Date :

S.No.	Date	Title	Page No.	Teacher's Sign/ Remarks

SIMULATION and MODELING

System:

It is a collection of entities that act and interact together towards the achievement of some logical result that is useful to human life.

Simulation:

It is the process of imitating some real world process. It is a reproducing of a system model and experimenting on it.

Model:

An abstract representation of system, usually containing structural, logical or mathematical relationship which describe the system in terms of state, entities and their attributes, set, process, events, activities etc.

A system has entities, attributes and activities.

Entity:

It is the object of interest in the system, which has physical existence

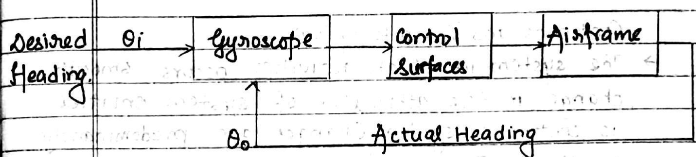
Attribute: Property of an entity.

Activity:
Any process that cause change in the system.

SYSTEM	ENTITIES	ATTRIBUTES	ACTIVITIES
SuperMarket	Staff, Customers	Shopping-list, manage items	Buying, Selling
Traffic.	Vehicles (Bus, Car etc)	speed, distance covered	Driving, Parking
Bank.			

System.
Example
\# Consider an artifact flying ~~through~~ under the control of an autopilot. A gyroscope is the autopilot that detects the difference between actual heading and desired heading. It sends the signal to move the control systems. It response to the control surface movement, the airframe, steers towards the desired heading.

Fig: An aircraft under autopilot control

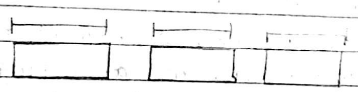


\# Consider a factory that makes an assembles parts into product. The major component of the system are the fabrication department making the parts and the assembly department maintains a supply of raw materials and a shipping department dispatches the finished products. A production control department receives order and assign work to the other departments.

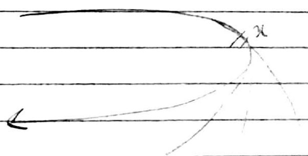
Continuos and Discrete System

→ The system in which activities occurs smooth change in the attributes of system entities is continuos system. Changes are predominantly smooth. E.g :- Turning of aircraft.

→ The system in which changes are predominantly discontinuos are discrete system. E.g :- Banking system



Time interval
(discrete)



(continuos)

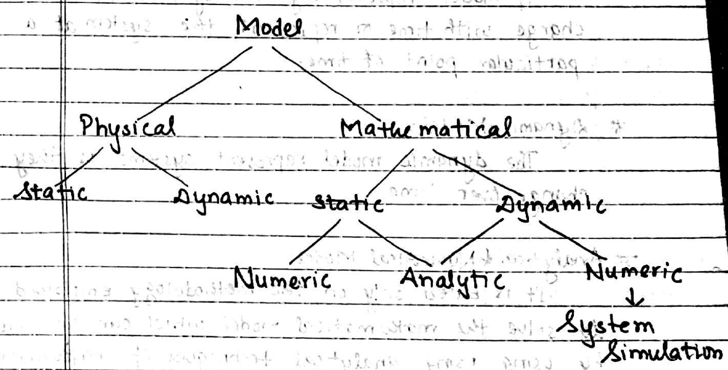
Endogeneous Activities:-

Activities occuring within the system (system [closed])

Exogeneous Activities:

Activities occuring outside the system which effects the system. (open system)

* Types of Model:



Physical Model.

In this model, physical objects are substituted for real things. This models may comprise of only physical objects without any dynamic interaction between them.

Mathematical Model:

This model comprise of symbolic notation and mathematical equation to represent the system.

Static Model:

A model representing the system, which doesn't change with time or represent the system at a particular point of time.

Dynamic Model:

The dynamic model represent systems as they change over time.

Analytical & Numerical Model:

It is based only on the methodology employed to solve the mathematical model which can be solved by using using analytical techniques of mathematics is analytical model.

The model which requires the application of numerical method are called numerical model.

System Simulation:

Simulation describes as a procedure of establishing a model and deriving solution from it, which covers the whole analysis of physical, analytical & numerical investigations.

Deterministic models:

It has a known set of inputs which results into an unique set of outputs.

Stochastic models:

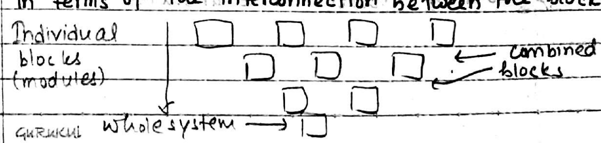
These are one or more random inputs variables, which gives many random outputs. The outputs in such cases are only estimates of true characteristics of system.

Principles Used in Modeling:

It is not possible to provide exact rules by which models are built but a number of guiding principles can be defined.

a) Block building:-

The description of system should be organized in a series of blocks. Each block describes the part of the system. The system as a whole can then be described in terms of the interconnection between the blocks.



b) Relevance: The model should include only those aspects of the system that are relevant or essential to the study objectives. Even though the unrelevant things does not harm the model it increases the complexity.

c) Accuracy:-

The accuracy of the information gathered for the model should be considered.

d) Aggregation:

Here, the number of individual entities can be grouped together into large entities.

Advantage of simulation:-

1) New policies, operating procedures, decision rules, information flow, and so on can be explored without disturbing ongoing operations on the real world.
2) Hypothesis about how or why certain phenomena occured can be tested for feasibility.
3) Bottle neck analysis can be performed indicating where in process information.
4) "What-If" questions can be answered.

Areas of application:

① Manufacturing Applications:
→ Analysis of assembly operations
→ Analysis of storage and revival strategies in a warehouse
→ Investigation of dynamics in a service-oriented supply chain

② Construction engineering
→ Construction of a dam embankment.
→ Activities scheduling in a dynamic, multiproject settings
→ Investigation of the structural steel erection process.
→ Special purpose template for utility tunnel construction.

③ Military Application:-
→ Modeling leadership effects and recruits type in an army recruiting station.
→ Using adaptive agents in Airforce pilot training.

④ IT field Applications:
→ Testing new designed hardware and software.
→ Testing of new generation networking Algorithm.
→ Testing of new design mobiles & its applications.

Simulation is used in every field now-a-days.

Unit-2.

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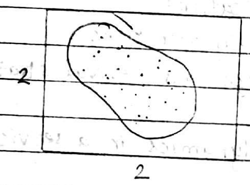
Monte-Carlo Method:



$$\frac{A}{4} = \frac{8}{20}$$

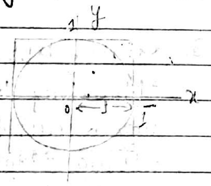
A = Area of irregular shape

$8 \to x$ = total samples within irregular shapes

4 = Area of regular shape

20 = Total no. of samples taken.

Calculate the value of pi(π) using Monte-Carlo.



Area of circle (A) $= \pi r^2$

We know: $x^2+y^2=r^2$

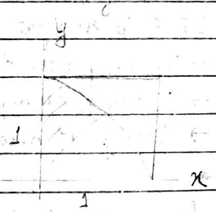
$x^2+y^2 \leq 1$

$y \leq \sqrt{1-x^2}$

Taking sample points = 20 (N)

fig (i)

$$A = \frac{\pi}{4} = \frac{n}{N}$$



$A = \frac{1}{4}\pi r^2$

$\pi = \frac{4.A}{r^2}$ $\therefore \pi = 4A$

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$$\frac{\text{Area of circle}}{\text{Area of sq.}} = \frac{\text{samples in shaded area}}{\text{Total no. of samples}}$$

$$\pi = \frac{4n}{N}$$

S.N	x	y	x^2	y^2	Is $y \leq \sqrt{1-x^2}$ Yes/No.
1	0.2	0.167	0.04	0.027	Yes
2	0.2	0.33	0.04	0.108	Yes
3	0.2	0.5	0.04	0.25	Yes
4	0.2	0.67	0.04	0.448	Yes
5	0.2	0.83	0.04	0.689	Yes
6	0.4	0.167	0.16	0.027	Yes
7	0.4	0.33	0.16	0.108	Yes
8	0.4	0.5	0.16	0.25	Yes
9	0.4	0.67	0.16	0.448	Yes
10	0.4	0.83	0.16	0.689	Yes
11	0.6	0.167	0.36	0.027	Yes
12	0.6	0.33	0.36	0.108	Yes
13	0.6	0.5	0.36	0.25	Yes
14	0.6	0.67	0.36	0.448	Yes
15	0.6	0.83	0.36	0.689	No
16	0.8	0.167	0.64	0.027	Yes
17	0.8	0.33	0.64	0.108	Yes
18	0.8	0.5	0.64	0.25	Yes
19	0.8	0.67	0.64	0.448	No
20	0.8	0.83	0.64	0.689	No

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$\pi = \frac{4 \times 17}{20} = 3.4$

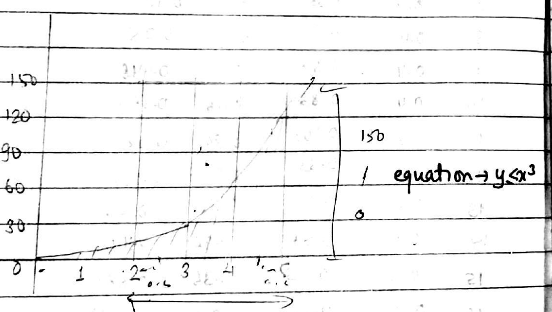
Numerical Integration by Monte-Carlo Method.

