ELECTROMAGNETIC WAVES

32

? Metal objects reflect not only visible light but also radio waves. What aspect of metals makes them so reflective?

LEARNING GOALS

By studying this chapter, you will learn:

- Why there are both electric and magnetic fields in a light wave.
- How the speed of light is related to the fundamental constants of electricity and magnetism.
- How to describe the propagation of a sinusoidal electromagnetic wave.
- What determines the amount of power carried by an electromagnetic wave.
- How to describe standing electromagnetic waves.

What is light? This question has been asked by humans for centuries, but there was no answer until electricity and magnetism were unified into *electromagnetism,* as described by Maxwell's equations. These equations show that a time-varying magnetic field acts as a source of electric field and that a time-varying electric field acts as a source of magnetic field. These $\vec{E}$ and $\vec{B}$ fields can sustain each other, forming an *electromagnetic wave* that propagates through space. Visible light emitted by the glowing filament of a light bulb is one example of an electromagnetic wave; other kinds of electromagnetic waves are produced by TV and radio stations, x-ray machines, and radioactive nuclei.

In this chapter we'll use Maxwell's equations as the theoretical basis for understanding electromagnetic waves. We'll find that these waves carry both energy and momentum. In sinusoidal electromagnetic waves, the $\vec{E}$ and $\vec{B}$ fields are sinusoidal functions of time and position, with a definite frequency and wavelength. Visible light, radio, x rays, and other types of electromagnetic waves differ only in their frequency and wavelength. Our study of optics in the following chapters will be based in part on the electromagnetic nature of light.

Unlike waves on a string or sound waves in a fluid, electromagnetic waves do not require a material medium; the light that you see coming from the stars at night has traveled without difficulty across tens or hundreds of light-years of (nearly) empty space. Nonetheless, electromagnetic waves and mechanical waves have much in common and are described in much the same language. Before reading further in this chapter, you should review the properties of mechanical waves as discussed in Chapters 15 and 16.

32.1 Maxwell's Equations and Electromagnetic Waves

In the last several chapters we studied various aspects of electric and magnetic fields. We learned that when the fields don't vary with time, such as an electric field produced by charges at rest or the magnetic field of a steady current, we can analyze the electric and magnetic fields independently without considering interactions between them. But when the fields vary with time, they are no longer independent. Faraday's law (see Section 29.2) tells us that a time-varying magnetic field acts as a source of electric field, as shown by induced emfs in inductors and transformers. Ampere's law, including the displacement current discovered by Maxwell (see Section 29.7), shows that a time-varying electric field acts as a source of magnetic field. This mutual interaction between the two fields is summarized in Maxwell's equations, presented in Section 29.7.

Thus, when *either* an electric or a magnetic field is changing with time, a field of the other kind is induced in adjacent regions of space. We are led (as Maxwell was) to consider the possibility of an electromagnetic disturbance, consisting of time-varying electric and magnetic fields, that can propagate through space from one region to another, even when there is no matter in the intervening region. Such a disturbance, if it exists, will have the properties of a *wave,* and an appropriate term is **electromagnetic wave.**

Such waves do exist; radio and television transmission, light, x rays, and many other kinds of radiation are examples of electromagnetic waves. Our goal in this chapter is to see how such waves are explained by the principles of electromagnetism that we have studied thus far and to examine the properties of these waves.

Electricity, Magnetism, and Light

As often happens in the development of science, the theoretical understanding of electromagnetic waves evolved along a considerably more devious path than the one just outlined. In the early days of electromagnetic theory (the early 19th century), two different units of electric charge were used: one for electrostatics and the other for magnetic phenomena involving currents. In the system of units used at that time, these two units of charge had different physical dimensions. Their *ratio* had units of velocity, and measurements showed that the ratio had a numerical value that was precisely equal to the speed of light, 3.00×10^8 m/s. At the time, physicists regarded this as an extraordinary coincidence and had no idea how to explain it.

In searching to understand this result, Maxwell (Fig. 32.1) proved in 1865 that an electromagnetic disturbance should propagate in free space with a speed equal to that of light and hence that light waves were likely to be electromagnetic in nature. At the same time, he discovered that the basic principles of electromagnetism can be expressed in terms of the four equations that we now call **Maxwell's equations,** which we discussed in Section 29.7. These four equations are (1) Gauss's law for electric fields; (2) Gauss's law for magnetic fields, showing the absence of magnetic monopoles; (3) Ampere's law, including displacement current; and (4) Faraday's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad \text{(Gauss's law)} \tag{29.18}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss's law for magnetism)} \tag{29.19}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0\left(i_C + \epsilon_0 \frac{d\Phi_E}{dt}\right)_{\text{encl}} \qquad \text{(Ampere's law)} \tag{29.20}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)} \tag{29.21}$$

32.1 James Clerk Maxwell (1831–1879) was the first person to truly understand the fundamental nature of light. He also made major contributions to thermodynamics, optics, astronomy, and color photography. Albert Einstein described Maxwell's accomplishments as "the most profound and the most fruitful that physics has experienced since the time of Newton."

These equations apply to electric and magnetic fields *in vacuum.* If a material is present, the permittivity ϵ_0 and permeability μ_0 of free space are replaced by the permittivity ϵ and permeability μ of the material. If the values of ϵ and μ are different at different points in the regions of integration, then ϵ and μ have to be transferred to the left sides of Eqs. (29.18) and (29.20), respectively, and placed inside the integrals. The ϵ in Eq. (29.20) also has to be included in the integral that gives $d\Phi_E/dt$.

According to Maxwell's equations, a point charge at rest produces a static $\vec{E}$ field but no $\vec{B}$ field; a point charge moving with a constant velocity (see Section 28.1) produces both $\vec{E}$ and $\vec{B}$ fields. Maxwell's equations can also be used to show that in order for a point charge to produce electromagnetic waves, the charge must *accelerate.* In fact, it's a general result of Maxwell's equations that *every* accelerated charge radiates electromagnetic energy (Fig. 32.2).

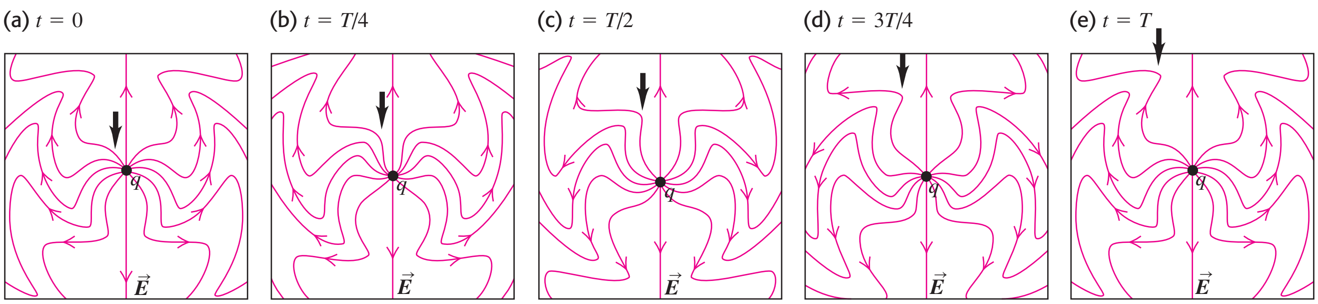
32.2 (Top) Every mobile phone, wireless modem, or radio transmitter emits signals in the form of electromagnetic waves that are made by accelerating charges. (Bottom) Power lines carry a strong alternating current, which means that a substantial amount of charge is accelerating back and forth and generating electromagnetic waves. These waves can produce a buzzing sound from your car radio when you drive near the lines.

Generating Electromagnetic Radiation

One way in which a point charge can be made to emit electromagnetic waves is by making it oscillate in simple harmonic motion, so that it has an acceleration at almost every instant (the exception is when the charge is passing through its equilibrium position). Figure 32.3 shows some of the electric field lines produced by such an oscillating point charge. Field lines are *not* material objects, but you may nonetheless find it helpful to think of them as behaving somewhat like strings that extend from the point charge off to infinity. Oscillating the charge up and down makes waves that propagate outward from the charge along these "strings." Note that the charge does not emit waves equally in all directions; the waves are strongest at 90° to the axis of motion of the charge, while there are *no* waves along this axis. This is just what the "string" picture would lead you to conclude. There is also a *magnetic* disturbance that spreads outward from the charge; this is not shown in Fig. 32.3. Because the electric and magnetic disturbances spread or radiate away from the source, the name **electromagnetic radiation** is used interchangeably with the phrase "electromagnetic waves."

Electromagnetic waves with macroscopic wavelengths were first produced in the laboratory in 1887 by the German physicist Heinrich Hertz. As a source of waves, he used charges oscillating in *L-C* circuits of the sort discussed in Section 30.5; he detected the resulting electromagnetic waves with other circuits tuned to the same frequency. Hertz also produced electromagnetic *standing waves* and measured the distance between adjacent nodes (one half-wavelength) to determine the wavelength. Knowing the resonant frequency of his circuits, he then found the speed of the waves from the wavelength–frequency relationship $v = \lambda f$. He established that their speed was the same as that of light; this verified Maxwell's theoretical prediction directly. The SI unit of frequency is named in honor of Hertz: One hertz (1 Hz) equals one cycle per second.

32.3 Electric field lines of a point charge oscillating in simple harmonic motion, seen at five instants during an oscillation period T. The charge's trajectory is in the plane of the drawings. At $t = 0$ the point charge is at its maximum upward displacement. The arrow shows one "kink" in the lines of $\vec{E}$ as it propagates outward from the point charge. The magnetic field (not shown) comprises circles that lie in planes perpendicular to these figures and concentric with the axis of oscillation.



The modern value of the speed of light, which we denote by the symbol c, is 299,792,458 m/s. (Recall from Section 1.3 that this value is the basis of our standard of length: One meter is defined to be the distance that light travels in 1/299,792,458 second.) For our purposes, $c = 3.00 \times 10^8$ m/s is sufficiently accurate.

The possible use of electromagnetic waves for long-distance communication does not seem to have occurred to Hertz. It was left to Marconi and others to make radio communication a familiar household experience. In a radio *transmitter,* electric charges are made to oscillate along the length of the conducting antenna, producing oscillating field disturbances like those shown in Fig. 32.3. Since many charges oscillate together in the antenna, the disturbances are much stronger than those of a single oscillating charge and can be detected at a much greater distance. In a radio *receiver* the antenna is also a conductor; the fields of the wave emanating from a distant transmitter exert forces on free charges within the receiver antenna, producing an oscillating current that is detected and amplified by the receiver circuitry.

For the remainder of this chapter our concern will be with electromagnetic waves themselves, not with the rather complex problem of how they are produced.

The Electromagnetic Spectrum

The **electromagnetic spectrum** encompasses electromagnetic waves of all frequencies and wavelengths. Figure 32.4 shows approximate wavelength and frequency ranges for the most commonly encountered portion of the spectrum. Despite vast differences in their uses and means of production, these are all electromagnetic waves with the same propagation speed (in vacuum) $c = 299{,}792{,}458$ m/s. Electromagnetic waves may differ in frequency f and wavelength λ, but the relationship $c = \lambda f$ in vacuum holds for each.

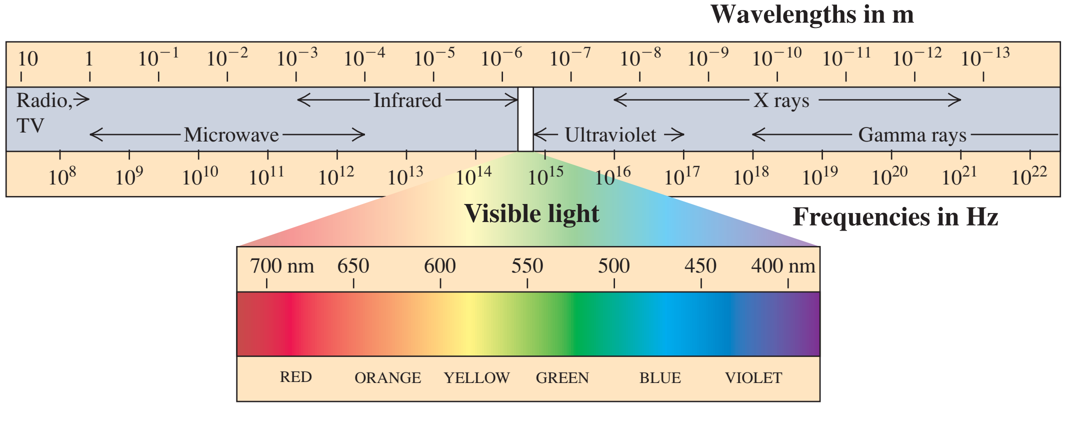
We can detect only a very small segment of this spectrum directly through our sense of sight. We call this range **visible light.** Its wavelengths range from about 380 to 750 nm $(380 \text{ to } 750 \times 10^{-9}\text{ m})$, with corresponding frequencies from about 790 to 400 THz $(7.9 \text{ to } 4.0 \times 10^{14}\text{ Hz})$. Different parts of the visible spectrum evoke in humans the sensations of different colors. Table 32.1 gives the approximate wavelengths for colors in the visible spectrum.

Table 32.1 Wavelengths of Visible Light

380–450 nm	Violet
450–495 nm	Blue
495–570 nm	Green
570–590 nm	Yellow
590–620 nm	Orange
620–750 nm	Red

Ordinary white light includes all visible wavelengths. However, by using special sources or filters, we can select a narrow band of wavelengths within a range of a few nm. Such light is approximately *monochromatic* (single-color) light. Absolutely monochromatic light with only a single wavelength is an unattainable idealization. When we use the expression “monochromatic light with $\lambda =$ 550 nm” with reference to a laboratory experiment, we really mean a small band

32.4 The electromagnetic spectrum. The frequencies and wavelengths found in nature extend over such a wide range that we have to use a logarithmic scale to show all important bands. The boundaries between bands are somewhat arbitrary.



of wavelengths *around* 550 nm. Light from a *laser* is much more nearly monochromatic than is light obtainable in any other way.

Invisible forms of electromagnetic radiation are no less important than visible light. Our system of global communication, for example, depends on radio waves: AM radio uses waves with frequencies from 5.4×10^5 Hz to 1.6×10^6 Hz, while FM radio broadcasts are at frequencies from 8.8×10^7 Hz to 1.08×10^8 Hz. (Television broadcasts use frequencies that bracket the FM band.) Microwaves are also used for communication (for example, by cellular phones and wireless networks) and for weather radar (at frequencies near 3×10^9 Hz). Many cameras have a device that emits a beam of infrared radiation; by analyzing the properties of the infrared radiation reflected from the subject, the camera determines the distance to the subject and automatically adjusts the focus. X rays are able to penetrate through flesh, which makes them invaluable in dentistry and medicine. Gamma rays, the shortest-wavelength type of electromagnetic radiation, are used in medicine to destroy cancer cells.

Test Your Understanding of Section 32.1 (a) Is it possible to have a purely electric wave propagate through empty space—that is, a wave made up of an electric field but no magnetic field? (b) What about a purely magnetic wave, with a magnetic field but no electric field?

Application Ultraviolet Vision
Many insects and birds can see ultraviolet wavelengths that humans cannot. As an example, the left-hand photo shows how black-eyed Susans (genus *Rudbeckia*) look to us. The right-hand photo (in false color), taken with an ultraviolet-sensitive camera, shows how these same flowers appear to the bees that pollinate them. Note the prominent central spot that is invisible to humans. Similarly, many birds with ultraviolet vision—including budgies, parrots, and peacocks—have ultraviolet patterns on their bodies that make them even more vivid to each other than they appear to us.

