# 35

# INTERFERENCE

**?** Soapy water is colorless, but when blown into bubbles it shows vibrant colors. How does the thickness of the bubble walls determine the particular colors that appear?

### LEARNING GOALS

*By studying this chapter, you will learn:*

- What happens when two waves combine, or interfere, in space.
- How to understand the interference pattern formed by the interference of two coherent light waves.
- How to calculate the intensity at various points in an interference pattern.
- How interference occurs when light reflects from the two surfaces of a thin film.
- How interference makes it possible to measure extremely small distances.

An ugly black oil spot on the pavement can become a thing of beauty after a rain, when the oil reflects a rainbow of colors. Multicolored reflections can also be seen from the surfaces of soap bubbles and DVDs. How is it possible for colorless objects to produce these remarkable colors?

In our discussion of lenses, mirrors, and optical instruments we used the model of *geometric optics,* in which we represent light as *rays,* straight lines that are bent at a reflecting or refracting surface. But many aspects of the behavior of light *can't* be understood on the basis of rays. We have already learned that light is fundamentally a *wave,* and in some situations we have to consider its wave properties explicitly. If two or more light waves of the same frequency overlap at a point, the total effect depends on the *phases* of the waves as well as their amplitudes. The resulting patterns of light are a result of the *wave* nature of light and cannot be understood on the basis of rays. Optical effects that depend on the wave nature of light are grouped under the heading **physical optics.**

In this chapter we'll look at *interference* phenomena that occur when two waves combine. The colors seen in oil films and soap bubbles are a result of interference between light reflected from the front and back surfaces of a thin film of oil or soap solution. Effects that occur when *many* sources of waves are present are called *diffraction* phenomena; we'll study these in Chapter 36. In that chapter we'll see that diffraction effects occur whenever a wave passes through an aperture or around an obstacle. They are important in practical applications of physical optics such as diffraction gratings, x-ray diffraction, and holography.

While our primary concern is with light, interference and diffraction can occur with waves of *any* kind. As we go along, we'll point out applications to other types of waves such as sound and water waves.

## 35.1 Interference and Coherent Sources

As we discussed in Chapter 15, the term **interference** refers to any situation in which two or more waves overlap in space. When this occurs, the total wave at any point at any instant of time is governed by the **principle of superposition,** which we introduced in Section 15.6 in the context of waves on a string. This principle also applies to electromagnetic waves and is the most important principle in all of physical optics. The principle of superposition states:

**When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.**

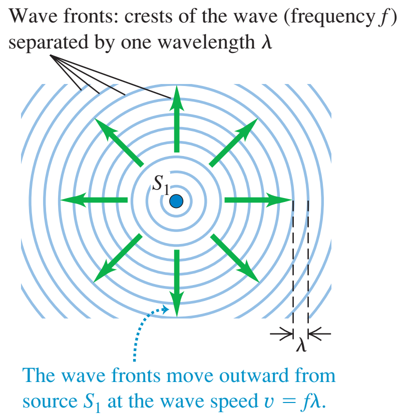
(In some special situations, such as electromagnetic waves propagating in a crystal, this principle may not apply. A discussion of these is beyond our scope.)

We use the term "displacement" in a general sense. With waves on the surface of a liquid, we mean the actual displacement of the surface above or below its normal level. With sound waves, the term refers to the excess or deficiency of pressure. For electromagnetic waves, we usually mean a specific component of electric or magnetic field.

### Interference in Two or Three Dimensions

We have already discussed one important case of interference, in which two identical waves propagating in opposite directions combine to produce a *standing wave*. We saw this in Chapters 15 and 16 for transverse waves on a string and for longitudinal waves in a fluid filling a pipe; we described the same phenomenon for electromagnetic waves in Section 32.5. In all of these cases the waves propagated along only a single axis: along a string, along the length of a fluid-filled pipe, or along the propagation direction of an electromagnetic plane wave. But light waves can (and do) travel in *two* or *three* dimensions, as can any kind of wave that propagates in a two- or three-dimensional medium. In this section we'll see what happens when we combine waves that spread out in two or three dimensions from a pair of identical wave sources.

**35.1** A "snapshot" of sinusoidal waves of frequency $f$ and wavelength $\lambda$ spreading out from source $S_1$ in all directions.




Interference effects are most easily seen when we combine *sinusoidal* waves with a single frequency $f$ and wavelength $\lambda$. Figure 35.1 shows a "snapshot" of a *single* source $S_1$ of sinusoidal waves and some of the wave fronts produced by this source. The figure shows only the wave fronts corresponding to wave *crests,* so the spacing between successive wave fronts is one wavelength. The material surrounding $S_1$ is uniform, so the wave speed is the same in all directions, and there is no refraction (and hence no bending of the wave fronts). If the waves are two-dimensional, like waves on the surface of a liquid, the circles in Fig. 35.1 represent circular wave fronts; if the waves propagate in three dimensions, the circles represent spherical wave fronts spreading away from $S_1$.

In optics, sinusoidal waves are characteristic of **monochromatic light** (light of a single color). While it's fairly easy to make water waves or sound waves of a single frequency, common sources of light *do not* emit monochromatic (single-frequency) light. For example, incandescent light bulbs and flames emit a continuous distribution of wavelengths. By far the most nearly monochromatic source that is available at present is the *laser.* An example is the helium–neon laser, which emits red light at 632.8 nm with a wavelength range of the order of $\pm 0.000001$ nm, or about one part in $10^9$. As we analyze interference and diffraction effects in this chapter and the next, we will assume that we are working with monochromatic waves (unless we explicitly state otherwise).

### Constructive and Destructive Interference

Two identical sources of monochromatic waves, $S_1$ and $S_2$, are shown in Fig. 35.2a. The two sources produce waves of the same amplitude and the same wavelength $\lambda$.

In addition, the two sources are permanently *in phase;* they vibrate in unison. They might be two loudspeakers driven by the same amplifier, two radio antennas powered by the same transmitter, or two small slits in an opaque screen, illuminated by the same monochromatic light source. We will see that if there were not a constant phase relationship between the two sources, the phenomena we are about to discuss would not occur. Two monochromatic sources of the same frequency and with a constant phase relationship (not necessarily in phase) are said to be **coherent.** We also use the term *coherent waves* (or, for light waves, *coherent light*) to refer to the waves emitted by two such sources.

If the waves emitted by the two coherent sources are *transverse,* like electromagnetic waves, then we will also assume that the wave disturbances produced by both sources have the same *polarization* (that is, they lie along the same line). For example, the sources $S_1$ and $S_2$ in Fig. 35.2a could be two radio antennas in the form of long rods oriented parallel to the $z$-axis (perpendicular to the plane of the figure); at any point in the $xy$-plane the waves produced by both antennas have $\vec{E}$ fields with only a $z$-component. Then we need only a single scalar function to describe each wave; this makes the analysis much easier.

We position the two sources of equal amplitude, equal wavelength, and (if the waves are transverse) the same polarization along the $y$-axis in Fig. 35.2a, equidistant from the origin. Consider a point $a$ on the $x$-axis. From symmetry the two distances from $S_1$ to $a$ and from $S_2$ to $a$ are *equal;* waves from the two sources thus require equal times to travel to $a$. Hence waves that leave $S_1$ and $S_2$ in phase arrive at $a$ in phase. The two waves add, and the total amplitude at $a$ is *twice* the amplitude of each individual wave. This is true for *any* point on the $x$-axis.

Similarly, the distance from $S_2$ to point $b$ is exactly two wavelengths *greater* than the distance from $S_1$ to $b$. A wave crest from $S_1$ arrives at $b$ exactly two cycles earlier than a crest emitted at the same time from $S_2$, and again the two waves arrive in phase. As at point $a$, the total amplitude is the sum of the amplitudes of the waves from $S_1$ and $S_2$.

In general, when waves from two or more sources arrive at a point *in phase,* they reinforce each other: The amplitude of the resultant wave is the *sum* of the amplitudes of the individual waves. This is called **constructive interference** (Fig. 35.2b). Let the distance from $S_1$ to any point $P$ be $r_1$, and let the distance from $S_2$ to $P$ be $r_2$. For constructive interference to occur at $P$, the path difference $r_2 - r_1$ for the two sources must be an integral multiple of the wavelength $\lambda$:

$$r_2 - r_1 = m\lambda \qquad (m = 0, \pm 1, \pm 2, \pm 3, \dots) \qquad \begin{array}{l}\text{(constructive}\\ \text{interference,}\\ \text{sources in phase)}\end{array} \tag{35.1}$$

In Fig. 35.2a, points $a$ and $b$ satisfy Eq. (35.1) with $m = 0$ and $m = +2$, respectively.

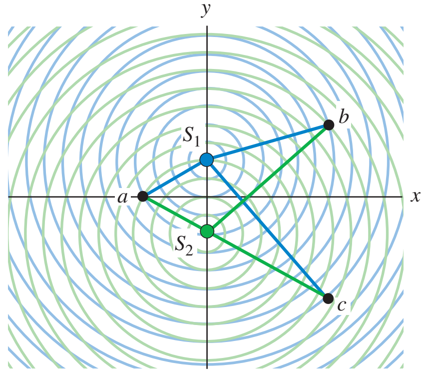
Something different occurs at point $c$ in Fig. 35.2a. At this point, the path difference $r_2 - r_1 = -2.50\lambda$, which is a *half-integral* number of wavelengths. Waves from the two sources arrive at point $c$ exactly a half-cycle out of phase. A crest of one wave arrives at the same time as a crest in the opposite direction (a "trough") of the other wave (Fig. 35.2c). The resultant amplitude is the *difference* between the two individual amplitudes. If the individual amplitudes are equal, then the total amplitude is *zero*! This cancellation or partial cancellation of the individual waves is called **destructive interference.** The condition for destructive interference in the situation shown in Fig. 35.2a is

$$r_2 - r_1 = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \dots) \quad \begin{array}{l}\text{(destructive}\\ \text{interference,}\\ \text{sources in phase)}\end{array} \tag{35.2}$$

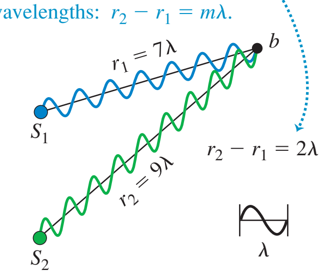
The path difference at point $c$ in Fig. 35.2a satisfies Eq. (35.2) with $m = -3$.

**35.2** (a) A "snapshot" of sinusoidal waves spreading out from two coherent sources $S_1$ and $S_2$. Constructive interference occurs at point $a$ (equidistant from the two sources) and (b) at point $b$. (c) Destructive interference occurs at point $c$.

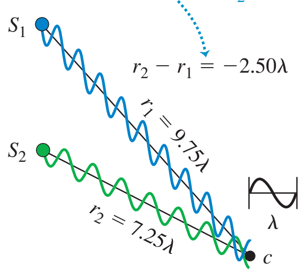
(a) Two coherent wave sources separated by a distance $4\lambda$




(b) Conditions for constructive interference: Waves interfere constructively if their path lengths differ by an integral number of wavelengths: $r_2 - r_1 = m\lambda$.




(c) Conditions for destructive interference: Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$.



**35.3** The same as Fig. 35.2a, but with red antinodal curves (curves of maximum amplitude) superimposed. All points on each curve satisfy Eq. (35.1) with the value of $m$ shown. The nodal curves (not shown) lie between each adjacent pair of antinodal curves.

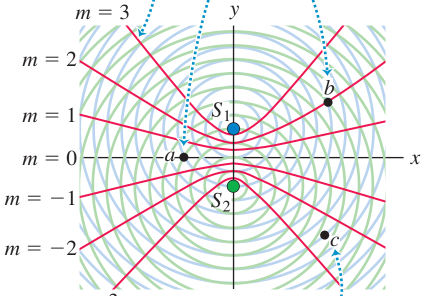



Figure 35.3 shows the same situation as in Fig. 35.2a, but with red curves that show all positions where *constructive* interference occurs. On each curve, the path difference $r_2 - r_1$ is equal to an integer $m$ times the wavelength, as in Eq. (35.1). These curves are called **antinodal curves.** They are directly analogous to *antinodes* in the standing-wave patterns described in Chapters 15 and 16 and Section 32.5. In a standing wave formed by interference between waves propagating in opposite directions, the antinodes are points at which the amplitude is maximum; likewise, the wave amplitude in the situation of Fig. 35.3 is maximum along the antinodal curves. Not shown in Fig. 35.3 are the **nodal curves,** which are the curves that show where *destructive* interference occurs in accordance with Eq. (35.2); these are analogous to the *nodes* in a standing-wave pattern. A nodal curve lies between each two adjacent antinodal curves in Fig. 35.3; one such curve, corresponding to $r_2 - r_1 = -2.50\lambda$, passes through point $c$.

In some cases, such as two loudspeakers or two radio-transmitter antennas, the interference pattern is three-dimensional. Think of rotating the color curves of Fig. 35.3 around the $y$-axis; then maximum constructive interference occurs at all points on the resulting surfaces of revolution.

**CAUTION** **Interference patterns are not standing waves** The interference patterns in Figs. 35.2a and 35.3 are *not* standing waves, though they have some similarities to the standing-wave patterns described in Chapters 15 and 16 and Section 32.5. In a standing wave, the interference is between two waves propagating in opposite directions; there is *no* net energy flow in either direction (the energy in the wave is left "standing"). In the situations shown in Figs. 35.2a and 35.3, there is likewise a stationary pattern of antinodal and nodal curves, but there is a net flow of energy *outward* from the two sources. All that interference does is to "channel" the energy flow so that it is greatest along the antinodal curves and least along the nodal curves.

For Eqs. (35.1) and (35.2) to hold, the two sources must have the same wavelength and must *always* be in phase. These conditions are rather easy to satisfy for sound waves. But with *light* waves there is no practical way to achieve a constant phase relationship (coherence) with two independent sources. This is because of the way light is emitted. In ordinary light sources, atoms gain excess energy by thermal agitation or by impact with accelerated electrons. Such an "excited" atom begins to radiate energy and continues until it has lost all the energy it can, typically in a time of the order of $10^{-8}$ s. The many atoms in a source ordinarily radiate in an unsynchronized and random phase relationship, and the light that is emitted from *two* such sources has no definite phase relationship.

However, the light from a single source can be split so that parts of it emerge from two or more regions of space, forming two or more *secondary sources.* Then any random phase change in the source affects these secondary sources equally and does not change their *relative* phase.

The distinguishing feature of light from a *laser* is that the emission of light from many atoms is synchronized in frequency and phase. As a result, the random phase changes mentioned above occur much less frequently. Definite phase relationships are preserved over correspondingly much greater lengths in the beam, and laser light is much more coherent than ordinary light.

**Test Your Understanding of Section 35.1** Consider a point in Fig. 35.3 on the positive $y$-axis above $S_1$. Does this point lie on (i) an antinodal curve; (ii) a nodal curve; or (iii) neither? (*Hint:* The distance between $S_1$ and $S_2$ is $4\lambda$.)

