a. Prove that: $\cot\theta + \tan\theta = \sec\theta . \csc\theta$

Solution.

Let $A = \cot\theta + \tan\theta$

and

$B = \sec\theta . \csc\theta$

Now,

$$A = \cot\theta + \tan\theta$$

$$A = (\cos\theta/\sin\theta) + (\sin\theta/\cos\theta)$$

$$A = \frac{(\cos^2\theta + \sin^2\theta)}{(\sin\theta \cdot \cos\theta)}$$

$\{$As we know, $\cos^2\theta + \sin^2\theta = 1\}$

$$A = \frac{1}{\sin\theta \cdot \cos\theta}$$

$$A = \left(\frac{1}{\cos\theta}\right) \cdot \left(\frac{1}{\sin\theta}\right)$$

$$A = \sec\theta . \csc\theta$$

$A = B$ proved//

$\therefore \cot\theta + \tan\theta = \sec\theta . \csc\theta$ proved//

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b. A boy is flying a kite .The thread is completely stretched by a strong wind it makes an angle of 600 with the ground surface .The length of the thread used between the kite and winder is 30 m. At what height from the ground is the kite flying?

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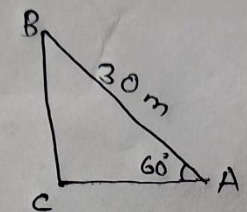
1.b.

Solution.

Let AB be the length of the thread use between kite and winder. BC be the height from the ground to the flying kite.



Now

$\angle BAC = 60°$ (Given)

$\sin\theta = \frac{BC}{AB} = \frac{P}{h}$

$\therefore \sin 60° = \frac{BC}{30}$

or, $30 \times \frac{\sqrt{3}}{2} = B \cdot \left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

$\therefore BC = 15\sqrt{3}$ m.

$\therefore$ The height from the ground to the flying kite is $15\sqrt{3}$ m.

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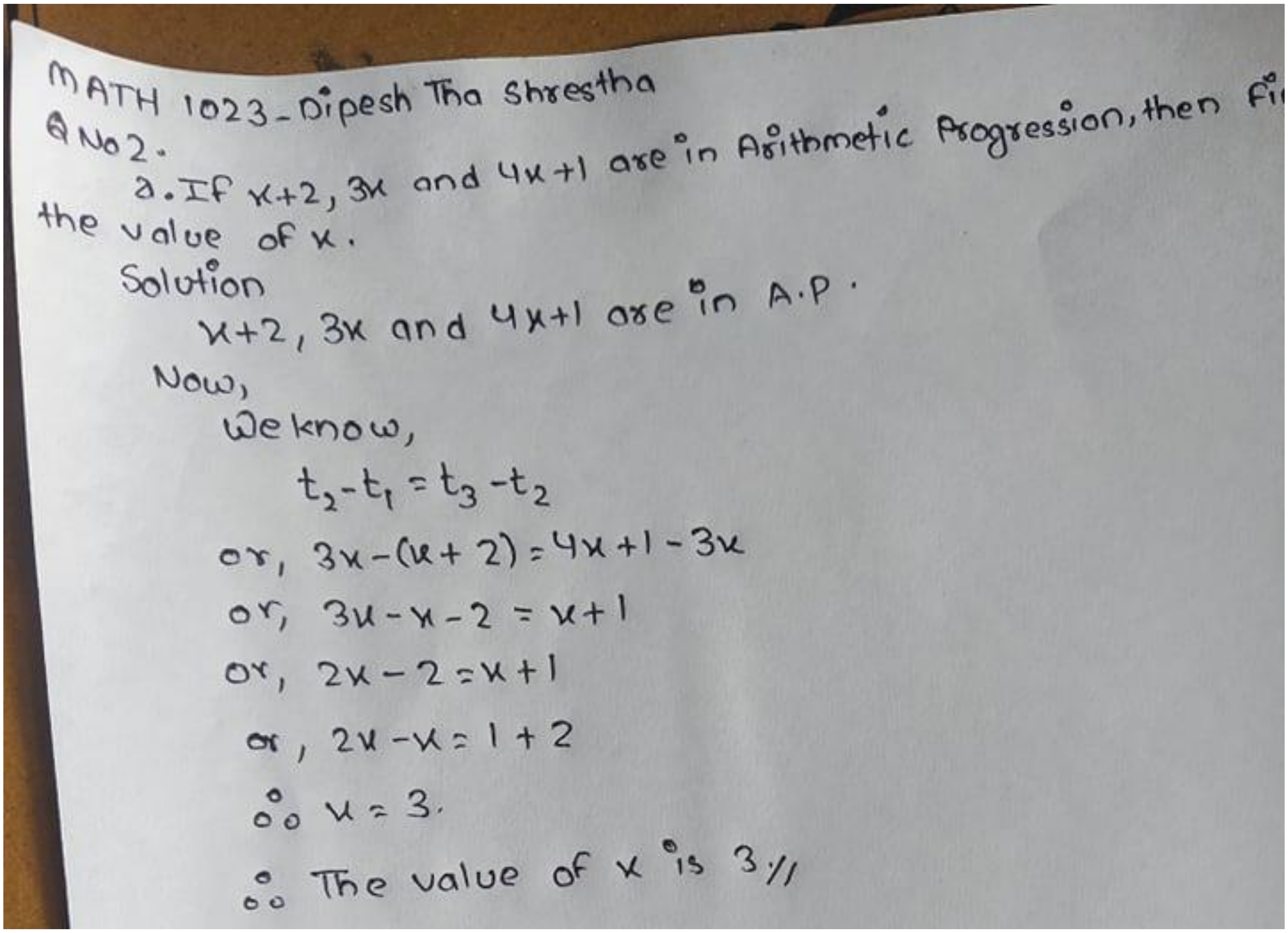
2. a. If x+2, 3x and 4x + 1 are in Arithmetic Progression then find the value of x.