32.2 Plane Electromagnetic Waves and the Speed of Light

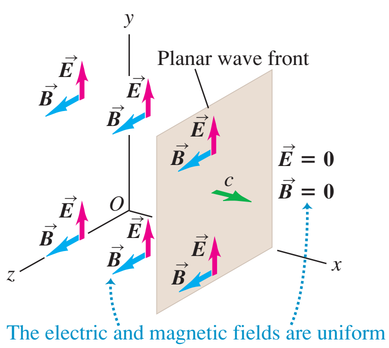
We are now ready to develop the basic ideas of electromagnetic waves and their relationship to the principles of electromagnetism. Our procedure will be to postulate a simple field configuration that has wavelike behavior. We'll assume an electric field $\vec{E}$ that has only a y-component and a magnetic field $\vec{B}$ with only a z-component, and we'll assume that both fields move together in the $+x$-direction with a speed c that is initially unknown. (As we go along, it will become clear why we choose $\vec{E}$ and $\vec{B}$ to be perpendicular to the direction of propagation as well as to each other.) Then we will test whether these fields are physically possible by asking whether they are consistent with Maxwell's equations, particularly Ampere's law and Faraday's law. We'll find that the answer is yes, provided that c has a particular value. We'll also show that the *wave equation,* which we encountered during our study of mechanical waves in Chapter 15, can be derived from Maxwell's equations.

A Simple Plane Electromagnetic Wave

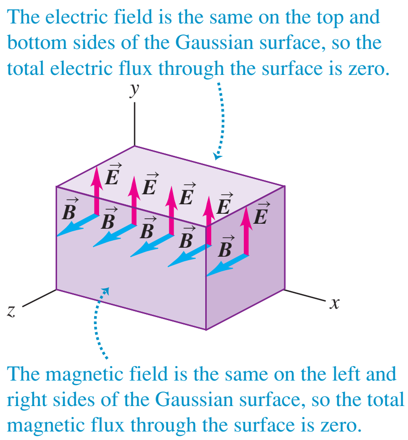
Using an xyz-coordinate system (Fig. 32.5), we imagine that all space is divided into two regions by a plane perpendicular to the x-axis (parallel to the yz-plane). At every point to the left of this plane there are a uniform electric field $\vec{E}$ in the $+y$-direction and a uniform magnetic field $\vec{B}$ in the $+z$-direction, as shown. Furthermore, we suppose that the boundary plane, which we call the *wave front,* moves to the right in the $+x$-direction with a constant speed c, the value of which we'll leave undetermined for now. Thus the $\vec{E}$ and $\vec{B}$ fields travel to the right into previously field-free regions with a definite speed. This is a rudimentary electromagnetic wave. A wave such as this, in which at any instant the fields are uniform over any plane perpendicular to the direction of propagation, is called a **plane wave.** In the case shown in Fig. 32.5, the fields are zero for planes to the right of the wave front and have the same values on all planes to the left of the wave front; later we will consider more complex plane waves.

We won't concern ourselves with the problem of actually *producing* such a field configuration. Instead, we simply ask whether it is consistent with the laws of electromagnetism—that is, with Maxwell's equations. We'll consider each of these four equations in turn.

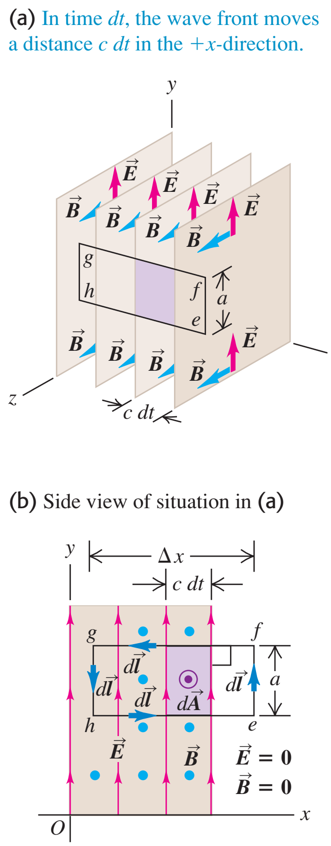
32.5 An electromagnetic wave front. The plane representing the wave front moves to the right (in the positive x-direction) with speed c.



32.6 Gaussian surface for a transverse plane electromagnetic wave.



32.7 (a) Applying Faraday's law to a plane wave. (b) In a time dt, the magnetic flux through the rectangle in the xy-plane increases by an amount $d\Phi_B$. This increase equals the flux through the shaded rectangle with area $ac\,dt$; that is, $d\Phi_B = Bac\,dt$. Thus $d\Phi_B/dt = Bac$.



Let us first verify that our wave satisfies Maxwell's first and second equations—that is, Gauss's laws for electric and magnetic fields. To do this, we take as our Gaussian surface a rectangular box with sides parallel to the xy, xz, and yz coordinate planes (Fig. 32.6). The box encloses no electric charge. The total electric flux and magnetic flux through the box are both zero, even if part of the box is in the region where $E = B = 0$. This would *not* be the case if $\vec{E}$ or $\vec{B}$ had an x-component, parallel to the direction of propagation; if the wave front were inside the box, there would be flux through the left-hand side of the box (at $x = 0$) but not the right-hand side (at $x > 0$). Thus to satisfy Maxwell's first and second equations, the electric and magnetic fields must be perpendicular to the direction of propagation; that is, the wave must be **transverse.**

The next of Maxwell's equations to be considered is Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \tag{32.1}$$

To test whether our wave satisfies Faraday's law, we apply this law to a rectangle $efgh$ that is parallel to the xy-plane (Fig. 32.7a). As shown in Fig. 32.7b, a cross section in the xy-plane, this rectangle has height a and width Δx. At the time shown, the wave front has progressed partway through the rectangle, and $\vec{E}$ is zero along the side ef. In applying Faraday's law we take the vector area $d\vec{A}$ of rectangle $efgh$ to be in the $+z$-direction. With this choice the right-hand rule requires that we integrate $\vec{E} \cdot d\vec{l}$ *counterclockwise* around the rectangle. At every point on side ef, $\vec{E}$ is zero. At every point on sides fg and he, $\vec{E}$ is either zero or perpendicular to $d\vec{l}$. Only side gh contributes to the integral. On this side, $\vec{E}$ and $d\vec{l}$ are opposite, and we obtain

$$\oint \vec{E} \cdot d\vec{l} = -Ea \tag{32.2}$$

Hence, the left-hand side of Eq. (32.1) is nonzero.

To satisfy Faraday's law, Eq. (32.1), there must be a component of $\vec{B}$ in the z-direction (perpendicular to $\vec{E}$) so that there can be a nonzero magnetic flux Φ_B through the rectangle $efgh$ and a nonzero derivative $d\Phi_B/dt$. Indeed, in our wave, $\vec{B}$ has *only* a z-component. We have assumed that this component is in the *positive* z-direction; let's see whether this assumption is consistent with Faraday's law. During a time interval dt the wave front moves a distance $c\,dt$ to the right in Fig. 32.7b, sweeping out an area $ac\,dt$ of the rectangle $efgh$. During this interval the magnetic flux Φ_B through the rectangle $efgh$ increases by $d\Phi_B = B(ac\,dt)$, so the rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = Bac \tag{32.3}$$

Now we substitute Eqs. (32.2) and (32.3) into Faraday's law, Eq. (32.1); we get

$$-Ea = -Bac$$

$$E = cB \qquad \text{(electromagnetic wave in vacuum)} \tag{32.4}$$

This shows that our wave is consistent with Faraday's law only if the wave speed c and the magnitudes of the perpendicular vectors $\vec{E}$ and $\vec{B}$ are related as in Eq. (32.4). Note that if we had assumed that $\vec{B}$ was in the *negative* z-direction, there would have been an additional minus sign in Eq. (32.4); since E, c, and B are all positive magnitudes, no solution would then have been possible. Furthermore, any component of $\vec{B}$ in the y-direction (parallel to $\vec{E}$) would not contribute to the changing magnetic flux Φ_B through the rectangle $efgh$ (which is parallel to the xy-plane) and so would not be part of the wave.

Finally, we carry out a similar calculation using Ampere's law, the remaining member of Maxwell's equations. There is no conduction current ($i_C = 0$), so Ampere's law is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{32.5}$$

To check whether our wave is consistent with Ampere's law, we move our rectangle so that it lies in the xz-plane, as shown in Fig. 32.8, and we again look at the situation at a time when the wave front has traveled partway through the rectangle. We take the vector area $d\vec{A}$ in the $+y$-direction, and so the right-hand rule requires that we integrate $\vec{B} \cdot d\vec{l}$ counterclockwise around the rectangle. The $\vec{B}$ field is zero at every point along side ef, and at each point on sides fg and he it is either zero or perpendicular to $d\vec{l}$. Only side gh, where $\vec{B}$ and $d\vec{l}$ are parallel, contributes to the integral, and we find

$$\oint \vec{B} \cdot d\vec{l} = Ba \tag{32.6}$$

Hence, the left-hand side of Ampere's law, Eq. (32.5), is nonzero; the right-hand side must be nonzero as well. Thus $\vec{E}$ must have a y-component (perpendicular to $\vec{B}$) so that the electric flux Φ_E through the rectangle and the time derivative $d\Phi_E/dt$ can be nonzero. We come to the same conclusion that we inferred from Faraday's law: In an electromagnetic wave, $\vec{E}$ and $\vec{B}$ must be mutually perpendicular.

In a time interval dt the electric flux Φ_E through the rectangle increases by $d\Phi_E = E(ac\,dt)$. Since we chose $d\vec{A}$ to be in the $+y$-direction, this flux change is positive; the rate of change of electric field is

$$\frac{d\Phi_E}{dt} = Eac \tag{32.7}$$

Substituting Eqs. (32.6) and (32.7) into Ampere's law, Eq. (32.5), we find

$$Ba = \epsilon_0 \mu_0 Eac$$

$$B = \epsilon_0 \mu_0 cE \qquad \text{(electromagnetic wave in vacuum)} \tag{32.8}$$

Thus our assumed wave obeys Ampere's law only if B, c, and E are related as in Eq. (32.8).

Our electromagnetic wave must obey *both* Ampere's law and Faraday's law, so Eqs. (32.4) and (32.8) must both be satisfied. This can happen only if $\epsilon_0 \mu_0 c = 1/c$, or

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \qquad \text{(speed of electromagnetic waves in vacuum)} \tag{32.9}$$

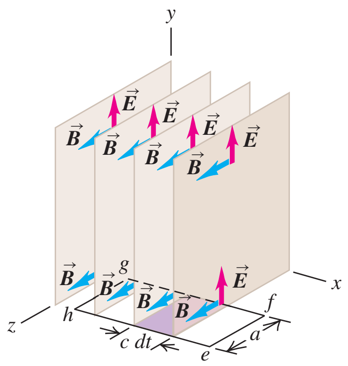
Inserting the numerical values of these quantities, we find

$$c = \frac{1}{\sqrt{(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(4\pi \times 10^{-7}\ \mathrm{N/A^2})}}$$
$$= 3.00 \times 10^8\ \mathrm{m/s}$$

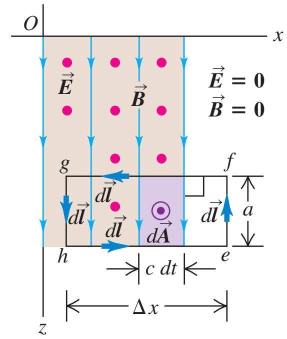
Our assumed wave is consistent with all of Maxwell's equations, provided that the wave front moves with the speed given above, which you should recognize as the speed of light! Note that the *exact* value of c is defined to be 299,792,458 m/s; the modern value of ϵ_0 is defined to agree with this when used in Eq. (32.9) (see Section 21.3).

32.8 (a) Applying Ampere's law to a plane wave. (Compare to Fig. 32.7a.) (b) In a time dt, the electric flux through the rectangle in the xz-plane increases by an amount $d\Phi_E$. This increase equals the flux through the shaded rectangle with area $ac\,dt$; that is, $d\Phi_E = Eac\,dt$. Thus $d\Phi_E/dt = Eac$.

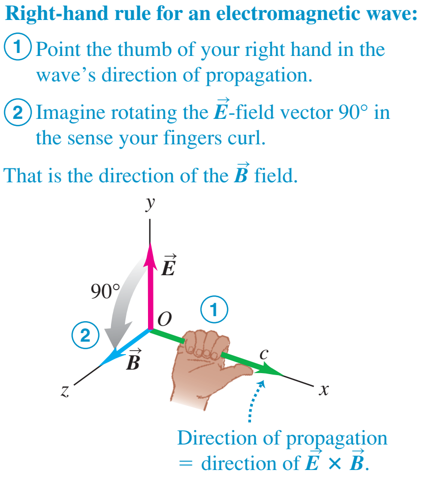
(a) In time dt, the wave front moves a distance $c\,dt$ in the $+x$-direction.



(b) Top view of situation in (a)



32.9 A right-hand rule for electromagnetic waves relates the directions of $\vec{E}$ and $\vec{B}$ and the direction of propagation.



Key Properties of Electromagnetic Waves

We chose a simple wave for our study in order to avoid mathematical complications, but this special case illustrates several important features of *all* electromagnetic waves:

1. The wave is *transverse;* both $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of propagation of the wave. The electric and magnetic fields are also perpendicular to each other. The direction of propagation is the direction of the vector product $\vec{E} \times \vec{B}$ (Fig. 32.9).
2. There is a definite ratio between the magnitudes of $\vec{E}$ and $\vec{B}$: $E = cB$.
3. The wave travels in vacuum with a definite and unchanging speed.
4. Unlike mechanical waves, which need the oscillating particles of a medium such as water or air to transmit a wave, electromagnetic waves require no medium.

We can generalize this discussion to a more realistic situation. Suppose we have several wave fronts in the form of parallel planes perpendicular to the x-axis, all of which are moving to the right with speed c. Suppose that the $\vec{E}$ and $\vec{B}$ fields are the same at all points within a single region between two planes, but that the fields differ from region to region. The overall wave is a plane wave, but one in which the fields vary in steps along the x-axis. Such a wave could be constructed by superposing several of the simple step waves we have just discussed (shown in Fig. 32.5). This is possible because the $\vec{E}$ and $\vec{B}$ fields obey the superposition principle in waves just as in static situations: When two waves are superposed, the total $\vec{E}$ field at each point is the vector sum of the $\vec{E}$ fields of the individual waves, and similarly for the total $\vec{B}$ field.

We can extend the above development to show that a wave with fields that vary in steps is also consistent with Ampere's and Faraday's laws, provided that the wave fronts all move with the speed c given by Eq. (32.9). In the limit that we make the individual steps infinitesimally small, we have a wave in which the $\vec{E}$ and $\vec{B}$ fields at any instant vary *continuously* along the x-axis. The entire field pattern moves to the right with speed c. In Section 32.3 we will consider waves in which $\vec{E}$ and $\vec{B}$ are *sinusoidal* functions of x and t. Because at each point the magnitudes of $\vec{E}$ and $\vec{B}$ are related by $E = cB$, the periodic variations of the two fields in any periodic traveling wave must be *in phase.*

Electromagnetic waves have the property of **polarization.** In the above discussion the choice of the y-direction for $\vec{E}$ was arbitrary. We could just as well have specified the z-axis for $\vec{E}$; then $\vec{B}$ would have been in the $-y$-direction. A wave in which $\vec{E}$ is always parallel to a certain axis is said to be **linearly polarized** along that axis. More generally, *any* wave traveling in the x-direction can be represented as a superposition of waves linearly polarized in the y- and z-directions. We will study polarization in greater detail in Chapter 33.

Derivation of the Electromagnetic Wave Equation

Here is an alternative derivation of Eq. (32.9) for the speed of electromagnetic waves. It is more mathematical than our other treatment, but it includes a derivation of the wave equation for electromagnetic waves. This part of the section can be omitted without loss of continuity in the chapter.

During our discussion of mechanical waves in Section 15.3, we showed that a function $y(x, t)$ that represents the displacement of any point in a mechanical wave traveling along the x-axis must satisfy a differential equation, Eq. (15.12):

$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x, t)}{\partial t^2} \tag{32.10}$$

This equation is called the **wave equation,** and v is the speed of propagation of the wave.

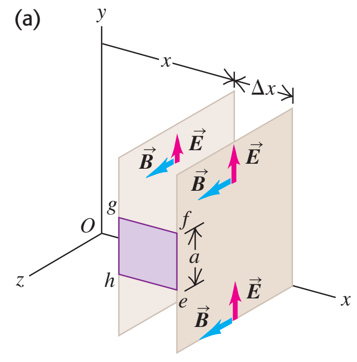
To derive the corresponding equation for an electromagnetic wave, we again consider a plane wave. That is, we assume that at each instant, E_y and B_z are uniform over any plane perpendicular to the x-axis, the direction of propagation. But now we let E_y and B_z vary continuously as we go along the x-axis; then each is a function of x and t. We consider the values of E_y and B_z on two planes perpendicular to the x-axis, one at x and one at $x + \Delta x$.