## 35.2 Two-Source Interference of Light

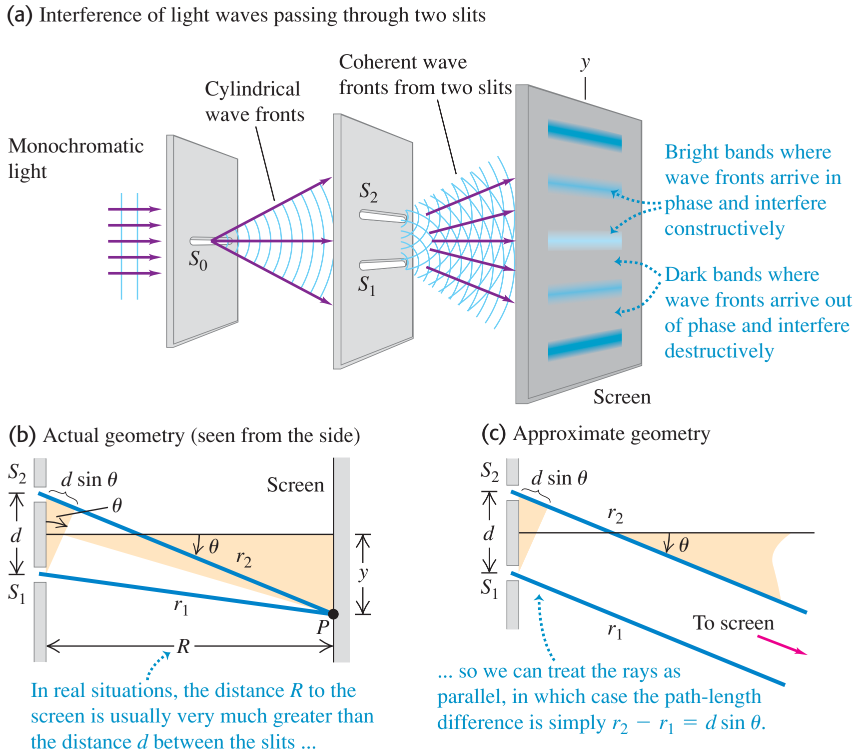
The interference pattern produced by two coherent sources of *water* waves of the same wavelength can be readily seen in a ripple tank with a shallow layer of water (Fig. 35.4). This pattern is not directly visible when the interference is between *light* waves, since light traveling in a uniform medium cannot be seen. (A shaft of afternoon sunlight in a room is made visible by scattering from airborne dust particles.)

One of the earliest quantitative experiments to reveal the interference of light from two sources was performed in 1800 by the English scientist Thomas Young. We will refer back to this experiment several times in this and later chapters, so it's important to understand it in detail. Young's apparatus is shown in perspective in Fig. 35.5a. A light source (not shown) emits monochromatic light; however, this light is not suitable for use in an interference experiment because emissions from different parts of an ordinary source are not synchronized. To remedy this, the light is directed at a screen with a narrow slit $S_0$, 1 $\mu$m or so wide. The light emerging from the slit originated from only a small region of the light source; thus slit $S_0$ behaves more nearly like the idealized source shown in Fig. 35.1. (In modern versions of the experiment, a laser is used as a source of coherent light, and the slit $S_0$ isn't needed.) The light from slit $S_0$ falls on a screen with two other narrow slits $S_1$ and $S_2$, each 1 $\mu$m or so wide and a few tens or hundreds of micrometers apart. Cylindrical wave fronts spread out from slit $S_0$ and reach slits $S_1$ and $S_2$ *in phase* because they travel equal distances from $S_0$. The waves *emerging* from slits $S_1$ and $S_2$ are therefore also always in phase, so $S_1$ and $S_2$ are *coherent* sources. The interference of waves from $S_1$ and $S_2$ produces a pattern in space like that to the right of the sources in Figs. 35.2a and 35.3.

To visualize the interference pattern, a screen is placed so that the light from $S_1$ and $S_2$ falls on it (Fig. 35.5b). The screen will be most brightly illuminated at points $P$, where the light waves from the slits interfere constructively, and will be darkest at points where the interference is destructive.

To simplify the analysis of Young's experiment, we assume that the distance $R$ from the slits to the screen is so large in comparison to the distance $d$ between the slits that the lines from $S_1$ and $S_2$ to $P$ are very nearly parallel, as in Fig. 35.5c. This is usually the case for experiments with light; the slit separation is typically a few millimeters, while the screen may be a meter or more away. The difference in path length is then given by

**35.4** The concepts of constructive interference and destructive interference apply to these water waves as well as to light waves and sound waves.




**35.5** (a) Young's experiment to show interference of light passing through two slits. A pattern of bright and dark areas appears on the screen (see Fig. 35.6). (b) Geometrical analysis of Young's experiment. For the case shown, $r_2 > r_1$ and both $y$ and $\theta$ are positive. If point $P$ is on the other side of the screen's center, $r_2 < r_1$ and both $y$ and $\theta$ are negative. (c) Approximate geometry when the distance $R$ to the screen is much greater than the distance $d$ between the slits.

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**ActivPhysics 16.3:** Two-Source Interference: Problems

$$r_2 - r_1 = d \sin\theta \tag{35.3}$$

where $\theta$ is the angle between a line from slits to screen (shown in blue in Fig. 35.5c) and the normal to the plane of the slits (shown as a thin black line).

## Constructive and Destructive Two-Slit Interference

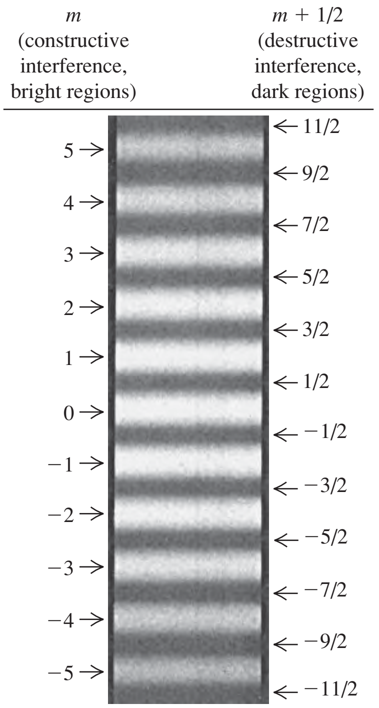
We found in Section 35.1 that constructive interference (reinforcement) occurs at points where the path difference is an integral number of wavelengths, $m\lambda$, where $m = 0, \pm 1, \pm 2, \pm 3, \ldots$. So the bright regions on the screen in Fig. 35.5a occur at angles $\theta$ for which

$$d \sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots) \qquad \text{(constructive interference, two slits)} \tag{35.4}$$

Similarly, destructive interference (cancellation) occurs, forming dark regions on the screen, at points for which the path difference is a half-integral number of wavelengths, $\left(m + \frac{1}{2}\right)\lambda$:

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots) \qquad \text{(destructive interference, two slits)} \tag{35.5}$$

**35.6** Photograph of interference fringes produced on a screen in Young's double-slit experiment.




Thus the pattern on the screen of Figs. 35.5a and 35.5b is a succession of bright and dark bands, or **interference fringes,** parallel to the slits $S_1$ and $S_2$. A photograph of such a pattern is shown in Fig. 35.6. The center of the pattern is a bright band corresponding to $m = 0$ in Eq. (35.4); this point on the screen is equidistant from the two slits.

We can derive an expression for the positions of the centers of the bright bands on the screen. In Fig. 35.5b, $y$ is measured from the center of the pattern, corresponding to the distance from the center of Fig. 35.6. Let $y_m$ be the distance from the center of the pattern $(\theta = 0)$ to the center of the $m$th bright band. Let $\theta_m$ be the corresponding value of $\theta$; then

$$y_m = R \tan\theta_m$$

In experiments such as this, the distances $y_m$ are often much smaller than the distance $R$ from the slits to the screen. Hence $\theta_m$ is very small, $\tan\theta_m$ is very nearly equal to $\sin\theta_m$, and

$$y_m = R \sin\theta_m$$

Combining this with Eq. (35.4), we find that *for small angles only,*

$$y_m = R\frac{m\lambda}{d} \qquad \text{(constructive interference in Young's experiment)} \tag{35.6}$$

We can measure $R$ and $d$, as well as the positions $y_m$ of the bright fringes, so this experiment provides a direct measurement of the wavelength $\lambda$. Young's experiment was in fact the first direct measurement of wavelengths of light.

**CAUTION** **Equation (35.6) is for small angles only** While Eqs. (35.4) and (35.5) are valid for any angle, Eq. (35.6) is valid only for *small* angles. It can be used *only* if the distance $R$ from slits to screen is much greater than the slit separation $d$ and if $R$ is much greater than the distance $y_m$ from the center of the interference pattern to the $m$th bright fringe.

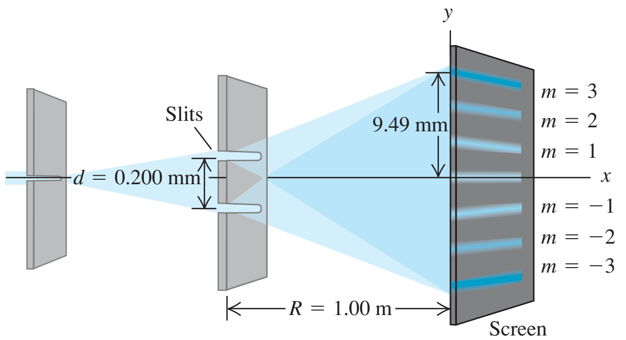
The distance between adjacent bright bands in the pattern is *inversely* proportional to the distance $d$ between the slits. The closer together the slits are, the more the pattern spreads out. When the slits are far apart, the bands in the pattern are closer together.

While we have described the experiment that Young performed with visible light, the results given in Eqs. (35.4) and (35.5) are valid for *any* type of wave, provided that the resultant wave from two coherent sources is detected at a point that is far away in comparison to the separation $d$.

## Example 35.1 Two-slit interference

Figure 35.7 shows a two-slit interference experiment in which the slits are 0.200 mm apart and the screen is 1.00 m from the slits. The $m = 3$ bright fringe in the figure is 9.49 mm from the central fringe. Find the wavelength of the light.

**35.7** Using a two-slit interference experiment to measure the wavelength of light.




**SOLUTION**

**IDENTIFY and SET UP:** Our target variable in this two-slit interference problem is the wavelength $\lambda$. We are given the slit separation $d = 0.200$ mm, the distance from slits to screen $R = 1.00$ m, and the distance $y_3 = 9.49$ mm on the screen from the center of the interference pattern to the $m = 3$ bright fringe. We may use Eq. (35.6) to find $\lambda$, since the value of $R$ is so much greater than the values of $d$ or $y_3$.

**EXECUTE:** We solve Eq. (35.6) for $\lambda$ for the case $m = 3$:

$$\lambda = \frac{y_m d}{mR} = \frac{(9.49 \times 10^{-3}\ \text{m})(0.200 \times 10^{-3}\ \text{m})}{(3)(1.00\ \text{m})}$$
$$= 633 \times 10^{-9}\ \text{m} = 633\ \text{nm}$$

**EVALUATE:** This bright fringe could also correspond to $m = -3$. Can you show that this gives the same result for $\lambda$?

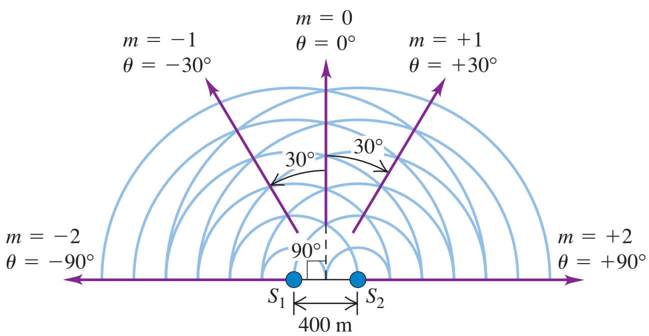
## Example 35.2 Broadcast pattern of a radio station

It is often desirable to radiate most of the energy from a radio transmitter in particular directions rather than uniformly in all directions. Pairs or rows of antennas are often used to produce the desired radiation pattern. As an example, consider two identical vertical antennas 400 m apart, operating at 1500 kHz $= 1.5 \times 10^6$ Hz (near the top end of the AM broadcast band) and oscillating in phase. At distances much greater than 400 m, in what directions is the intensity from the two antennas greatest?

**SOLUTION**

**IDENTIFY and SET UP:** The antennas, shown in Fig. 35.8, correspond to sources $S_1$ and $S_2$ in Fig. 35.5. Hence we can apply the ideas of two-slit interference to this problem. Since the resultant wave is detected at distances much greater than $d = 400$ m, we may use Eq. (35.4) to give the directions of the intensity maxima, the values of $\theta$ for which the path difference is zero or a whole number of wavelengths.

**35.8** Two radio antennas broadcasting in phase. The purple arrows indicate the directions of maximum intensity. The waves that are emitted toward the lower half of the figure are not shown.




**EXECUTE:** The wavelength is $\lambda = c/f = 200$ m. From Eq. (35.4) with $m = 0$, $\pm1$, and $\pm2$, the intensity maxima are given by

$$\sin\theta = \frac{m\lambda}{d} = \frac{m(200\ \text{m})}{400\ \text{m}} = \frac{m}{2} \qquad \theta = 0,\ \pm30°,\ \pm90°$$

In this example, values of $m$ greater than 2 or less than $-2$ give values of $\sin\theta$ greater than 1 or less than $-1$, which is impossible. There is *no* direction for which the path difference is three or more wavelengths, so values of $m$ of $\pm3$ or beyond have no meaning in this example.

**EVALUATE:** We can check our result by calculating the angles for *minimum* intensity, using Eq. (35.5). There should be one intensity minimum between each pair of intensity maxima, just as in Fig. 35.6. From Eq. (35.5), with $m = -2, -1, 0$, and 1,

$$\sin\theta = \frac{(m + \frac{1}{2})\lambda}{d} = \frac{m + \frac{1}{2}}{2} \qquad \theta = \pm14.5°,\ \pm48.6°$$

These angles fall between the angles for intensity maxima, as they should. The angles are not small, so the angles for the minima are *not* exactly halfway between the angles for the maxima.

**Test Your Understanding of Section 35.2** You shine a tunable laser (whose wavelength can be adjusted by turning a knob) on a pair of closely spaced slits. The light emerging from the two slits produces an interference pattern on a screen like that shown in Fig. 35.6. If you adjust the wavelength so that the laser light changes from red to blue, how will the spacing between bright fringes change? (i) The spacing increases; (ii) the spacing decreases; (iii) the spacing is unchanged; (iv) not enough information is given to decide.

## 35.3 Intensity in Interference Patterns

In Section 35.2 we found the positions of maximum and minimum intensity in a two-source interference pattern. Let's now see how to find the intensity at *any* point in the pattern. To do this, we have to combine the two sinusoidally varying fields (from the two sources) at a point $P$ in the radiation pattern, taking proper account of the phase difference of the two waves at point $P$, which results from the path difference. The intensity is then proportional to the square of the resultant electric-field amplitude, as we learned in Chapter 32.

