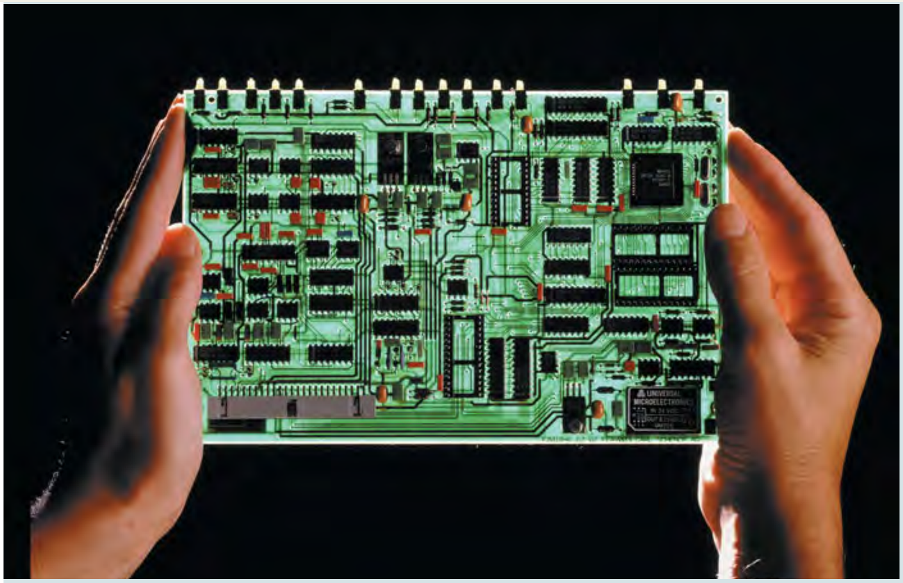
# 26 DIRECT-CURRENT CIRCUITS



? In a complex circuit like the one on this circuit board, is it possible to connect several resistors with different resistances so that they all have the same potential difference? If so, will the current be the same through all of the resistors?

**LEARNING GOALS**

*By studying this chapter, you will learn:*

- How to analyze circuits with multiple resistors in series or parallel.
- Rules that you can apply to any circuit with more than one loop.
- How to use an ammeter, voltmeter, ohmmeter, or potentiometer in a circuit.
- How to analyze circuits that include both a resistor and a capacitor.
- How electric power is distributed in the home.

If you look inside your TV, your computer, or under the hood of a car, you will find circuits of much greater complexity than the simple circuits we studied in Chapter 25. Whether connected by wires or integrated in a semiconductor chip, these circuits often include several sources, resistors, and other circuit elements interconnected in a *network*.

In this chapter we study general methods for analyzing such networks, including how to find voltages and currents of circuit elements. We'll learn how to determine the equivalent resistance for several resistors connected in series or in parallel. For more general networks we need two rules called *Kirchhoff's rules.* One is based on the principle of conservation of charge applied to a junction; the other is derived from energy conservation for a charge moving around a closed loop. We'll discuss instruments for measuring various electrical quantities. We'll also look at a circuit containing resistance and capacitance, in which the current varies with time.

Our principal concern in this chapter is with **direct-current** (dc) circuits, in which the direction of the current does not change with time. Flashlights and automobile wiring systems are examples of direct-current circuits. Household electrical power is supplied in the form of **alternating current** (ac), in which the current oscillates back and forth. The same principles for analyzing networks apply to both kinds of circuits, and we conclude this chapter with a look at household wiring systems. We'll discuss alternating-current circuits in detail in Chapter 31.

MasteringPHYSICS®

**ActivPhysics 12.1:** DC Series Circuits (Qualitative)

## 26.1 Resistors in Series and Parallel

Resistors turn up in all kinds of circuits, ranging from hair dryers and space heaters to circuits that limit or divide current or reduce or divide a voltage. Such circuits often contain several resistors, so it's appropriate to consider *combinations* of resistors. A simple example is a string of light bulbs used for holiday decorations;

each bulb acts as a resistor, and from a circuit-analysis perspective the string of bulbs is simply a combination of resistors.

Suppose we have three resistors with resistances $R_1$, $R_2$, and $R_3$. Figure 26.1 shows four different ways in which they might be connected between points $a$ and $b$. When several circuit elements such as resistors, batteries, and motors are connected in sequence as in Fig. 26.1a, with only a single current path between the points, we say that they are connected in **series.** We studied *capacitors* in series in Section 24.2; we found that, because of conservation of charge, capacitors in series all have the same charge if they are initially uncharged. In circuits we're often more interested in the *current,* which is charge flow per unit time.

The resistors in Fig. 26.1b are said to be connected in **parallel** between points $a$ and $b$. Each resistor provides an alternative path between the points. For circuit elements that are connected in parallel, the *potential difference* is the same across each element. We studied capacitors in parallel in Section 24.2.

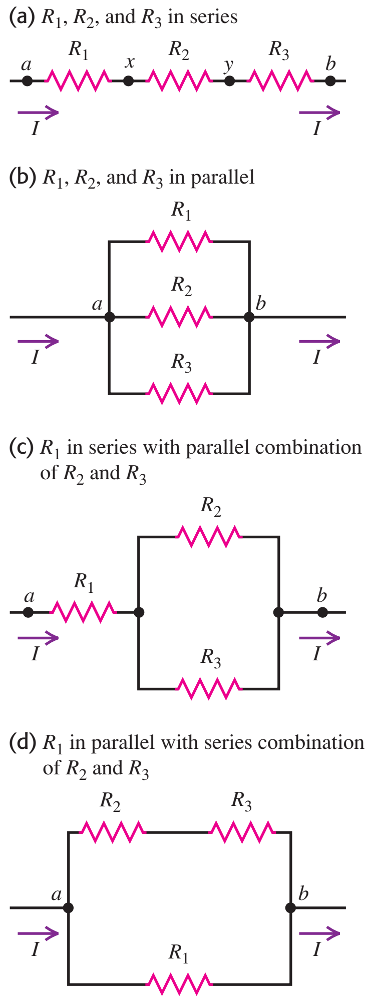
In Fig. 26.1c, resistors $R_2$ and $R_3$ are in parallel, and this combination is in series with $R_1$. In Fig. 26.1d, $R_2$ and $R_3$ are in series, and this combination is in parallel with $R_1$.

For any combination of resistors we can always find a *single* resistor that could replace the combination and result in the same total current and potential difference. For example, a string of holiday light bulbs could be replaced by a single, appropriately chosen light bulb that would draw the same current and have the same potential difference between its terminals as the original string of bulbs. The resistance of this single resistor is called the **equivalent resistance** of the combination. If any one of the networks in Fig. 26.1 were replaced by its equivalent resistance $R_{eq}$, we could write

$$V_{ab} = IR_{eq} \quad \text{or} \quad R_{eq} = \frac{V_{ab}}{I}$$

where $V_{ab}$ is the potential difference between terminals $a$ and $b$ of the network and $I$ is the current at point $a$ or $b$. To compute an equivalent resistance, we assume a potential difference $V_{ab}$ across the actual network, compute the corresponding current $I$, and take the ratio $V_{ab}/I$.

**26.1** Four different ways of connecting three resistors.




## Resistors in Series

We can derive general equations for the equivalent resistance of a series or parallel combination of resistors. If the resistors are in *series,* as in Fig. 26.1a, the current $I$ must be the same in all of them. (As we discussed in Section 25.4, current is *not* "used up" as it passes through a circuit.) Applying $V = IR$ to each resistor, we have

$$V_{ax} = IR_1 \qquad V_{xy} = IR_2 \qquad V_{yb} = IR_3$$

The potential differences across each resistor need not be the same (except for the special case in which all three resistances are equal). The potential difference $V_{ab}$ across the entire combination is the sum of these individual potential differences:

$$V_{ab} = V_{ax} + V_{xy} + V_{yb} = I(R_1 + R_2 + R_3)$$

and so

$$\frac{V_{ab}}{I} = R_1 + R_2 + R_3$$

The ratio $V_{ab}/I$ is, by definition, the equivalent resistance $R_{eq}$. Therefore

$$R_{eq} = R_1 + R_2 + R_3$$

It is easy to generalize this to any number of resistors:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad \text{(resistors in series)} \qquad (26.1)$$

**The equivalent resistance of *any number* of resistors in series equals the sum of their individual resistances.**

The equivalent resistance is *greater than* any individual resistance.

Let's compare this result with Eq. (24.5) for *capacitors* in series. Resistors in series add directly because the voltage across each is directly proportional to its resistance and to the common current. Capacitors in series add reciprocally because the voltage across each is directly proportional to the common charge but *inversely* proportional to the individual capacitance.

## Resistors in Parallel

If the resistors are in *parallel,* as in Fig. 26.1b, the current through each resistor need not be the same. But the potential difference between the terminals of each resistor must be the same and equal to $V_{ab}$ (Fig. 26.2). (Remember that the potential difference between any two points does not depend on the path taken between the points.) Let's call the currents in the three resistors $I_1$, $I_2$, and $I_3$. Then from $I = V/R$,

$$I_1 = \frac{V_{ab}}{R_1} \qquad I_2 = \frac{V_{ab}}{R_2} \qquad I_3 = \frac{V_{ab}}{R_3}$$

**26.2** A car's headlights and taillights are connected in parallel. Hence each light is exposed to the full potential difference supplied by the car's electrical system, giving maximum brightness. Another advantage is that if one headlight or taillight burns out, the other one keeps shining (see Example 26.2).

In general, the current is different through each resistor. Because charge is not accumulating or draining out of point $a$, the total current $I$ must equal the sum of the three currents in the resistors:

$$I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad \text{or}$$

$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But by the definition of the equivalent resistance $R_{eq}$, $I/V_{ab} = 1/R_{eq}$, so

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Again it is easy to generalize to *any number* of resistors in parallel:

MasteringPHYSICS®
**ActivPhysics 12.2:** DC Parallel Circuits

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad \text{(resistors in parallel)} \tag{26.2}$$

**For *any number* of resistors in parallel, the *reciprocal* of the equivalent resistance equals the *sum of the reciprocals* of their individual resistances.**

The equivalent resistance is always *less than* any individual resistance.

Compare this with Eq. (24.7) for *capacitors* in parallel. Resistors in parallel add reciprocally because the current in each is proportional to the common voltage across them and *inversely* proportional to the resistance of each. Capacitors in parallel add directly because the charge on each is proportional to the common voltage across them and *directly* proportional to the capacitance of each.

For the special case of *two* resistors in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \quad \text{and}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad \text{(two resistors in parallel)} \tag{26.3}$$

Because $V_{ab} = I_1R_1 = I_2R_2$, it follows that

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \qquad \text{(two resistors in parallel)} \qquad (26.4)$$

This shows that the currents carried by two resistors in parallel are *inversely proportional* to their resistances. More current goes through the path of least resistance.

## Problem-Solving Strategy 26.1 Resistors in Series and Parallel

**IDENTIFY** *the relevant concepts:* As in Fig. 26.1, many resistor networks are made up of resistors in series, in parallel, or a combination thereof. Such networks can be replaced by a single equivalent resistor. The logic is similar to that of Problem-Solving Strategy 24.1 for networks of capacitors.

**SET UP** *the problem* using the following steps:

1. Make a drawing of the resistor network.
2. Identify groups of resistors connected in series or parallel.
3. Identify the target variables. They could include the equivalent resistance of the network, the potential difference across each resistor, or the current through each resistor.

**EXECUTE** *the solution* as follows:

1. Use Eq. (26.1) or (26.2), respectively, to find the equivalent resistance for series or parallel combinations.
2. If the network is more complex, try reducing it to series and parallel combinations. For example, in Fig. 26.1c we first replace the parallel combination of $R_2$ and $R_3$ with its equivalent resistance; this then forms a series combination with $R_1$. In Fig. 26.1d, the combination of $R_2$ and $R_3$ in series forms a parallel combination with $R_1$.
3. Keep in mind that the total potential difference across resistors connected in series is the sum of the individual potential differences. The potential difference across resistors connected in parallel is the same for every resistor and equals the potential difference across the combination.
4. The current through resistors connected in series is the same through every resistor and equals the current through the combination. The total current through resistors connected in parallel is the sum of the currents through the individual resistors.

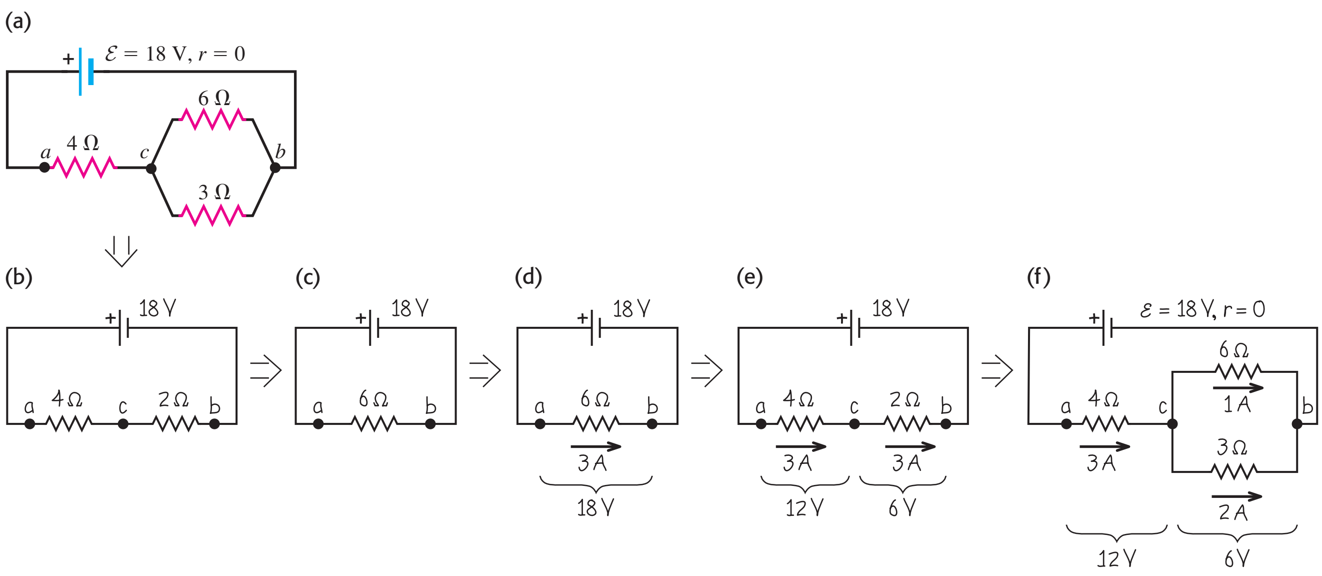
**EVALUATE** *your answer:* Check whether your results are consistent. The equivalent resistance of resistors connected in series should be greater than that of any individual resistor; that of resistors in parallel should be less than that of any individual resistor.

## Example 26.1 Equivalent resistance

Find the equivalent resistance of the network in Fig. 26.3a and the current in each resistor. The source of emf has negligible internal resistance.

**SOLUTION**

**IDENTIFY and SET UP:** This network of three resistors is a *combination* of series and parallel resistances, as in Fig. 26.1c. We determine




**26.3** Steps in reducing a combination of resistors to a single equivalent resistor and finding the current in each resistor.

*Continued*

the equivalent resistance of the parallel 6-Ω and 3-Ω resistors, and then that of their series combination with the 4-Ω resistor: This is the equivalent resistance $R_{eq}$ of the network as a whole. We then find the current in the emf, which is the same as that in the 4-Ω resistor. The potential difference is the same across each of the parallel 6-Ω and 3-Ω resistors; we use this to determine how the current is divided between these.

**EXECUTE:** Figures 26.3b and 26.3c show successive steps in reducing the network to a single equivalent resistance $R_{eq}$. From Eq. (26.2), the 6-Ω and 3-Ω resistors in parallel in Fig. 26.3a are equivalent to the single 2-Ω resistor in Fig. 26.3b:

$$\frac{1}{R_{6\,\Omega+3\,\Omega}} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega} = \frac{1}{2\ \Omega}$$

[Equation (26.3) gives the same result.] From Eq. (26.1) the series combination of this 2-Ω resistor with the 4-Ω resistor is equivalent to the single 6-Ω resistor in Fig. 26.3c.

We reverse these steps to find the current in each resistor of the original network. In the circuit shown in Fig. 26.3d (identical to Fig. 26.3c), the current is $I = V_{ab}/R = (18\ \text{V})/(6\ \Omega) = 3\ \text{A}$. So the current in the 4-Ω and 2-Ω resistors in Fig. 26.3e (identical to Fig. 26.3b) is also 3 A. The potential difference $V_{cb}$ across the 2-Ω resistor is therefore $V_{cb} = IR = (3\ \text{A})(2\ \Omega) = 6\ \text{V}$. This potential difference must also be 6 V in Fig. 26.3f (identical to Fig. 26.3a). From $I = V_{cb}/R$, the currents in the 6-Ω and 3-Ω resistors in Fig. 26.3f are respectively $(6\ \text{V})/(6\ \Omega) = 1\ \text{A}$ and $(6\ \text{V})/(3\ \Omega) = 2\ \text{A}$.

**EVALUATE:** Note that for the two resistors in parallel between points $c$ and $b$ in Fig. 26.3f, there is twice as much current through the 3-Ω resistor as through the 6-Ω resistor; more current goes through the path of least resistance, in accordance with Eq. (26.4). Note also that the total current through these two resistors is 3 A, the same as it is through the 4-Ω resistor between points $a$ and $c$.

## Example 26.2 Series versus parallel combinations

Two identical light bulbs, each with resistance $R = 2\ \Omega$, are connected to a source with $\mathcal{E} = 8\ \text{V}$ and negligible internal resistance. Find the current through each bulb, the potential difference across each bulb, and the power delivered to each bulb and to the entire network if the bulbs are connected (a) in series and (b) in parallel. (c) Suppose one of the bulbs burns out; that is, its filament breaks and current can no longer flow through it. What happens to the other bulb in the series case? In the parallel case?

**SOLUTION**

**IDENTIFY and SET UP:** The light bulbs are just resistors in simple series and parallel connections (Figs. 26.4a and 26.4b). Once we find the current $I$ through each bulb, we can find the power delivered to each bulb using Eq. (25.18), $P = I^2R = V^2/R$.

**EXECUTE:** (a) From Eq. (26.1) the equivalent resistance of the two bulbs between points $a$ and $c$ in Fig. 26.4a is $R_{eq} = 2R = 2(2\ \Omega) = 4\ \Omega$. In series, the current is the same through each bulb:

$$I = \frac{V_{ac}}{R_{eq}} = \frac{8\ \text{V}}{4\ \Omega} = 2\ \text{A}$$

Since the bulbs have the same resistance, the potential difference is the same across each bulb:

$$V_{ab} = V_{bc} = IR = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

From Eq. (25.18), the power delivered to each bulb is

$$P = I^2R = (2\ \text{A})^2(2\ \Omega) = 8\ \text{W} \qquad \text{or}$$

$$P = \frac{V_{ab}{}^2}{R} = \frac{V_{bc}{}^2}{R} = \frac{(4\ \text{V})^2}{2\ \Omega} = 8\ \text{W}$$

The total power delivered to both bulbs is $P_{tot} = 2P = 16\ \text{W}$.

