(I) PART 1: WRITTEN PROBLEMS

(1) Consider a set of integers $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots\}$.

(a) Both addition and Multiplication are commutative.

$$\boxed{\forall x, y \in Z \ (x + y \Leftrightarrow y + x) \land (x \cdot y \Leftrightarrow y \cdot x)}$$

... Using commutative law $(a + b = b + a)$

(b) Z is closed under addition and multiplication.

$$\boxed{\forall x, y \in Z \ (x + y \in Z) \land (x \cdot y \in Z)}$$

(c) Multiplication distributes over addition.

$$\boxed{\forall x, y, z \in Z \quad x \cdot (y + z) \Leftrightarrow (x \cdot y) + (x \cdot z)}$$

(d) Multiplication and addition are associative

$$\boxed{\begin{aligned} \forall x, y, z \in Z \ & [(x + y) + z \Leftrightarrow x + (y + z)] \\ & \land [(x \cdot y) \cdot z \Leftrightarrow x \cdot (y \cdot z)] \end{aligned}}$$

(e) Identify property :-

$$\boxed{\forall x \in Z \ [(x + 0) \Leftrightarrow x \ \land \ x \cdot 1 \Leftrightarrow x]}$$

(2) $\alpha = \forall x \, (P(x) \vee Q(x))$

$\beta = \forall x \, P(x) \vee \forall x \, Q(x)$

- FOR α :-

$x \in$ World of Red candies.

$P(x)$ = Not picking Red candies = $\neg$ Picking (Red)

$Q(x)$ = Picking Red candy = Picking (Red)

$\therefore \quad \alpha = \forall x \, [P(x) \vee Q(x)]$

$\equiv \forall x \in$ Red candies $[\neg$ Picking (Red) $\vee$ Picking (Red)$]$

$\alpha \equiv$ TRUE.

- FOR β :-

$\beta \equiv \forall x \, P(x) \vee \forall x \, Q(x)$.

considering $\forall x \, P(x)$ $\quad x \in$ world of Red candies.

$\forall y \, Q(y)$ $\quad y \in$ world of yellow candies

$\therefore \quad \beta \equiv \forall x \, P(x) \vee \forall y \, Q(y)$

we know that $P(x) = \neg$ Picking (Red)

$Q(y)$ = Picking (Red).

$\therefore$

$\beta \equiv \forall_{x \in \text{Red candies}} [\neg \text{Picking (Red)}]$

$\vee \; \forall_{y \in \text{Yellow candies}} [\text{Picking (Red)}]$

$\equiv$ FALSE.

$\therefore$ Specific condition where α is true but β is false.

$$\boxed{\alpha \not\models \beta}$$

Hence proved.

(3)

(a) Han Solo owns the millennium Falcon :

$\equiv$ $\boxed{\text{owns (Solo, Falcon)}}$

(b) Princess Leia is unhappy :

$\equiv$ $\boxed{\text{unhappy (Leia)}}$

(c) Princess Leia loves Han Solo .

$\equiv$ $\boxed{\text{loves (Leia , Solo)}}$

d) For all x, if x owns the millennium falcon or x is unhappy then x visits Obi-wan Kenobi .

$$\equiv \boxed{\forall x \, [\text{ owns } (x, \text{Falcon}) \vee \text{unhappy}(x)] \Rightarrow \text{visits } (x, \text{Obi-wan}}$$

e) For all x, if x visits Obi-wan kenobi then x is wise .

$$\boxed{\forall x \quad \text{visits } (x, \text{Obi-Wan}) \Rightarrow \text{wise}(x)}$$

f) For all x, if x owns Millennium Falcon and visits obi-wan kenobi, then Obi-wan Kenobi teaches x to use the lightsaber .

$$\boxed{\forall x \, [\text{ owns } (x, \text{Falcon}) \wedge \text{visits } (x, \text{Obi-wan})] \rightarrow \text{teaches } (\text{Obi-wan}, x, \text{lightsaber})}$$

(g) For all x, if x is unhappy or x owns the Millennium Falcon, and if Obi-wan Kenobi teaches x to use lightsaber, then x joins Rebel Alliance.

$$\boxed{\begin{aligned} \forall x \; & [[\text{unhappy}(x) \lor \text{owns}(x, \text{Falcon})] \\ & \quad \land \text{teaches}(\text{Obi-wan}, x, \text{lightsaber})] \\ & \quad \Rightarrow \text{joins}(x, \text{Rebel}) \end{aligned}}$$

