PHOTONS: LIGHT WAVES BEHAVING AS PARTICLES 38

? This plastic surgeon is using two light sources: a headlamp that emits a beam of visible light and a handheld laser that emits infrared light. The light from both sources is emitted in the form of packets of energy called photons. For which source are the photons more energetic: the headlamp or the laser?

LEARNING GOALS

By studying this chapter, you will learn:

- How experiments involving the photoelectric effect and x rays pointed the way to a radical reinterpretation of the nature of light.
- How Einstein's photon picture of light explains the photoelectric effect.
- How experiments with x rays and gamma rays helped confirm the photon picture of light.
- How the wave and particle pictures of light complement each other.
- How the Heisenberg uncertainty principle imposes fundamental limits on what can be measured.

In Chapter 32 we saw how Maxwell, Hertz, and others established firmly that light is an electromagnetic wave. Interference, diffraction, and polarization, discussed in Chapters 35 and 36, further demonstrate this *wave nature* of light.

When we look more closely at the emission, absorption, and scattering of electromagnetic radiation, however, we discover a completely different aspect of light. We find that the energy of an electromagnetic wave is *quantized;* it is emitted and absorbed in particle-like packages of definite energy, called *photons.* The energy of a single photon is proportional to the frequency of the radiation.

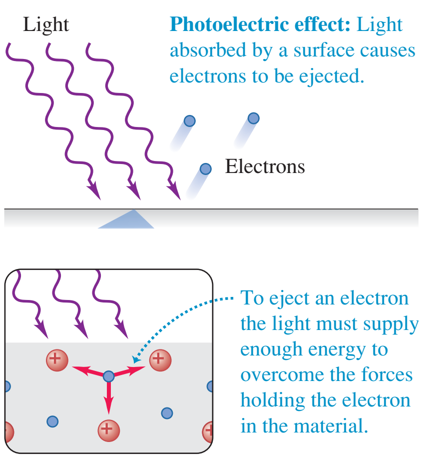
We'll find that light and other electromagnetic radiation exhibits *wave–particle duality:* Light acts sometimes like waves and sometimes like particles. Interference and diffraction demonstrate wave behavior, while emission and absorption of photons demonstrate the particle behavior. This radical reinterpretation of light will lead us in the next chapter to no less radical changes in our views of the nature of matter.

38.1 Light Absorbed as Photons: The Photoelectric Effect

A phenomenon that gives insight into the nature of light is the **photoelectric effect,** in which a material emits electrons from its surface when illuminated (Fig. 38.1). To escape from the surface, an electron must absorb enough energy from the incident light to overcome the attraction of positive ions in the material. These attractions constitute a potential-energy barrier; the light supplies the "kick" that enables the electron to escape.

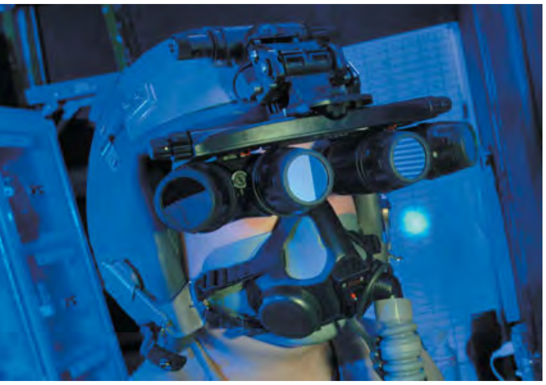
The photoelectric effect has a number of applications. Digital cameras and night-vision scopes use it to convert light energy into an electric signal that is

38.1 The photoelectric effect.

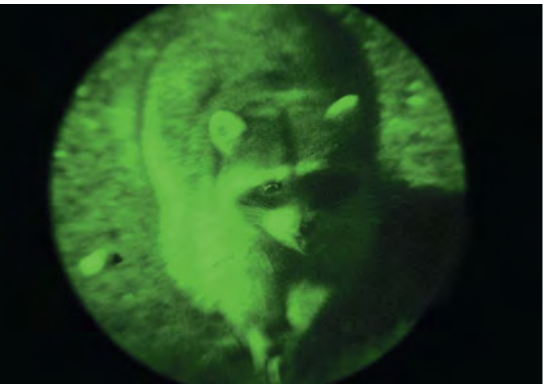


38.2 (a) A night-vision scope makes use of the photoelectric effect. Photons entering the scope strike a plate, ejecting electrons that pass through a thin disk in which there are millions of tiny channels. The current through each channel is amplified electronically and then directed toward a screen that glows when hit by electrons. (b) The image formed on the screen, which is a combination of these millions of glowing spots, is thousands of times brighter than the naked-eye view.

(a)



(b)



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PhET: Photoelectric Effect
ActivPhysics 17.3: Photoelectric Effect

reconstructed into an image (Fig. 38.2). On the moon, sunlight striking the surface causes surface dust to eject electrons, leaving the dust particles with a positive charge. The mutual electric repulsion of these charged dust particles causes them to rise above the moon's surface, a phenomenon that was observed from lunar orbit by the Apollo astronauts.

Threshold Frequency and Stopping Potential

In Section 32.1 we explored the wave model of light, which Maxwell formulated two decades before the photoelectric effect was observed. Is the photoelectric effect consistent with this model? Figure 38.3a shows a modern version of one of the experiments that explored this question. Two conducting electrodes are enclosed in an evacuated glass tube and connected by a battery, and the cathode is illuminated. Depending on the potential difference V_{AC} between the two electrodes, electrons emitted by the illuminated cathode (called *photoelectrons*) may travel across to the anode, producing a *photocurrent* in the external circuit. (The tube is evacuated to a pressure of 0.01 Pa or less to minimize collisions between the electrons and gas molecules.)

The illuminated cathode emits photoelectrons with various kinetic energies. If the electric field points toward the cathode, as in Fig. 38.3a, all the electrons are accelerated toward the anode and contribute to the photocurrent. But by reversing the field and adjusting its strength as in Fig. 38.3b, we can prevent the less energetic electrons from reaching the anode. In fact, we can determine the *maximum* kinetic energy K_{max} of the emitted electrons by making the potential of the anode relative to the cathode, V_{AC}, just negative enough so that the current stops. This occurs for $V_{AC} = -V_0$, where V_0 is called the **stopping potential.** As an electron moves from the cathode to the anode, the potential decreases by V_0 and negative work $-eV_0$ is done on the (negatively charged) electron. The most energetic electron leaves the cathode with kinetic energy $K_{max} = \frac{1}{2}mv_{max}{}^2$ and has zero kinetic energy at the anode. Using the work–energy theorem, we have

$$\begin{aligned} W_{tot} &= -eV_0 = \Delta K = 0 - K_{max} \\ K_{max} &= \tfrac{1}{2}mv_{max}{}^2 = eV_0 \end{aligned} \quad \text{(maximum kinetic energy of photoelectrons)} \tag{38.1}$$

Hence by measuring the stopping potential V_0, we can determine the maximum kinetic energy with which electrons leave the cathode. (We are ignoring any effects due to differences in the materials of the cathode and anode.)

In this experiment, how do we expect the photocurrent to depend on the voltage across the electrodes and on the frequency and intensity of the light? Based on Maxwell's picture of light as an electromagnetic wave, here is what we *would* expect:

Wave-Model Prediction 1: We saw in Section 32.4 that the intensity of an electromagnetic wave depends on its amplitude but not on its frequency. So the photoelectric effect should occur for light of any frequency, and *the magnitude of the photocurrent should not depend on the frequency of the light*.

Wave-Model Prediction 2: It takes a certain minimum amount of energy, called the **work function,** to eject a single electron from a particular surface (see Fig. 38.1). If the light falling on the surface is very faint, some time may elapse before the total energy absorbed by the surface equals the work function. Hence, for faint illumination, *we expect a time delay* between when we switch on the light and when photoelectrons appear.

Wave-Model Prediction 3: Because the energy delivered to the cathode surface depends on the intensity of illumination, *we expect the stopping potential to increase with increasing light intensity*. Since intensity does not depend on frequency, we further expect that *the stopping potential should not depend on the frequency of the light*.

The experimental results proved to be *very* different from these predictions. Here is what was found in the years between 1877 and 1905:

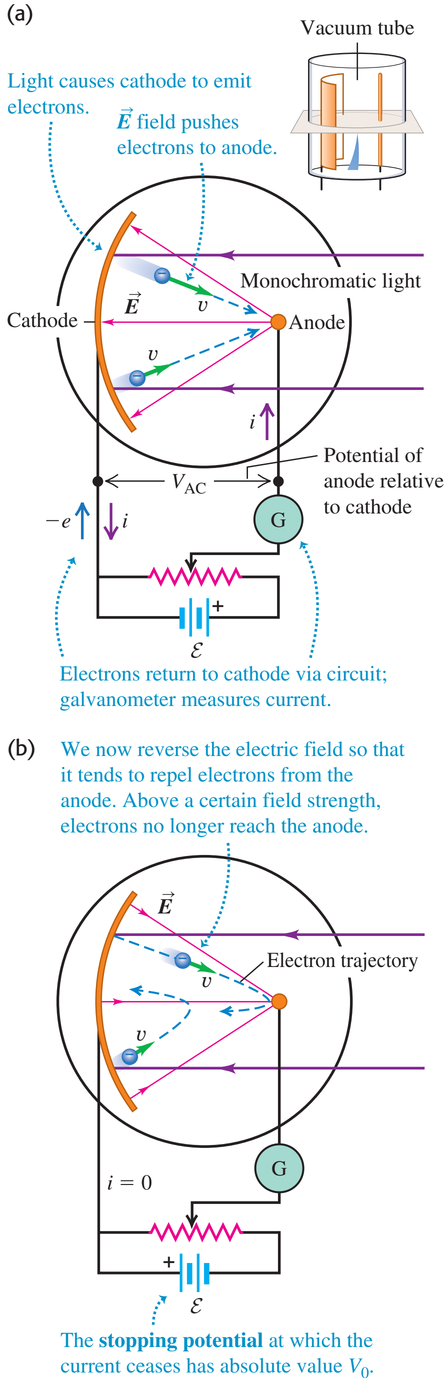
Experimental Result 1: *The photocurrent depends on the light frequency*. For a given material, monochromatic light with a frequency below a minimum **threshold frequency** produces *no* photocurrent, regardless of intensity. For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths λ between 200 and 300 nm), but for other materials like potassium oxide and cesium oxide it is in the visible spectrum (λ between 380 and 750 nm).

Experimental Result 2: There is *no measurable time delay* between when the light is turned on and when the cathode emits photoelectrons (assuming the frequency of the light exceeds the threshold frequency). This is true no matter how faint the light is.

Experimental Result 3: *The stopping potential does not depend on intensity, but does depend on frequency*. Figure 38.4 shows graphs of photocurrent as a function of potential difference V_{AC} for light of a given frequency and two different intensities. The reverse potential difference $-V_0$ needed to reduce the current to zero is the same for both intensities. The only effect of increasing the intensity is to increase the number of electrons per second and hence the photocurrent i. (The curves level off when V_{AC} is large and positive because at that point all the emitted electrons are being collected by the anode.) If the intensity is held constant but the frequency is increased, the stopping potential also increases. In other words, the greater the light frequency, the higher the energy of the ejected photoelectrons.

These results directly contradict Maxwell's description of light as an electromagnetic wave. A solution to this dilemma was provided by Albert Einstein in 1905. His proposal involved nothing less than a new picture of the nature of light.

38.3 An experiment testing whether the photoelectric effect is consistent with the wave model of light.



Einstein's Photon Explanation

Einstein made the radical postulate that a beam of light consists of small packages of energy called **photons** or *quanta*. This postulate was an extension of an idea developed five years earlier by Max Planck to explain the properties of blackbody radiation, which we discussed in Section 17.7. (We'll explore Planck's ideas in Section 39.5.) In Einstein's picture, the energy E of an individual photon is equal to a constant h times the photon frequency f. From the relationship $f = c/\lambda$ for electromagnetic waves in vacuum, we have

$$E = hf = \frac{hc}{\lambda} \quad \text{(energy of a photon)} \tag{38.2}$$

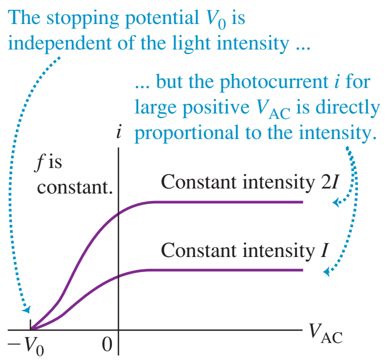
where h is a universal constant called **Planck's constant.** The numerical value of this constant, to the accuracy known at present, is

$$h = 6.62606896(33) \times 10^{-34}\ \text{J} \cdot \text{s}$$

CAUTION **Photons are not "particles" in the usual sense** It's common to envision photons as miniature billiard balls or pellets. While that's a convenient mental picture, it's not very accurate. For one thing, billiard balls and bullets have a rest mass and travel slower than the speed of light c, while photons travel at the speed of light and have *zero* rest mass. For another thing, photons have wave aspects (frequency and wavelength) that are easy to observe. The fact is that the photon concept is a very strange one, and the true nature of photons is difficult to visualize in a simple way. We'll discuss the dual personality of photons in more detail in Section 38.4.

In Einstein's picture, an individual photon arriving at the surface in Fig. 38.1a or 38.2 is absorbed by a single electron. This energy transfer is an all-or-nothing process, in contrast to the continuous transfer of energy in the wave theory of

38.4 Photocurrent i for a constant light frequency f as a function of the potential V_{AC} of the anode with respect to the cathode.



