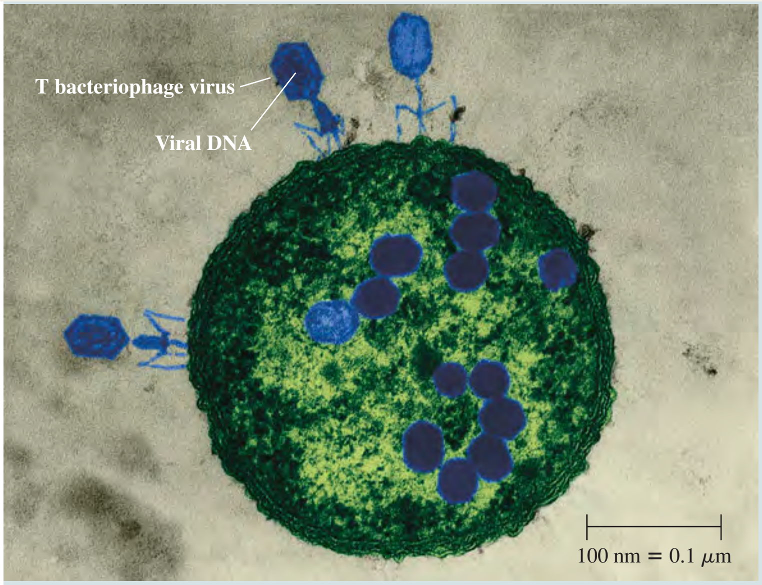
39 PARTICLES BEHAVING AS WAVES

LEARNING GOALS

By studying this chapter, you will learn:

- De Broglie's proposal that electrons, protons, and other particles can behave like waves.
- How electron diffraction experiments provided evidence for de Broglie's ideas.
- How electron microscopes can provide much higher magnification than visible-light microscopes.
- How physicists discovered the atomic nucleus.
- How Bohr's model of electron orbits explained the spectra of hydrogen and hydrogenlike atoms.
- How a laser operates.
- How the idea of energy levels, coupled with the photon model of light, explains the spectrum of light emitted by a hot, opaque object.
- What the uncertainty principle tells us about the nature of the atom.



? Viruses (shown in blue) have landed on an *E. coli* bacterium and injected their DNA, converting the bacterium into a virus factory. This false-color image was made using a beam of electrons rather than a light beam. What properties of electrons make them useful for imaging such fine details?

In Chapter 38 we discovered one aspect of nature's wave–particle duality: Light and other electromagnetic radiation act sometimes like waves and sometimes like particles. Interference and diffraction demonstrate wave behavior, while emission and absorption of photons demonstrate particle behavior.

If light waves can behave like particles, can the particles of matter behave like waves? As we will discover, the answer is a resounding yes. Electrons can be made to interfere and diffract just like other kinds of waves. We will see that the wave nature of electrons is not merely a laboratory curiosity: It is the fundamental reason why atoms, which according to classical physics should be profoundly unstable, are able to exist. In this chapter we'll use the wave nature of matter to help us understand the structure of atoms, the operating principles of a laser, and the curious properties of the light emitted by a heated, glowing object. Without the wave picture of matter, there would be no way to explain these phenomena.

In Chapter 40 we'll introduce an even more complete wave picture of matter called *quantum mechanics*. Through the remainder of this book we'll use the ideas of quantum mechanics to understand the nature of molecules, solids, atomic nuclei, and the fundamental particles that are the building blocks of our universe.

39.1 Electron Waves

In 1924 a French physicist and nobleman, Prince Louis de Broglie (pronounced "de broy"; Fig. 39.1), made a remarkable proposal about the nature of matter. His reasoning, freely paraphrased, went like this: Nature loves symmetry. Light is dualistic in nature, behaving in some situations like waves and in others like particles. If nature is symmetric, this duality should also hold for matter. Electrons and protons, which we usually think of as *particles,* may in some situations behave like *waves.*

If a particle acts like a wave, it should have a wavelength and a frequency. De Broglie postulated that a free particle with rest mass m, moving with nonrelativistic speed v, should have a wavelength λ related to its momentum $p = mv$ in exactly the same way as for a photon, as expressed by Eq. (38.5): $\lambda = h/p$. The **de Broglie wavelength** of a particle is then

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{(de Broglie wavelength of a particle)} \tag{39.1}$$

where h is Planck's constant. If the particle's speed is an appreciable fraction of the speed of light c, we use Eq. (37.27) to replace mv in Eq. (39.1) with $\gamma mv = mv/\sqrt{1 - v^2/c^2}$. The frequency f, according to de Broglie, is also related to the particle's energy E in the same way as for a photon—namely,

$$E = hf \tag{39.2}$$

Thus in de Broglie's hypothesis, the relationships of wavelength to momentum and of frequency to energy are exactly the same for free particles as for photons.

CAUTION **Not all photon equations apply to particles with mass** Be careful when applying the relationship $E = hf$ to particles with nonzero rest mass, such as electrons and protons. Unlike a photon, they do *not* travel at speed c, so the equations $f = c/\lambda$ and $E = pc$ do *not* apply to them!

39.1 Louis-Victor de Broglie, the seventh Duke de Broglie (1892–1987), broke with family tradition by choosing a career in physics rather than as a diplomat. His revolutionary proposal that particles have wave characteristics—for which de Broglie won the 1929 Nobel Prize in physics—was published in his doctoral thesis.

Observing the Wave Nature of Electrons

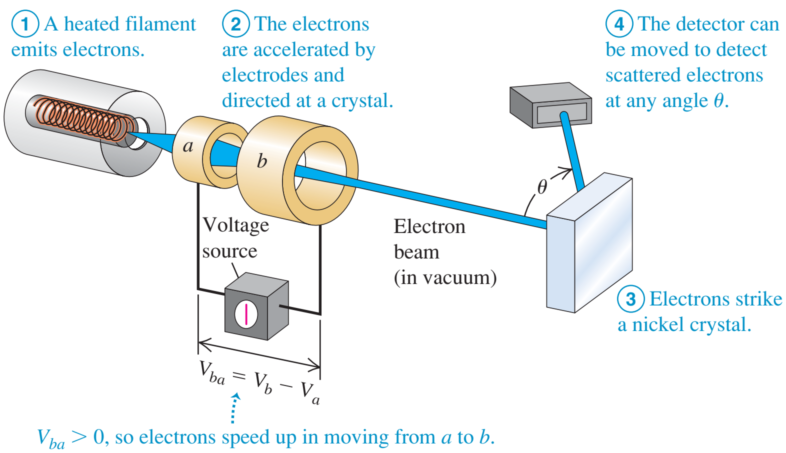
De Broglie's proposal was a bold one, made at a time when there was no direct experimental evidence that particles have wave characteristics. But within a few years of de Broglie's publication of his ideas, they were resoundingly verified by a diffraction experiment with electrons. This experiment was analogous to those we described in Section 36.6, in which atoms in a crystal act as a three-dimensional diffraction grating for x rays. An x-ray beam is strongly reflected when it strikes a crystal at an angle that gives constructive interference among the waves scattered from the various atoms in the crystal. These interference effects demonstrate the *wave* nature of x rays.

In 1927 the American physicists Clinton Davisson and Lester Germer, working at the Bell Telephone Laboratories, were studying the surface of a piece of nickel by directing a beam of *electrons* at the surface and observing how many electrons bounced off at various angles. Figure 39.2 shows an experimental setup like theirs. Like many ordinary metals, the sample was *polycrystalline:* It consisted of many randomly oriented microscopic crystals bonded together. As a result, the electron beam reflected diffusely, like light bouncing off a rough surface (see Fig. 33.6b), with a smooth distribution of intensity as a function of the angle θ.

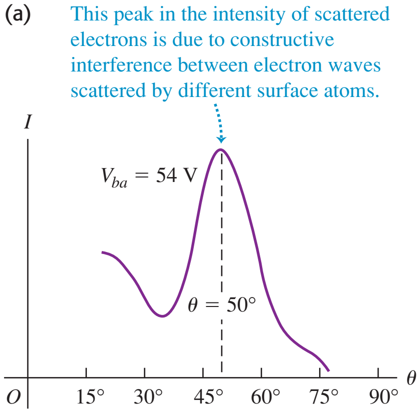
MasteringPHYSICS

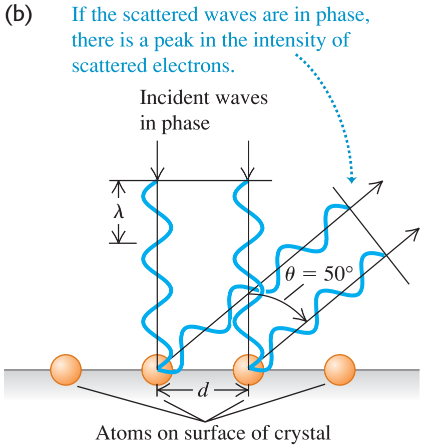
PhET: Davisson-Germer: Electron Diffraction
ActivPhysics 17.5: Electron Interference

39.2 An apparatus similar to that used by Davisson and Germer to discover electron diffraction.



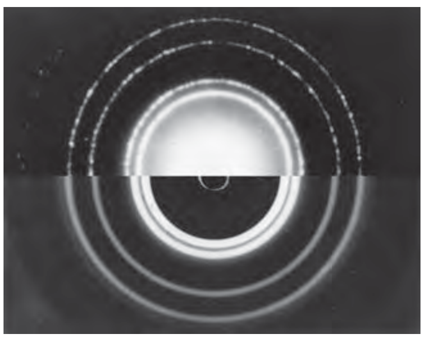
39.3 (a) Intensity of the scattered electron beam in Fig. 39.2 as a function of the scattering angle θ. (b) Electron waves scattered from two adjacent atoms interfere constructively when $d\sin\theta = m\lambda$. In the case shown here, $\theta = 50°$ and $m = 1$.





39.4 X-ray and electron diffraction. The upper half of the photo shows the diffraction pattern for 71-pm x rays passing through aluminum foil. The lower half, with a different scale, shows the diffraction pattern for 600-eV electrons from aluminum. The similarity shows that electrons undergo the same kind of diffraction as x rays.

Top: x-ray diffraction



Bottom: electron diffraction

During the experiment an accident occurred that permitted air to enter the vacuum chamber, and an oxide film formed on the metal surface. To remove this film, Davisson and Germer baked the sample in a high-temperature oven, almost hot enough to melt it. Unknown to them, this had the effect of creating large regions within the nickel with crystal planes that were continuous over the width of the electron beam. From the perspective of the electrons, the sample looked like a *single* crystal of nickel.

When the observations were repeated with this sample, the results were quite different. Now strong maxima in the intensity of the reflected electron beam occurred at specific angles (Fig. 39.3a), in contrast to the smooth variation of intensity with angle that Davisson and Germer had observed before the accident. The angular positions of the maxima depended on the accelerating voltage V_{ba} used to produce the electron beam. Davisson and Germer were familiar with de Broglie's hypothesis, and they noticed the similarity of this behavior to x-ray diffraction. This was not the effect they had been looking for, but they immediately recognized that the electron beam was being *diffracted.* They had discovered a very direct experimental confirmation of the wave hypothesis.

Davisson and Germer could determine the speeds of the electrons from the accelerating voltage, so they could compute the de Broglie wavelength from Eq. (39.1). If an electron is accelerated from rest at point a to point b through a potential increase $V_{ba} = V_b - V_a$ as shown in Fig. 39.2, the work done on the electron eV_{ba} equals its kinetic energy K. Using $K = (\frac{1}{2})mv^2 = p^2/2m$ for a nonrelativistic particle, we have

$$eV_{ba} = \frac{p^2}{2m} \qquad p = \sqrt{2meV_{ba}}$$

We substitute this into Eq. (39.1), the expression for the de Broglie wavelength of the electron:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ba}}} \qquad \text{(de Broglie wavelength of an electron)} \quad (39.3)$$

The greater the accelerating voltage V_{ba}, the shorter the wavelength of the electron.

To predict the angles at which strong reflection occurs, note that the electrons were scattered primarily by the planes of atoms near the surface of the crystal. Atoms in a surface plane are arranged in rows, with a distance d that can be measured by x-ray diffraction techniques. These rows act like a reflecting diffraction grating; the angles at which strong reflection occurs are the same as for a grating with center-to-center distance d between its slits (Fig. 39.3b). From Eq. (36.13) the angles of maximum reflection are given by

$$d\sin\theta = m\lambda \qquad (m = 1, 2, 3, \dots) \quad (39.4)$$

where θ is the angle shown in Fig. 39.2. (Note that the geometry in Fig. 39.3b is different from that for Fig. 36.22, so Eq. (39.4) is different from Eq. (36.16).) Davisson and Germer found that the angles predicted by this equation, using the de Broglie wavelength given by Eq. (39.3), agreed with the observed values (Fig. 39.3a). Thus the accidental discovery of **electron diffraction** was the first direct evidence confirming de Broglie's hypothesis.

In 1928, just a year after the Davisson–Germer discovery, the English physicist G. P. Thomson carried out electron-diffraction experiments using a thin, polycrystalline, metallic foil as a target. Debye and Sherrer had used a similar technique several years earlier to study x-ray diffraction from polycrystalline specimens. In these experiments the beam passes *through* the target rather than being reflected from it. Because of the random orientations of the individual microscopic crystals in the foil, the diffraction pattern consists of intensity maxima forming rings around the direction of the incident beam. Thomson's results again confirmed the de Broglie relationship. Figure 39.4 shows both x-ray and electron diffraction patterns for a polycrystalline aluminum foil. (G. P. Thomson was the son of J. J. Thomson, who 31 years earlier discovered the electron. Davisson and the younger Thomson shared the 1937 Nobel Prize in physics for their discoveries.)

Additional diffraction experiments were soon carried out in many laboratories using not only electrons but also various ions and low-energy neutrons. All of these are in agreement with de Broglie's bold predictions. Thus the wave nature of particles, so strange in 1924, became firmly established in the years that followed.

Problem-Solving Strategy 39.1 Wavelike Properties of Particles

IDENTIFY *the relevant concepts:* Particles have wavelike properties. A particle's (de Broglie) wavelength is inversely proportional to its momentum, and its frequency is proportional to its energy.

SET UP *the problem:* Identify the target variables and decide which equations you will use to calculate them.

EXECUTE *the solution* as follows:

1. Use Eq. (39.1) to relate a particle's momentum p to its wavelength λ; use Eq. (39.2) to relate its energy E to its frequency f.
2. Nonrelativistic kinetic energy may be expressed as either $K = \frac{1}{2}mv^2$ or (because $p = mv$) $K = p^2/2m$. The latter form is useful in calculations involving the de Broglie wavelength.
3. You may express energies in either joules or electron volts, using $h = 6.626 \times 10^{-34}\ \text{J}\cdot\text{s}$ or $h = 4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}$ as appropriate.

EVALUATE *your answer:* To check numerical results, it helps to remember some approximate orders of magnitude. Here's a partial list:

Size of an atom: $10^{-10}\ \text{m} = 0.1\ \text{nm}$

Mass of an atom: $10^{-26}\ \text{kg}$

Mass of an electron: $m = 10^{-30}\ \text{kg}$; $mc^2 = 0.511\ \text{MeV}$

Electron charge magnitude: $10^{-19}\ \text{C}$

kT at room temperature: $\frac{1}{40}\ \text{eV}$

Difference between energy levels of an atom (to be discussed in Section 39.3): 1 to 10 eV

Speed of an electron in the Bohr model of a hydrogen atom (to be discussed in Section 39.3): $10^6\ \text{m/s}$

Example 39.1 An electron-diffraction experiment

In an electron-diffraction experiment using an accelerating voltage of 54 V, an intensity maximum occurs for $\theta = 50°$ (see Fig. 39.3a). X-ray diffraction indicates that the atomic spacing in the target is $d = 2.18 \times 10^{-10}\ \text{m} = 0.218\ \text{nm}$. The electrons have negligible kinetic energy before being accelerated. Find the electron wavelength.

SOLUTION

IDENTIFY, SET UP, and EXECUTE: We'll determine λ from both de Broglie's equation, Eq. (39.3), and the diffraction equation, Eq. (39.4). From Eq. (39.3),

$$\lambda = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.109 \times 10^{-31}\ \text{kg})(1.602 \times 10^{-19}\ \text{C})(54\ \text{V})}}$$
$$= 1.7 \times 10^{-10}\ \text{m} = 0.17\ \text{nm}$$

Alternatively, using Eq. (39.4) and assuming $m = 1$,

$$\lambda = d\sin\theta = (2.18 \times 10^{-10}\ \text{m})\sin 50° = 1.7 \times 10^{-10}\ \text{m}$$

EVALUATE: The two numbers agree within the accuracy of the experimental results, which gives us an excellent check on our calculations. Note that this electron wavelength is less than the spacing between the atoms.

Example 39.2 Energy of a thermal neutron

Find the speed and kinetic energy of a neutron ($m = 1.675 \times 10^{-27}\ \text{kg}$) with de Broglie wavelength $\lambda = 0.200\ \text{nm}$, a typical interatomic spacing in crystals. Compare this energy with the average translational kinetic energy of an ideal-gas molecule at room temperature ($T = 20°\text{C} = 293\ \text{K}$).

SOLUTION

IDENTIFY and SET UP: This problem uses the relationships between particle speed and wavelength, between particle speed and kinetic energy, and between gas temperature and the average kinetic energy of a gas molecule. We'll find the neutron speed v using Eq. (39.1) and from that calculate the neutron kinetic energy $K = \frac{1}{2}mv^2$. We'll use Eq. (18.16) to find the average kinetic energy of a gas molecule.

EXECUTE: From Eq. (39.1), the neutron speed is

$$v = \frac{h}{\lambda m} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{(0.200 \times 10^{-9}\ \text{m})(1.675 \times 10^{-27}\ \text{kg})}$$
$$= 1.98 \times 10^3\ \text{m/s}$$

The neutron kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.675 \times 10^{-27}\ \text{kg})(1.98 \times 10^3\ \text{m/s})^2$$
$$= 3.28 \times 10^{-21}\ \text{J} = 0.0205\ \text{eV}$$

Continued

From Eq. (18.16), the average translational kinetic energy of an ideal-gas molecule at $T = 293$ K is

$$\tfrac{1}{2}m(v^2)_{av} = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38 \times 10^{-23}\ \mathrm{J/K})(293\ \mathrm{K})$$
$$= 6.07 \times 10^{-21}\ \mathrm{J} = 0.0379\ \mathrm{eV}$$

The two energies are comparable in magnitude, which is why a neutron with kinetic energy in this range is called a *thermal neutron.* Diffraction of thermal neutrons is used to study crystal and molecular structure in the same way as x-ray diffraction. Neutron diffraction has proved to be especially useful in the study of large organic molecules.

EVALUATE: Note that the calculated neutron speed is much less than the speed of light. This justifies our use of the nonrelativistic form of Eq. (39.1).

De Broglie Waves and the Macroscopic World

If the de Broglie picture is correct and matter has wave aspects, you might wonder why we don't see these aspects in everyday life. As an example, we know that waves diffract when sent through a single slit. Yet when we walk through a doorway (a kind of single slit), we don't worry about our body diffracting!

The principal reason we don't see these effects on human scales is that Planck's constant h has such a minuscule value. As a result, the de Broglie wavelengths of even the smallest ordinary objects that you can see are extremely small, and the wave effects are unimportant. For instance, what is the wavelength of a falling grain of sand? If the grain's mass is 5×10^{-10} kg and its diameter is $0.07\ \mathrm{mm} = 7 \times 10^{-5}$ m, it will fall in air with a terminal speed of about 0.4 m/s. The magnitude of its momentum is then $p = mv = (5 \times 10^{-10}\ \mathrm{kg}) \times (0.4\ \mathrm{m/s}) = 2 \times 10^{-10}\ \mathrm{kg \cdot m/s}$. The de Broglie wavelength of this falling sand grain is then

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{2 \times 10^{-10}\ \mathrm{kg \cdot m/s}} = 3 \times 10^{-24}\ \mathrm{m}$$

Not only is this wavelength far smaller than the diameter of the sand grain, but it's also far smaller than the size of a typical atom (about 10^{-10} m). A more massive, faster-moving object would have an even larger momentum and an even smaller de Broglie wavelength. The effects of such tiny wavelengths are so small that they are never noticed in daily life.

The Electron Microscope

The **electron microscope** offers an important and interesting example of the interplay of wave and particle properties of electrons. An electron beam can be used to form an image of an object in much the same way as a light beam. A ray of light can be bent by reflection or refraction, and an electron trajectory can be bent by an electric or magnetic field. Rays of light diverging from a point on an object can be brought to convergence by a converging lens or concave mirror, and electrons diverging from a small region can be brought to convergence by electric and/or magnetic fields.

The analogy between light rays and electrons goes deeper. The *ray* model of geometric optics is an approximate representation of the more general *wave* model. Geometric optics (ray optics) is valid whenever interference and diffraction effects can be neglected. Similarly, the model of an electron as a point particle following a line trajectory is an approximate description of the actual behavior of the electron; this model is useful when we can neglect effects associated with the wave nature of electrons.

? How is an electron microscope superior to an optical microscope? The *resolution* of an optical microscope is limited by diffraction effects, as we discussed in Section 36.7. Since an optical microscope uses wavelengths around 500 nm, it can't resolve objects smaller than a few hundred nanometers, no matter how carefully its lenses are made. The resolution of an electron microscope is similarly limited by the wavelengths of the electrons, but these wavelengths may be many thousands of times smaller than wavelengths of visible light. As a result, the useful magnification of an electron microscope can be thousands of times greater than that of an optical microscope.

Note that the ability of the electron microscope to form a magnified image *does not* depend on the wave properties of electrons. Within the limitations of the Heisenberg uncertainty principle (which we'll discuss in Section 39.6), we can compute the electron trajectories by treating them as classical charged particles under the action of electric and magnetic forces. Only when we talk about *resolution* do the wave properties become important.

Example 39.3 An electron microscope

In an electron microscope, the nonrelativistic electron beam is formed by a setup similar to the electron gun used in the Davisson–Germer experiment (see Fig. 39.2). The electrons have negligible kinetic energy before they are accelerated. What accelerating voltage is needed to produce electrons with wavelength 10 pm = 0.010 nm (roughly 50,000 times smaller than typical visible-light wavelengths)?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: We can use the same concepts we used to understand the Davisson–Germer experiment. The accelerating voltage is the quantity V_{ba} in Eq. (39.3). Rewrite this equation to solve for V_{ba}:

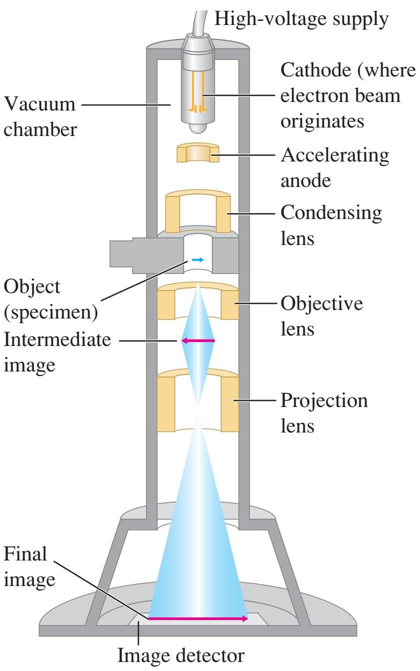
$$V_{ba} = \frac{h^2}{2me\lambda^2}$$
$$= \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.109 \times 10^{-31}\ \text{kg})(1.602 \times 10^{-19}\ \text{C})(10 \times 10^{-12}\ \text{m})^2}$$
$$= 1.5 \times 10^4\ \text{V} = 15{,}000\ \text{V}$$

EVALUATE: It is easy to attain 15-kV accelerating voltages from 120-V or 240-V line voltage using a step-up transformer (Section 31.6) and a rectifier (Section 31.1). The accelerated electrons have kinetic energy 15 keV; since the electron rest energy is 0.511 MeV = 511 keV, these electrons are indeed nonrelativistic.

Types of Electron Microscope

Figure 39.5 shows the design of a *transmission electron microscope,* in which electrons actually pass through the specimen being studied. The specimen to be viewed can be no more than 10 to 100 nm thick so the electrons are not slowed appreciably as they pass through. The electrons used in a transmission electron microscope are emitted from a hot cathode and accelerated by a potential difference, typically 40 to 400 kV. They then pass through a condensing "lens" that uses magnetic fields to focus the electrons into a parallel beam before they pass through the specimen. The beam then passes through two more magnetic lenses: an objective lens that forms an intermediate image of the specimen and a projection lens that produces a final real image of the intermediate image. The objective and projection lenses play the roles of the objective and eyepiece lenses, respectively, of a compound optical microscope (see Section 34.8). The final image is projected onto a fluorescent screen for viewing or photographing. The entire apparatus, including the specimen, must be enclosed in a vacuum container; otherwise, electrons would scatter off air molecules and muddle the image. The image that opens this chapter was made with a transmission electron microscope.

