Analysis

MOLECULAR COMMUNICATIONS: MODEL BASED AND DATA DRIVEN RECEIVER DESIGN AND OPTIMIZATION

Goal: To optimize the receiver design

Approach: 1) Model based (Analytical)
2) Data driven (ANN)

System Model

→ Transmitter is of point size (small enough).

→ The information particles follow the Brownian motion (diffuse randomly and independently of each other in medium)

→ Receiver is spherical absorbing type.

→ Assumptions are:
- Temperature is constant
- Viscosity remains unchanged
- Diffusion co-efficient remains constant
- No extra energy needed as particles diffuse freely.

→ 3D unbounded diffusion channel

→ As particles reach receiver in different time slots, it cause ISI.

Hitting rate of each particle will be

$$f_{hit}^{3D}(t) = \frac{r(d-r)}{d\sqrt{4\pi D t^3}} e^{-\frac{(d-r)^2}{4Dt}}$$

→ On-Off Keying (OOK) modulation scheme is taken

N_{TX} = No. of particles transmitter releases

If s_i (signal) $= \begin{cases} 1 & \text{Transmitter emits particles} \\ 0 & \text{Transmitter doesn't emit particles} \end{cases}$

So, hitting probability will be (after t seconds),

$$P_{hit}(t) = \int_0^t f_{hit}(t)\,dt = \frac{r}{d}\,\mathrm{erfc}\left(\frac{d-r}{\sqrt{4Dt}}\right)$$

where $erfc(y) = 1 - erf(y)$

$$erf(y) = \int_0^y \frac{2}{\sqrt{\pi}} e^{-x^2} dx$$

Therefore, the hitting probability during the $(i-1)^{th}$ time slot is

$$P_{i-1} = \frac{r}{d}\left\{ erfc\left(\frac{d-r}{\sqrt{4DiT}}\right) - erfc\left(\frac{d-r}{\sqrt{4D(i-1)T}}\right)\right\}$$

The average number of received particles after emission from the transmitter will be (for the jth time slot),

$$C_j = N_{TX} P_j$$

The particles are generated at a time plot, so, it is a time series particle generation, so the number of received particles for a particular time slot will follow the Poisson random variable distribution.

Let the received particles for the ith time slot be r_i,

$$r_i \sim Poisson\,(I_i + s_i C_0)$$

where

$$I_i = \lambda_0 T + \sum_{j=1}^{\infty} s_{i-j} C_j$$

I_i is the sum of ISI and the background noise

λ_0 is the background noise power per unit time

So, probability of receiving particles r_i is,

$$P(r_i \mid I_i + s_i C_0) = \frac{e^{-(I_i + s_i C_0)} (I_i + s_i C_0)^{r_i}}{r_i!}$$

The, signal to Noise ratio (SNR) will be

$$SNR = 10 \log_{10} \frac{C_0}{2\lambda_0 T}$$

Putting $C_0 = N_{TX} P_0$,

$$SNR = 10 \log_{10} \frac{N_{TX} P_0}{2\lambda_0 T}$$

$$\frac{SNR}{10} = \log_{10} \frac{N_{TX} P_0}{2\lambda_0 T}$$

$$\frac{N_{Tx} P_0}{2\lambda_0 T} = 10^{\frac{SNR}{10}}$$

$$N_{Tx} = \frac{2\lambda_0 T}{P_0} 10^{\frac{SNR}{10}}$$

For optimizing the BER, threshold based detector receiver design is proposed which uses the system model information.

The threshold based receivers consists of having apriori information of the previous bits and are of 3 types.

1) Zero-bit memory receiver
2) One-bit memory receiver
3) K-bit memory receiver

Zero-bit memory receiver

Let the demodulation threshold be τ.
Let the estimate of symbol s_i at i^{th} time slot be $\bar{s_i}$
The demodulation rule will be:

$$\bar{s_i} = \begin{cases} 0 & r_i \leq \tau \\ 1 & r_i > \tau \end{cases}$$

The sub-optimal threshold value can be obtained from

$$P_{app}(r_i | s_i) = \frac{e^{-\lambda|s_i} (\lambda|s_i)^{r_i}}{r_i!}$$

where $\lambda|s_i = C_0 s_i + \frac{\sum_{j=1}^{L} C_j + \lambda_0 T}{2}$.

