Long-range metal consumption forecasts using innovative methods

The case of aluminium in Brazil to the year 2000

Saul B. Suslick and DeVerle P. Harris

Brazil's aluminium consumption in the year 2000 is forecast to be in the range of 800 000 to 1 300 000 tonnes; these bounds correspond to annual rates of growth in GDP of 3.6% and 6.1% respectively. This range of consumption conforms closely to that produced by an extensive translog consumption model which includes GDP, price of aluminium and its chief substitute, copper, and time as a proxy for technical change in aluminium using products and product technology. Analysis of various models by *ex post* forecast errors on a test period showed the most accurate models to be a simple learning model for cumulative IU and a time varying coefficient consumption model, followed by the extensive translog consumption model. The models with the largest *ex post* errors included the linear IU, lognormal IU, and extensive learning IU models.

Saul B. Suslick is with the Departamento de Administração e Política de Recursos Minerais, Instituto de Geociências, State University of Campinas, Unicamp, PO Box 6152, 13081 Campinas Sp, Brazil; DeVerle P. Harris is Director of Mineral Economics, Department of Mining and Geological Engineering, College of Engineering and Mines, University of Arizona, Tucson, AZ 85721, USA.

The authors are indebted to FAPESP (Fundação de Amparo a Pesquisa do
Continued on p 185

The purpose of this study is to forecast Brazil's aluminium consumption for the year 2000 using methods and models especially selected for their long-range forecasting capability. These models are referred to as innovative because they combine various kinds of pattern analysis with highly simplified economic models relating aluminium consumption to price and major determinants.

Methods which have been used to make long-range forecasts of mineral consumption include trend projections of consumption or intensity of use, intensity of use models, econometric models and across time adjustments of technical coefficients of input–output models. The models employed in this study are simple in terms of economic structure. This is deliberate, because when the objective is long-range forecasting of mineral consumption, dynamics of demand and its major determinants, GDP and technology, may be more important than own price and prices of substitutes in explaining across time patterns of mineral consumption. The natural inclination to formulate econometric models rich in economic structure and to use them to simulate future consumption is moderated to some degree by the need ultimately to build dynamics into such models and to estimate the states of at least some economic determinants, using other economic variables to explore contingencies for selected scenarios.

The empirical work of Malenbaum,[1] who demonstrated a skewed bell-shaped pattern for intensity of use of some mineral commodities with increasing per capita GDP, was taken as a starting point because of its emphasis on across time changes in consumption, in contrast to short-run models which emphasize price theory. Accordingly, intensity of use is forecast by linear and logistic trend models and by linear and lognormal functions of per capita GDP. The lognormal model of intensity of use is shown to be equivalent to a translog model;

0301-4207/90/030184-16

Sources: Associação Brasileira de Alumínio (ABAL), *Anuário Estatístico*, São Paulo, 1988; A.C. Aranha, *A industria do Aluminio: Um Historico e Perspectiva para os Anos 90*, Conference, Instituto de Geociências UNICAMP, Campinas, SP, Brazil, 1988; Metallgesellschaft Aktiengesellschaft, *Metal Statistics*, Metallgesellschaft AG, Frankfurt, various issues.

[a]Adjusted for inventory change. Exports and imports data included primary metals and alloys, mill products and scrap.

Table 1. Production and consumption of primary aluminium in Brazil (in thousand tonnes).

	1979	1980	1981	1982	1983	1984	1985	1986	1987
Production	238	261	256	299	401	455	549	757	844
Recovery of secondary	59	50	42	43	45	47	52	58	61
Imports	79	69	37	15	6	10	7	7	14
Exports	9	12	21	18	170	250	215	357	463
Consumption[a]	257	285	262	282	271	295	347	424	430
Per capita consumption (kg/person)	3.0	3.0	2.4	2.5	2.2	2.2	2.6	3.1	2.9

consequently, intensity of use models are stated as translog models and expanded to include economic variables in addition to per capita GDP eg own price, substitute price and technical change. Since across time consumption reflects in part improvements in product and improvements in production technology, intensity of use is forecast by simple and extensive learning models. In the simple learning model cumulative per capita GDP serves as the measure of experience and the ratio of cumulative consumption to cumulative GDP is the measure that improves with experience. The extensive learning model includes additional variables to describe learning own price, substitute price and technical change.

Translog intensity of use models are shown to be special cases of across time consumption models; consequently consumption is forecast directly, instead of indirectly by intensity of use, by an extensive translog consumption model and by a linear factor model with time variable coefficients.

These models are demonstrated using Brazil's aluminium consumption in year 2000. Most models are initially estimated on a restricted data set to permit their evaluation and comparison by *ex post* forecast accuracy; then they are reestimated on all of the data in preparation for their use in forecasting.

The next section presents a brief description of aluminium production, consumption and intensity of use in Brazil as background for demonstration of the forecasting models. Because many, but not all, of these models employ the intensity of use measure, that section is followed by a general review of intensity of use concepts, modelling, and forecasting, which in turn is followed by a description of the estimated models, evaluation of the forecasts and conclusions.

Aluminium in Brazil

Supply

Brazil became self sufficient in aluminium in the early 1980s (Table 1) by combining huge hydroelectric energy potential with large bauxite ore reserves. Although Brazil is an important metal consumer with an internal market it exports about one-half of its primary production, making it one of the top ten aluminium world exporters. Moreover, Brazil seems to have good prospects for increasing aluminium production capacity in the near future.

There are two aluminium supply regions in Brazil, each differing considerably in vintage and economic setting.[2] The older region, which is in southern Brazil, was developed for the local domestic market. The supply capacity of this region grew under government protection to achieve an efficient scale and became internationally competitive. The northern region has large hydropower resources and is favoured by

Continued from p 184
Estado de São Paulo) for financial support during 1989, while Professor Suslick was a visiting professor in the Mineral Economics Program, Department of Mining and Geological Engineering, University of Arizona.

[1]W. Malenbaum, 'Law of demand for minerals', *Proceedings of the Council of Economics*, 104th Annual Meeting of the American Institute of Mining, Metallurgical and Petroleum Engineers, 1975, pp 147–155.