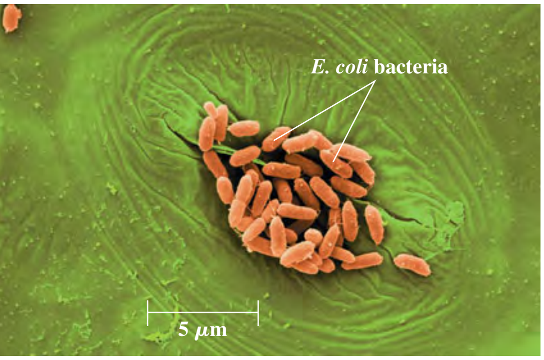
39.5 Schematic diagram of a transmission electron microscope (TEM).



We might think that when the electron wavelength is 0.01 nm (as in Example 39.3), the resolution would also be about 0.01 nm. In fact, it is seldom better than 0.1 nm, in part because the focal length of a magnetic lens depends on the electron speed, which is never exactly the same for all electrons in the beam.

An important variation is the *scanning electron microscope*. The electron beam is focused to a very fine line and scanned across the specimen. The beam knocks additional electrons off the specimen wherever it hits. These ejected electrons are collected by an anode that is kept at a potential a few hundred volts positive with respect to the specimen. The current of ejected electrons flowing to the collecting anode varies as the microscope beam sweeps across the specimen. The varying strength of the current is then used to create a "map" of the scanned specimen, and this map forms a greatly magnified image of the specimen.

This scheme has several advantages. The specimen can be thick because the beam does not need to pass through it. Also, the knock-off electron production depends on the *angle* at which the beam strikes the surface. Thus scanning electron



39.6 This scanning electron microscope image shows *Escherichia coli* bacteria crowded into a stoma, or respiration opening, on the surface of a lettuce leaf. (False color has been added.) If not washed off before the lettuce is eaten, these bacteria can be a health hazard. The *transmission* electron micrograph that opens this chapter shows a greatly magnified view of the surface of an *E. coli* bacterium.

micrographs have an appearance that is much more three-dimensional than conventional visible-light micrographs (Fig. 39.6). The resolution is typically of the order of 10 nm, not as good as a transmission electron microscope but still much finer than the best optical microscopes.

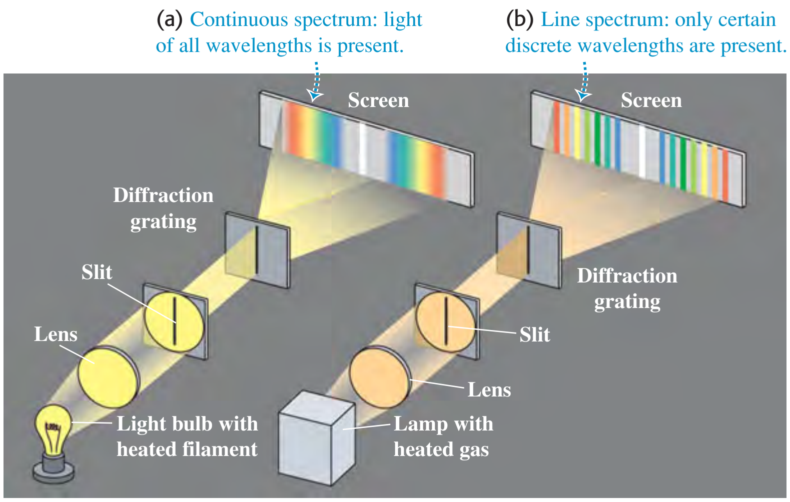
Test Your Understanding of Section 39.1 (a) A proton has a slightly smaller mass than a neutron. Compared to the neutron described in Example 39.2, would a proton of the same wavelength have (i) more kinetic energy; (ii) less kinetic energy; or (iii) the same kinetic energy? (b) Example 39.1 shows that to give electrons a wavelength of 1.7×10^{-10} m, they must be accelerated from rest through a voltage of 54 V and so acquire a kinetic energy of 54 eV. Does a photon of this same energy also have a wavelength of 1.7×10^{-10} m?

39.2 The Nuclear Atom and Atomic Spectra

Every neutral atom contains at least one electron. How does the wave aspect of electrons affect atomic structure? As we will see, it is crucial for understanding not only the structure of atoms but also how they interact with light. Historically, the quest to understand the nature of the atom was intimately linked with both the idea that electrons have wave characteristics and the notion that light has particle characteristics. Before we explore how these ideas shaped atomic theory, it's useful to look at what was known about atoms—as well as what remained mysterious—by the first decade of the twentieth century.

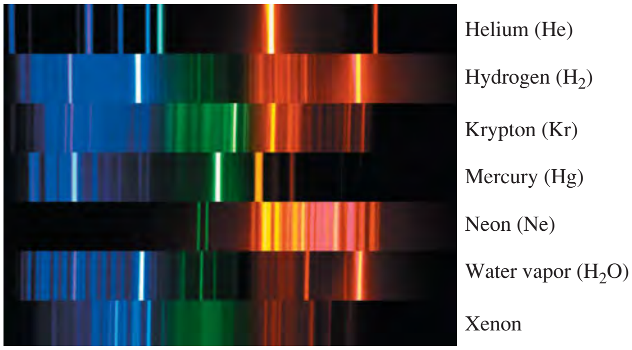
Line Spectra

Everyone knows that heated materials emit light, and that different materials emit different kinds of light. The coils of a toaster glow red when in operation, the flame of a match has a characteristic yellow color, and the flame from a gas range is a distinct blue. To analyze these different types of light, we can use a prism or a diffraction grating to separate the various wavelengths in a beam of light into a spectrum. If the light source is a hot solid (such as the filament of an incandescent light bulb) or liquid, the spectrum is *continuous;* light of all wavelengths is present (Fig. 39.7a). But if the source is a heated *gas*, such as the neon in an advertising sign or the sodium vapor formed when table salt is thrown into a campfire, the spectrum includes only a few colors in the form of isolated sharp parallel lines (Fig. 39.7b). (Each "line" is an image of the spectrograph slit, deviated through an angle that depends on the wavelength of the light forming that image; see Section 36.5.) A spectrum of this sort is called an **emission line spectrum,** and the lines are called **spectral lines.** Each spectral line corresponds to a definite wavelength and frequency.



39.7 (a) Continuous spectrum produced by a glowing light bulb filament.
(b) Emission line spectrum emitted by a lamp containing a heated gas.

39.8 The emission line spectra of several kinds of atoms and molecules. No two are alike. Note that the spectrum of water vapor (H_2O) is similar to that of hydrogen (H_2), but there are important differences that make it straightforward to distinguish these two spectra.



It was discovered early in the 19th century that each element in its gaseous state has a unique set of wavelengths in its line spectrum. The spectrum of hydrogen always contains a certain set of wavelengths; mercury produces a different set, neon still another, and so on (Fig. 39.8). Scientists find the use of spectra to identify elements and compounds to be an invaluable tool. For instance, astronomers have detected the spectra from more than 100 different molecules in interstellar space, including some that are not found naturally on earth.

While a *heated* gas selectively *emits* only certain wavelengths, a *cool* gas selectively *absorbs* certain wavelengths. If we pass white (continuous-spectrum) light through a gas and look at the *transmitted* light with a spectrometer, we find a series of dark lines corresponding to the wavelengths that have been absorbed (Fig. 39.9). This is called an **absorption line spectrum.** What's more, a given kind of atom or molecule absorbs the *same* characteristic set of wavelengths when it's cool as it emits when heated. Hence scientists can use absorption line spectra to identify substances in the same manner that they use emission line spectra.

As useful as emission line spectra and absorption line spectra are, they presented a quandary to scientists: *Why* does a given kind of atom emit and absorb only certain very specific wavelengths? To answer this question, we need to have a better idea of what the inside of an atom is like. We know that atoms are much smaller than the wavelengths of visible light, so there is no hope of actually *seeing* an atom using that light. But we can still describe how the mass and electric charge are distributed throughout the volume of the atom.

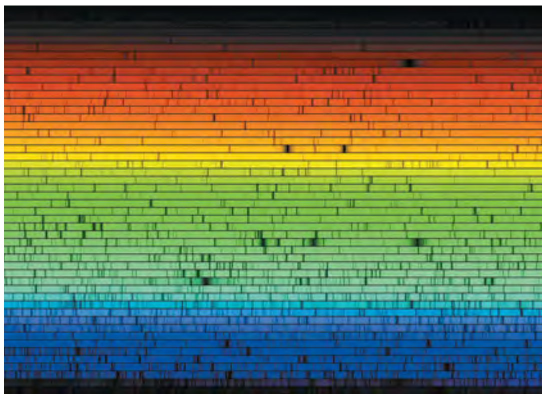
Here's where things stood in 1910. In 1897 the English physicist J. J. Thomson (Nobel Prize 1906) had discovered the electron and measured its charge-to-mass ratio e/m. By 1909, the American physicist Robert Millikan (Nobel Prize 1923) had made the first measurements of the electron charge $-e$. These and other experiments showed that almost all the mass of an atom had to be associated with the *positive* charge, not with the electrons. It was also known that the overall size of atoms is of the order of 10^{-10} m and that all atoms except hydrogen contain more than one electron.

In 1910 the best available model of atomic structure was one developed by Thomson. He envisioned the atom as a sphere of some as yet unidentified positively charged substance, within which the electrons were embedded like raisins in cake. This model offered an explanation for line spectra. If the atom collided with another atom, as in a heated gas, each electron would oscillate around its equilibrium position with a characteristic frequency and emit electromagnetic radiation with that frequency. If the atom were illuminated with light of many frequencies, each electron would selectively absorb only light whose frequency matched the electron's natural oscillation frequency. (This is the phenomenon of resonance that we discussed in Section 14.8.)

Application Using Spectra to Analyze an Interstellar Gas Cloud

The light from this glowing gas cloud—located in the Small Magellanic Cloud, a small satellite galaxy of the Milky Way some 200,000 light-years (1.9×10^{18} km) from earth—has an emission line spectrum. Despite its immense distance, astronomers can tell that this cloud is composed mostly of hydrogen because its spectrum is dominated by red light at a wavelength of 656.3 nm, a wavelength emitted by hydrogen and no other element.

39.9 The absorption line spectrum of the sun. (The spectrum "lines" read from left to right and from top to bottom, like text on a page.) The spectrum is produced by the sun's relatively cool atmosphere, which absorbs photons from deeper, hotter layers. The absorption lines thus indicate what kinds of atoms are present in the solar atmosphere.



39.10 Born in New Zealand, Ernest Rutherford (1871–1937) spent his professional life in England and Canada. Before carrying out the experiments that established the existence of atomic nuclei, he shared (with Frederick Soddy) the 1908 Nobel Prize in chemistry for showing that radioactivity results from the disintegration of atoms.

MasteringPHYSICS

PhET: Rutherford Scattering
ActivPhysics 19.1: Particle Scattering

Rutherford's Exploration of the Atom

The first experiments designed to test Thomson's model by probing the interior structure of the atom were carried out in 1910–1911 by Ernest Rutherford (Fig. 39.10) and two of his students, Hans Geiger and Ernest Marsden, at the University of Manchester in England. These experiments consisted of shooting a beam of charged particles at thin foils of various elements and observing how the foil deflected the particles.

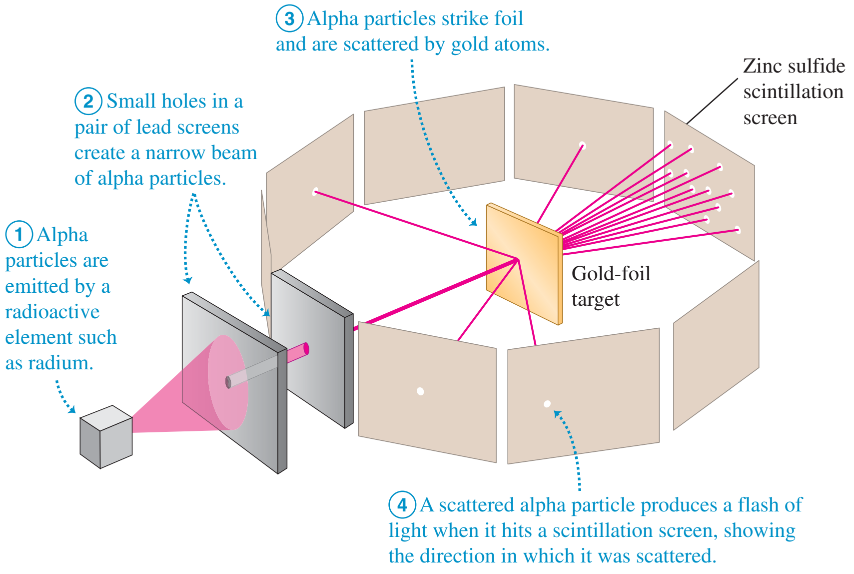
The particle accelerators now in common use in laboratories had not yet been invented, and Rutherford's projectiles were *alpha particles* emitted from naturally radioactive elements. The nature of these alpha particles was not completely understood, but it was known that they are ejected from unstable nuclei with speeds of the order of 10^7 m/s, are positively charged, and can travel several centimeters through air or 0.1 mm or so through solid matter before they are brought to rest by collisions.

Figure 39.11 is a schematic view of Rutherford's experimental setup. A radioactive substance at the left emits alpha particles. Thick lead screens stop all particles except those in a narrow beam. The beam passes through the foil target (consisting of gold, silver, or copper) and strikes screens coated with zinc sulfide, creating a momentary flash, or *scintillation*. Rutherford and his students counted the numbers of particles deflected through various angles.

The atoms in a metal foil are packed together like marbles in a box (not spaced apart). Because the particle beam passes through the foil, the alpha particles must pass through the interior of atoms. Within an atom, the charged alpha particle will interact with the electrons and the positive charge. (Because the *total* charge of the atom is zero, alpha particles feel little electrical force outside an atom.) An electron has about 7300 times less mass than an alpha particle, so momentum considerations indicate that the atom's electrons cannot appreciably deflect the alpha particle—any more than a swarm of gnats deflects a tossed pebble. Any deflection will be due to the positively charged material that makes up almost all of the atom's mass.

In the Thomson model, the positive charge and the negative electrons are distributed through the whole atom. Hence the electric field inside the atom should be quite small, and the electric force on an alpha particle that enters the atom should be quite weak. The maximum deflection to be expected is then only a few degrees (Fig. 39.12a). The results of the Rutherford experiments were *very* different

39.11 The Rutherford scattering experiments investigated what happens to alpha particles fired at a thin gold foil. The results of this experiment helped reveal the structure of atoms.



from the Thomson prediction. Some alpha particles were scattered by nearly 180°—that is, almost straight backward (Fig. 39.12b). Rutherford later wrote:

> **It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you.**

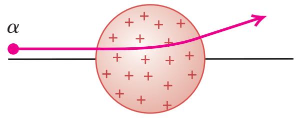
Clearly the Thomson model was wrong and a new model was needed. Suppose the positive charge, instead of being distributed through a sphere with atomic dimensions (of the order of 10^{-10} m), is all concentrated in a much *smaller* volume. Then it would act like a point charge down to much smaller distances. The maximum electric field repelling the alpha particle would be much larger, and the amazing large-angle scattering that Rutherford observed could occur. Rutherford developed this model and called the concentration of positive charge the **nucleus.** He again computed the numbers of particles expected to be scattered through various angles. Within the accuracy of his experiments, the computed and measured results agreed, down to distances of the order of 10^{-14} m. His experiments therefore established that the atom does have a nucleus—a very small, very dense structure, no larger than 10^{-14} m in diameter. The nucleus occupies only about 10^{-12} of the total volume of the atom or less, but it contains *all* the positive charge and at least 99.95% of the total mass of the atom.

Figure 39.13 shows a computer simulation of alpha particles with a kinetic energy of 5.0 MeV being scattered from a gold nucleus of radius 7.0×10^{-15} m (the actual value) and from a nucleus with a hypothetical radius ten times larger. In the second case there is *no* large-angle scattering. The presence of large-angle scattering in Rutherford's experiments thus attested to the small size of the nucleus.

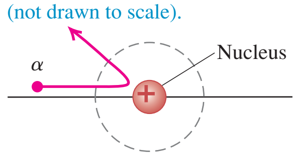
Later experiments showed that all nuclei are composed of positively charged protons (discovered in 1918) and electrically neutral neutrons (discovered in 1930). For example, the gold atoms in Rutherford's experiments have 79 protons and 118 neutrons. In fact, an alpha particle is itself the nucleus of a helium atom, with two protons and two neutrons. It is much more massive than an electron but only about 2% as massive as a gold nucleus, which helps explain why alpha particles are scattered by gold nuclei but not by electrons.

39.12 A comparison of Thomson's and Rutherford's models of the atom.

(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.



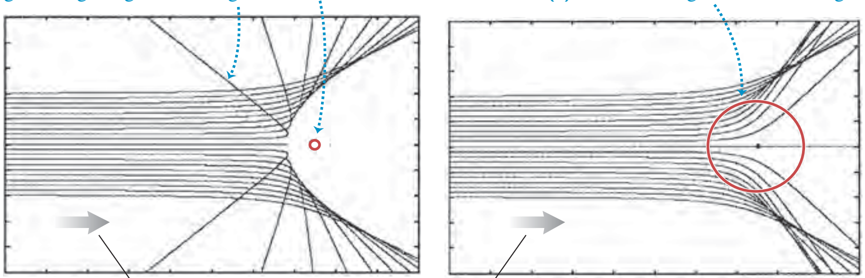
(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).



39.13 Computer simulation of scattering of 5.0-MeV alpha particles from a gold nucleus. Each curve shows a possible alpha-particle trajectory. (a) The scattering curves match Rutherford's experimental data if a radius of 7.0×10^{-15} m is assumed for a gold nucleus. (b) A model with a much larger radius for the gold nucleus does not match the data.

(a) A gold nucleus with radius 7.0×10^{-15} m gives large-angle scattering.

(b) A nucleus with 10 times the radius of the nucleus in (a) shows *no* large-scale scattering.



Motion of incident 5.0-MeV alpha particles

Example 39.4 A Rutherford experiment

An alpha particle (charge $2e$) is aimed directly at a gold nucleus (charge $79e$). What minimum initial kinetic energy must the alpha particle have to approach within 5.0×10^{-14} m of the center of the gold nucleus before reversing direction? Assume that the gold nucleus, which has about 50 times the mass of an alpha particle, remains at rest.

Continued

SOLUTION

IDENTIFY: The repulsive electric force exerted by the gold nucleus makes the the alpha particle slow to a halt as it approaches, then reverse direction. This force is conservative, so the total mechanical energy (kinetic energy of the alpha particle plus electric potential energy of the system) is conserved.

SET UP: Let point 1 be the initial position of the alpha particle, very far from the gold nucleus, and let point 2 be 5.0×10^{-14} m from the center of the gold nucleus. Our target variable is the kinetic energy K_1 of the alpha particle at point 1 that allows it to reach point 2 with $K_2 = 0$. To find this we'll use the law of conservation of energy and Eq. (23.9) for electric potential energy, $U = qq_0/4\pi\epsilon_0 r$.

EXECUTE: At point 1 the separation r of the alpha particle and gold nucleus is effectively infinite, so from Eq. (23.9) $U_1 = 0$. At point 2 the potential energy is

$$
\begin{aligned}
U_2 &= \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \\
&= (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(2)(79)(1.60 \times 10^{-19}\ \mathrm{C})^2}{5.0 \times 10^{-14}\ \mathrm{m}} \\
&= 7.3 \times 10^{-13}\ \mathrm{J} = 4.6 \times 10^6\ \mathrm{eV} = 4.6\ \mathrm{MeV}
\end{aligned}
$$

By energy conservation $K_1 + U_1 = K_2 + U_2$, so $K_1 = K_2 + U_2 - U_1 = 0 + 4.6\ \mathrm{MeV} - 0 = 4.6\ \mathrm{MeV}$. Thus, to approach within 5.0×10^{-14} m, the alpha particle must have initial kinetic energy $K_1 = 4.6$ MeV.

EVALUATE: Alpha particles emitted from naturally occurring radioactive elements typically have energies in the range 4 to 6 MeV. For example, the common isotope of radium, ^{226}Ra, emits an alpha particle with energy 4.78 MeV.

Was it valid to assume that the gold nucleus remains at rest? To find out, note that when the alpha particle stops momentarily, all of its initial momentum has been transferred to the gold nucleus. An alpha particle has a mass $m_\alpha = 6.64 \times 10^{-27}$ kg; if its initial kinetic energy $K_1 = \frac{1}{2}mv_1^2$ is 7.3×10^{-13} J, you can show that its initial speed is $v_1 = 1.5 \times 10^7$ m/s and its initial momentum is $p_1 = m_\alpha v_1 = 9.8 \times 10^{-20}\ \mathrm{kg \cdot m/s}$. A gold nucleus (mass $m_{\mathrm{Au}} = 3.27 \times 10^{-25}$ kg) with this much momentum has a much slower speed $v_{\mathrm{Au}} = 3.0 \times 10^5$ m/s and kinetic energy $K_{\mathrm{Au}} = \frac{1}{2}mv_{\mathrm{Au}}^2 = 1.5 \times 10^{-14}\ \mathrm{J} = 0.092$ MeV. This *recoil kinetic energy* of the gold nucleus is only 2% of the total energy in this situation, so we are justified in neglecting it.

The Failure of Classical Physics

Rutherford's discovery of the atomic nucleus raised a serious question: What prevented the negatively charged electrons from falling into the positively charged nucleus due to the strong electrostatic attraction? Rutherford suggested that perhaps the electrons *revolve* in orbits about the nucleus, just as the planets revolve around the sun.

But according to classical electromagnetic theory, any accelerating electric charge (either oscillating or revolving) radiates electromagnetic waves. An example is the radiation from an oscillating point charge that we depicted in Fig. 32.3 (Section 32.1). An electron orbiting inside an atom would always have a centripetal acceleration toward the nucleus, and so should be emitting radiation *at all times*. The energy of an orbiting electron should therefore decrease continuously, its orbit should become smaller and smaller, and it should spiral into the nucleus within a fraction of a second (Fig. 39.14). Even worse, according to classical theory the *frequency* of the electromagnetic waves emitted should equal the frequency of revolution. As the electrons radiated energy, their angular speeds would change continuously, and they would emit a *continuous* spectrum (a mixture of all frequencies), not the *line* spectrum actually observed.

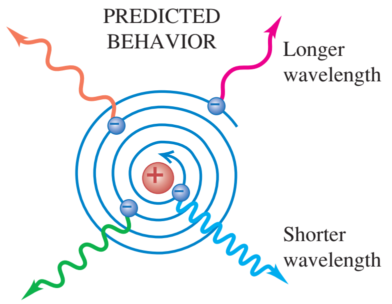
Thus Rutherford's model of electrons orbiting the nucleus, which is based on Newtonian mechanics and classical electromagnetic theory, makes three entirely *wrong* predictions about atoms: They should emit light continuously, they should be unstable, and the light they emit should have a continuous spectrum. Clearly a radical reappraisal of physics on the scale of the atom was needed. In the next section we will see the bold idea that led to an new understanding of the atom, and see how this idea meshes with de Broglie's no less bold notion that electrons have wave attributes.

39.14 Classical physics makes predictions about the behavior of atoms that do not match reality.

ACCORDING TO CLASSICAL PHYSICS:

- An orbiting electron is accelerating, so it should radiate electromagnetic waves.
- The waves would carry away energy, so the electron should lose energy and spiral inward.
- The electron's angular speed would increase as its orbit shrank, so the frequency of the radiated waves should increase.

Thus, classical physics says that atoms should collapse within a fraction of a second and should emit light with a continuous spectrum as they do so.



IN FACT:

- Atoms are stable.
- They emit light only when excited, and only at specific frequencies (as a line spectrum).

Test Your Understanding of Section 39.2 Suppose you repeated Rutherford's scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14.0 K.) The nucleus of a hydrogen atom is a single proton, with about one-fourth the mass of an alpha particle. Compared to the original experiment with gold foil, would you expect the alpha particles in this experiment to undergo (i) more large-angle scattering; (ii) the same amount of large-angle scattering; or (iii) less large-angle scattering?

