Ellipse

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***Abstract* - This document contains solution of sketching loci of the given equation.**

Problem
Vector-2, Example-4, Question No.-7

Question 7. Sketch the loci of the following equation

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \tag{0.0.1}$$

Solution :
Given equation is,

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \tag{0.0.2}$$

We can write equation (0.0.2) as,

$$9x^2 + 4y^2 - 36 = 0 \tag{0.0.3}$$

The general equation is given as,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.0.4}$$

Comparing (0.0.3) and (0.0.4) we get,

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -36 \tag{0.0.5}$$

The vertex of ellipse is given as **c** and can be obtained from,

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} \tag{0.0.6}$$

We know,

$$\mathbf{V}^{-1} = \frac{1}{\left|\mathbf{V}\right|} Adj\mathbf{V} \tag{0.0.7}$$

Substituting the values of $\left|\mathbf{V}\right|$ and Adj **V** we get,

$$\mathbf{V}^{-1} = \frac{1}{36}\begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix}^\top = \begin{pmatrix} \frac{4}{36} & 0 \\ 0 & \frac{9}{36} \end{pmatrix} \tag{0.0.8}$$

Substituting values in equation (0.0.6) we get the vertex of the ellipse,

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.9}$$

The length of semi major axis and semi minor axis are given by,

$$\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} = 3, \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} = 2 \tag{0.0.10}$$

Solving equation (0.0.10) we get,

$$\lambda_1 = 4, \lambda_2 = 9 \tag{0.0.11}$$

The eccentricity of ellipse is given by,

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \tag{0.0.12}$$

Substituting the values in equation (0.0.12) we get,

$$e = \frac{\sqrt{5}}{3} \tag{0.0.13}$$

The directrices of ellipse is given by,

$$c = \frac{e\mathbf{u}^\top \mathbf{n} \pm \sqrt{e^2(\mathbf{u}^\top n)^2 - \lambda_2(e^2-1)(\|\mathbf{u}\|^2 - \lambda_2 f)}}{\lambda_2 e(e^2-1)} \tag{0.0.14}$$

Where

$$\mathbf{n} = \sqrt{\lambda_2}\mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.0.15}$$

As

$$\mathbf{p_1} = \frac{1}{\sqrt{9}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.0.16}$$

Substituting the values in equation (0.0.14) we get directrices of the ellipse,

$$c = \pm\frac{9}{\sqrt{5}} \tag{0.0.17}$$

The foci of ellipse is given by,

$$\mathbf{F} = \frac{ce^2\mathbf{n} - \mathbf{u}}{\lambda_2} \tag{0.0.18}$$

Substituting the respective values in equation (0.0.18) we get,

$$\mathbf{F} = \begin{pmatrix} 0 \\ \frac{\sqrt{5}}{3} \end{pmatrix} \tag{0.0.19}$$

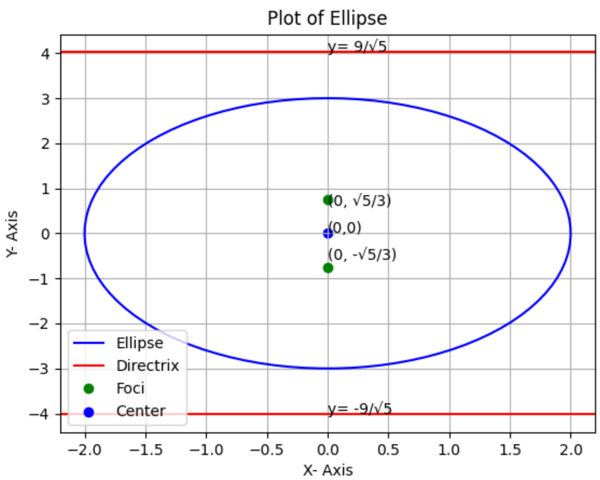

Figure 0: Plot of the Ellipse with vertex c=$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$