[2]E. Braz-Pereira, 'Brazil: the transition to an export industry', in Merton J. Peck, ed, *The World Aluminium Industry in a Changing Energy Era*, Resources for the Future, Washington, DC, 1988, pp 148–174

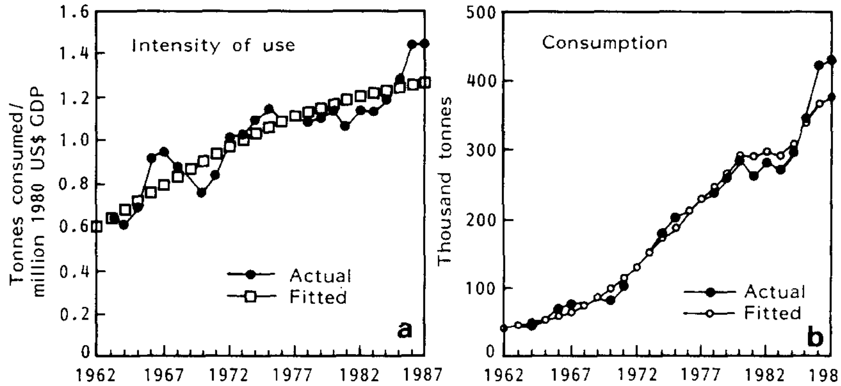

Figure 1. The logistic IU trend model: comparing actual and estimated aluminium consumption and intensity of use.

Source: For primary aluminium consumption: *Metallgesellschaft Aktiengesellschaft, Metal Statistics*, Metallgesellschaft AG, Frankfurt, various years. For GDP: constant 1980 values in million US$, taken from ECLA/UN, *Economic Survey of Latin America and Caribbean*, ECLA, various issues; ECLA/UN, *Statistical Yearbook for Latin America*, ECLA, various issues; IMF, *Yearbook 88: International Finance Statistics*, IMF, Washington, DC, 1988.

governmental policies (low electricity rates) to ensure that the Amazon region is competitive in the international market.

Completion of the projects Alumar and Albrás in 1995, both in northern Brazil, will increase primary aluminium capacity in Brazil to 1.25 million tons.[3]

Consumption and price

Aluminium consumption in Brazil grew exponentially from 1962 to 1974; consumption slowed and levelled off in the early 1980s but surged vigorously in 1984 and 1985 (Figure 1b). In addition to the impact of increased energy prices on economic growth, macroeconomic policies combined with price regulation caused an erratic consumption pattern and a decrease in intensity of use of aluminium.[4] Restrictive macroeconomic policies induced by the international debt crisis caused GDP per capita to decline from its 1980 peak. Industrial growth after 1980 is characterized by a marked stop and go pattern as policy induced recessions alternate with bursts of economic expansion.[5]

The domestic price of primary aluminium has been controlled by the Brazilian government since 1964. For most of the period domestic price followed international price; but since 1979 the two prices have behaved quite differently. Price declines in 1980 and 1983 reflect two 30% devaluations of the cruzeiro which took place in the preceding months, and in 1983 the domestic price increased due to adjustment to the devaluation.

During the 1980s metal price was regulated to combat overall price inflation; this caused a variety of problems. When the Brazilian aluminium price was set below international levels, supply scarcities tended to develop in the domestic market since exports were more profitable to producers.

Aluminium intensity of use

The ratio of consumption (D_t) to a measure of national income (Y), such as GNP or GDP, was defined by Malenbaum as intensity of use.[6] Malenbaum showed that IU plotted against per capita income, y, exhibits a skewed bell-shaped pattern for highly industrialized economies:

$$IU = f(y)$$

where the graph of y against IU is of lognormal-type shape.

Aluminium intensity of use in Brazil from 1962 to 1987, shown in

[3]Associação Brasileira de Alumínio (ABAL), *Anuário Estatístico*, São Paulo, 1988.
[4]S.B. Suslick and D.P. Harris, 'Latin American metal consumption – recent trends and determinants', *Materials and Society*, forthcoming.
[5]World Bank, *Brazil: Industrial Policies and Manufactured Exports*, a World Bank country study, 1983.
[6]W. Malenbaum, *Material Requirements in the United States and Abroad in the Year 2000*, A research project prepared for the National Commission on Materials Policy, University of Pennsylvania Press, Philadelphia, PA, 1973; *op cit*, Ref 1; and W. Malenbaum, *World Demand for Raw Materials in 1985 and 2000*, McGraw-Hill, New York, 1978.

Figure 1a, is still increasing, although at a decreasing rate. With respect to the idealized lognormal-type lifecycle, intensity of use in Brazil has not yet reached maturity. However, decreases in the rate of growth suggests a levelling off and an eventual decline of intensity of use in the future, as has been observed for other metals in more mature economies.

The IU model and economic implications

Although data on mineral consumption in the USA and other developed regions clearly substantiate a lognormal-type pattern of IU against y, rationalization of this pattern in terms of determinants of metal consumption or a 'law of mineral demand' has received notable criticism. A fundamental criticism is that Malenbaum's IU model does not describe the use of a material by a domestic economy in a comprehensive sense because it ignores material that is embodied in imported and exported products. Clearly, being based upon apparent consumption, Malenbaum's IU is a measure only of the intensity of use of raw material. Thus, depending upon whether a region is a net importer or exporter of products that embody the material, the traditional IU measure understates or overstates respectively the actual intensity of use of the material in the domestic economy.

A partial analysis of this effect on US copper IU has been made by Hutchison and Tilton,[7] and a comprehensive analysis of this effect on US sulphur IU has been made by Al-Rawahi and Rieber.[8]

A second major criticism of Malenbaum's model is that it ignores the effect of prices on materials substitution and of technology on materials savings and on product innovation, because it describes IU as a function of y only. In contrast, Canavan, Roberts and Considine[9] found that technical change, materials substitution and consumer preferences have strong impacts on IU. Tilton[10] provides an economic structure for IU by decomposing it into two components: *PCI* and *MCP*, where *PCI* is the product composition of income, and *MCP* is the materials composition of product:

$$IU = PCI \times MCP = (Q/Y) \times (D/Q)$$

where Q = material containing products.

