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CHAPTER - 1

INTRODUCTION

1.1 Data and Signal

1.1.1 Data

Data is an entity which conveys some meaning based on some mutually agreed rules or communication. For example, if a sender sends 01000001 to the receiver; it has no meaning unless the receiver understands. But if it is said that an ASCII character has been sent, the above string is 'A'. Hence, it can be considered as 'data'.

Data are of two types: *analog data* and *digital data*.

i. **Analog data:** Information that has continuous (infinite) values in some interval of time is analog data. Examples: Human voice, temperature, etc.

ii. **Digital data:** Information that has finite number of values within a certain time is digital data. Example: Information stored in memory of a computer.

1.1.2 Signal

Whenever data has to be sent over a communication medium, it has to be converted into signal. Since data cannot be sent as it is, through a transmission media; signal is an electric, electronic or optical representation of data which can be sent over a communication media. There are two types of signal: *analog signal* and *digital signal*.

1.2 Analog and Digital Signal

1.2.1 Analog signal

An *analog signal* is a signal with an amplitude (i.e., value of the signal at some fixed time) that varies continuously for all time. Electrical signals obtained from microphone, photo detector cell are examples of analog signal.

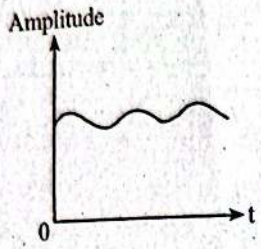


Fig.: Analog signal

1.2.2 Digital Signal

Digital signals are those signals that are obtained when discrete time signals are quantized and then coded. They have finite number of values over a period of time.

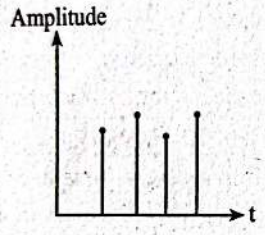


Fig.: Digital signal

1.3 Data Representation

Information today comes in different forms such as text, numbers, images, audio, and video.

i. **Text:**

In data communication, text is represented as a bit pattern, a sequence of bits (0s or 1s). Different sets of bit patterns have been designed to represent text symbols. Each set is called code and various coding techniques are Unicode, ASCII, etc.

ii. **Numbers:**

Numbers are also represented by bit pattern and used to simplify mathematical operation.

iii. **Images:**

In the simplest form, an image is composed of matrix of pixels (picture elements), where each pixel is a small dot and is assigned a bit pattern.

iv. **Audio:**

Audio refers to the recording or broadcasting of sound or music. Audio is by nature different from text, number, or images. It is continuous, not discrete.

v. **Video:**

Video refers to the recording or broadcasting of picture or movie. Video can either be produced as continuous entity or combination of images, each a discrete entity, arranged to convey the idea of motion.

1.3.1 Data Flow

Transmission mode between two devices can be either *simplex*, *half-duplex*, and *full-duplex* depending upon the flow the data.

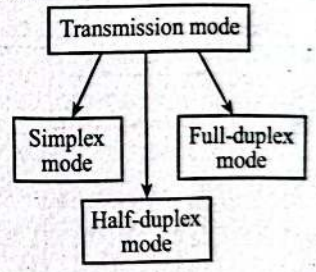


Fig.: Transmission mode

i. **Simplex Mode**

In simplex mode, the communication is unidirectional, as on a one-way street. Only one of the two devices on a link can transmit, the other can only receive. The simplex mode can use the entire capacity of the channel to send data in one direction. Example: Keyboard and traditional monitors. The keyboard can only introduce input, the monitor can only give the output.

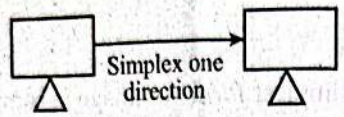


Fig.: Simplex mode

ii. Half-Duplex Mode

In half-duplex mode, each station can both transmit and receive, but not at the same time. When one device is sending, the other can only receive, and vice-versa. The half-duplex mode is used in cases where there is no need for communication in both direction at the same time. The entire capacity of the channel can be utilized for each direction.

Example: Walkie-talkie

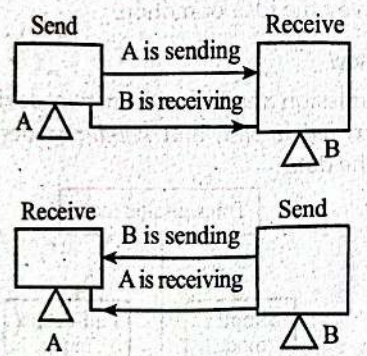


Fig.: Half-duplex mode

ii. **Full-Duplex Mode**

In full-duplex mode, both stations can transmit and receive simultaneously. In full-duplex mode, signals going in one direction share the capacity of the link with signals going in other direction, this sharing can occur in two ways: Either the link must contain two physically separate transmission paths, one for sending and other for receiving or the capacity is divided between signals travelling in both directions. Full-duplex mode is used when communication in both direction is required all the time. The capacity of

the channel, however must be divided between the two directions.

Example: Telephone network, in which there is communication between two persons by a telephone line, through which both can talk and listen at the same time.

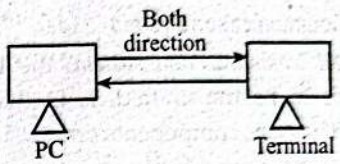


Fig.: Full-duplex mode

1.4 Communication System

In broad sense, the term "communication" refers to the sending receiving, and processing of information by electronic means. *Communication system* is a system designed to send information from a source generating that information to one (point-to-point communication) or more (broadcasting) receivers of that information.

1.4.1 Analog Communication System

In *analog communication system*, the baseband message, which is to be transmitted, is in the form of an analog signal. The basic components are:

i. **Transmitter:** This portion processes the message signal into a form suitable for transmission over the channel. Such an operation is called *modulation*.

ii. **Communication channel:** The function of the channel is to provide a physical connection between the transmitter output and the receiver input. Examples: wire, lines, optical fibers, microwave links, coaxial cables, etc.

iii. **Receiver:** This portion processes the received signal to get message signal. Such an operation is called *demodulation*.

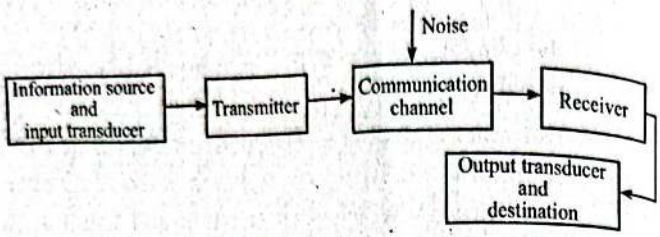


Fig.: Block diagram of an analog communication system

1.4.2 Digital Communication System

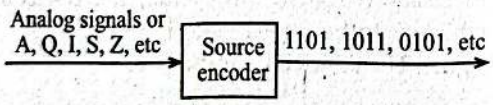
In *digital communication system*, the baseband message, which is to be transmitted, is in the form of a digital signal. The basic components are:

i. Source:

The source is where the information to be transmitted originates. This information/message may be available in digital form, as for instance, in the case of the output of a teletype system (sequence of symbols), or it may be available in an analog form (audio or video signal).

ii. Source encoder:

Source encoder converts the analog signals or sequence of symbols into binary sequence of 1s and 0s.



iii. Source decoder:

Source decoder converts the binary output of the channel decoder into its original analog form or sequence of symbols.

iv. Channel encoder:

The channel encoder adds some error control bits (redundancy) to the bit streams of source encoder output. This is done in order to provide some amount of error-correction capability to the data being transmitted.

v. Channel decoder:

The output sequence of digits from the channel demodulator are fed to the channel decoder. Using its knowledge of the type of coding performed by the channel encoder of the transmitting end, and using the redundancy introduced by the channel encoder, it produces as its output, the output of source coder of the transmitter with as few errors as possible.

vi. Channel modulator:

Channel modulator is intended to convert bit streams form channel encoder to electrical wave form suitable for transmission over the communication channel.

vii. Channel demodulator:

It converts received electrical signal into sequence for wired communication or free space for wireless communication.

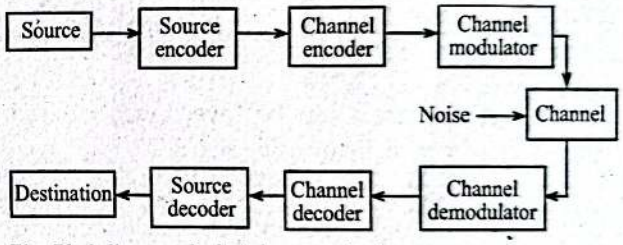


Fig.: Block diagram of a digital communication system

Advantages of digital communication:

As the signals are digitized, there are many advantages of digital communication over analog communication. These are:

- The effect of distortion, noise, and interference is much less in digital signals as they are less affected.
- Digital circuits are more reliable.
- Digital circuits are easy to design and cheaper than analog circuits.
- The hardware implementation in digital circuits, is more flexible than analog.

- The occurrence of cross-talk is very rare in digital communication.
- The signal is unaltered as the pulse needs a high disturbance to alter its properties, which is very difficult.
- Signal processing functions such as encryption and compression are employed in digital circuits to maintain the secrecy of the information.
- The probability of error occurrence is reduced by employing error detecting and error correcting codes.
- Spread spectrum technique is used to avoid signal jamming.
- Combining digital signals using time division multiplexing (TDM) is easier than combining analog signals using frequency division multiplexing (FDM).
- The configuring process of digital signals is easier than analog signals.
- Digital signals can be saved and retrieved more conveniently than analog signals.
- Many of the digital circuits have almost common encoding techniques and hence similar devices can be used for a number of purposes.
- The capacity of the channel is effectively utilized by digital signals.

However, there are some disadvantages of digital communication. These are:

- **Increased bandwidth:** The sampling of an analog waveform requires more bandwidth as samples are encoded in binary and transmitted with a pulse for every bit.
- **A/D and D/A conversion:** Because of the analog nature of voice, conversion between analog and digital forms is required. The electronics required to perform this task add additional cost to the digital systems.

1.4.3. Comparison of Analog and Digital Communication System

Parameters	Analog Communication	Digital Communication
Definiton	Analog communication is the technology which uses Analog signal for the transmission of information.	Digital communication is the technology which uses digital signal for the transmission of information.
Noise and distortion	Get affected by noise	Immune from noise and distortion
Error probability	Error probability is high due to parallax.	Error probability is low
Hardware	Hardware is complicated and less flexible than digital system.	Hardware is flexible and less complicated than analog system.
Cost	Low cost	High cost
Bandwidth requirement	Low bandwidth requirement	High bandwidth Requirement
Power requirement	High power is required	Low Power Requirement
Portability	Less portable as the components are heavy	More portable due to compact equipments.
Modulation used	Amplitude and angle modulation	Pulse coded modulation or PCM, DPCM etc.
Representation of signal	Analog signal can be represented by sine wave.	Digital signal is represented by square wave.

Parameters	Analog Communication	Digital Communication
Signal values	Consists of continuous values	Consists of discrete values
Example of signal	Analog signal comprises of voice, sound, etc.	Digital signals are used in computers.

1.4.4 Block Diagram of Analog and Digital Communication Systems on the Basis of Various Transmission Modes

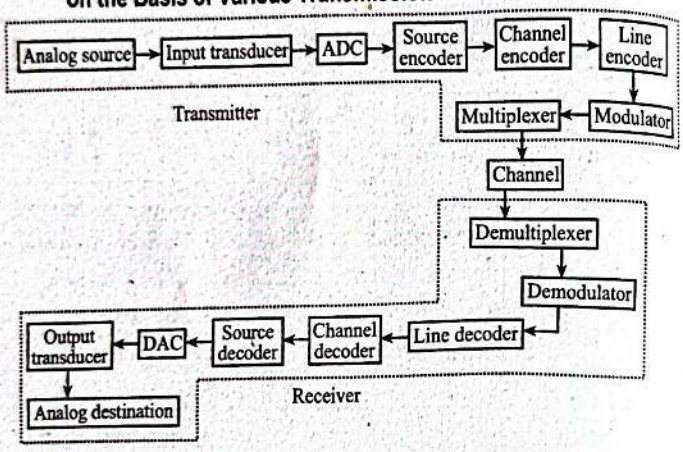


Fig.: Half-duplex digital communication system

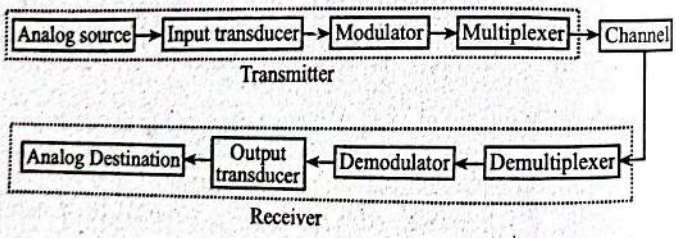


Fig.: Half-duplex analog communication system

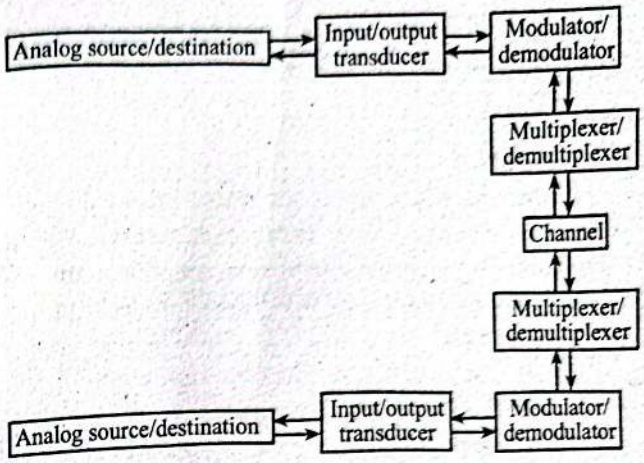


Fig.: Full-duplex analog communication system

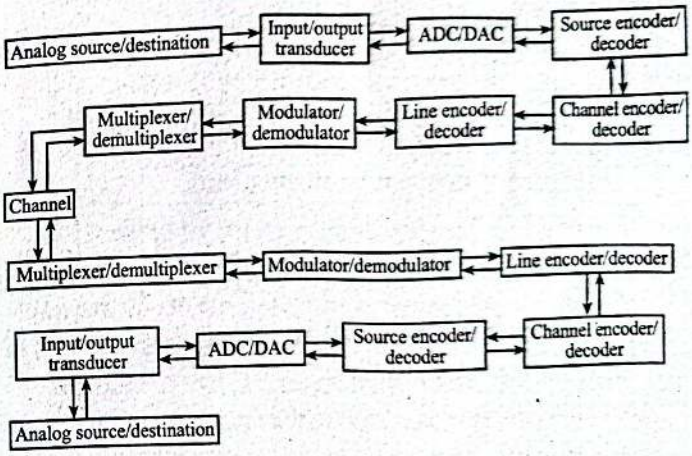


Fig.: Full-duplex digital communication system

1.5 Transmission Impairments

To send the data, we have to send it through a transmission medium which is not perfect or ideal. This imperfection causes transmission impairments. It means that the signal at the beginning of the medium is not the same as the signal at the end of the medium. The major causes of impairment are:

i. Attenuation

ii. Distortion

iii. Noise

1.5.1 Attenuation

It is the loss of energy as the signal passes through a medium. This is mainly because of resistance of the medium. To overcome attenuation, amplifiers are used. The loss or gain of energy is expressed in decibel (dB). It is calculated as:

$$(\text{Loss/Gain})_{dB} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

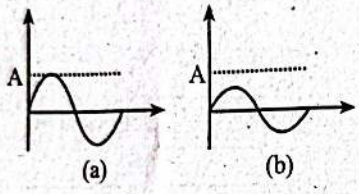


Fig.: Attenuation (a) original signal (b) attenuated signal

Challenges in overcoming attenuation:

- Received signal should not be over amplified so that it saturates.
- Signal amplification should maintain at levels insufficiently higher than noise at all times.

Remedy of above challenges is by setting proper amplification level in amplifiers. For example, if attenuation varies with frequency, equalizer is used to overcome this challenge.

1.5.2 Distortion

Distortion refers to the deviation in any parameter (like amplitude, frequency, shape) of a signal from that of an ideal signal. Distortion generates additional frequencies that were not present in the original signal.

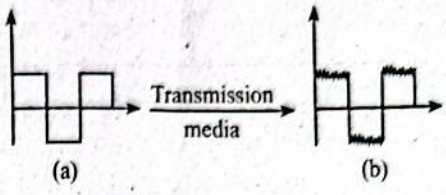


Fig.: Distortion (a) transmitted signal (b) distorted received signal

Distortion can be categorized as:

i. **Linear distortion**

The term "linear distortion" means any change to the signal that does not change the shape of the individual sine wave components of a signal. Linear distortion may be amplitude distortion or phase distortion.

Amplitude distortion: When different frequency components of the input signal are amplified or attenuated by different amounts, the output signal consists of distortion, known as *amplitude distortion.*

Phase distortion: If the phase of the output signal is different from the phase of the input signal, then such distortion is known as *phase distortion.* Phase distortion leads to delay in the transmission of the signal. Hence, it is also known as *delay distortion.*

ii. **Nonlinear distortion**

When the sine wave components of the signal are changed by nonlinear elements, then the distortion is called *nonlinear distortion.*

Distortion can be minimized by using equalizer.

1.5.3 Noise

Noise is any kind of disturbance which tries to obscure the information content of a signal. It is also defined as unwanted form of energy which tends to interfere with the transmission and reception of desired signal in a communication signal.

Noise

Internal Noise
a) Thermal noise
b) Shot noise
c) Partition noise
d) Flicker/low frequency noise
e) Transit-time or high frequency noise
f) Generation-recombination noise

External Noise
a) Atmospheric noise
b) Industrial noise

Fig.: Classification of noise

i. **Internal Noise**

Noise created within the receiver itself is *internal noise.*

a. **Thermal noise:** It occurs in all transmission medium and communication equipment including passive devices.

It arises from random motion of electrons. Every equipment, element and transmission medium itself contribute to thermal noise if the temperature of the element is above absolute 0 (0 K). This noise is also known as *Johnson noise* or *white noise.*

The noise power is calculated by using

$P_n = KTB$

where P_n = power of noise, K= Boltzmann's constant $= 1.38\times10^{23}$ J/K, T = absolute temperature, B = band-width over which noise is measured.

The corresponding equivalent rms noise voltage is given by

$E_n = \sqrt{4KTRB}$

b. **Shot noise:** It arises in active devices due to random behaviour of charge carriers. It is caused by random variation in arrival of electrons or holes at the output of amplifying device.

c. **Partition noise:** It is generated in a circuit when the current has to be divided between two or more paths.

d. **Flicker/low-frequency noise:** At low audio frequency, a particular type of noise appears at transistor which is known as *low-frequency noise.*

e. **Transit time/high-frequency noise:** It is generated in a semiconductor device when transit time of charge carrier crossing the junction (time between emitter to collector) is comparable to time period of a signal being amplified.

f. **Generation-recombination noise:** Random process of generation and recombination of free electrons in semiconductor devices due to random ionization of impurities produce this kind of noise.

ii. **External Noise:** Noise whose sources are external to receiver is *external noise.*

a. **Atmospheric noise:** It is also called *static noise.* This noise is produced by lightening discharge in thunder storm and other natural electrical disturbances.

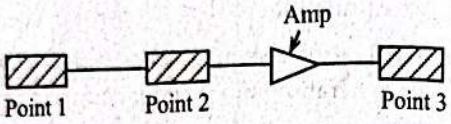
b. **Industrial noise:** This noise is produced by sources such as automobiles, electrical motors, etc.

Another example of external noise is extra-terrestrial noise which is defined as noise from sun and cosmos.

NUMERICAL PROBLEMS

1. **A signal has been sent from point 1 to point 3. But there is huge loss of signal; so an amplifier is placed at point 2. Find the overall loss or gain of system if $\left(\frac{P_2}{P_1}\right) = \frac{1}{10}$ and $\left(\frac{P_3}{P_2}\right) = 100$.**

Solution:



$$(\text{Loss})_{1\text{-}2} = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$$

$$= 10 \log\left(\frac{1}{10}\right) = -10 \text{ dB.}$$

$$(\text{Gain})_{2\text{-}3} = 10 \log_{10}\left(\frac{P_3}{P_2}\right)$$

$$= 10 \log_{10}(100) = 20 \text{ dB.}$$

$\therefore$ Total loss or gain = (20-10) dB = 10 dB.

2. **Suppose a signal travels through a transmission medium and its power is reduced to $\frac{1}{3}$ rd. Find the attenuation loss of power.**

Solution:

$$P_2 = \frac{1}{3} P_1$$

$$\therefore \text{Loss} = 10 \log_{10}\left(\frac{P_2}{P_1}\right) = 10 \log_{10}\left(\frac{1/3P_1}{P_1}\right)$$

$$= -4.771 \text{ dB.}$$

3. **Find the rms noise voltage at the input of amplifier at an ambient temperature of 17°C if we are using resistor of 8 KΩ and amplifier works in the frequency range of 12-15.5 MHz.**

Solution:

$T = 17°C = 17 + 273 = 290K$, $R = 8\times10^3\,\Omega$

Bandwidth (B) = $(15.5 - 12) \times 10^{-6}$ Hz = 3.5×10^{-6} Hz

$$E_n = \sqrt{4KTRB} = \sqrt{4\times1.38\times10^{-23}\times290\times8\times10^3\times3.5\times10^{-6}}$$

$$= 2.11\times10^{-11} \text{ Joule}$$

4. **Calculate the thermal noise power density in watts/Hz at a temperature of 17°C. The Boltzmann's constant is 1.38×10^{-23} J/K.**

Solution:

Temperature (T) = 17°C = 290K

Boltzmann's constant (K) = 1.38×10^{-23} J/K

Thermal noise power density $\left(\frac{P}{B}\right)$ is calculated using

$$P = KTB$$

$$\text{or, } \frac{P}{B} = KT$$

$$\therefore \frac{P}{B} = 290\times1.38\times10^{-23} = 4\times10^{-21} \text{ W/Hz}$$

CHAPTER - 2

SIGNALS AND SYSTEMS

2.1 Introduction to Some Signals

i. Unit Step Signal

Signal whose amplitude is of unity for the zero or positive value of time t and zero value for negative values of time t is *unit step signal*. A continuous time unit step signal and discrete time unit step signal are represented by u(t) and u[n] respectively.

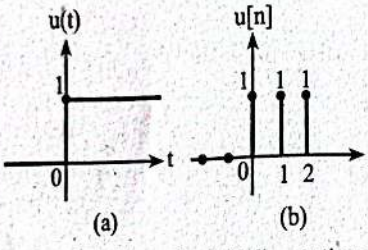


Fig.: (a) Continuous time unit step signal (b) discrete time unit step signal

Continuous time unit step signal is given by

$$u(t) = \begin{cases} 1 \text{ for } t \geq 0 \\ 0 \text{ for } t < 0 \end{cases}$$

Discrete time unit step signal is given by

$$u[n] = \begin{cases} 1 \text{ for } n \geq 0 \\ 0 \text{ for } n < 0 \end{cases}$$

ii. Signum Function

Signal whose amplitude is of positive unit value for positive value of time t, negative unit value for negative value of time t, and zero for zero value of time is *signum signal*. Signal representation for continuous time signal and digital time signal are sgn(t) and sgn[n] respectively.

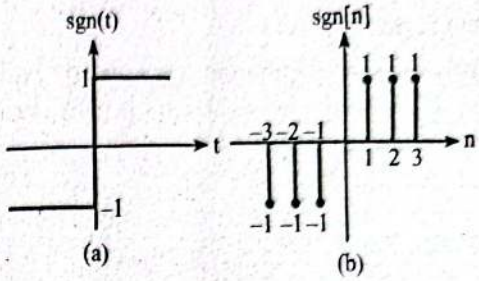


Fig.: (a) Continuous time signum signal (b) discrete time signum signal

Mathematically, continuous time signum signal is expressed as

$$sgn(t) = \begin{cases} 1 \text{ for } t > 0 \\ -1 \text{ for } t < 0 \end{cases}$$

and discrete time signum signal is expressed as

$$sgn[n] = \begin{cases} 1 \text{ for } n > 0 \\ -1 \text{ for } n < 0 \end{cases}$$

iii. Sinc Function or Signal

The sinc function sinc(t), also called the "sampling function", is a function that arises frequently in signal processing and the theory of Fourier transforms. The full name of the function is "sine cardinal", but it is commonly referred to by its abbreviation, "sinc".

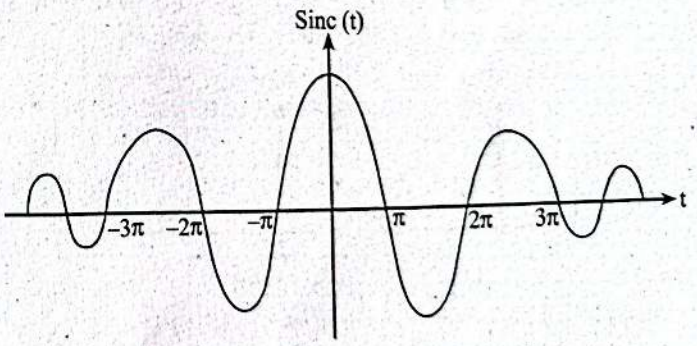


Fig.: Graphical representation of sinc signal.

$$sinc(t) = \frac{\sin \pi t}{\pi t} = \begin{cases} 1 \text{ for } t = 0 \\ 0 \text{ for } t = \pm 1 \pm 2 \end{cases}$$

iv. Ramp Signal

Signal whose amplitude increases linearly with positive value of time and remains zero for negative value of time is *ramp signal*.

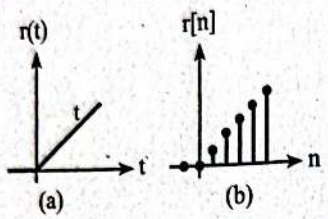


Fig.: (a) Continuous time ramp signal (b) Discrete time ramp signal

Mathematically, continuous time ramp signal is represented as

$$r(t) = \begin{cases} t \text{ for } t \geq 0 \\ 0 \text{ for } t < 0 \end{cases}$$

and discrete time ramp signal is represented as

$$r[n] = \begin{cases} n \text{ for } n \geq 0 \\ 0 \text{ for } n < 0 \end{cases}$$

v. Unit Impulse Signal

Signal that exists only at the time instant zero with amplitude of unity is called *unit impulse signal*. Unit impulse signal is represented by $\delta(t)$ and $\delta[n]$ in continuous time and discrete time respectively.

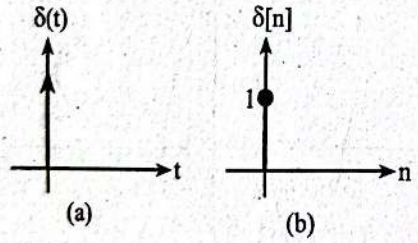


Fig.: (a) Continuous time unit impulse signal (b) Discrete time unit impulse signal

Mathemtically,

$$\delta(t) = \begin{cases} \infty \text{ for } t = 0 \\ 0 \text{ otherwise} \end{cases} \text{ and } \int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$\delta[n] = \begin{cases} 1 \text{ for } n = 0 \\ 0 \text{ otherwise} \end{cases}$$

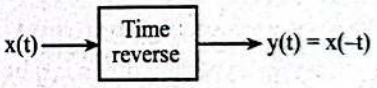
2.2 Signal Analysis and Processing

Signal processing focuses on analyzing, modifying, and synthesizing signals such as sound, images, and biological measurements. Signal processing techniques can be used to improve transmission, storage efficiency and subjective quality and to also emphasize or detect components of interest in a measured signal.

For signal analysis and processing, some transformations like time shifting, time scaling, and time inversion (or folding) of independent variable are needed.

i. Time reversal /folding /inversion

It is achieved by replacing independent variable t by –t. It is also called as reflection of signal about origin.



For drawing the time reversed version of a signal x(t), we simply draw its reflected form with respect to y-axis.

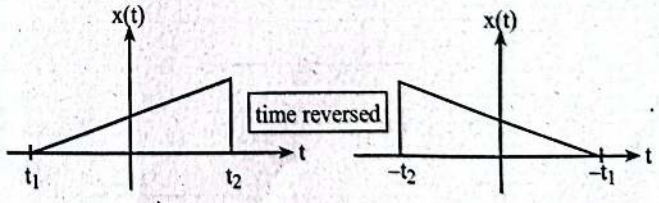
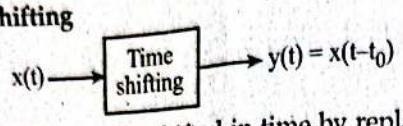


Fig.: Time reversal

ii. **Time shifting**



A signal x(t) may be shifted in time by replacing t by $(t-t_0)$. If $t_0 > 0$, it causes delay in the signal by t_0 time. If $t_0 < 0$, it causes advance in the signal by t_0 time.

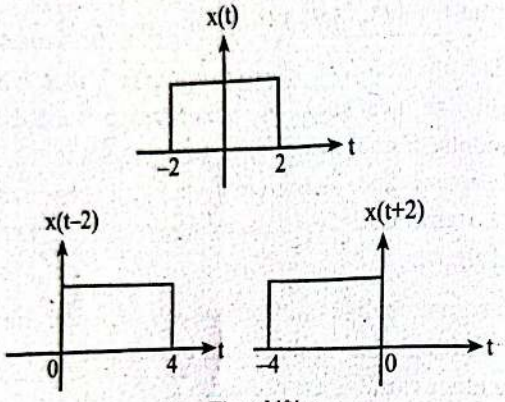


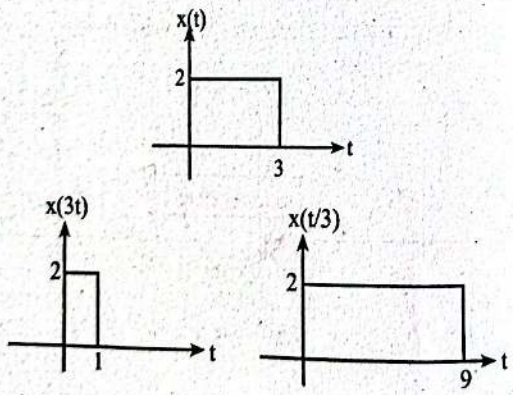
Fig.: Time shifting

iii. **Time scaling:** Scaling of independent variable in any signal x(t) is called as time scaling.

The signal y(t) = x(at) is called the time scaled version of x(t).

- If |a|>1, we are speeding up x(t) by a factor 'a'
- If |a|<1, we are speeding down x(t) by a factor 'a'

Example:



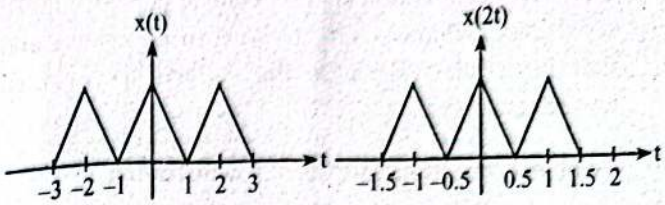


Fig.: Time scaling

iv. **Amplitude transformation**

Let us assume an equation y(t) = Ax(t) + B

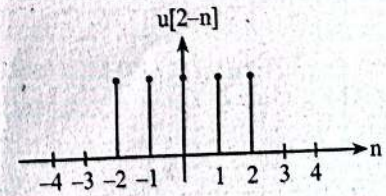
- If B>0, the signal is shifted upward.
- If B<0, the signal is shifted downward.
- If |A|>1, then there is gain in the signal.
- If |A|<1, then there is attenuation in the signal.
- If A>0, there is no reversal, and if A<0, there is reversal.

v. **Combination of reversal and shifting**

Let us see this transformation from the following example:

x[n] = u[2−n] = u[−n + 2]

n	2−n	u[2−n]
−2	4	u[4] = 1
−1	3	u[3] = 1
0	2	u[2] = 1
1	1	u[1] = 1
2	0	u[0] = 1
3	−1	u[−1] = 0

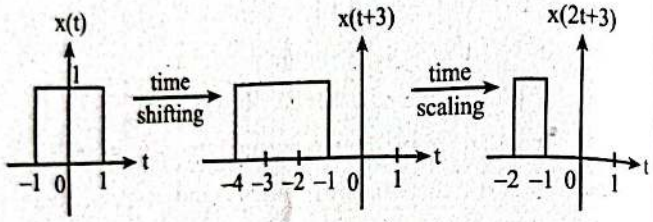


When n is negative and shift is positive, signal is flipped and then shifted towards right. When n is negative and shift is negative, signal is flipped and then shifted towards left.

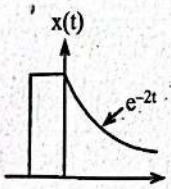
vi. Combination of time shifting and time scaling:

For this kind of transformation, we perform time shifting first which is followed by time scaling.

As an example, we wish to find $y(t) = x(2t + 3)$ for $x(t)$ shown below.

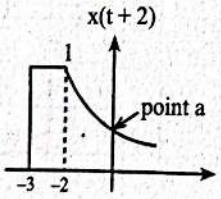


As another example, we wish to find $x(2 - t) + 2$ for $x(t)$ shown below.



The amplitude of the signal at $t = 0$ is $(e^{-2})^0 = e^0 = 1$.

$x(t + 2)$ is obtained by advancing signal $x(t)$ by 2 units as

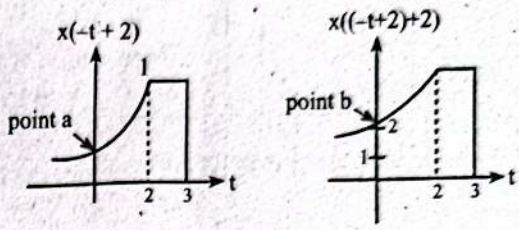


The point 'a' at which the curve crosses the x-axis is determined by putting $t = 0$.

$a = e^{-2(0+2)}$

$\therefore a = e^{-4} = 0.0183$

We fold the signal $x(t + 2)$ to obtain $(-t + 2)$ as



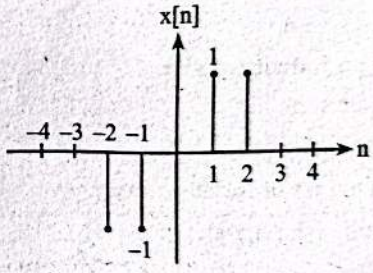
Then, $x(-t+ 2) + 2$ is obtained by adding 2 in amplitude of $x(-t + 2)$

where point $b = 2+e^{-4} = 2.0183$

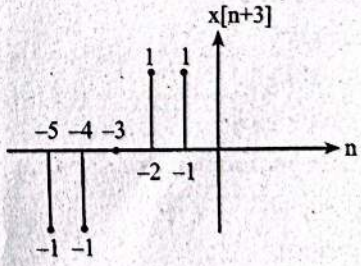
As a last example, consider

$$x[n] = \begin{cases} 1 \text{ for } n = 1, 2, \ldots\ldots \\ -1 \text{ for } n = -1, -2, \ldots\ldots \\ 0 \text{ for } n = 0 \text{ and } |n > 2 \end{cases}$$

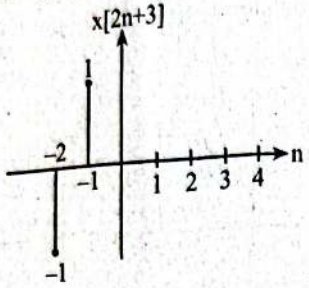
and find $y[n] = x[2n + 3]$.



First shift $x[n]$ by 3 units to get $x[n + 3]$ as



Then, scale x[n + 3] by 2 to get



2.2 Classification of Signals

Signals can be classified based on parameter used to classify them such as

i. **Nature of independent variable of signal**
   - Continuous time signal
   - Discrete time signal

ii. **Nature of dependent variable of signal**
   - Analog signal
   - Digital signal

iii. **Based on periodicity of signal**
   - Periodic signal
   - Aperiodic signal

iv. **Based on nature of indeterminacy**
   - Deterministic signal
   - Random signal

v. **Based on causality**
   - Casual signal
   - Anticasual signal
   - Noncasual signal

vi. **Based on energy content in the signal**
   - Energy signal
   - Power signal
   - Neither energy nor power signals.

vii. **Based on symmetricity about vertical axis**
   - Even signal
   - Odd signal

2.2.1 Continuous Time and Discrete Time Signals

Continuous signals are the signals or quantities that can be defined and represented at any instant of time in the sequence, that is, infinite or uncountable set of number sequence.

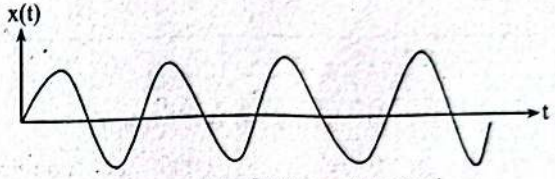


Fig.: Continuous time signal

Discrete signals are the signals or quantities that can be defined and represented at certain time instants of the sequence, that is, finite or countable set of number sequence. These are also called digitalized signals.

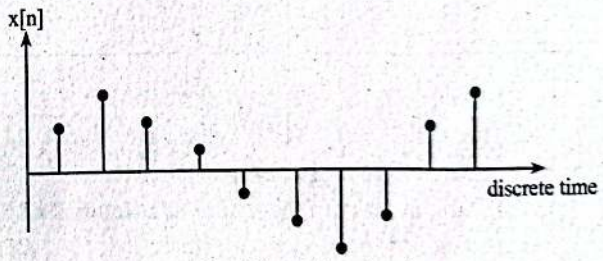


Fig.: Discrete time signal

2.2.2 Periodic and Non-Periodic Signals

A signal that repeats its shape at regular interval of time is called *periodic signal*.

Mathematically, a continuous time signal x(t) is called periodic in time if there exist a constant $T_0>0$ such that $x(t) = x(t+T_0)$ for $-\infty<t<\infty$, where T_0 is fundamental time period.

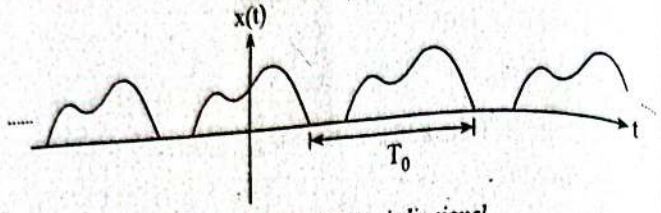


Fig.: Continuous time periodic signal

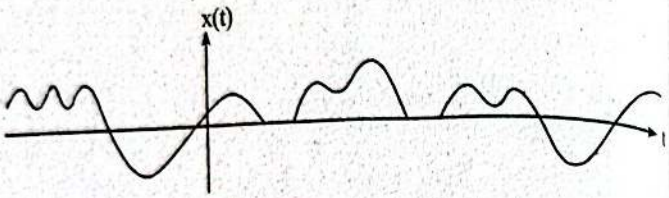


Fig.: Continuous time non-periodic signal

Similarly, for discrete time signal, the condition is $x[k]=x[k+K_0]$ for $-\infty<n<\infty$, where K_0 is fundamental time period.

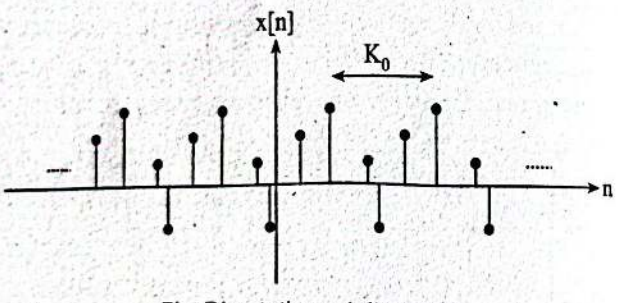


Fig.: Discrete time periodic signal

A signal that do not repeat at regular interval of time are called *non-periodic* or *aperiodic signal.*

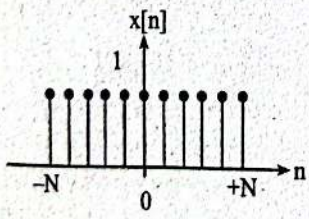


Fig.: Discrete time non-periodic signal

2.2.3 Deterministic and Random Signals

A signal can be classified as *random signal* or *probabilistic signal* if there is some degree of uncertainty before the signal actually occurs. The value of these signals cannot be predicted accurately. Random signals cannot be described by a mathematical equation. They are modelled in probabilistic terms.

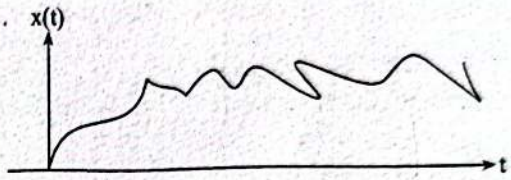


Fig.: Random signal

A signal is called *deterministic* if there is no uncertainty in determining its value at any time. These signals can be represented by some mathematical formula.

