21 ELECTRIC CHARGE AND ELECTRIC FIELD

? Water makes life possible: The cells of your body could not function without water in which to dissolve essential biological molecules. What electrical properties of water make it such a good solvent?

LEARNING GOALS

By studying this chapter, you will learn:

- The nature of electric charge, and how we know that electric charge is conserved.
- How objects become electrically charged.
- How to use Coulomb's law to calculate the electric force between charges.
- The distinction between electric force and electric field.
- How to calculate the electric field due to a collection of charges.
- How to use the idea of electric field lines to visualize and interpret electric fields.
- How to calculate the properties of electric dipoles.

In Chapter 5 we mentioned the four kinds of fundamental forces. To this point the only one of these forces that we have examined in any detail is gravity. Now we are ready to examine the force of *electromagnetism,* which encompasses both electricity and magnetism. Electromagnetic phenomena will occupy our attention for most of the remainder of this book.

Electromagnetic interactions involve particles that have a property called *electric charge,* an attribute that is as fundamental as mass. Just as objects with mass are accelerated by gravitational forces, so electrically charged objects are accelerated by electric forces. The shock you feel when you scuff your shoes across a carpet and then reach for a metal doorknob is due to charged particles leaping between your finger and the doorknob. Electric currents are simply streams of charged particles flowing within wires in response to electric forces. Even the forces that hold atoms together to form solid matter, and that keep the atoms of solid objects from passing through each other, are fundamentally due to electric interactions between the charged particles within atoms.

We begin our study of electromagnetism in this chapter by examining the nature of electric charge. We'll find that charge is quantized and obeys a conservation principle. When charges are at rest in our frame of reference, they exert *electrostatic* forces on each other. These forces are of tremendous importance in chemistry and biology and have many technological applications. Electrostatic forces are governed by a simple relationship known as *Coulomb's law* and are most conveniently described by using the concept of *electric field.* In later chapters we'll expand our discussion to include electric charges in motion. This will lead us to an understanding of magnetism and, remarkably, of the nature of light.

While the key ideas of electromagnetism are conceptually simple, applying them to practical problems will make use of many of your mathematical skills, especially your knowledge of geometry and integral calculus. For this reason you may find this chapter and those that follow to be more mathematically demanding than earlier chapters. The reward for your extra effort will be a deeper understanding of principles that are at the heart of modern physics and technology.

21.1 Electric Charge

The ancient Greeks discovered as early as 600 B.C. that after they rubbed amber with wool, the amber could attract other objects. Today we say that the amber has acquired a net **electric charge,** or has become *charged.* The word "electric" is derived from the Greek word *elektron,* meaning amber. When you scuff your shoes across a nylon carpet, you become electrically charged, and you can charge a comb by passing it through dry hair.

Plastic rods and fur (real or fake) are particularly good for demonstrating **electrostatics,** the interactions between electric charges that are at rest (or nearly so). After we charge both plastic rods in Fig. 21.1a by rubbing them with the piece of fur, we find that the rods repel each other.

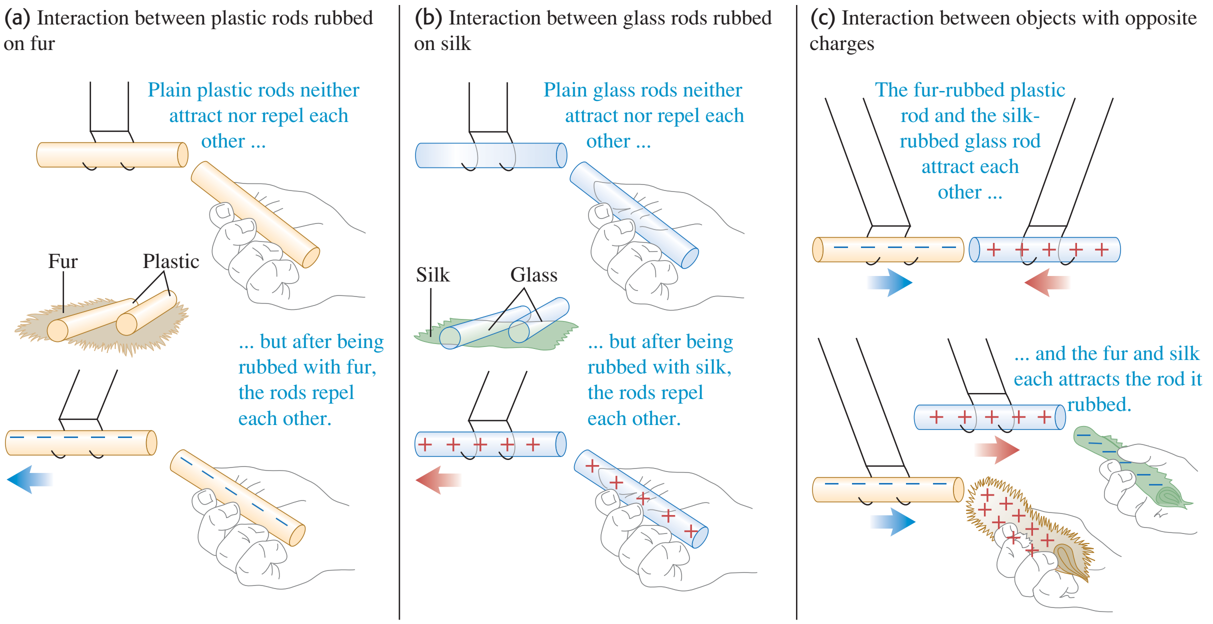
When we rub glass rods with silk, the glass rods also become charged and repel each other (Fig. 21.1b). But a charged plastic rod *attracts* a charged glass rod; furthermore, the plastic rod and the fur attract each other, and the glass rod and the silk attract each other (Fig. 21.1c).

These experiments and many others like them have shown that there are exactly two kinds of electric charge: the kind on the plastic rod rubbed with fur and the kind on the glass rod rubbed with silk. Benjamin Franklin (1706–1790) suggested calling these two kinds of charge *negative* and *positive,* respectively, and these names are still used. The plastic rod and the silk have negative charge; the glass rod and the fur have positive charge.

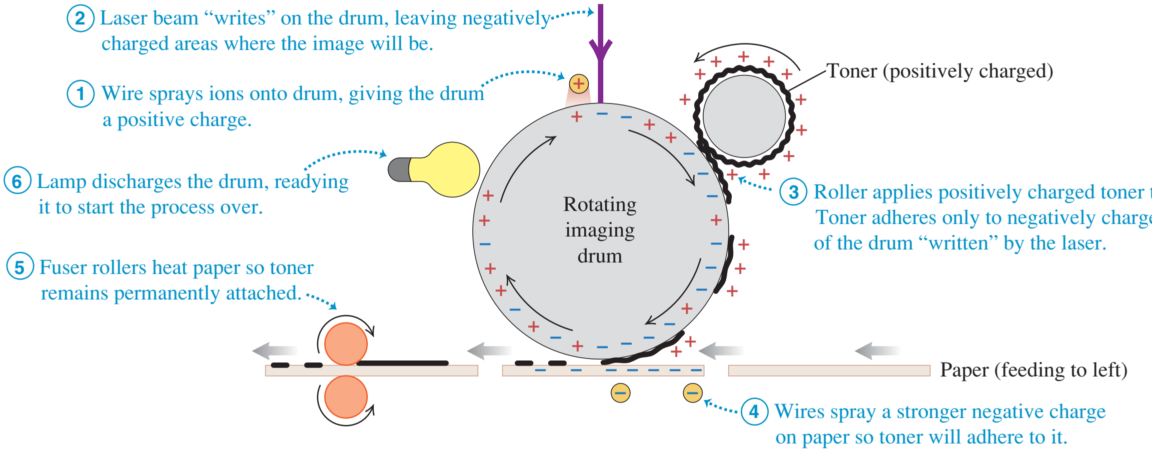
Two positive charges or two negative charges repel each other. A positive charge and a negative charge attract each other.

CAUTION **Electric attraction and repulsion** The attraction and repulsion of two charged objects are sometimes summarized as "Like charges repel, and opposite charges attract." But keep in mind that the phrase "like charges" does *not* mean that the two charges are exactly identical, only that both charges have the same algebraic *sign* (both positive or both negative). "Opposite charges" means that both objects have an electric charge, and those charges have different signs (one positive and the other negative).

21.1 Experiments in electrostatics. (a) Negatively charged objects repel each other. (b) Positively charged objects repel each other. (c) Positvely charged objects and negatively charged objects attract each other.



21.2 Schematic diagram of the operation of a laser printer.



One application of forces between charged bodies is in a laser printer (Fig. 21.2). The printer's light-sensitive imaging drum is given a positive charge. As the drum rotates, a laser beam shines on selected areas of the drum, leaving those areas with a *negative* charge. Positively charged particles of toner adhere only to the areas of the drum "written" by the laser. When a piece of paper is placed in contact with the drum, the toner particles stick to the paper and form an image.

Electric Charge and the Structure of Matter

When you charge a rod by rubbing it with fur or silk as in Fig. 21.1, there is no visible change in the appearance of the rod. What, then, actually happens to the rod when you charge it? To answer this question, we must look more closely at the structure of atoms, the building blocks of ordinary matter.

The structure of atoms can be described in terms of three particles: the negatively charged **electron**, the positively charged **proton,** and the uncharged **neutron** (Fig. 21.3). The proton and neutron are combinations of other entities called *quarks,* which have charges of $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ times the electron charge. Isolated quarks have not been observed, and there are theoretical reasons to believe that it is impossible in principle to observe a quark in isolation.

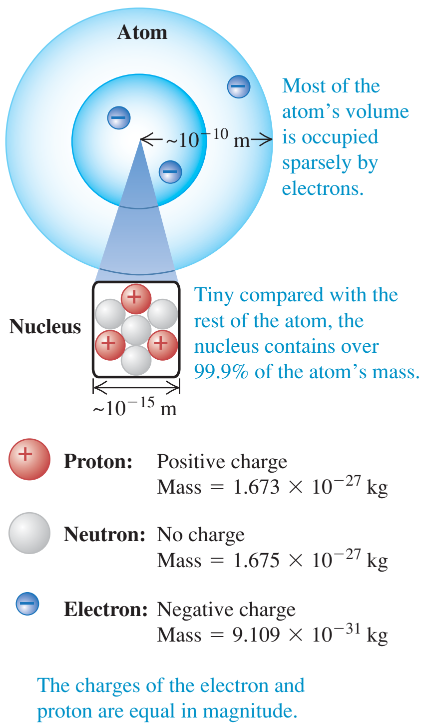
The protons and neutrons in an atom make up a small, very dense core called the **nucleus,** with dimensions of the order of 10^{-15} m. Surrounding the nucleus are the electrons, extending out to distances of the order of 10^{-10} m from the nucleus. If an atom were a few kilometers across, its nucleus would be the size of a tennis ball. The negatively charged electrons are held within the atom by the attractive electric forces exerted on them by the positively charged nucleus. (The protons and neutrons are held within stable atomic nuclei by an attractive interaction, called the *strong nuclear force,* that overcomes the electric repulsion of the protons. The strong nuclear force has a short range, and its effects do not extend far beyond the nucleus.)

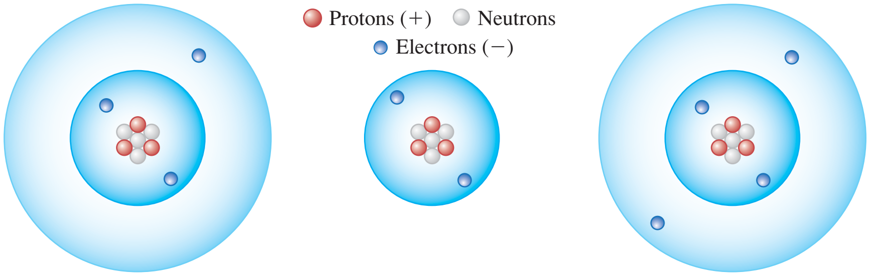
The masses of the individual particles, to the precision that they are presently known, are

$$\text{Mass of electron} = m_e = 9.10938215(45) \times 10^{-31}\ \text{kg}$$
$$\text{Mass of proton} = m_p = 1.672621637(83) \times 10^{-27}\ \text{kg}$$
$$\text{Mass of neutron} = m_n = 1.674927211(84) \times 10^{-27}\ \text{kg}$$

The numbers in parentheses are the uncertainties in the last two digits. Note that the masses of the proton and neutron are nearly equal and are roughly 2000 times

21.3 The structure of an atom. The particular atom depicted here is lithium (see Fig. 21.4a).





21.4 (a) A neutral atom has as many electrons as it does protons. (b) A positive ion has a deficit of electrons. (c) A negative ion has an excess of electrons. (The electron "shells" are a schematic representation of the actual electron distribution, a diffuse cloud many times larger than the nucleus.)

(a) **Neutral lithium atom (Li):**

3 protons (3+)
4 neutrons
3 electrons (3−)

Electrons equal protons:
Zero net charge

(b) **Positive lithium ion (Li^+):**

3 protons (3+)
4 neutrons
2 electrons (2−)

Fewer electrons than protons:
Positive net charge

(c) **Negative lithium ion (Li^-):**

3 protons (3+)
4 neutrons
4 electrons (4−)

More electrons than protons:
Negative net charge

the mass of the electron. Over 99.9% of the mass of any atom is concentrated in its nucleus.

The negative charge of the electron has (within experimental error) *exactly* the same magnitude as the positive charge of the proton. In a neutral atom the number of electrons equals the number of protons in the nucleus, and the net electric charge (the algebraic sum of all the charges) is exactly zero (Fig. 21.4a). The number of protons or electrons in a neutral atom of an element is called the **atomic number** of the element. If one or more electrons are removed from an atom, what remains is called a **positive ion** (Fig. 21.4b). A **negative ion** is an atom that has *gained* one or more electrons (Fig. 21.4c). This gain or loss of electrons is called **ionization.**

When the total number of protons in a macroscopic body equals the total number of electrons, the total charge is zero and the body as a whole is electrically neutral. To give a body an excess negative charge, we may either *add negative* charges to a neutral body or *remove positive* charges from that body. Similarly, we can create an excess positive charge by either *adding positive* charge or *removing negative* charge. In most cases, negatively charged (and highly mobile) electrons are added or removed, and a "positively charged body" is one that has lost some of its normal complement of electrons. When we speak of the charge of a body, we always mean its *net* charge. The net charge is always a very small fraction (typically no more than 10^{-12}) of the total positive charge or negative charge in the body.

Electric Charge Is Conserved

Implicit in the foregoing discussion are two very important principles. First is the **principle of conservation of charge:**

The algebraic sum of all the electric charges in any closed system is constant.

If we rub together a plastic rod and a piece of fur, both initially uncharged, the rod acquires a negative charge (since it takes electrons from the fur) and the fur acquires a positive charge of the *same* magnitude (since it has lost as many electrons as the rod has gained). Hence the total electric charge on the two bodies together does not change. In any charging process, charge is not created or destroyed; it is merely *transferred* from one body to another.

Conservation of charge is thought to be a *universal* conservation law. No experimental evidence for any violation of this principle has ever been observed. Even in high-energy interactions in which particles are created and destroyed, such as the creation of electron–positron pairs, the total charge of any closed system is exactly constant.

The second important principle is:

The magnitude of charge of the electron or proton is a natural unit of charge.

Every observable amount of electric charge is always an integer multiple of this basic unit. We say that charge is *quantized.* A familiar example of quantization is money. When you pay cash for an item in a store, you have to do it in one-cent increments. Cash can't be divided into amounts smaller than one cent, and electric charge can't be divided into amounts smaller than the charge of one electron or proton. (The quark charges, $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ of the electron charge, are probably not observable as isolated charges.) Thus the charge on any macroscopic body is always either zero or an integer multiple (negative or positive) of the electron charge.

Understanding the electric nature of matter gives us insight into many aspects of the physical world (Fig. 21.5). The chemical bonds that hold atoms together to form molecules are due to electric interactions between the atoms. They include the strong ionic bonds that hold sodium and chlorine atoms together to make table salt and the relatively weak bonds between the strands of DNA that record your body's genetic code. The normal force exerted on you by the chair in which you're sitting arises from electric forces between charged particles in the atoms of your seat and in the atoms of your chair. The tension force in a stretched string and the adhesive force of glue are likewise due to the electric interactions of atoms.

21.5 Most of the forces on this water skier are electric. Electric interactions between adjacent molecules give rise to the force of the water on the ski, the tension in the tow rope, and the resistance of the air on the skier's body. Electric interactions also hold the atoms of the skier's body together. Only one wholly nonelectric force acts on the skier: the force of gravity.

Test Your Understanding of Section 21.1 (a) Strictly speaking, does the plastic rod in Fig. 21.1 weigh more, less, or the same after rubbing it with fur? (b) What about the glass rod after rubbing it with silk? What about (c) the fur and (d) the silk?

21.2 Conductors, Insulators, and Induced Charges

MasteringPHYSICS®

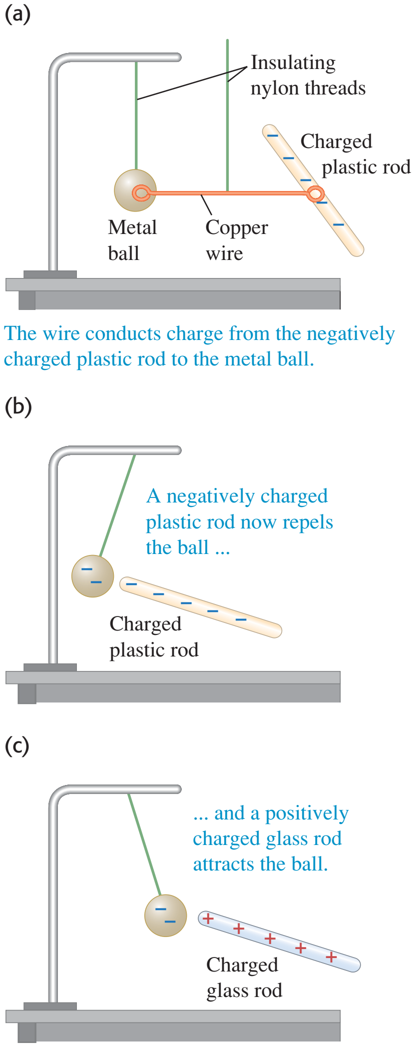
PhET: Balloons and Static Electricity
PhET: John Travoltage

Some materials permit electric charge to move easily from one region of the material to another, while others do not. For example, Fig. 21.6a shows a copper wire supported by a nylon thread. Suppose you touch one end of the wire to a charged plastic rod and attach the other end to a metal ball that is initially uncharged; you then remove the charged rod and the wire. When you bring another charged body up close to the ball (Figs. 21.6b and 21.6c), the ball is attracted or repelled, showing that the ball has become electrically charged. Electric charge has been transferred through the copper wire between the ball and the surface of the plastic rod.

The copper wire is called a **conductor** of electricity. If you repeat the experiment using a rubber band or nylon thread in place of the wire, you find that *no* charge is transferred to the ball. These materials are called **insulators.** Conductors permit the easy movement of charge through them, while insulators do not. (The supporting nylon threads shown in Fig. 21.6 are insulators, which prevents charge from leaving the metal ball and copper wire.)

As an example, carpet fibers on a dry day are good insulators. As you walk across a carpet, the rubbing of your shoes against the fibers causes charge to build up on you, and this charge remains on you because it can't flow through the insulating fibers. If you then touch a conducting object such as a doorknob, a rapid charge transfer takes place between your finger and the doorknob, and you feel a shock. One way to prevent this is to wind some of the carpet fibers around conducting cores so that any charge that builds up on you can be transferred harmlessly to the carpet. Another solution is to coat the carpet fibers with an antistatic layer that does not easily transfer electrons to or from your shoes; this prevents any charge from building up on you in the first place.

21.6 Copper is a good conductor of electricity; nylon is a good insulator. (a) The copper wire conducts charge between the metal ball and the charged plastic rod to charge the ball negatively. Afterward, the metal ball is (b) repelled by a negatively charged plastic rod and (c) attracted to a positively charged glass rod.



Most metals are good conductors, while most nonmetals are insulators. Within a solid metal such as copper, one or more outer electrons in each atom become detached and can move freely throughout the material, just as the molecules of a gas can move through the spaces between the grains in a bucket of sand. The other electrons remain bound to the positively charged nuclei, which themselves are bound in nearly fixed positions within the material. In an insulator there are no, or very few, free electrons, and electric charge cannot move freely through the material. Some materials called *semiconductors* are intermediate in their properties between good conductors and good insulators.

Charging by Induction

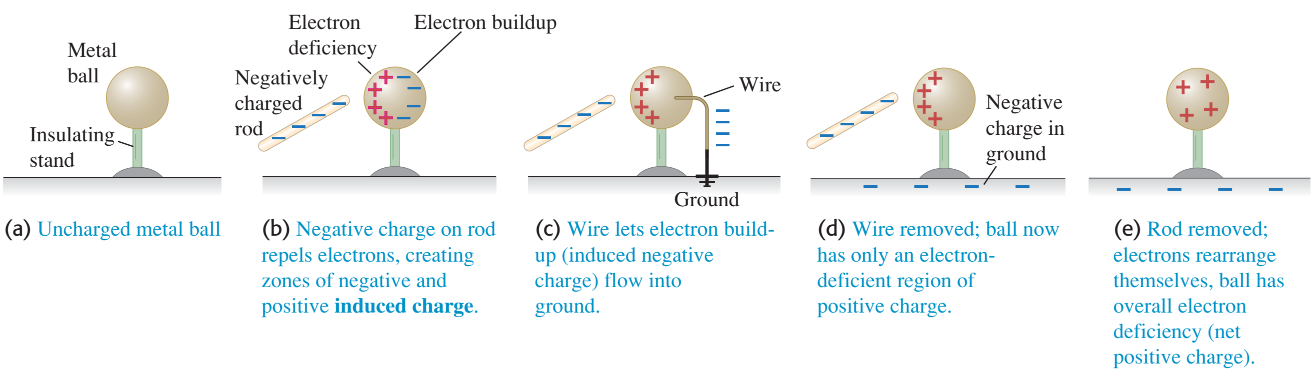
We can charge a metal ball using a copper wire and an electrically charged plastic rod, as in Fig. 21.6a. In this process, some of the excess electrons on the rod are transferred from it to the ball, leaving the rod with a smaller negative charge. But there is a different technique in which the plastic rod can give another body a charge of *opposite* sign without losing any of its own charge. This process is called charging by **induction.**

Figure 21.7 shows an example of charging by induction. An uncharged metal ball is supported on an insulating stand (Fig. 21.7a). When you bring a negatively charged rod near it, without actually touching it (Fig. 21.7b), the free electrons in the metal ball are repelled by the excess electrons on the rod, and they shift toward the right, away from the rod. They cannot escape from the ball because the supporting stand and the surrounding air are insulators. So we get excess negative charge at the right surface of the ball and a deficiency of negative charge (that is, a net positive charge) at the left surface. These excess charges are called **induced charges.**

Not all of the free electrons move to the right surface of the ball. As soon as any induced charge develops, it exerts forces toward the *left* on the other free electrons. These electrons are repelled by the negative induced charge on the right and attracted toward the positive induced charge on the left. The system reaches an equilibrium state in which the force toward the right on an electron, due to the charged rod, is just balanced by the force toward the left due to the induced charge. If we remove the charged rod, the free electrons shift back to the left, and the original neutral condition is restored.