To calculate the intensity, we will assume that the two sinusoidal functions (corresponding to waves from the two sources) have equal amplitude $E$ and that the $\vec{E}$ fields lie along the same line (have the same polarization). This assumes that the sources are identical and neglects the slight amplitude difference caused by the unequal path lengths (the amplitude decreases with increasing distance from the source). From Eq. (32.29), each source by itself would give an intensity $\frac{1}{2}\epsilon_0 cE^2$ at point $P$. If the two sources are in phase, then the waves that arrive at $P$ differ in phase by an amount proportional to the difference in their path lengths, $(r_2 - r_1)$. If the phase angle between these arriving waves is $\phi$, then we can use the following expressions for the two electric fields superposed at $P$:

$$E_1(t) = E\cos(\omega t + \phi)$$
$$E_2(t) = E\cos\omega t$$

The superposition of the two fields at $P$ is a sinusoidal function with some amplitude $E_P$ that depends on $E$ and the phase difference $\phi$. First we'll work on finding the amplitude $E_P$ if $E$ and $\phi$ are known. Then we'll find the intensity $I$ of the resultant wave, which is proportional to $E_P{}^2$. Finally, we'll relate the phase difference $\phi$ to the path difference, which is determined by the geometry of the situation.

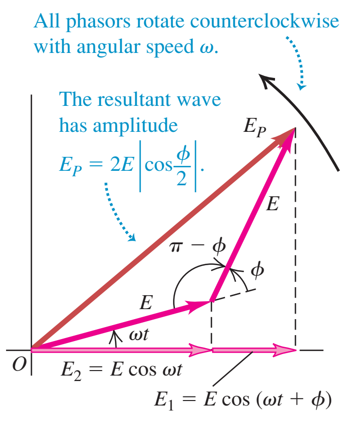
### Amplitude in Two-Source Interference

To add the two sinusoidal functions with a phase difference, we use the same *phasor* representation that we used for simple harmonic motion (see Section 14.2) and for voltages and currents in ac circuits (see Section 31.1). We suggest that you review these sections now. Each sinusoidal function is represented by a rotating vector (phasor) whose projection on the horizontal axis at any instant represents the instantaneous value of the sinusoidal function.

In Fig. 35.9, $E_1$ is the horizontal component of the phasor representing the wave from source $S_1$, and $E_2$ is the horizontal component of the phasor for the wave from $S_2$. As shown in the diagram, both phasors have the same magnitude $E$, but $E_1$ is *ahead* of $E_2$ in phase by an angle $\phi$. Both phasors rotate counterclockwise with constant angular speed $\omega$, and the sum of the projections on the horizontal axis at any time gives the instantaneous value of the total $E$ field at point $P$. Thus the amplitude $E_P$ of the resultant sinusoidal wave at $P$ is the magnitude of the dark red phasor in the diagram (labeled $E_P$); this is the *vector sum* of the other two phasors. To find $E_P$, we use the law of cosines and the trigonometric identity $\cos(\pi - \phi) = -\cos\phi$:

$$E_P{}^2 = E^2 + E^2 - 2E^2\cos(\pi - \phi)$$
$$= E^2 + E^2 + 2E^2\cos\phi$$

**35.9** Phasor diagram for the superposition at a point $P$ of two waves of equal amplitude $E$ with a phase difference $\phi$.



Then, using the identity $1 + \cos\phi = 2\cos^2(\phi/2)$, we obtain

$$E_P{}^2 = 2E^2(1 + \cos\phi) = 4E^2\cos^2\left(\frac{\phi}{2}\right)$$

$$E_P = 2E\left|\cos\frac{\phi}{2}\right| \qquad \text{(amplitude in two-source interference)} \qquad (35.7)$$

You can also obtain this result without using phasors (see Problem 35.50).

When the two waves are in phase, $\phi = 0$ and $E_P = 2E$. When they are exactly a half-cycle out of phase, $\phi = \pi$ rad $= 180°$, $\cos(\phi/2) = \cos(\pi/2) = 0$, and $E_P = 0$. Thus the superposition of two sinusoidal waves with the same frequency and amplitude but with a phase difference yields a sinusoidal wave with the same frequency and an amplitude between zero and twice the individual amplitudes, depending on the phase difference.

## Intensity in Two-Source Interference

To obtain the intensity $I$ at point $P$, we recall from Section 32.4 that $I$ is equal to the average magnitude of the Poynting vector, $S_{av}$. For a sinusoidal wave with electric-field amplitude $E_P$, this is given by Eq. (32.29) with $E_{max}$ replaced by $E_P$. Thus we can express the intensity in several equivalent forms:

$$I = S_{av} = \frac{E_P{}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_P{}^2 = \frac{1}{2}\epsilon_0 c E_P{}^2 \qquad (35.8)$$

The essential content of these expressions is that $I$ is proportional to $E_P{}^2$. When we substitute Eq. (35.7) into the last expression in Eq. (35.8), we get

$$I = \frac{1}{2}\epsilon_0 c E_P{}^2 = 2\epsilon_0 c E^2 \cos^2\frac{\phi}{2} \qquad (35.9)$$

In particular, the *maximum* intensity $I_0$, which occurs at points where the phase difference is zero ($\phi = 0$), is

$$I_0 = 2\epsilon_0 c E^2$$

Note that the maximum intensity $I_0$ is *four times* (not twice) as great as the intensity $\frac{1}{2}\epsilon_0 c E^2$ from each individual source.

Substituting the expression for $I_0$ into Eq. (35.9), we can express the intensity $I$ at any point very simply in terms of the maximum intensity $I_0$:

$$I = I_0\cos^2\frac{\phi}{2} \qquad \text{(intensity in two-source interference)} \qquad (35.10)$$

For some phase angles $\phi$ the intensity is $I_0$, four times as great as for an individual wave source, but for other phase angles the intensity is zero. If we average Eq. (35.10) over all possible phase differences, the result is $I_0/2 = \epsilon_0 c E^2$ (the average of $\cos^2(\phi/2)$ is $\frac{1}{2}$). This is just twice the intensity from each individual source, as we should expect. The total energy output from the two sources isn't changed by the interference effects, but the energy is redistributed (as we mentioned in Section 35.1).

## Phase Difference and Path Difference

Our next task is to find the phase difference $\phi$ between the two fields at any point $P$. We know that $\phi$ is proportional to the difference in path length from the two sources to point $P$. When the path difference is one wavelength, the phase difference is one cycle, and $\phi = 2\pi$ rad $= 360°$. When the path difference is

$\lambda/2$, $\phi = \pi$ rad $= 180°$, and so on. That is, the ratio of the phase difference $\phi$ to $2\pi$ is equal to the ratio of the path difference $r_2 - r_1$ to $\lambda$:

$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

Thus a path difference $(r_2 - r_1)$ causes a phase difference given by

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1) \qquad \text{(phase difference related to path difference)} \qquad (35.11)$$

where $k = 2\pi/\lambda$ is the *wave number* introduced in Section 15.3.

If the material in the space between the sources and $P$ is anything other than vacuum, we must use the wavelength *in the material* in Eq. (35.11). If the material has index of refraction $n$, then

$$\lambda = \frac{\lambda_0}{n} \qquad \text{and} \qquad k = nk_0 \qquad (35.12)$$

where $\lambda_0$ and $k_0$ are the wavelength and the wave number, respectively, in vacuum.

Finally, if the point $P$ is far away from the sources in comparison to their separation $d$, the path difference is given by Eq. (35.3):

$$r_2 - r_1 = d\sin\theta$$

Combining this with Eq. (35.11), we find

$$\phi = k(r_2 - r_1) = kd\sin\theta = \frac{2\pi d}{\lambda}\sin\theta \qquad (35.13)$$

When we substitute this into Eq. (35.10), we find

$$I = I_0\cos^2\left(\tfrac{1}{2}kd\sin\theta\right) = I_0\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) \qquad \text{(intensity far from two sources)} \qquad (35.14)$$

The directions of *maximum* intensity occur when the cosine has the values $\pm 1$—that is, when

$$\frac{\pi d}{\lambda}\sin\theta = m\pi \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

or

$$d\sin\theta = m\lambda$$

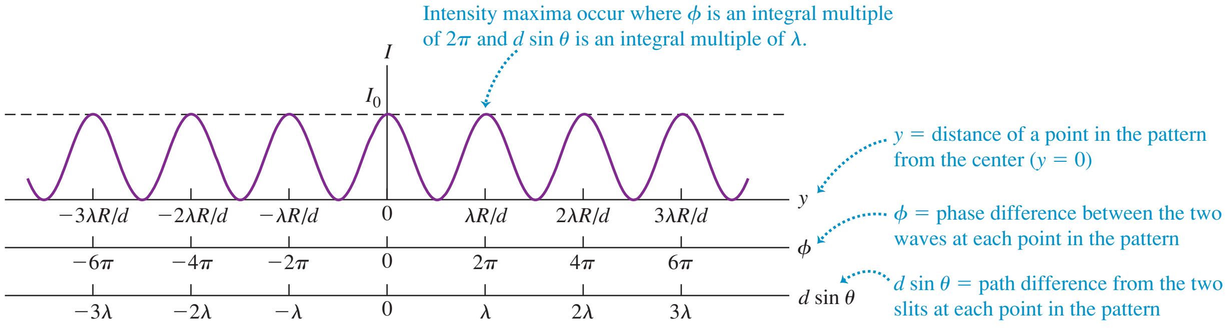
in agreement with Eq. (35.4). You can also derive Eq. (35.5) for the zero-intensity directions from Eq. (35.14).

As we noted in Section 35.2, in experiments with light we visualize the interference pattern due to two slits by using a screen placed at a distance $R$ from the slits. We can describe positions on the screen with the coordinate $y$; the positions of the bright fringes are given by Eq. (35.6), where ordinarily $y \ll R$. In that case, $\sin\theta$ is approximately equal to $y/R$, and we obtain the following expressions for the intensity at *any* point on the screen as a function of $y$:

$$I = I_0\cos^2\left(\frac{kdy}{2R}\right) = I_0\cos^2\left(\frac{\pi dy}{\lambda R}\right) \qquad \text{(intensity in two-slit interference)} \qquad (35.15)$$

Figure 35.10 shows a graph of Eq. (35.15); we can compare this with the photographically recorded pattern of Fig. 35.6. The peaks in Fig. 35.10 all have the same intensity, while those in Fig. 35.6 fade off as we go away from the center. We'll explore the reasons for this variation in peak intensity in Chapter 36.

**35.10** Intensity distribution in the interference pattern from two identical slits.




## Example 35.3 A directional transmitting antenna array

Suppose the two identical radio antennas of Fig. 35.8 are moved to be only 10.0 m apart and the broadcast frequency is increased to $f = 60.0$ MHz. At a distance of 700 m from the point midway between the antennas and in the direction $\theta = 0$ (see Fig. 35.8), the intensity is $I_0 = 0.020\ \text{W/m}^2$. At this same distance, find (a) the intensity in the direction $\theta = 4.0°$; (b) the direction near $\theta = 0$ for which the intensity is $I_0/2$; and (c) the directions in which the intensity is zero.

### SOLUTION

**IDENTIFY and SET UP:** This problem involves the intensity distribution as a function of angle. Because the 700-m distance from the antennas to the point at which the intensity is measured is much greater than the distance $d = 10.0$ m between the antennas, the amplitudes of the waves from the two antennas are very nearly equal. Hence we can use Eq. (35.14) to relate intensity $I$ and angle $\theta$.

**EXECUTE:** The wavelength is $\lambda = c/f = 5.00$ m. The spacing $d = 10.0$ m between the antennas is just twice the wavelength (as was the case in Example 35.2), so $d/\lambda = 2.00$ and Eq. (35.14) becomes

$$I = I_0\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = I_0\cos^2[(2.00\pi\ \text{rad})\sin\theta]$$

(a) When $\theta = 4.0°$,

$$I = I_0\cos^2[(2.00\pi\ \text{rad})\sin 4.0°] = 0.82 I_0$$
$$= (0.82)(0.020\ \text{W/m}^2) = 0.016\ \text{W/m}^2$$

(b) The intensity $I$ equals $I_0/2$ when the cosine in Eq. (35.14) has the value $\pm 1/\sqrt{2}$. The smallest angles at which this occurs correspond to $2.00\pi \sin\theta = \pm\pi/4$ rad, so that $\sin\theta = \pm(1/8.00) = \pm 0.125$ and $\theta = \pm 7.2°$.

(c) The intensity is zero when $\cos[(2.00\pi\ \text{rad})\sin\theta] = 0$. This occurs for $2.00\pi\sin\theta = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \ldots$, or $\sin\theta = \pm 0.250, \pm 0.750, \pm 1.25, \ldots$. Values of $\sin\theta$ greater than 1 have no meaning, so the answers are

$$\theta = \pm 14.5°, \pm 48.6°$$

**EVALUATE:** The condition in part (b) that $I = I_0/2$, so that $(2.00\pi\ \text{rad})\sin\theta = \pm\pi/4$ rad, is also satisfied when $\sin\theta = \pm 0.375, \pm 0.625$, or $\pm 0.875$ so that $\theta = \pm 22.0°, \pm 38.7°$, or $\pm 61.0°$. (Can you verify this?) It would be incorrect to include these angles in the solution, however, because the problem asked for the angle *near* $\theta = 0$ at which $I = I_0/2$. These additional values of $\theta$ aren't the ones we're looking for.

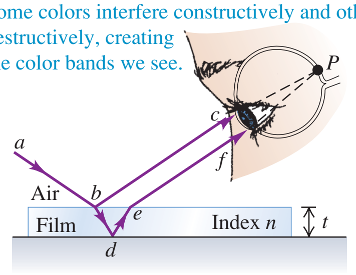
**Test Your Understanding of Section 35.3** A two-slit interference experiment uses coherent light of wavelength $5.00 \times 10^{-7}$ m. Rank the following points in the interference pattern according to the intensity at each point, from highest to lowest. (i) a point that is closer to one slit than the other by $4.00 \times 10^{-7}$ m; (ii) a point where the light waves received from the two slits are out of phase by 4.00 rad; (iii) a point that is closer to one slit than the other by $7.50 \times 10^{-7}$ m; (iv) a point where the light waves received by the two slits are out of phase by 2.00 rad.

## 35.4 Interference in Thin Films

You often see bright bands of color when light reflects from a thin layer of oil floating on water or from a soap bubble (see the photograph that opens this chapter). These are the results of interference. Light waves are reflected from the front and back surfaces of such thin films, and constructive interference between the two reflected waves (with different path lengths) occurs in different

**35.11** (a) A diagram and (b) a photograph showing interference of light reflected from a thin film.

(a) Interference between rays reflected from the two surfaces of a thin film




(b) The rainbow fringes of an oil slick on water

places for different wavelengths. Figure 35.11a shows the situation. Light shining on the upper surface of a thin film with thickness $t$ is partly reflected at the upper surface (path $abc$). Light *transmitted* through the upper surface is partly reflected at the lower surface (path $abdef$). The two reflected waves come together at point $P$ on the retina of the eye. Depending on the phase relationship, they may interfere constructively or destructively. Different colors have different wavelengths, so the interference may be constructive for some colors and destructive for others. That's why we see colored rings or fringes in Fig. 35.11b (which shows a thin film of oil floating on water) and in the photograph that opens this chapter (which shows thin films of soap solution that make up the bubble walls). The complex shapes of the colored rings in each photograph result from variations in the thickness of the film.