PCI is heavily influenced by consumer preferences and other determinants of demand for consumer durables, and *MCP* is influenced by product technology and its influence on derived demand for material and by relative prices of substitutes. Roberts[11] found for several metals in the USA that changes in *PCI* have been a more important factor in IU changes than have changes in *MCP*. Suslick and Harris[12] have found similar results for metals in Brazil.

The last criticism made of Malenbaum's IU is that as a forecasting model it is highly simplistic because it considers only per capita GDP. Of course, for long-range forecasting, simplicity is desirable, everything else being equal. But if technical change in metal consuming industries is at a different rate from in the economy generally, forecasts based strictly upon GDP may be biased. Moreover, a forecasting model based only on GDP clearly exaggerates the effect of GDP and obviates the exploration of scenarios of price or technical change. Finally, the simple IU model must be used with care when IU has peaked and is well out on the gently sloping, righthand tail of the model, because there is a fine

[7]R.S Hutchison and J.E. Tilton, 'Is the intensity of copper use still declining in the USA?', *Natural Resources Forum*, Vol 11, No 4, 1987, pp 325–334.
[8]K. Al-Rawahi and M. Rieber, 'Embodied net exports, the effects on intensity of use estimation and mineral demand forecasting: US sulphur 1965–1985' in this volume.
[9]P.D. Canavan, *The Determinants of Intensity-of-use: A Case Study of Tin Solder End Uses*, PhD dissertation, Pennsylvania State University, University Park, Philadelphia, PA, 1983; M.C. Roberts, *Theory and Practice of the Intensity of Use Method of Mineral Consumption Forecasting*, PhD Dissertation, The University of Arizona, Tucson, AZ, unpublished, 1985; and T.J. Considine, 'Economic and technologic determinants of materials intensity of use', *Land Economics*, in press, 1990.
[10]J.E. Tilton, 'Atrophy in metal demand', *Earth and Mineral Science*, Vol 54, No 2, 1985, pp 15–18.
[11]M.C. Roberts, 'What caused the slack demand for metals after 1974?', *Resources Policy*, Vol 14, No 3, 1988, pp 231–246.
[12]*Op cit*, Ref 4.

line between IU forecasts which translate into increasing versus decreasing consumption forecasts.

Attempting to preserve the robustness of the lognormal-type pattern demonstrated by Malenbaum and at the same time enrich the economics of the IU model, Harris and Jeon[13] show that representing the IU pattern by a lognormal model implies an income demand model in which income elasticity of demand varies with income level. This demonstration is based upon the equivalence of lognormal and translog models:

$$IU_t = \frac{K}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln y_t - \mu}{\sigma}\right)^2} \rightarrow IU_t = A\, y^{\beta_0 + \beta_1 \ln y_t} \qquad (1)$$

where K, σ and μ are lognormal curve parameters and A, β_0 and β_1 are translog parameters.[14]

Given

$$IU_t = \frac{D_t/POP_t}{Y_t/POP_t} = \frac{d_t}{y_t} = \frac{D_t}{Y_t} \qquad (2)$$

$$d_t = IU_t . y_t$$

where d_t and y_t are per capita consumption and per capita GDP respectively.

Therefore

$$d_t = A\, y^{\beta_0 + 1 + \beta_1 \ln y_t}$$

and (3)

$$IU_t = A\, y^{\beta_0 + \beta_1 \ln y_t}$$

Finally,

$$D_t = d_t . POP_t = IU_t . Y_t$$

Thus, the income elasticity of per capita demand is $\beta_0 + 1 + \beta_1 \ln y_t$, and the income elasticity of intensity of use is $\beta_0 + \beta_1 \ln y_t$.

The value of this demonstration is that it is a formal linkage of IU and demand. Designate the translog IU model by $T(y;\beta)$, where $\beta' = [A, \beta_1, \beta_2]$; then derived demand theory suggests that a more complete description of IU would include own price P, substitute price PS and technical change τ:

$$IU_t = T(y, P, PS, \tau; \Omega) \qquad (4)$$

where Ω is a vector of coefficients of the expanded translog model. In this paper, a translog IU model that contains variables in addition to per capita GDP is referred to as an extensive translog IU model.

Long-range forecasting models for Brazil's aluminium consumption

Perspectives on long-range forecasting

When the objective is long-range forecasting of mineral consumption, forecasting models that are more complex and complete in economic variables have not always provided better forecasts than simpler models that employ a robust empirical feature identified from the data, such as lifecycle models, trend models and learning models. Of course, disadvantages of forecasts by pattern analysis and projection are that they

[13] D.P. Harris and G.J. Jeon, *Improved Methods for Long-Range Forecasting.*, Research Project. Mineral Economics, Department of Mining and Geological Engineering, College of Engineering and Mines, University of Arizona, 1987.

[14] $\ln A = \ln K - \ln\sigma - \ln(\sqrt{\pi}) - \mu^2/2\sigma^2$
$\beta_0 = \mu/\sigma^2$
$\beta_1 = -1/2\sigma^2$

are not very interesting in an economic sense and they may miss future turning points if the structure of the economy in the future changes in ways that are not captured by the projection of present and past patterns. The approach used in this study is to employ several methods, ranging from projections of time trends or learning patterns to projection of extensive translog and factor models that contain a few economic variables and to performing an *ex post* evaluation of accuracy of forecast on a test period. These models are then reestimated on all of the available data and used to forecast Brazilian aluminium consumption for the year 2000.

IU is employed extensively as a vehicle for forecasting consumption because it affords a bridge from empirical patterns in across time materials use to a simple economic model. As used here IU is based upon apparent consumption, reflecting only intensity of use. Data were not available to permit the use of an IU net of aluminium embodied in imported and exported products. Consequently, forecasts made here assume that product imports and exports will increase or decrease according to historical patterns or relations.

Simple time–trend projection of IU

Trend projection is based on the premise that mineral demand over time presents a general regularity (growth or decline) that will persist in the future. Trend analysis and projection require a history of economic activity sufficient to describe clearly patterns of long-term growth or decline in the time series. Then, based upon a number of strong assumptions, a mathematical trend model is fitted and extrapolated to some future time horizon.