What happens if, while the plastic rod is nearby, you touch one end of a conducting wire to the right surface of the ball and the other end to the earth (Fig. 21.7c)? The earth is a conductor, and it is so large that it can act as a practically infinite source of extra electrons or sink of unwanted electrons. Some of the negative charge flows through the wire to the earth. Now suppose you disconnect the wire (Fig. 21.7d) and then remove the rod (Fig. 21.7e); a net positive charge is left on the ball. The charge on the negatively charged rod has not changed during this process. The earth acquires a negative charge that is equal in magnitude to the induced positive charge remaining on the ball.

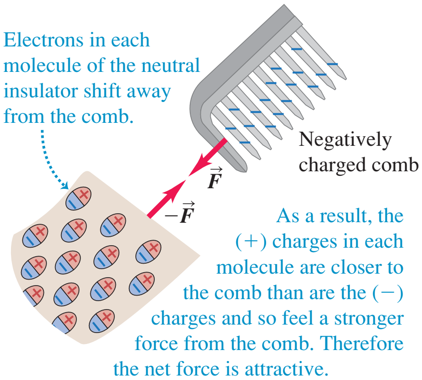
21.7 Charging a metal ball by induction.



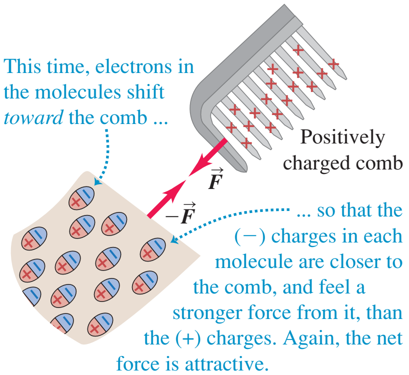
21.8 The charges within the molecules of an insulating material can shift slightly. As a result, a comb with either sign of charge attracts a neutral insulator. By Newton's third law the neutral insulator exerts an equal-magnitude attractive force on the comb.

(a) A charged comb picking up uncharged pieces of plastic

(b) How a negatively charged comb attracts an insulator



(c) How a positively charged comb attracts an insulator

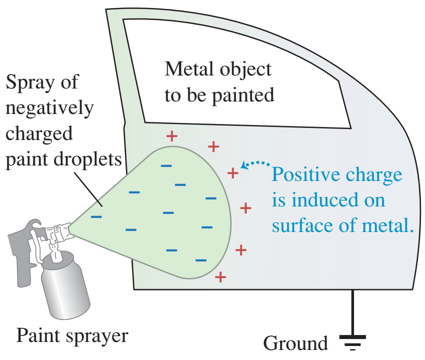


Electric Forces on Uncharged Objects

Finally, we note that a charged body can exert forces even on objects that are *not* charged themselves. If you rub a balloon on the rug and then hold the balloon against the ceiling, it sticks, even though the ceiling has no net electric charge. After you electrify a comb by running it through your hair, you can pick up uncharged bits of paper or plastic with the comb (Fig. 21.8a). How is this possible?

This interaction is an induced-charge effect. Even in an insulator, electric charges can shift back and forth a little when there is charge nearby. This is shown in Fig. 21.8b; the negatively charged plastic comb causes a slight shifting of charge within the molecules of the neutral insulator, an effect called *polarization.* The positive and negative charges in the material are present in equal amounts, but the positive charges are closer to the plastic comb and so feel an attraction that is stronger than the repulsion felt by the negative charges, giving a net attractive force. (In Section 21.3 we will study how electric forces depend on distance.) Note that a neutral insulator is also attracted to a *positively* charged comb (Fig. 21.8c). Now the charges in the insulator shift in the opposite direction; the negative charges in the insulator are closer to the comb and feel an attractive force that is stronger than the repulsion felt by the positive charges in the insulator. Hence a charged object of *either* sign exerts an attractive force on an uncharged insulator. Figure 21.9 shows an industrial application of this effect.

21.9 The electrostatic painting process (compare Figs. 21.7b and 21.7c). A metal object to be painted is connected to the earth ("ground"), and the paint droplets are given an electric charge as they exit the sprayer nozzle. Induced charges of the opposite sign appear in the object as the droplets approach, just as in Fig. 21.7b, and they attract the droplets to the surface. This process minimizes overspray from clouds of stray paint particles and gives a particularly smooth finish.



Test Your Understanding of Section 21.2 You have two lightweight metal spheres, each hanging from an insulating nylon thread. One of the spheres has a net negative charge, while the other sphere has no net charge. (a) If the spheres are close together but do not touch, will they (i) attract each other, (ii) repel each other, or (iii) exert no force on each other? (b) You now allow the two spheres to touch. Once they have touched, will the two spheres (i) attract each other, (ii) repel each other, or (iii) exert no force on each other?

21.3 Coulomb's Law

Charles Augustin de Coulomb (1736–1806) studied the interaction forces of charged particles in detail in 1784. He used a torsion balance (Fig. 21.10a) similar to the one used 13 years later by Cavendish to study the much weaker gravitational interaction, as we discussed in Section 13.1. For **point charges,** charged

MasteringPHYSICS

ActivPhysics 11.1: Electric Force: Coulomb's Law
ActivPhysics 11.2: Electric Force: Superposition Principle
ActivPhysics 11.3: Electric Force: Superposition (Quantitative)

Application **Electric Forces, Sweat, and Cystic Fibrosis**

One way to test for the genetic disease cystic fibrosis (CF) is by measuring the salt content of a person's sweat. Sweat is a mixture of water and ions, including the sodium (Na^+) and chloride (Cl^-) ions that make up ordinary salt (NaCl). When sweat is secreted by epithelial cells, some of the Cl^- ions flow from the sweat back into these cells (a process called reabsorption). The electric attraction between negative and positive charges pulls Na^+ ions along with the Cl^-. Water molecules cannot flow back into the epithelial cells, so sweat on the skin has a low salt content. However, in persons with CF the reabsorption of Cl^- ions is blocked. Hence the sweat of persons with CF is unusually salty, with up to four times the normal concentration of Cl^- and Na^+.

bodies that are very small in comparison with the distance r between them, Coulomb found that the electric force is proportional to $1/r^2$. That is, when the distance r doubles, the force decreases to one-quarter of its initial value; when the distance is halved, the force increases to four times its initial value.

The electric force between two point charges also depends on the quantity of charge on each body, which we will denote by q or Q. To explore this dependence, Coulomb divided a charge into two equal parts by placing a small charged spherical conductor into contact with an identical but uncharged sphere; by symmetry, the charge is shared equally between the two spheres. (Note the essential role of the principle of conservation of charge in this procedure.) Thus he could obtain one-half, one-quarter, and so on, of any initial charge. He found that the forces that two point charges q_1 and q_2 exert on each other are proportional to each charge and therefore are proportional to the *product* q_1q_2 of the two charges.

Thus Coulomb established what we now call **Coulomb's law:**

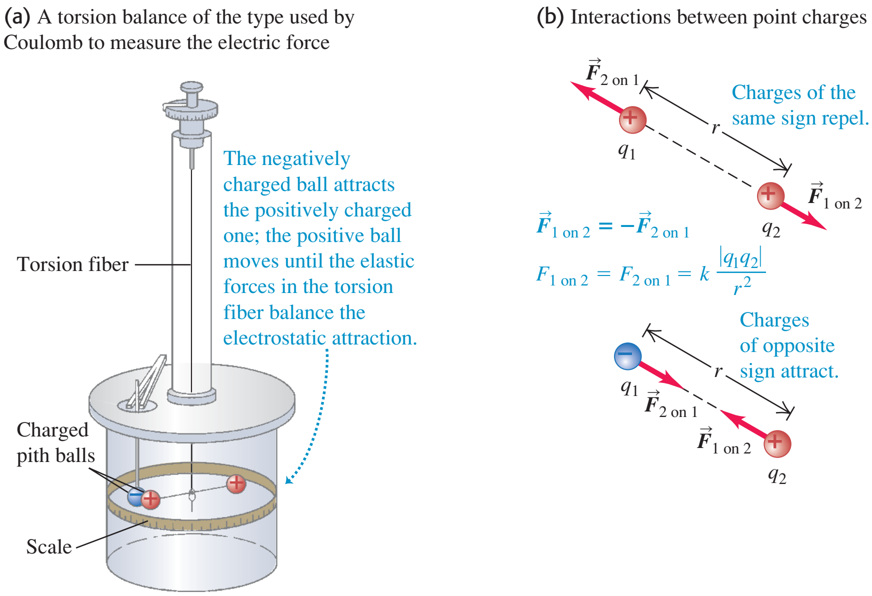
The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

In mathematical terms, the magnitude F of the force that each of two point charges q_1 and q_2 a distance r apart exerts on the other can be expressed as

$$F = k\frac{|q_1q_2|}{r^2} \tag{21.1}$$

where k is a proportionality constant whose numerical value depends on the system of units used. The absolute value bars are used in Eq. (21.1) because the charges q_1 and q_2 can be either positive or negative, while the force magnitude F is always positive.

The directions of the forces the two charges exert on each other are always along the line joining them. When the charges q_1 and q_2 have the same sign, either both positive or both negative, the forces are repulsive; when the charges have opposite signs, the forces are attractive (Fig. 21.10b). The two forces obey Newton's third law; they are always equal in magnitude and opposite in direction, even when the charges are not equal in magnitude.



21.10 (a) Measuring the electric force between point charges. (b) The electric forces between point charges obey Newton's third law: $\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$.

The proportionality of the electric force to $1/r^2$ has been verified with great precision. There is no reason to suspect that the exponent is different from precisely 2. Thus the form of Eq. (21.1) is the same as that of the law of gravitation. But electric and gravitational interactions are two distinct classes of phenomena. Electric interactions depend on electric charges and can be either attractive or repulsive, while gravitational interactions depend on mass and are always attractive (because there is no such thing as negative mass).

Fundamental Electric Constants

The value of the proportionality constant k in Coulomb's law depends on the system of units used. In our study of electricity and magnetism we will use SI units exclusively. The SI electric units include most of the familiar units such as the volt, the ampere, the ohm, and the watt. (There is *no* British system of electric units.) The SI unit of electric charge is called one **coulomb** (1 C). In SI units the constant k in Eq. (21.1) is

$$k = 8.987551787 \times 10^9\ \mathrm{N \cdot m^2/C^2} \cong 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

The value of k is known to such a large number of significant figures because this value is closely related to the speed of light in vacuum. (We will show this in Chapter 32 when we study electromagnetic radiation.) As we discussed in Section 1.3, this speed is *defined* to be exactly $c = 2.99792458 \times 10^8\ \mathrm{m/s}$. The numerical value of k is defined in terms of c to be precisely

$$k = (10^{-7}\ \mathrm{N \cdot s^2/C^2})c^2$$

You should check this expression to confirm that k has the right units.

In principle we can measure the electric force F between two equal charges q at a measured distance r and use Coulomb's law to determine the charge. Thus we could regard the value of k as an operational definition of the coulomb. For reasons of experimental precision it is better to define the coulomb instead in terms of a unit of electric *current* (charge per unit time), the *ampere,* equal to 1 coulomb per second. We will return to this definition in Chapter 28.

In SI units we usually write the constant k in Eq. (21.1) as $1/4\pi\epsilon_0$, where ϵ_0 ("epsilon-nought" or "epsilon-zero") is another constant. This appears to complicate matters, but it actually simplifies many formulas that we will encounter in later chapters. From now on, we will usually write Coulomb's law as

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \qquad \text{(Coulomb's law: force between two point charges)} \qquad (21.2)$$

The constants in Eq. (21.2) are approximately

$$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \quad \text{and} \quad \frac{1}{4\pi\epsilon_0} = k = 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

In examples and problems we will often use the approximate value

$$\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

which is within about 0.1% of the correct value.

As we mentioned in Section 21.1, the most fundamental unit of charge is the magnitude of the charge of an electron or a proton, which is denoted by e. The most precise value available as of the writing of this book is

$$e = 1.602176487(40) \times 10^{-19}\ \mathrm{C}$$

One coulomb represents the negative of the total charge of about 6×10^{18} electrons. For comparison, a copper cube 1 cm on a side contains about 2.4×10^{24}

electrons. About 10^{19} electrons pass through the glowing filament of a flashlight bulb every second.

In electrostatics problems (that is, problems that involve charges at rest), it's very unusual to encounter charges as large as 1 coulomb. Two 1-C charges separated by 1 m would exert forces on each other of magnitude 9×10^9 N (about 1 million tons)! The total charge of all the electrons in a copper one-cent coin is even greater, about 1.4×10^5 C, which shows that we can't disturb electric neutrality very much without using enormous forces. More typical values of charge range from about 10^{-9} to about 10^{-6} C. The microcoulomb $(1\ \mu\text{C} = 10^{-6}\ \text{C})$ and the nanocoulomb $(1\ \text{nC} = 10^{-9}\ \text{C})$ are often used as practical units of charge.

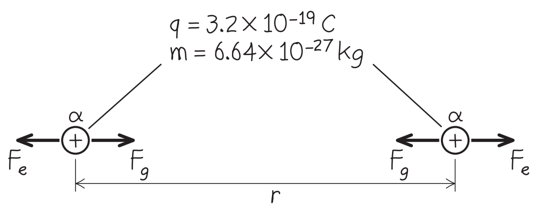
Example 21.1 Electric force versus gravitational force

An α particle (the nucleus of a helium atom) has mass $m = 6.64 \times 10^{-27}$ kg and charge $q = +2e = 3.2 \times 10^{-19}$ C. Compare the magnitude of the electric repulsion between two α ("alpha") particles with that of the gravitational attraction between them.

SOLUTION

IDENTIFY and SET UP: This problem involves Newton's law for the gravitational force F_g between particles (see Section 13.1) and Coulomb's law for the electric force F_e between point charges. To compare these forces, we make our target variable the ratio F_e/F_g. We use Eq. (21.2) for F_e and Eq. (13.1) for F_g.

21.11 Our sketch for this problem.



EXECUTE: Figure 21.11 shows our sketch. From Eqs. (21.2) and (13.1),

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} \qquad F_g = G\frac{m^2}{r^2}$$

These are both inverse-square forces, so the r^2 factors cancel when we take the ratio:

$$\frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0 G}\frac{q^2}{m^2}$$

$$= \frac{9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}\frac{(3.2 \times 10^{-19}\ \text{C})^2}{(6.64 \times 10^{-27}\ \text{kg})^2}$$

$$= 3.1 \times 10^{35}$$

EVALUATE: This astonishingly large number shows that the gravitational force in this situation is completely negligible in comparison to the electric force. This is always true for interactions of atomic and subnuclear particles. But within objects the size of a person or a planet, the positive and negative charges are nearly equal in magnitude, and the net electric force is usually much *smaller* than the gravitational force.

Superposition of Forces

Coulomb's law as we have stated it describes only the interaction of two *point* charges. Experiments show that when two charges exert forces simultaneously on a third charge, the total force acting on that charge is the *vector sum* of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces,** holds for any number of charges. By using this principle, we can apply Coulomb's law to *any* collection of charges. Two of the examples at the end of this section use the superposition principle.

Strictly speaking, Coulomb's law as we have stated it should be used only for point charges *in a vacuum.* If matter is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening material. We will describe this effect later. As a practical matter, though, we can use Coulomb's law unaltered for point charges in air. At normal atmospheric pressure, the presence of air changes the electric force from its vacuum value by only about one part in 2000.

Problem-Solving Strategy 21.1 Coulomb's Law

IDENTIFY *the relevant concepts:* Coulomb's law describes the electric force between charged particles.

SET UP *the problem* using the following steps:
1. Sketch the locations of the charged particles and label each particle with its charge.
2. If the charges do not all lie on a single line, set up an *xy*-coordinate system.
3. The problem will ask you to find the electric force on one or more particles. Identify which these are.

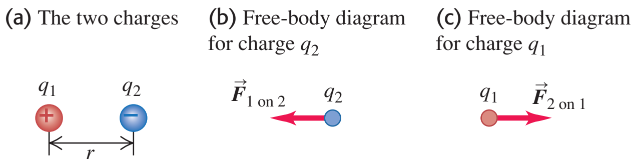
EXECUTE *the solution* as follows:
1. For each particle that exerts an electric force on a given particle of interest, use Eq. (21.2) to calculate the magnitude of that force.
2. Using those magnitudes, sketch a free-body diagram showing the electric force vectors acting on each particle of interest. The force exerted by particle 1 on particle 2 points from particle 2 toward particle 1 if the charges have opposite signs, but points from particle 2 directly away from particle 1 if the charges have the same sign.
3. Use the principle of superposition to calculate the total electric force—a *vector* sum—on each particle of interest. (Review the vector algebra in Sections 1.7 through 1.9. The method of components is often helpful.)
4. Use consistent units; SI units are completely consistent. With $1/4\pi\epsilon_0 = 9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$, distances must be in meters, charges in coulombs, and forces in newtons.
5. Some examples and problems in this and later chapters involve *continuous* distributions of charge along a line, over a surface, or throughout a volume. In these cases the vector sum in step 3 becomes a vector *integral*. We divide the charge distribution into infinitesimal pieces, use Coulomb's law for each piece, and integrate to find the vector sum. Sometimes this can be done without actual integration.
6. Exploit any symmetries in the charge distribution to simplify your problem solving. For example, two identical charges q exert zero net electric force on a charge Q midway between them, because the forces on Q have equal magnitude and opposite direction.

EVALUATE *your answer:* Check whether your numerical results are reasonable. Confirm that the direction of the net electric force agrees with the principle that charges of the same sign repel and charges of opposite sign attract.

Example 21.2 Force between two point charges

Two point charges, $q_1 = +25$ nC and $q_2 = -75$ nC, are separated by a distance $r = 3.0$ cm (Fig. 21.12a). Find the magnitude and direction of the electric force (a) that q_1 exerts on q_2 and (b) that q_2 exerts on q_1.

21.12 What force does q_1 exert on q_2, and what force does q_2 exert on q_1? Gravitational forces are negligible.



SOLUTION

IDENTIFY and SET UP: This problem asks for the electric forces that two charges exert on each other. We use Coulomb's law, Eq. (21.2), to calculate the magnitudes of the forces. The signs of the charges will determine the directions of the forces.

EXECUTE: (a) After converting the units of r to meters and the units of q_1 and q_2 to coulombs, Eq. (21.2) gives us

$$F_{1\text{ on }2} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2}$$
$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{|(+25 \times 10^{-9}\ \text{C})(-75 \times 10^{-9}\ \text{C})|}{(0.030\ \text{m})^2}$$
$$= 0.019\ \text{N}$$

The charges have opposite signs, so the force is attractive (to the left in Fig. 21.12b); that is, the force that acts on q_2 is directed toward q_1 along the line joining the two charges.

(b) Proceeding as in part (a), we have

$$F_{1\text{ on }2} = \frac{1}{4\pi\epsilon_0}\frac{|q_2q_1|}{r^2} = F_{2\text{ on }1} = 0.019\ \text{N}$$

The attractive force that acts on q_1 is to the right, toward q_2 (Fig. 21.12c).

EVALUATE: Newton's third law applies to the electric force. Even though the charges have different magnitudes, the magnitude of the force that q_2 exerts on q_1 is the same as the magnitude of the force that q_1 exerts on q_2, and these two forces are in opposite directions.

Example 21.3 Vector addition of electric forces on a line

Two point charges are located on the *x*-axis of a coordinate system: $q_1 = 1.0$ nC is at $x = +2.0$ cm, and $q_2 = -3.0$ nC is at $x = +4.0$ cm. What is the total electric force exerted by q_1 and q_2 on a charge $q_3 = 5.0$ nC at $x = 0$?

SOLUTION

IDENTIFY and SET UP: Figure 21.13a shows the situation. To find the total force on q_3, our target variable, we find the vector sum of the two electric forces on it.

Continued

EXECUTE: Figure 21.13b is a free-body diagram for q_3, which is repelled by q_1 (which has the same sign) and attracted to q_2 (which has the opposite sign): $\vec{F}_{1\text{ on }3}$ is in the $-x$-direction and $\vec{F}_{2\text{ on }3}$ is in the $+x$-direction. After unit conversions, we have from Eq. (21.2)

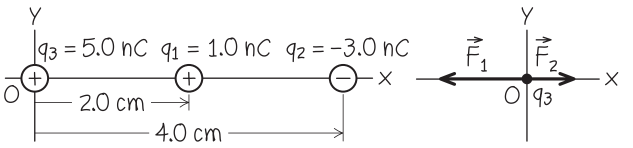
$$F_{1\text{ on }3} = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r_{13}^2}$$
$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.020\ \text{m})^2}$$
$$= 1.12 \times 10^{-4}\ \text{N} = 112\ \mu\text{N}$$

In the same way you can show that $F_{2\text{ on }3} = 84\ \mu\text{N}$. We thus have $\vec{F}_{1\text{ on }3} = (-112\ \mu\text{N})\hat{\imath}$ and $\vec{F}_{2\text{ on }3} = (84\ \mu\text{N})\hat{\imath}$. The net force on q_3 is

$$\vec{F}_3 = \vec{F}_{1\text{ on }3} + \vec{F}_{2\text{ on }3} = (-112\ \mu\text{N})\hat{\imath} + (84\ \mu\text{N})\hat{\imath} = (-28\ \mu\text{N})\hat{\imath}$$

21.13 Our sketches for this problem.

(a) Our diagram of the situation (b) Free-body diagram for q_3



EVALUATE: As a check, note that the magnitude of q_2 is three times that of q_1, but q_2 is twice as far from q_3 as q_1. Equation (21.2) then says that $F_{2\text{ on }3}$ must be $3/2^2 = 3/4 = 0.75$ as large as $F_{1\text{ on }3}$. This agrees with our calculated values: $F_{2\text{ on }3}/F_{1\text{ on }3} = (84\ \mu\text{N})/(112\ \mu\text{N}) = 0.75$. Because $F_{2\text{ on }3}$ is the weaker force, the direction of the net force is that of $\vec{F}_{1\text{ on }3}$—that is, in the negative x-direction.