(b) If the bulbs are in parallel, as in Fig. 26.4b, the potential difference $V_{de}$ across each bulb is the same and equal to 8 V, the terminal voltage of the source. Hence the current through each light bulb is

$$I = \frac{V_{de}}{R} = \frac{8\ \text{V}}{2\ \Omega} = 4\ \text{A}$$

and the power delivered to each bulb is

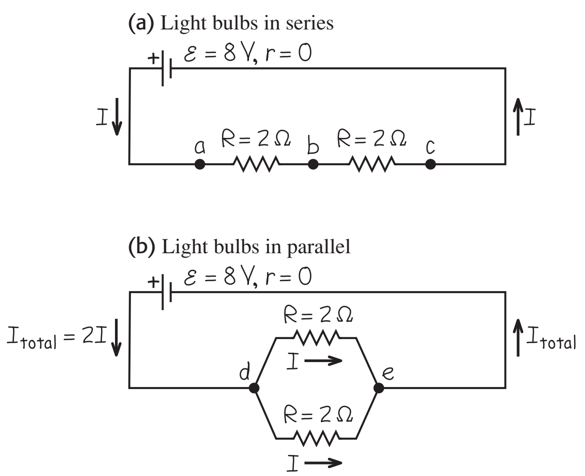
$$P = I^2R = (4\ \text{A})^2(2\ \Omega) = 32\ \text{W} \qquad \text{or}$$

$$P = \frac{V_{de}{}^2}{R} = \frac{(8\ \text{V})^2}{2\ \Omega} = 32\ \text{W}$$

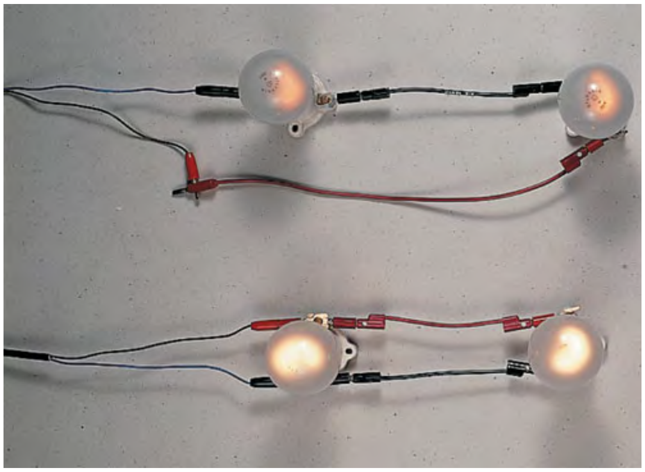
Both the potential difference across each bulb and the current through each bulb are twice as great as in the series case. Hence the power delivered to each bulb is *four* times greater, and each bulb is brighter.

The total power delivered to the parallel network is $P_{total} = 2P = 64\ \text{W}$, four times greater than in the series case. The

**26.4** Our sketches for this problem.



**26.5** When connected to the same source, two light bulbs in series (shown at top) draw less power and glow less brightly than when they are in parallel (shown at bottom).



increased power compared to the series case isn't obtained "for free"; energy is extracted from the source four times more rapidly in the parallel case than in the series case. If the source is a battery, it will be used up four times as fast.

(c) In the series case the same current flows through both bulbs. If one bulb burns out, there will be no current in the circuit, and neither bulb will glow.

In the parallel case the potential difference across either bulb is unchanged if a bulb burns out. The current through the functional bulb and the power delivered to it are unchanged.

**EVALUATE:** Our calculation isn't completely accurate, because the resistance $R = V/I$ of real light bulbs depends on the potential difference $V$ across the bulb. That's because the filament resistance increases with increasing operating temperature and therefore with increasing $V$. But bulbs connected in series across a source do in fact glow less brightly than when connected in parallel across the same source (Fig. 26.5).

**Test Your Understanding of Section 26.1** Suppose all three of the resistors shown in Fig. 26.1 have the same resistance, so $R_1 = R_2 = R_3 = R$. Rank the four arrangements shown in parts (a)–(d) of Fig. 26.1 in order of their equivalent resistance, from highest to lowest.

## 26.2 Kirchhoff's Rules

Many practical resistor networks cannot be reduced to simple series-parallel combinations. Figure 26.6a shows a dc power supply with emf $\mathcal{E}_1$ charging a battery with a smaller emf $\mathcal{E}_2$ and feeding current to a light bulb with resistance $R$. Figure 26.6b is a "bridge" circuit, used in many different types of measurement and control systems. (Problem 26.81 describes one important application of a "bridge" circuit.) To compute the currents in these networks, we'll use the techniques developed by the German physicist Gustav Robert Kirchhoff (1824–1887).

First, here are two terms that we will use often. A **junction** in a circuit is a point where three or more conductors meet. A **loop** is any closed conducting path. In Fig. 26.6a points $a$ and $b$ are junctions, but points $c$ and $d$ are not; in Fig. 26.6b the points $a$, $b$, $c$, and $d$ are junctions, but points $e$ and $f$ are not. The blue lines in Figs. 26.6a and 26.6b show some possible loops in these circuits.

Kirchhoff's rules are the following two statements:

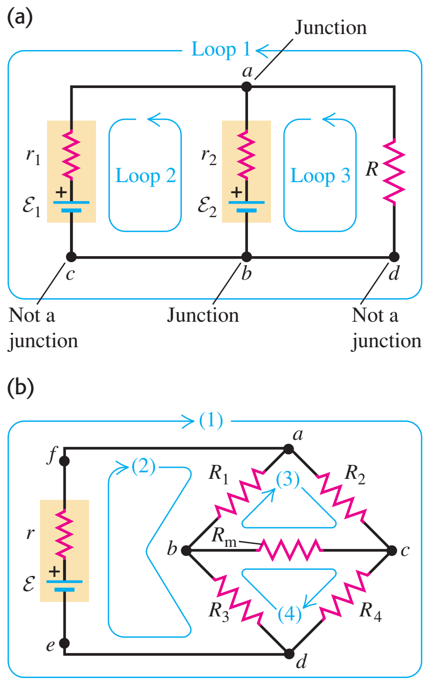
**Kirchhoff's junction rule:** *The algebraic sum of the currents into any junction is zero.* That is,

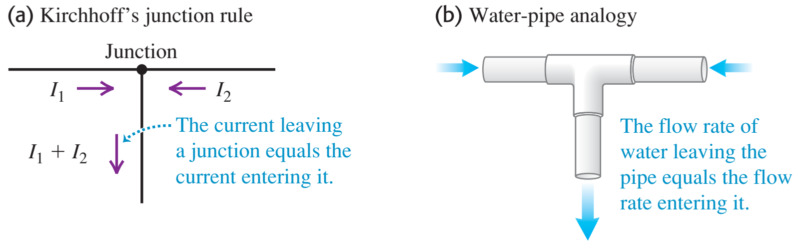
$$\sum I = 0 \qquad \text{(junction rule, valid at any junction)} \qquad (26.5)$$

**Kirchhoff's loop rule:** *The algebraic sum of the potential differences in any loop,* including those associated with emfs and those of resistive elements, *must equal zero.* That is,

$$\sum V = 0 \qquad \text{(loop rule, valid for any closed loop)} \qquad (26.6)$$

**26.6** Two networks that cannot be reduced to simple series-parallel combinations of resistors.






**26.7** Kirchhoff's junction rule states that as much current flows into a junction as flows out of it.

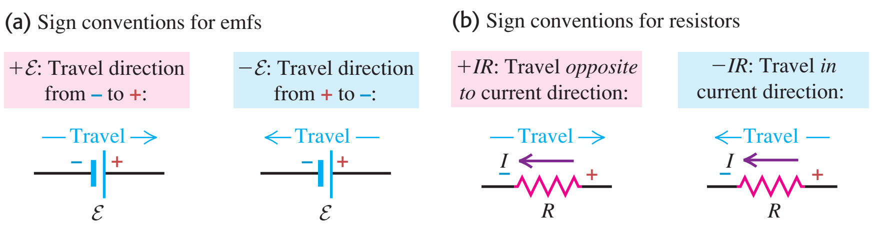
The junction rule is based on *conservation of electric charge*. No charge can accumulate at a junction, so the total charge entering the junction per unit time must equal the total charge leaving per unit time (Fig. 26.7a). Charge per unit time is current, so if we consider the currents entering a junction to be positive and those leaving to be negative, the algebraic sum of currents into a junction must be zero. It's like a T branch in a water pipe (Fig. 26.7b); if you have a total of 1 liter per minute coming in the two pipes, you can't have 3 liters per minute going out the third pipe. We may as well confess that we used the junction rule (without saying so) in Section 26.1 in the derivation of Eq. (26.2) for resistors in parallel.

The loop rule is a statement that the electrostatic force is *conservative*. Suppose we go around a loop, measuring potential differences across successive circuit elements as we go. When we return to the starting point, we must find that the *algebraic sum* of these differences is zero; otherwise, we could not say that the potential at this point has a definite value.

## Sign Conventions for the Loop Rule

In applying the loop rule, we need some sign conventions. Problem-Solving Strategy 26.2 describes in detail how to use these, but here's a quick overview. We first assume a direction for the current in each branch of the circuit and mark it on a diagram of the circuit. Then, starting at any point in the circuit, we imagine traveling around a loop, adding emfs and $IR$ terms as we come to them. When we travel through a source in the direction from $-$ to $+$, the emf is considered to be *positive;* when we travel from $+$ to $-$, the emf is considered to be *negative* (Fig. 26.8a). When we travel through a resistor in the *same* direction as the assumed current, the $IR$ term is *negative* because the current goes in the direction of decreasing potential. When we travel through a resistor in the direction *opposite* to the assumed current, the $IR$ term is *positive* because this represents a rise of potential (Fig. 26.8b).

Kirchhoff's two rules are all we need to solve a wide variety of network problems. Usually, some of the emfs, currents, and resistances are known, and others are unknown. We must always obtain from Kirchhoff's rules a number of independent equations equal to the number of unknowns so that we can solve the equations simultaneously. Often the hardest part of the solution is not understanding the basic principles but keeping track of algebraic signs!




**26.8** Use these sign conventions when you apply Kirchhoff's loop rule. In each part of the figure "Travel" is the direction that we imagine going around the loop, which is not necessarily the direction of the current.

## Problem-Solving Strategy 26.2 Kirchhoff's Rules

**IDENTIFY** *the relevant concepts:* Kirchhoff's rules are useful for analyzing any electric circuit.

**SET UP** *the problem* using the following steps:

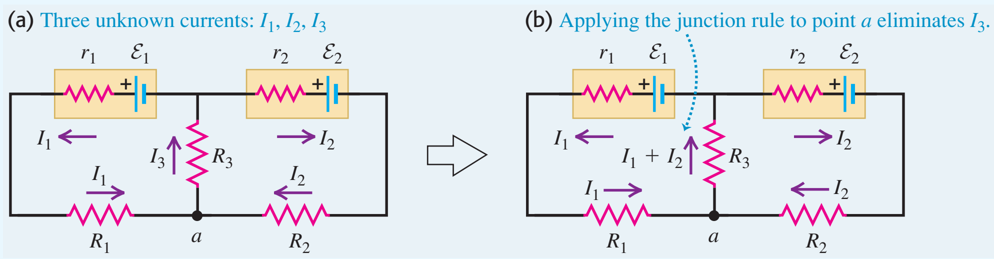
1. Draw a circuit diagram, leaving room to label all quantities, known and unknown. Indicate an assumed direction for each unknown current and emf. (Kirchhoff's rules will yield the magnitudes *and directions* of unknown currents and emfs. If the actual direction of a quantity is opposite to your assumption, the resulting quantity will have a negative sign.)
2. As you label currents, it helpful to use Kirchhoff's junction rule, as in Fig. 26.9, so as to express the currents in terms of as few quantities as possible.
3. Identify the target variables.

**EXECUTE** *the solution* as follows:

1. Choose any loop in the network and choose a direction (clockwise or counterclockwise) to travel around the loop as you apply Kirchhoff's loop rule. The direction need not be the same as any assumed current direction.
2. Travel around the loop in the chosen direction, adding potential differences algebraically as you cross them. Use the sign conventions of Fig. 26.8.
3. Equate the sum obtained in step 2 to zero in accordance with the loop rule.
4. If you need more independent equations, choose another loop and repeat steps 1–3; continue until you have as many independent equations as unknowns or until every circuit element has been included in at least one loop.
5. Solve the equations simultaneously to determine the unknowns.
6. You can use the loop-rule bookkeeping system to find the potential $V_{ab}$ of any point $a$ with respect to any other point $b$. Start at $b$ and add the potential changes you encounter in going from $b$ to $a$, using the same sign rules as in step 2. The algebraic sum of these changes is $V_{ab} = V_a - V_b$.

**EVALUATE** *your answer:* Check all the steps in your algebra. Apply steps 1 and 2 to a loop you have not yet considered; if the sum of potential drops isn't zero, you've made an error somewhere.

**26.9** Applying the junction rule to point $a$ reduces the number of unknown currents from three to two.




## Example 26.3 A single-loop circuit

The circuit shown in Fig. 26.10a contains two batteries, each with an emf and an internal resistance, and two resistors. Find (a) the current in the circuit, (b) the potential difference $V_{ab}$, and (c) the power output of the emf of each battery.

**SOLUTION**

**IDENTIFY and SET UP:** There are no junctions in this single-loop circuit, so we don't need Kirchhoff's junction rule. To apply Kirchhoff's loop rule, we first assume a direction for the current; let's assume a counterclockwise direction as shown in Fig. 26.10a.

**EXECUTE:** (a) Starting at $a$ and traveling counterclockwise with the current, we add potential increases and decreases and equate the sum to zero as in Eq. (26.6):

$$-I(4\ \Omega) - 4\text{ V} - I(7\ \Omega) + 12\text{ V} - I(2\ \Omega) - I(3\ \Omega) = 0$$

Collecting like terms and solving for $I$, we find

$$8\text{ V} = I(16\ \Omega) \quad \text{and} \quad I = 0.5\text{ A}$$

The positive result for $I$ shows that our assumed current direction is correct.

(b) To find $V_{ab}$, the potential at $a$ with respect to $b$, we start at $b$ and add potential changes as we go toward $a$. There are two paths from $b$ to $a$; taking the lower one, we find

$$V_{ab} = (0.5\text{ A})(7\ \Omega) + 4\text{ V} + (0.5\text{ A})(4\ \Omega) = 9.5\text{ V}$$

Point $a$ is at 9.5 V higher potential than $b$. All the terms in this sum, including the $IR$ terms, are positive because each represents an *increase* in potential as we go from $b$ to $a$. Taking the upper path, we find

$$V_{ab} = 12\text{ V} - (0.5\text{ A})(2\ \Omega) - (0.5\text{ A})(3\ \Omega) = 9.5\text{ V}$$

Here the $IR$ terms are negative because our path goes in the direction of the current, with potential decreases through the resistors. The results for $V_{ab}$ are the same for both paths, as they must be in order for the total potential change around the loop to be zero.

*Continued*

(c) The power outputs of the emf of the 12-V and 4-V batteries are

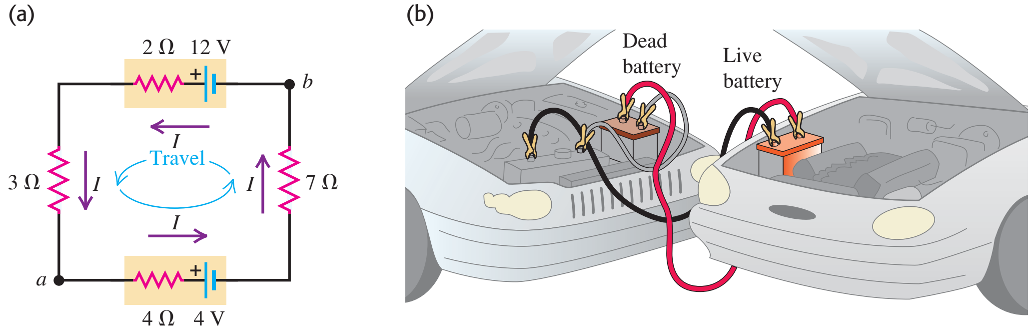
$$P_{12\text{V}} = \mathcal{E}I = (12\ \text{V})(0.5\ \text{A}) = 6\ \text{W}$$
$$P_{4\text{V}} = \mathcal{E}I = (-4\ \text{V})(0.5\ \text{A}) = -2\ \text{W}$$

The negative sign in $\mathcal{E}$ for the 4-V battery appears because the current actually runs from the higher-potential side of the battery to the lower-potential side. The negative value of $P$ means that we are *storing* energy in that battery; the 12-V battery is *recharging* it (if it is in fact rechargeable; otherwise, we're destroying it).

**EVALUATE:** By applying the expression $P = I^2R$ to each of the four resistors in Fig. 26.10a, you can show that the total power dissipated in all four resistors is 4 W. Of the 6 W provided by the emf of the 12-V battery, 2 W goes into storing energy in the 4-V battery and 4 W is dissipated in the resistances.

The circuit shown in Fig. 26.10a is much like that used when a fully charged 12-V storage battery (in a car with its engine running) is used to "jump-start" a car with a run-down battery (Fig. 26.10b). The run-down battery is slightly recharged in the process. The 3-Ω and 7-Ω resistors in Fig. 26.10a represent the resistances of the jumper cables and of the conducting path through the automobile with the run-down battery. (The values of the resistances in actual automobiles and jumper cables are considerably lower.)

**26.10** (a) In this example we travel around the loop in the same direction as the assumed current, so all the $IR$ terms are negative. The potential decreases as we travel from + to − through the bottom emf but increases as we travel from − to + through the top emf. (b) A real-life example of a circuit of this kind.



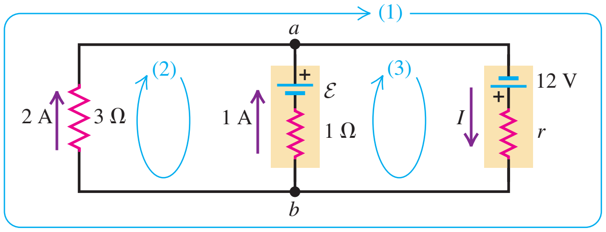

## Example 26.4 Charging a battery

In the circuit shown in Fig. 26.11, a 12-V power supply with unknown internal resistance $r$ is connected to a run-down rechargeable battery with unknown emf $\mathcal{E}$ and internal resistance 1 Ω and to an indicator light bulb of resistance 3 Ω carrying a current of 2 A. The current through the run-down battery is 1 A in the direction shown. Find $r$, $\mathcal{E}$, and the current $I$ through the power supply.

### SOLUTION

**IDENTIFY and SET UP:** This circuit has more than one loop, so we must apply both the junction and loop rules. We assume the direction of the current through the 12-V power supply, and the polarity of the run-down battery, to be as shown in Fig. 26.11. There are three target variables, so we need three equations.

**26.11** In this circuit a power supply charges a run-down battery and lights a bulb. An assumption has been made about the polarity of the emf $\mathcal{E}$ of the run-down battery. Is this assumption correct?




**EXECUTE:** We apply the junction rule, Eq. (26.5), to point $a$:

$$-I + 1\ \text{A} + 2\ \text{A} = 0 \qquad \text{so} \qquad I = 3\ \text{A}$$

To determine $r$, we apply the loop rule, Eq. (26.6), to the large, outer loop (1):

$$12\ \text{V} - (3\ \text{A})r - (2\ \text{A})(3\ \Omega) = 0 \qquad \text{so} \qquad r = 2\ \Omega$$

To determine $\mathcal{E}$, we apply the loop rule to the left-hand loop (2):

$$-\mathcal{E} + (1\ \text{A})(1\ \Omega) - (2\ \text{A})(3\ \Omega) = 0 \qquad \text{so} \qquad \mathcal{E} = -5\ \text{V}$$

The negative value for $\mathcal{E}$ shows that the actual polarity of this emf is opposite to that shown in Fig. 26.11. As in Example 26.3, the battery is being recharged.

**EVALUATE:** Try applying the junction rule at point $b$ instead of point $a$, and try applying the loop rule by traveling counterclockwise rather than clockwise around loop (1). You'll get the same results for $I$ and $r$. We can check our result for $\mathcal{E}$ by using the right-hand loop (3):

$$12\ \text{V} - (3\ \text{A})(2\ \Omega) - (1\ \text{A})(1\ \Omega) + \mathcal{E} = 0$$

which again gives us $\mathcal{E} = -5$ V.

As an additional check, we note that $V_{ba} = V_b - V_a$ equals the voltage across the 3-Ω resistance, which is $(2\ \text{A})(3\ \Omega) = 6$ V. Going from $a$ to $b$ by the top branch, we encounter potential differences $+12\ \text{V} - (3\ \text{A})(2\ \Omega) = +6$ V, and going by the middle branch, we find $-(-5\ \text{V}) + (1\ \text{A})(1\ \Omega) = +6$ V. The three ways of getting $V_{ba}$ give the same results.

## Example 26.5 Power in a battery-charging circuit

In the circuit of Example 26.4 (shown in Fig. 26.11), find the power delivered by the 12-V power supply and by the battery being recharged, and find the power dissipated in each resistor.

**SOLUTION**

**IDENTIFY and SET UP:** We use the results of Section 25.5, in which we found that the power delivered *from* an emf to a circuit is $\mathcal{E}I$ and the power delivered *to* a resistor from a circuit is $V_{ab}I = I^2R$. We know the values of all relevant quantities from Example 26.4.

