37 RELATIVITY

? At Brookhaven National Laboratory in New York, atomic nuclei are accelerated to 99.995% of the ultimate speed limit of the universe—the speed of light. Is there also an upper limit on the *kinetic energy* of a particle?

LEARNING GOALS

By studying this chapter, you will learn:

- The two postulates of Einstein's special theory of relativity, and what motivates these postulates.
- Why different observers can disagree about whether two events are simultaneous.
- How relativity predicts that moving clocks run slow, and that experimental evidence confirms this.
- How the length of an object changes due to the object's motion.
- How the velocity of an object depends on the frame of reference from which it is observed.
- How the theory of relativity modifies the relationship between velocity and momentum.
- How to solve problems involving work and kinetic energy for particles moving at relativistic speeds.
- Some of the key concepts of Einstein's general theory of relativity.

When the year 1905 began, Albert Einstein was an unknown 25-year-old clerk in the Swiss patent office. By the end of that amazing year he had published three papers of extraordinary importance. One was an analysis of Brownian motion; a second (for which he was awarded the Nobel Prize) was on the photoelectric effect. In the third, Einstein introduced his **special theory of relativity,** proposing drastic revisions in the Newtonian concepts of space and time.

The special theory of relativity has made wide-ranging changes in our understanding of nature, but Einstein based it on just two simple postulates. One states that the laws of physics are the same in all inertial frames of reference; the other states that the speed of light in vacuum is the same in all inertial frames. These innocent-sounding propositions have far-reaching implications. Here are three: (1) Events that are simultaneous for one observer may not be simultaneous for another. (2) When two observers moving relative to each other measure a time interval or a length, they may not get the same results. (3) For the conservation principles for momentum and energy to be valid in all inertial systems, Newton's second law and the equations for momentum and kinetic energy have to be revised.

Relativity has important consequences in *all* areas of physics, including electromagnetism, atomic and nuclear physics, and high-energy physics. Although many of the results derived in this chapter may run counter to your intuition, the theory is in solid agreement with experimental observations.

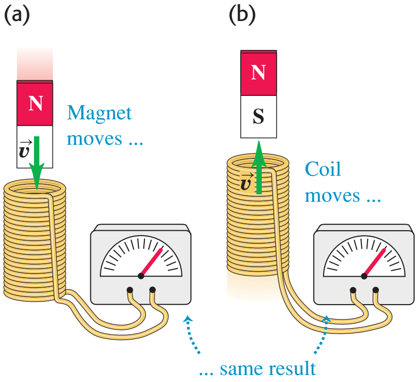
37.1 Invariance of Physical Laws

Let's take a look at the two postulates that make up the special theory of relativity. Both postulates describe what is seen by an observer in an *inertial frame of reference,* which we introduced in Section 4.2. The theory is "special" in the sense that it applies to observers in such special reference frames.

Einstein's First Postulate

Einstein's first postulate, called the **principle of relativity,** states: **The laws of physics are the same in every inertial frame of reference.** If the laws differed, that difference could distinguish one inertial frame from the others or make one frame somehow more "correct" than another. Here are two examples. Suppose you watch two children playing catch with a ball while the three of you are aboard a train moving with constant velocity. Your observations of the motion *of the ball,* no matter how carefully done, can't tell you how fast (or whether) the train is moving. This is because Newton's laws of motion are the same in every inertial frame.

37.1 The same emf is induced in the coil whether (a) the magnet moves relative to the coil or (b) the coil moves relative to the magnet.



Another example is the electromotive force (emf) induced in a coil of wire by a nearby moving permanent magnet. In the frame of reference in which the *coil* is stationary (Fig. 37.1a), the moving magnet causes a change of magnetic flux through the coil, and this induces an emf. In a different frame of reference in which the *magnet* is stationary (Fig. 37.1b), the motion of the coil through a magnetic field induces the emf. According to the principle of relativity, both of these frames of reference are equally valid. Hence the same emf must be induced in both situations shown in Fig. 37.1. As we saw in Chapter 29, this is indeed the case, so Faraday's law is consistent with the principle of relativity. Indeed, *all* of the laws of electromagnetism are the same in every inertial frame of reference.

Equally significant is the prediction of the speed of electromagnetic radiation, derived from Maxwell's equations (see Section 32.2). According to this analysis, light and all other electromagnetic waves travel in vacuum with a constant speed, now defined to equal exactly 299,792,458 m/s. (We often use the approximate value $c = 3.00 \times 10^8$ m/s, which is within one part in 1000 of the exact value.) As we will see, the speed of light in vacuum plays a central role in the theory of relativity.

Einstein's Second Postulate

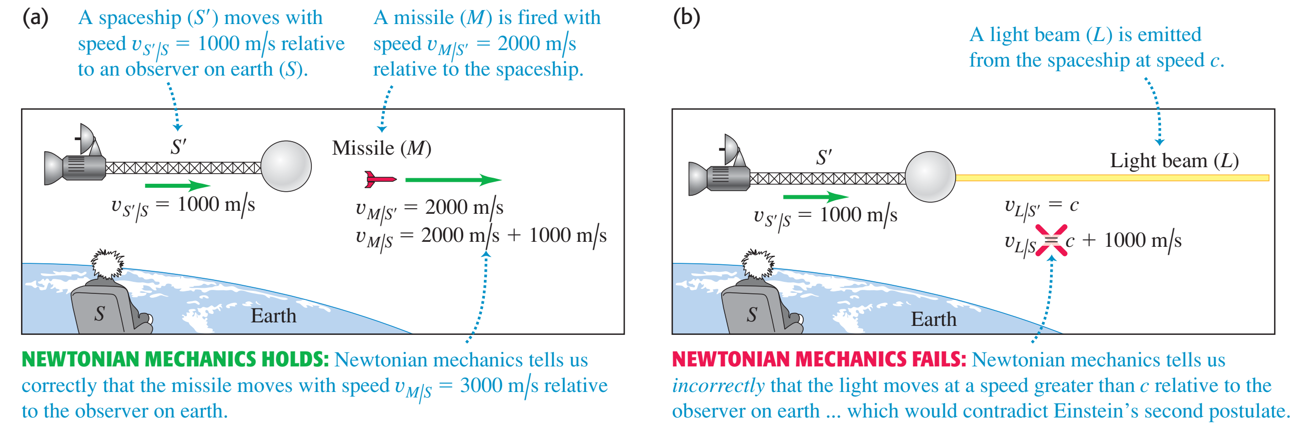
During the 19th century, most physicists believed that light traveled through a hypothetical medium called the *ether,* just as sound waves travel through air. If so, the speed of light measured by observers would depend on their motion relative to the ether and would therefore be different in different directions. The Michelson-Morley experiment, described in Section 35.5, was an effort to detect motion of the earth relative to the ether. Einstein's conceptual leap was to recognize that if Maxwell's equations are valid in all inertial frames, then the speed of light in vacuum should also be the same in all frames and in all directions. In fact, Michelson and Morley detected *no* ether motion across the earth, and the ether concept has been discarded. Although Einstein may not have known about this negative result, it supported his bold hypothesis of the constancy of the speed of light in vacuum.

Einstein's second postulate states: The speed of light in vacuum is the same in all inertial frames of reference and is independent of the motion of the source.

Let's think about what this means. Suppose two observers measure the speed of light in vacuum. One is at rest with respect to the light source, and the other is moving away from it. Both are in inertial frames of reference. According to the principle of relativity, the two observers must obtain the same result, despite the fact that one is moving with respect to the other.

If this seems too easy, consider the following situation. A spacecraft moving past the earth at 1000 m/s fires a missile straight ahead with a speed of 2000 m/s (relative to the spacecraft) (Fig. 37.2). What is the missile's speed relative to the earth? Simple, you say; this is an elementary problem in relative velocity (see Section 3.5). The correct answer, according to Newtonian mechanics, is 3000 m/s.

37.2 (a) Newtonian mechanics makes correct predictions about relatively slow-moving objects; (b) it makes incorrect predictions about the behavior of light.



But now suppose the spacecraft turns on a searchlight, pointing in the same direction in which the missile was fired. An observer on the spacecraft measures the speed of light emitted by the searchlight and obtains the value c. According to Einstein's second postulate, the motion of the light after it has left the source cannot depend on the motion of the source. So the observer on earth who measures the speed of this same light must also obtain the value c, *not* $c + 1000$ m/s. This result contradicts our elementary notion of relative velocities, and it may not appear to agree with common sense. But "common sense" is intuition based on everyday experience, and this does not usually include measurements of the speed of light.

The Ultimate Speed Limit

Einstein's second postulate immediately implies the following result:

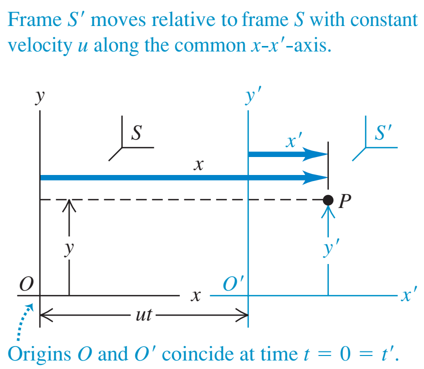
It is impossible for an inertial observer to travel at c, the speed of light in vacuum.

We can prove this by showing that travel at c implies a logical contradiction. Suppose that the spacecraft S' in Fig. 37.2b is moving at the speed of light relative to an observer on the earth, so that $v_{S'/S} = c$. If the spacecraft turns on a headlight, the second postulate now asserts that the earth observer S measures the headlight beam to be also moving at c. Thus this observer measures that the headlight beam and the spacecraft move together and are always at the same point in space. But Einstein's second postulate also asserts that the headlight beam moves at a speed c relative to the spacecraft, so they *cannot* be at the same point in space. This contradictory result can be avoided only if it is impossible for an inertial observer, such as a passenger on the spacecraft, to move at c. As we go through our discussion of relativity, you may find yourself asking the question Einstein asked himself as a 16-year-old student, "What would I see if I were traveling at the speed of light?" Einstein realized only years later that his question's basic flaw was that he could *not* travel at c.

The Galilean Coordinate Transformation

Let's restate this argument symbolically, using two inertial frames of reference, labeled S for the observer on earth and S' for the moving spacecraft, as shown in Fig. 37.3. To keep things as simple as possible, we have omitted the z-axes. The x-axes of the two frames lie along the same line, but the origin O' of frame S' moves relative to the origin O of frame S with constant velocity u along the common x-x'-axis. We on earth set our clocks so that the two origins coincide at time $t = 0$, so their separation at a later time t is ut.

37.3 The position of particle P can be described by the coordinates x and y in frame of reference S or by x' and y' in frame S'.



CAUTION **Choose your inertial frame coordinates wisely** Many of the equations derived in this chapter are true *only* if you define your inertial reference frames as stated in the preceding paragraph. For instance, the positive x-direction must be the direction in which the origin O' moves relative to the origin O. In Fig. 37.3 this direction is to the right; if instead O' moves to the left relative to O, you must define the positive x-direction to be to the left.

Now think about how we describe the motion of a particle P. This might be an exploratory vehicle launched from the spacecraft or a pulse of light from a laser. We can describe the *position* of this particle by using the earth coordinates (x, y, z) in S or the spacecraft coordinates (x', y', z') in S'. Figure 37.3 shows that these are simply related by

$$x = x' + ut \qquad y = y' \qquad z = z' \qquad \text{(Galilean coordinate transformation)} \tag{37.1}$$

These equations, based on the familiar Newtonian notions of space and time, are called the **Galilean coordinate transformation.**

If particle P moves in the x-direction, its instantaneous velocity v_x as measured by an observer stationary in S is $v_x = dx/dt$. Its velocity v'_x as measured by an observer stationary in S' is $v'_x = dx'/dt$. We can derive a relationship between v_x and v'_x by taking the derivative with respect to t of the first of Eqs. (37.1):

$$\frac{dx}{dt} = \frac{dx'}{dt} + u$$

Now dx/dt is the velocity v_x measured in S, and dx'/dt is the velocity v'_x measured in S', so we get the *Galilean velocity transformation* for one-dimensional motion:

$$v_x = v'_x + u \qquad \text{(Galilean velocity transformation)} \tag{37.2}$$

Although the notation differs, this result agrees with our discussion of relative velocities in Section 3.5.

Now here's the fundamental problem. Applied to the speed of light in vacuum, Eq. (37.2) says that $c = c' + u$. Einstein's second postulate, supported subsequently by a wealth of experimental evidence, says that $c = c'$. This is a genuine inconsistency, not an illusion, and it demands resolution. If we accept this postulate, we are forced to conclude that Eqs. (37.1) and (37.2) *cannot* be precisely correct, despite our convincing derivation. These equations have to be modified to bring them into harmony with this principle.

The resolution involves some very fundamental modifications in our kinematic concepts. The first idea to be changed is the seemingly obvious assumption that the observers in frames S and S' use the same *time scale,* formally stated as $t = t'$. Alas, we are about to show that this everyday assumption cannot be correct; the two observers *must* have different time scales. We must define the velocity v' in frame S' as $v' = dx'/dt'$, not as dx'/dt; the two quantities are not the same. The difficulty lies in the concept of *simultaneity,* which is our next topic. A careful analysis of simultaneity will help us develop the appropriate modifications of our notions about space and time.

Test Your Understanding of Section 37.1 As a high-speed spaceship flies past you, it fires a strobe light that sends out a pulse of light in all directions. An observer aboard the spaceship measures a spherical wave front that spreads away from the spaceship with the same speed c in all directions. (a) What is the shape of the wave front that *you* measure? (i) spherical; (ii) ellipsoidal, with the longest axis of the ellipsoid along the direction of the spaceship's motion; (iii) ellipsoidal, with the shortest axis of the ellipsoid along the direction of the spaceship's motion; (iv) not enough information is given to decide. (b) Is the wave front centered on the spaceship?

37.2 Relativity of Simultaneity

Measuring times and time intervals involves the concept of **simultaneity.** In a given frame of reference, an **event** is an occurrence that has a definite position and time (Fig. 37.4). When you say that you awoke at seven o'clock, you mean that two events (your awakening and your clock showing 7:00) occurred *simultaneously.* The fundamental problem in measuring time intervals is this: In general, two events that are simultaneous in one frame of reference are *not* simultaneous in a second frame that is moving relative to the first, even if both are inertial frames.

37.4 An event has a definite position and time—for instance, on the pavement directly below the center of the Eiffel Tower at midnight on New Year's Eve.

A Thought Experiment in Simultaneity

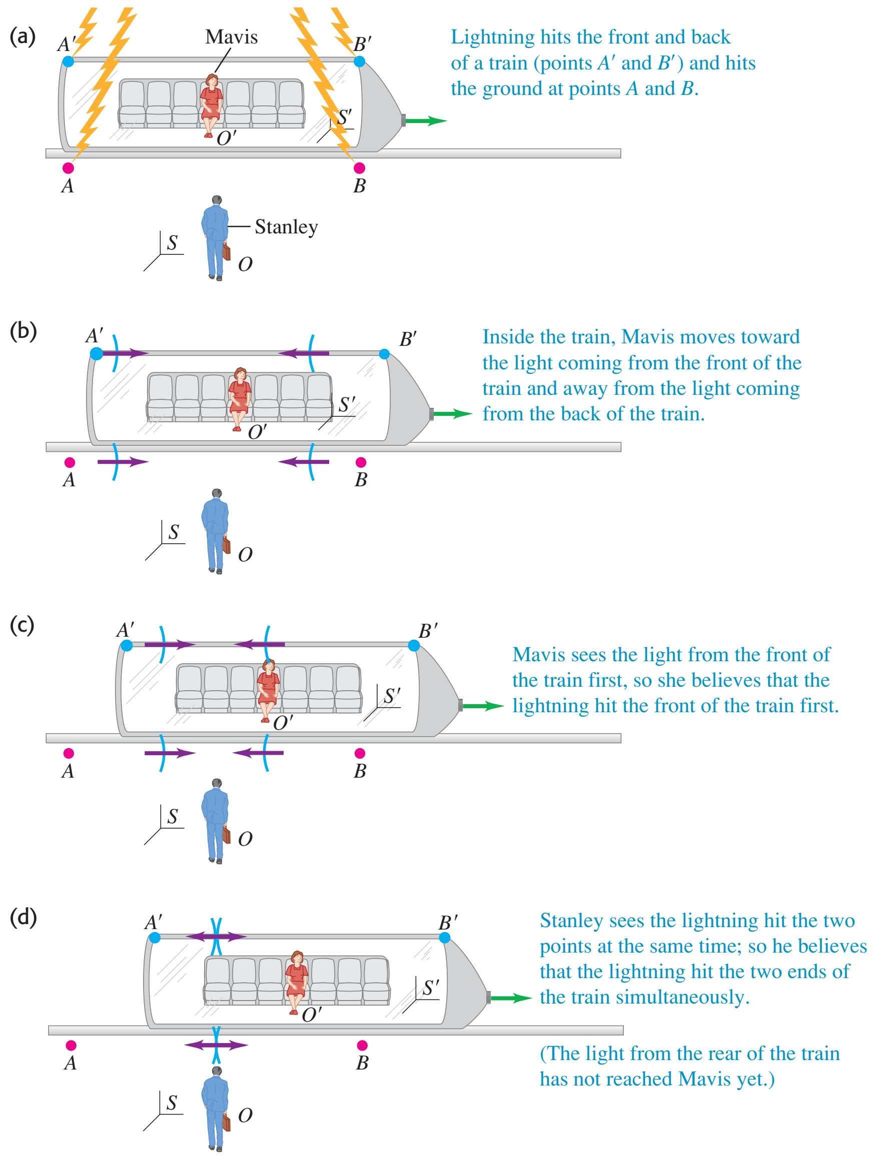
This may seem to be contrary to common sense. To illustrate the point, here is a version of one of Einstein's *thought experiments*—mental experiments that follow concepts to their logical conclusions. Imagine a train moving with a speed comparable to c, with uniform velocity (Fig. 37.5). Two lightning bolts strike a passenger car, one near each end. Each bolt leaves a mark on the car and one on the ground at the instant the bolt hits. The points on the ground are labeled A and B in the figure, and the corresponding points on the car are A' and B'. Stanley is stationary on the ground at O, midway between A and B. Mavis is moving with the train at O' in the middle of the passenger car, midway between A' and B'. Both Stanley and Mavis see both light flashes emitted from the points where the lightning strikes.

Suppose the two wave fronts from the lightning strikes reach Stanley at O simultaneously. He knows that he is the same distance from B and A, so Stanley concludes that the two bolts struck B and A simultaneously. Mavis agrees that the two wave fronts reached Stanley at the same time, but she disagrees that the flashes were emitted simultaneously.

Stanley and Mavis agree that the two wave fronts do not reach Mavis at the same time. Mavis at O' is moving to the right with the train, so she runs into the wave front from B' *before* the wave front from A' catches up to her. However, because she is in the middle of the passenger car equidistant from A' and B', her observation is that both wave fronts took the same time to reach her because both moved the same distance at the same speed c. (Recall that the speed of each wave front with respect to *either* observer is c.) Thus she concludes that the lightning bolt at B' struck *before* the one at A'. Stanley at O measures the two events to be simultaneous, but Mavis at O' does not! *Whether or not two events at different x-axis locations are simultaneous depends on the state of motion of the observer.*

You may want to argue that in this example the lightning bolts really *are* simultaneous and that if Mavis at O' could communicate with the distant points without the time delay caused by the finite speed of light, she would realize this. But that would be erroneous; the finite speed of information transmission is not the real issue. If O' is midway between A' and B', then in her frame of reference the time for a signal to travel from A' to O' is the same as that from B' to O'. Two signals arrive simultaneously at O' only if they were emitted simultaneously at A' and B'. In this example they *do not* arrive simultaneously at O', and so Mavis must conclude that the events at A' and B' were *not* simultaneous.

Furthermore, there is no basis for saying that Stanley is right and Mavis is wrong, or vice versa. According to the principle of relativity, no inertial frame of reference is more correct than any other in the formulation of physical laws. Each observer is correct *in his or her own frame of reference.* In other words, simultaneity is not an absolute concept. Whether two events are simultaneous depends on the frame of reference. As we mentioned at the beginning of this section, simultaneity plays an essential role in measuring time intervals. It follows that *the time interval between two events may be different in different frames of reference.* So our next task is to learn how to compare time intervals in different frames of reference.