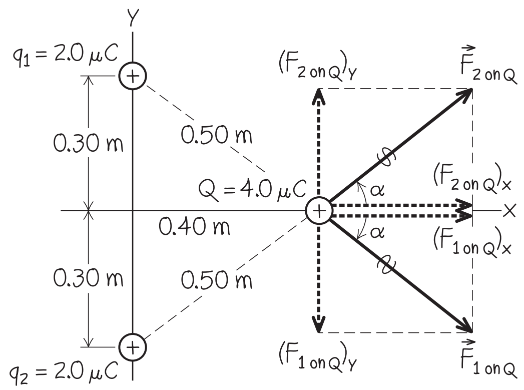
Example 21.4 Vector addition of electric forces in a plane

Two equal positive charges $q_1 = q_2 = 2.0\ \mu\text{C}$ are located at $x = 0$, $y = 0.30$ m and $x = 0$, $y = -0.30$ m, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0\ \mu\text{C}$ at $x = 0.40$ m, $y = 0$?

SOLUTION

IDENTIFY and SET UP: As in Example 21.3, we must compute the force that each charge exerts on Q and then find the vector sum of those forces. Figure 21.14 shows the situation. Since the three charges do not all lie on a line, the best way to calculate the forces is to use components.

21.14 Our sketch for this problem.



EXECUTE: Figure 21.14 shows the forces $\vec{F}_{1\text{ on }Q}$ and $\vec{F}_{2\text{ on }Q}$ due to the identical charges q_1 and q_2, which are at equal distances from Q. From Coulomb's law, *both* forces have magnitude

$$F_{1\text{ or }2\text{ on }Q} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \frac{(4.0 \times 10^{-6}\ \text{C})(2.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2} = 0.29\ \text{N}$$

The x-components of the two forces are equal:

$$(F_{1\text{ or }2\text{ on }Q})_x = (F_{1\text{ or }2\text{ on }Q})\cos\alpha = (0.29\ \text{N})\frac{0.40\ \text{m}}{0.50\ \text{m}} = 0.23\ \text{N}$$

From symmetry we see that the y-components of the two forces are equal and opposite. Hence their sum is zero and the total force $\vec{F}$ on Q has only an x-component $F_x = 0.23\ \text{N} + 0.23\ \text{N} = 0.46$ N. The total force on Q is in the $+x$-direction, with magnitude 0.46 N.

EVALUATE: The total force on Q points neither directly away from q_1 nor directly away from q_2. Rather, this direction is a compromise that points away from the *system* of charges q_1 and q_2. Can you see that the total force would *not* be in the $+x$-direction if q_1 and q_2 were not equal or if the geometrical arrangement of the changes were not so symmetric?

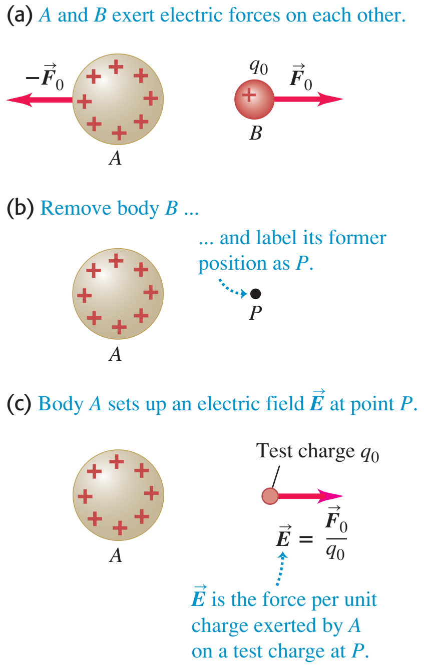
Test Your Understanding of Section 21.3 Suppose that charge q_2 in Example 21.4 were $-2.0\ \mu\text{C}$. In this case, the total electric force on Q would be (i) in the positive x-direction; (ii) in the negative x-direction; (iii) in the positive y-direction; (iv) in the negative y-direction; (v) zero; (vi) none of these.

21.4 Electric Field and Electric Forces

When two electrically charged particles in empty space interact, how does each one know the other is there? We can begin to answer this question, and at the same time reformulate Coulomb's law in a very useful way, by using the concept of *electric field.*

Electric Field

To introduce this concept, let's look at the mutual repulsion of two positively charged bodies A and B (Fig. 21.15a). Suppose B has charge q_0, and let $\vec{F}_0$ be the electric force of A on B. One way to think about this force is as an "action-at-a-distance" force—that is, as a force that acts across empty space without needing any matter (such as a push rod or a rope) to transmit it through the intervening space. (Gravity can also be thought of as an "action-at-a-distance" force.) But a more fruitful way to visualize the repulsion between A and B is as a two-stage process. We first envision that body A, as a result of the charge that it carries, somehow *modifies the properties of the space around it.* Then body B, as a result of the charge that *it* carries, senses how space has been modified at its position. The response of body B is to experience the force $\vec{F}_0$.



21.15 A charged body creates an electric field in the space around it.

To elaborate how this two-stage process occurs, we first consider body A by itself: We remove body B and label its former position as point P (Fig. 21.15b). We say that the charged body A produces or causes an **electric field** at point P (and at all other points in the neighborhood). This electric field is present at P even if there is no charge at P; it is a consequence of the charge on body A only. If a point charge q_0 is then placed at point P, it experiences the force $\vec{F}_0$. We take the point of view that this force is exerted on q_0 *by the field* at P (Fig. 21.15c). Thus the electric field is the intermediary through which A communicates its presence to q_0. Because the point charge q_0 would experience a force at *any* point in the neighborhood of A, the electric field that A produces exists at all points in the region around A.

We can likewise say that the point charge q_0 produces an electric field in the space around it and that this electric field exerts the force $-\vec{F}_0$ on body A. For each force (the force of A on q_0 and the force of q_0 on A), one charge sets up an electric field that exerts a force on the second charge. We emphasize that this is an *interaction* between *two* charged bodies. A single charge produces an electric field in the surrounding space, but this electric field cannot exert a net force on the charge that created it; as we discussed in Section 4.3, a body cannot exert a net force on itself. (If this wasn't true, you would be able to lift yourself to the ceiling by pulling up on your belt!)

The electric force on a charged body is exerted by the electric field created by *other* charged bodies.

Application Sharks and the "Sixth Sense"

Sharks have the ability to locate prey (such as flounder and other bottom-dwelling fish) that are completely hidden beneath the sand at the bottom of the ocean. They do this by sensing the weak electric fields produced by muscle contractions in their prey. Sharks derive their sensitivity to electric fields (a "sixth sense") from jelly-filled canals in their bodies. These canals end in pores on the shark's skin (shown in this photograph). An electric field as weak as 5×10^{-7} N/C causes charge flow within the canals and triggers a signal in the shark's nervous system. Because the shark has canals with different orientations, it can measure different components of the electric-field vector and hence determine the direction of the field.

To find out experimentally whether there is an electric field at a particular point, we place a small charged body, which we call a **test charge,** at the point (Fig. 21.15c). If the test charge experiences an electric force, then there is an electric field at that point. This field is produced by charges other than q_0.

Force is a vector quantity, so electric field is also a vector quantity. (Note the use of vector signs as well as boldface letters and plus, minus, and equals signs in the following discussion.) We define the *electric field* $\vec{E}$ at a point as the electric force $\vec{F}_0$ experienced by a test charge q_0 at the point, divided by the charge q_0. That is, the electric field at a certain point is equal to the *electric force per unit charge* experienced by a charge at that point:

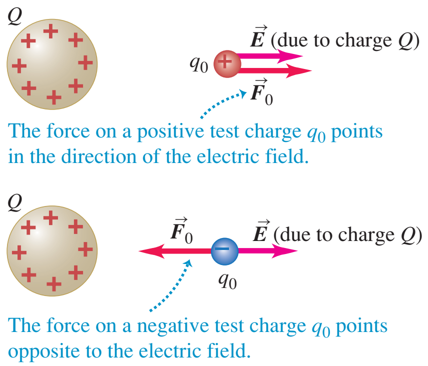
$$\vec{E} = \frac{\vec{F}_0}{q_0} \quad \text{(definition of electric field as electric force per unit charge)} \tag{21.3}$$

In SI units, in which the unit of force is 1 N and the unit of charge is 1 C, the unit of electric field magnitude is 1 newton per coulomb (1 N/C).

If the field $\vec{E}$ at a certain point is known, rearranging Eq. (21.3) gives the force $\vec{F}_0$ experienced by a point charge q_0 placed at that point. This force is just equal to the electric field $\vec{E}$ produced at that point by charges other than q_0, multiplied by the charge q_0:

$$\vec{F}_0 = q_0\vec{E} \quad \text{(force exerted on a point charge } q_0 \text{ by an electric field } \vec{E}\text{)} \tag{21.4}$$

21.16 The force $\vec{F}_0 = q_0\vec{E}$ exerted on a point charge q_0 placed in an electric field $\vec{E}$.



MasteringPHYSICS

ActivPhysics 11.4: Electric Field: Point Charge
ActivPhysics 11.9: Motion of a Charge in an Electric Field: Introduction
ActivPhysics 11.10: Motion in an Electric Field: Problems

The charge q_0 can be either positive or negative. If q_0 is *positive,* the force $\vec{F}_0$ experienced by the charge is the same direction as $\vec{E}$; if q_0 is *negative,* $\vec{F}_0$ and $\vec{E}$ are in opposite directions (Fig. 21.16).

While the electric field concept may be new to you, the basic idea—that one body sets up a field in the space around it and a second body responds to that field—is one that you've actually used before. Compare Eq. (21.4) to the familiar expression for the gravitational force $\vec{F}_g$ that the earth exerts on a mass m_0:

$$\vec{F}_g = m_0\vec{g} \tag{21.5}$$

In this expression, $\vec{g}$ is the acceleration due to gravity. If we divide both sides of Eq. (21.5) by the mass m_0, we obtain

$$\vec{g} = \frac{\vec{F}_g}{m_0}$$

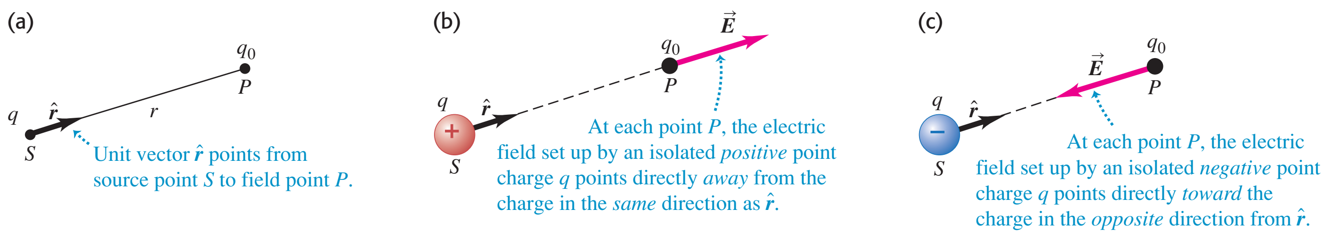
Thus $\vec{g}$ can be regarded as the gravitational force per unit mass. By analogy to Eq. (21.3), we can interpret $\vec{g}$ as the *gravitational field.* Thus we treat the gravitational interaction between the earth and the mass m_0 as a two-stage process: The earth sets up a gravitational field $\vec{g}$ in the space around it, and this gravitational field exerts a force given by Eq. (21.5) on the mass m_0 (which we can regard as a *test mass*). The gravitational field $\vec{g}$, or gravitational force per unit mass, is a useful concept because it does not depend on the mass of the body on which the gravitational force is exerted; likewise, the electric field $\vec{E}$, or electric force per unit charge, is useful because it does not depend on the charge of the body on which the electric force is exerted.

CAUTION **$\vec{F}_0 = q_0\vec{E}_0$ is for *point* test charges only** The electric force experienced by a test charge q_0 can vary from point to point, so the electric field can also be different at different points. For this reason, Eq. (21.4) can be used only to find the electric force on a *point* charge. If a charged body is large enough in size, the electric field $\vec{E}$ may be noticeably different in magnitude and direction at different points on the body, and calculating the net electric force on the body can become rather complicated.

Electric Field of a Point Charge

If the source distribution is a point charge q, it is easy to find the electric field that it produces. We call the location of the charge the **source point,** and we call the point P where we are determining the field the **field point.** It is also useful to introduce a *unit vector* $\hat{r}$ that points along the line from source point to field point (Fig. 21.17a). This unit vector is equal to the displacement vector $\vec{r}$ from the source point to the field point, divided by the distance $r = |\vec{r}|$ between these two points; that is, $\hat{r} = \vec{r}/r$. If we place a small test charge q_0 at the field point P, at a

21.17 The electric field $\vec{E}$ produced at point P by an isolated point charge q at S. Note that in both (b) and (c), $\vec{E}$ is *produced* by q [see Eq. (21.7)] but *acts* on the charge q_0 at point P [see Eq. (21.4)].



distance r from the source point, the magnitude F_0 of the force is given by Coulomb's law, Eq. (21.2):

$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{|qq_0|}{r^2}$$

From Eq. (21.3) the magnitude E of the electric field at P is

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \quad \text{(magnitude of electric field of a point charge)} \tag{21.6}$$

Using the unit vector $\hat{\boldsymbol{r}}$, we can write a *vector* equation that gives both the magnitude and direction of the electric field $\vec{\boldsymbol{E}}$:

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}} \quad \text{(electric field of a point charge)} \tag{21.7}$$

By definition, the electric field of a point charge always points *away from* a positive charge (that is, in the same direction as $\hat{\boldsymbol{r}}$; see Fig. 21.17b) but *toward* a negative charge (that is, in the direction opposite $\hat{\boldsymbol{r}}$; see Fig. 21.17c).

We have emphasized calculating the electric field $\vec{\boldsymbol{E}}$ at a certain point. But since $\vec{\boldsymbol{E}}$ can vary from point to point, it is not a single vector quantity but rather an *infinite* set of vector quantities, one associated with each point in space. This is an example of a **vector field.** Figure 21.18 shows a number of the field vectors produced by a positive or negative point charge. If we use a rectangular (x, y, z) coordinate system, each component of $\vec{\boldsymbol{E}}$ at any point is in general a function of the coordinates (x, y, z) of the point. We can represent the functions as $E_x(x, y, z)$, $E_y(x, y, z)$, and $E_z(x, y, z)$. Vector fields are an important part of the language of physics, not just in electricity and magnetism. One everyday example of a vector field is the velocity $\vec{\boldsymbol{v}}$ of wind currents; the magnitude and direction of $\vec{\boldsymbol{v}}$, and hence its vector components, vary from point to point in the atmosphere.

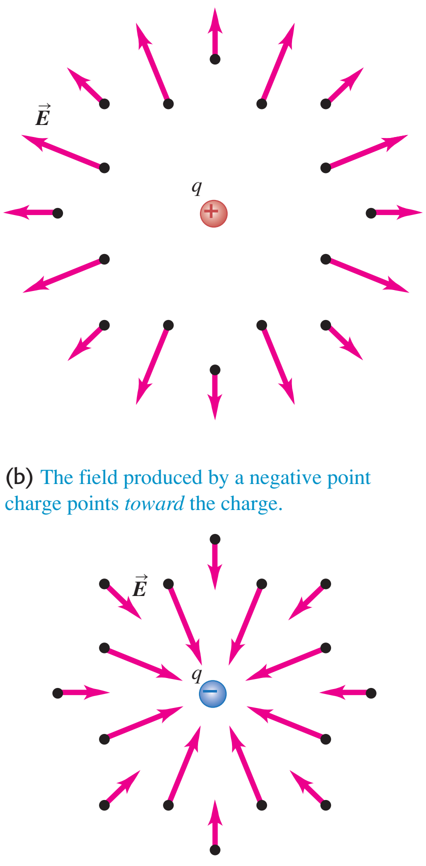
In some situations the magnitude and direction of the field (and hence its vector components) have the same values everywhere throughout a certain region; we then say that the field is *uniform* in this region. An important example of this is the electric field inside a *conductor.* If there is an electric field within a conductor, the field exerts a force on every charge in the conductor, giving the free charges a net motion. By definition an electrostatic situation is one in which the charges have *no* net motion. We conclude that *in electrostatics the electric field at every point within the material of a conductor must be zero.* (Note that we are not saying that the field is necessarily zero in a *hole* inside a conductor.)

In summary, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in the field. Our analysis often has two corresponding steps: first, calculating the field caused by a source charge distribution; second, looking at the effect of the field in terms of force and motion. The second step often involves Newton's laws as well as the principles of electric interactions. In the next section we show how to calculate fields caused by various source distributions, but first here are three examples of calculating the field due to a point charge and of finding the force on a charge due to a given field $\vec{\boldsymbol{E}}$.

21.18 A point charge q produces an electric field $\vec{\boldsymbol{E}}$ at *all* points in space. The field strength decreases with increasing distance.

(a) The field produced by a positive point charge points *away from* the charge.

(b) The field produced by a negative point charge points *toward* the charge.



Example 21.5 Electric-field magnitude for a point charge

What is the magnitude of the electric field $\vec{\boldsymbol{E}}$ at a field point 2.0 m from a point charge $q = 4.0$ nC?

SOLUTION

IDENTIFY and SET UP: This problem concerns the electric field due to a point charge. We are given the magnitude of the charge

Continued

and the distance from the charge to the field point, so we use Eq. (21.6) to calculate the field magnitude E.

EXECUTE: From Eq. (21.6),

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{4.0 \times 10^{-9}\ \mathrm{C}}{(2.0\ \mathrm{m})^2}$$
$$= 9.0\ \mathrm{N/C}$$

EVALUATE: Our result $E = 9.0\ \mathrm{N/C}$ means that if we placed a 1.0-C charge at a point 2.0 m from q, it would experience a 9.0-N force. The force on a 2.0-C charge at that point would be $(2.0\ \mathrm{C})(9.0\ \mathrm{N/C}) = 18\ \mathrm{N}$, and so on.

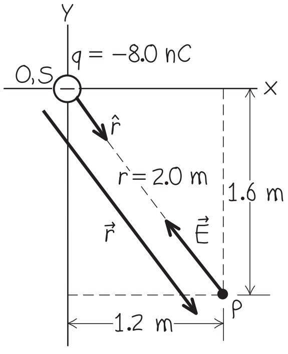
Example 21.6 Electric-field vector for a point charge

A point charge $q = -8.0$ nC is located at the origin. Find the electric-field vector at the field point $x = 1.2$ m, $y = -1.6$ m.

SOLUTION

IDENTIFY and SET UP: We must find the electric-field *vector* $\vec{E}$ due to a point charge. Figure 21.19 shows the situation. We use Eq. (21.7); to do this, we must find the distance r from the source point S (the position of the charge q, which in this example is at the origin O) to the field point P, and we must obtain an expression for the unit vector $\hat{r} = \vec{r}/r$ that points from S to P.

21.19 Our sketch for this problem.



EXECUTE: The distance from S to P is

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2\ \mathrm{m})^2 + (-1.6\ \mathrm{m})^2} = 2.0\ \mathrm{m}$$

The unit vector $\hat{r}$ is then

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{\imath} + y\hat{\jmath}}{r}$$
$$= \frac{(1.2\ \mathrm{m})\hat{\imath} + (-1.6\ \mathrm{m})\hat{\jmath}}{2.0\ \mathrm{m}} = 0.60\hat{\imath} - 0.80\hat{\jmath}$$

Then, from Eq. (21.7),

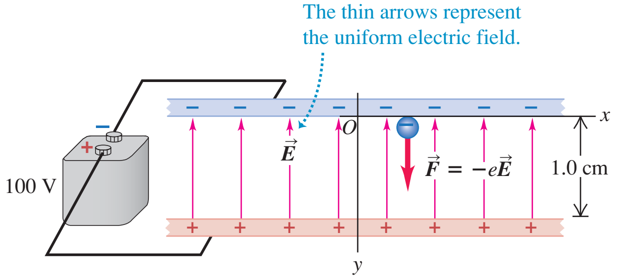
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$
$$= (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{(-8.0 \times 10^{-9}\ \mathrm{C})}{(2.0\ \mathrm{m})^2}(0.60\hat{\imath} - 0.80\hat{\jmath})$$
$$= (-11\ \mathrm{N/C})\hat{\imath} + (14\ \mathrm{N/C})\hat{\jmath}$$

EVALUATE: Since q is negative, $\vec{E}$ points from the field point to the charge (the source point), in the direction opposite to $\hat{r}$ (compare Fig. 21.17c). We leave the calculation of the magnitude and direction of $\vec{E}$ to you (see Exercise 21.36).

Example 21.7 Electron in a uniform field

When the terminals of a battery are connected to two parallel conducting plates with a small gap between them, the resulting charges on the plates produce a nearly uniform electric field $\vec{E}$ between the plates. (In the next section we'll see why this is.) If the plates are 1.0 cm apart and are connected to a 100-volt battery as shown in Fig. 21.20, the field is vertically upward and has magnitude $E = 1.00 \times 10^4\ \mathrm{N/C}$. (a) If an electron (charge $-e = -1.60 \times 10^{-9}$ C, mass $m = 9.11 \times 10^{-31}$ kg) is released from rest at the upper plate, what is its acceleration? (b) What speed and kinetic energy does it acquire while traveling 1.0 cm to the lower plate? (c) How long does it take to travel this distance?

21.20 A uniform electric field between two parallel conducting plates connected to a 100-volt battery. (The separation of the plates is exaggerated in this figure relative to the dimensions of the plates.)



SOLUTION

IDENTIFY and SET UP: This example involves the relationship between electric field and electric force. It also involves the relationship between force and acceleration, the definition of kinetic energy, and the kinematic relationships among acceleration, distance, velocity, and time. Figure 21.20 shows our coordinate system. We are given the electric field, so we use Eq. (21.4) to find the force on the electron and Newton's second law to find its acceleration. Because the field is uniform, the force is constant and we can use the constant-acceleration formulas from Chapter 2 to find the electron's velocity and travel time. We find the kinetic energy using $K = \frac{1}{2}mv^2$.