## Thin-Film Interference and Phase Shifts During Reflection

Let's look at a simplified situation in which *monochromatic* light reflects from two nearly parallel surfaces at nearly normal incidence. Figure 35.12 shows two plates of glass separated by a thin wedge, or film, of air. We want to consider interference between the two light waves reflected from the surfaces adjacent to the air wedge, as shown. (Reflections also occur at the top surface of the upper plate and the bottom surface of the lower plate; to keep our discussion simple, we won't include these.) The situation is the same as in Fig. 35.11a except that the film (wedge) thickness is not uniform. The path difference between the two waves is just twice the thickness $t$ of the air wedge at each point. At points where $2t$ is an integer number of wavelengths, we expect to see constructive interference and a bright area; where it is a half-integer number of wavelengths, we expect to see destructive interference and a dark area. Along the line where the plates are in contact, there is practically *no* path difference, and we expect a bright area.

When we carry out the experiment, the bright and dark fringes appear, but they are interchanged! Along the line where the plates are in contact, we find a *dark* fringe, not a bright one. This suggests that one or the other of the reflected waves has undergone a half-cycle phase shift during its reflection. In that case the two waves that are reflected at the line of contact are a half-cycle out of phase even though they have the same path length.

In fact, this phase shift can be predicted from Maxwell's equations and the electromagnetic nature of light. The details of the derivation are beyond our scope, but here is the result. Suppose a light wave with electric-field amplitude $E_\mathrm{i}$ is traveling in an optical material with index of refraction $n_a$. It strikes, at normal incidence, an interface with another optical material with index $n_b$. The amplitude $E_\mathrm{r}$ of the wave reflected from the interface is proportional to the amplitude $E_\mathrm{i}$ of the incident wave and is given by

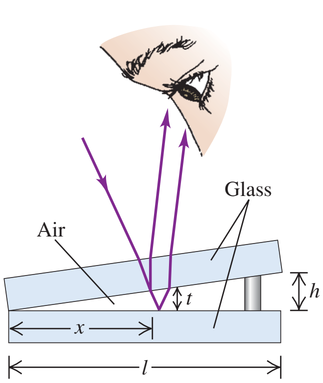
$$E_\mathrm{r} = \frac{n_a - n_b}{n_a + n_b} E_\mathrm{i} \quad \text{(normal incidence)} \tag{35.16}$$

This result shows that the incident and reflected amplitudes have the same sign when $n_a$ is larger than $n_b$ and opposite sign when $n_b$ is larger than $n_a$. We can distinguish three cases, as shown in Fig. 35.13:

*Figure 35.13a:* When $n_a > n_b$, light travels more slowly in the first material than in the second. In this case, $E_\mathrm{r}$ and $E_\mathrm{i}$ have the same sign, and the phase shift of the reflected wave relative to the incident wave is zero. This is analogous to reflection of a transverse mechanical wave on a heavy rope at a point where it is tied to a lighter rope or a ring that can move vertically without friction.

*Figure 35.13b:* When $n_a = n_b$, the amplitude $E_\mathrm{r}$ of the reflected wave is zero. The incident light wave can't "see" the interface, and there is *no* reflected wave.

**35.12** Interference between light waves reflected from the two sides of an air wedge separating two glass plates. The angles and the thickness of the air wedge have been exaggerated for clarity; in the text we assume that the light strikes the upper plate at normal incidence and that the distances $h$ and $t$ are much less than $l$.



**35.13** Upper figures: electromagnetic waves striking an interface between optical materials at normal incidence (shown as a small angle for clarity). Lower figures: mechanical wave pulses on ropes.

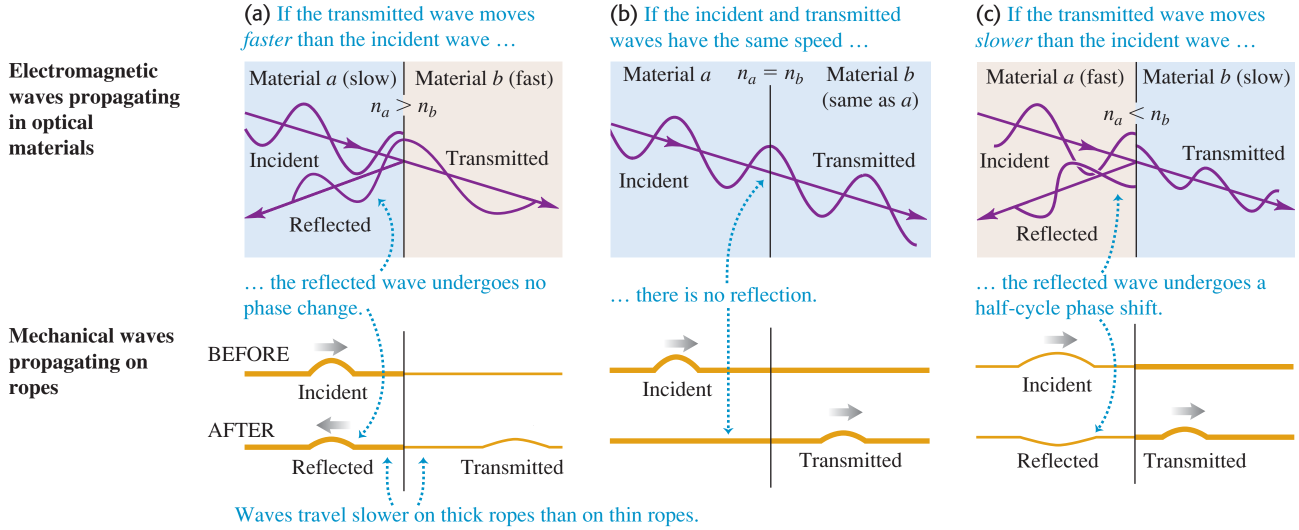



*Figure 35.13c:* When $n_a < n_b$, light travels more slowly in the second material than in the first. In this case, $E_r$ and $E_i$ have opposite signs, and the phase shift of the reflected wave relative to the incident wave is $\pi$ rad (180° or a half-cycle). This is analogous to reflection (with inversion) of a transverse mechanical wave on a light rope at a point where it is tied to a heavier rope or a rigid support.

Let's compare with the situation of Fig. 35.12. For the wave reflected from the upper surface of the air wedge, $n_a$ (glass) is greater than $n_b$, so this wave has zero phase shift. For the wave reflected from the lower surface, $n_a$ (air) is less than $n_b$ (glass), so this wave has a half-cycle phase shift. Waves that are reflected from the line of contact have no path difference to give additional phase shifts, and they interfere destructively; this is what we observe. You can use the above principle to show that for normal incidence, the wave reflected at point $b$ in Fig. 35.11a is shifted by a half-cycle, while the wave reflected at $d$ is not (if there is air below the film).

We can summarize this discussion mathematically. If the film has thickness $t$, the light is at normal incidence and has wavelength $\lambda$ in the film; if neither or both of the reflected waves from the two surfaces have a half-cycle reflection phase shift, the conditions for constructive and destructive interference are

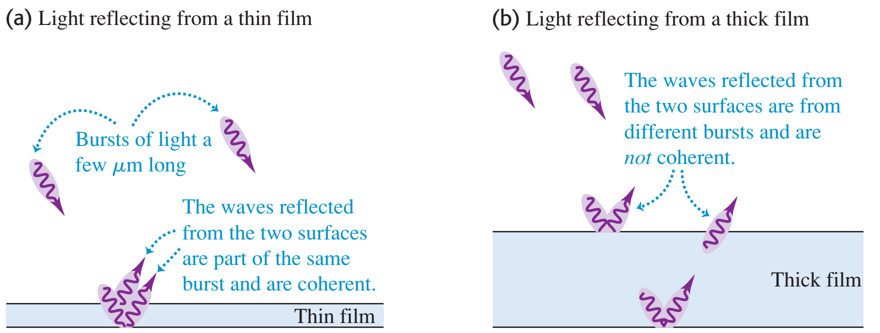
$$2t = m\lambda \qquad (m = 0, 1, 2, \dots)$$ (constructive reflection from thin film, no relative phase shift) (35.17a)

$$2t = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \dots)$$ (destructive reflection from thin film, no relative phase shift) (35.17b)

If *one* of the two waves has a half-cycle reflection phase shift, the conditions for constructive and destructive interference are reversed:

$$2t = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \dots)$$ (constructive reflection from thin film, half-cycle relative phase shift) (35.18a)

$$2t = m\lambda \qquad (m = 0, 1, 2, \dots)$$ (destructive reflection from thin film, half-cycle relative phase shift) (35.18b)




**35.14** (a) Light reflecting from a thin film produces a steady interference pattern, but (b) light reflecting from a thick film does not.

## Thin and Thick Films

We have emphasized *thin* films in our discussion because of a principle we introduced in Section 35.1: In order for two waves to cause a steady interference pattern, the waves must be *coherent,* with a definite and constant phase relationship. The sun and light bulbs emit light in a stream of short bursts, each of which is only a few micrometers long ($1 \text{ micrometer} = 1\ \mu\text{m} = 10^{-6}\text{ m}$). If light reflects from the two surfaces of a thin film, the two reflected waves are part of the same burst (Fig. 35.14a). Hence these waves are coherent and interference occurs as we have described. If the film is too thick, however, the two reflected waves will belong to different bursts (Fig. 35.14b). There is no definite phase relationship between different light bursts, so the two waves are incoherent and there is no fixed interference pattern. That's why you see interference colors in light reflected from an oil slick a few micrometers thick (see Fig. 35.11b), but you do *not* see such colors in the light reflected from a pane of window glass with a thickness of a few millimeters (a thousand times greater).

### Problem-Solving Strategy 35.1 Interference in Thin Films

**IDENTIFY** *the relevant concepts:* Problems with thin films involve interference of two waves, one reflected from the film's front surface and one reflected from the back surface. Typically you will be asked to relate the wavelength, the film thickness, and the index of refraction of the film.

**SET UP** *the problem* using the following steps:

1. Make a drawing showing the geometry of the film. Identify the materials that adjoin the film; their properties determine whether one or both of the reflected waves have a half-cycle phase shift.
2. Identify the target variable.

**EXECUTE** *the solution* as follows:

1. Apply the rule for phase changes to each reflected wave: There is a half-cycle phase shift when $n_b > n_a$ and none when $n_b < n_a$.
2. If *neither* reflected wave undergoes a phase shift, or if *both* do, use Eqs. (35.17). If only one reflected wave undergoes a phase shift, use Eqs. (35.18).
3. Solve the resulting equation for the target variable. Use the wavelength $\lambda = \lambda_0/n$ of light *in the film* in your calculations, where $n$ is the index of refraction of the film. (For air, $n = 1.000$ to four-figure precision.)
4. If you are asked about the wave that is transmitted through the film, remember that minimum intensity in the reflected wave corresponds to maximum *transmitted* intensity, and vice versa.

**EVALUATE** *your answer:* Interpret your results by examining what would happen if the wavelength were changed or if the film had a different thickness.

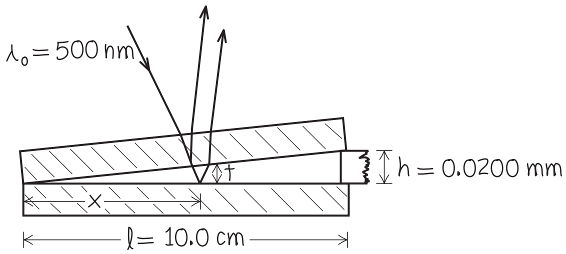
### Example 35.4 Thin-film interference I

Suppose the two glass plates in Fig. 35.12 are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Is the fringe at the line of contact bright or dark? Assume monochromatic light with a wavelength in air of $\lambda = \lambda_0 = 500$ nm.

**SOLUTION**

**IDENTIFY and SET UP:** Figure 35.15 depicts the situation. We'll consider only interference between the light reflected from the upper and lower surfaces of the air wedge between the microscope slides. [The top slide has a relatively great thickness, about 1 mm,

**35.15** Our sketch for this problem.




so we can ignore interference between the light reflected from its upper and lower surfaces (see Fig. 35.14b).] Light travels more slowly in the glass of the slides than it does in air. Hence the wave reflected from the upper surface of the air wedge has no phase shift (see Fig. 35.13a), while the wave reflected from the lower surface has a half-cycle phase shift (see Fig. 35.13c).

**EXECUTE:** Since only one of the reflected waves undergoes a phase shift, the condition for *destructive* interference (a dark fringe) is Eq. (35.18b):

$$2t = m\lambda_0 \qquad (m = 0, 1, 2, \dots)$$

From similar triangles in Fig. 35.15 the thickness $t$ of the air wedge at each point is proportional to the distance $x$ from the line of contact:

$$\frac{t}{x} = \frac{h}{l}$$

Combining this with Eq. (35.18b), we find

$$\frac{2xh}{l} = m\lambda_0$$

$$x = m\frac{l\lambda_0}{2h} = m\frac{(0.100 \text{ m})(500 \times 10^{-9} \text{ m})}{(2)(0.0200 \times 10^{-3} \text{ m})} = m(1.25 \text{ mm})$$

Successive dark fringes, corresponding to $m = 1, 2, 3, \dots$, are spaced 1.25 mm apart. Substituting $m = 0$ into this equation gives $x = 0$, which is where the two slides touch (at the left-hand side of Fig. 35.15). Hence there is a dark fringe at the line of contact.

**EVALUATE:** Our result shows that the fringe spacing is proportional to the wavelength of the light used; the fringes would be farther apart with red light (larger $\lambda_0$) than with blue light (smaller $\lambda_0$). If we use white light, the reflected light at any point is a mixture of wavelengths for which constructive interference occurs; the wavelengths that interfere destructively are weak or absent in the reflected light. (This same effect explains the colors seen when an oil film on water is illuminated by white light, as in Fig. 35.11b).

## Example 35.5 Thin-film interference II

Suppose the glass plates of Example 35.4 have $n = 1.52$ and the space between plates contains water ($n = 1.33$) instead of air. What happens now?

### SOLUTION

**IDENTIFY and SET UP:** The index of refraction of the water wedge is still less than that of the glass on either side of it, so the phase shifts are the same as in Example 35.4. Once again we use Eq. (35.18b) to find the positions of the dark fringes; the only difference is that the wavelength $\lambda$ in this equation is now the wavelength in water instead of in air.

**EXECUTE:** In the film of water ($n = 1.33$), the wavelength is $\lambda = \lambda_0/n = (500 \text{ nm})/(1.33) = 376$ nm. When we replace $\lambda_0$ by $\lambda$ in the expression from Example 35.4 for the position $x$ of the $m$th dark fringe, we find that the fringe spacing is reduced by the same factor of 1.33 and is equal to 0.940 mm. There is still a dark fringe at the line of contact.

**EVALUATE:** Can you see that to obtain the same fringe spacing as in Example 35.4, the dimension $h$ in Fig. 35.15 would have to be reduced to $(0.0200 \text{ mm})/1.33 = 0.0150$ mm? This shows that what matters in thin-film interference is the *ratio* $t/\lambda$ between film thickness and wavelength. [You can see this by considering Eqs. (35.17) and (35.18).]