(h) For all x and y, if x is unhappy and x loves y then x declares love for y.

$$\boxed{\begin{aligned} \forall x \forall y \; & [[\text{unhappy}(x) \land \text{loves}(x, y)] \\ & \quad \Rightarrow \text{declares\_love}(x, y)] \end{aligned}}$$

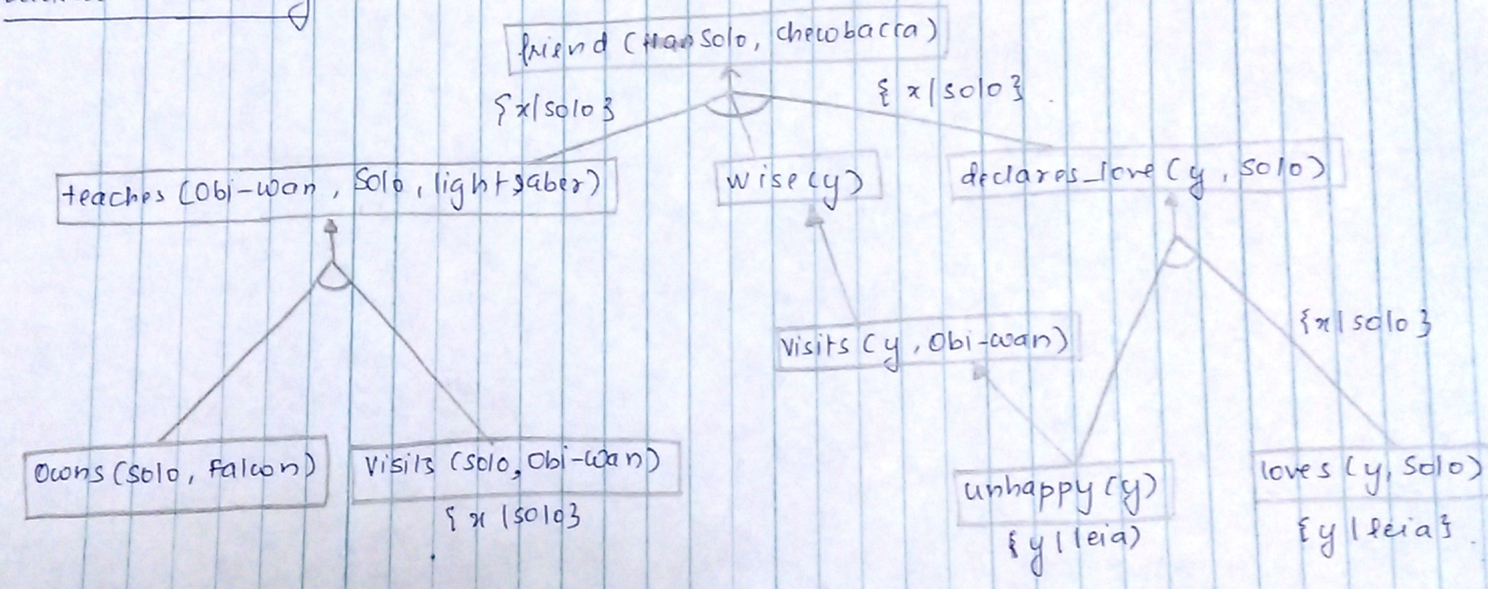
(i) For all x and y if Obi-wan Kenobi teaches x to use the lightsaber and y declares love for x and y is wise, then x has chewbacca as a friend.

$$\boxed{\begin{aligned} \forall x \forall y \; & [\text{teaches}(\text{Obi-wan}, x, \text{lightsaber}) \\ & \quad \land \text{declares\_love}(y, x) \land \text{wise}(y)] \\ & \quad \Rightarrow \text{friend}(x, \text{Chewbacca}) \end{aligned}}$$