Forecasting by trend projection requires the simultaneous identification of the preferred trend model and of the data base for the estimation of model parameters. Gregg *et al*[15] point out that selection of a model and data by visual inspection of fit and sum of squares of residuals is difficult when two or more models both fit the data reasonably well. Accordingly, they suggest the comparison of slope and intercept characteristics computed from time series data with those of known models. Similarly, Harris and Jeon[16] developed a computer program referred to as CARDMA to compute slope characteristics of mineral consumption data over time. The patterns of these slope characteristics are compared with patterns of some known models, as described in Table 2. Once the model form and data cut have been identified, the model is fitted using a computer program referred to as GOMES, which uses orthogonal polynomials to approximate a least squares fit of the selected model to the selected (cut) data.

Based upon the examination of slope characteristics of a five-period moving average of the IU data,[17] the logistic model (Figure 1a) is identified as an appropriate trend model for aluminium IU for the 1962–85 period. Program GOMES produced the following model:

$$IU_t = \frac{1}{0.73762 + 1.02268 \times 0.892382^t}$$

t = 0 for 1962, 1 for 1963, etc . . .

R^2 = 0.892

Estimated consumption derived from the logistic IU model fit actual consumption data very well except for the last seven years (Figure 1b).

[15] J.V. Gregg *et al*, *Mathematical Trend Curves: An Aid to Forecasting*, Monograph No 1, D. van Nostrand, Toronto, 1968.
[16] *Op cit*, Ref 13.
[17] Data source for primary aluminium consumption: Metallgesellschaft Aktiengesellschaft, *Metal Statistics*, annual, Metallgesellschaft AG, Frankfurt am Main, various issues. For GDP: constant 1980 values in million US$, taken from ECLA/UN, *Economic Survey of Latin America and Caribbean*, Economic Commission for Latin America and the Caribbean, several issues; ECLA/UN, *Statistical Yearbook for Latin America*, Economic Commission for Latin America and the Caribbean, several issues; IMF, *Yearbook 88: International Finance Statistics*, International Monetary Fund, Washington, DC, 1988. IU: expressed in tonnes/million 1980 US$ GDP.

Table 2. Slope characteristics and model identification adapted to mineral forecasting data.

Model	Data transformation[a]	$\theta(a,b)$[a]	Linear feature
Linear	$y_t = z_t$	$x_t = b_t$	Horizontal
Simple modified exponential	$y_t = z_t$	$x_t = 1nb_t$	Negative slope
Gompertz	$y_t = z_t$	$x_t = 1n(b_t/a_t)$	Negative slope
Logistic	$y_t = z_t$	$x_t = 1n(b_t/a_t^2)$	Negative slope
Derivative Gompertz[b]	$y_t = \sum_{i=1}^{t} z_i$	$x_t = 1n(b_t/a_t)$	Negative slope
Derivative logistic[b]	$y_t = \sum_{i=1}^{t} z_i$	$x_t = 1n(b_t/a_t^2)$	Negative slope
Normal curve[b]	$y_t = 1n(z_t)$	$x_t = b_t$	Negative slope

[a] z_t represents the raw data; y_t is a transformation of data eg y_t is the logarithm of z_t; $y_t = z_t$ means that the raw data are used; x_t is a characteristic based upon moving average a_t and moving slope b_t computed from data on y_t according to function $\theta(a, b)$.
[b] The normal curve, derivative Gompertz, and derivative logistic models were added by D.P. Harris, *Forecasting of Mineral Industries*, Lecture notes MnEc-665, Mineral Economics Program, Department of Mining and Geological Engineering, College of Mines, University of Arizona, 1989, and the others were proposed by J.V. Gregg *et al*, *Mathematical Trend Curves: An Aid to Forecasting*, Monograph No 1, D. van Nostrand, Toronto, 1968.

The reason for truncating the first 12 years of data is to increase the asymptote of the logistic curve when fitted by least squares.

For comparative purposes a linear trend model is fitted to the IU data (Figure 2a), giving the following equation:

$$IU_t = -64.964 + 0.332t$$
$$R^2 = 0.94$$

Except for the last seven years, consumption estimates based upon linear trend values of IU also fit the consumption data very well (Figure 2b).

The procedure for forecasting mineral consumption is very straightforward and can be accomplished in two additional steps. The second step is to obtain forecasts for GDP; in this study these were made by projecting a simple exponential trend model. In the third step, mineral consumption forecasts are made simply by multiplying forecast IU by forecast GDP.

Simple IU models

Malenbaum describes IU as a function of GDP per capita (y_t):

$$IU_t = \frac{D_t}{GDP_t} = f(y_t) \tag{5}$$

This study employs two different specifications of $f(y_t)$: linear and lognormal.

A frequently selected model for data on IU and y in developed economies is the lognormal model (Equation (1)). Accordingly, a lognormal model of IU was fitted to Brazil's intensity of use and per capita GDP data using transformations and least squares methods, giving the following equation:

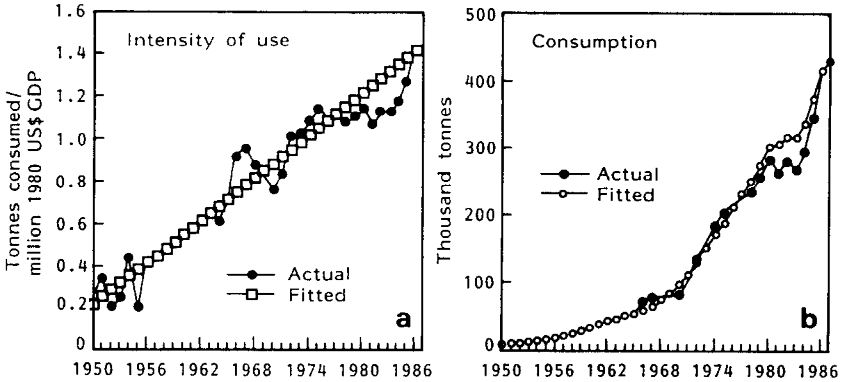

Figure 2. The linear IU trend model: comparing actual and estimated aluminium consumption and intensity of use.

