Points and Vectors

Diptasri Ghosh

EE21MTECH14004

***Abstract* - This document contains solution of finding the coordinates of a point which divides a line segment internally and Externally.**

Vector

Question 21 : Find the coordinates of the points which divide, internally and externally, the line joining the point (a+b, a-b) to the point (a-b, a+b) in the ratio a : b.

Solution :
Let us consider C be the point which divides the AB line segment in the ratio a : b internally and externally. Given that the coordinates of A point= (a+b, a-b) and coordinates of B point= (a-b, a+b).
Let

$$\mathbf{A} = \left[\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}\right], \mathbf{B} = \left[\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}\right] \tag{1}$$

Let us consider $\mathbf{A}$ and $\mathbf{B}$ as,

$$\mathbf{A} = \mathbf{P}u, \mathbf{B} = \mathbf{Q}u \tag{2}$$

1. **For Internal Section Coordinates:**

$$\mathbf{C} = \frac{m\mathbf{B} + n\mathbf{A}}{m+n} \tag{3}$$

For the given problem, $\mathbf{C}$ divides AB in the ratio a:b.
So, here

$$\frac{m}{n} = \frac{a}{b} \tag{4}$$

Putting the values of $\mathbf{A}$ and $\mathbf{B}$ in equation (3), we get,

$$\mathbf{C} = \frac{a\mathbf{Q}u + b\mathbf{P}u}{a+b} \tag{5}$$

Solving this we get,

$$\mathbf{C} = \left(\frac{a\mathbf{Q}}{a+b} + \frac{b\mathbf{P}}{a+b}\right)u \tag{6}$$

Putting the values of $\mathbf{P}$, $\mathbf{Q}$ and u in equation (5) we get,

$$\mathbf{C} = \frac{1}{a+b}\left[\begin{pmatrix} a & -a \\ a & a \end{pmatrix} + \begin{pmatrix} b & b \\ b & -b \end{pmatrix}\right]\begin{pmatrix} a \\ b \end{pmatrix} \tag{7}$$

So, internal section coordinates of $\mathbf{C}$,

$$\mathbf{C} = \frac{1}{a+b}\begin{pmatrix} a+b & b-a \\ a+b & a-b \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \tag{8}$$

2. **For External Section Coordinates:**

$$\mathbf{C} = \frac{m\mathbf{B} - n\mathbf{A}}{m-n} \tag{9}$$

Putting the values of $\mathbf{A}$ and $\mathbf{B}$ in equation (8), we get,

$$\mathbf{C} = \frac{a\mathbf{Q}u - b\mathbf{P}u}{a-b} \tag{10}$$

Solving this we get,

$$\mathbf{C} = \left(\frac{a\mathbf{Q}}{a-b} - \frac{b\mathbf{P}}{a-b}\right)u \tag{11}$$

Putting the values of $\mathbf{P}$, $\mathbf{Q}$ and u in equation (10) we get,

$$\mathbf{C} = \frac{1}{a-b}\left[\begin{pmatrix} a & -a \\ a & a \end{pmatrix} - \begin{pmatrix} b & b \\ b & -b \end{pmatrix}\right]\begin{pmatrix} a \\ b \end{pmatrix} \tag{12}$$

So, external section coordinates of $\mathbf{C}$,

$$\mathbf{C} = \frac{1}{a-b}\begin{pmatrix} a-b & -a-b \\ a-b & a+b \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \tag{13}$$

Result

Plot of coordinates of the points obtained from Python code considering a= 6, b=3 is shown below.

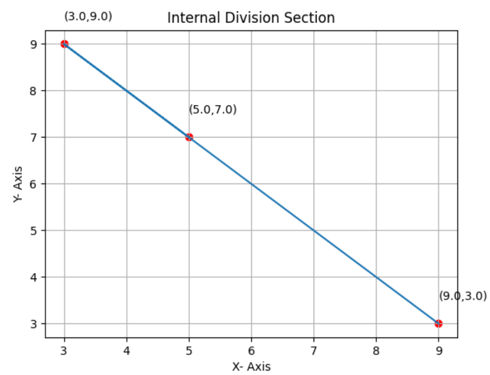

Figure 1: Internal Division Section

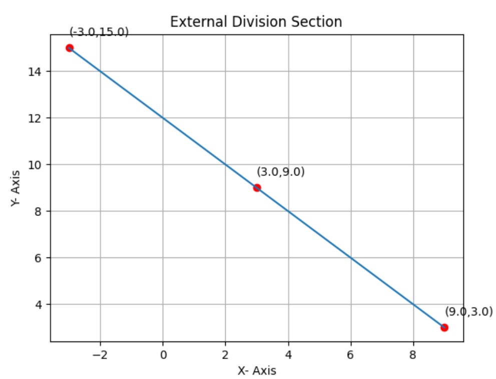

Figure 2: External Division Section