39.3 Energy Levels and the Bohr Model of the Atom

In 1913 a young Danish physicist working with Ernest Rutherford at the University of Manchester made a revolutionary proposal to explain both the stability of atoms and their emission and absorption line spectra. The physicist was Niels Bohr (Fig. 39.15), and his innovation was to combine the photon concept that we introduced in Chapter 38 with a fundamentally new idea: The energy of an atom can have only certain particular values. His hypothesis represented a clean break from 19th-century ideas.

Photon Emission and Absorption by Atoms

Bohr's reasoning went like this. The emission line spectrum of an element tells us that atoms of that element emit photons with only certain specific frequencies f and hence certain specific energies $E = hf$. During the emission of a photon, the internal energy of the atom changes by an amount equal to the energy of the photon. Therefore, said Bohr, each atom must be able to exist with only certain specific values of internal energy. Each atom has a set of possible **energy levels.** An atom can have an amount of internal energy equal to any one of these levels, but it *cannot* have an energy *intermediate* between two levels. All isolated atoms of a given element have the same set of energy levels, but atoms of different elements have different sets.

Suppose an atom is raised, or *excited*, to a high energy level. (In a hot gas this happens when fast-moving atoms undergo inelastic collisions with each other or with the walls of the gas container. In an electric discharge tube, such as those used in a neon light fixture, atoms are excited by collisions with fast-moving electrons.) According to Bohr, an excited atom can make a *transition* from one energy level to a lower level by emitting a photon with energy equal to the energy *difference* between the initial and final levels (Fig. 39.16). If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $hf = hc/\lambda$, then conservation of energy gives

$$hf = \frac{hc}{\lambda} = E_i - E_f \qquad \text{(energy of emitted photon)} \qquad (39.5)$$

For example, an excited lithium atom emits red light with wavelength $\lambda = 671\ \text{nm}$. The corresponding photon energy is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{671 \times 10^{-9}\ \text{m}}$$
$$= 2.96 \times 10^{-19}\ \text{J} = 1.85\ \text{eV}$$

This photon is emitted during a transition like that shown in Fig. 39.16 between two levels of the atom that differ in energy by $E_i - E_f = 1.85\ \text{eV}$.

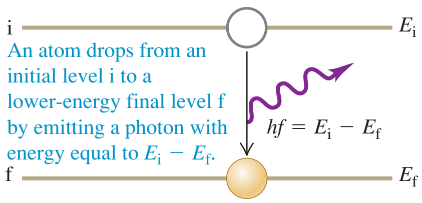
The emission line spectra shown in Fig. 39.8 show that many different wavelengths are emitted by each atom. Hence each kind of atom must have a number of energy levels, with different spacings in energy between them. Each wavelength in the spectrum corresponds to a transition between two specific energy levels of the atom.

CAUTION **Producing a line spectrum** The lines of an emission line spectrum, such as the helium spectrum shown at the top of Fig. 39.8, are *not* all produced by a single atom. The sample of helium gas that produced the spectrum in Fig. 39.8 contained a large number of helium atoms; these were excited in an electric discharge tube to various energy levels. The spectrum of the gas shows the light emitted from all the different transitions that occurred in different atoms of the sample.

The observation that atoms are stable means that each atom has a *lowest* energy level, called the **ground level.** Levels with energies greater than the

39.15 Niels Bohr (1885–1962) was a young postdoctoral researcher when he proposed the novel idea that the energy of an atom could have only certain discrete values. He won the 1922 Nobel Prize in physics for these ideas. Bohr went on to make seminal contributions to nuclear physics and to become a passionate advocate for the free exchange of scientific ideas among all nations.

39.16 An excited atom emitting a photon.



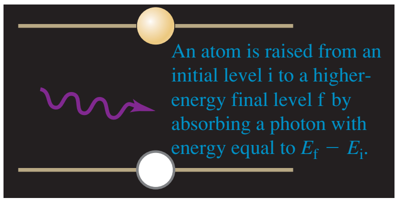
MasteringPHYSICS

PhET: Models of the Hydrogen Atom
ActivPhysics 18.1: The Bohr Model

ground level are called **excited levels.** An atom in an excited level, called an *excited atom*, can make a transition into the ground level by emitting a photon as in Fig. 39.16. But since there are no levels below the ground level, an atom in the ground level cannot lose energy and so cannot emit a photon.

Collisions are not the only way that an atom's energy can be raised from one level to a higher level. If an atom initially in the lower energy level in Fig. 39.16 is struck by a photon with just the right amount of energy, the photon can be *absorbed* and the atom will end up in the higher level (Fig. 39.17). As an example, we previously mentioned two levels in the lithium atom with an energy difference of 1.85 eV. For a photon to be absorbed and excite the atom from the lower level to the higher one, the photon must have an energy of 1.85 eV and a wavelength of 671 nm. In other words, an atom *absorbs* the same wavelengths that it *emits*. This explains the correspondence between an element's emission line spectrum and its absorption line spectrum that we described in Section 39.2.

39.17 An atom absorbing a photon. (Compare with Fig. 39.16.)

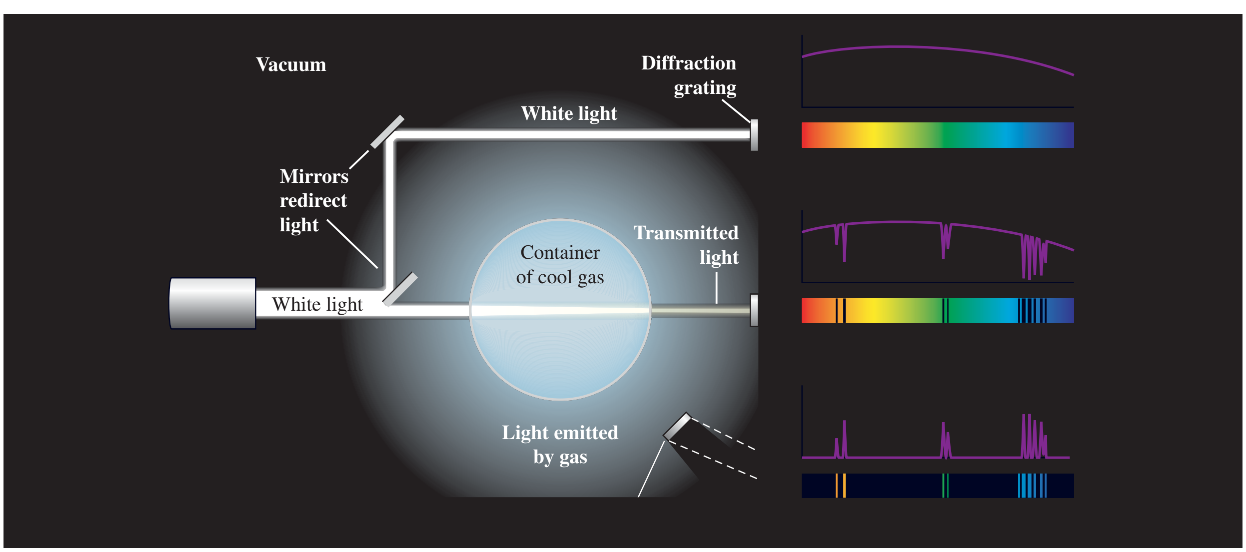


Note that a lithium atom *cannot* absorb a photon with a slightly longer wavelength (say, 672 nm) or one with a slightly shorter wavelength (say, 670 nm). That's because these photons have, respectively, slightly too little or slightly too much energy to raise the atom's energy from one level to the next, and an atom cannot have an energy that's intermediate between levels. This explains why absorption line spectra have distinct dark lines (see Fig. 39.9): Atoms can absorb only photons with specific wavelengths.

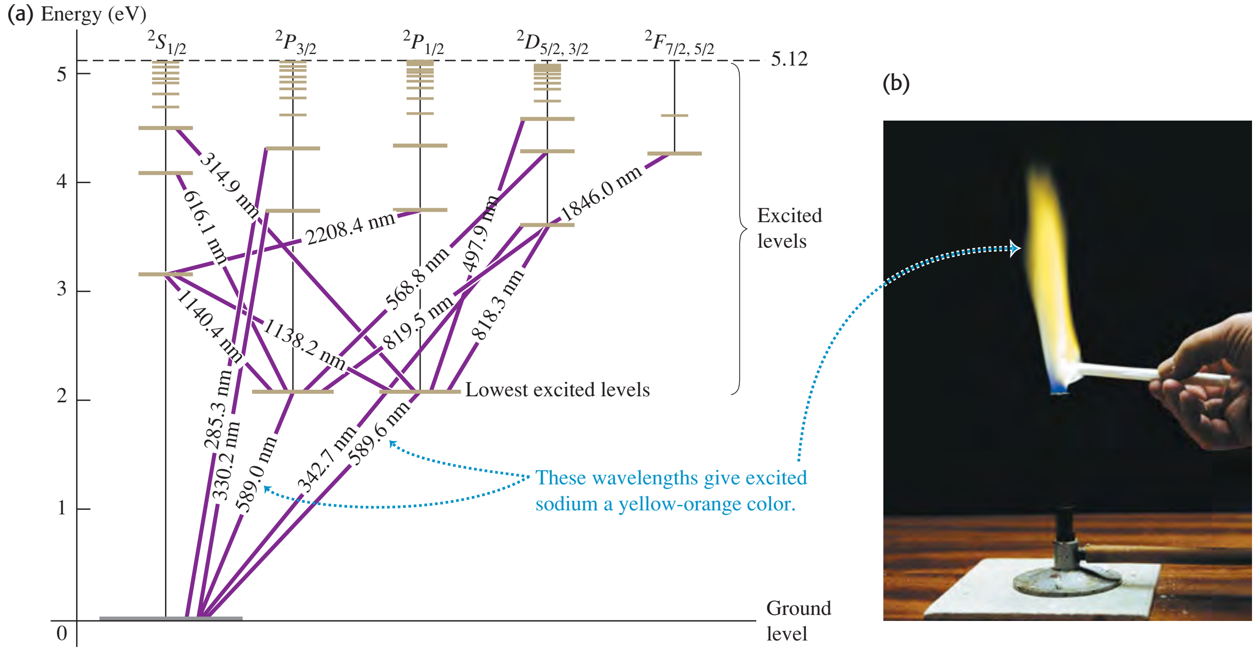
An atom that's been excited into a high energy level, either by photon absorption or by collisions, does not stay there for long. After a short time, called the *lifetime* of the level (typically around 10^{-8} s), the excited atom will emit a photon and make a transition into a lower excited level or the ground level. A cool gas that's illuminated by white light to make an *absorption* line spectrum thus also produces an *emission* line spectrum when viewed from the side, since when the atoms de-excite they emit photons in all directions (Fig. 39.18). To keep a gas of atoms glowing, you have to continually provide energy to the gas in order to re-excite atoms so that they can emit more photons. If you turn off the energy supply (for example, by turning off the electric current through a neon light fixture, or by shutting off the light source in Fig. 39.18), the atoms drop back into their ground levels and cease to emit light.

By working backward from the observed emission line spectrum of an element, physicists can deduce the arrangement of energy levels in an atom of that element. As an example, Fig. 39.19a shows some of the energy levels for a sodium atom. You may have noticed the yellow-orange light emitted by sodium

39.18 When a beam of white light with a continuous spectrum passes through a cool gas, the transmitted light has an absorption spectrum. The absorbed light energy excites the gas and causes it to emit light of its own, which has an emission spectrum.



39.19 (a) Energy levels of the sodium atom relative to the ground level. Numbers on the lines between levels are wavelengths of the light emitted or absorbed during transitions between those levels. The column labels, such as $^2S_{1/2}$, refer to some quantum states of the atom. (b) When a sodium compound is placed in a flame, sodium atoms are excited into the lowest excited levels. As they drop back to the ground level, the atoms emit photons of yellow-orange light with wavelengths 589.0 and 589.6 nm.



vapor street lights. Sodium atoms emit this characteristic yellow-orange light with wavelengths 589.0 and 589.6 nm when they make transitions from the two closely spaced levels labeled *lowest excited levels* to the ground level. A standard test for the presence of sodium compounds is to look for this yellow-orange light from a sample placed in a flame (Fig. 39.19b).

Example 39.5 Emission and absorption spectra

A hypothetical atom (Fig. 39.20a) has energy levels at 0.00 eV (the ground level), 1.00 eV, and 3.00 eV. (a) What are the frequencies and wavelengths of the spectral lines this atom can emit when excited? (b) What wavelengths can this atom absorb if it is in its ground level?

SOLUTION

IDENTIFY and SET UP: Energy is conserved when a photon is emitted or absorbed. In each transition the photon energy is equal to the difference between the energies of the levels involved in the transition.

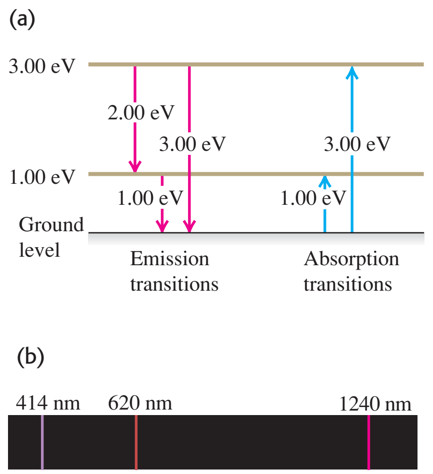
EXECUTE: (a) The possible energies of emitted photons are 1.00 eV, 2.00 eV, and 3.00 eV. For 1.00 eV, Eq. (39.2) gives

$$f = \frac{E}{h} = \frac{1.00\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV} \cdot \text{s}} = 2.42 \times 10^{14}\ \text{Hz}$$

For 2.00 eV and 3.00 eV, $f = 4.84 \times 10^{14}$ Hz and 7.25×10^{14} Hz, respectively. For 1.00-eV photons,

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\ \text{m/s}}{2.42 \times 10^{14}\ \text{Hz}} = 1.24 \times 10^{-6}\ \text{m} = 1240\ \text{nm}$$

39.20 (a) Energy-level diagram for the hypothetical atom, showing the possible transitions for emission from excited levels and for absorption from the ground level. (b) Emission spectrum of this hypothetical atom.



Continued

This is in the infrared region of the spectrum (Fig. 39.20b). For 2.00 eV and 3.00 eV, the wavelengths are 620 nm (red) and 414 nm (violet), respectively.

(b) From the ground level, only a 1.00-eV or a 3.00-eV photon can be absorbed (Fig. 39.20a); a 2.00-eV photon cannot be absorbed because the atom has no energy level 2.00 eV above the ground level. Passing light from a hot solid through a gas of these hypothetical atoms (almost all of which would be in the ground state if the gas were cool) would yield a continuous spectrum with dark absorption lines at 1240 nm and 414 nm.

EVALUATE: Note that if a gas of these atoms were at a sufficiently high temperature, collisions would excite a number of atoms into the 1.00-eV energy level. Such excited atoms *can* absorb 2.00-eV photons, as Fig. 39.20a shows, and an absorption line at 620 nm would appear in the spectrum. Thus the observed spectrum of a given substance depends on its energy levels and its temperature.

Application Fish Fluorescence
When illuminated by blue light, this tropical lizardfish (family *Synodontidae*) fluoresces and emits longer-wavelength green light. The fluorescence may be a sexual signal or a way for the fish to camouflage itself among coral (which also have a green fluorescence).

Suppose we take a gas of the hypothetical atoms in Example 39.5 and illuminate it with violet light of wavelength 414 nm. Atoms in the ground level can absorb this photon and make a transition to the 3.00-eV level. Some of these atoms will make a transition back to the ground level by emitting a 414-nm photon. But other atoms will return to the ground level in two steps, first emitting a 620-nm photon to transition to the 1.00-eV level, then a 1240-nm photon to transition back to the ground level. Thus this gas will emit longer-wavelength radiation than it absorbs, a phenomenon called *fluorescence*. For example, the electric discharge in a fluorescent lamp causes the mercury vapor in the tube to emit ultraviolet radiation. This radiation is absorbed by the atoms of the coating on the inside of the tube. The coating atoms then re-emit light in the longer-wavelength, visible portion of the spectrum. Fluorescent lamps are more efficient than incandescent lamps in converting electrical energy to visible light because they do not waste as much energy producing (invisible) infrared photons.

Our discussion of energy levels and spectra has concentrated on *atoms*, but the same ideas apply to *molecules*. Figure 39.8 shows the emission line spectra of two molecules, hydrogen (H_2) and water (H_2O). Just as for sodium or other atoms, physicists can work backward from these molecular spectra and deduce the arrangement of energy levels for each kind of molecule. We'll return to molecules and molecular structure in Chapter 42.

The Franck–Hertz Experiment: Are Energy Levels Real?

Are atomic energy levels real, or just a convenient fiction that helps us to explain spectra? In 1914, the German physicists James Franck and Gustav Hertz answered this question when they found direct experimental evidence for the existence of atomic energy levels.

Franck and Hertz studied the motion of electrons through mercury vapor under the action of an electric field. They found that when the electron kinetic energy was 4.9 eV or greater, the vapor emitted ultraviolet light of wavelength 250 nm. Suppose mercury atoms have an excited energy level 4.9 eV above the ground level. An atom can be raised to this level by collision with an electron; it later decays back to the ground level by emitting a photon. From the photon formula $E = hc/\lambda$, the wavelength of the photon should be

$$\lambda = \frac{hc}{E} = \frac{(4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{4.9\ \text{eV}}$$
$$= 2.5 \times 10^{-7}\ \text{m} = 250\ \text{nm}$$

This is equal to the wavelength that Franck and Hertz measured, which demonstrates that this energy level actually exists in the mercury atom. Similar experiments with other atoms yield the same kind of evidence for atomic energy levels. Franck and Hertz shared the 1925 Nobel Prize in physics for their research.

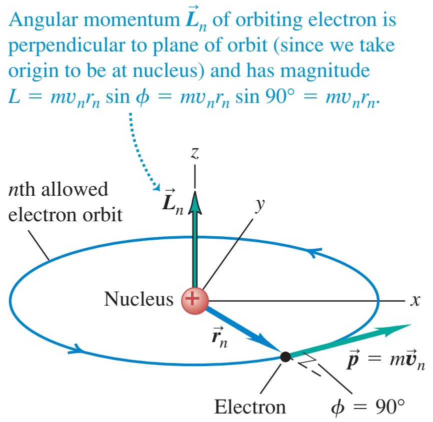
Electron Waves and the Bohr Model of Hydrogen

Bohr's hypothesis established the relationship between atomic spectra and energy levels. By itself, however, it provided no general principles for *predicting* the energy levels of a particular atom. Bohr addressed this problem for the case of the simplest atom, hydrogen, which has just one electron. Let's look at the ideas behind the **Bohr model** of the hydrogen atom.

Bohr postulated that each energy level of a hydrogen atom corresponds to a specific *stable* circular orbit of the electron around the nucleus. In a break with classical physics, Bohr further postulated that an electron in such an orbit does *not* radiate. Instead, an atom radiates energy only when an electron makes a transition from an orbit of energy E_i to a different orbit with lower energy E_f, emitting a photon of energy $hf = E_i - E_f$ in the process.

As a result of a rather complicated argument that related the angular frequency of the light emitted to the angular speed of the electron in highly excited energy levels, Bohr found that the magnitude of the electron's angular momentum is *quantized;* that is, this magnitude must be an integral multiple of $h/2\pi$. (Because $1\ \text{J} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$, the SI units of Planck's constant h, J·s, are the same as the SI units of angular momentum, usually written as $\text{kg}\cdot\text{m}^2/\text{s}$.) Let's number the orbits by an integer n, where $n = 1, 2, 3, \ldots$, and call the radius of orbit n r_n and the speed of the electron in that orbit v_n. The value of n for each orbit is called the **principal quantum number** for the orbit. From Section 10.5, Eq. (10.28), the magnitude of the angular momentum of an electron of mass m in such an orbit is $L_n = mv_nr_n$ (Fig. 39.21). So Bohr's argument led to

39.21 Calculating the angular momentum of an electron in a circular orbit around an atomic nucleus.



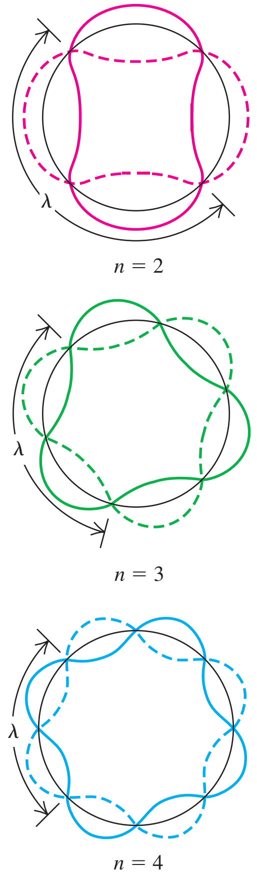
$$L_n = mv_nr_n = n\frac{h}{2\pi} \qquad \text{(quantization of angular momentum)} \qquad (39.6)$$

Instead of going through Bohr's argument to justify Eq. (39.6), we can use de Broglie's picture of electron waves. Rather then visualizing the orbiting electron as a particle moving around the nucleus in a circular path, think of it as a sinusoidal *standing wave* with wavelength λ that extends around the circle. A standing wave on a string transmits no energy (see Section 15.7), and electrons in Bohr's orbits radiate no energy. For the wave to "come out even" and join onto itself smoothly, the circumference of this circle must include some *whole number* of wavelengths, as Fig. 39.22 suggests. Hence for an orbit with radius r_n and circumference $2\pi r_n$, we must have $2\pi r_n = n\lambda_n$, where λ_n is the wavelength and $n = 1, 2, 3, \ldots$. According to the de Broglie relationship, Eq. (39.1), the wavelength of a particle with rest mass m moving with nonrelativistic speed v_n is $\lambda_n = h/mv_n$. Combining $2\pi r_n = n\lambda_n$ and $\lambda_n = h/mv_n$, we find $2\pi r_n = nh/mv_n$ or

$$mv_nr_n = n\frac{h}{2\pi}$$

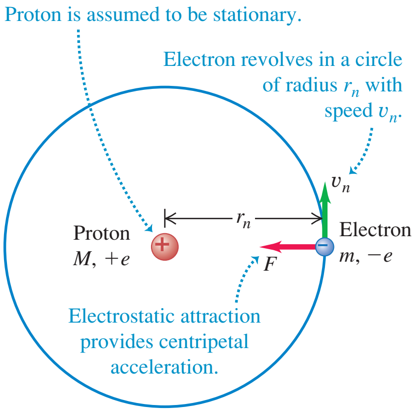
This is the same as Bohr's result, Eq. (39.6). Thus a wave picture of the electron leads naturally to the quantization of the electron's angular momentum.

39.22 These diagrams show the idea of fitting a standing electron wave around a circular orbit. For the wave to join onto itself smoothly, the circumference of the orbit must be an integral number n of wavelengths.



Now let's consider a model of the hydrogen atom that is Newtonian in spirit but incorporates this quantization assumption (Fig. 39.23). This atom consists of a single electron with mass m and charge $-e$ in a circular orbit around a single proton with charge $+e$. The proton is nearly 2000 times as massive as the electron, so we can assume that the proton does not move. We learned in Section 5.4 that when a particle with mass m moves with speed v_n in a circular orbit with radius r_n, its centripetal (inward) acceleration is v_n^2/r_n. According to Newton's second law, a radially inward net force with magnitude $F = mv_n^2/r_n$ is needed to cause this acceleration. We discussed in Section 12.4 how the gravitational attraction provides that inward force for satellite orbits. In hydrogen the force F is provided by the electrical attraction between the positive proton and the negative electron. From Coulomb's law, Eq. (21.2),

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n^2}$$



39.23 The Bohr model of the hydrogen atom.

Hence Newton's second law states that

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n^{\,2}} = \frac{mv_n^{\,2}}{r_n} \tag{39.7}$$

When we solve Eqs. (39.6) and (39.7) simultaneously for r_n and v_n, we get

$$r_n = \epsilon_0 \frac{n^2h^2}{\pi me^2} \qquad \text{(orbital radii in the Bohr model)} \tag{39.8}$$

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh} \qquad \text{(orbital speeds in the Bohr model)} \tag{39.9}$$

Equation (39.8) shows that the orbit radius r_n is proportional to n^2, so the smallest orbit radius corresponds to $n = 1$. We'll denote this minimum radius, called the *Bohr radius,* as a_0:

$$a_0 = \epsilon_0 \frac{h^2}{\pi me^2} \qquad \text{(Bohr radius)} \tag{39.10}$$

Then we can rewrite Eq. (39.8) as

$$r_n = n^2 a_0 \tag{39.11}$$

The permitted orbits have radii a_0, $4a_0$, $9a_0$, and so on.