Following the same procedure as previously, we apply Faraday's law to a rectangle lying parallel to the xy-plane, as in Fig. 32.10. This figure is similar to Fig. 32.7. Let the left end gh of the rectangle be at position x, and let the right end ef be at position $(x + \Delta x)$. At time t, the values of E_y on these two sides are $E_y(x, t)$ and $E_y(x + \Delta x, t)$, respectively. When we apply Faraday's law to this rectangle, we find that instead of $\oint \vec{E} \cdot d\vec{l} = -Ea$ as before, we have

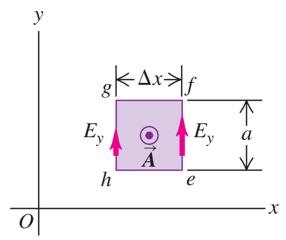
$$\oint \vec{E} \cdot d\vec{l} = -E_y(x, t)a + E_y(x + \Delta x, t)a$$
$$= a[E_y(x + \Delta x, t) - E_y(x, t)] \tag{32.11}$$

32.10 Faraday's law applied to a rectangle with height a and width Δx parallel to the xy-plane.

(a)



(b) Side view of the situation in (a)



To find the magnetic flux Φ_B through this rectangle, we assume that Δx is small enough that B_z is nearly uniform over the rectangle. In that case, $\Phi_B = B_z(x, t)A = B_z(x, t)a\,\Delta x$, and

$$\frac{d\Phi_B}{dt} = \frac{\partial B_z(x, t)}{\partial t} a\,\Delta x$$

We use partial-derivative notation because B_z is a function of both x and t. When we substitute this expression and Eq. (32.11) into Faraday's law, Eq. (32.1), we get

$$a[E_y(x + \Delta x, t) - E_y(x, t)] = -\frac{\partial B_z}{\partial t} a\,\Delta x$$
$$\frac{E_y(x + \Delta x, t) - E_y(x, t)}{\Delta x} = -\frac{\partial B_z}{\partial t}$$

Finally, imagine shrinking the rectangle down to a sliver so that Δx approaches zero. When we take the limit of this equation as $\Delta x \to 0$, we get

$$\frac{\partial E_y(x, t)}{\partial x} = -\frac{\partial B_z(x, t)}{\partial t} \tag{32.12}$$

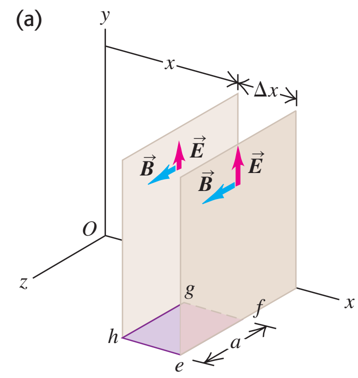
This equation shows that if there is a time-varying component B_z of magnetic field, there must also be a component E_y of electric field that varies with x, and conversely. We put this relationship on the shelf for now; we'll return to it soon.

Next we apply Ampere's law to the rectangle shown in Fig. 32.11. The line integral $\oint \vec{B} \cdot d\vec{l}$ becomes

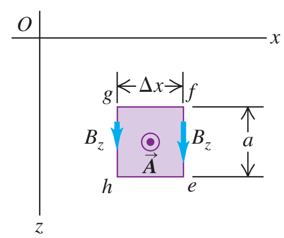
$$\oint \vec{B} \cdot d\vec{l} = -B_z(x + \Delta x, t)a + B_z(x, t)a \tag{32.13}$$

32.11 Ampere's law applied to a rectangle with height a and width Δx parallel to the xz-plane.

(a)



(b) Top view of the situation in (a)



Again assuming that the rectangle is narrow, we approximate the electric flux Φ_E through it as $\Phi_E = E_y(x, t)A = E_y(x, t)a\,\Delta x$. The rate of change of Φ_E, which we need for Ampere's law, is then

$$\frac{d\Phi_E}{dt} = \frac{\partial E_y(x, t)}{\partial t} a\,\Delta x$$

Now we substitute this expression and Eq. (32.13) into Ampere's law, Eq. (32.5):

$$-B_z(x + \Delta x, t)a + B_z(x, t)a = \epsilon_0\mu_0 \frac{\partial E_y(x, t)}{\partial t} a\,\Delta x$$

Again we divide both sides by $a\,\Delta x$ and take the limit as $\Delta x \to 0$. We find

$$-\frac{\partial B_z(x, t)}{\partial x} = \epsilon_0\mu_0 \frac{\partial E_y(x, t)}{\partial t} \tag{32.14}$$

Now comes the final step. We take the partial derivatives with respect to x of both sides of Eq. (32.12), and we take the partial derivatives with respect to t of both sides of Eq. (32.14). The results are

$$-\frac{\partial^2 E_y(x,t)}{\partial x^2} = \frac{\partial^2 B_z(x,t)}{\partial x \partial t}$$

$$-\frac{\partial^2 B_z(x,t)}{\partial x \partial t} = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

Combining these two equations to eliminate B_z, we finally find

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2} \quad \text{(electromagnetic wave equation in vacuum)} \tag{32.15}$$

This expression has the same form as the general wave equation, Eq. (32.10). Because the electric field E_y must satisfy this equation, it behaves as a wave with a pattern that travels through space with a definite speed. Furthermore, comparison of Eqs. (32.15) and (32.10) shows that the wave speed v is given by

$$\frac{1}{v^2} = \epsilon_0 \mu_0 \quad \text{or} \quad v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

This agrees with Eq. (32.9) for the speed c of electromagnetic waves.

We can show that B_z also must satisfy the same wave equation as E_y, Eq. (32.15). To prove this, we take the partial derivative of Eq. (32.12) with respect to t and the partial derivative of Eq. (32.14) with respect to x and combine the results. We leave this derivation for you to carry out.

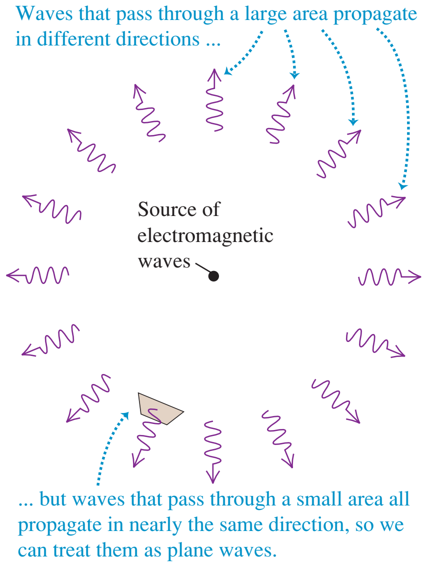
Test Your Understanding of Section 32.2 For each of the following electromagnetic waves, state the direction of the magnetic field. (a) The wave is propagating in the positive z-direction, and $\vec{E}$ is in the positive x-direction; (b) the wave is propagating in the positive y-direction, and $\vec{E}$ is in the negative z-direction; (c) the wave is propagating in the negative x-direction, and $\vec{E}$ is in the positive z-direction.

32.3 Sinusoidal Electromagnetic Waves

Sinusoidal electromagnetic waves are directly analogous to sinusoidal transverse mechanical waves on a stretched string, which we studied in Section 15.3. In a sinusoidal electromagnetic wave, $\vec{E}$ and $\vec{B}$ at any point in space are sinusoidal functions of time, and at any instant of time the *spatial* variation of the fields is also sinusoidal.

Some sinusoidal electromagnetic waves are *plane waves;* they share with the waves described in Section 32.2 the property that at any instant the fields are uniform over any plane perpendicular to the direction of propagation. The entire pattern travels in the direction of propagation with speed c. The directions of $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of propagation (and to each other), so the wave is *transverse.* Electromagnetic waves produced by an oscillating point charge, shown in Fig. 32.3, are an example of sinusoidal waves that are *not* plane waves. But if we restrict our observations to a relatively small region of space at a sufficiently great distance from the source, even these waves are well approximated by plane waves (Fig. 32.12). In the same way, the curved surface of the (nearly) spherical earth appears flat to us because of our small size relative to the earth's radius. In this section we'll restrict our discussion to plane waves.

32.12 Waves passing through a small area at a sufficiently great distance from a source can be treated as plane waves.



The frequency f, the wavelength λ, and the speed of propagation c of any periodic wave are related by the usual wavelength–frequency relationship $c = \lambda f$. If the frequency f is 10^8 Hz (100 MHz), typical of commercial FM radio broadcasts, the wavelength is

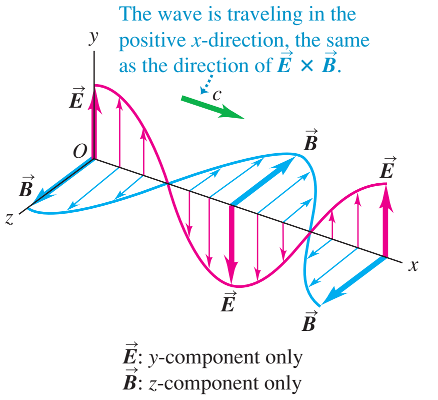
$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{10^8 \text{ Hz}} = 3 \text{ m}$$

Figure 32.4 shows the inverse proportionality between wavelength and frequency.

Fields of a Sinusoidal Wave

Figure 32.13 shows a linearly polarized sinusoidal electromagnetic wave traveling in the $+x$-direction. The $\vec{E}$ and $\vec{B}$ vectors are shown for only a few points on the positive x-axis. Note that the electric and magnetic fields oscillate in phase: $\vec{E}$ is maximum where $\vec{B}$ is maximum and $\vec{E}$ is zero where $\vec{B}$ is zero. Note also that where $\vec{E}$ is in the $+y$-direction, $\vec{B}$ is in the $+z$-direction; where $\vec{E}$ is in the $-y$-direction, $\vec{B}$ is in the $-z$-direction. At all points the vector product $\vec{E} \times \vec{B}$ is in the direction in which the wave is propagating (the $+x$-direction). We mentioned this in Section 32.2 in the list of characteristics of electromagnetic waves.

32.13 Representation of the electric and magnetic fields as functions of x for a linearly polarized sinusoidal plane electromagnetic wave. One wavelength of the wave is shown at time $t = 0$. The fields are shown only for points along the x-axis.



CAUTION **In a plane wave, $\vec{E}$ and $\vec{B}$ are everywhere** Figure 32.13 may give you the erroneous impression that the electric and magnetic fields exist only along the x-axis. In fact, in a sinusoidal plane wave there are electric and magnetic fields at *all* points in space. Imagine a plane perpendicular to the x-axis (that is, parallel to the yz-plane) at a particular point, at a particular time; the fields have the same values at all points in that plane. The values are different on different planes.

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ActivPhysics 10.1: Properties of Mechanical Waves

We can describe electromagnetic waves by means of *wave functions,* just as we did in Section 15.3 for waves on a string. One form of the wave function for a transverse wave traveling in the $+x$-direction along a stretched string is Eq. (15.7):

$$y(x, t) = A\cos(kx - \omega t)$$

where $y(x, t)$ is the transverse displacement from its equilibrium position at time t of a point with coordinate x on the string. The quantity A is the maximum displacement, or *amplitude,* of the wave; ω is its *angular frequency,* equal to 2π times the frequency f; and k is the *wave number,* equal to $2\pi/\lambda$, where λ is the wavelength.

Let $E_y(x, t)$ and $B_z(x, t)$ represent the instantaneous values of the y-component of $\vec{E}$ and the z-component of $\vec{B}$, respectively, in Fig. 32.13, and let E_{max} and B_{max} represent the maximum values, or *amplitudes,* of these fields. The wave functions for the wave are then

$$E_y(x, t) = E_{max}\cos(kx - \omega t) \qquad B_z(x, t) = B_{max}\cos(kx - \omega t) \tag{32.16}$$

(sinusoidal electromagnetic plane wave, propagating in $+x$-direction)

We can also write the wave functions in vector form:

$$\vec{E}(x, t) = \hat{j}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x, t) = \hat{k}B_{max}\cos(kx - \omega t) \tag{32.17}$$

CAUTION **The symbol k has two meanings** Note the two different k's: the unit vector $\hat{k}$ in the z-direction and the wave number k. Don't get these confused!

The sine curves in Fig. 32.13 represent instantaneous values of the electric and magnetic fields as functions of x at time $t = 0$—that is, $\vec{E}(x, t = 0)$ and $\vec{B}(x, t = 0)$. As time goes by, the wave travels to the right with speed c. Equations (32.16) and (32.17) show that at any point the sinusoidal oscillations of $\vec{E}$ and $\vec{B}$ are *in phase.* From Eq. (32.4) the amplitudes must be related by

$$E_{max} = cB_{max} \qquad \text{(electromagnetic wave in vacuum)} \tag{32.18}$$

These amplitude and phase relationships are also required for $E(x, t)$ and $B(x, t)$ to satisfy Eqs. (32.12) and (32.14), which came from Faraday's law and Ampere's law, respectively. Can you verify this statement? (See Problem 32.38.)

32.14 Representation of one wavelength of a linearly polarized sinusoidal plane electromagnetic wave traveling in the negative x-direction at $t = 0$. The fields are shown only for points along the x-axis. (Compare with Fig. 32.13.)

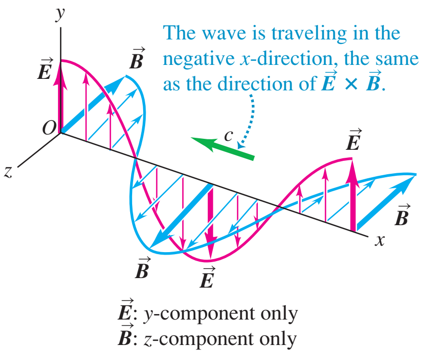


Figure 32.14 shows the electric and magnetic fields of a wave traveling in the *negative* x-direction. At points where $\vec{E}$ is in the positive y-direction, $\vec{B}$ is in the *negative* z-direction; where $\vec{E}$ is in the negative y-direction, $\vec{B}$ is in the *positive* z-direction. The wave functions for this wave are

$$E_y(x,t) = E_{\max}\cos(kx + \omega t) \quad B_z(x,t) = -B_{\max}\cos(kx + \omega t) \tag{32.19}$$

(sinusoidal electromagnetic plane wave, propagating in $-x$-direction)

As with the wave traveling in the $+x$-direction, at any point the sinusoidal oscillations of the $\vec{E}$ and $\vec{B}$ fields are *in phase,* and the vector product $\vec{E} \times \vec{B}$ points in the direction of propagation.

The sinusoidal waves shown in Figs. 32.13 and 32.14 are both linearly polarized in the y-direction; the $\vec{E}$ field is always parallel to the y-axis. Example 32.1 concerns a wave that is linearly polarized in the z-direction.

Problem-Solving Strategy 32.1 Electromagnetic Waves

IDENTIFY *the relevant concepts:* Many of the same ideas that apply to mechanical waves apply to electromagnetic waves. One difference is that electromagnetic waves are described by two quantities (in this case, electric field $\vec{E}$ and magnetic field $\vec{B}$), rather than by a single quantity, such as the displacement of a string.

SET UP *the problem* using the following steps:
1. Draw a diagram showing the direction of wave propagation and the directions of $\vec{E}$ and $\vec{B}$.
2. Identify the target variables.

EXECUTE *the solution* as follows:
1. Review the treatment of sinusoidal mechanical waves in Chapters 15 and 16, and particularly the four problem-solving strategies suggested there.
2. Keep in mind the basic relationships for periodic waves: $v = \lambda f$ and $\omega = vk$. For electromagnetic waves in vacuum, $v = c$. Distinguish between ordinary frequency f, usually expressed in hertz, and angular frequency $\omega = 2\pi f$, expressed in rad/s. Remember that the wave number is $k = 2\pi/\lambda$.
3. Concentrate on basic relationships, such as those between $\vec{E}$ and $\vec{B}$ (magnitude, direction, and relative phase), how the wave speed is determined, and the transverse nature of the waves.

