SUNWAY

INT'L BUSINESS SCHOOL

Programme Name: __________**BCS HONS**____

Course Code: __**MATH 1023**________

Course Name: __________**Additional Mathematics**_____________________________

Mathematics Individual Project Work

Date of Submission: _______**9/24/2020**_____________

Submitted By:

Student Name**: Dipesh Tha Shrestha**

IUKL ID: **041902900028**

Semester**: Second Semester**

Intake**: September 2019**

Submitted To:

Faculty Name**: SHANTA RAYAMJHI BASNET**

Department**: LMS**

1.

a. Three numbers are in the ratio of 1:2:4. If 3 is added to the first and 8 is subtracted from the third then the new numbers will be in Arithmetic Progression. Find the numbers.

Dipesh Jha Sh
Dipesh Jha Sh

Q.No. 1

a) Three numbers are in the ratio 1:2:4. If 3 is added to the first and 8 is subtracted from the third then the new numbers will be in Arithmetic Progression. Find the numbers.

Soln

Given,

The ratio = 1:2:4

Let the ratio be $x, 2x$ & $4x$ respectively.

By the questions,

If 3 is added to the first and 8 is subtracted from the third then the numbers are in A.P.

So, $x+3, 2x, 4x-8$ are in A.P.

$\therefore t_1 = x+3$

$t_2 = 2x$

$t_3 = 4x-8$

We know,

$$t_2 - t_1 = t_3 - t_2$$

or, $2x - (x+3) = 4x - 8 - 2x$

or, $x - 3 = 2x - 8$

or, $2x - x = -8 + 3$

$\therefore x = 5$

$\therefore$ The numbers are

$t_1 = x+3 = 5+3 = 8$

$t_2 = 2x = 2\times5 = 10$

$t_3 = 4x-8 = 4\times5-8 = 12$

$\therefore$ 8, 10, 12 are in A.P.

b. Find the sum of: $1^3 + 2^3 + 3^3 + \ldots \ldots + n^3$

b. Find the sum of : $1^3+2^3+3^3+\ldots\ldots+n^3$.

Dipesh Tha Shrestha

Solution

$$r^4-(r-1)^4=(r^2)^2-[(r-1)^2]^2$$
$$=[r^2+(r-1)^2][r^2-(r-1)^2]$$
$$=[r^2+r^2-2r+1][r^2-r^2+2r-1]$$

$\therefore r^4-(r-1)^4 = 4r^3-6r^2+4r-1$ which is an identity and it is true for all values of n.

when $n = 1, 2, 3, 4 \ldots, n$.

$$1^4-0^4 = 4.1^3-6.1^2+4.1-1$$
$$2^4-1^4 = 4.2^3-6.2^2+4.2-1$$
$$\ldots\ldots\ldots\ldots$$
$$n^4-(n-1)^4 = 4.n^3-6n^2+4.n-1$$

Adding above equations

$$n^4 = 4(1^3+2^3+3^3+\ldots\ldots+n^3)-6(1^2+2^2+3^2+\ldots+n^2)+4(1+2+3+\ldots\ldots+n)-n$$

or

$$4(1^3+2^3+\ldots+n^3) = n^4+6(1^2+2^2+3^2+\ldots+n^2)-4(1+2+3+\ldots+n)+n$$

$$4(1^3+2^3+3^3+\ldots\ldots+n^3) = n^4+6\left[\frac{n(n+1)(2n+1)}{6}\right]-4\left[\frac{n(n+1)}{2}\right]+n$$

$$\therefore 1^3+2^3+3^3+\ldots+n^3 = \frac{n^2(n+1)^2}{4}$$

//

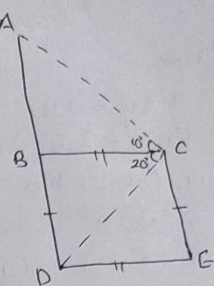
2. From the top of a 200 meters high building, the angle of depression to the bottom of a second building is 20 degrees. From the same point, the angle of elevation to the top of the second building is 10 degrees. Calculate the height of the second building.

Q.No.2.

Dipesh Tha Shrestha

From the top of a 200 m high building, the angle of depression to the bottom of a second building is 20 degrees. From the ~~surface~~ same point, the angle of elevation to the top of the second building is 10 degrees. Calculate the height of the second building.

Soln



Let the height of 1st building be. CE = 200m, and it forms the angle of depression on the bottom of 2nd building ie.

$\angle BCD = 20°$

From the same point the height of 1st building forms the angle of elevation on the top of 2nd building ie. $\angle ACB = 10°$

where (AB + BD) is the total height of 2nd building.

Now,

In $\triangle BCD$

$\tan 20° = \frac{BD}{BC}$

or, $\tan 20° = \frac{200}{BC}$

or, $BC = 89.39$

Again,

In $\triangle ABC$

$\tan 10° = \frac{AB}{BC}$

or, $\tan 10° \times 89.39 = AB$

$\therefore AB = 57.95$

$\therefore$ The height of 2nd building $AD = (AB + AD)$

$= 57.95 + 200$

$= 257.95m$

3. Prove that:

a. Log (a + b)/3=1/2(log a +log b) if $a^2+b^2=7ab$

Quadratic Equation

Dipesh Tha Shrestha

Q. No. 3

a. Prove $\log(a+b)/3 = 1/2(\log a + \log b)$ if $a^2+b^2=7ab$

Here

$a^2+b^2=7ab$

To prove : $\log\frac{(a+b)}{3} = \frac{1}{2}(\log a + \log b)$

We have,

$a^2+b^2=7ab$

$(a+b)^2-2ab=7ab$

or, $(a+b)^2=9ab$

or, $\sqrt{(a+b)^2}=\sqrt{9ab}$ (Taking square root on both sides)

or, $(a+b)=3(ab)^{1/2}$

Now,

Taking log on both sides, we get

$\log(a+b)=\log[3(ab)^{1/2}]$

or, $\log(a+b)=\log 3+\log(ab)^{1/2}$

or, $\log(a+b)=\log 3+\frac{1}{2}\log(ab)$

or, $\log(a+b)-\log 3=\frac{1}{2}\log(ab)$

~~or, log~~

By the ~~question~~, formula,

$\log\frac{(a+b)}{3}=\frac{1}{2}(\log a+\log b)$

proved,

b. $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1} = 1$ If x=$\log_a$bc, y=$\log_b$ca, z = $\log_c$ab