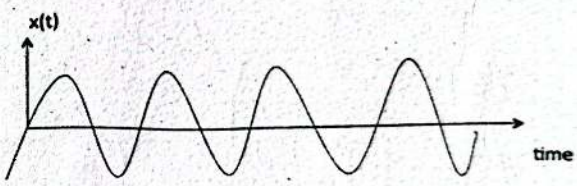


Fig.: Deterministic signal

2.2.4 Energy and Power Signals

Energy signal is one which have finite energy and zero average power i.e.,

$0< E <\infty$

$P = 0$

Energy is calculated by using following formula:

For continuous time signal,

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

For discrete time signal,

$$E = \sum_{n=-k}^{k} |x[n]|^2$$

Power signal is the one which has finite non-zero power but infinite energy i.e.,

$0 < P < \infty$

$E = \infty$

Generally, power signals are periodic signals.

Power is calculated by:

For continuous time signal,

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T} |x(t)|^2 dt$$

For discrete time signal,

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

$$P = \frac{1}{2k+1} \sum_{n=-k}^{k} |x[n]|^2$$

Comparison of Energy and Power Signal

Energy Signal	Power Signal
1. Total normalized energy is finite and non-zero.	1. The normalized average power is finite and non zero.
2. The energy is obtained by $E = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	2. The average power is obtained by $P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \lvert x(t)\rvert^2 dt$
3. Non-periodic signals are energy signals.	3. Practical periodic signals are power signals.

Energy Signal	Power Signal
4. These signals are time limited.	4. These signals can exist over infinite time.
5. Power of the energy signal is zero.	5. Energy of the power signal is infinite.

2.2.5 Causal, Anti-causal, and Non-causal Signal

Causal signals are signals that are zero for all negative time, while *anti-causal* are signals that are zero for all positive time. Non-causal signals are signals that have nonzero values in both positive and negative time.

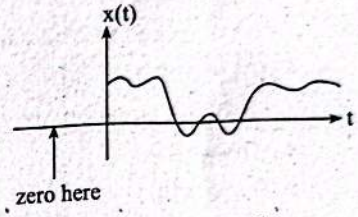


Fig.: Causal signal

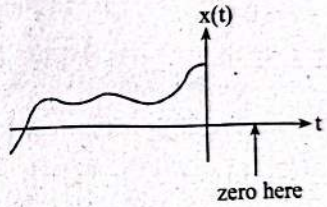


Fig.: Anti-causal signal

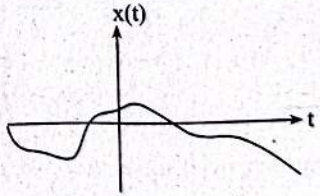


Fig.: Non-causal signal

2.2.6 Even and Odd Signals

A signal is called *even signal* if $x(t) = x(-t)$ and *odd signal* if $x(t) = -x(-t)$.

Even signal is symmetric about vertical axis whereas odd signal is antisymmetric about vertical axis. All signals can be expressed as the sum of even and odd signals.

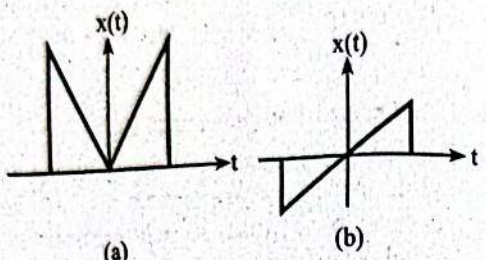


Fig.: (a) Even signal (b) odd signal

Any signal can be decomposed into odd and even signal.

Therefore, $x(t) = x_0(t) + x_e(t)$ ..... (i)

Replacing (t) by –t in above equation, we get

$x(-t) = x_o(-t) + x_e(-t)$

$= -x_o(t) + x_e(t)$ ..... (ii)

Adding equations (i) and (ii), we get

$x(t) + x(-t) = 2x_e(t)$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

Subtracting equation (ii) from (i), we get

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

Similarly, for discrete time signal, we can write

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

Some examples of even signal are: cost, t^2, t sint

Some examples of odd signal are: sint, t^3, t cost

2.2.7 Analog and Digital Signals

Those signals whose dependent and independent variables are continuous in nature are *analog signals.*

Those signals whose both dependent and independent variables are discrete in nature are *digital signals.*

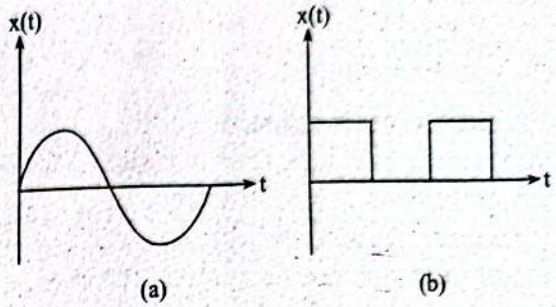
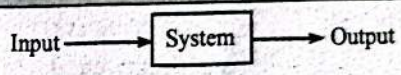


Fig.: (a) Analog signal (b) digital signal

2.3 System

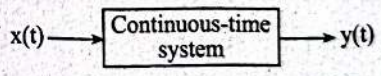


A *system* is defined as the set of elements or functional blocks which are connected together and produces an output in response to an input signal.

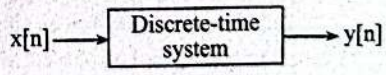
The input signals are transformed by the system. Amplifier, filter, etc. are examples of electrical system.

A system can be classified broadly into two types:

i. **Continuous time system:** It has input and output both continuous in nature.



ii. **Discrete time system:** It has input and output both discrete in nature.



These two broad types of system are further classified on the basis of system properties as follows:

- Linear and non-linear systems
- Time-variant and time-invariant systems

- Static and dynamic systems
- Causal and non-causal systems
- Invertible and non-invertible systems
- Stable and unstable systems

2.3.1 Linear and Non-Linear System

A system is said to be *linear* if it satisfies the principle of superposition.

Consider two systems with inputs as $x_1(t)$, $x_2(t)$, and outputs as $y_1(t)$, $y_2(t)$ respectively. Then, according to the superposition principle,

$$a\,x_1(t) + b\,x_2(t) \xrightarrow{\text{system}} a\,y_1(t) + b\,y_2(t)$$

Linearity test:

- Let $x_1(t)$ and $x_2(t)$ be the two inputs to the system 'H' and $y_1(t)$ and $y_2(t)$ be the corresponding responses.
- Consider a signal $x_3(t) = a_1x_1(t) + a_2x_2(t)$ which is weighted sum of $x_1(t)$ and $x_2(t)$.
- Let $y_3(t)$ be the response of $x_3(t)$.
- If $y_3(t) = a_1y_1(t) + a_2y_2(t)$, the system is linear.

  Example:

  $y(t) = tx(t)$ (linear system)

  $y(t) = x(t^2)$ (linear system)

  $y(t) = x^2(t)$ (non-linear system)

  $y(t) = Ax(t) + B$ (non-linear system)

2.3.2 Time-Invariant and Time-Variant System

A system is called *time-invariant system* if a time delay or time advance of input signal leads to an identical time shift in the output signal. That is, for every input signal x(t) and every time shift m,

$$x(t) \xrightarrow{H} y(t)$$

implies that

$$x(t-m) \xrightarrow{H} y(t-m)$$

Invariance test:

- Delay input by 'm' and find the response of the system for this delayed input. Let the output be $y_1(t)$.
- Delay the response of the system for unshifted input by m units.
- Let the output be $y_2(t)$.
- If $y_1(t) = y_2(t)$, the system is time-invariant system.

  Example:

  $y(t) = tx(t)$ (time-variant system)

  $y(t) = x^2(t)$ (time-invariant system)

2.3.3 Static and Dynamic System

A system in said to be *static* or *memoryless system* if its output at a given time is dependent only upon its input at the same time.

A system is said to be a *dynamic* or *system with memory* if its output at a given time is dependent on the past values of input.

Example:

$y(t) = 2x(t)$ (static system)

$y(t) = x(t-3) + x(t)$ (dynamic system)

2.3.4 Causal and Non causal Systems

A system in said to be *causal* if the output of the system at any time t depends only on the present input and past inputs but does not depend on the future inputs.

If the system output at time t depends on future inputs, the system is called *non-causal*. Non-causal systems can not be implemented practically. If the system output solely depends on future inputs, the system is *anticausal*.

Example:

$y(t) = x(t-2)$ (causal system)

$y(t) = e^{-x(t)}$ (causal system)

$y(t) = x(t+1) + x(t)$ (non-causal system)

$y(t) = x(t+1)$ (anticausal system)

2.3.5 Invertible and Non-Invertible System

A system is said to be *invertible* if the input can be recovered from its output. In invertible system, distinct inputs cause distinct outputs.

If the system is invertible, then there exists an inverse system which when cascaded with the original system gives the output equal to its input.

Example:

$y(t) = 6x(t) + 8$ (invertible system)

$y(t) = x^2(t)$ (non-invertible system)

2.3.6 Stable and Unstable Systems

An arbitrary relaxed system is said to be *BIBO stable* (bounded input bounded output stable) if and only if every bounded input produces a bounded output.

Let x(t) be the input of continuous time system and y(t) be the response or output for x(t).

The term *bounded input* refers to finite value of the input signal x(t) for any value of t. Hence if input x(t) is bounded, then there exists a constant M_2 such that $|x(t)| \leq M_2$ and $M_2 < \infty$, for all t.

Examples of *bounded input signals* are step signal, decaying exponential signal, and impulse signal.

Examples of *unbounded input signals* are ramp signal and increasing exponential signal.

The term *bounded output* refers to finite and predictable output for any value of t. Hence if output y(t) is bounded, then there exists a constant M_2 such that $|y(t)| \leq M_y$ and $M_y < \infty$, for all t.

In general, the test for stability of the system is performed by applying specific input. On applying a bounded input to a system, if the output is bounded, then the system is said to be BIBO stable.

Condition for stability of an LTI system:

For stability,

$$\int_{-\infty}^{+\infty} |h(t)|\, dt < \infty \text{ for continuous LTI system}$$

$$\sum_{n=-\infty}^{\infty} h[n] < \infty \text{ for discrete LTI system}$$

NUMERICAL PROBLEMS

1. **Check linearity, causality, stability, and time invariance characteristics of system $y(t) = 2x(t + 1)$.** [2073 Shrawan]

Solution:

$y(t) = 2x(t + 1)$

Causality check:

As the output depends on future input i.e., $x(t + 1)$, the system is noncausal.

Linearity check:

Let H be the system operating on $x(t)$ to produce output $y(t)$.

Now, let $x_1(t)$ and $x_2(t)$ be the two inputs to the system H.

Then, $y_1(t) = H\{x_1(t)\} = 2x_1(t + 1)$

$y_2(t) = H\{x_2(t)\} = 2x_2(t + 1)$

Now, consider a signal $x_3(t) = a_1x_1(t) + a_2x_2(t)$ which is weighted sum of $x_1(t)$ and $x_2(t)$. Let $y_3(t)$ be its response. Then,

$$y_3(t) = H\{x_3(t)\} = 2x_3(t + 1)$$
$$= 2\{a_1x_1(t + 1) + a_2x_2(t + 1)\}$$
$$= 2a_1x_1(t + 1) + 2a_2x_2(t + 1)$$
$$= a_1y_1(t) + a_2y_2(t)$$

Since $y_3(t) = a_1y_1(t) + a_2y_2(t)$, the system follows the property of superposition. Hence, given system is linear.

Time invariance check:

For time invariance test, let's first delay the input by t_0 and observe its output $y_1(t)$.

$$x(t) \xrightarrow{\text{delay}} x(t-t_0) \xrightarrow{\text{system}} y_1(t) = 2x(t - t_0 + 1)$$

Now, let's delay the normal output. Let output be $y_2(t)$.

$$x(t) \xrightarrow{\text{system}} 2x(t + 1) \xrightarrow{\text{delay}} y_2(t) = 2x(t + 1 - t_0) = 2x(t - t_0 + 1)$$

Since $y_1(t) = y_2(t)$, the system is time invariant.

Stability test:

The operation performed by the system on the input signal is shifting, a bounded input signal will remain bounded even after shifting. Therefore in given system, output will be bounded as long as input is bounded. Hence, the system is BIBO stable.

2. **Check whether the following system is linear, time invariant and causal or not:**

$y(t) = x(t - 2) + x(2 - t)$ [2074 Ashwin]

Solution:

Linearity test:

Let H be the system operating on $x(t)$ to produce

$$y(t) = H\{x(t)\} = x(t - 2) + x(2 - t)$$

Now, $y_1(t) = x_1(t - 2) + x_1(2 - t)$..... (i)

$y_2(t) = x_2(t - 2) + x_2(2 - t)$..... (ii)

Let $x_3(t) = a_1x_1(t) + a_2x_2(t)$

Then $y_3(t) = H\{x_3(t)\}$

$$= H\{a_1x_1(t) + a_2x_2(t)\}$$
$$= x_3(t-2) + x_3(2 - t)$$
$$= \{a_1x_1(t-2) + a_2x_2(t - 2)\}\ \{a_1x_1(2 - t) + a_2x_2(2 - t)\}$$
$$= a_1\{x_1(t - 2) + x_1(2 - t)\} + a_2\{x_2(t-2) + x_2(2 - t)\}$$
$$= a_1y_1(t) + a_2y_2(t)$$

Since the given system satisfies the property of superposition, the system is linear.

Time invariance test:

Let us delay the input $x(t)$ by t_0 and pass the delayed input to the system. Then the output will by $y_1(t)$.

$$x(t) \rightarrow \boxed{\text{Delay}} \rightarrow x(t-t_0) \rightarrow \boxed{\text{System}} \rightarrow y_1(t)$$

$$y_1(t) = x(t - t_0 - 2) + x(2 - t - t_0)$$

Now, let us delay the output and let the delayed output be $y_2(t)$.

$$y_2(t) = y(t - t_0)$$
$$= x(t - t_0 - 2) + x[2 - (t - t_0)]$$
$$= x(t - t_0 - 2) + x(2 - t + t_0)$$

Since $y_1(t) \neq y_2(t)$, the system is time variant.

Causality test:

In the given system, if we substitute $t = 0$, we get

$$y(0) = x(-2) + x(2)$$

which means that output at $t = 0$ depends upon future input. Hence, the given system is noncausal.

3. Check whether the given system is linear and time invariant or not.

i. $y(t) = \log[x(t)]$

ii. $y(t) = t\,x(t)$ [2070 Ashadh]

Solution:

i. $y(t) = \log[x(t)]$

Linearity test:

Let H be the sytem operating on $x(t)$ to produce

$y(t) = H\{x(t)\} = \log[x(t)]$

Now, $y_1(t) = \log[x_1(t)]$

$y_2(t) = \log[x_2(t)]$

Let, $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

Then, $y_3(t) = H\{x_3(t)\} = \log[x_3(t)]$

$= \log[a_1 x_1(t) + a_2 x_2(t)]$

Since $y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$, the system is non-linear.

Time-invariance test:

Let us delay the input $x(t)$ by t_0 and pass the delayed input to the system. Let the output be $y_1(t)$.

$$x(t) \rightarrow \boxed{\text{Delay}} \xrightarrow{x(t-t_0)} \boxed{\text{System}} \rightarrow y_1(t) = \log(x(t-t_0))$$

Again, let us delay the output and this delayed output be $y_2(t)$.

$y_2(t) = y(t-t_0) = \log[x(t-t_0)]$

Since $y_1(t) = y_2(t)$, it is time-invariant system.

ii. $y(t) = t\,x(t)$

Linearity test:

Let H be the sytem operating on $x(t)$ to produce

$y(t) = H\{x(t)\} = t\,x(t)$

$y_1(t) = t\,x_1(t)$

$y_2(t) = t\,x_2(t)$

Let, $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

Then, $y_3(t) = H\{x_3(t)\}$

$= H\{(a_1 x_1(t) + a_2 x_2(t))\}$

$= t(a_1 x_1(t) + a_2 x_2(t))$

$= a_1 t\,x_1(t) + a_2 t\,x_2(t)$

$= a_1 y_1(t) + a_2 y_2(t)$

Since $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$, the system is linear.

Time-invariance test:

$$x(t) \rightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{System}} \rightarrow y_1(t) = t\,x(t-m)$$

$$x(t) \rightarrow \boxed{\text{System}} \xrightarrow{y(t) = t\,x(t)} \boxed{\text{Delay}} \rightarrow y_2(t) = (t-m)\,x(t-m)$$

Since $y_1(t) \neq y_2(t)$, the system is time-variant.

4. Test the stability of the LTI system whose impulse responses are given as:

i. $h(t) = e^{\psi t}\,u(t)$

ii. $h(t) = e^{-\psi t}\,u(t)$ [2072 Chaitra]

Solution:

i. $h(t) = e^{\psi t}\,u(t)$

$$\int_{-\infty}^{\infty} h(t)\,dt = \int_{-\infty}^{\infty} e^{\psi t}\,u(t)\,dt$$

$$= \int_0^{\infty} e^{\psi t}\, dt$$

$$= \left[\frac{e^{\psi t}}{\psi}\right]_0^{\infty} = \frac{1}{\psi}(e^{\infty} - e^{0}) = \frac{1}{\psi}(\infty - 1) = \infty$$

Since $\int_{-\infty}^{\infty} h(t)dt$ is not finite, the given system is not stable system.

ii. $h(t) = e^{-\psi t} u(t)$

$$\int_{-\infty}^{\infty} h(t)\, dt = \int_{-\infty}^{\infty} e^{-\psi t} u(t)\, dt$$

$$= \int_0^{\infty} e^{-\psi t}\, dt$$

$$= \left[\frac{e^{-\psi t}}{-\psi}\right]_0^{\infty}$$

$$= -\frac{1}{\psi}(e^{-\infty} - e^{0}) = -\frac{1}{\psi}\left(\frac{1}{e^{\infty}} - 1\right) = -\frac{1}{\psi}(0 - 1) = \frac{1}{\psi}$$

Since $\int_{-\infty}^{\infty} h(t)\, dt$ is finite, the given system is a stable system.

5. **Determine the range of values of a and b for the stability of LTI system with impulse response $h(t) = e^{at} u(t) + e^{-bt} u(t)$.** [2074 Chaitra]

Solution:

For any system with impulse response h(t), the condition for stability is:

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \text{finite value}$$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |e^{at}u(t) + e^{-bt}u(t)|\, dt$$

$$= \left|\int_0^{\infty} e^{at} dt + \int_0^{\infty} e^{-bt} dt\right|$$

$$= \left|\left[\frac{e^{at}}{a}\right]_0^{\infty} + \left[\frac{e^{-bt}}{-b}\right]_0^{\infty}\right|$$

$$= \left|\frac{e^{a\times\infty}}{a} - \frac{e^{a\times 0}}{a} + \frac{e^{-b\times\infty}}{-b} - \frac{e^{-b\times 0}}{-b}\right|$$

$$= \left|\frac{e^{a\times\infty}}{a} - \frac{1}{a} + \frac{e^{-b\times\infty}}{-b} + \frac{1}{b}\right|$$

In the above equation, if a is negative and b is positive, then it converges to a finite value. That is, when a is negative and b is positive,

$$\int_0^{\infty} |h(t)|\, dt = \left|\frac{0}{a} - \frac{1}{a} + \frac{0}{-b} + \frac{1}{b}\right|$$

$$= \left|-\frac{1}{a} + \frac{1}{b}\right| = \left|\frac{a-b}{ab}\right| = \text{constant}$$

Here, the integral of impulse response is a constant when a is negative and b is positive.

Therefore, range of values of a and b for stability of LTI system are

$a < 0$ and $b > 0$.

6. **Determine whether the following system is time invertible system or not.**

   i. $y(t) = 6x(t) + 8$

   ii. $y(t) = x^2(t)$

Solution:

i. $y(t) = 6x(t) + 8$

or, $x(t) = \frac{y(t)-8}{6}$

For each input x(t), the value of y(t) is unique. Hence, the system is an invertible system .

ii. $y(t) = x^2(t)$

or, $x(t) = \pm\sqrt{y(t)}$

For each value of x(t), there are two possible values of y(t). Hence, the system is non-invertible system.

7. State whether the following systems are time invariant or not.

i. $y(t) = 2\,e^{x|t|}$

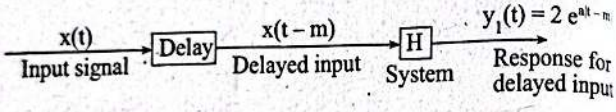
ii. $y(t) = x(t) + C$

iii. $y(t) = 3x^2(t)$

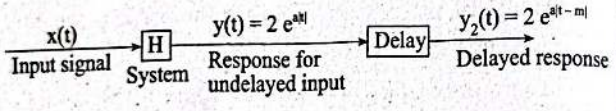
Solution:

i. $y(t) = 2\,e^{x|t|}$

Test 1: Response for delayed input



Test 2: Delayed response

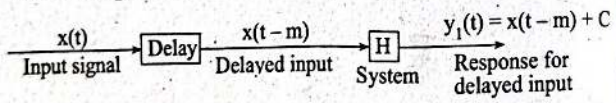


Conclusion:

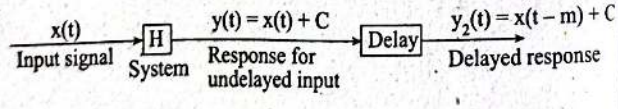
Here, $y_1(t) = y_2(t)$. Therefore, the system is time invariant.

ii. $y(t) = x(t) + C$

Test 1: Response for delayed input



Test 2: Delayed response

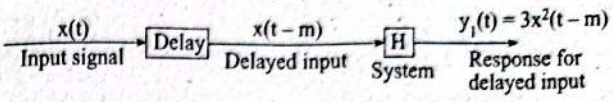


Conclusion:

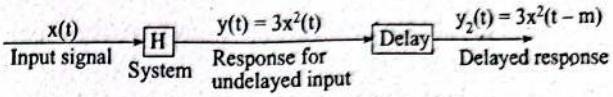
Here, $y_1(t) = y_2(t)$. Therefore, the system is time invariant.

iii. $y(t) = 3x^2(t)$

Test 1: Response for delayed input



Test 2: Delayed response



Conclusion: Here, $y_1(t) = y_2(t)$. Therefore, the system is time invariant

8. Test the causality of the following systems.

i. $y(t) = x(t) + 3\,x(t + 4)$

ii. $y(t) = x(t^2)$

Solution:

i. $y(t) = x(t) + 3\,x(t + 4)$

When $t = 0$, $y(0) = x(0) + 3\,x(4)$

The response at $t = 0$, i.e., $y(0)$ depends on the present input $x(0)$ and future input $x(4)$.

When $t = 1$, $y(1) = x(1) + 3x(5)$

The response at $t = 1$, i.e. $y(1)$ depends on the present input $x(1)$ and future input $x(5)$.

From the above analysis, we can say that the response for any value of t depends on present and future inputs. Hence, the system is noncausal.

ii. $y(t) = x(t^2)$

When $t = -1$, $y(-1) = x(1)$

The response at $t = -1$ i.e., $y(-1)$ depends on the future input $x(1)$.

When $t = 0$, $y(0) = x(0)$

The response at $t = 0$ i.e., $y(0)$ depends on the present input $x(0)$.

When $t = 1$, $y(1) = x(1)$

The response at $t = 1$ i.e., $y(1)$ depends on the present input $x(1)$.

When $t = 2$, $y(2) = x(4)$

The response at $t = 2$ i.e., $y(2)$ depends on the future input $x(4)$.

From the above analysis, we can say that the response for any value of t (except $t = 0$ and $t = 1$) depends on future inputs. Hence, the system is noncausal.

9. Determine whether the system $y[n] = x[-n]$ is causal or noncausal.

Solution:

$y[-1] = x[1]$

$y[-2] = x[2]$

The function depends upon future values. So, this is a noncausal system.

10. Test the following systems for linearity:

i. $y(t) = x^2(t)$

ii. $y(t) = x(t^2)$

Solution:

i. $y(t) = x^2(t)$

Let $x_1(t)$ and $x_2(t)$ be the two inputs to the system (H), and $y_1(t)$ and $y_2(t)$ be the corresponding responses.

$y(t) = H\{x(t)\} = x^2(t)$

$y_1(t) = H\{x_1(t)\} = x_1^2(t)$

$y_2(t) = H\{x_2(t)\} = x_2^2(t)$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$y_3(t) = H\{x_3(t)\}$

$= H\{a_1 x_1(t) + a_2 x_2(t)\}$

$= [a_1 x_1(t) + a_2 x_2(t)]^2$

$= a_1^2 x_1^2(t) + 2a_1 x_1(t)\, a_2 x_2(t) + a_2^2 x_2^2(t)$

$= a_1^2 y_1(t) + 2a_1 a_2 x_1(t)\, x_2(t) + a_2^2 y_2(t)$

Since $y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$, the given system is nonlinear.

ii. $y(t) = x(t^2)$

$y_1(t) = x_1(t^2)$

$y_2(t) = x_2(t^2)$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$y_3(t) = T[x_3(t)]$

$= x_3(t^2)$

$= a_1 x_1(t^2) + a_2 x_2(t^2)$

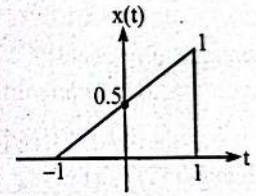
$= a_1 y_1(t) + a_2 y_2(t)$

Since $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$, the given system is linear.

11. Find even and odd part of a signal $x(t) = 0.5\,(t + 1)$ for $-1 \le t \le 1$. [2074 Chaitra]

Solution:

The given signal $x(t)$ can be drawn as:



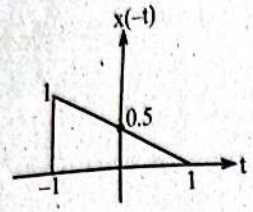
T	–1	0	1
x(t)	0	0.5	1

The even and odd parts of the signal are given by:

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

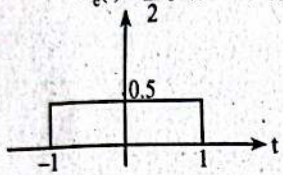
$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$x(-t)$ can be drawn as:



Thus, even part of the signal $x_e(t)$ can be drawn by adding x(t) and x(–t), and then reducing amplitude by half.

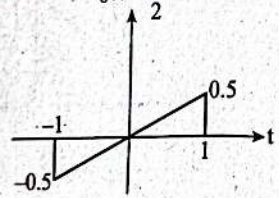
$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$



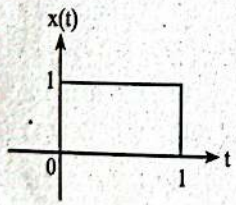
Similarly, odd part of the signal $x_o(t)$ can be drawn by subtracting signal x(–t) from x(t) and then reducing amplitude by half.

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

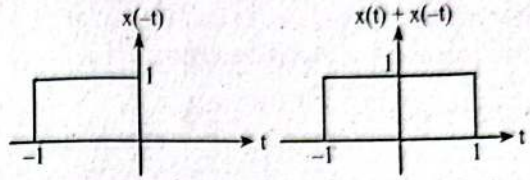


12. Find the even and odd function of function x(t) where the signal x(t) is given by:

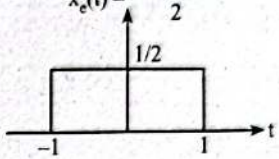


Solution:

$$x(t) = \begin{cases} 1 \text{ for } t \geq 0 \\ 0 \text{ for } t < 0 \end{cases}$$

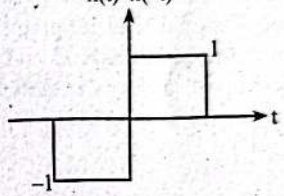


$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

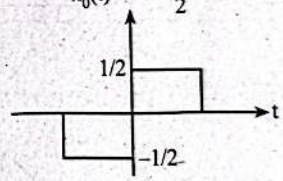


Even signal

x(t)–x(–t)

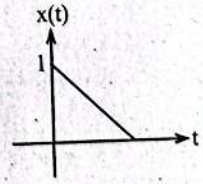


$$x_0(t) = \frac{x(t) - x(-t)}{2}$$



Odd signal

13. Find the even and odd function of function x(t) where the signal x(t) is given by:



Solution:

Even part of signal is given by $x_e(t)$

$$= \frac{1}{2}[x(t) + x(-t)]$$

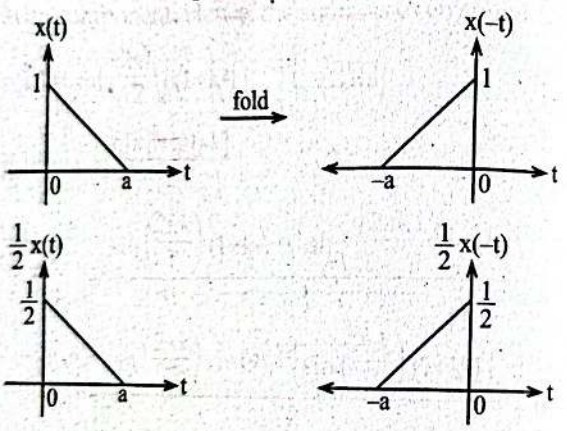
$$= \frac{1}{2}[x(t)] + \frac{1}{2}[x(-t)] \text{ ... (i)}$$

Odd part of signal is given by $x_o(t)$

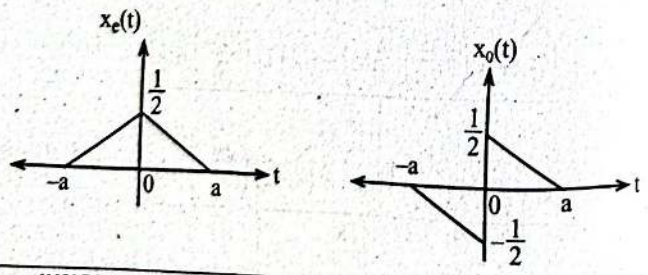
$$= \frac{1}{2}[x(t) - x(-t)]$$

$$= \frac{1}{2}x(t) - \frac{1}{2}x(-t) \text{..... (ii)}$$

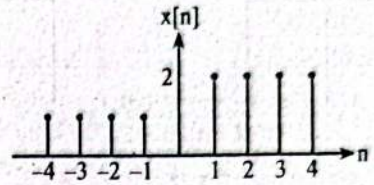
From equation (i) and (ii), it is observed that even and odd parts of signal com be obtained from the folded and scaled version of signal. Hence, given signal is folded, scaled and then graphically added and subtracted to get the even and odd parts as shown below:



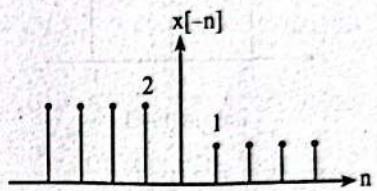
$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ $\quad$ $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$



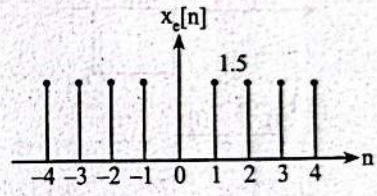
14. Find the even and odd function of function x(t) where the signal x[n] is given by:



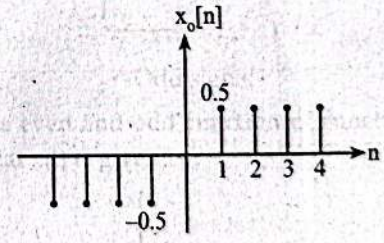
Solution:



Even signal: $x_e[n] = \frac{x[n] + x[-n]}{2}$



Odd signal: $x_o[n] = \frac{x[n] - x[-n]}{2}$



15. Express following function as a combination of odd and even signal.

(a) $g(t) = 4\cos(9\pi t)$

(b) $g[k] = \sin\left(\frac{2\pi k}{7}\right)(1+k^2)$

Solution:

(a) $g(t) = 4\cos(9\pi t)$

$$g_e(t) = \frac{g(t) + g(-t)}{2}$$

$$= \frac{4\cos(9\pi t) + 4\cos(-9\pi t)}{2}$$

$$= \frac{4\cos(9\pi t) + 4\cos(9\pi t)}{2} = 4\cos(9\pi t)$$

$$g_o(t) = \frac{g(t) - g(-t)}{2}$$

$$= \frac{4\cos(9\pi t) - 4\cos(-9\pi t)}{2}$$

$$= \frac{4\cos(9\pi t) - 4\cos(9\pi t)}{2} = 0$$

Given signal has zero odd component and some value of even component. Hence, the signal is even signal.

(b) $g[k] = \sin\left(\frac{2\pi k}{7}\right)(1+k^2)$

$$g_e[k] = \frac{g[k] + g[-k]}{2}$$

$$= \frac{\sin\left(\frac{2\pi k}{7}\right)(1+k^2) + \left(\sin -\left(\frac{2\pi k}{7}\right)(1+k^2)\right)}{2}$$

$$= \frac{\sin\left(\frac{2\pi k}{7}\right)(1+k^2) - \left(\sin\left(\frac{2\pi k}{7}\right)(1+k^2)\right)}{2} = 0$$

$$g_o[k] = \frac{g[k] - g[-k]}{2}$$

$$= \frac{\sin\left(\frac{2\pi k}{7}\right)(1+k^2) - \left(\sin -\left(\frac{2\pi k}{7}\right)(1+k^2)\right)}{2}$$

$$= \frac{\sin\left(\frac{2\pi k}{7}\right)(1+k^2) + \left(\sin\left(\frac{2\pi k}{7}\right)(1+k^2)\right)}{2}$$

$$= \sin\left(\frac{2\pi k}{7}\right)(1+k^2)$$

Given signal has zero even component and some value of odd component. Hence, the signal is odd signal.

16. Determine the odd and even part of the following continuous time signals.

(a) $x(t) = e^t$

(b) $x(t) = 3 + 2t + 5t^2$

(c) $x(t) = \sin 2t + \cos t + \sin t \cos 2t$

Solution:

(a) $x(t) = e^t$

$x(-t) = e^{-t}$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[e^t + e^{-t}]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[e^t - e^{-t}]$$

Since the given signal has both odd and even components, the signal is neither odd nor even.

(b) $x(t) = 3 + 2t + 5t^2$

$x(-t) = 3 + 2(-t) + 5(-t)^2$

$= 3 - 2t + 5t^2$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[3 + 2t + 5t^2 + 3 - 2t + 5t^2]$$

$$= \frac{1}{2}(6 + 10t^2) = 3 + 5t^2$$

$$x_e(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[3 + 2t + 5t^2 - 3 + 2t - 5t^2]$$

$$= 2t$$

Since the given signal has both odd and even components, the signal is neither odd nor even.

(c) $x(t) = \sin 2t + \cos t + \sin t \cos 2t$

$x(-t) = -\sin 2t + \cos t - \sin t \cos 2t$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$= \frac{1}{2}[\sin 2t + \cos t + \sin t \cos 2t - \sin 2t + \cos t - \sin t \cos 2t]$

$= \cos t$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$= \frac{1}{2}[\sin 2t + \cos t + \sin t \cos 2t + \sin 2t - \cos t + \sin t \cos 2t]$

$= \frac{1}{2}[2 \sin 2t + 2 \sin t \cos 2t] = \sin 2t + \sin t \cos 2t$

Since the given signal has both odd and even components, the signal is neither odd nor even.

17. Determine whether the following signals are periodic or not.

(a) $x(t) = \sin 15\pi t$

(b) $x(t) = \sin\sqrt{2}\pi t$ [2075 Ashwin]

Solution:

(a) $x(t) = \sin 15\pi t$

Since the given signal is a sinusoidal signal in continuous time domain, the given signal is a periodic signal.

The time period of this signal can be calculated by comparing this equation with $x(t) = A \sin\omega t$ to get

$\omega = 15\pi$

or, $2\pi f = 15\pi$

or, $f = \frac{15}{2}$

$\therefore$ Time period $(T) = \frac{1}{f} = \frac{2}{15}$ sec.

This can also be verified by satisfying condition of periodicity:

$x(t + T) = \sin 15\pi(t + T)$

$= \sin 15\pi\left(t + \frac{2}{15}\right)$

$= \sin(15\pi t + 2\pi)$

$= \sin 15\pi t$

$= x(t)$

Hence, given signal is periodic with period $\frac{2}{15}$ sec.

(b) $x(t) = \sin\sqrt{2}\pi t$

Since the given signal is a sinusoidal signal in continuous time domain, the given signal is a periodic signal.

The time period of this signal can be calculated by comparing this equation with $x(t) = A \sin\omega t$ to get

$\omega = \sqrt{2}\pi$

or, $2\pi f = \sqrt{2}\pi$

or, $f = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

or, $T = \frac{1}{f} = \sqrt{2}$

This can also be verified by satisfying condition of periodicity.

$x(t + T) = \sin(\sqrt{2}\pi(t + T))$

$= \sin(\sqrt{2}\pi(t + \sqrt{2}))$

$= \sin(\sqrt{2}\pi t + \sqrt{2}\pi)$

$= \sin\sqrt{2}\pi t = x(t)$

Hence, given signal is periodic with period $\sqrt{2}$.

18. **Check whether the signal x(t) is energy or power type.** [2071 Chaitra]

Solution:

a. $x(t) = u(t)$

The energy is calculated as

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} (1)^2\,dt = \lim_{T\to\infty} T = \infty$$

Power of signal is calculated as

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} (1)^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} [T]_0^{T/2} = \lim_{T\to\infty} \frac{1}{T} \times \frac{T}{2} = \frac{1}{2}$$

Since power is finite and energy is ∞. So, it is power signal.

b. $x(t) = \delta(t)$

$$x(t) = \delta(t) = \begin{cases} 1 & \text{at } t = 0 \\ 0 & \text{otherwise} \end{cases}$$

The energy of x(t) is given by

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} |\delta(t)|^2 dt = 1$$

[since area under curve of $\delta(t)$ is1]

Power is given by

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |\delta(t)|^2\,dt = 0$$

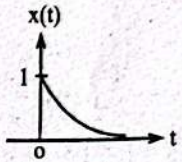
Since power is zero but energy is finite, it is energy signal.

19. **Determine the power and energy for a continuous time signal, $x(t) = e^{-2t}\,u(t)$ $(t \geq 0)$.** [2072 Kartik]

Solution:

The given signal is

$x(t) = e^{-2t}\,u(t)$



The energy of the signal is

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} e^{-4t}\,dt$$

$$= \int_{0}^{\infty} e^{-4t}\,dt$$

$$= -\frac{1}{4}[e^{-4t}]_0^{\infty} = -\frac{1}{4}(0-1) = \frac{1}{4}$$

The power of the signal is

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

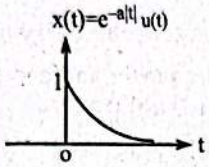
$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{-4t}\,dt = \lim_{T\to\infty} \frac{4}{T} = 0$$

Since energy is finite and average power is zero, the given signal is energy signal.

20. Justify whether a signal $x(t) = e^{-a|t|}\,u(t)$, a>0 is energy or power signal. [2072 Chaitra]

Solution:

The given signal is



The energy of the signal is given by

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{0}^{\infty} e^{-2at}\,dt = -\frac{1}{2a}\left[e^{-2at}\right]_0^{\infty} = -\frac{1}{2a}[0-1] = \frac{1}{2a}$$

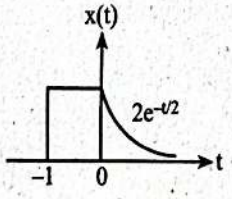
The power of the signal is

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x^2(t)|\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{\infty} e^{-2at}\,dt = \frac{\frac{1}{2a}}{\infty} = 0$$

Since the signal has finite energy and zero average power, it is energy signal.

21. Determine whether the signal is an energy signal or a power signal.



Solution:

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-\infty}^{0} x^2(t)\,dt + \int_{0}^{\infty} (2e^{-t/2})^2\,dt$$

$$= \int_{-1}^{0} x^2(t)\,dt + \int_{0}^{\infty} (2e^{-t/2})^2\,dt$$

$$= \int_{-1}^{0} (2)^2\,dt + 4\int_{0}^{\infty} e^{-t}\,dt$$

$$= 4\,[t]_{-1}^{0} + 4\left(\frac{e^{-t}}{-1}\right)_0^{\infty} = 4\,(0+1) + \frac{4}{(-1)}(0-1) = 8$$

Since the amplitude of the signal tends to zero as $t \to \infty$, the given signal is an energy signal.