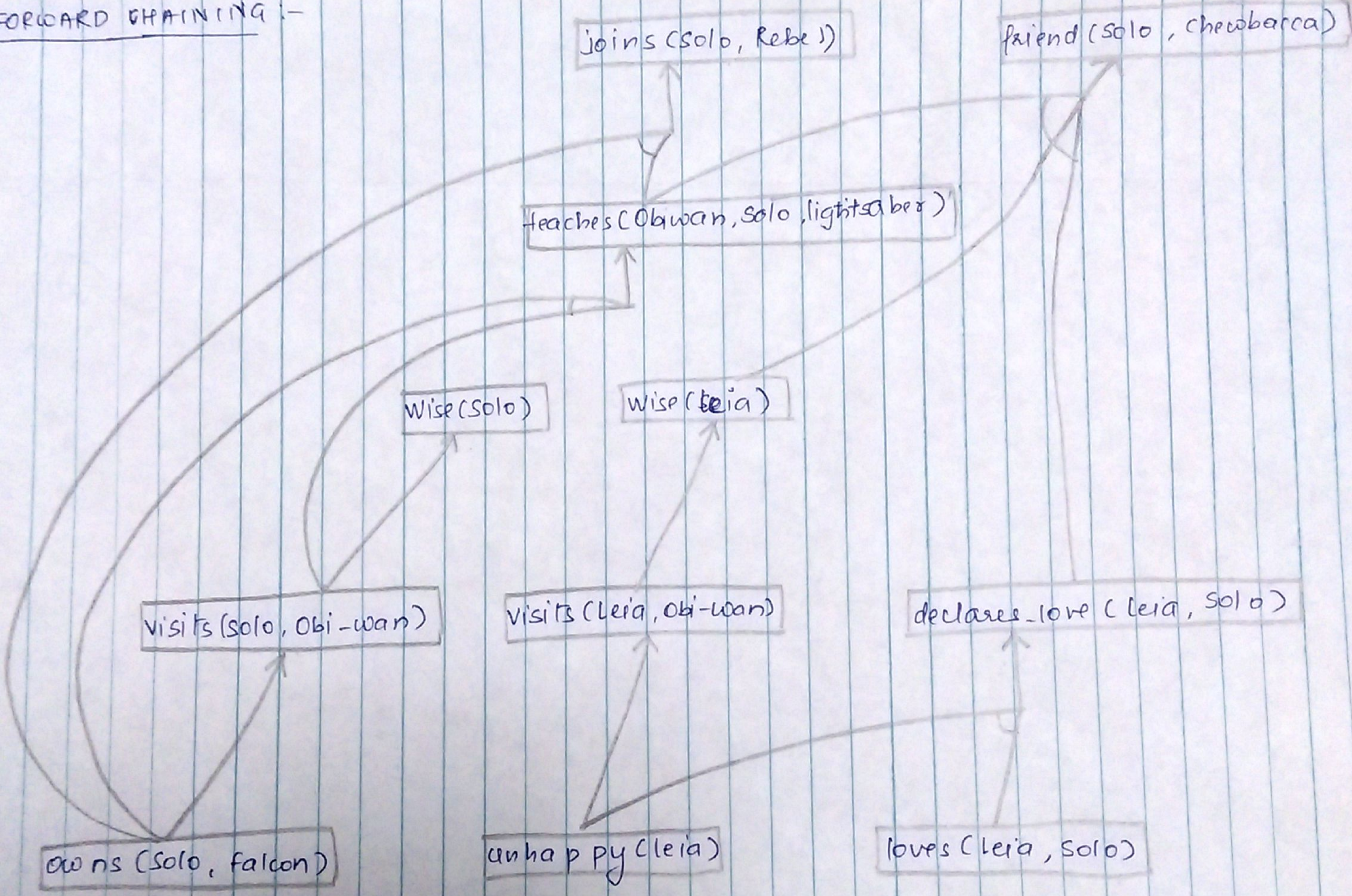
To prove

friend (Han Solo, Chewbacca)

Backward chaining :-



FORWARD CHAINING :-



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(4)

(a) If Zeus were able and willing to prevent evil, then he would so :-

$$A(Z) \land W(Z) \Rightarrow P(Z, Evil)$$

(b) If Zeus were unable to prevent evil, then her would be impotent.

$$\neg A(Z) \Rightarrow I(Z)$$

(c) If he were unwilling to prevent evil, then he would be malevolent.

$$\neg W(Z) \Rightarrow M(Z)$$

(d) Zeus does not prevent evil.

$$\neg P(Z, Evil)$$

(e) If Zeus Exists, he is neither impotent nor malevolent.

$$\neg E(Z) \lor [\neg I(Z) \land \neg M(Z)]$$

Prove :- $\neg E(Z)$

Denial :- $E(Z)$

... Able : A , Z : Zeus,
W : willing , P : Prevent
E : Exist , M : Malevolent.
I : Impotent.