**EXECUTE:** The power output $P_s$ from the emf of the power supply is

$$P_{\text{supply}} = \mathcal{E}_{\text{supply}} I_{\text{supply}} = (12\text{ V})(3\text{ A}) = 36\text{ W}$$

The power dissipated in the power supply's internal resistance $r$ is

$$P_{r\text{-supply}} = I_{\text{supply}}^2 r_{\text{supply}} = (3\text{ A})^2(2\ \Omega) = 18\text{ W}$$

so the power supply's *net* power output is $P_{\text{net}} = 36\text{ W} - 18\text{ W} = 18\text{ W}$. Alternatively, from Example 26.4 the terminal voltage of the battery is $V_{ba} = 6$ V, so the net power output is

$$P_{\text{net}} = V_{ba} I_{\text{supply}} = (6\text{ V})(3\text{ A}) = 18\text{ W}$$

The power output of the emf $\mathcal{E}$ of the battery being charged is

$$P_{\text{emf}} = \mathcal{E}I_{\text{battery}} = (-5\text{ V})(1\text{ A}) = -5\text{ W}$$

This is negative because the 1-A current runs through the battery from the higher-potential side to the lower-potential side. (As we mentioned in Example 26.4, the polarity assumed for this battery in Fig. 26.11 was wrong.) We are storing energy in the battery as we charge it. Additional power is dissipated in the battery's internal resistance; this power is

$$P_{r\text{-battery}} = I_{\text{battery}}^2 r_{\text{battery}} = (1\text{ A})^2(1\ \Omega) = 1\text{ W}$$

The total power input to the battery is thus $1\text{ W} + |-5\text{ W}| = 6\text{ W}$. Of this, 5 W represents useful energy stored in the battery; the remainder is wasted in its internal resistance.

The power dissipated in the light bulb is

$$P_{\text{bulb}} = I_{\text{bulb}}^2 R_{\text{bulb}} = (2\text{ A})^2(3\ \Omega) = 12\text{ W}$$

**EVALUATE:** As a check, note that all of the power from the supply is accounted for. Of the 18 W of net power from the power supply, 5 W goes to recharge the battery, 1 W is dissipated in the battery's internal resistance, and 12 W is dissipated in the light bulb.

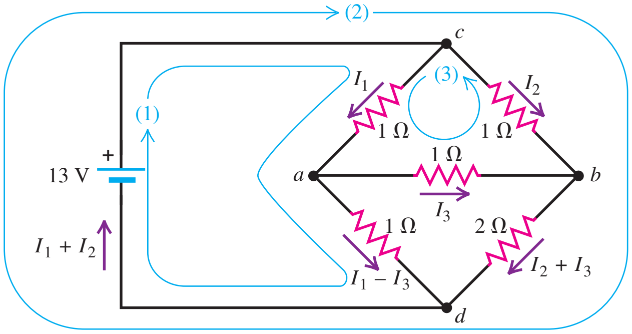
## Example 26.6 A complex network

Figure 26.12 shows a "bridge" circuit of the type described at the beginning of this section (see Fig. 26.6b). Find the current in each resistor and the equivalent resistance of the network of five resistors.

**SOLUTION**

**IDENTIFY and SET UP:** This network is neither a series combination nor a parallel combination. Hence we must use Kirchhoff's rules to find the values of the target variables. There are five unknown currents, but by applying the junction rule to junctions $a$ and $b$, we can represent them in terms of three unknown currents $I_1$, $I_2$, and $I_3$, as shown in Fig. 26.12.

**26.12** A network circuit with several resistors.




**EXECUTE:** We apply the loop rule to the three loops shown:

$$13\text{ V} - I_1(1\ \Omega) - (I_1 - I_3)(1\ \Omega) = 0 \qquad (1)$$

$$-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + 13\text{ V} = 0 \qquad (2)$$

$$-I_1(1\ \Omega) - I_3(1\ \Omega) + I_2(1\ \Omega) = 0 \qquad (3)$$

One way to solve these simultaneous equations is to solve Eq. (3) for $I_2$, obtaining $I_2 = I_1 + I_3$, and then substitute this expression into Eq. (2) to eliminate $I_2$. We then have

$$13\text{ V} = I_1(2\ \Omega) - I_3(1\ \Omega) \qquad (1')$$

$$13\text{ V} = I_1(3\ \Omega) + I_3(5\ \Omega) \qquad (2')$$

Now we can eliminate $I_3$ by multiplying Eq. (1′) by 5 and adding the two equations. We obtain

$$78\text{ V} = I_1(13\ \Omega) \qquad I_1 = 6\text{ A}$$

We substitute this result into Eq. (1′) to obtain $I_3 = -1$ A, and from Eq. (3) we find $I_2 = 5$ A. The negative value of $I_3$ tells us that its direction is opposite to the direction we assumed.

The total current through the network is $I_1 + I_2 = 11$ A, and the potential drop across it is equal to the battery emf, 13 V. The equivalent resistance of the network is therefore

$$R_{\text{eq}} = \frac{13\text{ V}}{11\text{ A}} = 1.2\ \Omega$$

**EVALUATE:** You can check our results for $I_1$, $I_2$, and $I_3$ by substituting them back into Eqs. (1)–(3). What do you find?

## Example 26.7 A potential difference in a complex network

In the circuit of Example 26.6 (Fig. 26.12), find the potential difference $V_{ab}$.

**SOLUTION**

**IDENTIFY and SET UP:** Our target variable $V_{ab} = V_a - V_b$ is the potential at point $a$ with respect to point $b$. To find it, we start at point $b$ and follow a path to point $a$, adding potential rises and drops as we go. We can follow any of several paths from $b$ to $a$; the result must be the same for all such paths, which gives us a way to check our result.

**EXECUTE:** The simplest path is through the center 1-Ω resistor. In Example 26.6 we found $I_3 = -1$ A, showing that the actual current direction through this resistor is from right to left. Thus, as we go from $b$ to $a$, there is a *drop* of potential with magnitude $|I_3|R = (1\text{ A})(1\ \Omega) = 1$ V. Hence $V_{ab} = -1$ V, and the potential at $a$ is 1 V less than at point $b$.

**EVALUATE:** To check our result, let's try a path from $b$ to $a$ that goes through the lower two resistors. The currents through these are

$$I_2 + I_3 = 5\text{ A} + (-1\text{ A}) = 4\text{ A} \quad \text{and}$$
$$I_1 - I_3 = 6\text{ A} - (-1\text{ A}) = 7\text{ A}$$

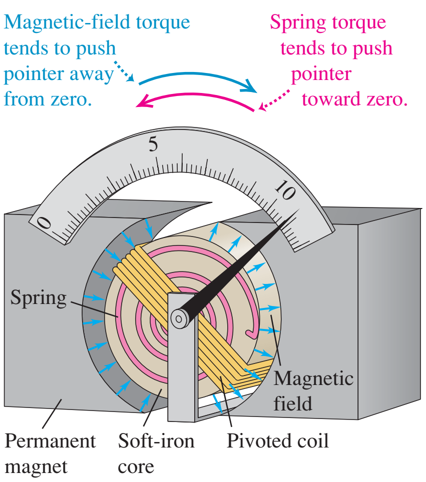
and so

$$V_{ab} = -(4\text{ A})(2\ \Omega) + (7\text{ A})(1\ \Omega) = -1\text{ V}$$

You can confirm this result using some other paths from $b$ to $a$.

**26.13** This ammeter (top) and voltmeter (bottom) are both d'Arsonval galvanometers. The difference has to do with their internal connections (see Fig. 26.15).


**26.14** A d'Arsonval galvanometer, showing a pivoted coil with attached pointer, a permanent magnet supplying a magnetic field that is uniform in magnitude, and a spring to provide restoring torque, which opposes magnetic-field torque.




**Test Your Understanding of Section 26.2** Subtract Eq. (1) from Eq. (2) in Example 26.6. To which loop in Fig. 26.12 does this equation correspond? Would this equation have simplified the solution of Example 26.6?

## 26.3 Electrical Measuring Instruments

We've been talking about potential difference, current, and resistance for two chapters, so it's about time we said something about how to *measure* these quantities. Many common devices, including car instrument panels, battery chargers, and inexpensive electrical instruments, measure potential difference (voltage), current, or resistance using a **d'Arsonval galvanometer** (Fig. 26.13). In the following discussion we'll often call it just a *meter*. A pivoted coil of fine wire is placed in the magnetic field of a permanent magnet (Fig. 26.14). Attached to the coil is a spring, similar to the hairspring on the balance wheel of a watch. In the equilibrium position, with no current in the coil, the pointer is at zero. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to the current. (We'll discuss this magnetic interaction in detail in Chapter 27.) As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement.

Thus the angular deflection of the coil and pointer is directly proportional to the coil current, and the device can be calibrated to measure current. The maximum deflection, typically 90° or so, is called *full-scale deflection*. The essential electrical characteristics of the meter are the current $I_{fs}$ required for full-scale deflection (typically on the order of 10 $\mu$A to 10 mA) and the resistance $R_c$ of the coil (typically on the order of 10 to 1000 Ω).

The meter deflection is proportional to the *current* in the coil. If the coil obeys Ohm's law, the current is proportional to the *potential difference* between the terminals of the coil, and the deflection is also proportional to this potential difference. For example, consider a meter whose coil has a resistance $R_c = 20.0\ \Omega$ and that deflects full scale when the current in its coil is $I_{fs} = 1.00$ mA. The corresponding potential difference for full-scale deflection is

$$V = I_{fs}R_c = (1.00 \times 10^{-3}\text{ A})(20.0\ \Omega) = 0.0200\text{ V}$$

### Ammeters

A current-measuring instrument is usually called an **ammeter** (or milliammeter, microammeter, and so forth, depending on the range). *An ammeter always measures the current passing through it.* An *ideal* ammeter, discussed in Section 25.4, would have *zero* resistance, so including it in a branch of a circuit would not affect the current in that branch. Real ammeters always have some finite resistance, but it is always desirable for an ammeter to have as little resistance as possible.

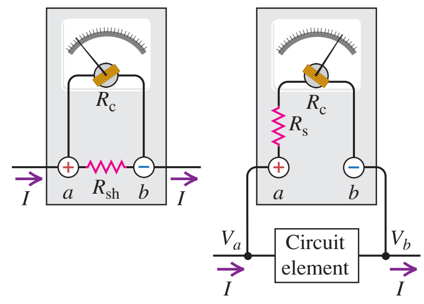
We can adapt any meter to measure currents that are larger than its full-scale reading by connecting a resistor in parallel with it (Fig. 26.15a) so that some of the current bypasses the meter coil. The parallel resistor is called a **shunt resistor** or simply a *shunt,* denoted as $R_{sh}$.

Suppose we want to make a meter with full-scale current $I_{fs}$ and coil resistance $R_c$ into an ammeter with full-scale reading $I_a$. To determine the shunt resistance $R_{sh}$ needed, note that at full-scale deflection the total current through the parallel combination is $I_a$, the current through the coil of the meter is $I_{fs}$, and the current through the shunt is the difference $I_a - I_{fs}$. The potential difference $V_{ab}$ is the same for both paths, so

$$I_{fs}R_c = (I_a - I_{fs})R_{sh} \qquad \text{(for an ammeter)} \tag{26.7}$$

**26.15** Using the same meter to measure (a) current and (b) voltage.

(a) Moving-coil ammeter (b) Moving-coil voltmeter




## Example 26.8 Designing an ammeter

What shunt resistance is required to make the 1.00-mA, 20.0-Ω meter described above into an ammeter with a range of 0 to 50.0 mA?

**SOLUTION**

**IDENTIFY and SET UP:** Since the meter is being used as an ammeter, its internal connections are as shown in Fig. 26.15a. Our target variable is the shunt resistance $R_{sh}$, which we will find using Eq. (26.7). The ammeter must handle a maximum current $I_a = 50.0 \times 10^{-3}$ A. The coil resistance is $R_c = 20.0\ \Omega$, and the meter shows full-scale deflection when the current through the coil is $I_{fs} = 1.00 \times 10^{-3}$ A.

**EXECUTE:** Solving Eq. (26.7) for $R_{sh}$, we find

$$R_{sh} = \frac{I_{fs}R_c}{I_a - I_{fs}} = \frac{(1.00 \times 10^{-3}\ \text{A})(20.0\ \Omega)}{50.0 \times 10^{-3}\ \text{A} - 1.00 \times 10^{-3}\ \text{A}}$$
$$= 0.408\ \Omega$$

**EVALUATE:** It's useful to consider the equivalent resistance $R_{eq}$ of the ammeter as a whole. From Eq. (26.2),

$$R_{eq} = \left(\frac{1}{R_c} + \frac{1}{R_{sh}}\right)^{-1} = \left(\frac{1}{20.0\ \Omega} + \frac{1}{0.408\ \Omega}\right)^{-1}$$
$$= 0.400\ \Omega$$

The shunt resistance is so small in comparison to the coil resistance that the equivalent resistance is very nearly equal to the shunt resistance. The result is an ammeter with a low equivalent resistance and the desired 0–50.0-mA range. At full-scale deflection, $I = I_a = 50.0$ mA, the current through the galvanometer is 1.00 mA, the current through the shunt resistor is 49.0 mA, and $V_{ab} = 0.0200$ V. If the current $I$ is *less* than 50.0 mA, the coil current and the deflection are proportionally less.

## Voltmeters

This same basic meter may also be used to measure potential difference or *voltage*. A voltage-measuring device is called a **voltmeter**. A voltmeter always measures the potential difference between two points, and its terminals must be connected to these points. (Example 25.6 in Section 25.4 described what can happen if a voltmeter is connected incorrectly.) As we discussed in Section 25.4, an ideal voltmeter would have *infinite* resistance, so connecting it between two points in a circuit would not alter any of the currents. Real voltmeters always have finite resistance, but a voltmeter should have large enough resistance that connecting it in a circuit does not change the other currents appreciably.

For the meter described in Example 26.8 the voltage across the meter coil at full-scale deflection is only $I_{fs}R_c = (1.00 \times 10^{-3}\ \text{A})(20.0\ \Omega) = 0.0200$ V. We can extend this range by connecting a resistor $R_s$ in *series* with the coil (Fig. 26.15b). Then only a fraction of the total potential difference appears across the coil itself, and the remainder appears across $R_s$. For a voltmeter with full-scale reading $V_V$, we need a series resistor $R_s$ in Fig. 26.15b such that

$$V_V = I_{fs}(R_c + R_s) \qquad \text{(for a voltmeter)} \tag{26.8}$$

**Application Electromyography**
A fine needle containing two electrodes is being inserted into a muscle in this patient's hand. By using a sensitive voltmeter to measure the potential difference between these electrodes, a physician can probe the muscle's electrical activity. This is an important technique for diagnosing neurological and neuromuscular diseases.

## Example 26.9 Designing a voltmeter

What series resistance is required to make the 1.00-mA, 20.0-$\Omega$ meter described above into a voltmeter with a range of 0 to 10.0 V?

**SOLUTION**

**IDENTIFY and SET UP:** Since this meter is being used as a voltmeter, its internal connections are as shown in Fig. 26.15b. Our target variable is the series resistance $R_s$. The maximum allowable voltage across the voltmeter is $V_V = 10.0$ V. We want this to occur when the current through the coil is $I_{fs} = 1.00 \times 10^{-3}$ A. Our target variable is the series resistance $R_s$, which we find using Eq. (26.8).

**EXECUTE:** From Eq. (26.8),

$$R_s = \frac{V_V}{I_{fs}} - R_c = \frac{10.0\ \text{V}}{0.00100\ \text{A}} - 20.0\ \Omega = 9980\ \Omega$$

**EVALUATE:** At full-scale deflection, $V_{ab} = 10.0$ V, the voltage across the meter is 0.0200 V, the voltage across $R_s$ is 9.98 V, and the current through the voltmeter is 0.00100 A. Most of the voltage appears across the series resistor. The meter's equivalent resistance is a desirably high $R_{eq} = 20.0\ \Omega + 9980\ \Omega = 10{,}000\ \Omega$. Such a meter is called a "1000 ohms-per-volt" meter, referring to the ratio of resistance to full-scale deflection. In normal operation the current through the circuit element being measured ($I$ in Fig. 26.15b) is much greater than 0.00100 A, and the resistance between points $a$ and $b$ in the circuit is much less than 10,000 $\Omega$. The voltmeter draws off only a small fraction of the current and thus disturbs the circuit being measured only slightly.

MasteringPHYSICS

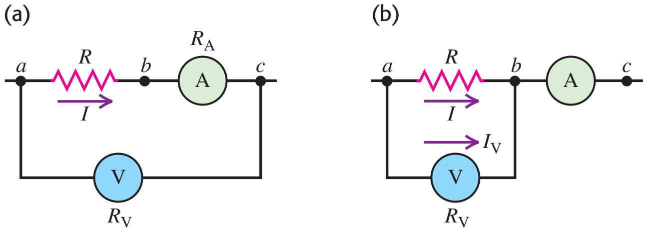
**ActivPhysics 12.4:** Using Ammeters and Voltmeters

## Ammeters and Voltmeters in Combination

A voltmeter and an ammeter can be used together to measure *resistance* and *power.* The resistance $R$ of a resistor equals the potential difference $V_{ab}$ between its terminals divided by the current $I$; that is, $R = V_{ab}/I$. The power input $P$ to any circuit element is the product of the potential difference across it and the current through it: $P = V_{ab}I$. In principle, the most straightforward way to measure $R$ or $P$ is to measure $V_{ab}$ and $I$ simultaneously.

With practical ammeters and voltmeters this isn't quite as simple as it seems. In Fig. 26.16a, ammeter A reads the current $I$ in the resistor $R$. Voltmeter V, however, reads the *sum* of the potential difference $V_{ab}$ across the resistor and the potential difference $V_{bc}$ across the ammeter. If we transfer the voltmeter terminal from $c$ to $b$, as in Fig. 26.16b, then the voltmeter reads the potential difference $V_{ab}$ correctly, but the ammeter now reads the *sum* of the current $I$ in the resistor and the current $I_V$ in the voltmeter. Either way, we have to correct the reading of one instrument or the other unless the corrections are small enough to be negligible.

**26.16** Ammeter–voltmeter method for measuring resistance.




## Example 26.10 Measuring resistance I

The voltmeter in the circuit of Fig. 26.16a reads 12.0 V and the ammeter reads 0.100 A. The meter resistances are $R_V = 10{,}000\ \Omega$ (for the voltmeter) and $R_A = 2.00\ \Omega$ (for the ammeter). What are the resistance $R$ and the power dissipated in the resistor?

**SOLUTION**

**IDENTIFY and SET UP:** The ammeter reads the current $I = 0.100$ A through the resistor, and the voltmeter reads the potential difference between $a$ and $c$. If the ammeter were *ideal* (that is, if $R_A = 0$), there would be zero potential difference between $b$ and $c$, the voltmeter reading $V = 12.0$ V would be equal to the potential difference $V_{ab}$ across the resistor, and the resistance would simply be equal to $R = V/I = (12.0\ \text{V})/(0.100\ \text{A}) = 120\ \Omega$. The ammeter is *not* ideal, however (its resistance is $R_A = 2.00\ \Omega$), so the voltmeter reading $V$ is actually the sum of the potential differences $V_{bc}$ (across the ammeter) and $V_{ab}$ (across the resistor). We use Ohm's law to find the voltage $V_{bc}$ from the known current and ammeter resistance. Then we solve for $V_{ab}$ and the resistance $R$. Given these, we are able to calculate the power $P$ into the resistor.

**EXECUTE:** From Ohm's law, $V_{bc} = IR_A = (0.100\ \text{A})(2.00\ \Omega) = 0.200\ \text{V}$ and $V_{ab} = IR$. The sum of these is $V = 12.0\ \text{V}$, so the potential difference across the resistor is $V_{ab} = V - V_{bc} = (12.0\ \text{V}) - (0.200\ \text{V}) = 11.8\ \text{V}$. Hence the resistance is

$$R = \frac{V_{ab}}{I} = \frac{11.8\ \text{V}}{0.100\ \text{A}} = 118\ \Omega$$

The power dissipated in this resistor is

$$P = V_{ab}I = (11.8\ \text{V})(0.100\ \text{A}) = 1.18\ \text{W}$$

**EVALUATE:** You can confirm this result for the power by using the alternative formula $P = I^2R$. Do you get the same answer?

## Example 26.11 Measuring resistance II

Suppose the meters of Example 26.10 are connected to a different resistor as shown in Fig. 26.16b, and the readings obtained on the meters are the same as in Example 26.10. What is the value of this new resistance $R$, and what is the power dissipated in the resistor?