38.5 Stopping potential as a function of frequency for a particular cathode material.

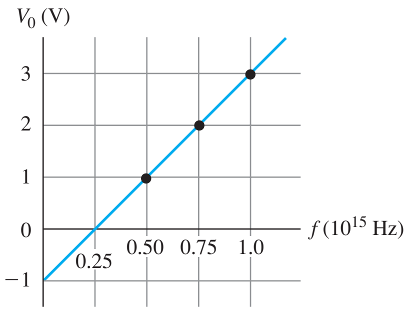
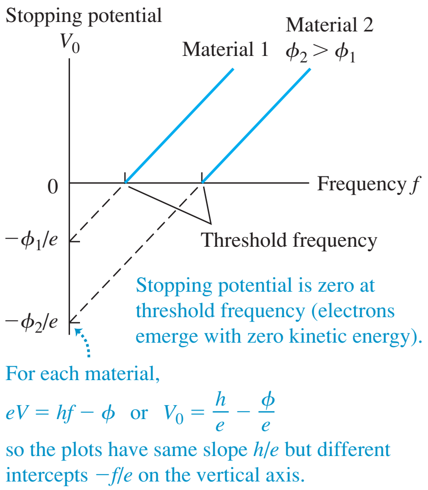


Table 38.1 Work Functions of Several Elements

Element	Work Function (eV)
Aluminum	4.3
Carbon	5.0
Copper	4.7
Gold	5.1
Nickel	5.1
Silicon	4.8
Silver	4.3
Sodium	2.7

38.6 Stopping potential as a function of frequency for two cathode materials having different work functions ϕ.



light; the electron gets all of the photon's energy or none at all. The electron can escape from the surface only if the energy it acquires is greater than the work function ϕ. Thus photoelectrons will be ejected only if $hf > \phi$, or $f > \phi/h$. Einstein's postulate therefore explains why the photoelectric effect occurs only for frequencies greater than a minimum threshold frequency. This postulate is also consistent with the observation that greater intensity causes a greater photocurrent (Fig. 38.4). Greater intensity at a particular frequency means a greater number of photons per second absorbed, and thus a greater number of electrons emitted per second and a greater photocurrent.

Einstein's postulate also explains why there is no delay between illumination and the emission of photoelectrons. As soon as photons of sufficient energy strike the surface, electrons can absorb them and be ejected.

Finally, Einstein's postulate explains why the stopping potential for a given surface depends only on the light frequency. Recall that ϕ is the *minimum* energy needed to remove an electron from the surface. Einstein applied conservation of energy to find that the *maximum* kinetic energy $K_{max} = \frac{1}{2}mv_{max}{}^2$ for an emitted electron is the energy hf gained from a photon minus the work function ϕ:

$$K_{max} = \tfrac{1}{2}mv_{max}{}^2 = hf - \phi \qquad (38.3)$$

Substituting $K_{max} = eV_0$ from Eq. (38.1), we find

$$eV_0 = hf - \phi \qquad \text{(photoelectric effect)} \qquad (38.4)$$

Equation (38.4) shows that the stopping potential V_0 increases with increasing frequency f. The intensity doesn't appear in Eq. (38.4), so V_0 is independent of intensity. As a check of Eq. (38.4), we can measure the stopping potential V_0 for each of several values of frequency f for a given cathode material (Fig. 38.5). A graph of V_0 as a function of f turns out to be a straight line, verifying Eq. (38.4), and from such a graph we can determine both the work function ϕ for the material and the value of the quantity h/e. After the electron charge $-e$ was measured by Robert Millikan in 1909, Planck's constant h could also be determined from these measurements.

Electron energies and work functions are usually expressed in electron volts (eV), defined in Section 23.2. To four significant figures,

$$1\text{ eV} = 1.602 \times 10^{-19}\text{ J}$$

To this accuracy, Planck's constant is

$$h = 6.626 \times 10^{-34}\text{ J}\cdot\text{s} = 4.136 \times 10^{-15}\text{ eV}\cdot\text{s}$$

Table 38.1 lists the work functions of several elements. These values are approximate because they are very sensitive to surface impurities. The greater the work function, the higher the minimum frequency needed to emit photoelectrons (Fig. 38.6).

The photon picture explains a number of other phenomena in which light is absorbed. One example is a *suntan*, which is caused when the energy in sunlight triggers a chemical reaction in skin cells that leads to increased production of the pigment melanin. This reaction can occur only if a specific molecule in the cell absorbs a certain minimum amount of energy. A short-wavelength ultraviolet photon has enough energy to trigger the reaction, but a longer-wavelength visible-light photon does not. Hence ultraviolet light causes tanning, while visible light cannot.

Photon Momentum

Einstein's photon concept applies to *all* regions of the electromagnetic spectrum, including radio waves, x rays, and so on. A photon of any electromagnetic radiation with frequency f and wavelength λ has energy E given by Eq. (38.2).

Furthermore, according to the special theory of relativity, every particle that has energy must also have momentum, even if it has no rest mass. Photons have zero rest mass. As we saw in Eq. (37.40), a particle with zero rest mass and energy E has momentum with magnitude p given by $E = pc$. Thus the wavelength λ of a photon and the magnitude of its momentum p are related simply by

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \qquad \text{(momentum of a photon)} \qquad (38.5)$$

The direction of the photon's momentum is simply the direction in which the electromagnetic wave is moving.

Problem-Solving Strategy 38.1 Photons

IDENTIFY *the relevant concepts:* The energy and momentum of an individual photon are proportional to the frequency and inversely proportional to the wavelength. Einstein's interpretation of the photoelectric effect is that energy is conserved as a photon ejects an electron from a material surface.

SET UP *the problem:* Identify the target variable. It could be the photon's wavelength λ, frequency f, energy E, or momentum p. If the problem involves the photoelectric effect, the target variable could be the maximum kinetic energy of photoelectrons K_{max}, the stopping potential V_0, or the work function ϕ.

EXECUTE *the solution* as follows:

1. Use Eqs. (38.2) and (38.5) to relate the energy and momentum of a photon to its wavelength and frequency. If the problem involves the photoelectric effect, use Eqs. (38.1), (38.3), and (38.4) to relate the photon frequency, stopping potential, work function, and maximum photoelectron kinetic energy.
2. The electron volt (eV), which we introduced in Section 23.2, is a convenient unit. It is the kinetic energy gained by an electron when it moves freely through an increase of potential of one volt: 1 eV $= 1.602 \times 10^{-19}$ J. If the photon energy E is given in electron volts, use $h = 4.136 \times 10^{-15}$ eV·s; if E is in joules, use $h = 6.626 \times 10^{-34}$ J·s.

EVALUATE *your answer:* In problems involving photons, at first the numbers will be unfamiliar to you and errors will not be obvious. It helps to remember that a visible-light photon with $\lambda = 600$ nm and $f = 5 \times 10^{14}$ Hz has an energy E of about 2 eV, or about 3×10^{-19} J.

Example 38.1 Laser-pointer photons

A laser pointer with a power output of 5.00 mW emits red light ($\lambda = 650$ nm). (a) What is the magnitude of the momentum of each photon? (b) How many photons does the laser pointer emit each second?

SOLUTION

IDENTIFY and SET UP: This problem involves the ideas of (a) photon momentum and (b) photon energy. In part (a) we'll use Eq. (38.5) and the given wavelength to find the magnitude of each photon's momentum. In part (b), Eq. (38.2) gives the energy per photon, and the power output tells us the energy emitted per second. We can combine these quantities to calculate the number of photons emitted per second.

EXECUTE: (a) We have $\lambda = 650$ nm $= 6.50 \times 10^{-7}$ m, so from Eq. (38.5) the photon momentum is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{6.50 \times 10^{-7}\ \text{m}}$$
$$= 1.02 \times 10^{-27}\ \text{kg}\cdot\text{m/s}$$

(Recall that 1 J $= 1$ kg·m^2/s^2.)

(b) From Eq. (38.2), the energy of a single photon is

$$E = pc = (1.02 \times 10^{-27}\ \text{kg}\cdot\text{m/s})(3.00 \times 10^8\ \text{m/s})$$
$$= 3.06 \times 10^{-19}\ \text{J} = 1.91\ \text{eV}$$

The laser pointer emits energy at the rate of 5.00×10^{-3} J/s, so it emits photons at the rate of

$$\frac{5.00 \times 10^{-3}\ \text{J/s}}{3.06 \times 10^{-19}\ \text{J/photon}} = 1.63 \times 10^{16}\ \text{photons/s}$$

EVALUATE: The result in part (a) is very small; a typical oxygen molecule in room-temperature air has 2500 times more momentum. As a check on part (b), we can calculate the photon energy using Eq. (38.2):

$$E = hf = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{6.50 \times 10^{-7}\ \text{m}}$$
$$= 3.06 \times 10^{-19}\ \text{J} = 1.91\ \text{eV}$$

Our result in part (b) shows that a huge number of photons leave the laser pointer each second, each of which has an infinitesimal amount of energy. Hence the discreteness of the photons isn't noticed, and the radiated energy appears to be a continuous flow.

Example 38.2 A photoelectric-effect experiment

While conducting a photoelectric-effect experiment with light of a certain frequency, you find that a reverse potential difference of 1.25 V is required to reduce the current to zero. Find (a) the maximum kinetic energy and (b) the maximum speed of the emitted photoelectrons.

SOLUTION

IDENTIFY and SET UP: The value of 1.25 V is the stopping potential V_0 for this experiment. We'll use this in Eq. (38.1) to find the maximum photoelectron kinetic energy K_{max}, and from this we'll find the maximum photoelectron speed.

EXECUTE: (a) From Eq. (38.1),

$$K_{max} = eV_0 = (1.60 \times 10^{-19}\ \mathrm{C})(1.25\ \mathrm{V}) = 2.00 \times 10^{-19}\ \mathrm{J}$$

(Recall that 1 V = 1 J/C.) In terms of electron volts,

$$K_{max} = eV_0 = e(1.25\ \mathrm{V}) = 1.25\ \mathrm{eV}$$

since the electron volt (eV) is the magnitude of the electron charge e times one volt (1 V).

(b) From $K_{max} = \frac{1}{2}mv_{max}{}^2$ we get

$$v_{max} = \sqrt{\frac{2K_{max}}{m}} = \sqrt{\frac{2(2.00 \times 10^{-19}\ \mathrm{J})}{9.11 \times 10^{-31}\ \mathrm{kg}}} = 6.63 \times 10^5\ \mathrm{m/s}$$

EVALUATE The value of v_{max} is about 0.2% of the speed of light, so we are justified in using the nonrelativistic expression for kinetic energy. (An equivalent justification is that the electron's 1.25-eV kinetic energy is much less than its rest energy $mc^2 = 0.511\ \mathrm{MeV} = 5.11 \times 10^5\ \mathrm{eV}$.)

Example 38.3 Determining ϕ and h experimentally

For a particular cathode material in a photoelectric-effect experiment, you measure stopping potentials $V_0 = 1.0$ V for light of wavelength $\lambda = 600$ nm, 2.0 V for 400 nm, and 3.0 V for 300 nm. Determine the work function ϕ for this material and the implied value of Planck's constant h.

SOLUTION

IDENTIFY and SET UP: This example uses the relationship among stopping potential V_0, frequency f, and work function ϕ in the photoelectric effect. According to Eq. (38.4), a graph of V_0 versus f should be a straight line as in Fig. 38.5 or 38.6. Such a graph is completely determined by its slope and the value at which it intercepts the vertical axis; we will use these to determine the values of the target variables ϕ and h.

EXECUTE: We rewrite Eq. (38.4) as

$$V_0 = \frac{h}{e}f - \frac{\phi}{e}$$

In this form we see that the slope of the line is h/e and the vertical-axis intercept (corresponding to $f = 0$) is $-\phi/e$. The frequencies, obtained from $f = c/\lambda$ and $c = 3.00 \times 10^8$ m/s, are 0.50×10^{15} Hz, 0.75×10^{15} Hz, and 1.0×10^{15} Hz, respectively. From a graph of these data (see Fig. 38.6), we find

$$-\frac{\phi}{e} = \text{vertical intercept} = -1.0\ \mathrm{V}$$

$$\phi = 1.0\ \mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J}$$

and

$$\text{Slope} = \frac{\Delta V_0}{\Delta f} - \frac{3.0\ \mathrm{V} - (-1.0\ \mathrm{V})}{1.00 \times 10^{15}\ \mathrm{s^{-1}} - 0} = 4.0 \times 10^{-15}\ \mathrm{J \cdot s/C}$$

$$h = \text{slope} \times e = (4.0 \times 10^{-15}\ \mathrm{J \cdot s/C})(1.60 \times 10^{-19}\ \mathrm{C})$$
$$= 6.4 \times 10^{-34}\ \mathrm{J \cdot s}$$

EVALUATE: The value of Planck's constant h determined from your experiment differs from the accepted value by only about 3%. The small value $\phi = 1.0$ eV tells us that the cathode surface is not composed solely of one of the elements in Table 38.1.

Application Sterilizing with High-Energy Photons
One technique for killing harmful microorganisms is to illuminate them with ultraviolet light with a wavelength shorter than 254 nm. If a photon of such short wavelength strikes a DNA molecule within a microorganism, the energy of the photon is great enough to break the bonds within the molecule. This renders the microorganism unable to grow or reproduce. Such ultraviolet germicidal irradiation is used for medical sanitation, to keep laboratories sterile (as shown here), and to treat both drinking water and wastewater.

