

Ellipse

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Abstract - This document contains solution of sketching loci of the given equation.

Problem

Vector-2, Example-4, Question No.-7

Question 7. Sketch the loci of the following equation

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \quad (1)$$

Solution :

Given equation is,

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \quad (2)$$

We can write equation (2) as,

$$9x^2 + 4y^2 - 36 = 0 \quad (3)$$

The general equation is given as,

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (4)$$

Comparing (3) and (4) we get,

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -36 \quad (5)$$

The vertex of Ellipse is given as \mathbf{c} and can be obtained from,

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} \quad (6)$$

We know,

$$\mathbf{V}^{-1} = \frac{1}{|\mathbf{V}|} \text{Adj } \mathbf{V} \quad (7)$$

Putting the values of $|\mathbf{V}|$ and $\text{Adj } \mathbf{V}$ we get,

$$\mathbf{V}^{-1} = \frac{1}{36} \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}^\top = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (8)$$

Putting values in equation (6) we get the vertex of the ellipse,

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (9)$$

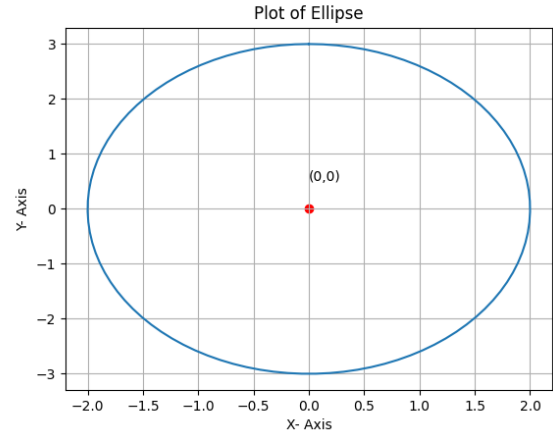


Figure 1: Plot of the Ellipse with vertex $\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$