$I = \int_2^5 x^3\,dx$ using 20 sample points

Given $f(x) = x^3$

say $y = x^3$

x	0	1	2	3	4	5	6
y	0	1	8	27	64	125	216



$I = \frac{n}{N} \cdot A$ → area of rectangle

area of shaded; ratio of points.

S.No	x	y	x^3	Is ($y \le x^3$)
1	2.5	30	15.625	No
2	2.5	60	15.625	No
3	2.5	90	15.625	No
4	2.5	120	15.625	No
5	3	30	27	No
6	3	60	27	No
7	3	90	27	No
8	3	120	27	No
9	3.5	30	42.875	Yes
10	3.5	60	42.875	No
11	3.5	90	42.875	No
12	3.5	120	42.875	No
13	4	30	64	Yes
14	4	60	64	Yes
15	4	90	64	No
16	4	120	64	No
17	4.5	30	91.125	Yes
18	4.5	60	91.125	Yes
19	4.5	90	91.125	Yes
20	4.5	120	91.125	No

$\left.\frac{x^4}{4}\right|_2^5$

$\frac{1}{4}\left(5^4 - 2^4\right)$

$= \frac{1}{4}\left[625 - 16\right]$

$= \frac{1}{4}\left[609\right]$

≈ 152

$150 \times 3 = 450$

$\therefore n = 6 \quad N = 20 \quad A = 3 \times 150$

$I = \frac{6}{20} \times 3 \times 150$

$= 135$

$\left[\ddot{x} = \frac{d^2x}{dt^2}, \; \dot{x} = \frac{dx}{dt}\right]$

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Note For better result adjust to make $n=7$ by replace S.No as (4.5, 70). The ans will be near to exact value.

Q) $I = \int_0^4 \sqrt{x}\, dx$ accurate ans $\left[\begin{array}{l} n=14 \\ N=20 \\ A=8 \end{array}\right.$ real value = 5.333

3rd April **Comparison of Simulation and Analytical Methods:-**

→ Simulation gives specific solution than general solution.

→ An analytical solution gives all the conditions.

→ For e.g.:- of Automobile wheel motion ($M\ddot{x} + D\dot{x} + Kx = KF(t)$)
Analytical gives all condition that can cause oscillation but simulation tells only whether a particular set of condition did or did not cause oscillation.

→ To find all such condition simulation must be repeated under different conditions.

→ To judge which method to apply, it is necessary to consider carefully what question need to be answered in the system study and to what degree of accuracy the answer need to be known.

→ The amount of detail in the model need to be limited to minimum level in simulation.

→ The ideal way of using simulation is an extension of mathematical solutions.

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→ There are certain limits in Analytical method like physical stops, finite time delay, non-linear forces etc

→ Such limitations are removed by simulation.

→ Simulation method provides a quicker or more convinient way of deriving results than analytical methods.

→ It is more convinient to use simulation to obtain results directly.

Experimental ~~Natural~~ Nature of Simulation.

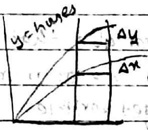
→ Simulation is essentially an experimental problem-solving techniques

→ Many simulation runs have to be made to understand the relationship involved in the system.

→ So the use of simulation in a study must be planned as a series of experiment.

→ To find the output or result in simulation the experiment or test or procedure must be implemented or run to we can see the appropriate output.

Types of System Simulation:



validation
calibration

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Distributed Lag Model:

→ Models that have the properties of changing only at fixed intervals of time and of basing current values of the variable on other current values and values that are occured in previous intervals are called distributed lag models.

→ They are used extensively in econometric studies.

Let C = consumption
I = Investment
T = Taxes
G = government expenditures
Y = National income.

$C = 20 + 0.7(Y - T)$
$I = 2 + 0.1Y$
$T = 0.2Y$
$Y = C + I + G$

→ This is a static mathematical model, but it can be made dynamic by taking a fixed interval, say one year.

→ Any variable that appears in the form of its current value and one or more previous intervals is said to be lagged variable.

The set of equation can be written in the form.

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$I = 2 + 0.1 Y_{-1}$
$Y = 45.45 + 2.27(I+G)$
$T = 0.2Y$
$C = 20 + 0.7(Y-T)$

Q)

Year	G
1	20
2	25
3	30
4	35
5	40

$Y_{-1} = 80$

For 1st Year:

$I = 2 + 0.1 Y_{-1}$
$= 2 + 0.1 \times 80$
$= 10.$

$Y = 45.45 + 2.27(I+G)$
$= 45.45 + 2.27(10 + 20)$
$= 113.55$

$T = 0.2Y$
$= 0.2 \times 113.55$
$= 22.71$

$C = 20 + 0.7(Y-T)$
$= 20 + 0.7(113.55 - 22.71)$
$= 83.588$

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For 2nd Year.

$Y_{-1} = 113.55$

$I = 2 + 0.1 \times Y_{-1}$
$= 2 + 0.1 \times 113.55$
$= 13.355$

$Y = 45.45 + 2.27(I + G)$
$= 45.45 + 2.27(13.355 + 25)$
$= 132.51585$

$T = 0.2 \times Y$
$= 0.2 \times 132.51585$
$= 26.50317$
$\simeq 26.5$

$C = 20 + 0.7(Y - T)$
$= 20 + 0.7(132.51585 - 26.50317)$
$= 94.2.$

For 3rd Year

$Y_{-1} = 132.515$

$I = 2 + 0.1 \times 132.515 = 15.25$ $T = 29.632$
$Y = 148.16$ $C = 102.9696$

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For 4th Year

$Y_{-1} = 148.16$

$I = 2 + 0.1 \times 148.16 = 16.816$
$Y = 45.45 + 2.27(16.816 + 35) = 163.07$
$T = 0.2 \times 163.07 = 32.61$
$C = 20 + 0.7(163.07 - 32.61) = 111.322$

For 5th Year

$Y_{-1} = 163.07$

$I = 2 + 0.1 \times 163.07 = 18.307$
$Y = 45.45 + 2.27(18.307 + 40) = 177.80$
$T = 0.2 \times Y = 35.56$
$C = 20 + 0.7(177.80 - 35.56) = 119.56$

5th April | Cobweb model:

Distributed lag model can be constructed from:

→ The static model.

→ This relates supply (S), demand (D) and market Price (P)

→ The supply should be made dependent upon the price from the previous marketing period.

→ The demand will respond to current price.

→ Again assuming the market in cleared, the market in distributed lag form is:

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S = Supply, D = Demand, P = Market Price.

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$Q = a - bP$ Q: Quantity

$S = c + dP_{-1}$ P: Price

$Q = S$

Initial value of P_0 will be given

1. Fluctuation of Market Price

Given data:-

$P_0 = 1.0$

$a = 12.4$ When $T = 0$, $P_0 = 1.0$

$b = 1.2$ $Q = 12.4 - (1.2 * 1.0) = 12.4 - 1.2$

$c = 1.0$ $Q = 11.2.$

$d = 0.9$ $S = 1.0 + (0.9 * 1.0) \Rightarrow 1 + 0.9 = 1.9$

$S = 1.9$

$Q = a - (bp)$ $Q = 1.9$

(ii) Now, $1.9 = 12.4 - (1.2 * P_1)$ $P_1 = ?$

$P_1 = 8.75 \Leftarrow \frac{12.4 - 1.9}{1.2}$

$S = 1.0 + (0.9 * 8.75) = 1 + 7.875 = 8.875$

$S = 8.875$

$Q = S = 8.875$

Also,

$8.875 = 12.4 - (1.2 * \cancel{8.875}\ P_2)$

$P_2 = \frac{12.4 - 8.875}{1.2} = 2.9375$

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Given data

$P_0 = 5$

$a = 10$

$b = 0.9$

~~c = 2.4~~

$d = 1.2$

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$S = 1.0 + 0.9 * 2.9375 = 3.64375$

$Q = S = 3.64375$

So, $3.64375 = 12.4 - (1.2 * P_3)$

$P_3 = 7.2968$

$S = 1.0 + 0.9 \times 7.2968$

$S = 7.567188$

Again, $7.567188 = 12.4 - (1.2 * P_4)$

$P_4 = 4.027$

$S = 1.0 + 0.9 \times 4.027$

$= 4.624609$

$Q = S = 4.624609$

Again, $4.624609 = 12.4 - (1.2 * P_5)$

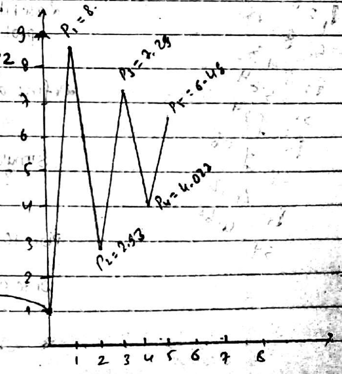
$P_5 = 6.479492$

$S = 1.0 + 0.9 \times 6.4792$

$S = 7.83128$

Fig: Fluctuation of market price.