You can find the numerical values of the quantities on the right-hand side of Eq. (39.10) in Appendix F. Using these values, we find that the radius a_0 of the smallest Bohr orbit is

$$a_0 = \frac{(8.854 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(6.626 \times 10^{-34}\ \mathrm{J\cdot s})^2}{\pi(9.109 \times 10^{-31}\ \mathrm{kg})(1.602 \times 10^{-19}\ \mathrm{C})^2}$$
$$= 5.29 \times 10^{-11}\ \mathrm{m}$$

This gives an atomic diameter of about $10^{-10}\ \mathrm{m} = 0.1\ \mathrm{nm}$, which is consistent with atomic dimensions estimated by other methods.

Equation (39.9) shows that the orbital speed v_n is proportional to $1/n$. Hence the greater the value of n, the larger the orbital radius of the electron and the slower its orbital speed. (We saw the same relationship between orbital radius and speed for satellite orbits in Section 13.4.) We leave it to you to calculate the speed in the $n = 1$ orbit, which is the greatest possible speed of the electron in the hydrogen atom (see Exercise 39.29); the result is $v_1 = 2.19 \times 10^6\ \mathrm{m/s}$. This is less than 1% of the speed of light, so relativistic considerations aren't significant.

Hydrogen Energy Levels in the Bohr Model

We can now use Eqs. (39.8) and (39.9) to find the kinetic and potential energies K_n and U_n when the electron is in the orbit with quantum number n:

$$K_n = \tfrac{1}{2}mv_n^{\,2} = \frac{1}{\epsilon_0^{\,2}}\frac{me^4}{8n^2h^2} \qquad \text{(kinetic energies in the Bohr model)} \tag{39.12}$$

$$U_n = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n} = -\frac{1}{\epsilon_0^{\,2}}\frac{me^4}{4n^2h^2} \qquad \text{(potential energies in the Bohr model)} \tag{39.13}$$

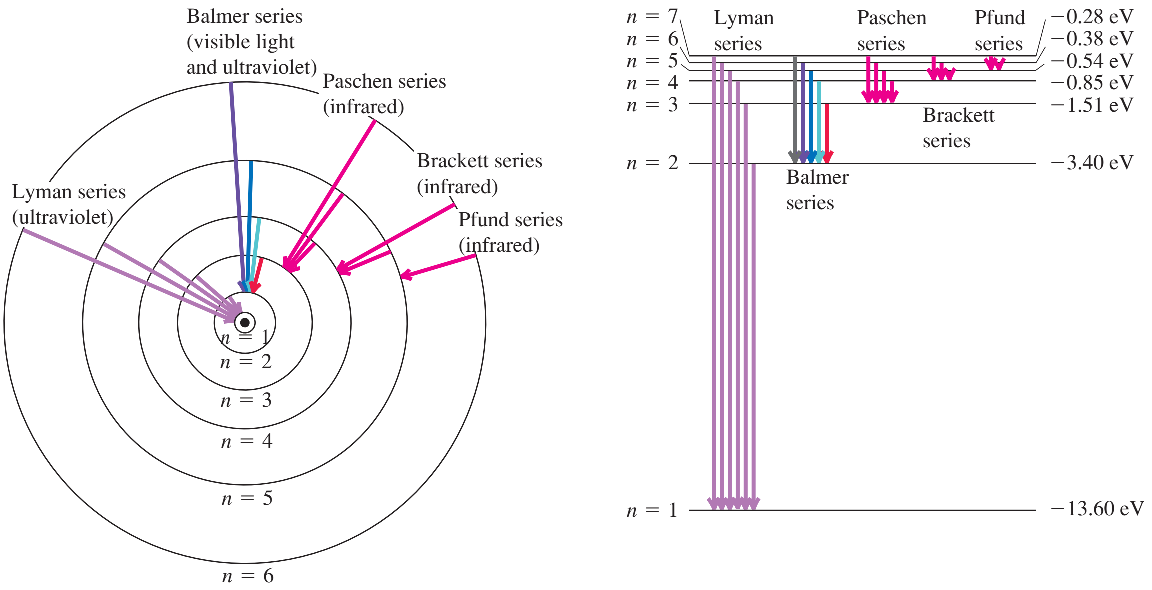
The potential energy has a negative sign because we have taken the electric potential energy to be zero when the electron is infinitely far from the nucleus. We are interested only in the *differences* in energy between orbits, so the reference position doesn't matter. The total energy E_n is the sum of the kinetic and potential energies:

$$E_n = K_n + U_n = -\frac{1}{\epsilon_0^{\,2}}\frac{me^4}{8n^2h^2} \qquad \text{(total energies in the Bohr model)} \tag{39.14}$$

39.24 Two ways to represent the energy levels of the hydrogen atom and the transitions between them. Note that the radius of the nth permitted orbit is actually n^2 times the radius of the $n = 1$ orbit.

(a) Permitted orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series



Since E_n in Eq. (39.14) has a different value for each n, you can see that this equation gives the *energy levels* of the hydrogen atom in the Bohr model. Each distinct orbit corresponds to a distinct energy level.

Figure 39.24 depicts the orbits and energy levels. We label the possible energy levels of the atom by values of the quantum number n. For each value of n there are corresponding values of orbit radius r_n, speed v_n, angular momentum $L_n = nh/2\pi$, and total energy E_n. The energy of the atom is least when $n = 1$ and E_n has its most negative value. This is the *ground level* of the hydrogen atom; it is the level with the smallest orbit, of radius a_0. For $n = 2, 3, \ldots,$ the absolute value of E_n is smaller and the energy is progressively larger (less negative).

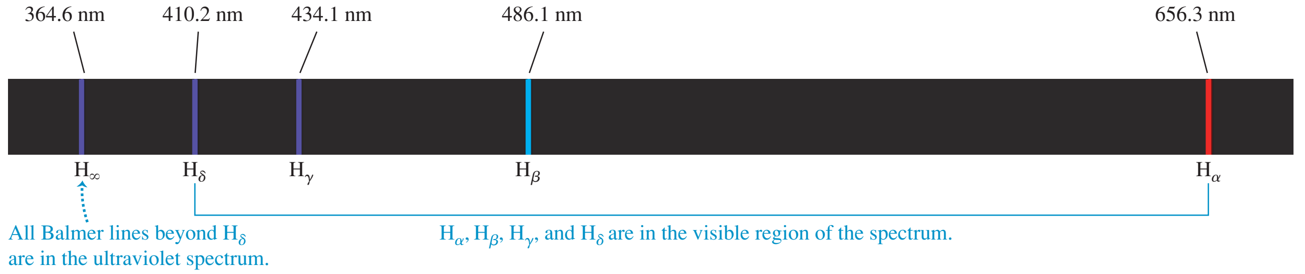
Figure 39.24 also shows some of the possible transitions from one electron orbit to an orbit of lower energy. Consider a transition from orbit n_U (for "upper") to a smaller orbit n_L (for "lower"), with $n_L < n_U$—or, equivalently, from *level* n_U to a lower *level* n_L. Then the energy hc/λ of the emitted photon of wavelength λ is equal to $E_{n_U} - E_{n_L}$. Before we use this relationship to solve for λ, it's convenient to rewrite Eq. (39.14) for the energies as

$$E_n = -\frac{hcR}{n^2}, \quad \text{where} \quad R = \frac{me^4}{8\epsilon_0^2 h^3 c} \quad \text{(total energies in the Bohr model)} \tag{39.15}$$

The quantity R in Eq. (39.15) is called the **Rydberg constant** (named for the Swedish physicist Johannes Rydberg, who did pioneering work on the hydrogen spectrum). When we substitute the numerical values of the fundamental physical constants m, c, e, h, and ϵ_0, all of which can be determined quite independently of the Bohr theory, we find that $R = 1.097 \times 10^7\ \text{m}^{-1}$. Now we solve for the wavelength of the photon emitted in a transition from level n_U to level n_L:

$$\frac{hc}{\lambda} = E_{n_U} - E_{n_L} = \left(-\frac{hcR}{n_U^2}\right) - \left(-\frac{hcR}{n_L^2}\right) = hcR\left(\frac{1}{n_L^2} - \frac{1}{n_U^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_L^2} - \frac{1}{n_U^2}\right) \quad \text{(hydrogen wavelengths in the Bohr model, } n_L < n_U\text{)} \tag{39.16}$$



39.25 The Balmer series of spectral lines for atomic hydrogen. You can see these same lines in the spectrum of *molecular* hydrogen (H_2) shown in Fig. 39.8, as well as additional lines that are present only when two hydrogen atoms are combined to make a molecule.

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Equation (39.16) is a *theoretical prediction* of the wavelengths found in the *emission* line spectrum of hydrogen atoms. When a hydrogen atom *absorbs* a photon, an electron makes a transition from a level n_L to a *higher* level n_U. This can happen only if the photon energy hc/λ is equal to $E_{n_U} - E_{n_L}$, which is the same condition expressed by Eq. (39.16). So this equation also predicts the wavelengths found in the *absorption* line spectrum of hydrogen.

How does this prediction compare with experiment? If $n_L = 2$, corresponding to transitions to the second energy level in Fig. 39.24, the wavelengths predicted by Eq. (39.16) are all in the visible and ultraviolet parts of the electromagnetic spectrum. These wavelengths are collectively called the *Balmer series* (Fig. 39.25). If we let $n_L = 2$ and $n_U = 3$ in Eq. (39.16) we obtain the wavelength of the H_α line:

$$\frac{1}{\lambda} = (1.097 \times 10^7\ \text{m}^{-1})\left(\tfrac{1}{4} - \tfrac{1}{9}\right) \qquad \text{or} \qquad \lambda = 656.3\ \text{nm}$$

With $n_L = 2$ and $n_U = 4$ we obtain the wavelength of the H_β line, and so on. With $n_L = 2$ and $n_U = \infty$ we obtain the shortest wavelength in the series, $\lambda = 364.6$ nm. These theoretical predictions are within 0.1% of the observed hydrogen wavelengths! This close agreement provides very strong and direct confirmation of Bohr's theory.

The Bohr model also predicts many other wavelengths in the hydrogen spectrum, as Fig. 39.24 shows. The observed wavelengths of all of these series, each of which is named for its discoverer, match the predicted values with the same percent accuracy as for the Balmer series. The *Lyman series* of spectral lines is caused by transitions between the ground level and the excited levels, corresponding to $n_L = 1$ and $n_U = 2, 3, 4, \ldots$ in Eq. (39.16). The energy difference between the ground level and any of the excited levels is large, so the emitted photons have wavelengths in the ultraviolet part of the electromagnetic spectrum. Transitions among the higher energy levels involve a much smaller energy difference, so the photons emitted in these transitions have little energy and long, infrared wavelengths. That's the case for both the *Brackett series* ($n_L = 3$ and $n_U = 4, 5, 6, \ldots$, corresponding to transitions between the third and higher energy levels) and the *Pfund series* ($n_L = 4$ and $n_U = 5, 6, 7, \ldots$, with transitions between the fourth and higher energy levels).

Figure 39.24 shows only transitions in which a hydrogen atom loses energy and a photon is emitted. But as we discussed previously, the wavelengths of those photons that an atom can *absorb* are the same as those that it can emit. For example, a hydrogen atom in the $n = 2$ level can absorb a 656.3-nm photon and end up in the $n = 3$ level.

One additional test of the Bohr model is its predicted value of the *ionization energy* of the hydrogen atom. This is the energy required to remove the electron completely from the atom. Ionization corresponds to a transition from the ground level $(n = 1)$ to an infinitely large orbit radius $(n = \infty)$, so the energy that must be added to the atom is $E_\infty - E_1 = 0 - E_1 = -E_1$ (recall that E_1 is negative).

Substituting the constants from Appendix F into Eq. (39.15) gives an ionization energy of 13.606 eV. The ionization energy can also be measured directly; the result is 13.60 eV. These two values agree within 0.1%.

Example 39.6 Exploring the Bohr model

Find the kinetic, potential, and total energies of the hydrogen atom in the first excited level, and find the wavelength of the photon emitted in a transition from that level to the ground level.

SOLUTION

IDENTIFY and SET UP: This problem uses the ideas of the Bohr model. We use simplified versions of Eqs. (39.12), (39.13), and (39.14) to find the energies of the atom, and Eq. (39.16), $hc/\lambda = E_{n_U} - E_{n_L}$, to find the photon wavelength λ in a transition from $n_U = 2$ (the first excited level) to $n_L = 1$ (the ground level).

EXECUTE: We could evaluate Eqs. (39.12), (39.13), and (39.14) for the nth level by substituting the values of m, e, ϵ_0, and h. But we can simplify the calculation by comparing with Eq. (39.15), which shows that the constant $me^4/8\epsilon_0^2h^2$ that appears in Eqs. (39.12), (39.13), and (39.14) is equal to hcR:

$$\begin{aligned}\frac{me^4}{8\epsilon_0^2h^2} &= hcR\\ &= (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})\\ &\quad\times (1.097 \times 10^7\ \text{m}^{-1})\\ &= 2.179 \times 10^{-18}\ \text{J} = 13.60\ \text{eV}\end{aligned}$$

This allows us to rewrite Eqs. (39.12), (39.13), and (39.14) as

$$K_n = \frac{13.60\ \text{eV}}{n^2} \qquad U_n = \frac{-27.20\ \text{eV}}{n^2} \qquad E_n = \frac{-13.60\ \text{eV}}{n^2}$$

For the first excited level ($n = 2$), we have $K_2 = 3.40$ eV, $U_2 = -6.80$ eV, and $E_2 = -3.40$ eV. For the ground level ($n = 1$), $E_1 = -13.60$ eV. The energy of the emitted photon is then $E_2 - E_1 = -3.40\ \text{eV} - (-13.60\ \text{eV}) = 10.20$ eV, and

$$\begin{aligned}\lambda &= \frac{hc}{E_2 - E_1} = \frac{(4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{10.20\ \text{eV}}\\ &= 1.22 \times 10^{-7}\ \text{m} = 122\ \text{nm}\end{aligned}$$

This is the wavelength of the Lyman-alpha (L_α) line, the longest-wavelength line in the Lyman series of ultraviolet lines in the hydrogen spectrum (see Fig. 39.24).

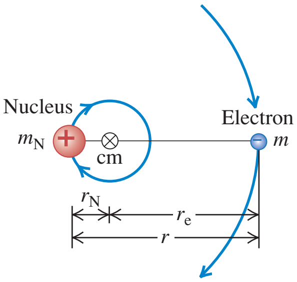
EVALUATE: The total mechanical energy for any level is negative and is equal to one-half the potential energy. We found the same energy relationship for Newtonian satellite orbits in Section 12.4. The situations are similar because both the electrostatic and gravitational forces are inversely proportional to $1/r^2$.

Nuclear Motion and the Reduced Mass of an Atom

The Bohr model is so successful that we can justifiably ask why its predictions for the wavelengths and ionization energy of hydrogen differ from the measured values by about 0.1%. The explanation is that we assumed that the nucleus (a proton) remains at rest. However, as Fig. 39.26 shows, the proton and electron *both* revolve in circular orbits about their common center of mass (see Section 8.5). It turns out that we can take this motion into account very simply by using in Bohr's equations not the electron rest mass m but a quantity called the **reduced mass** m_r of the system. For a system composed of two bodies of masses m_1 and m_2, the reduced mass is

$$m_r = \frac{m_1 m_2}{m_1 + m_2} \qquad (39.17)$$

39.26 The nucleus and the electron both orbit around their common center of mass. The distance r_N has been exaggerated for clarity; for ordinary hydrogen it actually equals $r_e/1836.2$.



For ordinary hydrogen we let m_1 equal m and m_2 equal the proton mass, $m_p = 1836.2m$. Thus the proton–electron system of ordinary hydrogen has a reduced mass of

$$m_r = \frac{m(1836.2m)}{m + 1836.2m} = 0.99946m$$

When this value is used instead of the electron mass m in the Bohr equations, the predicted values agree very well with the measured values.

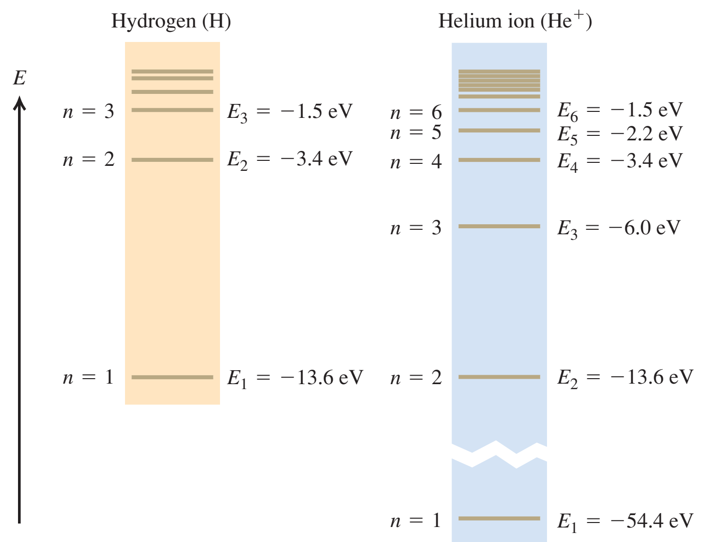
In an atom of deuterium, also called *heavy hydrogen,* the nucleus is not a single proton but a proton and a neutron bound together to form a composite body called the *deuteron.* The reduced mass of the deuterium atom turns out to be $0.99973m$. Equations (39.15) and (39.16) (with m replaced by m_r) show that all wavelengths are inversely proportional to m_r. Thus the wavelengths of the deuterium spectrum should be those of hydrogen divided by $(0.99973m)/(0.99946m) = 1.00027$. This is a small effect but well within the precision of modern spectrometers. This small wavelength shift led the American scientist Harold Urey to the discovery of deuterium in 1932, an achievement that earned him the 1934 Nobel Prize in chemistry.

Hydrogenlike Atoms

We can extend the Bohr model to other one-electron atoms, such as singly ionized helium (He^+), doubly ionized lithium (Li^{2+}), and so on. Such atoms are called *hydrogenlike* atoms. In such atoms, the nuclear charge is not e but Ze, where Z is the *atomic number,* equal to the number of protons in the nucleus. The effect in the previous analysis is to replace e^2 everywhere by Ze^2. In particular, the orbital radii r_n given by Eq. (39.8) become smaller by a factor of Z, and the energy levels E_n given by Eq. (39.14) are multiplied by Z^2. We invite you to verify these statements. The reduced-mass correction in these cases is even less than 0.1% because the nuclei are more massive than the single proton of ordinary hydrogen. Figure 39.27 compares the energy levels for H and for He^+, which has $Z = 2$.

Atoms of the alkali metals (at the far left-hand side of the periodic table; see Appendix D) have one electron outside a core consisting of the nucleus and the inner electrons, with net core charge $+e$. These atoms are approximately hydrogenlike, especially in excited levels. Physicists have studied alkali atoms in which the outer electron has been excited into a very large orbit with $n = 1000$. In accordance with Eq. (39.8), the radius of such a *Rydberg atom* with $n = 1000$ is $n^2 = 10^6$ times the Bohr radius, or about 0.05 mm—about the same size as a small grain of sand.

Although the Bohr model predicted the energy levels of the hydrogen atom correctly, it raised as many questions as it answered. It combined elements of classical physics with new postulates that were inconsistent with classical ideas. The model provided no insight into what happens during a transition from one orbit to another; the angular speeds of the electron motion were not in general the angular frequencies of the emitted radiation, a result that is contrary to classical electrodynamics. Attempts to extend the model to atoms with two or more electrons were not successful. An electron moving in one of Bohr's circular orbits forms a current loop and should produce a magnetic dipole moment (see Section 27.7). However, a hydrogen atom in its ground level has *no* magnetic moment due to orbital motion. In Chapters 40 and 41 we will find that an even more radical departure from classical concepts was needed before the understanding of atomic structure could progress further.



39.27 Energy levels of H and He^+. The energy expression, Eq. (39.14), is multiplied by $Z^2 = 4$ for He^+, so the energy of an He^+ ion with a given n is almost exactly four times that of an H atom with the same n. (There are small differences of the order of 0.05% because the reduced masses are slightly different.)

Test Your Understanding of Section 39.3 Consider the possible transitions between energy levels in a He^+ ion. For which of these transitions in He^+ will the wavelength of the emitted photon be nearly the same as one of the wavelengths emitted by excited H atoms? (i) $n = 2$ to $n = 1$; (ii) $n = 3$ to $n = 2$; (iii) $n = 4$ to $n = 3$; (iv) $n = 4$ to $n = 2$; (v) more than one of these; (vi) none of these.

39.4 The Laser

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PhET: Lasers

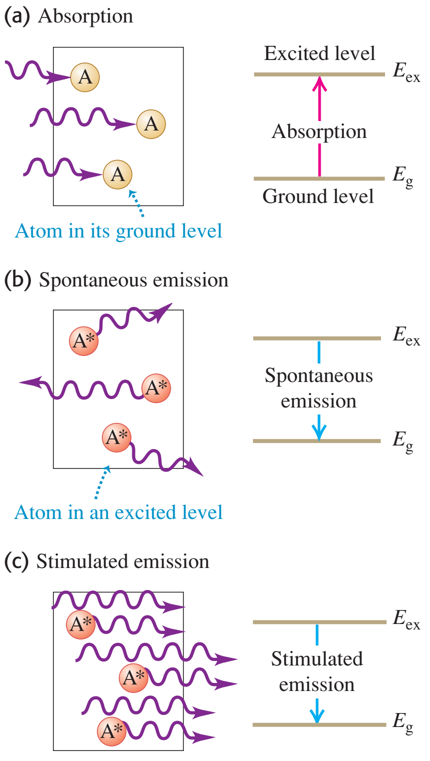
The **laser** is a light source that produces a beam of highly coherent and very nearly monochromatic light as a result of cooperative emission from many atoms. The name "laser" is an acronym for "light amplification by stimulated emission of radiation." We can understand the principles of laser operation from what we have learned about atomic energy levels and photons. To do this we'll have to introduce two new concepts: *stimulated emission* and *population inversion.*

Spontaneous and Stimulated Emission

Consider a gas of atoms in a transparent container. Each atom is initially in its ground level of energy E_g and also has an excited level of energy E_{ex}. If we shine light of frequency f on the container, an atom can absorb one of the photons provided the photon energy $E = hf$ equals the energy difference $E_{ex} - E_g$ between the levels. Figure 39.28a shows this process, in which three atoms A each absorb a photon and go into the excited level. Some time later, the excited atoms (which we denote as A*) return to the ground level by each emitting a photon with the same frequency as the one originally absorbed (Fig. 39.28b). This process is called **spontaneous emission.** The direction and phase of the spontaneously emitted photons are random.

In **stimulated emission** (Fig. 39.28c), each incident photon encounters a previously excited atom. A kind of resonance effect induces each atom to emit a second photon with the same frequency, direction, phase, and polarization as the incident photon, which is not changed by the process. For each atom there is one photon before a stimulated emission and two photons after—thus the name *light amplification.* Because the two photons have the same phase, they emerge together as *coherent* radiation. The laser makes use of stimulated emission to produce a beam consisting of a large number of such coherent photons.

39.28 Three processes in which atoms interact with light.



To discuss stimulated emission from atoms in excited levels, we need to know something about how many atoms are in each of the various energy levels. First, we need to make the distinction between the terms *energy level* and *state.* A system may have more than one way to attain a given energy level; each different way is a different **state.** For instance, there are two ways of putting an ideal unstretched spring in a given energy level. Remembering that the spring potential energy is $U = \frac{1}{2}kx^2$, we could compress the spring by $x = -b$ or we could stretch it by $x = +b$ to get the same $U = \frac{1}{2}kb^2$. The Bohr model had only one state in each energy level, but we will find in Chapter 41 that the hydrogen atom (Fig. 39.24b) actually has two states in its -13.60-eV ground level, eight states in its -3.40-eV first exited level, and so on.