Test Your Understanding of Section 38.1 Silicon films become better electrical conductors when illuminated by photons with energies of 1.14 eV or greater, an effect called *photoconductivity*. Which of the following wavelengths of electromagnetic radiation can cause photoconductivity in silicon films? (i) ultraviolet light with $\lambda = 300$ nm; (ii) red light with $\lambda = 600$ nm; (iii) infrared light with $\lambda = 1200$ nm.

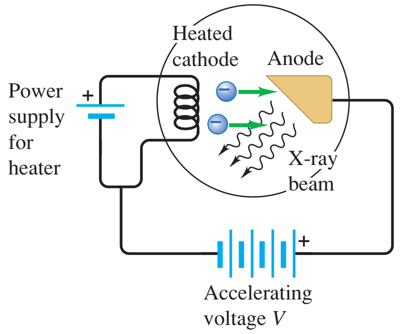
38.2 Light Emitted as Photons: X-Ray Production

The photoelectric effect provides convincing evidence that light is *absorbed* in the form of photons. For physicists to accept Einstein's radical photon concept, however, it was also necessary to show that light is *emitted* as photons. An experiment that demonstrates this convincingly is the inverse of the photoelectric effect: Instead of releasing electrons from a surface by shining electromagnetic radiation on it, we cause a surface to emit radiation—specifically, *x rays*—by bombarding it with fast-moving electrons.

X-Ray Photons

X rays were first produced in 1895 by the German physicist Wilhelm Röntgen, using an apparatus similar in principle to the setup shown in Fig. 38.7. Electrons are released from the cathode by *thermionic emission,* in which the escape energy is supplied by heating the cathode to a very high temperature. (As in the photoelectric effect, the minimum energy that an individual electron must be given to escape from the cathode's surface is equal to the work function for the surface. In this case the energy is provided to the electrons by heat rather than by light.) The electrons are then accelerated toward the anode by a potential difference V_{AC}. The bulb is evacuated (residual pressure 10^{-7} atm or less), so the electrons can travel from the cathode to the anode without colliding with air molecules. When V_{AC} is a few thousand volts or more, x rays are emitted from the anode surface.

38.7 An apparatus used to produce x rays, similar to Röntgen's 1895 apparatus.



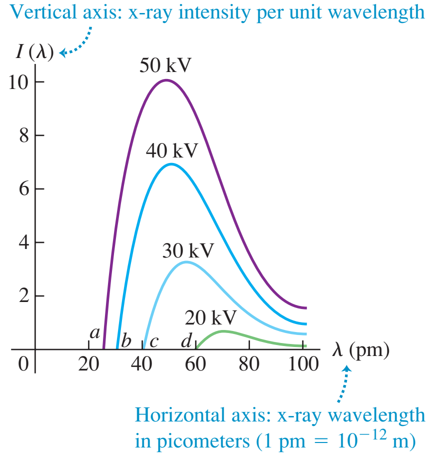
The anode produces x rays in part simply by slowing the electrons abruptly. (Recall from Section 32.1 that accelerated charges emit electromagnetic waves.) This process is called *bremsstrahlung* (German for "braking radiation"). Because the electrons undergo accelerations of very great magnitude, they emit much of their radiation at short wavelengths in the x-ray range, about 10^{-9} to 10^{-12} m (1 nm to 1 pm). (X-ray wavelengths can be measured quite precisely by crystal diffraction techniques, which we discussed in Section 36.6.) Most electrons are braked by a series of collisions and interactions with anode atoms, so bremsstrahlung produces a continuous spectrum of electromagnetic radiation.

Just as we did for the photoelectric effect in Section 38.1, let's compare what Maxwell's wave theory of electromagnetic radiation would predict about this radiation to what is observed experimentally.

Wave-Model Prediction: The electromagnetic waves produced when an electron slams into the anode should be analogous to the sound waves produced by crashing cymbals together. These waves include sounds of all frequencies. By analogy, the x rays produced by bremsstrahlung should have a spectrum that includes *all* frequencies and hence *all* wavelengths.

Experimental Result: Figure 38.8 shows bremsstrahlung spectra using the same cathode and anode with four different accelerating voltages. We see that *not* all x-ray frequencies and wavelengths are emitted: Each spectrum has a maximum frequency f_{max} and a corresponding minimum wavelength λ_{min}. The greater the potential difference V_{AC}, the higher the maximum frequency and the shorter the minimum wavelength.

38.8 The continuous spectrum of x rays produced when a tungsten target is struck by electrons accelerated through a voltage V_{AC}. The curves represent different values of V_{AC}; points a, b, c, and d show the minimum wavelength for each voltage.



The wave model of electromagnetic radiation cannot explain these experimental results. But we can readily understand them using the photon model. An electron has charge $-e$ and gains kinetic energy eV_{AC} when accelerated through a potential increase V_{AC}. The most energetic photon (highest frequency and shortest wavelength) is produced if the electron is braked to a stop all at once when it hits the anode, so that all of its kinetic energy goes to produce one photon; that is,

$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}} \qquad \text{(bremsstrahlung)} \qquad (38.6)$$

(In this equation we neglect the work function of the target anode and the initial kinetic energy of the electrons "boiled off" from the cathode. These energies are very small compared to the kinetic energy eV_{AC} gained due to the potential difference.) If only a portion of an electron's kinetic energy goes into producing a photon, the photon energy will be less than eV_{AC} and the wavelength will be greater than λ_{min}. As further support for the photon model, the measured values for λ_{min} for different values of eV_{AC} (see Fig. 38.8) agree with Eq. (38.6). Note that according to Eq. (38.6), the maximum frequency and minimum wavelength in the bremsstrahlung process do not depend on the target material; this also agrees with experiment. So we can conclude that the photon picture of electromagnetic radiation is valid for the *emission* as well as the absorption of radiation.

The apparatus shown in Fig. 38.7 can also produce x rays by a second process in which electrons transfer their kinetic energy partly or completely to individual atoms within the target. It turns out that this process not only is consistent with the photon model of electromagnetic radiation, but also provides insight into the structure of atoms. We'll return to this process in Section 41.5.

Example 38.4 Producing x rays

Electrons in an x-ray tube accelerate through a potential difference of 10.0 kV before striking a target. If an electron produces one photon on impact with the target, what is the minimum wavelength of the resulting x rays? Find the answer by expressing energies in both SI units and electron volts.

SOLUTION

IDENTIFY and SET UP: To produce an x-ray photon with minimum wavelength and hence maximum energy, all of the electron's kinetic energy must go into producing a single x-ray photon. We'll use Eq. (38.6) to determine the wavelength.

EXECUTE: From Eq. (38.6), using SI units we have

$$\lambda_{min} = \frac{hc}{eV_{AC}} = \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{(1.602 \times 10^{-19}\ \mathrm{C})(10.0 \times 10^{3}\ \mathrm{V})}$$

$$= 1.24 \times 10^{-10}\ \mathrm{m} = 0.124\ \mathrm{nm}$$

Using electron volts, we have

$$\lambda_{min} = \frac{hc}{eV_{AC}} = \frac{(4.136 \times 10^{-15}\ \mathrm{eV \cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{e(10.0 \times 10^{3}\ \mathrm{V})}$$

$$= 1.24 \times 10^{-10}\ \mathrm{m} = 0.124\ \mathrm{nm}$$

In the second calculation, the "e" for the magnitude of the electron charge cancels the "e" in the unit "eV," because the electron volt (eV) is the magnitude of the electron charge e times one volt (1 V).

EVALUATE: To check our result, recall from Example 38.1 that a 1.91-eV photon has a wavelength of 650 nm. Here the electron energy, and therefore the x-ray photon energy, is 10.0×10^{3} eV = 10.0 keV, about 5000 times greater than in Example 38.1, and the wavelength is about $\frac{1}{5000}$ as great as in Example 38.1. This makes sense, since wavelength and photon energy are inversely proportional.

Applications of X Rays

X rays have many practical applications in medicine and industry. Because x-ray photons are of such high energy, they can penetrate several centimeters of solid matter. Hence they can be used to visualize the interiors of materials that are opaque to ordinary light, such as broken bones or defects in structural steel. The object to be visualized is placed between an x-ray source and an electronic detector (like that used in a digital camera) or a piece of photographic film. The darker an area in the image recorded by such a detector, the greater the radiation exposure. Bones are much more effective x-ray absorbers than soft tissue, so bones appear as light areas. A crack or air bubble allows greater transmission and shows as a dark area.

38.9 This radiologist is operating a CT scanner (seen through the window) from a separate room to avoid repeated exposure to x rays.

A widely used and vastly improved x-ray technique is *computed tomography;* the corresponding instrument is called a *CT scanner.* The x-ray source produces a thin, fan-shaped beam that is detected on the opposite side of the subject by an array of several hundred detectors in a line. Each detector measures absorption along a thin line through the subject. The entire apparatus is rotated around the subject in the plane of the beam, and the changing photon-counting rates of the detectors are recorded digitally. A computer processes this information and reconstructs a picture of absorption over an entire cross section of the subject (see Fig. 38.9). Differences in absorption as small as 1% or less can be detected with CT scans, and tumors and other anomalies that are much too small to be seen with older x-ray techniques can be detected.

X rays cause damage to living tissues. As x-ray photons are absorbed in tissues, their energy breaks molecular bonds and creates highly reactive free radicals (such as neutral H and OH), which in turn can disturb the molecular structure of proteins and especially genetic material. Young and rapidly growing cells are particularly susceptible, which is why x rays are useful for selective destruction of cancer cells. Conversely, however, a cell may be damaged by radiation but survive, continue dividing, and produce generations of defective cells; thus x rays can *cause* cancer.

Even when the organism itself shows no apparent damage, excessive exposure to x rays can cause changes in the organism's reproductive system that will affect its offspring. A careful assessment of the balance between risks and benefits of radiation exposure is essential in each individual case.

Application X-Ray Absorption and Medical Imaging
Atomic electrons can absorb x rays. Hence materials with many electrons per atom tend to be better x-ray absorbers than materials with few electrons. In this x-ray image the lighter areas show where x rays are absorbed as they pass through the body, while the darker areas indicate regions that are relatively transparent to x rays. Bones contain large amounts of elements such as phosphorus and calcium, with 15 and 20 electrons per atom, respectively. In soft tissue, the predominant elements are hydrogen, carbon, and oxygen, with only 1, 6, and 8 electrons per atom, respectively. Hence x rays are absorbed by bone but can pass relatively easily through soft tissue.

Test Your Understanding of Section 38.2 In the apparatus shown in Fig. 38.7, suppose you increase the number of electrons that are emitted from the cathode per second while keeping the potential difference V_{AC} the same. How will this affect the intensity I and minimum wavelength λ_{min} of the emitted x rays? (i) I and λ_{min} will both increase; (ii) I will increase but λ_{min} will be unchanged; (iii) I will increase but λ_{min} will decrease; (iv) I will remain the same but λ_{min} will decrease; (v) none of these.

38.3 Light Scattered as Photons: Compton Scattering and Pair Production

The final aspect of light that we must test against Einstein's photon model is its behavior after the light is produced and before it is eventually absorbed. We can do this by considering the *scattering* of light. As we discussed in Section 33.6, scattering is what happens when light bounces off particles such as molecules in the air.

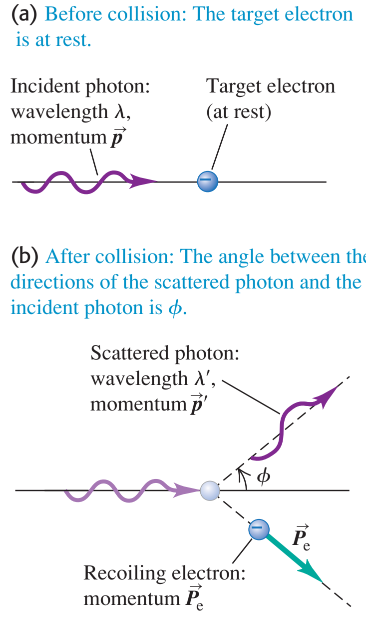
Compton Scattering

Let's see what Maxwell's wave model and Einstein's photon model predict for how light behaves when it undergoes scattering by a single electron, such as an individual electron within an atom.

