Points and Vectors

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***Abstract* - This document contains solution of finding the coordinates of a point which divides a line segment internally and Externally.**

Vector

Question 21 : Find the coordinates of the points which divide, internally and externally, the line joining the point (a+b, a-b) to the point (a-b, a+b) in the ratio a : b.

Solution :
Let us consider C be the point which divides the AB line segment in the ratio a : b internally and externally. Given that the coordinates of A point= (a+b, a-b) and coordinates of B point= (a-b, a+b).
Let

$$\mathbf{A} = \left[\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \right], \mathbf{B} = \left[\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \right] \quad (1)$$

Let us consider $\mathbf{A}$ and $\mathbf{B}$ as,

$$\mathbf{A} = \mathbf{P}u, \mathbf{B} = \mathbf{Q}u \quad (2)$$

1. **For Internal Section Coordinates:**

$$\mathbf{C} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (3)$$

For the given problem, $\mathbf{C}$ divides AB in the ratio a:b.
So, here k = a : b
Putting the values of $\mathbf{A}$ and $\mathbf{B}$ in equation (3), we get,

$$\mathbf{C} = \frac{\frac{a}{b}\mathbf{Q}u + \mathbf{P}u}{\frac{a}{b} + 1} \quad (4)$$

Solving this we get,

$$\mathbf{C} = \left(\frac{a\mathbf{Q}}{a + b} + \frac{b\mathbf{P}}{a + b} \right) u \quad (5)$$

Putting the values of $\mathbf{P}$, $\mathbf{Q}$ and u in equation (5) we get,

$$\mathbf{C} = \frac{1}{a + b} \left[\begin{pmatrix} a & -a \\ a & a \end{pmatrix} + \begin{pmatrix} b & b \\ b & -b \end{pmatrix} \right] \begin{pmatrix} a \\ b \end{pmatrix} \quad (6)$$

So, internal section coordinates of $\mathbf{C}$,

$$\mathbf{C} = \frac{1}{a + b} \begin{pmatrix} a + b & b - a \\ a + b & a - b \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad (7)$$

2. **For External Section Coordinates:**

$$\mathbf{C} = \frac{k\mathbf{B} - \mathbf{A}}{k - 1} \quad (8)$$

Putting the values of $\mathbf{A}$ and $\mathbf{B}$ in equation (8), we get,

$$\mathbf{C} = \frac{\frac{a}{b}\mathbf{Q}u - \mathbf{P}u}{\frac{a}{b} - 1} \quad (9)$$

Solving this we get,

$$\mathbf{C} = \left(\frac{a\mathbf{Q}}{a - b} - \frac{b\mathbf{P}}{a - b} \right) u \quad (10)$$

Putting the values of $\mathbf{P}$, $\mathbf{Q}$ and u in equation (10) we get,

$$\mathbf{C} = \frac{1}{a - b} \left[\begin{pmatrix} a & -a \\ a & a \end{pmatrix} - \begin{pmatrix} b & b \\ b & -b \end{pmatrix} \right] \begin{pmatrix} a \\ b \end{pmatrix} \quad (11)$$

So, external section coordinates of $\mathbf{C}$,

$$\mathbf{C} = \frac{1}{a - b} \begin{pmatrix} a - b & -a - b \\ a - b & a + b \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad (12)$$

Result

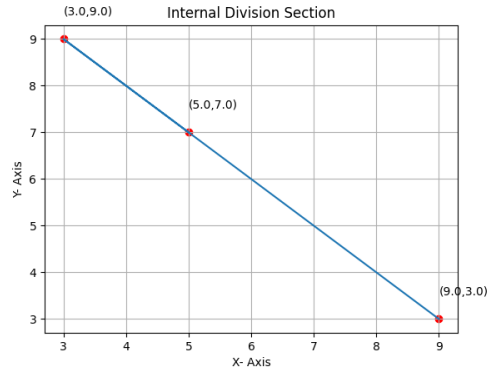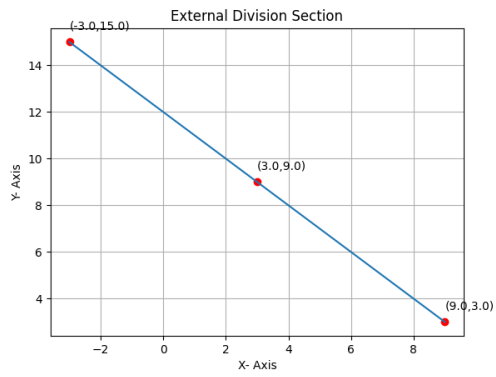Plot of coordinates of the points obtained from Python code considering a= 6, b=3 is shown below.

Figure 1: Internal Division Section

Figure 2: External Division Section