EXECUTE: (a) Although $\vec{E}$ is upward (in the $+y$-direction), $\vec{F}$ is downward (because the electron's charge is negative) and so F_y is negative. Because F_y is constant, the electron's acceleration is constant:

$$a_y = \frac{F_y}{m} = \frac{-eE}{m} = \frac{(-1.60 \times 10^{-19}\ \mathrm{C})(1.00 \times 10^4\ \mathrm{N/C})}{9.11 \times 10^{-31}\ \mathrm{kg}}$$
$$= -1.76 \times 10^{15}\ \mathrm{m/s^2}$$

(b) The electron starts from rest, so its motion is in the y-direction only (the direction of the acceleration). We can find the electron's speed at any position y using the constant-acceleration Eq. (2.13), $v_y{}^2 = v_{0y}{}^2 + 2a_y(y - y_0)$. We have $v_{0y} = 0$ and $y_0 = 0$, so at $y = -1.0$ cm $= -1.0 \times 10^{-2}$ m we have

$$|v_y| = \sqrt{2a_y y} = \sqrt{2(-1.76 \times 10^{15}\ \mathrm{m/s^2})(-1.0 \times 10^{-2}\ \mathrm{m})}$$
$$= 5.9 \times 10^6\ \mathrm{m/s}$$

The velocity is downward, so $v_y = -5.9 \times 10^6$ m/s. The electron's kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.11 \times 10^{-31}\ \mathrm{kg})(5.9 \times 10^6\ \mathrm{m/s})^2$$
$$= 1.6 \times 10^{-17}\ \mathrm{J}$$

(c) From Eq. (2.8) for constant acceleration, $v_y = v_{0y} + a_y t$,

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{(-5.9 \times 10^6\ \mathrm{m/s}) - (0\ \mathrm{m/s})}{-1.76 \times 10^{15}\ \mathrm{m/s^2}}$$
$$= 3.4 \times 10^{-9}\ \mathrm{s}$$

EVALUATE: Our results show that in problems concerning subatomic particles such as electrons, many quantities—including acceleration, speed, kinetic energy, and time—will have *very* different values from those typical of everyday objects such as baseballs and automobiles.

Test Your Understanding of Section 21.4 (a) A negative point charge moves along a straight-line path directly toward a stationary positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction. (b) A negative point charge moves along a circular orbit around a positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction.

21.5 Electric-Field Calculations

MasteringPHYSICS

ActivPhysics 11.5: Electric Field Due to a Dipole
ActivPhysics 11.6: Electric Field: Problems

Equation (21.7) gives the electric field caused by a single point charge. But in most realistic situations that involve electric fields and forces, we encounter charge that is *distributed* over space. The charged plastic and glass rods in Fig. 21.1 have electric charge distributed over their surfaces, as does the imaging drum of a laser printer (Fig. 21.2). In this section we'll learn to calculate electric fields caused by various distributions of electric charge. Calculations of this kind are of tremendous importance for technological applications of electric forces. To determine the trajectories of atomic nuclei in an accelerator for cancer radiotherapy or of charged particles in a semiconductor electronic device, you have to know the detailed nature of the electric field acting on the charges.

The Superposition of Electric Fields

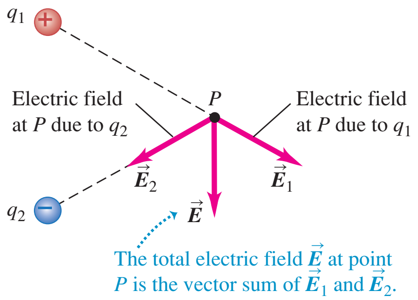
To find the field caused by a charge distribution, we imagine the distribution to be made up of many point charges $q_1, q_2, q_3, \ldots$. (This is actually quite a realistic description, since we have seen that charge is carried by electrons and protons that are so small as to be almost pointlike.) At any given point P, each point charge produces its own electric field $\vec{E}_1, \vec{E}_2, \vec{E}_3, \ldots$, so a test charge q_0 placed at P experiences a force $\vec{F}_1 = q_0\vec{E}_1$ from charge q_1, a force $\vec{F}_2 = q_0\vec{E}_2$ from charge q_2, and so on. From the principle of superposition of forces discussed in Section 21.3, the *total* force $\vec{F}_0$ that the charge distribution exerts on q_0 is the vector sum of these individual forces:

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = q_0\vec{E}_1 + q_0\vec{E}_2 + q_0\vec{E}_3 + \cdots$$

The combined effect of all the charges in the distribution is described by the *total* electric field $\vec{E}$ at point P. From the definition of electric field, Eq. (21.3), this is

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

21.21 Illustrating the principle of superposition of electric fields.



The total electric field at P is the vector sum of the fields at P due to each point charge in the charge distribution (Fig. 21.21). This is the **principle of superposition of electric fields.**

When charge is distributed along a line, over a surface, or through a volume, a few additional terms are useful. For a line charge distribution (such as a long, thin, charged plastic rod), we use λ (the Greek letter lambda) to represent the **linear charge density** (charge per unit length, measured in C/m). When charge is distributed over a surface (such as the surface of the imaging drum of a laser printer), we use σ (sigma) to represent the **surface charge density** (charge per unit area, measured in C/m^2). And when charge is distributed through a volume, we use ρ (rho) to represent the **volume charge density** (charge per unit volume, C/m^3).

Some of the calculations in the following examples may look fairly intricate. After you've worked through the examples one step at a time, the process will seem less formidable. We will use many of the calculational techniques in these examples in Chapter 28 to calculate the *magnetic* fields caused by charges in motion.

Problem-Solving Strategy 21.2 Electric-Field Calculations

IDENTIFY *the relevant concepts:* Use the principle of superposition to calculate the electric field due to a discrete or continuous charge distribution.

SET UP *the problem* using the following steps:
1. Make a drawing showing the locations of the charges and your choice of coordinate axes.
2. On your drawing, indicate the position of the *field point* P (the point at which you want to calculate the electric field $\vec{E}$).

EXECUTE *the solution* as follows:
1. Use consistent units. Distances must be in meters and charge must be in coulombs. If you are given centimeters or nanocoulombs, don't forget to convert.
2. Distinguish between the source point S and the field point P. The field produced by a point charge always points from S to P if the charge is positive, and from P to S if the charge is negative.
3. Use *vector* addition when applying the principle of superposition; review the treatment of vector addition in Chapter 1 if necessary.
4. Simplify your calculations by exploiting any symmetries in the charge distribution.
5. If the charge distribution is continuous, define a small element of charge that can be considered as a point, find its electric field at P, and find a way to add the fields of all the charge elements by doing an integral. Usually it is easiest to do this for each component of $\vec{E}$ separately, so you may need to evaluate more than one integral. Ensure that the limits on your integrals are correct; especially when the situation has symmetry, don't count a charge twice.

EVALUATE *your answer:* Check that the direction of $\vec{E}$ is reasonable. If your result for the electric-field magnitude E is a function of position (say, the coordinate x), check your result in any limits for which you know what the magnitude should be. When possible, check your answer by calculating it in a different way.

Example 21.8 Field of an electric dipole

Point charges $q_1 = +12$ nC and $q_2 = -12$ nC are 0.100 m apart (Fig. 21.22). (Such pairs of point charges with equal magnitude and opposite sign are called *electric dipoles*.) Compute the electric field caused by q_1, the field caused by q_2, and the total field (a) at point a; (b) at point b; and (c) at point c.

SOLUTION

IDENTIFY and SET UP: We must find the total electric field at various points due to two point charges. We use the principle of superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2$. Figure 21.22 shows the coordinate system and the locations of the field points a, b, and c.

EXECUTE: At each field point, $\vec{E}$ depends on $\vec{E}_1$ and $\vec{E}_2$ there; we first calculate the magnitudes E_1 and E_2 at each field point. At a the magnitude of the field $\vec{E}_{1a}$ caused by q_1 is

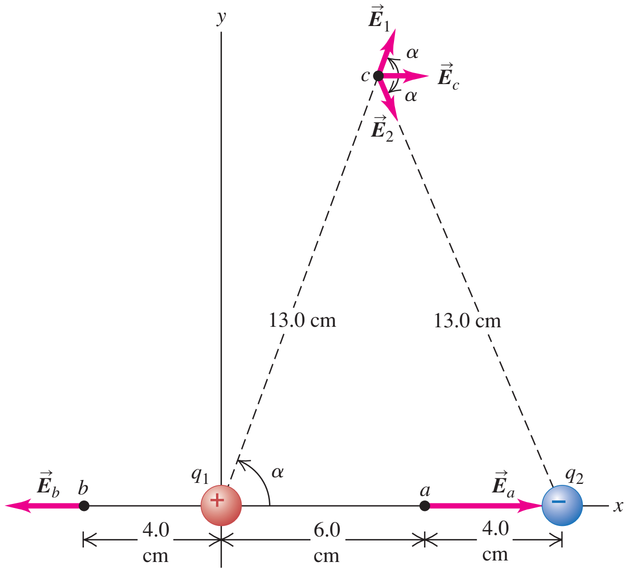
$$E_{1a} = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.060\ \text{m})^2}$$
$$= 3.0 \times 10^4\ \text{N/C}$$

We calculate the other field magnitudes in a similar way. The results are

$$E_{1a} = 3.0 \times 10^4\ \text{N/C} \qquad E_{1b} = 6.8 \times 10^4\ \text{N/C}$$
$$E_{1c} = 6.39 \times 10^3\ \text{N/C}$$
$$E_{2a} = 6.8 \times 10^4\ \text{N/C} \qquad E_{2b} = 0.55 \times 10^4\ \text{N/C}$$
$$E_{2c} = E_{1c} = 6.39 \times 10^3\ \text{N/C}$$

The *directions* of the corresponding fields are in all cases *away* from the positive charge q_1 and *toward* the negative charge q_2.

21.22 Electric field at three points, a, b, and c, set up by charges q_1 and q_2, which form an electric dipole.



(a) At a, $\vec{E}_{1a}$ and $\vec{E}_{2a}$ are both directed to the right, so

$$\vec{E}_a = E_{1a}\hat{\imath} + E_{2a}\hat{\imath} = (9.8 \times 10^4\ \text{N/C})\hat{\imath}$$

(b) At b, $\vec{E}_{1b}$ is directed to the left and $\vec{E}_{2b}$ is directed to the right, so

$$\vec{E}_b = -E_{1b}\hat{\imath} + E_{2b}\hat{\imath} = (-6.2 \times 10^4\ \text{N/C})\hat{\imath}$$

(c) Figure 21.22 shows the directions of $\vec{E}_1$ and $\vec{E}_2$ at c. Both vectors have the same x-component:

$$E_{1cx} = E_{2cx} = E_{1c}\cos\alpha = (6.39 \times 10^3\ \text{N/C})\left(\tfrac{5}{13}\right)$$
$$= 2.46 \times 10^3\ \text{N/C}$$

From symmetry, E_{1y} and E_{2y} are equal and opposite, so their sum is zero. Hence

$$\vec{E}_c = 2(2.46 \times 10^3\ \text{N/C})\hat{\imath} = (4.9 \times 10^3\ \text{N/C})\hat{\imath}$$

EVALUATE: We can also find $\vec{E}_c$ using Eq. (21.7) for the field of a point charge. The displacement vector $\vec{r}_1$ from q_1 to point c is $\vec{r}_1 = r\cos\alpha\,\hat{\imath} + r\sin\alpha\,\hat{\jmath}$. Hence the unit vector that points from q_1 to point c is $\hat{r}_1 = \vec{r}_1/r = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$. By symmetry, the unit vector that points from q_2 to point c has the opposite x-component but the same y-component: $\hat{r}_2 = -\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$. We can now use Eq. (21.7) to write the fields $\vec{E}_{1c}$ and $\vec{E}_{2c}$ at c in vector form, then find their sum. Since $q_2 = -q_1$ and the distance r to c is the same for both charges,

$$\vec{E}_c = \vec{E}_{1c} + \vec{E}_{2c} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}_1 + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}\hat{r}_2$$
$$= \frac{1}{4\pi\epsilon_0 r^2}(q_1\hat{r}_1 + q_2\hat{r}_2) = \frac{q_1}{4\pi\epsilon_0 r^2}(\hat{r}_1 - \hat{r}_2)$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(2\cos\alpha\,\hat{\imath})$$
$$= 2(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.13\ \text{m})^2}\left(\tfrac{5}{13}\right)\hat{\imath}$$
$$= (4.9 \times 10^3\ \text{N/C})\hat{\imath}$$

This is the same as we calculated in part (c).

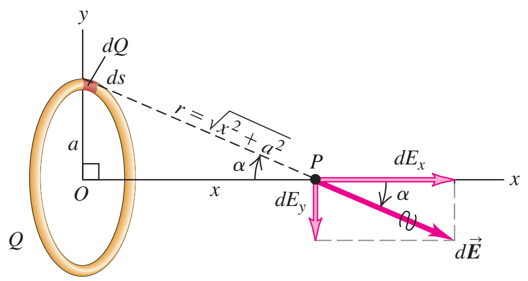
Example 21.9 Field of a ring of charge

Charge Q is uniformly distributed around a conducting ring of radius a (Fig. 21.23). Find the electric field at a point P on the ring axis at a distance x from its center.

SOLUTION

IDENTIFY and SET UP: This is a problem in the superposition of electric fields. Each bit of charge around the ring produces an electric field at an arbitrary point on the x-axis; our target variable is the total field at this point due to all such bits of charge.

21.23 Calculating the electric field on the axis of a ring of charge. In this figure, the charge is assumed to be positive.



EXECUTE: We divide the ring into infinitesimal segments ds as shown in Fig. 21.23. In terms of the linear charge density $\lambda = Q/2\pi a$, the charge in a segment of length ds is $dQ = \lambda\,ds$. Consider two identical segments, one as shown in the figure at $y = a$ and another halfway around the ring at $y = -a$. From Example 21.4, we see that the net force $d\vec{F}$ they exert on a point test charge at P, and thus their net field $d\vec{E}$, are directed along the x-axis. The same is true for any such pair of segments around the ring, so the *net* field at P is along the x-axis: $\vec{E} = E_x\hat{\imath}$.

To calculate E_x, note that the square of the distance r from a single ring segment to the point P is $r^2 = x^2 + a^2$. Hence the magnitude of this segment's contribution $d\vec{E}$ to the electric field at P is

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x^2 + a^2}$$

The x-component of this field is $dE_x = dE\cos\alpha$. We know $dQ = \lambda\,ds$ and Fig. 21.23 shows that $\cos\alpha = x/r = x/(x^2 + a^2)^{1/2}$, so

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x^2 + a^2}\frac{x}{\sqrt{x^2 + a^2}}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda x}{(x^2 + a^2)^{3/2}}ds$$

Continued

To find E_x we integrate this expression over the entire ring—that is, for s from 0 to $2\pi a$ (the circumference of the ring). The integrand has the same value for all points on the ring, so it can be taken outside the integral. Hence we get

$$E_x = \int dE_x = \frac{1}{4\pi\epsilon_0}\frac{\lambda x}{(x^2 + a^2)^{3/2}}\int_0^{2\pi a} ds$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda x}{(x^2 + a^2)^{3/2}}(2\pi a)$$

$$\vec{\boldsymbol{E}} = E_x\hat{\boldsymbol{\imath}} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}\hat{\boldsymbol{\imath}} \qquad (21.8)$$

EVALUATE: Equation (21.8) shows that $\vec{\boldsymbol{E}} = \mathbf{0}$ at the center of the ring ($x = 0$). This makes sense; charges on opposite sides of the ring push in opposite directions on a test charge at the center, and the vector sum of each such pair of forces is zero. When the field point P is much farther from the ring than the ring's radius, we have $x \gg a$ and the denominator in Eq. (21.8) becomes approximately equal to x^3. In this limit the electric field at P is

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\hat{\boldsymbol{\imath}}$$

That is, when the ring is so far away that its radius is negligible in comparison to the distance x, its field is the same as that of a point charge.

Example 21.10 Field of a charged line segment

Positive charge Q is distributed uniformly along the y-axis between $y = -a$ and $y = +a$. Find the electric field at point P on the x-axis at a distance x from the origin.

SOLUTION

IDENTIFY and SET UP: Figure 21.24 shows the situation. As in Example 21.9, we must find the electric field due to a continuous distribution of charge. Our target variable is an expression for the electric field at P as a function of x. The x-axis is a perpendicular bisector of the segment, so we can use a symmetry argument.

EXECUTE: We divide the line charge of length $2a$ into infinitesimal segments of length dy. The linear charge density is $\lambda = Q/2a$, and the charge in a segment is $dQ = \lambda\, dy = (Q/2a)dy$. The distance r from a segment at height y to the field point P is $r = (x^2 + y^2)^{1/2}$, so the magnitude of the field at P due to the segment at height y is

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{(x^2 + y^2)}$$

Figure 21.24 shows that the x- and y-components of this field are $dE_x = dE\cos\alpha$ and $dE_y = -dE\sin\alpha$, where $\cos\alpha = x/r$ and $\sin\alpha = y/r$. Hence

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{x\,dy}{(x^2 + y^2)^{3/2}}$$

$$dE_y = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{y\,dy}{(x^2 + y^2)^{3/2}}$$

To find the total field at P, we must sum the fields from all segments along the line—that is, we must integrate from $y = -a$ to $y = +a$. You should work out the details of the integration (a table of integrals will help). The results are

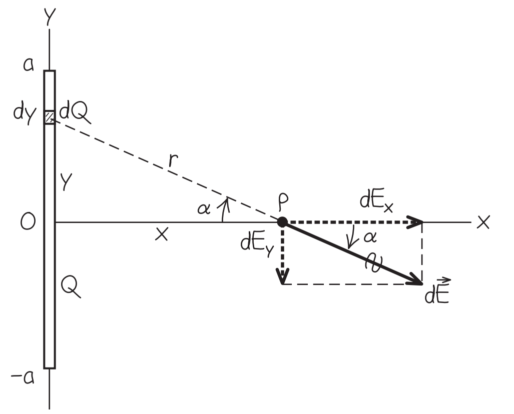
$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{x\,dy}{(x^2 + y^2)^{3/2}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{x\sqrt{x^2 + a^2}}$$

$$E_y = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{y\,dy}{(x^2 + y^2)^{3/2}} = 0$$

or, in vector form,

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2 + a^2}}\hat{\boldsymbol{\imath}} \qquad (21.9)$$

21.24 Our sketch for this problem.



$\vec{\boldsymbol{E}}$ points away from the line of charge if λ is positive and toward the line of charge if λ is negative.

EVALUATE: Using a symmetry argument as in Example 21.9, we could have guessed that E_y would be zero; if we place a positive test charge at P, the upper half of the line of charge pushes downward on it, and the lower half pushes up with equal magnitude. Symmetry also tells us that the upper and lower halves of the segment contribute equally to the total field at P.

If the segment is very *short* (or the field point is very far from the segment) so that $x \gg a$, we can neglect a in the denominator of Eq. (21.9). Then the field becomes that of a point charge, just as in Example 21.9:

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\hat{\boldsymbol{\imath}}$$

To see what happens if the segment is very *long* (or the field point is very close to it) so that $a \gg x$, we first rewrite Eq. (21.9) slightly:

$$\vec{\boldsymbol{E}} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x^2/a^2) + 1}}\hat{\boldsymbol{\imath}} \qquad (21.10)$$

In the limit $a \gg x$ we can neglect x^2/a^2 in the denominator of Eq. (21.10), so

$$\vec{\boldsymbol{E}} = \frac{\lambda}{2\pi\epsilon_0 x}\hat{\boldsymbol{\imath}}$$

This is the field of an *infinitely long* line of charge. At any point P at a perpendicular distance r from the line in *any* direction, $\vec{E}$ has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \qquad \text{(infinite line of charge)}$$

Note that this field is proportional to $1/r$ rather than to $1/r^2$ as for a point charge.

There's really no such thing in nature as an infinite line of charge. But when the field point is close enough to the line, there's very little difference between the result for an infinite line and the real-life finite case. For example, if the distance r of the field point from the center of the line is 1% of the length of the line, the value of E differs from the infinite-length value by less than 0.02%.

Example 21.11 Field of a uniformly charged disk

A nonconducting disk of radius R has a uniform positive surface charge density σ. Find the electric field at a point along the axis of the disk a distance x from its center. Assume that x is positive.

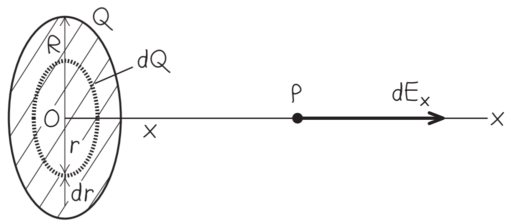
SOLUTION

IDENTIFY and SET UP: Figure 21.25 shows the situation. We represent the charge distribution as a collection of concentric rings of charge dQ. In Example 21.9 we obtained Eq. (21.8) for the field on the axis of a single uniformly charged ring, so all we need do here is integrate the contributions of our rings.

EXECUTE: A typical ring has charge dQ, inner radius r, and outer radius $r + dr$. Its area is approximately equal to its width dr times its circumference $2\pi r$, or $dA = 2\pi r\, dr$. The charge per unit area is $\sigma = dQ/dA$, so the charge of the ring is $dQ = \sigma\, dA = 2\pi\sigma r\, dr$. We use dQ in place of Q in Eq. (21.8), the expression for the field due to a ring that we found in Example 21.9, and replace the ring radius a with r. Then the field component dE_x at point P due to this ring is

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{2\pi\sigma r x\, dr}{(x^2 + r^2)^{3/2}}$$

21.25 Our sketch for this problem.



To find the total field due to all the rings, we integrate dE_x over r from $r = 0$ to $r = R$ (*not* from $-R$ to R):

$$E_x = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{(2\pi\sigma r\, dr)x}{(x^2 + r^2)^{3/2}} = \frac{\sigma x}{4\epsilon_0}\int_0^R \frac{2r\, dr}{(x^2 + r^2)^{3/2}}$$

You can evaluate this integral by making the substitution $t = x^2 + r^2$ (which yields $dt = 2r\, dr$); you can work out the details. The result is

$$E_x = \frac{\sigma x}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x}\right]$$
$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2) + 1}}\right] \qquad (21.11)$$

EVALUATE: If the disk is very large (or if we are very close to it), so that $R \gg x$, the term $1/\sqrt{(R^2/x^2) + 1}$ in Eq. (21.11) is very much less than 1. Then Eq. (21.11) becomes

$$E = \frac{\sigma}{2\epsilon_0} \qquad (21.12)$$

Our final result does not contain the distance x from the plane. Hence the electric field produced by an *infinite* plane sheet of charge is *independent of the distance from the sheet.* The field direction is everywhere perpendicular to the sheet, away from it. There is no such thing as an infinite sheet of charge, but if the dimensions of the sheet are much larger than the distance x of the field point P from the sheet, the field is very nearly given by Eq. (21.12).

If P is to the *left* of the plane $(x < 0)$, the result is the same except that the direction of $\vec{E}$ is to the left instead of the right. If the surface charge density is negative, the directions of the fields on both sides of the plane are toward it rather than away from it.

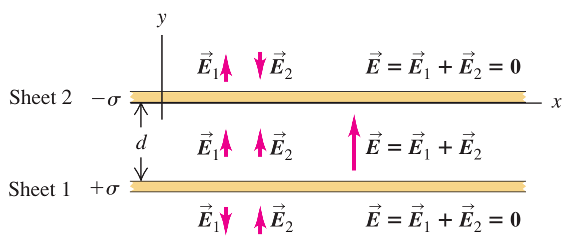
Example 21.12 Field of two oppositely charged infinite sheets

Two infinite plane sheets with uniform surface charge densities $+\sigma$ and $-\sigma$ are placed parallel to each other with separation d (Fig. 21.26). Find the electric field between the sheets, above the upper sheet, and below the lower sheet.