37.5 A thought experiment in simultaneity.

Test Your Understanding of Section 37.2 Stanley, who works for the rail system shown in Fig. 37.5, has carefully synchronized the clocks at all of the rail stations. At the moment that Stanley measures all of the clocks striking noon, Mavis is on a high-speed passenger car traveling from Ogdenville toward North Haverbrook. According to Mavis, when the Ogdenville clock strikes noon, what time is it in North Haverbrook? (i) noon; (ii) before noon; (iii) after noon.

37.3 Relativity of Time Intervals

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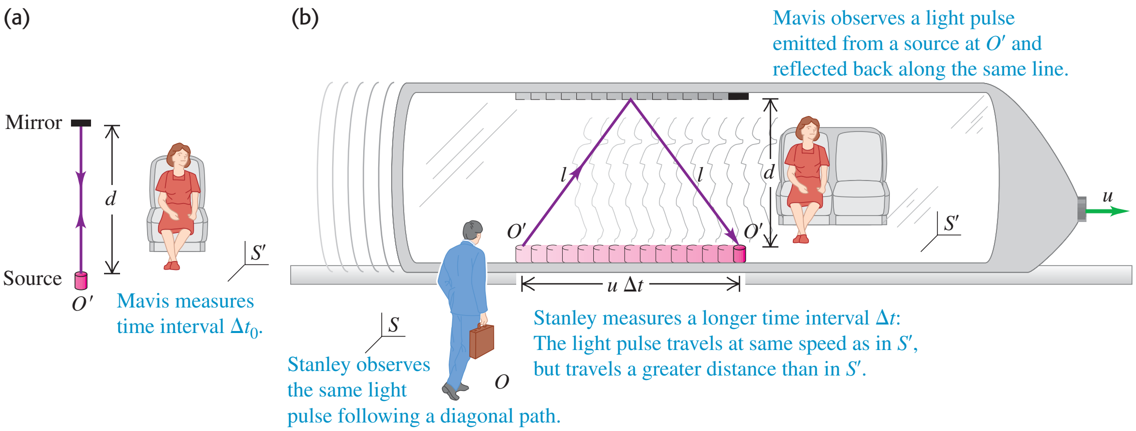
ActivPhysics 17.1: Relativity of Time

We can derive a quantitative relationship between time intervals in different coordinate systems. To do this, let's consider another thought experiment. As before, a frame of reference S' moves along the common x-x'-axis with constant speed u relative to a frame S. As discussed in Section 37.1, u must be less than the speed of light c. Mavis, who is riding along with frame S', measures the time interval between two events that occur at the *same* point in space. Event 1 is when a flash of light from a light source leaves O'. Event 2 is when the flash returns to O', having been reflected from a mirror a distance d away, as shown in Fig. 37.6a. We label the time interval Δt_0, using the subscript zero as a reminder that the apparatus is at rest, with zero velocity, in frame S'. The flash of light moves a total distance $2d$, so the time interval is

37.6 (a) Mavis, in frame of reference S', observes a light pulse emitted from a source at O' and reflected back along the same line. (b) How Stanley (in frame of reference S) and Mavis observe the same light pulse. The positions of O' at the times of departure and return of the pulse are shown.



$$\Delta t_0 = \frac{2d}{c} \tag{37.3}$$

The round-trip time measured by Stanley in frame S is a different interval Δt; in his frame of reference the two events occur at *different* points in space. During the time Δt, the source moves relative to S a distance $u\,\Delta t$ (Fig. 37.6b). In S' the round-trip distance is $2d$ perpendicular to the relative velocity, but the round-trip distance in S is the longer distance $2l$, where

$$l = \sqrt{d^2 + \left(\frac{u\,\Delta t}{2}\right)^2}$$

In writing this expression, we have assumed that both observers measure the same distance d. We will justify this assumption in the next section. The speed of light is the same for both observers, so the round-trip time measured in S is

$$\Delta t = \frac{2l}{c} = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\,\Delta t}{2}\right)^2} \tag{37.4}$$

We would like to have a relationship between Δt and Δt_0 that is independent of d. To get this, we solve Eq. (37.3) for d and substitute the result into Eq. (37.4), obtaining

$$\Delta t = \frac{2}{c}\sqrt{\left(\frac{c\,\Delta t_0}{2}\right)^2 + \left(\frac{u\,\Delta t}{2}\right)^2} \tag{37.5}$$

Now we square this and solve for Δt; the result is

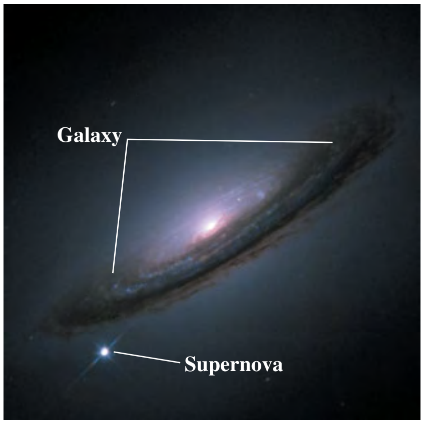
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$

Since the quantity $\sqrt{1 - u^2/c^2}$ is less than 1, Δt is greater than Δt_0: Thus Stanley measures a *longer* round-trip time for the light pulse than does Mavis.

Time Dilation

We may generalize this important result. In a particular frame of reference, suppose that two events occur at the same point in space. The time interval between these events, as measured by an observer at rest in this same frame (which we call the *rest frame* of this observer), is Δt_0. Then an observer in a second frame moving with constant speed u relative to the rest frame will measure the time interval to be Δt, where

37.7 This image shows an exploding star, called a *supernova,* within a distant galaxy. The brightness of a typical supernova decays at a certain rate. But supernovae that are moving away from us at a substantial fraction of the speed of light decay more slowly, in accordance with Eq. (37.6). The decaying supernova is a moving "clock" that runs slow.



$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} \quad \text{(time dilation)} \tag{37.6}$$

We recall that no inertial observer can travel at $u = c$ and we note that $\sqrt{1 - u^2/c^2}$ is imaginary for $u > c$. Thus Eq. (37.6) gives sensible results only when $u < c$. The denominator of Eq. (37.6) is always smaller than 1, so Δt is always *larger* than Δt_0. Thus we call this effect **time dilation.**

Think of an old-fashioned pendulum clock that has one second between ticks, as measured by Mavis in the clock's rest frame; this is Δt_0. If the clock's rest frame is moving relative to Stanley, he measures a time between ticks Δt that is longer than one second. In brief, *observers measure any clock to run slow if it moves relative to them* (Fig. 37.7). Note that this conclusion is a direct result of the fact that the speed of light in vacuum is the same in both frames of reference.

The quantity $1/\sqrt{1 - u^2/c^2}$ in Eq. (37.6) appears so often in relativity that it is given its own symbol γ (the Greek letter gamma):

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \tag{37.7}$$

In terms of this symbol, we can express the time dilation formula, Eq. (37.6), as

$$\Delta t = \gamma\, \Delta t_0 \quad \text{(time dilation)} \tag{37.8}$$

As a further simplification, u/c is sometimes given the symbol β (the Greek letter beta); then $\gamma = 1/\sqrt{1 - \beta^2}$.

37.8 The quantity $\gamma = 1/\sqrt{1 - u^2/c^2}$ as a function of the relative speed u of two frames of reference.

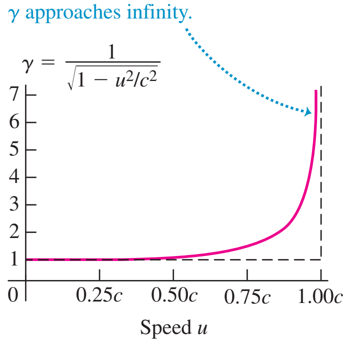


Figure 37.8 shows a graph of γ as a function of the relative speed u of two frames of reference. When u is very small compared to c, u^2/c^2 is much smaller than 1 and γ is very nearly *equal* to 1. In that limit, Eqs. (37.6) and (37.8) approach the Newtonian relationship $\Delta t = \Delta t_0$, corresponding to the same time interval in all frames of reference.

If the relative speed u is great enough that γ is appreciably greater than 1, the speed is said to be *relativistic;* if the difference between γ and 1 is negligibly small, the speed u is called *nonrelativistic.* Thus $u = 6.00 \times 10^7\ \text{m/s} = 0.200c$ (for which $\gamma = 1.02$) is a relativistic speed, but $u = 6.00 \times 10^4\ \text{m/s} = 0.000200c$ (for which $\gamma = 1.00000002$) is a nonrelativistic speed.

Proper Time

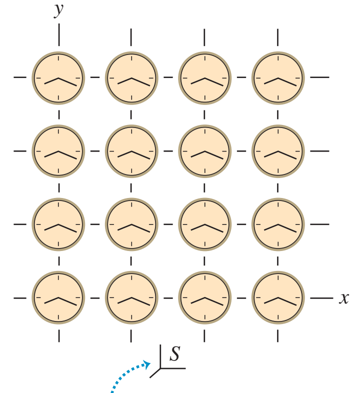
There is only one frame of reference in which a clock is at rest, and there are infinitely many in which it is moving. Therefore the time interval measured between two events (such as two ticks of the clock) that occur at the same point in a particular frame is a more fundamental quantity than the interval between events at different points. We use the term **proper time** to describe the time interval Δt_0 between two events that occur *at the same point.*

CAUTION **Measuring time intervals** It is important to note that the time interval Δt in Eq. (37.6) involves events that occur *at different space points* in the frame of reference *S*. Note also that any differences between Δt and the proper time Δt_0 are *not* caused by differences in the times required for light to travel from those space points to an observer at rest in *S*. We assume that our observer is able to correct for differences in light transit times, just as an astronomer who's observing the sun understands that an event seen now on earth actually occurred 500 s ago on the sun's surface. Alternatively, we can use *two* observers, one stationary at the location of the first event and the other at the second, each with his or her own clock. We can synchronize these two clocks without difficulty, as long as they are at rest in the same frame of reference. For example, we could send a light pulse simultaneously to the two clocks from a point midway between them. When the pulses arrive, the observers set their clocks to a prearranged time. (But note that clocks that are synchronized in one frame of reference *are not* in general synchronized in any other frame.)

In thought experiments, it's often helpful to imagine many observers with synchronized clocks at rest at various points in a particular frame of reference. We can picture a frame of reference as a coordinate grid with lots of synchronized clocks distributed around it, as suggested by Fig. 37.9. Only when a clock is moving relative to a given frame of reference do we have to watch for ambiguities of synchronization or simultaneity.

Throughout this chapter we will frequently use phrases like "Stanley *observes* that Mavis passes the point $x = 5.00$ m, $y = 0$, $z = 0$ at time 2.00 s." This means that Stanley is using a grid of clocks in his frame of reference, like the grid shown in Fig. 37.9, to record the time of an event. We could restate the phrase as "When Mavis passes the point at $x = 5.00$ m, $y = 0$, $z = 0$, the clock at that location in Stanley's frame of reference reads 2.00 s." We will avoid using phrases like "Stanley *sees* that Mavis is a certain point at a certain time," because there is a time delay for light to travel to Stanley's eye from the position of an event.

37.9 A frame of reference pictured as a coordinate system with a grid of synchronized clocks.



The grid is three dimensional; identical planes of clocks lie in front of and behind the page, connected by grid lines perpendicular to the page.

Problem-Solving Strategy 37.1 Time Dilation

IDENTIFY *the relevant concepts:* The concept of time dilation is used whenever we compare the time intervals between events as measured by observers in different inertial frames of reference.

SET UP *the problem* using the following steps:

1. First decide what two events define the beginning and the end of the time interval. Then identify the two frames of reference in which the time interval is measured.
2. Identify the target variable.

EXECUTE *the solution* as follows:

1. In many problems, the time interval as measured in one frame of reference is the *proper* time Δt_0. This is the time interval between two events in a frame of reference in which the two events occur at the same point in space. In a second frame of reference that has a speed u relative to that first frame, there is a longer time interval Δt between the same two events. In this second frame the two events occur at different points. You will need to decide in which frame the time interval is Δt_0 and in which frame it is Δt.
2. Use Eq. (37.6) or (37.8) to relate Δt_0 and Δt, and then solve for the target variable.

EVALUATE *your answer:* Note that Δt is never smaller than Δt_0, and u is never greater than c. If your results suggest otherwise, you need to rethink your calculation.

Example 37.1 Time dilation at 0.990*c*

High-energy subatomic particles coming from space interact with atoms in the earth's upper atmosphere, in some cases producing unstable particles called *muons.* A muon decays into other particles with a mean lifetime of 2.20 μs $= 2.20 \times 10^{-6}$ s as measured in a reference frame in which it is at rest. If a muon is moving at $0.990c$ relative to the earth, what will an observer on earth measure its mean lifetime to be?

SOLUTION

IDENTIFY and SET UP: The muon's lifetime is the time interval between two events: the production of the muon and its subsequent decay. Our target variable is the lifetime in your frame of reference on earth, which we call frame S. We are given the lifetime in a frame S' in which the muon is at rest; this is its *proper* lifetime, $\Delta t_0 = 2.20\ \mu\text{s}$. The relative speed of these two frames is $u = 0.990c$. We use Eq. (37.6) to relate the lifetimes in the two frames.

EXECUTE: The muon moves relative to the earth between the two events, so the two events occur at different positions as measured in S and the time interval in that frame is Δt (the target variable). From Eq. (37.6),

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \frac{2.20\ \mu\text{s}}{\sqrt{1 - (0.990)^2}} = 15.6\ \mu\text{s}$$

EVALUATE: Our result predicts that the mean lifetime of the muon in the earth frame (Δt) is about seven times longer than in the muon's frame (Δt_0). This prediction has been verified experimentally; indeed, this was the first experimental confirmation of the time dilation formula, Eq. (37.6).

Example 37.2 Time dilation at airliner speeds

An airplane flies from San Francisco to New York (about 4800 km, or 4.80×10^6 m) at a steady speed of 300 m/s (about 670 mi/h). How much time does the trip take, as measured by an observer on the ground? By an observer in the plane?

SOLUTION

IDENTIFY and SET UP: Here we're interested in the time interval between the airplane departing from San Francisco and landing in New York. The target variables are the time intervals as measured in the frame of reference of the ground S and in the frame of reference of the airplane S'.

EXECUTE: As measured in S the two events occur at different positions (San Francisco and New York), so the time interval measured by ground observers corresponds to Δt in Eq. (37.6). To find it, we simply divide the distance by the speed $u = 300$ m/s:

$$\Delta t = \frac{4.80 \times 10^6\ \text{m}}{300\ \text{m/s}} = 1.60 \times 10^4\ \text{s} \quad \left(\text{about } 4\tfrac{1}{2} \text{ hours}\right)$$

In the airplane's frame S', San Francisco and New York passing under the plane occur at the same point (the position of the plane). Hence the time interval in the airplane is a proper time, corresponding to Δt_0 in Eq. (37.6). We have

$$\frac{u^2}{c^2} = \frac{(300\ \text{m/s})^2}{(3.00 \times 10^8\ \text{m/s})^2} = 1.00 \times 10^{-12}$$

From Eq. (37.6),

$$\Delta t_0 = (1.60 \times 10^4\ \text{s})\sqrt{1 - 1.00 \times 10^{-12}}$$

The square root can't be evaluated with adequate precision with an ordinary calculator. But we can approximate it using the binomial theorem (see Appendix B):

$$(1 - 1.00 \times 10^{-12})^{1/2} = 1 - \left(\tfrac{1}{2}\right)(1.00 \times 10^{-12}) + \cdots$$

The remaining terms are of the order of 10^{-24} or smaller and can be discarded. The approximate result for Δt_0 is

$$\Delta t_0 = (1.60 \times 10^4\ \text{s})(1 - 0.50 \times 10^{-12})$$

The proper time Δt_0, measured in the airplane, is very slightly less (by less than one part in 10^{12}) than the time measured on the ground.

EVALUATE: We don't notice such effects in everyday life. But present-day atomic clocks (see Section 1.3) can attain a precision of about one part in 10^{13}. A cesium clock traveling a long distance in an airliner has been used to measure this effect and thereby verify Eq. (37.6) even at speeds much less than c.

Example 37.3 Just when is it proper?

Mavis boards a spaceship and then zips past Stanley on earth at a relative speed of $0.600c$. At the instant she passes him, they both start timers. (a) A short time later Stanley measures that Mavis has traveled 9.00×10^7 m beyond him and is passing a space station. What does Stanley's timer read as she passes the space station? What does Mavis's timer read? (b) Stanley starts to blink just as Mavis flies past him, and Mavis measures that the blink takes 0.400 s from beginning to end. According to Stanley, what is the duration of his blink?

SOLUTION

IDENTIFY and SET UP: This problem involves time dilation for two *different* sets of events measured in Stanley's frame of reference (which we call S) and in Mavis's frame of reference (which we call S'). The two events of interest in part (a) are when Mavis passes Stanley and when Mavis passes the space station; the target variables are the time intervals between these two events as measured in S and in S'. The two events in part (b) are the start and finish of Stanley's blink; the target variable is the time interval between these two events as measured in S.

EXECUTE: (a) The two events, Mavis passing the earth and Mavis passing the space station, occur at different positions in Stanley's frame but at the same position in Mavis's frame. Hence Stanley measures time interval Δt, while Mavis measures the *proper* time Δt_0. As measured by Stanley, Mavis moves at $0.600c = 0.600(3.00 \times 10^8\ \text{m/s}) = 1.80 \times 10^8\ \text{m/s}$ and travels 9.00×10^7 m in time $\Delta t = (9.00 \times 10^7\ \text{m})/(1.80 \times 10^8\ \text{m/s}) = 0.500$ s. From Eq. (37.6), Mavis's timer reads an elapsed time of

$$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = 0.500\ \text{s} \sqrt{1 - (0.600)^2} = 0.400\ \text{s}$$

(b) It is tempting to answer that Stanley's blink lasts 0.500 s in his frame. But this is wrong, because we are now considering a *different* pair of events than in part (a). The start and finish of Stanley's blink occur at the same point in his frame S but at different positions in Mavis's frame S', so the time interval of 0.400 s that she measures between these events is equal to Δt. The duration of the blink measured on Stanley's timer is the proper time Δt_0:

$$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = 0.400\ \text{s} \sqrt{1 - (0.600)^2} = 0.320\ \text{s}$$

EVALUATE: This example illustrates the relativity of simultaneity. In Mavis's frame she passes the space station at the same instant that Stanley finishes his blink, 0.400 s after she passed Stanley. Hence these two events are simultaneous to Mavis in frame S'. But these two events are *not* simultaneous to Stanley in his frame S: According to his timer, he finishes his blink after 0.320 s and Mavis passes the space station after 0.500 s.

The Twin Paradox

Equations (37.6) and (37.8) for time dilation suggest an apparent paradox called the **twin paradox.** Consider identical twin astronauts named Eartha and Astrid.

Eartha remains on earth while her twin Astrid takes off on a high-speed trip through the galaxy. Because of time dilation, Eartha observes Astrid's heartbeat and all other life processes proceeding more slowly than her own. Thus to Eartha, Astrid ages more slowly; when Astrid returns to earth she is younger (has aged less) than Eartha.

Now here is the paradox: All inertial frames are equivalent. Can't Astrid make exactly the same arguments to conclude that Eartha is in fact the younger? Then each twin measures the other to be younger when they're back together, and that's a paradox.

To resolve the paradox, we recognize that the twins are *not* identical in all respects. While Eartha remains in an approximately inertial frame at all times, Astrid must *accelerate* with respect to that inertial frame during parts of her trip in order to leave, turn around, and return to earth. Eartha's reference frame is always approximately inertial; Astrid's is often far from inertial. Thus there is a real physical difference between the circumstances of the two twins. Careful analysis shows that Eartha is correct; when Astrid returns, she *is* younger than Eartha.