22. Consider a periodic signal represented by the sequence {1, 2, 0, 5, 3, 7, 0, 9} repeating in a period of N = 8, find its average power.

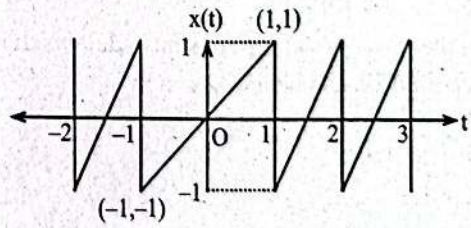
Solution:

$$P = \frac{1}{8}\sum_{n=-2}^{5} |x[n]|^2$$

$$= \frac{1}{8}[\{x[-2]\}^2 + \{x[-1]\}^2 + \{x[0]\}^2 + \{x[1]\}^2 + \{x[2]\}^2 + \{x[3]\}^2 + \{x[4]\}^2 + \{x[5]\}^2]$$

$$= \frac{1}{8}[1 + 4 + 0 + 25 + 9 + 49 + 0 + 81] = 21.125$$

23. Determine whether the following signal is energy as power signal and find the corresponding value.



Solution:

Since the signal amplitude does not tend to zero as $|t| \to \infty$, the signal is power signal.

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

Here, T = 1-(-1) = 2.

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

The equation can be written as

$$y - y_1 = m(x - x_1)$$

$$\text{or, } x(t) - 1 = \frac{1+1}{1+1}(t-1)$$

$$\text{or, } x(t) - 1 = (t-1)$$

$$\text{or, } x(t) = t$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} t^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left(\frac{t^3}{3}\right)_{-T/2}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{3T} [t^3]_{-1}^{1} = \frac{1}{2\times 3} \times (1+1) = \frac{1}{3}$$

$E = \infty$

Hence, the given signal is power signal.

24. Determine whether y = asint for $-\infty<t<+\infty$ is a power signal or an energy signal.

Solution:

Since the given signal is pure sinusoidal which is periodic with period 2π, its power is given by

$$P = \frac{1}{T} \int_{0}^{T} x^2(t)\, dt$$

$$= \frac{1}{2\pi} \left[\int_{0}^{2\pi} (a \sin t)^2\, dt \right]$$

$$= \frac{a^2}{2\pi} \int_{0}^{\pi} \left(\frac{1 - \cos 2t}{2} \right) dt$$

$$= \frac{a^2}{2\pi} \left[\frac{t}{2} - \frac{\sin 2t}{4} \right]_0^{2\pi}$$

$$= \frac{a^2}{8\pi} [2t - \sin 2t]_0^{2\pi}$$

$$= \frac{a^2}{8\pi} [(4\pi - \sin 4\pi) - (0 - 0)] = \frac{a^2}{2}$$

$E = \infty$

The given signal is power signal.

25. Find energy and power of the signal, $x(t) = e^{-a|t|}$ for a>0 and state whether it is energy signal, power signal or neither.

Solution:

$$x(t) = e^{-a|t|}$$

$$= \begin{cases} e^{-at}, & t > 0 \\ e^{at}, & t < 0 \end{cases}$$

We have,

$$E = \int_{-\infty}^{\infty} |x^2(t)|\, dt$$

$$= \int_{-\infty}^{0} e^{2at}\, dt + \int_{0}^{\infty} e^{-2at}\, dt$$

$$= \left[\frac{e^{2at}}{2a}\right]_{-\infty}^{0} + \left[\frac{e^{-2a}}{-2a}\right]_{0}^{\infty}$$

$$= \frac{1}{2a} + \frac{1}{2a} = \frac{1}{a}$$

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)^2|\, dt$$

$$= \lim_{T\to\infty} \frac{E}{T} = 0$$

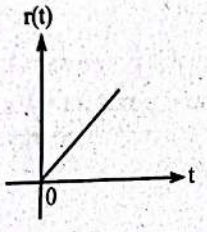
Since $E = \frac{1}{a} < \infty$ and $P_{av} = 0$, the given signal is an energy signal.

26. Determine the total energy and total average power of a continuous time ramp signal and decide whether it is energy signal or power signal or neither.

Solution:

The continuous time ramp signal is given by

$$r(t) = \begin{cases} 0, \text{ for } t < 0 \\ t, \text{ for } t \geq 0 \end{cases}$$



$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{t^3}{3}\right]_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T} \times \frac{T^3}{24} = \lim_{T\to\infty} \frac{T^2}{24} = \infty$$

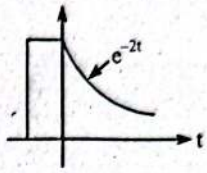
$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \int_0^{T/2} t^2\, dt$$

$$= \lim_{T\to\infty} \left[\frac{t^3}{3}\right]_0^{T/2} = \lim_{T\to\infty} \frac{T^3}{24} = \infty$$

Since average power and energy both are infinite, the given signal is neither energy nor power.

27. Find energy and power of the following signal.



Solution:

The energy of the signal is

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-1}^{0} 1^2\, dt + \int_{0}^{\infty} e^{-2t}\, dt$$

$$= 1 + \frac{e^{-2t}}{-2} \int_0^{\infty} = 0.5$$

The power is calculated as

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{E}{T} = 0$$

Since average power is zero ad energy is finite, the given signal is an energy signal.

29. Justify whether signal $t^{-1/4}$ u (t–1) is energy signal or power signal.

Solution:

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= \int_{-\infty}^{\infty} |t^{-1/4} u(t-1)|^2\, dt$$

$$u(t-1) = \begin{cases} 1; t > 1 \\ 0; t < 1 \end{cases}$$

$$\therefore E = \int_{1}^{\infty} t^{-1/2}\, dt$$

$$= \left[\frac{t^{1/2}}{1/2}\right]_1^{\infty} = \infty$$

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T}^{T} |x(t)^2|\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T}^{T} t^{-1/4}\, u(t-1)^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{1}^{T} t^{-1/2}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[2t^{1/2}\right]_1^T$$

$$= \lim_{T\to\infty}\left[\frac{T^{1/2}-1}{T}\right] = \lim_{T\to\infty}\left[\frac{1-\frac{1}{T^{1/2}}}{T^{1/2}}\right] = 0$$

Since average power is zero and energy is infinite, the signal is neither energy signal nor power signal.

CHAPTER - 3

SIGNAL ANALYSIS

3.1 Impulse Signal

The *impulse signal* is a signal with infinite magnitude and zero duration but with an area of A.

The *unit impulse signal* is a signal with infinite magnitude and zero duration but with unit area. Mathematically, a unit impulse signal is defined as

$$\delta(t) = \begin{cases} \infty \text{ for } t = 0 \\ 0 \text{ for } t \neq 0 \end{cases} \text{ and } \int_{-\infty}^{\infty} \delta(t)dt = 1$$

3.2 Linear Time Invariant System (LTI System)

Both continuous and discrete time signals have several basic properties. Out of these properties, *linearity and time invariant* plays a vital role in the analysis of signals and system. If the system has both linearity and time invariant properties, the system is called *linear time invariant system* or *LTI system*. The complete characterization of any LTI system in terms of its impulse response is performed by *convolution integral* in continuous time and by *convolution sum* in discrete time.

$$x(t) = \delta(t) \longrightarrow \boxed{\text{LTI system}} \longrightarrow y(t) = h(t)$$

In the given figure, $\delta(t)$ denotes unit impulse function whereas $h(t)$ denotes unit impulse response. The unit impulse response is the output of LTI system when applied input $x(t)$ is equal to the unit impulse signal $\delta(t)$.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

For discrete time signal,

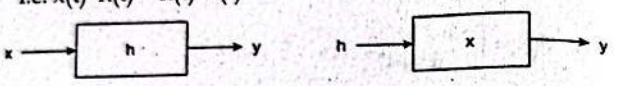
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Properties of Convolution:

1. **Commutative property**

According to this property, output of continuous time LTI system having input x(t) and unit impulse response h(t) is equal to the output of a continuous time LTI system having input h(t) and unit impulse response x(t).

i.e. $x(t)*h(t) = h(t)*x(t)$

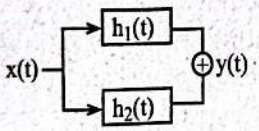


2. **Distributive property**

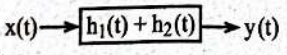
According to this property, the following relation holds true:

$x(t)*[h_1(t) + h_2(t)] = [x(t)*h_1(t)] + [x(t)*h_2(t)]$

Diagrammatically,



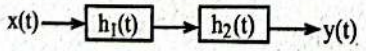
is identical to:



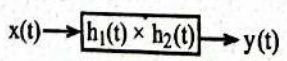
3. **Associative property**

According to this property, the following relation holds true:

$[x_1(t)*h_1(t)]*h_2(t) = x_1(t)*[h_1(t)*h_2(t)]$



is identical to



4. **Shifting property**

$x_1(t)*x_2(t) = g(t)$

$x_1(t-t_1)*x_2(t-t_2) = g(t-(t_1+t_2))$

3.3 Fourier Series Representation of Continuous Time Signal

A *Fourier series* is called a linear combination of hormonally related complex exponentials. It is used for the spectrum analysis of periodic signals. Spectrum analysis of continuous time periodic signal is called *continuous time Fourier series* (CTFS) and that of discrete time signal is called *discrete time Fourier series* (DTFS).

Fourier series is important because it allows us to model a periodic signal as a sum of distinct harmonic components.

Application of Fourier series:

1. File compression (JPG, MPEG, etc.)
2. Signal processing in communication
3. Cryptography

The pair of equation that defines the Fourier series of periodic continuous time signal are:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega t}$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega t} \rightarrow \text{synthesis equation} \quad \text{..... (i)}$$

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega t}\, dt \rightarrow \text{analysis equation} \quad \text{..... (ii)}$$

The set of coefficients a_k are called the *Fourier series coefficients* or the *spectral coefficients* of x(t).

The component $a_0 = \frac{1}{T}\int_0^T x(t)\, dt$ is called the dc component of signal x(t).

Trigonometric Fourier Series

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \text{ ..... (i)}$$

The values of a_0, a_n, and b_n are calculated as

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega t\,dt$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega t\,dt$$

where $T = \frac{2\pi}{\omega}$

Symmetric Condition:

1. If the signal x(t) is even, then $b_n = 0$,
2. If the signal x(t) is odd then $a_n = 0 = a_0$.

Properties of Fourier series:

i. **Linearity**

Let x(t) and y(t) be two periodic signals with Fourier coefficients a_k and b_k respectively.

$$x(t) \overset{FS}{\longleftrightarrow} a_k$$

$$y(t) \overset{FS}{\longleftrightarrow} b_k$$

If c_k be the Fourier coefficients of linear combination of x(t) and y(t), then

$$z(t)=Ax(t)+By(t) \overset{FS}{\longleftrightarrow} c_k=Aa_k+Bb_k$$

ii. **Time shifting**

If $x(t) \overset{FS}{\longleftrightarrow} a_k$, then

$$y(t)= x(t-t_0) \overset{FS}{\longleftrightarrow} b_k=e^{-jk\omega_o t_o}a_k$$

iii. **Time reversal**

If $x(t) \overset{FS}{\longleftrightarrow} a_k$, then

$$y(t)= x(-t) \overset{FS}{\longleftrightarrow} a_{-k}$$

iv. **Time scaling**

If $x(t) \overset{FS}{\longleftrightarrow} a_k$, then

$$x(\alpha t) \overset{FS}{\longleftrightarrow} a_k$$

v. **Multiplication**

If $x(t) \overset{FS}{\longleftrightarrow} a_k$

If $y(t) \overset{FS}{\longleftrightarrow} b_k$

If z(t) = x(t)y(t) and h_k be the Fourier series coefficients of z(t), then

$$h_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

vi. **Conjugation**

If $x(t) \overset{FS}{\longleftrightarrow} a_k$, then

$$x^*(t) \overset{FS}{\longleftrightarrow} a^*_{-k}$$

vii. **Integration**

If $x(t) \overset{FS}{\longleftrightarrow} a_k$, then

$$\int_{-\infty}^{t} x(\tau)d\tau \overset{FS}{\longleftrightarrow} \frac{1}{jk\omega_o} a_k$$

viii. **Differentiation**

If $x(t) \overset{FS}{\longleftrightarrow} a_k$, then

$$\frac{dx(t)}{dt} \overset{FS}{\longleftrightarrow} jk\omega_o a_k$$

ix. **Parseval's relation**

If $x(t) \overset{FS}{\longleftrightarrow} a_k$, then

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 = \sum_{k=-\infty}^{\infty} |a_k|^2$$

3.4 Fourier Transform for Continuous Time Signals

The *Fourier transform* is a tool that breaks a waveform (a function or signal) into an alternate representation, characterized by sine and cosines. The Fourier transform shows that any waveform can be re-written as the sum of sinusoidal functions.

The Fourier transform is an extension to Fourier series for which signals are aperiodic. Changing time period tends to infinite ($t \to \infty$). As contract to periodic signals of finite period in case of Fourier series. A Fourier Transform of a signal tells us what frequency components are present in a signal.

Applications:

i. Image compression

ii. Analysis of differential equation

iii. Signal processing and spectrum analysis

The pair of equations or Fourier transform pairs are given by:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad \text{..... (i)}$$

And, Anti-Fourier transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)\, e^{j\omega t}\, d\omega \quad \text{..... (ii)}$$

Equation (i) is called *Fourier transform* of x(t) denoted by $X(j\omega)$ and x(t) is the *inverse Fourier transform* of $X(j\omega)$.

Properties of continuous time Fourier transform:

i. **Linearity**

$x_1(t) \leftrightarrow X_1(j\omega)$

$x_2(t) \leftrightarrow X_2(j\omega)$

then, $A\, x_1(t) + B\, x_2(t) \rightarrow A\, X_1(j\omega) + B\, X_2(j\omega)$

ii. **Time shifting**

$x(t) \leftrightarrow X(j\omega)$

$x(t-t_0) \leftrightarrow X(j\omega)\, e^{-j\omega t_0}$

iii. **Frequency shifting**

$x(t) \leftrightarrow X(j\omega)$

$x(t)\, e^{j\omega_0 t} \leftrightarrow X(j\omega - j\omega_0)$

iv. **Time scaling**

$x(t) \leftrightarrow X(j\omega)$

$$x(at) \leftrightarrow \frac{1}{|a|}\frac{X(j\omega)}{a}$$

v. **Time differentiation**

$x(t) \leftrightarrow X(j\omega)$

$$\frac{dx(t)}{dt} \leftrightarrow j\omega\, X(j\omega)$$

vi. **Convolution**

$x(t) \leftrightarrow X(j\omega)$

$y(t) \leftrightarrow Y(j\omega)$

$x(t) * y(t) \leftrightarrow X(j\omega)Y(j\omega)$

vii. **Frequency convolution theorem**

$x(t) \leftrightarrow X(j\omega)$

$y(t) \leftrightarrow Y(j\omega)$

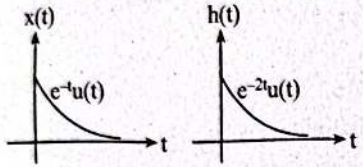
Then, $2\pi.\, x(t).\, y(t) \leftrightarrow X(j\omega) \times Y(j\omega)$

NUMERICAL PROBLEMS

1. Find the output response of an LTI system when input $x(t) = \exp(-t)\, u(t)$ and impulse response $h(t) = \exp(-2t)\, u(t)$.

Solution:

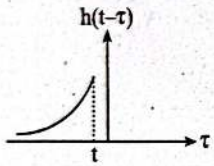
Given signals can be drawn as:



The convolution output is given by:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

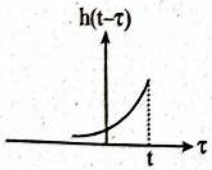
For $t < 0$,



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{0} x(\tau)\, h(t-\tau)\, d\tau + \int_{0}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = 0$$

For $t > 0$,



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{0} x(\tau)h(t-\tau)d\tau + \int_{0}^{t} x(\tau)\, h(t-\tau)d\tau + \int_{t}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{0}^{t} e^{-\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)\, d\tau$$

$$= e^{-2t}\int_{0}^{t} e^{\tau}\, d\tau = e^{-2t}(e^{t} - 1)$$

$$y(t) = \begin{cases} 0 \text{ for } t < 0 \\ e^{-2t}(e^{t}-1) \text{ for } t > 0 \end{cases}$$

Hence, $y(t) = e^{-2t}(e^{t}-1)\, u(t)$

2. Assume impulse response of the system is given by $h[k] = 0.5^k\, u[k]$. Find $y[k]$ when $x[k] = 0.8^k\, u[k]$.

Solution:

$$y[k] = \sum_{m=-\infty}^{\infty} x[m]\, h[k-m]$$

$$\text{So, } y[k] = \sum_{m=-\infty}^{\infty} 0.8^m\, u[m]\, (0.5)^{k-m}\, u[k-m]$$

$$\text{or, } y[k] = \sum_{m=-\infty}^{\infty} (0.8)^m\, (0.5)^{k-m}\, u(k-m)$$

Depending upon the value of k, the output of the system may take two different forms:

Case I: $k < 0$

$u[k-m] = u\,[-\text{ve value}] = 0$

$\therefore\ y[k] = 0$

Case II: $k \geq 0$

$$u[k-m] = \begin{cases} 1;\ m \leq k \\ 0;\ m > k \end{cases}$$

$$y[k] = \sum_{m=0}^{k} (0.8)^m\, (0.5)^{k-m}\, (1)$$

$$\text{or, } y[k] = \sum_{m=0}^{k} (0.8)^m\, (0.5)^{k-m}$$

$$y[k] = 0.5^k \sum_{m=0}^{k} \left(\frac{0.8}{0.5}\right)^m$$

For geometric series,

$$\sum_{n=0}^{N} (r)^n = \frac{1-(r)^{N+1}}{1-r}$$

$$\therefore y[k] = 0.5^k \frac{1-\left(\frac{0.8}{0.5}\right)^{k+1}}{1-\left(\frac{0.8}{0.5}\right)}$$

$$= \frac{10}{3}\{(0.8)^{k+1} - (0.5)^{k+1}\}$$

Combining two cases,

$$y[k] = \begin{cases} 0 & ; k < 0 \\ \frac{10}{3}\{(0.8)^{k+1} - (0.5)^{k+1}; & k \geq 0 \end{cases}$$

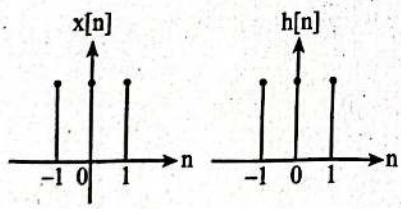
$$\therefore y[k] = \frac{10}{3}\{(0.8)^{k+1} - (0.5)^{k+1}\}\, u[k]$$

3. Find the output of the following LTI system where x[n] is given as input and h[n] is its unit impulse response.

$$x[n] = \begin{cases} 1 & \text{for } -1 \leq n \leq 1 \\ 0 & \text{for otherwise} \end{cases}$$

$$h[n] = \begin{cases} 1 & \text{for } -1 \leq n \leq 1 \\ 0 & \text{for otherwise} \end{cases}$$

Solution:



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$\text{For } n = 0,\ y[0] = \sum_{k=-\infty}^{\infty} x[k]\, h[-k]$$

$$= x[-1]\,h[1] + x[0]\,h[0] + x[1]\,h[-1]$$
$$= 1\times1 + 1\times1 + 1\times1 = 3$$

$$\text{For } n = 1,\ y[1] = \sum_{k=-\infty}^{\infty} x[k]\, h[1-k]$$
$$= x[-1]\,h[2] + x[0]\,h[1] + x[1]\,h[0]$$
$$= 1 + 1 = 2$$

$$\text{For } n = -1,\ y[-1] = \sum_{k=-\infty}^{\infty} x[k]\, h[-1-k]$$
$$= x[-1]\,h[0] + x[0]\,h[-1] + x[1]\,h[-2]$$
$$= 1 + 1 = 2$$

For $n = -2$, $y[-2] = 1$

For $n = 2$, $y[2] = 1$

For $n \leq -3$, $y[n] = 0$

For $n \geq 3$, $y[n] = 0$

Hence, $y[n] = \{\ldots, 0, 1, 2, 3, 2, 1, 0, \ldots\}$
$\uparrow$

4. Consider a periodic signal x(t) with fundamental frequency 2π i.e., $x(t) = \sum_{k=-3}^{3} a_k e^{jk2\pi t}$ with $a_0 = 1$, $a_{-1} = a_1 = \frac{1}{4}$, $a_2 = a_{-2} = \frac{1}{2}$ and $a_3 = a_{-3} = \frac{1}{3}$. Construct x(t) as a linear combination of harmonically related complex exponentials.

Solution:

The given equation is

$$x(t) = \sum_{k=-3}^{3} a_k e^{jk2\pi t}$$

$$= a_{-3}e^{j(-3)2\pi t} + a_{-2}e^{j(-2)2\pi t} + a_{-1}e^{j(-1)2\pi t} + a_0 e^{j(0)2\pi t} + a_1 e^{+j(1)2\pi t} + a_2 e^{+j(2)2\pi t} + a_3 e^{+j(3)2\pi t}.$$

$$= \frac{1}{3}(e^{-j6\pi t} + e^{j6\pi t}) + \frac{1}{2}(e^{-j4\pi t} + e^{j4\pi t}) + \frac{1}{4}(e^{-j2\pi t} + e^{j2\pi t}) + 1$$

$$= 1 + \frac{2}{3}\cos 6\pi t + \cos 4\pi t + \frac{1}{2}\cos 2\pi t$$

$$= 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t$$

5. **For the $x(t) = \sin\omega_0 t$ where ω_0 is fundamental frequency find the Fourier series coefficients.**

Solution:

$$x(t) = \sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t}$$

We have,

$$x(t) = \sum_{k=-3}^{3} a_k e^{jk\omega_0 t}$$

$$= \ldots\ldots\ldots\ldots 0 + a_{-1}e^{-j\omega_0 t} + a_0 e^{j\omega_0 t} + 0 + \ldots\ldots\ldots\ldots$$

On comparing, $a_{-1} = \frac{-1}{2j}$, $a_0 = 0$, $a_1 = \frac{1}{2j}$

6. **Find the Fourier series coefficients and plot the magnitude and phase spectra for the following signal.**

$x(t) = 1 + \sin\omega_0 t + \cos\omega_0 t + \cos(2\omega_0 t + \pi/4)$.

Solution:

$$x(t) = 1 + \sin\omega_0 t + \cos\omega_0 t + \cos(2\omega_0 t + \pi/4)$$

$$= 1 + \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} + \frac{e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}}{2}$$

$$= 1 + \left(\frac{1}{2} + \frac{1}{2j}\right)e^{j\omega_0 t} + \left(\frac{1}{2} - \frac{1}{2j}\right)e^{-j\omega_0 t} + \frac{1}{2}e^{j\pi/4}e^{j2\omega_0 t} + \frac{1}{2}e^{-j\pi/4}$$

$$e^{-j2\omega_0 t}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{-j\omega_0 t}$$

$$= \ldots\ldots + a_{-2}e^{-j2\omega_0 t} + a_{-1}e^{-j\omega_0 t} + a_0 + a_1 e^{j\omega_0 t} + a_2 e^{j2\omega_0 t} + \ldots$$

$$+ \ldots\ldots\ldots$$

$$\therefore a_{-2} = \frac{1}{2}e^{-j\pi/4}$$

$$= \frac{1}{2}\left(\cos\frac{\pi}{4} - j\sin\frac{\pi}{4}\right) = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = \frac{1}{2\sqrt{2}}(1-j)$$

$$a_{-1} = \frac{1}{2} - \frac{1}{2j} = \frac{1}{2} + j\frac{1}{2}$$

$$a_0 = 1$$

$$a_1 = \frac{1}{2} + \frac{1}{2j} = \frac{1}{2} - j\frac{1}{2}$$

$$a_2 = \frac{1}{2}e^{j\pi/4} = \frac{1}{2\sqrt{2}}(1+j)$$

For magnitude plot,

$$|a+ib| = \sqrt{a^2 + b^2}$$

$$|a_{-2}| = \sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{-1}{2\sqrt{2}}\right)^2} = 0.5$$

$$|a_{-1}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 0.70$$

$$|a_0| = 1$$

$$|a_1| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2} = 0.70$$

$$|a_2| = \sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2} = 0.5$$

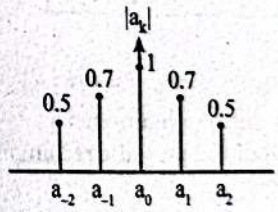


Fig.: Magnitude plot

For phase spectra,

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) \text{ for } (a + ib) \text{ form}$$

$$\angle a_{-2} = \tan^{-1}\left(\frac{\frac{-1}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}}}\right) = -45°$$

$$\angle a_{-1} = \tan^{-1}\frac{\frac{1}{2}}{\frac{1}{2}} = 45°$$

$$\angle a_0 = \tan^{-1}\frac{0}{1} = 0°$$

$$\angle a_1 = \tan^{-1}\left(\frac{\frac{-1}{2}}{\frac{1}{2}}\right) = -45°$$

$$\angle a_2 = \tan^{-1} = \left(\frac{\frac{1}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}}}\right) = 45°$$

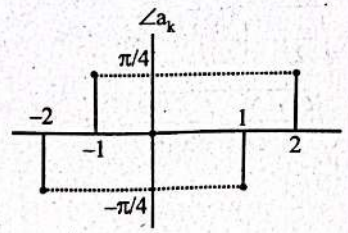
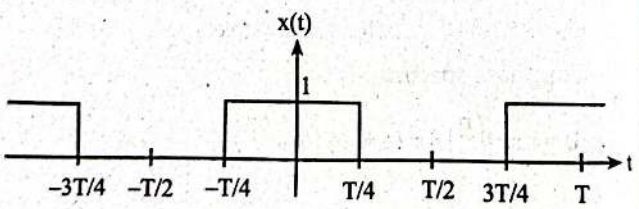


Fig.: Phase plot

7. Determine the trigonometric Fourier series representation of the periodic rectangular waveform x(t) as shown below:



Solution:

$$x(t) = \begin{cases} 0 & \text{for } \frac{-T}{2} < t < \frac{-T}{4} \\ 1 & \text{for } \frac{-T}{4} < t < \frac{T}{4} \\ 0 & \text{for } \frac{T}{4} < t < \frac{T}{2} \end{cases}$$

Since the given signal is even, $b_n = 0$

$$a_0 = \frac{2}{T}\int_0^{T/2} x(t)\,dt$$

$$= \frac{2}{T}\left[\int_0^{T/4} x(t)\,dt + \int_{T/4}^{T/2} x(t)\,dt\right]$$

$$= \frac{2}{T}\int_0^{T/4} 1\,dt + \frac{2}{T}\int_{T/4}^{T/2} 0\,dt$$

$$= \frac{2}{T}[t]_0^{T/4} = \frac{2}{T}\times\frac{T}{4} = \frac{1}{2}$$

$$b_n = 0$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega t\,dt$$

$$= \frac{4}{T}\int_0^{T/4} x(t)\cos n\omega t\,dt + \int_{T/4}^{T/2} x(t)\cos n\omega t\,dt$$

$$= \frac{4}{T}\int_0^{T/4} \cos n\omega t\,dt$$

$$= \frac{4}{T}\left[\frac{\sin n\omega t}{n\omega}\right]_0^{T/4}$$

$$= \frac{4}{T\times n\times\frac{2\pi}{T}}\left(\sin n\times\frac{2\pi}{T}\times\frac{T}{4}\right) = \frac{2}{n\pi}\sin\frac{n\pi}{2}$$

$$\text{Thus, } a_n = \begin{cases} 0 & \text{for } n = 2, 4, 6, \ldots\ldots \\ \dfrac{2}{n\pi} & \text{for } n = 1, 5, 9, \ldots\ldots \\ \dfrac{-2}{n\pi} & \text{for } n = 3, 7, 11, \ldots\ldots\ldots \end{cases}$$

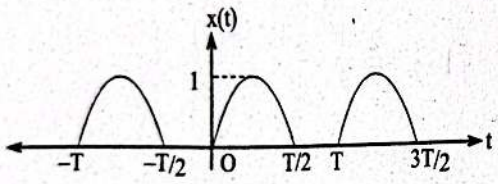
Hence, Fourier series representation of x(t) will be

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + 0)$$

$$= \frac{1}{2} + a_1 \cos\omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + a_4 \cos 4\omega t + a_5 \cos 5\omega t + \ldots\ldots\ldots$$

$$= \frac{1}{2} + \frac{2}{\pi}\cos\omega t - \frac{2}{3\pi}\cos 3\omega t + \frac{2}{5\pi}\cos 5\omega t - \frac{2}{7\pi}\cos 7\omega t + \ldots\ldots\ldots$$

8. Find the Fourier series representation of the half-wave rectified sine wave.



Solution:

$$x(t) = \begin{cases} \sin\omega t & \text{for } 0 \le t \le \dfrac{T}{2} \\ 0 & \text{for } \dfrac{T}{2} < t < T \end{cases}$$

The average value of signal x(t) is

$$a_0 = \frac{1}{T}\int_0^T x(t)\, dt$$

$$= \frac{1}{T}\int_0^{T/2} \sin\omega t\, dt = \frac{1}{T}\left[\frac{-\cos\omega t}{\omega}\right]_0^{T/2} = \frac{1}{\pi}$$

$$a_n = \frac{2}{T}\int_0^T x(t) \cos n\omega t\, dt$$

$$= \frac{2}{T}\int_0^{T/2} \sin\omega t \cos n\omega t\, dt$$

$$= \frac{2}{T}\int_0^{T/2} [\sin(\omega t + n\omega t) + \sin(\omega t - n\omega t)]\, dt$$

$$= \frac{1}{T}\int_0^{T/2} [\sin(\omega t + n\omega t)]\, dt + \frac{1}{T}\int_0^{T/2} [\sin(\omega t - n\omega t)]\, dt$$

$$= \frac{1}{T}\left[\frac{-\cos(1+n)\omega t}{(1+n)\omega} - \frac{\cos(1-n)\omega t}{(1-n)\omega}\right]_0^{T/2}$$

$$= \frac{1}{2\pi(n^2-1)}[(1-n)\cos(1+n)\pi + (1+n)\cos(1-n)\pi - 2]$$

For odd n, $a_n = 0$

For even n, $a_n = \dfrac{2}{\pi(1-n^2)}$

$$b_n = \frac{2}{T}\int_0^T x(t) \sin\omega t\, dt$$

$$= \frac{2}{T}\int_0^{T/2} \sin\omega t \sin n\omega t\, dt$$

$$= \frac{2}{T}\int_0^{T/2}\left[\frac{1}{2}\cos(\omega t - n\omega t) - \frac{1}{2}\cos(\omega t + n\omega t)\right] dt$$

$$= \frac{1}{T}\left[\frac{\sin(1-n)\omega t}{(1-n)\omega} - \frac{\sin(1+n)\omega t}{(1+n)\omega}\right]_0^{T/2}$$

$$= \left[\frac{\sin(1-n)\pi}{(1-n)} - \frac{\sin(1+n)\pi}{(1+n)}\right]$$

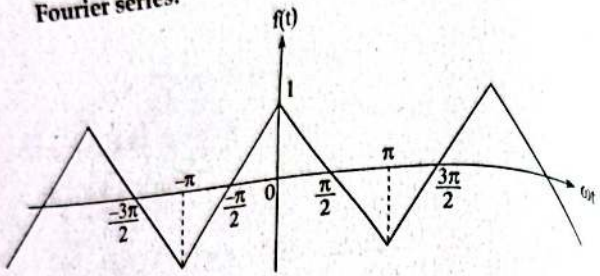
For $n \neq 1$, $b_n = 0$

For $n = 1$, $b_1 = \dfrac{1}{2\pi}$

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$= \frac{1}{\pi} + \frac{1}{2\pi}\sin\omega t - \sum_{n=\text{even}}^{\infty} \frac{2}{\pi(n^2-1)}\cos n\omega t$$

9. **For the given waveform, find the trigonometric form Fourier series.**



Solution:

$$f(t) = \frac{1}{\frac{\pi}{2}}\omega t + 1 \qquad -\pi \le \omega t \le 0$$

$$= -\frac{1}{\frac{\pi}{2}}\omega t + 1 \quad 0 \le \omega t \le \pi$$

The trigonometric Fourier series is given by

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t) \quad -\pi \le \omega t \le \pi \text{ ..... (1)}$$

Since the function f(t) is even, $b_n = 0$; and since its average value in a complete cycle is 0, $\frac{a_0}{2} = 0$.

Now,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos n\omega t \, d(\omega t)$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(t)\cos n\omega t \, d(\omega t) + \int_{0}^{\pi} f(t)\cos n\omega t \, d(\omega t)\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}\left(\frac{2}{\pi}\omega t + 1\right)\cos n\omega t \, d(\omega t) + \int_{0}^{\pi}\left(\frac{-2}{\pi}\omega t + 1\right)\cos n\omega t \, d(\omega t)\right]$$

$$= \frac{4}{n^2\pi^2}[1 - (-1)^n]$$

For even n, $a_n = 0$

For odd n, $a_n = \frac{8}{n^2\pi^2}$

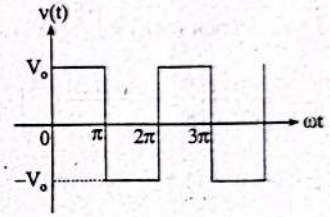
From equation (1),

$$f(t) = 0 + \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}\cos n\omega t \text{ ; for odd n}$$

$$= \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}\cos n\omega t$$

$$= \frac{8}{\pi^2}\left(\cos\omega t + \frac{1}{9}\cos 3\omega t + \frac{1}{25}\cos 5\omega t + \text{.....}\right)$$

10. **For the following waveform, find trigonometric Fourier series.**



Solution:

$$v = V_o \qquad 0 \le \omega t \le \pi$$

$$= -V_o \quad \pi \le \omega t \le 2\pi$$

The trigonometric Fourier series is given by

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t) \quad 0 \le \omega t \le 2\pi$$

..... (1)

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} v(t)\, d(\omega t)$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} v(t)\, d(\omega t) + \int_0^{\pi} v(t)\, d(\omega t)\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} V_o\, d(\omega t) + \int_0^{\pi} -V_o\, d(\omega t)\right]$$

$$= \frac{V_o}{\pi}\left[|\omega t|_0^{\pi} - |\omega t|_{\pi}^{2\pi} \right]$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(t) \cos n\omega t \, d(\omega t)$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} V_o \cos n\omega t \, (d\omega t) + \int_{\pi}^{2\pi} (-V_o) \cos n\omega t \, d(\omega t)\right]$$

$$= \frac{V_o}{\pi}\left[\left|\frac{\sin n\omega t}{n}\right|_0^{\pi} - \left|\frac{\sin n\,\omega t}{n}\right|_{\pi}^{2\pi}\right]$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v(t) \sin n\omega t \, d(\omega t)$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} V_o \sin n\omega t \, (d\omega t) + \int_{\pi}^{2\pi} (-V_o) \sin n\omega t \, d(\omega t)\right]$$

$$= \frac{V_o}{\pi}\left[-\left|\frac{\cos n\omega t}{n}\right|_0^{\pi} - \left|\frac{\cos n\,\omega t}{n}\right|_{\pi}^{2\pi}\right]$$

$$= \frac{V_o}{n\pi}[-(\cos n\pi - 1) + (1 - \cos n\pi)]$$

$$= \frac{2V_o}{n\pi}(1 - \cos n\pi)$$

$b_n = 0$ if n = even

$= \frac{4V_o}{n\pi}$ if n = odd

Hence, equation (1) becomes

$$v(t) = 0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$= \sum_{n=1,3,5,\ldots}^{\infty} b_n \sin n\omega t$$

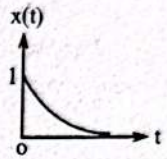
$$\therefore v(t) = \frac{4V_o}{\pi}\sin \omega t + \frac{4V_o}{3\pi}\sin 3\omega t + \frac{4V_o}{5\pi}\sin 5\omega t + \ldots\ldots\ldots$$

11. For the given signal $x(t) = e^{-at}\, u(t)$ $(a > 0)$, find the Fourier transform and plot the magnitude and phase spectra. [2072 Kartik]

Solution:

The given signal is

$x(t) = e^{-at}\, u(t)$



The Fourier transform of $x(t)$ is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= -\frac{1}{(a+j\omega)}\left[e^{-(a+j\omega)t}\right]_0^{\infty}$$

$$= \frac{1}{a+j\omega}\ ;\ a>0$$

The magnitude response is given by

$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

For $\omega = 0$, $|X(j\omega)| = \frac{1}{a}$

For $\omega = a$, $|X(j\omega)| = \frac{1}{\sqrt{2}a}$

For $\omega = -a$, $|X(j\omega)| = \frac{1}{\sqrt{2}a}$

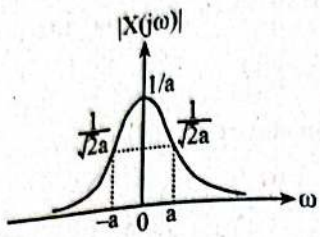


Fig.: Magnitude plot

The phase response is given by

$$\angle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

For $\omega = 0$, $\angle X(j\omega) = 0$

For $\omega = a$, $\angle X(j\omega) = \frac{-\pi}{4}$

For $\omega = -a$, $\angle X(j\omega) = \frac{\pi}{4}$

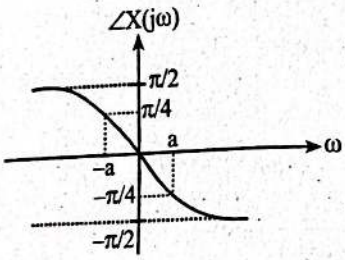


Fig.: Phase plot

12. Find the Fourier transform of the signal $x(t) = e^{-a|t|}$ where $(0<a<\infty)$ is real-valued and $|t|$ denotes the absolute value of t. [2071 Chaitra]

Solution:

The given signal $x(t) = e^{-a|t|}$ can be written as

$$x(t) = \begin{cases} e^{-at} \text{ for } t \geq 0 \\ e^{at} \text{ for } t < 0 \end{cases} \qquad 0 < a < \infty$$

The Fourier transform is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

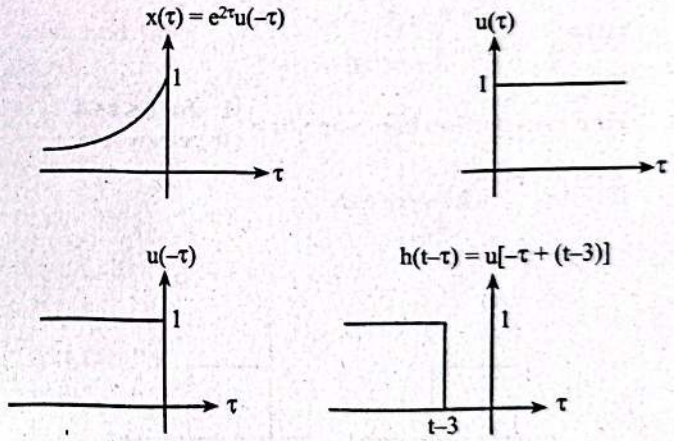
$$= \frac{1}{a-j\omega}\left[e^{(a-j\omega)t}\right]_{-\infty}^{0} + \frac{1}{-(a+j\omega)}\left[e^{-(a+j\omega)t}\right]_{0}^{\infty}$$

$$= \frac{1}{a-j\omega}[1-0] + \frac{1}{-(a+j\omega)}[0-1]$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{a+j\omega+a-j\omega}{(a-j\omega)(a+j\omega)} = \frac{2a}{a^2+\omega^2}$$

13. Find convolution between two signals $x(t) = e^{2t}\, u(-t)$ and $h(t) = u(t-3)$.

Solution:



For $t-3 \leq 0$,

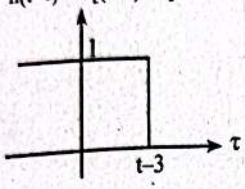
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{t-3} x(\tau)\, h(t-\tau)\, d\tau + \int_{t-3}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{t-3} e^{2\tau} 1\, d\tau + 0 = \frac{1}{2} e^{2(t-3)}$$

For $t-3 \geq 0$,

$h(t-\tau) = u[(t-3)-\tau]$



$$y(t) = \int_{-\infty}^{0} x(\tau)\, h(t-\tau)\, d\tau + \int_{0}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

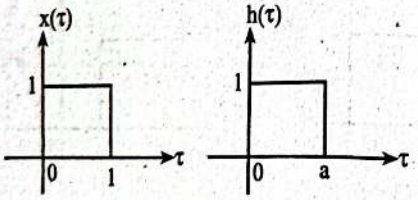
$$= \int_{-\infty}^{0} e^{2\tau} 1 d\tau + 0 = \int_{-\infty}^{0} e^{2\tau}\, d\tau = \frac{1}{2}$$

$$y(t) = \begin{cases} \frac{1}{2} e^{2(t-3)} & \text{for } t-3 \leq 0 \\ 0 & \text{for } t-3 > 0 \end{cases}$$

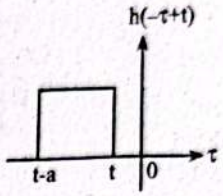
14. Find convolution between $x(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

$h(t) = x\left(\frac{t}{a}\right)$ **where,** $0 \leq a \leq 1$.