## Example 35.6 Thin-film interference III

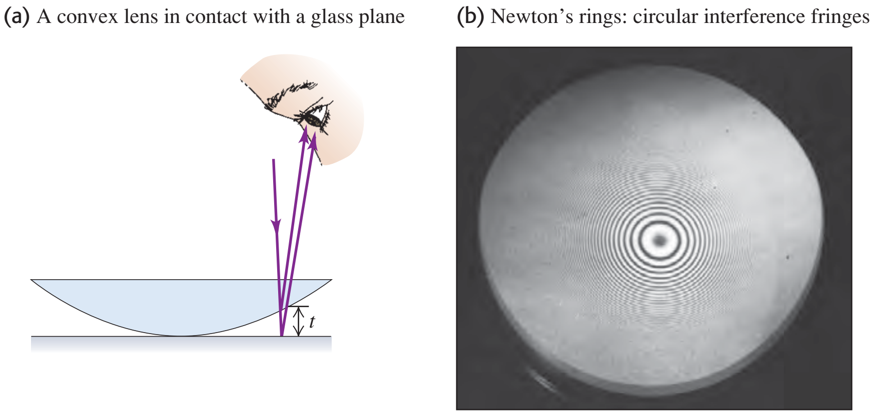
Suppose the upper of the two plates of Example 35.4 is a plastic with $n = 1.40$, the wedge is filled with a silicone grease with $n = 1.50$, and the bottom plate is a dense flint glass with $n = 1.60$. What happens now?

### SOLUTION

**IDENTIFY and SET UP:** The geometry is again the same as shown in Fig. 35.15, but now half-cycle phase shifts occur at *both* surfaces of the grease wedge (see Fig. 35.13c). Hence there is no *relative* phase shift and we must use Eq. (35.17b) to find the positions of the dark fringes.

**EXECUTE:** The value of $\lambda$ to use in Eq. (35.17b) is the wavelength in the silicone grease, $\lambda = \lambda_0/n = (500 \text{ nm})/1.50 = 333$ nm. You can readily show that the fringe spacing is 0.833 mm. Note that the two reflected waves from the line of contact are in phase (they both undergo the same phase shift), so the line of contact is at a *bright* fringe.

**EVALUATE:** What would happen if you carefully removed the upper microscope slide so that the grease wedge retained its shape? There would still be half-cycle phase changes at the upper and lower surfaces of the wedge, so the pattern of fringes would be the same as with the upper slide present.



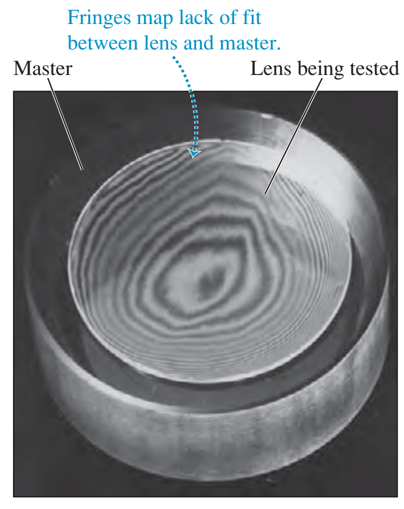

**35.16** (a) Air film between a convex lens and a plane surface. The thickness of the film $t$ increases from zero as we move out from the center, giving (b) a series of alternating dark and bright rings for monochromatic light.

## Newton's Rings

Figure 35.16a shows the convex surface of a lens in contact with a plane glass plate. A thin film of air is formed between the two surfaces. When you view the setup with monochromatic light, you see circular interference fringes (Fig. 35.16b). These were studied by Newton and are called **Newton's rings.**

We can use interference fringes to compare the surfaces of two optical parts by placing the two in contact and observing the interference fringes. Figure 35.17 is a photograph made during the grinding of a telescope objective lens. The lower, larger-diameter, thicker disk is the correctly shaped master, and the smaller, upper disk is the lens under test. The "contour lines" are Newton's interference fringes; each one indicates an additional distance between the specimen and the master of one half-wavelength. At 10 lines from the center spot the distance between the two surfaces is five wavelengths, or about 0.003 mm. This isn't very good; high-quality lenses are routinely ground with a precision of less than one wavelength. The surface of the primary mirror of the Hubble Space Telescope was ground to a precision of better than $\frac{1}{50}$ wavelength. Unfortunately, it was ground to incorrect specifications, creating one of the most precise errors in the history of optical technology (see Section 34.2).

**35.17** The surface of a telescope objective lens under inspection during manufacture.

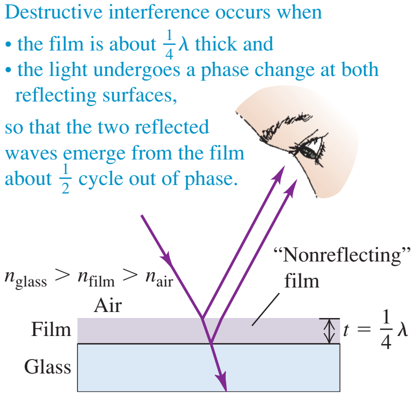



## Nonreflective and Reflective Coatings

**Nonreflective coatings** for lens surfaces make use of thin-film interference. A thin layer or film of hard transparent material with an index of refraction smaller than that of the glass is deposited on the lens surface, as in Fig. 35.18. Light is reflected from both surfaces of the layer. In both reflections the light is reflected from a medium of greater index than that in which it is traveling, so the same phase change occurs in both reflections. If the film thickness is a quarter (one-fourth) of the wavelength *in the film* (assuming normal incidence), the total path difference is a half-wavelength. Light reflected from the first surface is then a half-cycle out of phase with light reflected from the second, and there is destructive interference.

The thickness of the nonreflective coating can be a quarter-wavelength for only one particular wavelength. This is usually chosen in the central yellow-green portion of the spectrum ($\lambda = 550$ nm), where the eye is most sensitive. Then there is somewhat more reflection at both longer (red) and shorter (blue) wavelengths, and the reflected light has a purple hue. The overall reflection from a lens or prism surface can be reduced in this way from 4–5% to less than 1%. This also increases the net amount of light that is *transmitted* through the lens, since light that is not reflected will be transmitted. The same principle is used to minimize reflection from silicon photovoltaic solar cells ($n = 3.5$) by use of a thin surface layer of silicon monoxide ($SiO$, $n = 1.45$); this helps to increase the amount of light that actually reaches the solar cells.

**35.18** A nonreflective coating has an index of refraction intermediate between those of glass and air.



If a quarter-wavelength thickness of a material with an index of refraction *greater* than that of glass is deposited on glass, then the reflectivity is *increased,* and the deposited material is called a **reflective coating.** In this case there is a half-cycle phase shift at the air–film interface but none at the film–glass interface, and reflections from the two sides of the film interfere constructively. For example, a coating with refractive index 2.5 causes 38% of the incident energy to be reflected, compared with 4% or so with no coating. By use of multiple-layer coatings, it is possible to achieve nearly 100% transmission or reflection for particular wavelengths. Some practical applications of these coatings are for color separation in television cameras and for infrared "heat reflectors" in motion-picture projectors, solar cells, and astronauts' visors.

### Example 35.7 A nonreflective coating

A common lens coating material is magnesium fluoride ($MgF_2$), with $n = 1.38$. What thickness should a nonreflective coating have for 550-nm light if it is applied to glass with $n = 1.52$?

**SOLUTION**

**IDENTIFY and SET UP:** This coating is of the sort shown in Fig. 35.18. The thickness must be one-quarter of the wavelength of this light *in the coating*.

**EXECUTE:** The wavelength in air is $\lambda_0 = 550$ nm, so its wavelength in the $MgF_2$ coating is $\lambda = \lambda_0/n = (550\text{ nm})/1.38 = 400$ nm. The coating thickness should be one-quarter of this, or $\lambda/4 = 100$ nm.

**EVALUATE:** This is a very thin film, no more than a few hundred molecules thick. Note that this coating is *reflective* for light whose wavelength is *twice* the coating thickness; light of that wavelength reflected from the coating's lower surface travels one wavelength farther than light reflected from the upper surface, so the two waves are in phase and interfere constructively. This occurs for light with a wavelength in $MgF_2$ of 200 nm and a wavelength in air of $(200\text{ nm})(1.38) = 276$ nm. This is an ultraviolet wavelength (see Section 32.1), so designers of optical lenses with nonreflective coatings need not worry about such enhanced reflection.

**Test Your Understanding of Section 35.4** A thin layer of benzene ($n = 1.501$) ties on top of a sheet of fluorite ($n = 1.434$). It is illuminated from above with light whose wavelength in benzene is 400 nm. Which of the following possible thicknesses of the benzene layer will maximize the brightness of the reflected light? (i) 100 nm; (ii) 200 nm; (iii) 300 nm; (iv) 400 nm.

MP

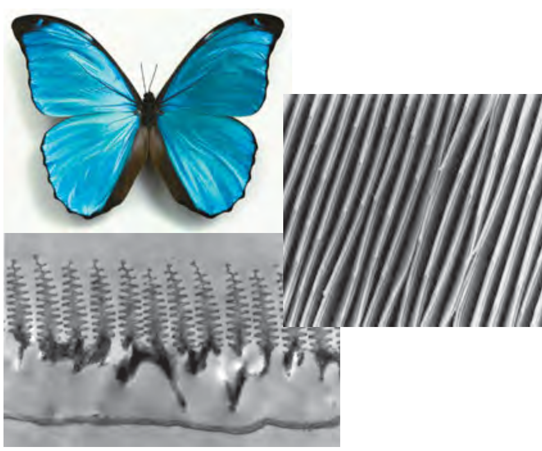
## 35.5 The Michelson Interferometer

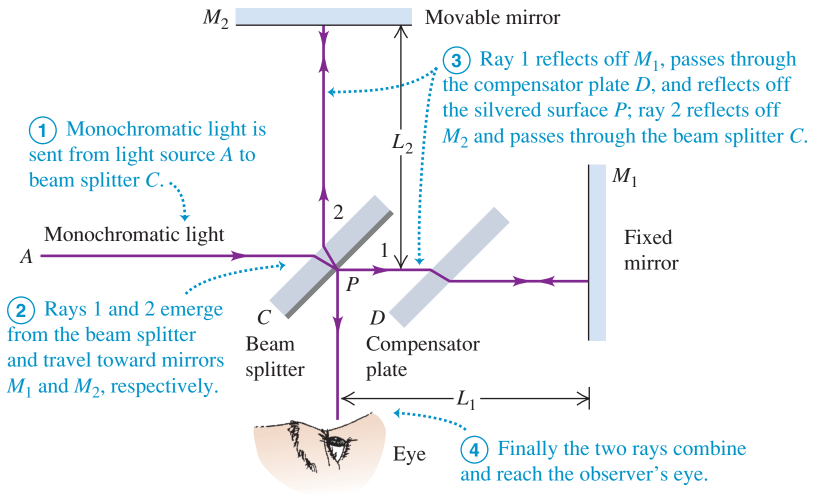
An important experimental device that uses interference is the **Michelson interferometer.** Michelson interferometers are used to make precise measurements of wavelengths and of very small distances, such as the minute changes in thickness of an axon when a nerve impulse propagates along its length. Like the Young two-slit experiment, a Michelson interferometer takes monochromatic light from a single source and divides it into two waves that follow different paths. In Young's experiment, this is done by sending part of the light through one slit and part through another; in a Michelson interferometer a device called a *beam splitter* is used. Interference occurs in both experiments when the two light waves are recombined.

### How a Michelson Interferometer Works

Figure 35.19 shows the principal components of a Michelson interferometer. A ray of light from a monochromatic source $A$ strikes the beam splitter $C$, which is a glass plate with a thin coating of silver on its right side. Part of the light (ray 1) passes through the silvered surface and the compensator plate $D$ and is reflected from mirror $M_1$. It then returns through $D$ and is reflected from the silvered surface of $C$ to the observer. The remainder of the light (ray 2) is reflected from the silvered surface at point $P$ to the mirror $M_2$ and back through $C$ to the observer's eye.

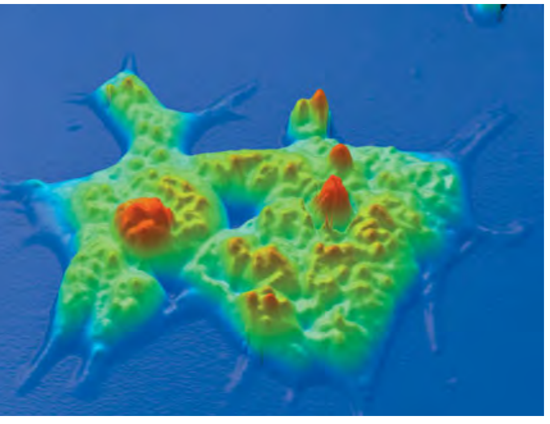
**Application Interference and Butterfly Wings**
Many of the most brilliant colors in the animal world are created by *interference* rather than by pigments. These photos show the butterfly *Morpho rhetenor* and the microscopic scales that cover the upper surfaces of its wings. The scales have a profusion of tiny ridges (middle photo); these carry regularly spaced flanges (bottom photo) that function as reflectors. These are spaced so that the reflections interfere constructively for blue light. The multilayered structure reflects 70% of the blue light that strikes it, giving the wings a mirror-like brilliance. (The undersides of the wings do not have these structures and are a dull brown.)






**35.19** A schematic Michelson interferometer. The observer sees an interference pattern that results from the difference in path lengths for rays 1 and 2.

**Application Imaging Cells with a Michelson Interferometer**
This false-color image of a human colon cancer cell was made using a microscope that was mated to a Michelson interferometer. The cell is in one arm of the interferometer, and light passing through the cell undergoes a phase shift that depends on the cell thickness and the organelles within the cell. The fringe pattern can then be used to construct a three-dimensional view of the cell. Scientists have used this technique to observe how different types of cells behave when prodded by microscopic probes. Cancer cells turn out to be "softer" than normal cells, a distinction that may make cancer stem cells easier to identify.



The purpose of the compensator plate $D$ is to ensure that rays 1 and 2 pass through the same thickness of glass; plate $D$ is cut from the same piece of glass as plate $C$, so their thicknesses are identical to within a fraction of a wavelength.

The whole apparatus in Fig. 35.19 is mounted on a very rigid frame, and the position of mirror $M_2$ can be adjusted with a fine, very accurate micrometer screw. If the distances $L_1$ and $L_2$ are exactly equal and the mirrors $M_1$ and $M_2$ are exactly at right angles, the virtual image of $M_1$ formed by reflection at the silvered surface of plate $C$ coincides with mirror $M_2$. If $L_1$ and $L_2$ are *not* exactly equal, the image of $M_1$ is displaced slightly from $M_2$; and if the mirrors are not exactly perpendicular, the image of $M_1$ makes a slight angle with $M_2$. Then the mirror $M_2$ and the virtual image of $M_1$ play the same roles as the two surfaces of a wedge-shaped thin film (see Section 35.4), and light reflected from these surfaces forms the same sort of interference fringes.

Suppose the angle between mirror $M_2$ and the virtual image of $M_1$ is just large enough that five or six vertical fringes are present in the field of view. If we now move the mirror $M_2$ slowly either backward or forward a distance $\lambda/2$, the difference in path length between rays 1 and 2 changes by $\lambda$, and each fringe moves to the left or right a distance equal to the fringe spacing. If we observe the fringe positions through a telescope with a crosshair eyepiece and $m$ fringes cross the crosshairs when we move the mirror a distance $y$, then

$$y = m\frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{2y}{m} \tag{35.19}$$

If $m$ is several thousand, the distance $y$ is large enough that it can be measured with good accuracy, and we can obtain an accurate value for the wavelength $\lambda$. Alternatively, if the wavelength is known, a distance $y$ can be measured by simply counting fringes when $M_2$ is moved by this distance. In this way, distances that are comparable to a wavelength of light can be measured with relative ease.