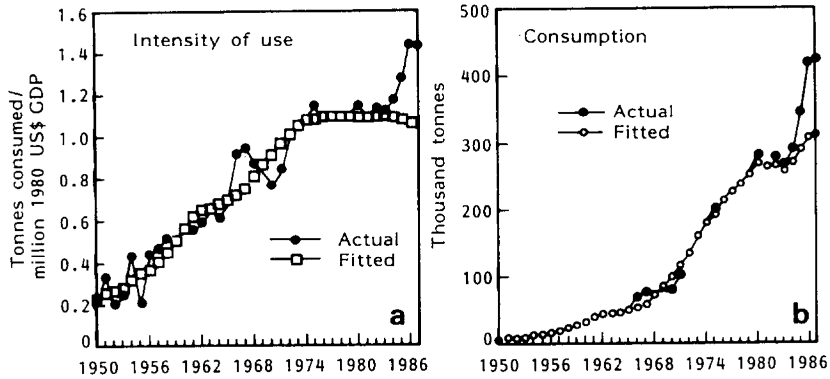

Figure 3. Comparing actual and estimated aluminium consumption and intensity of use based upon IU as a lognormal model of per capita GDP.

$$IU_t = \left(\frac{e^{-58.5603}}{1.0962\sqrt{2\pi}}\, e^{-1/2}\left(\frac{\ln y_t - 3.5046}{1.0962}\right)^2\right)$$

$$R^2 = 0.90$$

The fit of this model to the IU data is shown in Figure 3a, and the fit of consumption estimates based upon this model are shown in Figure 3b.

Using all of the data on IU and y, a simple linear model is estimated: $IU = \alpha_0 + \alpha_1 y_t$. Given the lognormal-type pattern demonstrated by Malenbaum, the use of a linear model is rationalized by the near linear pattern of the long tail for mature economies. The estimated linear GDP model (Figure 4a) is $IU_t = 0.2893 + 4.224 \times 10^{-6}\ GDP_t$ ($R^2 = 0.799$). Figure 4b depicts actual and estimated aluminium consumption, the latter being the product of GDP per capita and IU described by the linear model.

Extensive translog models of IU and consumption

Given that the lognormal is an acceptable model for the relation of IU to per capita GDP (y) established empirically by Malenbaum, it follows directly from mathematical transformation that:

$$IU_t = A\, y_t^{\beta_0 + \beta_1 \ln y_t}$$

and (6)

$$d = A\, y_t^{\beta_0 + 1 + \beta_1 \ln y_t}$$

Alternatively

$$D_t = K\, Y_t^{\alpha_0 + \alpha_1 \ln Y_t} \qquad (7)$$

where $D_t = d_t \cdot POP_t$, $Y_t = y_t \cdot POP_t$ and POP_t = population.

The simplest of possible derived demand models specifies demand to be a function of own price (P_t), price of substitute (PS_t), and income, Y_t. When this model is extended to describe across time consumption, it must also include technical change, τ_t. Thus, retaining the translog form, we have:

$$D_t = T(Y_t, P_t, \tau_t; \Omega)$$

where Ω is a vector of parameters. Alternatively, since $D_t/Y_t = IU_t$, $IU_t = T(Y_t, P_t, PS_t, \tau_t; \Omega')$, where Ω' is the same as Ω except for the exponent of y, which is decreased by 1.0. Stepwise usually, regression eliminates many of the cross product variables to give a simplified model.

Neither of these models could be applied directly to Brazilian

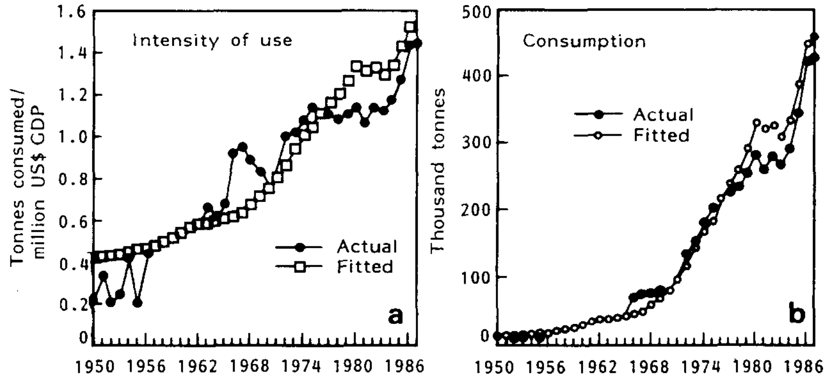

Figure 4. Comparing actual and estimated aluminium consumption and intensity of use based upon IU as a linear model of per capita GDP.

aluminium consumption because measures of technical change or productivity were not available for the time period of analysis. Moreover, the imposition of price regulation in the post-1979 period created a problem for the direct use of LME prices. To compensate for these problems time was used as a proxy for technical change and a constructed price was developed using dummy variables.

Before 1980 consumption of aluminium in Brazil was consistent with LME prices for aluminium and copper. But with the regulation of aluminium price, domestic aluminium consumption is not well explained by LME prices. Of course, if a reliable time series of post-1979 domestic price were available, that price could be employed. Instead, an artificial price variable, Pw, was constructed using LME prices for aluminium and copper and dummy variables x_1 and x_2. The dummy variables were defined in the following way:

$$x_{1t} = 1,\ 1950 \leq t \leq 1979 \text{ and } x_{1t} = 0,\ 1979 < t \leq 1987$$
$$x_{2t} = 0,\ 1950 \leq t \leq 1979 \text{ and } x_{2t} = 1,\ 1979 < t \leq 1987$$

The constructed price, Pw, was obtained through the following equation:

$$\ln Pw_t = -7.6227 + \ln Pal_t(2.0472 \,.\, x_{1t} + 2.1057 \,.\, x_{2t}) \\ -\ln Pal_t \,.\, \ln Pcu_t(0.1365 \,.\, x_{1t} + 0.1442 \,.\, x_{2t})$$

where:

Pal_t = LME aluminium price
Pcu_t = LME copper price as a substitute price[18]

Stepwise regression on data for 1950–80 produced the following consumption model:

$$D_t = 4.5277 \times 10^{-6}\, Y_t^{1.3587}\, Pw_t^{-0.0824}\, \tau_t^{0.3359}$$
$$R^2 = 0.971 \qquad F(3,30) = 343.5$$

where

D_t = aluminium consumption
Y_t = gross domestic product (billion 1980 US$)
τ_t = time as a proxy for technical change
Pw_t = constructed price

Thus, in this case the translog degenerates to a common log-linear model. Noteworthy is the very small price elasticity of consumption (0.08) and the large effect of the proxy (time) for technology. Figure 5 shows the fit of the translog consumption model for the base period

[18]Average annual LME prices in US$ tonnes. Source: *op cit*, Ref 17, Metallgesellschaft, various issues and IMF. Deflator 1980 = 100, implicit price index for metals Department of Commerce (1988).