T = 0
$P_0 = 1.0$



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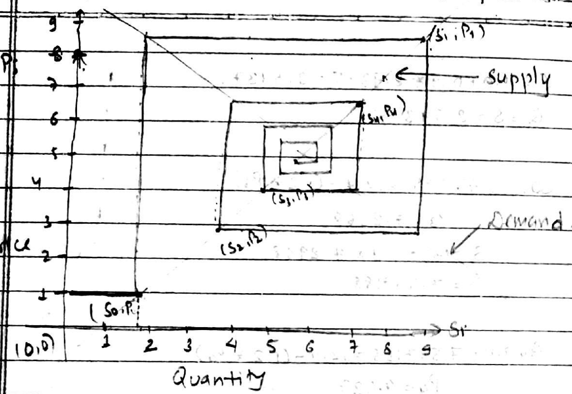
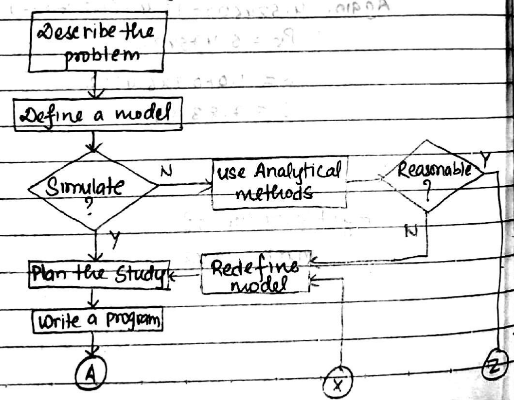


Fig: CobWeb graph.
Relation of demand and supply

P0 = 1
S0 = 1.9
P1 = 8.75
S1 = 8.83
P2 = 2.93
S2 = 3.69
P3 = 7.29
S3 = 4.56
P4 = 4.02
S4 = 9.62

Steps in a Simulation Study:-



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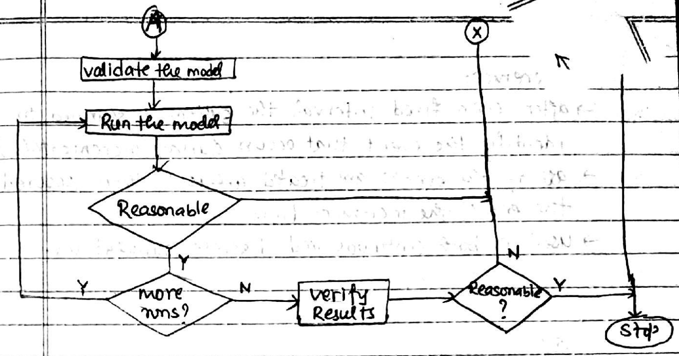

25th April.

Time Advancement Mechanism:

- is a component of any simulation model.
- while performing simulation after each event is executed, the time goes forward & find the state of the system at a new point of time.
- keeps track of passed time & end the action or experiment at the end of the specific simulation period.
- also called time flow mechanism, timing routines or simulation executives.

Two types:

(1) Fixed Time Increment Methods:

- also called time oriented simulation.
- a timer is simulated which is advanced by a fixed time

→ After each increment each event type is checked to see if it should have completed during the time of the last increment.

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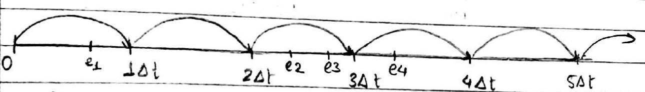
increment.

→ after each fixed interval, the system is scanned to identify the event that occurs during incremental time.

→ all of the events are treated ~~are~~ as if they occured at the end of the increment time.

→ used in both continuos and discrete simulations.



[ei (i=1,2,3,...) is the actual time of occurence of the ith event]

(2) Next Event Increment Methods:

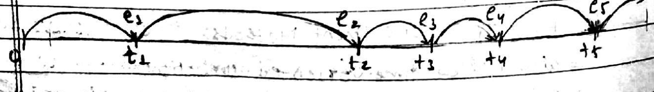
→ also called Event Oriented simulation

→ timer is advanced to event to event.

→ state of the system is updated at each event.

→ used in all discrete-event simulation models.

→ here the simulation clock is initialized to zero and the times of occurence of future events are determined.



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Queuing Model:

→ Waiting lines or queues are a common site in Real Life.

→ This situation occurs because:

(1) There is too much demand on the service facility. So, customer or entities or events have to wait for getting services

(2) There is to less demand, where service facility have to wait for the entities or customers to come.

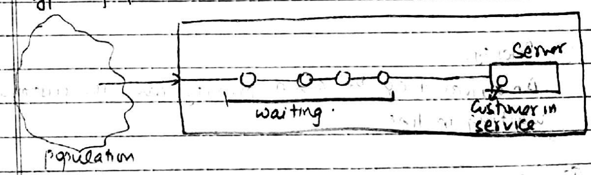
Components:

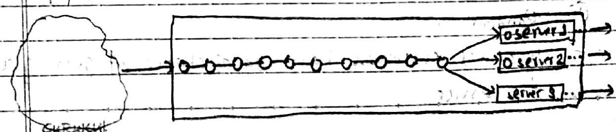
(1) Calling sources: Population from where customer are drawn or called.

(2) Waiting Line or queue: The number of customers waiting to be scared.

(3) Service facility: The number of service (counter) channels.

Types of Queues:





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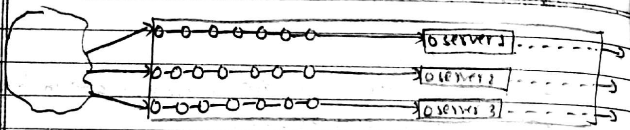

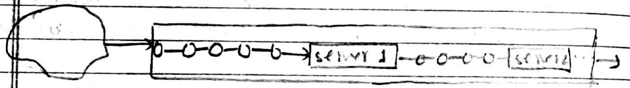

29th July:

Queue Discipline:

→ It determines how the customers are selected from the queue for services

① First-In First Out (FIFO):

→ Also called as First ~~In~~ Come First served

→ Customers are served in the order of their arivals.

② Last-In First out (LIFO):

Here the last arriver is first served.

③ Priority:

- An arrival may be given priority over the customer waiting in line

④ Random.

- When all customers have equal chance of getting selected for service.

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① FIFO

e.g:- Queue in ticket counter.

② LIFO

e.g:- Stack.

③ Priority

e.g:- Hospitals.

④ Random

e.g: Food Counter

Measures of queues:-

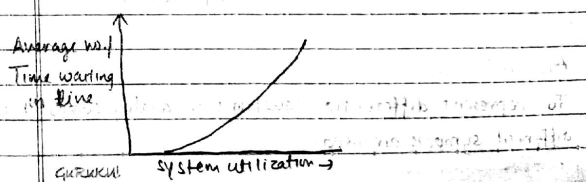
The performance of a queuing system can be evaluated in terms of a number of response parameters, as follows:

1) average no. of customers in the queue or in the system.

2) average waiting time of a customer in the queue or in the system.

3) system utilization

4) The cost of waiting time & idle time.



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System utilization factor (s) $= \frac{\lambda}{\mu}$ in single server

$(s) = \frac{\lambda}{\mu n}$ in multiple server.

λ = average arrival rate
μ = average service rate.

Continuos System:

→ A continuos system is one in which the predominant activities of the system cause smooth changes in the attributes of the system entities.

→ When such system is modeled mathematically, the variables of the models representing the attributes are controlled by continuos functions.

→ mathematical model consists of differential equations.

* Differential Equations:

E.g:- $M\ddot{x} + D\dot{x} + kx = kF(t)$

M, D, K = constants
x = dependent variable
t = independent variable

Analog Computer

To represent differential equations in analog computer many different symbols are used.

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Assignment RL circuit $E(t) = R\dot{q} + L\ddot{q}$.
RC " $E(t) = R\dot{q} + q/c$

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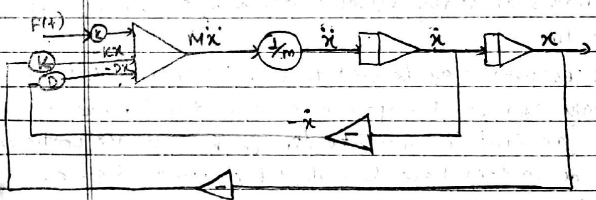
summer ( ) = for adding, sign changer.