Application Who's the Grandmother?
The answer to this question may seem obvious, but it could depend on which person had traveled to a distant planet at relativistic speeds. Imagine that a 20-year-old woman had given birth to a child and then immediately left on a 100-light-year trip (50 light-years out and 50 light-years back) at 99.5% the speed of light. Because of time dilation for the traveler, only 10 years would pass, and she would be 30 years old when she returned, even though 100 years had passed by for people on earth. Meanwhile, the child she left behind at home could have had a baby 20 years after her departure, and this grandchild would now be 80 years old!

Test Your Understanding of Section 37.3 Samir (who is standing on the ground) starts his stopwatch at the instant that Maria flies past him in her spaceship at a speed of $0.600c$. At the same instant, Maria starts her stopwatch. (a) As measured in Samir's frame of reference, what is the reading on Maria's stopwatch at the instant that Samir's stopwatch reads 10.0 s? (i) 10.0 s; (ii) less than 10.0 s; (iii) more than 10.0 s. (b) As measured in Maria's frame of reference, what is the reading on Samir's stopwatch at the instant that Maria's stopwatch reads 10.0 s? (i) 10.0 s; (ii) less than 10.0 s; (iii) more than 10.0 s.

37.4 Relativity of Length

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ActivPhysics 17.2: Relativity of Length

Not only does the time interval between two events depend on the observer's frame of reference, but the *distance* between two points may also depend on the observer's frame of reference. The concept of simultaneity is involved. Suppose you want to measure the length of a moving car. One way is to have two assistants make marks on the pavement at the positions of the front and rear bumpers. Then you measure the distance between the marks. But your assistants have to make their marks *at the same time*. If one marks the position of the front bumper at one time and the other marks the position of the rear bumper half a second later, you won't get the car's true length. Since we've learned that simultaneity isn't an absolute concept, we have to proceed with caution.

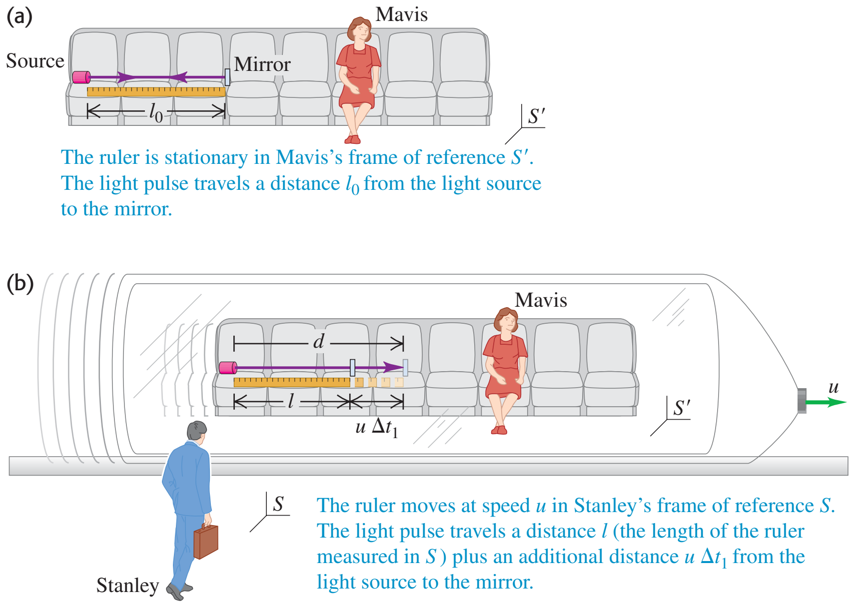
Lengths Parallel to the Relative Motion

To develop a relationship between lengths that are measured parallel to the direction of motion in various coordinate systems, we consider another thought experiment. We attach a light source to one end of a ruler and a mirror to the other end. The ruler is at rest in reference frame S', and its length in this frame is l_0 (Fig. 37.10a). Then the time Δt_0 required for a light pulse to make the round trip from source to mirror and back is

$$\Delta t_0 = \frac{2l_0}{c} \tag{37.9}$$

This is a proper time interval because departure and return occur at the same point in S'.

In reference frame S the ruler is moving to the right with speed u during this travel of the light pulse (Fig. 37.10b). The length of the ruler in S is l, and the time of travel from source to mirror, as measured in S, is Δt_1. During this interval



37.10 (a) A ruler is at rest in Mavis's frame S'. A light pulse is emitted from a source at one end of the ruler, reflected by a mirror at the other end, and returned to the source position. (b) Motion of the light pulse as measured in Stanley's frame S.

the ruler, with source and mirror attached, moves a distance $u\,\Delta t_1$. The total length of path d from source to mirror is not l, but rather

$$d = l + u\,\Delta t_1 \tag{37.10}$$

The light pulse travels with speed c, so it is also true that

$$d = c\,\Delta t_1 \tag{37.11}$$

Combining Eqs. (37.10) and (37.11) to eliminate d, we find

$$c\,\Delta t_1 = l + u\,\Delta t_1 \quad \text{or}$$

$$\Delta t_1 = \frac{l}{c - u} \tag{37.12}$$

(Dividing the distance l by $c - u$ does *not* mean that light travels with speed $c - u$, but rather that the distance the pulse travels in S is greater than l.)

In the same way we can show that the time Δt_2 for the return trip from mirror to source is

$$\Delta t_2 = \frac{l}{c + u} \tag{37.13}$$

The *total* time $\Delta t = \Delta t_1 + \Delta t_2$ for the round trip, as measured in S, is

$$\Delta t = \frac{l}{c - u} + \frac{l}{c + u} = \frac{2l}{c(1 - u^2/c^2)} \tag{37.14}$$

We also know that Δt and Δt_0 are related by Eq. (37.6) because Δt_0 is a proper time in S'. Thus Eq. (37.9) for the round-trip time in the rest frame S' of the ruler becomes

$$\Delta t\sqrt{1 - \frac{u^2}{c^2}} = \frac{2l_0}{c} \tag{37.15}$$

Finally, combining Eqs. (37.14) and (37.15) to eliminate Δt and simplifying, we obtain

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma} \qquad \text{(length contraction)} \tag{37.16}$$

[We have used the quantity $\gamma = 1/\sqrt{1 - u^2/c^2}$ defined in Eq. (37.7).] Thus the length l measured in S, in which the ruler is moving, is *shorter* than the length l_0 measured in its rest frame S'.

CAUTION **Length contraction is real** This is *not* an optical illusion! The ruler really is shorter in reference frame S than it is in S'.

A length measured in the frame in which the body is at rest (the rest frame of the body) is called a **proper length;** thus l_0 is a proper length in S', and the length measured in any other frame moving relative to S' is *less than* l_0. This effect is called **length contraction.**

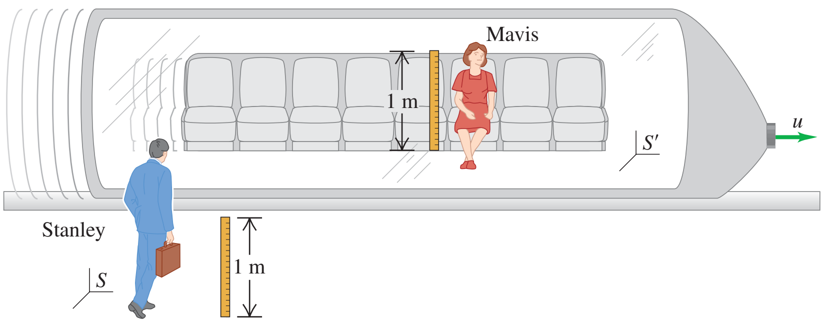
When u is very small in comparison to c, γ approaches 1. Thus in the limit of small speeds we approach the Newtonian relationship $l = l_0$. This and the corresponding result for time dilation show that Eqs. (37.1), the Galilean coordinate transformation, are usually sufficiently accurate for relative speeds much smaller than c. If u is a reasonable fraction of c, however, the quantity $\sqrt{1 - u^2/c^2}$ can be appreciably less than 1. Then l can be substantially smaller than l_0, and the effects of length contraction can be substantial (Fig. 37.11).

37.11 The speed at which electrons traverse the 3-km beam line of the SLAC National Accelerator Laboratory is slower than c by less than 1 cm/s. As measured in the reference frame of such an electron, the beam line (which extends from the top to the bottom of this photograph) is only about 15 cm long!

Lengths Perpendicular to the Relative Motion

We have derived Eq. (37.16) for lengths measured in the direction *parallel* to the relative motion of the two frames of reference. Lengths that are measured *perpendicular* to the direction of motion are *not* contracted. To prove this, consider two identical meter sticks. One stick is at rest in frame S and lies along the positive y-axis with one end at O, the origin of S. The other is at rest in frame S' and lies along the positive y'-axis with one end at O', the origin of S'. Frame S' moves in the positive x-direction relative to frame S. Observers Stanley and Mavis, at rest in S and S' respectively, station themselves at the 50-cm mark of their sticks. At the instant the two origins coincide, the two sticks lie along the same line. At this instant, Mavis makes a mark on Stanley's stick at the point that coincides with her own 50-cm mark, and Stanley does the same to Mavis's stick.

Suppose for the sake of argument that Stanley observes Mavis's stick as longer than his own. Then the mark Stanley makes on her stick is *below* its center. In that case, Mavis will think Stanley's stick has become shorter, since half of its length coincides with *less* than half her stick's length. So Mavis observes moving sticks getting shorter and Stanley observes them getting longer. But this implies an asymmetry between the two frames that contradicts the basic postulate of relativity that tells us all inertial frames are equivalent. We conclude that consistency with the postulates of relativity requires that both observers measure the rulers as having the *same* length, even though to each observer one of them is stationary and the other is moving (Fig. 37.12). So *there is no length contraction perpendicular to the direction of relative motion of the coordinate systems.* We used this result in our derivation of Eq. (37.6) in assuming that the distance d is the same in both frames of reference.



37.12 The meter sticks are perpendicular to the relative velocity. For any value of u, both Stanley and Mavis measure either meter stick to have a length of 1 meter.

For example, suppose a moving rod of length l_0 makes an angle θ_0 with the direction of relative motion (the x-axis) as measured in its rest frame. Its length component in that frame parallel to the motion, $l_0 \cos\theta_0$, is contracted to $(l_0 \cos\theta_0)/\gamma$. However, its length component perpendicular to the motion, $l_0 \sin\theta_0$, remains the same.

Problem-Solving Strategy 37.2 Length Contraction

IDENTIFY *the relevant concepts:* The concept of length contraction is used whenever we compare the length of an object as measured by observers in different inertial frames of reference.

SET UP *the problem* using the following steps:

1. Decide what defines the length in question. If the problem describes an object such as a ruler, it is just the distance between the ends of the object. If the problem is about a distance between two points in space, it helps to envision an object like a ruler that extends from one point to the other.
2. Identify the target variable.

EXECUTE *the solution* as follows:

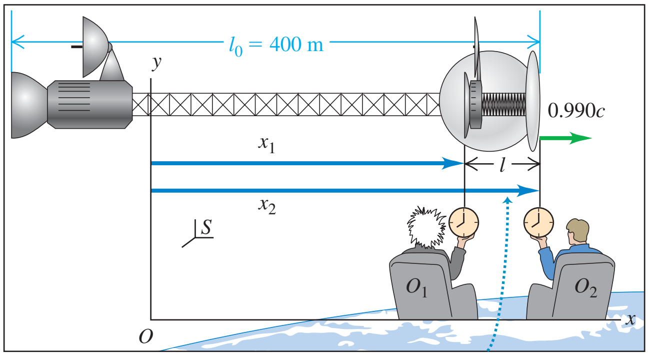
1. Determine the reference frame in which the object in question is at rest. In this frame, the length of the object is its proper length l_0. In a second reference frame moving at speed u relative to the first frame, the object has contracted length l.
2. Keep in mind that length contraction occurs only for lengths parallel to the direction of relative motion of the two frames. Any length that is perpendicular to the relative motion is the same in both frames.
3. Use Eq. (37.16) to relate l and l_0, and then solve for the target variable.

EVALUATE *your answer:* Check that your answers make sense: l is never larger than l_0, and u is never greater than c.

Example 37.4 How long is the spaceship?

A spaceship flies past earth at a speed of $0.990c$. A crew member on board the spaceship measures its length, obtaining the value 400 m. What length do observers measure on earth?

37.13 Measuring the length of a moving spaceship.



The two observers on earth (S) must measure x_2 and x_1 simultaneously to obtain the correct length $l = x_2 - x_1$ in their frame of reference.

SOLUTION

IDENTIFY and SET UP: This problem is about the nose-to-tail length of the spaceship as measured on the spaceship and on earth. This length is along the direction of relative motion (Fig. 37.13), so there will be length contraction. The spaceship's 400-m length is the *proper* length l_0 because it is measured in the frame in which the spaceship is at rest. Our target variable is the length l measured in the earth frame, relative to which the spaceship is moving at $u = 0.990c$.

EXECUTE: From Eq. (37.16), the length in the earth frame is

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = (400\text{ m})\sqrt{1 - (0.990)^2} = 56.4\text{ m}$$

EVALUATE: The spaceship is shorter in a frame in which it is in motion than in a frame in which it is at rest. To measure the length l, two earth observers with synchronized clocks could measure the positions of the two ends of the spaceship simultaneously in the earth's reference frame, as shown in Fig. 37.13. (These two measurements will *not* appear simultaneous to an observer in the spaceship.)

Example 37.5 How far apart are the observers?

Observers O_1 and O_2 in Fig. 37.13 are 56.4 m apart on the earth. How far apart does the spaceship crew measure them to be?

SOLUTION

IDENTIFY and SET UP: In this example the 56.4-m distance is the *proper* length l_0. It represents the length of a ruler that extends from O_1 to O_2 and is at rest in the earth frame in which the observers are at rest. Our target variable is the length l of this ruler measured in the spaceship frame, in which the earth and ruler are moving at $u = 0.990c$.

EXECUTE: As in Example 37.4, but with $l_0 = 56.4$ m,

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = (56.4\ \text{m})\sqrt{1 - (0.990)^2} = 7.96\ \text{m}$$

EVALUATE: This answer does *not* say that the crew measures their spaceship to be both 400 m long and 7.96 m long. As measured on earth, the tail of the spacecraft is at the position of O_1 at the same instant that the nose of the spacecraft is at the position of O_2. Hence the length of the spaceship measured on earth equals the 56.4-m distance between O_1 and O_2. But in the spaceship frame O_1 and O_2 are only 7.96 m apart, and the nose (which is 400 m in front of the tail) passes O_2 before the tail passes O_1.

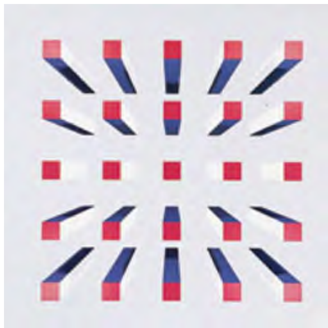
How an Object Moving Near c Would Appear

Let's think a little about the visual appearance of a moving three-dimensional body. If we could see the positions of all points of the body simultaneously, it would appear to shrink only in the direction of motion. But we *don't* see all the points simultaneously; light from points farther from us takes longer to reach us than does light from points near to us, so we see the farther points at the positions they had at earlier times.

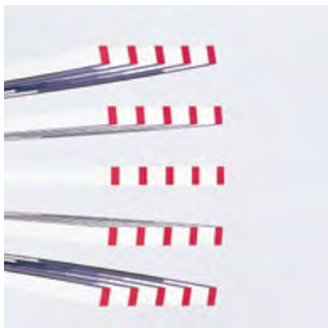
Suppose we have a rectangular rod with its faces parallel to the coordinate planes. When we look end-on at the center of the closest face of such a rod at rest, we see only that face. (See the center rod in computer-generated Fig. 37.14a.) But when that rod is moving past us toward the right at an appreciable fraction of the speed of light, we may also see its left side because of the earlier-time effect just described. That is, we can see some points that we couldn't see when the rod was at rest because the rod moves out of the way of the light rays from those points to us. Conversely, some light that can get to us when the rod is at rest is blocked by the moving rod. Because of all this, the rods in Figs. 37.14b and 37.14c appear rotated and distorted.

37.14 Computer simulation of the appearance of an array of 25 rods with square cross section. The center rod is viewed end-on. The simulation ignores color changes in the array caused by the Doppler effect (see Section 37.6).

(a) Array at rest



(b) Array moving to the right at $0.2c$



(c) Array moving to the right at $0.9c$

Test Your Understanding of Section 37.4 A miniature spaceship is flying past you, moving horizontally at a substantial fraction of the speed of light. At a certain instant, you observe that the nose and tail of the spaceship align exactly with the two ends of a meter stick that you hold in your hands. Rank the following distances in order from longest to shortest: (i) the proper length of the meter stick; (ii) the proper length of the spaceship; (iii) the length of the spaceship measured in your frame of reference; (iv) the length of the meter stick measured in the spaceship's frame of reference.

37.5 The Lorentz Transformations

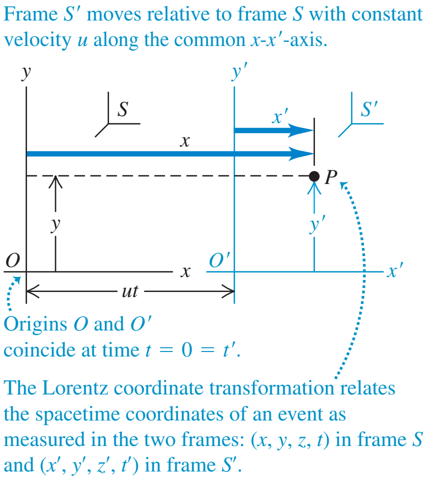
In Section 37.1 we discussed the Galilean coordinate transformation equations, Eqs. (37.1). They relate the coordinates (x, y, z) of a point in frame of reference S to the coordinates (x', y', z') of the point in a second frame S'. The second frame moves with constant speed u relative to S in the positive direction along the common x-x'-axis. This transformation also assumes that the time scale is the same in the two frames of reference, as expressed by the additional relationship $t = t'$. This Galilean transformation, as we have seen, is valid only in the limit when u approaches zero. We are now ready to derive more general transformations that are consistent with the principle of relativity. The more general relationships are called the **Lorentz transformations.**

The Lorentz Coordinate Transformation

Our first question is this: When an event occurs at point (x, y, z) at time t, as observed in a frame of reference S, what are the coordinates (x', y', z') and time t' of the event as observed in a second frame S' moving relative to S with constant speed u in the $+x$-direction?

To derive the coordinate transformation, we refer to Fig. 37.15 (next page), which is the same as Fig. 37.3. As before, we assume that the origins coincide at the initial time $t = 0 = t'$. Then in S the distance from O to O' at time t is

37.15 As measured in frame of reference S, x' is contracted to x'/γ, so $x = ut + x'/\gamma$ and $x' = \gamma(x - ut)$.



still ut. The coordinate x' is a *proper length* in S', so in S it is contracted by the factor $1/\gamma = \sqrt{1 - u^2/c^2}$, as in Eq. (37.16). Thus the distance x from O to P, as seen in S, is not simply $x = ut + x'$, as in the Galilean coordinate transformation, but

$$x = ut + x'\sqrt{1 - \frac{u^2}{c^2}} \tag{37.17}$$

Solving this equation for x', we obtain

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} \tag{37.18}$$

Equation (37.18) is part of the Lorentz coordinate transformation; another part is the equation giving t' in terms of x and t. To obtain this, we note that the principle of relativity requires that the *form* of the transformation from S to S' be identical to that from S' to S. The only difference is a change in the sign of the relative velocity component u. Thus from Eq. (37.17) it must be true that

$$x' = -ut' + x\sqrt{1 - \frac{u^2}{c^2}} \tag{37.19}$$

We now equate Eqs. (37.18) and (37.19) to eliminate x'. This gives us an equation for t' in terms of x and t. We leave the algebraic details for you to work out; the result is

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} \tag{37.20}$$

As we discussed previously, lengths perpendicular to the direction of relative motion are not affected by the motion, so $y' = y$ and $z' = z$.