Solution:



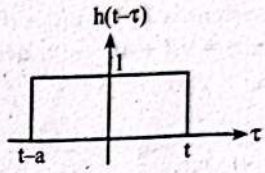
For $t < 0$,



Since there is no overlapping between $x(\tau)$ and $h(-\tau+t)$,

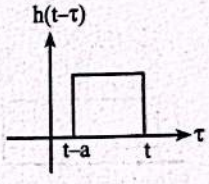
$y(t) = 0$

For $0 \leq t < 1$,



$$y(t) = \int_{0}^{t} 1 \times 1\, d\tau = t$$

For $a \leq t < 1$,



$$y(t) = \int_{t-a}^{t} 1 \times 1\, d\tau = t - t + a = a$$

For $1 \leq t < 1+a$,

$$y(t) = \int_{t-a}^{1} 1 \times 1\, d\tau = \int_{t-a}^{1} d\tau = 1 - t + a$$

For $1+a \leq t < 2$, there is no overlapping between $x(\tau)$ and $h(t-\tau)$. So,

$y(t) = 0$

For $t>2$,

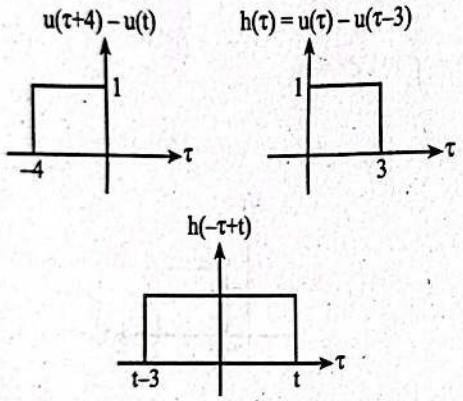
$y(t) = 0$

Hence,

$$y(t) = \begin{cases} 0 \text{ for } t<0 \\ t \text{ for } 0\le t<1 \\ a \text{ for } a\le t<1 \\ 1-t+a \text{ for } 1\le t<1+a \\ 0 \text{ for } 1+a\le t<2 \\ 0 \text{ for } t>2 \end{cases}$$

15. If impulse response of continuous time linear time invariant system is $h(t) = u(t) - u(t-3)$ and input to the system is $x(t) = u(t + 4) - u(t)$, determine output $y(t)$ of the system.

Solution:



For $t \ge 3$,

$y(t) = 0$ (no overlapping)

For $t < 3$ and $t \ge -1$,

$$y(t) = \int_{t-3}^{0} d\tau = 3-t$$

For $t \le -1$ and $t \ge -4$,

$$y(t) = \int_{-4}^{t} d\tau = t + 4$$

For $t \le -4$,

$y(t) = 0$ (no overlapping)

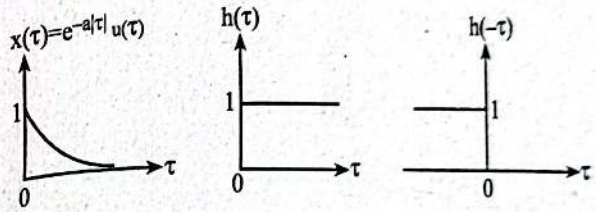
$$y(t) = \begin{cases} 0; t\ge 3 \\ 3-t; t<3 \text{ and } t\ge -1 \\ t+4; t\le -1 \text{ and } t\ge -4 \\ 0; t\le -4 \end{cases}$$

16. Find the convolution between $x(t) = e^{-at} u(t)$ $(a > 0)$ and $h(t) = u(t)$. [2070 Chaitra]

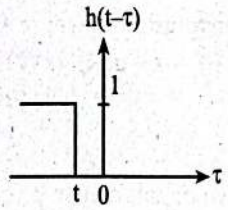
Solution:

Convolution is given by

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$



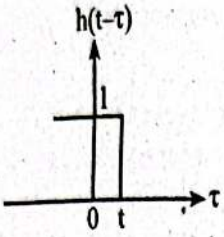
For $t < 0$,



Since there is no overlap (intersection) between $x(\tau)$ and $h(t-\tau)$,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = 0$$

For $t \geq 0$,



There is overlap between $x(\tau)$ and $h(t-\tau)$ from 0 to t. So,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{0}^{t} e^{-a\tau}(1)d\tau$$

$$= -\frac{1}{a}[e^{-a\tau}]_0^t$$

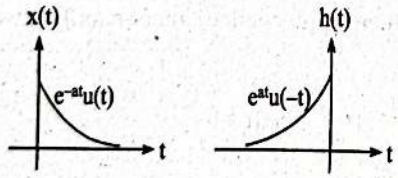
$$= -\frac{1}{a}[e^{-at}-1] = \frac{1}{a}[1-e^{-at}]$$

$$y(t) = \begin{cases} \frac{1}{a}(1-e^{-at}); & t \geq 0 \\ 0 & ; t<0 \end{cases}$$

17. Compute convolution between two signals $x(t) = e^{-at}u(t)$ (a>0) and $h(t) = e^{at}u(-t)$ (a>0) and plot the resulting signal. [2073 Shrawan]

Solution:

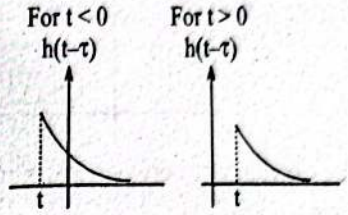
The given two signals can be plotted as



The convolution between two signals x(t) and h(t) can be found out by using

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)d\tau$$

Depending upon the value of t, $h(t-\tau)$ can be graphically shown as



Thus, the convolution y(t) is:

For $t < 0$,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)d\tau$$

$$= \int_{0}^{\infty} e^{-a\tau} u(\tau)\, e^{a(t-\tau)} u(-t+\tau)\, d\tau$$

$$= e^{at}\int_{0}^{\infty} e^{-2a\tau} d\tau = \frac{e^{at}}{2a}$$

For $t \geq 0$,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)d\tau$$

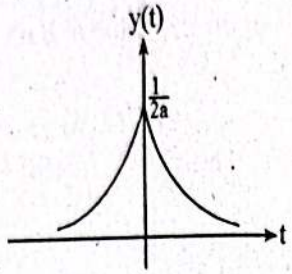
$$= \int_{t}^{\infty} e^{-a\tau} u(\tau)\, e^{a(t-\tau)} u(-t+\tau)\, d\tau$$

$$= e^{at}\int_{t}^{\infty} e^{-2a\tau} d\tau$$

$$= e^{at}\frac{e^{-2a\tau}}{-2a}\Big|_t^{\infty} = \frac{e^{at}}{-2a}(0 - e^{-2at}) = \frac{e^{-at}}{2a}$$

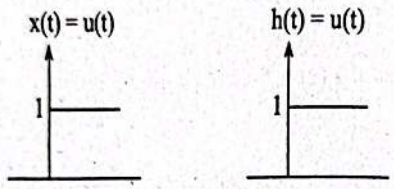
$$\text{Hence, } y(t) = \begin{cases} \frac{e^{at}}{2a} & \text{for } t<0 \\ \frac{e^{-at}}{2a} & \text{for } t \geq 0 \end{cases}$$

It can be plotted as shown:



18. Find the convolution between x(t) = u(t) and h(t) = u(t). Comment on result. [2073 Chaitra]

Solution:

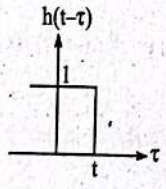


The convolution between two signals x(t) and h(t) is given by

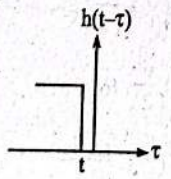
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$h(t-\tau)$ can be represent graphically as shown below for two different cases:

For $t \geq 0$,



For $t < 0$,



Hence, the convolution output is:

For $t \geq 0$,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{0}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{0}^{t} u(t)\, u(t-\tau)\, d\tau = \int_{0}^{t} 1\, d\tau = t$$

For $t < 0$,

From the above figures, it is observed that there is overlap between signals $x(\tau)$ and $h(t-\tau)$ when $t < 0$.

Hence, the convolution output is zero i.e.,

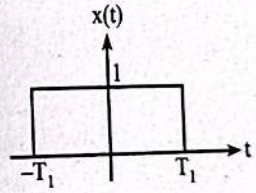
For $t < 0$,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = 0$$

Hence, $y(t) = \begin{cases} 0 \text{ for } t<0 \\ t \text{ for } t\geq 0 \end{cases}$

19. Determine the Fourier transform of the rectangular pulse signal, $x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases}$.

Solution:



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

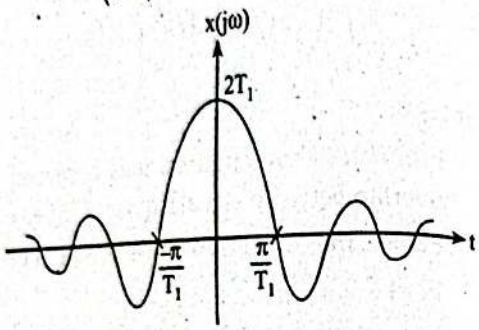
$$= \int_{-T_1}^{T_1} e^{-j\omega t}\, dt$$

$$= \frac{1}{-j\omega}\left(e^{-j\omega T_1} - e^{j\omega T_1}\right)$$

$$= \frac{2}{\omega}\left(\frac{e^{j\omega T_1} - e^{-j\omega T_1}}{2j}\right)$$

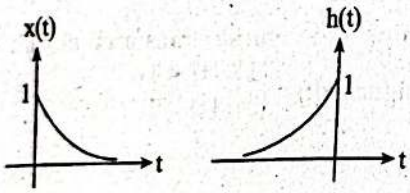
$$= \frac{2}{\omega}\sin(\omega T_1)$$

$$= 2T_1 \frac{\sin(\omega T_1)}{(\omega T_1)} = 2T_1 \operatorname{sinc}(\omega T_1)$$



20. An LTI system has input $x(t) = e^{-t}\,u(t)$ and impulse response $h(t) = e^{t}\,u(-t)$. Find output $y(t)$ of system using Fourier transform of $x(t)$ and $h(t)$.

Solution:



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-t} e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-(1+j\omega)t}\, dt = \frac{1}{1+j\omega}$$

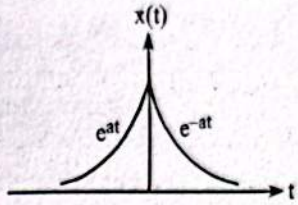
$$H(j\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{t} e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{(1-j\omega)t}\, dt = \frac{1}{1-j\omega}$$

$$Y(j\omega) = H(j\omega)\, X(j\omega) = \frac{1}{1-j\omega}\,\frac{1}{1+j\omega} = \frac{1}{1+\omega^2}$$

21. Find the Fourier transform of $x(t) = e^{-a|t|}$ where $a>0$.

Solution:



The Fourier transform is given by:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} x(t)\, e^{-j\omega t}\, dt + \int_{0}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

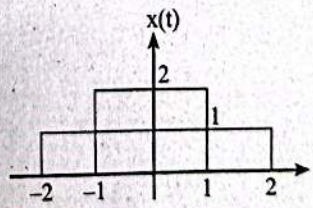
$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left[\frac{e^{(a-j\omega)t}}{(a-j\omega)}\right]_{-\infty}^{0} + \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)}\left(e^{0} - e^{-\infty}\right) + \frac{1}{-(a+j\omega)}\left(e^{-\infty} - e^{0}\right)$$

$$= \frac{1}{a-j\omega} + \frac{1}{(a+j\omega)} = \frac{2a}{a^2\omega^2}$$

22. Determine Fourier transform of the signal shown below:



Solution:

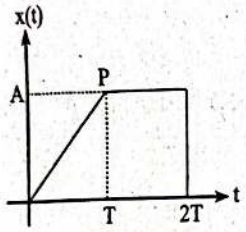
$$x(t) = \begin{cases} 2 & \text{for } -1 \le t \le 1 \\ 1 & \text{for } -2 \le t \le -1 \\ 1 & \text{for } 1 \le t \le 2 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} 2\, e^{-j\omega t}\, dt + \int_{-2}^{-1} 1\, e^{-j\omega t}\, dt + \int_{1}^{2} e^{-j\omega t}\, dt$$

$$= \frac{1}{-j\omega}\left[2(e^{-j\omega} - e^{j\omega}) + (e^{j\omega} - e^{2j\omega}) + (e^{-j2\omega} - e^{-j\omega})\right]$$

23. Determine Fourier transform of the signal shown below.



Solution:

The equation of line OP

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$$

$$\text{or, } \frac{x(t)-0}{t-0} = \frac{0-A}{0-T}$$

$$\text{or, } x(t) = \frac{A}{T}t$$

$$\therefore\ x(t) = \begin{cases} \frac{A}{T}t & \text{for } 0 < t < T \\ A & \text{for } T < t < 2T \end{cases}$$

$$X(j\omega) = \int_{0}^{T} \frac{A}{T} t e^{-j\omega t}\, dt + \int_{T}^{2T} A e^{-j\omega t}\, dt$$

$$= \frac{A}{j\omega}\left(e^{-j\omega t T/2} \operatorname{sinc}\frac{\omega T}{2} - e^{-2\omega t}\right)$$

CHAPTER - 4

TRANSMISSION MEDIA

4.1 Introduction

Transmission media is a pathway that carries the information from sender to receiver. We use different types of cables or waves to transmit data. Data is transmitted normally through electrical or electromagnetic signals. An electrical signal is in the form of current. An electromagnetic signal is series of electromagnetic energy pulses at various frequencies. These signals can be transmitted through copper wires, optical fibers, atmosphere, water and vacuum. Different medias have different properties like bandwidth, delay, cost and ease of installation and maintenance. Transmission media is also called *communication channel.*

Transmission media is broadly classified into two groups.

i. Wired or guided media or bound transmission media

ii. Wireless or unguided media or unbound transmission media

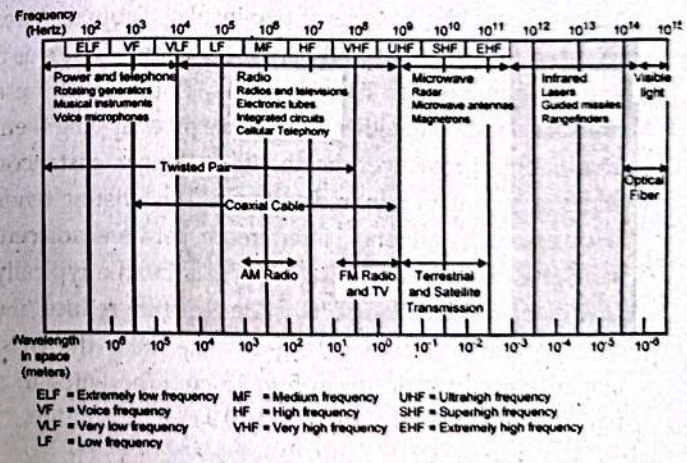


Fig.: Electromagnetic wave chart

Design Factors

Key design concern are *data rate* and *distance*. Greater the data rate and distance, the better. Number of factors that determine the limit on data rate and distance are:

1. **Bandwidth** - Higher the bandwidth, higher the data rate.
2. **Transmission impairments** - Attenuation, distortion, etc.
3. **Interference** - Intersymbol interference (ISI)
4. **Number of receivers** - In guided media, more receivers (multi-point) introduce more attenuation (need more amplifies or repeaters)

4.2 Guided Transmission Media

Bound transmission media are the cables that are tangible or have physical existence and are limited by the physical geography. Popular bound transmission media in use are twisted pair cable, co-axial cable, and fiber optical cable. Each of them has its own characteristics like transmission speed, effect of noise, physical appearance, cost, etc.

1. **Twisted Pair Cable**

Physical description:

A *twisted pair cable*, consists of two insulated copper wires arranged in a regular spiral pattern. A wire pair acts as a single communication link. Typically, a number of these pairs are bundled together into a cable by wrapping them in a tough protective sheath. Over longer distances, cables may contain hundreds of pairs. The twisting tends to decrease the crosstalk interference between adjacent pairs in a cable. Neighboring pairs in a bundle typically have somewhat different twist lengths to reduce the crosstalk interference. On long-distance links, the twist length typically varies from 5 to 15 cm. The wires in a pair have thicknesses of from 0.4 to 0.9 mm.

Fig.: Twisted pair cable

- Separately insulated
- Twisted together
- Often "bundled' into cables
- Usually installed in building during construction

Applications:

By far the most common guided transmission medium for both analog and digital signals is twisted pair.

- It is the most commonly used medium in the telephone network and is the workhorse for communications within buildings.
- These twisted-pair installations were designed to support voice traffic using analog signaling. However, by means of a modem, these facilities can handle digital data traffic at modest data rates.
- Twisted pair is also the most common medium used for digital signaling. For connections to a digital data switch or digital PBX within a building, a data rate of 64 kbps is common.
- Twisted pair is also commonly used within a building for local area networks supporting personal computers. Data rates for such products are typically in the neighborhood of 100 Mbps.
- For long-distance applications, twisted pair can be used at data rates of 4 Mbps or more.
- Twisted pair is much less expensive than the other commonly used guided transmission media (coaxial cable, optical fiber) and is easier to work with.

Transmission characteristics:

Twisted pair may be used to transmit both analog and digital transmission. For analog signals, amplifiers are required about every 5 to 6 km. For digital transmission, repeaters are required every 2 or 3 km. Compared to

other commonly used guided transmission media (coaxial cable, optical fiber), twisted pair is limited in distance, bandwidth, and data rate.

The attenuation for twisted pair is a very strong function of frequency. Other impairments are also severe for twisted pair. The medium is quite susceptible to interference and noise because of its easy coupling with electromagnetic fields. Impulse noise also easily intrudes into twisted pair. Several measures are taken to reduce impairments. Shielding the wire with metallic braid or sheathing reduces interference. The twisting of the wire reduces low-frequency interference, and the use of different twist lengths in adjacent pairs reduces crosstalk. For point-to-point analog signaling, a bandwidth of up to about 1 MHz is possible. This accommodates a number of voice channels. For long-distance digital point-to-point signaling, data rates of up to a few Mbps are possible; for very short distances, data rates of up to 10 Gbps have been achieved in commercially available products.

Classification of twisted pair:

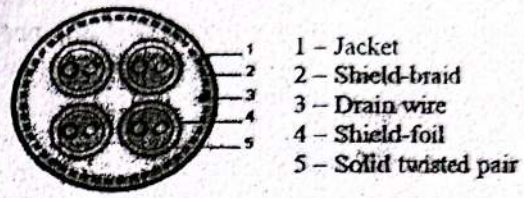
It is classified as *unshielded* and *unshielded twisted pair*.

Unshielded twisted pair(UTP):

- Ordinary telephone wire
- Cheapest
- Easiest to install
- Suffers from external EM interference

Shielded Twisted Pair (STP):

- Metal braid or sheathing that reduces interference
- More expensive
- Harder to handle (thick, heavy)



UTP

Categories:

CAT

- up to 16 MHz
- Voice grade-found in most offices
- Twist length of 7.5 cm to 10 cm

CAT 4

- up to 20 MHz

CAT 5

- up to 100 MHz
- Commonly pre-installed in new office buildings
- Twist length 0.6 cm to 0.85 cm

CAT 5E (enhanced)

CAT 6

- upto 200 MHz

CAT 7

- Upto 600 MHz

2. Coaxial Cable

Physical description:

Coaxial cable, like twisted pair, consists of two conductors, but is constructed differently to permit it to operate over a wider range of frequencies. It consists of a hollow outer cylindrical conductor that surrounds a single inner wire conductor. The inner conductor is held in place by either regularly spaced insulating rings or a solid dielectric material. The outer conductor is covered with a jacket or shield. A single coaxial cable has a diameter of from 1 to 2.5 cm. Coaxial cable can be used over longer distances and support more stations on a shared line than twisted pair.

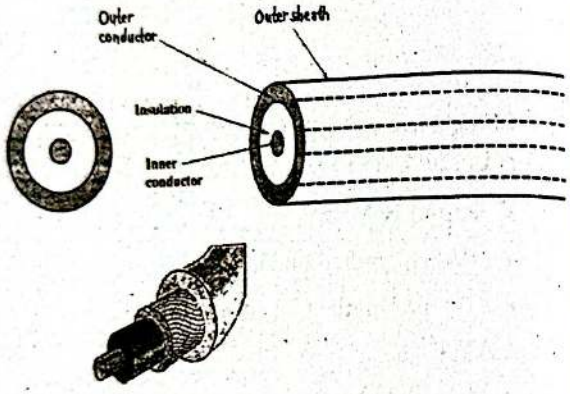


Fig.: Coaxial cable

- Outer conductor is braided shield.
- Inner conductor is solid metal.
- Separated by insulating material.
- Covered by padding

Applications:

Coaxial cable is a versatile transmission medium, used in a wide variety of applications. The most important of these are:

- **Television distribution** – Coaxial cable is widely used as a means of distributing TV signals to individual homes—cable TV. A cable TV system can carry dozens or even hundreds of TV channels at ranges up to a few tens of kilometers.
- **Long-distance telephone transmission** – Coaxial cable has traditionally been an important part of the long-distance telephone network. Today, it faces increasing competition from optical fiber, terrestrial microwave, and satellite. A coaxial cable can carry over 10,000 voice channels simultaneously.
- **Short-run computer system links** – Coaxial cable is also commonly used for short-range connections between devices. Using digital signaling, coaxial cable can be used to provide high-speed I/O channels on computer systems.

Transmission characteristics:

Coaxial cable is used to transmit both analog and digital signals. A coaxial cable has frequency characteristics that are superior to those of twisted pair and can hence be used effectively at higher frequencies and data rates. Because of its shielded, concentric construction, coaxial cable is much less susceptible to interference and crosstalk than twisted pair. The principal constraints on performance are attenuation, thermal noise, and intermodulation noise. The latter is present only when several channels or frequency bands are in use on the cable. For long-distance transmission of analog signals, amplifiers are needed every few kilometers, with closer spacing required if higher frequencies are used. The usable spectrum for analog signaling extends to about 500 MHz. For digital signaling, repeaters are needed every kilometer or so, with closer spacing needed for higher data rates.

3. Optical Fiber

Optical fiber already enjoys considerable use in long-distance telecommunications, and its use in military applications is growing. The continuing improvements in performance and decline in prices, together with the inherent advantages of optical fiber, have made it increasingly attractive for local area networking.

The following characteristics distinguish optical fiber from twisted pair or coaxial cable:

- **Greater capacity:** Data rates of hundreds of Gbps over tens of kilometers have been demonstrated.
- **Smaller size and lighter weight:** The corresponding reduction in weight reduces structural support requirements.
- **Lower attenuation:** Attenuation is significantly lower for optical fiber and is constant over a wide range.
- **Electromagnetic isolation:** Optical fiber systems are not affected by external electromagnetic fields. Thus the system is not vulnerable to interference, impulse noise, or crosstalk. By the same token, fibers do not radiate energy, so there is little interference with other equipment and there is a high degree of security from eavesdropping.
- **Greater repeater spacing:** Fewer repeaters mean lower cost and fewer sources of error. The performance of optical fiber systems from this point of view has been steadily improving.

Physical description:

An optical fiber is a thin (2 to 125 μm), flexible medium capable of guiding an optical ray. Various glasses and plastics can be used to make optical fibers. The lowest losses have been obtained using fibers of ultrapure fused silica. Ultrapure fiber is difficult to manufacture; higher-

loss multicomponent glass fibers are more economical and still provide good performance. Plastic fiber is even less costly and can be used for short-haul links, for which moderately high losses are acceptable. An optical fiber cable has a cylindrical shape and consists of three concentric sections: the *core*, the *cladding*, and the *jacket*. The core is the innermost section and consists of one or more very thin strands, or fibers, made of glass or plastic; the core has a diameter in the range of 8 to 50 μm. Each fiber is surrounded by its own cladding, a glass or plastic coating that has optical properties different from those of the core and a diameter of 125 μm. The interface between the core and cladding acts as a reflector to confine light that would otherwise escape the core. The outermost layer, surrounding one or a bundle of cladded fibers, is the jacket. The jacket is composed of plastic and other material layered to protect against moisture, abrasion, crushing, and other environmental dangers.

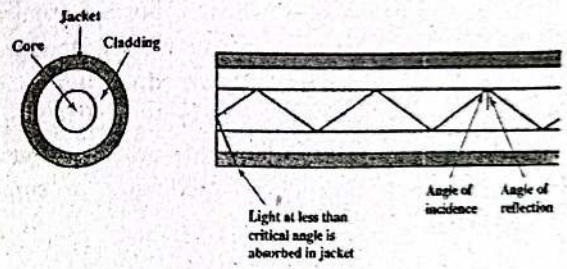


Fig.: Optical fibre

- Glass or plastic core
- Laser or light emitting diode
- Specially designed jacket
- Small size and weight.

Applications:

Five basic categories of application have become important for optical fiber:

Long-haul trunks – Long-haul fiber transmission is becoming increasingly common in the telephone network. Long-haul routes average about 1500 km in length and offer high capacity (typically 20,000 to 60,000 voice channels).

Metropolitan trunks – Metropolitan trunking circuits have an average length of 12 km and may have as many as 100,000 voice channels in a trunk group.

Rural exchange trunks – It have circuit lengths ranging from 40 to 160 km and link towns and villages. Most of these systems have fewer than 5000 voice channels.

Subscriber loops – It circuits are fibers that run directly from the central exchange to a subscriber as the telephone networks evolve into full-service networks capable of handling not only voice and data, but also image and video.

Local area networks – A final important application of optical fiber is for local area networks. Standards have been developed and products introduced for optical fiber networks that have a total capacity of 100 Mbps to 10 Gbps and can support hundreds or even thousands of stations in a large office building or a complex of buildings.

Transmission characteristics:

Optical fiber transmits a signal-encoded beam of light by means of total internal reflection. Total internal reflection can occur in any transparent medium that has a higher index of refraction than the surrounding medium. Figure shows the principle of optical fiber transmission. Light from a source enters the cylindrical glass or plastic core. Rays at shallow angles are reflected and propagated along the fiber; other rays are absorbed by the surrounding material. This form of propagation is called

step-index multimode, referring to the variety of angles that will reflect.

With multimode transmission, multiple propagation paths exist, each with a different path length and hence time to traverse the fiber. This causes signal elements (light pulses) to spread out in time, which limits the rate at which data can be accurately received. This type of fiber is best suited for transmission over very short distances. When the fiber core radius is reduced, fewer angles will reflect. By reducing the radius of the core to the order of a wavelength, only a single angle or mode can pass: the axial ray. This single-mode propagation provides superior performance for the following reason. Because there is a single transmission path with single-mode transmission, the distortion found in multimode cannot occur. Single-mode is typically used for long-distance applications, including telephone and cable television. Finally, by varying the index of refraction of the core, a third type of transmission, known as *graded-index multimode*, is possible. This type is intermediate between the other two in characteristics. Graded-index fibers are often used in local area networks.

Modes of transmission:

1. **Single mode:** Single transmission path
2. **Multimode:** Multiple transmission path
   i. **Step-index multimode**
   ii. **Graded-index multimode**

Light source:

i. **Light emitting diode (LED)**
   - Less costly
   - Operates over a greater temperature range
   - Longer operational life

ii. **Injection laser diode (ILD)**

- More efficient
- Can sustain greater data rates

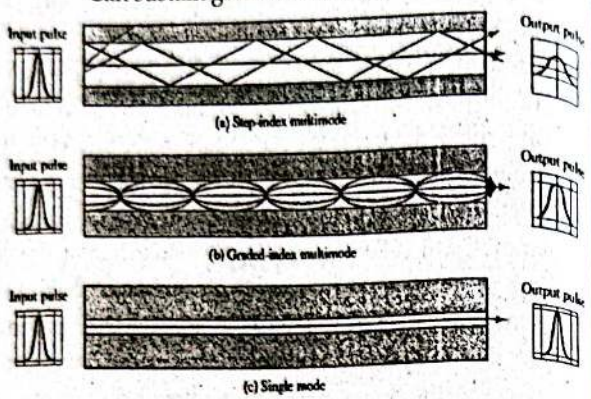


Fig.: Optical fibre transmission modes

4.3 Unguided Transmission Media

Unguided transmission media are the ways of transmitting data without using any cables. These media are not bounded by physical geography. This type of transmission is called *wireless communication*. Nowadays, wireless communication is becoming popular. Wireless LANs are being installed in office and college campuses. Microwave, radio wave, infrared wave are some of popular unbound transmission media. Wireless propagation is possible in following ways:

1. **Ground wave propagation**

   In *ground wave propagation*, radio waves travel through the lowest portion of the atmosphere. Ground wave propagation takes place below the frequency of 2 MHz. These low frequency signals radiate in all directions from the transmitting antenna and follow the curvature of the earth.

2. **Sky wave propagation**

   In *sky wave propagation*, radio waves radiate up into the ionosphere and gets reflected back to the earth. Sky wave propagation takes place for frequencies between 2-30 MHz. This type of transmission allows communication at a greater distance.

3. **Line-of-sight (LOS) propagation**

   In *LOS propagation*, very high frequency signals are transmitted in straight line from antenna to antenna. Antennas must be directional and facing each other. LOS propagation takes place for frequencies above 30 MHz. Distance of signal propagation is limited due to the curvature of the earth.

Few of the unguided transmission media are explained below:

1. **Terrestrial Microwave**

   Physical description:

   The most common type of microwave antenna is the parabolic "dish."A typical size is about 3 m in diameter. The antenna is fixed rigidly and focuses a narrow beam to achieve line-of-sight transmission to the receiving antenna. Microwave antennas are usually located at substantial heights above ground level to extend the range between antennas and to be able to transmit over intervening obstacles.

   Applications:

   - The primary use for terrestrial microwave systems is in long-haul telecommunications service.
   - The microwave facility requires far fewer amplifiers or repeaters than coaxial cable over the same distance but requires line-of-sight transmission.
   - Microwave is commonly used for both voice and television transmission.
   - Another increasingly common use of microwave is for short point-to-point links between buildings. This can be used for closed-circuit TV or as a data link between local area networks.

- Short-haul microwave can also be used for the so called bypass application.
- A business can establish a microwave link to a long distance telecommunications facility in the same city, bypassing the local telephone company.
- Another important use of microwave is in cellular systems.

Transmission characteristics:

Microwave transmission covers a substantial portion of the electromagnetic spectrum. Common frequencies used for transmission are in the range 1 to 40 GHz. The higher the frequency used, the higher the potential bandwidth and therefore the higher the potential data rate.

As with any transmission system, a main source of loss is attenuation. For microwave (and radio frequencies), the loss can be expressed as:

$$L = 10 \log \left(\frac{4\pi d}{\lambda}\right)^2 \text{ dB}$$

Where d is the distance and λ is the wavelength, in the same units. Thus, loss varies as the square of the distance. In contrast, for twisted-pair and coaxial cable, loss varies exponentially with distance (linear in decibels). Thus repeaters or amplifiers may be placed farther apart for microwave systems—10 to 100 km is typical. Attenuation is increased with rainfall. The effects of rainfall become especially noticeable above 10 GHz. Another source of impairment is interference. With the growing popularity of microwave, transmission areas overlap and interference is always a danger shorter distances. In addition, at the higher frequencies, the antennas are smaller and cheaper.

2. Satellite Communication

In this type of communication, satellite is relay station. The satellite receives one frequency *(uplink)*, amplifies or

repeats signal and transmits on another frequency *(downlink)*. It is a line-of-sight communication where the earth station has to be in the line of sight with the communicating satellite.

The path in which satellite moves is called orbit which can be categorized on the basis of its distance from the earth:

- Lower earth orbit (LEO)
- Medium earth orbit (MEO)
- Geostationary earth orbit (GEO)

Lower earth orbit (LEO):

LEO satellites are closer to the earth (500-2000 km) and have rotation period 90-120 minutes. Large number of LEO satellites are needed for a complete system. It is mostly used for voice communication and Internet services.

Medium earth orbit (MEO)

MEO satellites are located at a distance 6000-20000 km from the earth surface and they take 6-8 hours to circle the earth depending on its orbit height above the earth surface. Examples of MEO satellites are GPS satellites which are orbiting at around 18000 km above the earth surface. It is mostly used for navigation and military service.

Geostationary earth orbit (GEO)

GEO satellites use line-of-sight propagation which requires that sending and receiving antennas are locked onto each other's location at all times. To ensure constant communication, the satellite must move at the same speed as the earth so that it seems to remain fixed above a certain spot. It takes a minimum of 3 satellites equidistant from each other in geostationary orbit to provide full global transmission. These satellites are located at a distance of 36000 km from the earth surface.

Satellite communication can be of two types:

1. Point-to-point satellite communication
2. Satellite broadcast communication

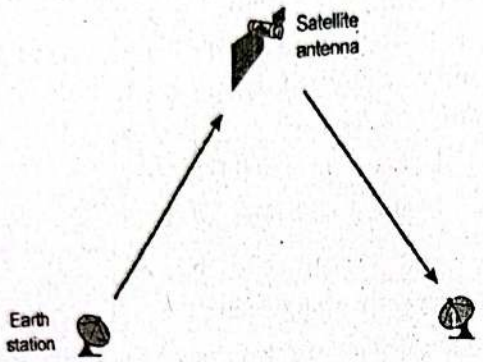


Fig.: Point-to-point satellite communication

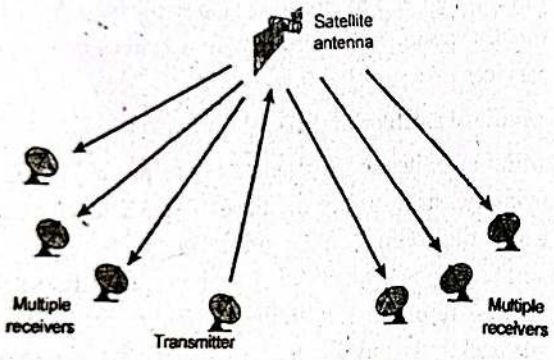


Fig.: Satellite broadcast communication

Applications:

- **Television distribution:** Broadcast nature satellites are well suited to television distribution.
- **Long-distance telephone transmission:** It is the optimum medium for high usage, international trunks and is competitive with terrestrial systems for many long distance international links.
- **Global positioning system (GPS)**

Transmission characteristics:

- The optimum frequency range for satellite transmission is in the range 1 to 10 GHz. Below 1 GHz, there is significant noise from natural sources, including galactic, solar, and atmospheric noise, and human made interference from various electronic devices. Above 10 GHz, the signal is severely attenuated by atmospheric absorption and precipitation.
- The 4/6 GHz band is within the optimum zone of 1 to 10 GHz but has become saturated. 12/14 GHz band has been developed recently.
- As it is a long distance communication, there is a propagation delay while transmitting signals from from one earth station to another earth station. This delay is noticeable in ordinary telephone conversations.

3. Cellular Systems

Before the introduction of cellular system, a large antenna was used for providing mobile communication in a large geographical area. However, it had some serious limitations like interference, large transmission power requirement, limited capacity, etc. To remove these limitations, cellular system was introduced.

Cellular system uses multiple low-power transmitters. The area where mobile communication has to be done is divided into cells, each one served by own antenna. The range of such antenna is small (within a cell). Each cell is allocated a band of frequencies and is served by a base station, consisting transmitter, receiver, and control unit. The main advantage of using a cellular structure for mobile communication is frequency reuse. Adjacent cells are assigned different frequencies to avoid interference. However, cells sufficiently distant from each other can reuse frequency band called co-channel.

4.4 Data Rate Limits

A very important consideration in data communications is how fast we can send data, in bits per second, over a channel. Data rate depends on three factors:

1. Available bandwidth
2. The level of the signals we use
3. The quality of the channel (the level of noise).

Two theoretical formulas were developed to calculate the data rate:

1. Nyquist bit rate for a noiseless channel
2. Shannon capacity for noisy channel

1. Nyquist Bit Rate for a Noiseless Channel

For a noiseless channel, the Nyquist bit rate formula defines the theoretical maximum bit rate:

Bit rate (R) = $2 \times \text{bandwidth} \times \log_2 L$ bits/sec

Increasing L increases the bit rate but in the same time reduces the reliability of the system.

Example:

Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. Calculate the maximum bit rate.

Solution:

Bit rate (R) = $2 \times 3000 \times \log_2 2 = 6000$ bps

Example:

Suppose we need to send 265 kbps over a noiseless channel with a bandwidth of 20kHz. How many signal levels do we need?

Solution:

$265000 = 2 \times 20000 \times \log_2 L$

or, $\log_2 L = 6.625$

or, L = 98.7 levels

Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

2. Shannon Capacity for Noisy Channel

In reality, we cannot have a noiseless channel; the channel is always noisy. In 1944, Claude Shannon introduced a formula called the *Shannon capacity*, to determine the theoretical-highest data rate for a noisy channel. This formula is expressed as:

Capacity = $\text{Bandwidth} \times \log_2(1 + \text{SNR})$ bits/sec

Example:

Consider an extremely noisy channel which in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel, calculate the channel capacity C.

Solution:

Capacity = $\text{Bandwidth} \times \log_2(1 + \text{SNR})$ bits/sec

= $\text{Bandwidth} \times \log_2(1 + 0)$ bits/sec

= 0 bits/sec

This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

Example:

Calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000 Hz. The signal-to-noise ratio is usually 3162.

Solution:

The channel capacity can be calculated as

Capacity = $\text{bandwidth (B)} \times \log_2(1 + \text{SNR})$ bits/sec

= $3000 \times \log_2(1 + 3162)$ bits/sec

= 34,860 bits/sec or bps

This means that the highest bit rate for a telephone line is 34.860 kbps. If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

Example:

The signal-to noise ratio is often given in decibels =3.6 and the channel bandwidth is 2MHz. Calculate the theoretical channel capacity:

Solution:

SNR in dB = $10 \log_{10}$SNR

or, 3.6 = $10 \log_{10}$SNR

or, SNR = $10^{3.6/10}$

or, SNR = 2.29

C = B×log 2(1 + SNR)

= $2\times10^6\times\log_2(1+2.29)$ = 3.436 Mbps

4.5 Performance of Channel

The different parameters on which the performance of a channel is measured are:

i. **Bandwidth**

The term *bandwidth* can be used in two different contexts with two different measuring values: *bandwidth in Hz* and *bandwidth in bits per second*. Bandwidth in Hz is the range of frequencies contained in a composite signal or the range of frequencies a channel can pass. Bandwidth in bits per second refers to the number of bits per second that a channel, a link, or even a network can transmit.

ii. **Throughput**

Throughput is a measure of how fast, we can actually send data through a network. Bandwidth in bits per second and throughput are different. A link may have a bandwidth of B bps, but we can only send T bps through this link with T always less than B. Example: we may

have a link with a bandwidth of 1 Mbps, but the devices connected to the end of the link may handle only 200 kbps. This means that we cannot send more than 200 kbps through this link.

iii. **Latency (delay)**

The *latency* defines how long it takes for an entire message to completely arrive at the destination from the time the first bit is sent out from the source. We can say that latency is made of four components: propagation time, transmission time, queuing time and processing delay.