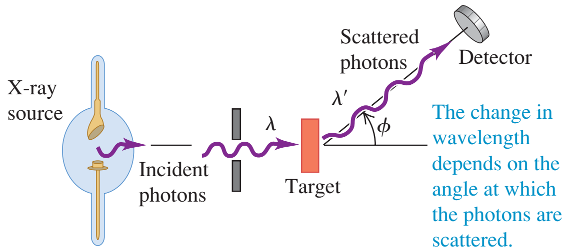
Wave-Model Prediction: In the wave description, scattering would be a process of absorption and re-radiation. Part of the energy of the light wave would be absorbed by the electron, which would oscillate in response to the oscillating electric field of the wave. The oscillating electron would act like a miniature antenna (see Section 32.1), re-radiating its acquired energy as *scattered* waves in a variety of directions. The frequency at which the electron oscillates would be the same as the frequency of the incident light, and the re-radiated light would have the same frequency as the oscillations of the electron. So, *in the wave model, the scattered light and incident light have the same frequency and same wavelength.*

Photon-Model Prediction: In the photon model we imagine the scattering process as a collision of two *particles,* the incident photon and an electron that is initially at rest (Fig. 38.10a). The incident photon would give up part of its energy and momentum to the electron, which recoils as a result of this impact. The scattered photon that remains can fly off at a variety of angles ϕ with respect to the incident direction, but it has less energy and less momentum than the incident photon (Fig. 38.10b). The energy and momentum of a photon are given by $E = hf = hc/\lambda$ (Eq. 38.2) and $p = hf/c = h/\lambda$ (Eq. 38.5). Therefore, *in the photon model, the scattered light has a lower frequency f and longer wavelength λ than the incident light.*

38.10 The photon model of light scattering by an electron.



38.11 A Compton-effect experiment.



The definitive experiment that tested these predictions of the wave and photon models was carried out in 1922 by the American physicist Arthur H. Compton. In his experiment Compton aimed a beam of x rays at a solid target and measured the wavelength of the radiation scattered from the target (Fig. 38.11). He discovered that some of the scattered radiation has smaller frequency (longer wavelength) than the incident radiation and that the change in wavelength depends on the angle through which the radiation is scattered. This is precisely what the photon model predicts for light scattered from electrons in the target, a process that is now called **Compton scattering.**

Specifically, if the scattered radiation emerges at an angle ϕ with respect to the incident direction, as shown in Fig. 38.11, and if λ and λ' are the wavelengths of the incident and scattered radiation, respectively, Compton found that

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \qquad \text{(Compton scattering)} \tag{38.7}$$

where m is the electron rest mass. In other words, λ' is greater than λ. The quantity h/mc that appears in Eq. (38.7) has units of length. Its numerical value is

$$\frac{h}{mc} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.109 \times 10^{-31}\ \text{kg})(2.998 \times 10^{8}\ \text{m/s})} = 2.426 \times 10^{-12}\ \text{m}$$

Compton showed that Einstein's photon theory, combined with the principles of conservation of energy and conservation of momentum, provides a beautifully clear explanation of his experimental results. We outline the derivation below. The electron recoil energy may be in the relativistic range, so we have to use the relativistic energy–momentum relationships, Eqs. (37.39) and (37.40). The incident photon has momentum $\vec{p}$, with magnitude p and energy pc. The scattered photon has momentum $\vec{p}\,'$, with magnitude p' and energy $p'c$. The electron is initially at rest, so its initial momentum is zero and its initial energy is its rest energy mc^2. The final electron momentum $\vec{P}_e$ has magnitude P_e, and the final electron energy is given by $E_e{}^2 = (mc^2)^2 + (P_e c)^2$. Then energy conservation gives us the relationship

$$pc + mc^2 = p'c + E_e$$

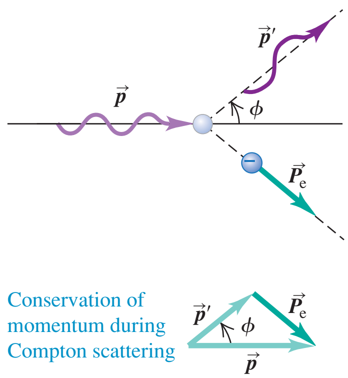
Rearranging, we find

$$(pc - p'c + mc^2)^2 = E_e{}^2 = (mc^2)^2 + (P_e c)^2 \tag{38.8}$$

We can eliminate the electron momentum $\vec{P}_e$ from Eq. (38.8) by using momentum conservation. From Fig. 38.12 we see that $\vec{p} = \vec{p}\,' + \vec{P}_e$, or

$$\vec{P}_e = \vec{p} - \vec{p}\,' \tag{38.9}$$

38.12 Vector diagram showing conservation of momentum in Compton scattering.



By taking the scalar product of each side of Eq. (38.9) with itself, we find

$$P_e^2 = p^2 + p'^2 - 2pp' \cos\phi \tag{38.10}$$

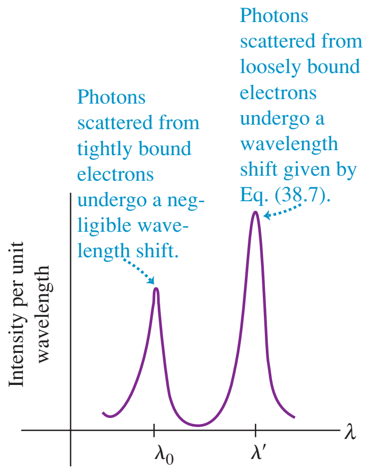
We now substitute this expression for P_e^2 into Eq. (38.8) and multiply out the left side. We divide out a common factor c^2; several terms cancel, and when the resulting equation is divided through by (pp'), the result is

$$\frac{mc}{p'} - \frac{mc}{p} = 1 - \cos\phi \tag{38.11}$$

Finally, we substitute $p' = h/\lambda'$ and $p = h/\lambda$, then multiply by h/mc to obtain Eq. (38.7).

When the wavelengths of x rays scattered at a certain angle are measured, the curve of intensity per unit wavelength as a function of wavelength has two peaks (Fig. 38.13). The longer-wavelength peak represents Compton scattering. The shorter-wavelength peak, labeled λ_0, is at the wavelength of the incident x rays and corresponds to x-ray scattering from tightly bound electrons. In such scattering processes the entire atom must recoil, so the m in Eq. (38.7) is the mass of the entire atom rather than of a single electron. The resulting wavelength shifts are negligible.

38.13 Intensity as a function of wavelength for photons scattered at an angle of 135° in a Compton-scattering experiment.



Example 38.5 Compton scattering

You use 0.124-nm x-ray photons in a Compton-scattering experiment. (a) At what angle is the wavelength of the scattered x rays 1.0% longer than that of the incident x rays? (b) At what angle is it 0.050% longer?

SOLUTION

IDENTIFY and SET UP: We'll use the relationship between scattering angle and wavelength shift in the Compton effect. In each case our target variable is the angle ϕ (see Fig. 38.10b). We solve for ϕ using Eq. (38.7).

EXECUTE: (a) In Eq. (38.7) we want $\Delta\lambda = \lambda' - \lambda$ to be 1.0% of 0.124 nm, so $\Delta\lambda = 0.00124\text{ nm} = 1.24 \times 10^{-12}$ m. Using the value $h/mc = 2.426 \times 10^{-12}$ m, we find

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$

$$\cos\phi = 1 - \frac{\Delta\lambda}{h/mc} = 1 - \frac{1.24 \times 10^{-12}\text{ m}}{2.426 \times 10^{-12}\text{ m}} = 0.4889$$

$$\phi = 60.7°$$

(b) For $\Delta\lambda$ to be 0.050% of 0.124 nm, or 6.2×10^{-14} m,

$$\cos\phi = 1 - \frac{6.2 \times 10^{-14}\text{ m}}{2.426 \times 10^{-12}\text{ m}} = 0.9744$$

$$\phi = 13.0°$$

EVALUATE: Our results show that smaller scattering angles give smaller wavelength shifts. Thus in a grazing collision the photon energy loss and the electron recoil energy are smaller than when the scattering angle is larger. This is just what we would expect for an elastic collision, whether between a photon and an electron or between two billiard balls.

Pair Production

Another effect that can be explained only with the photon picture involves *gamma rays,* the shortest-wavelength and highest-frequency variety of electromagnetic radiation. If a gamma-ray photon of sufficiently short wavelength is fired at a target, it may not scatter. Instead, as depicted in Fig. 38.14, it may disappear completely and be replaced by two new particles: an electron and a **positron** (a particle that has the same rest mass m as an electron but has a positive charge $+e$ rather than the negative charge $-e$ of the electron). This process, called **pair production,** was first observed by the physicists Patrick Blackett and Giuseppe Occhialini in 1933. The electron and positron have to be produced in pairs in order to conserve electric charge: The incident photon has zero charge, and the electron–positron pair has net charge $(-e) + (+e) = 0$. Enough energy must be available to account for the rest energy $2mc^2$ of the two particles. To four significant figures, this minimum energy is

$$E_{\text{min}} = 2mc^2 = 2(9.109 \times 10^{-31}\ \text{kg})(2.998 \times 10^8\ \text{m/s})^2$$
$$= 1.637 \times 10^{-13}\ \text{J} = 1.022\ \text{MeV}$$

Thus the photon must have at least this much energy to produce an electron–positron pair. From Eq. (38.2), $E = hc/\lambda$, the photon wavelength has to be shorter than

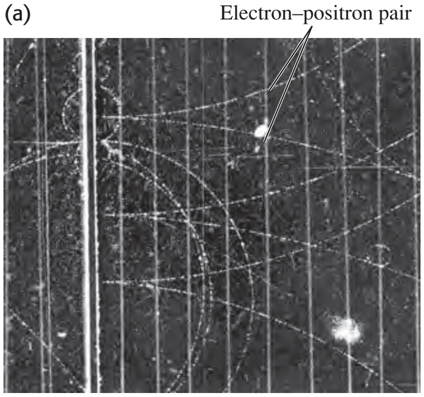
$$\lambda_{\text{max}} = \frac{hc}{E_{\text{min}}} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})}{1.637 \times 10^{-13}\ \text{J}}$$
$$= 1.213 \times 10^{-12}\ \text{m} = 1.213 \times 10^{-3}\ \text{nm} = 1.213\ \text{pm}$$

This is a very short wavelength, about $\frac{1}{1000}$ as large as the x-ray wavelengths that Compton used in his scattering experiments. (The requisite minimum photon energy is actually a bit higher than 1.022 MeV, so the photon wavelength must be a bit shorter than 1.213 pm. The reason is that when the incident photon encounters an atomic nucleus in the target, some of the photon energy goes into the kinetic energy of the recoiling nucleus.) Just as for the photoelectric effect, the wave model of electromagnetic radiation cannot explain why pair production occurs only when very short wavelengths are used.

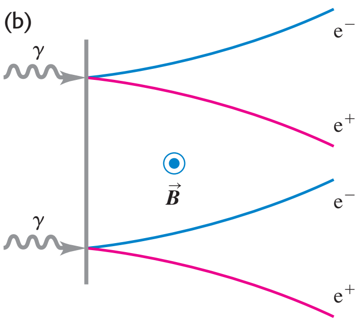
The inverse process, *electron–positron pair annihilation,* occurs when a positron and an electron collide. Both particles disappear, and two (or occasionally three) photons can appear, with total energy of at least $2m_ec^2 = 1.022$ MeV. Decay into a *single* photon is impossible because such a process could not conserve both energy and momentum. It's easiest to analyze this annihilation process in the frame of reference called the *center-of-momentum system,* in which the total momentum is zero. It is the relativistic generalization of the center-of-mass system that we discussed in Section 8.5.

38.14 (a) Photograph of bubble-chamber tracks of electron–positron pairs that are produced when 300-MeV photons strike a lead sheet. A magnetic field directed out of the photograph made the electrons (e^-) and positrons (e^+) curve in opposite directions. (b) Diagram showing the pair-production process for two of the gamma-ray photons (γ).

(a)



(b)



Example 38.6 Pair annihilation

An electron and a positron, initially far apart, move toward each other with the same speed. They collide head-on, annihilating each other and producing two photons. Find the energies, wavelengths, and frequencies of the photons if the initial kinetic energies of the electron and positron are (a) both negligible and (b) both 5.000 MeV. The electron rest energy is 0.511 MeV.

SOLUTION

IDENTIFY and SET UP: Just as in the elastic collisions we studied in Chapter 8, both momentum and energy are conserved in pair annihilation. The electron and positron are initially far apart, so the initial electric potential energy is zero and the initial energy is the sum of the particle kinetic and rest energies. The final energy is the sum of the photon energies. The total initial momentum is zero; the total momentum of the two photons must likewise be zero. We find the photon energy E using conservation of energy, conservation of momentum, and the relationship $E = pc$ (see Section 38.1). We then calculate the wavelengths and frequencies using $E = hc/\lambda = hf$.

EXECUTE: If the total momentum of the two photons is to be zero, their momenta must have equal magnitudes p and opposite directions. From $E = pc = hc/\lambda = hf$, the two photons must also have the same energy E, wavelength λ, and frequency f.

Before the collision the energy of each electron is $K + mc^2$, where K is its kinetic energy and $mc^2 = 0.511$ MeV. Conservation of energy then gives

$$(K + mc^2) + (K + mc^2) = E + E$$

Hence the energy of each photon is $E = K + mc^2$.

(a) In this case the electron kinetic energy K is negligible compared to its rest energy mc^2, so each photon has energy $E = mc^2 = 0.511$ MeV. The corresponding photon wavelength and frequency are

$$\lambda = \frac{hc}{E} = \frac{(4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{0.511 \times 10^6\ \text{eV}}$$
$$= 2.43 \times 10^{-12}\ \text{m} = 2.43\ \text{pm}$$
$$f = \frac{E}{h} = \frac{0.511 \times 10^6\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}} = 1.24 \times 10^{20}\ \text{Hz}$$

(b) In this case $K = 5.000$ MeV, so each photon has energy $E = 5.000\ \text{MeV} + 0.511\ \text{MeV} = 5.511$ MeV. Proceeding as in part (a), you can show that the photon wavelength is 0.2250 pm and the frequency is 1.333×10^{21} Hz.

EVALUATE: As a check, recall from Example 38.1 that a 650-nm visible-light photon has energy 1.91 eV and frequency 4.62×10^{14} Hz. The photon energy in part (a) is about 2.5×10^5 times greater. As expected, the photon's wavelength is shorter and its frequency higher than those for a visible-light photon by the same factor. You can check the results for part (b) in the same way.

Test Your Understanding of Section 38.3 If you used visible-light photons in the experiment shown in Fig. 38.11, would the photons undergo a wavelength shift due to the scattering? If so, is it possible to detect the shift with the human eye?

38.4 Wave–Particle Duality, Probability, and Uncertainty

We have studied many examples of the behavior of light and other electromagnetic radiation. Some, including the interference and diffraction effects described in Chapters 35 and 36, demonstrate conclusively the *wave* nature of light. Others, the subject of the present chapter, point with equal force to the *particle* nature of light. At first glance these two aspects seem to be in direct conflict. How can light be a wave and a particle at the same time?