The Maxwell–Boltzmann distribution function (see Section 18.5) determines the number of atoms in a given state in a gas. The function tells us that when the gas is in thermal equilibrium at absolute temperature T, the number n_i of atoms in a state with energy E_i equals $Ae^{-E_i/kT}$, where k is Boltzmann's constant and A is another constant determined by the total number of atoms in the gas. (In Section 18.5, E was the kinetic energy $\frac{1}{2}mv^2$ of a gas molecule; here we're talking about the internal energy of an atom.) Because of the negative exponent, fewer atoms are in higher-energy states, as we should expect. If E_g is a ground-state energy and E_{ex} is the energy of an excited state, then the ratio of numbers of atoms in the two states is

$$\frac{n_{ex}}{n_g} = \frac{Ae^{-E_{ex}/kT}}{Ae^{-E_g/kT}} = e^{-(E_{ex}-E_g)/kT} \tag{39.18}$$

For example, suppose $E_{ex} - E_g = 2.0\text{ eV} = 3.2 \times 10^{-19}\text{ J}$, the energy of a 620-nm visible-light photon. At $T = 3000\text{ K}$ (the temperature of the filament in an incandescent light bulb),

$$\frac{E_{ex} - E_g}{kT} = \frac{3.2 \times 10^{-19}\text{ J}}{(1.38 \times 10^{-23}\text{ J/K})(3000\text{ K})} = 7.73$$

and

$$e^{-(E_{ex}-E_g)/kT} = e^{-7.73} = 0.00044$$

That is, the fraction of atoms in a state 2.0 eV above a ground state is extremely small, even at this high temperature. The point is that at any reasonable temperature there aren't enough atoms in excited states for any appreciable amount of stimulated emission from these states to occur. Rather, a photon emitted by one of the rare excited atoms will almost certainly be absorbed by an atom in the ground state rather than encountering another excited atom.

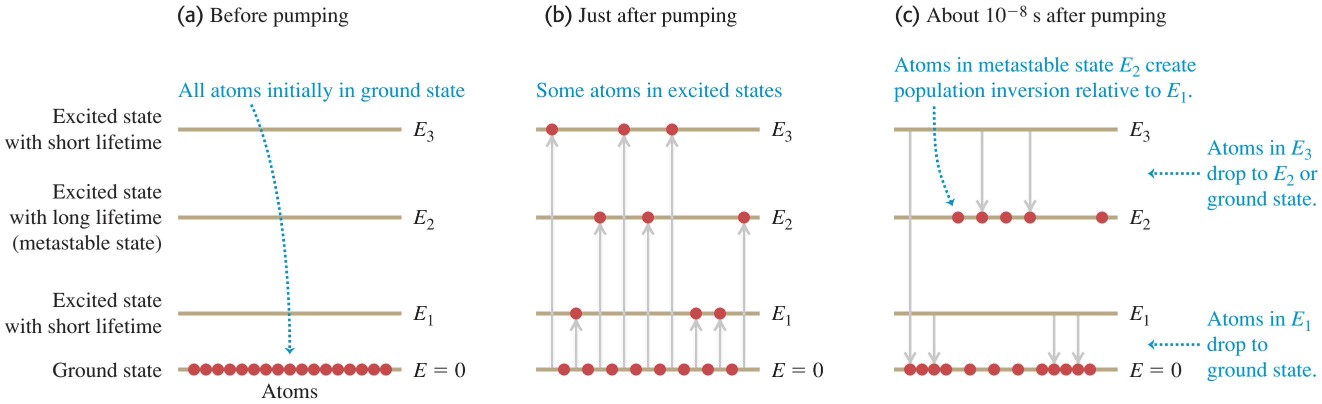
Enhancing Stimulated Emission: Population Inversions

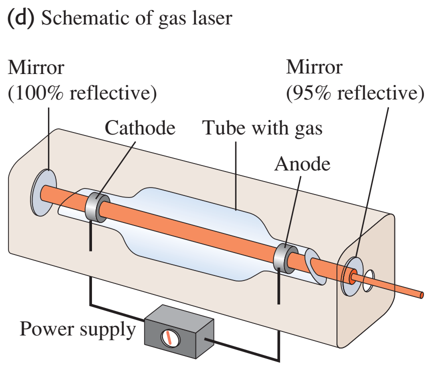
To make a laser, we need to promote stimulated emission by increasing the number of atoms in excited states. Can we do that simply by illuminating the container with radiation of frequency $f = E/h$ corresponding to the energy difference $E = E_{ex} - E_g$, as in Fig. 39.28a? Some of the atoms absorb photons of energy E and are raised to the excited state, and the population ratio n_{ex}/n_g momentarily increases. But because n_g is originally so much larger than n_{ex}, an enormously intense beam of light would be required to momentarily increase n_{ex} to a value comparable to n_g. The rate at which energy is *absorbed* from the beam by the n_g ground-state atoms far exceeds the rate at which energy is added to the beam by stimulated emission from the relatively rare (n_{ex}) excited atoms.

We need to create a *nonequilibrium* situation in which the number of atoms in a higher-energy state is greater than the number in a lower-energy state. Such a situation is called a **population inversion.** Then the rate of energy radiation by stimulated emission can *exceed* the rate of absorption, and the system will act as a net *source* of radiation with photon energy E. It turns out that we can achieve a population inversion by starting with atoms that have the right kinds of excited states. Figure 39.29a shows an energy-level diagram for such an atom with a ground state and *three* excited states of energies E_1, E_2, and E_3. A laser that uses a material with energy levels like these is called a *four-level laser*. For the laser action to work, the states of energies E_1 and E_3 must have ordinary short lifetimes of about 10^{-8} s, while the state of energy E_2 must have an unusually long lifetime of 10^{-3} s or so. Such a long-lived **metastable state** can occur if, for instance, there are restrictions imposed by conservation of angular momentum that hinder photon emission from this state. (We'll discuss these restrictions in Chapter 41.) The metastable state is the one that we want to populate.

To produce a population inversion, we *pump* the material to excite the atoms out of the ground state into the states of energies E_1, E_2, and E_3 (Fig. 39.29b). If the atoms are in a gas, this pumping can be done by inserting two electrodes into the gas container. When a burst of sufficiently high voltage is applied to the electrodes, an electric discharge occurs. Collisions between ionized atoms and electrons carrying the discharge current then excite the atoms to various energy states. Within about 10^{-8} s the atoms that are excited to states E_1 and E_3 undergo spontaneous photon emission, so these states end up depopulated. But atoms "pile up" in the metastable state with energy E_2. The number of atoms in the metastable state is *less* than the number in the ground state, but is *much greater* than in the nearly unoccupied state of energy E_1. Hence there is a population inversion of state E_2 relative to state E_1 (Fig. 39.29c). You can see why we need

39.29 (a), (b), (c) Stages in the operation of a four-level laser. (d) The light emitted by atoms making spontaneous transitions from state E_2 to state E_1 is reflected between mirrors, so it continues to stimulate emission and gives rise to coherent light. One mirror is partially transmitting and allows the high-intensity light beam to escape.





the two levels E_1 and E_3: atoms that undergo spontaneous emission from the E_3 level help to populate the E_2 level, and the presence of the E_1 level makes a population inversion possible.

Over the next 10^{-3} s, some of the atoms in the long-lived metastable state E_2 transition to state E_1 by spontaneous emission. The emitted photons of energy $hf = E_2 - E_1$ are sent back and forth through the gas many times by a pair of parallel mirrors (Fig. 39.29d), so that they can *stimulate* emission from as many of the atoms in state E_2 as possible. The net result of all these processes is a beam of light of frequency f that can be quite intense, has parallel rays, is highly monochromatic, and is spatially *coherent* at all points within a given cross section—that is, a laser beam. One of the mirrors is partially transparent, so a portion of the beam emerges.

What we've described is a *pulsed* laser that produces a burst of coherent light every time the atoms are pumped. Pulsed lasers are used in LASIK eye surgery (an acronym for *laser-assisted in situ keratomileusis*) to reshape the cornea and correct for nearsightedness, farsightedness, or astigmatism. In a *continuous* laser, such as those found in the barcode scanners used at retail checkout counters, energy is supplied to the atoms continuously (for instance, by having the power supply in Fig. 39.29d provide a steady voltage to the electrodes) and a steady beam of light emerges from the laser. For such a laser the pumping must be intense enough to sustain the population inversion, so that the rate at which atoms are added to level E_2 through pumping equals the rate at which atoms in this level emit a photon and transition to level E_1.

Since a special arrangement of energy levels is needed for laser action, it's not surprising that only certain materials can be used to make a laser. Some types of laser use a solid, transparent material such as neodymium glass rather than a gas. The most common kind of laser—used in laser printers (Section 21.2), laser pointers, and to read the data on the disc in a DVD player or Blu-ray player—is a *semiconductor laser,* which doesn't use atomic energy levels at all. As we'll discuss in Chapter 42, these lasers instead use the energy levels of electrons that are free to roam throughout the volume of the semiconductors.

Test Your Understanding of Section 39.4 An ordinary neon light fixture like those used in advertising signs emits red light of wavelength 632.8 nm. Neon atoms are also used in a helium–neon laser (a type of gas laser). The light emitted by a neon light fixture is (i) spontaneous emission; (ii) stimulated emission; (iii) both spontaneous and stimulated emission.

MasteringPHYSICS

PhET: Blackbody Spectrum
PhET: The Greenhouse Effect

39.5 Continuous Spectra

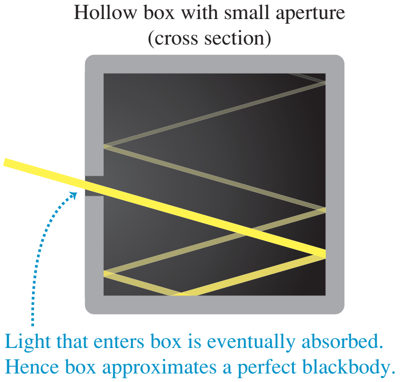
Emission line spectra come from matter in the gaseous state, in which the atoms are so far apart that interactions between them are negligible and each atom behaves as an isolated system. By contrast, a heated solid or liquid (in which atoms are close to each other) nearly always emits radiation with a *continuous* distribution of wavelengths rather than a line spectrum.

Here's an analogy that suggests why there is a difference. A tuning fork emits sound waves of a single definite frequency (a pure tone) when struck. But if you tightly pack a suitcase full of tuning forks and then shake the suitcase, the proximity of the tuning forks to each other affects the sound that they produce. What you hear is mostly noise, which is sound with a continuous distribution of all frequencies. In the same manner, isolated atoms in a gas emit light of certain distinct frequencies when excited, but if the same atoms are crowded together in a solid or liquid they produce a continuous spectrum of light.

In this section we'll study an idealized case of continuous-spectrum radiation from a hot, dense object. Just as was the case for the emission line spectrum of light from an atom, we'll find that we can understand the continuous spectrum only if we use the ideas of energy levels and photons.

In the same way that an atom's emission spectrum has the same lines as its absorption spectrum, the ideal surface for *emitting* light with a continuous spectrum is one that also *absorbs* all wavelengths of electromagnetic radiation. Such an ideal surface is called a *blackbody* because it would appear perfectly black when illuminated; it would reflect no light at all. The continuous-spectrum radiation that a blackbody emits is called **blackbody radiation.** Like a perfectly frictionless incline or a massless rope, a perfect blackbody does not exist but is nonetheless a useful idealization.

39.30 A hollow box with a small aperture behaves like a blackbody. When the box is heated, the electromagnetic radiation that emerges from the aperture has a blackbody spectrum.

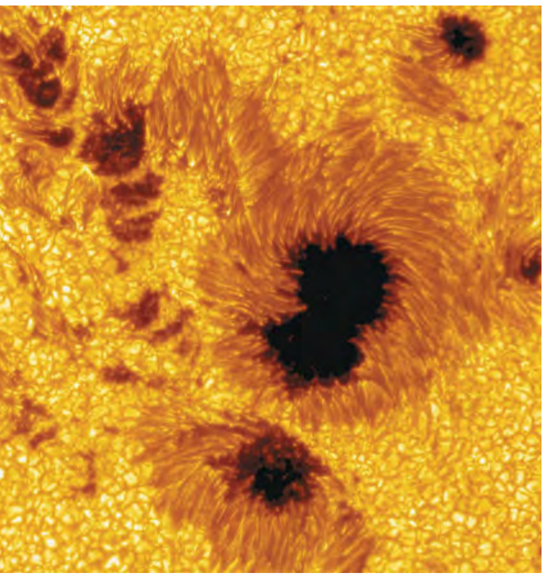


A good approximation to a blackbody is a hollow box with a small aperture in one wall (Fig. 39.30). Light that enters the aperture will eventually be absorbed by the walls of the box, so the box is a nearly perfect absorber. Conversely, when we heat the box, the light that emanates from the aperture is nearly ideal blackbody radiation with a continuous spectrum.

By 1900 blackbody radiation had been studied extensively, and three characteristics had been established. First, the total intensity I (the average rate of radiation of energy per unit surface area or average power per area) emitted from the surface of an ideal radiator is proportional to the fourth power of the absolute temperature (Fig. 39.31). We studied this relationship in Section 17.7 during our study of heat-transfer mechanisms. This total intensity I emitted at absolute temperature T is given by the **Stefan–Boltzmann law:**

$$I = \sigma T^4 \qquad \text{(Stefan–Boltzmann law for a blackbody)} \tag{39.19}$$

39.31 This close-up view of the sun's surface shows two dark sunspots. Their temperature is about 4000 K, while the surrounding solar material is at $T =$ 5800 K. From the Stefan–Boltzmann law, the intensity from a given area of sunspot is only $(4000\text{ K}/5800\text{ K})^4 = 0.23$ as great as the intensity from the same area of the surrounding material—which is why sunspots appear dark.



where σ is a fundamental physical constant called the *Stefan–Boltzmann constant.* In SI units,

$$\sigma = 5.670400(40) \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$$

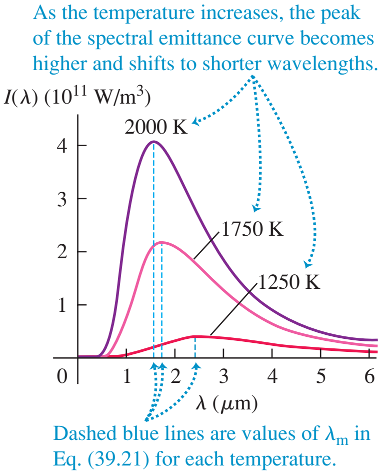
Second, the intensity is not uniformly distributed over all wavelengths. Its distribution can be measured and described by the intensity per wavelength interval $I(\lambda)$, called the *spectral emittance.* Thus $I(\lambda)\,d\lambda$ is the intensity corresponding to wavelengths in the interval from λ to $\lambda + d\lambda$. The *total* intensity I, given by Eq. (39.19), is the *integral* of the distribution function $I(\lambda)$ over all wavelengths, which equals the area under the $I(\lambda)$ versus λ curve:

$$I = \int_0^\infty I(\lambda)\,d\lambda \tag{39.20}$$

CAUTION **Spectral emittance vs. intensity** Although we use the symbol $I(\lambda)$ for spectral emittance, keep in mind that spectral emittance is *not* the same thing as intensity I. Intensity is power per unit area, with units W/m^2. Spectral emittance is power per unit area *per unit wavelength interval*, with units W/m^3.

Figure 39.32 shows the measured spectral emittances $I(\lambda)$ for blackbody radiation at three different temperatures. Each has a peak wavelength λ_m at which the emitted intensity per wavelength interval is largest. Experiment shows that λ_m is inversely proportional to T, so their product is constant. This observation is called the **Wien displacement law.** The experimental value of the constant is $2.90 \times 10^{-3}\ \text{m}\cdot\text{K}$:

$$\lambda_m T = 2.90 \times 10^{-3}\ \text{m}\cdot\text{K} \qquad \text{(Wien displacement law)} \tag{39.21}$$



39.32 These graphs show the spectral emittance $I(\lambda)$ for radiation from a blackbody at three different temperatures.

As the temperature rises, the peak of $I(\lambda)$ becomes higher and shifts to shorter wavelengths. Yellow light has shorter wavelengths than red light, so a body that glows yellow is hotter and brighter than one of the same size that glows red.

Third, experiments show that the *shape* of the distribution function is the same for all temperatures. We can make a curve for one temperature fit any other temperature by simply changing the scales on the graph.

Rayleigh and the "Ultraviolet Catastrophe"

During the last decade of the 19th century, many attempts were made to derive these empirical results about blackbody radiation from basic principles. In one attempt, the English physicist Lord Rayleigh considered the light enclosed within a rectangular box like that shown in Fig. 39.30. Such a box, he reasoned, has a series of possible *normal modes* for electromagnetic waves, as we discussed in Section 32.5. It also seemed reasonable to assume that the distribution of energy among the various modes would be given by the equipartition principle (see Section 18.4), which had been used successfully in the analysis of heat capacities.

Including both the electric- and magnetic-field energies, Rayleigh assumed that the total energy of each normal mode was equal to kT. Then by computing the *number* of normal modes corresponding to a wavelength interval $d\lambda$, Rayleigh calculated the expected distribution of wavelengths in the radiation within the box. Finally, he computed the predicted intensity distribution $I(\lambda)$ for the radiation emerging from the hole. His result was quite simple:

$$I(\lambda) = \frac{2\pi ckT}{\lambda^4} \qquad \text{(Rayleigh's calculation)} \tag{39.22}$$

At large wavelengths this formula agrees quite well with the experimental results shown in Fig. 39.32, but there is serious disagreement at small wavelengths. The experimental curves in Fig. 39.32 fall toward zero at small λ. By contrast, Rayleigh's prediction in Eq. (39.22) goes in the opposite direction, approaching infinity at $1/\lambda^4$, a result that was called in Rayleigh's time the "ultraviolet catastrophe." Even worse, the integral of Eq. (39.22) over all λ is infinite, indicating an infinitely large *total* radiated intensity. Clearly, something is wrong.

Planck and the Quantum Hypothesis

Finally, in 1900, the German physicist Max Planck succeeded in deriving a function, now called the **Planck radiation law,** that agreed very well with experimental intensity distribution curves. In his derivation he made what seemed at the time to be a crazy assumption. Planck assumed that electromagnetic oscillators (electrons) in the walls of Rayleigh's box vibrating at a frequency f could have only certain values of energy equal to nhf, where $n = 0, 1, 2, 3, \ldots$ and h turned out to be the constant that now bears Planck's name. These oscillators were in equilibrium with the electromagnetic waves in the box, so they both emitted and absorbed light. His assumption gave quantized energy levels and said that the energy in each normal mode was also a multiple of hf. This was in sharp contrast to Rayleigh's point of view that each normal mode could have any amount of energy.

Planck was not comfortable with this quantum hypothesis; he regarded it as a calculational trick rather than a fundamental principle. In a letter to a friend, he called it "an act of desperation" into which he was forced because "a theoretical explanation had to be found at any cost, whatever the price." But five years later, Einstein identified the energy change hf between levels as the energy of a photon to explain the photoelectric effect (see Section 38.1), and other evidence quickly mounted. By 1915 there was little doubt about the validity of the quantum concept and the existence of photons. By discussing atomic spectra *before* continuous spectra, we have departed from the historical order of things. The credit for inventing the concept of quantization of energy levels goes to Planck, even though he didn't believe it at first. He received the 1918 Nobel Prize in physics for his achievements.

39.33 Energy levels for two of the oscillators that Planck envisioned in the walls of a blackbody like that shown in Fig. 39.30. The spacing between adjacent energy levels for each oscillator is hf, which is smaller for the low-frequency oscillator.

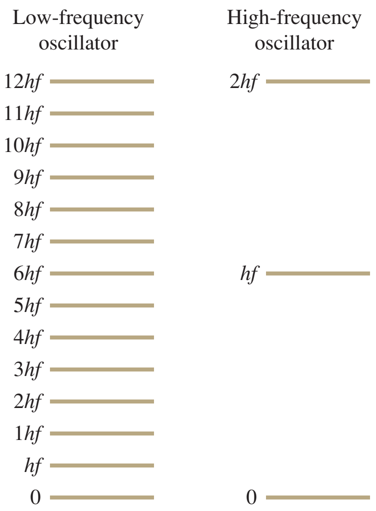


Figure 39.33 shows energy-level diagrams for two of the oscillators that Planck envisioned in the walls of the rectangular box, one with a low frequency and the other with a high frequency. The spacing in energy between adjacent levels is hf. This spacing is small for the low-frequency oscillator that emits and absorbs photons of low frequency f and long wavelength $\lambda = c/f$. The energy spacing is greater for the high-frequency oscillator, which emits high-frequency photons of short wavelength.

According to Rayleigh's picture, both of these oscillators have the same amount of energy kT and are equally effective at emitting radiation. In Planck's model, however, the high-frequency oscillator is very ineffective as a source of light. To see why, we can use the ideas from Section 39.4 about the populations of various energy states. If we consider all the oscillators of a given frequency f in a box at temperature T, the number of oscillators that have energy nhf is $Ae^{-nhf/kT}$. The ratio of the number of oscillators in the first excited state ($n = 1$, energy hf) to the number of oscillators in the ground state ($n = 0$, energy zero) is

$$\frac{n_1}{n_0} = \frac{Ae^{-hf/kT}}{Ae^{-(0)/kT}} = e^{-hf/kT} \qquad (39.23)$$

Let's evaluate Eq. (39.23) for $T = 2000$ K, one of the temperatures shown in Fig. 39.32. At this temperature $kT = 2.76 \times 10^{-20}$ J $= 0.172$ eV. For an oscillator that emits photons of wavelength $\lambda = 3.00\ \mu\text{m}$, we can show that $hf = hc/\lambda = 0.413$ eV; for a higher-frequency oscillator that emits photons of wavelength $\lambda = 0.500\ \mu\text{m}$, $hf = hc/\lambda = 2.48$ eV. For these two cases Eq. (39.23) gives

$$\frac{n_1}{n_0} = e^{-hf/kT} = 0.0909 \text{ for } \lambda = 3.00\ \mu\text{m}$$

$$\frac{n_1}{n_0} = e^{-hf/kT} = 5.64 \times 10^{-7} \text{ for } \lambda = 0.500\ \mu\text{m}$$

The value for $\lambda = 3.00\ \mu\text{m}$ means that of all the oscillators that can emit light at this wavelength, 0.0909 of them—about one in 11—are in the first excited state. These excited oscillators can each emit a 3.00-μm photon and contribute it to the radiation inside the box. Hence we would expect that this radiation would be rather plentiful in the spectrum of radiation from a 2000 K blackbody. By contrast, the value for $\lambda = 0.500\ \mu\text{m}$ means that only 5.64×10^{-7} (about one in two million) of the oscillators that can emit this wavelength are in the first excited state. An oscillator can't emit if it's in the ground state, so the amount of radiation in the box at this wavelength is *tremendously* suppressed compared to Rayleigh's prediction. That's why the spectral emittance curve for 2000 K in Fig. 39.32 has such a low value at $\lambda = 0.500\ \mu\text{m}$ and shorter wavelengths. So Planck's quantum hypothesis provided a natural way to suppress the spectral emittance of a blackbody at short wavelengths, and hence averted the ultraviolet catastrophe that plagued Rayleigh's calculations.

We won't go into all the details of Planck's derivation of the spectral emittance. Here is his result:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)} \qquad \text{(Planck radiation law)} \tag{39.24}$$

where h is Planck's constant, c is the speed of light, k is Boltzmann's constant, T is the absolute temperature, and λ is the wavelength. This function turns out to agree well with experimental emittance curves such as those in Fig. 39.32.

The Planck radiation law also contains the Wien displacement law and the Stefan–Boltzmann law as consequences. To derive the Wien law, we find the value of λ at which $I(\lambda)$ is maximum by taking the derivative of Eq. (39.24) and setting it equal to zero. We leave it to you to fill in the details; the result is

$$\lambda_{\text{m}} = \frac{hc}{4.965kT} \tag{39.25}$$

To obtain this result, you have to solve the equation

$$5 - x = 5e^{-x} \tag{39.26}$$

The root of this equation, found by trial and error or more sophisticated means, is 4.965 to four significant figures. You should evaluate the constant $hc/4.965k$ and show that it agrees with the experimental value of $2.90 \times 10^{-3}\ \text{m}\cdot\text{K}$ given in Eq. (39.21).