Latency = propagation time +transmission time +queuing time + processing delay

iv. **Jitter**

Another performance issue that is related to delay is *jitter*. Jitter is a problem in which different packets of data encounter different delays and the application using the data at the receiver site is time-sensitive (audio and video data, for example).

v. Bit Error Rate (BER)

As the name implies, a *bit error rate* is defined as the rate at which errors occur in transmission system. This can be directly translated into the number of errors that occur in string of a stated number of bits. The definition of bit error rate can be translated into simple formula:

BER = number of errors / total number of bits sent

NUMERICAL PROBLEMS

1. **Find the capacity of a channel for a signal with a bandwidth of 3.1 KHz and signal to noise ratio of 0 dB and comment on it.** [2074 Ashwin]

Solution:

BW = 3.1 KHz, $(SNR)_{dB} = 0$ dB

We know,

$(SNR)_{dB} = 10 \log_{10} SNR$

or, $0 = 10 \log_{10} SNR$

$\therefore$ SNR = 1

Now,

$C = BW \times \log_2 (1 + SNR)$

$= 3.1 \times \log_2 (1 + 1)$

$= 3.1$ Kbps

2. **Calculate the channel capacity having bandwidth and SNR of 6 Hz and 6 dB respectively.** [2071 Shrawan]

Solution:

Bandwidth (B) = 6Hz

SNR = 6 dB

or, $10\log\left(\frac{S}{N}\right) = 6$

$\therefore \frac{S}{N} = 10^{0.6} = 3.981$

The channel capacity is given by

$C = B \log_2 (1+SNR)$

$= 6\times\log_2(1 + 3.981)$

$= 13.92$ bits/sec

3. **The spectrum of a channel is in between 3 MHz and 4 MHz, and SNR = 24 dB. Calculate the bandwidth, the maximum channel capacity using Shannon's formula. Then based on Nyquist's formula, find the number of signal levels required to reach the maximum channel capacity.** [2069 Chaitra]

Solution:

Bandwidth (B) = 4 MHz – 3 MHz = 1 MHz = 10^6 Hz

We have,

$(SNR)_{dB} = 10 \log_{10}(SNR)$

or, $24 = 10 \log_{10}(SNR)$

or, SNR = 251.18

Now,

$C = B \log_2(1 + SNR)$

$= 10^6\times\log_2(1 + 251.18) \approx 10^6\times 8 = 8$ Mbps

From Nyquist formula,

$R = 2B \log_2 L$

or, $8\times10^6 = 2\times10^6 \log_2 L$

or, $4 = \log_2 L$

$\therefore L = 16$

CHAPTER • 5

DATA ENCODING AND MODULATION

5.1 Baseband Communication

A message signal generated from an information source is called a *baseband signal*. The original information from source may be analog or digital. Analog signal varies continuously with time. Digital baseband signal are discrete in time and amplitude. Baseband signal has frequency component close to 0 Hz. In case of baseband transmission, the information signal is sent as it is, without modulation.

The limitations of baseband communication are:

- It only works for short distance.
- It does not work if channel is passband.

5.2 Modulation

Modulation is the process of transformation of the message signal (baseband) into a form suitable for transmission over the channel with acceptable degradation. The modulating signals (baseband signal or information signal), and carrier signals which are of high frequency signals are used in modulating process. In this process, the parameter of carrier wave such as amplitude, frequency, or phase is varied in accordance with a modulating signal. A device that performs modulation is known as a *modulator* and a device that performs the inverse operation of modulation is known as a *demodulator* (also known as detector). A device that can do both operations is a *modem* (modulator-demodulator).

Need for modulation:

i. **Reduction in height of an antenna:**

For transmission of radio signals, height of an antenna must be multiple of $\frac{\lambda}{4}$, where λ is the wavelength ($\lambda = \frac{c}{f}$),

c is the velocity of light, and f is the frequency of signal to be transmitted.

For frequency f = 10 KHz, minimum antenna height $= \frac{\lambda}{4} = \frac{c}{4f} = 7500$ m.

For frequency f = 1 MHz, minimum antenna height $= \frac{\lambda}{4} =$ 75 m.

The antenna of height 7500 m is practically impossible to install compared to antenna of height 75 m. Hence modulation reduces height of antenna.

ii. **Avoids mixing of signals:**

If baseband sound signals are transmitted without using modulation by more than one transmitter then all signals will be in same frequency range i.e., 0 to 20 KHz. Therefore, all signals get mixed together and a receiver cannot separate them from each other.

Modulated signal occupy different slots in frequency domain.

iii. **Increases the range of communication:**

The frequency of baseband signal is low and low frequency signals cannot travel a long distance when they are transmitted. They get heavily attenuated (suppressed). The attenuation reduces with increase in frequency of transmitted signal and travel longer distance.

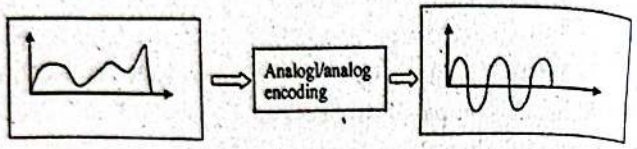
iv. **Multiplexing is possible:**

Multiplexing is the process in which two or more signals can be transmitted over same communication channel simultaneously. This is possible only with modulation. This allows same channel to be used by many signals. Hence, many TV channels can use same frequency range without getting mixed.

v. **Improves quality of reception:**

With frequency modulation (FM), digital communication techniques like PCM, effect of noise is reduced to a great extent. This improves quality of reception.

5.3 Analog Modulation



Analog modulation also known as *analog-to-analog encoding* is the representation of analog information by an analog signal (e.g., radio communication).

Analog-to-analog modulation can be accomplished in three ways: *amplitude modulation (AM), frequency modulation (FM),* and *phase modulation (PM).* In analog to analog modulation, analog message signal with analog carrier signal results in analog modulated signal.

1. Amplitude Modulation

In AM transmission, the carrier signal is modulated so that its amplitude varies with the amplitude of the modulating signal. The frequency and phase of the carrier remain the same; only the amplitude changes to follow variations in the information. The modulating signal becomes the envelope of the carrier. AM is used in radio and TV broadcasting applications.

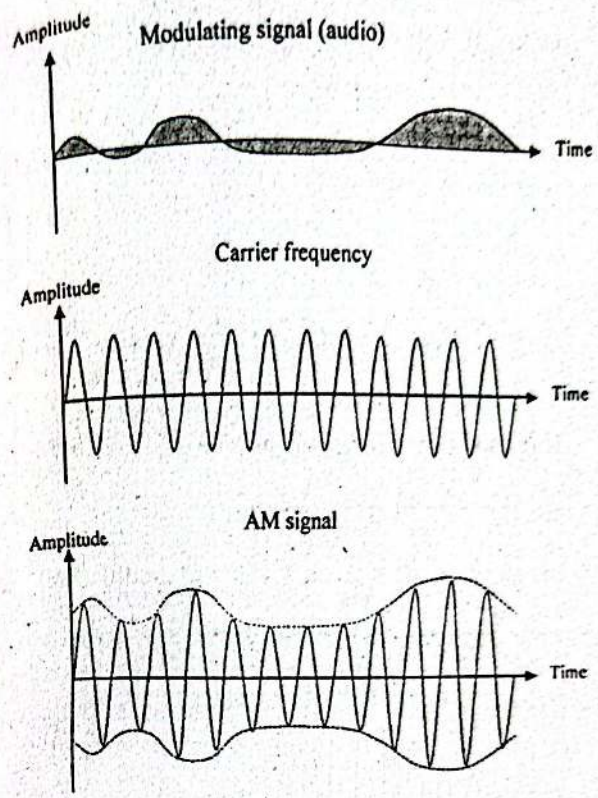


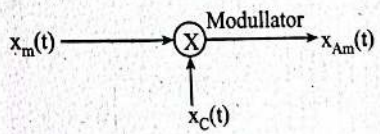
Fig.: Amplitude modulation

The message signal is given by

$x_m(t) = V_{DC} + V_m \sin(\omega_m t)$

The carrier signal is

$x_c(t) = V_c \sin \omega_c t$



$x_{AM}(t) = [x_m(t)]\,[x_c(t)]$

or, $x_{AM}(t) = V_{DC}V_c \sin(2\pi f_c t) + \frac{V_m}{2}[\cos\{2\pi(f_c - f_m)t - \cos\{2\pi(f_C + f_m)t\}]$

or, $x_{AM}(t) = V_{DC}V_c\ [\sin(2\pi f_c t) + \frac{1}{2}\left(\frac{V_m}{V_{DC}}\right)\{\cos\ 2\pi(f_c - f_m) - \cos 2\pi(f_c + f_m)t\}]$

The modulation degree (i.e., depth of modulation) is measured by *modulation index* (m).

$$m = \frac{V_m}{V_{DC}}$$

$$x_{AM}(t) = V_{DC}V_c\ [\sin(2\pi f_c t) + \frac{m}{2}\ \{\cos(2\pi(f_c - f_m)t - \cos\{2\pi(f_c + f_m)t\}]$$

Thus modulation index can also be obtained as

$$m = \frac{V_m}{V_{DC}} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{V_m}{V_c}$$

Usually $0 < m < 1$. **$m > 1$(over modulation)** causes distorted signal.

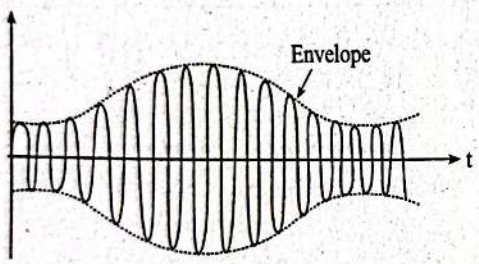


Fig.: AM wave (0<m<1)

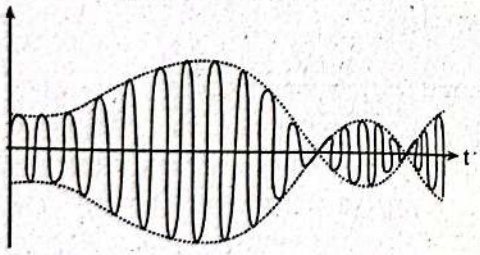


Fig.: AM wave (m>1)

Bandwidth of AM:

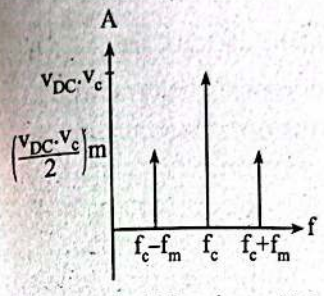
f_c = frequency of carrier signal

$f_c - f_m$ = lower side band = f_{LSB}

$f_c + f_m$ = upper side band = f_{USB}

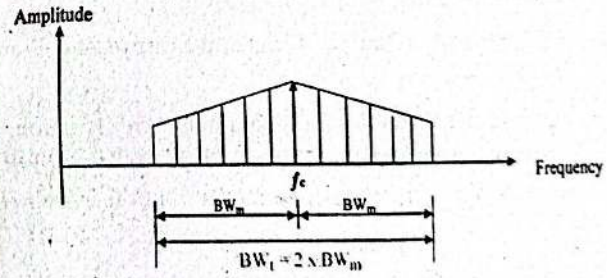
The bandwidth is

$BW = f_{USB} - f_{LSB} = (f_c - f_m) - (f_c - f_m) = 2f_m$



The bandwidth of an AM signal is equal to twice the bandwidth of the modulating signal and covers a range centered on the carrier frequency. The total bandwidth required for AM can determined from the bandwidth of the audio signal:

$BW_t = 2 \times BW_m$



Advantages of AM:

- AM transmitter are less complex.
- AM receivers are simple, and detection is easy.
- AM receivers are cost efficient.

- AM wave can travel a longer distance.
- Low bandwidth.

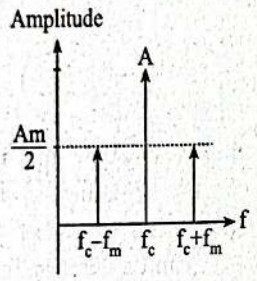
Applications of AM:

- Radio broadcasting
- Picture transmission in TV system

Types of AM:

1. **DSB-FC-AM (Double Sideband-Full Carrier-AM)**

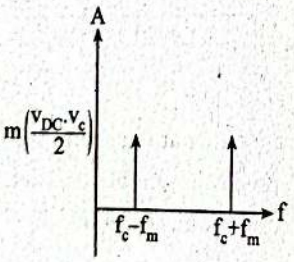
Amplitude modulation discussed till now is also called as double side band full carrier AM.



The various disadvantages of DSB-FC-AM are power wastage in transmission, bandwidth inefficiency, and performance degradtion with noise.

2. **DSB-SC-AM (Double Sideband-Suppressed Carrier-AM)**

In AM signal, the carrier contains no information. Any transmitted information lies solely in sideband. For that reason, carrier may be suppressed and not transmitted.



Due to suppression of carrier, a lot of power saving takes place in DSB-SC. At 100% modulation i.e., m=1, the percent power saving is 66.66%. Transmission bandwidth is same as DSB-FC.

3. **SSB-SC (Single Sideband-Suppressed Carrier-AM)**

DSB-FC and DSB-SC modulation are wasteful of bandwidth, that is, they have a bandwidth of $2f_m$ (where f_m is message signal bandwidth). One half of bandwidth is occupied by both upper side band and lower side band in both cases. Since upper sideband and lower sideband both are uniquely related to each other by virtue of symmetry about carrier frequency, suppression of carrier and any one of sideband will not lead to loss in information. An AM signal with no carrier and no sideband saves bandwidth. This technique is employed in SSB-SC.

Drawbacks:

- Expensive and highly complex to implement.
- The SSB transmitter and receiver need to have an excellent frequency stability. A slight change in frequency will hamper the quality of transmitted and received signal (not generally used for good quality music).

Advantages:

- Less bandwidth
- Lots of power saving

  At 100% modulation, the percent power saving is 83.33%.
- Reduced interference of noise

Applications:

Land and air mobile communication, telemetry, military communication, navigation and amateur radios, point to point communication system.

4. **Vestigial Sideband Transmission (VSB)**

Incase of SSB modulation, when a sideband is passed through the filters, the bandpass filter may not pass perfectly in practice. As a result of which, some of the information may get lost. Hence, to avoid this loss, a technique is chosen which is a compromise between DSB-SC and SSB-SC, called as *vestigial sideband (VSB)* technique.

VSB modulation requires a channel bandwidth that is between that required for SSB and DSB-SC systems and saving in bandwidth can be significant if modulating signals with large bandwidth are being handled, as in the case of television signals and high speed data.

Bandwidth (B)= $f_m + f_v$

where f_m= message bandwidth, f_v = width of vestigeal sideband.

Advantages:

- Highly efficient
- Reduction in bandwidth
- Practically easy to design filter.

Applications:

Standard for transmission of television signals

2. **Frequency Modulation (FM)**

In FM transmission, the frequency of the carrier signal is modulated to follow the changing voltage level (amplitude) of the modulating signal. The peak amplitude and phase of the carrier signal remain constant, but as the amplitude of the information signal changes, the frequency of the carrier changes correspondingly. FM is used for TV, sound, radio broadcasting.

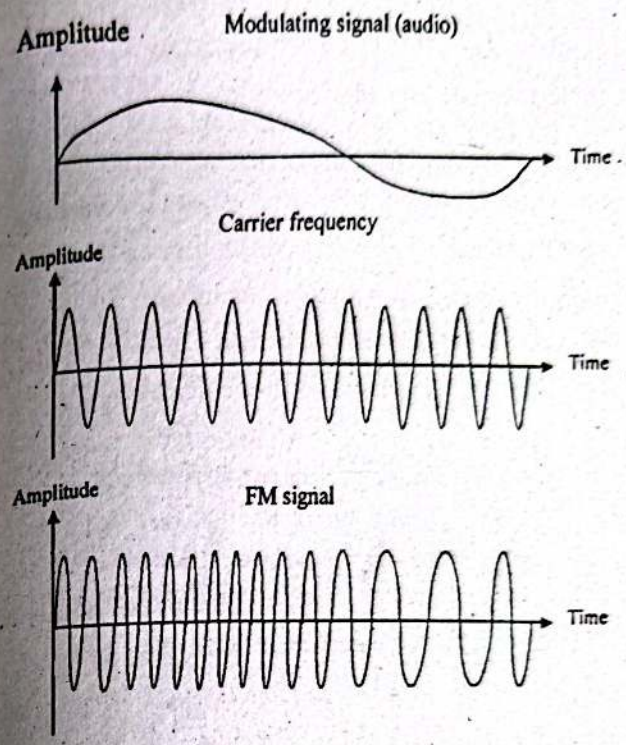


Fig.: FM modulation

$x_c(t) = V_c \sin(\omega_c t + \theta_c(t))$

The instantaneous phase,

$\theta_i(t) = \omega_c t + \theta_c(t)$

The instantaneous frequency is

$$\omega_i(t) = \omega_c + \frac{d\theta_c(t)}{dt}$$

The deviation in instantaneous frequency is proportional to the amplitude of message signal.

$$\frac{d\theta_c(t)}{dt} \propto x_m(t)$$

$$\frac{d\theta_c(t)}{dt} = k_f\, x_m(t)$$

Let, $x_m(t) = V_m \sin(\omega_m t)$

$$\frac{d\theta_c(t)}{dt} = k_f V_m \sin(\omega_m t)$$

Integrating on both sides,

$$\theta_c(t) = \int_0^t k_f V_m \sin(\omega_m t)\, dt$$

So, FM signal is

$$x_{FM}(t) = V_c \sin(\omega_c t + \theta_c(t))$$

$$= V_c \sin\left[\omega_c t + \int_0^t k_f V_m \sin(\omega_m t)\, dt\right]$$

$$= V_c \sin\left[\omega_c t - \frac{k_f V_m}{\omega_m}(\cos \omega_m t - 1)\right]$$

$$= V_c \sin\left[\omega_c t - \frac{k_f V_m}{2\pi f_m}\cos\omega_m t + \frac{k_f V_m}{2\pi f_m}\right]$$

Let, $\Delta f = \frac{k_f V_m}{2\pi}$ = frequency deviation

$$\therefore \quad x_{FM}(t) = V_c \sin\left[\omega_c t - \frac{\Delta f}{f_m}\cos\omega_m t + \frac{\Delta f}{f_m}\right]$$

and frequency modulation index is

$$\beta = \frac{\Delta f}{f_m}$$

Thus, $x_{FM}(t) = V_c \sin[\omega_c t - \beta\cos(\omega_m t) + \beta]$

$\therefore \quad x_{FM}(t) = V_c \sin\{\omega_c t - \beta(\cos(\omega_m t) - 1)\}$

Bandwidth for FM:

$$BW = 2(\Delta f + f_m) = 2f_m\left(\frac{\Delta f}{f_m} + 1\right)$$

$BW = 2f_m(\beta + 1)$..... (i)

For narrowband FM,

$BW = 2f_m$ (since $\beta << 1$)

For wideband FM,

$BW = 2f_m\beta$ (since $\beta >> 1$)

$$= 2 f_m \times \frac{\Delta f}{f_m} = 2\Delta f$$

Types of FM:

i. **Narrowband FM**

- Maximum deviation of frequency is 5 KHz.
- Range of modulating frequency is 30Hz-3KHz.
- Maximum modulation index is slightly greater than 1.

Applications:

FM mobile communication like police wireless, ambulance, etc.

ii. **Wideband FM**

- Maximum deviation is 75 KHz.
- Range of modulating frequency is 30Hz-15KHz.
- Maximum modulation index is 5-2500.

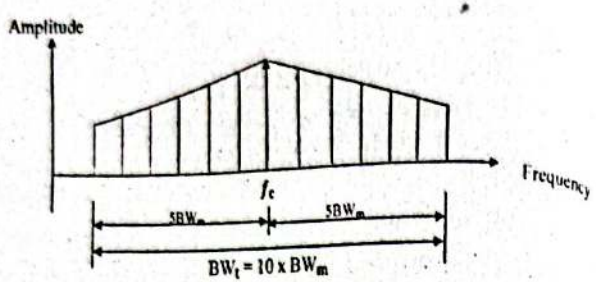
Applications:

Entertainment broadcasting (can be used for high quality music transmission)

FM Bandwidth:

The bandwidth of an FM signal is equal to 10 times the bandwidth of the modulating signal and, like AM bandwidth, covers a range centered on the carrier frequency. The total bandwidth required for FM can be determined from the bandwidth of the audio signal:

$$BW_t = 10 \times BW_m$$



Advantages of FM over AM:

- The primary advantage of FM over AM is its immunity to noise.
- Noise is short-duration amplitude variations caused by lightning, motors, auto ignitions, power transients and other sources.
- Another advantage of FM over AM is the capture effect that allows the strongest signal on frequency to dominate without interference from the other signal.
- FM has greater transmitter efficiency since class C amplifier may be used.
- FM receivers may be fitted with amplitude limiters to remove the amplitude variations caused by noise. This makes FM reception a good deal more immune to noise than AM reception.
- It is possible to reduce noise still further by increasing frequency deviation. This is a feature which AM do not have because it is not possible to exceed 100% modulation without causing severe distortion.
- Standard frequency allocation provides a guard band between commercial FM stations. Due to this, there is less adjacent channel interference than in AM.
- FM broadcasts operate in upper VHF and UHF frequency ranges at which there happens to be less noise than in MF and HF ranges occupied by AM broadcasts.

- The amplitude of FM wave is constant. It is thus independent of modulation depth, whereas in AM, modulation depth governs transmitted power. In AM transmitter the maximum power is four times the average power. FM transmitted power is useful whereas in AM, most of power is carrier power which does not contain any information.

Disadvantages of FM over AM:

- FM has wide bandwidth compared to AM.
- The circuits used in FM to produce and demodulate FM is more complex and expensive than AM circuits.

iii. Phase Modulation (PM)

Due to simpler hardware requirements, PM is used in some systems as an alternative to FM. In PM transmission the phase of the carrier signal is modulated to follow the changing voltage level of the modulating signal. The peak amplitude and frequency of the carrier signal remain constant, but as the amplitude of the information signal changes, the phase of carrier changes correspondingly. The analysis and final result (modulating signal) are similar to those of FM.

Let, $x_c(t) = V_c \sin(\omega_c t + \theta_c(t))$

$x_m(t) = V_m \sin \omega_m t$

Phase,

$\theta_c(t) \propto x_m(t)$

or, $\theta_c(t) = k_p\, x_m(t)$

where k_p = phase sensitivity

Then,

$x_{PM}(t) = V_c \sin[\omega_c t + k_p x_m(t)] = V_c \sin(\omega_c t + k_p V_m \sin \omega_m t)$

$x_{PM}(t) = V_c \sin(\omega_c t + m_p \sin \omega_m t)$

where m_p (= $k_p V_m$) is modulation index.

Bandwidth of PM signal,

$BW = 2f_m(m_p + 1) = 2_{fm}(k_p V_m + 1)$

5.4 Pulse Modulation

The process of transmitting signals in form of pulses is pulse *modulation*. The pulse modulation are classified as: *analog pulse modulation* and *digital pulse modulation*. Analog pulse modulation are *pulse amplitude modulation (PAM), pulse width modulation (PWM), pulse position modulation (PPM)*. Digital pulse modulation are *pulse code modulation (PCM), delta modulation*, etc.

5.4.1 Analog Pulse Modulation (PAM, PWM, PPM)

For AM, FM, PM; the carrier is a sinusoidal signal. However, in pulse modulation, the carrier is in the form of pulse instead of being sine wave. In *PAM (pulse amplitude modulation)*, amplitude of pulsed carrier is varied in accordance with amplitude of modulating signal. In PWM (pulse width modulation), width of pulses and in *PPM (pulse position modulation)*, the position of pulses is varied in accordance with modulating signal.

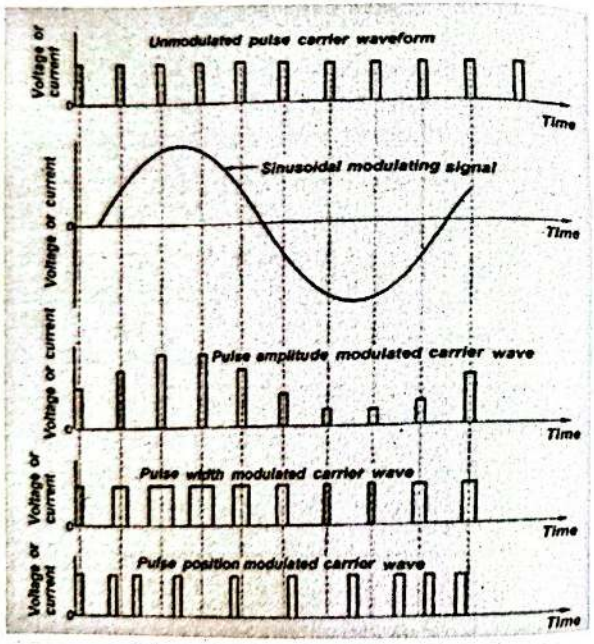


Fig.: Analog pulse modulation

Pulse Amplitude Modulation (PAM)

Drawbacks:

- Bandwidth required is maximum
- Since amplitude of pulse varies, interference of noise is maximum.

Pulse Width Modulation (PWM)

Advantages:

- Less effect of noise.
- Synchronization between transmitter and receiver is not essential which is essential in case of PPM.
- It is possible to reconstruct PWM signal from noise, contaminated PWM which is not possible in PAM.

Disadvantages:

- Variable pulse width have variable power contents.
- To avoid distortion, bandwidth required for PWM communication is large as compared to bandwidth of PAM

Pulse Position Modulation (PPM)

Advantages:

- Due to constant amplitude of PPM, the information is not contaminated in amplitude. Hence, noise added to PPM signal does not distort information.
- It is possible to reconstruct PPM signal.
- Due to constant amplitude, transmitted power always remains constant.

Disadvantages:

- Needs synchronization between transmitter and receiver.
- Large bandwidth is required to ensure of undistorted pulses.

5.4.2 Digital Pulse Modulation

It is of two types:

1. Pulsce code modulation (PCM)
2. Delta modulation

Pulse Code Modulation (PCM)

It is an analog-to-digital encoding method. In analog-to-digital encoding, the information contained in a continuous waveform are represented as a series of digital pulses (1s and 0s).

In pulsce code modulation (PCM), at first, sampling of analog message signal is done and then it is quantized. *Quantization* is the process of rounding-off the values of the flat-top samples to certain predetermined levels. These quantized values are encoded as combination of 0s and 1s. The binary digits are then transformed into a digital signal using one of the digital encoding techniques. Thus, PCM is made up of four separate processes: *sampling, quantization, binary encoding,* and *digital-to-digital encoding*.

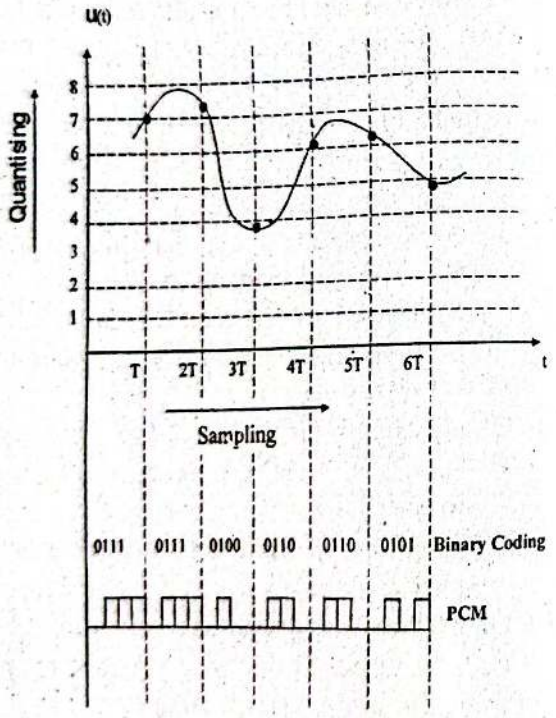


Fig.: Pulse code modulation

Advantages of PCM:

- Very high noise immunity
- Due to digital nature of the signal, repeaters can be placed between the transmitter and the receivers. The repeaters actually regenerate the received PCM signal. This is not possible in analog systems. Repeaters further reduce the effect of noise.
- It is possible to store the PCM signal due to its digital nature.
- It is possible to use various coding techniques so that only the desired person can decode the received signal.

Disadvantages:

- The encoding, decoding, and quantizing circuitry is complex.
- PCM system needs a large bandwidth as compared to other systems.

Applications:

- In telephony
- Due to high noise immunity, only PCM systems can be used in space communication.

5.5 Digital-to-Digital Conversion

Digital to digital conversion involves three techniques: *line coding, block coding,* and *scrambling*. Line coding is always needed; block coding and scrambling may or may not be needed.

Line Coding

Line encoding is the process of converting digital data to digital signals. Line coding converts a sequence of bits to a digital signal. At the sender, digital data are encoded into a digital signal; at the receiver, the digital data are recreated by

decoding the digital signal. The types of line encoding are *polar*, *unipolar*, and *bipolar*.

The following properties are desirable for a line code:

- It is important that the pulses stream to be transmitted does not have a DC component.
- It should be relatively easy to recover the data block.
- The line coding scheme should be bandwidth efficient.
- The line code should be robust in the presence of noise.
- It should be possible to recognize a line coding error, sometimes called a line violation. (In some signaling protocols, a line violation is deliberately generated to mark the start of a frame.)

5.6 Line Coding Schemes

The type of line encoding with different format are as:

1. **Unipolar**
   - Non-return-to-zero (NRZ)
   - Return-to-zero (RZ)
2. **Polar**
   - Non-return-to zero-level (NRZ-L)
   - Non-return-to-zero-invert (NRZ-I)
3. **Bipolar**
   - Alternate mark inversion (AMI)
   - Pseudo ternary
4. **Biphase**
   - Manchester encoding
   - Differential manchester encoding

1. Unipolar encoding

Digital transmission systems work by sending voltage pulses along a media link, usually a wire or a cable. In must types of encoding, one voltage level stands for

binary 0 and another level stands for binary 1. The polarity of a pulse refers to whether it is positive or negative. Unipolar encoding is so named because it uses only one polarity. Therefore, only one of the two binary states is encoded, usually the 1. The other state, usually 0, is represented by zero voltage, or an idle line.

Unipolar encoding uses only one level of value.

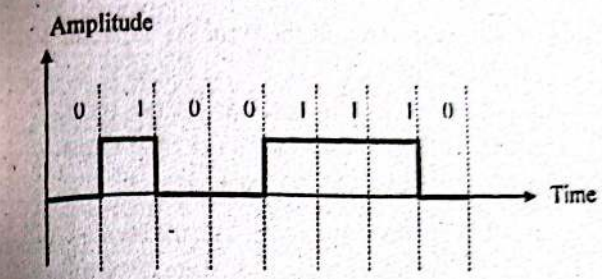


Fig.: Unipolar NRZ format

1's encoded as positive, 0's are idle. Unipolar encoding is straight forward and inexpensive to implement. However, it has two problems that make it unusable: DC component and synchronization.

i. **DC component**

Average amplitude is nonzero creates a-direct current (DC) component, when a single contains a DC component it cannot travel through media that cannot handle DC components: e.g. microwaves or transformers.

ii. **Synchronization**

When a signal is unvarying, the receiver cannot determine the beginning and ending of each bit. Therefore, synchronization problem in unipolar encoding can occur whenever the data stream includes a long uninterrupted series of 1's or 0's.

2. Polar encoding

Polar encoding uses two voltage levels: one positive and one negative. In most polar encoding methods the

average voltage level on the line is reduced and the DC component problem of unipolar encoding is alleviated.

Non-return-to-zero (NRZ) encoding

In NRZ encoding, the level of the signal is always either positive or negative. In NRZ system if the line is idle it means no transmission is occurring at all.

- **Non-return-to-zero-level (NRZ-L)**

  In NRZ-L, the level of the signal is dependent upon the state of the bit.

  A positive voltage usually means the bit is 0, and negative voltage means the bit is a 1 (and vice-versa).

- **Non-return-to-zero-invert (NRZ-I)**

  In NRZ-I, an inversion of the voltage level represents a 1 bit. It is the transition of voltage level for every 1 bit.

  An advantage of NRZ-I over NRZ-L is that because the signal changes every time a 1 bit is encountered, it provides some synchronization. Each inversion allows the receiver to synchronize its timer to the actual arrival of the transmission.

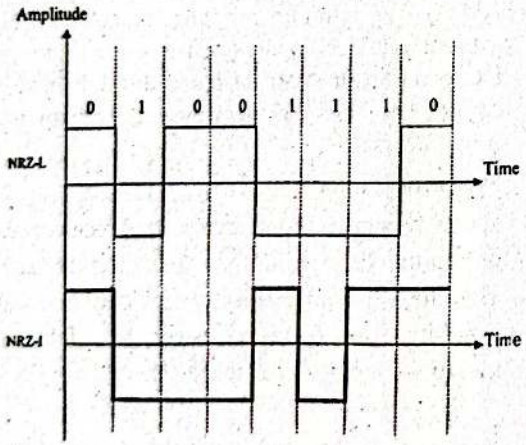


Fig.: NRZ encoding

Return-to-zero (RZ) encoding

To assure synchronization, there must be a signal change for each bit. The receiver can use these changes to built up, update, and synchronize its clock.

One solution is return to zero (RZ) encoding, which uses three values: positive, negative, and zero.

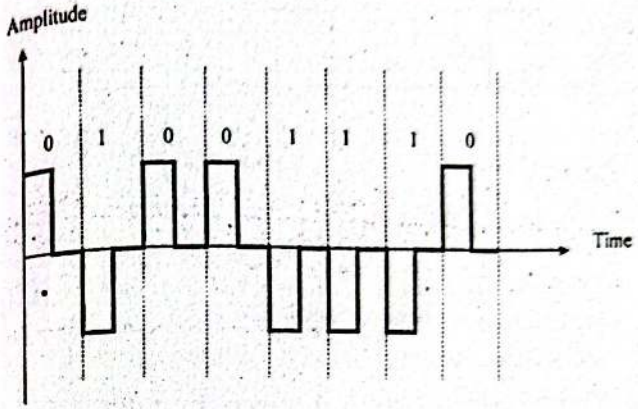


Fig.: RZ encoding

The main disadvantage of RZ encoding is that it requires two signal changes to encode one bit and therefore occupies more bandwidth. But of the three alternatives discussed above, it is the most effective. Because a good encoded digital signal must contain a provision for synchronization.

3. **Bipolar encoding**

Bipolar encoding uses three voltage levels: positive, negative, and zero. The zero level is used to represent binary 0, positive and negative voltages represent alternating 1s.

Three types of bipolar encoding are popular in use by the data communications industry: AMI, B8ZS, and HDB3.

i. **Alternate mark inversion**

Bipolar AMI is the simplest type of bipolar encoding. The word mark comes from telegraphy and means 1.

AMI means alternate 1 inversion. A neutral, zero voltage represents binary 0. Binary 1s are represented by alternating positive and negative voltages.

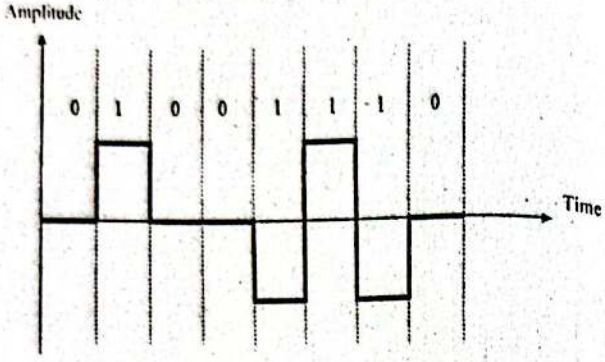


Fig.: Alternate mark inversion

By inverting on each occurrence of a 1, bipolar AMI accomplishes two things: first, the DC component is zero, and second, a sequence of is stays synchronized.

Two variations of bipolar AMI have been developed to solve the problem of synchronization sequential 0s. The first used in North America, is called *bipolar 8-zero substitution (B8ZS)*; the second, used in Europe and Japan, is called *high-density bipolar 3 (HDB3)*. Both are adaptations of bipolar AMI that modify the original pattern only in the case of multiple consecutive 0s.

4. Biphase encoding

Probably the best existing solution to the problem of synchronization is *biphase encoding*. In this method, the signal changes at the middle of the bit interval but doesnot return to zero. Instead it continues to the opposite pole. As in RZ, these mid-interval transitions allow for synchronization.

Biphase encoding is implemented in two different ways: *Manchester* and *differential Manchester*.

i. Manchester

Manchester encoding uses the inversion at the middle of each bit interval for both synchronization and bit representation. A negative-to-positive transition represents binary 1 and a positive-to-negative transition represents binary 0.

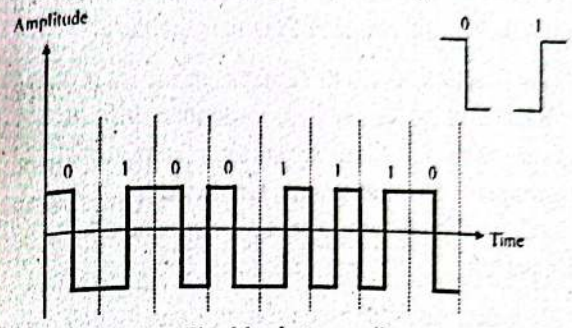


Fig.: Manchester encoding

ii. Differential Manchester

In this method, the inversion at the middle of the bit is used for synchronization, but the presence or absence of an additional transition at the beginning of the interval is used to identify the bit. A transition means binary 1and no transition means binary 0. The bit representation is shown by the inversion and non-inversion at the beginning of the bit.

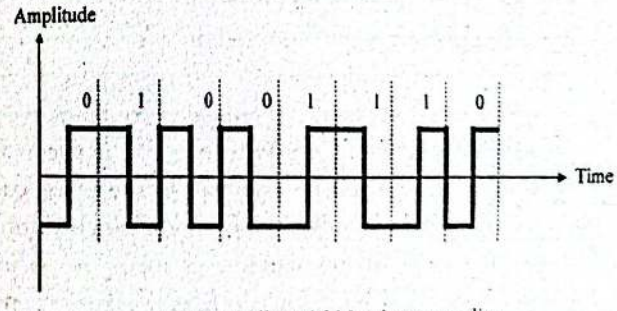


Fig.: Differential Manchester encoding

5.7 Block Coding, Scrambling

Block coding helps in error detection and re-transmission of the signal. It is normally referred to as mB/nB coding as it replaces each m-bit data group with an n-bit data group (where n>m). Thus, its adds extra bits (redundancy bits) which helps in synchronization at receiver's and sender's end and also providing some kind of error detecting capability.

It normally involves three steps: *division*, *substitution*, and *combination*. In the division step, a sequence of bits is divided into groups of m-bits. In the substitution step, we substitute an m-bit group for an n-bit group. Finally, the n-bit groups are combined together to form a stream which has more bits than the original bits.

Scrambling is a technique that does not increase the number of bits and does provide synchronization. Problem with technique like bipolar AMI (alternate mark inversion) is that continuous sequence of zeros create synchronization problems. One solution to this is scrambling.

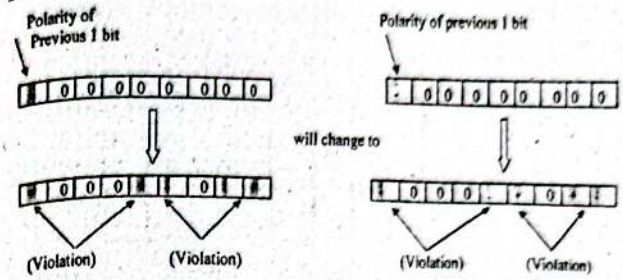
There are two common scrambling techniques:

1. B8ZS (bipolar with 8-zero substitution)
2. HDB3 (high-density bipolar 3)

i. Bipolar 8-zero substitution (B8ZS)

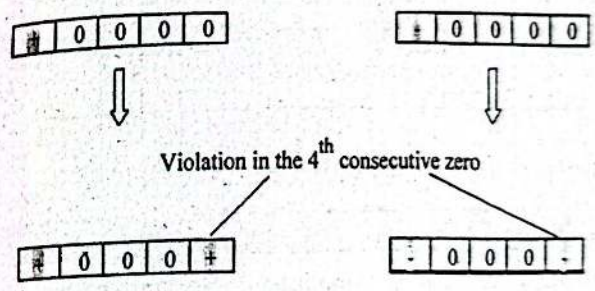
B8ZS is the convention adopted in North America to provide synchronization of long strings of 0s. In most situations B8ZS functions identically to bipolar AMI. Bipolar AMI changes poles with every 1 it encounters. These changes provide the synchronization needed by the receiver, but the signal does not change during a string of 0s, so synchronization is lost. The solution provided by B8ZS is to force artificial signal changes, called violations.

In B8ZS, if eight 0s come one after another, we change the pattern in one of two ways based on the polarity of previous I.

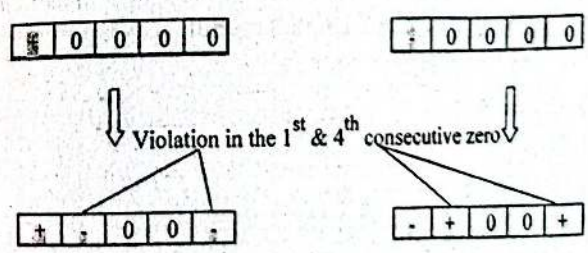


ii. High-density bipolar 3 (HDB3)

In HDB3, when four 0s appear simultaneously, they are replaced with one of the four ways based on the polarity of the previous 1 and the number of 0 since the last substitution.