SOLUTION

IDENTIFY and SET UP: Equation (21.12) gives the electric field due to a single infinite plane sheet of charge. To find the field due to *two* such sheets, we combine the fields using the principle of superposition (Fig. 21.26).

21.26 Finding the electric field due to two oppositely charged infinite sheets. The sheets are seen edge-on; only a portion of the infinite sheets can be shown!



Continued

EXECUTE: From Eq. (21.12), both $\vec{E}_1$ and $\vec{E}_2$ have the same magnitude at all points, independent of distance from either sheet:

$$E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$$

From Example 21.11, $\vec{E}_1$ is everywhere directed away from sheet 1, and $\vec{E}_2$ is everywhere directed toward sheet 2.

Between the sheets, $\vec{E}_1$ and $\vec{E}_2$ reinforce each other; above the upper sheet and below the lower sheet, they cancel each other. Thus the total field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} \mathbf{0} & \text{above the upper sheet} \\ \dfrac{\sigma}{\epsilon_0}\hat{\boldsymbol{j}} & \text{between the sheets} \\ \mathbf{0} & \text{below the lower sheet} \end{cases}$$

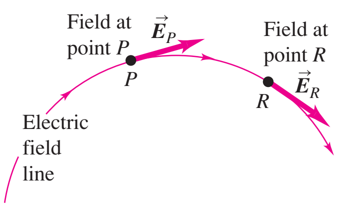
EVALUATE: Because we considered the sheets to be infinite, our result does not depend on the separation d. Our result shows that the field between oppositely charged plates is essentially uniform if the plate separation is much smaller than the dimensions of the plates. We actually used this result in Example 21.7 (Section 21.4).

CAUTION **Electric fields are not "flows"** You may have thought that the field $\vec{E}_1$ of sheet 1 would be unable to "penetrate" sheet 2, and that field $\vec{E}_2$ caused by sheet 2 would be unable to "penetrate" sheet 1. You might conclude this if you think of the electric field as some kind of physical substance that "flows" into or out of charges. But in fact there is no such substance, and the electric fields $\vec{E}_1$ and $\vec{E}_2$ depend only on the individual charge distributions that create them. The *total* field at every point is just the vector sum of $\vec{E}_1$ and $\vec{E}_2$.

Test Your Understanding of Section 21.5 Suppose that the line of charge in Fig. 21.25 (Example 21.11) had charge $+Q$ distributed uniformly between $y = 0$ and $y = +a$ and had charge $-Q$ distributed uniformly between $y = 0$ and $y = -a$. In this situation, the electric field at P would be (i) in the positive x-direction; (ii) in the negative x-direction; (iii) in the positive y-direction; (iv) in the negative y-direction; (v) zero; (vi) none of these.

21.6 Electric Field Lines

21.27 The direction of the electric field at any point is tangent to the field line through that point.



MasteringPHYSICS

PhET: Charges and Fields
PhET: Electric Field of Dreams
PhET: Electric Field Hockey

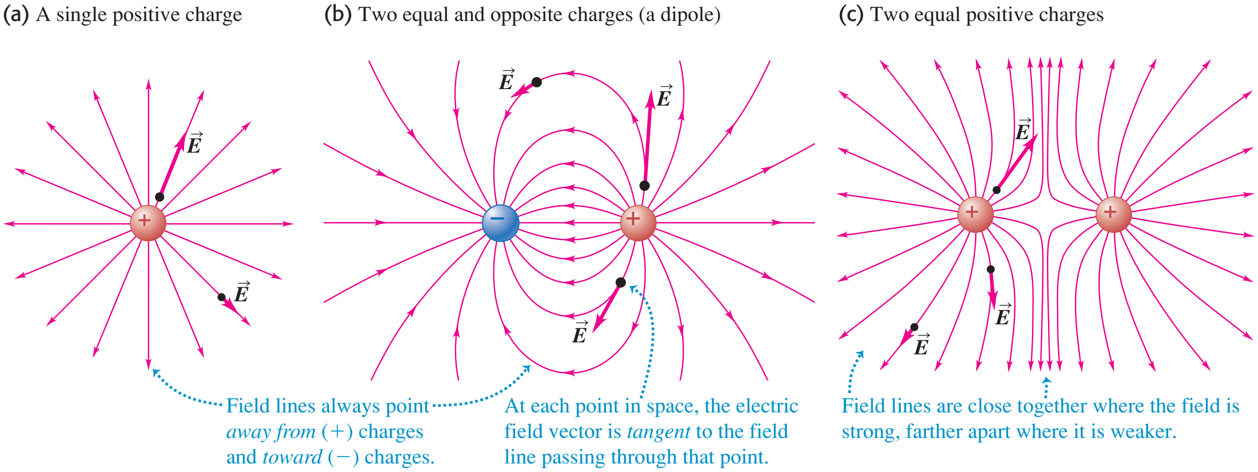
The concept of an electric field can be a little elusive because you can't see an electric field directly. Electric field *lines* can be a big help for visualizing electric fields and making them seem more real. An **electric field line** is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric-field vector at that point. Figure 21.27 shows the basic idea. (We used a similar concept in our discussion of fluid flow in Section 12.5. A *streamline* is a line or curve whose tangent at any point is in the direction of the velocity of the fluid at that point. However, the similarity between electric field lines and fluid streamlines is a mathematical one only; there is nothing "flowing" in an electric field.) The English scientist Michael Faraday (1791–1867) first introduced the concept of field lines. He called them "lines of force," but the term "field lines" is preferable.

Electric field lines show the direction of $\vec{E}$ at each point, and their spacing gives a general idea of the *magnitude* of $\vec{E}$ at each point. Where $\vec{E}$ is strong, we draw lines bunched closely together; where $\vec{E}$ is weaker, they are farther apart. At any particular point, the electric field has a unique direction, so only one field line can pass through each point of the field. In other words, *field lines never intersect.*

Figure 21.28 shows some of the electric field lines in a plane containing (a) a single positive charge; (b) two equal-magnitude charges, one positive and one negative (a dipole); and (c) two equal positive charges. Diagrams such as these are sometimes called *field maps;* they are cross sections of the actual three-dimensional patterns. The direction of the total electric field at every point in each diagram is along the tangent to the electric field line passing through the point. Arrowheads indicate the direction of the $\vec{E}$-field vector along each field line. The actual field vectors have been drawn at several points in each pattern. Notice that in general, the magnitude of the electric field is different at different points on a given field line; a field line is *not* a curve of constant electric-field magnitude!

Figure 21.28 shows that field lines are directed *away* from positive charges (since close to a positive point charge, $\vec{E}$ points away from the charge) and

21.28 Electric field lines for three different charge distributions. In general, the magnitude of $\vec{E}$ is different at different points along a given field line.



toward negative charges (since close to a negative point charge, $\vec{E}$ points toward the charge). In regions where the field magnitude is large, such as between the positive and negative charges in Fig. 21.28b, the field lines are drawn close together. In regions where the field magnitude is small, such as between the two positive charges in Fig. 21.28c, the lines are widely separated. In a *uniform* field, the field lines are straight, parallel, and uniformly spaced, as in Fig. 21.20.

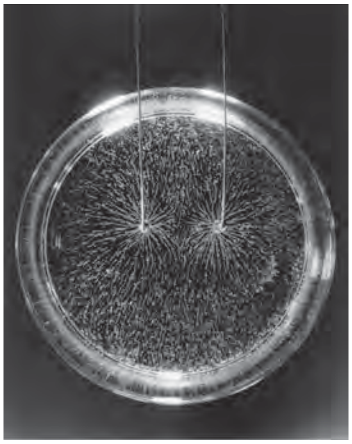
Figure 21.29 is a view from above of a demonstration setup for visualizing electric field lines. In the arrangement shown here, the tips of two positively charged wires are inserted in a container of insulating liquid, and some grass seeds are floated on the liquid. The grass seeds are electrically neutral insulators, but the electric field of the two charged wires causes *polarization* of the grass seeds; there is a slight shifting of the positive and negative charges within the molecules of each seed, like that shown in Fig. 21.8. The positively charged end of each grass seed is pulled in the direction of $\vec{E}$ and the negatively charged end is pulled opposite $\vec{E}$. Hence the long axis of each grass seed tends to orient parallel to the electric field, in the direction of the field line that passes through the position of the seed (Fig. 21.29b).

CAUTION **Electric field lines are not the same as trajectories** It's a common misconception that if a charged particle of charge q is in motion where there is an electric field, the particle must move along an electric field line. Because $\vec{E}$ at any point is tangent to the field line that passes through that point, it is indeed true that the *force* $\vec{F} = q\vec{E}$ on the particle, and hence the particle's acceleration, are tangent to the field line. But we learned in Chapter 3 that when a particle moves on a curved path, its acceleration *cannot* be tangent to the path. So in general, the trajectory of a charged particle is *not* the same as a field line.

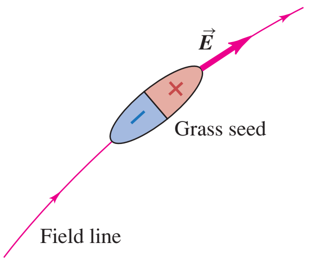
Test Your Understanding of Section 21.6 Suppose the electric field lines in a region of space are straight lines. If a charged particle is released from rest in that region, will the trajectory of the particle be along a field line?

21.29 (a) Electric field lines produced by two equal point charges. The pattern is formed by grass seeds floating on a liquid above two charged wires. Compare this pattern with Fig. 21.28c. (b) The electric field causes polarization of the grass seeds, which in turn causes the seeds to align with the field.

(a)



(b)

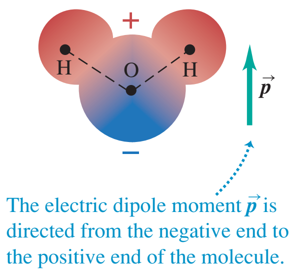


21.7 Electric Dipoles

An **electric dipole** is a pair of point charges with equal magnitude and opposite sign (a positive charge q and a negative charge $-q$) separated by a distance d. We introduced electric dipoles in Example 21.8 (Section 21.5); the concept is worth exploring further because many physical systems, from molecules to TV antennas, can be described as electric dipoles. We will also use this concept extensively in our discussion of dielectrics in Chapter 24.

21.30 **(a)** A water molecule is an example of an electric dipole. **(b)** Each test tube contains a solution of a different substance in water. The large electric dipole moment of water makes it an excellent solvent.

(a) A water molecule, showing positive charge as red and negative charge as blue



(b) Various substances dissolved in water

Figure 21.30 a shows a molecule of water (H_2O), which in many ways behaves like an electric dipole. The water molecule as a whole is electrically neutral, but the chemical bonds within the molecule cause a displacement of charge; the result is a net negative charge on the oxygen end of the molecule and a net positive charge on the hydrogen end, forming an electric dipole. The effect is equivalent to shifting one electron only about 4×10^{-11} m (about the radius of a hydrogen atom), but the consequences of this shift are profound. Water is an excellent solvent for ionic substances such as table salt (sodium chloride, NaCl) precisely because the water molecule is an electric dipole (Fig. 21.30b). When dissolved in water, salt dissociates into a positive sodium ion (Na^+) and a negative chlorine ion (Cl^-), which tend to be attracted to the negative and positive ends, respectively, of water molecules; this holds the ions in solution. If water molecules were not electric dipoles, water would be a poor solvent, and almost all of the chemistry that occurs in aqueous solutions would be impossible. This includes all of the biochemical reactions that occur in all of the life on earth. In a very real sense, your existence as a living being depends on electric dipoles!

We examine two questions about electric dipoles. First, what forces and torques does an electric dipole experience when placed in an external electric field (that is, a field set up by charges outside the dipole)? Second, what electric field does an electric dipole itself produce?

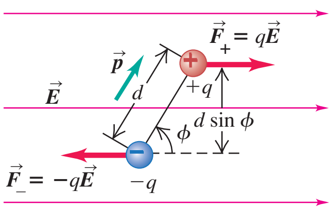
Force and Torque on an Electric Dipole

To start with the first question, let's place an electric dipole in a *uniform* external electric field $\vec{E}$, as shown in Fig. 21.31. The forces $\vec{F}_+$ and $\vec{F}_-$ on the two charges both have magnitude qE, but their directions are opposite, and they add to zero. *The net force on an electric dipole in a uniform external electric field is zero.*

However, the two forces don't act along the same line, so their *torques* don't add to zero. We calculate torques with respect to the center of the dipole. Let the angle between the electric field $\vec{E}$ and the dipole axis be ϕ; then the lever arm for both $\vec{F}_+$ and $\vec{F}_-$ is $(d/2)\sin\phi$. The torque of $\vec{F}_+$ and the torque of $\vec{F}_-$ both have the same magnitude of $(qE)(d/2)\sin\phi$, and both torques tend to rotate the dipole clockwise (that is, $\vec{\tau}$ is directed into the page in Fig. 21.31). Hence the magnitude of the net torque is twice the magnitude of either individual torque:

$$\tau = (qE)(d\sin\phi) \tag{21.13}$$

21.31 The net force on this electric dipole is zero, but there is a torque directed into the page that tends to rotate the dipole clockwise.



where $d \sin \phi$ is the perpendicular distance between the lines of action of the two forces.

The product of the charge q and the separation d is the magnitude of a quantity called the **electric dipole moment,** denoted by p:

$$p = qd \qquad \text{(magnitude of electric dipole moment)} \tag{21.14}$$

The units of p are charge times distance ($C \cdot m$). For example, the magnitude of the electric dipole moment of a water molecule is $p = 6.13 \times 10^{-30}\ C \cdot m$.

CAUTION **The symbol p has multiple meanings** Be careful not to confuse dipole moment with momentum or pressure. There aren't as many letters in the alphabet as there are physical quantities, so some letters are used several times. The context usually makes it clear what we mean, but be careful.

We further define the electric dipole moment to be a *vector* quantity $\vec{p}$. The magnitude of $\vec{p}$ is given by Eq. (21.14), and its direction is along the dipole axis from the negative charge to the positive charge as shown in Fig. 21.31.

In terms of p, Eq. (21.13) for the magnitude τ of the torque exerted by the field becomes

$$\tau = pE\sin\phi \qquad \text{(magnitude of the torque on an electric dipole)} \tag{21.15}$$

Since the angle ϕ in Fig. 21.31 is the angle between the directions of the vectors $\vec{p}$ and $\vec{E}$, this is reminiscent of the expression for the magnitude of the *vector product* discussed in Section 1.10. (You may want to review that discussion.) Hence we can write the torque on the dipole in vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad \text{(torque on an electric dipole, in vector form)} \tag{21.16}$$

You can use the right-hand rule for the vector product to verify that in the situation shown in Fig. 21.31, $\vec{\tau}$ is directed into the page. The torque is greatest when $\vec{p}$ and $\vec{E}$ are perpendicular and is zero when they are parallel or antiparallel. The torque always tends to turn $\vec{p}$ to line it up with $\vec{E}$. The position $\phi = 0$, with $\vec{p}$ parallel to $\vec{E}$, is a position of stable equilibrium, and the position $\phi = \pi$, with $\vec{p}$ and $\vec{E}$ antiparallel, is a position of unstable equilibrium. The polarization of a grass seed in the apparatus of Fig. 21.29b gives it an electric dipole moment; the torque exerted by $\vec{E}$ then causes the seed to align with $\vec{E}$ and hence with the field lines.

Potential Energy of an Electric Dipole

MasteringPHYSICS®

PhET: Microwaves

When a dipole changes direction in an electric field, the electric-field torque does *work* on it, with a corresponding change in potential energy. The work dW done by a torque τ during an infinitesimal displacement $d\phi$ is given by Eq. (10.19): $dW = \tau\, d\phi$. Because the torque is in the direction of decreasing ϕ, we must write the torque as $\tau = -pE\sin\phi$, and

$$dW = \tau\, d\phi = -pE\sin\phi\, d\phi$$

In a finite displacement from ϕ_1 to ϕ_2 the total work done on the dipole is

$$W = \int_{\phi_1}^{\phi_2} (-pE\sin\phi)\, d\phi$$
$$= pE\cos\phi_2 - pE\cos\phi_1$$

The work is the negative of the change of potential energy, just as in Chapter 7: $W = U_1 - U_2$. So a suitable definition of potential energy U for this system is

$$U(\phi) = -pE\cos\phi \tag{21.17}$$

In this expression we recognize the *scalar product* $\vec{p} \cdot \vec{E} = pE\cos\phi$, so we can also write

$$U = -\vec{p} \cdot \vec{E} \qquad \text{(potential energy for a dipole in an electric field)} \tag{21.18}$$

The potential energy has its minimum (most negative) value $U = -pE$ at the stable equilibrium position, where $\phi = 0$ and $\vec{p}$ is parallel to $\vec{E}$. The potential energy is maximum when $\phi = \pi$ and $\vec{p}$ is antiparallel to $\vec{E}$; then $U = +pE$. At $\phi = \pi/2$, where $\vec{p}$ is perpendicular to $\vec{E}$, U is zero. We could define U differently so that it is zero at some other orientation of $\vec{p}$, but our definition is simplest.

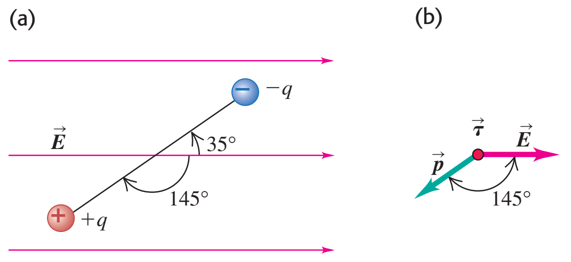
Equation (21.18) gives us another way to look at the effect shown in Fig. 21.29. The electric field $\vec{E}$ gives each grass seed an electric dipole moment, and the grass seed then aligns itself with $\vec{E}$ to minimize the potential energy.

Example 21.13 Force and torque on an electric dipole

Figure 21.32a shows an electric dipole in a uniform electric field of magnitude 5.0×10^5 N/C that is directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19}$ C; both lie in the plane and are separated by 0.125 nm $= 0.125 \times 10^{-9}$ m. Find (a) the net force exerted by the field on the dipole; (b) the magnitude and

Continued

21.32 (a) An electric dipole. (b) Directions of the electric dipole moment, electric field, and torque ($\vec{\tau}$ points out of the page).



direction of the electric dipole moment; (c) the magnitude and direction of the torque; (d) the potential energy of the system in the position shown.

SOLUTION

IDENTIFY and SET UP: This problem uses the ideas of this section about an electric dipole placed in an electric field. We use the relationship $\vec{F} = q\vec{E}$ for each point charge to find the force on the dipole as a whole. Equation (21.14) gives the dipole moment, Eq. (21.16) gives the torque on the dipole, and Eq. (21.18) gives the potential energy of the system.

EXECUTE: (a) The field is uniform, so the forces on the two charges are equal and opposite. Hence the total force on the dipole is zero.

(b) The magnitude p of the electric dipole moment $\vec{p}$ is

$$\begin{aligned} p &= qd = (1.6 \times 10^{-19}\ \text{C})(0.125 \times 10^{-9}\ \text{m}) \\ &= 2.0 \times 10^{-29}\ \text{C} \cdot \text{m} \end{aligned}$$

The direction of $\vec{p}$ is from the negative to the positive charge, 145° clockwise from the electric-field direction (Fig. 21.32b).

(c) The magnitude of the torque is

$$\begin{aligned} \tau &= pE\sin\phi = (2.0 \times 10^{-29}\ \text{C})(5.0 \times 10^{5}\ \text{N/C})(\sin 145°) \\ &= 5.7 \times 10^{-24}\ \text{N} \cdot \text{m} \end{aligned}$$

From the right-hand rule for vector products (see Section 1.10), the direction of the torque $\vec{\tau} = \vec{p} \times \vec{E}$ is out of the page. This corresponds to a counterclockwise torque that tends to align $\vec{p}$ with $\vec{E}$.

(d) The potential energy

$$\begin{aligned} U &= -pE\cos\phi \\ &= -(2.0 \times 10^{-29}\ \text{C} \cdot \text{m})(5.0 \times 10^{5}\ \text{N/C})(\cos 145°) \\ &= 8.2 \times 10^{-24}\ \text{J} \end{aligned}$$

EVALUATE: The charge magnitude, the distance between the charges, the dipole moment, and the potential energy are all very small, but are all typical of molecules.

In this discussion we have assumed that $\vec{E}$ is uniform, so there is no net force on the dipole. If $\vec{E}$ is not uniform, the forces at the ends may not cancel completely, and the net force may not be zero. Thus a body with zero net charge but an electric dipole moment can experience a net force in a nonuniform electric field. As we mentioned in Section 21.1, an uncharged body can be polarized by an electric field, giving rise to a separation of charge and an electric dipole moment. This is how uncharged bodies can experience electrostatic forces (see Fig. 21.8).

Field of an Electric Dipole

Now let's think of an electric dipole as a *source* of electric field. What does the field look like? The general shape of things is shown by the field map of Fig. 21.28b. At each point in the pattern the total $\vec{E}$ field is the vector sum of the fields from the two individual charges, as in Example 21.8 (Section 21.5). Try drawing diagrams showing this vector sum for several points.

To get quantitative information about the field of an electric dipole, we have to do some calculating, as illustrated in the next example. Notice the use of the principle of superposition of electric fields to add up the contributions to the field of the individual charges. Also notice that we need to use approximation techniques even for the relatively simple case of a field due to two charges. Field calculations often become very complicated, and computer analysis is typically used to determine the field due to an arbitrary charge distribution.

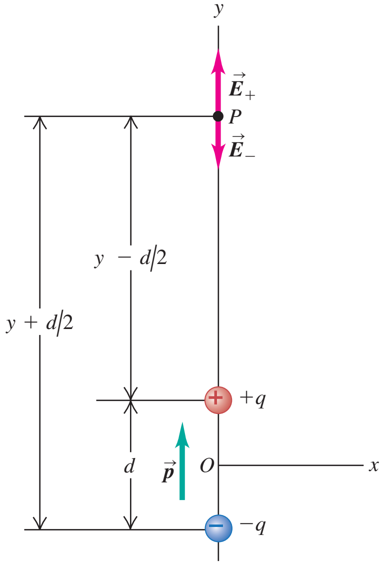
Example 21.14 Field of an electric dipole, revisited

An electric dipole is centered at the origin, with $\vec{p}$ in the direction of the $+y$-axis (Fig. 21.33). Derive an approximate expression for the electric field at a point P on the y-axis for which y is much larger than d. To do this, use the binomial expansion $(1 + x)^n \cong 1 + nx + n(n-1)x^2/2 + \cdots$ (valid for the case $|x| < 1$).