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Q No 2.

a. If $x+2$, $3x$ and $4x+1$ are in Arithmetic Progression, then fi[nd] the value of x.

Solution

$x+2$, $3x$ and $4x+1$ are in A.P.

Now,

We know,

$$t_2 - t_1 = t_3 - t_2$$

or, $3x-(x+2) = 4x+1-3x$

or, $3x - x - 2 = x+1$

or, $2x - 2 = x + 1$

or, $2x - x = 1 + 2$

$\therefore x = 3.$

$\therefore$ The value of x is 3.//

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b. Find the sum of 2+ 6+18+54+ ...up to 15th term.

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2b. Find the sum of 2+6+18+54+...up to 15th term

Solution

Given,

$a = 2.$

$r = \frac{6}{2} = 3$

$n = 15$

Now,

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{15} = \frac{2(3^{15} - 1)}{3 - 1}$

$S_{15} = \frac{2(3^{15} - 1)}{2}$

$\therefore S_{15} = 14348906$

2. a. Find root of the equation using completion of square: 2x2 - 12x - 9 = 0.

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QNo 3 a) Find root of the equation using completion of square: $2x^2-12x-9=0$

Solution

Here, By using completion of square in the given equation.

i) Dividing the equation by 2, we get

$$x^2-6x-9/2=0$$

ii) Subtracting the constant term from both sides, we get,

$$x^2-6x-9/2+9/2=0+9/2$$

or $x^2-6x=9/2$

iii) Adding the square of one-half of variable 'x' to both sides.

$$x^2-6x+(1/2\times6)^2=9/2+(1/2\times6)^2$$

or, $x^2-6x+9=9/2+9$

or $x^2-6x+9=\frac{9+18}{2}$

or $x^2-6x+9=27/2$

or $(x-3)^2=27/2$

or $x-3=\sqrt{27/2}$

~~or x = 3 ± 3√~~

or $x=3\pm\frac{3\sqrt{6}}{2}$

Hence, the root of the equation are $3\pm\frac{3\sqrt{6}}{2}$.

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b. If α and β be the roots of x2 + px + q = 0, find the quadratic equation whose roots are α/β and β/α.

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3b. If $\alpha = \beta$ be the roots of $x^2+px+q=0$, find the quadratic equation whose roots are α/β and β/α.

Solution,

If α, β are the roots of the equation $x^2-px+q=$

Then, sum of the roots $= \alpha+\beta = p \ldots$ (i)

Similarly, product of the roots $= \alpha\beta = q \ldots$ (ii)

If $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are roots of the a quadratic equation, then the quadratic equation is given ~~below~~ by:

$$\left(x-\frac{\alpha}{\beta}\right)\left(x-\frac{\beta}{\alpha}\right)=0$$

or $x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+1=0$

or $\alpha\beta x^2-(\alpha^2+\beta^2)x+1=0$

or $\alpha\beta x^2-((\alpha+\beta)^2-2\alpha\beta)x+1=0 \ldots$ (iii)

Subtracting (i), (ii) in (iii),

we get

$$qx^2-(p^2-2q)x+q=0$$

$$\Rightarrow qx^2+(2q-p^2)x+q=0$$

$\therefore$ $qx^2+(2q-p^2)x+q$ is the quadratic equation whose roots are α/β and β/α.

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4. a. Find the derivative of tanx using first principle

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4 a) Find the derivative of tanx using first principle.