EVALUATE *your answer:* Check that your result is reasonable. For electromagnetic waves in vacuum, the magnitude of the magnetic field in teslas is much smaller (by a factor of 3.00×10^8) than the magnitude of the electric field in volts per meter. If your answer suggests otherwise, you probably made an error using the relationship $E = cB$. (We'll see later in this section that the relationship between E and B is different for electromagnetic waves in a material medium.)

Example 32.1 Electric and magnetic fields of a laser beam

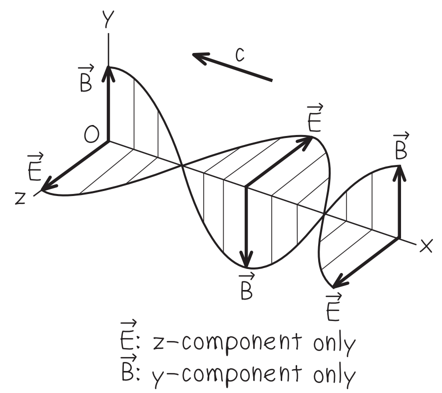
A carbon dioxide laser emits a sinusoidal electromagnetic wave that travels in vacuum in the negative x-direction. The wavelength is 10.6 μm (in the infrared; see Fig. 32.4) and the $\vec{E}$ field is parallel to the z-axis, with $E_{\max} = 1.5$ MV/m. Write vector equations for $\vec{E}$ and $\vec{B}$ as functions of time and position.

SOLUTION

IDENTIFY and SET UP: Equations (32.19) describe a wave traveling in the negative x-direction with $\vec{E}$ along the y-axis—that is, a wave that is linearly polarized along the y-axis. By contrast, the wave in this example is linearly polarized along the z-axis. At points where $\vec{E}$ is in the positive z-direction, $\vec{B}$ must be in the positive y-direction for the vector product $\vec{E} \times \vec{B}$ to be in the negative x-direction (the direction of propagation). Figure 32.15 shows a wave that satisfies these requirements.

EXECUTE: A possible pair of wave functions that describe the wave shown in Fig. 32.15 are

32.15 Our sketch for this problem.



$$\vec{E}(x,t) = \hat{k}E_{\max}\cos(kx + \omega t)$$
$$\vec{B}(x,t) = \hat{j}B_{\max}\cos(kx + \omega t)$$

The plus sign in the arguments of the cosine functions indicates that the wave is propagating in the negative x-direction, as it should. Faraday's law requires that $E_{max} = cB_{max}$ [Eq. (32.18)], so

$$B_{max} = \frac{E_{max}}{c} = \frac{1.5 \times 10^6 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 5.0 \times 10^{-3} \text{ T}$$

To check unit consistency, note that $1 \text{ V} = 1 \text{ Wb/s}$ and $1 \text{ Wb/m}^2 = 1 \text{ T}$.

We have $\lambda = 10.6 \times 10^{-6}$ m, so the wave number and angular frequency are

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{10.6 \times 10^{-6} \text{ m}} = 5.93 \times 10^5 \text{ rad/m}$$

$$\omega = ck = (3.00 \times 10^8 \text{ m/s})(5.93 \times 10^5 \text{ rad/m}) = 1.78 \times 10^{14} \text{ rad/s}$$

Substituting these values into the above wave functions, we get

$$\vec{E}(x, t) = \hat{k}(1.5 \times 10^6 \text{ V/m}) \times \cos[(5.93 \times 10^5 \text{ rad/m})x + (1.78 \times 10^{14} \text{ rad/s})t]$$

$$\vec{B}(x, t) = \hat{j}(5.0 \times 10^{-3} \text{ T}) \times \cos[(5.93 \times 10^5 \text{ rad/m})x + (1.78 \times 10^{14} \text{ rad/s})t]$$

EVALUATE: As we expect, the magnitude B_{max} in teslas is much smaller than the magnitude E_{max} in volts per meter. To check the directions of $\vec{E}$ and $\vec{B}$, note that $\vec{E} \times \vec{B}$ is in the direction of $\hat{k} \times \hat{j} = -\hat{\imath}$. This is as it should be for a wave that propagates in the negative x-direction.

Our expressions for $\vec{E}(x, t)$ and $\vec{B}(x, t)$ are not the only possible solutions. We could always add a phase angle ϕ to the arguments of the cosine function, so that $kx + \omega t$ would become $kx + \omega t + \phi$. To determine the value of ϕ we would need to know $\vec{E}$ and $\vec{B}$ either as functions of x at a given time t or as functions of t at a given coordinate x. However, the statement of the problem doesn't include this information.

Electromagnetic Waves in Matter

So far, our discussion of electromagnetic waves has been restricted to waves in *vacuum.* But electromagnetic waves can also travel in *matter;* think of light traveling through air, water, or glass. In this subsection we extend our analysis to electromagnetic waves in nonconducting materials—that is, *dielectrics.*

In a dielectric the wave speed is not the same as in vacuum, and we denote it by v instead of c. Faraday's law is unaltered, but in Eq. (32.4), derived from Faraday's law, the speed c is replaced by v. In Ampere's law the displacement current is given not by $\epsilon_0 \, d\Phi_E/dt$, where Φ_E is the flux of $\vec{E}$ through a surface, but by $\epsilon \, d\Phi_E/dt = K\epsilon_0 \, d\Phi_E/dt$, where K is the dielectric constant and ϵ is the permittivity of the dielectric. (We introduced these quantities in Section 24.4.) Also, the constant μ_0 in Ampere's law must be replaced by $\mu = K_m\mu_0$, where K_m is the relative permeability of the dielectric and μ is its permeability (see Section 28.8). Hence Eqs. (32.4) and (32.8) are replaced by

$$E = vB \qquad \text{and} \qquad B = \epsilon\mu v E \tag{32.20}$$

Following the same procedure as for waves in vacuum, we find that the wave speed v is

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}}\frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{KK_m}} \quad \text{(speed of electromagnetic waves in a dielectric)} \tag{32.21}$$

For most dielectrics the relative permeability K_m is very nearly equal to unity (except for insulating ferromagnetic materials). When $K_m \cong 1$,

$$v = \frac{1}{\sqrt{K}}\frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{K}}$$

Because K is always greater than unity, the speed v of electromagnetic waves in a dielectric is always *less* than the speed c in vacuum by a factor of $1/\sqrt{K}$ (Fig. 32.16). The ratio of the speed c in vacuum to the speed v in a material is known in optics as the **index of refraction** n of the material. When $K_m \cong 1$,

$$\frac{c}{v} = n = \sqrt{KK_m} \cong \sqrt{K} \tag{32.22}$$

Usually, we can't use the values of K in Table 24.1 in this equation because those values are measured using *constant* electric fields. When the fields oscillate rapidly,

32.16 The dielectric constant K of water is about 1.8 for visible light, so the speed of visible light in water is slower than in vacuum by a factor of $1/\sqrt{K} = 1/\sqrt{1.8} = 0.75$.

there is usually not time for the reorientation of electric dipoles that occurs with steady fields. Values of K with rapidly varying fields are usually much *smaller* than the values in the table. For example, K for water is 80.4 for steady fields but only about 1.8 in the frequency range of visible light. Thus the dielectric "constant" K is actually a function of frequency (the *dielectric function*).

Example 32.2 Electromagnetic waves in different materials

(a) Visiting a jewelry store one evening, you hold a diamond up to the light of a sodium-vapor street lamp. The heated sodium vapor emits yellow light with a frequency of 5.09×10^{14} Hz. Find the wavelength in vacuum and the wave speed and wavelength in diamond, for which $K = 5.84$ and $K_m = 1.00$ at this frequency. (b) A 90.0-MHz radio wave (in the FM radio band) passes from vacuum into an insulating ferrite (a ferromagnetic material used in computer cables to suppress radio interference). Find the wavelength in vacuum and the wave speed and wavelength in the ferrite, for which $K = 10.0$ and $K_m = 1000$ at this frequency.

SOLUTION

IDENTIFY and SET UP: In each case we find the wavelength in vacuum using $c = \lambda f$. To use the corresponding equation $v = \lambda f$ to find the wavelength in a material medium, we find the speed v of electromagnetic waves in the medium using Eq. (32.21), which relates v to the values of dielectric constant K and relative permeability K_m for the medium.

EXECUTE: (a) The wavelength in vacuum of the sodium light is

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}} = 5.89 \times 10^{-7} \text{ m} = 589 \text{ nm}$$

The wave speed and wavelength in diamond are

$$v_{\text{diamond}} = \frac{c}{\sqrt{KK_m}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(5.84)(1.00)}} = 1.24 \times 10^8 \text{ m/s}$$

$$\lambda_{\text{diamond}} = \frac{v_{\text{diamond}}}{f} = \frac{1.24 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}}$$
$$= 2.44 \times 10^{-7} \text{ m} = 244 \text{ nm}$$

(b) Following the same steps as in part (a), we find

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}} = 3.33 \text{ m}$$

$$v_{\text{ferrite}} = \frac{c}{\sqrt{KK_m}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(10.0)(1000)}} = 3.00 \times 10^6 \text{ m/s}$$

$$\lambda_{\text{ferrite}} = \frac{v_{\text{ferrite}}}{f} = \frac{3.00 \times 10^6 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}}$$
$$= 3.33 \times 10^{-2} \text{ m} = 3.33 \text{ cm}$$

EVALUATE: The speed of light in transparent materials is typically between $0.2c$ and c; our result in part (a) shows that $v_{\text{diamond}} = 0.414c$. As our results in part (b) show, the speed of electromagnetic waves in dense materials like ferrite (for which $v_{\text{ferrite}} = 0.010c$) can be *far* slower than in vacuum.

Test Your Understanding of Section 32.3 The first of Eqs. (32.17) gives the electric field for a plane wave as measured at points along the x-axis. For this plane wave, how does the electric field at points *off* the x-axis differ from the expression in Eqs. (32.17)? (i) The amplitude is different; (ii) the phase is different; (iii) both the amplitude and phase are different; (iv) none of these.

32.4 Energy and Momentum in Electromagnetic Waves

It is a familiar fact that energy is associated with electromagnetic waves; think of the energy in the sun's radiation. Microwave ovens, radio transmitters, and lasers for eye surgery all make use of the energy that these waves carry. To understand how to utilize this energy, it's helpful to derive detailed relationships for the energy in an electromagnetic wave.

We begin with the expressions derived in Sections 24.3 and 30.3 for the **energy densities** in electric and magnetic fields; we suggest you review those derivations now. Equations (24.11) and (30.10) show that in a region of empty space where $\vec{E}$ and $\vec{B}$ fields are present, the total energy density u is given by

$$u = \tfrac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \tag{32.23}$$

where ϵ_0 and μ_0 are, respectively, the permittivity and permeability of free space. For electromagnetic waves in vacuum, the magnitudes E and B are related by

$$B = \frac{E}{c} = \sqrt{\epsilon_0 \mu_0}\,E \tag{32.24}$$

Combining Eqs. (32.23) and (32.24), we can also express the energy density u in a simple electromagnetic wave in vacuum as

$$u = \tfrac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}(\sqrt{\epsilon_0\mu_0}\,E)^2 = \epsilon_0 E^2 \tag{32.25}$$

This shows that in vacuum, the energy density associated with the $\vec{E}$ field in our simple wave is equal to the energy density of the $\vec{B}$ field. In general, the electric-field magnitude E is a function of position and time, as for the sinusoidal wave described by Eqs. (32.16); thus the energy density u of an electromagnetic wave, given by Eq. (32.25), also depends in general on position and time.

Electromagnetic Energy Flow and the Poynting Vector

Electromagnetic waves such as those we have described are *traveling* waves that transport energy from one region to another. We can describe this energy transfer in terms of energy transferred *per unit time per unit cross-sectional area,* or *power per unit area,* for an area perpendicular to the direction of wave travel.

To see how the energy flow is related to the fields, consider a stationary plane, perpendicular to the x-axis, that coincides with the wave front at a certain time. In a time dt after this, the wave front moves a distance $dx = c\,dt$ to the right of the plane. Considering an area A on this stationary plane (Fig. 32.17), we note that the energy in the space to the right of this area must have passed through the area to reach the new location. The volume dV of the relevant region is the base area A times the length $c\,dt$, and the energy dU in this region is the energy density u times this volume:

$$dU = u\,dV = (\epsilon_0 E^2)(Ac\,dt)$$

This energy passes through the area A in time dt. The energy flow per unit time per unit area, which we will call S, is

$$S = \frac{1}{A}\frac{dU}{dt} = \epsilon_0 c E^2 \qquad \text{(in vacuum)} \tag{32.26}$$

Using Eqs. (32.4) and (32.9), we can derive the alternative forms

$$S = \frac{\epsilon_0}{\sqrt{\epsilon_0\mu_0}}E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}}E^2 = \frac{EB}{\mu_0} \qquad \text{(in vacuum)} \tag{32.27}$$

We leave the derivation of Eq. (32.27) from Eq. (32.26) as an exercise for you. The units of S are energy per unit time per unit area, or power per unit area. The SI unit of S is 1 $\text{J/s}\cdot\text{m}^2$ or 1 W/m^2.

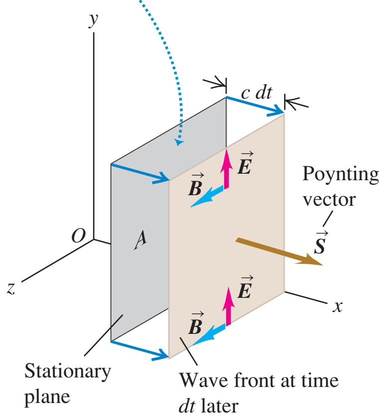
We can define a *vector* quantity that describes both the magnitude and direction of the energy flow rate:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad \text{(Poynting vector in vacuum)} \tag{32.28}$$

The vector $\vec{S}$ is called the **Poynting vector;** it was introduced by the British physicist John Poynting (1852–1914). Its direction is in the direction of propagation of the wave (Fig. 32.18). Since $\vec{E}$ and $\vec{B}$ are perpendicular, the magnitude of $\vec{S}$ is $S = EB/\mu_0$; from Eqs. (32.26) and (32.27) this is the energy flow per unit area and per unit time through a cross-sectional area perpendicular to the propagation direction. The total energy flow per unit time (power, P) out of any closed surface is the integral of $\vec{S}$ over the surface:

$$P = \oint \vec{S}\cdot d\vec{A}$$

32.17 A wave front at a time dt after it passes through the stationary plane with area A.



32.18 These rooftop solar panels are tilted to be face-on to the sun—that is, face-on to the Poynting vector of electromagnetic waves from the sun, so that the panels can absorb the maximum amount of wave energy.

For the sinusoidal waves studied in Section 32.3, as well as for other more complex waves, the electric and magnetic fields at any point vary with time, so the Poynting vector at any point is also a function of time. Because the frequencies of typical electromagnetic waves are very high, the time variation of the Poynting vector is so rapid that it's most appropriate to look at its *average* value. The magnitude of the average value of $\vec{S}$ at a point is called the **intensity** of the radiation at that point. The SI unit of intensity is the same as for S, 1 W/m^2 (watt per square meter).