**SOLUTION**

**IDENTIFY and SET UP:** In Example 26.10 the ammeter read the actual current through the resistor, but the voltmeter reading was not the same as the potential difference across the resistor. Now the situation is reversed: The voltmeter reading $V = 12.0\ \text{V}$ shows the actual potential difference $V_{ab}$ across the resistor, but the ammeter reading $I_A = 0.100\ \text{A}$ is *not* equal to the current $I$ through the resistor. Applying the junction rule at $b$ in Fig. 26.16b shows that $I_A = I + I_V$, where $I_V$ is the current through the voltmeter. We find $I_V$ from the given values of $V$ and the voltmeter resistance $R_V$, and we use this value to find the resistor current $I$. We then determine the resistance $R$ from $I$ and the voltmeter reading, and calculate the power as in Example 26.10.

**EXECUTE:** We have $I_V = V/R_V = (12.0\ \text{V})/(10{,}000\ \Omega) = 1.20\ \text{mA}$. The actual current $I$ in the resistor is $I = I_A - I_V = 0.100\ \text{A} - 0.0012\ \text{A} = 0.0988\ \text{A}$, and the resistance is

$$R = \frac{V_{ab}}{I} = \frac{12.0\ \text{V}}{0.0988\ \text{A}} = 121\ \Omega$$

The power dissipated in the resistor is

$$P = V_{ab}I = (12.0\ \text{V})(0.0988\ \text{A}) = 1.19\ \text{W}$$

**EVALUATE:** Had the meters been ideal, our results would have been $R = 12.0\ \text{V}/0.100\ \text{A} = 120\ \Omega$ and $P = VI = (12.0\ \text{V}) \times (0.100\ \text{A}) = 1.2\ \text{W}$ both here and in Example 26.10. The actual (correct) results are not too different in either case. That's because the ammeter and voltmeter are nearly ideal: Compared with the resistance $R$ under test, the ammeter resistance $R_A$ is very small and the voltmeter resistance $R_V$ is very large. Under these conditions, treating the meters as ideal yields pretty good results; accurate work requires calculations as in these two examples.

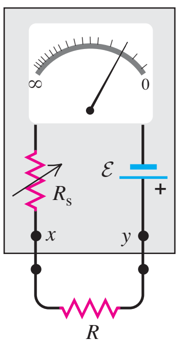
## Ohmmeters

An alternative method for measuring resistance is to use a d'Arsonval meter in an arrangement called an **ohmmeter.** It consists of a meter, a resistor, and a source (often a flashlight battery) connected in series (Fig. 26.17). The resistance $R$ to be measured is connected between terminals $x$ and $y$.

The series resistance $R_s$ is variable; it is adjusted so that when terminals $x$ and $y$ are short-circuited (that is, when $R = 0$), the meter deflects full scale. When nothing is connected to terminals $x$ and $y$, so that the circuit between $x$ and $y$ is *open* (that is, when $R \to \infty$), there is no current and hence no deflection. For any intermediate value of $R$ the meter deflection depends on the value of $R$, and the meter scale can be calibrated to read the resistance $R$ directly. Larger currents correspond to smaller resistances, so this scale reads backward compared to the scale showing the current.

In situations in which high precision is required, instruments containing d'Arsonval meters have been supplanted by electronic instruments with direct digital readouts. Digital voltmeters can be made with extremely high internal resistance, of the order of 100 M$\Omega$. Figure 26.18 shows a digital *multimeter,* an instrument that can measure voltage, current, or resistance over a wide range.

**26.17** Ohmmeter circuit. The resistor $R_s$ has a variable resistance, as is indicated by the arrow through the resistor symbol. To use the ohmmeter, first connect $x$ directly to $y$ and adjust $R_s$ until the meter reads zero. Then connect $x$ and $y$ across the resistor $R$ and read the scale.

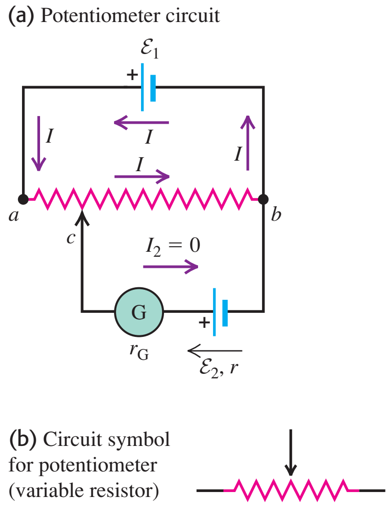



## The Potentiometer

The *potentiometer* is an instrument that can be used to measure the emf of a source without drawing any current from the source; it also has a number of other useful applications. Essentially, it balances an unknown potential difference against an adjustable, measurable potential difference.

**26.18** This digital multimeter can be used as a voltmeter (red arc), ammeter (yellow arc), or ohmmeter (green arc).

**26.19** A potentiometer.




The principle of the potentiometer is shown schematically in Fig. 26.19a. A resistance wire *ab* of total resistance $R_{ab}$ is permanently connected to the terminals of a source of known emf $\mathcal{E}_1$. A sliding contact *c* is connected through the galvanometer G to a second source whose emf $\mathcal{E}_2$ is to be measured. As contact *c* is moved along the resistance wire, the resistance $R_{cb}$ between points *c* and *b* varies; if the resistance wire is uniform, $R_{cb}$ is proportional to the length of wire between *c* and *b*. To determine the value of $\mathcal{E}_2$, contact *c* is moved until a position is found at which the galvanometer shows no deflection; this corresponds to zero current passing through $\mathcal{E}_2$. With $I_2 = 0$, Kirchhoff's loop rule gives

$$\mathcal{E}_2 = IR_{cb}$$

With $I_2 = 0$, the current $I$ produced by the emf $\mathcal{E}_1$ has the same value no matter what the value of the emf $\mathcal{E}_2$. We calibrate the device by replacing $\mathcal{E}_2$ by a source of known emf; then any unknown emf $\mathcal{E}_2$ can be found by measuring the length of wire *cb* for which $I_2 = 0$. Note that for this to work, $V_{ab}$ must be greater than $\mathcal{E}_2$.

The term *potentiometer* is also used for any variable resistor, usually having a circular resistance element and a sliding contact controlled by a rotating shaft and knob. The circuit symbol for a potentiometer is shown in Fig. 26.19b.

**Test Your Understanding of Section 26.3** You want to measure the current through and the potential difference across the 2-Ω resistor shown in Fig. 26.12 (Example 26.6 in Section 26.2). (a) How should you connect an ammeter and a voltmeter to do this? (i) ammeter and voltmeter both in series with the 2-Ω resistor; (ii) ammeter in series with the 2-Ω resistor and voltmeter connected between points *b* and *d*; (iii) ammeter connected between points *b* and *d* and voltmeter in series with the 2-Ω resistor; (iv) ammeter and voltmeter both connected between points *b* and *d*. (b) What resistances should these meters have? (i) Ammeter and voltmeter resistances should both be much greater than 2 Ω; (ii) ammeter resistance should be much greater than 2 Ω and voltmeter resistance should be much less than 2 Ω; (iii) ammeter resistance should be much less than 2 Ω and voltmeter resistance should be much greater than 2 Ω; (iv) ammeter and voltmeter resistances should both be much less than 2 Ω.

## 26.4 *R-C* Circuits

In the circuits we have analyzed up to this point, we have assumed that all the emfs and resistances are *constant* (time independent) so that all the potentials, currents, and powers are also independent of time. But in the simple act of charging or discharging a capacitor we find a situation in which the currents, voltages, and powers *do* change with time.

Many devices incorporate circuits in which a capacitor is alternately charged and discharged. These include flashing traffic lights, automobile turn signals, and electronic flash units. Understanding what happens in such circuits is thus of great practical importance.

### Charging a Capacitor

Figure 26.20 shows a simple circuit for charging a capacitor. A circuit such as this that has a resistor and a capacitor in series is called an ***R-C* circuit.** We idealize the battery (or power supply) to have a constant emf $\mathcal{E}$ and zero internal resistance $(r = 0)$, and we neglect the resistance of all the connecting conductors.

We begin with the capacitor initially uncharged (Fig. 26.20a); then at some initial time $t = 0$ we close the switch, completing the circuit and permitting current around the loop to begin charging the capacitor (Fig. 26.20b). For all practical purposes, the current begins at the same instant in every conducting part of the circuit, and at each instant the current is the same in every part.

**CAUTION** **Lowercase means time-varying** Up to this point we have been working with constant potential differences (voltages), currents, and charges, and we have used *capital* letters $V$, $I$, and $Q$, respectively, to denote these quantities. To distinguish between quantities that vary with time and those that are constant, we will use *lowercase* letters $v$, $i$, and $q$ for time-varying voltages, currents, and charges, respectively. We suggest that you follow this same convention in your own work.

Because the capacitor in Fig. 26.20 is initially uncharged, the potential difference $v_{bc}$ across it is zero at $t = 0$. At this time, from Kirchhoff's loop law, the voltage $v_{ab}$ across the resistor $R$ is equal to the battery emf $\mathcal{E}$. The initial ($t = 0$) current through the resistor, which we will call $I_0$, is given by Ohm's law: $I_0 = v_{ab}/R = \mathcal{E}/R$.

As the capacitor charges, its voltage $v_{bc}$ increases and the potential difference $v_{ab}$ across the resistor decreases, corresponding to a decrease in current. The sum of these two voltages is constant and equal to $\mathcal{E}$. After a long time the capacitor becomes fully charged, the current decreases to zero, and the potential difference $v_{ab}$ across the resistor becomes zero. Then the entire battery emf $\mathcal{E}$ appears across the capacitor and $v_{bc} = \mathcal{E}$.

Let $q$ represent the charge on the capacitor and $i$ the current in the circuit at some time $t$ after the switch has been closed. We choose the positive direction for the current to correspond to positive charge flowing onto the left-hand capacitor plate, as in Fig. 26.20b. The instantaneous potential differences $v_{ab}$ and $v_{bc}$ are

$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C}$$

Using these in Kirchhoff's loop rule, we find

$$\mathcal{E} - iR - \frac{q}{C} = 0 \tag{26.9}$$

The potential drops by an amount $iR$ as we travel from $a$ to $b$ and by $q/C$ as we travel from $b$ to $c$. Solving Eq. (26.9) for $i$, we find

$$i = \frac{\mathcal{E}}{R} - \frac{q}{RC} \tag{26.10}$$

At time $t = 0$, when the switch is first closed, the capacitor is uncharged, and so $q = 0$. Substituting $q = 0$ into Eq. (26.10), we find that the *initial* current $I_0$ is given by $I_0 = \mathcal{E}/R$, as we have already noted. If the capacitor were not in the circuit, the last term in Eq. (26.10) would not be present; then the current would be *constant* and equal to $\mathcal{E}/R$.

As the charge $q$ increases, the term $q/RC$ becomes larger and the capacitor charge approaches its final value, which we will call $Q_f$. The current decreases and eventually becomes zero. When $i = 0$, Eq. (26.10) gives

$$\frac{\mathcal{E}}{R} = \frac{Q_f}{RC} \qquad Q_f = C\mathcal{E} \tag{26.11}$$

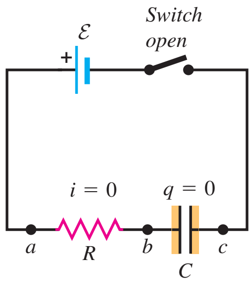
Note that the final charge $Q_f$ does not depend on $R$.

Figure 26.21 shows the current and capacitor charge as functions of time. At the instant the switch is closed ($t = 0$), the current jumps from zero to its initial value $I_0 = \mathcal{E}/R$; after that, it gradually approaches zero. The capacitor charge starts at zero and gradually approaches the final value given by Eq. (26.11), $Q_f = C\mathcal{E}$.

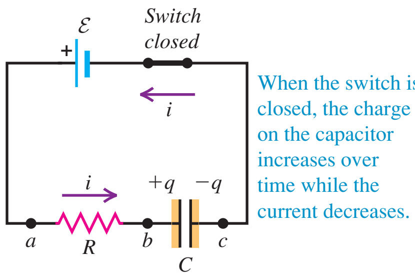
We can derive general expressions for the charge $q$ and current $i$ as functions of time. With our choice of the positive direction for current (Fig. 26.20b), $i$ equals the rate at which positive charge arrives at the left-hand (positive)

**26.20** Charging a capacitor. (a) Just before the switch is closed, the charge $q$ is zero. (b) When the switch closes (at $t = 0$), the current jumps from zero to $\mathcal{E}/R$. As time passes, $q$ approaches $Q_f$ and the current $i$ approaches zero.

(a) Capacitor initially uncharged

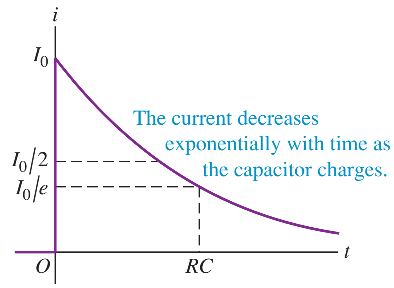



(b) Charging the capacitor




**26.21** Current $i$ and capacitor charge $q$ as functions of time for the circuit of Fig. 26.20. The initial current is $I_0$ and the initial capacitor charge is zero. The current asymptotically approaches zero, and the capacitor charge asymptotically approaches a final value of $Q_f$.

(a) Graph of current versus time for a charging capacitor




(b) Graph of capacitor charge versus time for a charging capacitor

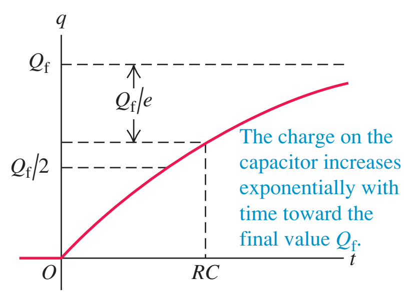


plate of the capacitor, so $i = dq/dt$. Making this substitution in Eq. (26.10), we have

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\mathcal{E})$$

We can rearrange this to

$$\frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC}$$

and then integrate both sides. We change the integration variables to $q'$ and $t'$ so that we can use $q$ and $t$ for the upper limits. The lower limits are $q' = 0$ and $t' = 0$:

$$\int_0^q \frac{dq'}{q' - C\mathcal{E}} = -\int_0^t \frac{dt'}{RC}$$

When we carry out the integration, we get

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

Exponentiating both sides (that is, taking the inverse logarithm) and solving for $q$, we find

$$\frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$$

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_{\mathrm{f}}(1 - e^{-t/RC}) \quad \text{(}R\text{-}C\text{ circuit, charging capacitor)} \tag{26.12}$$

The instantaneous current $i$ is just the time derivative of Eq. (26.12):

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \quad \text{(}R\text{-}C\text{ circuit, charging capacitor)} \tag{26.13}$$

The charge and current are both *exponential* functions of time. Figure 26.21a is a graph of Eq. (26.13) and Fig. 26.21b is a graph of Eq. (26.12).

## Time Constant

After a time equal to $RC$, the current in the $R$-$C$ circuit has decreased to $1/e$ (about 0.368) of its initial value. At this time, the capacitor charge has reached $(1 - 1/e) = 0.632$ of its final value $Q_{\mathrm{f}} = C\mathcal{E}$. The product $RC$ is therefore a measure of how quickly the capacitor charges. We call $RC$ the **time constant,** or the **relaxation time,** of the circuit, denoted by $\tau$:

$$\tau = RC \qquad \text{(time constant for } R\text{-}C \text{ circuit)} \tag{26.14}$$

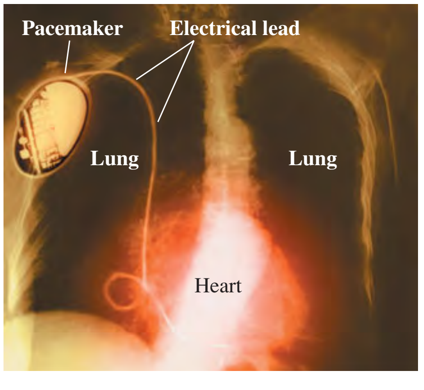
When $\tau$ is small, the capacitor charges quickly; when it is larger, the charging takes more time. If the resistance is small, it's easier for current to flow, and the capacitor charges more quickly. If $R$ is in ohms and $C$ in farads, $\tau$ is in seconds.

In Fig. 26.21a the horizontal axis is an *asymptote* for the curve. Strictly speaking, $i$ never becomes exactly zero. But the longer we wait, the closer it gets. After a time equal to $10RC$, the current has decreased to 0.000045 of its initial value. Similarly, the curve in Fig. 26.21b approaches the horizontal dashed line labeled $Q_{\mathrm{f}}$ as an asymptote. The charge $q$ never attains exactly this value, but after a time equal to $10RC$, the difference between $q$ and $Q_{\mathrm{f}}$ is only 0.000045 of $Q_{\mathrm{f}}$. We invite you to verify that the product $RC$ has units of time.

MasteringPHYSICS®

**PhET:** Circuit Construction Kit (AC+DC)
**PhET:** Circuit Construction Kit (DC Only)
**ActivPhysics 12.6:** Capacitance
**ActivPhysics 12.7:** Series and Parallel Capacitors
**ActivPhysics 12.8:** Circuit Time Constants

**Application Pacemakers and Capacitors**
This x-ray image shows a pacemaker implanted in a patient with a malfunctioning sinoatrial node, the part of the heart that generates the electrical signal to trigger heartbeats. The pacemaker circuit contains a battery, a capacitor, and a computer-controlled switch. To maintain regular beating, once per second the switch discharges the capacitor and sends an electrical pulse along the lead to the heart. The switch then flips to allow the capacitor to recharge for the next pulse.



## Discharging a Capacitor

Now suppose that after the capacitor in Fig. 26.21b has acquired a charge $Q_0$, we remove the battery from our *R-C* circuit and connect points *a* and *c* to an open switch (Fig. 26.22a). We then close the switch and at the same instant reset our stopwatch to $t = 0$; at that time, $q = Q_0$. The capacitor then *discharges* through the resistor, and its charge eventually decreases to zero.

Again let $i$ and $q$ represent the time-varying current and charge at some instant after the connection is made. In Fig. 26.22b we make the same choice of the positive direction for current as in Fig. 26.20b. Then Kirchhoff's loop rule gives Eq. (26.10) but with $\mathcal{E} = 0$; that is,

$$i = \frac{dq}{dt} = -\frac{q}{RC} \qquad (26.15)$$

The current $i$ is now negative; this is because positive charge $q$ is leaving the left-hand capacitor plate in Fig. 26.22b, so the current is in the direction opposite to that shown in the figure. At time $t = 0$, when $q = Q_0$, the initial current is $I_0 = -Q_0/RC$.

To find $q$ as a function of time, we rearrange Eq. (26.15), again change the names of the variables to $q'$ and $t'$, and integrate. This time the limits for $q'$ are $Q_0$ to $q$. We get

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt'$$

$$\ln\frac{q}{Q_0} = -\frac{t}{RC}$$

$$q = Q_0 e^{-t/RC} \quad \text{(}R\text{-}C\text{ circuit, discharging capacitor)} \qquad (26.16)$$

The instantaneous current $i$ is the derivative of this with respect to time:

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \quad \text{(}R\text{-}C\text{ circuit, discharging capacitor)} \qquad (26.17)$$

We graph the current and the charge in Fig. 26.23; both quantities approach zero exponentially with time. Comparing these results with Eqs. (26.12) and (26.13), we note that the expressions for the current are identical, apart from the sign of $I_0$. The capacitor charge approaches zero asymptotically in Eq. (26.16), while the *difference* between $q$ and $Q$ approaches zero asymptotically in Eq. (26.12).

Energy considerations give us additional insight into the behavior of an *R-C* circuit. While the capacitor is charging, the instantaneous rate at which the battery delivers energy to the circuit is $P = \mathcal{E}i$. The instantaneous rate at which electrical energy is dissipated in the resistor is $i^2R$, and the rate at which energy is stored in the capacitor is $iv_{bc} = iq/C$. Multiplying Eq. (26.9) by $i$, we find

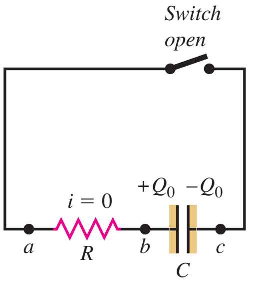
$$\mathcal{E}i = i^2R + \frac{iq}{C} \qquad (26.18)$$

This means that of the power $\mathcal{E}i$ supplied by the battery, part $(i^2R)$ is dissipated in the resistor and part $(iq/C)$ is stored in the capacitor.