→ scale factor

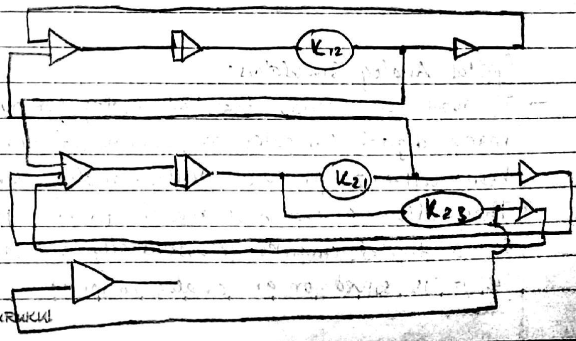
Integrator

$M\ddot{x} = (kF(t) - D\dot{x} - kx)$

$\ddot{x} = \frac{1}{m}(\quad)$



2nd May. Draw Analog Computer



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$\dot{x}_1 = -k_{12}x_1 + k_{21}x_2$

$\dot{x}_2 = k_{12}x_1 + (k_{21}+k_{23})x_2$

$\dot{x}_3 = k_{23}x_2$

Hybrid Computers:-

→ Hybrid computers describe the combination of ~~of~~ transition of analog-computer elements giving smooth, continuous outputs and elements carrying out such non-linear digital operations as storing values, switching & performing logical operations.

- This computer may be used to simulate systems that are mainly continuous do-infact have some digital elements. E.g:- an artificial satellite, for which both the continuous but do in fact have some digital elements. E.g.- an artificial satellite for which both the continuous eq^n of motion and the digital control system.

Digital Analog Simulators: (analog – analog → digital)

→ To avoid the disadvantages of analog computer, many digital computer programming languages have been written to produce digital-analog simulators.

→ They allow a continuous model to be programmed on a digital computer in essentially the same way as it is solved on an analog computer.

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→ The languages contain micro-instruction that carry out the action of adder, integrators & sign changers.

CSSL (Continuous System Simulation Language)

→ They use the familiar statement-type of input for digital computer, allowing a ~~progra~~ problem to be programmed directly from the equation of a mathematical model.

→ CSSL include a variety of algebric & logical expressions to describe the relations between variables.

CSMP-III (Continuous System Modeling Program Version-3)

→ A CSMP-III program is constructed from three general type of statements:

① Structural statements:

It defines the model. They consist of FORTRAN like statements and functional blocks designed for operations that frequently occur in the model definitions.

9th May ② Data statements:

- which assigns numerical values to parameters, constants, and initial conditions.

③ Control statements:

- which specify options in the assembly and execution of the program, and the choice of output.

Structural statement can make use of the operation of

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addition, subtraction, multiplication, division and exponential using the same notation and rules as used in FORTRAN.

E.g:- $X = \frac{6Y}{W} + (Z-2)^2$

$= X = 6.0 * Y/W + (Z-2.0) ** 2$ square

Y = INTGRL(IC, X) $\quad Y = \int_0^1 x\,dt + IC$

Y = EXP(X) $\quad Y = e^x$

Y = SIN(X) $\quad Y = \sin(x)$

Y = ALOG(X) $\quad Y = \ln(x)$

Y = ABS(X) $\quad Y = |x|$

e.g:- $M\ddot{x} + D\dot{x} + kx = KF(t)$

CSMP-III Program.

```
TITLE Automobile Suspension System.
*
PARAM D = (5.656, 16.968, 39.592, 56.56, 113.12)
X2DOT = (1.0/M)*(K*F-K*X-D*XDOT).
XDOT = INTGRL(0.0, X2DOT)
```