Let's work out the intensity of the sinusoidal wave described by Eqs. (32.17). We first substitute $\vec{E}$ and $\vec{B}$ into Eq. (32.28):

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t) \times \vec{B}(x,t)$$
$$= \frac{1}{\mu_0}[\hat{\jmath}E_{\max}\cos(kx - \omega t)] \times [\hat{k}B_{\max}\cos(kx - \omega t)]$$

The vector product of the unit vectors is $\hat{\jmath} \times \hat{k} = \hat{\imath}$ and $\cos^2(kx - \omega t)$ is never negative, so $\vec{S}(x,t)$ always points in the positive x-direction (the direction of wave propagation). The x-component of the Poynting vector is

$$S_x(x,t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx - \omega t) = \frac{E_{\max}B_{\max}}{2\mu_0}[1 + \cos 2(kx - \omega t)]$$

The time average value of $\cos 2(kx - \omega t)$ is zero because at any point, it is positive during one half-cycle and negative during the other half. So the average value of the Poynting vector over a full cycle is $\vec{S}_{\text{av}} = \hat{\imath}S_{\text{av}}$, where

$$S_{\text{av}} = \frac{E_{\max}B_{\max}}{2\mu_0}$$

That is, the magnitude of the average value of $\vec{S}$ for a sinusoidal wave (the intensity I of the wave) is $\frac{1}{2}$ the maximum value. By using the relationships $E_{\max} = B_{\max}c$ and $\epsilon_0\mu_0 = 1/c^2$, we can express the intensity in several equivalent forms:

$$I = S_{\text{av}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}{}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_{\max}{}^2 = \frac{1}{2}\epsilon_0 c E_{\max}{}^2 \qquad \text{(intensity of a sinusoidal wave in vacuum)} \quad (32.29)$$

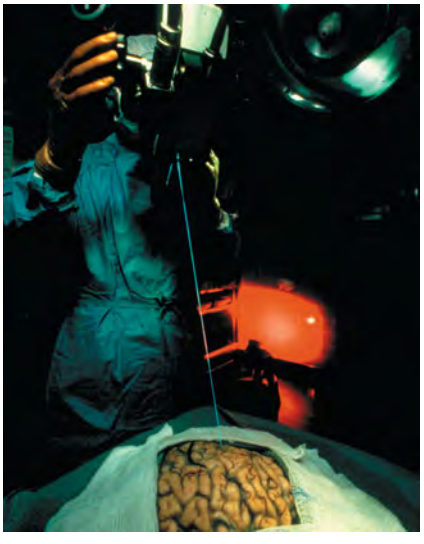
We invite you to verify that these expressions are all equivalent.

For a wave traveling in the $-x$-direction, represented by Eqs. (32.19), the Poynting vector is in the $-x$-direction at every point, but its magnitude is the same as for a wave traveling in the $+x$-direction. Verifying these statements is left to you (see Exercise 32.24).

CAUTION **Poynting vector vs. intensity** At any point x, the magnitude of the Poynting vector varies with time. Hence, the *instantaneous* rate at which electromagnetic energy in a sinusoidal plane wave arrives at a surface is not constant. This may seem to contradict everyday experience; the light from the sun, a light bulb, or the laser in a grocery-store scanner appears steady and unvarying in strength. In fact the Poynting vector from these sources *does* vary in time, but the variation isn't noticeable because the oscillation frequency is so high (around 5×10^{14} Hz for visible light). All that you sense is the *average* rate at which energy reaches your eye, which is why we commonly use intensity (the average value of S) to describe the strength of electromagnetic radiation.

Throughout this discussion we have considered only electromagnetic waves propagating in vacuum. If the waves are traveling in a dielectric medium, however,

Application Laser Surgery
Lasers are used widely in medicine as ultra-precise, bloodless "scalpels." They can reach and remove tumors with minimal damage to neighboring healthy tissues, as in the brain surgery shown here. The power output of the laser is typically below 40 W, less than that of a typical light bulb. However, this power is concentrated into a spot from 0.1 to 2.0 mm in diameter, so the intensity of the light (equal to the average value of the Poynting vector) can be as high as 5×10^9 W/m^2.



the expressions for energy density [Eq. (32.23)], the Poynting vector [Eq. (32.28)], and the intensity of a sinusoidal wave [Eq. (32.29)] must be modified. It turns out that the required modifications are quite simple: Just replace ϵ_0 with the permittivity ϵ of the dielectric, replace μ_0 with the permeability μ of the dielectric, and replace c with the speed v of electromagnetic waves in the dielectric. Remarkably, the energy densities in the $\vec{E}$ and $\vec{B}$ fields are equal even in a dielectric.

Example 32.3 Energy in a nonsinusoidal wave

For the nonsinusoidal wave described in Section 32.2, suppose that $E = 100\ \text{V/m} = 100\ \text{N/C}$. Find the value of B, the energy density u, and the rate of energy flow per unit area S.

SOLUTION

IDENTIFY and SET UP: In this wave $\vec{E}$ and $\vec{B}$ are uniform behind the wave front (and zero ahead of it). Hence the target variables B, u, and S must also be uniform behind the wave front. Given the magnitude E, we use Eq. (32.4) to find B, Eq. (32.25) to find u, and Eq. (32.27) to find S. (We cannot use Eq. (32.29), which applies to sinusoidal waves only.)

EXECUTE: From Eq. (32.4),

$$B = \frac{E}{c} = \frac{100\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}} = 3.33 \times 10^{-7}\ \text{T}$$

From Eq. (32.25),

$$\begin{aligned} u &= \epsilon_0 E^2 = (8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(100\ \text{N/C})^2 \\ &= 8.85 \times 10^{-8}\ \text{N/m}^2 = 8.85 \times 10^{-8}\ \text{J/m}^3 \end{aligned}$$

The magnitude of the Poynting vector is

$$\begin{aligned} S &= \frac{EB}{\mu_0} = \frac{(100\ \text{V/m})(3.33 \times 10^{-7}\ \text{T})}{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}} \\ &= 26.5\ \text{V}\cdot\text{A/m}^2 = 26.5\ \text{W/m}^2 \end{aligned}$$

EVALUATE: We can check our result for S by using Eq. (32.26):

$$\begin{aligned} S &= \epsilon_0 c E^2 = (8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00 \times 10^8\ \text{m/s}) \\ &\quad \times (100\ \text{N/C})^2 = 26.5\ \text{W/m}^2 \end{aligned}$$

Since $\vec{E}$ and $\vec{B}$ have the same values at all points behind the wave front, u and S likewise have the same value everywhere behind the wave front. In front of the wave front, $\vec{E} = \mathbf{0}$ and $\vec{B} = \mathbf{0}$, and so $u = 0$ and $S = 0$; where there are no fields, there is no field energy.

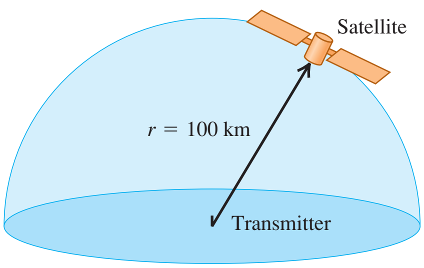
Example 32.4 Energy in a sinusoidal wave

A radio station on the earth's surface emits a sinusoidal wave with average total power 50 kW (Fig. 32.19). Assuming that the transmitter radiates equally in all directions above the ground (which is unlikely in real situations), find the electric-field and magnetic-field amplitudes E_{max} and B_{max} detected by a satellite 100 km from the antenna.

SOLUTION

IDENTIFY and SET UP: We are given the transmitter's average total power P. The intensity I is just the average power per unit area, so to find I at 100 km from the transmitter we divide P by the surface area of the hemisphere shown in Fig. 32.19. For a sinusoidal wave, I is also equal to the magnitude of the average value S_{av} of the Poynting vector, so we can use Eqs. (32.29) to find E_{max}; Eq. (32.4) then yields B_{max}.

32.19 A radio station radiates waves into the hemisphere shown.



EXECUTE: The surface area of a hemisphere of radius $r = 100\ \text{km} = 1.00 \times 10^5\ \text{m}$ is

$$A = 2\pi R^2 = 2\pi(1.00 \times 10^5\ \text{m})^2 = 6.28 \times 10^{10}\ \text{m}^2$$

All the radiated power passes through this surface, so the average power per unit area (that is, the intensity) is

$$I = \frac{P}{A} = \frac{P}{2\pi R^2} = \frac{5.00 \times 10^4\ \text{W}}{6.28 \times 10^{10}\ \text{m}^2} = 7.96 \times 10^{-7}\ \text{W/m}^2$$

From Eqs. (32.29), $I = S_{av} = E_{max}{}^2/2\mu_0 c$, so

$$\begin{aligned} E_{max} &= \sqrt{2\mu_0 c S_{av}} \\ &= \sqrt{2(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(3.00 \times 10^8\ \text{m/s})(7.96 \times 10^{-7}\ \text{W/m}^2)} \\ &= 2.45 \times 10^{-2}\ \text{V/m} \end{aligned}$$

Then from Eq. (32.4),

$$B_{max} = \frac{E_{max}}{c} = 8.17 \times 10^{-11}\ \text{T}$$

EVALUATE: Note that E_{max} is comparable to fields commonly seen in the laboratory, but B_{max} is extremely small in comparison to $\vec{B}$ fields we saw in previous chapters. For this reason, most detectors of electromagnetic radiation respond to the effect of the electric field, not the magnetic field. Loop radio antennas are an exception (see the Bridging Problem at the end of this chapter).

Electromagnetic Momentum Flow and Radiation Pressure

By using the observation that energy is required to establish electric and magnetic fields, we have shown that electromagnetic waves transport energy. It can also be shown that electromagnetic waves carry *momentum* p, with a corresponding momentum density (momentum dp per volume dV) of magnitude

$$\frac{dp}{dV} = \frac{EB}{\mu_0 c^2} = \frac{S}{c^2} \tag{32.30}$$

This momentum is a property of the field; it is not associated with the mass of a moving particle in the usual sense.

There is also a corresponding momentum flow rate. The volume dV occupied by an electromagnetic wave (speed c) that passes through an area A in time dt is $dV = Ac\,dt$. When we substitute this into Eq. (32.30) and rearrange, we find that the momentum flow rate per unit area is

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c} \quad \text{(flow rate of electromagnetic momentum)} \tag{32.31}$$

This is the momentum transferred per unit surface area per unit time. We obtain the *average* rate of momentum transfer per unit area by replacing S in Eq. (32.31) by $S_{av} = I$.

This momentum is responsible for **radiation pressure.** When an electromagnetic wave is completely absorbed by a surface, the wave's momentum is also transferred to the surface. For simplicity we'll consider a surface perpendicular to the propagation direction. Using the ideas developed in Section 8.1, we see that the rate dp/dt at which momentum is transferred to the absorbing surface equals the *force* on the surface. The average force per unit area due to the wave, or *radiation pressure* p_{rad}, is the average value of dp/dt divided by the absorbing area A. (We use the subscript "rad" to distinguish pressure from momentum, for which the symbol p is also used.) From Eq. (32.31) the radiation pressure is

$$p_{rad} = \frac{S_{av}}{c} = \frac{I}{c} \quad \text{(radiation pressure, wave totally absorbed)} \tag{32.32}$$

If the wave is totally reflected, the momentum change is twice as great, and the pressure is

$$p_{rad} = \frac{2S_{av}}{c} = \frac{2I}{c} \quad \text{(radiation pressure, wave totally reflected)} \tag{32.33}$$

For example, the value of I (or S_{av}) for direct sunlight, before it passes through the earth's atmosphere, is approximately 1.4 kW/m^2. From Eq. (32.32) the corresponding average pressure on a completely absorbing surface is

$$p_{rad} = \frac{I}{c} = \frac{1.4 \times 10^3\ \text{W/m}^2}{3.0 \times 10^8\ \text{m/s}} = 4.7 \times 10^{-6}\ \text{Pa}$$

From Eq. (32.33) the average pressure on a totally *reflecting* surface is twice this: $2I/c$ or 9.4×10^{-6} Pa. These are very small pressures, of the order of 10^{-10} atm, but they can be measured with sensitive instruments.

The radiation pressure of sunlight is much greater *inside* the sun than at the earth (see Problem 32.45). Inside stars that are much more massive and luminous than the sun, radiation pressure is so great that it substantially augments the gas pressure within the star and so helps to prevent the star from collapsing under its own gravity. In some cases the radiation pressure of stars can have dramatic effects on the material surrounding them (Fig. 32.20).

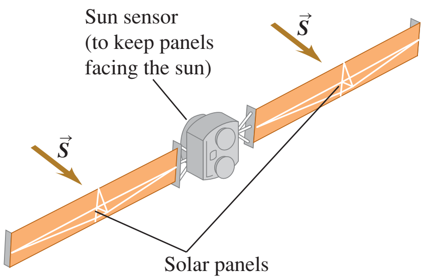
32.20 At the center of this interstellar gas cloud is a group of intensely luminous stars that exert tremendous radiation pressure on their surroundings. Aided by a "wind" of particles emanating from the stars, over the past million years the radiation pressure has carved out a bubble within the cloud 70 light-years across.

Example 32.5 Power and pressure from sunlight

An earth-orbiting satellite has solar energy–collecting panels with a total area of 4.0 m^2 (Fig. 32.21). If the sun's radiation is perpendicular to the panels and is completely absorbed, find the average solar power absorbed and the average radiation-pressure force.

SOLUTION

IDENTIFY and SET UP: This problem uses the relationships among intensity, power, radiation pressure, and force. In the above discussion we calculated the intensity I (power per unit area) of sunlight as well as the radiation pressure p_{rad} (force per unit area) of sunlight on an absorbing surface. (We calculated these values for points above the atmosphere, which is where the satellite orbits.) Multiplying each value by the area of the solar panels gives the average power absorbed and the net radiation force on the panels.



32.21 Solar panels on a satellite.

EXECUTE: The intensity I (power per unit area) is 1.4×10^3 W/m^2. Although the light from the sun is not a simple sinusoidal wave, we can still use the relationship that the average power P is the intensity I times the area A:

$$P = IA = (1.4 \times 10^3 \text{ W/m}^2)(4.0 \text{ m}^2)$$
$$= 5.6 \times 10^3 \text{ W} = 5.6 \text{ kW}$$

The radiation pressure of sunlight on an absorbing surface is $p_{rad} = 4.7 \times 10^{-6}$ Pa $= 4.7 \times 10^{-6}$ N/m^2. The total force F is the pressure p_{rad} times the area A:

$$F = p_{rad}A = (4.7 \times 10^{-6} \text{ N/m}^2)(4.0 \text{ m}^2) = 1.9 \times 10^{-5} \text{ N}$$

EVALUATE: The absorbed power is quite substantial. Part of it can be used to power the equipment aboard the satellite; the rest goes into heating the panels, either directly or due to inefficiencies in the photocells contained in the panels.

The total radiation force is comparable to the weight (on earth) of a single grain of salt. Over time, however, this small force can have a noticeable effect on the orbit of a satellite like that in Fig. 32.21, and so radiation pressure must be taken into account.

Test Your Understanding of Section 32.4 Figure 32.13 shows one wavelength of a sinusoidal electromagnetic wave at time $t = 0$. For which of the following four values of x is (a) the energy density a maximum; (b) the energy density a minimum; (c) the magnitude of the instantaneous (not average) Poynting vector a maximum; (d) the magnitude of the instantaneous (not average) Poynting vector a minimum? (i) $x = 0$; (ii) $x = \lambda/4$; (iii) $x = \lambda/2$; (iv) $x = 3\lambda/4$.

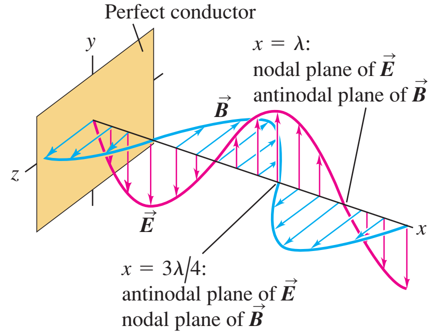
32.5 Standing Electromagnetic Waves

Electromagnetic waves can be *reflected;* the surface of a conductor (like a polished sheet of metal) or of a dielectric (such as a sheet of glass) can serve as a reflector. The superposition principle holds for electromagnetic waves just as for electric and magnetic fields. The superposition of an incident wave and a reflected wave forms a **standing wave.** The situation is analogous to standing waves on a stretched string, discussed in Section 15.7; you should review that discussion.

Suppose a sheet of a perfect conductor (zero resistivity) is placed in the yz-plane of Fig. 32.22 and a linearly polarized electromagnetic wave, traveling in the negative x-direction, strikes it. As we discussed in Section 23.4, $\vec{E}$ cannot have a component parallel to the surface of a perfect conductor. Therefore in the present situation, $\vec{E}$ must be zero everywhere in the yz-plane. The electric field of the *incident* electromagnetic wave is *not* zero at all times in the yz-plane. But this incident wave induces oscillating currents on the surface of the conductor, and these currents give rise to an additional electric field. The *net* electric field, which is the vector sum of this field and the incident $\vec{E}$, *is* zero everywhere inside and on the surface of the conductor.