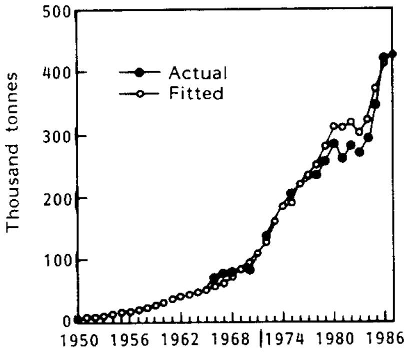

Figure 5. Actual and estimated consumption: based upon extensive translog model.

(1950–80) and of the *ex post* forecasts by the model for years 1981–87. Although the model fits the data very well on most of the base period, the *ex post* forecasts fit poorly the actual consumption for the years 1981–83.

Learning IU models

Improvement of system performance that is repetitive and predictable is considered to be the result of learning. A learning model describes the empirical relations between output quantities and quantities of certain inputs, where improvement by learning is present.

Learning models are important management tools and have been used to assist in planning, budgeting and goal setting, and economic analysis.[19] Examples of learning in mine and metallurgical plants are given by Pierson.[20]

There are two motivations or rationalizations for using learning models for forecasting mineral consumption. The first motivation is that mineral consumption must reflect to some degree the materials savings created by technical changes in materials utilization in the manufacturing of consumer durables, and these changes are evidence of a type of learning. Of course, even when each end-use shows a learning effect, if learning rates differ markedly, a single learning model for aggregate consumption may not be effective; moreover, extrapolation of the aggregate learning pattern requries strong assumptions. However, when data reveal a strong learning relation its extrapolation may be very useful and provide a robust forecast.

The second motivation or rationalization does not derive from learning and improved application of a given technology at all but from the nature of time series data and their use for long-range forecasting. There are two elements to this rationale. The first is that the only realistic goal for long-range forecasting is to forecast the overall level of consumption, ignoring the cyclical components. The second element is the recognition that time series are analogous to time derivatives and as such emphasize short-term and cyclical movements. Thus, if

$$D(t) = \frac{\mathrm{d}[\overset{*}{D}(t)]}{\mathrm{d}t}$$

we should examine the time trends of

$$\overset{*}{D}(t) = \int^{t} D(t)\mathrm{d}t$$

Then, given D^{*}_{t}, our estimate of $D(t)$ is

$$\frac{\mathrm{d}[D^{*}(t)]}{\mathrm{d}t}$$

Simply stated, this approach forecasts the integral series as an indirect means of forecasting the derivatives, rather than forecasting the time derivatives directly, because pattern analysis and projections of the integral series are more robust than that of the derivative. Finally, if comparison of one integral series with another which is a determinant of the former reveals a learning pattern, a learning model may be an effective forecasting method.

A simple learning model was defined in the following way:

$$\frac{\overset{*}{D}_t}{\overset{*}{Y}_t} = L(\overset{*}{Y}_t) \qquad (8)$$

[19] A. Belkaouni, *The Learning Curve: A Management Accounting Tool*, Quorum Books, London, 1986.
[20] G. Pierson, 'Learning curves make productivity gains predictable', *Engineering and Mining Journal*, August 1981, pp 56–64.

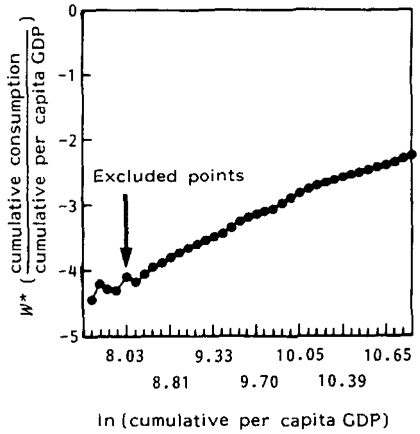

Figure 6. The simple learning relation.

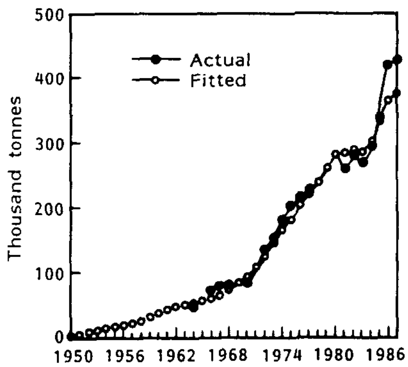

Figure 7. Actual and estimated consumption: based upon a simple learning model.

where

$\overset{*}{D}_t$ is cumulative mineral consumption
$\overset{*}{Y}_t$ is cumulative per capita income
$L(\ )$ is a learning function

Once parameters of the learning model $L(\ )$ for $\overset{*}{W}_t$ have been estimated, the model can be transformed to describe annual consumption, D_t:

$$D_t = [L'(\overset{*}{Y}_t)\overset{*}{Y}_t + L(\overset{*}{Y}_t)]Y_t \qquad (9)$$

where

$$L'(Y_t) = \frac{d[\overset{*}{W}_t]}{dt} \quad \frac{d[L(\overset{*}{Y}_t)]}{dt} \text{ and } Y_t = \frac{d[\overset{*}{Y}_t]}{dt}$$