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```
X = INTGRL (0.0, XDOT)
*
CONST M=2.0, F=1.0, K=400.0
TIMER DELT=0.005, FINITIM=1.5, PRDEL=0.05, OUTDEL=0.05
PRINT X, XDOT, X2DOT
PRTPLT X
LABEL DISPLACEMENT VERSUS TIME
END
STOP.
```

DELT: Integration Interval
FINTIM: Finish Time
PRDEL: Interval at which to print result
OUTDEL: Interval at which to print plot

Hybrid Simulation:

Systems are either continuos or discrete in nature. But, there are times when an analog and digital computer are combined to provide a hybrid simulation.

→ It is also possible that the system being simulated is an interconnection of continuos & discrete sub-systems, which can be modeled by an analog and digital computer being linked together.

→ Here, high-speed converters are needed to transform signals from one form to another.

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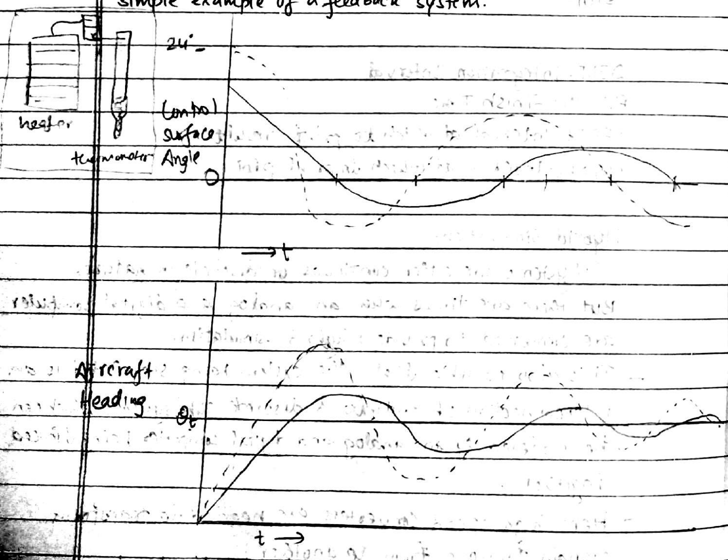
E.g:- Satellite System
Mobile Communication.

Feedback System

In this system, coupling occurs between the input and output of the system. The term feedback is used to describe the phenomenon.

A home heating system controlled by a thermostat is a simple example of a feedback system.



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Interactive System

→ is concerned with the response of a system over time.

→ the observation of the response over time is important for continuous system simulation.

→ A user can judge whether the output is developing satisfactory and if not interrupt a simulation run to try for better conditions.

Real Time Simulation

→ Here preliminary work could be avoided if an actual device can be used rather than constructing a model.

→ This approach followed when using real time simulation.

→ With this technique, actual devices, which are part of system, are used in conjunction with either a digital or hybrid computer, providing a simulation of the part of the system that do not exist or that cannot conveniently be used in experiment.

→ The computer model should run at the same rate as the actual physical system runs.
E.g:- In computer gaming.

→ The aerospace industry in particular makes extensive use of real time simulation.

→ There are devices called simulators, whose main functions is to provide human beings with a substitute for some environment or situation.

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Training of pilots before using actual plane.
~~Training for~~

Predator-Prey Model

Let $x(t)$ = Number of individual in the prey at the time t

$y(t)$ = Number of individual in the predator at time t.

Suppose now there is sufficient food (grain) for prey animals & in the absence of predator, here,

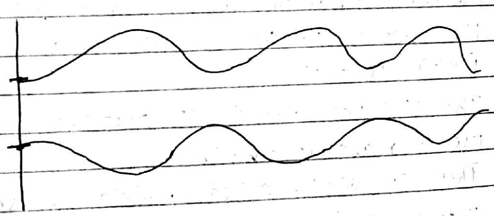
$r\,x(t)$ = rate of growth of prey animals.

(We take 'r' as: natural birth rate - natural death rate)
minus

The death rate of the prey due to interaction with predator can be assumed to be propotional to the product of two population sizes, $x(t) \cdot y(t)$

∴ The overall rate of change of the prey population is given by:

$$\frac{dx}{dt} = r\,x(t) - a \cdot x(t) \cdot y(t), \quad a = \text{positive constant of propotionality.}$$



Chapter:

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Random Numbers:

Random Numbers are a necessary basic ingredients in the simulation of almost all discrete system. Most computer languages have a subroutine, object or function that will generate a random numbers.

Similarly, simulation language generate random numbers that are used to generate event times & other random variables.

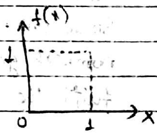
Properties:-

A sequence of random numbers has two important properties.

1. Uniformity
2. Independence.

Each random number is an independent sample drawn from a continuous uniform distribution between an interval 0 to 1. The probability density function is given by:

$$f(x) = \begin{cases} 1, & 0 \le x \le 1, \\ 0, & \text{otherwise} \end{cases}$$



The expected value of each Random Number R_i is given by

$$E(R) = \int_0^1 x \cdot dx = \frac{x^2}{2}\Big]_0^1 = \frac{1}{2}$$

and its variance is given by:

$$V(R) = \int_0^1 x^2\,dx - E(R)^2 = \frac{x^3}{3}\Big]_0^1 - \left(\tfrac{1}{2}\right)^2$$

$$= \frac{1}{3} - \frac{1}{4}$$

$$\therefore V(R) = \frac{1}{12}$$

Random Number Table.

Table gives random number produced by certain physical process. The digits have been arranged in column and rows.

Pseudo Random Numbers:

The word Pseudo means false. But here word pseudo implies that the random number are generated by using some kind of known arithmetic ~~process~~ operations. If the method is known, the set of random number are not truly random. So, we call pseudo random numbers. However, the pseudo random numbers generated by many computer routines vary closely fulfill the requirement of desired randomness.

Some problems while generating pseudo random numbers:

1. The generated ~~num~~ numbers may not be uniformly distributed
2. The generated number may not be continuous.

3. The mean and varience of generated numbers may be too high or low.
4. There may be cyclic pattern in the generated number.

Generation of Random Numbers:

Congruence Method or Residue Method:-

Expressed by: $r_{i+1} = (ar_i + b) \text{ MOD } m$

where a, b, m are constants.

r_{i+1} & r_i are the $(i+1)$ and ith term random numbers.

a) Mixed Multiplicative Congruential (MMC) method

$r_{i+1} = (ar_i + b) \text{ MOD } m$

Given,

$a = 13$

$b = 1$

$m = 19$

$r_0 = 1$

$r_1 = (a r_0 + b) \bmod m$

$= (13 \times 1 + 1) \bmod 19$

$= 14$

$r_3 = 13 \times 12 +$

$r_2 = (13 \times 14 + 1) \bmod 19$

$= (182 + 1) \bmod 19$

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$r_3 = 3$
$r_4 = 2$
$r_5 = 6$
$r_6 = 1$
$r_7 = 12$

b) Multiplicative congruential (MC) method

$$r_{i+1} = a r_i \bmod m$$

find same $r_0, r_1, r_2, r_3 \ldots$

c) Additive

$$r_{i+1} = (r_i + b) \bmod m$$

Testing of Randomness:

Uniformity Testing

(i) The Kolmogorov-Smirnov Test

(ii) Chi-squared test

(i) The Kolmogorov-Smirnov Test

Steps:

1) Rank the data from the smallest to largest

Let $R(i)$ denote ith smallest observation

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$R(1) \leq R(2) \leq R(3) \leq \ldots \leq R(N)$

3) Calculate:

$$D = Max\{D^+, D^-\}$$

2) Calculate

$$D^+ = max\{i/N - R(i)\}$$
$$D^- = max\{R(i) - \frac{i-1}{N}\}$$

4) Determine the critical value from standard table chart.

5) If the sample statistic D is greater than the critical value D_α then it is rejected. If D is smaller than D_α then it is accepted.

Q) Given Random numbers 0.44, 0.81, 0.14, 0.05, 0.93. Perform a K-S test with a level of significance α is 0.05.

⇒ Given, N = 5

	i = 1	i = 2	i = 3	i = 4	i = 5
$R(i)$	0.05	0.14	0.44	0.81	0.93
i/N	1/5	2/5	3/5	4/5	5/5
$D^+ = i/N - R(i)$	0.15	0.26	0.16	−0.01	0.07
$D^- = R(i) - \frac{i-1}{N}$	0.05	−0.06	0.04	0.21	0.13

$D^+ = 0.26$

$D^- = 0.21$

So,

$D = 0.26$

critical value $D_{0.05} = 0.565$ (from table)

$D < D_{0.05}$ So accepted.

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Chi-squared Test: (Uniformity Testing)

→ The chi-square test uses the sample statistics:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

O_i = Observed number in the i^{th} class

E_i = Expected number in the ith class

n = number of class.

Degree of Freedom (DoF) = n-1

Q) Use the chi-square test with 2 of 95% to test the data shown below are uniformly distributed.

23-34

10	63	1	64	18	8	9	1	17	21
31	40	99	20	30	55	2	27	47	58
81	3	7	9	11	22	72	87	92	9
4	16	23	38	41	53	68	71	88	92
3	33	24	7	53	59	25	13	46	16
75	2	34	58	8	24	54	26	14	92
57	73	3	23	35	9	45	17	55	15
81	22	28	4	36	18	10	37	30	72
21	83	27	43	5	19	32	11	91	95
14	41	84	29	70	6	93	31	12	56

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Interval	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1 - (1-10)	21	10	11	121	121/10 = 12.1
2 - (11-20)	15	10	5	25	2.5
3 (21-30)	15	10	5	25	2.5
4 (31-40)	10	10	0	0	0
5 (41-50)	6	10	-4	16	1.6
6 (51-60)	10	10	0	0	0
7 (61-70)	3	10	-7	49	4.9
8 (71-80)	6	10	-4	16	1.6
9 (81-90)	7	10	-3	9	0.9
10 (91-100)	7	10	-3	9	0.9
	100		0		27

interval no

100-95 = 5%

$\frac{5}{100} = 0.05$ — (Table value)

0.10, 0.20, 0.01, 0.11

DoF = 10-1 = 9

Chi-Square Distribution with degree at 0.05

D is 16.919

D < Table value

D > T.V → Reject.