First order logic : conversion to CNF Form

(I) Eliminate bi-condition and Implication.

(a) $A(z) \wedge W(z) \Rightarrow P(z, Evil)$

$\equiv \boxed{\neg(A(z) \wedge W(z)) \vee P(z, Evil)}$

2 (b) $\neg A(z) \Rightarrow I(z)$

$\equiv \boxed{A(z) \vee I(z)}$

3 (c) $\neg W(z) \Rightarrow M(z)$

$\equiv \boxed{W(z) \vee M(z)}$

4 (d) $\boxed{\neg P(z, Evil)}$

(e) $\neg E(z) \Rightarrow [\neg I(z) \wedge \neg M(z)]$

$\equiv \boxed{\neg E(z) \vee [\neg I(z) \wedge \neg M(z)]}$

(II) Using DeMorgan's law :-

(a) $\neg(A(z) \wedge W(z)) \vee P(z, Evil)$

1 $\equiv \boxed{\neg A(z) \vee \neg W(z) \vee P(z, Evil)}$

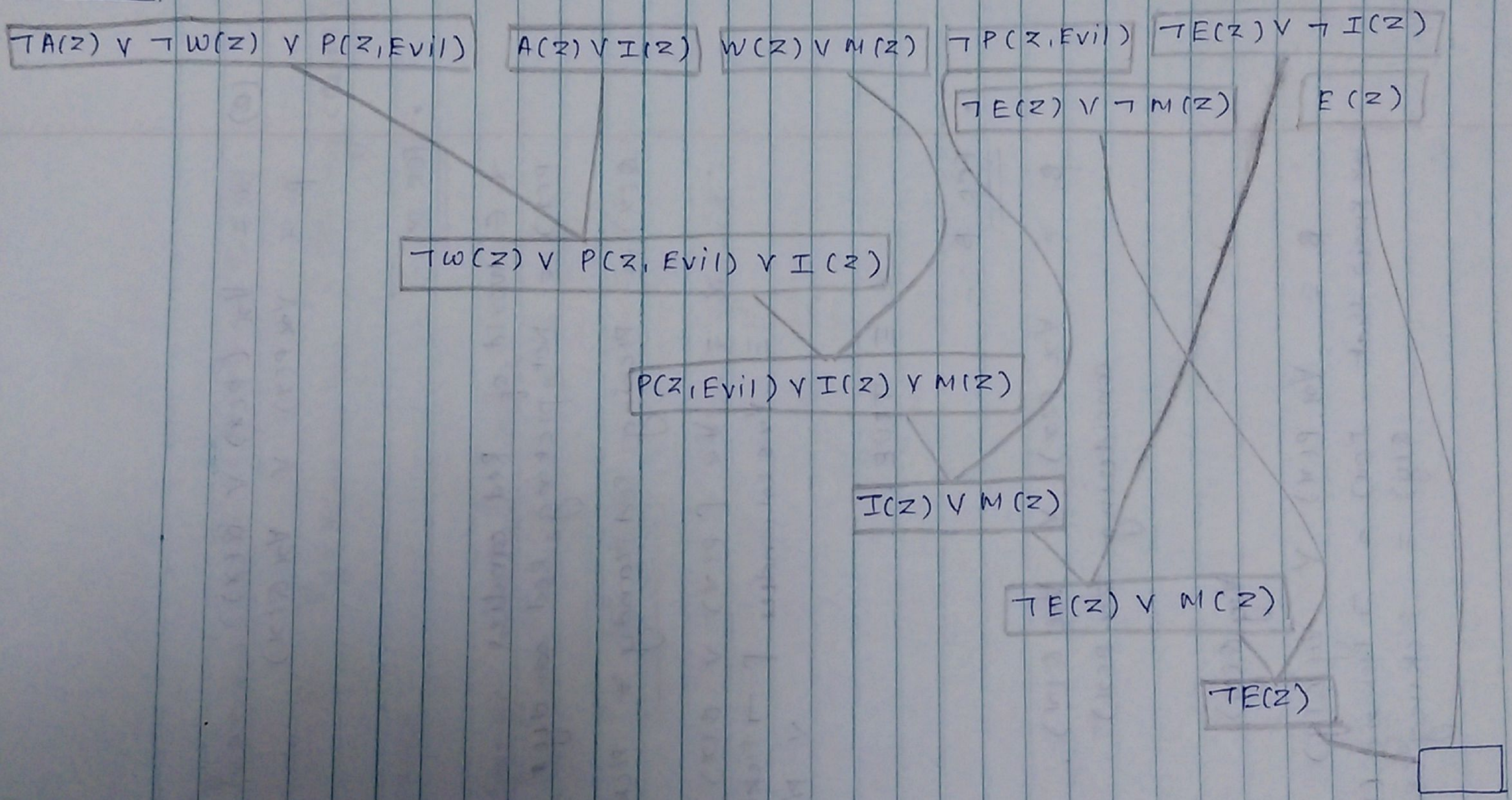
(III) Distribute over '$\wedge$'