## The Michelson-Morley Experiment

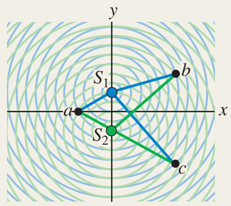
The original application of the Michelson interferometer was to the historic **Michelson-Morley experiment.** Before the electromagnetic theory of light became established, most physicists thought that the propagation of light waves occurred in a medium called the **ether,** which was believed to permeate all space. In 1887 the American scientists Albert Michelson and Edward Morley used the Michelson interferometer in an attempt to detect the motion of the earth through the ether. Suppose the interferometer in Fig. 35.19 is moving from left to right relative to the ether. According to the ether theory, this would lead to changes in the speed of light in the portions of the path shown as horizontal lines in the figure. There would be fringe shifts relative to the positions that the fringes would have if the instrument were at rest in the ether. Then when the entire instrument was rotated 90°, the other portions of the paths would be similarly affected, giving a fringe shift in the opposite direction.

Michelson and Morley expected that the motion of the earth through the ether would cause a shift of about four-tenths of a fringe when the instrument was rotated. The shift that was actually observed was less than a hundredth of a fringe and, within the limits of experimental uncertainty, appeared to be exactly zero. Despite its orbital motion around the sun, the earth appeared to be *at rest* relative to the ether. This negative result baffled physicists until 1905, when Albert Einstein developed the special theory of relativity (which we will study in detail in Chapter 37). Einstein postulated that the speed of a light wave in vacuum has the same magnitude $c$ relative to *all* inertial reference frames, no matter what their velocity may be relative to each other. The presumed ether then plays no role, and the concept of an ether has been abandoned.

**Test Your Understanding of Section 35.5** You are observing the pattern of fringes in a Michelson interferometer like that shown in Fig. 35.19. If you change the index of refraction (but not the thickness) of the compensator plate, will the pattern change?

# CHAPTER 35 SUMMARY

**Interference and coherent sources:** Monochromatic light is light with a single frequency. Coherence is a definite, unchanging phase relationship between two waves. The overlap of waves from two coherent sources of monochromatic light forms an interference pattern. The principle of superposition states that the total wave disturbance at any point is the sum of the disturbances from the separate waves.

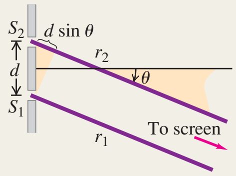



**Two-source interference of light:** When two sources are in phase, constructive interference occurs where the difference in path length from the two sources is zero or an integer number of wavelengths; destructive interference occurs where the path difference is a half-integer number of wavelengths. If two sources separated by a distance $d$ are both very far from a point $P$, and the line from the sources to $P$ makes an angle $\theta$ with the line perpendicular to the line of the sources, then the condition for constructive interference at $P$ is Eq. (35.4). The condition for destructive interference is Eq. (35.5). When $\theta$ is very small, the position $y_m$ of the $m$th bright fringe on a screen located a distance $R$ from the sources is given by Eq. (35.6). (See Examples 35.1 and 35.2.)

$$d\sin\theta = m\lambda \quad (m = 0, \pm1, \pm2, \ldots)$$
(constructive interference) (35.4)

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$
$$(m = 0, \pm1, \pm2, \ldots)$$ (35.5)
(destructive interference)

$$y_m = R\frac{m\lambda}{d}$$ (35.6)
(bright fringes)

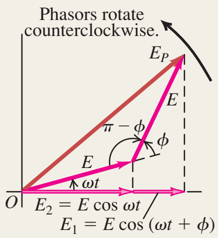



**Intensity in interference patterns:** When two sinusoidal waves with equal amplitude $E$ and phase difference $\phi$ are superimposed, the resultant amplitude $E_P$ and intensity $I$ are given by Eqs. (35.7) and (35.10), respectively. If the two sources emit in phase, the phase difference $\phi$ at a point $P$ (located a distance $r_1$ from source 1 and a distance $r_2$ from source 2) is directly proportional to the difference in path length $r_2 - r_1$. (See Example 35.3.)

$$E_P = 2E\left|\cos\frac{\phi}{2}\right|$$ (35.7)

$$I = I_0\cos^2\frac{\phi}{2}$$ (35.10)

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1)$$ (35.11)



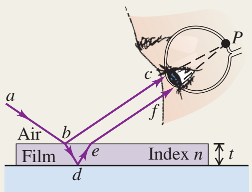

**Interference in thin films:** When light is reflected from both sides of a thin film of thickness $t$ and no phase shift occurs at either surface, constructive interference between the reflected waves occurs when $2t$ is equal to an integral number of wavelengths. If a half-cycle phase shift occurs at one surface, this is the condition for destructive interference. A half-cycle phase shift occurs during reflection whenever the index of refraction in the second material is greater than that in the first. (See Examples 35.4–35.7.)

$$2t = m\lambda \quad (m = 0, 1, 2, \ldots)$$
(constructive reflection from thin film, no relative phase shift) (35.17a)

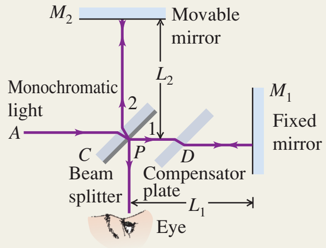
$$2t = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$$
(destructive reflection from thin film, no relative phase shift) (35.17b)

$$2t = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$$
(constructive reflection from thin film, half-cycle relative phase shift) (35.18a)

$$2t = m\lambda \quad (m = 0, 1, 2, \ldots)$$
(destructive reflection from thin film, half-cycle relative phase shift) (35.18b)



**Michelson interferometer:** The Michelson interferometer uses a monochromatic light source and can be used for high-precision measurements of wavelengths. Its original purpose was to detect motion of the earth relative to a hypothetical ether, the supposed medium for electromagnetic waves. The ether has never been detected, and the concept has been abandoned; the speed of light is the same relative to all observers. This is part of the foundation of the special theory of relativity.




## BRIDGING PROBLEM Modifying a Two-Slit Experiment

An oil tanker spills a large amount of oil ($n = 1.45$) into the sea ($n = 1.33$). (a) If you look down onto the oil spill from overhead, what predominant wavelength of light do you see at a point where the oil is 380 nm thick? What color is the light? (*Hint:* See Table 32.1.) (b) In the water under the slick, what visible wavelength (as measured in air) is predominant in the transmitted light at the same place in the slick as in part (a)?

### SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.

### IDENTIFY and SET UP

1. The oil layer acts as a thin film, so we must consider interference between light that reflects from the top and bottom surfaces of the oil. If a wavelength is prominent in the *transmitted* light, there is destructive interference for that wavelength in the *reflected* light.
2. Choose the appropriate interference equations that relate the thickness of the oil film and the wavelength of light. Take account of the indexes of refraction of the air, oil, and water.

### EXECUTE

3. For part (a), find the wavelengths for which there is constructive interference as seen from above the oil film. Which of these are in the visible spectrum?
4. For part (b), find the visible wavelength for which there is destructive interference as seen from above the film. (This will ensure that there is substantial transmitted light at the wavelength.)

### EVALUATE

5. If a diver below the water's surface shines a light up at the bottom of the oil film, at what wavelengths would there be constructive interference in the light that reflects back downward?

# Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

## DISCUSSION QUESTIONS

**Q35.1** A two-slit interference experiment is set up, and the fringes are displayed on a screen. Then the whole apparatus is immersed in the nearest swimming pool. How does the fringe pattern change?

**Q35.2** Could an experiment similar to Young's two-slit experiment be performed with sound? How might this be carried out? Does it matter that sound waves are longitudinal and electromagnetic waves are transverse? Explain.

**Q35.3** Monochromatic coherent light passing through two thin slits is viewed on a distant screen. Are the bright fringes equally spaced on the screen? If so, why? If not, which ones are closest to being equally spaced?

**Q35.4** In a two-slit interference pattern on a distant screen, are the bright fringes midway between the dark fringes? Is this ever a good approximation?

**Q35.5** Would the headlights of a distant car form a two-source interference pattern? If so, how might it be observed? If not, why not?

**Q35.6** The two sources $S_1$ and $S_2$ shown in Fig. 35.3 emit waves of the same wavelength $\lambda$ and are in phase with each other. Suppose $S_1$ is a weaker source, so that the waves emitted by $S_1$ have half the amplitude of the waves emitted by $S_2$. How would this affect the positions of the antinodal lines and nodal lines? Would there be total reinforcement at points on the antinodal curves? Would there be total cancellation at points on the nodal curves? Explain your answers.

**Q35.7** Could the Young two-slit interference experiment be performed with gamma rays? If not, why not? If so, discuss differences in the experimental design compared to the experiment with visible light.

**Q35.8** Coherent red light illuminates two narrow slits that are 25 cm apart. Will a two-slit interference pattern be observed when the light from the slits falls on a screen? Explain.

**Q35.9** Coherent light with wavelength $\lambda$ falls on two narrow slits separated by a distance $d$. If $d$ is less than some minimum value, no dark fringes are observed. Explain. In terms of $\lambda$, what is this minimum value of $d$?

**Q35.10** A fellow student, who values memorizing equations above understanding them, combines Eqs. (35.4) and (35.13) to "prove" that $\phi$ can *only* equal $2\pi m$. How would you explain to this student that $\phi$ can have values other than $2\pi m$?

**Q35.11** If the monochromatic light shown in Fig. 35.5a were replaced by white light, would a two-slit interference pattern be seen on the screen? Explain.

**Q35.12** In using the superposition principle to calculate intensities in interference patterns, could you add the intensities of the waves instead of their amplitudes? Explain.

**Q35.13** A glass windowpane with a thin film of water on it reflects less than when it is perfectly dry. Why?

**Q35.14** A *very* thin soap film ($n = 1.33$), whose thickness is much less than a wavelength of visible light, looks black; it appears to reflect no light at all. Why? By contrast, an equally thin layer of soapy water ($n = 1.33$) on glass ($n = 1.50$) appears quite shiny. Why is there a difference?

**Q35.15** Interference can occur in thin films. Why is it important that the films be *thin*? Why don't you get these effects with a relatively *thick* film? Where should you put the dividing line between "thin" and "thick"? Explain your reasoning.

**Q35.16** If we shine white light on an air wedge like that shown in Fig. 35.12, the colors that are weak in the light *reflected* from any point along the wedge are strong in the light *transmitted* through the wedge. Explain why this should be so.

**Q35.17** Monochromatic light is directed at normal incidence on a thin film. There is destructive interference for the reflected light, so the intensity of the reflected light is very low. What happened to the energy of the incident light?

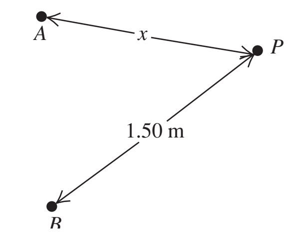
**Q35.18** When a thin oil film spreads out on a puddle of water, the thinnest part of the film looks dark in the resulting interference pattern. What does this tell you about the relative magnitudes of the refractive indexes of oil and water?

## EXERCISES

### Section 35.1 Interference and Coherent Sources

**35.1** • Two small stereo speakers $A$ and $B$ that are 1.40 m apart are sending out sound of wavelength 34 cm in all directions and all in phase. A person at point $P$ starts out equidistant from both speakers and walks so that he is always 1.50 m from speaker $B$ (Fig. E35.1). For what values of $x$ will the sound this person hears be (a) maximally reinforced, (b) cancelled? Limit your solution to the cases where $x \leq 1.50$ m.

Figure **E35.1**




**35.2** •• Two speakers that are 15.0 m apart produce in-phase sound waves of frequency 250.0 Hz in a room where the speed of sound is 340.0 m/s. A woman starts out at the midpoint between the two speakers. The room's walls and ceiling are covered with absorbers to eliminate reflections, and she listens with only one ear for best precision. (a) What does she hear: constructive or destructive interference? Why? (b) She now walks slowly toward one of the speakers. How far from the center must she walk before she first hears the sound reach a minimum intensity? (c) How far from the center must she walk before she first hears the sound maximally enhanced?

**35.3** •• Two identical audio speakers connected to the same amplifier produce in-phase sound waves with a single frequency that can be varied between 300 and 600 Hz. The speed of sound is 340 m/s. You find that where you are standing, you hear minimum-intensity sound. (a) Explain why you hear minimum-intensity sound. (b) If one of the speakers is moved 39.8 cm toward you, the sound you hear has maximum intensity. What is the frequency of the sound? (c) How much closer to you from the position in part (b) must the speaker be moved to the next position where you hear maximum intensity?

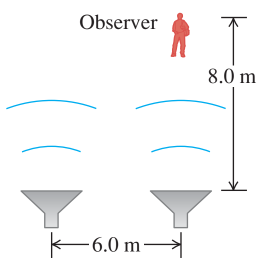
**35.4** • **Radio Interference.** Two radio antennas $A$ and $B$ radiate in phase. Antenna $B$ is 120 m to the right of antenna $A$. Consider point $Q$ along the extension of the line connecting the antennas, a horizontal distance of 40 m to the right of antenna $B$. The frequency, and hence the wavelength, of the emitted waves can be varied. (a) What is the longest wavelength for which there will be destructive interference at point $Q$? (b) What is the longest wavelength for which there will be constructive interference at point $Q$?

**35.5** •• A radio transmitting station operating at a frequency of 120 MHz has two identical antennas that radiate in phase. Antenna $B$ is 9.00 m to the right of antenna $A$. Consider point $P$ between the antennas and along the line connecting them, a horizontal distance $x$ to the right of antenna $A$. For what values of $x$ will constructive interference occur at point $P$?

**35.6** • Two light sources can be adjusted to emit monochromatic light of any visible wavelength. The two sources are coherent, 2.04 $\mu$m apart, and in line with an observer, so that one source is 2.04 $\mu$m farther from the observer than the other. (a) For what visible wavelengths (380 to 750 nm) will the observer see the brightest light, owing to constructive interference? (b) How would your answers to part (a) be affected if the two sources were not in line with the observer, but were still arranged so that one source is 2.04 $\mu$m farther away from the observer than the other? (c) For what visible wavelengths will there be *destructive* interference at the location of the observer?

**35.7** • Two speakers, emitting identical sound waves of wavelength 2.0 m in phase with each other, and an observer are located as shown in Fig. E35.7. (a) At the observer's location, what is the path difference for waves from the two speakers? (b) Will the sound waves interfere constructively or destructively at the observer's location—or something in between constructive and destructive? (c) Suppose the observer now increases her distance from the closest speaker to 17.0 m, staying directly in front of the same speaker as initially. Answer the questions of parts (a) and (b) for this new situation.

Figure **E35.7**




**35.8** •• Figure 35.3 shows the wave pattern produced by two identical, coherent sources emitting waves with wavelength $\lambda$ and separated by a distance $d = 4\lambda$. (a) Explain why the positive $y$-axis above $S_1$ constitutes an antinodal curve with $m = +4$ and why the negative $y$-axis below $S_2$ constitutes an antinodal curve with $m = -4$. (b) Draw the wave pattern produced when the separation between the sources is reduced to $3\lambda$. In your drawing, sketch all antinodal curves—that is, the curves on which $r_2 - r_1 = m\lambda$. Label each curve by its value of $m$. (c) In general, what determines the maximum (most positive) and minimum (most negative) values of the integer $m$ that labels the antinodal lines? (d) Suppose the separation between the sources is increased to $7\frac{1}{2}\lambda$. How many antinodal curves will there be? To what values of $m$ do they correspond? Explain your reasoning. (You should not have to make a drawing to answer these questions.)