Figure 6 shows a plot of $\overset{*}{W}_t$ against $\overset{*}{Y}_t$. By cutting the data so as to exclude the five leading points a robust pattern is defined. Regression analysis produced the following linear equation:

$$\ln\overset{*}{W}_t = -10.6487 + 0.7763\ln\overset{*}{Y}_t$$
$$R^2 = 0.995$$

where

$\overset{*}{W}_t$ = cumulative consumption/cumulative per capita GDP
$\overset{*}{Y}_t$ = cumulative per capita GDP

Alternatively, $\overset{*}{W}_t = 2.373 \times 10^{-5}\ \overset{*}{Y}_t$ and according to (9),

$$D_t = 4.2147 \times 10^{-5}\ Y_t\ .\ \overset{*}{Y}_t^{\,0.77631}$$

Consumption is forecast for a future year by substituting the per capita GDP projected for that year and projected cumulative per capita GDP into this equation. Figure 7 shows the fit of consumption derived from the learning model to the actual aluminium data.

Harris[21] suggested the examination of an extensive learning model. The following is a possible form for such a model to forecast aluminium consumption in Brazil:

$$\overset{*}{Z}_t = T(\overset{*}{Y}_t, \overset{*}{Pw}_t, \overset{*}{\Gamma}_t) \qquad (10)$$

where

$\overset{*}{Z}_t = \overset{*}{V}_t / \overset{*}{Y}_t$

$\overset{*}{V}_t = \sum_i^t D_i\ .\ Pw_i$ cumulative value

$\overset{*}{Y}_t = \sum_i^t Y_i$ cumulative per capita income

$\overset{*}{Pw}_t = \sum_i^t Pw_i$ cumulative weighted price

$\overset{*}{\Gamma}_t = \sum_i^t \Gamma_i$ cumulative technological factor

Figure 8 shows aluminium consumption estimated by the following extensive learning model:

$$\ln\overset{*}{Z}_t = -7.3798 - 0.4515\ .\ \ln\overset{*}{Y}_t + 0.0272\ .\ \overset{*}{Pw}_t + 2.585\ln\overset{*}{\Gamma}_t$$
$$R^2 = 0.96$$

[21]D.P. Harris, *Mineral Resources Appraisal – Mineral Endowment, Resources and Potential Supply: Concepts, Methods and Cases*, Clarendon Press, Oxford, 1984.

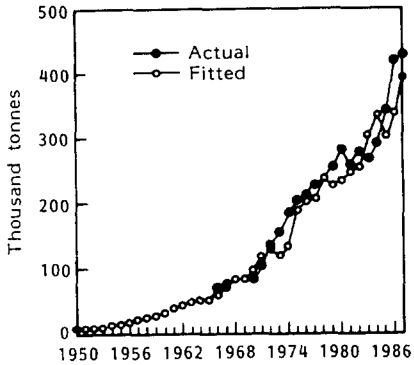

Figure 8. Actual and estimated consumption: based upon an extensive learning model.

Exponentiating both sides of this equation,

$$\overset{*}{Z}_t = 6.23726 \times 10^{-4}\, \overset{*}{Y}_t^{-0.4515}\, \overset{*}{Pw}_t^{0.0272}\, \overset{*}{\Gamma}_t^{2.5851}$$

Alternatively, since $\overset{*}{Z}_t = \overset{*}{V}_t/\overset{*}{Y}_t$

$$\overset{*}{Z}_t = 6.23726 \times 10^{-4}\, \overset{*}{Y}_t^{-0.5485}\, \overset{*}{Pw}_t^{0.0272}\, \overset{*}{\Gamma}_t^{2.5851}$$

Given $\dfrac{d\overset{*}{V}_t}{dt} = V_t = D_t \,.\, PW_t$

$$D_t = \frac{-0.4515\overset{*}{Y}_t^{-0.4515}\, \overset{*}{Pw}_t^{0.0272}\, \overset{*}{\Gamma}_t^{2.5851}\, \overset{*}{Y}_t + 2.5851\, \overset{*}{Y}_t^{-0.5485}\, \overset{*}{Pw}_t^{-0.0272}\, \overset{*}{\Gamma}_t^{1.5851}\, \Gamma_t}{Pw_t\,[1603.27 - 0.0272\, \overset{*}{Y}_t^{0.5485}\, \overset{*}{Pw}_t^{-0.9728}\, \overset{*}{\Gamma}_t^{2.5851}]}$$

Vector projections

Long-range forecasting models of consumption or production of minerals that contain economic variables must either forecast the economic variables or generate scenario forecasts. As most economic variables are correlated to some extent, forecasting them as a means to forecasting consumption presents a statistical problem. When economic variables are highly correlated, forecasting a dependent variable using separate projections of each explanatory variable is problematic because the predicted values may not be consistent.

One solution is to replace the original m explanatory variables by m especially constructed variables, referred to as factor scores, which are linearly independent of each other so that each factor score can be independently projected. The advantage of this technique is that it honours the correlation between the variables.[22]

Given the m factor scores and consumption for each of the time periods in the data base, a linear model can be estimated by conventional regression analysis which relates consumption to the factor scores:

$$D_t = \alpha_0 + \alpha_1 F_{1t} + \ldots\ldots + \alpha_m F_{mt} \tag{11}$$

Given parameter estimates $\hat{\alpha}_i$, $i = 0, \ldots\ldots, m$, consumption D_{t^*} in future time period t^* is forecast by evaluating the consumption model on projected values of the factor scores, F_{it^*}. Of course, this approach requires the prior estimation of factor scores equations and the transformations to the factor space, meaning the replacement of the observed values of the original variables by factor scores. A factor score equation is a linear equation in the original variables:

$$F_{jt} = b_1\left(\frac{X_{1t} - \bar{X}_1}{s_1}\right) + \ldots\ldots + b_m\left(\frac{X_{mt} - \bar{X}_m}{s_m}\right)$$

where b_i is the factor score coefficient of the ith explanatory variable for the jth factor and X_i, sj are mean and standard deviations respectively of the ith explanatory variable for the jth factor.

These factor score coefficients are obtained by processing the data on the original explanatory variables through a multivariate technique known as factor analysis when commonalities are specified to be 1.0, meaning that there are no factors unique to any one variable, all factors being in common.