We can find the answer to this apparent wave–particle conflict in the **principle of complementarity,** first stated by the Danish physicist Niels Bohr in 1928. The wave descriptions and the particle descriptions are complementary. That is, we need both to complete our model of nature, but we will never need to use both at the same time to describe a single part of an occurrence.

Diffraction and Interference in the Photon Picture

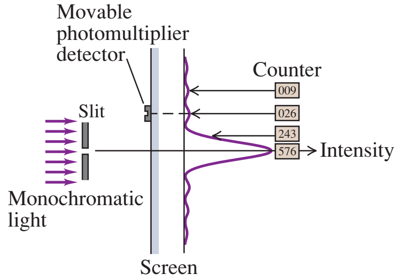
Let's start by considering again the diffraction pattern for a single slit, which we analyzed in Sections 36.2 and 36.3. Instead of recording the pattern on a digital camera chip or photographic film, we use a detector called a *photomultiplier* that can actually detect individual photons. Using the setup shown in Fig. 38.15, we place the photomultiplier at various positions for equal time intervals, count the photons at each position, and plot out the intensity distribution.

We find that, on average, the distribution of photons agrees with our predictions from Section 36.3. At points corresponding to the maxima of the pattern, we count many photons; at minimum points, we count almost none; and so on. The graph of the counts at various points gives the same diffraction pattern that we predicted with Eq. (36.7).

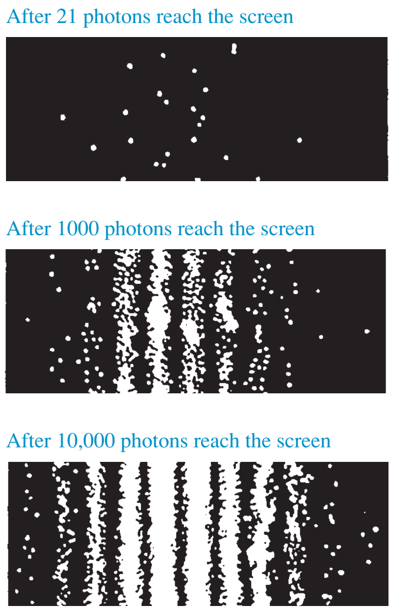
But suppose we now reduce the intensity to such a low level that only a few photons per second pass through the slit. We now record a series of discrete strikes, each representing a single photon. While we *cannot predict* where any given photon will strike, over time the accumulating strikes build up the familiar diffraction pattern we expect for a wave. To reconcile the wave and particle aspects of this pattern, we have to regard the pattern as a *statistical* distribution that tells us how many photons, on average, go to each spot. Equivalently, the pattern tells us the *probability* that any individual photon will land at a given spot. If we shine our faint light beam on a two-slit apparatus, we get an analogous result (Fig. 38.16). Again we can't predict exactly where an individual photon will go; the interference pattern is a statistical distribution.

How does the principle of complementarity apply to these diffraction and interference experiments? The wave description, not the particle description, explains the single- and double-slit patterns. But the particle description, not the wave description, explains why the photomultiplier records discrete packages of energy. The two descriptions complete our understanding of the results. For instance, suppose we consider an individual photon and ask how it knows "which way to go" when passing through the slit. This question seems like a conundrum, but that is because it is framed in terms of a *particle* description—whereas it is the *wave* nature of light that determines the distribution of photons. Conversely,

38.15 Single-slit diffraction pattern of light observed with a movable photomultiplier. The curve shows the intensity distribution predicted by the wave picture. The photon distribution is shown by the numbers of photons counted at various positions.



38.16 These images record the positions where individual photons in a two-slit interference experiment strike the screen. As more photons reach the screen, a recognizable interference pattern appears.



the fact that the photomultiplier detects faint light as a sequence of individual "spots" can't be explained in wave terms.

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Probability and Uncertainty

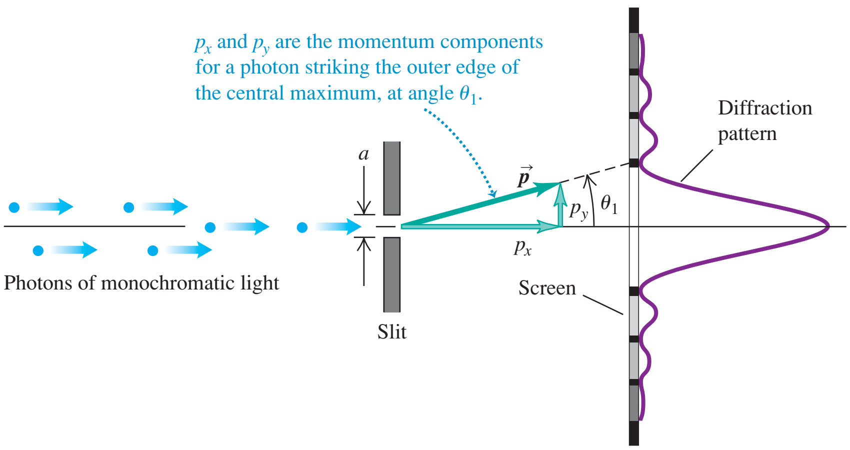
Although photons have energy and momentum, they are nonetheless very different from the particle model we used for Newtonian mechanics in Chapters 4 through 8. The Newtonian particle model treats an object as a point mass. We can describe the location and state of motion of such a particle at any instant with three spatial coordinates and three components of momentum, and we can then predict the particle's future motion. This model doesn't work at all for photons, however: We *cannot* treat a photon as a point object. This is because there are fundamental limitations on the precision with which we can simultaneously determine the position and momentum of a photon. Many aspects of a photon's behavior can be stated only in terms of *probabilities.* (In Chapter 39 we will find that the non-Newtonian ideas we develop for photons in this section also apply to particles such as electrons.)

To get more insight into the problem of measuring a photon's position and momentum simultaneously, let's look again at the single-slit diffraction of light (Fig. 38.17). Suppose the wavelength λ is much less than the slit width a. Then most (85%) of the photons go into the central maximum of the diffraction pattern, and the remainder go into other parts of the pattern. We use θ_1 to denote the angle between the central maximum and the first minimum. Using Eq. (36.2) with $m = 1$, we find that θ_1 is given by $\sin\theta_1 = \lambda/a$. Since we assume $\lambda = a$, it follows that θ_1 is very small, $\sin\theta_1$ is very nearly equal to θ_1 (in radians), and

$$\theta_1 = \frac{\lambda}{a} \tag{38.12}$$

Even though the photons all have the same initial state of motion, they don't all follow the same path. We can't predict the exact trajectory of any individual photon from knowledge of its initial state; we can only describe the *probability* that an individual photon will strike a given spot on the screen. This fundamental indeterminacy has no counterpart in Newtonian mechanics.

Furthermore, there are fundamental *uncertainties* in both the position and the momentum of an individual particle, and these uncertainties are related inseparably. To clarify this point, let's go back to Fig. 38.17. A photon that strikes the screen at the outer edge of the central maximum, at angle θ_1, must have a component of momentum p_y in the y-direction, as well as a component p_x in the x-direction, despite the fact that initially the beam was directed along the x-axis. From the geometry of the situation the two components are related by $p_y/p_x = \tan\theta_1$. Since θ_1 is small, we may use the approximation $\tan\theta_1 = \theta_1$, and



38.17 Interpreting single-slit diffraction in terms of photon momentum.

$$p_y = p_x\theta_1 \tag{38.13}$$

Substituting Eq. (38.12), $\theta_1 = \lambda/a$, into Eq. (38.13) gives

$$p_y = p_x\frac{\lambda}{a} \tag{38.14}$$

Equation (38.14) says that for the 85% of the photons that strike the detector within the central maximum (that is, at angles between $-\lambda/a$ and $+\lambda/a$), the y-component of momentum is spread out over a range from $-p_x\lambda/a$ to $+p_x\lambda/a$. Now let's consider *all* the photons that pass through the slit and strike the screen. Again, they may hit above or below the center of the pattern, so their component p_y may be positive or negative. However the symmetry of the diffraction pattern shows us the average value $(p_y)_{av} = 0$. There will be an *uncertainty* Δp_y in the y-component of momentum at least as great as $p_x\lambda/a$. That is,

$$\Delta p_y \geq p_x\frac{\lambda}{a} \tag{38.15}$$

The narrower the slit width a, the broader is the diffraction pattern and the greater is the uncertainty in the y-component of momentum p_y.

The photon wavelength λ is related to the momentum p_x by Eq. (38.5), which we can rewrite as $\lambda = h/p_x$. Using this relationship in Eq. (38.15) and simplifying, we find

$$\Delta p_y \geq p_x\frac{h}{p_x a} = \frac{h}{a}$$

$$\Delta p_y a \geq h \tag{38.16}$$

What does Eq. (38.16) mean? The slit width a represents an uncertainty in the y-component of the *position* of a photon as it passes through the slit. We don't know exactly *where* in the slit each photon passes through. So both the y-position and the y-component of momentum have uncertainties, and the two uncertainties are related by Eq. (38.16). We can reduce the *momentum* uncertainty Δp_y only by reducing the width of the diffraction pattern. To do this, we have to increase the slit width a, which increases the *position* uncertainty. Conversely, when we *decrease* the position uncertainty by narrowing the slit, the diffraction pattern broadens and the corresponding momentum uncertainty *increases.*

You may protest that it doesn't seem to be consistent with common sense for a photon not to have a definite position and momentum. We reply that what we call *common sense* is based on familiarity gained through experience. Our usual experience includes very little contact with the microscopic behavior of particles like photons. Sometimes we have to accept conclusions that violate our intuition when we are dealing with areas that are far removed from everyday experience.

The Uncertainty Principle

In more general discussions of uncertainty relationships, the uncertainty of a quantity is usually described in terms of the statistical concept of *standard deviation,* which is a measure of the spread or dispersion of a set of numbers around their average value. Suppose we now begin to describe uncertainties in this way [neither Δp_y nor a in Eq. (38.16) is a standard deviation]. If a coordinate x has an uncertainty Δx and if the corresponding momentum component p_x has an uncertainty Δp_x, then those standard-deviation uncertainties are found to be related in general by the inequality

$$\Delta x\,\Delta p_x \geq \hbar/2 \quad \text{(Heisenberg uncertainty principle for position and momentum)} \tag{38.17}$$

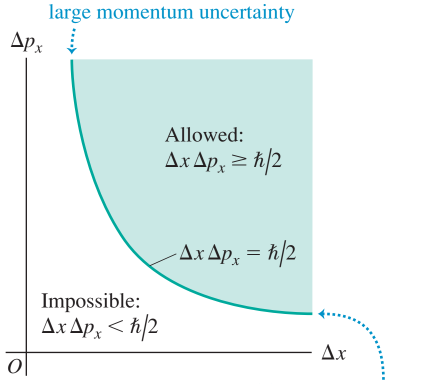
In this expression the quantity $\hbar$ (pronounced "h-bar") is Planck's constant divided by 2π:

$$\hbar = \frac{h}{2\pi} = 1.054571628(53) \times 10^{-34}\ \mathrm{J \cdot s}$$

We will use this quantity frequently to avoid writing a lot of factors of 2π in later equations.

CAUTION **h versus h-bar** It's common for students to plug in the value of h when what they really wanted was $\hbar = h/2\pi$, or vice versa. Be careful not to make the same mistake, or you'll find yourself wondering why your answer is off by a factor of 2π!

38.18 The Heisenberg uncertainty principle for position and momentum components. It is impossible for the product $\Delta x \Delta p_x$ to be less than $\hbar/2 = h/4\pi$.



Equation (38.17) is one form of the **Heisenberg uncertainty principle,** first discovered by the German physicist Werner Heisenberg (1901–1976). It states that, in general, it is impossible to simultaneously determine both the position and the momentum of a particle with arbitrarily great precision, as classical physics would predict. Instead, the uncertainties in the two quantities play complementary roles, as we have described. Figure 38.18 shows the relationship between the two uncertainties. Our derivation of Eq. (38.16), a less refined form of the uncertainty principle given by Eq. (38.17), shows that this principle has its roots in the wave aspect of photons. We will see in Chapter 39 that electrons and other subatomic particles also have a wave aspect, and the same uncertainty principle applies to them as well.

It is tempting to suppose that we could get greater precision by using more sophisticated detectors of position and momentum. This turns out not to be possible. To detect a particle, the detector must *interact* with it, and this interaction unavoidably changes the state of motion of the particle, introducing uncertainty about its original state. For example, we could imagine placing an electron at a certain point in the middle of the slit in Fig. 38.17. If the photon passes through the middle, we would see the electron recoil. We would then know that the photon passed through that point in the slit, and we would be much more certain about the x-coordinate of the photon. However, the collision between the photon and the electron would change the photon momentum, giving us greater uncertainty in the value of that momentum. A more detailed analysis of such hypothetical experiments shows that the uncertainties we have described are fundamental and intrinsic. They *cannot* be circumvented *even in principle* by any experimental technique, no matter how sophisticated.

There is nothing special about the x-axis. In a three-dimensional situation with coordinates (x, y, z) there is an uncertainty relationship for each coordinate and its corresponding momentum component: $\Delta x \Delta p_x \geq \hbar/2$, $\Delta y \Delta p_y \geq \hbar/2$, and $\Delta z \Delta p_z \geq \hbar/2$. However, the uncertainty in one coordinate is *not* related to the uncertainty in a different component of momentum. For example, Δx is not related directly to Δp_y.

