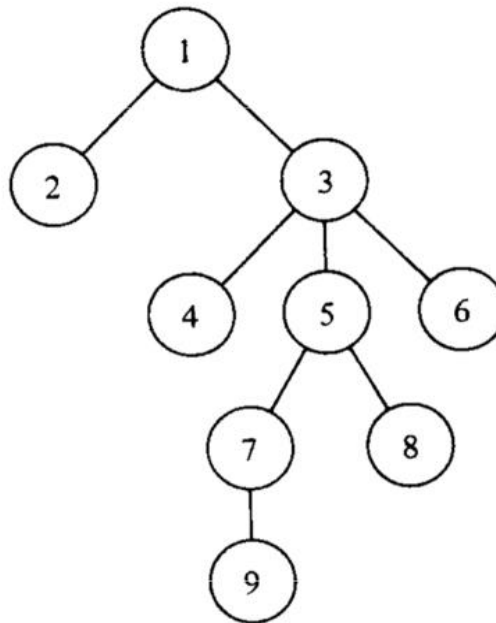


## Herramientas Básicas

Fuente: J.E Hopcroft, J.D. Ullman. Introduction to Automata Theory, Languages, and Computation, Addison-Wesley, (1979), pp. 10-11

### 1.1 In the tree of Fig. 1.4,

- a) Which vertices are leaves and which are interior vertices?
  - a. Leaves: 2,4,6,8,9.
  - b. Interior: 1,3,5,7.
- b) Which vertices are the sons of 5?
  - a. Sons of 5: 7,8.
- c) Which vertex is the father of 5?
  - a. Father of 5: 3.
- d) What is the length of the path from 1 to 9?
  - a. Length path from 1 to 9: 4.
- e) Which vertex is the root?
  - a. Root: 1.



**Fig. 1.4** A tree.

1.2.- Prove by induction that N on:

$$a) \sum_{i=0}^n i = \frac{n(n+1)}{2}$$

$$a) \sum_{i=0}^n i = \frac{n(n+1)}{2} \Rightarrow \textcircled{1} \text{ Caso Base } n=1 \Rightarrow 1 = \frac{1(1+1)}{2} \Rightarrow 1=1$$

$$n=3 \Rightarrow 1+2+3 = \frac{3(3+1)}{2} = 6=6$$


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$$\textcircled{2} \text{ Caso donde } n=k \quad \sum_{i=0}^k i = \frac{k(k+1)}{2}$$


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$$\textcircled{3} \text{ Caso } n=k+1 \quad \sum_{i=0}^{k+1} i = \frac{k+1(k+2)}{2} \Rightarrow \sum_{i=0}^k i + k+1 = \frac{k+1(k+2)}{2}$$

$$\frac{k(k+1)}{2} + \frac{k+1}{1} = \frac{(k+1)(k+2)}{2} \Rightarrow \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$$\frac{k^2 + k + 2k + 2}{2} = \frac{k^2 + 2k + k + 2}{2} = \frac{k^2 + 3k + 2}{2} = \frac{k^2 + 3k + 2}{2}$$