(e) $\neg E(z) \vee [\neg I(z) \wedge \neg M(z)]$

5

$\equiv \boxed{[\neg E(z) \vee \neg I(z)] \wedge [\neg E(z) \vee \neg M(z)]}$

Resolution :-



∴ E(z) does not Exist

5(a) $\forall x \; [P(x) \Rightarrow P(x)]$

$A \equiv \forall x \; [P(x) \Rightarrow P(x)]$

Eliminating Implication

$A \equiv \forall x \; [\neg P(x) \vee P(x)]$

$\equiv \forall x \neg P(x) \vee \forall x P(x)$ [Removing $\forall x$]

$\equiv \neg P(x) \vee P(x)$

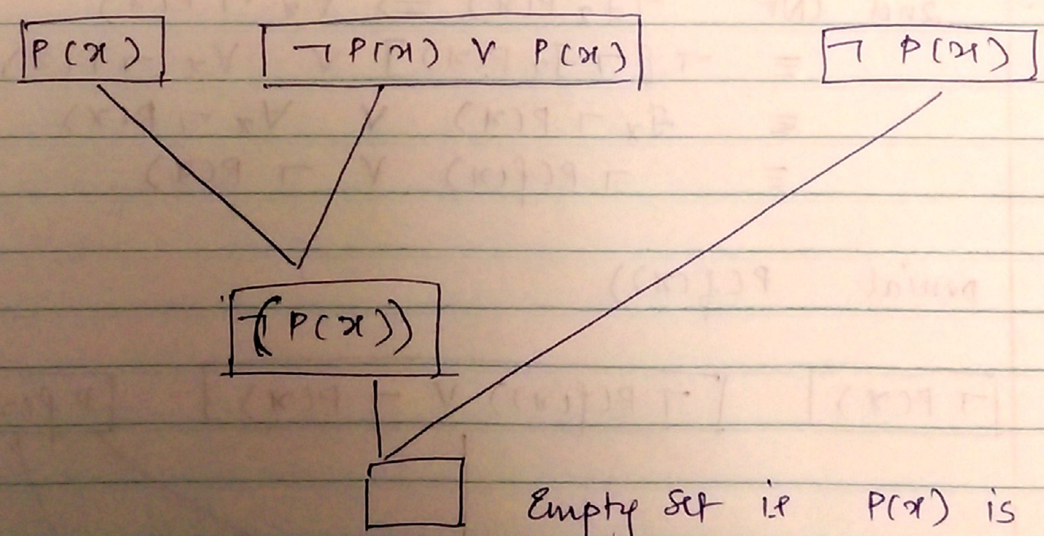
→ A is a part of our Knowledge Base

→ $P(x)$ is a part of KB. $[\forall x \; \underset{\text{i}}{\underline{P(x)}} \Rightarrow \underset{\text{(ii)}}{\underline{P(x)}}]$

We have to prove (ii) is also a part of KB.

∴ Denial of (ii) → To prove by contradiction $\equiv \neg P(x)$

∴



Empty set i.e. $P(x)$ is ~~par~~ correctly implied

5 (b) $(\neg \exists x\, P(x)) \Rightarrow (\forall x\, \neg P(x))$

* We know that, $\neg \exists x\, P(x)$ is a part of KB and CNF of $[\neg \exists x\, P(x)) \Rightarrow \forall x\, \neg P(x)]$ is also a part of KB.