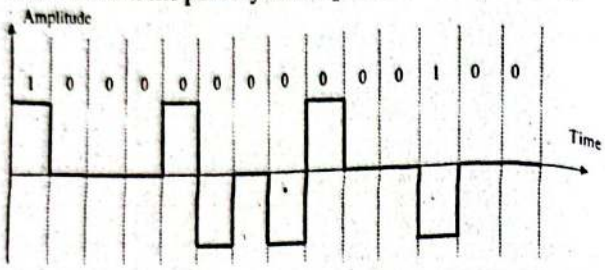
If the number of 1s since the last substitution is odd



If the number of 1s since the last substitution is even

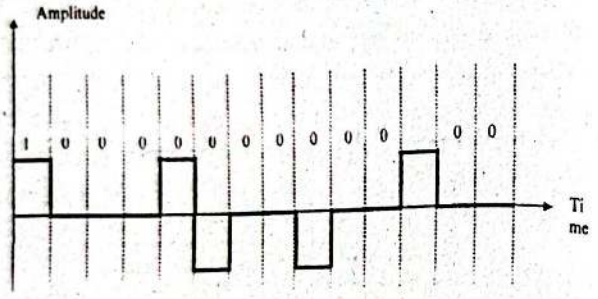
Example:

Using B8ZS, encode the hit stream 10000000000100. Assume that the polarity of the previous 1 is positive.



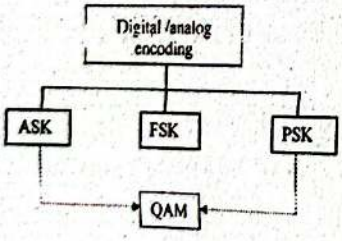
Example:

Using HDB3, encode 10000000000100. Assume that the number of Is so far is odd and the previous 1 is positive.



5.8 Digital Modulation

Digital modulation is a modulation technique that uses discrete signals to modulate a carrier wave. The different types of digital modulation are: *amplitude shift keying (ASK), frequency shift keying (FSK), phase shift keying (PSK),* and *quadrature amplitude modulation (QAM).* Digital modulation is a type of *digital-to-analog encoding* technique.



Bit Rate and Baud Rate

- *Bit rate* is the number bits transmitted in one second.
- *Baud rate* refers to the number of signal units per second that are required to represent those bits.
- For computer efficiency, bit rate is more important.
- For data transmission, baud rate is more important. The fewer the signal units required, the more efficient the system, and the less bandwidth required to transmit more bits.

In analog transmission, the sending device produces high frequency signal that acts as a basis for the information signal. The base signal is called the carrier signal or carrier frequency. The receiving device is tuned to the frequency of the carrier signal that it expects from the sender. Digital information is then encoded onto the carrier signal by modifying one or more of its characteristics (amplitude, frequency or phase). This kind of modification is called *modulation* (or *shift keying*) and the information signal is called a *modulating signal.*

5.8.1 Amplitude Shift Keying (ASK)

In *amplitude shift keying (ASK),* the amplitude of the carier signal is varied to create signal elements. Both frequency and phase remain constant while the amplitude changes.

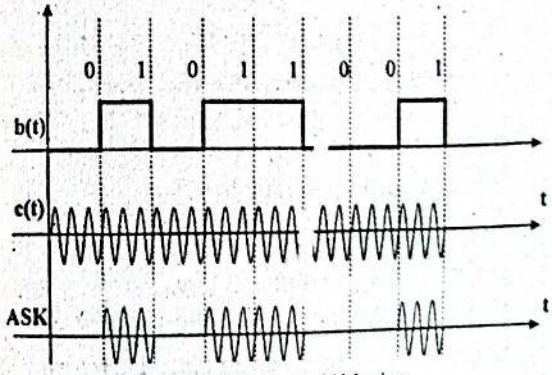


Fig.: Amplitude shift keying

5.8.2 Frequency Shift Keying (FSK)

In *frequency shift keying (FSK)*, the frequency of the signal is varied to represent data. The frequency of the modulated signal is constant for the duration of one signal element but changes for the next signal element if the data element changes.Both peak amplitude and phase remain constant for all signal elements.

FSK avoids most of the noise problems of ASK. The limiting factors of FSK are the physical capabilities of the carrier.

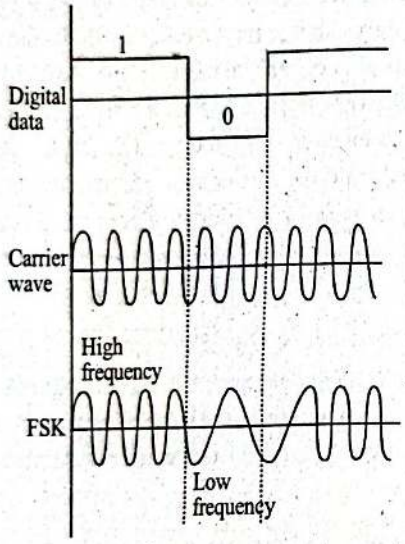


Fig.: Frequency shift keying

Salient features of FSK:

- Typically used up to 1200 bps
- Easy to implement
- Better noise immunity than ASK, hence the probability of error free reception of data is high.
- High bandwidth requirement. Therefore, FSK is extensively used in low speed modem having bit rates below 1200 bits/sec.

5.9.3 Phase Shift Keying (PSK)

In *phase shift keying (PSK)*, the phase of the carrier is varied to represent two or more different signal elements. Both peak amplitude and frequency remain constant as the phase changes. PSK is less susceptible to noise than ASK. PSK is superior to FSK because we do not need two carrier signals. Nowadays, PSK is more common than ASK or FSK.

Advantages:

- It has bandwidth lower than that of FSK signal.
- Very good noise immunity.

Disadvantage:

- It is more complex than ASK and FSK.

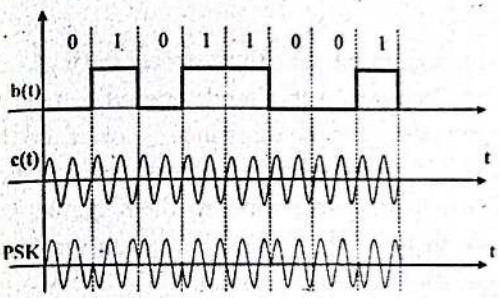


Fig.: Phase shift keying

Binary Phase Shift Keying (BPSK)

The simplest PSK is *binary PSK* in which we have only two signal elements one with a phase of 0° and other with a phase of 180°. Figure above shows the BPSK waveform.

Quadrature Phase Shift Keying (QPSK)

The simplicity of BPSK enticed designers to use two bits at a time in each signal element, thereby decreasing the baud rate and eventually the required bandwidth. This kind of scheme is called *QPSK* because it uses two separate BPSK modulations; one is in phase, the other one is quadrature (out of phase).

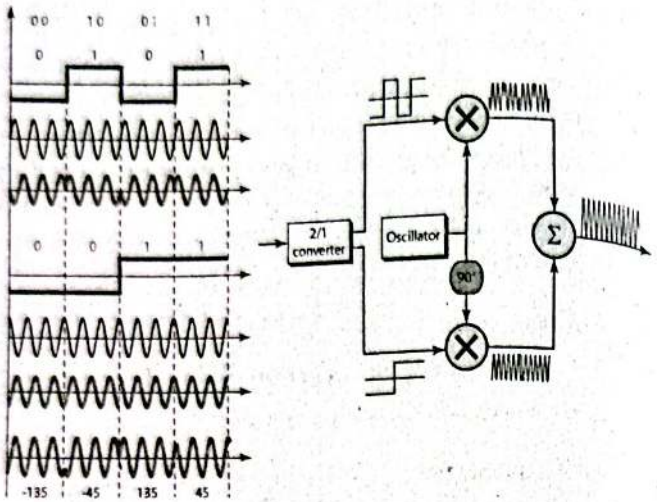


Fig.: Waveform of QPSK and circuit diagram

The incoming bits are first passed through a serial-to-parallel conversion that sends one bit to one modulator and the next bit to the other modulator. If the duration of each bit in the incoming singal is T, the duration of each bit sent to the corresponding BPSK signal is 2T. This means that the bit to each BPSK signal has one half the frequency of the original signal.

The two composite signals created by each multiplier are sinewaves with the same frequency but different phases. When they are added, the result is another sine wave with one of four possible phases: 45°, –45°, 135°, –135°. There are four kinds of signal elements in the output signal. So, we can send two bits per signal elements.

PSK Constellation

BPSK:

Bit	Phase
0	0°
1	180°

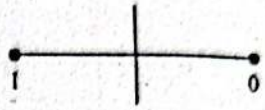


Fig.: Constellation diagram for BPSK

QPSK:

Dibit	Phase
00	0°
01	90°
10	180°
11	270°

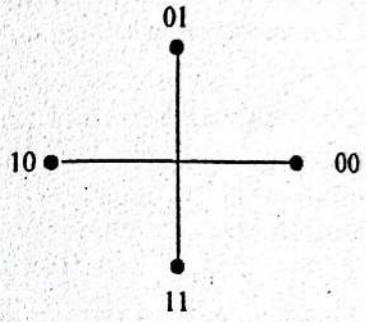


Fig.: Constellation diagram for QPSK

8-PSK:

Tribit	Phase
000	0°
001	45°
010	90°
011	135°
100	180°
101	225°
110	270°
111	315°

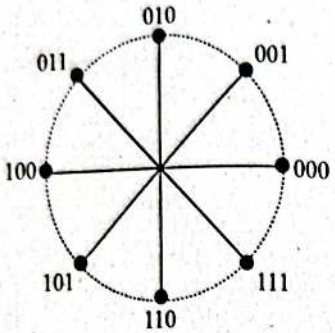


Fig.: Constellation diagram for 8-PSK

Example:

Find the bandwidth for a 4-PSK signal transmitting at 2,000 bps. Transmission is in half-duplex mode.

Solution:

For 4-PSK, the baud rate is half of the bit rate.

The baud rate is therefore 1,000. A PSK signal requires a bandwidth equal to its baud rate. Therefore, the bandwidth is 1,000 Hz.

Example:

Given a bandwidth of 5,000 Hz for an 8-PSK signal. What are the baud rate and bit rate?

Solution:

For PSK, the baud rate is the same as the bandwidth, which means the baud rate is 5,000. But in 8-PSK, the bit rate is three times the baud rate. So, the bit rate is 15,000 bps.

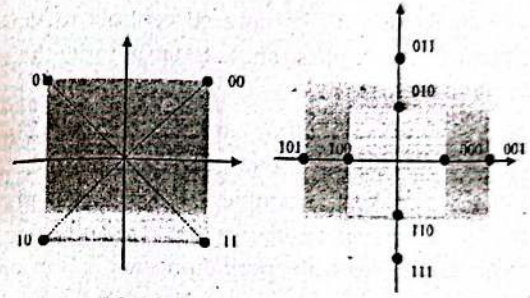
5.8.4 Quadrature Amplitude Modulation (QAM)

QAM means combining ASK and PSK in such a way that we have maximum contrast between each bit, dibit, tribit, quadbit, and so on.

Possible variation of QAM is numerous: theoretically any measurable number of changes in amplitude can, be

combined with any measurable number of changes in phase.

The figure below shows the constellation diagrams of 4-QAM and 8-QAM. In both cases the number of amplitude shifts is less than the number of phase shifts. Because amplitude changes are susceptible to noise and require greater shift differences than do phase changes, the number of phase shifts used by a QAM system is always larger than the number of amplitude shifts.



4-QAM Constellation Diagram

8-QAM Constellation Diagram

Fig.: Constellation diagram for 4-QAM and 8-QAM

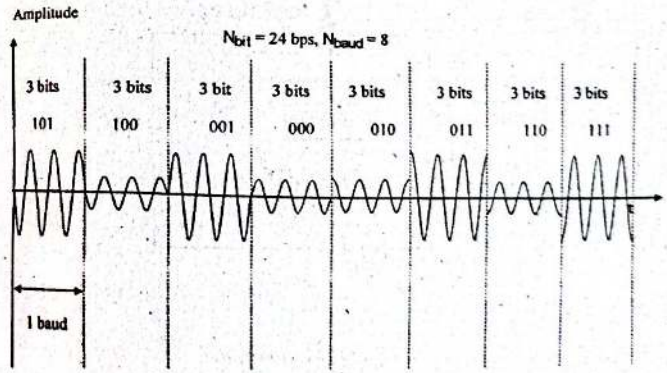


Fig.: 8-QAM waveform

Three popular 16-QAM configurations are shown bellow:

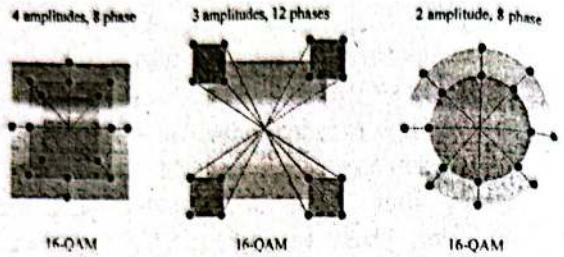


Fig.: Constellation diagram for 16-QAM

Since amplitude shift is more susceptible to noise, the greater the ratio of phase shifts to amplitude, the greater the immunity to noise.

The second example, three amplitudes and 12 phases, handles noise best because of greater ratio of phase shift to amplitude. The first example, 4 amplitudes, 8 phases, is the OSI recommendation. Several QAM designs link specific amplitudes with specific phases. This means that even with noise problems associated with amplitude shifting, the meaning of a shift can be recovered from phase information.

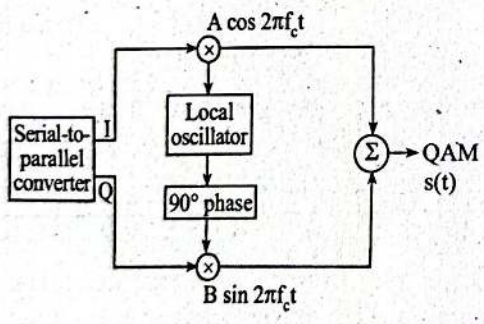


Fig.: QAM transmitter

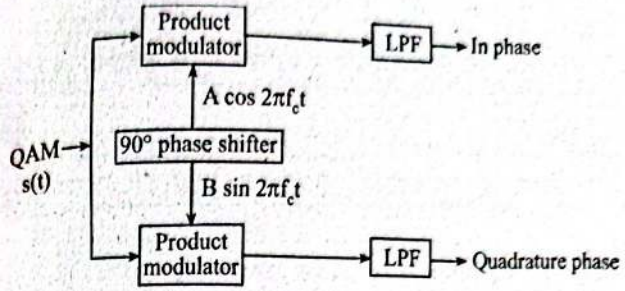


Fig.: QAM receiver

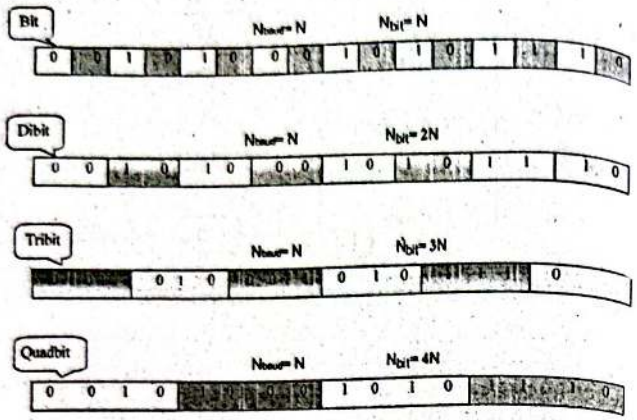
Bandwidth for QAM:

The minimum bandwidth required for QAM transmission is the same as that required for ASK and PSK transmission. QAM has the same advantages of PSK over ASK.

Bit/baud comparison

Assuming that an FSK signal over voice-grade phone lines can send 1200 bps, the bit rate is 1200 bps. Each frequency shift represents a single bit; so it requires 1200 signal elements to send 1200 bits. Its baud rate, therefore, is also 1200. Each signal variation in 8-QAM system, however, represents three bits. So a bit rate of 1200 bps, using 8-QAM, has a bad rate of only 400.

As the figure below shows, a dibit system has a baud rate of one-half the bit rate, a tribit system has a baud rate of one-third the bit rate, a quadbit system has a baud rate of one-fourth the bit rate.



Bit/ Baud Rate Comparison

Modulation	Units	Bits/baud	Baud rate	Bit rate
ASK, FSK, 2-PSK	Bit	1	N	N
4-PSK, 4-QAM	Dibit	2	N	2N
8-PSK, 8-QAM	Tribit	3	N	3N
16-QAM	Quadbit	4	N	4N
32-QAM	Pentabit	5	N	5N
64-QAM	Hexabit	6	N	6N
128-QAM	Septabit	7	N	7N
256-QAM	Octabit	8	N	8N

Example:

A constellation diagram consists of eight equally spaced points on a circle. If the bit rate is 4800 bps, what is the baud rate?

Solution:

The constellation indicates 8-PSK with points 45° apart. Since $2^3 = 8$, three bits are transmitted with each signal element. Therefore, the baud rate is

4800/3 = 1600 baud

Example:

Compute the baud rate for a 72,000 bps 64-QAM.

Solution:

A 64-QAM signal means that there are six bits per signal elements since $2^6 = 64$. Thus, 72,000/6 = 12,000 baud

Example:

Compute the bit rate for a 1,000 baud16-QAM signal.

Solution:

A 16-QAM signal means that there are four bits per signal elements since $2^4 = 16$.

Thus, (1,000)(4) = 4,000 bps

NUMERICAL PROBLEMS

1. An audio signal of 10 sin 1000πt is used for AM with a carrier of 50 sin 200000πt. Calculate:

i. Modulation index

ii. Required bandwidth

iii. Total power using load resistance of 800 ohm.

iv. Efficiency of AM [2073 Chaitra]

Solution:

Message signal, $e_m = 10 \sin 2\pi \times 500\, t$

Carrier signal, $e_c = 50 \sin 2\pi \times 10^5 t$

i. Modulation index $(m) = \frac{E_m}{E_c} = \frac{10}{50} = \frac{1}{5}$

ii. Comparing given message signal with e_m

$= E_m \sin 2\pi f_m t$, we get

$f_m = 500$ Hz.

$\therefore$ Required bandwidth $= 2f_m = 2 \times 500 = 1000$ Hz

iii. Total power delivered is calculated as:

$$P_T = P_C\left(1 + \frac{m^2}{2}\right)$$

$$= \frac{E_c^2}{2R}\left(1 + \frac{m^2}{2}\right)$$

$$= \frac{(50)^2}{2 \times 800}\left(1 + \frac{(1/5)^2}{2}\right)$$

$$= 1.593 \text{ watt}$$

iv. The efficiency of AM is given by

$$\eta = \frac{\text{Power in sideband}}{\text{Total power}} = \frac{P_{LSB} + P_{USB}}{P_T}$$

But,

$$P_{LSB} = P_{USB} = \frac{E_{sideband}^2}{R} = \left(\frac{\frac{mE_c}{2}}{\sqrt{2}}\right)^2 \times \frac{1}{R} = \frac{m^2 E_c^2}{8R}$$

$$= \frac{\left(\frac{1}{5}\right)^2 \times (50)^2}{8 \times 800} = \frac{\frac{1}{25} \times 50 \times 50}{8 \times 800} = 0.015625$$

$$\eta = \frac{P_{LSB} + P_{USB}}{P_T} = \frac{0.015625 + 0.015625}{1.593} = 0.0196$$

2. For a binary data sequence 1111000111, sketch:

a. NRZ-I waveform

b. AMI waveform

c. RZ waveform

d. Manchester waveform. [2073 Chaitra]

Solution:

NRZ-I

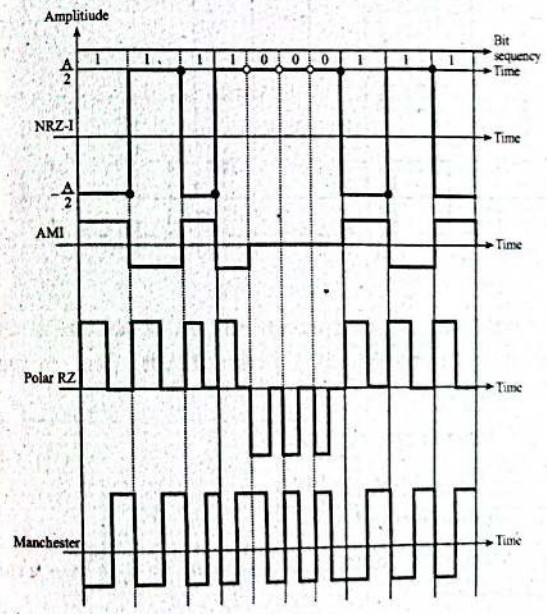
When next bit is 0 ⇒ no inversion

When next bit is 1 ⇒ inversion

Manchester

0 ⇒ ⅃ ⇒ high to low

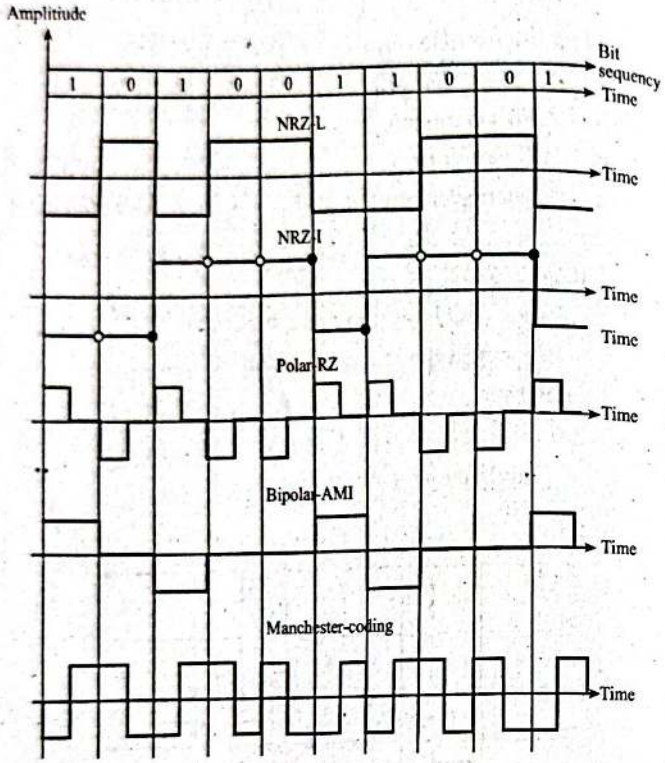
1 ⇒ ʃ ⇒ low to high



3. **Encode the bit stream 1010011001 using NRZ-L, NRZ-I, RZ, Manchester, Bipolar AMI encoding technique.**

[2074 Ashwin]

Solution:



4. **A single tone FM is represented by the voltage equation as $v(t) = 12 \cos (6 \times 10^8 t + 5\sin 1250t)$. Determine the following:**

   i. **Carrier frequency**

   ii. **Modulating frequency**

   iii. **Modulation index**

   iv. **Maximum frequency deviation** [2074 Chaitra]

Solution:

The equation of standard FM voltage wave is given by

$s(t) = E_c \cos (\omega_c t + m_f \sin\omega_m t)$ ..... (i)

Comparing above equation with given voltage equation, we have

i. Carrier frequency,

$$f_c = \frac{\omega_c}{2\pi} = \frac{6 \times 10^8}{2\pi} = 95.5 \times 10^6 \text{ Hz}$$

ii. Modulating frequency (message frequency),

$$f_m = \frac{\omega_m}{2\pi} = \frac{1250}{2\pi}$$

$$= 199 \text{ Hz}$$

iii. Modulation index, $m_f = 5$

iv. Maximum frequency deviation $(\Delta f) = m_f \times f_m$

$$[\because m_f = \frac{\Delta f}{f_m}]$$

$$= 5 \times 199 = 995 \text{ Hz}$$

5. **An audio frequency signal 20 sin 2π500t is used for AM with a carrier of 60 sin2π10⁶ t. Calculate: (i) modulation index (ii) sideband frequencies (iii) bandwidth requirement (iv) total power delivered with a load of 75 ohm.**

Solution:

Given, message signal is

$m(t) = 20 \sin 2\pi\, 500t$

Comparing, $A_m = 20$ and $f_m = 500$ Hz

The carrier signal is

$c(t) = 60 \sin 2\pi 10^6 t$

Comparing, $A_c = 60$ and $f_c = 10^6$ Hz

i) Modulation index, $m = \frac{A_m}{A_c} = \frac{20}{60} = 0.33$

ii) Sideband frequencies are:

a. $f_c + f_m = 10^6 + 500Hz$

b. $f_c - f_m = 10^6 - 500Hz$

iii) Bandwidth requirement:

$B = 2f_m = 2\times500 = 1$ KHz

iv) Total power delivered:

$$P_c = \frac{A_c^2}{2R} = \frac{60^2}{2\times75} = 24 \text{ W}$$

$$\text{Total power delivered } (P_T) = P_c\left(1 + \frac{m^2}{2}\right)$$

$$= 24\left(1+\frac{0.33^2}{2}\right)$$

$$= 25.306 \text{ W}$$

CHAPTER - 6

MULTIPLEXING AND SPREADING

6.1 Multiplexing and Application

Whenever the bandwidth of a medium linking two devices is greater than the bandwidth need of the devices, the link can be shared. The technique of sharing is called *multiplexing*. *Multiplexing* (also known as *muxing*) is a method by which multiple message signals or digital data streams are combined into one signal over a shared medium. For example, in telecommunications, servers telephone calls may be carried by using one wire.

In a multiplexed system, n lines share the bandwidth of one link i.e., there are n inputs to a multiplexer and a signal data link to de multiplexer. The link is able to carry 'n' separate channels of data. The multiplexer combines (multiplexes) data from 'n' inputs and transmits over a higher capacity data link. The demultiplexer (DEMUX) accepts the multiplexer data stream and separates them and delivers data to appropriate line.

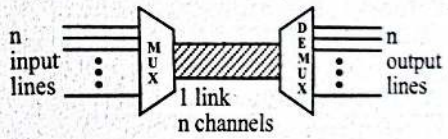


Fig.: Multiplexing

Applications of multiplexer:

The most common application of multiplexing is that in long distance communication, trunks (lines) used are high capacity optical fiber, coaxial cables or microwave links. These links can carry large number of voice and data simultaneously, all because of multiplexing. So, multiplexing is used almost everywhere from TV station to FM station or ISP etc.

This is because, multiplexing helps increasing the efficiency by per utilization of bandwidth.

6.2 Multiplexing Techniques

The various types of multiplexing techniques are:

i. FDM (frequency division multiplexing)

ii. TDM (time division multiplexing)

iii. WDM (wavelength division multiplexing)

6.2.1 FDM (Frequency Division Multiplexing)

FDM is an analog technique that can be applied when the bandwidth of the link is greater than the combined bandwidths of signals to be transmitted. In FDM, many signals are transmitted simultaneously where each occupies a different frequency slot within a common bandwidth.

The concept of FDM can be understood by the figure given below.

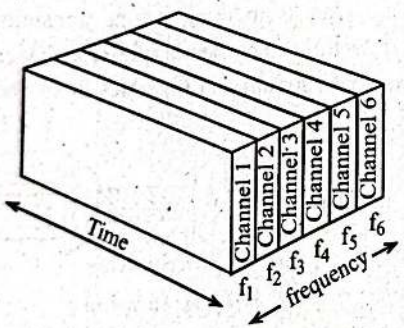


Fig.: Frequency division multiplexing

Here six signals are fed into a MUX, which modulates each signal into different frequency (f_1, f_2, f_3, f_4, f_5, f_6). Each modulated signal requires a BW central on its carrier frequency, referred to as channel. To prevent interference, channels are separated by certain *guard band*.

FDM process can be better understood by the following multiplexing and demultiplexing process shown below:

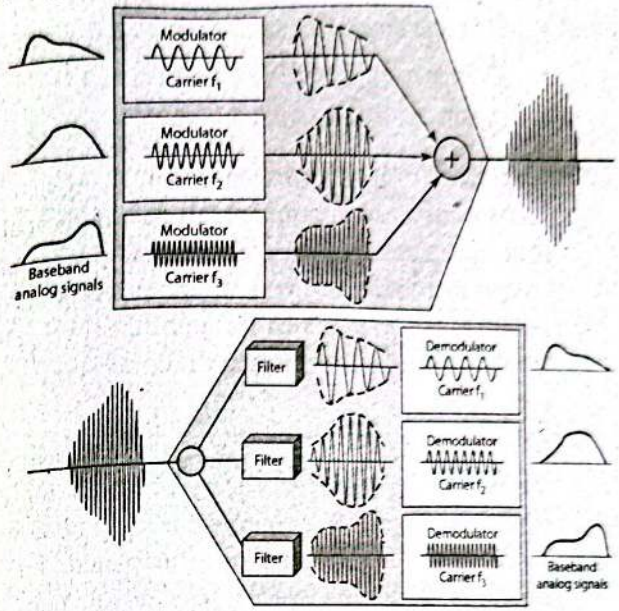


Fig.: FDM multiplexing and demultiplexing process

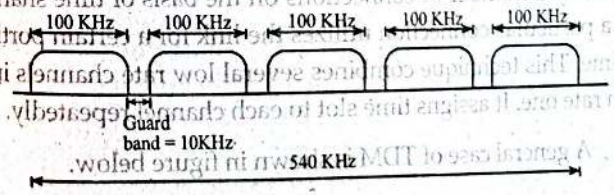
Example:

Five channels, each with a 100KHz bandwidth, are to be multiplexed together. What is minimum BW of the link if there is a need a guard band of 10 KHz between channels to prevent interference?

Solution:

For five channels, there should be four guard bands. So, the BW will be 10×4 = 40 KHz (for guard band) and 100×5 = 500 KHz for five channels.

∴ Total BW = 500 + 40 = 540 KHz



Applications of FDM:

- AM radio broadcasting (530 Hz to 1700 KHz band)
- FM radio broadcasting (88 MHz to 108 MHz)
- TV broadcasting (6 MHz channels)
- Cellular communication

6.2.2 Wavelength Division Multiplexing (WDM)

WDM is designed to use the high-data rate capability of fiber optic cable. WDM is conceptually same as FDM, except that multiplexing and demultiplexing involve optical signals transmitted through fiber-optic channels. In other words, WDM refers to the application of FDM in optical fiber.

In WDM, a number of sources generate a laser beam at different wavelengths which are sent to a multiplexer, which consolidates the sources for transmission over a single fiber line. Signals are amplified by optical amplifiers which are spaced tens of kilometers apart. Finally, they are demultiplexed and are separated and sent to the receiver.

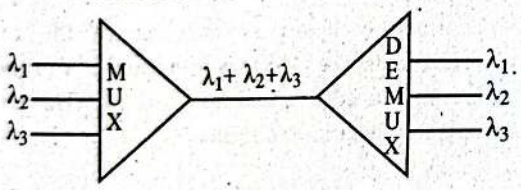


Fig.: WDM

6.3 Time Division Multiplexing (TDM)

It is a process in which a high bandwidth of a link is shared by a number of connections on the basis of time sharing i.e., a particular connection utilizes the link for a certain portion of time. This technique combines several low rate channels into high rate one. It assigns time slot to each channel repeatedly.

A general case of TDM is shown in figure below.

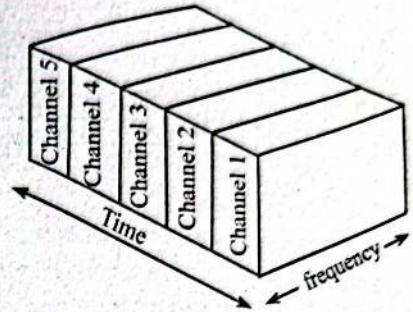


Fig.: TDM

Hence, TDM is a digital multiplexing technique where digital data from different sources are combined into one time shared link.

In TDM, all the signals to be transmitted are not transmitted simultaneously. Instead, they are transmitted one by one. Thus, each signal will be transmitted for a very short time. One cycle or frame is said to be complete when all the signals are transmitted once on the transmission channel. The TDM signal in the form of frames is transmitted on the common communication medium.

Types of TDM:

i. Synchronous TDM
ii. Statistical TDM (asynchronous TDM)

i. Synchronous TDM (STDM)

In *synchronous TDM*, each device is given same time slot to transmit the data over the link, irrespective of the fact that the device has any data to transmit or not. Each device places its data onto the link when its time slot arrives i.e., each device is given the possession of line turn by turn. If any device does not have data to send, then its time slot remains empty. The various time slots are organized into frames and each frame consists of one or more time slots dedicated to each sending device. If there are n sending devices, there will be n slots in a frame i.e., one slot for each device.

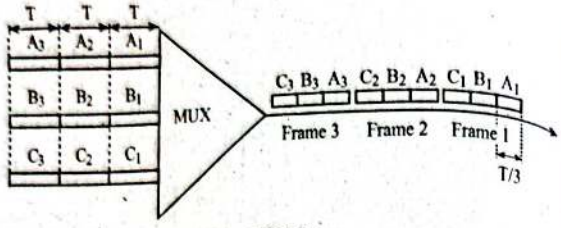


Fig.: STDM

Multiplexing process in STDM:

In STDM, each device is given the opportunity to transmit a specific amount of data onto the link. Each device gets its turn in fixed order and for fixed amount of time. This process is known as *interleaving*. We can say that the operation of STDM is similar to that of a fast interleaved switch. The switch opens in front of device; the device gets a chance to place the date onto the link. Such an interleaving may be done on the basis of a bit, byte, or by any other data unit. If one device sends a byte, other will also send a byte and so on.

As shown in figure interleaving is done characterwise in given example.

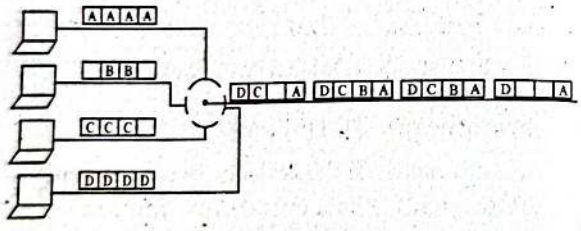


Fig.: Multiplexing in STDM

Disadvantages of synchronous TDM:

- Channel capacity is not fully utilized. Some of the slots go empty in certain frames.
- Capacity of single line that is used to carry various transmission should be greater than the total speed of input lines.

ii. **Asynchronous TDM (Statistical TDM)**

In asynchronous TDM, time slots are not fixed i.e., slots are flexible. The total speed of input lines can be greater than capacity of path. In synchronous TDM, if we have n input lines, there are n slots in one frame. But in asynchronous TDM, it is not so. In asynchronous TDM, if we have n input lines, then frame contains not more than m slots, where m<n. In this system, slots are not predefined, slots are allocated to any of the device that has data to send. If there are not enough data to fill all the slots in a frame, then the frames are transmitted partially filled.

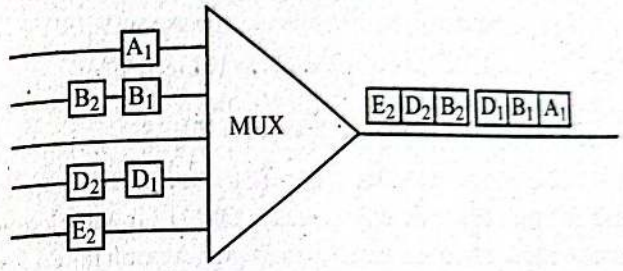


Fig.: Statistical TDM

6.4 Spread Spectrum

Spread spectrum is an important encoding method for wireless communications. Basic idea of spread spectrum is to modulate the signal so as to increase significantly the bandwidth (spread the spectrum) of the signal to be transmitted. It makes jamming (decrease of noise) and interception harder & provides improved reception.

Consider the general model of spread spectrum system shown below.

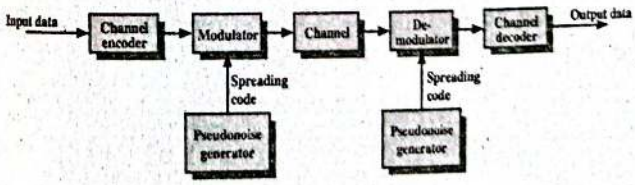


Fig.: General model of spread spectrum system

This signal is further modulated using a sequence of digits known as a spreading code which is generated by a pseudonoise generator. The effect of this modulation is to increase significantly the bandwidth (spread the spectrum) of the signal to be transmitted. On the receiving end, the same digit sequence is used to demodulate the spread spectrum signal. Finally, the signal is fed into a channel decoder to recover the data. Input is fed into a channel encoder that produces an analog signal with a relatively narrow bandwidth around some center frequency.

The two approaches of spread spectrum are:

i. Frequency hopping spread spectrum (FHSS)

ii. Direct sequence spread spectrum (DSSS)

Frequency Hopping Spread Spectrum (FHSS)

The *frequency hopping spread spectrum (FHSS)* technique uses M different carrier frequencies that are modulated by the source signal. At one moment, the signal modulates one carrier frequency; at the next moment, the signal modulates another carrier frequency. Although the modulation is done using the one carrier frequency at a time, M frequencies are used in the long run. The bandwidth occupied by a source after spreading is $B_{FHSS} >> B$.

Figure shows the general layout for FHSS. A pseudorandom code generator, called *pseudorandom noise (PN)*, creates a k-bit pattern for every hopping period T_h. The frequency table uses the pattern to find the frequency to be used for this hopping period and passes it to the frequency synthesizer. The frequency synthesizer creates a carrier signal of that frequency, and the source signal modulates the carrier signal.

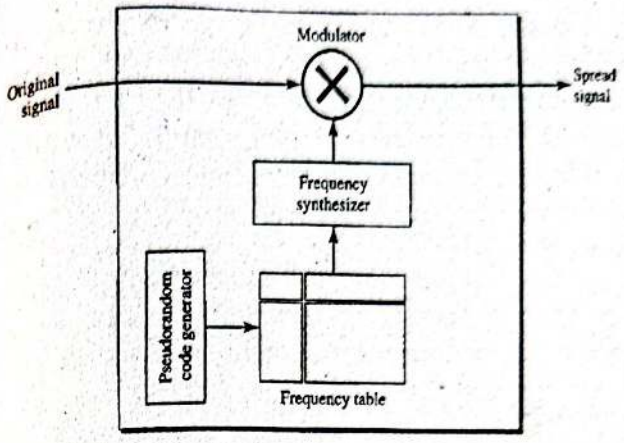


Fig.: Frequency hopping spread spectrum (FHSS)

Suppose we have decided to have eight hopping frequencies. This is extremely low for real applications and is just for illustration. In this case, M is 8 and k is 3. The pseudorandom code generator will create eight different 3-bit patterns. These are mapped to eight different frequencies in the frequency table.

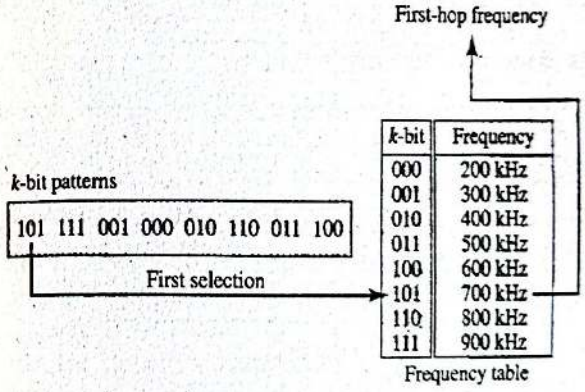


Fig.: Frequency selection in FHSS

The pattern for this station is 101, 111, 001, 000, 010, 011, 100. Note that the pattern is pseudorandom and is repeated after eight hoppings. This means that at hopping period 1, the pattern is 101. The frequency selected is 700 KHz; the source

signal modulates this carrier frequency. The second k-bit pattern selected is 111, which selects the 900-KHz carrier; the eighth pattern is 100, the frequency is 600 KHz. After eight hoppings, the pattern repeats, starting from 101 again. Figure below shows how the signal hops around from carrier to carrier. We assume the required bandwidth of the original signal is 100 KHz.