$$E_i = \frac{\text{total no. of pairing}}{\text{total no. of class}} = \frac{99}{9} = 11$$

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Testing of Independence (Auto correlation)

Consider 10×10 table.

make a group of 25, 25 or 33, 33. but make a bigger gap

(0 − 33), (34 − 66), (67 − 100)

(10, 63), (63, 1), (1, 64) . . . (21, 31), . . . (12, 56)

R_1 R_2 R_1 R_2 R_2 R_3

Class	Count	Frequency (O_i)	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
$R_1 \le 33 - R_2 \le 33$	1+1+1+1+1+1+ 1+1+1+1+1+1+1+1+	27	11	256	
$R_1 \le 66 - R_2 \le 33$	1+1+1+1+1+1+1 +1+1+1+1+1+1+1	16	11	25	
$R_1 \le 100 - R_2 \le 33$	1+1+1+1+1+1+1 +1+1+1	10	11	1	
$R_1 \le 33 - R_2 \le 66$	1+1+1+1+1+1+ 1+1+1+1+1+	19	11	64	
$R_1 \le 66 - R_2 \le 66$	1+1+1+1+1+	[illegible]	11	36	
$R_1 \le 100 - R_2 \le 66$	1+1+	2	11	81	
$R_1 \le 33 - R_2 \le 100$	[illegible]	8	11	9	
$R_1 \le 66 - R_2 \le 100$	[illegible]	[illegible]	11	36	
$R_1 \le 100 - R_2 \le 100$	[illegible]	[illegible]	11	16	
		99		524	48

$DoF = 9 - 2 = 7$ (R_1 & R_2)

$\alpha = 0.5$

1+1+1+1+1+1+1+1+1+1+1+1+1+1+1

1+1+1+1+1+1+1+1

1+1+1+

A 67 ✓

D

$n - 1$ ($n - 2$)

$9 - 2 = \boxed{7}$

α < D → Acc.

→ Rej.

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Discrete System Simulation

→ Discrete Events: A number used to represent some aspect of the system state is called a state description.

→ We define discrete event as a set of circumstances that cause an instanteneous change in one or more system state descriptor.

Representation of Time:

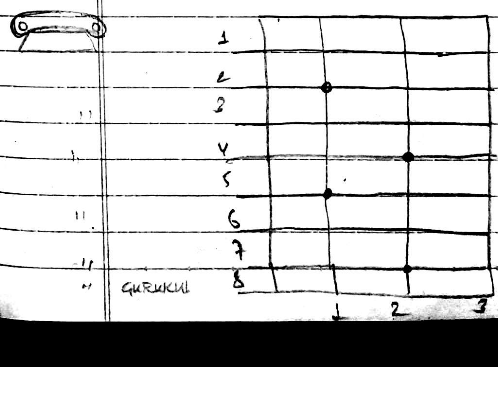
→ The passage of time is recorded by a number & subsequent indicates how many units of simulation time have passed since the beginning of the simulation.

Two basic method exists for updating clock time.

1. To advance the clock to the time at which next event due to occur. This is referred as event oriented.
2. To advance clock by small (uniform) interval of time.

20th May

Simulation of a Telephone system.



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a) System state 1

Next call:

From	To	Length
1	7	20

Arrival Time: 1057

Lines	
1	0
2	1
3	0
4	1
5	1
6	0
7	1
8	0

Max No	3
In use	2

Clock: 1027

Call in progress:

From	To	End
4	7	1075
2	5	1053

Processed	Completed	Blocked	Busy
131	98	5	28

b) System state 2

Next call:

From	To	Length
1	7	20

Arrival Time: 1057

Lines	
1	0
2	0
3	0
4	1
5	0
6	0
7	1
8	0

Links

Max no	3
In use	1

Clock: 1053

From	To	End
4	7	1075

Processed	Completed	Blocked	Busy
132	99	5	28

Next call:

From	To	length
3	6	98

Arrival Time: 1063

c) System state 3

Lost call system

Lines	
1	0
2	0
3	0
4	1
5	0
6	0
7	1
8	0

Links

Max no	3
In use	1

Clock: 1057

From	To	End
4	7	1075

Processed	Completed	Blocked	Busy
132	99	5	29

Next call:

From	To	length
1	5	132

Arrival Time: 1082

d) System state 4

Links	
1	0
2	0
3	1
4	1
5	0
6	1
7	1
8	0

Links

Max no	3
In use	2

Clock: 1063

From	To	End
3	6	1161
4	7	1075

1063
+98
1161

Processed	Completed	Blocked	Busy
133	99	5	29

A>

Next Call

From	To	length
3	6	198

Arrival Time: 1063

Lines	
1	0
2	0
3	0
4	1
5	0
6	0
7	1
8	0

Links

Max no	3
In use	1

Delay call

From	To	length
1	7	20

Clock: 1057

From	To	End
4	7	1075

Processed	Completed	Blocked	Busy
133	99	5	29

Fig: System state - 3A [From lost call]

Delay

B>

Next Call

From	To	length
1	5	132

Arrival Time: 1083

Lines	
1	0
2	0
3	1
4	1
5	0
6	1
7	1
8	0

Links

Max No	3
In Use	2

Delay call

From	To	length
1	7	20

Clock: 1063

From	To	End
3	6	1161
4	7	1075

System state 4A

Processed	Completed	Blocked	Busy
133	99	5	29

C>

Next Call

From	To	Length
1	5	132

Arrival Time: 1083

Lines	
1	1
2	0
3	1
4	0
5	0
6	1
7	1
8	0

Links

Max No	3
In use	2

Delay call

From	To	length

Clock: 1075

From	To	End
3	6	1161
1	7	1095

System state 5A

Procured	Completed	Blocked	Busy
133	100	5	29

22nd May

Gathering Statistics:

Most simulation programming system includes a report generator to print out statistics gathered during the run. The exact statistics required from a model depends upon the study being performed.

The commonly needed statistics are:-

a) Counts: It gives the number of entities of a particular type or the number of times some event occured.

b) Summary measures: It measures such as extreme values, mean values and standard deviation.

c) Utilization: It is defined as the function (or percentage) of time some entity is engaged.

d) Occupancy: It is defined as the fraction (or percentage) of a group of entities in use on the average.

e) Distribution: It is of important variable such as queue lengths or waiting times.

f) Transit times: It is defined as the time taken for an entity to move from one part of the system to another part.

[2068, 2069, 2070, 2071]

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Chapter:

B.D → Blocks → Represent activities
→ lines joining the blocks → Indicate the sequence in which the activities can be exec.

General Purpose Simulation System (GPSS):-

→ It is a process oriented language designed for discrete event modeling.

→ It was designed by Geoffrey Gordon in 1961 at IBM

→ It has in-built mechanisms to collect statistics, analyze them, produce tabulated outputs & execute a number of different tasks.

GPSS Block Diagram

→ The development of a simulation model in GPSS is a block by block construction.

→ A set of standard block is arranged in the form of block diagram that represent the flow of entities through the various parts of the system.

→ Each block represent a step in a action of the system, and links, representing the sequence of events that can occur.

→ Each block must be assigned a block time.

→ Each block can be used any number of times in a block diagram of a system.

→ Total number of blocks used in a system should not be more than 2047

→ There are about 48 specific block types.

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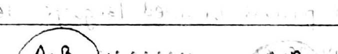
Block Types:

① Generate:

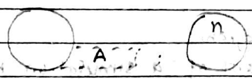


→ used for creation of transaction

→ employs action time.

syntax: GENERATE A, B.

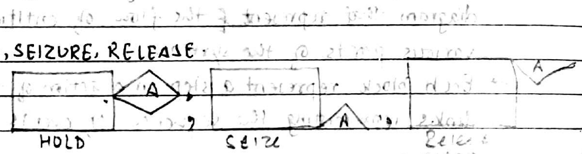
② Terminate:



→ is concerned with the removal of the transactions.

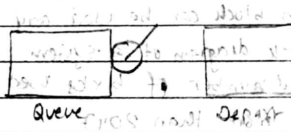
Syntax: TERMINATE A

③ HOLD, SEIZURE, RELEASE:



→ are concerned with the use of facilities

④ QUEUE & DEPART:



→ Queue Symbol is used at the position where over crowding in flow of transaction is expected.

→ Depart is used for the release of queued object.

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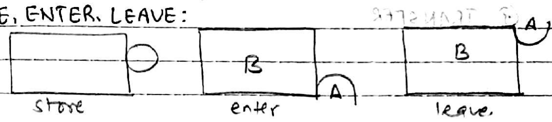
⑤ ADVANCED: $\boxed{A, B}$

→ It is concerned with the flow of transactions.

→ It represent any action required time.

23rd May.

⑥ STORE, ENTER, LEAVE:



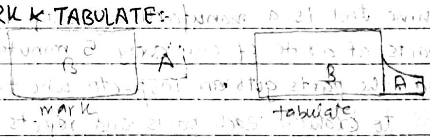
→ The flag on side of each block is for indicating the number of store.

→ STORE block allows a transaction to occupy space in the system.

→ ENTER & LEAVE block allows a transaction to occupy a space or vacate a space in store.

→ ENTER will allow a transaction to enter a store if space is available

⑦ MARK & TABULATE:



→ These blocks are used to measure the time spent by a transaction in the system or part of the system.

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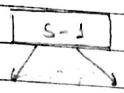
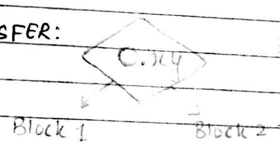
⑧ BRANCH:



- It is concerned with the movement of transaction & 127 exists are possible

⑨ TRANSFER:



→ is used when the transaction are moved in a non-sequential manner.

e.g:- TRANSFER 0.xy, Block1, Block 2.

⑩ SIMULATE. START. END:

→ SIMULATE is generally first statement of the program

→ START will start the simulation

→ END is always the last statement.

Q> A machine tool is a manufacturing shop is turning out parts at a rate of one every 5 minutes. As they are finished, the parts goto an Inspector who takes 4±3 minutes to examine each parts and rejects about 10% of the parts. Each part will be represented by one transaction and the time unit selected for the problem will be 1 minute. Draw GPSS block diagram & write

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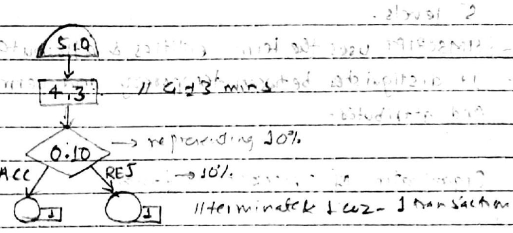
File new model.
student GPSS eg.

edit s Insert gps.

GPSS program.



SIMULATE
GENERATE 5,0
ADVANCE 4,3 denotes this.
TRANSFER 0.1, ACC, REJ and this is remaining part.
ACC TERMINATE 1
REJ TERMINATE 1
START 100
END

26th May SIMSCRIPT