## Section 35.2 Two-Source Interference of Light

**35.9** • Young's experiment is performed with light from excited helium atoms ($\lambda = 502$ nm). Fringes are measured carefully on a screen 1.20 m away from the double slit, and the center of the 20th fringe (not counting the central bright fringe) is found to be 10.6 mm from the center of the central bright fringe. What is the separation of the two slits?

**35.10** •• Coherent light with wavelength 450 nm falls on a double slit. On a screen 1.80 m away, the distance between dark fringes is 4.20 mm. What is the separation of the slits?

**35.11** •• Two slits spaced 0.450 mm apart are placed 75.0 cm from a screen. What is the distance between the second and third dark lines of the interference pattern on the screen when the slits are illuminated with coherent light with a wavelength of 500 nm?

**35.12** •• If the entire apparatus of Exercise 35.11 (slits, screen, and space in between) is immersed in water, what then is the distance between the second and third dark lines?

**35.13** •• Two thin parallel slits that are 0.0116 mm apart are illuminated by a laser beam of wavelength 585 nm. (a) On a very large distant screen, what is the *total* number of bright fringes (those indicating complete constructive interference), including the central fringe and those on both sides of it? Solve this problem without calculating all the angles! (*Hint:* What is the largest that $\sin\theta$ can be? What does this tell you is the largest value of $m$?) (b) At what angle, relative to the original direction of the beam, will the fringe that is most distant from the central bright fringe occur?

**35.14** • Coherent light with wavelength 400 nm passes through two very narrow slits that are separated by 0.200 mm, and the interference pattern is observed on a screen 4.00 m from the slits. (a) What is the width (in mm) of the central interference maximum? (b) What is the width of the first-order bright fringe?

**35.15** •• Two very narrow slits are spaced 1.80 $\mu$m apart and are placed 35.0 cm from a screen. What is the distance between the first and second dark lines of the interference pattern when the slits are illuminated with coherent light with $\lambda = 550$ nm? (*Hint:* The angle $\theta$ in Eq. (35.5) is *not* small.)

**35.16** •• Coherent light that contains two wavelengths, 660 nm (red) and 470 nm (blue), passes through two narrow slits separated by 0.300 mm, and the interference pattern is observed on a screen 5.00 m from the slits. What is the distance on the screen between the first-order bright fringes for the two wavelengths?

**35.17** •• Coherent light with wavelength 600 nm passes through two very narrow slits and the interference pattern is observed on a screen 3.00 m from the slits. The first-order bright fringe is at 4.84 mm from the center of the central bright fringe. For what wavelength of light will the first-order dark fringe be observed at this same point on the screen?

**35.18** •• Coherent light of frequency $6.32 \times 10^{14}$ Hz passes through two thin slits and falls on a screen 85.0 cm away. You observe that the third bright fringe occurs at $\pm 3.11$ cm on either side of the central bright fringe. (a) How far apart are the two slits? (b) At what distance from the central bright fringe will the third dark fringe occur?

## Section 35.3 Intensity in Interference Patterns

**35.19** •• In a two-slit interference pattern, the intensity at the peak of the central maximum is $I_0$. (a) At a point in the pattern where the phase difference between the waves from the two slits is 60.0°, what is the intensity? (b) What is the path difference for 480-nm light from the two slits at a point where the phase angle is 60.0°?

**35.20** • Coherent sources $A$ and $B$ emit electromagnetic waves with wavelength 2.00 cm. Point $P$ is 4.86 m from $A$ and 5.24 m from $B$. What is the phase difference at $P$ between these two waves?

**35.21** • Coherent light with wavelength 500 nm passes through narrow slits separated by 0.340 mm. At a distance from the slits large compared to their separation, what is the phase difference (in radians) in the light from the two slits at an angle of 23.0° from the centerline?

**35.22** • Two slits spaced 0.260 mm apart are placed 0.700 m from a screen and illuminated by coherent light with a wavelength of 660 nm. The intensity at the center of the central maximum ($\theta = 0°$) is $I_0$. (a) What is the distance on the screen from the center of the central maximum to the first minimum? (b) What is the distance on the screen from the center of the central maximum to the point where the intensity has fallen to $I_0/2$?

**35.23** •• Points $A$ and $B$ are 56.0 m apart along an east-west line. At each of these points, a radio transmitter is emitting a 12.5-MHz signal horizontally. These transmitters are in phase with each other and emit their beams uniformly in a horizontal plane. A receiver is taken 0.500 km north of the $AB$ line and initially placed at point $C$, directly opposite the midpoint of $AB$. The receiver can be moved only along an east-west direction but, due to its limited sensitivity, it must always remain within a range so that the intensity of the signal it receives from the transmitter is no less than $\frac{1}{4}$ of its maximum value. How far from point $C$ (along an east-west line) can the receiver be moved and always be able to pick up the signal?

**35.24** • Consider two antennas separated by 9.00 m that radiate in phase at 120 MHz, as described in Exercise 35.5. A receiver placed 150 m from both antennas measures an intensity $I_0$. The receiver is moved so that it is 1.8 m closer to one antenna than to the other. (a) What is the phase difference $\phi$ between the two radio waves produced by this path difference? (b) In terms of $I_0$, what is the intensity measured by the receiver at its new position?

## Section 35.4 Interference in Thin Films

**35.25** • What is the thinnest film of a coating with $n = 1.42$ on glass ($n = 1.52$) for which destructive interference of the red component (650 nm) of an incident white light beam in air can take place by reflection?

**35.26** •• **Nonglare Glass.** When viewing a piece of art that is behind glass, one often is affected by the light that is reflected off the front of the glass (called *glare*), which can make it difficult to see the art clearly. One solution is to coat the outer surface of the glass with a film to cancel part of the glare. (a) If the glass has a refractive index of 1.62 and you use $TiO_2$, which has an index of refraction of 2.62, as the coating, what is the minimum film thickness that will cancel light of wavelength 505 nm? (b) If this coating is too thin to stand up to wear, what other thickness would also work? Find only the three thinnest ones.

**35.27** •• Two rectangular pieces of plane glass are laid one upon the other on a table. A thin strip of paper is placed between them at one edge so that a very thin wedge of air is formed. The plates are illuminated at normal incidence by 546-nm light from a mercury-vapor lamp. Interference fringes are formed, with 15.0 fringes per centimeter. Find the angle of the wedge.

**35.28** •• A plate of glass 9.00 cm long is placed in contact with a second plate and is held at a small angle with it by a metal strip

0.0800 mm thick placed under one end. The space between the plates is filled with air. The glass is illuminated from above with light having a wavelength in air of 656 nm. How many interference fringes are observed per centimeter in the reflected light?

**35.29** •• A uniform film of $TiO_2$, 1036 nm thick and having index of refraction 2.62, is spread uniformly over the surface of crown glass of refractive index 1.52. Light of wavelength 520.0 nm falls at normal incidence onto the film from air. You want to increase the thickness of this film so that the reflected light cancels. (a) What is the *minimum* thickness of $TiO_2$ that you must *add* so the reflected light cancels as desired? (b) After you make the adjustment in part (a), what is the path difference between the light reflected off the top of the film and the light that cancels it after traveling through the film? Express your answer in (i) nanometers and (ii) wavelengths of the light in the $TiO_2$ film.

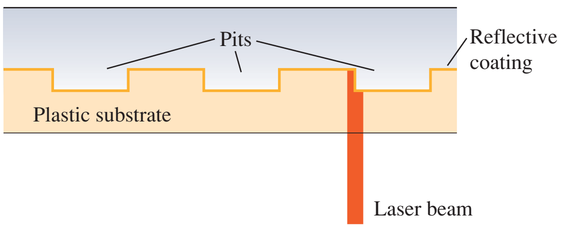
**35.30** • A plastic film with index of refraction 1.85 is put on the surface of a car window to increase the reflectivity and thus to keep the interior of the car cooler. The window glass has index of refraction 1.52. (a) What minimum thickness is required if light with wavelength 550 nm in air reflected from the two sides of the film is to interfere constructively? (b) It is found to be difficult to manufacture and install coatings as thin as calculated in part (a). What is the next greatest thickness for which there will also be constructive interference?

**35.31** • The walls of a soap bubble have about the same index of refraction as that of plain water, $n = 1.33$. There is air both inside and outside the bubble. (a) What wavelength (in air) of visible light is most strongly reflected from a point on a soap bubble where its wall is 290 nm thick? To what color does this correspond (see Fig. 32.4 and Table 32.1)? (b) Repeat part (a) for a wall thickness of 340 nm.

**35.32** •• Light with wavelength 648 nm in air is incident perpendicularly from air on a film 8.76 $\mu$m thick and with refractive index 1.35. Part of the light is reflected from the first surface of the film, and part enters the film and is reflected back at the second surface, where the film is again in contact with air. (a) How many waves are contained along the path of this second part of the light in its round trip through the film? (b) What is the phase difference between these two parts of the light as they leave the film?

**35.33** •• **Compact Disc Player.** A compact disc (CD) is read from the bottom by a semiconductor laser with wavelength 790 nm passing through a plastic substrate of refractive index 1.8. When the beam encounters a pit, part of the beam is reflected from the pit and part from the flat region between the pits, so these two beams interfere with each other (Fig. E35.33). What must the minimum pit depth be so that the part of the beam reflected from a pit cancels the part of the beam reflected from the flat region? (It is this cancellation that allows the player to recognize the beginning and end of a pit.)

Figure **E35.33**




**35.34** • What is the thinnest soap film (excluding the case of zero thickness) that appears black when illuminated with light with wavelength 480 nm? The index of refraction of the film is 1.33, and there is air on both sides of the film.

### Section 35.5 The Michelson Interferometer

**35.35** • How far must the mirror $M_2$ (see Fig. 35.19) of the Michelson interferometer be moved so that 1800 fringes of He-Ne laser light ($\lambda = 633$ nm) move across a line in the field of view?

**35.36** • Jan first uses a Michelson interferometer with the 606-nm light from a krypton-86 lamp. He displaces the movable mirror away from him, counting 818 fringes moving across a line in his field of view. Then Linda replaces the krypton lamp with filtered 502-nm light from a helium lamp and displaces the movable mirror toward her. She also counts 818 fringes, but they move across the line in her field of view opposite to the direction they moved for Jan. Assume that both Jan and Linda counted to 818 correctly. (a) What distance did each person move the mirror? (b) What is the resultant displacement of the mirror?

## PROBLEMS

**35.37** ••• The radius of curvature of the convex surface of a planoconvex lens is 68.4 cm. The lens is placed convex side down on a perfectly flat glass plate that is illuminated from above with red light having a wavelength of 580 nm. Find the diameter of the second bright ring in the interference pattern.

**35.38** •• Newton's rings can be seen when a planoconvex lens is placed on a flat glass surface. For a particular lens with an index of refraction of $n = 1.50$ and a glass plate with an index of $n = 1.80$, the diameter of the third bright ring is 0.720 mm. If water ($n = 1.33$) now fills the space between the lens and the plate, what is the new diameter of this ring?

**35.39** • BIO **Coating Eyeglass Lenses.** Eyeglass lenses can be coated on the *inner* surfaces to reduce the reflection of stray light to the eye. If the lenses are medium flint glass of refractive index 1.62 and the coating is fluorite of refractive index 1.432, (a) what minimum thickness of film is needed on the lenses to cancel light of wavelength 550 nm reflected toward the eye at normal incidence? (b) Will any other wavelengths of visible light be cancelled or enhanced in the reflected light?

**35.40** •• BIO **Sensitive Eyes.** After an eye examination, you put some eyedrops on your sensitive eyes. The cornea (the front part of the eye) has an index of refraction of 1.38, while the eyedrops have a refractive index of 1.45. After you put in the drops, your friends notice that your eyes look red, because red light of wavelength 600 nm has been reinforced in the reflected light. (a) What is the minimum thickness of the film of eyedrops on your cornea? (b) Will any other wavelengths of visible light be reinforced in the reflected light? Will any be cancelled? (c) Suppose you had contact lenses, so that the eyedrops went on them instead of on your corneas. If the refractive index of the lens material is 1.50 and the layer of eyedrops has the same thickness as in part (a), what wavelengths of visible light will be reinforced? What wavelengths will be cancelled?

**35.41** •• Two flat plates of glass with parallel faces are on a table, one plate on the other. Each plate is 11.0 cm long and has a refractive index of 1.55. A very thin sheet of metal foil is inserted under the end of the upper plate to raise it slightly at that end, in a manner similar to that discussed in Example 35.4. When you view the glass plates from above with reflected white light, you observe that, at 1.15 mm from the line where the sheets are in contact, the violet light of wavelength 400.0 nm is enhanced in this reflected light, but no visible light is enhanced closer to the line of contact.

(a) How far from the line of contact will green light (of wavelength 550 nm) and orange light (of wavelength 600.0 nm) first be enhanced? (b) How far from the line of contact will the violet, green, and orange light again be enhanced in the reflected light? (c) How thick is the metal foil holding the ends of the plates apart?

**35.42** •• In a setup similar to that of Problem 35.41, the glass has an index of refraction of 1.53, the plates are each 8.00 cm long, and the metal foil is 0.015 mm thick. The space between the plates is filled with a jelly whose refractive index is not known precisely, but is known to be greater than that of the glass. When you illuminate these plates from above with light of wavelength 525 nm, you observe a series of equally spaced dark fringes in the reflected light. You measure the spacing of these fringes and find that there are 10 of them every 6.33 mm. What is the index of refraction of the jelly?

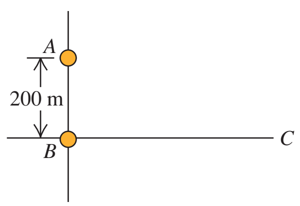
**35.43** ••• Suppose you illuminate two thin slits by monochromatic coherent light in air and find that they produce their first interference *minima* at $\pm 35.20°$ on either side of the central bright spot. You then immerse these slits in a transparent liquid and illuminate them with the same light. Now you find that the first minima occur at $\pm 19.46°$ instead. What is the index of refraction of this liquid?

**35.44** •• **CP CALC** A very thin sheet of brass contains two thin parallel slits. When a laser beam shines on these slits at normal incidence and room temperature $(20.0°\text{C})$, the first interference dark fringes occur at $\pm 32.5°$ from the original direction of the laser beam when viewed from some distance. If this sheet is now slowly heated up to 135°C, by how many degrees do these dark fringes change position? Do they move closer together or get farther apart? See Table 17.1 for pertinent information, and ignore any effects that might occur due to change in the thickness of the slits. (*Hint:* Since thermal expansion normally produces very small changes in length, you can use differentials to find the change in the angle.)

**35.45** •• Two speakers, 2.50 m apart, are driven by the same audio oscillator so that each one produces a sound consisting of *two* distinct frequencies, 0.900 kHz and 1.20 kHz. The speed of sound in the room is 344 m/s. Find all the angles relative to the usual centerline in front of (and far from) the speakers at which *both* frequencies interfere constructively.