Collecting all these transformation equations, we have

$$\begin{aligned} x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut) \\ y' &= y \\ z' &= z \\ t' &= \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2) \end{aligned} \qquad \text{(Lorentz coordinate transformation)} \tag{37.21}$$

These equations are the *Lorentz coordinate transformation,* the relativistic generalization of the Galilean coordinate transformation, Eqs. (37.1) and $t = t'$. For values of u that approach zero, the radicals in the denominators and γ approach 1, and the ux/c^2 term approaches zero. In this limit, Eqs. (37.21) become identical to Eqs. (37.1) along with $t = t'$. In general, though, both the coordinates and time of an event in one frame depend on its coordinates and time in another frame. *Space and time have become intertwined; we can no longer say that length and time have absolute meanings independent of the frame of reference.* For this reason, we refer to time and the three dimensions of space collectively as a four-dimensional entity called **spacetime,** and we call (x, y, z, t) together the **spacetime coordinates** of an event.

The Lorentz Velocity Transformation

We can use Eqs. (37.21) to derive the relativistic generalization of the Galilean velocity transformation, Eq. (37.2). We consider only one-dimensional motion along the x-axis and use the term "velocity" as being short for the "x-component of the velocity." Suppose that in a time dt a particle moves a distance dx, as measured in frame S. We obtain the corresponding distance dx' and time dt' in S' by taking differentials of Eqs. (37.21):

$$dx' = \gamma(dx - u\,dt)$$
$$dt' = \gamma(dt - u\,dx/c^2)$$

We divide the first equation by the second and then divide the numerator and denominator of the result by dt to obtain

$$\frac{dx'}{dt'} = \frac{\dfrac{dx}{dt} - u}{1 - \dfrac{u}{c^2}\dfrac{dx}{dt}}$$

Now dx/dt is the velocity v_x in S, and dx'/dt' is the velocity v'_x in S', so we finally obtain the relativistic generalization

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} \qquad \text{(Lorentz velocity transformation)} \qquad (37.22)$$

When u and v_x are much smaller than c, the denominator in Eq. (37.22) approaches 1, and we approach the nonrelativistic result $v'_x = v_x - u$. The opposite extreme is the case $v_x = c$; then we find

$$v'_x = \frac{c - u}{1 - uc/c^2} = \frac{c(1 - u/c)}{1 - u/c} = c$$

This says that anything moving with velocity $v_x = c$ measured in S also has velocity $v'_x = c$ measured in S', despite the relative motion of the two frames. So Eq. (37.22) is consistent with Einstein's postulate that the speed of light in vacuum is the same in all inertial frames of reference.

The principle of relativity tells us there is no fundamental distinction between the two frames S and S'. Thus the expression for v_x in terms of v'_x must have the same form as Eq. (37.22), with v_x changed to v'_x, and vice versa, and the sign of u reversed. Carrying out these operations with Eq. (37.22), we find

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} \qquad \text{(Lorentz velocity transformation)} \qquad (37.23)$$

This can also be obtained algebraically by solving Eq. (37.22) for v_x. Both Eqs. (37.22) and (37.23) are *Lorentz velocity transformations* for one-dimensional motion.

CAUTION **Use the correct reference frame coordinates** Keep in mind that the Lorentz transformation equations given by Eqs. (37.21), (37.22), and (37.23) assume that frame S' is moving in the positive x-direction with velocity u relative to frame S. You should always set up your coordinate system to follow this convention.

When u is less than c, the Lorentz velocity transformations show us that a body moving with a speed less than c in one frame of reference always has a speed less than c in *every other* frame of reference. This is one reason for concluding that no material body may travel with a speed equal to or greater than that of light in vacuum, relative to *any* inertial frame of reference. The relativistic generalizations of energy and momentum, which we will explore later, give further support to this hypothesis.

Problem-Solving Strategy 37.3 Lorentz Transformations

IDENTIFY *the relevant concepts:* The Lorentz *coordinate* transformation equations relate the spacetime coordinates of an event in one inertial reference frame to the coordinates of the same event in a second inertial frame. The Lorentz *velocity* transformation equations relate the velocity of an object in one inertial reference frame to its velocity in a second inertial frame.

SET UP *the problem* using the following steps:
1. Identify the target variable.
2. Define the two inertial frames S and S'. Remember that S' moves relative to S at a constant velocity u in the $+x$-direction.
3. If the coordinate transformation equations are needed, make a list of spacetime coordinates in the two frames, such as x_1, x_1', t_1, t_1', and so on. Label carefully which of these you know and which you don't.
4. In velocity-transformation problems, clearly identify u (the relative velocity of the two frames of reference), v_x (the velocity of the object relative to S), and v_x' (the velocity of the object relative to S').

EXECUTE *the solution* as follows:
1. In a coordinate-transformation problem, use Eqs. (37.21) to solve for the spacetime coordinates of the event as measured in S' in terms of the corresponding values in S. (If you need to solve for the spacetime coordinates in S in terms of the corresponding values in S', you can easily convert the expressions in Eqs. (37.21): Replace all of the primed quantities with unprimed ones, and vice versa, and replace u with $-u$.)
2. In a velocity-transformation problem, use either Eq. (37.22) or Eq. (37.23), as appropriate, to solve for the target variable.

EVALUATE *your answer:* Don't be discouraged if some of your results don't seem to make sense or if they disagree with "common sense." It takes time to develop intuition about relativity; you'll gain it with experience.

Example 37.6 Was it received before it was sent?

Winning an interstellar race, Mavis pilots her spaceship across a finish line in space at a speed of $0.600c$ relative to that line. A "hooray" message is sent from the back of her ship (event 2) at the instant (in her frame of reference) that the front of her ship crosses the line (event 1). She measures the length of her ship to be 300 m. Stanley is at the finish line and is at rest relative to it. When and where does he measure events 1 and 2 to occur?

SOLUTION

IDENTIFY and SET UP: This example involves the Lorentz coordinate transformation. Our derivation of this transformation assumes that the origins of frames S and S' coincide at $t = 0 = t'$. Thus for simplicity we fix the origin of S at the finish line and the origin of S' at the front of the spaceship so that Stanley and Mavis measure event 1 to be at $x = 0 = x'$ and $t = 0 = t'$.

Mavis in S' measures her spaceship to be 300 m long, so she has the "hooray" sent from 300 m behind her spaceship's front at the instant she measures the front to cross the finish line. That is, she measures event 2 at $x' = -300$ m and $t' = 0$.

Our target variables are the coordinate x and time t of event 2 that Stanley measures in S.

EXECUTE: To solve for the target variables, we modify the first and last of Eqs. (37.21) to give x and t as functions of x' and t'. We do so in the same way that we obtained Eq. (37.23) from Eq. (37.22). We remove the primes from x' and t', add primes to x and t, and replace each u with $-u$. The results are

$$x = \gamma(x' + ut') \quad \text{and} \quad t = \gamma(t' + ux'/c^2)$$

From Eq. (37.7), $\gamma = 1.25$ for $u = 0.600c = 1.80 \times 10^8$ m/s. We also substitute $x' = -300$ m, $t' = 0$, $c = 3.00 \times 10^8$ m/s, and $u = 1.80 \times 10^8$ m/s in the equations for x and t to find $x = -375$ m at $t = -7.50 \times 10^{-7}$ s $= -0.750\ \mu$s for event 2.

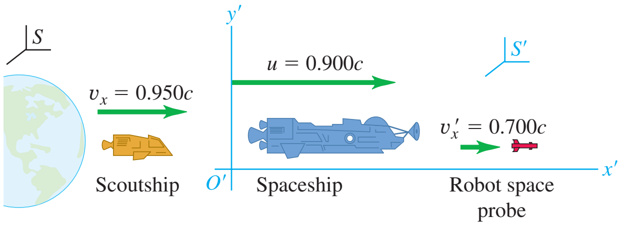
EVALUATE: Mavis says that the events are simultaneous, but Stanley says that the "hooray" was sent *before* Mavis crossed the finish line. This does not mean that the effect preceded the cause. The fastest that Mavis can send a signal the length of her ship is $300\text{ m}/(3.00 \times 10^8\text{ m/s}) = 1.00\ \mu$s. She cannot send a signal from the front at the instant it crosses the finish line that would cause a "hooray" to be broadcast from the back at the same instant. She would have to send that signal from the front at least 1.00 μs before then, so she had to slightly anticipate her success.

Example 37.7 Relative velocities

(a) A spaceship moving away from the earth at $0.900c$ fires a robot space probe in the same direction as its motion at $0.700c$ relative to the spaceship. What is the probe's velocity relative to the earth? (b) A scoutship is sent to catch up with the spaceship by traveling at $0.950c$ relative to the earth. What is the velocity of the scoutship relative to the spaceship?

SOLUTION

IDENTIFY and SET UP: This example uses the Lorentz velocity transformation. Let the earth and spaceship reference frames be S and S', respectively (Fig. 37.16); their relative velocity is $u = 0.900c$. In part (a) we are given the probe velocity $v_x' = 0.700c$ with respect to S', and the target variable is the velocity v_x of the



37.16 The spaceship, robot space probe, and scoutship.

probe relative to S. In part (b) we are given the velocity $v_x = 0.950c$ of the scoutship relative to S, and the target variable is its velocity v'_x relative to S'.

EXECUTE: (a) We use Eq. (37.23) to find the probe velocity relative to the earth:

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{0.700c + 0.900c}{1 + (0.900c)(0.700c)/c^2} = 0.982c$$

(b) We use Eq. (37.22) to find the scoutship velocity relative to the spaceship:

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} = \frac{0.950c - 0.900c}{1 - (0.900c)(0.950c)/c^2} = 0.345c$$

EVALUATE: What would the Galilean velocity transformation formula, Eq. (37.2), say? In part (a) we would have found the probe's velocity relative to the earth to be $v_x = v'_x + u = 0.700c + 0.900c = 1.600c$, which is greater than c and hence impossible. In part (b), we would have found the scoutship's velocity relative to the spaceship to be $v'_x = v_x - u = 0.950c - 0.900c = 0.050c$; the relativistically correct value, $v'_x = 0.345c$, is almost seven times greater than the incorrect Galilean value.

Test Your Understanding of Section 37.5 (a) In frame S events P_1 and P_2 occur at the same x-, y-, and z-coordinates, but event P_1 occurs before event P_2. In frame S', which event occurs first? (b) In frame S events P_3 and P_4 occur at the same time t and the same y- and z-coordinates, but event P_3 occurs at a less positive x-coordinate than event P_4. In frame S', which event occurs first?

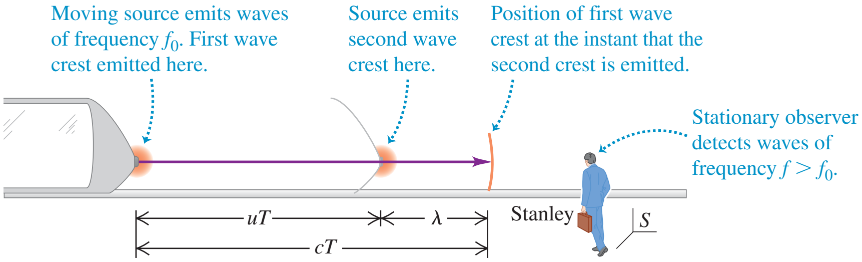
37.6 The Doppler Effect for Electromagnetic Waves

An additional important consequence of relativistic kinematics is the Doppler effect for electromagnetic waves. In our previous discussion of the Doppler effect (see Section 16.8) we quoted without proof the formula, Eq. (16.30), for the frequency shift that results from motion of a source of electromagnetic waves relative to an observer. We can now derive that result.

Here's a statement of the problem. A source of light is moving with constant speed u toward Stanley, who is stationary in an inertial frame (Fig. 37.17). As measured in its rest frame, the source emits light waves with frequency f_0 and period $T_0 = 1/f_0$. What is the frequency f of these waves as received by Stanley?

Let T be the time interval between *emission* of successive wave crests as observed in Stanley's reference frame. Note that this is *not* the interval between the *arrival* of successive crests at his position, because the crests are emitted at different points in Stanley's frame. In measuring only the frequency f he receives, he does not take into account the difference in transit times for successive crests. Therefore the frequency he receives is *not* $1/T$. What is the equation for f?

During a time T the crests ahead of the source move a distance cT, and the source moves a shorter distance uT in the same direction. The distance λ between



37.17 The Doppler effect for light. A light source moving at speed u relative to Stanley emits a wave crest, then travels a distance uT toward an observer and emits the next crest. In Stanley's reference frame S, the second crest is a distance λ behind the first crest.

successive crests—that is, the wavelength—is thus $\lambda = (c - u)T$, as measured in Stanley's frame. The frequency that he measures is c/λ. Therefore

$$f = \frac{c}{(c - u)T} \tag{37.24}$$

So far we have followed a pattern similar to that for the Doppler effect for sound from a moving source (see Section 16.8). In that discussion our next step was to equate T to the time T_0 between emissions of successive wave crests by the source. However, due to time dilation it is *not* relativistically correct to equate T to T_0. The time T_0 is measured in the rest frame of the source, so it is a proper time. From Eq. (37.6), T_0 and T are related by

$$T = \frac{T_0}{\sqrt{1 - u^2/c^2}} = \frac{cT_0}{\sqrt{c^2 - u^2}}$$

or, since $T_0 = 1/f_0$,

$$\frac{1}{T} = \frac{\sqrt{c^2 - u^2}}{cT_0} = \frac{\sqrt{c^2 - u^2}}{c} f_0$$

Remember, $1/T$ is not equal to f. We must substitute this expression for $1/T$ into Eq. 37.24 to find f:

$$f = \frac{c}{c - u} \frac{\sqrt{c^2 - u^2}}{c} f_0$$

Using $c^2 - u^2 = (c - u)(c + u)$ gives

$$f = \sqrt{\frac{c + u}{c - u}} f_0 \quad \text{(Doppler effect, electromagnetic waves, source approaching observer)} \tag{37.25}$$

This shows that when the source moves *toward* the observer, the observed frequency f is *greater* than the emitted frequency f_0. The difference $f - f_0 = \Delta f$ is called the Doppler frequency shift. When u/c is much smaller than 1, the fractional shift $\Delta f/f$ is also small and is approximately equal to u/c:

$$\frac{\Delta f}{f} = \frac{u}{c}$$

When the source moves *away from* the observer, we change the sign of u in Eq. (37.25) to get

$$f = \sqrt{\frac{c - u}{c + u}} f_0 \quad \text{(Doppler effect, electromagnetic waves, source moving away from observer)} \tag{37.26}$$

This agrees with Eq. (16.30), which we quoted previously, with minor notation changes.

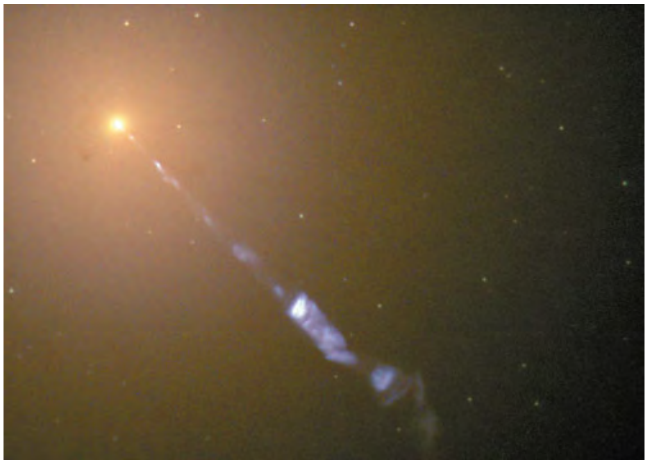
With light, unlike sound, there is no distinction between motion of source and motion of observer; only the *relative* velocity of the two is significant. The last four paragraphs of Section 16.8 discuss several practical applications of the Doppler effect with light and other electromagnetic radiation; we suggest you review those paragraphs now. Figure 37.18 shows one common application.

37.18 This handheld radar gun emits a radio beam of frequency f_0, which in the frame of reference of an approaching car has a higher frequency f given by Eq. (37.25). The reflected beam also has frequency f in the car's frame, but has an even higher frequency f' in the police officer's frame. The radar gun calculates the car's speed by comparing the frequencies of the emitted beam and the doubly Doppler-shifted reflected beam. (Compare Example 16.18 in Section 16.8.)

Example 37.8 A jet from a black hole

Many galaxies have supermassive black holes at their centers (see Section 13.8). As material swirls around such a black hole, it is heated, becomes ionized, and generates strong magnetic fields. The resulting magnetic forces steer some of the material into high-speed jets that blast out of the galaxy and into intergalactic space (Fig. 37.19). The light we observe from the jet in Fig. 37.19 has a

37.19 This image shows a fast-moving jet 5000 light-years in length emanating from the center of the galaxy M87. The light from the jet is emitted by fast-moving electrons spiraling around magnetic field lines (see Fig. 27.18).



frequency of 6.66×10^{14} Hz (in the far ultraviolet region of the electromagnetic spectrum; see Fig. 32.4), but in the reference frame of the jet material the light has a frequency of 5.55×10^{13} Hz (in the infrared). What is the speed of the jet material with respect to us?

SOLUTION

IDENTIFY and SET UP: This problem involves the Doppler effect for electromagnetic waves. The frequency we observe is $f = 6.66 \times 10^{14}$ Hz, and the frequency in the frame of the source is $f_0 = 5.55 \times 10^{13}$ Hz. Since $f > f_0$, the jet is approaching us and we use Eq. (37.25) to find the target variable u.

EXECUTE: We need to solve Eq. (37.25) for u. We'll leave it as an exercise for you to show that the result is

$$u = \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1}c$$

We have $f/f_0 = (6.66 \times 10^{14}\ \text{Hz})/(5.55 \times 10^{13}\ \text{Hz}) = 12.0$, so

$$u = \frac{(12.0)^2 - 1}{(12.0)^2 + 1}c = 0.986c$$

EVALUATE: Because the frequency shift is quite substantial, it would have been erroneous to use the approximate expression $\Delta f/f = u/c$. Had you done so, you would have found $u = c(\Delta f/f_0) = c(6.66 \times 10^{14}\ \text{Hz} - 5.55 \times 10^{13}\ \text{Hz})/(5.55 \times 10^{13}\ \text{Hz}) = 11.0c$. This result cannot be correct because the jet material cannot travel faster than light.

37.7 Relativistic Momentum

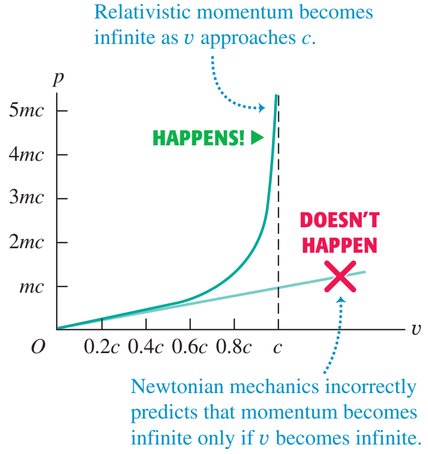
Newton's laws of motion have the same form in all inertial frames of reference. When we use transformations to change from one inertial frame to another, the laws should be *invariant* (unchanging). But we have just learned that the principle of relativity forces us to replace the Galilean transformations with the more general Lorentz transformations. As we will see, this requires corresponding generalizations in the laws of motion and the definitions of momentum and energy.

The principle of conservation of momentum states that *when two bodies interact, the total momentum is constant,* provided that the net external force acting on the bodies in an inertial reference frame is zero (for example, if they form an isolated system, interacting only with each other). If conservation of momentum is a valid physical law, it must be valid in *all* inertial frames of reference. Now, here's the problem: Suppose we look at a collision in one inertial coordinate system S and find that momentum is conserved. Then we use the Lorentz transformation to obtain the velocities in a second inertial system S'. We find that if we use the Newtonian definition of momentum ($\vec{p} = m\vec{v}$), momentum is *not* conserved in the second system! If we are convinced that the principle of relativity and the Lorentz transformation are correct, the only way to save momentum conservation is to generalize the *definition* of momentum.