SOLUTION

IDENTIFY and SET UP: We use the principle of superposition: The total electric field is the vector sum of the field produced by the positive charge and the field produced by the negative charge. At the field point P shown in Fig. 21.33, the field $\vec{E}_+$ of the positive charge has a positive (upward) y-component and the field $\vec{E}_-$ of

21.33 Finding the electric field of an electric dipole at a point on its axis.



the negative charge has a negative (downward) y-component. We add these components to find the total field and then apply the approximation that y is much greater than d.

EXECUTE: The total y-component E_y of electric field from the two charges is

$$E_y = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(y - d/2)^2} - \frac{1}{(y + d/2)^2}\right]$$
$$= \frac{q}{4\pi\epsilon_0 y^2}\left[\left(1 - \frac{d}{2y}\right)^{-2} - \left(1 + \frac{d}{2y}\right)^{-2}\right]$$

We used this same approach in Example 21.8 (Section 21.5). Now the approximation: When we are far from the dipole compared to its size, so $y \gg d$, we have $d/2y \ll 1$. With $n = -2$ and with $d/2y$ replacing x in the binomial expansion, we keep only the first two terms (the terms we discard are much smaller). We then have

$$\left(1 - \frac{d}{2y}\right)^{-2} \cong 1 + \frac{d}{y} \quad \text{and} \quad \left(1 + \frac{d}{2y}\right)^{-2} \cong 1 - \frac{d}{y}$$

Hence E_y is given approximately by

$$E_y \cong \frac{q}{4\pi\epsilon_0 y^2}\left[1 + \frac{d}{y} - \left(1 - \frac{d}{y}\right)\right] = \frac{qd}{2\pi\epsilon_0 y^3} = \frac{p}{2\pi\epsilon_0 y^3}$$

EVALUATE: An alternative route to this result is to put the fractions in the first expression for E_y over a common denominator, add, and then approximate the denominator $(y - d/2)^2(y + d/2)^2$ as y^4. We leave the details to you (see Exercise 21.60).

For points P off the coordinate axes, the expressions are more complicated, but at *all* points far away from the dipole (in any direction) the field drops off as $1/r^3$. We can compare this with the $1/r^2$ behavior of a point charge, the $1/r$ behavior of a long line charge, and the independence of r for a large sheet of charge. There are charge distributions for which the field drops off even more quickly. At large distances, the field of an *electric quadrupole,* which consists of two equal dipoles with opposite orientation, separated by a small distance, drops off as $1/r^4$.

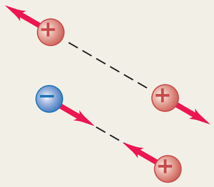
Test Your Understanding of Section 21.7 An electric dipole is placed in a region of uniform electric field $\vec{E}$, with the electric dipole moment $\vec{p}$, pointing in the direction opposite to $\vec{E}$. Is the dipole (i) in stable equilibrium, (ii) in unstable equilibrium, or (iii) neither? (*Hint:* You many want to review Section 7.5.)

CHAPTER 21 SUMMARY

Electric charge, conductors, and insulators: The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

All ordinary matter is made of protons, neutrons, and electrons. The positive protons and electrically neutral neutrons in the nucleus of an atom are bound together by the nuclear force; the negative electrons surround the nucleus at distances much greater than the nuclear size. Electric interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Conductors are materials in which charge moves easily; in insulators, charge does not move easily. Most metals are good conductors; most nonmetals are insulators.

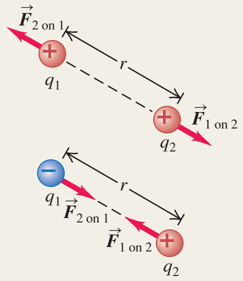


Coulomb's law: For charges q_1 and q_2 separated by a distance r, the magnitude of the electric force on either charge is proportional to the product q_1q_2 and inversely proportional to r^2. The force on each charge is along the line joining the two charges—repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. In SI units the unit of electric charge is the coulomb, abbreviated C. (See Examples 21.1 and 21.2.)

When two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges. (See Examples 21.3 and 21.4.)

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} \quad (21.2)$$

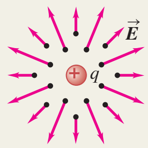
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \mathrm{N\cdot m^2/C^2}$$



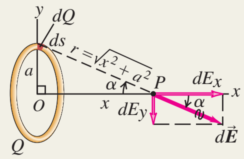
Electric field: Electric field $\vec{E}$, a vector quantity, is the force per unit charge exerted on a test charge at any point. The electric field produced by a point charge is directed radially away from or toward the charge. (See Examples 21.5–21.7.)

$$\vec{E} = \frac{\vec{F}_0}{q_0} \quad (21.3)$$

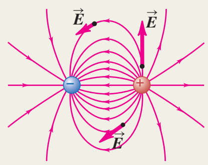
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \quad (21.7)$$



Superposition of electric fields: The electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges. To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements, calculate the field caused by each element, and then carry out the vector sum, usually by integrating. Charge distributions are described by linear charge density λ, surface charge density σ, and volume charge density ρ. (See Examples 21.8–21.12.)



Electric field lines: Field lines provide a graphical representation of electric fields. At any point on a field line, the tangent to the line is in the direction of $\vec{E}$ at that point. The number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point.

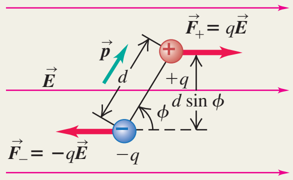


Electric dipoles: An electric dipole is a pair of electric charges of equal magnitude q but opposite sign, separated by a distance d. The electric dipole moment $\vec{p}$ has magnitude $p = qd$. The direction of $\vec{p}$ is from negative toward positive charge. An electric dipole in an electric field $\vec{E}$ experiences a torque $\vec{\tau}$ equal to the vector product of $\vec{p}$ and $\vec{E}$. The magnitude of the torque depends on the angle ϕ between $\vec{p}$ and $\vec{E}$. The potential energy U for an electric dipole in an electric field also depends on the relative orientation of $\vec{p}$ and $\vec{E}$. (See Examples 21.13 and 21.14.)

$$\tau = pE\sin\phi \quad (21.15)$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (21.16)$$

$$U = -\vec{p}\cdot\vec{E} \quad (21.18)$$



BRIDGING PROBLEM — Calculating Electric Field: Half a Ring of Charge

Positive charge Q is uniformly distributed around a semicircle of radius a as shown in Fig. 21.34. Find the magnitude and direction of the resulting electric field at point P, the center of curvature of the semicircle.

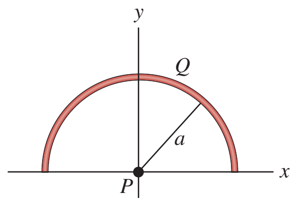
SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.

IDENTIFY and SET UP

1. The target variables are the components of the electric field at P.
2. Divide the semicircle into infinitesimal segments, each of which is a short circular arc of radius a and angle $d\theta$. What is the length of such a segment? How much charge is on a segment?

21.34



3. Consider an infinitesimal segment located at an angular position θ on the semicircle, measured from the lower right corner of the semicircle at $x = a$, $y = 0$. (Thus $\theta = \pi/2$ at $x = 0$, $y = a$ and $\theta = \pi$ at $x = -a$, $y = 0$.) What are the x- and y-components of the electric field at P (dE_x and dE_y) produced by just this segment?

EXECUTE

4. Integrate your expressions for dE_x and dE_y from $\theta = 0$ to $\theta = \pi$. The results will be the x-component and y-component of the electric field at P.
5. Use your results from step 4 to find the magnitude and direction of the field at P.

EVALUATE

6. Does your result for the electric-field magnitude have the correct units?
7. Explain how you could have found the x-component of the electric field using a symmetry argument.
8. What would be the electric field at P if the semicircle were extended to a full circle centered at P?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q21.1 If you peel two strips of transparent tape off the same roll and immediately let them hang near each other, they will repel each other. If you then stick the sticky side of one to the shiny side of the other and rip them apart, they will attract each other. Give a plausible explanation, involving transfer of electrons between the strips of tape, for this sequence of events.

Q21.2 Two metal spheres are hanging from nylon threads. When you bring the spheres close to each other, they tend to attract. Based on this information alone, discuss all the possible ways that the spheres could be charged. Is it possible that after the spheres touch, they will cling together? Explain.

Q21.3 The electric force between two charged particles becomes weaker with increasing distance. Suppose instead that the electric force were *independent* of distance. In this case, would a charged comb still cause a neutral insulator to become polarized as in Fig. 21.8? Why or why not? Would the neutral insulator still be attracted to the comb? Again, why or why not?

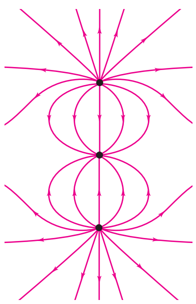
Q21.4 Your clothing tends to cling together after going through the dryer. Why? Would you expect more or less clinging if all your clothing were made of the same material (say, cotton) than if you dried different kinds of clothing together? Again, why? (You may want to experiment with your next load of laundry.)

Q21.5 An uncharged metal sphere hangs from a nylon thread. When a positively charged glass rod is brought close to the metal sphere, the sphere is drawn toward the rod. But if the sphere touches the rod, it suddenly flies away from the rod. Explain why the sphere is first attracted and then repelled.

Q21.6 The free electrons in a metal are gravitationally attracted toward the earth. Why, then, don't they all settle to the bottom of the conductor, like sediment settling to the bottom of a river?

Q21.7 • Figure Q21.7 shows some of the electric field lines due to three point charges arranged along the vertical axis. All three charges have the same magnitude. (a) What are the signs of the three charges? Explain your reasoning. (b) At what point(s) is the magnitude of the electric field the smallest? Explain your reasoning. Explain how the fields produced by each individual point charge combine to give a small net field at this point or points.

Figure **Q21.7**



Q21.8 Good electrical conductors, such as metals, are typically good conductors of heat; electrical insulators, such as wood, are typically poor conductors of heat. Explain why there should be a relationship between electrical conduction and heat conduction in these materials.

Q21.9 • Suppose the charge shown in Fig. 21.28a is fixed in position. A small, positively charged particle is then placed at some point in the figure and released. Will the trajectory of the particle follow an electric field line? Why or why not? Suppose instead that the particle is placed at some point in Fig. 21.28b and released (the positive and negative charges shown in the figure are fixed in position). Will its trajectory follow an electric field line? Again, why or why not? Explain any differences between your answers for the two different situations.

Q21.10 Two identical metal objects are mounted on insulating stands. Describe how you could place charges of opposite sign but exactly equal magnitude on the two objects.

Q21.11 You can use plastic food wrap to cover a container by stretching the material across the top and pressing the overhanging material against the sides. What makes it stick? (*Hint:* The answer involves the electric force.) Does the food wrap stick to itself with equal tenacity? Why or why not? Does it work with metallic containers? Again, why or why not?

Q21.12 If you walk across a nylon rug and then touch a large metal object such as a doorknob, you may get a spark and a shock. Why does this tend to happen more on dry days than on humid days? (*Hint:* See Fig. 21.30.) Why are you less likely to get a shock if you touch a *small* metal object, such as a paper clip?

Q21.13 You have a negatively charged object. How can you use it to place a net negative charge on an insulated metal sphere? To place a net positive charge on the sphere?

Q21.14 When two point charges of equal mass and charge are released on a frictionless table, each has an initial acceleration a_0. If instead you keep one fixed and release the other one, what will be its initial acceleration: a_0, $2a_0$, or $a_0/2$? Explain.

Q21.15 A point charge of mass m and charge Q and another point charge of mass m but charge $2Q$ are released on a frictionless table. If the charge Q has an initial acceleration a_0, what will be the acceleration of $2Q$: a_0, $2a_0$, $4a_0$, $a_0/2$, or $a_0/4$? Explain.

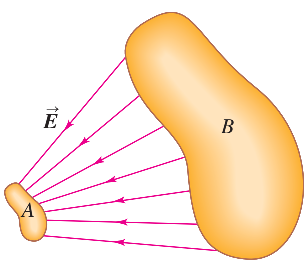
Q21.16 A proton is placed in a uniform electric field and then released. Then an electron is placed at this same point and released. Do these two particles experience the same force? The same acceleration? Do they move in the same direction when released?

Q21.17 In Example 21.1 (Section 21.3) we saw that the electric force between two α particles is of the order of 10^{35} times as strong as the gravitational force. So why do we readily feel the gravity of the earth but no electrical force from it?

Q21.18 What similarities do electrical forces have with gravitational forces? What are the most significant differences?

Q21.19 Two irregular objects A and B carry charges of opposite sign. Figure Q21.19 shows the electric field lines near each of these objects. (a) Which object is positive, A or B? How do you know? (b) Where is the electric field stronger, close to A or close to B? How do you know?

Figure **Q21.19**

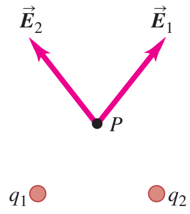


Q21.20 Atomic nuclei are made of protons and neutrons. This shows that there must be another kind of interaction in addition to gravitational and electric forces. Explain.

Q21.21 Sufficiently strong electric fields can cause atoms to become positively ionized—that is, to lose one or more electrons. Explain how this can happen. What determines how strong the field must be to make this happen?

Q21.22 The electric fields at point P due to the positive charges q_1 and q_2 are shown in Fig. Q21.22. Does the fact that they cross each other violate the statement in Section 21.6 that electric field lines never cross? Explain.

Figure **Q21.22**



Q21.23 The air temperature and the velocity of the air have different values at different places in the earth's atmosphere. Is the air velocity a vector field? Why or why not? Is the air temperature a vector field? Again, why or why not?

EXERCISES

Section 21.3 Coulomb's Law

21.1 •• Excess electrons are placed on a small lead sphere with mass 8.00 g so that its net charge is -3.20×10^{-9} C. (a) Find the number of excess electrons on the sphere. (b) How many excess electrons are there per lead atom? The atomic number of lead is 82, and its atomic mass is 207 g/mol.

21.2 • Lightning occurs when there is a flow of electric charge (principally electrons) between the ground and a thundercloud. The maximum rate of charge flow in a lightning bolt is about 20,000 C/s; this lasts for 100 μs or less. How much charge flows between the ground and the cloud in this time? How many electrons flow during this time?

21.3 •• **BIO** Estimate how many electrons there are in your body. Make any assumptions you feel are necessary, but clearly state what they are. (*Hint:* Most of the atoms in your body have equal numbers of electrons, protons, and neutrons.) What is the combined charge of all these electrons?

21.4 • **Particles in a Gold Ring.** You have a pure (24-karat) gold ring with mass 17.7 g. Gold has an atomic mass of 197 g/mol and an atomic number of 79. (a) How many protons are in the ring, and what is their total positive charge? (b) If the ring carries no net charge, how many electrons are in it?

21.5 • **BIO** **Signal Propagation in Neurons.** *Neurons* are components of the nervous system of the body that transmit signals as electrical impulses travel along their length. These impulses propagate when charge suddenly rushes into and then out of a part of the neuron called an *axon.* Measurements have shown that, during the inflow part of this cycle, approximately 5.6×10^{11} Na^+ (sodium ions) per meter, each with charge $+e$, enter the axon. How many coulombs of charge enter a 1.5-cm length of the axon during this process?

21.6 • Two small spheres spaced 20.0 cm apart have equal charge. How many excess electrons must be present on each sphere if the magnitude of the force of repulsion between them is 4.57×10^{-21} N?

21.7 •• An average human weighs about 650 N. If two such generic humans each carried 1.0 coulomb of excess charge, one positive and one negative, how far apart would they have to be for the electric attraction between them to equal their 650-N weight?

21.8 •• Two small aluminum spheres, each having mass 0.0250 kg, are separated by 80.0 cm. (a) How many electrons does each sphere contain? (The atomic mass of aluminum is 26.982 g/mol, and its atomic number is 13.) (b) How many electrons would have to be removed from one sphere and added to the other to cause an attractive force between the spheres of magnitude 1.00×10^4 N (roughly 1 ton)? Assume that the spheres may be treated as point charges. (c) What fraction of all the electrons in each sphere does this represent?

21.9 •• Two small plastic spheres are given positive electrical charges. When they are 15.0 cm apart, the repulsive force between them has magnitude 0.220 N. What is the charge on each sphere (a) if the two charges are equal and (b) if one sphere has four times the charge of the other?

21.10 •• What If We Were Not Neutral? A 75-kg person holds out his arms so that his hands are 1.7 m apart. Typically, a person's hand makes up about 1.0% of his or her body weight. For round numbers, we shall assume that all the weight of each hand is due to the calcium in the bones, and we shall treat the hands as point charges. One mole of Ca contains 40.18 g, and each atom has 20 protons and 20 electrons. Suppose that only 1.0% of the positive charges in each hand were unbalanced by negative charge. (a) How many Ca atoms does each hand contain? (b) How many coulombs of unbalanced charge does each hand contain? (c) What force would the person's arms have to exert on his hands to prevent them from flying off? Does it seem likely that his arms are capable of exerting such a force?

21.11 •• Two very small 8.55-g spheres, 15.0 cm apart from center to center, are charged by adding equal numbers of electrons to each of them. Disregarding all other forces, how many electrons would you have to add to each sphere so that the two spheres will accelerate at 25.0*g* when released? Which way will they accelerate?

21.12 •• Just How Strong Is the Electric Force? Suppose you had two small boxes, each containing 1.0 g of protons. (a) If one were placed on the moon by an astronaut and the other were left on the earth, and if they were connected by a very light (and very long!) string, what would be the tension in the string? Express your answer in newtons and in pounds. Do you need to take into account the gravitational forces of the earth and moon on the protons? Why? (b) What gravitational force would each box of protons exert on the other box?

21.13 • In an experiment in space, one proton is held fixed and another proton is released from rest a distance of 2.50 mm away. (a) What is the initial acceleration of the proton after it is released? (b) Sketch qualitative (no numbers!) acceleration–time and velocity–time graphs of the released proton's motion.

21.14 • A negative charge of $-0.550\ \mu\text{C}$ exerts an upward 0.200-N force on an unknown charge 0.300 m directly below it. (a) What is the unknown charge (magnitude and sign)? (b) What are the magnitude and direction of the force that the unknown charge exerts on the $-0.550\text{-}\mu\text{C}$ charge?

21.15 •• Three point charges are arranged on a line. Charge $q_3 = +5.00$ nC and is at the origin. Charge $q_2 = -3.00$ nC and is at $x = +4.00$ cm. Charge q_1 is at $x = +2.00$ cm. What is q_1 (magnitude and sign) if the net force on q_3 is zero?

21.16 •• In Example 21.4, suppose the point charge on the *y*-axis at $y = -0.30$ m has negative charge $-2.0\ \mu\text{C}$, and the other charges remain the same. Find the magnitude and direction of the net force on Q. How does your answer differ from that in Example 21.4? Explain the differences.

21.17 •• In Example 21.3, calculate the net force on charge q_1.

21.18 •• In Example 21.4, what is the net force (magnitude and direction) on charge q_1 exerted by the other two charges?

21.19 •• Three point charges are arranged along the *x*-axis. Charge $q_1 = +3.00\ \mu\text{C}$ is at the origin, and charge $q_2 = -5.00\ \mu\text{C}$ is at $x = 0.200$ m. Charge $q_3 = -8.00\ \mu\text{C}$. Where is q_3 located if the net force on q_1 is 7.00 N in the $-x$-direction?

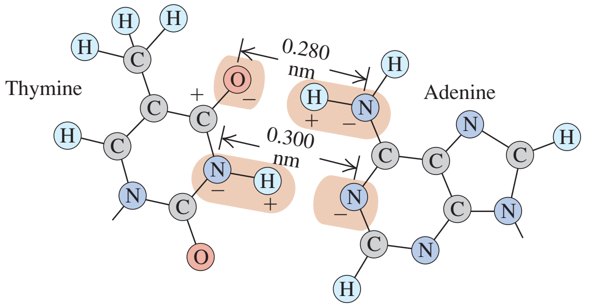
21.20 •• Repeat Exercise 21.19 for $q_3 = +8.00\ \mu\text{C}$.

21.21 •• Two point charges are located on the *y*-axis as follows: charge $q_1 = -1.50$ nC at $y = -0.600$ m, and charge $q_2 = +3.20$ nC at the origin ($y = 0$). What is the total force (magnitude and direction) exerted by these two charges on a third charge $q_3 = +5.00$ nC located at $y = -0.400$ m?

21.22 •• Two point charges are placed on the *x*-axis as follows: Charge $q_1 = +4.00$ nC is located at $x = 0.200$ m, and charge $q_2 = +5.00$ nC is at $x = -0.300$ m. What are the magnitude and direction of the total force exerted by these two charges on a negative point charge $q_3 = -6.00$ nC that is placed at the origin?

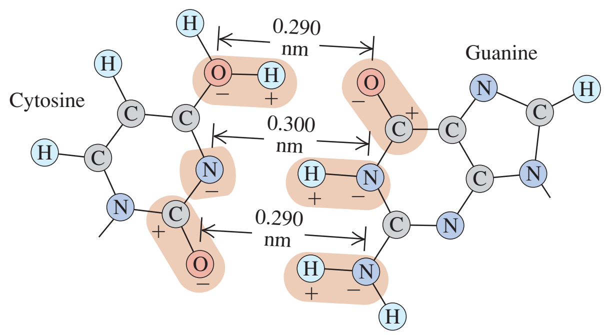
21.23 •• BIO Base Pairing in DNA, I. The two sides of the DNA double helix are connected by pairs of bases (adenine, thymine, cytosine, and guanine). Because of the geometric shape of these molecules, adenine bonds with thymine and cytosine bonds with guanine. Figure E21.23 shows the thymine–adenine bond. Each charge shown is $\pm e$, and the H—N distance is 0.110 nm. (a) Calculate the *net* force that thymine exerts on adenine. Is it attractive or repulsive? To keep the calculations fairly simple, yet reasonable, consider only the forces due to the O—H—N and the N—H—N combinations, assuming that these two combinations are parallel to each other. Remember, however, that in the O—H—N set, the O^- exerts a force on both the H^+ and the N^-, and likewise along the N—H—N set. (b) Calculate the force on the electron in the hydrogen atom, which is 0.0529 nm from the proton. Then compare the strength of the bonding force of the electron in hydrogen with the bonding force of the adenine–thymine molecules.

Figure **E21.23**



21.24 •• BIO Base Pairing in DNA, II. Refer to Exercise 21.23. Figure E21.24 shows the bonding of the cytosine and guanine molecules. The O—H and H—N distances are each 0.110 nm. In this case, assume that the bonding is due only to the forces along the O—H—O, N—H—N, and O—H—N combinations, and assume also that these three combinations are parallel to each other. Calculate the *net* force that cytosine exerts on guanine due to the preceding three combinations. Is this force attractive or repulsive?