Dipesh Tha Shrestha

Q.No 3

b) Prove $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=1$ if $x=\log_a bc$, $y=\log_b ca$, $z=\log_c ab$

Given,

$x=\log_a bc$

$x=\frac{\log bc}{\log a}=\frac{\log b+\log c}{\log a}$

$y=\log_b ca$

$=\frac{\log c+\log a}{\log b}$

$z=\log_c ab$

$=\frac{\log a+\log b}{\log c}$

Now,

$$LHS=\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$$

$$=\frac{1}{\frac{\log b+\log c}{\log a}+1}+\frac{1}{\frac{\log c+\log a}{\log b}+1}+\frac{1}{\frac{\log a+\log b}{\log c}+1}$$

$$=\frac{1}{\frac{\log b+\log c+\log a}{\log a}}+\frac{1}{\frac{\log c+\log a+\log b}{\log b}}+\frac{1}{\frac{\log a+\log b+\log c}{\log c}}$$

$$=\frac{\log a}{\log a+\log b+\log c}+\frac{\log b}{\log a+\log b+\log c}+\frac{\log c}{\log a+\log b+\log c}$$

$$=\frac{\log a+\log b+\log c}{\log a+\log b+\log c}$$

$$=1$$

RHS proved,

4.

a. Drive Quadratic Equation Formula.

Q.No 4 Dipesh Tha Shres

a. Drive Quradratic Equation Formula.

Soln: let the equation be $an^2+bn+c=0$

or, $an^2+bn=-c$

Dividing both sides by a, we get.

$$\frac{an^2}{a}+\frac{bn}{b}+\frac{c}{a}$$

or, $n^2+\left(\frac{b}{a}\right)n+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$

or, $\left(n+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$

or, $\left(n+\frac{b}{2a}\right)^2=-\frac{4ac}{4a^2}+\frac{b^2}{4a^2}$

or, $\left(n+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

or, $n+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

or, $n=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{2a}}$

or, $n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Which is the required formula of quadratic equation.

b. Discuss the nature of the roots of the quadratic equation $2x^2 - 8x + 3 = 0$.

Q.No 4. Dipesh Jha Shrestha

b) Discuss the nature of the roots of the quadratic equation $2x^2-8x+3=0$.

Soln:

Here,

$2x^2-8x+3=0$

Comparing the given eqn with $ax^2+bx+c=0$

We get,

$a=2, b=-8$ & $c=3$

Now,

The nature of the roots can be find out by

b^2-4ac

$=(-8)^2-4\times 2\times 3$

$=64-24$

$=40$

$b^2-4ac>0$

Since, b^2-4ac is greater than 0. It is not a perfect square.

So, the nature of root is real, irrational and equal.

5.

a. Prove that the roots of the equation $x^2-2(a+b)x+a^2+b^2+2c^2 = 0$ will be imaginary.

Dipesh Tha Shrestha

Q.No.5.

a) Prove that the roots of the equation $\cancel{x^2+}\ x^2-2(a+b)x+a^2+b^2+2c^2=0$ will be imaginary.

Soln:

Given,

$$x^2-2(a+b)x+a^2+b^2+2c^2=0$$

Comparing above eqn with $ax^2+bx+c=0$

we get,

$$a=1,\ b=-2(a+b),\ c=a^2+b^2+2c^2$$

Now,

$$b^2-4ac$$

$$=[-2(a+b)]^2-4.1.(a^2+b^2+2c^2)$$

$$=4(a+b)^2-4(a^2+b^2+2c^2)$$

$$=4[\cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}-\cancel{b^2}-2c^2)$$

$$=4(2ab-2c^2)$$

$$=4\times2(-ab+c^2)$$

$$=-8(c^2-ab)<0$$

Hence, $b^2-4ac<0$. So the roots of given equation are imaginary.

b. If α and β be the roots of $x^2 + px + q = 0$, find the quadratic equation whose roots are α/β and β/α

Dipesh Tha Shrestha.

Q.N 5b

b). If α and β be the roots of $x^2+px+q=0$, find the quadratic equation whose roots are α/β and β/α.

Sol:-

Given,

$x^2+px+q=0$ —— eq^n (1)

If α and β are the root of eq^n (1)

Then

Sum of roots = $\frac{\text{coefficient of } x}{\text{coeff of } x^2}$

$\alpha+\beta = \frac{p}{1}$

$\alpha+\beta = -p$

Product of roots = $\frac{\text{Constant term}}{\text{Coeff of } x^2}$

$\alpha\beta = q$

Quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$x^2 - \left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x + \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 0$

$x^2 - \left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x + 1 = 0$ —— (ii)

Now,

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2}{\alpha\beta}+\frac{\beta^2}{\alpha\beta}$

or, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

or, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

or, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{p^2-2q}{q}$

So, the req. quad. equation is

$x^2 - \left[\frac{(p^2-2q)}{q}\right]x + 1 = 0$ (multiplying each sides by q)

or, $qx^2-(p^2-2q)x+q=0$

which is the required equation whose roots are $\frac{\alpha}{\beta}$ and β/α.