We won't derive the correct relativistic generalization of momentum, but here is the result. Suppose we measure the mass of a particle to be m when it is at rest relative to us: We often call m the **rest mass.** We will use the term *material particle* for a particle that has a nonzero rest mass. When such a particle has a velocity $\vec{v}$, its **relativistic momentum** $\vec{p}$ is

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} \qquad \text{(relativistic momentum)} \qquad (37.27)$$

37.20 Graph of the magnitude of the momentum of a particle of rest mass m as a function of speed v. Also shown is the Newtonian prediction, which gives correct results only at speeds much less than c.



When the particle's speed v is much less than c, this is approximately equal to the Newtonian expression $\vec{p} = m\vec{v}$, but in general the momentum is greater in magnitude than mv (Fig. 37.20). In fact, as v approaches c, the momentum approaches infinity.

Relativity, Newton's Second Law, and Relativistic Mass

What about the relativistic generalization of Newton's second law? In Newtonian mechanics the most general form of the second law is

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{37.28}$$

That is, the net force $\vec{F}$ on a particle equals the time rate of change of its momentum. Experiments show that this result is still valid in relativistic mechanics, provided that we use the relativistic momentum given by Eq. 37.27. That is, the relativistically correct generalization of Newton's second law is

$$\vec{F} = \frac{d}{dt}\frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} \tag{37.29}$$

Because momentum is no longer directly proportional to velocity, the rate of change of momentum is no longer directly proportional to the acceleration. As a result, *constant force does not cause constant acceleration.* For example, when the net force and the velocity are both along the x-axis, Eq. 37.29 gives

$$F = \frac{m}{(1 - v^2/c^2)^{3/2}}a \qquad (\vec{F} \text{ and } \vec{v} \text{ along the same line}) \tag{37.30}$$

where a is the acceleration, also along the x-axis. Solving Eq. (37.30) for the acceleration a gives

$$a = \frac{F}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2}$$

We see that as a particle's speed increases, the acceleration caused by a given force continuously *decreases.* As the speed approaches c, the acceleration approaches zero, no matter how great a force is applied. Thus it is impossible to accelerate a particle with nonzero rest mass to a speed equal to or greater than c. We again see that the speed of light in vacuum represents an ultimate speed limit.

Equation (37.27) for relativistic momentum is sometimes interpreted to mean that a rapidly moving particle undergoes an increase in mass. If the mass at zero velocity (the rest mass) is denoted by m, then the "relativistic mass" m_{rel} is given by

$$m_{\text{rel}} = \frac{m}{\sqrt{1 - v^2/c^2}}$$

Indeed, when we consider the motion of a system of particles (such as rapidly moving ideal-gas molecules in a stationary container), the total rest mass of the system is the sum of the relativistic masses of the particles, not the sum of their rest masses.

However, if blindly applied, the concept of relativistic mass has its pitfalls. As Eq. (37.29) shows, the relativistic generalization of Newton's second law is *not* $\vec{F} = m_{\text{rel}}\vec{a}$, and we will show in Section 37.8 that the relativistic kinetic energy of a particle is *not* $K = \frac{1}{2}m_{\text{rel}}v^2$. The use of relativistic mass has its supporters and detractors, some quite strong in their opinions. We will mostly deal with individual particles, so we will sidestep the controversy and use Eq. (37.27) as the generalized definition of momentum with m as a constant for each particle, independent of its state of motion.

We will use the abbreviation

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

We used this abbreviation in Section 37.3 with v replaced by u, the relative speed of two coordinate systems. Here v is the speed of a particle in a particular coordinate system—that is, the speed of the particle's *rest frame* with respect to that system. In terms of γ, Eqs. (37.27) and (37.30) become

$$\vec{p} = \gamma m \vec{v} \qquad \text{(relativistic momentum)} \qquad (37.31)$$

$$F = \gamma^3 ma \qquad (\vec{F} \text{ and } \vec{v} \text{ along the same line}) \qquad (37.32)$$

In linear accelerators (used in medicine as well as nuclear and elementary-particle physics; see Fig. 37.11) the net force $\vec{F}$ and the velocity $\vec{v}$ of the accelerated particle are along the same straight line. But for much of the path in most *circular* accelerators the particle moves in uniform circular motion at constant speed v. Then the net force and velocity are perpendicular, so the force can do no work on the particle and the kinetic energy and speed remain constant. Thus the denominator in Eq. (37.29) is constant, and we obtain

$$F = \frac{m}{(1 - v^2/c^2)^{1/2}} a = \gamma ma \qquad (\vec{F} \text{ and } \vec{v} \text{ perpendicular}) \qquad (37.33)$$

Recall from Section 3.4 that if the particle moves in a circle, the net force and acceleration are directed inward along the radius r, and $a = v^2/r$.

What about the general case in which $\vec{F}$ and $\vec{v}$ are neither along the same line nor perpendicular? Then we can resolve the net force $\vec{F}$ at any instant into components parallel to and perpendicular to $\vec{v}$. The resulting acceleration will have corresponding components obtained from Eqs. (37.32) and (37.33). Because of the different γ^3 and γ factors, the acceleration components will not be proportional to the net force components. That is, *unless the net force on a relativistic particle is either along the same line as the particle's velocity or perpendicular to it, the net force and acceleration vectors are not parallel.*

Example 37.9 Relativistic dynamics of an electron

An electron (rest mass 9.11×10^{-31} kg, charge -1.60×10^{-19} C) is moving opposite to an electric field of magnitude $E = 5.00 \times 10^5$ N/C. All other forces are negligible in comparison to the electric-field force. (a) Find the magnitudes of momentum and of acceleration at the instants when $v = 0.010c$, $0.90c$, and $0.99c$. (b) Find the corresponding accelerations if a net force of the same magnitude is perpendicular to the velocity.

SOLUTION

IDENTIFY and SET UP: In addition to the expressions from this section for relativistic momentum and acceleration, we need the relationship between electric force and electric field from Chapter 21. In part (a) we use Eq. (37.31) to determine the magnitude of momentum; the force acts along the same line as the velocity, so we use Eq. (37.32) to determine the magnitude of acceleration. In part (b) the force is perpendicular to the velocity, so we use Eq. (37.33) rather than Eq. (37.32).

EXECUTE: (a) For $v = 0.010c$, $0.90c$, and $0.99c$ we have $\gamma = \sqrt{1 - v^2/c^2} = 1.00, 2.29$, and 7.09, respectively. The values of the momentum magnitude $p = \gamma mv$ are

$$p_1 = (1.00)(9.11 \times 10^{-31}\text{ kg})(0.010)(3.00 \times 10^8 \text{m/s})$$
$$= 2.7 \times 10^{-24}\text{ kg} \cdot \text{m/s at } v_1 = 0.010c$$

$$p_2 = (2.29)(9.11 \times 10^{-31}\text{ kg})(0.90)(3.00 \times 10^8\text{ m/s})$$
$$= 5.6 \times 10^{-22}\text{ kg} \cdot \text{m/s at } v_2 = 0.90c$$

$$p_3 = (7.09)(9.11 \times 10^{-31}\text{ kg})(0.99)(3.00 \times 10^8\text{ m/s})$$
$$= 1.9 \times 10^{-21}\text{ kg} \cdot \text{m/s at } v_3 = 0.99c$$

From Eq. (21.4), the magnitude of the force on the electron is

$$F = |q|E = (1.60 \times 10^{-19}\text{ C})(5.00 \times 10^5\text{ N/C})$$
$$= 8.00 \times 10^{-14}\text{ N}$$

Continued

From Eq. (37.32), $a = F/\gamma^3 m$. For $v = 0.010c$ and $\gamma = 1.00$,

$$a_1 = \frac{8.00 \times 10^{-14}\ \text{N}}{(1.00)^3(9.11 \times 10^{-31}\ \text{kg})} = 8.8 \times 10^{16}\ \text{m/s}^2$$

The accelerations at the two higher speeds are smaller than the nonrelativistic value by factors of $\gamma^3 = 12.0$ and 356, respectively:

$$a_2 = 7.3 \times 10^{15}\ \text{m/s}^2 \qquad a_3 = 2.5 \times 10^{14}\ \text{m/s}^2$$

(b) From Eq. (37.33), $a = F/\gamma m$ if $\vec{F}$ and $\vec{v}$ are perpendicular. When $v = 0.010c$ and $\gamma = 1.00$,

$$a_1 = \frac{8.00 \times 10^{-14}\ \text{N}}{(1.00)(9.11 \times 10^{-31}\ \text{kg})} = 8.8 \times 10^{16}\ \text{m/s}^2$$

Now the accelerations at the two higher speeds are smaller by factors of $\gamma = 2.29$ and 7.09, respectively:

$$a_2 = 3.8 \times 10^{16}\ \text{m/s}^2 \qquad a_3 = 1.2 \times 10^{16}\ \text{m/s}^2$$

These accelerations are larger than the corresponding ones in part (a) by factors of γ^2.

EVALUATE: Our results in part (a) show that at higher speeds, the relativistic values of momentum differ more and more from the nonrelativistic values calculated from $p = mv$. The momentum at $0.99c$ is more than three times as great as at $0.90c$ because of the increase in the factor γ. Our results also show that the acceleration drops off very quickly as v approaches c.

Test Your Understanding of Section 37.7 According to relativistic mechanics, when you double the speed of a particle, the magnitude of its momentum increases by (i) a factor of 2; (ii) a factor greater than 2; (iii) a factor between 1 and 2 that depends on the mass of the particle.

37.8 Relativistic Work and Energy

When we developed the relationship between work and kinetic energy in Chapter 6, we used Newton's laws of motion. When we generalize these laws according to the principle of relativity, we need a corresponding generalization of the equation for kinetic energy.

Relativistic Kinetic Energy

We use the work–energy theorem, beginning with the definition of work. When the net force and displacement are in the same direction, the work done by that force is $W = \int F\,dx$. We substitute the expression for F from Eq. (37.30), the applicable relativistic version of Newton's second law. In moving a particle of rest mass m from point x_1 to point x_2,

$$W = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} \frac{ma\,dx}{(1 - v^2/c^2)^{3/2}} \tag{37.34}$$

To derive the generalized expression for kinetic energy K as a function of speed v, we would like to convert this to an integral on v. To do this, first remember that the kinetic energy of a particle equals the net work done on it in moving it from rest to the speed v: $K = W$. Thus we let the speeds be zero at point x_1 and v at point x_2. So as not to confuse the variable of integration with the final speed, we change v to v_x in Eq. 37.34. That is, v_x is the varying x-component of the velocity of the particle as the net force accelerates it from rest to a speed v. We also realize that dx and dv_x are the infinitesimal changes in x and v_x, respectively, in the time interval dt. Because $v_x = dx/dt$ and $a = dv_x/dt$, we can rewrite $a\,dx$ in Eq. (37.34) as

$$a\,dx = \frac{dv_x}{dt}dx = dx\frac{dv_x}{dt} = \frac{dx}{dt}dv_x = v_x dv_x$$

Making these substitutions gives us

$$K = W = \int_0^v \frac{mv_x dv_x}{(1 - v_x{}^2/c^2)^{3/2}} \tag{37.35}$$

We can evaluate this integral by a simple change of variable; the final result is

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2 \qquad \text{(relativistic kinetic energy)} \tag{37.36}$$

As v approaches c, the kinetic energy approaches infinity. If Eq. (37.36) is correct, it must also approach the Newtonian expression $K = \frac{1}{2}mv^2$ when v is much smaller than c (Fig. 37.21). To verify this, we expand the radical, using the binomial theorem in the form

$$(1 + x)^n = 1 + nx + n(n - 1)x^2/2 + \cdots$$

In our case, $n = -\frac{1}{2}$ and $x = -v^2/c^2$, and we get

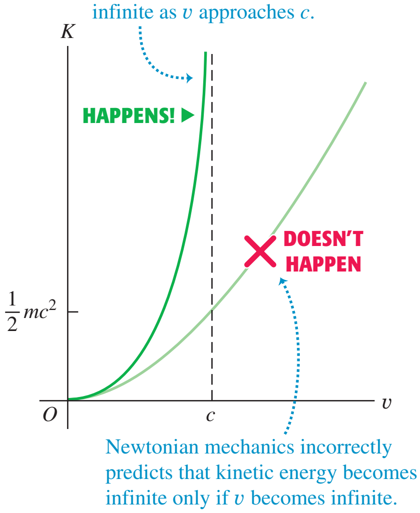
$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots$$

Combining this with $K = (\gamma - 1)mc^2$, we find

$$K = \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots - 1\right)mc^2$$

$$= \frac{1}{2}mv^2 + \frac{3}{8}\frac{mv^4}{c^2} + \cdots \tag{37.37}$$

When v is much smaller than c, all the terms in the series in Eq. (37.37) except the first are negligibly small, and we obtain the Newtonian expression $\frac{1}{2}mv^2$.



37.21 Graph of the kinetic energy of a particle of rest mass m as a function of speed v. Also shown is the Newtonian prediction, which gives correct results only at speeds much less than c.

Rest Energy and $E = mc^2$

Equation (37.36) for the kinetic energy of a moving particle includes a term $mc^2/\sqrt{1 - v^2/c^2}$ that depends on the motion and a second energy term mc^2 that is independent of the motion. It seems that the kinetic energy of a particle is the difference between some **total energy** E and an energy mc^2 that it has even when it is at rest. Thus we can rewrite Eq. (37.36) as

$$E = K + mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma mc^2 \quad \text{(total energy of a particle)} \tag{37.38}$$

For a particle at rest $(K = 0)$, we see that $E = mc^2$. The energy mc^2 associated with rest mass m rather than motion is called the **rest energy** of the particle.

There is in fact direct experimental evidence that rest energy really does exist. The simplest example is the decay of a neutral *pion*. This is an unstable subatomic particle of rest mass m_π; when it decays, it disappears and electromagnetic radiation appears. If a neutral pion has no kinetic energy before its decay, the total energy of the radiation after its decay is found to equal exactly $m_\pi c^2$. In many other fundamental particle transformations the sum of the rest masses of the particles changes. In every case there is a corresponding energy change, consistent with the assumption of a rest energy mc^2 associated with a rest mass m.

Historically, the principles of conservation of mass and of energy developed quite independently. The theory of relativity shows that they are actually two special cases of a single broader conservation principle, the *principle of conservation of mass and energy*. In some physical phenomena, neither the sum of the rest masses of the particles nor the total energy other than rest energy is separately conserved, but there is a more general conservation principle: In an isolated system, when the sum of the rest masses changes, there is always a change in $1/c^2$ times the total energy other than the rest energy. This change is equal in magnitude but opposite in sign to the change in the sum of the rest masses.

This more general mass-energy conservation law is the fundamental principle involved in the generation of power through nuclear reactions. When a uranium nucleus undergoes fission in a nuclear reactor, the sum of the rest masses of the resulting fragments is *less than* the rest mass of the parent nucleus. An amount of energy is released that equals the mass decrease multiplied by c^2. Most of this energy can be used to produce steam to operate turbines for electric power generators.

Application Monitoring Mass–Energy Conversion
Although the control room of a nuclear power plant is very complex, the physical principle on which such a plant operates is a simple one: Part of the rest energy of atomic nuclei is converted to thermal energy, which in turn is used to produce steam to drive electric generators.

We can also relate the total energy E of a particle (kinetic energy plus rest energy) directly to its momentum by combining Eq. (37.27) for relativistic momentum and Eq. (37.38) for total energy to eliminate the particle's velocity. The simplest procedure is to rewrite these equations in the following forms:

$$\left(\frac{E}{mc^2}\right)^2 = \frac{1}{1 - v^2/c^2} \quad \text{and} \quad \left(\frac{p}{mc}\right)^2 = \frac{v^2/c^2}{1 - v^2/c^2}$$

Subtracting the second of these from the first and rearranging, we find

$$E^2 = (mc^2)^2 + (pc)^2 \quad \text{(total energy, rest energy, and momentum)} \tag{37.39}$$

Again we see that for a particle at rest $(p = 0)$, $E = mc^2$.

Equation (37.39) also suggests that a particle may have energy and momentum even when it has no rest mass. In such a case, $m = 0$ and

$$E = pc \quad \text{(zero rest mass)} \tag{37.40}$$

In fact, zero rest mass particles do exist. Such particles always travel at the speed of light in vacuum. One example is the *photon,* the quantum of electromagnetic radiation (to be discussed in Chapter 38). Photons are emitted and absorbed during changes of state of an atomic or nuclear system when the energy and momentum of the system change.

Example 37.10 Energetic electrons

(a) Find the rest energy of an electron $(m = 9.109 \times 10^{-31}\ \text{kg},\ q = -e = -1.602 \times 10^{-19}\ \text{C})$ in joules and in electron volts. (b) Find the speed of an electron that has been accelerated by an electric field, from rest, through a potential increase of 20.0 kV or of 5.00 MV (typical of a high-voltage x-ray machine).

SOLUTION

IDENTIFY and SET UP: This problem uses the ideas of rest energy, relativistic kinetic energy, and (from Chapter 23) electric potential energy. We use $E = mc^2$ to find the rest energy and Eqs. (37.7) and (37.38) to find the speed that gives the stated total energy.

EXECUTE: (a) The rest energy is

$$\begin{aligned} mc^2 &= (9.109 \times 10^{-31}\ \text{kg})(2.998 \times 10^8\ \text{m/s})^2 \\ &= 8.187 \times 10^{-14}\ \text{J} \end{aligned}$$

From the definition of the electron volt in Section 23.2, 1 eV $= 1.602 \times 10^{-19}$ J. Using this, we find

$$\begin{aligned} mc^2 &= (8.187 \times 10^{-14}\ \text{J})\frac{1\ \text{eV}}{1.602 \times 10^{-19}\ \text{J}} \\ &= 5.11 \times 10^5\ \text{eV} = 0.511\ \text{MeV} \end{aligned}$$

(b) In calculations such as this, it is often convenient to work with the quantity $\gamma = 1/\sqrt{1 - v^2/c^2}$ from Eq. (37.38). Solving this for v, we find

$$v = c\sqrt{1 - (1/\gamma)^2}$$

The total energy E of the accelerated electron is the sum of its rest energy mc^2 and the kinetic energy eV_{ba} that it gains from the work done on it by the electric field in moving from point a to point b:

$$E = \gamma mc^2 = mc^2 + eV_{ba} \quad \text{or}$$

$$\gamma = 1 + \frac{eV_{ba}}{mc^2}$$

An electron accelerated through a potential increase of $V_{ba} = 20.0$ kV gains 20.0 keV of energy, so for this electron

$$\gamma = 1 + \frac{20.0 \times 10^3\ \text{eV}}{0.511 \times 10^6\ \text{eV}} = 1.039$$

and

$$v = c\sqrt{1 - (1/1.039)^2} = 0.272c = 8.15 \times 10^7\ \text{m/s}$$

Repeating the calculation for $V_{ba} = 5.00$ MV, we find $eV_{ba}/mc^2 = 9.78$, $\gamma = 10.78$, and $v = 0.996c$.

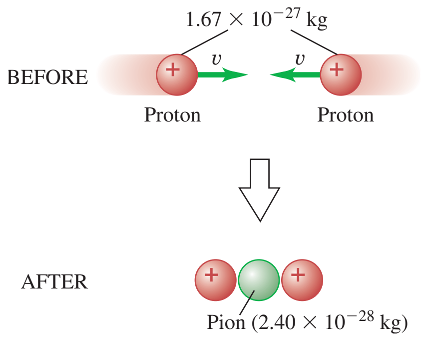
EVALUATE: With $V_{ba} = 20.0$ kV, the added kinetic energy of 20.0 keV is less than 4% of the rest energy of 0.511 MeV, and the final speed is about one-fourth the speed of light. With $V_{ba} = 5.00$ MV, the added kinetic energy of 5.00 MeV is much greater than the rest energy and the speed is close to c.

CAUTION **Three electron energies** All electrons have *rest* energy 0.511 MeV. An electron accelerated from rest through a 5.00-MeV potential increase has *kinetic* energy 5.00 MeV (we call it a "5.00-MeV electron") and *total* energy 5.51 MeV. Be careful to distinguish these energies from one another.