$\therefore$ Denial of Implication

$\equiv \neg [\forall x\, \neg (P(x))]$

$\equiv \exists x\, P(x)$

Skolemizing

$\equiv P(f(x))$

① so 1st KB $\neg \exists x\, P(x) \equiv \forall x\, \neg P(x)$

$\equiv \neg P(x)$

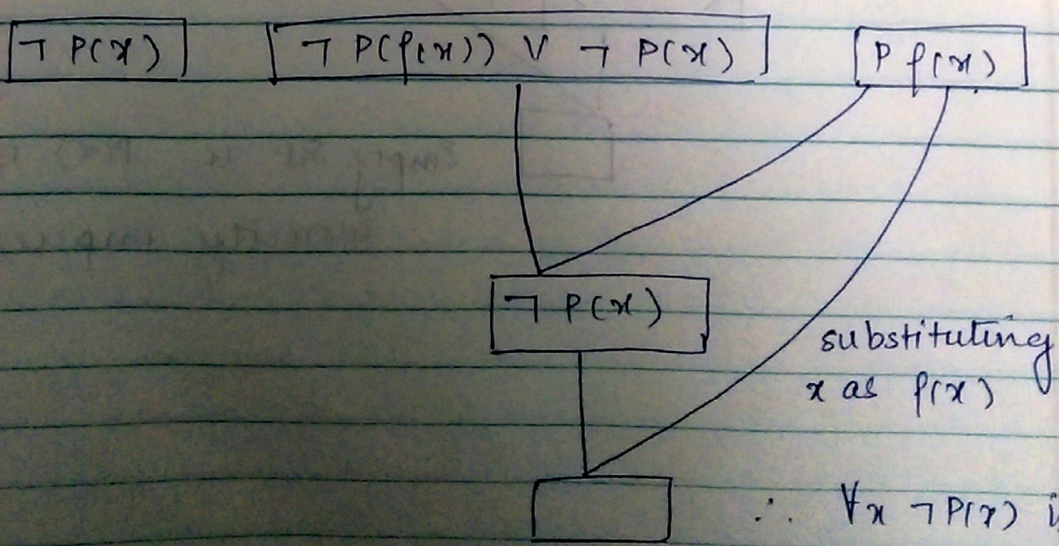
②. 2nd CNF $\neg \exists x\, P(x) \Rightarrow \forall x\, \neg P(x)$

$\equiv \neg [\neg \exists x\, P(x)] \vee \forall x\, \neg P(x)$

$\equiv \exists x\, \neg P(x) \vee \forall x\, \neg P(x)$

$\equiv \neg P(f(x)) \vee \neg P(x)$

③ Denial $P(f(x))$



$\therefore \forall x\, \neg P(x)$ is apart of KB.

(c) $(\forall x (P(x) \vee Q(x))) \Rightarrow ((\forall x P(x) \vee \exists_x Q(x)))$.

We know that.

→ $\forall x (P(x) \vee Q(x))$ is part of KB

→ CNF $[(\forall x (P(x) \vee Q(x))) \Rightarrow ((\forall x P(x) \vee \exists_x Q(x)))]$ is part of KB.

→ Prove that :- $\forall_x P(x) \vee \exists_x Q(x)$ is part of KB.

* Denial :- $\neg[\forall x P(x) \vee \exists_x Q(x)]$

$\equiv \exists x \neg P(x) \wedge \forall x \neg Q(x)$ Removing $\forall$

$\equiv \exists x \neg P(x) \wedge \neg Q(x)$ ... Skolemizing

$\equiv \boxed{\neg P(f(x)) \wedge \neg Q(x)}$

* ① $\forall x (P(x) \vee Q(x)) \equiv \boxed{P(x) \vee Q(x)}$

② $(\forall x (P(x) \vee Q(x)) \Rightarrow ((\forall x P(x) \vee \exists_x Q(x))$

$\equiv \neg(\forall x (P(x) \vee Q(x))) \vee [\forall x P(x) \vee \exists_x Q(x)]$

$\equiv [\exists x (\neg P(x) \wedge \neg Q(x))] \vee \forall x P(x) \vee \exists x Q(x)$

$\equiv [\exists x \neg P(x) \vee \forall x P(x) \vee \exists x Q(x)] \wedge$
$[\exists x \neg Q(x) \vee \forall x P(x) \vee \exists_x Q(x)]$

