Assignment 1

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1 Assignment 1

Question 21 : Find the coordinates of the points which divide, internally and externally, the line joining the point (a+b, a-b) to the point (a-b, a+b) in the ratio a: b.

Solution :
Suppose $\vec{a}$ and $\vec{b}$ are the position vectors of the points **A** and **B** respectively referred to the origin **O** and $\vec{c}$ be the position vector of point **P** which we have to find out. So,

$$\vec{a} = \begin{bmatrix} (a+b & a-b) \begin{pmatrix} \hat{i} \\ \hat{j} \end{pmatrix} \end{bmatrix}^\top \tag{1}$$

$$\vec{a} = (a+b)\hat{i} + (a-b)\hat{j} \tag{2}$$

$$\vec{b} = \begin{bmatrix} (a-b & a+b) \begin{pmatrix} \hat{i} \\ \hat{j} \end{pmatrix} \end{bmatrix}^\top \tag{3}$$

$$\vec{b} = (a-b)\hat{i} + (a+b)\hat{j} \tag{4}$$

and

$$\frac{AP}{PB} = \frac{a}{b} \tag{5}$$

$$b(AP) = a(PB) \tag{6}$$

For Internal Division Section:

$$b(\vec{c} - \vec{a}) = a(\vec{b} - \vec{c}) \tag{7}$$

Solving this equation we get,

$$\vec{c}(a+b) = a\vec{b} + b\vec{a} \tag{8}$$

Putting the values of $\vec{a}$ and $\vec{b}$:

$$\vec{c} = \frac{a}{a+b}\left((a-b)\hat{i} + (a+b)\hat{j}\right) + \frac{b}{a+b}\left((a+b)\hat{i} + (a-b)\hat{j}\right) \tag{9}$$

$$\vec{c} = \left(\frac{a(a-b) + b(a+b)}{a+b}\right)\hat{i} + \left(\frac{a(a+b) + b(a-b)}{a+b}\right)\hat{j} \tag{10}$$

Coordinates of P=

$$\left(\frac{a(a-b) + b(a+b)}{a+b}, \frac{a(a+b) + b(a-b)}{a+b}\right) \tag{11}$$

For External Division Section:
From equation (6):

$$b(\vec{c} - \vec{a}) = a(\vec{c} - \vec{b}) \tag{12}$$

Solving this equation we get,

$$\vec{c}(a-b) = a\vec{b} - b\vec{a} \tag{13}$$

Putting the values of $\vec{a}$ and $\vec{b}$:

$$\vec{c} = \frac{a}{a-b}\left((a-b)\hat{i} + (a+b)\hat{j}\right) - \frac{b}{a-b}\left((a+b)\hat{i} + (a-b)\hat{j}\right) \tag{14}$$

$$\vec{c} = \left(\frac{a(a-b) - b(a+b)}{a-b}\right)\hat{i} + \left(\frac{a(a+b) - b(a-b)}{a-b}\right)\hat{j} \tag{15}$$

Coordinates of P=

$$\left(\frac{a(a-b) - b(a+b)}{a-b}, \frac{a(a+b) - b(a-b)}{a-b}\right) \tag{16}$$

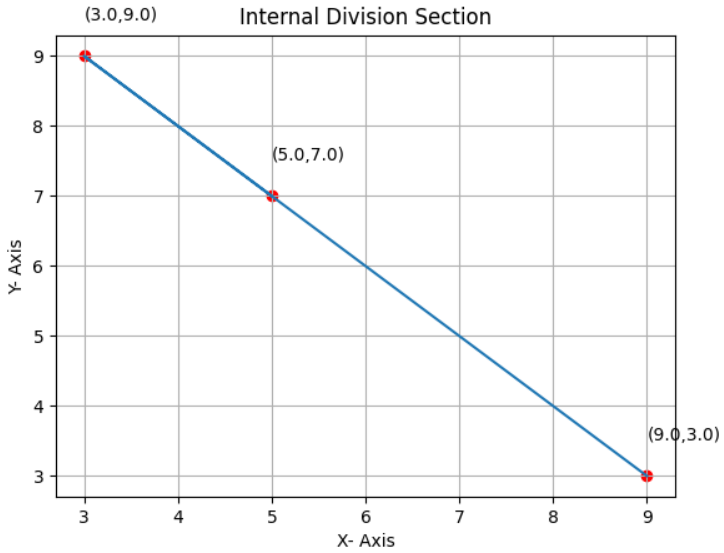

Figure 1: Internal Division Section

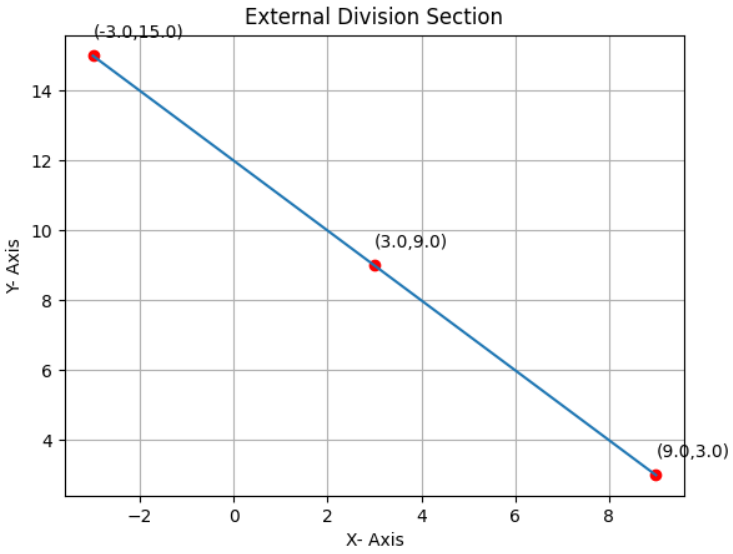

Figure 2: External Division Section