Waves and Uncertainty

Here's an alternative way to understand the Heisenberg uncertainty principle in terms of the properties of waves. Consider a sinusoidal electromagnetic wave propagating in the positive x-direction with its electric field polarized in the y-direction. If the wave has wavelength λ, frequency f, and amplitude A, we can write the wave function as

$$E_y(x, t) = A \sin(kx - \omega t) \tag{38.18}$$

In this expression the wave number is $k = 2\pi/\lambda$ and the angular frequency is $\omega = 2\pi f$. We can think of the wave function in Eq. (38.18) as a description of a photon with a definite wavelength and a definite frequency. In terms of k and ω we can express the momentum and energy of the photon as

$$p_x = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k \qquad \text{(photon momentum in terms of wave number)} \tag{38.19a}$$

$$E = hf = \frac{h}{2\pi} 2\pi f = \hbar\omega \qquad \text{(photon energy in terms of angular frequency)} \qquad (38.19b)$$

Using Eqs. (38.19) in Eq. (38.18), we can rewrite our photon wave equation as

$$E_y(x, t) = A\sin[(p_x x - Et)/\hbar] \qquad \text{(wave function for a photon with } x\text{-momentum } p_x \text{ and energy } E) \qquad (38.20)$$

Since this wave function has a definite value of x-momentum p_x, there is *no* uncertainty in the value of this quantity: $\Delta p_x = 0$. The Heisenberg uncertainty principle, Eq. (38.17), says that $\Delta x \Delta p_x \geq \hbar/2$. If Δp_x is zero, then Δx must be infinite. Indeed, the wave described by Eq. (38.20) extends along the entire x-axis and has the same amplitude everywhere. The price we pay for knowing the photon's momentum precisely is that we have no idea *where* the photon is!

In practical situations we always have *some* idea where a photon is. To describe this situation, we need a wave function that is more localized in space. We can create one by superimposing two or more sinusoidal functions. To keep things simple, we'll consider only waves propagating in the positive x-direction. For example, let's add together two sinusoidal wave functions like those in Eqs. (38.18) and (38.20), but with slightly different wavelengths and frequencies and hence slightly different values p_{x1} and p_{x2} of x-momentum and slightly different values E_1 and E_2 of energy. The total wave function is

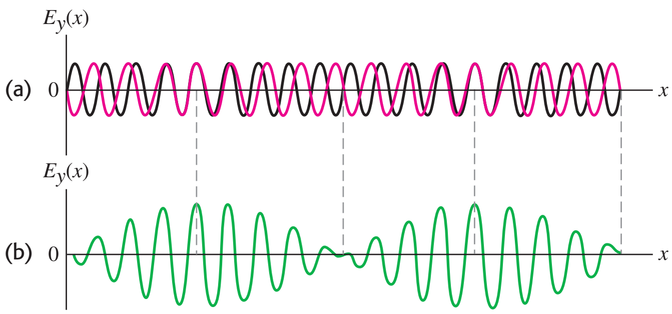
$$E_y(x, t) = A_1\sin[(p_{1x}x - E_1t)/\hbar] + A_2\sin[(p_{2x}x - E_2t)/\hbar] \qquad (38.21)$$

Consider what this wave function looks like at a particular instant of time, say $t = 0$. At this instant Eq. (38.21) becomes

$$E_y(x, t = 0) = A_1\sin(p_{1x}x/\hbar) + A_2\sin(p_{2x}x/\hbar) \qquad (38.22)$$

Figure 38.19a is a graph of the individual wave functions at $t = 0$ for the case $A_2 = -A_1$, and Fig. 38.19b graphs the combined wave function $E_y(x, t = 0)$ given by Eq. (38.22). We saw something very similar to Fig. 38.19b in our discussion of beats in Section 16.7: When we superimposed two sinusoidal waves with slightly different frequencies (see Fig. 16.24), the resulting wave exhibited amplitude variations not present in the original waves. In the same way, a photon represented by the wave function in Eq. (38.21) is most likely to be found in the regions where the wave function's amplitude is greatest. That is, the photon is *localized.* However, the photon's momentum no longer has a definite value because we began with two different x-momentum values, p_{x1} and p_{x2}. This agrees with the Heisenberg uncertainty principle: By decreasing the uncertainty in the photon's position, we have increased the uncertainty in its momentum.

38.19 (a) Two sinusoidal waves with slightly different wave numbers k and hence slightly different values of momentum $p_x = \hbar k$ shown at one instant of time. (b) The superposition of these waves has a momentum equal to the average of the two individual values of momentum. The amplitude varies, giving the total wave a lumpy character not possessed by either individual wave.



Uncertainty in Energy

Our discussion of combining waves also shows that there is an uncertainty principle that involves *energy* and *time*. To see why this is so, imagine measuring the combined wave function described by Eq. (38.21) at a certain position, say $x = 0$, over a period of time. At $x = 0$, the wave function from Eq. (38.21) becomes

$$\begin{aligned} E_y(x, t) &= A_1 \sin(-E_1 t/\hbar) + A_2 \sin(-E_2 t/\hbar) \\ &= -A_1 \sin(E_1 t/\hbar) - A_2 \sin(E_2 t/\hbar) \end{aligned} \tag{38.23}$$

What we measure at $x = 0$ is a combination of two oscillating electric fields with slightly different angular frequencies $\omega_1 = E_1/\hbar$ and $\omega_2 = E_2/\hbar$. This is exactly the phenomenon of beats that we discussed in Section 16.7 (compare Fig. 16.24). The amplitude of the combined field rises and falls, so the photon described by this field is localized in *time* as well as in position. The photon is most likely to be found at the times when the amplitude is large. The price we pay for localizing the photon in time is that the wave does not have a definite energy. By contrast, if the photon is described by a sinousoidal wave like that in Eq. (38.20) that *does* have a definite energy E but that has the same amplitude at all times, we have no idea when the photon will appear at $x = 0$. So the better we know the photon's energy, the less certain we are of when we will observe the photon.

Just as for the momentum–position uncertainty principle, we can write a mathematical expression for the uncertainty principle that relates energy and time. In fact, except for an overall minus sign, Eq. (38.23) is identical to Eq. (38.22) if we replace the x-momentum p_x by energy E and the position x by time t. This tells us that in the momentum–position uncertainty relation, Eq. (38.17), we can replace the momentum uncertainty Δp_x with the energy uncertainty ΔE and replace the position uncertainty Δx with the time uncertainty Δt. The result is

$$\Delta t \, \Delta E \geq \hbar/2 \quad \text{(Heisenberg uncertainty principle for energy and time)} \tag{38.24}$$

In practice, any real photon has a limited spatial extent and hence passes any point in a limited amount of time. The following example illustrates how this affects the momentum and energy of the photon.

Example 38.7 Ultrashort laser pulses and the uncertainty principle

Many varieties of lasers emit light in the form of pulses rather than a steady beam. A tellurium–sapphire laser can produce light at a wavelength of 800 nm in ultrashort pulses that last only 4.00×10^{-15} s (4.00 femtoseconds, or 4.00 fs). The energy in a single pulse produced by one such laser is $2.00\ \mu\text{J} = 2.00 \times 10^{-6}$ J, and the pulses propagate in the positive x-direction. Find (a) the frequency of the light; (b) the energy and minimum energy uncertainty of a single photon in the pulse; (c) the minimum frequency uncertainty of the light in the pulse; (d) the spatial length of the pulse, in meters and as a multiple of the wavelength; (e) the momentum and minimum momentum uncertainty of a single photon in the pulse; and (f) the approximate number of photons in the pulse.

SOLUTION

IDENTIFY and SET UP: It's important to distinguish between the light pulse as a whole (which contains a very large number of photons) and an individual photon within the pulse. The 5.00-fs pulse duration represents the time it takes the pulse to emerge from the laser; it is also the time *uncertainty* for an individual photon within the pulse, since we don't know when during the pulse that photon emerges. Similarly, the position uncertainty of a photon is the spatial length of the pulse, since a given photon could be found anywhere within the pulse. To find our target variables, we'll use the relationships for photon energy and momentum from Section 38.1 and the two Heisenberg uncertainty principles, Eqs. (38.17) and (38.24).

EXECUTE: (a) From the relationship $c = \lambda f$, the frequency of 800-nm light is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{8.00 \times 10^{-7}\ \text{m}} = 3.75 \times 10^{14}\ \text{Hz}$$

(b) From Eq. (38.2) the energy of a single 800-nm photon is

$$\begin{aligned} E = hf &= (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.75 \times 10^{14}\ \text{Hz}) \\ &= 2.48 \times 10^{-19}\ \text{J} \end{aligned}$$

The time uncertainty equals the pulse duration, $\Delta t = 4.00 \times 10^{-15}$ s. From Eq. (38.24) the minimum uncertainty in energy corresponds to the case $\Delta t \, \Delta E = \hbar/2$, so

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}{2(4.00 \times 10^{-15}\ \text{s})} = 1.32 \times 10^{-20}\ \text{J}$$

This is 5.3% of the photon energy $E = 2.48 \times 10^{-19}$ J, so the energy of a given photon is uncertain by at least 5.3%. The uncertainty could be greater, depending on the shape of the pulse.

(c) From the relationship $f = E/h$, the minimum frequency uncertainty is

$$\Delta f = \frac{\Delta E}{h} = \frac{1.32 \times 10^{-20}\ \text{J}}{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}} = 1.99 \times 10^{13}\ \text{Hz}$$

This is 5.3% of the frequency $f = 3.75 \times 10^{14}$ Hz we found in part (a). Hence these ultrashort pulses do not have a definite frequency; the average frequency of many such pulses will be 3.75×10^{14} Hz, but the frequency of any individual pulse can be anywhere from 5.3% higher to 5.3% lower.

(d) The spatial length Δx of the pulse is the distance that the front of the pulse travels during the time $\Delta t = 4.00 \times 10^{-15}$ s it takes the pulse to emerge from the laser:

$$\Delta x = c\Delta t = (3.00 \times 10^{8}\ \text{m/s})(4.00 \times 10^{-15}\ \text{s}) = 1.20 \times 10^{-6}\ \text{m}$$

$$\Delta x = \frac{1.20 \times 10^{-6}\ \text{m}}{8.00 \times 10^{-7}\ \text{m/wavelength}} = 1.50\ \text{wavelengths}$$

This justifies the term *ultrashort.* The pulse is less than two wavelengths long!

(e) From Eq. (38.5), the momentum of an average photon in the pulse is

$$p_x = \frac{E}{c} = \frac{2.48 \times 10^{-19}\ \text{J}}{3.00 \times 10^{8}\ \text{m/s}} = 8.28 \times 10^{-28}\ \text{kg}\cdot\text{m/s}$$

The spatial uncertainty is $\Delta x = 1.20 \times 10^{-6}$ m. From Eq. (38.17) minimum momentum uncertainty corresponds to $\Delta x\,\Delta p_x = \hbar/2$, so

$$\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}{2(1.20 \times 10^{-6}\ \text{m})} = 4.40 \times 10^{-29}\ \text{kg}\cdot\text{m/s}$$

This is 5.3% of the average photon momentum p_x. An individual photon within the pulse can have a momentum that is 5.3% greater or less than the average.

(f) To estimate the number of photons in the pulse, we divide the total pulse energy by the average photon energy:

$$\frac{2.00 \times 10^{-6}\ \text{J/pulse}}{2.48 \times 10^{-19}\ \text{J/photon}} = 8.06 \times 10^{12}\ \text{photons/pulse}$$

The energy of an individual photon is uncertain, so this is the *average* number of photons per pulse.

EVALUATE: The percentage uncertainties in energy and momentum are large because this laser pulse is so short. If the pulse were longer, both Δt and Δx would be greater and the corresponding uncertainties in photon energy and photon momentum would be smaller.

Our calculation in part (f) shows an important distinction between photons and other kinds of particles. In principle it is possible to make an exact count of the number of electrons, protons, and neutrons in an object such as this book. If you repeated the count, you would get the same answer as the first time. By contrast, if you counted the number of photons in a laser pulse you would *not* necessarily get the same answer every time! The uncertainty in photon energy means that on each count there could be a different number of photons whose individual energies sum to 2.00×10^{-6} J. That's yet another of the many strange properties of photons.

Test Your Understanding of Section 38.4 Through which of the following angles is a photon of wavelength λ most likely to be deflected after passing through a slit of width a? Assume that λ is much less than a. (i) $\theta = \lambda/a$; (ii) $\theta = 3\lambda/2a$; (iii) $\theta = 2\lambda/a$; (iv) $\theta = 3\lambda/a$; (v) not enough information given to decide.

CHAPTER 38 SUMMARY

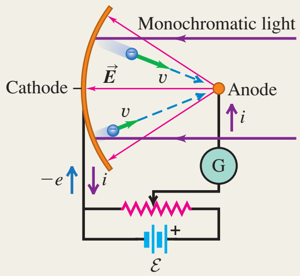
Photons: Electromagnetic radiation behaves as both waves and particles. The energy in an electromagnetic wave is carried in units called photons. The energy E of one photon is proportional to the wave frequency f and inversely proportional to the wavelength λ, and is proportional to a universal quantity h called Planck's constant. The momentum of a photon has magnitude E/c. (See Example 38.1.)

$$E = hf = \frac{hc}{\lambda} \quad (38.2)$$

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \quad (38.5)$$

The photoelectric effect: In the photoelectric effect, a surface can eject an electron by absorbing a photon whose energy hf is greater than or equal to the work function ϕ of the material. The stopping potential V_0 is the voltage required to stop a current of ejected electrons from reaching an anode. (See Examples 38.2 and 38.3.)