Example 37.11 A relativistic collision

Two protons (each with mass $M_p = 1.67 \times 10^{-27}$ kg) are initially moving with equal speeds in opposite directions. They continue to exist after a head-on collision that also produces a neutral pion of mass $M_\pi = 2.40 \times 10^{-28}$ kg (Fig. 37.22). If all three particles are at rest after the collision, find the initial speed of the protons. Energy is conserved in the collision.

37.22 In this collision the kinetic energy of two protons is transformed into the rest energy of a new particle, a pion.



SOLUTION

IDENTIFY and SET UP: Relativistic total energy is conserved in the collision, so we can equate the (unknown) total energy of the two protons before the collision to the combined rest energies of the two protons and the pion after the collision. We then use Eq. (37.38) to find the speed of each proton.

EXECUTE: The total energy of each proton before the collision is γMc^2. By conservation of energy,

$$2(\gamma M_p c^2) = 2(M_p c^2) + M_\pi c^2$$

$$\gamma = 1 + \frac{M_\pi}{2M_p} = 1 + \frac{2.40 \times 10^{-28}\ \text{kg}}{2(1.67 \times 10^{-27}\ \text{kg})} = 1.072$$

From Eq. (37.38), the initial proton speed is

$$v = c\sqrt{1 - (1/\gamma)^2} = 0.360c$$

EVALUATE: The proton rest energy is 938 MeV, so the initial kinetic energy of each proton is $(\gamma - 1)Mc^2 = 0.072Mc^2 = (0.072)(938\ \text{MeV}) = 67.5$ MeV. You can verify that the rest energy $M_\pi c^2$ of the pion is twice this, or 135 MeV. All the kinetic energy "lost" in this completely inelastic collision is transformed into the rest energy of the pion.

Test Your Understanding of Section 37.8 A proton is accelerated from rest by a constant force that always points in the direction of the particle's motion. Compared to the amount of kinetic energy that the proton gains during the first meter of its travel, how much kinetic energy does the proton gain during one meter of travel while it is moving at 99% of the speed of light? (i) the same amount; (ii) a greater amount; (iii) a smaller amount.

37.9 Newtonian Mechanics and Relativity

The sweeping changes required by the principle of relativity go to the very roots of Newtonian mechanics, including the concepts of length and time, the equations of motion, and the conservation principles. Thus it may appear that we have destroyed the foundations on which Newtonian mechanics is built. In one sense this is true, yet the Newtonian formulation is still accurate whenever speeds are small in comparison with the speed of light in vacuum. In such cases, time dilation, length contraction, and the modifications of the laws of motion are so small that they are unobservable. In fact, every one of the principles of Newtonian mechanics survives as a special case of the more general relativistic formulation.

The laws of Newtonian mechanics are not *wrong;* they are *incomplete.* They are a limiting case of relativistic mechanics. They are *approximately* correct when all speeds are small in comparison to c, and they become exactly correct in the limit when all speeds approach zero. Thus relativity does not completely destroy the laws of Newtonian mechanics but *generalizes* them. This is a common pattern in the development of physical theory. Whenever a new theory is in partial conflict with an older, established theory, the new must yield the same predictions as the old in areas in which the old theory is supported by experimental evidence. Every new physical theory must pass this test, called the **correspondence principle.**

The General Theory of Relativity

At this point we may ask whether the special theory of relativity gives the final word on mechanics or whether *further* generalizations are possible or necessary.

For example, inertial frames have occupied a privileged position in our discussion. Can the principle of relativity be extended to noninertial frames as well?

Here's an example that illustrates some implications of this question. A student decides to go over Niagara Falls while enclosed in a large wooden box. During her free fall she doesn't fall to the floor of the box because both she and the box are in free fall with a downward acceleration of 9.8 $\mathrm{m/s^2}$. But an alternative interpretation, from her point of view, is that she doesn't fall to the floor because her gravitational interaction with the earth has suddenly been turned off. As long as she remains in the box and it remains in free fall, she cannot tell whether she is indeed in free fall or whether the gravitational interaction has vanished.

A similar problem occurs in a space station in orbit around the earth. Objects in the space station *seem* to be weightless, but without looking outside the station there is no way to determine whether gravity has been turned off or whether the station and all its contents are accelerating toward the center of the earth. Figure 37.23 makes a similar point for a spaceship that is not in free fall but may be accelerating relative to an inertial frame or be at rest on the earth's surface.

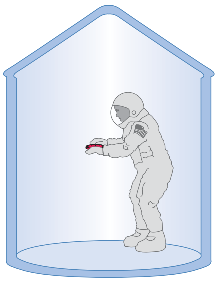
These considerations form the basis of Einstein's **general theory of relativity.** If we cannot distinguish experimentally between a uniform gravitational field at a particular location and a uniformly accelerated reference frame, then there cannot be any real distinction between the two. Pursuing this concept, we may try to represent *any* gravitational field in terms of special characteristics of the coordinate system. This turns out to require even more sweeping revisions of our space-time concepts than did the special theory of relativity. In the general theory of relativity the geometric properties of space are affected by the presence of matter (Fig. 37.24).

The general theory of relativity has passed several experimental tests, including three proposed by Einstein. One test has to do with understanding the rotation of the axes of the planet Mercury's elliptical orbit, called the *precession of the perihelion.* (The perihelion is the point of closest approach to the sun.) A second test concerns the apparent bending of light rays from distant stars when they pass near the sun. The third test is the *gravitational red shift,* the increase in wavelength of light proceeding outward from a massive source. Some details of the general theory are more difficult to test, but this theory has played a central role in investigations of the formation and evolution of stars, black holes, and studies of the evolution of the universe.

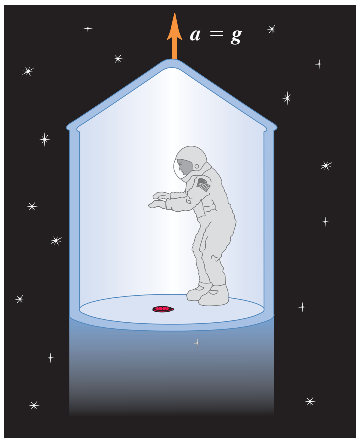
The general theory of relativity may seem to be an exotic bit of knowledge with little practical application. In fact, this theory plays an essential role in the

37.23 Without information from outside the spaceship, the astronaut cannot distinguish situation (b) from situation (c).

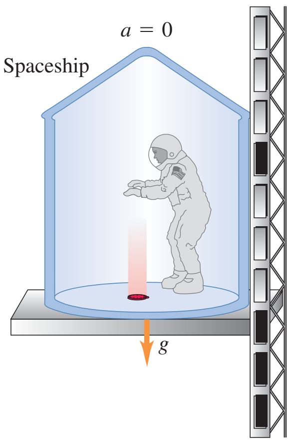
(a) An astronaut is about to drop her watch in a spaceship.



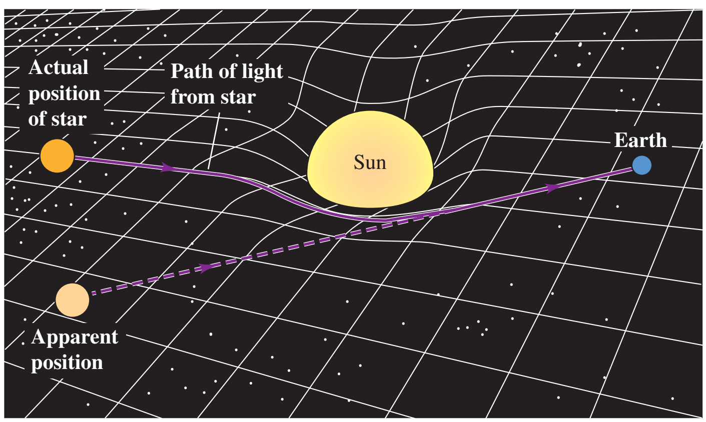
(b) In gravity-free space, the floor accelerates upward at $a = g$ and hits the watch.



(c) On the earth's surface, the watch accelerates downward at $a = g$ and hits the floor.



37.24 A two-dimensional representation of curved space. We imagine the space (a plane) as being distorted as shown by a massive object (the sun). Light from a distant star (solid line) follows the distorted surface on its way to the earth. The dashed line shows the direction from which the light *appears* to be coming. The effect is greatly exaggerated; for the sun, the maximum deviation is only 0.00048°.

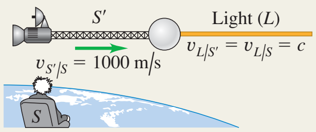


global positioning system (GPS), which makes it possible to determine your position on the earth's surface to within a few meters using a handheld receiver (Fig. 37.25). The heart of the GPS system is a collection of more than two dozen satellites in very precise orbits. Each satellite emits carefully timed radio signals, and a GPS receiver simultaneously detects the signals from several satellites. The receiver then calculates the time delay between when each signal was emitted and when it was received, and uses this information to calculate the receiver's position. To ensure the proper timing of the signals, it's necessary to include corrections due to the special theory of relativity (because the satellites are moving relative to the receiver on earth) as well as the general theory (because the satellites are higher in the earth's gravitational field than the receiver). The corrections due to relativity are small—less than one part in 10^9—but are crucial to the superb precision of the GPS system.

37.25 A GPS receiver uses radio signals from the orbiting GPS satellites to determine its position. To account for the effects of relativity, the receiver must be tuned to a slightly higher frequency (10.23 MHz) than the frequency emitted by the satellites (10.22999999543 MHz).

CHAPTER 37 SUMMARY

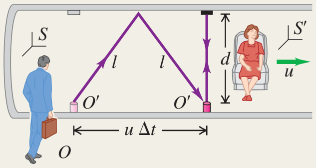
Invariance of physical laws, simultaneity: All of the fundamental laws of physics have the same form in all inertial frames of reference. The speed of light in vacuum is the same in all inertial frames and is independent of the motion of the source. Simultaneity is not an absolute concept; events that are simultaneous in one frame are not necessarily simultaneous in a second frame moving relative to the first.



Time dilation: If two events occur at the same space point in a particular frame of reference, the time interval Δt_0 between the events as measured in that frame is called a proper time interval. If this frame moves with constant velocity u relative to a second frame, the time interval Δt between the events as observed in the second frame is longer than Δt_0. (See Examples 37.1–37.3.)

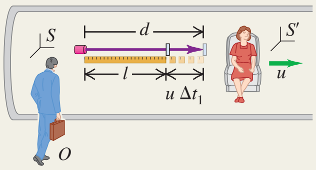
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \gamma\, \Delta t_0 \qquad (37.6), (37.8)$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \qquad (37.7)$$



Length contraction: If two points are at rest in a particular frame of reference, the distance l_0 between the points as measured in that frame is called a proper length. If this frame moves with constant velocity u relative to a second frame and the distances are measured parallel to the motion, the distance l between the points as measured in the second frame is shorter than l_0. (See Examples 37.4 and 37.5.)

$$l = l_0\sqrt{1 - u^2/c^2} = \frac{l_0}{\gamma} \qquad (37.16)$$



The Lorentz transformations: The Lorentz coordinate transformations relate the coordinates and time of an event in an inertial frame S to the coordinates and time of the same event as observed in a second inertial frame S' moving at velocity u relative to the first. For one-dimensional motion, a particle's velocities v_x in S and v_x' in S' are related by the Lorentz velocity transformation. (See Examples 37.6 and 37.7.)

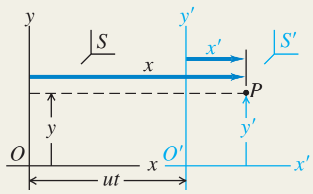
$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut)$$

$$y' = y \qquad z' = z \qquad (37.21)$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2)$$

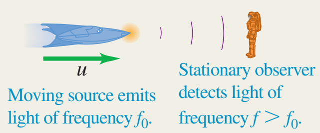
$$v_x' = \frac{v_x - u}{1 - uv_x/c^2} \qquad (37.22)$$

$$v_x = \frac{v_x' + u}{1 + uv_x'/c^2} \qquad (37.23)$$



The Doppler effect for electromagnetic waves: The Doppler effect is the frequency shift in light from a source due to the relative motion of source and observer. For a source moving toward the observer with speed u, Eq. (37.25) gives the received frequency f in terms of the emitted frequency f_0. (See Example 37.8.)

$$f = \sqrt{\frac{c + u}{c - u}}\, f_0 \qquad (37.25)$$



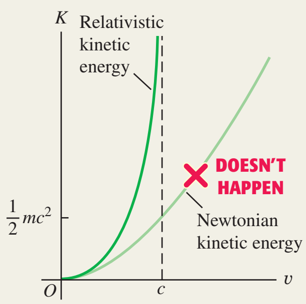
Relativistic momentum and energy: For a particle of rest mass m moving with velocity $\vec{v}$, the relativistic momentum $\vec{p}$ is given by Eq. (37.27) or (37.31) and the relativistic kinetic energy K is given by Eq. (37.36). The total energy E is the sum of the kinetic energy and the rest energy mc^2. The total energy can also be expressed in terms of the magnitude of momentum p and rest mass m. (See Examples 37.9–37.11.)

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \gamma m\vec{v} \qquad (37.27), (37.31)$$

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2 \qquad (37.36)$$

$$E = K + mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma mc^2 \qquad (37.38)$$

$$E^2 = (mc^2)^2 + (pc)^2 \qquad (37.39)$$



BRIDGING PROBLEM Colliding Protons

In an experiment, two protons are shot directly toward each other. Their speeds are such that in the frame of reference of each proton, the other proton is moving at $0.500c$. (a) What does an observer in the laboratory measure for the speed of each proton? (b) What is the kinetic energy of each proton as measured by an observer in the laboratory? (c) What is the kinetic energy of each proton as measured by the other proton?

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.

IDENTIFY and SET UP

1. This problem uses the Lorentz velocity transformation, which allows us to relate the velocity v_x of a proton in one frame to its velocity v_x' in a different frame. It also uses the idea of relativistic kinetic energy.
2. Take the x-axis to be the line of motion of the protons, and take the $+x$-direction to be to the right. In the frame in which the left-hand proton is at rest, the right-hand proton has velocity $-0.500c$. In the laboratory frame the two protons have velocities $-\alpha c$ and $+\alpha c$, where α (each proton's laboratory speed as a fraction of c) is our first target variable. Given this we can find the laboratory kinetic energy of each proton.

EXECUTE

3. Write a Lorentz velocity-transformation equation that relates the velocity of the right-hand proton in the laboratory frame to its velocity in the frame of the left-hand proton. Solve this equation for α. (*Hint:* Remember that α cannot be greater than 1. Why?)
4. Use your result from step 3 to find the laboratory kinetic energy of each proton.
5. Find the kinetic energy of the right-hand proton as measured in the frame of the left-hand proton.

EVALUATE

6. How much total kinetic energy must be imparted to the protons by a scientist in the laboratory? If the experiment were to be repeated with one proton stationary, what kinetic energy would have to be given to the other proton for the collision to be equivalent?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

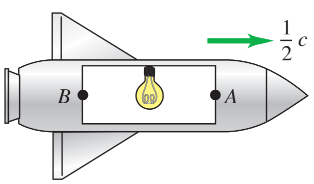
DISCUSSION QUESTIONS

Q37.1 You are standing on a train platform watching a high-speed train pass by. A light inside one of the train cars is turned on and then a little later it is turned off. (a) Who can measure the proper time interval for the duration of the light: you or a passenger on the train? (b) Who can measure the proper length of the train car: you or a passenger on the train? (c) Who can measure the proper length of a sign attached to a post on the train platform: you or a passenger on the train? In each case explain your answer.

Q37.2 If simultaneity is not an absolute concept, does that mean that we must discard the concept of causality? If event A is to *cause* event B, A must occur first. Is it possible that in some frames A appears to be the cause of B, and in others B appears to be the cause of A? Explain.

Q37.3 A rocket is moving to the right at $\frac{1}{2}$ the speed of light relative to the earth. A light bulb in the center of a room inside the rocket suddenly turns on. Call the light hitting the front end of the room event A and the light hitting the back of the room event B (Fig. Q37.3). Which event occurs first, A or B, or are they simultaneous, as viewed by (a) an astronaut riding in the rocket and (b) a person at rest on the earth?

Figure **Q37.3**



Q37.4 What do you think would be different in everyday life if the speed of light were 10 m/s instead of 3.00×10^8 m/s?

Q37.5 The average life span in the United States is about 70 years. Does this mean that it is impossible for an average person to travel a distance greater than 70 light-years away from the earth? (A light-year is the distance light travels in a year.) Explain.

Q37.6 You are holding an elliptical serving platter. How would you need to travel for the serving platter to appear round to another observer?

Q37.7 Two events occur at the same space point in a particular inertial frame of reference and are simultaneous in that frame. Is it possible that they may not be simultaneous in a different inertial frame? Explain.

Q37.8 A high-speed train passes a train platform. Larry is a passenger on the train, Adam is standing on the train platform, and David is riding a bicycle toward the platform in the same direction as the train is traveling. Compare the length of a train car as measured by Larry, Adam, and David.

Q37.9 The theory of relativity sets an upper limit on the speed that a particle can have. Are there also limits on the energy and momentum of a particle? Explain.

Q37.10 A student asserts that a material particle must always have a speed slower than that of light, and a massless particle must always move at exactly the speed of light. Is she correct? If so, how do massless particles such as photons and neutrinos acquire this speed? Can't they start from rest and accelerate? Explain.

Q37.11 The speed of light relative to still water is 2.25×10^8 m/s. If the water is moving past us, the speed of light we measure depends on the speed of the water. Do these facts violate Einstein's second postulate? Explain.

Q37.12 When a monochromatic light source moves toward an observer, its wavelength appears to be shorter than the value measured when the source is at rest. Does this contradict the hypothesis that the speed of light is the same for all observers? Explain.

Q37.13 In principle, does a hot gas have more mass than the same gas when it is cold? Explain. In practice, would this be a measurable effect? Explain.

Q37.14 Why do you think the development of Newtonian mechanics preceded the more refined relativistic mechanics by so many years?

EXERCISES

Section 37.2 Relativity of Simultaneity

37.1 • Suppose the two lightning bolts shown in Fig. 37.5a are simultaneous to an observer on the train. Show that they are *not* simultaneous to an observer on the ground. Which lightning strike does the ground observer measure to come first?

Section 37.3 Relativity of Time Intervals

37.2 • The positive muon (μ^+), an unstable particle, lives on average 2.20×10^{-6} s (measured in its own frame of reference) before decaying. (a) If such a particle is moving, with respect to the laboratory, with a speed of $0.900c$, what average lifetime is measured in the laboratory? (b) What average distance, measured in the laboratory, does the particle move before decaying?

37.3 • How fast must a rocket travel relative to the earth so that time in the rocket "slows down" to half its rate as measured by earth-based observers? Do present-day jet planes approach such speeds?

37.4 • A spaceship flies past Mars with a speed of $0.985c$ relative to the surface of the planet. When the spaceship is directly overhead, a signal light on the Martian surface blinks on and then off. An observer on Mars measures that the signal light was on for 75.0 μs. (a) Does the observer on Mars or the pilot on the spaceship measure the proper time? (b) What is the duration of the light pulse measured by the pilot of the spaceship?

37.5 • The negative pion (π^-) is an unstable particle with an average lifetime of 2.60×10^{-8} s (measured in the rest frame of the pion). (a) If the pion is made to travel at very high speed relative to a laboratory, its average lifetime is measured in the laboratory to be 4.20×10^{-7} s. Calculate the speed of the pion expressed as a fraction of c. (b) What distance, measured in the laboratory, does the pion travel during its average lifetime?

37.6 •• As you pilot your space utility vehicle at a constant speed toward the moon, a race pilot flies past you in her spaceracer at a constant speed of $0.800c$ relative to you. At the instant the spaceracer passes you, both of you start timers at zero. (a) At the instant when you measure that the spaceracer has traveled 1.20×10^8 m past you, what does the race pilot read on her timer? (b) When the race pilot reads the value calculated in part (a) on her timer, what does she measure to be your distance from her? (c) At the instant when the race pilot reads the value calculated in part (a) on her timer, what do you read on yours?