**35.46** •• Two radio antennas radiating in phase are located at points $A$ and $B$, 200 m apart (Fig. P35.46). The radio waves have a frequency of 5.80 MHz. A radio receiver is moved out from point $B$ along a line perpendicular to the line connecting $A$ and $B$ (line $BC$ shown in Fig. P35.46). At what distances from $B$ will there be *destructive* interference? (*Note:* The distance of the receiver from the sources is not large in comparison to the separation of the sources, so Eq. (35.5) does not apply.)

Figure **P35.46**

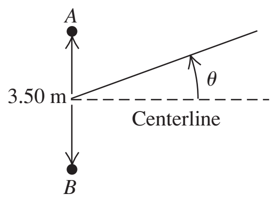



**35.47** •• One round face of a 3.25-m, solid, cylindrical plastic pipe is covered with a thin black coating that completely blocks light. The opposite face is covered with a fluorescent coating that glows when it is struck by light. Two straight, thin, parallel scratches, 0.225 mm apart, are made in the center of the black face. When laser light of wavelength 632.8 nm shines through the slits perpendicular to the black face, you find that the central bright fringe on the opposite face is 5.82 mm wide, measured between the dark fringes that border it on either side. What is the index of refraction of the plastic?

**35.48** • A uniform thin film of material of refractive index 1.40 coats a glass plate of refractive index 1.55. This film has the proper thickness to cancel normally incident light of wavelength 525 nm that strikes the film surface from air, but it is somewhat greater than the minimum thickness to achieve this cancellation. As time goes by, the film wears away at a steady rate of 4.20 nm per year. What is the minimum number of years before the reflected light of this wavelength is now enhanced instead of cancelled?

**35.49** •• Two speakers $A$ and $B$ are 3.50 m apart, and each one is emitting a frequency of 444 Hz. However, because of signal delays in the cables, speaker $A$ is one-fourth of a period *ahead* of speaker $B$. For points far from the speakers, find all the angles relative to the centerline (Fig. P35.49) at which the sound from these speakers cancels. Include angles on *both* sides of the centerline. The speed of sound is 340 m/s.

Figure **P35.49**




**35.50** ••• **CP** The electric fields received at point $P$ from two identical, coherent wave sources are $E_1(t) = E\cos(\omega t + \phi)$ and $E_2(t) = E\cos\omega t$. (a) Use the trigonometric identities in Appendix B to show that the resultant wave is $E_P(t) = 2E\cos(\phi/2)\cos(\omega t + \phi/2)$. (b) Show that the amplitude of this resultant wave is given by Eq. (35.7). (c) Use the result of part (a) to show that at an interference maximum, the amplitude of the resultant wave is in phase with the original waves $E_1(t)$ and $E_2(t)$. (d) Use the result of part (a) to show that near an interference minimum, the resultant wave is approximately $\frac{1}{4}$ cycle out of phase with either of the original waves. (e) Show that the *instantaneous* Poynting vector at point $P$ has magnitude $S = 4\epsilon_0 cE^2\cos^2(\phi/2)\cos^2(\omega t + \phi/2)$ and that the *time-averaged* Poynting vector is given by Eq. (35.9).

**35.51** •• **CP** A thin uniform film of refractive index 1.750 is placed on a sheet of glass of refractive index 1.50. At room temperature $(20.0°\text{C})$, this film is just thick enough for light with wavelength 582.4 nm reflected off the top of the film to be cancelled by light reflected from the top of the glass. After the glass is placed in an oven and slowly heated to 170°C, you find that the film cancels reflected light with wavelength 588.5 nm. What is the coefficient of linear expansion of the film? (Ignore any changes in the refractive index of the film due to the temperature change.)

**35.52** ••• **GPS Transmission.** The GPS (Global Positioning System) satellites are approximately 5.18 m across and transmit two low-power signals, one of which is at 1575.42 MHz (in the UHF band). In a series of laboratory tests on the satellite, you put two 1575.42-MHz UHF transmitters at opposite ends of the satellite. These broadcast in phase uniformly in all directions. You measure the intensity at points on a circle that is several hundred meters in radius and centered on the satellite. You measure angles on this circle relative to a point that lies along the centerline of the satellite (that is, the perpendicular bisector of a line that extends from one transmitter to the other). At this point on the circle, the measured intensity is $2.00\ \text{W/m}^2$. (a) At how many other angles in the range $0° < \theta < 90°$ is the intensity also $2.00\ \text{W/m}^2$? (b) Find the four smallest angles in the range $0° < \theta < 90°$ for which the intensity is $2.00\ \text{W/m}^2$. (c) What is the intensity at a point on the circle at an angle of 4.65° from the centerline?

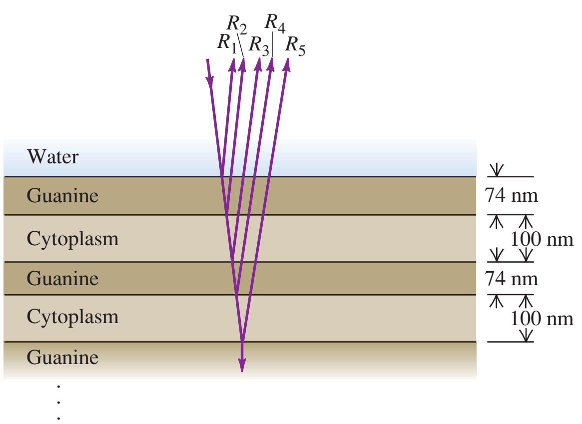
**35.53** •• Consider a two-slit interference pattern, for which the intensity distribution is given by Eq. (35.14). Let $\theta_m$ be the angular position of the $m$th bright fringe, where the intensity is $I_0$. Assume that $\theta_m$ is small, so that $\sin\theta_m \cong \theta_m$. Let $\theta_m{}^+$ and $\theta_m{}^-$ be the two angles on either side of $\theta_m$ for which $I = \frac{1}{2}I_0$. The quantity $\Delta\theta_m = |\theta_m{}^+ - \theta_m{}^-|$ is the half-width of the $m$th fringe. Calculate $\Delta\theta_m$. How does $\Delta\theta_m$ depend on $m$?

**35.54** •• White light reflects at normal incidence from the top and bottom surfaces of a glass plate ($n = 1.52$). There is air above and below the plate. Constructive interference is observed for light whose wavelength in air is 477.0 nm. What is the thickness of the plate if the next longer wavelength for which there is constructive interference is 540.6 nm?

**35.55** ••• A source $S$ of monochromatic light and a detector $D$ are both located in air a distance $h$ above a horizontal plane sheet of glass and are separated by a horizontal distance $x$. Waves reaching $D$ directly from $S$ interfere with waves that reflect off the glass. The distance $x$ is small compared to $h$ so that the reflection is at close to normal incidence. (a) Show that the condition for constructive interference is $\sqrt{x^2 + 4h^2} - x = \left(m + \frac{1}{2}\right)\lambda$, and the condition for destructive interference is $\sqrt{x^2 + 4h^2} - x = m\lambda$. (*Hint:* Take into account the phase change on reflection.) (b) Let $h = 24$ cm and $x = 14$ cm. What is the longest wavelength for which there will be constructive interference?

**35.56** •• BIO **Reflective Coatings and Herring.** Herring and related fish have a brilliant silvery appearance that camouflages them while they are swimming in a sunlit ocean. The silveriness is due to *platelets* attached to the surfaces of these fish. Each platelet is made up of several alternating layers of crystalline guanine ($n = 1.80$) and of cytoplasm ($n = 1.333$, the same as water), with a guanine layer on the outside in contact with the surrounding water (Fig. P35.56). In one typical platelet, the guanine layers are 74 nm thick and the cytoplasm layers are 100 nm thick. (a) For light striking the platelet surface at normal incidence, for which vacuum wavelengths of visible light will all of the reflections $R_1$, $R_2$, $R_3$, $R_4$, and $R_5$, shown in Fig. P35.56, be approximately in phase? If white light is shone on this platelet, what color will be most strongly reflected (see Fig. 32.4)? The surface of a herring has very many platelets side by side with layers of different thickness, so that *all* visible wavelengths are reflected. (b) Explain why such a "stack" of layers is more reflective than a single layer of guanine with cytoplasm underneath. (A stack of five guanine layers separated by cytoplasm layers reflects more than 80% of incident light at the wavelength for which it is "tuned.") (c) The color that is most strongly reflected from a platelet depends on the angle at which it is viewed. Explain why this should be so. (You can see these changes in color by examining a herring from different angles. Most of the platelets on these fish are oriented in the same way, so that they are vertical when the fish is swimming.)

Figure **P35.56**




**35.57** • Two thin parallel slits are made in an opaque sheet of film. When a monochromatic beam of light is shone through them at normal incidence, the first bright fringes in the transmitted light occur in air at $\pm 18.0°$ with the original direction of the light beam on a distant screen when the apparatus is in air. When the apparatus is immersed in a liquid, the same bright fringes now occur at $\pm 12.6°$. Find the index of refraction of the liquid.

**35.58** •• Red light with wavelength 700 nm is passed through a two-slit apparatus. At the same time, monochromatic visible light with another wavelength passes through the same apparatus. As a result, most of the pattern that appears on the screen is a mixture of two colors; however, the center of the third bright fringe ($m = 3$) of the red light appears pure red, with none of the other color. What are the possible wavelengths of the second type of visible light? Do you need to know the slit spacing to answer this question? Why or why not?

**35.59** ••• In a Young's two-slit experiment a piece of glass with an index of refraction $n$ and a thickness $L$ is placed in front of the upper slit. (a) Describe qualitatively what happens to the interference pattern. (b) Derive an expression for the intensity $I$ of the light at points on a screen as a function of $n$, $L$, and $\theta$. Here $\theta$ is the usual angle measured from the center of the two slits. That is, determine the equation analogous to Eq. (35.14) but that also involves $L$ and $n$ for the glass plate. (c) From your result in part (b) derive an expression for the values of $\theta$ that locate the maxima in the interference pattern [that is, derive an equation analogous to Eq. (35.4)].

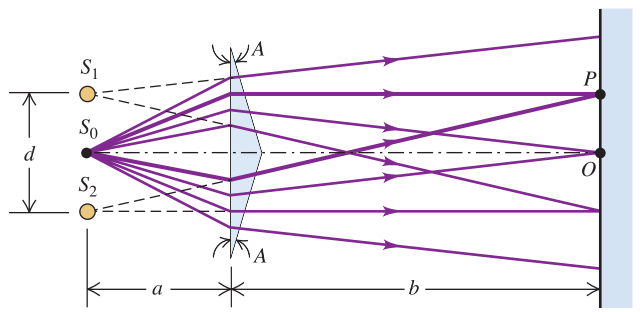
**35.60** •• After a laser beam passes through two thin parallel slits, the first completely dark fringes occur at $\pm 19.0°$ with the original direction of the beam, as viewed on a screen far from the slits. (a) What is the ratio of the distance between the slits to the wavelength of the light illuminating the slits? (b) What is the smallest angle, relative to the original direction of the laser beam, at which the intensity of the light is $\frac{1}{10}$ the maximum intensity on the screen?

## CHALLENGE PROBLEMS

**35.61** ••• CP The index of refraction of a glass rod is 1.48 at $T = 20.0°\text{C}$ and varies linearly with temperature, with a coefficient of $2.50 \times 10^{-5}/\text{C°}$. The coefficient of linear expansion of the glass is $5.00 \times 10^{-6}/\text{C°}$. At 20.0°C the length of the rod is 3.00 cm. A Michelson interferometer has this glass rod in one arm, and the rod is being heated so that its temperature increases at a rate of $5.00\ \text{C°/min}$. The light source has wavelength $\lambda = 589$ nm, and the rod initially is at $T = 20.0°\text{C}$. How many fringes cross the field of view each minute?

**35.62** ••• CP Figure P35.62 shows an interferometer known as *Fresnel's biprism.* The magnitude of the prism angle $A$ is

Figure **P35.62**



extremely small. (a) If $S_0$ is a very narrow source slit, show that the separation of the two virtual coherent sources $S_1$ and $S_2$ is given by $d = 2aA(n - 1)$, where $n$ is the index of refraction of the material of the prism. (b) Calculate the spacing of the fringes of green light with wavelength 500 nm on a screen 2.00 m from the biprism. Take $a = 0.200$ m, $A = 3.50$ mrad, and $n = 1.50$.

# Answers

## Chapter Opening Question ?

The colors appear due to constructive interference between light waves reflected from the outer and inner surfaces of the soap bubble. The thickness of the bubble walls at each point determines the wavelength of light for which the most constructive interference occurs and hence the color that appears the brightest at that point (see Example 35.4 in Section 35.4).

## Test Your Understanding Questions

**35.1** **Answer: (i)** At any point $P$ on the positive $y$-axis above $S_1$, the distance $r_2$ from $S_2$ to $P$ is greater than the distance $r_1$ from $S_1$ to $P$ by $4\lambda$. This corresponds to $m = 4$ in Eq. (35.1), the equation for constructive interference. Hence all such points make up an antinodal curve.

**35.2** **Answer: (ii)** Blue light has a shorter wavelength than red light (see Section 32.1). Equation (35.6) tells us that the distance $y_m$ from the center of the pattern to the $m$th bright fringe is proportional to the wavelength $\lambda$. Hence all of the fringes will move toward the center of the pattern as the wavelength decreases, and the spacing between fringes will decrease.

**35.3** **Answer: (i), (iv), (ii), (iii)** In cases (i) and (iii) we are given the wavelength $\lambda$ and path difference $d\sin\theta$. Hence we use Eq. (35.14), $I = I_0\cos^2[(\pi d\sin\theta)/\lambda]$. In parts (ii) and (iii) we are given the phase difference $\phi$ and we use Eq. (35.10), $I = I_0\cos^2(\phi/2)$. We find:
(i) $I = I_0\cos^2[\pi(4.00 \times 10^{-7}\text{ m})/(5.00 \times 10^{-7}\text{ m})] = I_0\cos^2(0.800\pi\text{ rad}) = 0.655I_0$;
(ii) $I = I_0\cos^2[(4.00\text{ rad})/2] = I_0\cos^2(2.00\text{ rad}) = 0.173I_0$;
(iii) $I = I_0\cos^2[\pi(7.50 \times 10^{-7}\text{ m})/(5.00 \times 10^{-7}\text{ m})] = I_0\cos^2(1.50\pi\text{ rad}) = 0$;
(iv) $I = I_0\cos^2[(2.00\text{ rad})/2] = I_0\cos^2(1.00\text{ rad}) = 0.292I_0$.

**35.4** **Answers: (i) and (iii)** Benzene has a larger index of refraction than air, so light that reflects off the upper surface of the benzene undergoes a half-cycle phase shift. Fluorite has a *smaller* index of refraction than benzene, so light that reflects off the benzene–fluorite interface does not undergo a phase shift. Hence the equation for constructive reflection is Eq. (35.18a), $2t = \left(m + \frac{1}{2}\right)\lambda$, which we can rewrite as $t = \left(m + \frac{1}{2}\right)\lambda/2 = \left(m + \frac{1}{2}\right)(400\text{ mm})/2 = 100\text{ nm}, 300\text{ nm}, 500\text{ nm}, \ldots$.

**35.5** **Answer: yes** Changing the index of refraction changes the wavelength of the light inside the compensator plate, and so changes the number of wavelengths within the thickness of the plate. Hence this has the same effect as changing the distance $L_1$ from the beam splitter to mirror $M_1$, which would change the interference pattern.

## Bridging Problem

**Answers: (a)** 441 nm **(b)** 551 nm