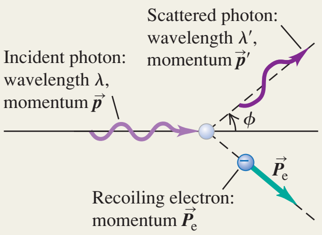
$$eV_0 = hf - \phi \quad (38.4)$$



Photon production, photon scattering, and pair production: X rays can be produced when electrons accelerated to high kinetic energy across a potential increase V_{AC} strike a target. The photon model explains why the maximum frequency and minimum wavelength produced are given by Eq. (38.6). (See Example 38.4.) In Compton scattering a photon transfers some of its energy and momentum to an electron with which it collides. For free electrons (mass m), the wavelengths of incident and scattered photons are related to the photon scattering angle ϕ by Eq. (38.7). (See Example 38.5.) In pair production a photon of sufficient energy can disappear and be replaced by an electron–positron pair. In the inverse process, an electron and a positron can annihilate and be replaced by a pair of photons. (See Example 38.6.)

$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}} \quad (38.6)$$
(bremsstrahlung)

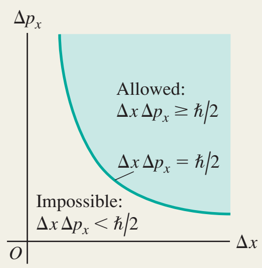
$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \quad (38.7)$$
(Compton scattering)



The Heisenberg uncertainty principle: It is impossible to determine both a photon's position and its momentum at the same time to arbitrarily high precision. The precision of such measurements for the x-components is limited by the Heisenberg uncertainty principle, Eq. (38.17); there are corresponding relationships for the y- and z-components. The uncertainty ΔE in the energy of a state that is occupied for a time Δt is given by Eq. (38.24). In these expressions, $\hbar = h/2\pi$. (See Example 38.7.)

$$\Delta x \, \Delta p_x \geq \hbar/2 \quad (38.17)$$
(Heisenberg uncertainty principle for position and momentum)

$$\Delta t \, \Delta E \geq \hbar/2 \quad (38.24)$$
(Heisenberg uncertainty principle for energy and time)



BRIDGING PROBLEM Compton Scattering and Electron Recoil

An incident x-ray photon is scattered from a free electron that is initially at rest. The photon is scattered straight back at an angle of 180° from its initial direction. The wavelength of the scattered photon is 0.0830 nm. (a) What is the wavelength of the incident photon? (b) What are the magnitude of the momentum and the speed of the electron after the collision? (c) What is the kinetic energy of the electron after the collision?

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.

IDENTIFY and SET UP

1. In this problem a photon is scattered by an electron initially at rest. In Section 38.3 you learned how to relate the wavelengths of the incident and scattered photons; in this problem you must also find the momentum, speed, and kinetic energy of the recoiling electron. You can find these because momentum and energy are conserved in the collision.
2. Which key equation can be used to find the incident photon wavelength? What is the photon scattering angle ϕ in this problem?

EXECUTE

3. Use the equation you selected in step 2 to find the wavelength of the incident photon.
4. Use momentum conservation and your result from step 3 to find the momentum of the recoiling electron. (*Hint:* All of the momentum vectors are along the same line, but not all point in the same direction. Be careful with signs.)
5. Find the speed of the recoiling electron from your result in step 4. (*Hint:* Assume that the electron is nonrelativistic, so you can use the relationship between momentum and speed from Chapter 8. This is acceptable if the speed of the electron is less than about $0.1c$. Is it?)
6. Use your result from step 4 or step 5 to find the electron kinetic energy.

EVALUATE

7. You can check your answer in step 6 by finding the difference between the energies of the incident and scattered photons. Is your result consistent with conservation of energy?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q38.1 In what ways do photons resemble other particles such as electrons? In what ways do they differ? Do photons have mass? Do they have electric charge? Can they be accelerated? What mechanical properties do they have?

Q38.2 There is a certain probability that a single electron may simultaneously absorb *two* identical photons from a high-intensity laser. How would such an occurrence affect the threshold frequency and the equations of Section 38.1? Explain.

Q38.3 According to the photon model, light carries its energy in packets called quanta or photons. Why then don't we see a series of flashes when we look at things?

Q38.4 Would you expect effects due to the photon nature of light to be generally more important at the low-frequency end of the electromagnetic spectrum (radio waves) or at the high-frequency end (x rays and gamma rays)? Why?

Q38.5 During the photoelectric effect, light knocks electrons out of metals. So why don't the metals in your home lose their electrons when you turn on the lights?

Q38.6 Most black-and-white photographic film (with the exception of some special-purpose films) is less sensitive to red light than blue light and has almost no sensitivity to infrared. How can these properties be understood on the basis of photons?

Q38.7 Human skin is relatively insensitive to visible light, but ultraviolet radiation can cause severe burns. Does this have anything to do with photon energies? Explain.

Q38.8 Explain why Fig. 38.4 shows that most photoelectrons have kinetic energies less than $hf - \phi$, and also explain how these smaller kinetic energies occur.

Q38.9 In a photoelectric-effect experiment, the photocurrent i for large positive values of V_{AC} has the same value no matter what the light frequency f (provided that f is higher than the threshold frequency f_0). Explain why.

Q38.10 In an experiment involving the photoelectric effect, if the intensity of the incident light (having frequency higher than the threshold frequency) is reduced by a factor of 10 without changing anything else, which (if any) of the following statements about this process will be true? (a) The number of photoelectrons will most likely be reduced by a factor of 10. (b) The maximum kinetic energy of the ejected photoelectrons will most likely be reduced by a factor of 10. (c) The maximum speed of the ejected photoelectrons will most likely be reduced by a factor of 10. (d) The maximum speed of the ejected photoelectrons will most likely be reduced by a factor of $\sqrt{10}$. (e) The time for the first photoelectron to be ejected will be increased by a factor of 10.

Q38.11 The materials called *phosphors* that coat the inside of a fluorescent lamp convert ultraviolet radiation (from the mercury-vapor discharge inside the tube) into visible light. Could one also make a phosphor that converts visible light to ultraviolet? Explain.

Q38.12 In a photoelectric-effect experiment, which of the following will increase the maximum kinetic energy of the photoelectrons? (a) Use light of greater intensity; (b) use light of higher frequency; (c) use light of longer wavelength; (d) use a metal surface with a larger work function. In each case justify your answer.

Q38.13 A photon of frequency f undergoes Compton scattering from an electron at rest and scatters through an angle ϕ. The frequency of the scattered photon is f'. How is f' related to f? Does your answer depend on ϕ? Explain.

Q38.14 Can Compton scattering occur with protons as well as electrons? For example, suppose a beam of x rays is directed at a target of liquid hydrogen. (Recall that the nucleus of hydrogen consists of a single proton.) Compared to Compton scattering with electrons, what similarities and differences would you expect? Explain.

Q38.15 Why must engineers and scientists shield against x-ray production in high-voltage equipment?

Q38.16 In attempting to reconcile the wave and particle models of light, some people have suggested that the photon rides up and down on the crests and troughs of the electromagnetic wave. What things are *wrong* with this description?

Q38.17 Some lasers emit light in pulses that are only 10^{-12} s in duration. The length of such a pulse is $(3 \times 10^8 \text{ m/s})(10^{-12} \text{ s}) = 3 \times 10^{-4} \text{ m} = 0.3$ mm. Can pulsed laser light be as monochromatic as light from a laser that emits a steady, continuous beam? Explain.

EXERCISES

Section 38.1 Light Absorbed as Photons: The Photoelectric Effect

38.1 •• (a) A proton is moving at a speed much slower than the speed of light. It has kinetic energy K_1 and momentum p_1. If the momentum of the proton is doubled, so $p_2 = 2p_1$, how is its new kinetic energy K_2 related to K_1? (b) A photon with energy E_1 has momentum p_1. If another photon has momentum p_2 that is twice p_1, how is the energy E_2 of the second photon related to E_1?

38.2 • **BIO** **Response of the Eye.** The human eye is most sensitive to green light of wavelength 505 nm. Experiments have found that when people are kept in a dark room until their eyes adapt to the darkness, a *single* photon of green light will trigger receptor cells in the rods of the retina. (a) What is the frequency of this photon? (b) How much energy (in joules and electron volts) does it deliver to the receptor cells? (c) To appreciate what a small amount of energy this is, calculate how fast a typical bacterium of mass 9.5×10^{-12} g would move if it had that much energy.

38.3 • A photon of green light has a wavelength of 520 nm. Find the photon's frequency, magnitude of momentum, and energy. Express the energy in both joules and electron volts.

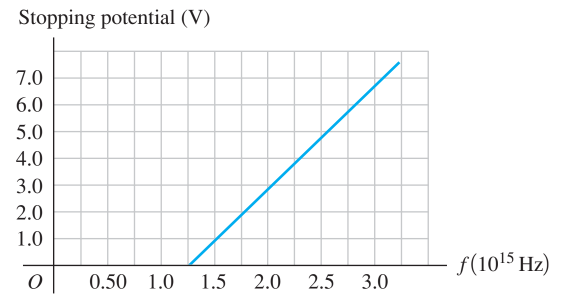
38.4 • **BIO** A laser used to weld detached retinas emits light with a wavelength of 652 nm in pulses that are 20.0 ms in duration. The average power during each pulse is 0.600 W. (a) How much energy is in each pulse in joules? In electron volts? (b) What is the energy of one photon in joules? In electron volts? (c) How many photons are in each pulse?

38.5 • A 75-W light source consumes 75 W of electrical power. Assume all this energy goes into emitted light of wavelength 600 nm. (a) Calculate the frequency of the emitted light. (b) How many photons per second does the source emit? (c) Are the answers to parts (a) and (b) the same? Is the frequency of the light the same thing as the number of photons emitted per second? Explain.

38.6 • A photon has momentum of magnitude 8.24×10^{-28} kg·m/s. (a) What is the energy of this photon? Give your answer in joules and in electron volts. (b) What is the wavelength of this photon? In what region of the electromagnetic spectrum does it lie?

38.7 • The graph in Fig. E38.7 shows the stopping potential as a function of the frequency of the incident light falling on a metal surface. (a) Find the photoelectric work function for this metal. (b) What value of Planck's constant does the graph yield? (c) Why does the graph *not* extend below the x-axis? (d) If a different metal were used, which characteristics of the graph would you expect to be the same and which ones would be different?

Figure **E38.7**



38.8 • The photoelectric threshold wavelength of a tungsten surface is 272 nm. Calculate the maximum kinetic energy of the electrons ejected from this tungsten surface by ultraviolet radiation of frequency 1.45×10^{15} Hz. Express the answer in electron volts.

38.9 •• A clean nickel surface is exposed to light of wavelength 235 nm. What is the maximum speed of the photoelectrons emitted from this surface? Use Table 38.1.

38.10 •• What would the minimum work function for a metal have to be for visible light (380–750 nm) to eject photoelectrons?

38.11 •• When ultraviolet light with a wavelength of 400.0 nm falls on a certain metal surface, the maximum kinetic energy of the emitted photoelectrons is measured to be 1.10 eV. What is the maximum kinetic energy of the photoelectrons when light of wavelength 300.0 nm falls on the same surface?

38.12 •• The photoelectric work function of potassium is 2.3 eV. If light having a wavelength of 250 nm falls on potassium, find (a) the stopping potential in volts; (b) the kinetic energy in electron volts of the most energetic electrons ejected; (c) the speed of these electrons.

38.13 • When ultraviolet light with a wavelength of 254 nm falls on a clean copper surface, the stopping potential necessary to stop emission of photoelectrons is 0.181 V. (a) What is the photoelectric threshold wavelength for this copper surface? (b) What is the work function for this surface, and how does your calculated value compare with that given in Table 38.1?

Section 38.2 Light Emitted as Photons: X-Ray Production

38.14 • The cathode-ray tubes that generated the picture in early color televisions were sources of x rays. If the acceleration voltage in a television tube is 15.0 kV, what are the shortest-wavelength x rays produced by the television? (Modern televisions contain shielding to stop these x rays.)

38.15 • Protons are accelerated from rest by a potential difference of 4.00 kV and strike a metal target. If a proton produces one photon on impact, what is the minimum wavelength of the resulting x rays? How does your answer compare to the minimum wavelength if 4.00-keV electrons are used instead? Why do x-ray tubes use electrons rather than protons to produce x rays?

38.16 •• (a) What is the minimum potential difference between the filament and the target of an x-ray tube if the tube is to produce x rays with a wavelength of 0.150 nm? (b) What is the shortest wavelength produced in an x-ray tube operated at 30.0 kV?

Section 38.3 Light Scattered as Photons: Compton Scattering and Pair Production

38.17 • An x ray with a wavelength of 0.100 nm collides with an electron that is initially at rest. The x ray's final wavelength is 0.110 nm. What is the final kinetic energy of the electron?

38.18 • X rays are produced in a tube operating at 18.0 kV. After emerging from the tube, x rays with the minimum wavelength produced strike a target and are Compton-scattered through an angle of 45.0°. (a) What is the original x-ray wavelength? (b) What is the wavelength of the scattered x rays? (c) What is the energy of the scattered x rays (in electron volts)?

38.19 •• X rays with initial wavelength 0.0665 nm undergo Compton scattering. What is the longest wavelength found in the scattered x rays? At which scattering angle is this wavelength observed?

38.20 • A beam of x rays with wavelength 0.0500 nm is Compton-scattered by the electrons in a sample. At what angle from the incident beam should you look to find x rays with a wavelength of (a) 0.0542 nm; (b) 0.0521 nm; (c) 0.0500 nm?