37.7 •• A spacecraft flies away from the earth with a speed of 4.80×10^6 m/s relative to the earth and then returns at the same speed. The spacecraft carries an atomic clock that has been carefully synchronized with an identical clock that remains at rest on earth. The spacecraft returns to its starting point 365 days (1 year) later, as measured by the clock that remained on earth. What is the difference in the elapsed times on the two clocks, measured in hours? Which clock, the one in the spacecraft or the one on earth, shows the shorter elapsed time?

37.8 • An alien spacecraft is flying overhead at a great distance as you stand in your backyard. You see its searchlight blink on for 0.190 s. The first officer on the spacecraft measures that the searchlight is on for 12.0 ms. (a) Which of these two measured times is the proper time? (b) What is the speed of the spacecraft relative to the earth expressed as a fraction of the speed of light c?

Section 37.4 Relativity of Length

37.9 • A spacecraft of the Trade Federation flies past the planet Coruscant at a speed of $0.600c$. A scientist on Coruscant measures the length of the moving spacecraft to be 74.0 m. The spacecraft later lands on Coruscant, and the same scientist measures the length of the now stationary spacecraft. What value does she get?

37.10 • A meter stick moves past you at great speed. Its motion relative to you is parallel to its long axis. If you measure the length of the moving meter stick to be 1.00 ft (1 ft = 0.3048 m)—for example, by comparing it to a 1-foot ruler that is at rest relative to you—at what speed is the meter stick moving relative to you?

37.11 •• **Why Are We Bombarded by Muons?** Muons are unstable subatomic particles that decay to electrons with a mean lifetime of 2.2 μs. They are produced when cosmic rays bombard the upper atmosphere about 10 km above the earth's surface, and they travel very close to the speed of light. The problem we want to address is why we see any of them at the earth's surface. (a) What is the greatest distance a muon could travel during its 2.2-μs lifetime? (b) According to your answer in part (a), it would seem that muons could never make it to the ground. But the 2.2-μs lifetime is measured in the frame of the muon, and muons are moving very fast. At a speed of $0.999c$, what is the mean lifetime of a muon as measured by an observer at rest on the earth? How far would the muon travel in this time? Does this result explain why we find muons in cosmic rays? (c) From the point of view of the muon, it still lives for only 2.2 μs, so how does it make it to the ground? What is the thickness of the 10 km of atmosphere through which the muon must travel, as measured by the muon? Is it now clear how the muon is able to reach the ground?

37.12 • An unstable particle is created in the upper atmosphere from a cosmic ray and travels straight down toward the surface of the earth with a speed of $0.99540c$ relative to the earth. A scientist at rest on the earth's surface measures that the particle is created at an altitude of 45.0 km. (a) As measured by the scientist, how much time does it take the particle to travel the 45.0 km to the surface of the earth? (b) Use the length-contraction formula to calculate the distance from where the particle is created to the surface of the earth as measured in the particle's frame. (c) In the particle's frame, how much time does it take the particle to travel from where it is created to the surface of the earth? Calculate this time both by the time dilation formula and from the distance calculated in part (b). Do the two results agree?

37.13 • As measured by an observer on the earth, a spacecraft runway on earth has a length of 3600 m. (a) What is the length of the runway as measured by a pilot of a spacecraft flying past at a speed of 4.00×10^7 m/s relative to the earth? (b) An observer on earth measures the time interval from when the spacecraft is directly over one end of the runway until it is directly over the other end. What result does she get? (c) The pilot of the spacecraft measures the time it takes him to travel from one end of the runway to the other end. What value does he get?

37.14 • A rocket ship flies past the earth at 85.0% of the speed of light. Inside, an astronaut who is undergoing a physical examination is having his height measured while he is lying down parallel to the direction the rocket ship is moving. (a) If his height is measured to be 2.00 m by his doctor inside the ship, what height would a person watching this from earth measure for his height? (b) If the earth-based person had measured 2.00 m, what would the doctor in the spaceship have measured for the astronaut's height? Is this a reasonable height? (c) Suppose the astronaut in part (a) gets up after the examination and stands with his body perpendicular to the direction of motion. What would the doctor in the rocket and the observer on earth measure for his height now?

Section 37.5 The Lorentz Transformations

37.15 • An observer in frame S' is moving to the right ($+x$-direction) at speed $u = 0.600c$ away from a stationary observer in frame S. The observer in S' measures the speed v' of a particle moving to the right away from her. What speed v does the observer in S measure for the particle if (a) $v' = 0.400c$; (b) $v' = 0.900c$; (c) $v' = 0.990c$?

37.16 • Space pilot Mavis zips past Stanley at a constant speed relative to him of $0.800c$. Mavis and Stanley start timers at zero when the front of Mavis's ship is directly above Stanley. When Mavis reads 5.00 s on her timer, she turns on a bright light under the front of her spaceship. (a) Use the Lorentz coordinate transformation derived in Example 37.6 to calculate x and t as measured by Stanley for the event of turning on the light. (b) Use the time dilation formula, Eq. (37.6), to calculate the time interval between the two events (the front of the spaceship passing overhead and turning on the light) as measured by Stanley. Compare to the value of t you calculated in part (a). (c) Multiply the time interval by Mavis's speed, both as measured by Stanley, to calculate the distance she has traveled as measured by him when the light turns on. Compare to the value of x you calculated in part (a).

37.17 •• A pursuit spacecraft from the planet Tatooine is attempting to catch up with a Trade Federation cruiser. As measured by an observer on Tatooine, the cruiser is traveling away from the planet with a speed of $0.600c$. The pursuit ship is traveling at a speed of $0.800c$ relative to Tatooine, in the same direction as the cruiser. (a) For the pursuit ship to catch the cruiser, should the velocity of the cruiser relative to the pursuit ship be directed toward or away from the pursuit ship? (b) What is the speed of the cruiser relative to the pursuit ship?

37.18 • An extraterrestrial spaceship is moving away from the earth after an unpleasant encounter with its inhabitants. As it departs, the spaceship fires a missile toward the earth. An observer on earth measures that the spaceship is moving away with a speed of $0.600c$. An observer in the spaceship measures that the missile is moving away from him at a speed of $0.800c$. As measured by an observer on earth, how fast is the missile approaching the earth?

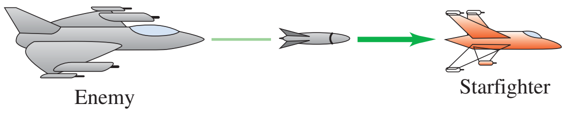
37.19 •• Two particles are created in a high-energy accelerator and move off in opposite directions. The speed of one particle, as measured in the laboratory, is $0.650c$, and the speed of each particle relative to the other is $0.950c$. What is the speed of the second particle, as measured in the laboratory?

37.20 •• Two particles in a high-energy accelerator experiment are approaching each other head-on, each with a speed of $0.9520c$ as measured in the laboratory. What is the magnitude of the velocity of one particle relative to the other?

37.21 •• Two particles in a high-energy accelerator experiment approach each other head-on with a relative speed of $0.890c$. Both particles travel at the same speed as measured in the laboratory. What is the speed of each particle, as measured in the laboratory?

37.22 •• An enemy spaceship is moving toward your starfighter with a speed, as measured in your frame, of $0.400c$. The enemy ship fires a missile toward you at a speed of $0.700c$ relative to the enemy ship (Fig. E37.22). (a) What is the speed of the missile relative to you? Express your answer in terms of the speed of light. (b) If you measure that the enemy ship is 8.00×10^6 km away from you when the missile is fired, how much time, measured in your frame, will it take the missile to reach you?

Figure **E37.22**



37.23 • An imperial spaceship, moving at high speed relative to the planet Arrakis, fires a rocket toward the planet with a speed of $0.920c$ relative to the spaceship. An observer on Arrakis measures that the rocket is approaching with a speed of $0.360c$. What is the speed of the spaceship relative to Arrakis? Is the spaceship moving toward or away from Arrakis?

Section 37.6 The Doppler Effect for Electromagnetic Waves

37.24 • Electromagnetic radiation from a star is observed with an earth-based telescope. The star is moving away from the earth with a speed of $0.600c$. If the radiation has a frequency of 8.64×10^{14} Hz in the rest frame of the star, what is the frequency measured by an observer on earth?

37.25 • **Tell It to the Judge.** (a) How fast must you be approaching a red traffic light ($\lambda = 675$ nm) for it to appear yellow ($\lambda = 575$ nm)? Express your answer in terms of the speed of light. (b) If you used this as a reason not to get a ticket for running a red light, how much of a fine would you get for speeding? Assume that the fine is \$1.00 for each kilometer per hour that your speed exceeds the posted limit of 90 km/h.

37.26 • A source of electromagnetic radiation is moving in a radial direction relative to you. The frequency you measure is 1.25 times the frequency measured in the rest frame of the source. What is the speed of the source relative to you? Is the source moving toward you or away from you?

Section 37.7 Relativistic Momentum

37.27 • A proton has momentum with magnitude p_0 when its speed is $0.400c$. In terms of p_0, what is the magnitude of the proton's momentum when its speed is doubled to $0.800c$?

37.28 • **When Should You Use Relativity?** As you have seen, relativistic calculations usually involve the quantity γ. When γ is appreciably greater than 1, we must use relativistic formulas instead of Newtonian ones. For what speed v (in terms of c) is the value of γ (a) 1.0% greater than 1; (b) 10% greater than 1; (c) 100% greater than 1?

37.29 • (a) At what speed is the momentum of a particle twice as great as the result obtained from the nonrelativistic expression mv? Express your answer in terms of the speed of light. (b) A force is applied to a particle along its direction of motion. At what speed is the magnitude of force required to produce a given acceleration twice as great as the force required to produce the same acceleration when the particle is at rest? Express your answer in terms of the speed of light.

37.30 • As measured in an earth-based frame, a proton is moving in the $+x$-direction at a speed of 2.30×10^8 m/s. (a) What force (magnitude and direction) is required to produce an acceleration in the $-x$-direction that has magnitude 2.30×10^8 m/s^2? (b) What magnitude of acceleration does the force calculated in part (a) give to a proton that is initially at rest?

37.31 • An electron is acted upon by a force of 5.00×10^{-15} N due to an electric field. Find the acceleration this force produces in each case: (a) The electron's speed is 1.00 km/s. (b) The electron's speed is 2.50×10^{8} m/s and the force is parallel to the velocity.

37.32 • **Relativistic Baseball.** Calculate the magnitude of the force required to give a 0.145-kg baseball an acceleration $a = 1.00 \text{ m/s}^2$ in the direction of the baseball's initial velocity when this velocity has a magnitude of (a) 10.0 m/s; (b) $0.900c$; (c) $0.990c$. (d) Repeat parts (a), (b), and (c) if the force and acceleration are perpendicular to the velocity.

Section 37.8 Relativistic Work and Energy

37.33 •• What is the speed of a particle whose kinetic energy is equal to (a) its rest energy and (b) five times its rest energy?

37.34 • If a muon is traveling at $0.999c$, what are its momentum and kinetic energy? (The mass of such a muon at rest in the laboratory is 207 times the electron mass.)

37.35 • A proton (rest mass 1.67×10^{-27} kg) has total energy that is 4.00 times its rest energy. What are (a) the kinetic energy of the proton; (b) the magnitude of the momentum of the proton; (c) the speed of the proton?

37.36 •• (a) How much work must be done on a particle with mass m to accelerate it (a) from rest to a speed of $0.090c$ and (b) from a speed of $0.900c$ to a speed of $0.990c$? (Express the answers in terms of mc^2.) (c) How do your answers in parts (a) and (b) compare?

37.37 • **CP** (a) By what percentage does your rest mass increase when you climb 30 m to the top of a ten-story building? Are you aware of this increase? Explain. (b) By how many grams does the mass of a 12.0-g spring with force constant 200 N/cm change when you compress it by 6.0 cm? Does the mass increase or decrease? Would you notice the change in mass if you were holding the spring? Explain.

37.38 • A 60.0-kg person is standing at rest on level ground. How fast would she have to run to (a) double her total energy and (b) increase her total energy by a factor of 10?

37.39 • **An Antimatter Reactor.** When a particle meets its antiparticle, they annihilate each other and their mass is converted to light energy. The United States uses approximately 1.0×10^{20} J of energy per year. (a) If all this energy came from a futuristic antimatter reactor, how much mass of matter and antimatter fuel would be consumed yearly? (b) If this fuel had the density of iron (7.86 g/cm^3) and were stacked in bricks to form a cubical pile, how high would it be? (Before you get your hopes up, antimatter reactors are a *long* way in the future—if they ever will be feasible.)

37.40 •• Electrons are accelerated through a potential difference of 750 kV, so that their kinetic energy is 7.50×10^{5} eV. (a) What is the ratio of the speed v of an electron having this energy to the speed of light, c? (b) What would the speed be if it were computed from the principles of classical mechanics?

37.41 • A particle has rest mass 6.64×10^{-27} kg and momentum 2.10×10^{-18} kg · m/s. (a) What is the total energy (kinetic plus rest energy) of the particle? (b) What is the kinetic energy of the particle? (c) What is the ratio of the kinetic energy to the rest energy of the particle?

37.42 •• A 0.100-μg speck of dust is accelerated from rest to a speed of $0.900c$ by a constant 1.00×10^{6} N force. (a) If the nonrelativistic mechanics is used, how far does the object travel to reach its final speed? (b) Using the correct relativistic treatment of Section 37.8, how far does the object travel to reach its final speed? (c) Which distance is greater? Why?

37.43 • Compute the kinetic energy of a proton (mass 1.67×10^{-27} kg) using both the nonrelativistic and relativistic expressions, and compute the ratio of the two results (relativistic divided by nonrelativistic) for speeds of (a) 8.00×10^{7} m/s and (b) 2.85×10^{8} m/s.

37.44 • What is the kinetic energy of a proton moving at (a) $0.100c$; (b) $0.500c$; (c) $0.900c$? How much work must be done to (d) increase the proton's speed from $0.100c$ to $0.500c$ and (e) increase the proton's speed from $0.500c$ to $0.900c$? (f) How do the last two results compare to results obtained in the nonrelativistic limit?

37.45 • (a) Through what potential difference does an electron have to be accelerated, starting from rest, to achieve a speed of $0.980c$? (b) What is the kinetic energy of the electron at this speed? Express your answer in joules and in electron volts.

37.46 • **Creating a Particle.** Two protons (each with rest mass $M = 1.67 \times 10^{-27}$ kg) are initially moving with equal speeds in opposite directions. The protons continue to exist after a collision that also produces an η^0 particle (see Chapter 44). The rest mass of the η^0 is $m = 9.75 \times 10^{-28}$ kg. (a) If the two protons and the η^0 are all at rest after the collision, find the initial speed of the protons, expressed as a fraction of the speed of light. (b) What is the kinetic energy of each proton? Express your answer in MeV. (c) What is the rest energy of the η^0, expressed in MeV? (d) Discuss the relationship between the answers to parts (b) and (c).

37.47 • The sun produces energy by nuclear fusion reactions, in which matter is converted into energy. By measuring the amount of energy we receive from the sun, we know that it is producing energy at a rate of 3.8×10^{26} W. (a) How many kilograms of matter does the sun lose each second? Approximately how many tons of matter is this (1 ton = 2000 lbs)? (b) At this rate, how long would it take the sun to use up all its mass?

PROBLEMS

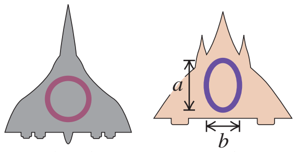
37.48 • Inside a spaceship flying past the earth at three-fourths the speed of light, a pendulum is swinging. (a) If each swing takes 1.50 s as measured by an astronaut performing an experiment inside the spaceship, how long will the swing take as measured by a person at mission control on earth who is watching the experiment? (b) If each swing takes 1.50 s as measured by a person at mission control on earth, how long will it take as measured by the astronaut in the spaceship?

37.49 • After being produced in a collision between elementary particles, a positive pion (π^+) must travel down a 1.90-km-long tube to reach an experimental area. A π^+ particle has an average lifetime (measured in its rest frame) of 2.60×10^{-8} s; the π^+ we are considering has this lifetime. (a) How fast must the π^+ travel if it is not to decay before it reaches the end of the tube? (Since u will be very close to c, write $u = (1 - \Delta)c$ and give your answer in terms of Δ rather than u.) (b) The π^+ has a rest energy of 139.6 MeV. What is the total energy of the π^+ at the speed calculated in part (a)?

37.50 •• A cube of metal with sides of length a sits at rest in a frame S with one edge parallel to the x-axis. Therefore, in S the cube has volume a^3. Frame S' moves along the x-axis with a speed u. As measured by an observer in frame S', what is the volume of the metal cube?

37.51 ••• The starships of the Solar Federation are marked with the symbol of the federation, a circle, while starships of the Denebian Empire are marked with the empire's symbol, an ellipse whose major axis is 1.40 times longer than its minor axis ($a = 1.40b$ in Fig. P37.51). How fast, relative to an observer, does an empire ship have to travel for its marking to be confused with the marking of a federation ship?

Figure **P37.51**



37.52 •• A space probe is sent to the vicinity of the star Capella, which is 42.2 light-years from the earth. (A light-year is the distance light travels in a year.) The probe travels with a speed of $0.9930c$. An astronaut recruit on board is 19 years old when the probe leaves the earth. What is her biological age when the probe reaches Capella?

37.53 • A particle is said to be *extremely relativistic* when its kinetic energy is much greater than its rest energy. (a) What is the speed of a particle (expressed as a fraction of c) such that the total energy is ten times the rest energy? (b) What is the percentage difference between the left and right sides of Eq. (37.39) if $(mc^2)^2$ is neglected for a particle with the speed calculated in part (a)?

37.54 •• **Everyday Time Dilation.** Two atomic clocks are carefully synchronized. One remains in New York, and the other is loaded on an airliner that travels at an average speed of 250 m/s and then returns to New York. When the plane returns, the elapsed time on the clock that stayed behind is 4.00 h. By how much will the readings of the two clocks differ, and which clock will show the shorter elapsed time? (*Hint:* Since $u \ll c$, you can simplify $\sqrt{1 - u^2/c^2}$ by a binomial expansion.)

37.55 • **The Large Hadron Collider (LHC).** Physicists and engineers from around the world have come together to build the largest accelerator in the world, the Large Hadron Collider (LHC) at the CERN Laboratory in Geneva, Switzerland. The machine will accelerate protons to kinetic energies of 7 TeV in an underground ring 27 km in circumference. (For the latest news and more information on the LHC, visit www.cern.ch.) (a) What speed v will protons reach in the LHC? (Since v is very close to c, write $v = (1 - \Delta)c$ and give your answer in terms of Δ.) (b) Find the relativistic mass, m_{rel}, of the accelerated protons in terms of their rest mass.

37.56 • **CP** A nuclear bomb containing 12.0 kg of plutonium explodes. The sum of the rest masses of the products of the explosion is less than the original rest mass by one part in 10^4. (a) How much energy is released in the explosion? (b) If the explosion takes place in 4.00 μs, what is the average power developed by the bomb? (c) What mass of water could the released energy lift to a height of 1.00 km?

37.57 • **CP Čerenkov Radiation.** The Russian physicist P. A. Čerenkov discovered that a charged particle traveling in a solid with a speed exceeding the speed of light in that material radiates electromagnetic radiation. (This is analogous to the sonic boom produced by an aircraft moving faster than the speed of sound in air; see Section 16.9. Čerenkov shared the 1958 Nobel Prize for this discovery.) What is the minimum kinetic energy (in electron volts) that an electron must have while traveling inside a slab of crown glass ($n = 1.52$) in order to create this Čerenkov radiation?

37.58 •• A photon with energy E is emitted by an atom with mass m, which recoils in the opposite direction. (a) Assuming that the motion of the atom can be treated nonrelativistically, compute the recoil speed of the atom. (b) From the result of part (a), show that the recoil speed is much less than c whenever E is much less than the rest energy mc^2 of the atom.

37.59 •• In an experiment, two protons are shot directly toward each other, each moving at half the speed of light relative to the laboratory. (a) What speed does one proton measure for the other proton? (b) What would be the answer to part (a) if we used only nonrelativistic Newtonian mechanics? (c) What is the kinetic energy of each proton as measured by (i) an observer at rest in the laboratory and (ii) an observer riding along with one of the protons? (d) What would be the answers to part (c) if we used only nonrelativistic Newtonian mechanics?