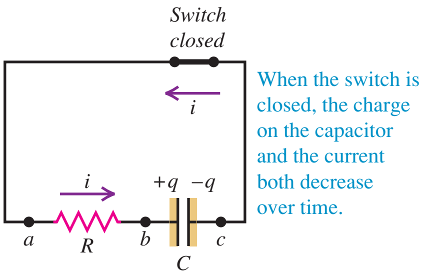
The *total* energy supplied by the battery during charging of the capacitor equals the battery emf $\mathcal{E}$ multiplied by the total charge $Q_f$, or $\mathcal{E}Q_f$. The total energy stored in the capacitor, from Eq. (24.9), is $Q_f\mathcal{E}/2$. Thus, of the energy supplied by the battery, *exactly half* is stored in the capacitor, and the other half is dissipated in the resistor. This half-and-half division of energy doesn't depend on $C$, $R$, or $\mathcal{E}$. You can verify this result by taking the integral over time of each of the power quantities in Eq. (26.18) (see Problem 26.88).

**26.22** Discharging a capacitor. (a) Before the switch is closed at time $t = 0$, the capacitor charge is $Q_0$ and the current is zero. (b) At time $t$ after the switch is closed, the capacitor charge is $q$ and the current is $i$. The actual current direction is opposite to the direction shown; $i$ is negative. After a long time, $q$ and $i$ both approach zero.

(a) Capacitor initially charged

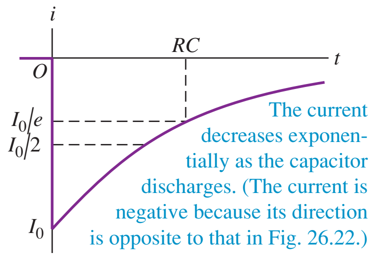



(b) Discharging the capacitor

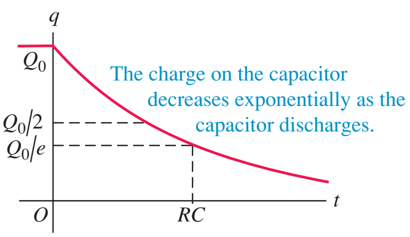



**26.23** Current $i$ and capacitor charge $q$ as functions of time for the circuit of Fig. 26.22. The initial current is $I_0$ and the initial capacitor charge is $Q_0$. Both $i$ and $q$ asymptotically approach zero.

(a) Graph of current versus time for a discharging capacitor




(b) Graph of capacitor charge versus time for a discharging capacitor



## Example 26.12 Charging a capacitor

A 10-MΩ resistor is connected in series with a 1.0-$\mu$F capacitor and a battery with emf 12.0 V. Before the switch is closed at time $t = 0$, the capacitor is uncharged. (a) What is the time constant? (b) What fraction of the final charge $Q_f$ is on the capacitor at $t = 46$ s? (c) What fraction of the initial current $I_0$ is still flowing at $t = 46$ s?

**SOLUTION**

**IDENTIFY and SET UP:** This is the same situation as shown in Fig. 26.20, with $R = 10$ MΩ, $C = 1.0$ $\mu$F, and $\mathcal{E} = 12.0$ V. The charge $q$ and current $i$ vary with time as shown in Fig. 26.21. Our target variables are (a) the time constant $\tau$, (b) the ratio $q/Q_f$ at $t = 46$ s, and (c) the ratio $i/I_0$ at $t = 46$ s. Equation (26.14) gives $\tau$. For a capacitor being charged, Eq. (26.12) gives $q$ and Eq. (26.13) gives $i$.

**EXECUTE:** (a) From Eq. (26.14),

$$\tau = RC = (10 \times 10^6\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 10\ \text{s}$$

(b) From Eq. (26.12),

$$\frac{q}{Q_f} = 1 - e^{-t/RC} = 1 - e^{-(46\ \text{s})/(10\ \text{s})} = 0.99$$

(c) From Eq. (26.13),

$$\frac{i}{I_0} = e^{-t/RC} = e^{-(46\ \text{s})/(10\ \text{s})} = 0.010$$

**EVALUATE:** After 4.6 time constants the capacitor is 99% charged and the charging current has decreased to 1.0% of its initial value. The circuit would charge more rapidly if we reduced the time constant by using a smaller resistance.

## Example 26.13 Discharging a capacitor

The resistor and capacitor of Example 26.12 are reconnected as shown in Fig. 26.22. The capacitor has an initial charge of 5.0 $\mu$C and is discharged by closing the switch at $t = 0$. (a) At what time will the charge be equal to 0.50 $\mu$C? (b) What is the current at this time?

**SOLUTION**

**IDENTIFY and SET UP:** Now the capacitor is being discharged, so $q$ and $i$ vary with time as in Fig. 26.23, with $Q_0 = 5.0 \times 10^{-6}$ C. Again we have $RC = \tau = 10$ s. Our target variables are (a) the value of $t$ at which $q = 0.50$ $\mu$C and (b) the value of $i$ at this time. We first solve Eq. (26.16) for $t$, and then solve Eq. (26.17) for $i$.

**EXECUTE:** (a) Solving Eq. (26.16) for the time $t$ gives

$$t = -RC\ln\frac{q}{Q_0} = -(10\ \text{s})\ln\frac{0.50\ \mu\text{C}}{5.0\ \mu\text{C}} = 23\ \text{s} = 2.3\tau$$

(b) From Eq. (26.17), with $Q_0 = 5.0\ \mu\text{C} = 5.0 \times 10^{-6}$ C,

$$i = -\frac{Q_0}{RC}e^{-t/RC} = -\frac{5.0 \times 10^{-6}\ \text{C}}{10\ \text{s}}e^{-2.3} = -5.0 \times 10^{-8}\ \text{A}$$

**EVALUATE:** The current in part (b) is negative because $i$ has the opposite sign when the capacitor is discharging than when it is charging. Note that we could have avoided evaluating $e^{-t/RC}$ by noticing that at the time in question, $q = 0.10Q_0$; from Eq. (26.16) this means that $e^{-t/RC} = 0.10$.

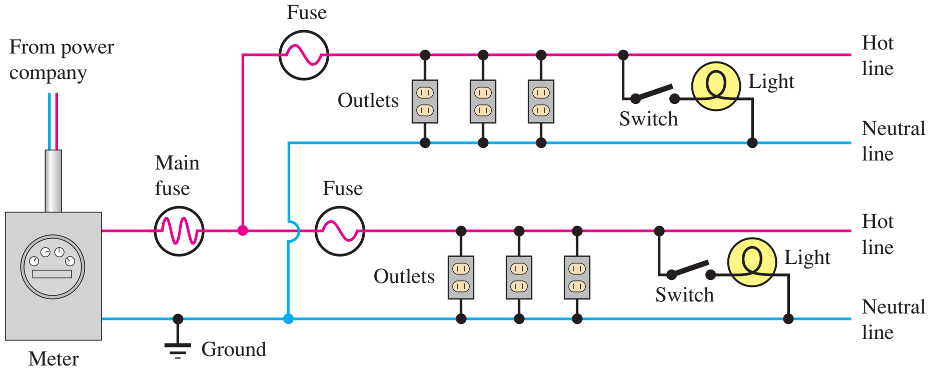
**Test Your Understanding of Section 26.4** The energy stored in a capacitor is equal to $q^2/2C$. When a capacitor is discharged, what fraction of the initial energy remains after an elapsed time of one time constant? (i) $1/e$; (ii) $1/e^2$; (iii) $1 - 1/e$; (iv) $(1 - 1/e)^2$; (v) answer depends on how much energy was stored initially.

# 26.5 Power Distribution Systems

We conclude this chapter with a brief discussion of practical household and automotive electric-power distribution systems. Automobiles use direct-current (dc) systems, while nearly all household, commercial, and industrial systems use alternating current (ac) because of the ease of stepping voltage up and down with transformers. Most of the same basic wiring concepts apply to both. We'll talk about alternating-current circuits in greater detail in Chapter 31.

The various lamps, motors, and other appliances to be operated are always connected in *parallel* to the power source (the wires from the power company for houses, or from the battery and alternator for a car). If appliances were connected in series, shutting one appliance off would shut them all off (see Example 26.2 in Section 26.1). Figure 26.24 shows the basic idea of house wiring. One side of the "line," as the pair of conductors is called, is called the *neutral* side; it is always connected to

**26.24** Schematic diagram of part of a house wiring system. Only two branch circuits are shown; an actual system might have four to thirty branch circuits. Lamps and appliances may be plugged into the outlets. The grounding wires, which normally carry no current, are not shown.




"ground" at the entrance panel. For houses, *ground* is an actual electrode driven into the earth (which is usually a good conductor) or sometimes connected to the household water pipes. Electricians speak of the "hot" side and the "neutral" side of the line. Most modern house wiring systems have *two* hot lines with opposite polarity with respect to the neutral. We'll return to this detail later.

Household voltage is nominally 120 V in the United States and Canada, and often 240 V in Europe. (For alternating current, which varies sinusoidally with time, these numbers represent the *root-mean-square* voltage, which is $1/\sqrt{2}$ times the peak voltage. We'll discuss this further in Section 31.1.) The amount of current $I$ drawn by a given device is determined by its power input $P$, given by Eq. (25.17): $P = VI$. Hence $I = P/V$. For example, the current in a 100-W light bulb is

$$I = \frac{P}{V} = \frac{100\ \text{W}}{120\ \text{V}} = 0.83\ \text{A}$$

The power input to this bulb is actually determined by its resistance $R$. Using Eq. (25.18), which states that $P = VI = I^2R = V^2/R$ for a resistor, the resistance of this bulb at operating temperature is

$$R = \frac{V}{I} = \frac{120\ \text{V}}{0.83\ \text{A}} = 144\ \Omega \qquad \text{or} \qquad R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{100\ \text{W}} = 144\ \Omega$$

Similarly, a 1500-W waffle iron draws a current of $(1500\ \text{W})/(120\ \text{V}) = 12.5\ \text{A}$ and has a resistance, at operating temperature, of $9.6\ \Omega$. Because of the temperature dependence of resistivity, the resistances of these devices are considerably less when they are cold. If you measure the resistance of a 100-W light bulb with an ohmmeter (whose small current causes very little temperature rise), you will probably get a value of about $10\ \Omega$. When a light bulb is turned on, this low resistance causes an initial surge of current until the filament heats up. That's why a light bulb that's ready to burn out nearly always does so just when you turn it on.

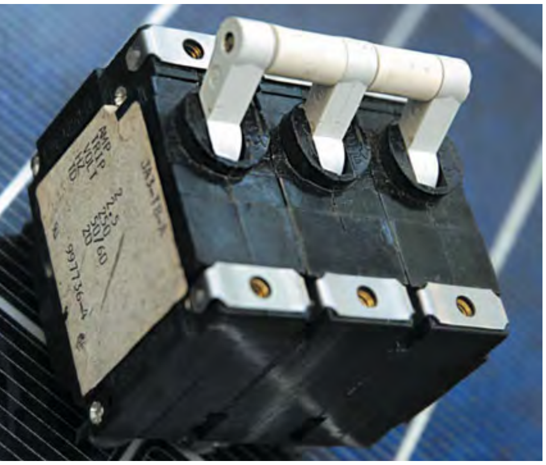
## Circuit Overloads and Short Circuits

The maximum current available from an individual circuit is limited by the resistance of the wires. As we discussed in Section 25.5, the $I^2R$ power loss in the wires causes them to become hot, and in extreme cases this can cause a fire or melt the wires. Ordinary lighting and outlet wiring in houses usually uses 12-gauge wire. This has a diameter of 2.05 mm and can carry a maximum current of 20 A safely (without overheating). Larger-diameter wires of the same length have lower resistance [see Eq. (25.10)]. Hence 8-gauge (3.26 mm) or 6-gauge (4.11 mm) are used for high-current appliances such as clothes dryers, and 2-gauge (6.54 mm) or larger is used for the main power lines entering a house.

**26.25** (a) Excess current will melt the thin wire of lead–tin alloy that runs along the length of a fuse, inside the transparent housing. (b) The switch on this circuit breaker will flip if the maximum allowable current is exceeded.

(a)

(b)



Protection against overloading and overheating of circuits is provided by fuses or circuit breakers. A *fuse* contains a link of lead–tin alloy with a very low melting temperature; the link melts and breaks the circuit when its rated current is exceeded (Fig. 26.25a). A *circuit breaker* is an electromechanical device that performs the same function, using an electromagnet or a bimetallic strip to "trip" the breaker and interrupt the circuit when the current exceeds a specified value (Fig. 26.25b). Circuit breakers have the advantage that they can be reset after they are tripped, while a blown fuse must be replaced.

If your system has fuses and you plug too many high-current appliances into the same outlet, the fuse blows. *Do not* replace the fuse with one of larger rating; if you do, you risk overheating the wires and starting a fire. The only safe solution is to distribute the appliances among several circuits. Modern kitchens often have three or four separate 20-A circuits.

Contact between the hot and neutral sides of the line causes a *short circuit.* Such a situation, which can be caused by faulty insulation or by any of a variety of mechanical malfunctions, provides a very low-resistance current path, permitting a very large current that would quickly melt the wires and ignite their insulation if the current were not interrupted by a fuse or circuit breaker (see Example 25.10 in Section 25.5). An equally dangerous situation is a broken wire that interrupts the current path, creating an *open circuit.* This is hazardous because of the sparking that can occur at the point of intermittent contact.

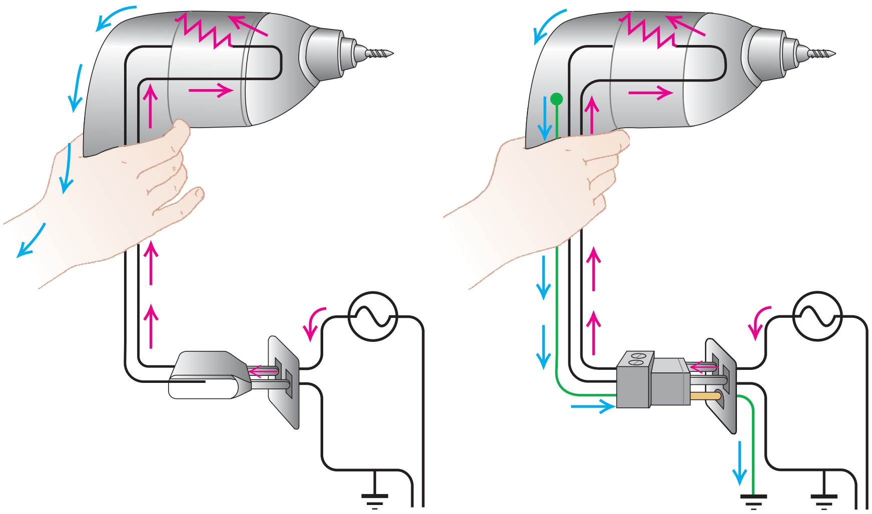
In approved wiring practice, a fuse or breaker is placed *only* in the hot side of the line, never in the neutral side. Otherwise, if a short circuit should develop because of faulty insulation or other malfunction, the ground-side fuse could blow. The hot side would still be live and would pose a shock hazard if you touched the live conductor and a grounded object such as a water pipe. For similar reasons the wall switch for a light fixture is always in the hot side of the line, never the neutral side.

Further protection against shock hazard is provided by a third conductor called the *grounding wire,* included in all present-day wiring. This conductor corresponds to the long round or U-shaped prong of the three-prong connector plug on an appliance or power tool. It is connected to the neutral side of the line at the entrance panel. The grounding wire normally carries no current, but it connects the metal case or frame of the device to ground. If a conductor on the hot side of the line accidentally contacts the frame or case, the grounding conductor provides a current path, and the fuse blows. Without the ground wire, the frame could become "live"—that is, at a potential 120 V above ground. Then if you touched it and a water pipe (or even a damp basement floor) at the same time, you could get a dangerous shock (Fig. 26.26). In some situations, especially outlets located outdoors or near a sink or other water pipes, a special kind of circuit breaker called a *ground-fault interrupter* (GFI or GFCI) is used. This device senses the difference in current between the hot and neutral conductors (which is normally zero) and trips when this difference exceeds some very small value, typically 5 mA.

## Household and Automotive Wiring

Most modern household wiring systems actually use a slight elaboration of the system described above. The power company provides *three* conductors. One is neutral; the other two are both at 120 V with respect to the neutral but with opposite polarity, giving a voltage between them of 240 V. The power company calls this a *three-wire line,* in contrast to the 120-V two-wire (plus ground wire) line described above. With a three-wire line, 120-V lamps and appliances can be connected between neutral and either hot conductor, and high-power devices requiring 240 V, such as electric ranges and clothes dryers, are connected between the two hot lines.

All of the above discussion can be applied directly to automobile wiring. The voltage is about 13 V (direct current); the power is supplied by the battery and by




**26.26** (a) If a malfunctioning electric drill is connected to a wall socket via a two-prong plug, a person may receive a shock. (b) When the drill malfunctions when connected via a three-prong plug, a person touching it receives no shock, because electric charge flows through the ground wire (shown in green) to the third prong and into the ground rather than into the person's body. If the ground current is appreciable, the fuse blows.

the alternator, which charges the battery when the engine is running. The neutral side of each circuit is connected to the body and frame of the vehicle. For this low voltage a separate grounding conductor is not required for safety. The fuse or circuit breaker arrangement is the same in principle as in household wiring. Because of the lower voltage (less energy per charge), more current (a greater number of charges per second) is required for the same power; a 100-W headlight bulb requires a current of about $(100\text{ W})/(13\text{ V}) = 8\text{ A}$.

Although we spoke of *power* in the above discussion, what we buy from the power company is *energy.* Power is energy transferred per unit time, so energy is average power multiplied by time. The usual unit of energy sold by the power company is the kilowatt-hour $(1\text{ kW}\cdot\text{h})$:

$$1\text{ kW}\cdot\text{h} = (10^3\text{ W})(3600\text{ s}) = 3.6\times10^6\text{ W}\cdot\text{s} = 3.6\times10^6\text{ J}$$

In the United States, one kilowatt-hour typically costs 8 to 27 cents, depending on the location and quantity of energy purchased. To operate a 1500-W (1.5-kW) waffle iron continuously for 1 hour requires 1.5 kW·h of energy; at 10 cents per kilowatt-hour, the energy cost is 15 cents. The cost of operating any lamp or appliance for a specified time can be calculated in the same way if the power rating is known. However, many electric cooking utensils (including waffle irons) cycle on and off to maintain a constant temperature, so the average power may be less than the power rating marked on the device.

## Example 26.14 A kitchen circuit

An 1800-W toaster, a 1.3-kW electric frying pan, and a 100-W lamp are plugged into the same 20-A, 120-V circuit. (a) What current is drawn by each device, and what is the resistance of each device? (b) Will this combination trip the circuit breaker?

**SOLUTION**

**IDENTIFY and SET UP:** When plugged into the same circuit, the three devices are connected in parallel, so the voltage across each appliance is $V = 120\text{ V}$. We find the current $I$ drawn by each device using the relationship $P = VI$, where $P$ is the power input of the device. To find the resistance $R$ of each device we use the relationship $P = V^2/R$.

**EXECUTE:** (a) To simplify the calculation of current and resistance, we note that $I = P/V$ and $R = V^2/P$. Hence

$$I_{\text{toaster}} = \frac{1800\text{ W}}{120\text{ V}} = 15\text{ A} \qquad R_{\text{toaster}} = \frac{(120\text{ V})^2}{1800\text{ W}} = 8\ \Omega$$

$$I_{\text{frying pan}} = \frac{1300\text{ W}}{120\text{ V}} = 11\text{ A} \qquad R_{\text{frying pan}} = \frac{(120\text{ V})^2}{1300\text{ W}} = 11\ \Omega$$

$$I_{\text{lamp}} = \frac{100\text{ W}}{120\text{ V}} = 0.83\text{ A} \qquad R_{\text{lamp}} = \frac{(120\text{ V})^2}{100\text{ W}} = 144\ \Omega$$

For constant voltage the device with the *least* resistance (in this case the toaster) draws the most current and receives the most power.

*Continued*

(b) The total current through the line is the sum of the currents drawn by the three devices:

$$I = I_{\text{toaster}} + I_{\text{frying pan}} + I_{\text{lamp}}$$
$$= 15\ \text{A} + 11\ \text{A} + 0.83\ \text{A} = 27\ \text{A}$$

This exceeds the 20-A rating of the line, and the circuit breaker will indeed trip.