Figure **E21.24**



Section 21.4 Electric Field and Electric Forces

21.25 • CP A proton is placed in a uniform electric field of 2.75×10^3 N/C. Calculate: (a) the magnitude of the electric force felt by the proton; (b) the proton's acceleration; (c) the proton's speed after 1.00 μs in the field, assuming it starts from rest.

21.26 • A particle has charge -3.00 nC. (a) Find the magnitude and direction of the electric field due to this particle at a point 0.250 m directly above it. (b) At what distance from this particle does its electric field have a magnitude of 12.0 N/C?

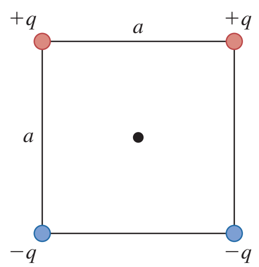
21.27 • **CP** A proton is traveling horizontally to the right at 4.50×10^6 m/s. (a) Find the magnitude and direction of the weakest electric field that can bring the proton uniformly to rest over a distance of 3.20 cm. (b) How much time does it take the proton to stop after entering the field? (c) What minimum field (magnitude and direction) would be needed to stop an electron under the conditions of part (a)?

21.28 • **CP** An electron is released from rest in a uniform electric field. The electron accelerates vertically upward, traveling 4.50 m in the first 3.00 μs after it is released. (a) What are the magnitude and direction of the electric field? (b) Are we justified in ignoring the effects of gravity? Justify your answer quantitatively.

21.29 •• (a) What must the charge (sign and magnitude) of a 1.45-g particle be for it to remain stationary when placed in a downward-directed electric field of magnitude 650 N/C? (b) What is the magnitude of an electric field in which the electric force on a proton is equal in magnitude to its weight?

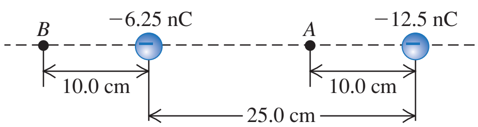
21.30 •• A point charge is placed at each corner of a square with side length a. The charges all have the same magnitude q. Two of the charges are positive and two are negative, as shown in Fig. E21.30. What is the direction of the net electric field at the center of the square due to the four charges, and what is its magnitude in terms of q and a?

Figure **E21.30**



21.31 • Two point charges are separated by 25.0 cm (Fig. E21.31). Find the net electric field these charges produce at (a) point A and (b) point B. (c) What would be the magnitude and direction of the electric force this combination of charges would produce on a proton at A?

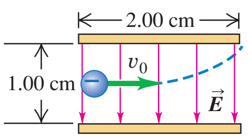
Figure **E21.31**



21.32 •• **Electric Field of the Earth.** The earth has a net electric charge that causes a field at points near its surface equal to 150 N/C and directed in toward the center of the earth. (a) What magnitude and sign of charge would a 60-kg human have to acquire to overcome his or her weight by the force exerted by the earth's electric field? (b) What would be the force of repulsion between two people each with the charge calculated in part (a) and separated by a distance of 100 m? Is use of the earth's electric field a feasible means of flight? Why or why not?

21.33 •• **CP** An electron is projected with an initial speed $v_0 = 1.60 \times 10^6$ m/s into the uniform field between the parallel plates in Fig. E21.33. Assume that the field between the plates is uniform and directed vertically downward, and that the field outside the plates is zero. The electron enters the field at a point midway between the plates. (a) If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field. (b) Suppose that in Fig. E21.33 the electron is replaced by a proton with the same initial speed v_0. Would the proton hit one of the plates? If the proton would not hit one of the plates, what would be the magnitude and direction of its vertical displacement as it exits the region between the plates? (c) Compare the paths traveled by the electron and the proton and explain the differences. (d) Discuss whether it is reasonable to ignore the effects of gravity for each particle.

Figure **E21.33**



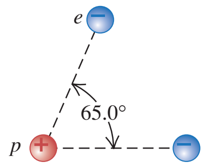
21.34 •• Point charge $q_1 = -5.00$ nC is at the origin and point charge $q_2 = +3.00$ nC is on the x-axis at $x = 3.00$ cm. Point P is on the y-axis at $y = 4.00$ cm. (a) Calculate the electric fields $\vec{E}_1$ and $\vec{E}_2$ at point P due to the charges q_1 and q_2. Express your results in terms of unit vectors (see Example 21.6). (b) Use the results of part (a) to obtain the resultant field at P, expressed in unit vector form.

21.35 •• **CP** In Exercise 21.33, what is the speed of the electron as it emerges from the field?

21.36 • (a) Calculate the magnitude and direction (relative to the $+x$-axis) of the electric field in Example 21.6. (b) A -2.5-nC point charge is placed at point P in Fig. 21.19. Find the magnitude and direction of (i) the force that the -8.0-nC charge at the origin exerts on this charge and (ii) the force that this charge exerts on the -8.0-nC charge at the origin.

21.37 •• If two electrons are each 1.50×10^{-10} m from a proton, as shown in Fig. E21.37, find the magnitude and direction of the net electric force they will exert on the proton.

Figure **E21.37**

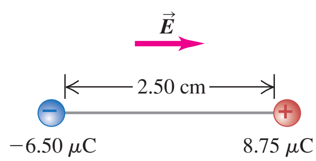


21.38 •• **CP** A uniform electric field exists in the region between two oppositely charged plane parallel plates. A proton is released from rest at the surface of the positively charged plate and strikes the surface of the opposite plate, 1.60 cm distant from the first, in a time interval of 1.50×10^{-6} s. (a) Find the magnitude of the electric field. (b) Find the speed of the proton when it strikes the negatively charged plate.

21.39 • A point charge is at the origin. With this point charge as the source point, what is the unit vector $\hat{\boldsymbol{r}}$ in the direction of (a) the field point at $x = 0$, $y = -1.35$ m; (b) the field point at $x = 12.0$ cm, $y = 12.0$ cm; (c) the field point at $x = -1.10$ m, $y = 2.60$ m? Express your results in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

21.40 •• A $+8.75$-μC point charge is glued down on a horizontal frictionless table. It is tied to a -6.50-μC point charge by a light, nonconducting 2.50-cm wire. A uniform electric field of magnitude 1.85×10^8 N/C is directed parallel to the wire, as shown in Fig. E21.40. (a) Find the tension in the wire. (b) What would the tension be if both charges were negative?

Figure **E21.40**

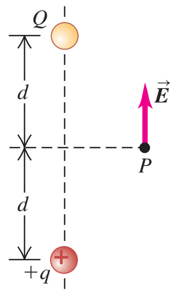


21.41 •• (a) An electron is moving east in a uniform electric field of 1.50 N/C directed to the west. At point A, the velocity of the electron is 4.50×10^5 m/s toward the east. What is the speed of the electron when it reaches point B, 0.375 m east of point A? (b) A proton is moving in the uniform electric field of part (a). At point A, the velocity of the proton is 1.90×10^4 m/s, east. What is the speed of the proton at point B?

Section 21.5 Electric-Field Calculations

21.42 • Two point charges Q and $+q$ (where q is positive) produce the net electric field shown at point P in Fig. E21.42. The field points parallel to the line connecting the two charges. (a) What can you conclude about the sign and magnitude of Q? Explain your reasoning. (b) If the lower charge were negative instead, would it be possible for the field to have the direction shown in the figure? Explain your reasoning.

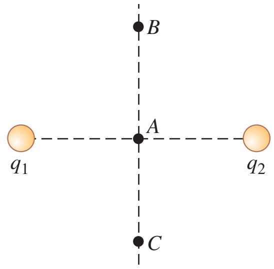
Figure **E21.42**



21.43 •• Two positive point charges q are placed on the x-axis, one at $x = a$ and one at $x = -a$. (a) Find the magnitude and direction of the electric field at $x = 0$. (b) Derive an expression for the electric field at points on the x-axis. Use your result to graph the x-component of the electric field as a function of x, for values of x between $-4a$ and $+4a$.

21.44 • The two charges q_1 and q_2 shown in Fig. E21.44 have equal magnitudes. What is the direction of the net electric field due to these two charges at points A (midway between the charges), B, and C if (a) both charges are negative, (b) both charges are positive, (c) q_1 is positive and q_2 is negative.

Figure **E21.44**

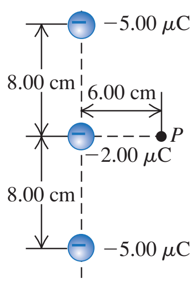


21.45 • A $+2.00$-nC point charge is at the origin, and a second -5.00-nC point charge is on the x-axis at $x = 0.800$ m. (a) Find the electric field (magnitude and direction) at each of the following points on the x-axis: (i) $x = 0.200$ m; (ii) $x = 1.20$ m; (iii) $x = -0.200$ m. (b) Find the net electric force that the two charges would exert on an electron placed at each point in part (a).

21.46 •• Repeat Exercise 21.44, but now let $q_1 = -4.00$ nC.

21.47 • Three negative point charges lie along a line as shown in Fig. E21.47. Find the magnitude and direction of the electric field this combination of charges produces at point P, which lies 6.00 cm from the -2.00-μC charge measured perpendicular to the line connecting the three charges.

Figure **E21.47**



21.48 •• **BIO Electric Field of Axons.** A nerve signal is transmitted through a neuron when an excess of Na^+ ions suddenly enters the axon, a long cylindrical part of the neuron. Axons are approximately 10.0 μm in diameter, and measurements show that about 5.6×10^{11} Na^+ ions per meter (each of charge $+e$) enter during this process. Although the axon is a long cylinder, the charge does not all enter everywhere at the same time. A plausible model would be a series of point charges moving along the axon. Let us look at a 0.10-mm length of the axon and model it as a point charge. (a) If the charge that enters each meter of the axon gets distributed uniformly along it, how many coulombs of charge enter a 0.10-mm length of the axon? (b) What electric field (magnitude and direction) does the sudden influx of charge produce at the surface of the body if the axon is 5.00 cm below the skin? (c) Certain sharks can respond to electric fields as weak as 1.0 μN/C. How far from this segment of axon could a shark be and still detect its electric field?

21.49 • In a rectangular coordinate system a positive point charge $q = 6.00 \times 10^{-9}$ C is placed at the point $x = +0.150$ m, $y = 0$, and an identical point charge is placed at $x = -0.150$ m, $y = 0$. Find the x- and y-components, the magnitude, and the direction of the electric field at the following points: (a) the origin; (b) $x = 0.300$ m, $y = 0$; (c) $x = 0.150$ m, $y = -0.400$ m; (d) $x = 0$, $y = 0.200$ m.

21.50 •• A point charge $q_1 = -4.00$ nC is at the point $x = 0.600$ m, $y = 0.800$ m, and a second point charge $q_2 = +6.00$ nC is at the point $x = 0.600$ m, $y = 0$. Calculate the magnitude and direction of the net electric field at the origin due to these two point charges.

21.51 •• Repeat Exercise 21.49 for the case where the point charge at $x = +0.150$ m, $y = 0$ is positive and the other is negative, each with magnitude 6.00×10^{-9} C.

21.52 •• A very long, straight wire has charge per unit length 1.50×10^{-10} C/m. At what distance from the wire is the electric-field magnitude equal to 2.50 N/C?

21.53 • A ring-shaped conductor with radius $a = 2.50$ cm has a total positive charge $Q = +0.125$ nC uniformly distributed around it, as shown in Fig. 21.23. The center of the ring is at the origin of coordinates O. (a) What is the electric field (magnitude and direction) at point P, which is on the x-axis at $x = 40.0$ cm? (b) A point charge $q = -2.50$ μC is placed at the point P described in part (a). What are the magnitude and direction of the force exerted *by* the charge q *on* the ring?

21.54 •• A straight, nonconducting plastic wire 8.50 cm long carries a charge density of $+175$ nC/m distributed uniformly along its length. It is lying on a horizontal tabletop. (a) Find the magnitude and direction of the electric field this wire produces at a point 6.00 cm directly above its midpoint. (b) If the wire is now bent into a circle lying flat on the table, find the magnitude and direction of the electric field it produces at a point 6.00 cm directly above its center.

21.55 •• A charge of -6.50 nC is spread uniformly over the surface of one face of a nonconducting disk of radius 1.25 cm. (a) Find the magnitude and direction of the electric field this disk produces at a point P on the axis of the disk a distance of 2.00 cm from its center. (b) Suppose that the charge were all pushed away from the center and distributed uniformly on the outer rim of the disk. Find the magnitude and direction of the electric field at point P. (c) If the charge is all brought to the center of the disk, find the magnitude and direction of the electric field at point P. (d) Why is the field in part (a) stronger than the field in part (b)? Why is the field in part (c) the strongest of the three fields?

Section 21.7 Electric Dipoles

21.56 • The ammonia molecule (NH_3) has a dipole moment of 5.0×10^{-30} C·m. Ammonia molecules in the gas phase are placed in a uniform electric field $\vec{E}$ with magnitude 1.6×10^6 N/C. (a) What is the change in electric potential energy when the dipole moment of a molecule changes its orientation with respect to $\vec{E}$ from parallel to perpendicular? (b) At what absolute temperature T is the average translational kinetic energy $\frac{3}{2}kT$ of a molecule equal to the change in potential energy calculated in part (a)? (*Note:* Above this temperature, thermal agitation prevents the dipoles from aligning with the electric field.)

21.57 • Point charges $q_1 = -4.5$ nC and $q_2 = +4.5$ nC are separated by 3.1 mm, forming an electric dipole. (a) Find the electric dipole moment (magnitude and direction). (b) The charges are in a uniform electric field whose direction makes an angle of 36.9° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude 7.2×10^{-9} N·m?

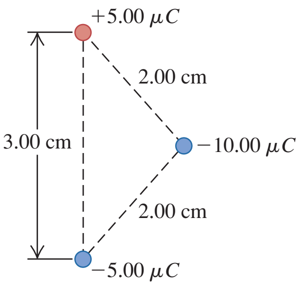
21.58 • The dipole moment of the water molecule (H_2O) is 6.17×10^{-30} C·m. Consider a water molecule located at the origin whose dipole moment $\vec{p}$ points in the $+x$-direction. A chlorine ion (Cl^-), of charge -1.60×10^{-19} C, is located at $x = 3.00 \times 10^{-9}$ m. Find the magnitude and direction of the electric force that the water molecule exerts on the chlorine ion. Is this force attractive or repulsive? Assume that x is much larger than the separation d between the charges in the dipole, so that the approximate expression for the electric field along the dipole axis derived in Example 21.14 can be used.

21.59 • **Torque on a Dipole.** An electric dipole with dipole moment $\vec{p}$ is in a uniform electric field $\vec{E}$. (a) Find the orientations of the dipole for which the torque on the dipole is zero. (b) Which of the orientations in part (a) is stable, and which is unstable? (*Hint:* Consider a small displacement away from the equilibrium position and see what happens.) (c) Show that for the stable orientation in part (b), the dipole's own electric field tends to oppose the external field.

21.60 •• Consider the electric dipole of Example 21.14. (a) Derive an expression for the magnitude of the electric field produced by the dipole at a point on the x-axis in Fig. 21.33. What is the direction of this electric field? (b) How does the electric field at points on the x-axis depend on x when x is very large?

21.61 • Three charges are at the corners of an isosceles triangle as shown in Fig. E21.61. The ± 5.00-μC charges form a dipole. (a) Find the force (magnitude and direction) the -10.00-μC charge exerts on the dipole. (b) For an axis perpendicular to the line connecting the ± 5.00-μC charges at the midpoint of this line, find the torque (magnitude and direction) exerted on the dipole by the -10.00-μC charge.

Figure **E21.61**



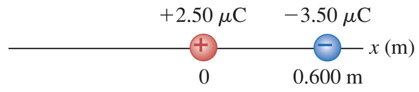
21.62 • A dipole consisting of charges $\pm e$, 220 nm apart, is placed between two very large (essentially infinite) sheets carrying equal but opposite charge densities of 125 $\mu C/m^2$. (a) What is the maximum potential energy this dipole can have due to the sheets, and how should it be oriented relative to the sheets to attain this value? (b) What is the maximum torque the sheets can exert on the dipole, and how should it be oriented relative to the sheets to attain this value? (c) What net force do the two sheets exert on the dipole?

PROBLEMS

21.63 ••• Four identical charges Q are placed at the corners of a square of side L. (a) In a free-body diagram, show all of the forces that act on one of the charges. (b) Find the magnitude and direction of the total force exerted on one charge by the other three charges.

21.64 ••• Two charges, one of 2.50 μC and the other of -3.50 μC, are placed on the x-axis, one at the origin and the other at $x = 0.600$ m, as shown in Fig. P21.64. Find the position on the x-axis where the net force on a small charge $+q$ would be zero.

Figure **P21.64**



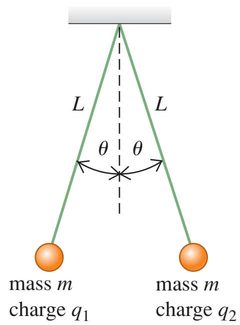
21.65 •• Three point charges are arranged along the x-axis. Charge $q_1 = -4.50$ nC is located at $x = 0.200$ m, and charge $q_2 = +2.50$ nC is at $x = -0.300$ m. A positive point charge q_3 is located at the origin. (a) What must the value of q_3 be for the net force on this point charge to have magnitude 4.00 μN? (b) What is the direction of the net force on q_3? (c) Where along the x-axis can q_3 be placed and the net force on it be zero, other than the trivial answers of $x = +\infty$ and $x = -\infty$?

21.66 •• A charge $q_1 = +5.00$ nC is placed at the origin of an xy-coordinate system, and a charge $q_2 = -2.00$ nC is placed on the positive x-axis at $x = 4.00$ cm. (a) If a third charge $q_3 = +6.00$ nC is now placed at the point $x = 4.00$ cm, $y = 3.00$ cm, find the x- and y-components of the total force exerted on this charge by the other two. (b) Find the magnitude and direction of this force.

21.67 •• **CP** Two positive point charges Q are held fixed on the x-axis at $x = a$ and $x = -a$. A third positive point charge q, with mass m, is placed on the x-axis away from the origin at a coordinate x such that $|x| \ll a$. The charge q, which is free to move along the x-axis, is then released. (a) Find the frequency of oscillation of the charge q. (*Hint:* Review the definition of simple harmonic motion in Section 14.2. Use the binomial expansion $(1 + z)^n = 1 + nz + n(n - 1)z^2/2 + \cdots$, valid for the case $|z| < 1$.) (b) Suppose instead that the charge q were placed on the y-axis at a coordinate y such that $|y| \ll a$, and then released. If this charge is free to move anywhere in the xy-plane, what will happen to it? Explain your answer.

21.68 •• **CP** Two identical spheres with mass m are hung from silk threads of length L, as shown in Fig. P21.68. Each sphere has the same charge, so $q_1 = q_2 = q$. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. Show that if the angle θ is small, the equilibrium separation d between the spheres is $d = (q^2L/2\pi\epsilon_0 mg)^{1/3}$. (*Hint:* If θ is small, then $\tan\theta \cong \sin\theta$.)

Figure **P21.68**



21.69 ••• **CP** Two small spheres with mass $m = 15.0$ g are hung by silk threads of length $L = 1.20$ m from a common point (Fig. P21.68). When the spheres are given equal quantities of negative charge, so that $q_1 = q_2 = q$, each thread hangs at $\theta = 25.0°$ from the vertical. (a) Draw a diagram showing the forces on each sphere. Treat the spheres as point charges. (b) Find the magnitude of q. (c) Both threads are now shortened to length $L = 0.600$ m, while the charges q_1 and q_2 remain unchanged. What new angle will each thread make with the vertical? (*Hint:* This part of the problem can be solved numerically by using trial values for θ and adjusting the values of θ until a self-consistent answer is obtained.)

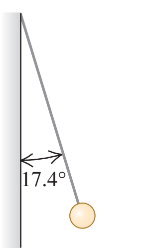
21.70 •• **CP** Two identical spheres are each attached to silk threads of length $L = 0.500$ m and hung from a common point (Fig. P21.68). Each sphere has mass $m = 8.00$ g. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. One sphere is given positive charge q_1, and the other a different positive charge q_2; this causes the spheres to separate so that when the spheres are in equilibrium, each thread makes an angle $\theta = 20.0°$ with the vertical. (a) Draw a free-body diagram for each sphere when in equilibrium, and label all the forces that act on each sphere. (b) Determine the magnitude of the electrostatic force that acts on each sphere, and determine the tension in each thread. (c) Based on the information you have been given, what can you say about the magnitudes of q_1 and q_2? Explain your answers. (d) A small wire is now connected between the spheres, allowing charge to be transferred from one sphere to the other until the two spheres have equal charges; the wire is then removed. Each thread now makes an angle of 30.0° with the vertical. Determine the original charges. (*Hint:* The total charge on the pair of spheres is conserved.)

21.71 •• Sodium chloride (NaCl, ordinary table salt) is made up of positive sodium ions (Na^+) and negative chloride ions (Cl^-). (a) If a point charge with the same charge and mass as all the Na^+ ions in 0.100 mol of NaCl is 2.00 cm from a point charge with the same charge and mass as all the Cl^- ions, what is the magnitude of the attractive force between these two point charges? (b) If the positive point charge in part (a) is held in place and the negative point charge is released from rest, what is its initial acceleration? (See Appendix D for atomic masses.) (c) Does it seem reasonable that the ions in NaCl could be separated in this way? Why or why not? (In fact, when sodium chloride dissolves in water, it breaks up into Na^+ and Cl^- ions. However, in this situation there are additional electric forces exerted by the water molecules on the ions.)

21.72 •• A -5.00-nC point charge is on the x-axis at $x = 1.20$ m. A second point charge Q is on the x-axis at -0.600 m. What must be the sign and magnitude of Q for the resultant electric field at the origin to be (a) 45.0 N/C in the $+x$-direction, (b) 45.0 N/C in the $-x$-direction?

21.73 •• **CP** A small 12.3-g plastic ball is tied to a very light 28.6-cm string that is attached to the vertical wall of a room (Fig. P21.73). A uniform horizontal electric field exists in this room. When the ball has been given an excess charge of $-1.11\ \mu C$, you observe that it remains suspended, with the string making an angle of 17.4° with the wall. Find the magnitude and direction of the electric field in the room.

Figure **P21.73**

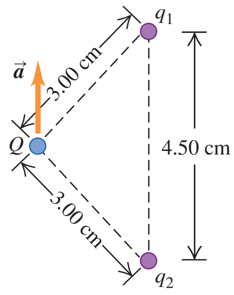


21.74 •• **CP** At $t = 0$ a very small object with mass 0.400 mg and charge $+9.00\ \mu C$ is traveling at 125 m/s in the $-x$-direction. The charge is moving in a uniform electric field that is in the $+y$-direction and that has magnitude $E = 895$ N/C. The gravitational force on the particle can be neglected. How far is the particle from the origin at $t = 7.00$ ms?