The currents induced on the surface of the conductor also produce a *reflected* wave that travels out from the plane in the $+x$-direction. Suppose the incident wave is described by the wave functions of Eqs. (32.19) (a sinusoidal wave traveling in the $-x$-direction) and the reflected wave by the negative of Eqs. (32.16) (a sinusoidal wave traveling in the $+x$-direction). We take the *negative* of the

32.22 Representation of the electric and magnetic fields of a linearly polarized electromagnetic standing wave when $\omega t = 3\pi/4$ rad. In any plane perpendicular to the x-axis, E is maximum (an antinode) where B is zero (a node), and vice versa. As time elapses, the pattern does *not* move along the x-axis; instead, at every point the $\vec{E}$ and $\vec{B}$ vectors simply oscillate.



wave given by Eqs. (32.16) so that the incident and reflected electric fields cancel at $x = 0$ (the plane of the conductor, where the total electric field must be zero). The superposition principle states that the total $\vec{E}$ field at any point is the vector sum of the $\vec{E}$ fields of the incident and reflected waves, and similarly for the $\vec{B}$ field. Therefore the wave functions for the superposition of the two waves are

$$E_y(x, t) = E_{\max}[\cos(kx + \omega t) - \cos(kx - \omega t)]$$

$$B_z(x, t) = B_{\max}[-\cos(kx + \omega t) - \cos(kx - \omega t)]$$

We can expand and simplify these expressions, using the identities

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

The results are

$$E_y(x, t) = -2E_{\max} \sin kx \sin \omega t \qquad (32.34)$$

$$B_z(x, t) = -2B_{\max} \cos kx \cos \omega t \qquad (32.35)$$

Equation (32.34) is analogous to Eq. (15.28) for a stretched string. We see that at $x = 0$ the electric field $E_y(x = 0, t)$ is *always* zero; this is required by the nature of the ideal conductor, which plays the same role as a fixed point at the end of a string. Furthermore, $E_y(x, t)$ is zero at *all* times at points in those planes perpendicular to the x-axis for which $\sin kx = 0$—that is, $kx = 0, \pi, 2\pi, \ldots$. Since $k = 2\pi/\lambda$, the positions of these planes are

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \qquad (\text{nodal planes of } \vec{E}) \qquad (32.36)$$

These planes are called the **nodal planes** of the $\vec{E}$ field; they are the equivalent of the nodes, or nodal points, of a standing wave on a string. Midway between any two adjacent nodal planes is a plane on which $\sin kx = \pm 1$; on each such plane, the magnitude of $E(x, t)$ equals the maximum possible value of $2E_{\max}$ twice per oscillation cycle. These are the **antinodal planes** of $\vec{E}$, corresponding to the antinodes of waves on a string.

The total magnetic field is zero at all times at points in planes on which $\cos kx = 0$. This occurs where

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \qquad (\text{nodal planes of } \vec{B}) \qquad (32.37)$$

These are the nodal planes of the $\vec{B}$ field; there is an antinodal plane of $\vec{B}$ midway between any two adjacent nodal planes.

Figure 32.22 shows a standing-wave pattern at one instant of time. The magnetic field is *not* zero at the conducting surface $(x = 0)$. The surface currents that must be present to make $\vec{E}$ exactly zero at the surface cause magnetic fields at the surface. The nodal planes of each field are separated by one half-wavelength. The nodal planes of one field are midway between those of the other; hence the nodes of $\vec{E}$ coincide with the antinodes of $\vec{B}$, and conversely. Compare this situation to the distinction between pressure nodes and displacement nodes in Section 16.4.

The total electric field is a *sine* function of t, and the total magnetic field is a *cosine* function of t. The sinusoidal variations of the two fields are therefore 90° out of phase at each point. At times when $\sin \omega t = 0$, the electric field is zero *everywhere,* and the magnetic field is maximum. When $\cos \omega t = 0$, the magnetic field is zero everywhere, and the electric field is maximum. This is in contrast to a wave traveling in one direction, as described by Eqs. (32.16) or (32.19) separately, in which the sinusoidal variations of $\vec{E}$ and $\vec{B}$ at any particular point are *in phase.* You can show that Eqs. (32.34) and (32.35) satisfy the wave equation, Eq. (32.15). You can also show that they satisfy Eqs. (32.12) and (32.14), the equivalents of Faraday's and Ampere's laws (see Exercise 32.36).

Standing Waves in a Cavity

Let's now insert a second conducting plane, parallel to the first and a distance L from it, along the $+x$-axis. The cavity between the two planes is analogous to a stretched string held at the points $x = 0$ and $x = L$. Both conducting planes must be nodal planes for $\vec{E}$; a standing wave can exist only when the second plane is placed at one of the positions where $E(x, t) = 0$, so L must be an integer multiple of $\lambda/2$. The wavelengths that satisfy this condition are

$$\lambda_n = \frac{2L}{n} \qquad (n = 1, 2, 3, \ldots) \tag{32.38}$$

The corresponding frequencies are

$$f_n = \frac{c}{\lambda_n} = n\frac{c}{2L} \qquad (n = 1, 2, 3, \ldots) \tag{32.39}$$

Thus there is a set of *normal modes,* each with a characteristic frequency, wave shape, and node pattern (Fig. 32.23). By measuring the node positions, we can measure the wavelength. If the frequency is known, the wave speed can be determined. This technique was first used by Hertz in the 1880s in his pioneering investigations of electromagnetic waves.

Conducting surfaces are not the only reflectors of electromagnetic waves. Reflections also occur at an interface between two insulating materials with different dielectric or magnetic properties. The mechanical analog is a junction of two strings with equal tension but different linear mass density. In general, a wave incident on such a boundary surface is partly transmitted into the second material and partly reflected back into the first. For example, light is transmitted through a glass window, but its surfaces also reflect light.

32.23 A typical microwave oven sets up a standing electromagnetic wave with $\lambda = 12.2$ cm, a wavelength that is strongly absorbed by the water in food. Because the wave has nodes spaced $\lambda/2 = 6.1$ cm apart, the food must be rotated while cooking. Otherwise, the portion that lies at a node—where the electric-field amplitude is zero—will remain cold.

Example 32.6 Intensity in a standing wave

Calculate the intensity of the standing wave represented by Eqs. (32.34) and (32.35).

SOLUTION

IDENTIFY and SET UP: The intensity I of the wave is the time-averaged value S_{av} of the magnitude of the Poynting vector $\vec{S}$. To find S_{av}, we first use Eq. (32.28) to find the instantaneous value of $\vec{S}$ and then average it over a whole number of cycles of the wave.

EXECUTE: Using the wave functions of Eqs. (32.34) and (32.35) in Eq. (32.28) for the Poynting vector $\vec{S}$, we find

$$\begin{aligned}\vec{S}(x, t) &= \frac{1}{\mu_0}\vec{E}(x, t) \times \vec{B}(x, t)\\ &= \frac{1}{\mu_0}[-2\hat{\jmath}E_{max}\sin kx\cos\omega t] \times [-2\hat{k}B_{max}\cos kx\sin\omega t]\\ &= \hat{\imath}\frac{E_{max}B_{max}}{\mu_0}(2\sin kx\cos kx)(2\sin\omega t\cos\omega t)\\ &= \hat{\imath}S_x(x, t)\end{aligned}$$

Using the identity $\sin 2A = 2\sin A\cos A$, we can rewrite $S_x(x, t)$ as

$$S_x(x, t) = \frac{E_{max}B_{max}\sin 2kx\sin 2\omega t}{\mu_0}$$

The average value of a sine function over any whole number of cycles is zero. Thus *the time average of $\vec{S}$ at any point is zero;* $I = S_{av} = 0$.

EVALUATE: This result is what we should expect. The standing wave is a superposition of two waves with the same frequency and amplitude, traveling in opposite directions. All the energy transferred by one wave is cancelled by an equal amount transferred in the opposite direction by the other wave. When we use electromagnetic waves to transmit power, it is important to avoid reflections that give rise to standing waves.

Example 32.7 Standing waves in a cavity

Electromagnetic standing waves are set up in a cavity with two parallel, highly conducting walls 1.50 cm apart. (a) Calculate the longest wavelength λ and lowest frequency f of these standing waves. (b) For a standing wave of this wavelength, where in the cavity does $\vec{E}$ have maximum magnitude? Where is $\vec{E}$ zero? Where does $\vec{B}$ have maximum magnitude? Where is $\vec{B}$ zero?

Continued

SOLUTION

IDENTIFY and SET UP: Only certain normal modes are possible for electromagnetic waves in a cavity, just as only certain normal modes are possible for standing waves on a string. The longest possible wavelength and lowest possible frequency correspond to the $n = 1$ mode in Eqs. (32.38) and (32.39); we use these to find λ and f. Equations (32.36) and (32.37) then give the locations of the nodal planes of $\vec{E}$ and $\vec{B}$. The antinodal planes of each field are midway between adjacent nodal planes.

EXECUTE: (a) From Eqs. (32.38) and (32.39), the $n = 1$ wavelength and frequency are

$$\lambda_1 = 2L = 2(1.50\text{ cm}) = 3.00\text{ cm}$$

$$f_1 = \frac{c}{2L} = \frac{3.00 \times 10^8\text{ m/s}}{2(1.50 \times 10^{-2}\text{ m})} = 1.00 \times 10^{10}\text{ Hz} = 10\text{ GHz}$$

(b) With $n = 1$ there is a single half-wavelength between the walls. The electric field has nodal planes ($\vec{E} = \mathbf{0}$) at the walls and an antinodal plane (where $\vec{E}$ has its maximum magnitude) midway between them. The magnetic field has *antinodal* planes at the walls and a nodal plane midway between them.

EVALUATE: One application of such standing waves is to produce an oscillating $\vec{E}$ field of definite frequency, which is used to probe the behavior of a small sample of material placed in the cavity. To subject the sample to the strongest possible field, it should be placed near the center of the cavity, at the antinode of $\vec{E}$.

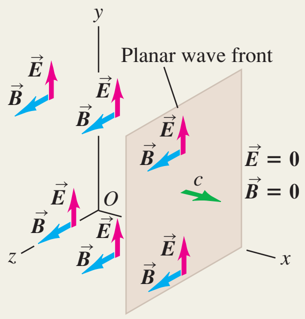
Test Your Understanding of Section 32.5 In the standing wave described in Example 32.7, is there any point in the cavity where the energy density is zero at all times? If so, where? If not, why not?

Maxwell's equations and electromagnetic waves: Maxwell's equations predict the existence of electromagnetic waves that propagate in vacuum at the speed of light c. The electromagnetic spectrum covers frequencies from at least 1 to 10^{24} Hz and a correspondingly broad range of wavelengths. Visible light, with wavelengths from 380 to 750 nm, is only a very small part of this spectrum. In a plane wave, $\vec{E}$ and $\vec{B}$ are uniform over any plane perpendicular to the propagation direction. Faraday's law and Ampere's law both give relationships between the magnitudes of $\vec{E}$ and $\vec{B}$; requiring both of these relationships to be satisfied gives an expression for c in terms of ϵ_0 and μ_0. Electromagnetic waves are transverse; the $\vec{E}$ and $\vec{B}$ fields are perpendicular to the direction of propagation and to each other. The direction of propagation is the direction of $\vec{E} \times \vec{B}$.

$$E = cB \quad (32.4)$$

$$B = \epsilon_0\mu_0 cE \quad (32.8)$$

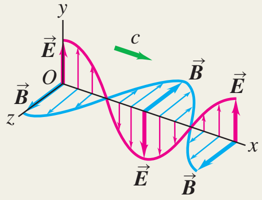
$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \quad (32.9)$$



Sinusoidal electromagnetic waves: Equations (32.17) and (32.18) describe a sinusoidal plane electromagnetic wave traveling in vacuum in the $+x$-direction. If the wave is propagating in the $-x$-direction, replace $kx - \omega t$ by $kx + \omega t$. (See Example 32.1.)

$$\vec{E}(x,t) = \hat{\jmath}E_{\max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{\max}\cos(kx - \omega t) \quad (32.17)$$

$$E_{\max} = cB_{\max} \quad (32.18)$$



Electromagnetic waves in matter: When an electromagnetic wave travels through a dielectric, the wave speed v is less than the speed of light in vacuum c. (See Example 32.2.)

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}}\frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{KK_m}} \quad (32.21)$$

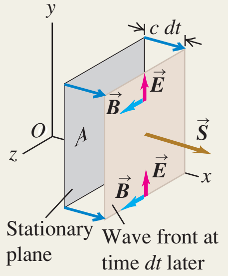
Energy and momentum in electromagnetic waves: The energy flow rate (power per unit area) in an electromagnetic wave in vacuum is given by the Poynting vector $\vec{S}$. The magnitude of the time-averaged value of the Poynting vector is called the intensity I of the wave. Electromagnetic waves also carry momentum. When an electromagnetic wave strikes a surface, it exerts a radiation pressure p_{rad}. If the surface is perpendicular to the wave propagation direction and is totally absorbing, $p_{\text{rad}} = I/c$; if the surface is a perfect reflector, $p_{\text{rad}} = 2I/c$. (See Examples 32.3–32.5.)

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \quad (32.28)$$

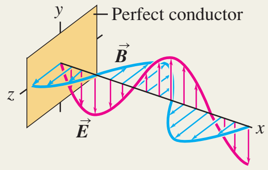
$$I = S_{\text{av}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}{}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_{\max}{}^2 = \frac{1}{2}\epsilon_0 c E_{\max}{}^2 \quad (32.29)$$

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c} \quad (32.31)$$

(flow rate of electromagnetic momentum)



Standing electromagnetic waves: If a perfect reflecting surface is placed at $x = 0$, the incident and reflected waves form a standing wave. Nodal planes for $\vec{E}$ occur at $kx = 0, \pi, 2\pi, \ldots,$ and nodal planes for $\vec{B}$ at $kx = \pi/2, 3\pi/2, 5\pi/2, \ldots$. At each point, the sinusoidal variations of $\vec{E}$ and $\vec{B}$ with time are 90° out of phase. (See Examples 32.6 and 32.7.)



BRIDGING PROBLEM Detecting Electromagnetic Waves

A circular loop of wire can be used as a radio antenna. If an 18.0-cm-diameter antenna is located 2.50 km from a 95.0-MHz source with a total power of 55.0 kW, what is the maximum emf induced in the loop? Assume that the plane of the antenna loop is perpendicular to the direction of the radiation's magnetic field and that the source radiates uniformly in all directions.

SOLUTION GUIDE

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IDENTIFY and SET UP:

1. The electromagnetic wave has an oscillating magnetic field. This causes a magnetic flux through the loop antenna that varies sinusoidally with time. By Faraday's law, this produces an emf equal in magnitude to the rate of change of the flux. The target variable is the magnitude of this emf.
2. Select the equations that you will need to find (i) the intensity of the wave at the position of the loop, a distance $r = 2.50$ km from the source of power $P = 55.0$ kW; (ii) the amplitude of the sinusoidally varying magnetic field at that position; (iii) the magnetic flux through the loop as a function of time; and (iv) the emf produced by the flux.

EXECUTE

3. Find the wave intensity at the position of the loop.
4. Use your result from step 3 to write expressions for the time-dependent magnetic field at this position and the time-dependent magnetic flux through the loop.
5. Use the results of step 4 to find the time-dependent induced emf in the loop. The amplitude of this emf is your target variable.

EVALUATE

6. Is the induced emf large enough to detect? (If it is, a receiver connected to this antenna will be able to pick up signals from the source.)

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q32.1 By measuring the electric and magnetic fields at a point in space where there is an electromagnetic wave, can you determine the direction from which the wave came? Explain.

Q32.2 According to Ampere's law, is it possible to have both a conduction current and a displacement current at the same time? Is it possible for the effects of the two kinds of current to cancel each other exactly so that *no* magnetic field is produced? Explain.

Q32.3 Give several examples of electromagnetic waves that are encountered in everyday life. How are they all alike? How do they differ?

Q32.4 Sometimes neon signs located near a powerful radio station are seen to glow faintly at night, even though they are not turned on. What is happening?

Q32.5 Is polarization a property of all electromagnetic waves, or is it unique to visible light? Can sound waves be polarized? What fundamental distinction in wave properties is involved? Explain.