We can obtain the Stefan–Boltzmann law for a blackbody by integrating Eq. (39.24) over all λ to find the *total* radiated intensity (see Problem 39.67). This is not a simple integral; the result is

$$I = \int_0^\infty I(\lambda)\, d\lambda = \frac{2\pi^5 k^4}{15c^2h^3}T^4 = \sigma T^4 \tag{39.27}$$

in agreement with Eq. (39.19). Our result in Eq. (39.27) also shows that the constant σ in that law can be expressed as a combination of other fundamental constants:

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} \tag{39.28}$$

You should substitute the values of k, c, and h from Appendix F and verify that you obtain the value $\sigma = 5.6704 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$ for the Stefan–Boltzmann constant.

The Planck radiation law, Eq. (39.24), looks so different from the unsuccessful Rayleigh expression, Eq. (39.22), that it may seem unlikely that they would agree at large values of λ. But when λ is large, the exponent in the denominator of Eq. (39.24) is very small. We can then use the approximation $e^x \approx 1 + x$ (for $x = 1$). You should verify that when this is done, the result approaches Eq. (39.22), showing that the two expressions do agree in the limit of very large λ. We also note that the Rayleigh expression does not contain h. At very long wavelengths (very small photon energies), quantum effects become unimportant.

Example 39.7 Light from the sun

To a good approximation, the sun's surface is a blackbody with a surface temperature of 5800 K. (We are ignoring the absorption produced by the sun's atmosphere, shown in Fig. 39.9.) (a) At what wavelength does the sun emit most strongly? (b) What is the total radiated power per unit surface area?

SOLUTION

IDENTIFY and SET UP: Our target variables are the peak-intensity wavelength λ_m and the radiated power per area I. Hence we'll use the Wien displacement law, Eq. (39.21) (which relates λ_m to the blackbody temperature T), and the Stefan–Boltzmann law, Eq. (39.19) (which relates I to T).

EXECUTE: (a) From Eq. (39.21),

$$\lambda_m = \frac{2.90 \times 10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.90 \times 10^{-3}\ \text{m}\cdot\text{K}}{5800\ \text{K}}$$
$$= 0.500 \times 10^{-6}\ \text{m} = 500\ \text{nm}$$

(b) From Eq. (39.19),

$$I = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(5800\ \text{K})^4$$
$$= 6.42 \times 10^7\ \text{W/m}^2 = 64.2\ \text{MW/m}^2$$

EVALUATE: The 500-nm wavelength found in part (a) is near the middle of the visible spectrum. This should not be a surprise: The human eye evolved to take maximum advantage of natural light.

The enormous value $I = 64.2\ \text{MW/m}^2$ found in part (b) is the intensity at the *surface* of the sun, a sphere of radius 6.96×10^8 m. When this radiated energy reaches the earth, 1.50×10^{11} m away, the intensity has decreased by the factor $[(6.96 \times 10^8\ \text{m})/(1.50 \times 10^{11}\ \text{m})]^2 = 2.15 \times 10^{-5}$ to the still-impressive 1.4 kW/m^2.

Example 39.8 A slice of sunlight

Find the power per unit area radiated from the sun's surface in the wavelength range 600.0 to 605.0 nm.

SOLUTION

IDENTIFY and SET UP: This question concerns the power emitted by a blackbody over a narrow range of wavelengths, and so involves the spectral emittance $I(\lambda)$ given by the Planck radiation law, Eq. (39.24). This requires that we find the area under the $I(\lambda)$ curve between 600.0 and 605.0 nm. We'll *approximate* this area as the product of the height of the curve at the median wavelength $\lambda = 602.5$ nm and the width of the interval, $\Delta\lambda = 5.0$ nm. From Example 39.7, $T = 5800$ K.

EXECUTE: To obtain the height of the $I(\lambda)$ curve at $\lambda = 602.5\ \text{nm} = 6.025 \times 10^{-7}$ m, we first evaluate the quantity $hc/\lambda kT$ in Eq. (39.24) and then substitute the result into Eq. (39.24):

$$\frac{hc}{\lambda kT} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})}{(6.025 \times 10^{-7}\ \text{m})(1.381 \times 10^{-23}\ \text{J/K})(5800\ \text{K})} = 4.116$$

$$I(\lambda) = \frac{2\pi(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})^2}{(6.025 \times 10^{-7}\ \text{m})^5(e^{4.116} - 1)}$$
$$= 7.81 \times 10^{13}\ \text{W/m}^3$$

The intensity in the 5.0-nm range from 600.0 to 605.0 nm is then approximately

$$I(\lambda)\Delta\lambda = (7.81 \times 10^{13}\ \text{W/m}^3)(5.0 \times 10^{-9}\ \text{m})$$
$$= 3.9 \times 10^5\ \text{W/m}^2 = 0.39\ \text{MW/m}^2$$

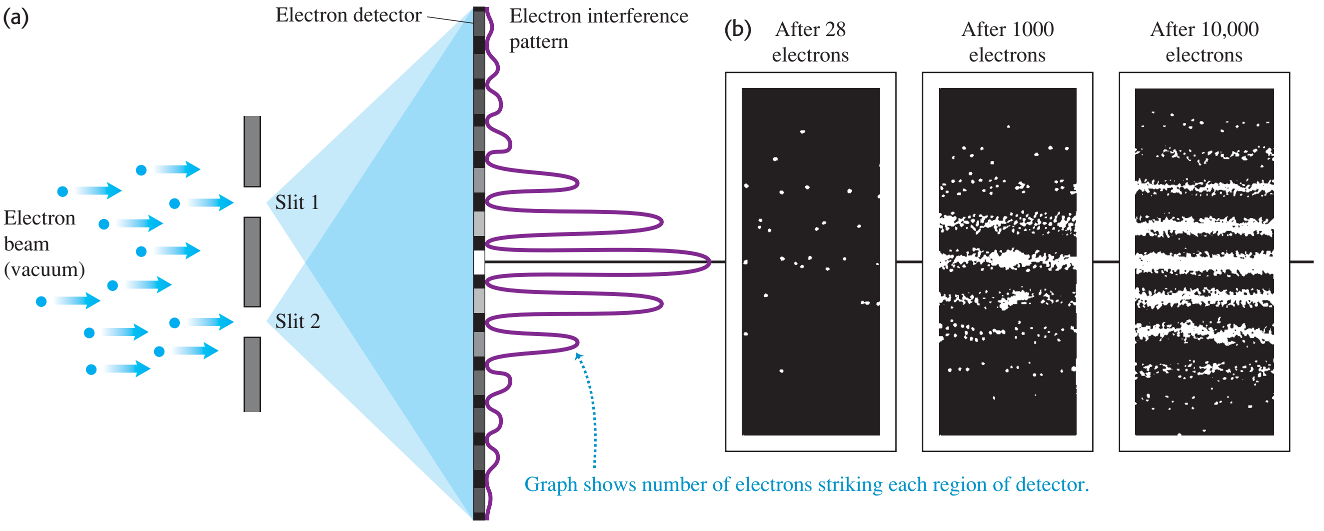
EVALUATE: In part (b) of Example 39.7, we found the power radiated per unit area by the sun at *all* wavelengths to be $I = 64.2\ \text{MW/m}^2$; here we have found that the power radiated per unit area in the wavelength range from 600 to 605 nm is $I(\lambda)\Delta\lambda = 0.39\ \text{MW/m}^2$, about 0.6% of the total.

Test Your Understanding of Section 39.5 (a) Does a blackbody at 2000 K emit x rays? (b) Does it emit radio waves?

39.6 The Uncertainty Principle Revisited

The discovery of the dual wave–particle nature of matter forces us to reevaluate the kinematic language we use to describe the position and motion of a particle. In classical Newtonian mechanics we think of a particle as a point. We can describe its location and state of motion at any instant with three spatial coordinates and three components of velocity. But because matter also has a wave aspect, when we look at the behavior on a small enough scale—comparable to the de Broglie wavelength of the particle—we can no longer use the Newtonian description. Certainly no Newtonian particle would undergo diffraction like electrons do (Section 39.1).

39.34 (a) A two-slit interference experiment for electrons. (b) The interference pattern after 28, 1000, and 10,000 electrons.



To demonstrate just how non-Newtonian the behavior of matter can be, let's look at an experiment involving the two-slit interference of electrons (Fig. 39.34). We aim an electron beam at two parallel slits, just as we did for light in Section 38.4. (The electron experiment has to be done in vacuum so that the electrons don't collide with air molecules.) What kind of pattern appears on the detector on the other side of the slits? The answer is: *exactly the same* kind of interference pattern we saw for photons in Section 38.4! Moreover, the principle of complementarity, which we introduced in Section 38.4, tells us that we cannot apply the wave and particle models simultaneously to describe any single element of this experiment. Thus we *cannot* predict exactly where in the pattern (a wave phenomenon) any individual electron (a particle) will land. We can't even ask which slit an individual electron passes through. If we tried to look at where the electrons were going by shining a light on them—that is, by scattering photons off them—the electrons would recoil, which would modify their motions so that the two-slit interference pattern would not appear.

CAUTION **Electron two-slit interference is not interference between two electrons** It's a common misconception that the pattern in Fig. 39.34b is due to the interference between *two* electron waves, each representing an electron passing through one slit. To show that this cannot be the case, we can send just one electron at a time through the apparatus. It makes no difference; we end up with the same interference pattern. In a sense, each electron wave interferes with itself.

The Heisenberg Uncertainty Principles for Matter

Just as electrons and photons show the same behavior in a two-slit interference experiment, electrons and other forms of matter obey the same Heisenberg uncertainty principles as photons do:

$$\begin{aligned} \Delta x \Delta p_x &\geq \hbar/2 \\ \Delta y \Delta p_y &\geq \hbar/2 \\ \Delta z \Delta p_z &\geq \hbar/2 \end{aligned} \qquad \text{(Heisenberg uncertainty principle for position and momentum)} \qquad (39.29)$$

$$\Delta t\, \Delta E \geq \hbar/2 \qquad \text{(Heisenberg uncertainty principle for energy and time interval)} \qquad (39.30)$$

In these equations $\hbar = h/2\pi = 1.055 \times 10^{-34}\ \mathrm{J \cdot s}$. The uncertainty principle for energy and time interval has a direct application to energy levels. We have assumed that each energy level in an atom has a very definite energy. However, Eq. (39.30) says that this is not true for all energy levels. A system that remains in a metastable state for a very long time (large Δt) can have a very well-defined energy (small ΔE), but if it remains in a state for only a short time (small Δt) the uncertainty in energy must be correspondingly greater (large ΔE). Figure 39.35 illustrates this idea.

Example 39.9 The uncertainty principle: position and momentum

An electron is confined within a region of width 5.000×10^{-11} m (roughly the Bohr radius). (a) Estimate the minimum uncertainty in the x-component of the electron's momentum. (b) What is the kinetic energy of an electron with this magnitude of momentum? Express your answer in both joules and electron volts.

SOLUTION

IDENTIFY and SET UP: This problem uses the Heisenberg uncertainty principle for position and momentum and the relationship between a particle's momentum and its kinetic energy. The electron could be anywhere within the region, so we take $\Delta x = 5.000 \times 10^{-11}$ m as its position uncertainty. We then find the momentum uncertainty Δp_x using Eq. (39.29) and the kinetic energy using the relationships $p = mv$ and $K = \frac{1}{2}mv^2$.

EXECUTE: (a) From Eqs. (39.29), for a given value of Δx, the uncertainty in momentum is minimum when the product $\Delta x \Delta p_x$ equals $\hbar$. Hence

$$\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \mathrm{J \cdot s}}{2(5.000 \times 10^{-11}\ \mathrm{m})} = 1.055 \times 10^{-24}\ \mathrm{J \cdot s/m}$$
$$= 1.055 \times 10^{-24}\ \mathrm{kg \cdot m/s}$$

(b) We can rewrite the nonrelativistic expression for kinetic energy as

$$K = \tfrac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

Hence an electron with a magnitude of momentum equal to Δp_x from part (a) has kinetic energy

$$K = \frac{p^2}{2m} = \frac{(1.055 \times 10^{-24}\ \mathrm{kg \cdot m/s})^2}{2(9.11 \times 10^{-31}\ \mathrm{kg})}$$
$$= 6.11 \times 10^{-19}\ \mathrm{J} = 3.81\ \mathrm{eV}$$

EVALUATE: This energy is typical of electron energies in atoms. This agreement suggests that the uncertainty principle is deeply involved in atomic structure.

A similar calculation explains why electrons in atoms do not fall into the nucleus. If an electron were confined to the interior of a nucleus, its position uncertainty would be $\Delta x \approx 10^{-14}$ m. This would give the electron a momentum uncertainty about 5000 times greater than that of the electron in this example, and a kinetic energy so great that the electron would immediately be ejected from the nucleus.

Example 39.10 The uncertainty principle: energy and time

A sodium atom in one of the states labeled "Lowest excited levels" in Fig. 39.19a remains in that state, on average, for 1.6×10^{-8} s before it makes a transition to the ground level, emitting a photon with wavelength 589.0 nm and energy 2.105 eV. What is the uncertainty in energy of that excited state? What is the wavelength spread of the corresponding spectral line?

SOLUTION

IDENTIFY and SET UP: This problem uses the Heisenberg uncertainty principle for energy and time interval and the relationship between photon energy and wavelength. The average time that the atom spends in this excited state is equal to Δt in Eq. (39.30). We find the minimum uncertainty in the energy of the excited level by replacing the $\geq$ sign in Eq. (39.30) with an equals sign and solving for ΔE.

EXECUTE: From Eq. (39.30),

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{1.055 \times 10^{-34}\ \mathrm{J \cdot s}}{2(1.6 \times 10^{-8}\ \mathrm{s})}$$
$$= 3.3 \times 10^{-27}\ \mathrm{J} = 2.1 \times 10^{-8}\ \mathrm{eV}$$

The atom remains in the ground level indefinitely, so that level has *no* associated energy uncertainty. The fractional uncertainty of the *photon* energy is therefore

$$\frac{\Delta E}{E} = \frac{2.1 \times 10^{-8}\ \mathrm{eV}}{2.105\ \mathrm{eV}} = 1.0 \times 10^{-8}$$

You can use some simple calculus and the relation $E = hc/\lambda$ to show that $\Delta\lambda/\lambda \approx \Delta E/E$, so that the corresponding spread in wavelength, or "width," of the spectral line is approximately

$$\Delta\lambda = \lambda\frac{\Delta E}{E} = (589.0\ \mathrm{nm})(1.0 \times 10^{-8}) = 0.0000059\ \mathrm{nm}$$

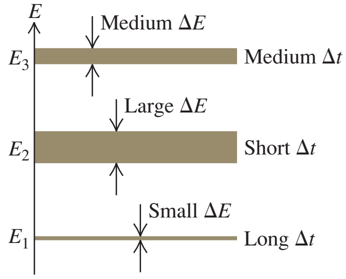
EVALUATE: This irreducible uncertainty $\Delta\lambda$ is called the *natural line width* of this particular spectral line. Though very small, it is within the limits of resolution of present-day spectrometers. Ordinarily, the natural line width is much smaller than the line width arising from other causes such as the Doppler effect and collisions among the rapidly moving atoms.

The Uncertainty Principle and the Limits of the Bohr Model

We saw in Section 39.3 that the Bohr model of the hydrogen atom was tremendously successful. However, the Heisenberg uncertainty principle for position and momentum shows that this model *cannot* be a correct description of how an electron in an atom behaves. Figure 39.22 shows that in the Bohr model as interpreted by de Broglie, an electron wave moves in a plane around the nucleus. Let's call this the xy-plane, so the z-axis is perpendicular to the plane. Hence the Bohr model says that an electron is always found at $z = 0$, and its z-momentum p_z is always zero (the electron does not move out of the xy-plane). But this implies that there are *no* uncertainties in either z or p_z, so $\Delta z = 0$ and $\Delta p_z = 0$. This directly contradicts Eq. (39.29), which says that the product $\Delta z \Delta p_z$ must be greater than or equal to $\hbar$.

This conclusion isn't too surprising, since the electron in the Bohr model is a mix of particle and wave ideas (the electron moves in an orbit like a miniature planet, but has a wavelength). To get an accurate picture of how electrons behave inside an atom and elsewhere, we need a description that is based *entirely* on the electron's wave properties. Our goal in Chapter 40 will be to develop this description, which we call *quantum mechanics*. To do this we'll introduce the *Schrödinger equation*, the fundamental equation that describes the dynamics of matter waves. This equation, as we will see, is as fundamental to quantum mechanics as Newton's laws are to classical mechanics or as Maxwell's equations are to electromagnetism.

39.35 The longer the lifetime Δt of a state, the smaller is its spread in energy (shown by the width of the energy levels).



Test Your Understanding of Section 39.6 Rank the following situations according to the uncertainty in x-momentum, from largest to smallest. The mass of the proton is 1836 times the mass of the electron. (i) an electron whose x-coordinate is known to within 2×10^{-15} m; (ii) an electron whose x-coordinate is known to within 4×10^{-15} m; (iii) a proton whose x-coordinate is known to within 2×10^{-15} m; (iv) a proton whose x-coordinate is known to within 4×10^{-15} m.

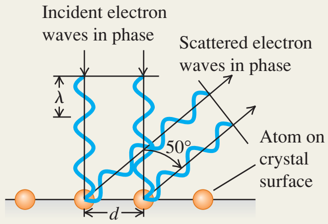
CHAPTER 39 SUMMARY

De Broglie waves and electron diffraction: Electrons and other particles have wave properties. A particle's wavelength depends on its momentum in the same way as for photons. A nonrelativistic electron accelerated from rest through a potential difference V_{ba} has a wavelength given by Eq. (39.3). Electron microscopes use the very small wavelengths of fast-moving electrons to make images with resolution thousands of times finer than is possible with visible light. (See Examples 39.1–39.3.)

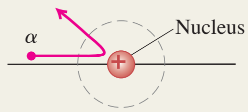
$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad (39.1)$$

$$E = hf \qquad (39.2)$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ba}}} \qquad (39.3)$$

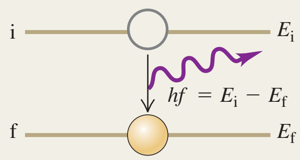


The nuclear atom: The Rutherford scattering experiments show that most of an atom's mass and all of its positive charge are concentrated in a tiny, dense nucleus at the center of the atom. (See Example 39.4.)



Atomic line spectra and energy levels: The energies of atoms are quantized: They can have only certain definite values, called energy levels. When an atom makes a transition from an energy level E_i to a lower level E_f, it emits a photon of energy $E_\mathrm{i} - E_\mathrm{f}$. The same photon can be absorbed by an atom in the lower energy level, which excites the atom to the upper level. (See Example 39.5.)

$$hf = \frac{hc}{\lambda} = E_\mathrm{i} - E_\mathrm{f} \qquad (39.5)$$



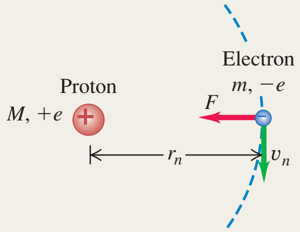
The Bohr model: In the Bohr model of the hydrogen atom, the permitted values of angular momentum are integral multiples of $h/2\pi$. The integer multiplier n is called the principal quantum number for the level. The orbital radii are proportional to n^2 and the orbital speeds are proportional to $1/n$. The energy levels of the hydrogen atom are given by Eq. (39.15), where R is the Rydberg constant. (See Example 39.6.)

$$L_n = mv_n r_n = n\frac{h}{2\pi}$$
$$(n = 1, 2, 3, \dots) \qquad (39.6)$$

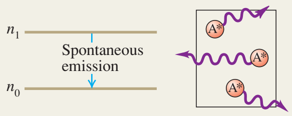
$$r_n = \epsilon_0 \frac{n^2h^2}{\pi m e^2} = n^2 a_0 \qquad (39.8)$$
$$= n^2(5.29 \times 10^{-11}\ \mathrm{m}) \qquad (39.10)$$

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh} = \frac{2.19 \times 10^6\ \mathrm{m/s}}{n} \qquad (39.9)$$

$$E_n = -\frac{hcR}{n^2} = -\frac{13.60\ \mathrm{eV}}{n^2} \qquad (39.15)$$
$$(n = 1, 2, 3, \dots)$$



The laser: The laser operates on the principle of stimulated emission, by which many photons with identical wavelength and phase are emitted. Laser operation requires a nonequilibrium condition called a population inversion, in which more atoms are in a higher-energy state than are in a lower-energy state.

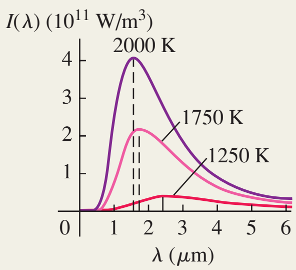


Blackbody radiation: The total radiated intensity (average power radiated per area) from a blackbody surface is proportional to the fourth power of the absolute temperature T. The quantity $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$ is called the Stefan–Boltzmann constant. The wavelength λ_m at which a blackbody radiates most strongly is inversely proportional to T. The Planck radiation law gives the spectral emittance $I(\lambda)$ (intensity per wavelength interval in blackbody radiation). (See Examples 39.7 and 39.8.)

$$I = \sigma T^4$$
(Stefan–Boltzmann law) (39.19)

$$\lambda_\mathrm{m} T = 2.90 \times 10^{-3}\ \mathrm{m \cdot K}$$
(Wien displacement law) (39.21)

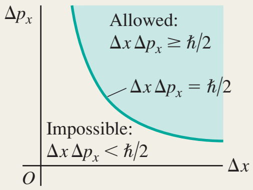
$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}$$
(Planck radiation law) (39.24)



The Heisenberg uncertainty principle for particles: The same uncertainty considerations that apply to photons also apply to particles such as electrons. The uncertainty ΔE in the energy of a state that is occupied for a time Δt is given by Eq. (39.30). (See Examples 39.9 and 39.10.)

$$\Delta x \Delta p_x \geq \hbar/2$$
$$\Delta y \Delta p_y \geq \hbar/2 \quad \text{(Heisenberg uncertainty principle for position and momentum)} \quad (39.29)$$
$$\Delta z \Delta p_z \geq \hbar/2$$

$$\Delta t \Delta E \geq \hbar/2 \quad \text{(Heisenberg uncertainty principle for energy and time interval)} \quad (39.30)$$



BRIDGING PROBLEM Hot Stars and Hydrogen Clouds

Figure 39.36 shows a cloud, or *nebula,* of glowing hydrogen in interstellar space. The atoms in this cloud are excited by short-wavelength radiation emitted by the bright blue stars at the center of the nebula. (a) The blue stars act as blackbodies and emit light with a continuous spectrum. What is the wavelength at which a star with a surface temperature of 15,100 K (about $2\frac{1}{2}$ times the surface temperature of the sun) has the maximum spectral emittance? In what region of the electromagnetic spectrum is this? (b) Figure 39.32 shows that most of the energy radiated by a blackbody is at wavelengths between about one half and three times the wavelength of maximum emittance. If a hydrogen atom near the star in part (a) is initially in the ground level, what is the principal quantum number of the highest energy level to which it could be excited by a photon in this wavelength range? (c) The red color of the nebula is primarily due to hydrogen atoms making a transition from $n = 3$ to $n = 2$ and emitting photons of wavelength 656.3 nm. In the Bohr model as interpreted by de Broglie, what are the *electron* wavelengths in the $n = 2$ and $n = 3$ levels?

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.

39.36 The Rosette Nebula.

IDENTIFY and SET UP

1. To solve this problem you need to use your knowledge of both blackbody radiation (Section 39.5) and the Bohr model of the hydrogen atom (Section 39.3).
2. In part (a) the target variable is the wavelength at which the star emits most strongly; in part (b) the target variable is a principal quantum number, and in part (c) it is the de Broglie wavelength of an electron in the $n = 2$ and $n = 3$ Bohr orbits (see Fig. 39.24). Select the equations you will need to find the target variables. (*Hint:* In Section 39.5 you learned how to find the energy change involved in a transition between two given levels of a hydrogen atom. Part (b) is a variation on this: You are to find the final level in a transition that starts in the $n = 1$ level and involves the absorption of a photon of a given wavelength and hence a given energy.)

EXECUTE

3. Use the Wien displacement law to find the wavelength at which the star has maximum spectral emittance. In what part of the electromagnetic spectrum is this wavelength?
4. Use your result from step 3 to find the range of wavelengths in which the star radiates most of its energy. Which end of this range corresponds to a photon with the greatest energy?
5. Write an expression for the wavelength of a photon that must be absorbed to cause an electron transition from the ground level ($n = 1$) to a higher level n. Solve for the value of n that corresponds to the highest-energy photon in the range you calculated in step 4. (*Hint:* Remember than n must be an integer.)
6. Find the electron wavelengths that correspond to the $n = 2$ and $n = 3$ orbits shown in Fig. 39.22.

