

Assignment 1

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1 Assignment 1

Question 21 : Find the coordinates of the points which divide, internally and externally, the line joining the point $(a+b, a-b)$ to the point $(a-b, a+b)$ in the ratio $a: b$.

Solution :

Given Vectors:

$$\mathbf{B} = \begin{pmatrix} a+b \\ a-b \end{pmatrix} \quad (1)$$

$$\mathbf{A} = \begin{pmatrix} a-b \\ a+b \end{pmatrix} \quad (2)$$

$$\frac{BP}{PA} = \frac{a}{b} \quad (3)$$

For Internal Division Section:

$$b(BP) = a(PA) \quad (4)$$

$$b(\mathbf{B}-\mathbf{P}) = a(\mathbf{P}-\mathbf{A}) \quad (5)$$

$$b\mathbf{B} - b\mathbf{P} = a\mathbf{P} - a\mathbf{A} \quad (6)$$

$$a\mathbf{P} + b\mathbf{P} = b\mathbf{B} + a\mathbf{A} \quad (7)$$

$$\mathbf{P}(a+b) = b\mathbf{B} + a\mathbf{A} \quad (8)$$

$$\mathbf{P}(a+b) = b \begin{pmatrix} a+b \\ a-b \end{pmatrix} + a \begin{pmatrix} a-b \\ a+b \end{pmatrix} \quad (9)$$

$$\mathbf{P} = \frac{b}{a+b} \begin{pmatrix} a+b \\ a-b \end{pmatrix} + \frac{a}{a+b} \begin{pmatrix} a-b \\ a+b \end{pmatrix} \quad (10)$$

$$\mathbf{P} = \left(\frac{\frac{b}{a+b}(a+b)}{\frac{a}{a+b}(a-b)} \right) + \left(\frac{\frac{a}{a+b}(a-b)}{\frac{a}{a+b}(a+b)} \right) \quad (11)$$

So, coordinates of \mathbf{P} for Internal Division Section:

$$\mathbf{P} = \left(\frac{\frac{a(a-b)+b(a+b)}{a+b}}{\frac{a(a+b)+b(a-b)}{a+b}} \right) \quad (12)$$

For External Division Section:

$$\frac{BP}{AP} = \frac{a}{b} \quad (13)$$

$$b(BP) = a(AP) \quad (14)$$

$$b(\mathbf{B}-\mathbf{P}) = a(\mathbf{A}-\mathbf{P}) \quad (15)$$

$$b\mathbf{B} - b\mathbf{P} = a\mathbf{A} - a\mathbf{P} \quad (16)$$

$$a\mathbf{P} - b\mathbf{P} = a\mathbf{A} - b\mathbf{B} \quad (17)$$

$$\mathbf{P}(a-b) = a\mathbf{A} - b\mathbf{B} \quad (18)$$

$$\mathbf{P}(a-b) = a \begin{pmatrix} a-b \\ a+b \end{pmatrix} - b \begin{pmatrix} a+b \\ a-b \end{pmatrix} \quad (19)$$

$$\mathbf{P} = \frac{a}{a-b} \begin{pmatrix} a-b \\ a+b \end{pmatrix} - \frac{b}{a-b} \begin{pmatrix} a+b \\ a-b \end{pmatrix} \quad (20)$$

$$\mathbf{P} = \left(\frac{\frac{a}{a-b}(a-b)}{\frac{a}{a-b}(a+b)} \right) - \left(\frac{\frac{b}{a-b}(a+b)}{\frac{a}{a-b}(a-b)} \right) \quad (21)$$

So, coordinates of \mathbf{P} for External Division Section:

$$\mathbf{P} = \left(\frac{\frac{a(a-b)-b(a+b)}{a-b}}{\frac{a(a+b)-b(a-b)}{a-b}} \right) \quad (22)$$

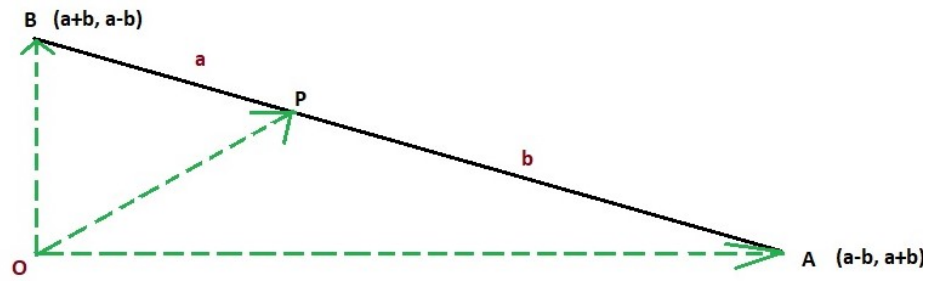


Figure 1: Internal Division Section

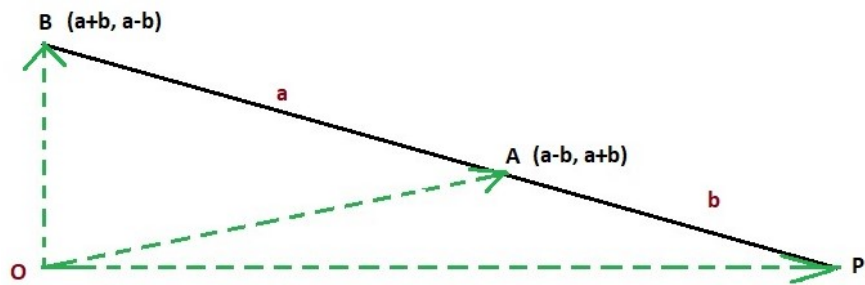


Figure 2: External Division Section