(L = slot length)

By using the equality $P_{app}(r_i | s_i = 0) = P_{app}(r_i | s_i = 1)$, we get,

$$\frac{e^{\lambda|s_i=0} (\lambda|s_i=0)^{r_i}}{r_i!} = \frac{e^{\lambda|s_i=1} (\lambda|s_i=1)^{r_i}}{r_i!}$$

Here $r_i = \tau$

$$\frac{e^{\lambda|s_i=0} (\lambda|s_i=0)^{\tau}}{\tau!} = \frac{e^{\lambda|s_i=1} (\lambda|s_i=1)^{\tau}}{\tau!}$$

$$e^{\lambda|s_i=0}(\lambda|s_i=0)^{\tau} = e^{\lambda|s_i=1}(\lambda|s_i=1)^{\tau}$$

$$e^{-\frac{\Sigma C_j}{2}+\lambda_0 T}\left(\frac{\Sigma C_j}{2}+\lambda_0 T\right)^{\tau} = e^{C_0+\frac{\Sigma C_j}{2}\lambda_0 T}\left(C_0+\frac{\Sigma C_j}{2}+\lambda_0 T\right)^{\tau}$$

$$e^{C_0} = \left(\frac{C_0+\frac{\Sigma C_j}{2}+\lambda_0 T}{\frac{\Sigma C_j}{2}+\lambda_0 T}\right)^{\tau}$$

$$\tau = \frac{C_0}{\ln\left(1+\frac{C_0}{\frac{\Sigma C_j}{2}+\lambda_0 T}\right)}$$

This will be the sub optimal threshold value.

The optimal threshold value will be as follows:

$$(\tau^*, P_e^*) = \arg\min_{\tau} P_e(\tau)$$

where $P_e(\tau)$ is the BER (function of τ)

$$P_e(\tau) = \frac{1}{2^L}\sum_{s_{i-1}} P_e(s_{i-1}, \tau)$$

where, $P_e(s_{i-1}, \tau) = \frac{1}{2}\left[Q(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j, \lceil\tau\rceil) + 1 - Q(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j + C_0, \lceil\tau\rceil)\right]$

$$= \frac{1}{2}\left[P(r_i \geq \tau | s_i = 0, s_{i-1}) + P(r_i < \tau | s_i = 1, s_{i-1})\right]$$

Proof of $P_e(s_{i-1}, \tau)$ is :

$$P(r_i \geq \tau | s_i = 0, s_{i-1}) = P(r_i \geq \tau | \lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j)$$

$$= \sum_{k=\lceil\tau\rceil}^{\infty} \frac{e^{-(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j)}(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j)^k}{k!}$$

$$= Q(\lambda_0 T + \sum_{j=0}^{L} s_{i-j} C_j, \lceil\tau\rceil)$$

$$P(r_i < \tau | s_i = 1, s_{i-1}) = P(r_i < \tau | \lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j + C_0)$$

$$= \sum_{k=0}^{\lceil\tau\rceil - 1} \frac{e^{-(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j + C_0)}(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j + C_0)^k}{k!}$$

$$= 1 - \sum_{k=\lceil\tau\rceil}^{\infty} \frac{e^{-(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} c_j + c_0)} (\lambda_0 T + \sum_{j=1}^{L} s_{i-j} c_j + c_0)^k}{k!}$$

$$= 1 - Q(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} c_j + c_0, \lceil\tau\rceil)$$

On putting this values in the equation, we get BER.

One bit receiver design

It has more apriori information compared to zero-bit receiver.
The estimate of symbol s_i at ith time slot be $\bar{s}_i$
The demodulation rule will be:

$$\bar{s}_i = \begin{cases} 0, & r_i \leq \tau | s_{i-1} \\ 1 & r_i > \tau | s_{i-1} \end{cases}$$

where $\tau | s_{i-1}$ denotes the threshold for ith symbol when previously transmitted symbol is s_{i-1}

The optimal threshold can be calculated as follows:

$$\tau^* | s_{i-1} = \arg\min_{\tau} P_e(\tau, s_{i-1})$$

$$\text{where, } P_e(\tau, s_{i-1}) = \frac{1}{2^{L-1}} \sum_{s_{i-2} \dots s_{i-L}} P_e(s_{i-1}, \tau)$$

So, P_e can be calculated as follows:

$$P_e = \frac{m+n}{2}$$

$$P_e = \frac{1}{2} [P(\bar{s}_i = 1 \mid s_i = 0) + P(\bar{s}_i = 0 \mid s_i = 1)]$$

$$m = \frac{1}{2^L} \sum_{s_{i-1}} \sum_{\bar{s}_{i-1}} Q(\lambda | s_{i-1}, s_i = 0, \lceil\tau\rceil_{\bar{s}_i}) \psi(s_{i-1}, \bar{s}_{i-1}, m, n)$$

$$n = \frac{1}{2^L} \sum_{s_{i-1}} \sum_{\bar{s}_{i-1}} (1 - Q(\lambda | s_{i-1}, s_i = 1, \lceil\tau\rceil_{\bar{s}_i}) \psi(s_{i-1}, \bar{s}_{i-1}, m, n)$$

where

$$\psi(s_{i-1}, \bar{s}_{i-1}, m, n) = \begin{cases} m & (s_{i-1} = 0, \bar{s}_{i-1} = 1) \\ 1-m & (s_{i-1} = 0, \bar{s}_{i-1} = 0) \\ n & (s_{i-1} = 1, \bar{s}_{i-1} = 0) \\ 1-n & (s_{i-1} = 1, \bar{s}_{i-1} = 1) \end{cases}$$