Q32.6 Suppose that a positive point charge q is initially at rest on the x-axis, in the path of the electromagnetic plane wave described in Section 32.2. Will the charge move after the wave front reaches it? If not, why not? If the charge does move, describe its motion qualitatively. (Remember that $\vec{E}$ and $\vec{B}$ have the same value at all points behind the wave front.)

Q32.7 The light beam from a searchlight may have an electric-field magnitude of 1000 V/m, corresponding to a potential difference of 1500 V between the head and feet of a 1.5-m-tall person on whom the light shines. Does this cause the person to feel a strong electric shock? Why or why not?

Q32.8 For a certain sinusoidal wave of intensity I, the amplitude of the magnetic field is B. What would be the amplitude (in terms of B) in a similar wave of twice the intensity?

Q32.9 The magnetic-field amplitude of the electromagnetic wave from the laser described in Example 32.1 (Section 32.3) is about 100 times greater than the earth's magnetic field. If you illuminate a compass with the light from this laser, would you expect the compass to deflect? Why or why not?

Q32.10 Most automobiles have vertical antennas for receiving radio broadcasts. Explain what this tells you about the direction of polarization of $\vec{E}$ in the radio waves used in broadcasting.

Q32.11 If a light beam carries momentum, should a person holding a flashlight feel a recoil analogous to the recoil of a rifle when it is fired? Why is this recoil not actually observed?

Q32.12 A light source radiates a sinusoidal electromagnetic wave uniformly in all directions. This wave exerts an average pressure p on a perfectly reflecting surface a distance R away from it. What average pressure (in terms of p) would this wave exert on a perfectly absorbing surface that was twice as far from the source?

Q32.13 Does an electromagnetic *standing* wave have energy? Does it have momentum? Are your answers to these questions the same as for a *traveling* wave? Why or why not?

Q32.14 When driving on the upper level of the Bay Bridge, westbound from Oakland to San Francisco, you can easily pick up a number of radio stations on your car radio. But when driving eastbound on the lower level of the bridge, which has steel girders on either side to support the upper level, the radio reception is much worse. Why is there a difference?

EXERCISES

Section 32.2 Plane Electromagnetic Waves and the Speed of Light

32.1 • (a) How much time does it take light to travel from the moon to the earth, a distance of 384,000 km? (b) Light from the star Sirius takes 8.61 years to reach the earth. What is the distance from earth to Sirius in kilometers?

32.2 • Consider each of the electric- and magnetic-field orientations given next. In each case, what is the direction of propagation of the wave? (a) $\vec{E}$ in the $+x$-direction, $\vec{B}$ in the $+y$-direction; (b) $\vec{E}$ in the $-y$-direction, $\vec{B}$ in the $+x$-direction; (c) $\vec{E}$ in the $+z$-direction, $\vec{B}$ in the $-x$-direction; (d) $\vec{E}$ in the $+y$-direction, $\vec{B}$ in the $-z$-direction.

32.3 • A sinusoidal electromagnetic wave is propagating in vacuum in the $+z$-direction. If at a particular instant and at a certain point in space the electric field is in the $+x$-direction and has magnitude 4.00 V/m, what are the magnitude and direction of the magnetic field of the wave at this same point in space and instant in time?

32.4 • Consider each of the following electric- and magnetic-field orientations. In each case, what is the direction of propagation of the wave? (a) $\vec{E} = E\hat{\imath}$, $\vec{B} = -B\hat{\jmath}$; (b) $\vec{E} = E\hat{\jmath}$, $\vec{B} = B\hat{\imath}$; (c) $\vec{E} = -E\hat{k}$, $\vec{B} = -B\hat{\imath}$; (d) $\vec{E} = E\hat{\imath}$, $\vec{B} = -B\hat{k}$.

Section 32.3 Sinusoidal Electromagnetic Waves

32.5 • BIO **Medical X rays.** Medical x rays are taken with electromagnetic waves having a wavelength of around 0.10 nm. What are the frequency, period, and wave number of such waves?

32.6 • BIO **Ultraviolet Radiation.** There are two categories of ultraviolet light. Ultraviolet A (UVA) has a wavelength ranging from 320 nm to 400 nm. It is not harmful to the skin and is necessary for the production of vitamin D. UVB, with a wavelength between 280 nm and 320 nm, is much more dangerous because it causes skin cancer. (a) Find the frequency ranges of UVA and UVB. (b) What are the ranges of the wave numbers for UVA and UVB?

32.7 • A sinusoidal electromagnetic wave having a magnetic field of amplitude 1.25 μT and a wavelength of 432 nm is traveling in the $+x$-direction through empty space. (a) What is the frequency of this wave? (b) What is the amplitude of the associated electric field? (c) Write the equations for the electric and magnetic fields as functions of x and t in the form of Eqs. (32.17).

32.8 • An electromagnetic wave of wavelength 435 nm is traveling in vacuum in the $-z$-direction. The electric field has amplitude 2.70×10^{-3} V/m and is parallel to the x-axis. What are (a) the frequency and (b) the magnetic-field amplitude? (c) Write the vector equations for $\vec{E}(z, t)$ and $\vec{B}(z, t)$.

32.9 • Consider electromagnetic waves propagating in air. (a) Determine the frequency of a wave with a wavelength of (i) 5.0 km, (ii) 5.0 μm, (iii) 5.0 nm. (b) What is the wavelength (in meters and nanometers) of (i) gamma rays of frequency 6.50×10^{21} Hz and (ii) an AM station radio wave of frequency 590 kHz?

32.10 • The electric field of a sinusoidal electromagnetic wave obeys the equation $E = (375\ \text{V/m})\cos[(1.99 \times 10^{7}\ \text{rad/m})x + (5.97 \times 10^{15}\ \text{rad/s})t]$. (a) What are the amplitudes of the electric and magnetic fields of this wave? (b) What are the frequency, wavelength, and period of the wave? Is this light visible to humans? (c) What is the speed of the wave?

32.11 • An electromagnetic wave has an electric field given by $\vec{E}(y, t) = (3.10 \times 10^{5}\ \text{V/m})\hat{k}\cos[ky - (12.65 \times 10^{12}\ \text{rad/s})t]$. (a) In which direction is the wave traveling? (b) What is the wavelength of the wave? (c) Write the vector equation for $\vec{B}(y, t)$.

32.12 • An electromagnetic wave has a magnetic field given by $\vec{B}(x, t) = -(8.25 \times 10^{-9}\ \text{T})\hat{\jmath}\cos[(1.38 \times 10^{4}\ \text{rad/m})x + \omega t]$. (a) In which direction is the wave traveling? (b) What is the frequency f of the wave? (c) Write the vector equation for $\vec{E}(x, t)$.

32.13 • Radio station WCCO in Minneapolis broadcasts at a frequency of 830 kHz. At a point some distance from the transmitter, the magnetic-field amplitude of the electromagnetic wave from WCCO is 4.82×10^{-11} T. Calculate (a) the wavelength; (b) the wave number; (c) the angular frequency; (d) the electric-field amplitude.

32.14 • An electromagnetic wave with frequency 65.0 Hz travels in an insulating magnetic material that has dielectric constant 3.64 and relative permeability 5.18 at this frequency. The electric field has amplitude 7.20×10^{-3} V/m. (a) What is the speed of propagation of the wave? (b) What is the wavelength of the wave? (c) What is the amplitude of the magnetic field?

32.15 • An electromagnetic wave with frequency 5.70×10^{14} Hz propagates with a speed of 2.17×10^{8} m/s in a certain piece of glass. Find (a) the wavelength of the wave in the glass; (b) the wavelength of a wave of the same frequency propagating in air; (c) the index of refraction n of the glass for an electromagnetic wave with this frequency; (d) the dielectric constant for glass at this frequency, assuming that the relative permeability is unity.

Section 32.4 Energy and Momentum in Electromagnetic Waves

32.16 • BIO **High-Energy Cancer Treatment.** Scientists are working on a new technique to kill cancer cells by zapping them with ultrahigh-energy (in the range of 10^{12} W) pulses of light that last for an extremely short time (a few nanoseconds). These short pulses scramble the interior of a cell without causing it to explode, as long pulses would do. We can model a typical such cell as a disk 5.0 μm in diameter, with the pulse lasting for 4.0 ns with an average power of 2.0×10^{12} W. We shall assume that the energy is spread uniformly over the faces of 100 cells for each pulse. (a) How much energy is given to the cell during this pulse? (b) What is the intensity (in W/m^2) delivered to the cell? (c) What are the maximum values of the electric and magnetic fields in the pulse?

32.17 •• **Fields from a Light Bulb.** We can reasonably model a 75-W incandescent light bulb as a sphere 6.0 cm in diameter. Typically, only about 5% of the energy goes to visible light; the rest goes largely to nonvisible infrared radiation. (a) What is the visible-light intensity (in W/m^2) at the surface of the bulb? (b) What are the amplitudes of the electric and magnetic fields at this surface, for a sinusoidal wave with this intensity?

32.18 •• A sinusoidal electromagnetic wave from a radio station passes perpendicularly through an open window that has area $0.500\ \text{m}^2$. At the window, the electric field of the wave has rms value 0.0200 V/m. How much energy does this wave carry through the window during a 30.0-s commercial?

32.19 •• **Testing a Space Radio Transmitter.** You are a NASA mission specialist on your first flight aboard the space shuttle. Thanks to your extensive training in physics, you have been assigned to evaluate the performance of a new radio transmitter on board the International Space Station (ISS). Perched on the shuttle's movable arm, you aim a sensitive detector at the ISS, which is 2.5 km away. You find that the electric-field amplitude of the radio waves coming from the ISS transmitter is 0.090 V/m and that the frequency of the waves is 244 MHz. Find the following: (a) the intensity of the radio wave at your location; (b) the magnetic-field amplitude of the wave at your location; (c) the total power output of the ISS radio transmitter. (d) What assumptions, if any, did you make in your calculations?

32.20 • The intensity of a cylindrical laser beam is 0.800 $\mathrm{W/m^2}$. The cross-sectional area of the beam is $3.0 \times 10^{-4}\ \mathrm{m^2}$ and the intensity is uniform across the cross section of the beam. (a) What is the average power output of the laser? (b) What is the rms value of the electric field in the beam?

32.21 • A space probe 2.0×10^{10} m from a star measures the total intensity of electromagnetic radiation from the star to be $5.0 \times 10^{3}\ \mathrm{W/m^2}$. If the star radiates uniformly in all directions, what is its total average power output?

32.22 • A sinusoidal electromagnetic wave emitted by a cellular phone has a wavelength of 35.4 cm and an electric-field amplitude of 5.40×10^{-2} V/m at a distance of 250 m from the phone. Calculate (a) the frequency of the wave; (b) the magnetic-field amplitude; (c) the intensity of the wave.

32.23 • A monochromatic light source with power output 60.0 W radiates light of wavelength 700 nm uniformly in all directions. Calculate $E_{\max}$ and $B_{\max}$ for the 700-nm light at a distance of 5.00 m from the source.

32.24 • For the electromagnetic wave represented by Eqs. (32.19), show that the Poynting vector (a) is in the same direction as the propagation of the wave and (b) has average magnitude given by Eqs. (32.29).

32.25 •• An intense light source radiates uniformly in all directions. At a distance of 5.0 m from the source, the radiation pressure on a perfectly absorbing surface is 9.0×10^{-6} Pa. What is the total average power output of the source?

32.26 • **Television Broadcasting.** Public television station KQED in San Francisco broadcasts a sinusoidal radio signal at a power of 316 kW. Assume that the wave spreads out uniformly into a hemisphere above the ground. At a home 5.00 km away from the antenna, (a) what average pressure does this wave exert on a totally reflecting surface, (b) what are the amplitudes of the electric and magnetic fields of the wave, and (c) what is the average density of the energy this wave carries? (d) For the energy density in part (c), what percentage is due to the electric field and what percentage is due to the magnetic field?

32.27 •• BIO **Laser Safety.** If the eye receives an average intensity greater than $1.0 \times 10^{2}\ \mathrm{W/m^2}$, damage to the retina can occur. This quantity is called the *damage threshold* of the retina. (a) What is the largest average power (in mW) that a laser beam 1.5 mm in diameter can have and still be considered safe to view head-on? (b) What are the maximum values of the electric and magnetic fields for the beam in part (a)? (c) How much energy would the beam in part (a) deliver per second to the retina? (d) Express the damage threshold in $\mathrm{W/cm^2}$.

32.28 • In the 25-ft Space Simulator facility at NASA's Jet Propulsion Laboratory, a bank of overhead arc lamps can produce light of intensity 2500 $\mathrm{W/m^2}$ at the floor of the facility. (This simulates the intensity of sunlight near the planet Venus.) Find the average radiation pressure (in pascals and in atmospheres) on (a) a totally absorbing section of the floor and (b) a totally reflecting section of the floor. (c) Find the average momentum density (momentum per unit volume) in the light at the floor.

32.29 • **Laboratory Lasers.** He–Ne lasers are often used in physics demonstrations. They produce light of wavelength 633 nm and a power of 0.500 mW spread over a cylindrical beam 1.00 mm in diameter (although these quantities can vary). (a) What is the intensity of this laser beam? (b) What are the maximum values of the electric and magnetic fields? (c) What is the average energy density in the laser beam?

32.30 •• **Solar Sail 1.** During 2004, Japanese scientists successfully tested two solar sails. One had a somewhat complicated shape that we shall model as a disk 9.0 m in diameter and 7.5 μm thick. The intensity of solar energy at that location was about 1400 $\mathrm{W/m^2}$. (a) What force did the sun's light exert on this sail, assuming that it struck perpendicular to the sail and that the sail was perfectly reflecting? (b) If the sail was made of magnesium, of density 1.74 $\mathrm{g/cm^3}$, what acceleration would the sun's radiation give to the sail? (c) Does the acceleration seem large enough to be feasible for space flight? In what ways could the sail be modified to increase its acceleration?

Section 32.5 Standing Electromagnetic Waves

32.31 • **Microwave Oven.** The microwaves in a certain microwave oven have a wavelength of 12.2 cm. (a) How wide must this oven be so that it will contain five antinodal planes of the electric field along its width in the standing-wave pattern? (b) What is the frequency of these microwaves? (c) Suppose a manufacturing error occurred and the oven was made 5.0 cm longer than specified in part (a). In this case, what would have to be the frequency of the microwaves for there still to be five antinodal planes of the electric field along the width of the oven?

32.32 • An electromagnetic standing wave in air of frequency 750 MHz is set up between two conducting planes 80.0 cm apart. At which positions between the planes could a point charge be placed at rest so that it would *remain* at rest? Explain.

32.33 • A standing electromagnetic wave in a certain material has frequency 2.20×10^{10} Hz. The nodal planes of $\vec{B}$ are 3.55 mm apart. Find (a) the wavelength of the wave in this material; (b) the distance between adjacent nodal planes of the $\vec{E}$ field; (c) the speed of propagation of the wave.

32.34 • An electromagnetic standing wave in air has frequency 75.0 MHz. (a) What is the distance between nodal planes of the $\vec{E}$ field? (b) What is the distance between a nodal plane of $\vec{E}$ and the closest nodal plane of $\vec{B}$?

32.35 • An electromagnetic standing wave in a certain material has frequency 1.20×10^{10} Hz and speed of propagation 2.10×10^{8} m/s. (a) What is the distance between a nodal plane of $\vec{B}$ and the closest antinodal plane of $\vec{B}$? (b) What is the distance between an antinodal plane of $\vec{E}$ and the closest antinodal plane of $\vec{B}$? (c) What is the distance between a nodal plane of $\vec{E}$ and the closest nodal plane of $\vec{B}$?

32.36 • CALC Show that the electric and magnetic fields for standing waves given by Eqs. (32.34) and (32.35) (a) satisfy the wave equation, Eq. (32.15), and (b) satisfy Eqs. (32.12) and (32.14).