$\equiv [\neg P(f(x)) \vee P(x) \vee Q(f(x))]$
$\wedge [\neg Q(f(x)) \vee P(x) \vee Q(f(x))]$

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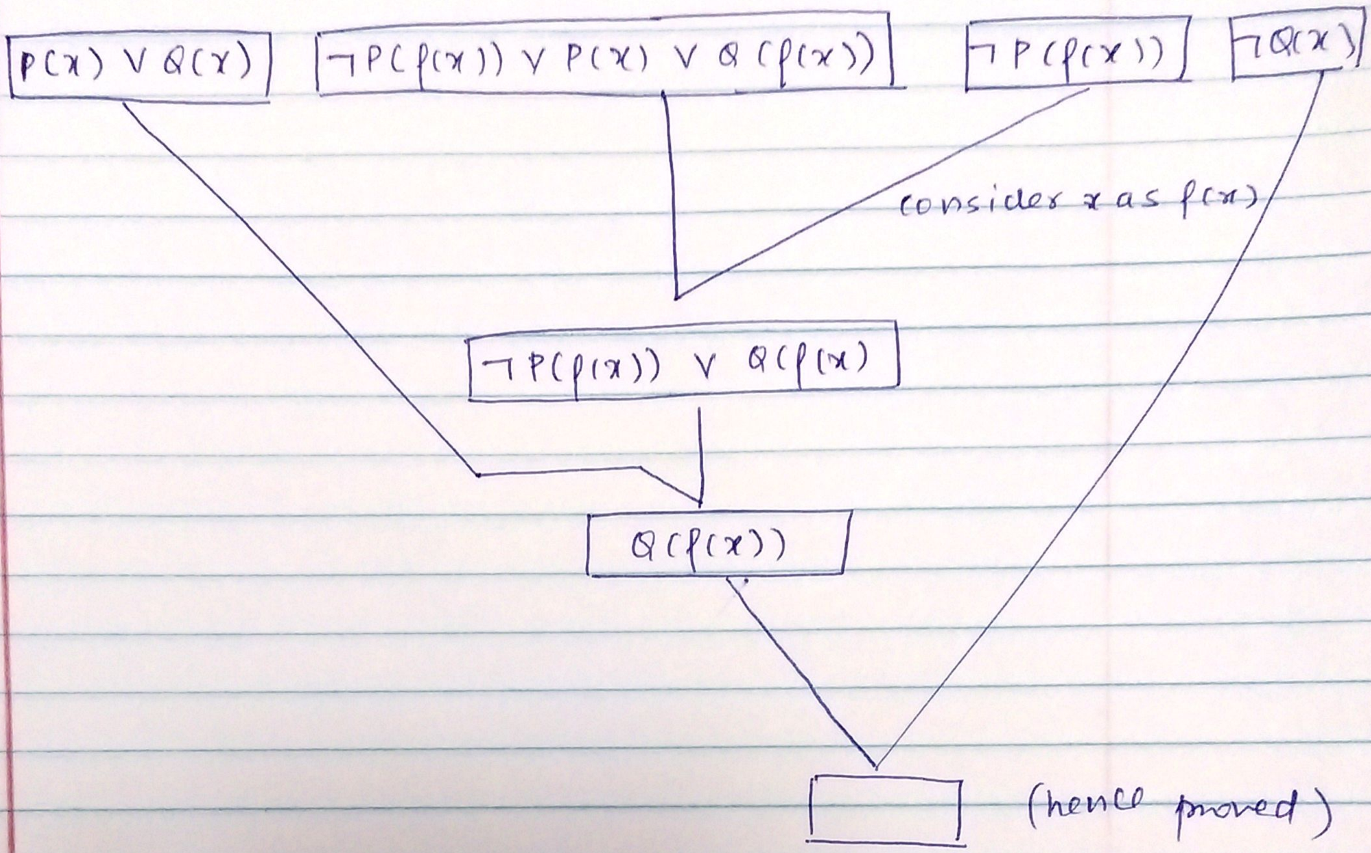

P(x) ∨ Q(x)
¬P(f(x)) ∨ P(x) ∨ Q(f(x))
¬P(f(x))
¬Q(x)
consider x as f(x)
¬P(f(x)) ∨ Q(f(x)
Q(f(x))
(hence proved)