21.75 •• Two particles having charges $q_1 = 0.500$ nC and $q_2 = 8.00$ nC are separated by a distance of 1.20 m. At what point along the line connecting the two charges is the total electric field due to the two charges equal to zero?

21.76 ••• Two point charges q_1 and q_2 are held in place 4.50 cm apart. Another point charge $Q = -1.75\ \mu C$ of mass 5.00 g is initially located 3.00 cm from each of these charges (Fig. P21.76) and released from rest. You observe that the initial acceleration of Q is 324 m/s^2 upward, parallel to the line connecting the two point charges. Find q_1 and q_2.

Figure **P21.76**



21.77 • Three identical point charges q are placed at each of three corners of a square of side L. Find the magnitude and direction of the net force on a point charge $-3q$ placed (a) at the center of the square and (b) at the vacant corner of the square. In each case, draw a free-body diagram showing the forces exerted on the $-3q$ charge by each of the other three charges.

21.78 ••• Three point charges are placed on the y-axis: a charge q at $y = a$, a charge $-2q$ at the origin, and a charge q at $y = -a$. Such an arrangement is called an electric quadrupole. (a) Find the magnitude and direction of the electric field at points on the positive x-axis. (b) Use the binomial expansion to find an approximate expression for the electric field valid for $x \gg a$. Contrast this behavior to that of the electric field of a point charge and that of the electric field of a dipole.

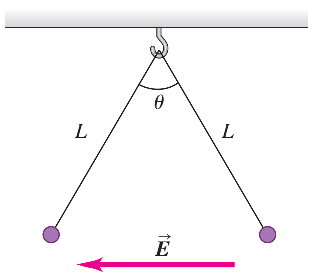
21.79 •• **CP** **Strength of the Electric Force.** Imagine two 1.0-g bags of protons, one at the earth's north pole and the other at the south pole. (a) How many protons are in each bag? (b) Calculate the gravitational attraction and the electrical repulsion that each bag exerts on the other. (c) Are the forces in part (b) large enough for you to feel if you were holding one of the bags?

21.80 • **Electric Force Within the Nucleus.** Typical dimensions of atomic nuclei are of the order of 10^{-15} m (1 fm). (a) If two protons in a nucleus are 2.0 fm apart, find the magnitude of the electric force each one exerts on the other. Express the answer in newtons and in pounds. Would this force be large enough for a person to feel? (b) Since the protons repel each other so strongly, why don't they shoot out of the nucleus?

21.81 •• **If Atoms Were Not Neutral . . .** Because the charges on the electron and proton have the same absolute value, atoms are electrically neutral. Suppose this were not precisely true, and the absolute value of the charge of the electron were less than the charge of the proton by 0.00100%. (a) Estimate what the net charge of this textbook would be under these circumstances. Make any assumptions you feel are justified, but state clearly what they are. (*Hint:* Most of the atoms in this textbook have equal numbers of electrons, protons, and neutrons.) (b) What would be the magnitude of the electric force between two textbooks placed 5.0 m apart? Would this force be attractive or repulsive? Estimate what the acceleration of each book would be if the books were 5.0 m apart and there were no non-electric forces on them. (c) Discuss how the fact that ordinary matter is stable shows that the absolute values of the charges on the electron and proton must be identical to a *very* high level of accuracy.

21.82 ••• **CP** Two tiny spheres of mass 6.80 mg carry charges of equal magnitude,

Figure **P21.82**



72.0 nC, but opposite sign. They are tied to the same ceiling hook by light strings of length 0.530 m. When a horizontal uniform electric field E that is directed to the left is turned on, the spheres hang at rest with the angle θ between the strings equal to 50.0° (Fig. P21.82). (a) Which ball (the one on the right or the one on the left) has positive charge? (b) What is the magnitude E of the field?

21.83 •• CP Consider a model of a hydrogen atom in which an electron is in a circular orbit of radius $r = 5.29 \times 10^{-11}$ m around a stationary proton. What is the speed of the electron in its orbit?

21.84 •• CP A small sphere with mass 9.00 μg and charge $-4.30\ \mu$C is moving in a circular orbit around a stationary sphere that has charge $+7.50\ \mu$C. If the speed of the small sphere is 5.90×10^3 m/s, what is the radius of its orbit? Treat the spheres as point charges and ignore gravity.

21.85 •• Two small copper spheres each have radius 1.00 mm. (a) How many atoms does each sphere contain? (b) Assume that each copper atom contains 29 protons and 29 electrons. We know that electrons and protons have charges of exactly the same magnitude, but let's explore the effect of small differences (see also Problem 21.81). If the charge of a proton is $+e$ and the magnitude of the charge of an electron is 0.100% smaller, what is the net charge of each sphere and what force would one sphere exert on the other if they were separated by 1.00 m?

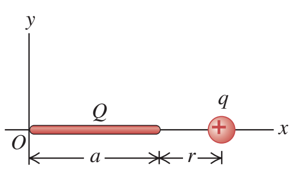
21.86 ••• CP Operation of an Inkjet Printer. In an inkjet printer, letters are built up by squirting drops of ink at the paper from a rapidly moving nozzle. The ink drops, which have a mass of 1.4×10^{-8} g each, leave the nozzle and travel toward the paper at 20 m/s, passing through a charging unit that gives each drop a positive charge q by removing some electrons from it. The drops then pass between parallel deflecting plates 2.0 cm long where there is a uniform vertical electric field with magnitude 8.0×10^4 N/C. If a drop is to be deflected 0.30 mm by the time it reaches the end of the deflection plates, what magnitude of charge must be given to the drop?

21.87 •• CP A proton is projected into a uniform electric field that points vertically upward and has magnitude E. The initial velocity of the proton has a magnitude v_0 and is directed at an angle α below the horizontal. (a) Find the maximum distance h_{max} that the proton descends vertically below its initial elevation. You can ignore gravitational forces. (b) After what horizontal distance d does the proton return to its original elevation? (c) Sketch the trajectory of the proton. (d) Find the numerical values of h_{max} and d if $E = 500$ N/C, $v_0 = 4.00 \times 10^5$ m/s, and $\alpha = 30.0°$.

21.88 • A negative point charge $q_1 = -4.00$ nC is on the x-axis at $x = 0.60$ m. A second point charge q_2 is on the x-axis at $x = -1.20$ m. What must the sign and magnitude of q_2 be for the net electric field at the origin to be (a) 50.0 N/C in the $+x$-direction and (b) 50.0 N/C in the $-x$-direction?

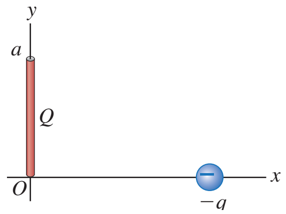
21.89 •• CALC Positive charge Q is distributed uniformly along the x-axis from $x = 0$ to $x = a$. A positive point charge q is located on the positive x-axis at $x = a + r$, a distance r to the right of the end of Q (Fig. P21.89). (a) Calculate the x- and y-components of the electric field produced by the charge distribution Q at points on the positive x-axis where $x > a$. (b) Calculate the force (magnitude and direction) that the charge distribution Q exerts on q. (c) Show that if $r \gg a$, the magnitude of the force in part (b) is approximately $Qq/4\pi\epsilon_0 r^2$. Explain why this result is obtained.

Figure **P21.89**



21.90 •• CALC Positive charge Q is distributed uniformly along the positive y-axis between $y = 0$ and $y = a$. A negative point charge $-q$ lies on the positive x-axis, a distance x from the origin (Fig. P21.90). (a) Calculate the x- and y-components of the electric field produced by the charge distribution Q at points on the positive x-axis. (b) Calculate the x- and y-components of the force that the charge distribution Q exerts on q. (c) Show that if $x \gg a$, $F_x \cong -Qq/4\pi\epsilon_0 x^2$ and $F_y \cong +Qqa/8\pi\epsilon_0 x^3$. Explain why this result is obtained.

Figure **P21.90**



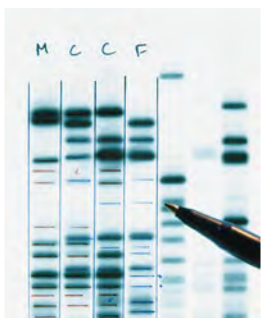
21.91 •• A charged line like that shown in Fig. 21.24 extends from $y = 2.50$ cm to $y = -2.50$ cm. The total charge distributed uniformly along the line is -7.00 nC. (a) Find the electric field (magnitude and direction) on the x-axis at $x = 10.0$ cm. (b) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 10.0 cm from a point charge that has the same total charge as this finite line of charge? In terms of the approximation used to derive $E = Q/4\pi\epsilon_0 x^2$ for a point charge from Eq. (21.9), explain why this is so. (c) At what distance x does the result for the finite line of charge differ by 1.0% from that for the point charge?

21.92 • CP A Parallel Universe. Imagine a parallel universe in which the electric force has the same properties as in our universe but there is no gravity. In this parallel universe, the sun carries charge Q, the earth carries charge $-Q$, and the electric attraction between them keeps the earth in orbit. The earth in the parallel universe has the same mass, the same orbital radius, and the same orbital period as in our universe. Calculate the value of Q. (Consult Appendix F as needed.)

21.93 ••• A uniformly charged disk like the disk in Fig. 21.25 has radius 2.50 cm and carries a total charge of 7.0×10^{-12} C. (a) Find the electric field (magnitude and direction) on the x-axis at $x = 20.0$ cm. (b) Show that for $x \gg R$, Eq. (21.11) becomes $E = Q/4\pi\epsilon_0 x^2$, where Q is the total charge on the disk. (c) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 20.0 cm from a point charge that has the same total charge as this disk? In terms of the approximation used in part (b) to derive $E = Q/4\pi\epsilon_0 x^2$ for a point charge from Eq. (21.11), explain why this is so. (d) What is the percent difference between the electric fields produced by the finite disk and by a point charge with the same charge at $x = 20.0$ cm and at $x = 10.0$ cm?

21.94 •• BIO Electrophoresis. Electrophoresis is a process used by biologists to separate different biological molecules (such as proteins) from each other according to their ratio of charge to size. The materials to be separated are in a viscous solution that produces a drag force F_D proportional to the size and speed of the molecule. We can express this relationship as $F_D = KRv$, where R is the radius of the molecule (modeled as being spherical), v is its speed, and K is a constant that depends on the viscosity of the

Figure **P21.94**



solution. The solution is placed in an external electric field E so that the electric force on a particle of charge q is $F = qE$. (a) Show that when the electric field is adjusted so that the two forces (electric and viscous drag) just balance, the ratio of q to R is Kv/E. (b) Show that if we leave the electric field on for a time T, the distance x that the molecule moves during that time is $x = (ET/k)(q/R)$. (c) Suppose you have a sample containing three different biological molecules for which the molecular ratio q/R for material 2 is twice that of material 1 and the ratio for material 3 is three times that of material 1. Show that the distances migrated by these molecules after the same amount of time are $x_2 = 2x_1$ and $x_3 = 3x_1$. In other words, material 2 travels twice as far as material 1, and material 3 travels three times as far as material 1. Therefore, we have separated these molecules according to their ratio of charge to size. In practice, this process can be carried out in a special gel or paper, along which the biological molecules migrate. (Fig. P21.94). The process can be rather slow, requiring several hours for separations of just a centimeter or so.

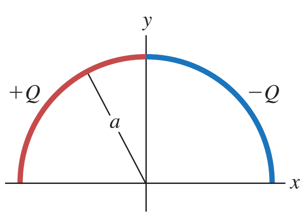
21.95 • **CALC** Positive charge $+Q$ is distributed uniformly along the $+x$-axis from $x = 0$ to $x = a$. Negative charge $-Q$ is distributed uniformly along the $-x$-axis from $x = 0$ to $x = -a$. (a) A positive point charge q lies on the positive y-axis, a distance y from the origin. Find the force (magnitude and direction) that the positive and negative charge distributions together exert on q. Show that this force is proportional to y^{-3} for $y \gg a$. (b) Suppose instead that the positive point charge q lies on the positive x-axis, a distance $x > a$ from the origin. Find the force (magnitude and direction) that the charge distribution exerts on q. Show that this force is proportional to x^{-3} for $x \gg a$.

21.96 •• **CP** A small sphere with mass m carries a positive charge q and is attached to one end of a silk fiber of length L. The other end of the fiber is attached to a large vertical insulating sheet that has a positive surface charge density σ. Show that when the sphere is in equilibrium, the fiber makes an angle equal to arctan $(q\sigma/2mg\epsilon_0)$ with the vertical sheet.

21.97 •• **CALC** Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius a that lies in the first quadrant, with the center of curvature at the origin. Find the x- and y-components of the net electric field at the origin.

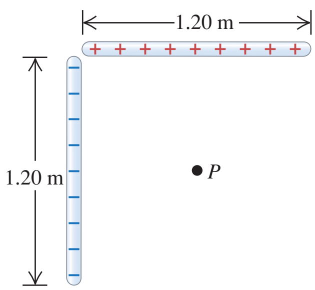
21.98 •• **CALC** A semicircle of radius a is in the first and second quadrants, with the center of curvature at the origin. Positive charge $+Q$ is distributed uniformly around the left half of the semicircle, and negative charge $-Q$ is distributed uniformly around the right half of the semicircle (Fig. P21.98). What are the magnitude and direction of the net electric field at the origin produced by this distribution of charge?

Figure **P21.98**



21.99 •• Two 1.20-m nonconducting wires meet at a right angle. One segment carries $+2.50\ \mu C$ of charge distributed uniformly along its length, and the other carries $-2.50\ \mu C$ distributed uniformly along it, as shown in Fig. P21.99. (a) Find the magnitude and direction of the electric field these wires produce at point P, which is 60.0 cm from each wire. (b) If an electron is released at P, what are the magnitude and direction of the net force that these wires exert on it?

Figure **P21.99**



21.100 • Two very large parallel sheets are 5.00 cm apart. Sheet A carries a uniform surface charge density of $-9.50 \mu C/m^2$, and sheet B, which is to the right of A, carries a uniform charge density of $-11.6\ \mu C/m^2$. Assume the sheets are large enough to be treated as infinite. Find the magnitude and direction of the net electric field these sheets produce at a point (a) 4.00 cm to the right of sheet A; (b) 4.00 cm to the left of sheet A; (c) 4.00 cm to the right of sheet B.

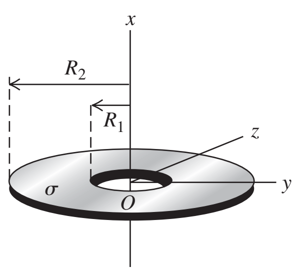
21.101 • Repeat Problem 21.100 for the case where sheet B is positive.

21.102 • Two very large horizontal sheets are 4.25 cm apart and carry equal but opposite uniform surface charge densities of magnitude σ. You want to use these sheets to hold stationary in the region between them an oil droplet of mass 324 μg that carries an excess of five electrons. Assuming that the drop is in vacuum, (a) which way should the electric field between the plates point, and (b) what should σ be?

21.103 •• An infinite sheet with positive charge per unit area σ lies in the xy-plane. A second infinite sheet with negative charge per unit area $-\sigma$ lies in the yz-plane. Find the net electric field at all points that do not lie in either of these planes. Express your answer in terms of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

21.104 •• **CP** A thin disk with a circular hole at its center, called an *annulus,* has inner radius R_1 and outer radius R_2 (Fig. P21.104). The disk has a uniform positive surface charge density σ on its surface. (a) Determine the total electric charge on the annulus. (b) The annulus lies in the yz-plane, with its center at the origin. For an arbitrary point on the x-axis (the axis of the annulus), find the magnitude and direction of the electric field $\vec{E}$. Consider points both above and below the annulus in Fig. P21.104. (c) Show that at points on the x-axis that are sufficiently close to the origin, the magnitude of the electric field is approximately proportional to the distance between the center of the annulus and the point. How close is "sufficiently close"? (d) A point particle with mass m and negative charge $-q$ is free to move along the x-axis (but cannot move off the axis). The particle is originally placed at rest at $x = 0.01R_1$ and released. Find the frequency of oscillation of the particle. (*Hint:* Review Section 14.2. The annulus is held stationary.)

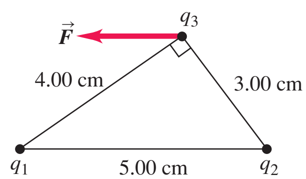
Figure **P21.104**



CHALLENGE PROBLEMS

21.105 ••• Three charges are placed as shown in Fig. P21.105. The magnitude of q_1 is 2.00 μC, but its sign and the value of the charge q_2 are not known. Charge q_3 is $+4.00\ \mu C$, and the net force $\vec{F}$ on q_3 is entirely in the negative x-direction. (a) Considering the different possible signs of q_1, there are four possible force diagrams representing the forces $\vec{F}_1$ and $\vec{F}_2$ that q_1 and q_2 exert on q_3. Sketch these four possible force configurations.

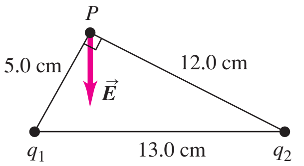
Figure **P21.105**



(b) Using the sketches from part (a) and the direction of $\vec{F}$, deduce the signs of the charges q_1 and q_2. (c) Calculate the magnitude of q_2. (d) Determine F, the magnitude of the net force on q_3.

21.106 ••• Two charges are placed as shown in Fig. P21.106. The magnitude of q_1 is 3.00 μC, but its sign and the value of the charge q_2 are not known. The direction of the net electric field $\vec{E}$ at point P is entirely in the negative y-direction. (a) Considering the different possible signs of q_1 and q_2, there are four possible diagrams that could represent the electric fields $\vec{E}_1$ and $\vec{E}_2$ produced by q_1 and q_2. Sketch the four possible electric-field configurations. (b) Using the sketches from part (a) and the direction of $\vec{E}$, deduce the signs of q_1 and q_2. (c) Determine the magnitude of $\vec{E}$.

Figure **P21.106**



21.107 ••• **CALC** Two thin rods of length L lie along the x-axis, one between $x = a/2$ and $x = a/2 + L$ and the other between $x = -a/2$ and $x = -a/2 - L$. Each rod has positive charge Q distributed uniformly along its length. (a) Calculate the electric field produced by the second rod at points along the positive x-axis. (b) Show that the magnitude of the force that one rod exerts on the other is

$$F = \frac{Q^2}{4\pi\epsilon_0 L^2}\ln\left[\frac{(a + L)^2}{a(a + 2L)}\right]$$

(c) Show that if $a \gg L$, the magnitude of this force reduces to $F = Q^2/4\pi\epsilon_0 a^2$. (*Hint:* Use the expansion $\ln(1 + z) = z - z^2/2 + z^3/3 - \cdots$, valid for $|z| \ll 1$. Carry *all* expansions to at least order L^2/a^2.) Interpret this result.

Answers

Chapter Opening Question ?

Water molecules have a permanent electric dipole moment: One end of the molecule has a positive charge and the other end has a negative charge. These ends attract negative and positive ions, respectively, holding the ions apart in solution. Water is less effective as a solvent for materials whose molecules do not ionize (called *nonionic* substances), such as oils.

Test Your Understanding Questions

21.1 Answers: (a) the plastic rod weighs more, (b) the glass rod weighs less, (c) the fur weighs less, (d) the silk weighs more The plastic rod gets a negative charge by taking electrons from the fur, so the rod weighs a little more and the fur weighs a little less after the rubbing. By contrast, the glass rod gets a positive charge by giving electrons to the silk. Hence, after they are rubbed together, the glass rod weighs a little less and the silk weighs a little more. The weight change is *very* small: The number of electrons transferred is a small fraction of a mole, and a mole of electrons has a mass of only $(6.02 \times 10^{23}\ \text{electrons})(9.11 \times 10^{-31}\ \text{kg/electron}) = 5.48 \times 10^{-7}\ \text{kg} = 0.548$ milligram!

21.2 Answers: (a) (i), (b) (ii) Before the two spheres touch, the negatively charged sphere exerts a repulsive force on the electrons in the other sphere, causing zones of positive and negative induced charge (see Fig. 21.7b). The positive zone is closer to the negatively charged sphere than the negative zone, so there is a net force of attraction that pulls the spheres together, like the comb and insulator in Fig. 21.8b. Once the two metal spheres touch, some of the excess electrons on the negatively charged sphere will flow onto the other sphere (because metals are conductors). Then both spheres will have a net negative charge and will repel each other.

21.3 Answer: (iv) The force exerted by q_1 on Q is still as in Example 21.4. The magnitude of the force exerted by q_2 on Q is still equal to $F_{1\ \text{on}\ Q}$, but the direction of the force is now *toward* q_2 at an angle α below the x-axis. Hence the x-components of the two forces cancel while the (negative) y-components add together, and the total electric force is in the negative y-direction.

21.4 Answers: (a) (ii), (b) (i) The electric field $\vec{E}$ produced by a positive point charge points directly away from the charge (see Fig. 21.18a) and has a magnitude that depends on the distance r from the charge to the field point. Hence a second, negative point charge $q < 0$ will feel a force $\vec{F} = q\vec{E}$ that points directly toward the positive charge and has a magnitude that depends on the distance r between the two charges. If the negative charge moves directly toward the positive charge, the direction of the force remains the same but the force magnitude increases as the distance r decreases. If the negative charge moves in a circle around the positive charge, the force magnitude stays the same (because the distance r is constant) but the force direction changes.

21.5 Answer: (iv) Think of a pair of segments of length dy, one at coordinate $y > 0$ and the other at coordinate $-y < 0$. The upper segment has a positive charge and produces an electric field $d\vec{E}$ at P that points away from the segment, so this $d\vec{E}$ has a positive x-component and a negative y-component, like the vector $d\vec{E}$ in Fig. 21.24. The lower segment has the same amount of negative charge. It produces a $d\vec{E}$ that has the same magnitude but points *toward* the lower segment, so it has a negative x-component and a negative y-component. By symmetry, the two x-components are equal but opposite, so they cancel. Thus the total electric field has only a negative y-component.

21.6 Answer: yes If the field lines are straight, $\vec{E}$ must point in the same direction throughout the region. Hence the force $\vec{F} = q\vec{E}$ on a particle of charge q is always in the same direction. A particle released from rest accelerates in a straight line in the direction of $\vec{F}$, and so its trajectory is a straight line along a field line.

21.7 Answer: (ii) Equations (21.17) and (21.18) tell us that the potential energy for a dipole in an electric field is $U = -\vec{p} \cdot \vec{E} = -pE\cos\phi$, where ϕ is the angle between the directions of $\vec{p}$ and $\vec{E}$. If $\vec{p}$ and $\vec{E}$ point in opposite directions, so that $\phi = 180°$, we have $\cos\phi = -1$ and $U = +pE$. This is the maximum value that U can have. From our discussion of energy diagrams in Section 7.5, it follows that this is a situation of unstable equilibrium.

Bridging Problem

Answer: $E = 2kQ/\pi a^2$ in the $-y$-direction