

# Assignment 1

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## 1 Assignment 1

**Question 21 :** Find the coordinates of the points which divide, internally and externally, the line joining the point  $(a+b, a-b)$  to the point  $(a-b, a+b)$  in the ratio  $a: b$ .

**Solution :**

Let us consider a line segment as  $\vec{BA}$ . The coordinates of point B is  $(a+b, a-b)$  and coordinates of point A is  $(a-b, a+b)$ .

Now considering another point say P, which divides the  $\vec{BA}$  internally or externally in the ratio  $a: b$ .

'O' be the origin.

$\vec{OB}, \vec{OP}, \vec{OA}$  be the position vectors

We have to find out the coordinates of P point when it divides  $\vec{BA}$  internally and also externally.

**Using Vectors:**

**For Internal Division Section:**

Say  $\hat{v}$  = a unit vector in the direction of  $\vec{BA}$

So,  $\vec{BA} = \hat{v}$

$\vec{BP} = a\hat{v}$  - (i)

$\vec{PA} = b\hat{v}$  - (ii)

Multiplying both sides of (i) with b we get,

$b\vec{BP} = ab\hat{v}$

Multiplying both sides of (ii) with a we get,

$a\vec{PA} = ab\hat{v}$

So,  $b\vec{BP} = a\vec{PA}$  - (iii)

$b(\vec{OP} - \vec{OB}) = a(\vec{OA} - \vec{OP})$

$b\vec{OP} - b\vec{OB} = a\vec{OA} - a\vec{OP}$

$b\vec{OP} + a\vec{OP} = a\vec{OA} + b\vec{OB}$

$\vec{OP}(a+b) = a\vec{OA} + b\vec{OB}$

$$\vec{OP} = \frac{a}{a+b}\vec{OA} + \frac{b}{a+b}\vec{OB}$$

$$\vec{OP} = \frac{a}{a+b}(a-b, a+b) + \frac{b}{a+b}(a+b, a-b)$$

$$= \left( \frac{a(a-b)}{a+b}, \frac{a(a+b)}{a+b} \right) + \left( \frac{b(a+b)}{a+b}, \frac{b(a-b)}{a+b} \right)$$

$$= \left( \frac{a(a-b)}{a+b} + \frac{b(a+b)}{a+b} \right), \left( \frac{a(a+b)}{a+b} + \frac{b(a-b)}{a+b} \right)$$

$$= \left( \frac{a(a-b) + b(a+b)}{a+b} \right), \left( \frac{a(a+b) + b(a-b)}{a+b} \right)$$

So, coordinates of P

$$\vec{OP} = \left( \frac{(a^2 + b^2)}{a+b} \right), \left( \frac{(a^2 + 2ab - b^2)}{a+b} \right)$$

**For External Division Section:**

From equation (iii)  $a\vec{PA} = b\vec{BP}$

So,  $a(\vec{OP} - \vec{OA}) = b(\vec{OP} - \vec{OB})$

$a\vec{OP} - a\vec{OA} = b\vec{OP} - b\vec{OB}$

$a\vec{OP} - b\vec{OP} = a\vec{OA} - b\vec{OB}$

$\vec{OP}(a-b) = a\vec{OA} - b\vec{OB}$

$$\vec{OP} = \frac{a}{a-b}\vec{OA} - \frac{b}{a-b}\vec{OB}$$

$$\vec{OP} = \frac{a}{a-b}(a-b, a+b) - \frac{b}{a-b}(a+b, a-b)$$

$$= \left( \frac{a(a-b)}{a-b}, \frac{a(a+b)}{a-b} \right) - \left( \frac{b(a+b)}{a-b}, \frac{b(a-b)}{a-b} \right)$$

$$= \left( \frac{a(a-b)}{a-b} - \frac{b(a+b)}{a-b} \right), \left( \frac{a(a+b)}{a-b} - \frac{b(a-b)}{a-b} \right)$$

$$= \left( \frac{a(a-b) - b(a+b)}{a-b} \right), \left( \frac{a(a+b) - b(a-b)}{a-b} \right)$$

So, coordinates of P

$$\vec{OP} = \left( \frac{(a^2 - 2ab - b^2)}{a-b} \right), \left( \frac{(a^2 + b^2)}{a-b} \right)$$

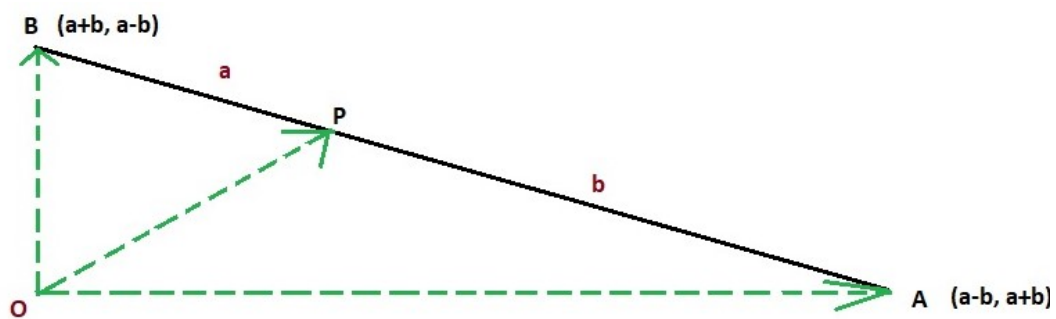


Figure 1: Internal Division Section

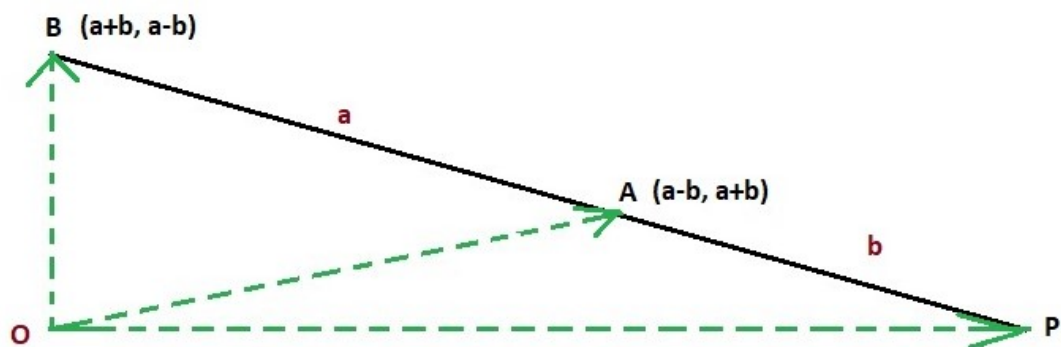


Figure 2: External Division Section