Assignment 1

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1 Assignment 1

Question 21 : Find the coordinates of the points which divide, internally and externally, the line joining the point (a+b, a-b) to the point (a-b, a+b) in the ratio a: b.

Solution :
Let us consider a line segment as $\vec{BA}$. The coordinates of point B is (a+b, a-b) and coordinates of point A is (a-b, a+b).

Now considering another point say P, which divides the $\vec{BA}$ internally or externally in the ratio a: b.

'O' be the origin.

$\vec{OB}, \vec{OP}, \vec{OA}$ be the position vectors

We have to find out the coordinates of P point when it divides $\vec{BA}$ internally and also externally.

Using Vectors:

For Internal Division Section:

Say $\hat{v}$= a unit vector in the direction of $\vec{BA}$

So, $\vec{BA}$= $\hat{v}$

$\vec{BP}$= a$\hat{v}$ – (i)

$\vec{PA}$= b$\hat{v}$ – (ii)

Multiplying both sides of (i) with b we get, b$\vec{BP}$= ab$\hat{v}$

Multiplying both sides of (ii) with a we get, a$\vec{PA}$= ab$\hat{v}$

So, b$\vec{BP}$= a$\vec{PA}$ –(iii)

b($\vec{OP}$- $\vec{OB}$)= a($\vec{OA}$- $\vec{OP}$)

b$\vec{OP}$- b$\vec{OB}$= a$\vec{OA}$- a$\vec{OP}$

b$\vec{OP}$+ a$\vec{OP}$= a$\vec{OA}$+ b$\vec{OB}$

$\vec{OP}$(a+b)= a$\vec{OA}$+ b$\vec{OB}$

$$\vec{OP} = \frac{a}{a+b}\vec{OA} + \frac{b}{a+b}\vec{OB}$$

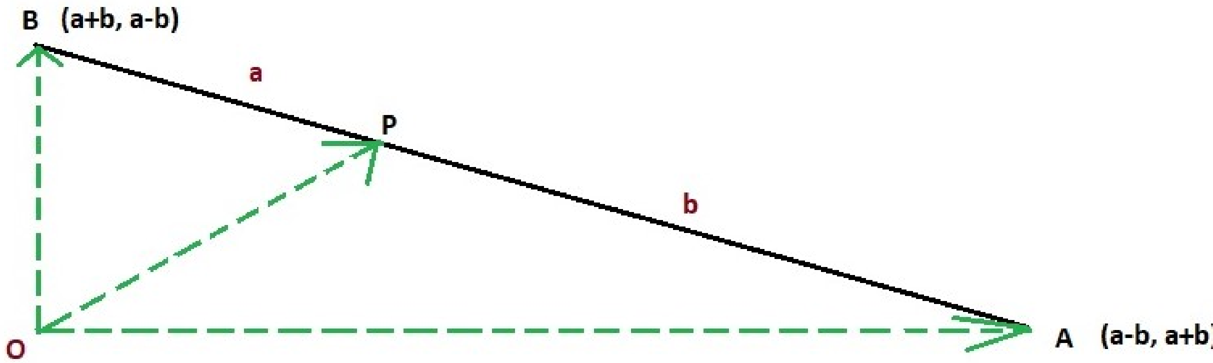

Figure 1: Internal Division Section

$$\vec{OP} = \frac{a}{a+b}(a-b, a+b) + \frac{b}{a+b}(a+b, a-b)$$

$$= \left(\frac{a(a-b)}{a+b}, \frac{a(a+b)}{a+b}\right) + \left(\frac{b(a+b)}{a+b}, \frac{b(a-b)}{a+b}\right)$$

$$= \left(\frac{a(a-b)}{a+b} + \frac{b(a+b)}{a+b}\right), \left(\frac{a(a+b)}{a+b} + \frac{b(a-b)}{a+b}\right)$$

$$= \left(\frac{a(a-b)+b(a+b)}{a+b}\right), \left(\frac{a(a+b)+b(a-b)}{a+b}\right)$$

So, coordinates of P

$$\vec{OP} = \left(\frac{(a^2+b^2)}{a+b}\right), \left(\frac{(a^2+2ab-b^2)}{a+b}\right)$$

For External Division Section:

From equation (iii) a$\vec{PA}$= b$\vec{BP}$
So, a($\vec{OP}$- $\vec{OA}$)= b($\vec{OP}$- $\vec{OB}$)
a$\vec{OP}$- a$\vec{OA}$= b$\vec{OP}$- b$\vec{OB}$
a$\vec{OP}$- b$\vec{OP}$= a$\vec{OA}$- b$\vec{OB}$
$\vec{OP}$(a-b)= a$\vec{OA}$- b$\vec{OB}$

$$\vec{OP} = \frac{a}{a-b}\vec{OA} - \frac{b}{a-b}\vec{OB}$$

$$\vec{OP} = \frac{a}{a-b}(a-b, a+b) - \frac{b}{a-b}(a+b, a-b)$$

$$= \left(\frac{a(a-b)}{a-b}, \frac{a(a+b)}{a-b}\right) - \left(\frac{b(a+b)}{a-b}, \frac{b(a-b)}{a-b}\right)$$

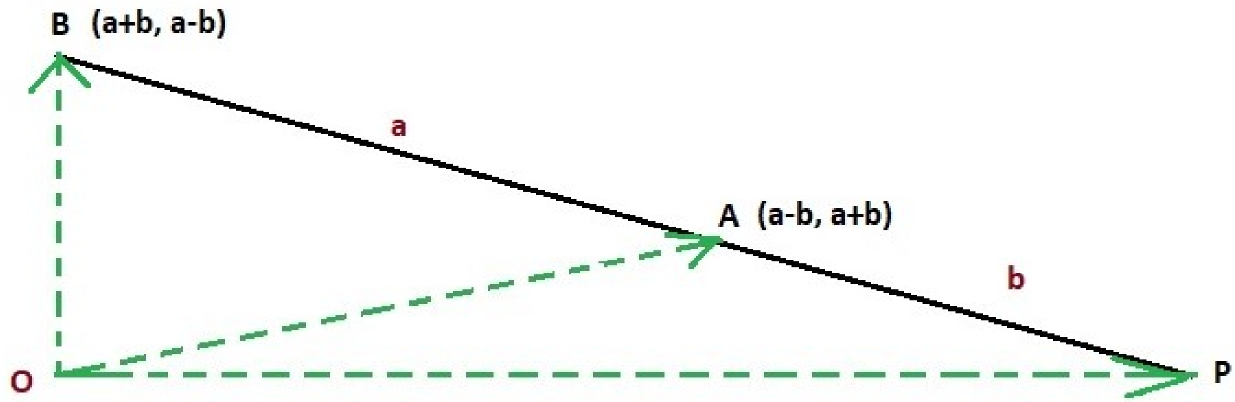

Figure 2: External Division Section

$$= \left(\frac{a(a-b)}{a-b} - \frac{b(a+b)}{a-b}\right), \left(\frac{a(a+b)}{a-b} - \frac{b(a-b)}{a-b}\right)$$

$$= \left(\frac{a(a-b)-b(a+b)}{a-b}\right), \left(\frac{a(a+b)-b(a-b)}{a-b}\right)$$

So, coordinates of P

$$\vec{OP} = \left(\frac{(a^2-2ab-b^2)}{a-b}\right), \left(\frac{(a^2+b^2)}{a-b}\right)$$