**EVALUATE:** We could also find the total current by using $I = P/V$ and dividing the total power $P$ delivered to all three devices by the voltage:

$$I = \frac{P_{\text{toaster}} + P_{\text{frying pan}} + P_{\text{lamp}}}{V}$$
$$= \frac{1800\ \text{W} + 1300\ \text{W} + 100\ \text{W}}{120\ \text{V}} = 27\ \text{A}$$

A third way to determine $I$ is to use $I = V/R_{\text{eq}}$, where $R_{\text{eq}}$ is the equivalent resistance of the three devices in parallel:

$$I = \frac{V}{R_{\text{eq}}} = (120\ \text{V})\left(\frac{1}{8\ \Omega} + \frac{1}{11\ \Omega} + \frac{1}{144\ \Omega}\right) = 27\ \text{A}$$

Appliances with such current demands are common, so modern kitchens have more than one 20-A circuit. To keep currents safely below 20 A, the toaster and frying pan should be plugged into different circuits.

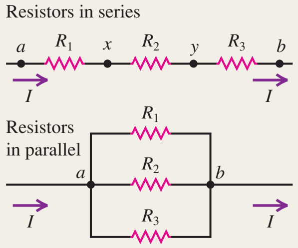
**Test Your Understanding of Section 26.5** To prevent the circuit breaker in Example 26.14 from blowing, a home electrician replaces the circuit breaker with one rated at 40 A. Is this a reasonable thing to do?

# CHAPTER 26 SUMMARY

**Resistors in series and parallel:** When several resistors $R_1, R_2, R_3, \ldots$ are connected in series, the equivalent resistance $R_{eq}$ is the sum of the individual resistances. The same *current* flows through all the resistors in a series connection. When several resistors are connected in parallel, the reciprocal of the equivalent resistance $R_{eq}$ is the sum of the reciprocals of the individual resistances. All resistors in a parallel connection have the same *potential difference* between their terminals. (See Examples 26.1 and 26.2.)

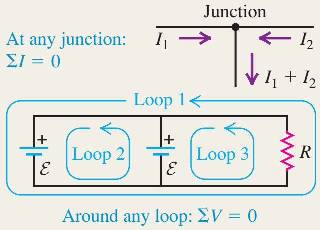
$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad (26.1)$$
(resistors in series)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad (26.2)$$
(resistors in parallel)

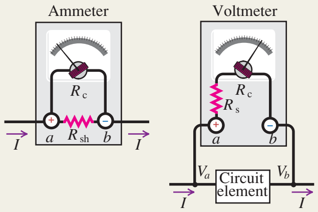



**Kirchhoff's rules:** Kirchhoff's junction rule is based on conservation of charge. It states that the algebraic sum of the currents into any junction must be zero. Kirchhoff's loop rule is based on conservation of energy and the conservative nature of electrostatic fields. It states that the algebraic sum of potential differences around any loop must be zero. Careful use of consistent sign rules is essential in applying Kirchhoff's rules. (See Examples 26.3–26.7.)

$$\sum I = 0 \quad \text{(junction rule)} \qquad (26.5)$$
$$\sum V = 0 \quad \text{(loop rule)} \qquad (26.6)$$




**Electrical measuring instruments:** In a d'Arsonval galvanometer, the deflection is proportional to the current in the coil. For a larger current range, a shunt resistor is added, so some of the current bypasses the meter coil. Such an instrument is called an ammeter. If the coil and any additional series resistance included obey Ohm's law, the meter can also be calibrated to read potential difference or voltage. The instrument is then called a voltmeter. A good ammeter has very low resistance; a good voltmeter has very high resistance. (See Examples 26.8–26.11.)




***R-C* circuits:** When a capacitor is charged by a battery in series with a resistor, the current and capacitor charge are not constant. The charge approaches its final value asymptotically and the current approaches zero asymptotically. The charge and current in the circuit are given by Eqs. (26.12) and (26.13). After a time $\tau = RC$, the charge has approached within $1/e$ of its final value. This time is called the time constant or relaxation time of the circuit. When the capacitor discharges, the charge and current are given as functions of time by Eqs. (26.16) and (26.17). The time constant is the same for charging and discharging. (See Examples 26.12 and 26.13.)

**Capacitor charging:**

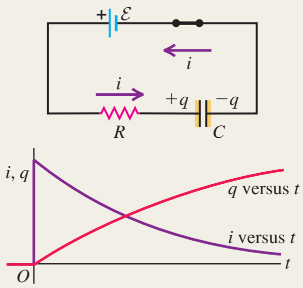
$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right) \qquad (26.12)$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \qquad (26.13)$$

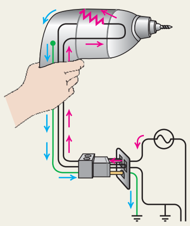
**Capacitor discharging:**

$$q = Q_0 e^{-t/RC} \qquad (26.16)$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \qquad (26.17)$$




**Household wiring:** In household wiring systems, the various electrical devices are connected in parallel across the power line, which consists of a pair of conductors, one "hot" and the other "neutral." An additional "ground" wire is included for safety. The maximum permissible current in a circuit is determined by the size of the wires and the maximum temperature they can tolerate. Protection against excessive current and the resulting fire hazard is provided by fuses or circuit breakers. (See Example 26.14.)



## BRIDGING PROBLEM Two Capacitors and Two Resistors

A 2.40-$\mu$F capacitor and a 3.60-$\mu$F capacitor are connected in series. (a) A charge of 5.20 mC is placed on each capacitor. What is the energy stored in the capacitors? (b) A 655-$\Omega$ resistor is connected to the terminals of the capacitor combination, and a voltmeter with resistance $4.58 \times 10^4\ \Omega$ is connected across the resistor. What is the rate of change of the energy stored in the capacitors just after the connection is made? (c) How long after the connection is made has the energy stored in the capacitors decreased to $1/e$ of its initial value? (d) At the instant calculated in part (c), what is the rate of change of the energy stored in the capacitors?

**SOLUTION GUIDE**

See MasteringPhysics® study area for a Video Tutor solution.

**IDENTIFY and SET UP**

1. The two capacitors act as a single equivalent capacitor (see Section 24.2), and the resistor and voltmeter act as a single equivalent resistor. Select equations that will allow you to calculate the values of these equivalent circuit elements.
2. Equation (24.9) gives the energy stored in a capacitor. Equations (26.16) and (26.17) give the capacitor charge and current as functions of time. Use these to set up the solutions to the various parts of this problem. (*Hint:* The rate at which energy is lost by the capacitors equals the rate at which energy is dissipated in the resistances.)

**EXECUTE**

3. Find the stored energy at $t = 0$.
4. Find the rate of change of the stored energy at $t = 0$.
5. Find the value of $t$ at which the stored energy has $1/e$ of the value you found in step 3.
6. Find the rate of change of the stored energy at the time you found in step 5.

**EVALUATE**

7. Check your results from steps 4 and 6 by calculating the rate of change in a different way. (*Hint:* The rate of change of the stored energy $U$ is $dU/dt$.)

## Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

### DISCUSSION QUESTIONS

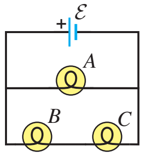
**Q26.1** In which 120-V light bulb does the filament have greater resistance: a 60-W bulb or a 120-W bulb? If the two bulbs are connected to a 120-V line in series, through which bulb will there be the greater voltage drop? What if they are connected in parallel? Explain your reasoning.

**Q26.2** Two 120-V light bulbs, one 25-W and one 200-W, were connected in series across a 240-V line. It seemed like a good idea at the time, but one bulb burned out almost immediately. Which one burned out, and why?

**Q26.3** You connect a number of identical light bulbs to a flashlight battery. (a) What happens to the brightness of each bulb as more and more bulbs are added to the circuit if you connect them (i) in series and (ii) in parallel? (b) Will the battery last longer if the bulbs are in series or in parallel? Explain your reasoning.

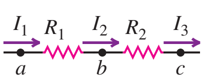
**Q26.4** In the circuit shown in Fig. Q26.4, three identical light bulbs are connected to a flashlight battery. How do the brightnesses of the bulbs compare? Which light bulb has the greatest current passing through it? Which light bulb has the greatest potential difference between its terminals? What happens if bulb $A$ is unscrewed? Bulb $B$? Bulb $C$? Explain your reasoning.

Figure **Q26.4**



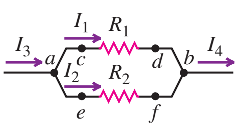

**Q26.5** If two resistors $R_1$ and $R_2$ ($R_2 > R_1$) are connected in series as shown in Fig. Q26.5, which of the following must be true? In each case justify your answer. (a) $I_1 = I_2 = I_3$. (b) The current is greater in $R_1$ than in $R_2$. (c) The electrical power consumption is the same for both resistors. (d) The electrical power consumption is greater in $R_2$ than in $R_1$. (e) The potential drop is the same across both resistors. (f) The potential at point $a$ is the same as at point $c$. (g) The potential at point $b$ is lower than at point $c$. (h) The potential at point $c$ is lower than at point $b$.

Figure **Q26.5**




**Q26.6** If two resistors $R_1$ and $R_2$ ($R_2 > R_1$) are connected in parallel as shown in Fig. Q26.6, which of the following must be true? In each case justify your answer. (a) $I_1 = I_2$. (b) $I_3 = I_4$. (c) The current is greater in $R_1$ than in $R_2$. (d) The rate of electrical energy consumption is the same for both resistors. (e) The rate of electrical energy consumption is greater in $R_2$ than in $R_1$. (f) $V_{cd} = V_{ef} = V_{ab}$. (g) Point $c$ is at higher potential than point $d$. (h) Point $f$ is at higher potential than point $e$. (i) Point $c$ is at higher potential than point $e$.

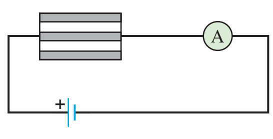
Figure **Q26.6**




**Q26.7** Why do the lights on a car become dimmer when the starter is operated?

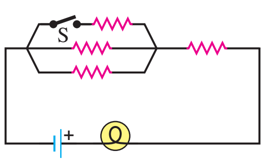
**Q26.8** A resistor consists of three identical metal strips connected as shown in Fig. Q26.8. If one of the strips is cut out, does the ammeter reading increase, decrease, or stay the same? Why?

Figure **Q26.8**



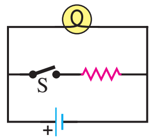

**Q26.9** A light bulb is connected in the circuit shown in Fig. Q26.9. If we close the switch S, does the bulb's brightness increase, decrease, or remain the same? Explain why.

Figure **Q26.9**



**Q26.10** A real battery, having nonnegligible internal resistance, is connected across a light bulb as shown in Fig. Q26.10. When the switch S is closed, what happens to the brightness of the bulb? Why?

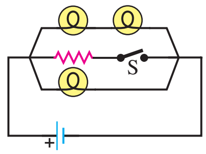
Figure **Q26.10**




**Q26.11** If the battery in Discussion Question Q26.10 is ideal with no internal resistance, what will happen to the brightness of the bulb when S is closed? Why?

**Q26.12** For the circuit shown in Fig. Q26.12 what happens to the brightness of the bulbs when the switch S is closed if the battery (a) has no internal resistance and (b) has nonnegligible internal resistance? Explain why.

Figure **Q26.12**




**Q26.13** Is it possible to connect resistors together in a way that cannot be reduced to some combination of series and parallel combinations? If so, give examples. If not, state why not.

**Q26.14** The direction of current in a battery can be reversed by connecting it to a second battery of greater emf with the positive terminals of the two batteries together. When the direction of current is reversed in a battery, does its emf also reverse? Why or why not?

**Q26.15** In a two-cell flashlight, the batteries are usually connected in series. Why not connect them in parallel? What possible advantage could there be in connecting several identical batteries in parallel?

**Q26.16** The greater the diameter of the wire used in household wiring, the greater the maximum current that can safely be carried by the wire. Why is this? Does the maximum permissible current depend on the length of the wire? Does it depend on what the wire is made of? Explain your reasoning.

**Q26.17** The emf of a flashlight battery is roughly constant with time, but its internal resistance increases with age and use. What sort of meter should be used to test the freshness of a battery?

**Q26.18** Is it possible to have a circuit in which the potential difference across the terminals of a battery in the circuit is zero? If so, give an example. If not, explain why not.

**Q26.19** Verify that the time constant *RC* has units of time.

**Q26.20** For very large resistances it is easy to construct *R-C* circuits that have time constants of several seconds or minutes. How might this fact be used to measure very large resistances, those that are too large to measure by more conventional means?

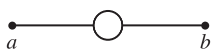
**Q26.21** When a capacitor, battery, and resistor are connected in series, does the resistor affect the maximum charge stored on the capacitor? Why or why not? What purpose does the resistor serve?

## EXERCISES

### Section 26.1 Resistors in Series and Parallel

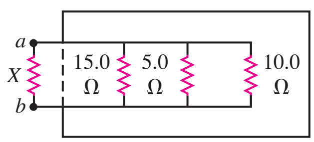
**26.1** •• A uniform wire of resistance $R$ is cut into three equal lengths. One of these is formed into a circle and connected between the other two (Fig. E26.1). What is the resistance between the opposite ends $a$ and $b$?

Figure **E26.1**




**26.2** •• A machine part has a resistor $X$ protruding from an opening in the side. This resistor is connected to three other resistors, as shown in Fig. E26.2. An ohmmeter connected across $a$ and $b$ reads 2.00 Ω. What is the resistance of $X$?

Figure **E26.2**

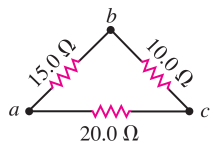



**26.3** • A resistor with $R_1 = 25.0\ \Omega$ is connected to a battery that has negligible internal resistance and electrical energy is dissipated by $R_1$ at a rate of 36.0 W. If a second resistor with $R_2 = 15.0\ \Omega$ is connected in series with $R_1$, what is the total rate at which electrical energy is dissipated by the two resistors?

**26.4** • A 32-Ω resistor and a 20-Ω resistor are connected in parallel, and the combination is connected across a 240-V dc line. (a) What is the resistance of the parallel combination? (b) What is the total current through the parallel combination? (c) What is the current through each resistor?

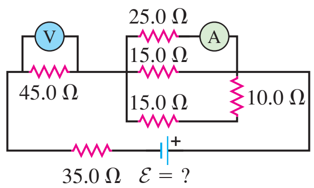
**26.5** • A triangular array of resistors is shown in Fig. E26.5. What current will this array draw from a 35.0-V battery having negligible internal resistance if we connect it across (a) *ab*; (b) *bc*; (c) *ac*? (d) If the battery has an internal resistance of 3.00 Ω, what current will the array draw if the battery is connected across *bc*?

Figure **E26.5**



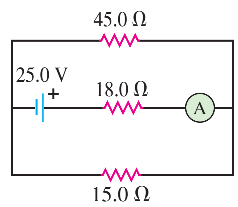

**26.6** •• For the circuit shown in Fig. E26.6 both meters are idealized, the battery has no appreciable internal resistance, and the ammeter reads 1.25 A. (a) What does the voltmeter read? (b) What is the emf $\mathcal{E}$ of the battery?

Figure **E26.6**




**26.7** •• For the circuit shown in Fig. E26.7 find the reading of the idealized ammeter if the battery has an internal resistance of 3.26 Ω.

Figure **E26.7**



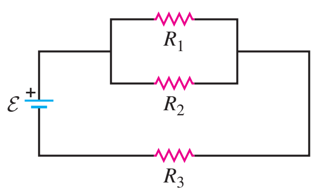

**26.8** • Three resistors having resistances of 1.60 Ω, 2.40 Ω, and 4.80 Ω are connected in parallel to a 28.0-V battery that has negligible internal resistance. Find (a) the equivalent resistance of the combination; (b) the current in each resistor; (c) the total current through the battery; (d) the voltage across each resistor; (e) the power dissipated in each resistor. (f) Which resistor dissipates the most power: the one with the greatest resistance or the least resistance? Explain why this should be.

**26.9** • Now the three resistors of Exercise 26.8 are connected in series to the same battery. Answer the same questions for this situation.

**26.10** •• **Power Rating of a Resistor.** The *power rating* of a resistor is the maximum power the resistor can safely dissipate without too great a rise in temperature and hence damage to the resistor. (a) If the power rating of a 15-kΩ resistor is 5.0 W, what is the maximum allowable potential difference across the terminals of the resistor? (b) A 9.0-kΩ resistor is to be connected across a 120-V potential difference. What power rating is required? (c) A 100.0-Ω and a 150.0-Ω resistor, both rated at 2.00 W, are connected in series across a variable potential difference. What is the greatest this potential difference can be without overheating either resistor, and what is the rate of heat generated in each resistor under these conditions?

**26.11** • In Fig. E26.11, $R_1 = 3.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 5.00\ \Omega$. The battery has negligible internal resistance. The current $I_2$ through $R_2$ is 4.00 A. (a) What are the currents $I_1$ and $I_3$? (b) What is the emf of the battery?

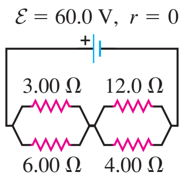
Figure **E26.11**



**26.12** •• In Fig. E26.11 the battery has emf 25.0 V and negligible internal resistance. $R_1 = 5.00\ \Omega$. The current through $R_1$ is 1.50 A and the current through $R_3 = 4.50$ A. What are the resistances $R_2$ and $R_3$?

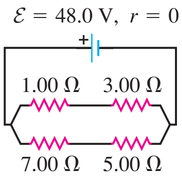
**26.13** • Compute the equivalent resistance of the network in Fig. E26.13, and find the current in each resistor. The battery has negligible internal resistance.

Figure **E26.13**



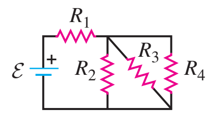

**26.14** • Compute the equivalent resistance of the network in Fig. E26.14, and find the current in each resistor. The battery has negligible internal resistance.

Figure **E26.14**



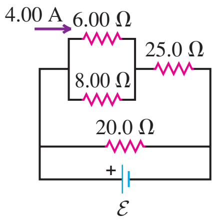

**26.15** • In the circuit of Fig. E26.15, each resistor represents a light bulb. Let $R_1 = R_2 = R_3 = R_4 = 4.50\ \Omega$ and $\mathcal{E} = 9.00$ V. (a) Find the current in each bulb. (b) Find the power dissipated in each bulb. Which bulb or bulbs glow the brightest? (c) Bulb $R_4$ is now removed from the circuit, leaving a break in the wire at its position. Now what is the current in each of the remaining bulbs $R_1$, $R_2$, and $R_3$? (d) With bulb $R_4$ removed, what is the power dissipated in each of the remaining bulbs? (e) Which light bulb(s) glow brighter as a result of removing $R_4$? Which bulb(s) glow less brightly? Discuss why there are different effects on different bulbs.

Figure **E26.15**



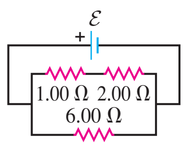

**26.16** • Consider the circuit shown in Fig. E26.16. The current through the 6.00-Ω resistor is 4.00 A, in the direction shown. What are the currents through the 25.0-Ω and 20.0-Ω resistors?

Figure **E26.16**




**26.17** • In the circuit shown in Fig. E26.17, the voltage across the 2.00-Ω resistor is 12.0 V. What are the emf of the battery and the current through the 6.00-Ω resistor?

Figure **E26.17**

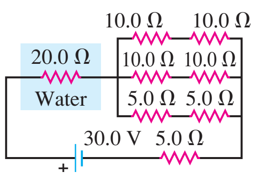



**26.18** • **A Three-Way Light Bulb.** A three-way light bulb has three brightness settings (low, medium, and high) but only two filaments. (a) A particular three-way light bulb connected across a 120-V line can dissipate 60 W, 120 W, or 180 W. Describe how the two filaments are arranged in the bulb, and calculate the resistance of each filament. (b) Suppose the filament with the higher resistance burns out. How much power will the bulb dissipate on each of the three brightness settings? What will be the brightness (low, medium, or high) on each setting? (c) Repeat part (b) for the situation in which the filament with the lower resistance burns out.

**26.19** •• **Working Late!** You are working late in your electronics shop and find that you need various resistors for a project. But alas, all you have is a big box of 10.0-Ω resistors. Show how you can make each of the following equivalent resistances by a combination of your 10.0-Ω resistors: (a) 35 Ω, (b) 1.0 Ω, (c) 3.33 Ω, (d) 7.5 Ω.