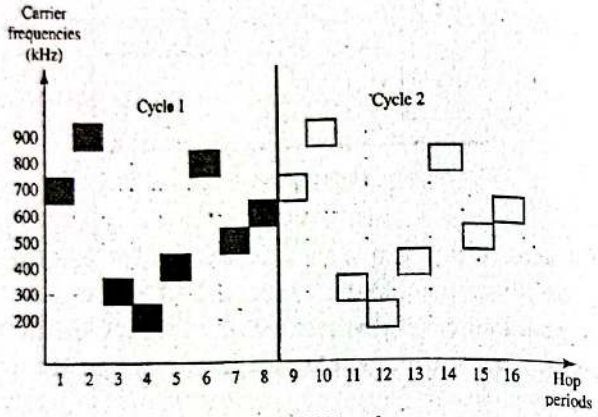


Fig.: FHSS cycles

Direct Sequence Spread Spectrum

The direct sequence spread spectrum (DSSS) technique also expands the bandwidth of the original signal, but the process is different. In DSSS, we replace each data bit with n bits using a spreading code. In other words, each bit is assigned a code of n bits, called chips, where the chip rate is n times that of the data bit. Figure below shows the concept of DSSS.

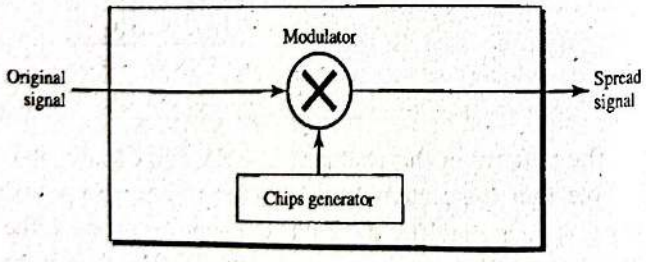


Fig.: DSSS

As an example, let us consider the sequence used in a wireless LAN, the famous Barker sequence where n is 11. We assume that the original signal and the chips in the chip generator use polar NRZ encoding. Figure below shows the chips and the result of multiplying the original data by the chips to get the spread signal. The spreading code is 11 chips having the pattern 10110111000 (in this case). If the original signal rate is N, the rate of the spread signal is 11N. This means that the required bandwidth for the spread signal is 11 times larger than the bandwidth of the original signal. The spread signal can provide privacy if the intruder does not know the code. It can also provide immunity against interference if each station uses a different code.

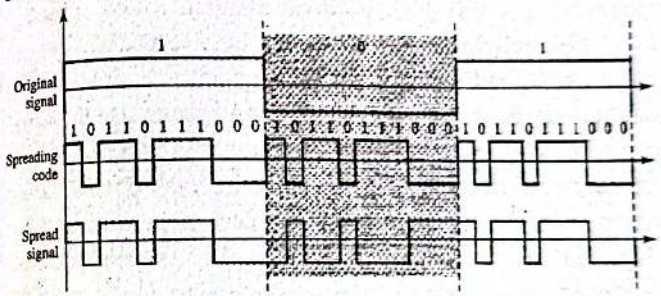


Fig.: DSSS example

The advantages of spread spectrum are:

- Immunity from noise and multipath distortion
- It can hide/encrypt signals i.e., Secure communication
- Several users can share same higher bandwidth with little interference
  - CDMA mobile telephones

6.5 Channelization

Channelization is a multiple access technique in which available bandwidth of a link is shared in time, frequency, or through code among different stations. Basically, there are 3 channelization protocols:

i. FDMA (frequency division multiple access)

ii. TDMA (time division multiple access)

iii. CDMA (code division multiple access)

6.6 CDMA

CDMA stands for *code division multiple access*. It is a multiplexing technique used with spread spectrum. CDMA means communicating with multiple codes. It is based on coding theory and uses sequences of numbers called *chips* which are generated by using orthogonal codes using Walsh table.

In CDMA systems, the narrowband message signal is multiplied by a very high bandwidth signal called *spreading signal*. The spreading signal is a pseudonoise code sequence that has a chip rate which is orders of magnitudes greater than data rate of message. All users in a CDMA system use the same carrie frequency and may transmit simultaneously. Each user has its own pseudorandom codeword which is approximately orthogonal to all other codewords. The receiver performs a time correlation operation to detect only the specific desired codeword. For detection of the message signal, the receiver needs to know the codeword used by the transmitter. Each user operates independently with no knowledge of the other users.

Properties of Sequence generated by Walsh table:

i. Each sequence is made of N elements, where N is no. of stations.

ii. Multiplying a sequence by a scalar results multiplying each element.

e.g., 2[+1 +1 -1 -1] = [+2 +2 -2 -2]

iii. If we multiply two equal sequences, element by element, and add the result, we get N which is no. of elements in sequence.

e.g., [+1 +1 -1 -1] [+1 +1 -1 -1] = 1 + 1 + 1 + 1 = 4 = N

iv. If we multiply two different sequences, element by element, and add the result, we get 0.

e.g., [+1 +1 -1 -1] [+1 +1 +1 +1] = 1 + 1 - 1 - 1 = 0

Generation of sequence:

- Sequence is generated by using Walsh table.
- General rule for creating walsh table is as shown:

$$W_1 = [+1]$$

$$W_{2N} = \begin{bmatrix} W_N & W_N \\ W_N & \overline{W_N} \end{bmatrix}$$

For example,

$$W_2 = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}, W_4 = \begin{bmatrix} W_2 & W_2 \\ W_2 & \overline{W_2} \end{bmatrix} = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}$$

- Each row of Walsh table indicates chip sequence.

Example: Find the chips for a network with four stations.

$$W_4 = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}$$

So, the chips are [+1 +1 +1 +1], [+1 -1 +1 -1], [+1 +1 -1 -1] and [+1 -1 -1 +1]

Working principle:

- If four stations 1, 2, 3, 4 are connected to the same channel, let data from stations 1, 2, 3, 4 be d_1, d_2, d_3, and d_4. Similarly, their codes assigned be c_1, c_2, c_3, and c_4.
- Any station that wants to receive data from one of other three multiplies the data on the channel by the code of the sender.

- In this case, data on the common channel will be $c_1d_1 + c_2d_2 + c_3d_3 + c_4d_4$.
- Now, if station 1 sends data to station 2, station 2 detects the message of 1 by multiplying the channel in the date by code of A.

  i.e., $(c_1d_1 + c_2d_2 + c_3d_3 + c_4d_4)\, c_1$

  $= d_1c_1c_1 + d_2c_2c_1 + d_3c_3c_1 + d_4c_4c_1$

  $= 4d_1$

[since $c_1c_1 = N$ according to property (3) and $c_1c_2 = c_3c_1 = c_4c_1 = 0$ according to property (4)]

- Hence, d_1 can be obtained by dividing the result by 4.

Example:

Suppose there are 4 stations 1, 2, 3, 4. Assume station 1 and 2 are sending 0 bits.

3 is silent (not sending anything)

4 is sending 1 bit.

Lets encode the bits as shown:

bit 0 → -1

bit 1 → +1

silence → 0

So, $d_1 = -1$

$d_2 = -1$

$d_3 = 0$

$d_4 = +1$

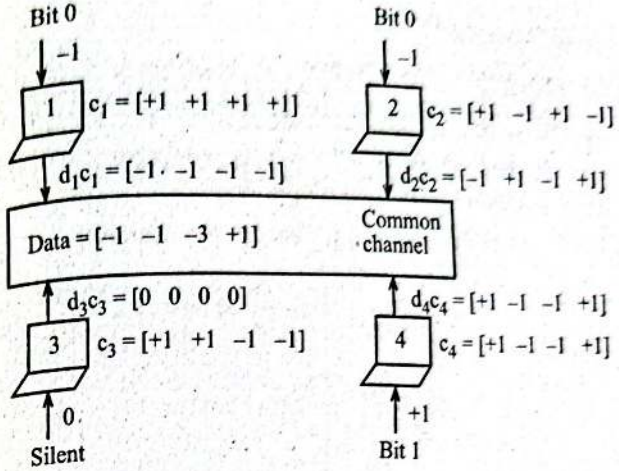
The codes of station 1, 2, 3, 4 are [+1 +1 +1 +1], [+1 -1 +1 -1], [+1 +1 -1 -1] and [+1 -1 -1 +1] respectively.

Then, $d_1c_1 = [-1\ -1\ -1\ -1]$

$d_2c_2 = [-1\ +1\ -1\ +1]$

$d_3c_3 = [0\ 0\ 0\ 0]$

$d_4c_4 = [+1\ -1\ -1\ +1]$



Therefore, data in the common channel

$= d_1c_1 + d_2c_2 + d_3c_3 + d_4c_4$

$= [-1\ -1\ -1\ -1]+[-1\ +1\ -1\ +1]+[0\ 0\ 0\ 0]+[+1\ -1\ -1\ +1]$

$= [-1\ -1\ -3\ +1]$

If station 3 has to detect the data sent by station 2, it uses the code for station 2. It multiples the data in common channel by code of station 2 and add each of the element.

So, $[-1\ -1\ -3\ +1]\ [+1\ -1\ +1\ -1]$

$= -1 +1 -3 -1 = -4$

which is divided by 4 to get -1 which is interpreted as bit 0.

Disadvantage of CDMA:

1. System is complicated
2. Performance degrades with increase in no. of users, due to limited orthogonal codes.
3. Requires large bandwidth

Advantages of CDMA:

1. Good noise rejection capability.
2. High immunity against interference and jamming.
3. CDMA does not require synchronization between sending and receiving stations.
4. CDMA offers highly secure communication.

CHAPTER - 7

SWITCHING

7.1 Unswitched/Unmultiplexed Network

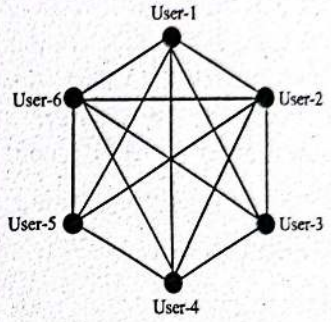


Fig.: Unswitched network

In *unswitched network*, there is dedicated link between each user. There is use of lot of wires and network ports. So, this method is less efficient. In such network, addition of user is even more difficult.

7.2 Switched Network

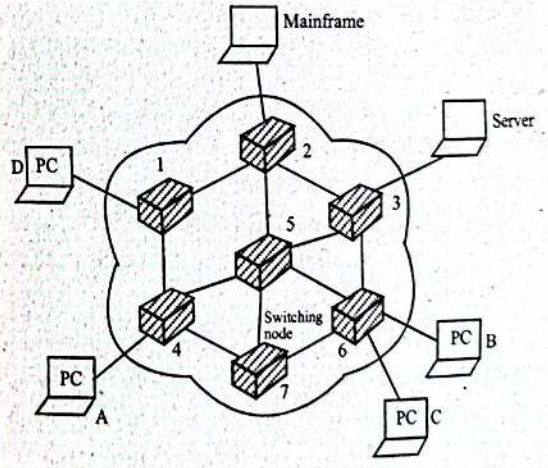


Fig.: Switched network

In a *switched network*, no dedicated link is needed between the users. It consists of a series of interlinked nodes called *switches*. Switches are devices capable of creating temporary connections between two or more devices linked to the switch. The switching nodes are not concerned with the content of data; rather their purpose is to provide a switching facility only, which will move data from node to node until the data reaches destination. The devices such as computer, telephone, etc. connected to the network are called *stations*. In a switched network, data entering the network from a station are routed to the destination by being switched from node to node.

There are three methods of switching:

- Circuit switching
- Packet switching
  - Virtual circuit approach
  - Datagram approach
- Message switching

Among these three, circuit switching and packet switching are commonly used today. Message switching has only network application these days. Compared to unswitched network, switched network has additional overhead to ensure data goes to the right place. So, this network is little bit more complicated.

7.2.1 Circuit-Switched Network

A *circuit-switched network* consists of a set of switches connected by physical links. It is used in public telephone networks. It was developed to handle voice traffic but it can also handle digital data.

With circuit switching, a dedicated path is established between two stations for communication. However, each connection uses only one dedicated channel on each link. Each link is normally divided into n channels using FDM or TDM. The routing decision is made when the path is

set up across the network. After the link has been set up between sender and receiver, information is forwarded continuously over the link.

The circuit switched network operates in three phases:

i. Setup phase

Before any signals can be transmitted, an end to end circuit must be established. Consider figure of switched network. If station A sends request to node 4 for connection with station B, a link from station A to node 4 becomes a dedicated line. Then, next node is selected by node 4 to reach the destination (station B). Let the next node be 5 (on the basis of routing information and measures of availability and cost). So far, connection has been established from station A to node 5. Now, remainder of the process proceeds similarly where node 5 allocates a channel to node 6 and eventually to station B. In completing the connection, a test is made to determine if station B is busy or is ready for connection. Hence, a dedicated path is created between A and B using the route A-4-5-6-B. This is called *setup phase*.

ii. Data transfer phase

After the dedicated path made of connected circuits is established, the *data transfer phase* can take place. In this phase, data can be transmitted from station A through the network to station B.

iii. Terminate phase

After some period of data transfer, connection is terminated. Signals are sent to nodes 4, 5, 6 to deallocate the dedicated resources.

Circuit switching is inefficient as channel capacity is dedicated for the entire duration even if there is no data being transferred. However, in terms of performance, it is a good option because information is transmitted at a

fixed data rate with no delay other than propagation delay through the transmission links. A simple example of circuit-switched network is public telephone network or a private branch exchange (PBX) which is used to interconnect telephones within a building or office.

7.2.2 Packet-Switched Network

In *packet switching*, a message is broken into short packets, each of which includes a header with source and destination address information and control information. At each node, the packets are received, stored briefly and passed onto the next node. There is no resource reservation – resources are allocated on demand.

The various advantages of packet switching over circuit switching are:

i. **Line efficiency is greater**

Unlike circuit switching, resources are allocated on demand. This is highly advantageous as much of the time, links may be unused or idle. In this method, a single node-to-node link can be dynamically shared by many packets over time which are queued up and transmitted over the link.

ii. **Data rate conversion is possible**

Unlike circuit switching, it can perform data rate conversion. Two stations at different data rates can exchange packets because each connects to its node at its own data rate.

iii. **Guaranteed delivery of packets**

In circuit switching, incase of heavy traffic, some calls may be blocked due to unavailability of link but, on packet switching, packets are still accepted. They are queued up and delivered later with delay.

iv. **Packets can be prioritized**

Packets can be delivered to the destination on the basis of their priority.

We can have two types of packet-switched network:

i. Datagram network

ii. Virtual circuit network

i. Datagram Network

In this approach, a message is divided into stream of packets. Each packet is treated independently, with no reference to packets that have gone before. So, even if a packet is a part of multipacket transmission, the network treats it as if it existed alone.

Each node chooses the next node on the packet's path taking into account information received from neighboring nodes on traffic, line failures, and so on. So, even though all the packets belong to the same message, they may or may not follow the same route to reach the destination. However, this approach can cause the datagram or packets arrive at the destination in out of order with different delays. It is the responsibility of upper-layer protocol to rearrange them or to ensure that all packets are received.

In datagram network, there is no dedicated path established between sender and receiver. There are no setup or teardown phase. Datagram network is robust to network problems as there are alternative routes for packet to reach the destination incase of any link failure.

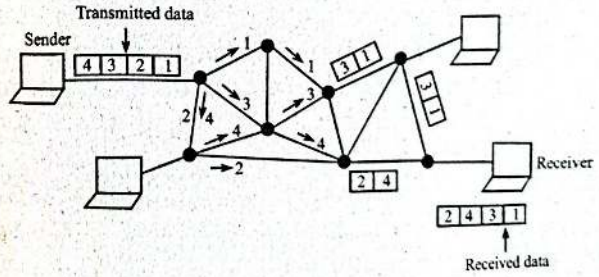


Fig.: Datagram network

ii. **Virtual Circuit Network**

In this approach, a preplanned route is established before any packets are sent. Once the route is established, all the packets follow the same route. It is a cross between circuit-switched network and datagram network. As in circuit-switched network, there is setup, data transfer, and terminate phase. Resources can be allocated during setup phase as in circuit-switched network or on demand as in datagram network. All packets follow the same route as in circuit-switched network.

A preplanned route is established prior to data transfer does not mean that it is a dedicated path, as in circuit switching. A transmitted packet is buffered at each node, and queued for output over a line, while other packets on other virtual circuits may share the use of the line.

Unlike datagram, the node need not make a routing decision for each packet and the packets arrive in order.

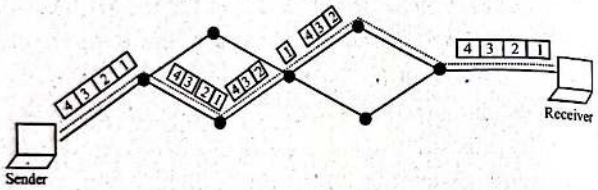


Fig.: Virtual circuit network

Advantages of datagram approach over virtual circuit approach:

- One of the advantages is that the call setup phase is not present in datagram. Thus if a station wants to send few packets, datagram approach will be quicker.
- Datagram delivery is more flexible as incase of congestion, the incoming packets can be routed away from congestion. But, it is not possible in virtual circuits as the packets follow a predefined route.

- Datagram delivery is more reliable. If a node fails in datagram network, the subsequent packets may find an alternate route that bypasses that node. But, in virtual circuits, if a node fails, all packets that pass through that node are lost.

7.3 Delays in a Switched Network

The three types of delay involved in a switched network are:

i. **Propagation delay**

It is the time taken by signal to travel from one node to another.

ii. **Transmission time**

It is the time taken by transmitter to send out a block of data.

iii. **Node delay**

It is the time involved in processing a data at each node.

The figure given below compares the relative performance of each switching technique along with delays involved:

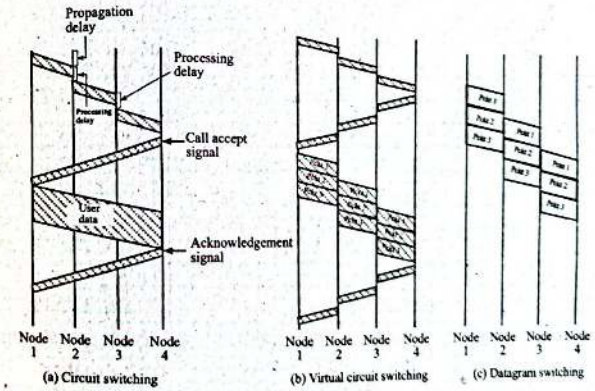


Fig.: Timing diagram

- In circuit switching, a processing delay is incurred at each node during the call request. This time is the time spent at each node for setting up route of connection. During the return of call accept signal, processing is not needed as connection is already set up. After connection, message is sent as a single block.
- Virtual circuit switching is similar to circuit switching except that the call request packet incurs delay at each node. The messages are sent in packets which faces delay at each node.
- Datagram switching doesn't require a call setup. It effective for short messages as each individual packet (datagram) is routed independently. Hence, it will be faster. But for longer message, virtual circuit may be the better option.

7.4 Comparison of Circuit Switching, Datagram Switching, and Virtual Circuit Switching

S.No.	Circuit Switching	Datagram Packet Switching	Virtual Circuit Switching
1.	Circuit setup is needed.	Circuit setup is not needed.	Circuit setup is needed.
2.	Dedicated transmission path	No dedicated path	No dedicated path
3.	Circuit-switched network is generally implemented in physical layer.	It is implemented in network layer.	It is implemented in data link layer.
4.	Path is established for entire conversation.	Route is established for each packet.	Route is established for entire conversation.
5.	The major delay is call setup delay. There is negligible transmission delay.	The delay involved is packet transmission delay.	The delays involved are setup delay and packet transmission delay.
6.	Messages are not stored.	Packets are queued or stored until delivered.	Packets are stored until delivered.
7.	Overhead bits are not added after connection establishment (call setup).	Overhead bits are added in each packet.	Overhead bits are added in each packet.

7.5 X.25, Frame Relay, ATM

7.5.1 X.25

X.25 is a standard protocol for packet switched wide area network (WAN) communication. It was developed by ITU-T (International Telecommunication Union - Telecommunication standardization sector). It is one of the oldest packet-switched service which was developed before the OSI reference model. In X.25, WAN consists of packet-switching exchange (PSE) nodes as the networking hardware, and leased lines or telephone lines as physical links.

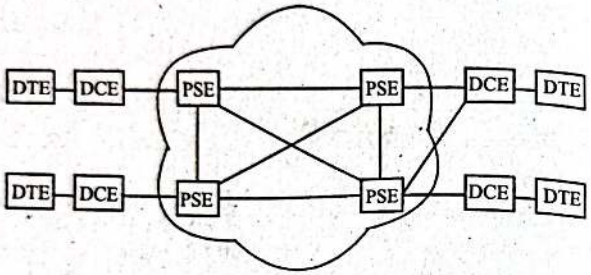
The functionality of X.25 is specified on three levels (or layers):

i. Physical layer
ii. Link layer
iii. Packet layer

- The *physical layer* deals with the physical interface between an attached station (DTE) and the link that attaches that station to the packet-switching node, This layer specifies the physical, electrical, functional,

and procedural characteristics to control the physical link between DTE and DCE.

- The *data link layer* consists of link access procedure for data interchange on the link between DTE and DCE. The link level standardization is referred to as LAPB (link-access-protocol-balanced).
- The *packet layer* defines a packet layer protocol for exchanging control and user data packets to form a packet switching network.



DTE – Data Terminal Equipment

DCE – Data Communication Equipment

PSE – Packet Switching Exchange node

***Fig.:** X.25 network diagram*

7.5.2 Frame Relay

Frame relay is relatively high-speed protocol as compared to DSL, cable TV, etc. It uses virtual circuit approach of circuit switching for data packet transfer in a WAN.

X.25 was earlier protocol used by internet for wide area networking (WAN). But it had many shortcomings like:

i. Low data rate (64 kbps)

ii. Large error and flow control at both data link and network layers leading to large overhead

iii. Use of leased lines by same service providers was found expensive.

Frame relay for WAN was the answer to the drawbacks of X.25 and leased lines.

Features of frame relay:

i. Higher speed (1.544 Mbps and 44.376 Mbps)

ii. Operates in physical and data link layer

iii. Relatively less expensive

iv. Has error detection in data link layer

v. Has no error or flow control

vi. Provides congestion control and quality of service.

Frame relay layers:

It has only two layers:

i. Physical layer

ii. Data link layer

Physical layer - The protocol is not specified, but left to the implementer. Frame relay supports any of the recognized protocols.

Data link layer - Frame relay has error defection mechanism, but not flow or error control mechanism.

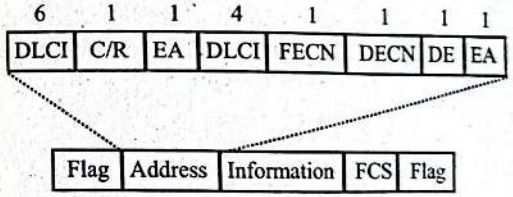


***Fig.:** Frame relay format*

Various subfields in the address are:

DLCI (Data Link Connection Identifier)

C/R (Command/Response): It allows upper layer to identify a frame as a command or response.

EA (Extended Address): EA bit '0' means another addres byte to follow and EA bit '1' means current address byte is final.

FECN (Forward Explicit Congestion Notification): With this information, the destination can expect delay or loss of packet.

BECN (Backward Explicit Congestion Notification): When BECN bit is set, it indicates traffic congestion. With this information, source inputs data at a slower rate to prevent data loss.

DE (Discard Eligibility): DE bit set to '1' tells the network to discard the frame incase of congestion.

7.5.3 ATM

Asynchronous transfer mode (ATM) is a cell relay protocol which in combination with SONET is supposed to provide an information highway. It is a streamlined packet transfer interface using fixed size packets. It can support wide range of traffic as it combines the benefits of circuit switching (guaranteed capacity and constant time delay) with those of packet switching (flexibility and efficiency). ATM has been quite successful in handling various types of services irrespective of their bit rates, bursty nature or whether they are connection-oriented or connectionless. In ATM, all traffic that flows in the network is converted into 53 byte blocks called cells. Each cell has 48 bytes of payload and a 5 byte header used for forwarding each cell into destination.

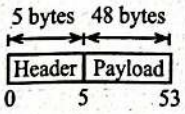


Fig.: ATM cell

The recommended speed for ATM network is 155 Mbps.

Advantages of ATM network:

- ATM is not limited by speed or distance.
- The switched nature of ATM allows it to operate over LANs and global backbone networks at speeds from few Mbps to several Gbps.

- ATM network carries voice, data and video with good QoS.

The ATM protocol reference model involves three separate planes.

i. **User plane** - Provides user information transfer, along with associated controls (e.g., flow control, error control).

ii. **Control plane** - Performs call control and connection control functions.

iii. **Management plane** - Includes plane management which performs management functions related to a system as a whole and provides coordination between all the planes, and layer management, which provides management functions relating to resources and parameters residing in its protocol entities.

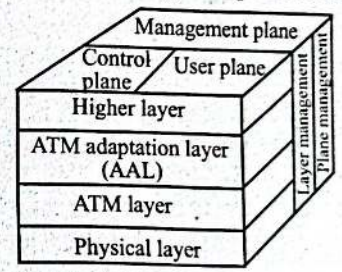


Fig.: ATM protocol architecture

CHAPTER - 8

INFORMATION THEORY AND CODING

8.1 Information

Information is the source of a communication system, whether it is analog or digital. *Information theory* is a mathematical approach to the study of coding of information along with the quantification, storage, and communication of information.

The purpose of communication system is to carry information bearing baseband signals from one place to another over a communication channel. The amount of information is directly proportional to amount of surprise of a particular event. The less probable an event is, more surprising is the event. Information is inversely proportional to the probability of an event.

Sun rises from east has no information. Sun rises from west has information. Car on road has no surprise, so no information but car in sky has surprise so has high information.

Consider an example. You are planning to go to Biratnagar during summer vacation. You call your friend in Biratnagar to know the weather condition of Biratnager. Assume that you received one of the following messages regarding the weather condition of Biratnagar on summer.

a. It is sunny and hot

b. It is cold

c. Snow is falling down

You know that on summer, it is sunny and hot in Biratnagar and therefore that first message would contain almost no information for you.

The information content on message (b) is relatively high because your do not expect, in general, cold weather during summer in Biratnagar, but may happen rarely.

Of course among the three possible messages you could have received, the third one has the highest information content as snow during summer in Biratnagar is almost next to impossible (but still could happen).

Therefore the information content of any message is closely related to the past knowledge of the occurrence of event and the level of uncertainty it contains with respect to the recipient of message. The same message, if given to a person living in Switzerland and who never has no idea of Biratnagar and have no plan to go there, would have contained almost zero information.

Thus in general, "on an intuitive basis, the amount of information received from the knowledge of occurrence of an event is related to the probability or the likelihood of occurrence of the event". In other words, the message related to an event least likely to occur (with respect to the knowledge of occurring such events in the past) contains more information.

Let $m_1, m_2, \ldots m_q$ be the q possible messages emitted by a source with probability of occurrence $P_1, P_2, \ldots, P_q$

$$P_1 + P_2 + \ldots P_q = 1$$

Let $I(m_k)$ be the Amount of information contained in k-th message.

Then for $I(m_k)$ to represent information content of m_k message, based on uncertainty, the following conditions should be met.

i. $I(m_k) > I(m_j)$ if $P_k < P_j$

ii. $I(m_k) \rightarrow 0$ if $P_K \rightarrow 1$

iii. $I(m_k) \rightarrow 1$ if $P_k \rightarrow 0$

iv. $I(m_k) \geq 0$ when $0 \leq P_k \leq 1$

We could relate $I(m_k)$ and P_k in the following manner.

$$I(m_k) = \log\left(\frac{1}{P_k}\right) = -\log(P_k)$$

The unit of $I(m_k)$ depends upon the base assigned to log:

a. For base 'e', i.e. natural logarithm; the unit of information is nat.

b. b. For base 10 - the unit of information is Hartley or deceit.

c. base is 2 - the unit of information is bit

Example:

If two binary digits 1 and 0 occurs with equal probability and are correctly detected at the receiving end, then the information content in each digit $I(0) = I(1) = -\log\left(\frac{1}{2}\right) = 1$

8.1.1 Self Information and Average Information (Entropy)

We define an *information source (S)* as a collection of discrete symbols (S_i) representing an indication or event that the source transmits. The information provided by individual symbols is known as *self information* and it is denoted by $I(S_i)$.

$$I(S_i) = \log_2 \frac{1}{P(S_i)} \text{ bits}$$

From the definition of information, following conclusions can be drawn:

i. $I(S_k) > I(S_i)$ if $P(S_k) < P(S_i)$

ii. $I(S_k) < I(S_i)$ if $P(S_k) > P(S_i)$

iii. $I(S_k) \to 1$ if $P(S_k) \to 0$

iv. $I(S_k) \to 0$ if $P(S_k) \to 1$

The average information content of a sequence of symbols is called *entropy* and is denoted by

$$H(s) = \sum_{i=1}^{n} P(S_i)\log_2\left(\frac{1}{P(S_i)}\right) \text{ bits}$$

The *information rate (R)* is given by

$R = r\,H(S)$ bits/sec

where r = no. of symbols per second emitted by the source.

8.2 Source Coding

The process by which data that is generated by a source is represented efficiently is source coding. Appropriate way to convey information is to reduce number of bits required for conveying information, this is purpose of source coding. It reduces redundancy and promotes compression of data.

8.2.1 Binary Huffman Coding

Huffman coding is an efficient method of compressing data without losing information. It provides an efficient, unambiguous code by analyzing the frequencies that certain symbols appear in a message. Symbols that appear more often will be encoded as a shorter-bit string while symbols that aren't used as much will be encoded as longer-bit string.

Algorithm:

The Huffman algorithm is stated as follows:

1. List the source symbols in order of decreasing probability.
2. Combine the probabilities of the two symbols having the lowest probabilities, and reorder the resultant probabilities, this step is called reduction 1. The same procedure is repeated until there are two ordered probabilities remaining.
3. Start encoding with last reduction, which consist of exactly two ordered probabilities assign 0 as first digit in the codewords for all the source symbols

associated with the first probability, assign 1 to the second probability.

4. Now, go back and assign 0 and 1 to the second digit for the two probabilities that were combined in the previous reduction step, retaining all assignments made in step 3.

5. Keep regressing this way until the first column is reached.

8.2.2 Shannon-Fano Coding

Shannon–Fano coding, named after Claude Elwood Shannon and Robert Fano, is a technique for constructing a prefix code based on a set of symbols and their probabilities.

Shannon–Fano does not always produce optimal prefix codes. For this reason, Shannon–Fano is almost never used; Huffman coding is almost as computationally simple and produces prefix codes that always achieve the lowest expected code word length. Shannon–Fano coding is used in the IMPLODE compression method, which is part of the ZIP file format, where it is desired to apply a simple algorithm with high performance and minimum requirements for programming.

Shannon-Fano Algorithm:

1. For a given list of symbols, develop a corresponding list of probabilities or frequency counts so that each symbol's relative frequency of occurrence is known.
2. Sort the lists of symbols according to frequency, with the most frequently occurring symbols at the top and the least common at the bottom.
3. Divide the list into two parts, with the total frequency counts of the top part being as close to the total of the bottom as possible.
4. The top part of the list is assigned the binary digit 0, and the bottom part is assigned the digit 1. This means that the codes for the symbols in the first part will all start with 0, and the codes in the second part will all start with 1.
5. Recursively apply the steps 3 and 4 to each of the two halves, subdividing groups and adding bits to the codes until each symbol has become a corresponding code leaf on the tree.

8.3 Error Detection and Correction Codes

Networks must be able to transfer data from one device to another with acceptable accuracy. For most applications, a system must guarantee that the data received are identical to the data transmitted. Whenever data are transmitted from one node to the next, they can become corrupted. Many factors can alter one or more bits of a message. Some applications require a mechanism for detecting and correcting errors.

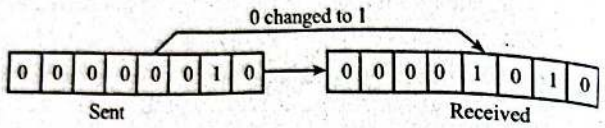
The correction of errors is more difficult than the detection. In error detection, we look only if any error has occurred. We are not even interested in the number of errors. A single-bit error is the same for us as a burst error. In error correction, we need to know the exact number of bits that are corrupted and more importantly, their location in the message. The number of the errors and the size of the message are important factors. If we need to correct one single error in an 8-bit data unit, we need to consider eight possible error locations; if we need to correct two errors in a data unit of the same size, we need to consider 2^8 possibilities.

The central concept in detecting or correcting errors is *redundancy*. To be able to detect or correct errors, we need to send some extra bits with our data. These redundant bits are added at transmitter and removed by the receiver. Their presence allows the receiver to detect or correct corrupted bits. To detect or correct errors, we need to send extra (redundant) bits with data.

Errors can be broadly categorized into two types:

1. **Single-bit error**

The term *single-bit error* means that only 1 bit of a given data unit (such as a byte, character, or packet) is changed from 1 to 0 or from 0 to 1.



Single-bit errors are the least likely type of error in serial data transmission.

2. **Burst error**

The term *burst error* means that 2 or more bits in the data unit have changed from 1 to 0 or from 0 to 1. A burst error means that 2 or more bits in the data unit have changed.

Figure below shows the effect of a burst error on a data unit. In this case, 0100010001000011 was sent, but 0101110101100011 was received. Note that a burst error does not necessarily mean that the errors occur in consecutive bits. The length of the burst is measured from the first corrupted bit to the last corrupted bit. Some bits in between may not have been corrupted:

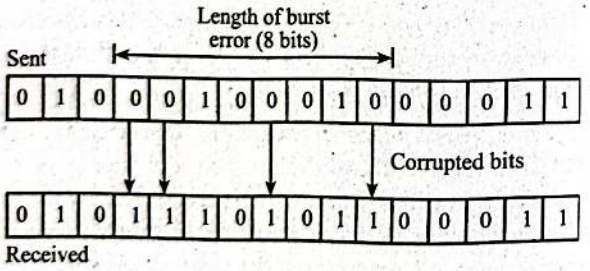


Fig.: Burst error of length 8

A burst error is more likely to occur than a single-bit error. The duration of noise is normally longer than the

duration of 1 bit, which means that when noise affects data, it affects a set of bits. The number of bits affected depends on the data rate and duration of noise.

8.3.1 Overview of Error Control Coding Techniques

The figure below shows the overview of error control coding techniques.

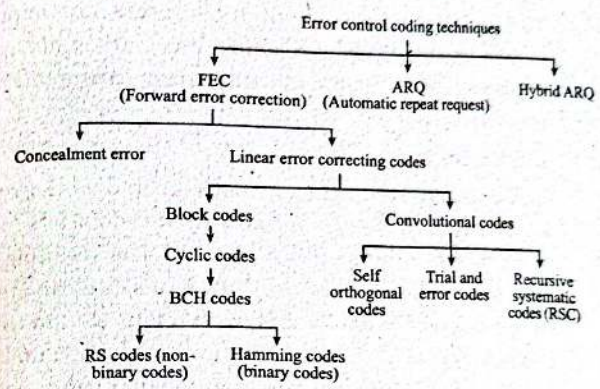


Fig.: Overview of error control coding techniques

Types of error control techniques:

1. **Automatic repeat request technique (ARQ)**

In an *ARQ technique*, the receiver requests for the retransmission of the complete or a part of message if it finds some errors in the received message. This requires an additional channel called feedback channel to send the receiver's request for retransmission.

2. **Forward error correction (FEC) technique**

In an *FEC technique*, there is no such feedback path and no request is made for transmission. It is the process in which the receiver tries to guess the message by using redundant bits.

8.3.2 Error Correcting Codes

Redundancy is achieved through various coding schemes. The sender adds redundant bits through a process that creates a relationship between the redundant bits and the actual data bits. The receiver checks the relationships between the two sets of bits to detect or correct the errors. The ratio of redundant bits to the data bits and the robustness of the process are important factors in any coding scheme. We can divide coding schemes into two broad categories: *block coding* and *convolution coding*.

1. Block Coding

In *block coding*, we divide our message into blocks, each of k bits, called *datawords*. We add r redundant bits to each block to make the length n = k + r. The resulting n-bit blocks are called *codewords*. With k bits, we can create a combination of 2^k datawords; with n bits, we can create a combination of 2^n codewords. Since n>k the number of possible codewords is larger than the number of possible datawords. The block coding process is one-to-one; the same dataword is always encoded as the same codeword. This means that we have $2^n - 2^k$ codewords that are not used, we call these codewords invalid or illegal.

Error detection

If the following two conditions are met, the receiver can detect a change in the original codeword:

1. The receiver has (or can find) a list of valid codewords.
2. The original codeword has changed to an invalid one.

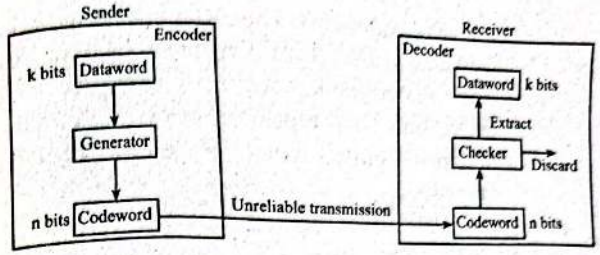


Fig.: Structure of encoder and decoder in error detection.

The sender creates codewords out of datawords by using a generator that applies the rules and procedures of encoding. Each codeword sent to the receiver may change during transmission. If the received codeword is the same as one of the valid codewords, the word is accepted and the corresponding dataword is extracted for use. If the received codeword is not valid, it is discarded. However, if the codeword is corrupted during transmission but the received word still matches a valid codeword, the error remains undetected. This type of coding can detect only single errors. Two or more errors may remain undetected.

Datawords	Codewords
00	000
01	011
10	101
11	110

Assume the sender encodes the dataword 01 as 011 and sends it to the receiver. Consider the following cases:

1. The receiver receives 011. It is a valid codeword. The receiver extracts the dataword 01 from it.
2. The codeword is corrupted during transmission, and 111 is received (the leftmost bit is corrupted). This is not a valid codeword and is discarded.
3. The codeword is corrupted during transmission, and 000 is received (the right two bits are corrupted). This

is a valid codeword. The receiver incorrectly extracts the dataword 00. Two corrupted bits have made the error undetectable. An error-detecting code can detect only the types of errors for which it is designed; other types of errors may remain undetected.

Error correction

In error detection, the receiver needs to know only that the received codeword is invalid. In error correction, the receiver needs to find the original codeword sent. Thus, we need more redundant bits for error correction than for error detection. Figure below shows the role of block coding in error correction.

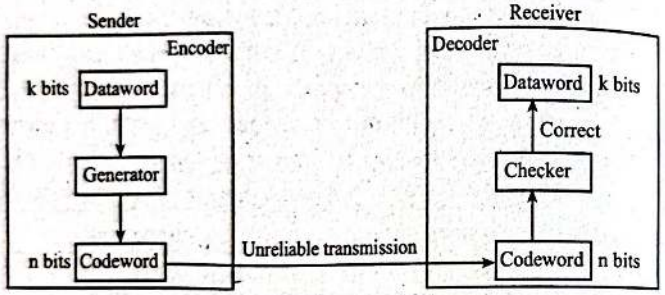


Fig.: Structure of encoder and decoder in error correction

Example:

Let us add more redundant bits to above example to see if the receiver can correct an error without knowing what was actually sent. We add 3 redundant bits to the 2-bit dataword to make 5-bit codewords. Table below shows the datawords and codewords. Assume the dataword is 01. The sender consults the table (or uses an algorithm) to create the codeword 01011. The codeword is corrupted during transmission, and 01001 is received (error in the second bit from the right). First, the receiver finds that the received codeword is not in the table.This means an error

has occurred. (Detection must come before correction.) The receiver, assuming that there is only 1 bit corrupted, uses the following strategy to guess the correct dataword.

Dataward	Codeword
00	00000
01	01011
10	10101
11	11110

1. Comparing the received codeword with the first codeword in the table (01001 versus 00000), the receiver decides that the first codeword is not the one that was sent because there are two different bits.
2. By the same reasoning, the original codeword cannot be the third or fourth one In the table.
3. The original codeword must be the second one in the table because this is the only one that differs from the received codeword by 1 bit. The receiver replaces 01001 with 01011 and consults the table to find the dataword 01.

8.4 Hamming Distance

One of the central concept in coding for error control is the idea of the *Hamming distance*. The Hamming distance between two words (of the same size) is the number of differences between the corresponding bits. We show the Hamming distance between two words x and y as d(x, y) The Hamming distance can easily be found if we apply the XOR operation (⊕) on thetwo words and count the number of 1 s in the result. Note that the Hamming distance is a value greater than zero.

Example:

Let us find the Hamming distance between two pairs of words.