PROBLEMS

32.37 •• BIO **Laser Surgery.** Very short pulses of high-intensity laser beams are used to repair detached portions of the retina of the eye. The brief pulses of energy absorbed by the retina weld the detached portions back into place. In one such procedure, a laser beam has a wavelength of 810 nm and delivers 250 mW of power spread over a circular spot 510 μm in diameter. The vitreous humor (the transparent fluid that fills most of the eye) has an index of refraction of 1.34. (a) If the laser pulses are each 1.50 ms long, how much energy is delivered to the retina with each pulse? (b) What average pressure does the pulse of the laser beam exert on the retina as it is fully absorbed by the circular spot? (c) What are the wavelength and frequency of the laser light inside the vitreous humor of the eye? (d) What are the maximum values of the electric and magnetic fields in the laser beam?

32.38 •• CALC Consider a sinusoidal electromagnetic wave with fields $\vec{E} = E_{\max}\hat{\jmath}\cos(kx - \omega t)$ and $\vec{B} = B_{\max}\hat{k}\cos(kx - \omega t + \phi)$, with $-\pi \le \phi \le \pi$. Show that if $\vec{E}$ and $\vec{B}$ are to satisfy Eqs. (32.12) and (32.14), then $E_{max} = cB_{max}$ and $\phi = 0$. (The result $\phi = 0$ means the $\vec{E}$ and $\vec{B}$ fields oscillate in phase.)

32.39 •• You want to support a sheet of fireproof paper horizontally, using only a vertical upward beam of light spread uniformly over the sheet. There is no other light on this paper. The sheet measures 22.0 cm by 28.0 cm and has a mass of 1.50 g. (a) If the paper is black and hence absorbs all the light that hits it, what must be the intensity of the light beam? (b) For the light in part (a), what are the amplitudes of its electric and magnetic fields? (c) If the paper is white and hence reflects all the light that hits it, what intensity of light beam is needed to support it? (d) To see if it is physically reasonable to expect to support a sheet of paper this way, calculate the intensity in a typical 0.500-mW laser beam that is 1.00 mm in diameter, and compare this value with your answer in part (a).

32.40 •• For a sinusoidal electromagnetic wave in vacuum, such as that described by Eq. (32.16), show that the *average* energy density in the electric field is the same as that in the magnetic field.

32.41 • A satellite 575 km above the earth's surface transmits sinusoidal electromagnetic waves of frequency 92.4 MHz uniformly in all directions, with a power of 25.0 kW. (a) What is the intensity of these waves as they reach a receiver at the surface of the earth directly below the satellite? (b) What are the amplitudes of the electric and magnetic fields at the receiver? (c) If the receiver has a totally absorbing panel measuring 15.0 cm by 40.0 cm oriented with its plane perpendicular to the direction the waves travel, what average force do these waves exert on the panel? Is this force large enough to cause significant effects?

32.42 • A plane sinusoidal electromagnetic wave in air has a wavelength of 3.84 cm and an $\vec{E}$-field amplitude of 1.35 V/m. (a) What is the frequency? (b) What is the $\vec{B}$-field amplitude? (c) What is the intensity? (d) What average force does this radiation exert on a totally absorbing surface with area 0.240 m^2 perpendicular to the direction of propagation?

32.43 • A small helium–neon laser emits red visible light with a power of 4.60 mW in a beam that has a diameter of 2.50 mm. (a) What are the amplitudes of the electric and magnetic fields of the light? (b) What are the average energy densities associated with the electric field and with the magnetic field? (c) What is the total energy contained in a 1.00-m length of the beam?

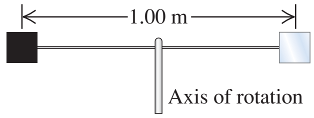
32.44 •• The electric-field component of a sinusoidal electromagnetic wave traveling through a plastic cylinder is given by the equation $E = (5.35\ \text{V/m})\cos[(1.39 \times 10^7\ \text{rad/m})x - (3.02 \times 10^{15}\ \text{rad/s})t]$. (a) Find the frequency, wavelength, and speed of this wave in the plastic. (b) What is the index of refraction of the plastic? (c) Assuming that the amplitude of the electric field does not change, write a comparable equation for the electric field if the light is traveling in air instead of in plastic.

32.45 • The sun emits energy in the form of electromagnetic waves at a rate of 3.9×10^{26} W. This energy is produced by nuclear reactions deep in the sun's interior. (a) Find the intensity of electromagnetic radiation and the radiation pressure on an absorbing object at the surface of the sun (radius $r = R = 6.96 \times 10^5$ km) and at $r = R/2$, in the sun's interior. Ignore any scattering of the waves as they move radially outward from the center of the sun. Compare to the values given in Section 32.4 for sunlight just before it enters the earth's atmosphere. (b) The gas pressure at the sun's surface is about 1.0×10^4 Pa; at $r = R/2$, the gas pressure is calculated from solar models to be about 4.7×10^{13} Pa. Comparing with your results in part (a), would you expect that radiation pressure is an important factor in determining the structure of the sun? Why or why not?

32.46 •• A source of sinusoidal electromagnetic waves radiates uniformly in all directions. At 10.0 m from this source, the amplitude of the electric field is measured to be 1.50 N/C. What is the electric-field amplitude at a distance of 20.0 cm from the source?

32.47 •• **CP** Two square reflectors, each 1.50 cm on a side and of mass 4.00 g, are located at opposite ends of a thin, extremely light, 1.00-m rod that can rotate without friction and in vacuum about an axle perpendicular to it through its center (Fig. P32.47). These reflectors are small enough to be treated as point masses in moment-of-inertia calculations. Both reflectors are illuminated on one face by a sinusoidal light wave having an electric field of amplitude 1.25 N/C that falls uniformly on both surfaces and always strikes them perpendicular to the plane of their surfaces. One reflector is covered with a perfectly absorbing coating, and the other is covered with a perfectly reflecting coating. What is the angular acceleration of this device?

Figure **P32.47**



32.48 •• **CP** A circular loop of wire has radius 7.50 cm. A sinusoidal electromagnetic plane wave traveling in air passes through the loop, with the direction of the magnetic field of the wave perpendicular to the plane of the loop. The intensity of the wave at the location of the loop is 0.0195 W/m^2, and the wavelength of the wave is 6.90 m. What is the maximum emf induced in the loop?

32.49 • **CALC** **CP** A cylindrical conductor with a circular cross section has a radius a and a resistivity ρ and carries a constant current I. (a) What are the magnitude and direction of the electric-field vector $\vec{E}$ at a point just inside the wire at a distance a from the axis? (b) What are the magnitude and direction of the magnetic-field vector $\vec{B}$ at the same point? (c) What are the magnitude and direction of the Poynting vector $\vec{S}$ at the same point? (The direction of $\vec{S}$ is the direction in which electromagnetic energy flows into or out of the conductor.) (d) Use the result in part (c) to find the rate of flow of energy into the volume occupied by a length l of the conductor. (*Hint:* Integrate $\vec{S}$ over the surface of this volume.) Compare your result to the rate of generation of thermal energy in the same volume. Discuss why the energy dissipated in a current-carrying conductor, due to its resistance, can be thought of as entering through the cylindrical sides of the conductor.

32.50 • In a certain experiment, a radio transmitter emits sinusoidal electromagnetic waves of frequency 110.0 MHz in opposite directions inside a narrow cavity with reflectors at both ends, causing a standing-wave pattern to occur. (a) How far apart are the nodal planes of the magnetic field? (b) If the standing-wave pattern is determined to be in its eighth harmonic, how long is the cavity?

32.51 •• **CP** **Flashlight to the Rescue.** You are the sole crew member of the interplanetary spaceship *T:1339 Vorga,* which makes regular cargo runs between the earth and the mining colonies in the asteroid belt. You are working outside the ship one day while at a distance of 2.0 AU from the sun. [1 AU (astronomical unit) is the average distance from the earth to the sun, 149,600,000 km.] Unfortunately, you lose contact with the ship's hull and begin to drift away into space. You use your spacesuit's rockets to try to push yourself back toward the ship, but they run out of fuel and stop working before you can return to the ship. You find yourself in an awkward position, floating 16.0 m from the spaceship with zero velocity relative to it. Fortunately, you are carrying a 200-W flashlight. You turn on the flashlight and use its beam as a "light rocket" to push yourself back toward the ship. (a) If you, your spacesuit, and the flashlight have a combined mass of 150 kg, how long will it take you to get back to the ship? (b) Is there another way you could use the flashlight to accomplish the same job of returning you to the ship?

32.52 • The 19th-century inventor Nikola Tesla proposed to transmit electric power via sinusoidal electromagnetic waves. Suppose power is to be transmitted in a beam of cross-sectional area 100 m^2. What electric- and magnetic-field amplitudes are required to transmit an amount of power comparable to that handled by modern transmission lines (that carry voltages and currents of the order of 500 kV and 1000 A)?

32.53 •• **CP** **Global Positioning System (GPS).** The GPS network consists of 24 satellites, each of which makes two orbits around the earth per day. Each satellite transmits a 50.0-W (or even less) sinusoidal electromagnetic signal at two frequencies, one of which is 1575.42 MHz. Assume that a satellite transmits half of its power at each frequency and that the waves travel uniformly in a downward hemisphere. (a) What average intensity does a GPS receiver on the ground, directly below the satellite, receive? (*Hint:* First use Newton's laws to find the altitude of the satellite.) (b) What are the amplitudes of the electric and magnetic fields at the GPS receiver in part (a), and how long does it take the signal to reach the receiver? (c) If the receiver is a square panel 1.50 cm on a side that absorbs all of the beam, what average pressure does the signal exert on it? (d) What wavelength must the receiver be tuned to?

32.54 •• **CP** **Solar Sail 2.** NASA is giving serious consideration to the concept of *solar sailing.* A solar sailcraft uses a large, low-mass sail and the energy and momentum of sunlight for propulsion. (a) Should the sail be absorbing or reflective? Why? (b) The total power output of the sun is 3.9×10^{26} W. How large a sail is necessary to propel a 10,000-kg spacecraft against the gravitational force of the sun? Express your result in square kilometers. (c) Explain why your answer to part (b) is independent of the distance from the sun.

32.55 •• **CP** Interplanetary space contains many small particles referred to as *interplanetary dust.* Radiation pressure from the sun sets a lower limit on the size of such dust particles. To see the origin of this limit, consider a spherical dust particle of radius R and mass density ρ. (a) Write an expression for the gravitational force exerted on this particle by the sun (mass M) when the particle is a distance r from the sun. (b) Let L represent the luminosity of the sun, equal to the rate at which it emits energy in electromagnetic radiation. Find the force exerted on the (totally absorbing) particle due to solar radiation pressure, remembering that the intensity of the sun's radiation also depends on the distance r. The relevant area is the cross-sectional area of the particle, *not* the total surface area of the particle. As part of your answer, explain why this is so. (c) The mass density of a typical interplanetary dust particle is about 3000 kg/m^3. Find the particle radius R such that the gravitational and radiation forces acting on the particle are equal in magnitude. The luminosity of the sun is 3.9×10^{26} W. Does your answer depend on the distance of the particle from the sun? Why or why not? (d) Explain why dust particles with a radius less than that found in part (c) are unlikely to be found in the solar system. [*Hint:* Construct the ratio of the two force expressions found in parts (a) and (b).]

CHALLENGE PROBLEMS

32.56 ••• **CALC** Electromagnetic waves propagate much differently in *conductors* than they do in dielectrics or in vacuum. If the resistivity of the conductor is sufficiently low (that is, if it is a sufficiently good conductor), the oscillating electric field of the wave gives rise to an oscillating conduction current that is much larger than the displacement current. In this case, the wave equation for an electric field $\vec{E}(x, t) = E_y(x, t)\hat{j}$ propagating in the $+x$-direction within a conductor is

$$\frac{\partial^2 E_y(x, t)}{\partial x^2} = \frac{\mu}{\rho}\frac{\partial E_y(x, t)}{\partial t}$$

where μ is the permeability of the conductor and ρ is its resistivity. (a) A solution to this wave equation is

$$E_y(x, t) = E_{\max} e^{-k_C x} \cos(k_C x - \omega t)$$

where $k_C = \sqrt{\omega\mu/2\rho}$. Verify this by substituting $E_y(x, t)$ into the above wave equation. (b) The exponential term shows that the electric field decreases in amplitude as it propagates. Explain why this happens. (*Hint:* The field does work to move charges within the conductor. The current of these moving charges causes i^2R heating within the conductor, raising its temperature. Where does the energy to do this come from?) (c) Show that the electric-field amplitude decreases by a factor of $1/e$ in a distance $1/k_C = \sqrt{2\rho/\omega\mu}$, and calculate this distance for a radio wave with frequency f =1.0 MHz in copper (resistivity $1.72 \times 10^{-8}\ \Omega \cdot m$; permeability $\mu = \mu_0$). Since this distance is so short, electromagnetic waves of this frequency can hardly propagate at all into copper. Instead, they are reflected at the surface of the metal. This is why radio waves cannot penetrate through copper or other metals, and why radio reception is poor inside a metal structure.

32.57 ••• **CP** Electromagnetic radiation is emitted by accelerating charges. The rate at which energy is emitted from an accelerating charge that has charge q and acceleration a is given by

$$\frac{dE}{dt} = \frac{q^2a^2}{6\pi\epsilon_0 c^3}$$

where c is the speed of light. (a) Verify that this equation is dimensionally correct. (b) If a proton with a kinetic energy of 6.0 MeV is traveling in a particle accelerator in a circular orbit of radius 0.750 m, what fraction of its energy does it radiate per second? (c) Consider an electron orbiting with the same speed and radius. What fraction of its energy does it radiate per second?

32.58 ••• **CP** **The Classical Hydrogen Atom.** The electron in a hydrogen atom can be considered to be in a circular orbit with a radius of 0.0529 nm and a kinetic energy of 13.6 eV. If the electron behaved classically, how much energy would it radiate per second (see Challenge Problem 32.57)? What does this tell you about the use of classical physics in describing the atom?

Answers

Chapter Opening Question ?

Metals are reflective because they are good conductors of electricity. When an electromagnetic wave strikes a conductor, the electric field of the wave sets up currents on the conductor surface that generate a reflected wave. For a perfect conductor, this reflected wave is just as intense as the incident wave. Tarnished metals are less shiny because their surface is oxidized and less conductive; polishing the metal removes the oxide and exposes the conducting metal.

Test Your Understanding Questions

32.1 Answers: (a) no, (b) no A purely electric wave would have a varying electric field. Such a field necessarily generates a magnetic field through Ampere's law, Eq. (29.20), so a purely electric wave is impossible. In the same way, a purely magnetic wave is impossible: The varying magnetic field in such a wave would automatically give rise to an electric field through Faraday's law, Eq. (29.21).

32.2 Answers: (a) positive y-direction, (b) negative x-direction, (c) positive y-direction You can verify these answers by using the right-hand rule to show that $\vec{E} \times \vec{B}$ in each case is in the direction of propagation, or by using the rule shown in Fig. 32.9.

32.3 Answer: (iv) In an ideal electromagnetic plane wave, at any instant the fields are the same anywhere in a plane perpendicular to the direction of propagation. The plane wave described by Eqs. (32.17) is propagating in the x-direction, so the fields depend on the coordinate x and time t but do *not* depend on the coordinates y and z.

32.4 Answers: (a) (i) and (iii), (b) (ii) and (iv), (c) (i) and (iii), (d) (ii) and (iv) Both the energy density u and the Poynting vector magnitude S are maximum where the $\vec{E}$ and $\vec{B}$ fields have their maximum magnitudes. (The directions of the fields doesn't matter.) From Fig. 32.13, this occurs at $x = 0$ and $x = \lambda/2$. Both u and S have a minimum value of zero; that occurs where $\vec{E}$ and $\vec{B}$ are both zero. From Fig. 32.13, this occurs at $x = \lambda/4$ and $x = 3\lambda/4$.

32.5 Answer: no There are places where $\vec{E} = \mathbf{0}$ at all times (at the walls) and the electric energy density $\frac{1}{2}\epsilon_0 E^2$ is always zero. There are also places where $\vec{B} = \mathbf{0}$ at all times (on the plane midway between the walls) and the magnetic energy density $B^2/2\mu_0$ is always zero. However, there are *no* locations where both $\vec{E}$ and $\vec{B}$ are always zero. Hence the energy density at any point in the standing wave is always nonzero.

Bridging Problem

Answer: 0.0368 V