SIMSCRIPT is a very widely used language for simulating discrete systems.

- There are different versions of SIMSCRIPT
- The language be considered as more than just a simulation language since it can be applied to a general programming problems.
- The description of the language given is organized in

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5 levels.

- SIMSCRIPT uses the term entities & attributes.
- It distiguishes between temporary and permanent entities and attributes.

Organization of SIMSCRIPT Program:

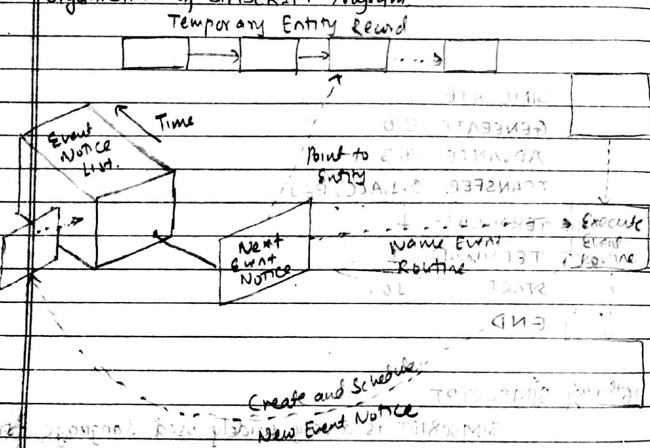

Fig: SIMSCRIPT Execution Cycle.

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Name - for Documentation purpose name can be of any length whereas for specific machine implementation the name should consists certain no. of initial char. (typically e.g. ...).

Names and Labels:

- The user must describe all the entities by giving name to each entity & its attributes.
- Name may consists of any combination of letters & digits provided there is at least one letter.
- We can use dot operator also.
- E.g:- PERSON
  PERSON_AGE
- Labels used for identifying programming statement similarly consists of any combination of letters & numbers, without the restriction that at least one be a letter.

SIMSCRIPT statement

- are written in a form closely resembling the English Language
- A statement calling 'n' lines of the text to be printed could be written in either of the ways.

PRINT n LINES AS FOLLOWS

PRINT n LINES THUS

PRINT n LINES WITH X AND Y LIKE THIS

X&Y Represent the variables or expression to be printed.

E.g: A 3 digit Integer indicated by ***

" 4 " real number " " ***.*

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Defining the System:

- A SIMSCRIPT program begins with a preamble. that defines the system. structure and establishes the condition under which the simulation will be run. Variables can be real, integer or alphabetical.
- It is possible to pack variables so that more than one value is placed in one word.
- It is possible to process character strings. called text mode of operation

Telephone System Model:

1. "TELEPHONE SYSTEM MODEL-I"
2. PREAMBLE
3. NORMALLY, MODE IS INTEGER
4. EVENT NOTICES INCLUDE ARRIVAL AND CLOSING
5. EVERY DISCONNECT HAS A DIS.CALL
6. TEMPORARY ENTITIES
7. EVERY CALL HAS AN ORIGIN AND A DESTINATION
8. DEFINE LINK.IN.USE, MAX.LINKS, BLOCKED, BUSY, FINISHED
9. AND STOP.TIME AS VARIABLES
10. DEFINE INTER.ARRIVAL.TIME AND MEAN.LENGTH AS REAL VARIABLES
11. PERMANENT ENTITIES
12. EVERY TLINE HAS A STATE
13. DEFINE SECS TO MEAN 1/60 MINUTES'
14. END

Pg 256
THE MAIN ROUTINE
(Defining First Figure of Telephone System)

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Analysis of Simulation output

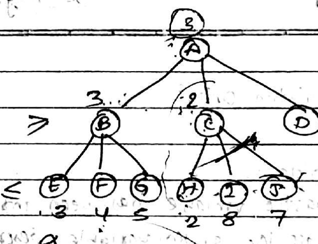
① Nature of the problem:

→ Once the stochastic variable has been introduced into a simulation model, all the system variable describing the system behaviour also becomes stochastic.

→ Many observation of variable value must be made in order to make a statistical estimates of its true value

→ Because of the experimental nature of the system simulation, it seems natural to the attempt applying these methods to simulation results.

(From Photocopy Paper Remaining)



	α	β
	$-\infty$	$+\infty$
E(3)	$-\infty$	3
F(4)	$-\infty$	3
G(5)	$-\infty$	3
(2)	$-\infty$	2

Date: Page:

Simulation Run Statistics:-

Let us see an examples that explains this analysis in simple way of ~~simple~~ single server system.

→ single server → service time → exponential dis:

→ FIFO Queing displine → interarrival time → expon " "

In simulation run the simplest way to find sample mean.

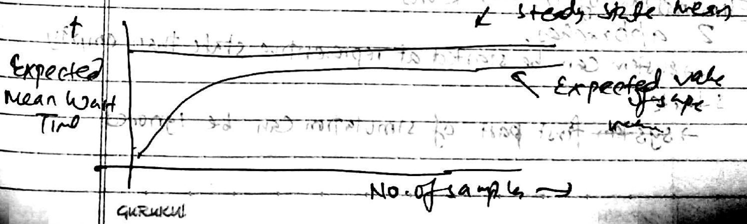
$$\rightarrow \bar{x}(n) = \frac{1}{n} \sum_{n-1}^{n} x_i \qquad (x_i, i = 1, 2, 2, - - n)$$

$$\text{or } \bar{x}(n) = \frac{1}{n} \sum_{n-1}^{n} x_i, \quad i = 1, 2, 2, - - n$$

2 problems:-

① → waiting time is not independent coz when waiting ~~time~~ line forms, waiting time depends on the waiting time of predecessor

② Distribution may not be stationary coz simulation run is started with the system in some initial state, then in idle state in which no entities are waiting & thus early arrivals have high probability of obtaining service quickly. sample mean ~~included~~ that includes the early arrivals ~~will~~ is biased



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Replication of runs:-

→ One problem during simulation run is that ~~while~~ while measuring statistics the results are dependent.

→ But in simulation indep result is required

→ Soln to this problem (obtain indep result) is ~~dup~~ replicate repeating simulation

→ Repeating same experiment with diff random no of size n give independent result of sample $\bar{X}_i(x)$.

→ Suppose:

x_{ij} = ith observation for jth run

Then $\bar{x}_j(n) = \frac{1}{n}\sum_{i=1}^{n} x_{ij}$

Then variance is

$$s_j^2(n) = \frac{1}{n-1}\sum_{i=1}^{n}(x_{ij} - \bar{x}_j(n))^2$$

Now combining results of p indep measurement gives

$$\bar{X} = \frac{1}{P}\sum_{j=1}^{P}\bar{x}_j(n) \quad \bar{x} = \sum_{j=1}^{P}\bar{x}_j(n)$$

$$s^2 = \frac{1}{P}\sum_{j=1}^{P} s_j^2(n) \quad s^2 = \sum_{j=1}^{P} s_j^2(n)$$

Elimination of Initial Runs

2 approaches.

→ System can be started at representive state than empty state

→ ~~system~~ first part of simulation can be ignored

(1) It is necessary to know the steady state destinction. so we can select initial state distinction. In simulation process it is necessary to select better initial condition, ~~for there may be information available on expected~~ cond^n thus removing initial bias.

→ 2nd part is common approach.

→ Here the first part of initial section is ~~remo~~ containing highly biased result is eliminated. The process ~~contr~~ starts at an idle state and continues until a certain period of time. The entities existing until that pt are left as they are and this pt is the restarting pt ~~of~~ ~~to~~ for other to repeat simulation run & statistic is also being gathered from here.

Disadv:

→ No simple rules can determine how long for an interval should be eliminated.

Simulink Lib.

Sink → O/P, Source → I/P

File → New → Model

Source → clk.

Commonly used blocks → gain.

gain → 2t. (2) Parameter.

math operation → trignometric function $\boxed{Co}$

Commonly used blocks → sum.

Commonly used blocks → $\frac{1}{s}$ (Integrator

Sink → scope → O/P.

Run → Scope double click.

To change the graph click on graph and Autoscale

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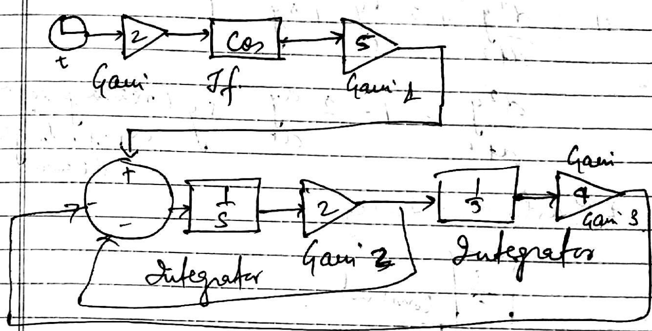
$$\frac{d^2x}{dt^2} = 5\cos(2t) - 2\frac{dx}{dt} - 4x$$



→ Random No. Congruencial Method.

3 types. Cmo Prog.

→ GPSS

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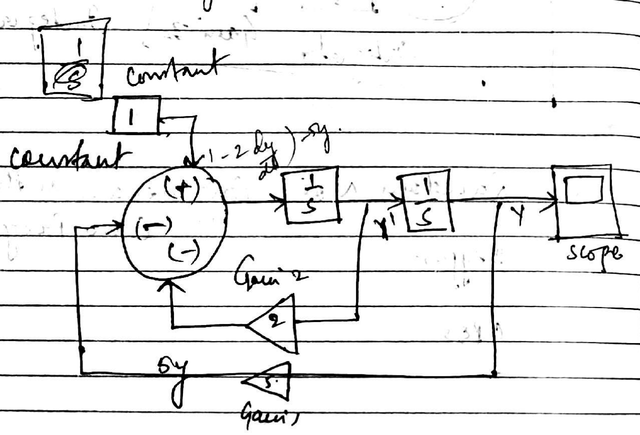
Integrator → $\frac{1}{s}$

1) $$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 1$$

$$\frac{d^2y}{dt^2} = 1 - 2\frac{dy}{dt} - 5y$$ (rearrange eqn. exp. for highest derivative)

2nd order → 2 integrator



eliminate END & START

file new → Program

1) Edit → Insert GPSS block → for system generated program

2) Command → Create simulation → Ready (By default should be shown) minimize

3) window → simulation window → Block window
↓
Table

Command start 100 (value) (no. of times it will run)
↓
o/p screen analysis

4th April

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c) Mixed congruential method of generating random numbers a, b & m are constant the values of which are suitably selected and entered nn is the random numbers to be generated. Seed is the starting random no. which is also to be entered.

→
```
int a,b,k,i,j,m,nn, seed r[50];
pf ("\n Enter int values for a,b,m = ");
sf ("%d %d %d", &a, &b, &m);
pf ("\n Enter SEED value = ");
sf ("%d", & seed);
pf ("\n Enter no. of random no. to be generated = ");
sf ("%d", &nn);          seed=145
r[0] = seed;   // a= 21, b=53, m=1000
for (i=1; i<=nn; i++)    nn=30
{
  r[i] = (a*r(i-1)+b) %m;
  printf ("%4d", r[i]);
}
getch ();
}
```

O/P

145 98 111 384 117 570 768 76 649
682 375 918 541 414 747 740 599 506
679 312 605
936 709 942
27 366 779
83 796

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```
Generate 4,0
Queue system
Queue ins_queue
Seize inspector
Depart ins_queue
Advance 5,2
Release inspector
Depart system
Transfer 0.15, ACC, Rej
Acc Terminate 1
Rej Terminate 1
```