38.21 •• If a photon of wavelength 0.04250 nm strikes a free electron and is scattered at an angle of 35.0° from its original direction, find (a) the change in the wavelength of this photon; (b) the wavelength of the scattered light; (c) the change in energy of the photon (is it a loss or a gain?); (d) the energy gained by the electron.

38.22 •• A photon scatters in the backward direction ($\phi = 180°$) from a free proton that is initially at rest. What must the wavelength of the incident photon be if it is to undergo a 10.0% change in wavelength as a result of the scattering?

38.23 •• X rays with an initial wavelength of 0.900×10^{-10} m undergo Compton scattering. For what scattering angle is the wavelength of the scattered x rays greater by 1.0% than that of the incident x rays?

38.24 •• A photon with wavelength $\lambda = 0.1385$ nm scatters from an electron that is initially at rest. What must be the angle between the direction of propagation of the incident and scattered photons if the speed of the electron immediately after the collision is 8.90×10^6 m/s?

38.25 • An electron and a positron are moving toward each other and each has speed $0.500c$ in the lab frame. (a) What is the kinetic energy of each particle? (b) The e^+ and e^- meet head-on and annihilate. What is the energy of each photon that is produced? (c) What is the wavelength of each photon? How does the wavelength compare to the photon wavelength when the initial kinetic energy of the e^+ and e^- is negligibly small (see Example 38.6)?

Section 38.4 Wave–Particle Duality, Probability, and Uncertainty

38.26 • A laser produces light of wavelength 625 nm in an ultrashort pulse. What is the minimum duration of the pulse if the minimum uncertainty in the energy of the photons is 1.0%?

38.27 • An ultrashort pulse has a duration of 9.00 fs and produces light at a wavelength of 556 nm. What are the momentum and momentum uncertainty of a single photon in the pulse?

38.28 • A horizontal beam of laser light of wavelength 585 nm passes through a narrow slit that has width 0.0620 mm. The intensity of the light is measured on a vertical screen that is 2.00 m from the slit. (a) What is the minimum uncertainty in the vertical component of the momentum of each photon in the beam after the photon has passed through the slit? (b) Use the result of part (a) to estimate the width of the central diffraction maximum that is observed on the screen.

PROBLEMS

38.29 • **Exposing Photographic Film.** The light-sensitive compound on most photographic films is silver bromide, AgBr. A film is "exposed" when the light energy absorbed dissociates this molecule into its atoms. (The actual process is more complex, but the quantitative result does not differ greatly.) The energy of dissociation of AgBr is 1.00×10^5 J/mol. For a photon that is just able to dissociate a molecule of silver bromide, find (a) the photon energy in electron volts; (b) the wavelength of the photon; (c) the frequency of the photon. (d) What is the energy in electron volts of a photon having a frequency of 100 MHz? (e) Light from a firefly can expose photographic film, but the radiation from an FM station broadcasting 50,000 W at 100 MHz cannot. Explain why this is so.

38.30 •• (a) If the average frequency emitted by a 200-W light bulb is 5.00×10^{14} Hz, and 10.0% of the input power is emitted as visible light, approximately how many visible-light photons are emitted per second? (b) At what distance would this correspond to 1.00×10^{11} visible-light photons per square centimeter per second if the light is emitted uniformly in all directions?

38.31 • When a certain photoelectric surface is illuminated with light of different wavelengths, the following stopping potentials are observed:

Wavelength (nm)	Stopping potential (V)
366	1.48
405	1.15
436	0.93
492	0.62
546	0.36
579	0.24

Plot the stopping potential on the vertical axis against the frequency of the light on the horizontal axis. Determine (a) the threshold frequency; (b) the threshold wavelength; (c) the photoelectric work function of the material (in electron volts); (d) the value of Planck's constant h (assuming that the value of e is known).

38.32 • A 2.50-W beam of light of wavelength 124 nm falls on a metal surface. You observe that the maximum kinetic energy of the ejected electrons is 4.16 eV. Assume that each photon in the beam ejects a photoelectron. (a) What is the work function (in electron volts) of this metal? (b) How many photoelectrons are ejected each second from this metal? (c) If the power of the light beam, but not its wavelength, were reduced by half, what would be the answer to part (b)? (d) If the wavelength of the beam, but not its power, were reduced by half, what would be the answer to part (b)?

38.33 •• CP BIO **Removing Vascular Lesions.** A pulsed dye laser emits light of wavelength 585 nm in 450-μs pulses. Because this wavelength is strongly absorbed by the hemoglobin in the blood, the method is especially effective for removing various types of blemishes due to blood, such as port-wine–colored birthmarks. To get a reasonable estimate of the power required for such laser surgery, we can model the blood as having the same specific heat and heat of vaporization as water (4190 J/kg·K, 2.256×10^6 J/kg). Suppose that each pulse must remove 2.0 μg of blood by evaporating it, starting at 33°C. (a) How much energy must each pulse deliver to the blemish? (b) What must be the power output of this laser? (c) How many photons does each pulse deliver to the blemish?

38.34 • The photoelectric work functions for particular samples of certain metals are as follows: cesium, 2.1 eV; copper, 4.7 eV;

potassium, 2.3 eV; and zinc, 4.3 eV. (a) What is the threshold wavelength for each metal surface? (b) Which of these metals could *not* emit photoelectrons when irradiated with visible light (380–750 nm)?

38.35 •• An incident x-ray photon of wavelength 0.0900 nm is scattered in the backward direction from a free electron that is initially at rest. (a) What is the magnitude of the momentum of the scattered photon? (b) What is the kinetic energy of the electron after the photon is scattered?

38.36 •• **CP** A photon with wavelength $\lambda = 0.0900$ nm is incident on an electron that is initially at rest. If the photon scatters in the backward direction, what is the magnitude of the linear momentum of the electron just after the collision with the photon?

38.37 •• **CP** A photon with wavelength $\lambda = 0.1050$ nm is incident on an electron that is initially at rest. If the photon scatters at an angle of 60.0° from its original direction, what are the magnitude and direction of the linear momentum of the electron just after the collision with the photon?

38.38 •• **CP** An x-ray tube is operating at voltage V and current I. (a) If only a fraction p of the electric power supplied is converted into x rays, at what rate is energy being delivered to the target? (b) If the target has mass m and specific heat c (in J/kg · K), at what average rate would its temperature rise if there were no thermal losses? (c) Evaluate your results from parts (a) and (b) for an x-ray tube operating at 18.0 kV and 60.0 mA that converts 1.0% of the electric power into x rays. Assume that the 0.250-kg target is made of lead ($c = 130$ J/kg · K). (d) What must the physical properties of a practical target material be? What would be some suitable target elements?

38.39 •• Nuclear fusion reactions at the center of the sun produce gamma-ray photons with energies of about 1 MeV (10^6 eV). By contrast, what we see emanating from the sun's surface are visible-light photons with wavelengths of about 500 nm. A simple model that explains this difference in wavelength is that a photon undergoes Compton scattering many times—in fact, about 10^{26} times, as suggested by models of the solar interior—as it travels from the center of the sun to its surface. (a) Estimate the increase in wavelength of a photon in an average Compton-scattering event. (b) Find the angle in degrees through which the photon is scattered in the scattering event described in part (a). (*Hint:* A useful approximation is $\cos\phi \approx 1 - \phi^2/2$, which is valid for $\phi \ll 1$. Note that ϕ is in radians in this expression.) (c) It is estimated that a photon takes about 10^6 years to travel from the core to the surface of the sun. Find the average distance that light can travel within the interior of the sun without being scattered. (This distance is roughly equivalent to how far you could see if you were inside the sun and could survive the extreme temperatures there. As your answer shows, the interior of the sun is *very* opaque.)

38.40 •• (a) Derive an expression for the total shift in photon wavelength after two successive Compton scatterings from electrons at rest. The photon is scattered by an angle θ_1 in the first scattering and by θ_2 in the second. (b) In general, is the total shift in wavelength produced by two successive scatterings of an angle $\theta/2$ the same as by a single scattering of θ? If not, are there any specific values of θ, other than $\theta = 0°$, for which the total shifts are the same? (c) Use the result of part (a) to calculate the total wavelength shift produced by two successive Compton scatterings of 30.0° each. Express your answer in terms of h/mc. (d) What is the wavelength shift produced by a single Compton scattering of 60.0°? Compare to the answer in part (c).

38.41 •• A photon with wavelength 0.1100 nm collides with a free electron that is initially at rest. After the collision the wavelength is 0.1132 nm. (a) What is the kinetic energy of the electron after the collision? What is its speed? (b) If the electron is suddenly stopped (for example, in a solid target), all of its kinetic energy is used to create a photon. What is the wavelength of this photon?

38.42 •• An x-ray photon is scattered from a free electron (mass m) at rest. The wavelength of the scattered photon is λ', and the final speed of the struck electron is v. (a) What was the initial wavelength λ of the photon? Express your answer in terms of λ', v, and m. (*Hint:* Use the relativistic expression for the electron kinetic energy.) (b) Through what angle ϕ is the photon scattered? Express your answer in terms of λ, λ', and m. (c) Evaluate your results in parts (a) and (b) for a wavelength of 5.10×10^{-3} nm for the scattered photon and a final electron speed of 1.80×10^8 m/s. Give ϕ in degrees.

38.43 •• (a) Calculate the maximum increase in photon wavelength that can occur during Compton scattering. (b) What is the energy (in electron volts) of the lowest-energy x-ray photon for which Compton scattering could result in doubling the original wavelength?

CHALLENGE PROBLEM

38.44 ••• Consider Compton scattering of a photon by a *moving* electron. Before the collision the photon has wavelength λ and is moving in the $+x$-direction, and the electron is moving in the $-x$-direction with total energy E (including its rest energy mc^2). The photon and electron collide head-on. After the collision, both are moving in the $-x$-direction (that is, the photon has been scattered by 180°). (a) Derive an expression for the wavelength λ' of the scattered photon. Show that if $E \gg mc^2$, where m is the rest mass of the electron, your result reduces to

$$\lambda' = \frac{hc}{E}\left(1 + \frac{m^2c^4\lambda}{4hcE}\right)$$

(b) A beam of infrared radiation from a CO_2 laser ($\lambda = 10.6\ \mu$m) collides head-on with a beam of electrons, each of total energy $E = 10.0$ GeV ($1\text{ GeV} = 10^9$ eV). Calculate the wavelength λ' of the scattered photons, assuming a 180° scattering angle. (c) What kind of scattered photons are these (infrared, microwave, ultraviolet, etc.)? Can you think of an application of this effect?

Answers

Chapter Opening Question ?

The energy of a photon E is inversely proportional to its wavelength λ: The shorter the wavelength, the more energetic is the photon. Since visible light has shorter wavelengths than infrared light, the headlamp emits photons of greater energy. However, the light from the infrared laser is far more *intense* (delivers much more energy per second per unit area to the patient's skin) because it emits many more photons per second than does the headlamp and concentrates them onto a very small spot.

Test Your Understanding Questions

38.1 Answers: (i) and (ii) From Eq. (38.2), a photon of energy $E = 1.14\,\text{eV}$ has wavelength $\lambda = hc/E = (4.136 \times 10^{-15}\ \text{eV}) \cdot (3.00 \times 10^{8}\ \text{m/s})/(1.14\,\text{eV}) = 1.09 \times 10^{-6}\ \text{m} = 1090\ \text{nm}$. This is in the infrared part of the spectrum. Since wavelength is inversely proportional to photon energy, the *minimum* photon energy of 1.14 eV corresponds to the *maximum* wavelength that causes photoconductivity in silicon. Thus the wavelength must be 1090 nm or less.

38.2 Answer: (ii) Equation (38.6) shows that the minimum wavelength of x rays produced by bremsstrahlung depends on the potential difference V_{AC} but does *not* depend on the rate at which electrons strike the anode. Each electron produces at most one photon, so increasing the number of electrons per second causes an increase in the number of x-ray photons emitted per second (that is, the x-ray intensity).

38.3 Answers: yes, no Equation (38.7) shows that the wavelength shift $\Delta\lambda = \lambda' - \lambda$ depends only on the photon scattering angle ϕ, not on the wavelength of the incident photon. So a visible-light photon scattered through an angle ϕ undergoes the same wavelength shift as an x-ray photon. Equation (38.7) also shows that this shift is of the order of $h/mc = 2.426 \times 10^{-12}\ \text{m} = 0.002426\ \text{nm}$. This is a few percent of the wavelength of x rays (see Example 38.5), so the effect is noticeable in x-ray scattering. However, h/mc is a tiny fraction of the wavelength of visible light (between 380 and 750 nm). The human eye cannot distinguish such minuscule differences in wavelength (that is, differences in color).

38.4 Answer: (ii) There is *zero* probability that a photon will be deflected by one of the angles where the diffraction pattern has zero intensity. These angles are given by $a \sin\theta = m\lambda$ with $m = \pm 1, \pm 2, \pm 3, \ldots$. Since λ is much less than a, we can write these angles as $\theta = m\lambda/a = \pm\lambda/a, \pm 2\lambda/a, \pm 3\lambda/a, \ldots$. These values include answers (i), (iii), and (iv), so it is impossible for a photon to be deflected through any of these angles. The intensity is not zero at $\theta = 3\lambda/2a$ (located between two zeros in the diffraction pattern), so there is some probability that a photon will be deflected through this angle.

Bridging Problem

Answers: **(a)** 0.0781 nm
(b) $1.65 \times 10^{-23}\ \text{kg} \cdot \text{m/s}$, $1.81 \times 10^{7}\ \text{m/s}$
(c) $1.49 \times 10^{-16}\ \text{J}$