**26.20** • In the circuit shown in Fig. E26.20, the rate at which $R_1$ is dissipating electrical energy is 20.0 W. (a) Find $R_1$ and $R_2$. (b) What is the emf of the battery? (c) Find the current through both $R_2$ and the 10.0-Ω resistor. (d) Calculate the total electrical power consumption in all the resistors and the electrical power delivered by the battery. Show that your results are consistent with conservation of energy.

Figure **E26.20**

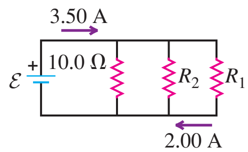



**26.21** • **Light Bulbs in Series and in Parallel.** Two light bulbs have resistances of 400 Ω and 800 Ω. If the two light bulbs are connected in series across a 120-V line, find (a) the current through each bulb; (b) the power dissipated in each bulb; (c) the total power dissipated in both bulbs. The two light bulbs are now connected in parallel across the 120-V line. Find (d) the current through each bulb; (e) the power dissipated in each bulb; (f) the total power dissipated in both bulbs. (g) In each situation, which of the two bulbs glows the brightest? (h) In which situation is there a greater total light output from both bulbs combined?

**26.22** • **Light Bulbs in Series.** A 60-W, 120-V light bulb and a 200-W, 120-V light bulb are connected in series across a 240-V line. Assume that the resistance of each bulb does not vary with current. (*Note:* This description of a light bulb gives the power it dissipates when connected to the stated potential difference; that is, a 25-W, 120-V light bulb dissipates 25 W when connected to a 120-V line.) (a) Find the current through the bulbs. (b) Find the power dissipated in each bulb. (c) One bulb burns out very quickly. Which one? Why?

**26.23** •• **CP** In the circuit in Fig. E26.23, a 20.0-Ω resistor is inside 100 g of pure water that is surrounded by insulating styrofoam. If the water is initially at 10.0°C, how long will it take for its temperature to rise to 58.0°C?

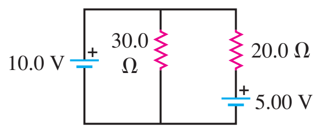
Figure **E26.23**




## Section 26.2 Kirchhoff's Rules

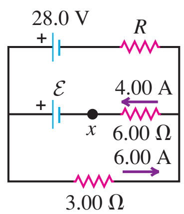
**26.24** •• The batteries shown in the circuit in Fig. E26.24 have negligibly small internal resistances. Find the current through (a) the 30.0-Ω resistor; (b) the 20.0-Ω resistor; (c) the 10.0-V battery.

Figure **E26.24**



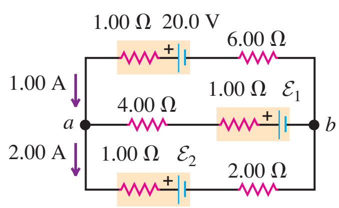

**26.25** • In the circuit shown in Fig. E26.25 find (a) the current in resistor $R$; (b) the resistance $R$; (c) the unknown emf $\mathcal{E}$. (d) If the circuit is broken at point $x$, what is the current in resistor $R$?

Figure **E26.25**



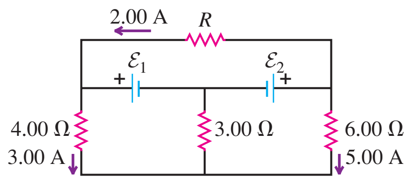

**26.26** • Find the emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ in the circuit of Fig. E26.26, and find the potential difference of point $b$ relative to point $a$.

Figure **E26.26**



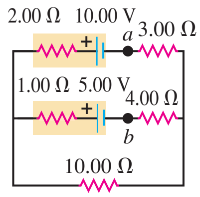

**26.27** • In the circuit shown in Fig. E26.27, find (a) the current in the 3.00-Ω resistor; (b) the unknown emfs $\mathcal{E}_1$ and $\mathcal{E}_2$; (c) the resistance $R$. Note that three currents are given.

Figure **E26.27**




**26.28** •• In the circuit shown in Fig. E26.28, find (a) the current in each branch and (b) the potential difference $V_{ab}$ of point $a$ relative to point $b$.

Figure **E26.28**

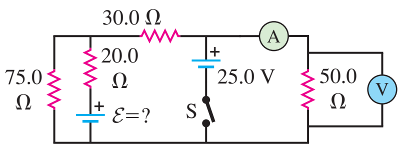



**26.29** • The 10.00-V battery in Fig. E26.28 is removed from the circuit and reinserted with the opposite polarity, so that its positive terminal is now next to point $a$. The rest of the circuit is as shown in the figure. Find (a) the current in each branch and (b) the potential difference $V_{ab}$ of point $a$ relative to point $b$.

**26.30** • The 5.00-V battery in Fig. E26.28 is removed from the circuit and replaced by a 20.00-V battery, with its negative terminal next to point $b$. The rest of the circuit is as shown in the figure. Find (a) the current in each branch and (b) the potential difference $V_{ab}$ of point $a$ relative to point $b$.

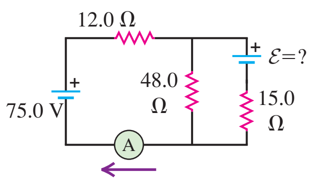
**26.31** •• In the circuit shown in Fig. E26.31 the batteries have negligible internal resistance and the meters are both idealized. With the switch S open, the voltmeter reads 15.0 V. (a) Find the emf $\mathcal{E}$ of the battery. (b) What will the ammeter read when the switch is closed?

Figure **E26.31**



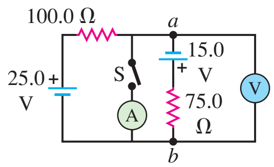

**26.32** •• In the circuit shown in Fig. E26.32 both batteries have insignificant internal resistance and the idealized ammeter reads 1.50 A in the direction shown. Find the emf $\mathcal{E}$ of the battery. Is the polarity shown correct?

Figure **E26.32**



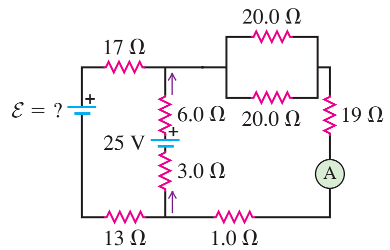

**26.33** • In the circuit shown in Fig. E26.33 all meters are idealized and the batteries have no appreciable internal resistance. (a) Find the reading of the voltmeter with the switch S open. Which point is at a higher potential: $a$ or $b$? (b) With the switch closed, find the reading of the voltmeter and the ammeter. Which way (up or down) does the current flow through the switch?

Figure **E26.33**




**26.34** •• In the circuit shown in Fig. E26.34, the 6.0-Ω resistor is consuming energy at a rate of 24 J/s when the current through it flows as shown. (a) Find the current through the ammeter A. (b) What are the polarity and emf $\mathcal{E}$ of the battery, assuming it has negligible internal resistance?

Figure **E26.34**

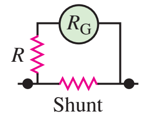



## Section 26.3 Electrical Measuring Instruments

**26.35** • The resistance of a galvanometer coil is 25.0 Ω, and the current required for full-scale deflection is 500 $\mu$A. (a) Show in a diagram how to convert the galvanometer to an ammeter reading 20.0 mA full scale, and compute the shunt resistance. (b) Show how to convert the galvanometer to a voltmeter reading 500 mV full scale, and compute the series resistance.

**26.36** • The resistance of the coil of a pivoted-coil galvanometer is 9.36 Ω, and a current of 0.0224 A causes it to deflect full scale. We want to convert this galvanometer to an ammeter reading 20.0 A full scale. The only shunt available has a resistance of 0.0250 Ω. What resistance $R$ must be connected in series with the coil (Fig. E26.36)?

Figure **E26.36**

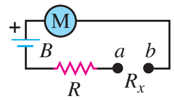



**26.37** • A circuit consists of a series combination of 6.00-kΩ and 5.00-kΩ resistors connected across a 50.0-V battery having negligible internal resistance. You want to measure the true potential difference (that is, the potential difference without the meter present) across the 5.00-kΩ resistor using a voltmeter having an internal resistance of 10.0 kΩ. (a) What potential difference does the voltmeter measure across the 5.00-kΩ resistor? (b) What is the *true* potential difference across this resistor when the meter is not present? (c) By what percentage is the voltmeter reading in error from the true potential difference?

**26.38** • A galvanometer having a resistance of 25.0 Ω has a 1.00-Ω shunt resistance installed to convert it to an ammeter. It is then used to measure the current in a circuit consisting of a 15.0-Ω resistor connected across the terminals of a 25.0-V battery having no appreciable internal resistance. (a) What current does the ammeter measure? (b) What should be the *true* current in the circuit (that is, the current without the ammeter present)? (c) By what percentage is the ammeter reading in error from the *true* current?

**26.39** • In the ohmmeter in Fig. E26.39 M is a 2.50-mA meter of resistance 65.0 Ω. (A 2.50-mA meter deflects full scale when the current through it is 2.50 mA.) The battery $B$ has an emf of 1.52 V and negligible internal resistance. $R$ is chosen so that when the terminals $a$ and $b$ are shorted ($R_x = 0$), the meter reads full scale. When $a$ and $b$ are open ($R_x = \infty$), the meter reads zero. (a) What is the resistance of the resistor $R$? (b) What current indicates a resistance $R_x$ of 200 Ω? (c) What values of $R_x$ correspond to meter deflections of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of full scale if the deflection is proportional to the current through the galvanometer?

Figure **E26.39**




## Section 26.4 *R-C* Circuits

**26.40** • A 4.60-$\mu F$ capacitor that is initially uncharged is connected in series with a 7.50-kΩ resistor and an emf source with $\mathcal{E}$ = 245 V and negligible internal resistance. Just after the circuit is completed, what are (a) the voltage drop across the capacitor;

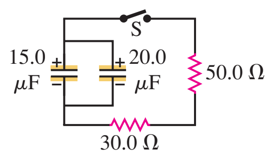
(b) the voltage drop across the resistor; (c) the charge on the capacitor; (d) the current through the resistor? (e) A long time after the circuit is completed (after many time constants) what are the values of the quantities in parts (a)–(d)?

**26.41** • A capacitor is charged to a potential of 12.0 V and is then connected to a voltmeter having an internal resistance of 3.40 MΩ. After a time of 4.00 s the voltmeter reads 3.0 V. What are (a) the capacitance and (b) the time constant of the circuit?

**26.42** • A 12.4-$\mu$F capacitor is connected through a 0.895-MΩ resistor to a constant potential difference of 60.0 V. (a) Compute the charge on the capacitor at the following times after the connections are made: 0, 5.0 s, 10.0 s, 20.0 s, and 100.0 s. (b) Compute the charging currents at the same instants. (c) Graph the results of parts (a) and (b) for $t$ between 0 and 20 s.

**26.43** •• **CP** In the circuit shown in Fig. E26.43 both capacitors are initially charged to 45.0 V. (a) How long after closing the switch S will the potential across each capacitor be reduced to 10.0 V, and (b) what will be the current at that time?

Figure **E26.43**



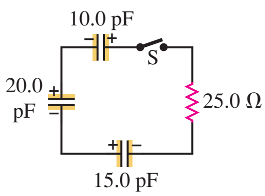

**26.44** • A resistor and a capacitor are connected in series to an emf source. The time constant for the circuit is 0.870 s. (a) A second capacitor, identical to the first, is added in series. What is the time constant for this new circuit? (b) In the original circuit a second capacitor, identical to the first, is connected in parallel with the first capacitor. What is the time constant for this new circuit?

**26.45** • An emf source with $\mathcal{E} = 120$ V, a resistor with $R = 80.0\ \Omega$, and a capacitor with $C = 4.00\ \mu$F are connected in series. As the capacitor charges, when the current in the resistor is 0.900 A, what is the magnitude of the charge on each plate of the capacitor?

**26.46** • A 1.50-$\mu$F capacitor is charging through a 12.0-Ω resistor using a 10.0-V battery. What will be the current when the capacitor has acquired $\frac{1}{4}$ of its maximum charge? Will it be $\frac{1}{4}$ of the maximum current?

**26.47** •• **CP** In the circuit shown in Fig. E26.47 each capacitor initially has a charge of magnitude 3.50 nC on its plates. After the switch S is closed, what will be the current in the circuit at the instant that the capacitors have lost 80.0% of their initial stored energy?

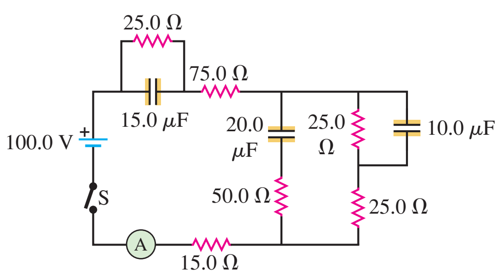
Figure **E26.47**




**26.48** • A 12.0-$\mu$F capacitor is charged to a potential of 50.0 V and then discharged through a 175-Ω resistor. How long does it take the capacitor to lose (a) half of its charge and (b) half of its stored energy?

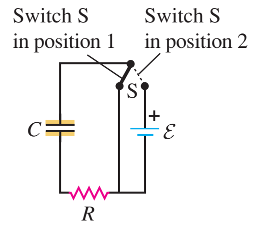
**26.49** • In the circuit in Fig. E26.49 the capacitors are all initially uncharged, the battery has no internal resistance, and the ammeter is idealized. Find the reading of the ammeter (a) just after the switch S is closed and (b) after the switch has been closed for a very long time.

Figure **E26.49**




**26.50** • In the circuit shown in Fig. E26.50, $C = 5.90\ \mu$F, $\mathcal{E} = 28.0$ V, and the emf has negligible resistance. Initially the capacitor is uncharged and the switch S is in position 1. The switch is then moved to position 2, so that the capacitor begins to charge. (a) What will be the charge on the capacitor a long time after the switch is moved to position 2? (b) After the switch has been in position 2 for 3.00 ms, the charge on the capacitor is measured to be 110 $\mu$C. What is the value of the resistance $R$? (c) How long after the switch is moved to position 2 will the charge on the capacitor be equal to 99.0% of the final value found in part (a)?

Figure **E26.50**




**26.51** • A capacitor with $C = 1.50 \times 10^{-5}$ F is connected as shown in Fig. E26.50 with a resistor with $R = 980\ \Omega$ and an emf source with $\mathcal{E} = 18.0$ V and negligible internal resistance. Initially the capacitor is uncharged and the switch S is in position 1. The switch is then moved to position 2, so that the capacitor begins to charge. After the switch has been in position 2 for 10.0 ms, the switch is moved back to position 1 so that the capacitor begins to discharge. (a) Compute the charge on the capacitor just *before* the switch is thrown from position 2 back to position 1. (b) Compute the voltage drops across the resistor and across the capacitor at the instant described in part (a). (c) Compute the voltage drops across the resistor and across the capacitor just *after* the switch is thrown from position 2 back to position 1. (d) Compute the charge on the capacitor 10.0 ms after the switch is thrown from position 2 back to position 1.

## Section 26.5 Power Distribution Systems

**26.52** • The heating element of an electric dryer is rated at 4.1 kW when connected to a 240-V line. (a) What is the current in the heating element? Is 12-gauge wire large enough to supply this current? (b) What is the resistance of the dryer's heating element at its operating temperature? (c) At 11 cents per kWh, how much does it cost per hour to operate the dryer?

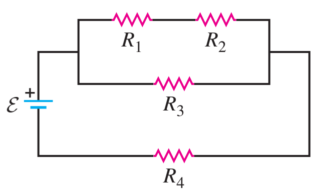
**26.53** • A 1500-W electric heater is plugged into the outlet of a 120-V circuit that has a 20-A circuit breaker. You plug an electric hair dryer into the same outlet. The hair dryer has power settings of 600 W, 900 W, 1200 W, and 1500 W. You start with the hair dryer on the 600-W setting and increase the power setting until the circuit breaker trips. What power setting caused the breaker to trip?

**26.54** • **CP** The heating element of an electric stove consists of a heater wire embedded within an electrically insulating material, which in turn is inside a metal casing. The heater wire has a resistance of 20 Ω at room temperature (23.0°C) and a temperature coefficient of resistivity $\alpha = 2.8 \times 10^{-3}\,(\text{C}°)^{-1}$. The heating element operates from a 120-V line. (a) When the heating element is first turned on, what current does it draw and what electrical power does it dissipate? (b) When the heating element has reached an operating temperature of 280°C (536°F), what current does it draw and what electrical power does it dissipate?

## PROBLEMS

**26.55** •• In Fig. P26.55, the battery has negligible internal resistance and $\mathcal{E} = 48.0$ V. $R_1 = R_2 = 4.00\ \Omega$ and $R_4 = 3.00\ \Omega$. What must the resistance $R_3$ be for the resistor network to dissipate electrical energy at a rate of 295 W?

Figure **P26.55**



**26.56** • A 400-Ω, 2.4-W resistor is needed, but only several 400-Ω, 1.2-W resistors are available (see Exercise 26.10). (a) What two different combinations of the available units give the required resistance and power rating? (b) For each of the resistor networks from part (a), what power is dissipated in each resistor when 2.4 W is dissipated by the combination?

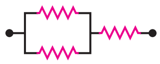
**26.57** • **CP** A 20.0-m-long cable consists of a solid-inner, cylindrical, nickel core 10.0 cm in diameter surrounded by a solid-outer cylindrical shell of copper 10.0 cm in inside diameter and 20.0 cm in outside diameter. The resistivity of nickel is $7.8 \times 10^{-8}\ \Omega \cdot \text{m}$. (a) What is the resistance of this cable? (b) If we think of this cable as a single material, what is its equivalent resistivity?

**26.58** • Two identical 3.00-Ω wires are laid side by side and soldered together so they touch each other for half of their lengths. What is the equivalent resistance of this combination?

**26.59** • The two identical light bulbs in Example 26.2 (Section 26.1) are connected in parallel to a different source, one with $\mathcal{E} = 8.0$ V and internal resistance 0.8 Ω. Each light bulb has a resistance $R = 2.0\ \Omega$ (assumed independent of the current through the bulb). (a) Find the current through each bulb, the potential difference across each bulb, and the power delivered to each bulb. (b) Suppose one of the bulbs burns out, so that its filament breaks and current no longer flows through it. Find the power delivered to the remaining bulb. Does the remaining bulb glow more or less brightly after the other bulb burns out than before?

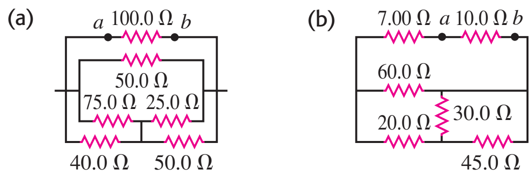
**26.60** •• Each of the three resistors in Fig. P26.60 has a resistance of 2.4 Ω and can dissipate a maximum of 48 W without becoming excessively heated. What is the maximum power the circuit can dissipate?

Figure **P26.60**



**26.61** • If an ohmmeter is connected between points $a$ and $b$ in each of the circuits shown in Fig. P26.61, what will it read?

Figure **P26.61**




**26.62** •• **CP** For the circuit shown in Fig. P26.62 a 20.0-Ω resistor is embedded in a large block of ice at 0.00°C, and the battery has negligible internal resistance. At what rate (in g/s) is this circuit melting the ice? (The latent heat of fusion for ice is $3.34 \times 10^5$ J/kg.)

Figure **P26.62**

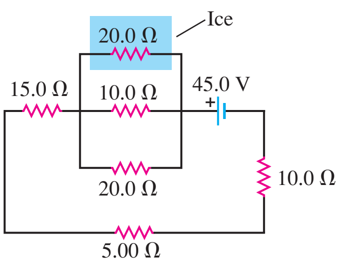
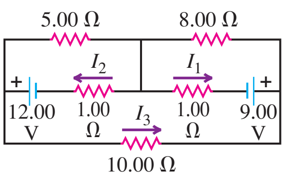



Figure **P26.63**




**26.63** • Calculate the three currents $I_1$, $I_2$, and $I_3$ indicated in the circuit diagram shown in Fig. P26.63.

**26.64** ••• What must the emf $\mathcal{E}$ in Fig. P26.64 be in order for the current through the 7.00-Ω resistor to be 1.80 A? Each emf source has negligible internal resistance.

Figure **P26.64**

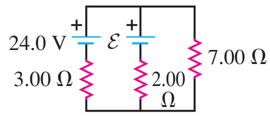
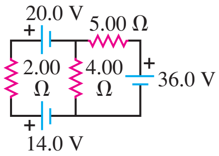


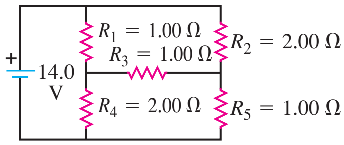

Figure **P26.65**




**26.65** • Find the current through each of the three resistors of the circuit shown in Fig. P26.65. The emf sources have negligible internal resistance.