Proof of Pe is:

$$P_e = \frac{1}{2}[P(\bar{s}_i = 1 | s_i = 0) + P(\bar{s}_i = 0 | s_i = 1)]$$

$$P(\bar{s}_i = 1 | s_i = 0) = \sum_{s_{i-1}, \bar{s}_{i-1}} P(\bar{s}_i = 1 | s_i = 0, s_{i-1}, \bar{s}_{i-1}) P(\bar{s}_{i-1}, s_{i-1}, \dots, s_{i-L})$$

$$= \sum_{s_{i-1}, \bar{s}_{i-1}} P(r_i \geq \tau|_{s_{i-1}} | s_i = 0, s_{i-1}, \bar{s}_{i-1}) P(\bar{s}_{i-1} | s_{i-1}) P(s_{i-1}) P(s_{i-2}, \dots, s_{i-L})$$

$$= \frac{1}{2^L} \sum_{s_{i-1}, \bar{s}_{i-1}} P(\bar{s}_{i-1} | s_{i-1}) Q(\lambda|_{s_{i-1}, s_i = 0}, \lceil \tau \rceil_{\bar{s}_{i-1}})$$

$$= \frac{1}{2^L} \sum_{s_{i-1}, \bar{s}_{i-1}} Q(\lambda|_{s_{i-1}, s_i = 0}, \lceil \tau \rceil_{\bar{s}_{i-1}}) \psi(s_{i-1}, \bar{s}_{i-1}, m, n)$$

$$= m.$$

where $\lambda|_{s_{i-1}, s_i = 0} = \lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j$

$$P(\bar{s}_i = 0 | s_i = 1) = \sum_{s_{i-1}, \bar{s}_{i-1}} P(\bar{s}_i = 0 | s_i = 1, s_{i-1}, \bar{s}_{i-1}) P(\bar{s}_{i-1}, s_{i-1}, \dots, s_{i-L})$$

$$= \sum_{s_{i-1}, \bar{s}_{i-1}} P(r_i < \tau|_{\bar{s}_{i-1}} | s_i = 1, s_{i-1}, \bar{s}_{i-1}) P(\bar{s}_{i-1} | s_{i-1}) P(s_{i-1}) P(s_{i-2}, \dots s_{i-L})$$

$$= \frac{1}{2^L} \sum_{s_{i-1}, \bar{s}_{i-1}} P(\bar{s}_{i-1} | s_{i-1}) (1 - Q(\lambda|_{s_{i-1}, s_i = 1}, \lceil \tau \rceil_{\bar{s}_{i-1}}))$$

$$= \frac{1}{2^L} \sum_{s_{i-1}, \bar{s}_{i-1}} (1 - Q(\lambda|_{s_{i-1}, s_i = 1}, \lceil \tau \rceil_{\bar{s}_{i-1}})) \psi(s_{i-1}, \bar{s}_{i-1}, m, n)$$

where $\lambda|_{s_{i-1}, s_i = 1} = \lambda_0 T + \sum_{j=1}^{L} s_{i-j} C_j + C_0$

By putting this values in the equation, we get BER.

K-bit memory receiver

It has more apriori information as compared to one-bit receiver
The estimate of symbol s_i at ith time slot be $\bar{s}_i$
The demodulation rule will be

$$\bar{s}_i = \begin{cases} 0 & r_i \leq \tau | \bar{s}_{i-1}, \dots, \bar{s}_{i-K} \\ 1 & r_i > \tau | \bar{s}_{i-1}, \dots, \bar{s}_{i-K} \end{cases}$$

where $\bar{s}_{i-1}, \dots \bar{s}_{i-K}$ are the K previously transmitted symbols

The optimal threshold can be calculated as follows.

$$\tau^*|_{s_{i-1},\dots,s_{i-K}} = \arg\min_{\tau} P_e(\tau, s_{i-1}, \dots, s_{i-K})$$

where,

$$P_e(\tau, s_{i-1}, \dots, s_{i-K}) = \frac{1}{2^{L-K}} \sum_{s_{i-K+1},\dots,s_{i-L}} P_e(s_{i-1}, \tau)$$

Here, $P_e = \frac{m+n}{2}$

where, $$m = \frac{1}{2^L} \sum_{s_{i-1}} \sum_{\bar{s}_{i-1},\dots,\bar{s}_{i-K}} Q(\lambda|_{s_{i-1}, s_i=0}, \lceil\tau\rceil_{\bar{s}_{i-1}\dots\bar{s}_{i-K}}) \prod_{j=1}^{K} \psi(s_{i-j}, \bar{s}_{i-j}, m, n)$$

$$n = \frac{1}{2^L} \sum_{s_{i-1}} \sum_{\bar{s}_{i-1},\dots,\bar{s}_{i-K}} (1-Q(\lambda|_{s_{i-1}, s_i=1}, \lceil\tau\rceil_{\bar{s}_{i-1}\dots\bar{s}_{i-K}}) \prod_{j=1}^{K} \psi(s_{i-j}, \bar{s}_{i-j}, m, n)$$