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Chapter -4

ANYLISIS OF SIMULATION OUTPUT

In the previous chapters we have learn how to produce a model of a system, how to simulate and produce a computer program for simulating the system such as predator prey system. By which of this program, we can performed the desired experiment on the system being studied and can analyze the output. In those programs, we have a freedom to specify initial condition, run length or replication runs in order to analyze the output simulation.

A large body statistical method has been developing over the years to analyze results in science, engineering and other fields where experimental observation are made. So, because of experimental measure of system of simulation for these statistical methods can be adapted to simulation results to analyze.

The newly developing statistical methodology concerns;

1) To ensure that the statistical estimates are consistent, meaning that as the sample size increases the estimate tends to true value.
2) To control biasing in measure of both new values of variance. Bias causes the distinction of estimate to differ significantly from the true population statistics, even though the estimate may be consistent.
3) To develop sequential testing methods, to determine how long a simulation should be run in order to obtain a confidence in its return.

ESTIMATION METHOD

- Statistical methods are commonly used on random variable. Usually a random variable is drawn from an infinite population with a finite mean 'μ' and finite variance 'σ²'.

- These random variable are independently and identically distributed (i.e. iid variables).

Let, x_i=iid random variables. (i=1, 2,......, n), then according central limit theorem and applying transformation, approximate normal variance,

$$Z=\frac{\sum_{i=1}^{n} x_i - n\mu}{\sqrt{n}\sigma}$$

Var.> In Prob. theory & statistics, Var. is the expectation of the squared deviation of a random variable from its mean and it informally measures how far a set of random nos are spread out from their mean.

$$Z=\frac{\dot{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

Where $\dot{x}$ = sample mean

- It can be shown to be a consistent estimator for the mean of population from which the sample is done.
- Since the sample mean is some of random variable, it is itself a random variable. So a confidence interval about its computed value needs to be established.
- The probability density function on the standard normal variable (Z) is shown in figure (1).

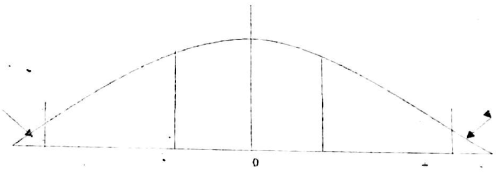


Fig. (1)

- The integral formed - ∞ to μ is the probability that Z is less than or equal to μ and is denoted by ϕ (u).
- Let us consider the value of u ($u_{\alpha/2}$) such that ϕ(u) = 1- α/2 where α = some constant < 1.
- Then probability of Z for Z> $u_{\alpha/2}$ = α/2.
- Probability of Z for ($-u_{\alpha/2} \le Z \le + u_{\alpha/2}$) = 1- α.
- In terms of sample mean μ, the probability statement can be written as

$$\text{Prob}\left\{\dot{x}+\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}} \ge \mu \ge \dot{x}-\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}\right\}=1-\alpha$$

Here the constant $1-\alpha$ is a confidence level (usually expressed in %) and the interval

$\dot{x}\pm\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}$ is the confidence interval.

Estimation population variance s2 (not σ^2 which is actual population variance) is given by:

$$s^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\dot{x})^2$$

Replacing $u_{\frac{\alpha}{2}}$ by $t_{n-2,\frac{\alpha}{2}}$, then the estimated variance s2, the confidence interval for $\dot{x}$ is given by

$$C=\dot{x}\pm\frac{s}{\sqrt{n}}t_{n-1,\alpha/2}$$

SIMULATION RUN STATISTICS

On every simulation run, some statistic are measure based on some assumption, for example - on establishing confidence interval it is assume that the observation are mutually independent and distinction from which they are drawn is stationary. But many statistics are interest in a simulation don't meet this condition.

Let us illustrate the problems that arise in measuring statistic from simulation run with the example of single server system.

Consider occurrence of arrivals has a Poisson distribution.

- The service time has an exponential distribution.
- The queuing discipline is FIFO
- The inter-arrival time is distributed exponentially
- System has a single server.

Then in a simulation run, the simplest way to estimate the mean waiting is to accumulate the waiting time of n successive entities and dividing it by 'n'. This gives sample mean denoted by $\dot{x}(n)$.

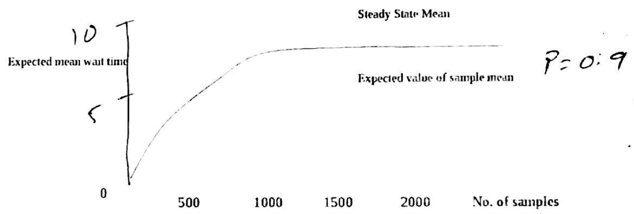
If x_i (i = 1, 2, 3, ...,n) are the individual waiting times, then

$$\dot{x}(n)=\frac{1}{n}\sum_{n-1}^{n}x_i$$

The 1st problem, here is that, the waiting times measure this way are not independent because whenever a waiting line forms, the waiting time of each entity on the line depends upon the waiting time of its predecessor (i.e. the entities are auto co-related).

The usual formula for estimating the mean value of the distribution remains on satisfactory estimate for the mean of auto co-related data. However the variance of auto-correlated data is not related to the population variance by simple expression $\frac{\sigma^2}{n}$ as occurs for independent data.

The 2nd problem is that the distribution may not be stationary; it is because a simulation run is started with the system in some initial state, frequently the idle state, in which no service is being given and no entities are waiting, thus the early arrivals have a more probability of obtaining service quickly. So a sample mean that includes the early arrivals will be biased. As the length of simulation run extended and the sample size increases, the effect of bias will be minimum. This is shown in below fig.



REPLICATION OF RUNS

One problem in measuring the statistic in the simulation run is that the results are dependent. But it is required, in simulation, to get the independent result. The one way of obtaining independent result is to repeat the simulation.

Repeating the experiment with different random numbers for the same sample size 'n' gives a set of independent determination of sample mean $\dot{x}(n)$.

Even though the distribution of sample mean depends upon the degree of auto correlation, this independent determination of sample mean can be used estimate the variance of distribution.

Suppose,

- Experiment is repeated p-times with independent random numbers.
- $x_{ij} = i^{th}$ observation of j^{th} run. Then estimates for:
- Sample mean $\dot{x}_j(n) = \frac{1}{n}\sum_{i=1}^{n} x_{ij}$
- Then estimate for variance for j^{th} run $s^2_j(n)$ is given by

$$x_{ij} - \dot{x}_j(n)$$

$$s^2_j(n) = \frac{1}{n-1}\sum_{i=1}^{n}$$

Now, combining the results of p independent measurement gives the following estimate for the mean waiting time $\dot{x}$, and variance s^2:

$$\dot{x} = \frac{1}{p}\sum_{j=1}^{p} \dot{x}_j(n)$$

$$s^2 = \frac{1}{p}\sum_{j=1}^{p} s^2_j(n)$$

Here, the value of $\dot{x}$ is an estimate for mean waiting time and the value of s^2 can be used to establish a confidence of intervals.

ELIMINATION OF INITIAL BIAS

There two general approaches that can be used to remove the initial bias;

1) The system can be started in more representative states rather than in the empty state.

2) The first part of the simulation run can be ignored.

- In the first approach, it is necessary to know the steady state distinction for the system and we then select the initial state distinction. In the study of simulation, particularly existing system, there may be information available on the expected condition which makes it feasible to select better initial condition and thus eliminating the initial bias.
- The second approach that is used to remove the initial bias is the most common approach. In this method, the initial section of the run which has highly bias (simulation) result is eliminated. First, the run is started from an idle state and stopped after a certain period of time (the time at which the bias is satisfactory). The entities existing in the system at that are left as they are and this point is the point of restart for other repeating simulation run. Then the run is restarted with statistics being gathered from the point of restarted. These approaches have following difficulties:

1) No simple rules can be given to decide how long an interval should be eliminated. For this we have to use some pilot run starting from the ideal state to judge how long the initial bias remains. These can be done by plotting the measured, statistics against the run length.
2) Another disadvantage of eliminating the first part of simulation run is that the estimate of variance will be based on less information affecting the establishing of confidence limit. These will then cause to increase in confidence internal size.