Ellipse

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***Abstract* - This document contains solution of sketching loci of the given equation.**

Problem

Vector-2, Example-4, Question No.-7

Question 7. Sketch the loci of the following equation

$$\frac{x^2}{4}+\frac{y^2}{9}=1 \tag{0.0.1}$$

Solution :

Given equation is,

$$\frac{x^2}{4}+\frac{y^2}{9}=1 \tag{0.0.2}$$

We can write equation (0.0.2) as,

$$9x^2+4y^2-36=0 \tag{0.0.3}$$

The general equation is given as,

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{0.0.4}$$

Comparing (0.0.3) and (0.0.4) we get,

$$\mathbf{V}=\begin{pmatrix}9 & 0\\ 0 & 4\end{pmatrix}, \mathbf{u}=\begin{pmatrix}0\\0\end{pmatrix}, f=-36 \tag{0.0.5}$$

The vertex of ellipse is given as $\mathbf{c}$ and can be obtained from,

$$\mathbf{c}=-\mathbf{V}^{-1}\mathbf{u} \tag{0.0.6}$$

We know,

$$\mathbf{V}^{-1}=\frac{1}{\left|\mathbf{V}\right|}Adj\mathbf{V} \tag{0.0.7}$$

Putting the values of $\left|\mathbf{V}\right|$ and Adj $\mathbf{V}$ we get,

$$\mathbf{V}^{-1}=\frac{1}{36}\begin{pmatrix}4 & 0\\ 0 & 9\end{pmatrix}^\top=\begin{pmatrix}\frac{4}{36} & 0\\ 0 & \frac{9}{36}\end{pmatrix} \tag{0.0.8}$$

Putting values in equation (0.0.6) we get the vertex of the ellipse,

$$\mathbf{c}=\begin{pmatrix}0\\0\end{pmatrix} \tag{0.0.9}$$

The length of semi major axis and semi minor axis are given by,

$$\sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1}}=3,\ \sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2}}=2 \tag{0.0.10}$$

Solving equation (0.0.10) we get,

$$\lambda_1=4, \lambda_2=9 \tag{0.0.11}$$

The eccentricity of ellipse is given by,

$$e=\sqrt{1-\frac{\lambda_1}{\lambda_2}} \tag{0.0.12}$$

Putting the values in equation (0.0.12) we get,

$$e=\frac{\sqrt{5}}{3} \tag{0.0.13}$$

The directrices of ellipse is given by,

$$c=\frac{e\mathbf{u}^\top\mathbf{n}\pm\sqrt{e^2(\mathbf{u}^\top n)^2-\lambda_2(e^2-1)(\|\mathbf{u}\|^2-\lambda_2 f)}}{\lambda_2 e(e^2-1)} \tag{0.0.14}$$

Where

$$\mathbf{n}=\sqrt{\lambda_2}\mathbf{p}_1=\begin{pmatrix}0\\1\end{pmatrix} \tag{0.0.15}$$

As

$$\mathbf{p_1}=\frac{1}{\sqrt{9}}\begin{pmatrix}0\\1\end{pmatrix} \tag{0.0.16}$$

Putting the values in equation (0.0.14) we get directrices of the ellipse,

$$c=\pm\frac{9}{\sqrt{5}} \tag{0.0.17}$$

The foci of ellipse is given by,

$$\mathbf{F} = \frac{ce^2\mathbf{n} - \mathbf{u}}{\lambda_2} \tag{0.0.18}$$

Putting the respective values in equation (0.0.18) we get,

$$\mathbf{F} = \begin{pmatrix} 0 \\ \frac{\sqrt{5}}{3} \end{pmatrix} \tag{0.0.19}$$

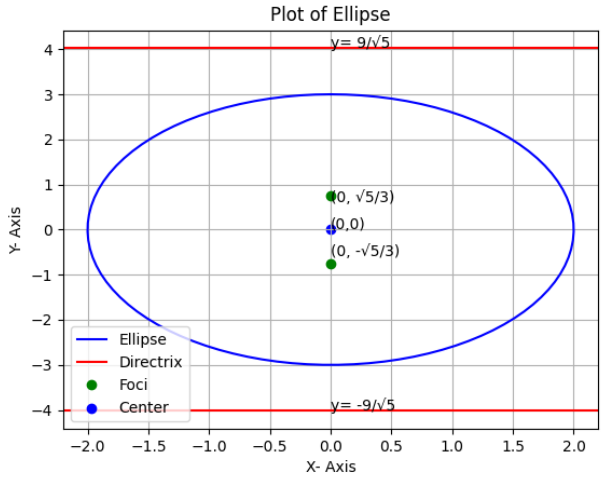

Figure 0: Plot of the Ellipse with vertex c=$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$