1. The Hamming distance d(000, 011) is 2 because 000⊕011 is 011 (two 1 s).
2. The Hamming distance d(10101, 11110) is 3 because 10101⊕11110 is 01011 (three 1 s).

Minimum Hamming Distance

Although the concept of the Hamming distance is the central point in dealing with error detection and correction codes, the measurement that is used for designing a code is the minimum Hamming distance. In a set of words, the *minimum Hamming distance* is the smallest Hamming distance between all possible pairs.. To find minimum Hamming distance, we find the Hamming distances between all words and select the smallest one.

The error detection and correction capabilities of a coding technique depend on the minimum distance d_{min} as shown in table.

Table: *Error detection and correction capabilities*

Description	Expression
Detect upto s errors per word	$d_{min} \geq (s + 1)$
Correct upto t errors per word	$d_{min} \geq (2t + 1)$
Correct upto t errors and detect s > t errors per word	$d_{min} \geq (t + s + 1)$

Example:

For a Hamming distance of 5, how many errors can be detected ? How many errors can be corrected?

Solution:

Assuming minimum Hamming distance i.e., $d_{min} = 5$.

i. Number of errors that can be detected (s) can be obtained as

$(s + 1) \leq d_{min}$

or, $(s + 1) \leq 5$

or, $s \leq 4$

Thus, atmost 4 errors can be detected.

ii. Number of errors that can be corrected (t) can be obtained as

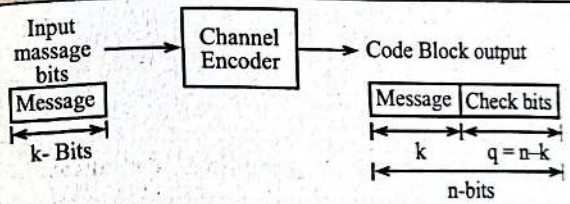
$(2t + 1) \leq d_{min}$

or, $(2t + 1) \leq 5$

or, $t \leq 2$

Thus, atmost 2 errors can be corrected.

8.5 Linear Block Coding



For the block of k message bits, q = n–k parity bits or check bits are added. This means that the total bits at the output of channel encoder are 'n'. Such types of code are known as *(n, k) block codes.*

A code is known as *linear* if the sum of any two code vectors produces another code vector. Here, sum is performed according to mod 2 additional rule. In general, linear block codes are described in a matrix form. The steps for determining code word for linear block code are as follows:

1. Let the particular code vector consists of $m_1, m_2 \ldots\ldots m_k$ message bits and check bits $c_1, c_2 \ldots\ldots c_q$. Then code vector may be ..... as

$$X = (m_1, m_2 \ldots\ldots m_k, c_1, c_2 \ldots\ldots c_q)$$

= Total size

= n

2. Here, the check bits play the role of error de..... and correction. The function of linear block code is to generate these check bits. The code vector represented in terms of generator matrix 'G' as

$[X]_{1xn} = [M]_{1xk}\,[G]_{kxn}$

Cohere, $G = [I_k / P_{kxq}]_{kxn}$

I_k = Identity matrix

$$P_{kxq} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ P_{k1} & P_{k2} & \cdots & P_{kq} \end{bmatrix}_{kxq}$$

3. Now, check vector may be obtained as c=MP

or, $[c_1, c_2, \ldots\ldots c_k] = [m_1, m_2, \ldots\ldots m_k]$

$$\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ P_{k1} & P_{k2} & \cdots & P_{kq} \end{bmatrix}$$

Modulo-2 Arithmetic

Of particular interest is modulo-2 arithmetic. In this arithmetic, the modulus can use only 0 and 1. Operations in this arithmetic are very, The following shows how we can add or subtract 2 bits.

0 + 0 = 0	0 + 1 = 1	I + 0 = I	1 + 1 = 0
0 - 1 = 1	0 – 0 = 0	1 - 0 = 1	1 - 1 = 0

Some linear block codes:

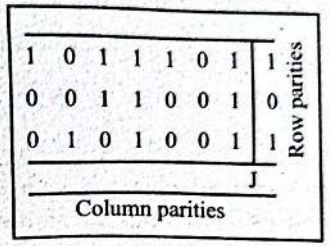
1. **Simple parity-check code**

Perhaps the most familiar error-detecting code is the *simple parity-check code*. In this code, a k-bit dataword is changed to an n-bit codeword where n = k + 1. The extra bit, called the parity bit, is selected to make the total number of 1s in the codeword even. Although some implementations specify odd number of 1s, we discuss

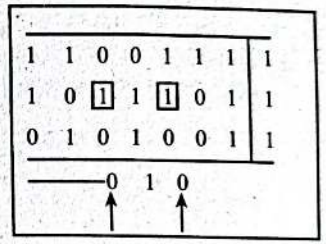
the even case. The minimum Hamming distance for this category is d_{min}=2, which means that the code is a single-bit error-detecting code; it cannot correct any error.

A simple parity-check code is a single-bit error-detecting code in which n = k + 1 with d_{min} = 2.

2. **Two-dimensional parity-check code**



Design of row and column parities



A better approach is the two-dimensional parity check. In this method, the data word is organized in a table (rows and columns). In Figure ,the data to be sent, five 7-bit bytes are put in separate rows. For each row and each column, 1 parity-check bit is calculated. The whole table is then sent to the receiver. The two-dimensional parity check can detect up to three errors that occuranywhere in the table.

3. **Hamming codes**

In telecommunication, Hamming codes are a family of linear error-correcting codes. It is one of the category under binary code which is used to detect and correct single bit error in digital data transmission. Hence,

Hamming code is single error correcting code. In this code, no. of parity bits can be obtained from the formula:

$2^p \geq p+m+1$

where m = no. of message bits, p = no. of parity bits.

If we want to transit 4-bit digital data, the no. of parity bits added is calculated as follows:

$2^p \geq p+m+1$

or, $2^p \geq p+4+1$

or, $2^p \geq p+5$

This equation is satisfied when p = 3. So, no. of parity bits added = 3. And, total no. of bits transmitted = 4 + 3 = 7.

The parity bits are added in fixed positions like 2^0 position, 2^1 position, 2^2 position, and so on. For above case, when m = 4 & n = 7, bit position and placement of parity bits are as shown:

	2^0	2^1		2^2			
Bit no.	1	2	3	4	5	6	7
	p_1	p_2	m_1	p_3	m_2	m_3	m_4

Example:

Generate Hamming code for message 1110.

Solution:

Here, m = 4. So, no. of parity bits added = 3.

	2^0	2^1		2^2			
Bit no.	1	2	3	4	5	6	7
	p_1	p_2	m_1	p_3	m_2	m_3	m_4
	p_1	p_2	1	p_3	1	1	0

Now, p_1, p_2, p_3 can be found out by either using even parity or odd parity as asked in the problem. Here, we have found p_1, p_2, p_3 by considering even parity.

For finding the reference position of p_1, p_2, p_3, we should write binary code from 0 to 7 (total no. of bits). Then, reference position is found out as shown below:

0	→	0	0	0
1	→	0	0	1
2	→	0	1	0
3	→	0	1	1
4	→	1	0	0
5	→	1	0	1
6	→	1	1	0
7	→	1	1	1
		↑	↑	↑
		Reference column 3	Reference column 2	Reference column 1

The reference position for p_1, p_2, p_3 can be found by observing position of 1, in reference column 1, 2, 3 respectively.

So, reference position for p_1 = 1, 3, 5, 7

Reference position for p_2 = 2, 3, 6, 7

Reference position for p_3 = 4, 5, 6, 7

Now, p_1, p_2, p_3 is finally calculated by making no. of 1s even in these positions i.e.,

Parity bit added

$p_1 \rightarrow 1, 3, 5, 7 \rightarrow p_1\,110 \rightarrow p_1 = 0$

$p_2 \rightarrow 2, 3, 6, 7 \rightarrow p_2\,110 \rightarrow p_2 = 0$

$p_3 \rightarrow 4, 5, 6, 7 \rightarrow p_3\,110 \rightarrow p_3 = 0$

Hence, Hamming code for message 1110 considering even parity is 0010110.

If we had considered odd parity, it would be 11111110.

Example:

Construct (7, 4) Hamming code for the message (1000). Consider even parity.

Solution:

Here, n = 7, m = 4

	2^0	2^1		2^2			
Bit no.	1	2	3	4	5	6	7
	p_1	p_2	1	p_3	0	0	0

Finding p_1, p_2, p_3:

$p_1 \rightarrow$ Check 1, 3, 5, 7 position for even parity = $p_1100 \rightarrow p_1 = 1$

$p_2 \rightarrow$ Check 2, 3, 6, 7 position = $p_2100 \rightarrow p_2 = 1$

$p_3 \rightarrow$ Check 4, 5, 6, 7 position = $p_3000 \rightarrow p_3 = 0$

$\therefore$ Encoded form of message = 1110000

Error correction in Hamming code:

The error correction method in hamming code is explained by the example given below:

Example:

A (7, 4) Hamming code is received as 1110000. Determine the corrected code when even parity is there.

Solution:

The error position is found out by finding E_3, E_2, E_1 as shown below:

Bit position	1	2	3	4	5	6	7
Received code	1	1	1	0	0	0	0

Now,

$E_3 \rightarrow$ Check 4, 5, 6, 7 position $\rightarrow 0000 \rightarrow E_3 = 0$

$E_2 \rightarrow$ Check 2, 3, 6, 7 position $\rightarrow 1100 \rightarrow E_2 = 0$

$E_1 \rightarrow$ Check 1, 3, 5, 7 position $\rightarrow 1100 \rightarrow E_1 = 0$

$\therefore$ Error is present at $E_3E_2E_1 = 000 \approx 0^{th}$ position i.e., no error in received Hamming code.

If the received code was

1	2	3	4	5	6	7
1	1	1	0	0	0	1

error

Then,

$E_3 \rightarrow 4, 5, 6, 7 \rightarrow 0001 \rightarrow E_3 = 1$ using even parity.

$E_2 \rightarrow 2, 3, 6, 7 \rightarrow 1101 \rightarrow E_2 = 1$ using even parity.

$E_1 \rightarrow 1, 3, 5, 7 \rightarrow 1101 \rightarrow E_1 = 1$ using even parity.

$\therefore$ Error position = $E_3E_2E_1 = 111$ i.e., 7th position.

So, corrected Hamming code would be 1110000.

8.6 Binary Cyclic Code

Cyclic codes may be described as the sub class of linear block codes. They have the property that a cyclic shift of one code word produces another code word. As an example let us consider an 'n' bit code vector as $X = (n_{n-1}, n_{n-2}, \ldots\ldots\ldots\ldots n_1, n_0)$

or

$$X(P) = n_{n-1}P^{n-1} + n_{n-2}P^{n-2} + \ldots\ldots\ldots\ldots\ldots n_1P + n_0$$

Where,

X_{n-1}, X_{n-2}, ... X_1, X_0 represents bits of code vector, A linear code is known as cyclic code if every cyclic shift of the code vector produces some other code vector if we shift the above code cyclically than we get another code vector.

$$X_1 = (n_{n-2}, n_{n-3}, \ldots\ldots n_1, n_0, n_{n-1})$$

or,

$$X^1(P) = n_{n-2}P^{n-1} + n_{n-3}P^{n-2} + \ldots\ldots\ldots\ldots + n_1P^2 + n_0P + n_{n-1}$$

This is also a code vector. The code vector is expressed in terms of generator polynomial as

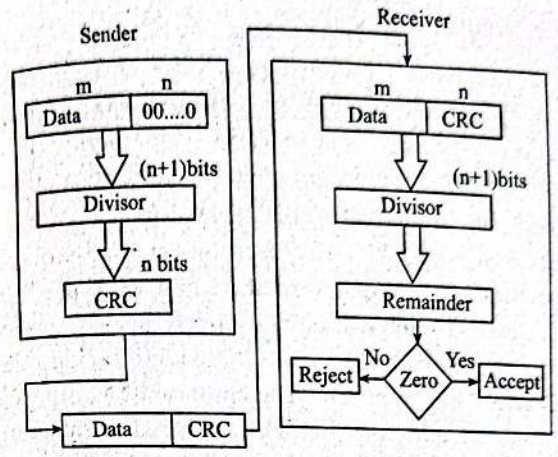
$$X(P) = M(P).\ G(P)$$

8.7 Cyclic Redundancy Check

The *cyclic redundancy check* is the most powerful and easy to implement technique.Unlike checksum scheme, which is based on addition, CRC is based on binary division. In CRC, a sequence of redundant bits, called *cyclic redundancy check bits*, are appended to the end of data unit so that the resulting data unit becomes exactly divisible by a second, predetermined binary number. At the destination, the incoming data unit is divided by the same number. If at this step there is no remainder, the data unit is assumed to be correct and is therefore accepted. A remainder indicates that the data unit has been damaged in transit and therefore must be rejected. The generalized technique can be explained as follows.

If a k bit message is to be transmitted, the transmitter generates an r-bit sequence, known as *frame check sequence* (FCS) so that the (k+r) bits are actually being transmitted. Now this r-bit FCS is generated by dividing the original number, appended by r zeros, by a predetermined number. This number, which is (r+1) bit in length, can also be considered as the coefficients of a polynomial, called *generator polynomial*. The remainder of this division process generates the r-bit FCS. On receiving the packet, the receiver divides the (k+r) bit frame by the same predetermined number and if it produces no remainder, it can be assumed that no error has occurred during the transmission.

Operations at both the sender and receiver end are shown in figure below:



This mathematical operation performed is illustrated in figure below by dividing a sample 4-bit number by the coefficient of the generator polynomial x^3+x+1, which is 1011, using the modulo-2 arithmetic. Modulo-2 arithmetic is a binary addition process without any carry over, which is just the exclusive-OR operation.

Consider the case where k=1101. Hence we have to divide 1101000 (i.e., k appended by 3 zeros) by 1011, which produces the remainder r=001, so that the bit frame (k+r) =1101001 is actually being transmitted through the communication channel. At the receiving end, if the received number, i.e., 1101001 is divided by the same generator polynomial 1011 to get the remainder as 000, it can be assumed that the data is free of errors.

```
        1111
1011 ) 1101000
       1011
        1100
        1011
         1110
         1011
          1010
          1011
           001 <------ r
```

8.8 Convolutional Codes

The difference between block codes and convolutional codes is the encoding principle. In the block codes, the information bits are followed by the parity bits. In convolutional codes, the information bits are spread along the sequence. That means that the convolutional codes map information to code bits not block wise, but sequentially convolve the sequence of information bits according to some rule. The code is defined by the circuit, which consists of different number of shift registers allowing building different codes in terms of complexity. The constraint length of a shift register encoder is defined as the number of symbols it can store in its memory.

Convolutional codes are commonly described using two parameters: the code rate and the constraint length. The code rate, k/n, is expressed as a ratio of the number of bits into the convolutional encoder (k) to the number of channel symbols output by the convolutional encoder (n) in a given encoder cycle. For the encoder with k=1, n=2 and m=2, the code rate is 1/2.

Convolutional codes are widely used as channel codes in practical communication systems for error correction. The encoded bits depend on the current k input bits and a few past input bits.

Numerical Problems

1. **Consider the binary source S = {0, 1} with probabilities P(0) = 0.5, P(1) = 0.5, then calculate self information.**

Solution:

$$I(0) = \log_2\frac{1}{P(0)}$$

$$= \log_2\frac{1}{0.5} = 1$$

$$I(1) = \log_2\frac{1}{P(1)}$$

$$= \log_2\frac{1}{0.5} = 1$$

2. **The probability of 5 possible outcomes of an experiment are $P(x_1) = \frac{1}{2}$, $P(x_2) = \frac{1}{4}$, $P(x_3) = \frac{1}{8}$, $P(x_4) = P(x_5) = \frac{1}{16}$. Determine entropy and information rate, if there are 16 outcomes per second.**

Solution:

$$H(S) = \sum_{i=1}^{5} P(S_i)\log_2\frac{1}{P(S_i)}$$

$$= P(x_1)\log_2\frac{1}{P(x_1)} + P(x_2)\log_2\frac{1}{P(x_2)} + \ldots + P(x_5)\log_2\frac{1}{P(x_5)}$$

$$= \left(0.5\log_2\frac{1}{0.5}\right) + \left(0.25\log_2\frac{1}{0.25}\right) + \left(0.125\log_2\frac{1}{0.125}\right) + \left(0.0625\log_2\frac{1}{0.0625}\right) + \left(0.0625\log_2\frac{1}{0.0625}\right)$$

$$= 0.5 + 0.5 + 0.375 + 0.5 = 1.875 \text{ bits}$$

Now, information rate is

$$R = r\,H(S)$$

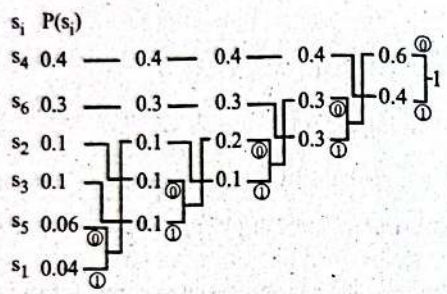
$$= (16\times1.875) = 30 \text{ bits/sec}$$

3. **Design a binary Huffman code with a six symbol source with probability assignment as $P(s_1) = 0.04$, $P(s_2) = 0.1$, $P(s_3) = 0.1$, $P(s_4) = 0.4$, $P(s_5) = 0.06$, $P(s_6) = 0.3$.**

[2073 Shrawan]

Solution:

The binary Huffman tree can be constructed as shown. (First arranging the symbols on the basis of decreasing order of probability).



From the above Huffman tree, the encoded signals are as shown below:

$s_4 = 1$

$s_6 = 01$

$s_2 = 0000$

$s_3 = 0001$

$s_5 = 0010$

$s_1 = 0011$

4. **Generate a codeword of a c(6, 3) block code with any suitable generator matrix.** [2073 Chaitra]

Solution:

Let the generator matrix be

$$G = \begin{bmatrix} 1\,0\,0 : 0\,1\,1 \\ 0\,1\,0 : 1\,0\,1 \\ 0\,0\,1 : 1\,1\,0 \end{bmatrix}$$

The general pattern of a block code is (n, k). Hence, in this case n = 6 and k = 3 i.e., message block size = 3, length of code vector = 6.

We know that, for finding codeword, we need to separate parity matrix from generator matrix.

Also, we know that generator matrix is given by

$G = [I_k/P]$

$$\therefore\ I_k = I_{3\times3} = \begin{bmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{bmatrix}$$

$$\text{and } P = P_{3\times3} = \begin{bmatrix} 0\,1\,1 \\ 1\,0\,1 \\ 1\,1\,0 \end{bmatrix}$$

As size of message block is k = 3, there are eight possible message blocks: (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1).

Since we need to find a codeword, we can take any one message and generate its codeword.

Let's take message word (0, 0, 0) and find its codeword.

Here, $(m_0, m_1, m_2) = (0, 0, 0)$ i.e., $m_0 = 0$, $m_1 = 0$, $m_2 = 0$

The relation between code vector c, message vector M and coefficient matrix P is given by

$C = M.P$

$$\text{i.e., } [c_0, c_1, c_2] = [m_0, m_1, m_2] \begin{bmatrix} 0\,1\,1 \\ 1\,0\,1 \\ 1\,1\,0 \end{bmatrix}$$

$c_0 = (m_0 \times 0) \oplus (m_1 \times 1) \oplus (m_2 \times 1) = m_1 \oplus m_2$

$c_1 = (m_0 \times 1) \oplus (m_1 \times 0) \oplus (m_2 \times 1) = m_0 \oplus m_2$

$c_2 = (m_0 \times 1) \oplus (m_1 \times 1) \oplus (m_2 \times 0) = m_0 \oplus m_1$

Substituting $m_0 = 0$, $m_1 = 0$, and $m_2 = 0$ in above equations, we get codeword for a message word (0, 0, 0).

The codeword for (0, 0, 0) is (0 0 0 0 0 0).

5. **A message source generates 8 symbols with the following probabilities: $P(x_1) = \frac{1}{2}$, $P(x_2) = \frac{1}{4}$, $P(x_3) = \frac{1}{8}$, $P(x_4) = \frac{1}{16}$, $P(x_5) = \frac{1}{32}$, $P(x_6) = \frac{1}{64}$, $P(x_7) = \frac{1}{128}$, $P(x_8) = \frac{1}{128}$. Encode the message with variable length binary codes using Shannon-Fano procedure. Find the transmission efficiency.** [2073 Chaitra]

Solution:

$P(x_1) = \frac{1}{2} = 0.5,$

$P(x_2) = \frac{1}{4} = 0.25,$

$P(x_3) = \frac{1}{8} = 0.125,$

$P(x_4) = \frac{1}{16} = 0.0625,$

$P(x_5) = \frac{1}{32} = 0.03125,$

$P(x_6) = \frac{1}{64} = 0.015625,$

$P(x_7) = \frac{1}{128} = 0.0078125,$

$P(x_8) = \frac{1}{128} = 0.0078125$

The Shannon-Fano code can be constructed by using following table:

x_i	$P(x_i)$	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Code	(l_i) Code length
x_1	$\frac{1}{2}$	0							$x_1 \to 0$	1
x_2	$\frac{1}{4}$	1	0						$x_2 \to 10$	2
x_3	$\frac{1}{8}$	1	1	0					$x_3 \to 110$	3
x_4	$\frac{1}{16}$	1	1	1	0				$x_4 \to 1110$	4
x_5	$\frac{1}{32}$	1	1	1	1	0			$x_5 \to 11110$	5
x_6	$\frac{1}{64}$	1	1	1	1	1	0		$x_6 \to 111110$	6
x_7	$\frac{1}{128}$	1	1	1	1	1	1	0	$x_7 \to 1111110$	7
x_8	$\frac{1}{128}$	1	1	1	1	1	1	1	$x_8 \to 1111111$	7

The average length of Shannon-Fano code is:

$$L_{avg} = \Sigma P_i l_i = \frac{1}{2}\times1 + \frac{1}{4}\times2 + \frac{1}{8}\times3 + \frac{1}{16}\times4 + \frac{1}{32}\times5 + \frac{1}{64}\times6 + \frac{1}{128}\times7 + \frac{1}{128}\times7$$

$$= 1.98$$

$$\text{Entropy (H)} = \Sigma P(x_i)\log_2\frac{1}{P(x_i)}$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + \frac{1}{32}\log_2 32 + \frac{1}{64}\log_2 64 + \frac{1}{128}\log_2 128 + \frac{1}{128}\log_2 128$$

$$= \frac{1}{2} + \frac{1}{4}\times2 + \frac{1}{8}\times3 + \frac{1}{16}\times4 + \frac{1}{32}\times5 + \frac{1}{64}\times6 + \frac{1}{128}\times7 + \frac{1}{128}\times7$$

$$= 1.98$$

$$\text{Efficiency } (\eta) = \frac{H}{L_{avg}} = \frac{1.98}{1.98} = 1$$

6. **Generate a CRC-3 transmission code and analyze its error detection performance with example.** [2074 Ashwin]

Solution:

Since it is CRC-3 transmission, let the divisor polynomial be $x^3 + 1$ i.e,. 1001 in bitwise.

Let the data word be 10011101.

Since divisor has 4 bits, we append one less number of zeros in the dataword to make the dividend. So, the dividend will be: 10011101000

Now, let us carry out modulo-2 arithmetic division as shown:

```
        1000110
1001 ) 10011101000
       1001
       ØØ001
        0000
       ØØ011
        0000
       ØØ110
        0000
       Ø1101
        1001
       Ø1000
        1001
       ØØ010
        0000
       ØØ100
        0000
       Ø100
```

So, the transmitted bit sequence will be dataword followed by remainder bits i.e., 10011101100.

Let us assume that one of the bits is corrupted during transmission i.e., 10011101001 is received.

Now, in the receiver side, the received codeword is again divided by the divisor. If the remainder comes zero; it is guaranteed that there is error free transmission.

Similarly, if there is some reminder, we can guarantee that there is some error as in case of our example. Let us explain this by dividing the received codeword by the divisor.

i.e.,


```
        10001100
1001 ) 10011101101
       1001
       ØØ001
        0000
       ØØ011
        0000
       ØØ110
        0000
       Ø1101
        1001
       Ø1001
        1001
       ØØ000
        0000
       ØØ001
        0000
       Ø001
```

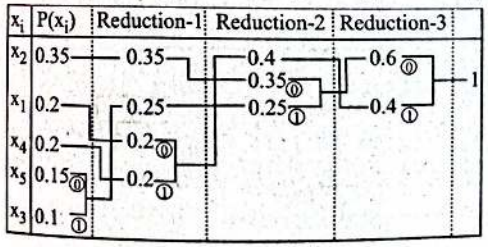
Since the remainder is not zero, we can assure that there is error during transmission.

7. **Consider a five symbol source with probability assignment as $P(x_1) = 0.2$, $P(x_2) = 0.35$, $P(x_4) = 0.2$, $P(x_5) = 0.15$. By using Huffman algorithm, find the source code for these symbols and determine efficiency of the code.**

[2074 Ashwin]

Solution:

First we list the source symbols in order of decreasing probability. Then, we combine the probabilities of the two symbols having the lowest probabilities and recorder the resultant probabilities. This procedure is repeated until there are two probabilities remaining. Then, Huffman code is generated by using following table.



From the table, the codes assigned are as shown.

$x_2 = 00$ i.e., code length of $x_2 = l_2 = 2$

$x_1 = 10$ i.e., code length of $x_1 = l_1 = 2$

$x_4 = 11$ i.e., code length of $x_4 = l_4 = 2$

$x_5 = 010$ i.e., code length of $x_5 = l_5 = 3$

$x_3 = 011$ i.e., code length of $x_3 = l_3 = 3$

Average code length,

$$L_{avg} = \Sigma P_i l_i$$

$$= (0.35\times2) + (0.2 \times 2) + (0.2 \times 2) + (0.15 \times 3) + (0.1 \times 3)$$

$$= 2.25$$

$$\text{Entropy (H)} = \Sigma P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$= 0.35 \times \log_2\left(\frac{1}{0.35}\right) + 0.2 \times \log_2\left(\frac{1}{0.2}\right) + \left(0.2 \times \log_2 \frac{1}{0.2}\right) + 0.15 \times \log_2\left(\frac{1}{0.15}\right) + 0.1 \times \log_2\left(\frac{1}{0.1}\right)$$

$$= 2.2013$$

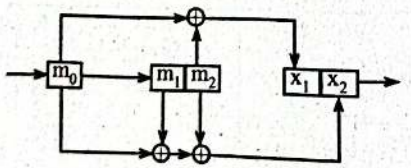
$$\text{Efficiency } (\eta) = \frac{H}{L_{avg}} = 0.978$$

8. Applying a $\frac{1}{2}$ rate, 4 state convolutional code correct errors of two bits with the help of its trellis diagram.

[2074 Chaitra]

Solution:

Consider the 4-state, binary, convolutional encoder shown below.



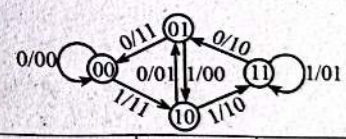
There are 2 output binary digits for each input binary digit. This gives a rate of $\frac{1}{2}$.

At any moment, the flip-flops could be in any one state: 00, 01, 10 or 11.

The state table can be drawn as shown below:

Input bit	Present state	New state	x_1	x_2	Output (x_1x_2)
0	0	0	0	0	00
1	0	0	1	1	11
0	0	1	1	1	11
1	0	1	0	0	00
0	1	0	0	1	01
1	1	0	1	0	10
0	1	1	1	0	10
1	1	1	0	1	01

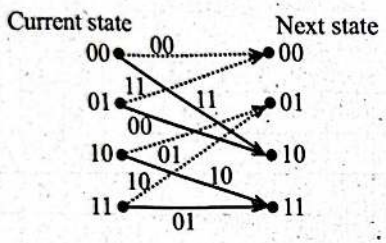
This can also be represented by using state diagram as shown:



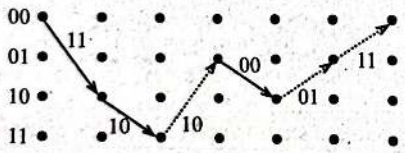
Present state	Input bit	New state
00	0	00
00	1	10
01	0	00
01	1	10
10	0	01
10	1	11
11	0	01
11	1	11

The information in state diagram can be conveyed in terms of graph called trellis diagram. Since the rate of encoder is $\frac{1}{2}$, only one bit is processes by the encoder. The incoming bit can be either 0 or 1. So, there are two branches coming out from each node.

The trellis encoding is done by trellis tree as shown:



Let us encode input bit stream 110100 by the help of above trellis diagram.

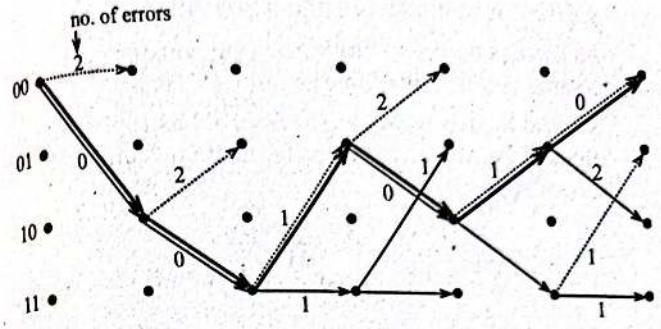


Hence, the data 110100 is encoded as: (111010000111).

Let us suppose that there are 2 bits during transmission and bits are received as

11 10 11 00 11 11

Now by using the viterbi decoding, these errors can be detected and corrected as shown:



Hence, the decoder output as decoded by viterbi algorithm and trellis diagram is 11 10 10 00 01 11 which is the transmitted encoded bitstream.

9. **Explain 3-bit CRC generator and decoder with example of no error case.** [2074 Chaitra]

Solution:

For 3-bit CRC, we add 3 bits to the message bits. So, the divisor should have 4 bits. Let the divisor polynomial be $x^3 + 1$ i.e., 1001 bitwise and the data word be 10011101. Since divisor has 4 bits, we append 3 zeros in the dataword to make the dividend and perform modulo-2 arithmetic at the transmitter side.

```
        1000110
1001 ) 10011101000
       1001
       Ø0001
        0000
       Ø0011
        0000
       Ø0110
        0000
       Ø1101
        1001
       Ø1000
        1001
       Ø0010
        0000
       Ø0100
        0000
        Ø100 ← Remainder
```

So, the transmitted bits is 10011101100

As there is no error during transmission, the received bit is same as transmitted one i.e., 10011101100

Now, at the receiver side, the received bit stream is again divided by the divisor using modulo-2 arithmetic as shown.

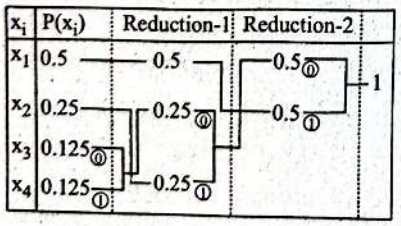
```
          10001100
1001 ) 10011101100
       1001
       ---------
       Ø0001
        0000
       ---------
       Ø0011
        0000
       ---------
       Ø0110
        0000
       ---------
       Ø1101
        1001
       ---------
       Ø1001
        1001
       ---------
       Ø0000
        0000
       ---------
       Ø0000
        0000
       ---------
       Ø000
```

Since the reminder is zero, we can confirm that it is error free transmission.

10. Find an efficient code word and calculate efficiency that can be assigned to the symbols using Huffman Algorithm using probabilities: $P(x_1) = 0.5$, $P(x_2) = 0.25$, $P(x_3) = 0.125$, $P(x_4) = 0.125$. [2074 Chaitra]

Solution:

The most efficient coding is Huffman coding. So, the given symbols can be coded as shown below:



Hence, the codewords are:

$x_1 \rightarrow 1$ i.e. $l_1 = 1$

$x_2 \rightarrow 01$ i.e. $l_2 = 2$

$x_3 \rightarrow 000$ i.e. $l_3 = 3$

$x_4 \rightarrow 001$ i.e. $l_4 = 3$

Average code length is

$$L_{avg} = \Sigma P_i l_i = \Sigma P(x_i)\, l(x_i)$$

$$= (0.5 \times 1) + (0.25 \times 2) + ((0.125 \times 3) + (0.125 \times 3)$$

$$= 0.5 + 0.5 + 0.375 + 0.375 = 1.75$$

Entropy (H)

$$= \Sigma P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$= 0.5 \times \log_2\left(\frac{1}{0.5}\right) + 0.25 \times \log_2\left(\frac{1}{0.25}\right) + 0.125 \times \log_2\left(\frac{1}{0.125}\right) + 0.125 \times \log_2\left(\frac{1}{0.125}\right)$$

$$= 0.5 \times \log_2 2 + 0.25 \times \log_2 4 + 0.125 \times \log_2 8 + 0.125 \times \log_2 8$$

$$= 0.5 \times 1 + 0.25 \times 2 + 0.125 \times 3 + 0.125 \times 3 = 1.75$$

$$\therefore \text{ Efficiency } (\eta) = \frac{H}{L_{avg}} = \frac{1.75}{1.75} = 100\%$$

11. Generate a convolution code for a input bit streams of (111011) using a C(3, 1, 3) architecture. [2075 Chaitra]

Solution:

In the given convolution encoder, we have

(n, k, m) = (3, 1, 3)

n = no. of output bits at a particular interval of time = 3

k = no. of input bits at a particular interval of time = 1

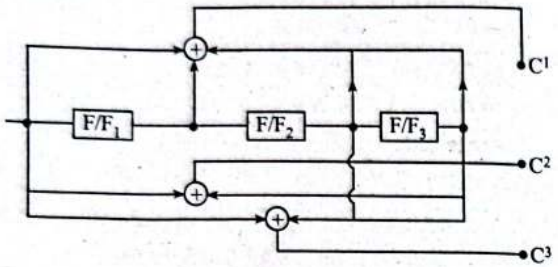
m = no. of flip-flop stages = 3.

Here, length of message (L) = 6.

Total length of encoded message is given by:

$$n(L + m) = 3(6 + 3) = 27$$

Let the convolution encoder of (3, 1, 3) architecture be as shown.



From above diagram, we can write

$g^{(1)} = [1111]$

$g^{(2)} = [1001]$

$g^{(3)} = [1011]$

Hence,

$[G]_{L\times[n(L+m)]} =$

$$\begin{bmatrix} 111 & 100 & 101 & 111 & 000 & 000 & \cdots & \cdots & 000 \\ 000 & 111 & 100 & 101 & 111 & 000 & \cdots & \cdots & 000 \\ 000 & 000 & 111 & 100 & 101 & 111 & 000 & \cdots & 000 \\ 000 & 000 & 000 & 111 & 100 & 101 & 111 & \cdots & 000 \\ 000 & 000 & 000 & 000 & 111 & 100 & 101 & 111 & 000 \\ 000 & 000 & 000 & 000 & 000 & 111 & 100 & 101 & 111 \end{bmatrix}_{6\times 27}$$

∴ The encoded convolution encoder output is [C]=[d] [G]

$= [111011]_{1\times 6}$ *

$$\begin{bmatrix} 111 & 100 & 101 & 111 & 000 & 000 & 000 & 000 & 000 \\ 000 & 111 & 100 & 101 & 111 & 000 & 000 & 000 & 000 \\ 000 & 000 & 111 & 100 & 101 & 111 & 000 & 000 & 000 \\ 000 & 000 & 000 & 111 & 100 & 101 & 111 & 000 & 000 \\ 000 & 000 & 000 & 000 & 111 & 100 & 101 & 111 & 000 \\ 000 & 000 & 000 & 000 & 000 & 111 & 100 & 101 & 111 \end{bmatrix}_{6\times 27}$$

= [111 011 110 110 101 100 001 010 111]

12. **A generator polynomial of X cyclic code is G(P) of $P^3 + P + 1$. Obtain all the code vectors for the code in non-systematic and systematic form with message vector 0101.**

Solution:

Here, n=7, k=4

$q = n-k = 3$

$M = (m_3\, m_2\, m_1\, m_0) = 0101$

$= m_3P^3 + m_2P^2 + m_1P + m_0$

$\therefore M(P) = P^2 + 1$

$G(P) = P^3 + P + 1$

i. For non-systematic code,

$X(P) = M(P).\,(G(P)$

$= (P^2 + 1)\,(P^3 + P + 1)$

$= P^5 + P^3 + P^2 + P^3 + P + 1$

$= P^5 + (1 \oplus 1)P^3 + P^2 + P + 1$

$\therefore X(P) = P^5 + P^2 + P + 1$

$= 0.P^6 + 1.P^5 + 0.P^4 + 0.P^3 + 1.P^2 + 1.P + 1$

Non-systematic code is X = (0 1 0 0 1 1 1) i.e., message not following by check bits.

ii. For systematic code vector, we first find check bits individually using the equation:

$$C(P) = \text{reminder}\left[\frac{P^qM(P)}{G(P)}\right]$$

Here, q = 3.

So, $P^qM(P) = P^3(P^2 + 1) = P^5 + P^3 = P^5 + 0.P^4 + P^3 + 0P^2 + 0P + 0$

and $G(P) = P^3 + 0.P^2 + P + 1$

$$\begin{array}{r|l} & P^2 \\ \hline P^3 + 0.P^2 + P + 1 & P^5 + 0.P^4 + P^3 + 0.P^2 + 0.P + 0 \\ & P^5 + 0.P^4 + P^3 + P^2 \\ \hline & 0 + 0 + 0 + P^2 + 0.P + 0 \end{array}$$

$\therefore C(P) = P^2 + 0.P + 0$

or, C = (1 0 0)

Hence, the systematic code vector is X = (0101100)

13. The generator matrix for a (6, 3) block code is shown below. Obtain all codewords.

$$G = \begin{bmatrix} 1\,0\,0 : 1\,1\,1 \\ 0\,1\,0 : 1\,1\,0 \\ 0\,0\,1 : 1\,0\,1 \end{bmatrix}$$ [2072 Chaitra]

Solution:

The given generator matrix is

$$G = \begin{bmatrix} 1\,0\,0 : 1\,1\,1 \\ 0\,1\,0 : 1\,1\,0 \\ 0\,0\,1 : 1\,0\,1 \end{bmatrix}$$

Separating the coefficient matrix,

$$P = \begin{bmatrix} 1\,1\,1 \\ 1\,1\,0 \\ 1\,0\,1 \end{bmatrix}$$

Relation between parity vector c message vector M and coefficient matrix P

$c = MP$

$$[c_0\, c_1\, c_2] = [m_0\, m_1\, m_2] \begin{bmatrix} 1\,1\,1 \\ 1\,1\,0 \\ 1\,0\,1 \end{bmatrix}$$

The solution of this equation is

$c_0 = m_0 \oplus m_1 \oplus m_2$

$c_1 = m_0 \oplus m_1$

$c_2 = m_0 \oplus m_2$

By substituting the values of m_0, m_1 and m_2 into these equations, we can obtain parity c_0, c_1, c_2

For the message word $(m_0\, m_1\, m_2) = (0\,0\,0)$, we have

$c_0 = m_0 \oplus m_1 \oplus m_2 = 0 \oplus 0 \oplus 0 = 0$

$c_1 = m_0 \oplus m_1 = 0 \oplus 0 = 0$

$c_2 = m_0 \oplus m_2 = 0 \oplus 0 = 0$

Codeword = $\begin{matrix} m_0 & m_1 & m_2 & c_0 & c_1 & c_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$

Similarly, codeword for other possible inputs (i.e., message word) can be tabulated as

m_0	m_1	m_2	c_0	c_1	c_2
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	1	1	0
0	1	1	0	1	1
1	0	0	1	1	1
1	0	1	0	1	0
1	1	0	0	0	1
1	1	1	1	0	0

14. Suppose there are 9 balls in a bin. Out of the 9 balls, 4 balls are red, 2 balls are yellow, and 3 balls are green. Calculate the entropy of the system each time a ball is picked from the bin. [2070 Ashadh]

Solution:

The repetitive probabilities can be calculated as

$$P(R) = \frac{4}{9},\ P(Y) = \frac{2}{9},\ P(G) = \frac{3}{9}$$

The entropy of the system can be calculated as

$$H(x) = \sum_{i=1}^{3} P(x_i)\, I(x_i) = -\sum_{i=1}^{3} P(x_i) \log_2 [P(x_i)]$$

$$= -[P(R) \log_2[P(R)] + P(Y) \log_2[P(Y)] + P(G) \log_2[P(G)]]$$

$$= -\left[\frac{4}{9}\log_2\left(\frac{4}{9}\right) + \frac{2}{9}\log_2\left(\frac{2}{9}\right) + \frac{3}{9}\log_2\left(\frac{3}{9}\right)\right]$$

$$= 1.5304$$