EVALUATE

7. Check your result in step 5 by calculating the wavelength needed to excite a hydrogen atom from the ground level into the level *above* the highest-energy level that you found in step 5. Is it possible for light in the range of wavelengths you found in step 4 to excite hydrogen atoms from the ground level into this level?
8. How do the electron wavelengths you found in step 6 compare to the wavelength of a *photon* emitted in a transition from the $n = 3$ level to the $n = 2$ level?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q39.1 If a proton and an electron have the same speed, which has the longer de Broglie wavelength? Explain.

Q39.2 If a proton and an electron have the same kinetic energy, which has the longer de Broglie wavelength? Explain.

Q39.3 Does a photon have a de Broglie wavelength? If so, how is it related to the wavelength of the associated electromagnetic wave? Explain.

Q39.4 When an electron beam goes through a very small hole, it produces a diffraction pattern on a screen, just like that of light. Does this mean that an electron spreads out as it goes through the hole? What does this pattern mean?

Q39.5 Galaxies tend to be strong emitters of Lyman-α photons (from the $n = 2$ to $n = 1$ transition in atomic hydrogen). But the intergalactic medium—the very thin gas between the galaxies—tends to *absorb* Lyman-α photons. What can you infer from these observations about the temperature in these two environments? Explain.

Q39.6 A doubly ionized lithium atom (Li^{++}) is one that has had two of its three electrons removed. The energy levels of the remaining single-electron ion are closely related to those of the hydrogen atom. The nuclear charge for lithium is $\pm 3e$ instead of just $+e$. How are the energy levels related to those of hydrogen? How is the *radius* of the ion in the ground level related to that of the hydrogen atom? Explain.

Q39.7 The emission of a photon by an isolated atom is a recoil process in which momentum is conserved. Thus Eq. (39.5) should include a recoil kinetic energy K_r for the atom. Why is this energy negligible in that equation?

Q39.8 How might the energy levels of an atom be measured directly—that is, without recourse to analysis of spectra?

Q39.9 Elements in the gaseous state emit line spectra with well-defined wavelengths. But hot solid bodies always emit a continuous spectrum—that is, a continuous smear of wavelengths. Can you account for this difference?

Q39.10 As a body is heated to a very high temperature and becomes self-luminous, the apparent color of the emitted radiation shifts from red to yellow and finally to blue as the temperature increases. Why does the color shift? What other changes in the character of the radiation occur?

Q39.11 The peak-intensity wavelength of red dwarf stars, which have surface temperatures around 3000 K, is about 1000 nm, which is beyond the visible spectrum. So why are we able to see these stars, and why do they appear red?

Q39.12 You have been asked to design a magnet system to steer a beam of 54-eV electrons like those described in Example 39.1 (Section 39.1). The goal is to be able to direct the electron beam to a specific target location with an accuracy of ± 1.0 mm. In your design, do you need to take the wave nature of electrons into account? Explain.

Q39.13 Why go through the expense of building an electron microscope for studying very small objects such as organic molecules? Why not just use extremely short electromagnetic waves, which are much cheaper to generate?

Q39.14 Which has more total energy: a hydrogen atom with an electron in a high shell (large n) or in a low shell (small n)? Which is moving faster: the high-shell electron or the low-shell electron? Is there a contradiction here? Explain.

Q39.15 Does the uncertainty principle have anything to do with marksmanship? That is, is the accuracy with which a bullet can be aimed at a target limited by the uncertainty principle? Explain.

Q39.16 Suppose a two-slit interference experiment is carried out using an electron beam. Would the same interference pattern result if one slit at a time is uncovered instead of both at once? If not, why not? Doesn't each electron go through one slit or the other? Or does every electron go through both slits? Discuss the latter possibility in light of the principle of complementarity.

Q39.17 Equation (39.30) states that the energy of a system can have uncertainty. Does this mean that the principle of conservation of energy is no longer valid? Explain.

Q39.18 Laser light results from transitions from long-lived metastable states. Why is it more monochromatic than ordinary light?

Q39.19 Could an electron-diffraction experiment be carried out using three or four slits? Using a grating with many slits? What sort of results would you expect with a grating? Would the uncertainty principle be violated? Explain.

Q39.20 As the lower half of Fig. 39.4 shows, the diffraction pattern made by electrons that pass through aluminum foil is a series of concentric rings. But if the aluminum foil is replaced by a single crystal of aluminum, only certain points on these rings appear in the pattern. Explain.

Q39.21 Why can an electron microscope have greater magnification than an ordinary microscope?

Q39.22 When you check the air pressure in a tire, a little air always escapes; the process of making the measurement changes the quantity being measured. Think of other examples of measurements that change or disturb the quantity being measured.

EXERCISES

Section 39.1 Electron Waves

39.1 • (a) An electron moves with a speed of 4.70×10^6 m/s. What is its de Broglie wavelength? (b) A proton moves with the same speed. Determine its de Broglie wavelength.

39.2 •• For crystal diffraction experiments (discussed in Section 39.1), wavelengths on the order of 0.20 nm are often appropriate. Find the energy in electron volts for a particle with this wavelength if the particle is (a) a photon; (b) an electron; (c) an alpha particle $(m = 6.64 \times 10^{-27}\ \mathrm{kg})$.

39.3 • An electron has a de Broglie wavelength of 2.80×10^{-10} m. Determine (a) the magnitude of its momentum and (b) its kinetic energy (in joules and in electron volts).

39.4 •• **Wavelength of an Alpha Particle.** An alpha particle $(m = 6.64 \times 10^{-27}\ \mathrm{kg})$ emitted in the radioactive decay of uranium-238 has an energy of 4.20 MeV. What is its de Broglie wavelength?

39.5 • In the Bohr model of the hydrogen atom, what is the de Broglie wavelength for the electron when it is in (a) the $n = 1$

level and (b) the $n = 4$ level? In each case, compare the de Broglie wavelength to the circumference $2\pi r_n$ of the orbit.

39.6 • (a) A nonrelativistic free particle with mass m has kinetic energy K. Derive an expression for the de Broglie wavelength of the particle in terms of m and K. (b) What is the de Broglie wavelength of an 800-eV electron?

39.7 • **Why Don't *We* Diffract?** (a) Calculate the de Broglie wavelength of a typical person walking through a doorway. Make reasonable approximations for the necessary quantities. (b) Will the person in part (a) exhibit wavelike behavior when walking through the "single slit" of a doorway? Why?

39.8 •• What is the de Broglie wavelength for an electron with speed (a) $v = 0.480c$ and (b) $v = 0.960c$? (*Hint:* Use the correct relativistic expression for linear momentum if necessary.)

39.9 • (a) If a photon and an electron each have the same energy of 20.0 eV, find the wavelength of each. (b) If a photon and an electron each have the same wavelength of 250 nm, find the energy of each. (c) You want to study an organic molecule that is about 250 nm long using either a photon or an electron microscope. Approximately what wavelength should you use, and which probe, the electron or the photon, is likely to damage the molecule the least?

39.10 • How fast would an electron have to move so that its de Broglie wavelength is 1.00 mm?

39.11 • **Wavelength of a Bullet.** Calculate the de Broglie wavelength of a 5.00-g bullet that is moving at 340 m/s. Will the bullet exhibit wavelike properties?

39.12 •• Find the wavelengths of a photon and an electron that have the same energy of 25 eV. (*Note:* The energy of the electron is its kinetic energy.)

39.13 •• (a) What accelerating potential is needed to produce electrons of wavelength 5.00 nm? (b) What would be the energy of photons having the same wavelength as these electrons? (c) What would be the wavelength of photons having the same energy as the electrons in part (a)?

39.14 •• Through what potential difference must electrons be accelerated so they will have (a) the same wavelength as an x ray of wavelength 0.150 nm and (b) the same energy as the x ray in part (a)?

39.15 • (a) Approximately how fast should an electron move so it has a wavelength that makes it useful to measure the distance between adjacent atoms in typical crystals (about 0.10 nm)? (b) What is the kinetic energy of the electron in part (a)? (c) What would be the energy of a photon of the same wavelength as the electron in part (b)? (d) Which would make a more effective probe of small-scale structures: electrons or photons? Why?

39.16 •• **CP** A beam of electrons is accelerated from rest through a potential difference of 0.100 kV and then passes through a thin slit. The diffracted beam shows its first diffraction minima at $\pm 11.5°$ from the original direction of the beam when viewed far from the slit. (a) Do we need to use relativity formulas? How do you know? (b) How wide is the slit?

39.17 •• A beam of neutrons that all have the same energy scatters from atoms that have a spacing of 0.0910 nm in the surface plane of a crystal. The $m = 1$ intensity maximum occurs when the angle θ in Fig. 39.2 is 28.6°. What is the kinetic energy (in electron volts) of each neutron in the beam?

39.18 • A beam of 188-eV electrons is directed at normal incidence onto a crystal surface as shown in Fig. 39.3b. The $m = 2$ intensity maximum occurs at an angle $\theta = 60.6°$. (a) What is the spacing between adjacent atoms on the surface? (b) At what other angle or angles is there an intensity maximum? (c) For what electron energy (in electron volts) would the $m = 1$ intensity maximum occur at $\theta = 60.6°$? For this energy, is there an $m = 2$ intensity maximum? Explain.

39.19 • A CD-ROM is used instead of a crystal in an electron-diffraction experiment. The surface of the CD-ROM has tracks of tiny pits with a uniform spacing of 1.60 μm. (a) If the speed of the electrons is 1.26×10^4 m/s, at which values of θ will the $m = 1$ and $m = 2$ intensity maxima appear? (b) The scattered electrons in these maxima strike at normal incidence a piece of photographic film that is 50.0 cm from the CD-ROM. What is the spacing on the film between these maxima?

39.20 • (a) In an electron microscope, what accelerating voltage is needed to produce electrons with wavelength 0.0600 nm? (b) If protons are used instead of electrons, what accelerating voltage is needed to produce protons with wavelength 0.0600 nm? (*Hint:* In each case the initial kinetic energy is negligible.)

39.21 •• You want to study a biological specimen by means of a wavelength of 10.0 nm, and you have a choice of using electromagnetic waves or an electron microscope. (a) Calculate the ratio of the energy of a 10.0-nm-wavelength photon to the kinetic energy of a 10.0-nm-wavelength electron. (b) In view of your answer to part (a), which would be less damaging to the specimen you are studying: photons or electrons?

Section 39.2 The Nuclear Atom and Atomic Spectra

39.22 •• **CP** A 4.78-MeV alpha particle from a ^{226}Ra decay makes a head-on collision with a uranium nucleus. A uranium nucleus has 92 protons. (a) What is the distance of closest approach of the alpha particle to the center of the nucleus? Assume that the uranium nucleus remains at rest and that the distance of closest approach is much greater than the radius of the uranium nucleus. (b) What is the force on the alpha particle at the instant when it is at the distance of closest approach?

39.23 • A beam of alpha particles is incident on a target of lead. A particular alpha particle comes in "head-on" to a particular lead nucleus and stops 6.50×10^{-14} m away from the center of the nucleus. (This point is well outside the nucleus.) Assume that the lead nucleus, which has 82 protons, remains at rest. The mass of the alpha particle is 6.64×10^{-27} kg. (a) Calculate the electrostatic potential energy at the instant that the alpha particle stops. Express your result in joules and in MeV. (b) What initial kinetic energy (in joules and in MeV) did the alpha particle have? (c) What was the initial speed of the alpha particle?

Section 39.3 Energy Levels and the Bohr Model of the Atom

39.24 • The silicon–silicon single bond that forms the basis of the mythical silicon-based creature the Horta has a bond strength of 3.80 eV. What wavelength of photon would you need in a (mythical) phasor disintegration gun to destroy the Horta?

39.25 •• A hydrogen atom is in a state with energy -1.51 eV. In the Bohr model, what is the angular momentum of the electron in the atom, with respect to an axis at the nucleus?

39.26 • A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of the photon.

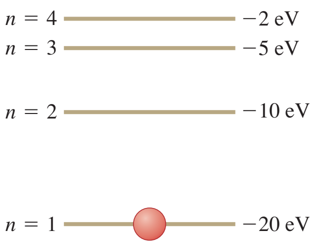
39.27 • A triply ionized beryllium ion, Be^{3+} (a beryllium atom with three electrons removed), behaves very much like a hydrogen atom except that the nuclear charge is four times as great. (a) What is the ground-level energy of Be^{3+}? How does this compare to the ground-level energy of the hydrogen atom? (b) What is the ionization energy of Be^{3+}? How does this compare to the ionization energy of the hydrogen atom? (c) For the hydrogen atom, the wavelength of the photon emitted in the $n = 2$ to $n = 1$ transition is 122 nm (see Example 39.6). What is the wavelength of the photon emitted when a Be^{3+} ion undergoes this transition? (d) For a given value of n, how does the radius of an orbit in Be^{3+} compare to that for hydrogen?

39.28 •• (a) Show that, as n gets very large, the energy levels of the hydrogen atom get closer and closer together in energy. (b) Do the radii of these energy levels also get closer together?

39.29 • (a) Using the Bohr model, calculate the speed of the electron in a hydrogen atom in the $n = 1$, 2, and 3 levels. (b) Calculate the orbital period in each of these levels. (c) The average lifetime of the first excited level of a hydrogen atom is 1.0×10^{-8} s. In the Bohr model, how many orbits does an electron in the $n = 2$ level complete before returning to the ground level?

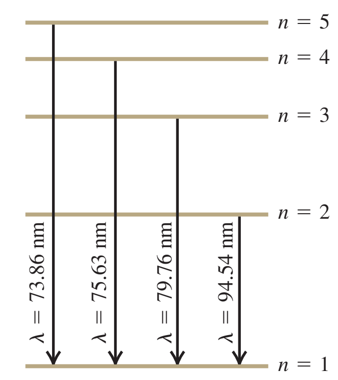
39.30 • **CP** The energy-level scheme for the hypothetical one-electron element Searsium is shown in Fig. E39.30. The potential energy is taken to be zero for an electron at an infinite distance from the nucleus. (a) How much energy (in electron volts) does it take to ionize an electron from the ground level? (b) An 18-eV photon is absorbed by a Searsium atom in its ground level. As the atom returns to its ground level, what possible energies can the emitted photons have? Assume that there can be transitions between all pairs of levels. (c) What will happen if a photon with an energy of 8 eV strikes a Searsium atom in its ground level? Why? (d) Photons emitted in the Searsium transitions $n = 3 \rightarrow n = 2$ and $n = 3 \rightarrow n = 1$ will eject photoelectrons from an unknown metal, but the photon emitted from the transition $n = 4 \rightarrow n = 3$ will not. What are the limits (maximum and minimum possible values) of the work function of the metal?

Figure **E39.30**



39.31 • In a set of experiments on a hypothetical one-electron atom, you measure the wavelengths of the photons emitted from transitions ending in the ground state ($n = 1$), as shown in the energy-level diagram in Fig. E39.31. You also observe that it takes 17.50 eV to ionize this atom. (a) What is the energy of the atom in each of the levels ($n = 1$, $n = 2$, etc.) shown in the figure? (b) If an electron made a transition from the $n = 4$ to the $n = 2$ level, what wavelength of light would it emit?

Figure **E39.31**



39.32 • Find the longest and shortest wavelengths in the Lyman and Paschen series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

39.33 • (a) An atom initially in an energy level with $E = -6.52$ eV absorbs a photon that has wavelength 860 nm. What is the internal energy of the atom after it absorbs the photon? (b) An atom initially in an energy level with $E = -2.68$ eV emits a photon that has wavelength 420 nm. What is the internal energy of the atom after it emits the photon?

39.34 •• Use Balmer's formula to calculate (a) the wavelength, (b) the frequency, and (c) the photon energy for the H_γ line of the Balmer series for hydrogen.

Section 39.4 The Laser

39.35 • **BIO** **Laser Surgery.** Using a mixture of CO_2, N_2, and sometimes He, CO_2 lasers emit a wavelength of 10.6 μm. At power outputs of 0.100 kW, such lasers are used for surgery. How many photons per second does a CO_2 laser deliver to the tissue during its use in an operation?

39.36 • **BIO** **Removing Birthmarks.** Pulsed dye lasers emit light of wavelength 585 nm in 0.45-ms pulses to remove skin blemishes such as birthmarks. The beam is usually focused onto a circular spot 5.0 mm in diameter. Suppose that the output of one such laser is 20.0 W. (a) What is the energy of each photon, in eV? (b) How many photons per square millimeter are delivered to the blemish during each pulse?

39.37 • How many photons per second are emitted by a 7.50-mW CO_2 laser that has a wavelength of 10.6 μm?

39.38 • **BIO** **PRK Surgery.** Photorefractive keratectomy (PRK) is a laser-based surgical procedure that corrects near- and farsightedness by removing part of the lens of the eye to change its curvature and hence focal length. This procedure can remove layers 0.25 μm thick using pulses lasting 12.0 ns from a laser beam of wavelength 193 nm. Low-intensity beams can be used because each individual photon has enough energy to break the covalent bonds of the tissue. (a) In what part of the electromagnetic spectrum does this light lie? (b) What is the energy of a single photon? (c) If a 1.50-mW beam is used, how many photons are delivered to the lens in each pulse?

39.39 • A large number of neon atoms are in thermal equilibrium. What is the ratio of the number of atoms in a 5s state to the number in a 3p state at (a) 300 K; (b) 600 K; (c) 1200 K? The energies of these states, relative to the ground state, are $E_{5s} = 20.66$ eV and $E_{3p} = 18.70$ eV. (d) At any of these temperatures, the rate at which a neon gas will spontaneously emit 632.8-nm radiation is quite low. Explain why.

39.40 • Figure 39.19a shows the energy levels of the sodium atom. The two lowest excited levels are shown in columns labeled $^2P_{3/2}$ and $^2P_{1/2}$. Find the ratio of the number of atoms in a $^2P_{3/2}$ state to the number in a $^2P_{1/2}$ state for a sodium gas in thermal equilibrium at 500 K. In which state are more atoms found?

Section 39.5 Continuous Spectra

39.41 •• A 100-W incandescent light bulb has a cylindrical tungsten filament 30.0 cm long, 0.40 mm in diameter, and with an emissivity of 0.26. (a) What is the temperature of the filament? (b) For what wavelength does the spectral emittance of the bulb peak? (c) Incandescent light bulbs are not very efficient sources of visible light. Explain why this is so.

39.42 • Determine λ_m, the wavelength at the peak of the Planck distribution, and the corresponding frequency f, at these temperatures: (a) 3.00 K; (b) 300 K; (c) 3000 K.

39.43 • Radiation has been detected from space that is characteristic of an ideal radiator at $T = 2.728$ K. (This radiation is a relic of the Big Bang at the beginning of the universe.) For this temperature, at what wavelength does the Planck distribution peak? In what part of the electromagnetic spectrum is this wavelength?

39.44 • The shortest visible wavelength is about 400 nm. What is the temperature of an ideal radiator whose spectral emittance peaks at this wavelength?

39.45 •• Two stars, both of which behave like ideal blackbodies, radiate the same total energy per second. The cooler one has a surface temperature T and a diameter 3.0 times that of the hotter star. (a) What is the temperature of the hotter star in terms of T? (b) What is the ratio of the peak-intensity wavelength of the hot star to the peak-intensity wavelength of the cool star?

39.46 • **Sirius B.** The brightest star in the sky is Sirius, the Dog Star. It is actually a binary system of two stars, the smaller one (Sirius B) being a white dwarf. Spectral analysis of Sirius B indicates that its surface temperature is 24,000 K and that it radiates energy at a total rate of 1.0×10^{25} W. Assume that it behaves like an ideal blackbody. (a) What is the total radiated intensity of Sirius B? (b) What is the peak-intensity wavelength? Is this wavelength visible to humans? (c) What is the radius of Sirius B? Express your answer in kilometers and as a fraction of our sun's radius. (d) Which star radiates more *total* energy per second, the hot Sirius B or the (relatively) cool sun with a surface temperature of 5800 K? To find out, calculate the ratio of the total power radiated by our sun to the power radiated by Sirius B.

39.47 •• **Blue Supergiants.** A typical blue supergiant star (the type that explodes and leaves behind a black hole) has a surface temperature of 30,000 K and a visual luminosity 100,000 times that of our sun. Our sun radiates at the rate of 3.86×10^{26} W. (Visual luminosity is the total power radiated at visible wavelengths.) (a) Assuming that this star behaves like an ideal blackbody, what is the principal wavelength it radiates? Is this light visible? Use your answer to explain why these stars are blue. (b) If we assume that the power radiated by the star is also 100,000 times that of our sun, what is the radius of this star? Compare its size to that of our sun, which has a radius of 6.96×10^5 km. (c) Is it really correct to say that the visual luminosity is proportional to the total power radiated? Explain.

Section 39.6 The Uncertainty Principle Revisited

39.48 • A pesky 1.5-mg mosquito is annoying you as you attempt to study physics in your room, which is 5.0 m wide and 2.5 m high. You decide to swat the bothersome insect as it flies toward you, but you need to estimate its speed to make a successful hit. (a) What is the maximum uncertainty in the horizontal position of the mosquito? (b) What limit does the Heisenberg uncertainty principle place on your ability to know the horizontal velocity of this mosquito? Is this limitation a serious impediment to your attempt to swat it?

39.49 • By extremely careful measurement, you determine the x-coordinate of a car's center of mass with an uncertainty of only 1.00 μm. The car has a mass of 1200 kg. (a) What is the minimum uncertainty in the x-component of the velocity of the car's center of mass as prescribed by the Heisenberg uncertainty principle? (b) Does the uncertainty principle impose a practical limit on our ability to make simultaneous measurements of the positions and velocities of ordinary objects like cars, books, and people? Explain.

39.50 • A 10.0-g marble is gently placed on a horizontal tabletop that is 1.75 m wide. (a) What is the maximum uncertainty in the horizontal position of the marble? (b) According to the Heisenberg uncertainty principle, what is the minimum uncertainty in the horizontal velocity of the marble? (c) In light of your answer to part (b), what is the longest time the marble could remain on the table? Compare this time to the age of the universe, which is approximately 14 billion years. (*Hint:* Can you know that the horizontal velocity of the marble is *exactly* zero?)

39.51 • A scientist has devised a new method of isolating individual particles. He claims that this method enables him to detect simultaneously the position of a particle along an axis with a standard deviation of 0.12 nm and its momentum component along this axis with a standard deviation of 3.0×10^{-25} kg · m/s. Use the Heisenberg uncertainty principle to evaluate the validity of this claim.

39.52 • (a) The x-coordinate of an electron is measured with an uncertainty of 0.20 mm. What is the x-component of the electron's velocity, v_x, if the minimum percentage uncertainty in a simultaneous measurement of v_x is 1.0%? (b) Repeat part (a) for a proton.

39.53 • An atom in a metastable state has a lifetime of 5.2 ms. What is the uncertainty in energy of the metastable state?

39.54 • (a) The uncertainty in the y-component of a proton's position is 2.0×10^{-12} m. What is the minimum uncertainty in a simultaneous measurement of the y-component of the proton's velocity? (b) The uncertainty in the z-component of an electron's velocity is 0.250 m/s. What is the minimum uncertainty in a simultaneous measurement of the z-coordinate of the electron?

PROBLEMS

39.55 •• The negative muon has a charge equal to that of an electron but a mass that is 207 times as great. Consider a hydrogenlike atom consisting of a proton and a muon. (a) What is the reduced mass of the atom? (b) What is the ground-level energy (in electron volts)? (c) What is the wavelength of the radiation emitted in the transition from the $n = 2$ level to the $n = 1$ level?

39.56 • An atom with mass m emits a photon of wavelength λ. (a) What is the recoil speed of the atom? (b) What is the kinetic energy K of the recoiling atom? (c) Find the ratio K/E, where E is the energy of the emitted photon. If this ratio is much less than unity, the recoil of the atom can be neglected in the emission process. Is the recoil of the atom more important for small or large atomic masses? For long or short wavelengths? (d) Calculate K (in electron volts) and K/E for a hydrogen atom (mass 1.67×10^{-27} kg) that emits an ultraviolet photon of energy 10.2 eV. Is recoil an important consideration in this emission process?

39.57 • (a) What is the smallest amount of energy in electron volts that must be given to a hydrogen atom initially in its ground level so that it can emit the H_α line in the Balmer series? (b) How many different possibilities of spectral-line emissions are there for this atom when the electron starts in the $n = 3$ level and eventually ends up in the ground level? Calculate the wavelength of the emitted photon in each case.