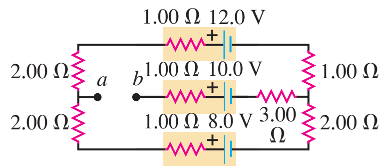
**26.66** • (a) Find the current through the battery and each resistor in the circuit shown in Fig. P26.66. (b) What is the equivalent resistance of the resistor network?

Figure **P26.66**



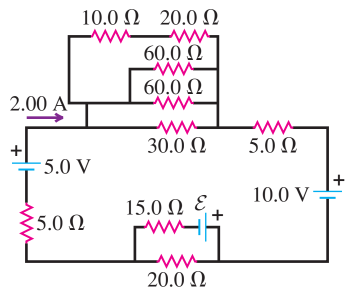

**26.67** •• (a) Find the potential of point $a$ with respect to point $b$ in Fig. P26.67. (b) If points $a$ and $b$ are connected by a wire with negligible resistance, find the current in the 12.0-V battery.

Figure **P26.67**



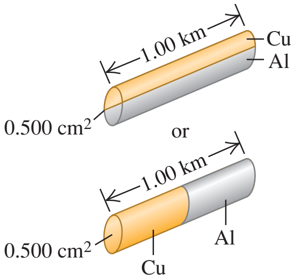

**26.68** •• Consider the circuit shown in Fig. P26.68. (a) What must the emf $\mathcal{E}$ of the battery be in order for a current of 2.00 A to flow through the 5.00-V battery as shown? Is the polarity of the battery correct as shown? (b) How long does it take for 60.0 J of thermal energy to be produced in the 10.0-Ω resistor?

Figure **P26.68**



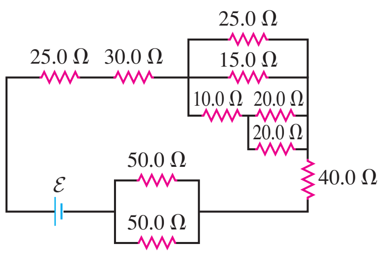

**26.69** •• **CP** A 1.00-km cable having a cross-sectional area of $0.500\ \text{cm}^2$ is to be constructed out of equal lengths of copper and aluminum. This could be accomplished either by making a 0.50-km cable of each one and welding them together end to end or by making two parallel 1.00-km cables, one of each metal (Fig. P26.69). Calculate the resistance of the 1.00-km cable for both designs to see which one provides the least resistance.

Figure **P26.69**



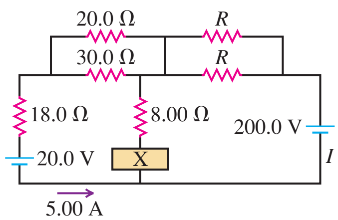

**26.70** ••• In the circuit shown in Fig. P26.70 all the resistors are rated at a maximum power of 2.00 W. What is the maximum emf $\mathcal{E}$ that the battery can have without burning up any of the resistors?

Figure **P26.70**




**26.71** • In the circuit shown in Fig. P26.71, the current in the 20.0-V battery is 5.00 A in the direction shown and the voltage across the 8.00-Ω resistor is 16.0 V, with the lower end of the resistor at higher potential. Find (a) the emf (including its polarity) of the battery X; (b) the current $I$ through the 200.0-V battery (including its direction); (c) the resistance $R$.

Figure **P26.71**

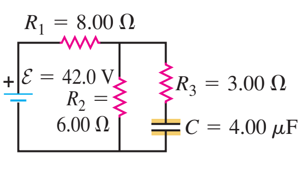



**26.72** •• Three identical resistors are connected in series. When a certain potential difference is applied across the combination, the total power dissipated is 36 W. What power would be dissipated if the three resistors were connected in parallel across the same potential difference?

**26.73** • A resistor $R_1$ consumes electrical power $P_1$ when connected to an emf $\mathcal{E}$. When resistor $R_2$ is connected to the same emf, it consumes electrical power $P_2$. In terms of $P_1$ and $P_2$, what is the total electrical power consumed when they are both connected to this emf source (a) in parallel and (b) in series?

**26.74** • The capacitor in Fig. P26.74 is initially uncharged. The switch is closed at $t = 0$. (a) Immediately after the switch is closed, what is the current through each resistor? (b) What is the final charge on the capacitor?

Figure **P26.74**

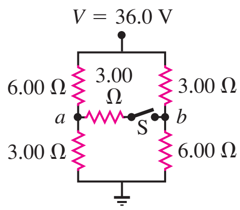



**26.75** •• A 2.00-$\mu$F capacitor that is initially uncharged is connected in series with a 6.00-kΩ resistor and an emf source with $\mathcal{E} = 90.0$ V and negligible internal resistance. The circuit is completed at $t = 0$. (a) Just after the circuit is completed, what is the rate at which electrical energy is being dissipated in the resistor? (b) At what value of $t$ is the rate at which electrical energy is being dissipated in the resistor equal to the rate at which electrical energy is being stored in the capacitor? (c) At the time calculated in part (b), what is the rate at which electrical energy is being dissipated in the resistor?

**26.76** •• A 6.00-$\mu$F capacitor that is initially uncharged is connected in series with a 5.00-Ω resistor and an emf source with $\mathcal{E} = 50.0$ V and negligible internal resistance. At the instant when the resistor is dissipating electrical energy at a rate of 250 W, how much energy has been stored in the capacitor?

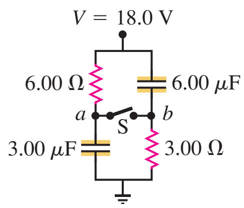
**26.77** • Figure P26.77 employs a convention often used in circuit diagrams. The battery (or other power supply) is not shown explicitly. It is understood that the point at the top, labeled "36.0 V," is connected to the positive terminal of a 36.0-V battery having negligible internal resistance, and that the "ground" symbol at the bottom is connected to the negative terminal of the battery. The circuit is completed through the battery, even though it is not shown on the diagram. (a) What is the potential difference $V_{ab}$, the potential of point $a$ relative to point $b$, when the switch S is open? (b) What is the current through switch S when it is closed? (c) What is the equivalent resistance when switch S is closed?

Figure **P26.77**



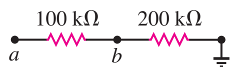

**26.78** • (See Problem 26.77.) (a) What is the potential of point $a$ with respect to point $b$ in Fig. P26.78 when switch S is open? (b) Which point, $a$ or $b$, is at the higher potential? (c) What is the final potential of point $b$ with respect to ground when switch S is closed? (d) How much does the charge on each capacitor change when S is closed?

Figure **P26.78**




**26.79** • Point $a$ in Fig. P26.79 is maintained at a constant potential of 400 V above ground. (See Problem 26.77.) (a) What is the reading of a voltmeter with the proper range and with resistance $5.00 \times 10^4$ Ω when connected between point $b$ and ground? (b) What is the reading of a voltmeter with resistance $5.00 \times 10^6$ Ω? (c) What is the reading of a voltmeter with infinite resistance?

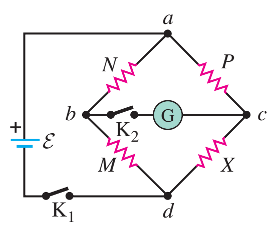
Figure **P26.79**




**26.80** •• A 150-V voltmeter has a resistance of 30,000 Ω. When connected in series with a large resistance $R$ across a 110-V line, the meter reads 74 V. Find the resistance $R$.

**26.81** •• **The Wheatstone Bridge.** The circuit shown in Fig. P26.81, called a *Wheatstone bridge,* is used to determine the value of an unknown resistor $X$ by comparison with three resistors $M$, $N$, and $P$ whose resistances can be varied. For each setting, the resistance of each resistor is precisely known. With switches $K_1$ and $K_2$ closed,

Figure **P26.81**



these resistors are varied until the current in the galvanometer G is zero; the bridge is then said to be *balanced.* (a) Show that under this condition the unknown resistance is given by $X = MP/N$. (This method permits very high precision in comparing resistors.) (b) If the galvanometer G shows zero deflection when $M = 850.0\ \Omega$, $N = 15.00\ \Omega$, and $P = 33.48\ \Omega$, what is the unknown resistance $X$?

**26.82** • A 2.36-$\mu$F capacitor that is initially uncharged is connected in series with a 5.86-$\Omega$ resistor and an emf source with $\mathcal{E} = 120$ V and negligible internal resistance. (a) Just after the connection is made, what are (i) the rate at which electrical energy is being dissipated in the resistor; (ii) the rate at which the electrical energy stored in the capacitor is increasing; (iii) the electrical power output of the source? How do the answers to parts (i), (ii), and (iii) compare? (b) Answer the same questions as in part (a) at a long time after the connection is made. (c) Answer the same questions as in part (a) at the instant when the charge on the capacitor is one-half its final value.

**26.83** • A 224-$\Omega$ resistor and a 589-$\Omega$ resistor are connected in series across a 90.0-V line. (a) What is the voltage across each resistor? (b) A voltmeter connected across the 224-$\Omega$ resistor reads 23.8 V. Find the voltmeter resistance. (c) Find the reading of the same voltmeter if it is connected across the 589-$\Omega$ resistor. (d) The readings on this voltmeter are lower than the "true" voltages (that is, without the voltmeter present). Would it be possible to design a voltmeter that gave readings *higher* than the "true" voltages? Explain.

**26.84** • A resistor with $R = 850\ \Omega$ is connected to the plates of a charged capacitor with capacitance $C = 4.62\ \mu$F. Just before the connection is made, the charge on the capacitor is 6.90 mC. (a) What is the energy initially stored in the capacitor? (b) What is the electrical power dissipated in the resistor just after the connection is made? (c) What is the electrical power dissipated in the resistor at the instant when the energy stored in the capacitor has decreased to half the value calculated in part (a)?

**26.85** • A capacitor that is initially uncharged is connected in series with a resistor and an emf source with $\mathcal{E} = 110$ V and negligible internal resistance. Just after the circuit is completed, the current through the resistor is $6.5 \times 10^{-5}$ A. The time constant for the circuit is 5.2 s. What are the resistance of the resistor and the capacitance of the capacitor?

**26.86** ••• An *R-C* circuit has a time constant *RC*. (a) If the circuit is discharging, how long will it take for its stored energy to be reduced to $1/e$ of its initial value? (b) If it is charging, how long will it take for the stored energy to reach $1/e$ of its maximum value?

**26.87** • Strictly speaking, Eq. (26.16) implies that an *infinite* amount of time is required to discharge a capacitor completely. Yet for practical purposes, a capacitor may be considered to be fully discharged after a finite length of time. To be specific, consider a capacitor with capacitance $C$ connected to a resistor $R$ to be fully discharged if its charge $q$ differs from zero by no more than the charge of one electron. (a) Calculate the time required to reach this state if $C = 0.920\ \mu$F, $R = 670$ k$\Omega$, and $Q_0 = 7.00\ \mu$C. How many time constants is this? (b) For a given $Q_0$, is the time required to reach this state always the same number of time constants, independent of the values of $C$ and $R$? Why or why not?

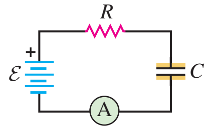
**26.88** • **CALC** The current in a charging capacitor is given by Eq. (26.13). (a) The instantaneous power supplied by the battery is $\mathcal{E}i$. Integrate this to find the total energy supplied by the battery. (b) The instantaneous power dissipated in the resistor is $i^2R$. Integrate this to find the total energy dissipated in the resistor. (c) Find the final energy stored in the capacitor, and show that this equals the total energy supplied by the battery less the energy dissipated in the resistor, as obtained in parts (a) and (b). (d) What fraction of the energy supplied by the battery is stored in the capacitor? How does this fraction depend on $R$?

**26.89** ••• **CALC** (a) Using Eq. (26.17) for the current in a discharging capacitor, derive an expression for the instantaneous power $P = i^2R$ dissipated in the resistor. (b) Integrate the expression for $P$ to find the total energy dissipated in the resistor, and show that this is equal to the total energy initially stored in the capacitor.

## CHALLENGE PROBLEMS

**26.90** ••• **A Capacitor Burglar Alarm.** The capacitance of a capacitor can be affected by dielectric material that, although not inside the capacitor, is near enough to the capacitor to be polarized by the fringing electric field that exists near a charged capacitor. This effect is usually of the order of picofarads (pF), but it can be used with appropriate electronic circuitry to detect a change in the dielectric material surrounding the capacitor. Such a dielectric material might be the human body, and the effect described above might be used in the design of a burglar alarm. Consider the simplified circuit shown in Fig. P26.90. The voltage source has emf $\mathcal{E} = 1000$ V, and the capacitor has capacitance $C = 10.0$ pF. The electronic circuitry for detecting the current, represented as an ammeter in the diagram, has negligible resistance and is capable of detecting a current that persists at a level of at least 1.00 $\mu$A for at least 200 $\mu$s after the capacitance has changed abruptly from $C$ to $C'$. The burglar alarm is designed to be activated if the capacitance changes by 10%. (a) Determine the charge on the 10.0-pF capacitor when it is fully charged. (b) If the capacitor is fully charged before the intruder is detected, assuming that the time taken for the capacitance to change by 10% is short enough to be ignored, derive an equation that expresses the current through the resistor $R$ as a function of the time $t$ since the capacitance has changed. (c) Determine the range of values of the resistance $R$ that will meet the design specifications of the burglar alarm. What happens if $R$ is too small? Too large? (*Hint:* You will not be able to solve this part analytically but must use numerical methods. Express $R$ as a logarithmic function of $R$ plus known quantities. Use a trial value of $R$ and calculate from the expression a new value. Continue to do this until the input and output values of $R$ agree to within three significant figures.)

Figure **P26.90**

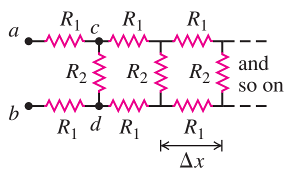



**26.91** ••• **An Infinite Network.** As shown in Fig. P26.91, a network of resistors of resistances $R_1$ and $R_2$ extends to infinity toward the right. Prove that the total resistance $R_T$ of the infinite network is equal to

$$R_T = R_1 + \sqrt{R_1^2 + 2R_1R_2}$$

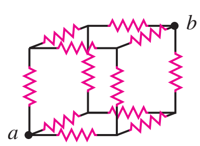
(*Hint:* Since the network is infinite, the resistance of the network to the right of points $c$ and $d$ is also equal to $R_T$.)

Figure **P26.91**




**26.92** ••• Suppose a resistor $R$ lies along each edge of a cube (12 resistors in all) with connections at the corners. Find the equivalent resistance between two diagonally opposite corners of the cube (points $a$ and $b$ in Fig. P26.92).

Figure **P26.92**




**26.93** ••• **BIO** **Attenuator Chains and Axons.** The infinite network of resistors shown in Fig. P26.91 is known as an *attenuator chain,* since this chain of resistors causes the potential difference between the upper and lower wires to decrease, or attenuate, along the length of the chain. (a) Show that if the potential difference between the points $a$ and $b$ in Fig. 26.91 is $V_{ab}$, then the potential difference between points $c$ and $d$ is $V_{cd} = V_{ab}/(1 + \beta)$, where $\beta = 2R_1(R_T + R_2)/R_T R_2$ and $R_T$, the total resistance of the network, is given in Challenge Problem 26.91. (See the hint given in that problem.) (b) If the potential difference between terminals $a$ and $b$ at the left end of the infinite network is $V_0$, show that the potential difference between the upper and lower wires $n$ segments from the left end is $V_n = V_0/(1 + \beta)^n$. If $R_1 = R_2$, how many segments are needed to decrease the potential difference $V_n$ to less than 1.0% of $V_0$? (c) An infinite attenuator chain provides a model of the propagation of a voltage pulse along a nerve fiber, or axon. Each segment of the network in Fig. P26.91 represents a short segment of the axon of length $\Delta x$. The resistors $R_1$ represent the resistance of the fluid inside and outside the membrane wall of the axon. The resistance of the membrane to current flowing through the wall is represented by $R_2$. For an axon segment of length $\Delta x = 1.0\ \mu$m, $R_1 = 6.4 \times 10^3\ \Omega$ and $R_2 = 8.0 \times 10^8\ \Omega$ (the membrane wall is a good insulator). Calculate the total resistance $R_T$ and $\beta$ for an infinitely long axon. (This is a good approximation, since the length of an axon is much greater than its width; the largest axons in the human nervous system are longer than 1 m but only about $10^{-7}$ m in radius.) (d) By what fraction does the potential difference between the inside and outside of the axon decrease over a distance of 2.0 mm? (e) The attenuation of the potential difference calculated in part (d) shows that the axon cannot simply be a passive, current-carrying electrical cable; the potential difference must periodically be reinforced along the axon's length. This reinforcement mechanism is slow, so a signal propagates along the axon at only about 30 m/s. In situations where faster response is required, axons are covered with a segmented sheath of fatty myelin. The segments are about 2 mm long, separated by gaps called the *nodes of Ranvier.* The myelin increases the resistance of a 1.0-$\mu$m-long segment of the membrane to $R_2 = 3.3 \times 10^{12}\ \Omega$. For such a myelinated axon, by what fraction does the potential difference between the inside and outside of the axon decrease over the distance from one node of Ranvier to the next? This smaller attenuation means the propagation speed is increased.

# Answers

## Chapter Opening Question ?

The potential difference $V$ is the same across resistors connected in parallel. However, there is a different current $I$ through each resistor if the resistances $R$ are different: $I = V/R$.

## Test Your Understanding Questions

**26.1 Answer: (a), (c), (d), (b)** Here's why: The three resistors in Fig. 26.1a are in series, so $R_{eq} = R + R + R = 3R$. In Fig. 26.1b the three resistors are in parallel, so $1/R_{eq} = 1/R + 1/R + 1/R = 3/R$ and $R_{eq} = R/3$. In Fig. 26.1c the second and third resistors are in parallel, so their equivalent resistance $R_{23}$ is given by $1/R_{23} = 1/R + 1/R = 2/R$; hence $R_{23} = R/2$. This combination is in series with the first resistor, so the three resistors together have equivalent resistance $R_{eq} = R + R/2 = 3R/2$. In Fig. 26.1d the second and third resistors are in series, so their equivalent resistance is $R_{23} = R + R = 2R$. This combination is in parallel with the first resistor, so the equivalent resistance of the three-resistor combination is given by $1/R_{eq} = 1/R + 1/2R = 3/2R$. Hence $R_{eq} = 2R/3$.

**26.2 Answer: loop *cbdac*** Equation (2) minus Eq. (1) gives $-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + (I_1 - I_3)(1\ \Omega) + I_1(1\ \Omega) = 0$. We can obtain this equation by applying the loop rule around the path from $c$ to $b$ to $d$ to $a$ to $c$ in Fig. 26.12. This isn't a new equation, so it would not have helped with the solution of Example 26.6.

**26.3 Answers: (a) (ii), (b) (iii)** An ammeter must always be placed in series with the circuit element of interest, and a voltmeter must always be placed in parallel. Ideally the ammeter would have zero resistance and the voltmeter would have infinite resistance so that their presence would have no effect on either the resistor current or the voltage. Neither of these idealizations is possible, but the ammeter resistance should be much less than 2 $\Omega$ and the voltmeter resistance should be much greater than 2 $\Omega$.

**26.4 Answer: (ii)** After one time constant, $t = RC$ and the initial charge $Q_0$ has decreased to $Q_0 e^{-t/RC} = Q_0 e^{-RC/RC} = Q_0 e^{-1} = Q_0/e$. Hence the stored energy has decreased from $Q_0^2/2C$ to $(Q_0/e)^2/2C = Q_0^2/2Ce^2$, a fraction $1/e^2 = 0.135$ of its initial value. This result doesn't depend on the initial value of the energy.

**26.5 Answer: no** This is a very dangerous thing to do. The circuit breaker will allow currents up to 40 A, double the rated value of the wiring. The amount of power $P = I^2R$ dissipated in a section of wire can therefore be up to four times the rated value, so the wires could get very warm and start a fire.

## Bridging Problem

**Answers: (a)** 9.39 J **(b)** $2.02 \times 10^4$ W **(c)** $4.65 \times 10^{-4}$ s **(d)** $7.43 \times 10^3$ W