Solution

let, $f(x) = \tan x$

$f(x+h) = \tan(x+h)$

By the first principle

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) \cdot \cos x - \sin x \cdot \cos(x+h)}{h \cdot \cos(x+h) \cdot \cos x}$$

$$= \lim_{h \to 0} \frac{\sin(x+h-x)}{h\cos(x+h) \cdot \cos x}$$

$$= \lim_{h \to 0} \frac{\sin h}{h\cos(x+h) \cdot \cos x}$$

$$= \lim_{h \to 0} \frac{\sin h}{h} \cdot \lim_{x \to 0} \frac{1}{\cos(x+h) \cdot \cos x}$$

$$= 1 \times \frac{1}{\cos x \cdot \cos x}$$

$$= \frac{1}{\cos^2 x}$$

$\therefore f'(x) = \sec^2 x$

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b. Find the maximum and the minimum values, if any, without using derivatives of the following functions: f (x) = 4x2 – 4x + 4.

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Q No 4

b.

Given,

$$f(x) = 4x^2 - 4x + 4$$
$$= 4x^2 - 4x + 1 + 3$$
$$= (2x)^2 - 2.2x.1 + (1)^2 + 3$$
$$\therefore f(x) = (2x-1)^2 + 3.$$

Here,

$$(2x-1)^2 \geq 0$$
$$\text{or } (2x-1)^2 + 3 \geq 3$$
$$\text{or } f(x) \geq f(1/2)$$

Hence, the maximum value of $f(x)$ is 3 at $x = 1/2$

Clearly, $f(x)$ can be made large.

So, maximum value does not exist.

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Section B

A

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Sec B QNo 1

a, Evaluate : $f(x) = 3\cos x + \frac{2x^4 - \sqrt{x}}{x}$

Solution

$$f(x) = 3\cos x + \frac{2x^4 - \sqrt{x}}{x}$$

$$f(x) = \frac{d}{dx}(3\cos x) + \frac{d}{dx}\left(\frac{2x^4 - \sqrt{x}}{x}\right)$$

$$= -3\sin x + \frac{d}{dx}\left(2x^{4-1} - x^{1/2-1}\right)$$

$$= -3\sin x + \frac{d}{dx}\left(2x^3 - x^{-1/2}\right)$$

$$= -3\sin x + 6x^2 - \frac{d}{dx}\left(x^{-1/2}\right)$$

$$= -3\sin x + 6x^2 - \left(-\tfrac{1}{2} x^{-3/2}\right)$$

$$= -3\sin x + 6x^2 + \tfrac{1}{2} x^{-3/2} + c //$$

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B

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Sec B

1 B

Evaluate : $\int \frac{x\,dx}{x^4+3}$

Substitute

$$u = \frac{x^2}{\sqrt{3}} \rightarrow \frac{du}{dx}$$

$$= \frac{2x}{\sqrt{3}} \Rightarrow dx = \frac{\sqrt{3}}{2x}\,du$$

$$= \int \frac{\sqrt{3}}{2(3u^2+3)\,dx}$$

Simplify:

$$= \frac{1}{2\sqrt{3}} \int \frac{1}{u^2+1}\,du$$

Now, Solving

$\int \frac{1}{u^2+1}\,du$ is a standard integral $= \arctan(u)$

$$\frac{1}{2\sqrt{3}} \int \frac{1}{u^2+1}\,du$$

$$= \frac{\arctan u}{2\sqrt{3}}$$

= Undo Substitution $u = \frac{x^2}{\sqrt{3}}$

$$= \frac{\arctan\left(x^2/\sqrt{3}\right)}{2\sqrt{3}}$$

Add constant C,

$$\frac{\arctan\left(x^2/\sqrt{3}\right)}{2\sqrt{3}} + C$$

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c. Find the area between f(x)=-x 2+4x and g(x)= x2 -6x+5 over the interval $0 \le x \le 1$

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Sec B
Q No 1 c).

Find the area between $f(x) = -x^2+4x$ and $g(x) = x^2-6x+5$ over the interval $0 \le x \le 1$.

Solution

$\rightarrow f(x) = -x^2+4x$

$g(x) = x^2-6x+5.$

$$\rightarrow \int_0^1 [4x - x^2 - (x^2 - 6x + 5)]\, dx$$

$$\text{or } \int_0^1 (4x - x^2 - x^2 + 6x - 5)\, dx$$

$$\text{or } \int_0^1 (10x - 2x^2 - 5)\, dx$$

$$\text{or } 10\frac{x^2}{2} - \frac{2x^3}{3} - 5x \Big|_0^1$$

$$\text{or } 5x^2 - \frac{2}{3}x^3 - 5x \Big|_0^1$$

$$\text{or } 5(1)^2 - 2/3 - 5(1) - 5(0)^2 - 1/3(0)^3 - 5(0)$$

$$\text{or } 5 - 2/3 - 5$$

$$\text{or, } -2/3$$

$$\therefore \text{Area} = \left|\frac{-2}{3}\right|$$

$$= 2/3$$

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