39.58 • A large number of hydrogen atoms are in thermal equilibrium. Let n_2/n_1 be the ratio of the number of atoms in an $n = 2$ excited state to the number of atoms in an $n = 1$ ground state. At what temperature is n_2/n_1 equal to (a) 10^{-12}; (b) 10^{-8}; (c) 10^{-4}? (d) Like the sun, other stars have continuous spectra with dark absorption lines (see Fig. 39.9). The absorption takes place in the star's atmosphere, which in all stars is composed primarily of hydrogen. Explain why the Balmer absorption lines are relatively weak in stars with low atmospheric temperatures such as the sun (atmosphere temperature 5800 K) but strong in stars with higher atmospheric temperatures.

39.59 ••• A sample of hydrogen atoms is irradiated with light with wavelength 85.5 nm, and electrons are observed leaving the gas. (a) If each hydrogen atom were initially in its ground level, what would be the maximum kinetic energy in electron volts of these photoelectrons? (b) A few electrons are detected with energies as much as 10.2 eV greater than the maximum kinetic energy calculated in part (a). How can this be?

39.60 • CP Bohr Orbits of a Satellite. A 20.0-kg satellite circles the earth once every 2.00 h in an orbit having a radius of 8060 km. (a) Assuming that Bohr's angular-momentum result $(L = nh/2\pi)$ applies to satellites just as it does to an electron in the hydrogen atom, find the quantum number n of the orbit of the satellite. (b) Show from Bohr's angular momentum result and Newton's law of gravitation that the radius of an earth-satellite orbit is directly proportional to the square of the quantum number, $r = kn^2$, where k is the constant of proportionality. (c) Using the result from part (b), find the distance between the orbit of the satellite in this problem and its next "allowed" orbit. (Calculate a numerical value.) (d) Comment on the possibility of observing the separation of the two adjacent orbits. (e) Do quantized and classical orbits correspond for this satellite? Which is the "correct" method for calculating the orbits?

39.61 •• The Red Supergiant Betelgeuse. The star Betelgeuse has a surface temperature of 3000 K and is 600 times the diameter of our sun. (If our sun were that large, we would be inside it!) Assume that it radiates like an ideal blackbody. (a) If Betelgeuse were to radiate all of its energy at the peak-intensity wavelength, how many photons per second would it radiate? (b) Find the ratio of the power radiated by Betelgeuse to the power radiated by our sun (at 5800 K).

39.62 •• CP Light from an ideal spherical blackbody 15.0 cm in diameter is analyzed using a diffraction grating having 3850 lines/cm. When you shine this light through the grating, you observe that the peak-intensity wavelength forms a first-order bright fringe at $\pm 11.6°$ from the central bright fringe. (a) What is the temperature of the blackbody? (b) How long will it take this sphere to radiate 12.0 MJ of energy?

39.63 • What must be the temperature of an ideal blackbody so that photons of its radiated light having the peak-intensity wavelength can excite the electron in the Bohr-model hydrogen atom from the ground state to the third excited state?

39.64 • CP An ideal spherical blackbody 24.0 cm in diameter is maintained at 225°C by an internal electrical heater and is immersed in a very large open-faced tank of water that is kept boiling by the energy radiated by the sphere. You can neglect any heat transferred by conduction and convection. Consult Table 17.4 as needed. (a) At what rate, in g/s, is water evaporating from the tank? (b) If a physics-wise thermophile organism living in the hot water is observing this process, what will it measure for the peak-intensity (i) wavelength and (ii) frequency of the electromagnetic waves emitted by the sphere?

39.65 ••• When a photon is emitted by an atom, the atom must recoil to conserve momentum. This means that the photon and the recoiling atom share the transition energy. (a) For an atom with mass m, calculate the correction $\Delta\lambda$ due to recoil to the wavelength of an emitted photon. Let λ be the wavelength of the photon if recoil is not taken into consideration. (*Hint:* The correction is very small, as Problem 39.56 suggests, so $|\Delta\lambda|/\lambda \ll 1$. Use this fact to obtain an approximate but very accurate expression for $\Delta\lambda$.) (b) Evaluate the correction for a hydrogen atom in which an electron in the nth level returns to the ground level. How does the answer depend on n?

39.66 •• An Ideal Blackbody. A large cavity with a very small hole and maintained at a temperature T is a good approximation to an ideal radiator or blackbody. Radiation can pass into or out of the cavity only through the hole. The cavity is a perfect absorber, since any radiation incident on the hole becomes trapped inside the cavity. Such a cavity at 200°C has a hole with area 4.00 mm^2. How long does it take for the cavity to radiate 100 J of energy through the hole?

39.67 •• CALC (a) Write the Planck distribution law in terms of the frequency f, rather than the wavelength λ, to obtain $I(f)$. (b) Show that

$$\int_0^{\infty} I(\lambda)\, d\lambda = \frac{2\pi^5 k^4}{15c^2h^3}T^4$$

where $I(\lambda)$ is the Planck distribution formula of Eq. (39.24). (*Hint:* Change the integration variable from λ to f. You will need to use the following tabulated integral:

$$\int_0^{\infty} \frac{x^3}{e^{\alpha x} - 1} dx = \frac{1}{240}\left(\frac{2\pi}{\alpha}\right)^4$$

(c) The result of part (b) is I and has the form of the Stefan–Boltzmann law, $I = \sigma T^4$ (Eq. 39.19). Evaluate the constants in part (b) to show that σ has the value given in Section 39.5.

39.68 •• CP A beam of 40-eV electrons traveling in the $+x$-direction passes through a slit that is parallel to the y-axis and 5.0 μm wide. The diffraction pattern is recorded on a screen 2.5 m from the slit. (a) What is the de Broglie wavelength of the electrons? (b) How much time does it take the electrons to travel from the slit to the screen? (c) Use the width of the central diffraction pattern to calculate the uncertainty in the y-component of momentum of an electron just after it has passed through the slit. (d) Use the result of part (c) and the Heisenberg uncertainty principle (Eq. 39.29 for y) to estimate the minimum uncertainty in the y-coordinate of an electron just after it has passed through the slit. Compare your result to the width of the slit.

39.69 • (a) What is the energy of a photon that has wavelength 0.10 μm? (b) Through approximately what potential difference must electrons be accelerated so that they will exhibit wave nature in passing through a pinhole 0.10 μm in diameter? What is the speed of these electrons? (c) If protons rather than electrons were used, through what potential difference would protons have to be accelerated so they would exhibit wave nature in passing through this pinhole? What would be the speed of these protons?

39.70 • CP Electrons go through a single slit 150 nm wide and strike a screen 24.0 cm away. You find that at angles of $\pm 20.0°$ from the center of the diffraction pattern, no electrons hit the screen but electrons hit at all points closer to the center. (a) How fast were these electrons moving when they went through the slit? (b) What will be the next larger angles at which no electrons hit the screen?

39.71 •• CP A beam of electrons is accelerated from rest and then passes through a pair of identical thin slits that are 1.25 nm apart. You observe that the first double-slit interference dark fringe occurs at $\pm 18.0°$ from the original direction of the beam when viewed on a distant screen. (a) Are these electrons relativistic? How do you know? (b) Through what potential difference were the electrons accelerated?

39.72 •• CP A beam of protons and a beam of alpha particles (of mass 6.64×10^{-27} kg and charge $+2e$) are accelerated from rest through the same potential difference and pass through identical circular holes in a very thin, opaque film. When viewed far from the hole, the diffracted proton beam forms its first dark ring at 15° with respect to its original direction. When viewed similarly, at what angle will the alpha particle form its first dark ring?

39.73 •• CP An electron beam and a photon beam pass through identical slits. On a distant screen, the first dark fringe occurs at the same angle for both of the beams. The electron speeds are much slower than that of light. (a) Express the energy of a photon in terms of the kinetic energy K of one of the electrons. (b) Which is greater, the energy of a photon or the kinetic energy of an electron?

39.74 • **CP** Coherent light is passed through two narrow slits whose separation is 40.0 μm. The second-order bright fringe in the interference pattern is located at an angle of 0.0300 rad. If electrons are used instead of light, what must the kinetic energy (in electron volts) of the electrons be if they are to produce an interference pattern for which the second-order maximum is also at 0.0300 rad?

39.75 • **BIO** What is the de Broglie wavelength of a red blood cell, with mass 1.00×10^{-11} g, that is moving with a speed of 0.400 cm/s? Do we need to be concerned with the wave nature of the blood cells when we describe the flow of blood in the body?

39.76 • Calculate the energy in electron volts of (a) an electron that has de Broglie wavelength 400 nm and (b) a photon that has wavelength 400 nm.

39.77 • High-speed electrons are used to probe the interior structure of the atomic nucleus. For such electrons the expression $\lambda = h/p$ still holds, but we must use the relativistic expression for momentum, $p = mv/\sqrt{1 - v^2/c^2}$. (a) Show that the speed of an electron that has de Broglie wavelength λ is

$$v = \frac{c}{\sqrt{1 + (mc\lambda/h)^2}}$$

(b) The quantity h/mc equals 2.426×10^{-12} m. (As we saw in Section 38.3, this same quantity appears in Eq. (38.7), the expression for Compton scattering of photons by electrons.) If λ is small compared to h/mc, the denominator in the expression found in part (a) is close to unity and the speed v is very close to c. In this case it is convenient to write $v = (1 - \Delta)c$ and express the speed of the electron in terms of Δ rather than v. Find an expression for Δ valid when $\lambda \ll h/mc$. [*Hint:* Use the binomial expansion $(1 + z)^n = 1 + nz + n(n-1)z^2/2 + \cdots$, valid for the case $|z| < 1$.] (c) How fast must an electron move for its de Broglie wavelength to be 1.00×10^{-15} m, comparable to the size of a proton? Express your answer in the form $v = (1 - \Delta)c$, and state the value of Δ.

39.78 • Suppose that the uncertainty of position of an electron is equal to the radius of the $n = 1$ Bohr orbit for hydrogen. Calculate the simultaneous minimum uncertainty of the corresponding momentum component, and compare this with the magnitude of the momentum of the electron in the $n = 1$ Bohr orbit. Discuss your results.

39.79 • **CP** (a) A particle with mass m has kinetic energy equal to three times its rest energy. What is the de Broglie wavelength of this particle? (*Hint:* You must use the relativistic expressions for momentum and kinetic energy: $E^2 = (pc)^2 + (mc^2)^2$ and $K = E - mc^2$.) (b) Determine the numerical value of the kinetic energy (in MeV) and the wavelength (in meters) if the particle in part (a) is (i) an electron and (ii) a proton.

39.80 • **Proton Energy in a Nucleus.** The radii of atomic nuclei are of the order of 5.0×10^{-15} m. (a) Estimate the minimum uncertainty in the momentum of a proton if it is confined within a nucleus. (b) Take this uncertainty in momentum to be an estimate of the magnitude of the momentum. Use the relativistic relationship between energy and momentum, Eq. (37.39), to obtain an estimate of the kinetic energy of a proton confined within a nucleus. (c) For a proton to remain bound within a nucleus, what must the magnitude of the (negative) potential energy for a proton be within the nucleus? Give your answer in eV and in MeV. Compare to the potential energy for an electron in a hydrogen atom, which has a magnitude of a few tens of eV. (This shows why the interaction that binds the nucleus together is called the "strong nuclear force.")

39.81 • **Electron Energy in a Nucleus.** The radii of atomic nuclei are of the order of 5.0×10^{-15} m. (a) Estimate the minimum uncertainty in the momentum of an electron if it is confined within a nucleus. (b) Take this uncertainty in momentum to be an estimate of the magnitude of the momentum. Use the relativistic relationship between energy and momentum, Eq. (37.39), to obtain an estimate of the kinetic energy of an electron confined within a nucleus. (c) Compare the energy calculated in part (b) to the magnitude of the Coulomb potential energy of a proton and an electron separated by 5.0×10^{-15} m. On the basis of your result, could there be electrons within the nucleus? (*Note:* It is interesting to compare this result to that of Problem 39.80.)

39.82 • In a TV picture tube the accelerating voltage is 15.0 kV, and the electron beam passes through an aperture 0.50 mm in diameter to a screen 0.300 m away. (a) Calculate the uncertainty in the component of the electron's velocity perpendicular to the line between aperture and screen. (b) What is the uncertainty in position of the point where the electrons strike the screen? (c) Does this uncertainty affect the clarity of the picture significantly? (Use nonrelativistic expressions for the motion of the electrons. This is fairly accurate and is certainly adequate for obtaining an estimate of uncertainty effects.)

39.83 • The neutral pion (π^0) is an unstable particle produced in high-energy particle collisions. Its mass is about 264 times that of the electron, and it exists for an average lifetime of 8.4×10^{-17} s before decaying into two gamma-ray photons. Using the relationship $E = mc^2$ between rest mass and energy, find the uncertainty in the mass of the particle and express it as a fraction of the mass.

39.84 • **Quantum Effects in Daily Life?** A 1.25-mg insect flies through a 4.00-mm-diameter hole in an ordinary window screen. The thickness of the screen is 0.500 mm. (a) What should be the approximate wavelength and speed of the insect for her to show wave behavior as she goes through the hole? (b) At the speed found in part (a), how long would it take the insect to pass through the 0.500-mm thickness of the hole in the screen? Compare this time to the age of the universe (about 14 billion years). Would you expect to see "insect diffraction" in daily life?

39.85 • **Doorway Diffraction.** If your wavelength were 1.0 m, you would undergo considerable diffraction in moving through a doorway. (a) What must your speed be for you to have this wavelength? (Assume that your mass is 60.0 kg.) (b) At the speed calculated in part (a), how many years would it take you to move 0.80 m (one step)? Will you notice diffraction effects as you walk through doorways?

39.86 • **Atomic Spectra Uncertainties.** A certain atom has an energy level 2.58 eV above the ground level. Once excited to this level, the atom remains in this level for 1.64×10^{-7} s (on average) before emitting a photon and returning to the ground level. (a) What is the energy of the photon (in electron volts)? What is its wavelength (in nanometers)? (b) What is the smallest possible uncertainty in energy of the photon? Give your answer in electron volts. (c) Show that $|\Delta E/E| = |\Delta\lambda/\lambda|$ if $|\Delta\lambda/\lambda| \ll 1$. Use this to calculate the magnitude of the smallest possible uncertainty in the wavelength of the photon. Give your answer in nanometers.

39.87 •• You intend to use an electron microscope to study the structure of some crystals. For accurate resolution, you want the electron wavelength to be 1.00 nm. (a) Are these electrons relativistic? How do you know? (b) What accelerating potential is needed? (c) What is the kinetic energy of the electrons you are using? To see if it is great enough to damage the crystals you are studying, compare it to the potential energy of a typical NaCl molecule, which is about 6.0 eV. (d) If you decided to use electromagnetic waves as your probe, what energy should their photons have to provide the same resolution as the electrons? Would this energy damage the crystal?

39.88 •• For x rays with wavelength 0.0300 nm, the $m = 1$ intensity maximum for a crystal occurs when the angle θ in Fig. 39.2 is 35.8°. At what angle θ does the $m = 1$ maximum occur when a beam of 4.50-keV electrons is used instead? Assume that the electrons also scatter from the atoms in the surface plane of this same crystal.

39.89 •• **CP** Electron diffraction can also take place when there is interference between electron waves that scatter from atoms on the surface of a crystal and waves that scatter from atoms in the next plane below the surface, a distance d from the surface (see Fig. 36.23c). (a) Find an equation for the angles θ at which there is an intensity maximum for electron waves of wavelength λ. (b) The spacing between crystal planes in a certain metal is 0.091 nm. If 71.0-eV electrons are used, find the angle at which there is an intensity maximum due to interference between scattered waves from adjacent crystal planes. The angle is measured as shown in Fig. 36.23c. (c) The actual angle of the intensity maximum is slightly different from your result in part (b). The reason is the work function ϕ of the metal (see Section 38.1), which changes the electron potential energy by $-e\phi$ when it moves from vacuum into the metal. If the effect of the work function is taken into account, is the angle of the intensity maximum larger or smaller than the value found in part (b)? Explain.

39.90 •• A certain atom has an energy level 3.50 eV above the ground state. When excited to this state, it remains 4.0 μs, on the average, before emitting a photon and returning to the ground state. (a) What is the energy of the photon? What is its wavelength? (b) What is the smallest possible uncertainty in energy of the photon?

39.91 •• **BIO** **Structure of a Virus.** To investigate the structure of extremely small objects, such as viruses, the wavelength of the probing wave should be about one-tenth the size of the object for sharp images. But as the wavelength gets shorter, the energy of a photon of light gets greater and could damage or destroy the object being studied. One alternative is to use electron matter waves instead of light. Viruses vary considerably in size, but 50 nm is not unusual. Suppose you want to study such a virus, using a wave of wavelength 5.00 nm. (a) If you use light of this wavelength, what would be the energy (in eV) of a single photon? (b) If you use an electron of this wavelength, what would be its kinetic energy (in eV)? Is it now clear why matter waves (such as in the electron microscope) are often preferable to electromagnetic waves for studying microscopic objects?

39.92 •• **CALC** **Zero-Point Energy.** Consider a particle with mass m moving in a potential $U = \frac{1}{2}kx^2$, as in a mass–spring system. The total energy of the particle is $E = p^2/2m + \frac{1}{2}kx^2$. Assume that p and x are approximately related by the Heisenberg uncertainty principle, so $px \approx h$. (a) Calculate the minimum possible value of the energy E, and the value of x that gives this minimum E. This lowest possible energy, which is not zero, is called the *zero-point energy.* (b) For the x calculated in part (a), what is the ratio of the kinetic to the potential energy of the particle?

39.93 •• **CALC** A particle with mass m moves in a potential $U(x) = A|x|$, where A is a positive constant. In a simplified picture, quarks (the constituents of protons, neutrons, and other particles, as will be described in Chapter 44) have a potential energy of interaction of approximately this form, where x represents the separation between a pair of quarks. Because $U(x) \rightarrow \infty$ as $x \rightarrow \infty$, it's not possible to separate quarks from each other (a phenomenon called *quark confinement*). (a) Classically, what is the force acting on this particle as a function of x? (b) Using the uncertainty principle as in Problem 39.92, determine approximately the zero-point energy of the particle.

39.94 •• Imagine another universe in which the value of Planck's constant is 0.0663 J·s, but in which the physical laws and all other physical constants are the same as in our universe. In this universe, two physics students are playing catch. They are 12 m apart, and one throws a 0.25-kg ball directly toward the other with a speed of 6.0 m/s. (a) What is the uncertainty in the ball's horizontal momentum, in a direction perpendicular to that in which it is being thrown, if the student throwing the ball knows that it is located within a cube with volume 125 cm^3 at the time she throws it? (b) By what horizontal distance could the ball miss the second student?

CHALLENGE PROBLEMS

39.95 ••• (a) Show that in the Bohr model, the frequency of revolution of an electron in its circular orbit around a stationary hydrogen nucleus is $f = me^4/4\epsilon_0^2n^3h^3$. (b) In classical physics, the frequency of revolution of the electron is equal to the frequency of the radiation that it emits. Show that when n is very large, the frequency of revolution does indeed equal the radiated frequency calculated from Eq. (39.5) for a transition from $n_1 = n + 1$ to $n_2 = n$. (This illustrates Bohr's *correspondence principle,* which is often used as a check on quantum calculations. When n is small, quantum physics gives results that are very different from those of classical physics. When n is large, the differences are not significant, and the two methods then "correspond." In fact, when Bohr first tackled the hydrogen atom problem, he sought to determine f as a function of n such that it would correspond to classical results for large n.)

39.96 ••• **CP** **CALC** You have entered a contest in which the contestants drop a marble with mass 20.0 g from the roof of a building onto a small target 25.0 m below. From uncertainty considerations, what is the typical distance by which you will miss the target, given that you aim with the highest possible precision? (*Hint:* The uncertainty Δx_f in the x-coordinate of the marble when it reaches the ground comes in part from the uncertainty Δx_i in the x-coordinate initially and in part from the initial uncertainty in v_x. The latter gives rise to an uncertainty Δv_x in the horizontal motion of the marble as it falls. The values of Δx_i and Δv_x are related by the uncertainty principle. A small Δx_i gives rise to a large Δv_x, and vice versa. Find the value of Δx_i that gives the smallest total uncertainty in x at the ground. Ignore any effects of air resistance.)

Answers

Chapter Opening Question ?

The smallest detail visible in an image is comparable to the wavelength used to make the image. Electrons can easily be given a large momentum p and hence a short wavelength $\lambda = h/p$, and so can be used to resolve extremely fine details. (See Section 39.1.)

Test Your Understanding Questions

39.1 Answers: (a) (i), (b) no From Example 39.2, the speed of a particle is $v = h/\lambda m$ and the kinetic energy is $K = \frac{1}{2}mv^2 = (m/2)(h/\lambda m)^2 = h^2/2\lambda^2 m$. This shows that for a given wavelength, the kinetic energy is inversely proportional to the mass. Hence the proton, with a smaller mass, has more kinetic energy than the neutron. For part (b), the energy of a photon is $E = hf$, and the frequency of a photon is $f = c/\lambda$. Hence $E = hc/\lambda$ and $\lambda = hc/E = (4.136 \times 10^{-15}\ \text{eV} \cdot \text{s})(2.998 \times 10^8\ \text{m/s})/(54\ \text{eV}) = 2.3 \times 10^{-8}\ \text{m}$. This is more than 100 times greater than the wavelength of an electron of the same energy. While both photons and electrons have wavelike properties, they have different relationships between their energy and momentum and hence between their frequency and wavelength.

39.2 Answer: (iii) Because the alpha particle is more massive, it won't bounce back in even a head-on collision with a proton that's initially at rest, any more than a bowling ball would when colliding with a Ping-Pong ball at rest (see Fig. 8.22b). Thus there would be *no* large-angle scattering in this case. Rutherford saw large-angle scattering in his experiment because gold nuclei are more massive than alpha particles (see Fig. 8.22a).

39.3 Answer: (iv) Figure 39.27 shows that many (though *not* all) of the energy levels of He^+ are the same as those of H. Hence photons emitted during transitions between corresponding pairs of levels in He^+ and H have the same energy E and the same wavelength $\lambda = hc/E$. An H atom that drops from the $n = 2$ level to the $n = 1$ level emits a photon of energy 10.20 eV and wavelength 122 nm (see Example 39.6); a He^+ ion emits a photon of the same energy and wavelength when it drops from the $n = 4$ level to the $n = 2$ level. Inspecting Fig. 39.27 will show you that every even-numbered level in He^+ matches a level in H, while none of the odd-numbered He^+ levels do. The first three He^+ transitions given in the question ($n = 2$ to $n = 1$, $n = 3$ to $n = 2$, and $n = 4$ to $n = 3$) all involve an odd-numbered level, so none of their wavelengths match a wavelength emitted by H atoms.

39.4 Answer: (i) In a neon light fixture, a large potential difference is applied between the ends of a neon-filled glass tube. This ionizes some of the neon atoms, allowing a current of electrons to flow through the gas. Some of the neon atoms are struck by fast-moving electrons, making them transition to an excited level. From this level the atoms undergo *spontaneous* emission, as depicted in Fig. 39.28b, and emit 632.8-nm photons in the process. No population inversion occurs and the photons are not trapped by mirrors as shown in Fig. 39.29d, so there is no stimulated emission. Hence there is no laser action.

39.5 Answers: (a) yes, (b) yes The Planck radiation law, Eq. (39.24), shows that an ideal blackbody emits radiation at *all* wavelengths: The spectral emittance $I(\lambda)$ is equal to zero only for $\lambda = 0$ and in the limit $\lambda \to \infty$. So a blackbody at 2000 K does indeed emit both x rays and radio waves. However, Fig. 39.32 shows that the spectral emittance for this temperature is very low for wavelengths much shorter than 1 μm (including x rays) and for wavelengths much longer than a few μm (including radio waves). Hence such a blackbody emits very little in the way of x rays or radio waves.

39.6 Answer: (i) and (iii) (tie), (ii) and (iv) (tie) According to the Heisenberg uncertainty principle, the smaller the uncertainty Δx in the x-coordinate, the greater the uncertainty Δp_x in the x-momentum. The relationship between Δx and Δp_x does not depend on the mass of the particle, and so is the same for a proton as for an electron.

Bridging Problem

Answers: **(a)** 192 nm; ultraviolet **(b)** $n = 4$

(c) $\lambda_2 = 0.665\ \text{nm}$, $\lambda_3 = 0.997\ \text{nm}$