37.60 •• Two protons are moving away from each other. In the frame of each proton, the other proton has a speed of $0.600c$. What does an observer in the rest frame of the earth measure for the speed of each proton?

37.61 •• Frame S' has an x-component of velocity u relative to frame S, and at $t = t' = 0$ the two frames coincide (see Fig. 37.3). A light pulse with a spherical wave front is emitted at the origin of S' at time $t' = 0$. Its distance x' from the origin after a time t' is given by $x'^2 = c^2t'^2$. Use the Lorentz coordinate transformation to transform this equation to an equation in x and t, and show that the result is $x^2 = c^2t^2$; that is, the motion appears exactly the same in frame of reference S as it does in S'; the wave front is observed to be spherical in both frames.

37.62 • In certain radioactive beta decay processes, the beta particle (an electron) leaves the atomic nucleus with a speed of 99.95% the speed of light relative to the decaying nucleus. If this nucleus is moving at 75.00% the speed of light in the laboratory reference frame, find the speed of the emitted electron relative to the laboratory reference frame if the electron is emitted (a) in the same direction that the nucleus is moving and (b) in the opposite direction from the nucleus's velocity. (c) In each case in parts (a) and (b), find the kinetic energy of the electron as measured in (i) the laboratory frame and (ii) the reference frame of the decaying nucleus.

37.63 •• **CALC** A particle with mass m accelerated from rest by a constant force F will, according to Newtonian mechanics, continue to accelerate without bound; that is, as $t \to \infty$, $v \to \infty$. Show that according to relativistic mechanics, the particle's speed approaches c as $t \to \infty$. [*Note:* A useful integral is $\int (1 - x^2)^{-3/2} dx = x/\sqrt{1 - x^2}$.]

37.64 •• Two events are observed in a frame of reference S to occur at the same space point, the second occurring 1.80 s after the first. In a second frame S' moving relative to S, the second event is observed to occur 2.35 s after the first. What is the difference between the positions of the two events as measured in S'?

37.65 ••• Two events observed in a frame of reference S have positions and times given by (x_1, t_1) and (x_2, t_2), respectively. (a) Frame S' moves along the x-axis just fast enough that the two events occur at the same position in S'. Show that in S', the time interval $\Delta t'$ between the two events is given by

$$\Delta t' = \sqrt{(\Delta t)^2 - \left(\frac{\Delta x}{c}\right)^2}$$

where $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$. Hence show that if $\Delta x > c\,\Delta t$, there is *no* frame S' in which the two events occur at the same point. The interval $\Delta t'$ is sometimes called the *proper time interval* for the events. Is this term appropriate? (b) Show that if $\Delta x > c\,\Delta t$, there is a different frame of reference S' in which the two events occur *simultaneously.* Find the distance between the two events in S'; express your answer in terms of Δx, Δt, and c. This distance is sometimes called a *proper length.* Is this term appropriate? (c) Two events are observed in a frame of reference S' to occur simultaneously at points separated by a distance of 2.50 m. In a second frame S moving relative to S' along the line joining the two points in S', the two events appear to be separated by 5.00 m. What is the time interval between the events as measured in S? [*Hint:* Apply the result obtained in part (b).]

37.66 •• **Albert in Wonderland.** Einstein and Lorentz, being avid tennis players, play a fast-paced game on a court where they stand 20.0 m from each other. Being very skilled players, they play without a net. The tennis ball has mass 0.0580 kg. You can ignore gravity and assume that the ball travels parallel to the ground as it travels between the two players. Unless otherwise specified, all measurements are made by the two men. (a) Lorentz serves the ball at 80.0 m/s. What is the ball's kinetic energy? (b) Einstein slams a return at 1.80×10^8 m/s. What is the ball's kinetic energy? (c) During Einstein's return of the ball in part (a), a white rabbit runs beside the court in the direction from Einstein to Lorentz. The rabbit has a speed of 2.20×10^8 m/s relative to the two men. What is the speed of the rabbit relative to the ball? (d) What does the rabbit measure as the distance from Einstein to Lorentz? (e) How much time does it take for the rabbit to run 20.0 m, according to the players? (f) The white rabbit carries a pocket watch. He uses this watch to measure the time (as he sees it) for the distance from Einstein to Lorentz to pass by under him. What time does he measure?

37.67 • One of the wavelengths of light emitted by hydrogen atoms under normal laboratory conditions is $\lambda = 656.3$ nm, in the red portion of the electromagnetic spectrum. In the light emitted from a distant galaxy this same spectral line is observed to be Doppler-shifted to $\lambda = 953.4$ nm, in the infrared portion of the spectrum. How fast are the emitting atoms moving relative to the earth? Are they approaching the earth or receding from it?

37.68 •• **Measuring Speed by Radar.** A baseball coach uses a radar device to measure the speed of an approaching pitched baseball. This device sends out electromagnetic waves with frequency f_0 and then measures the shift in frequency Δf of the waves reflected from the moving baseball. If the fractional frequency shift produced by a baseball is $\Delta f/f_0 = 2.86 \times 10^{-7}$, what is the baseball's speed in km/h? (*Hint:* Are the waves Doppler-shifted a second time when reflected off the ball?)

37.69 • **Space Travel?** Travel to the stars requires hundreds or thousands of years, even at the speed of light. Some people have suggested that we can get around this difficulty by accelerating the rocket (and its astronauts) to very high speeds so that they will age less due to time dilation. The fly in this ointment is that it takes a great deal of energy to do this. Suppose you want to go to the immense red giant Betelgeuse, which is about 500 light-years away. (A light-year is the distance that light travels in a year.) You plan to travel at constant speed in a 1000-kg rocket ship (a little over a ton), which, in reality, is far too small for this purpose. In each case that follows, calculate the time for the trip, as measured by people on earth and by astronauts in the rocket ship, the energy needed in joules, and the energy needed as a percentage of U.S. yearly use (which is 1.0×10^{20} J). For comparison, arrange your results in a table showing v_{rocket}, t_{earth}, t_{rocket}, E (in J), and E (as % of U.S. use). The rocket ship's speed is (a) 0.50c; (b) 0.99c; (c) 0.9999c. On the basis of your results, does it seem likely that any government will invest in such high-speed space travel any time soon?

37.70 •• A spaceship moving at constant speed u relative to us broadcasts a radio signal at constant frequency f_0. As the spaceship approaches us, we receive a higher frequency f; after it has passed, we receive a lower frequency. (a) As the spaceship passes by, so it is instantaneously moving neither toward nor away from us, show that the frequency we receive is not f_0, and derive an expression for the frequency we do receive. Is the frequency we receive higher or lower than f_0? (*Hint:* In this case, successive wave crests move the same distance to the observer and so they have the same transit time. Thus f equals $1/T$. Use the time dilation formula to relate the periods in the stationary and moving frames.) (b) A spaceship emits electromagnetic waves of frequency $f_0 = 345$ MHz as measured in a frame moving with the ship. The spaceship is moving at a constant speed $0.758c$ relative to us. What frequency f do we receive when the spaceship is approaching us? When it is moving away? In each case what is the shift in frequency, $f - f_0$? (c) Use the result of part (a) to calculate the frequency f and the frequency shift $(f - f_0)$ we receive at the instant that the ship passes by us. How does the shift in frequency calculated here compare to the shifts calculated in part (b)?

37.71 • **CP** In a particle accelerator a proton moves with constant speed $0.750c$ in a circle of radius 628 m. What is the net force on the proton?

37.72 •• **CP** The French physicist Armand Fizeau was the first to measure the speed of light accurately. He also found experimentally that the speed, relative to the lab frame, of light traveling in a tank of water that is itself moving at a speed V relative to the lab frame is

$$v = \frac{c}{n} + kV$$

where $n = 1.333$ is the index of refraction of water. Fizeau called k the dragging coefficient and obtained an experimental value of $k = 0.44$. What value of k do you calculate from relativistic transformations?

CHALLENGE PROBLEMS

37.73 ••• **CALC** **Lorentz Transformation for Acceleration.** Using a method analogous to the one in the text to find the Lorentz transformation formula for velocity, we can find the Lorentz transformation for *acceleration.* Let frame S' have a constant x-component of velocity u relative to frame S. An object moves relative to frame S along the x-axis with instantaneous velocity v_x and instantaneous acceleration a_x. (a) Show that its instantaneous acceleration in frame S' is

$$a'_x = a_x\left(1 - \frac{u^2}{c^2}\right)^{3/2}\left(1 - \frac{uv_x}{c^2}\right)^{-3}$$

[*Hint:* Express the acceleration in S' as $a'_x = dv'_x/dt'$. Then use Eq. (37.21) to express dt' in terms of dt and dx, and use Eq. (37.22) to express dv'_x in terms of u and dv_x. The velocity of the object in S is $v_x = dx/dt$.] (b) Show that the acceleration in frame S can be expressed as

$$a_x = a'_x\left(1 - \frac{u^2}{c^2}\right)^{3/2}\left(1 + \frac{uv'_x}{c^2}\right)^{-3}$$

where $v'_x = dx'/dt'$ is the velocity of the object in frame S'.

37.74 ••• **CALC** **A Realistic Version of the Twin Paradox.** A rocket ship leaves the earth on January 1, 2100. Stella, one of a pair of twins born in the year 2075, pilots the rocket (reference frame S'); the other twin, Terra, stays on the earth (reference frame S). The rocket ship has an acceleration of constant magnitude g in its own reference frame (this makes the pilot feel at home, since it simulates the earth's gravity). The path of the rocket ship is a straight line in the $+x$-direction in frame S. (a) Using the results of Challenge Problem 37.73, show that in Terra's earth frame S, the rocket's acceleration is

$$\frac{du}{dt} = g\left(1 - \frac{u^2}{c^2}\right)^{3/2}$$

where u is the rocket's instantaneous velocity in frame S. (b) Write the result of part (a) in the form $dt = f(u)\,du$, where $f(u)$ is a function of u, and integrate both sides. (*Hint:* Use the integral given in Problem 37.63.) Show that in Terra's frame, the time when Stella attains a velocity v_{1x} is

$$t_1 = \frac{v_{1x}}{g\sqrt{1 - v_{1x}^2/c^2}}$$

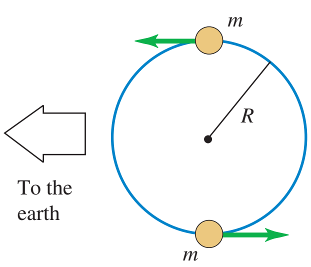
(c) Use the time dilation formula to relate dt and dt' (infinitesimal time intervals measured in frames S and S', respectively). Combine this result with the result of part (a) and integrate as in part (b) to show the following: When Stella attains a velocity v_{1x} relative to Terra, the time t_1' that has elapsed in frame S' is

$$t_1' = \frac{c}{g}\operatorname{arctanh}\left(\frac{v_{1x}}{c}\right)$$

Here arctanh is the inverse hyperbolic tangent. (*Hint:* Use the integral given in Challenge Problem 5.124.) (d) Combine the results of parts (b) and (c) to find t_1 in terms of t_1', g, and c alone. (e) Stella accelerates in a straight-line path for five years (by her clock), slows down at the same rate for five years, turns around, accelerates for five years, slows down for five years, and lands back on the earth. According to Stella's clock, the date is January 1, 2120. What is the date according to Terra's clock?

37.75 ••• CP Determining the Masses of Stars. Many of the stars in the sky are actually *binary stars,* in which two stars orbit about their common center of mass. If the orbital speeds of the stars are high enough, the motion of the stars can be detected by the Doppler shifts of the light they emit. Stars for which this is the case are called *spectroscopic binary stars.* Figure P37.75 shows the simplest case of a spectroscopic binary star: two identical stars, each with mass m, orbiting their center of mass in a circle of radius R. The plane of the stars' orbits is edge-on to the line of sight of an observer on the earth. (a) The light produced by heated hydrogen gas in a laboratory on the earth has a frequency of 4.568110×10^{14} Hz. In the light received from the stars by a telescope on the earth, hydrogen light is observed to vary in frequency between 4.567710×10^{14} Hz and 4.568910×10^{14} Hz. Determine whether the binary star system as a whole is moving toward or away from the earth, the speed of this motion, and the orbital speeds of the stars. (*Hint:* The speeds involved are much less than c, so you may use the approximate result $\Delta f/f = u/c$ given in Section 37.6.) (b) The light from each star in the binary system varies from its maximum frequency to its minimum frequency and back again in 11.0 days. Determine the orbital radius R and the mass m of each star. Give your answer for m in kilograms and as a multiple of the mass of the sun, 1.99×10^{30} kg. Compare the value of R to the distance from the earth to the sun, 1.50×10^{11} m. (This technique is actually used in astronomy to determine the masses of stars. In practice, the problem is more complicated because the two stars in a binary system are usually not identical, the orbits are usually not circular, and the plane of the orbits is usually tilted with respect to the line of sight from the earth.)

Figure **P37.75**



37.76 ••• CP CALC Relativity and the Wave Equation. (a) Consider the Galilean transformation along the x-direction: $x' = x - vt$ and $t' = t$. In frame S the wave equation for electromagnetic waves in a vacuum is

$$\frac{\partial^2 E(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2} = 0$$

where E represents the electric field in the wave. Show that by using the Galilean transformation the wave equation in frame S' is found to be

$$\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 E(x',t')}{\partial x'^2} + \frac{2v}{c^2}\frac{\partial^2 E(x',t')}{\partial x'\partial t'} - \frac{1}{c^2}\frac{\partial^2 E(x',t')}{\partial t'^2} = 0$$

This has a different form than the wave equation in S. Hence the Galilean transformation *violates* the first relativity postulate that all physical laws have the same form in all inertial reference frames. (*Hint:* Express the derivatives $\partial/\partial x$ and $\partial/\partial t$ in terms of $\partial/\partial x'$ and $\partial/\partial t'$ by use of the chain rule.) (b) Repeat the analysis of part (a), but use the Lorentz coordinate transformations, Eqs. (37.21), and show that in frame S' the wave equation has the same form as in frame S:

$$\frac{\partial^2 E(x',t')}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E(x',t')}{\partial t'^2} = 0$$

Explain why this shows that the speed of light in vacuum is c in both frames S and S'.

37.77 ••• CP Kaon Production. In high-energy physics, new particles can be created by collisions of fast-moving projectile particles with stationary particles. Some of the kinetic energy of the incident particle is used to create the mass of the new particle. A proton–proton collision can result in the creation of a negative kaon (K^-) and a positive kaon (K^+)

$$p + p \rightarrow p + p + K^- + K^+$$

(a) Calculate the minimum kinetic energy of the incident proton that will allow this reaction to occur if the second (target) proton is initially at rest. The rest energy of each kaon is 493.7 MeV, and the rest energy of each proton is 938.3 MeV. (*Hint:* It is useful here to work in the frame in which the total momentum is zero. But note that the Lorentz transformation must be used to relate the velocities in the laboratory frame to those in the zero-total-momentum frame.) (b) How does this calculated minimum kinetic energy compare with the total rest mass energy of the created kaons? (c) Suppose that instead the two protons are both in motion with velocities of equal magnitude and opposite direction. Find the minimum combined kinetic energy of the two protons that will allow the reaction to occur. How does this calculated minimum kinetic energy compare with the total rest mass energy of the created kaons? (This example shows that when colliding beams of particles are used instead of a stationary target, the energy requirements for producing new particles are reduced substantially.)

Answers

Chapter Opening Question ?

No. While the speed of light c is the ultimate "speed limit" for any particle, there is *no* upper limit on a particle's kinetic energy (see Fig. 37.21). As the speed approaches c, a small increase in speed corresponds to a large increase in kinetic energy.

Test Your Understanding Questions

37.1 Answers: (a) (i), (b) no You, too, will measure a spherical wave front that expands at the same speed c in all directions. This is a consequence of Einstein's second postulate. The wave front that you measure is *not* centered on the current position of the spaceship; rather, it is centered on the point P where the spaceship was located at the instant that it emitted the light pulse. For example, suppose the spaceship is moving at speed $c/2$. When your watch shows that a time t has elapsed since the pulse of light was emitted, your measurements will show that the wave front is a sphere of radius ct centered on P and that the spaceship is a distance $ct/2$ from P.

37.2 Answer: (iii) In Mavis's frame of reference, the two events (the Ogdenville clock striking noon and the North Haverbrook clock striking noon) are not simultaneous. Figure 37.5 shows that the event toward the front of the rail car occurs first. Since the rail car is moving toward North Haverbrook, that clock struck noon before the one on Ogdenville. So, according to Mavis, it is after noon in North Haverbrook.

37.3 Answers: (a) (ii), (b) (ii) The statement that moving clocks run slow refers to any clock that is moving relative to an observer. Maria and her stopwatch are moving relative to Samir, so Samir measures Maria's stopwatch to be running slow and to have ticked off fewer seconds than his own stopwatch. Samir and his stopwatch are moving relative to Maria, so she likewise measures Samir's stopwatch to be running slow. Each observer's measurement is correct for his or her own frame of reference. *Both* observers conclude that a moving stopwatch runs slow. This is consistent with the principle of relativity (see Section 37.1), which states that the laws of physics are the same in all inertial frames of reference.

37.4 Answer: (ii), (i) and (iii) (tie), (iv) You measure the rest length of the stationary meter stick and the contracted length of the moving spaceship to both be 1 meter. The rest length of the spaceship is greater than the contracted length that you measure, and so must be greater than 1 meter. A miniature observer on board the spaceship would measure a contracted length for the meter stick of less than 1 meter. Note that in your frame of reference the nose and tail of the spaceship can simultaneously align with the two ends of the meter stick, since in your frame of reference they have the same length of 1 meter. In the spaceship's frame these two alignments cannot happen simultaneously because the meter stick is shorter than the spaceship. Section 37.2 tells us that this shouldn't be a surprise; two events that are simultaneous to one observer may not be simultaneous to a second observer moving relative to the first one.

37.5 Answers: (a) P_1, (b) P_4 (a) The last of Eqs. (37.21) tells us the times of the two events in S': $t'_1 = \gamma(t_1 - ux_1/c^2)$ and $t'_2 = \gamma(t_2 - ux_2/c^2)$. In frame S the two events occur at the same x-coordinate, so $x_1 = x_2$, and event P_1 occurs before event P_2, so $t_1 < t_2$. Hence you can see that $t'_1 < t'_2$ and event P_1 happens before P_2 in frame S', too. This says that if event P_1 happens before P_2 in a frame of reference S where the two events occur at the same position, then P_1 happens before P_2 in any other frame moving relative to S. (b) In frame S the two events occur at different x-coordinates such that $x_3 < x_4$, and events P_3 and P_4 occur at the same time, so $t_3 = t_4$. Hence you can see that $t'_3 = \gamma(t_3 - ux_3/c^2)$ is greater than $t'_4 = \gamma(t_4 - ux_4/c^2)$, so event P_4 happens before P_3 in frame S'. This says that even though the two events are simultaneous in frame S, they need not be simultaneous in a frame moving relative to S.

37.7 Answer: (ii) Equation (37.27) tells us that the magnitude of momentum of a particle with mass m and speed v is $p = mv/\sqrt{1 - v^2/c^2}$. If v increases by a factor of 2, the numerator mv increases by a factor of 2 *and* the denominator $\sqrt{1 - v^2/c^2}$ decreases. Hence p increases by a factor greater than 2. (Note that in order to double the speed, the initial speed must be less than $c/2$. That's because the speed of light is the ultimate speed limit.)

37.8 Answer: (i) As the proton moves a distance s, the constant force of magnitude F does work $W = Fs$ and increases the kinetic energy by an amount $\Delta K = W = Fs$. This is true no matter what the speed of the proton before moving this distance. Thus the constant force increases the proton's kinetic energy by the same amount during the first meter of travel as during any subsequent meter of travel. (It's true that as the proton approaches the ultimate speed limit of c, the increase in the proton's *speed* is less and less with each subsequent meter of travel. That's not what the question is asking, however.)

Bridging Problem

Answers: (a) $0.268c$ **(b)** 35.6 MeV **(c)** 145 MeV