As an example consider a model of Brazil's aluminium consumption which consists of two explanatory variables, Y_t (per capita income) and

[22]D.P. Harris, *Forecasting of Mineral Industries*, Lecture notes MnEc–665, Mineral Economics Program, Department of Mining and Geological Engineering, College of Mines, University of Arizona, 1989.

Pw_t (the constructed price variable for aluminium). Processing observations on these two variables through factor analysis, given commonalities of 1.0, yielded the following factor score equations:

$$F_{1t} = -0.1728\left(\frac{Y_t-1280}{542.885}\right) + 1.0423\left(\frac{Pw_t+0.4522}{0.2978}\right)$$

$$F_{2t} = 1.0423\left(\frac{Y_t-1280}{542.885}\right) - 0.1728\left(\frac{Pw_t+0.4522}{0.2978}\right)$$

Using these two equations, observations for the original variables (Y_t and Pw_t) can be replaced by the linearly independent scores (F_{1t} and F_{2t}). Then, given these scores and data on aluminium consumption, the following linear consumption function is estimated:

$$D_t = -125.924689 + 10.3038F_{1t} + 32.6694F_{2t} \tag{12}$$

To forecast consumption for some future time, t^*, requires projecting the factor scores for that time, giving F_{1t^*} and F_{2t^*}. Evaluation of Equation (12) gives D_{t^*} a forecast of consumption in year t^*. Of course, F_{1t^*} and F_{2t^*} could be projected by different means, including objective and subjective trend projection. In the case study of Brazil's aluminium consumption in the year 2000, projections were made by expressing each factor score as a linear model of per capita GDP and previous values of the factor:

$$F_{it^*} = \Phi(Y_{t^*}, F_{it^*}-1, \ldots), i = 1, 2 \tag{13}$$

Thus projection of the factor scores requires first the projection of per capita GDP and then iterative evaluation of the projection equations on forecast per capita GDP and previous factor scores. Substitution of these projected factor scores into (12) provides an estimate of future consumption for each year of interest.

Variable coefficients

Consider again the foregoing consumption function:

$$D_t = \alpha_0 + \alpha_1 F_{1t} + \alpha_2 F_{2t} \tag{14}$$

where F_{1t} and F_{2t} are factor scores.

This is the classicial fixed coefficients model. Now, suppose that α_0, α_1 and α_2 vary across time and express this variation as a function of time: $\alpha_0 = f_0(t)$, $\alpha_1 = f_1(t)$, and $\alpha_2 = f_2(t)$. When these functions are known, we have the following consumption model:[23]

$$D_t = a_0 \mathrm{f}_0(t) + a_1 \mathrm{f}_1(t) F_{1t} + a_2 \mathrm{f}_2(t) F_{2t} \tag{15}$$

where a_0, a_1 and a_2 are scaling constants.

Forecasting by this model is done in three steps. First forecast the factor scores F_{1t^*} and F_{2t^*}; forecast $f_0(t^*)$, $f_1(t^*)$ and $f_2(t^*)$; and then forecast D_t by evaluating expression (15) on $f_0(t^*)$, $f_1(t^*)$, $f_2(t^*)$, F_{1t^*} and F_{2t^*}. Of course, applied analysis must first determine if a variable coefficients model is required. For the two-factor model of (15) this is done by computing $\triangle D_t$, $\triangle D_t/\triangle F_{1t}$, and $\triangle D_t/\triangle F_{2t}$ for each observation. When one or more of these shows a significant trend a time variable coefficient model may be indicated. Fluctuation around a mean value indicates that the classical fixed coefficient model is appropriate.

Plots of the coefficients for the factor model of (15) for Brazilian aluminium consumption are shown in Figures 9, 10 and 11. The

[23] *Ibid.*

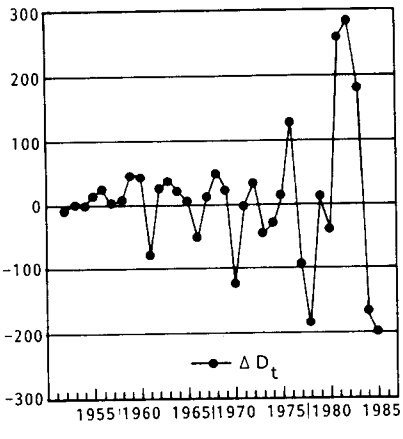

Figure 9. Time coefficient – ΔD_t.

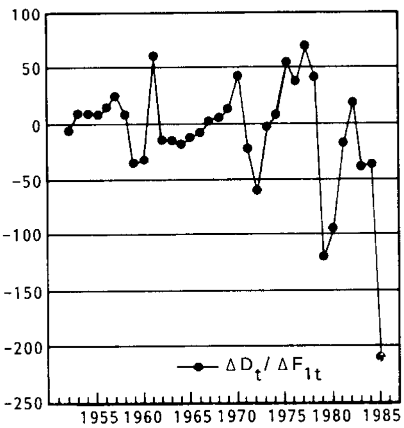

Figure 10. Time coefficient – $\Delta D_t / \Delta F_{1t}$.

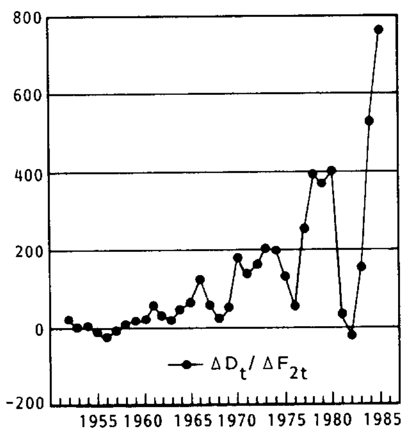

Figure 11. Time coefficient – $\Delta D_t / F_{2t}$.

intercept term comes closest to meeting the conditions for a fixed (constant) value, namely that estimates fluctuate around a mean value (Figure 9). Clearly, the presence of strong cycles makes the judgement difficult; however, there is a suggestion of a downward trend in the estimates. There can be little doubt about the presence of trends in the coefficients of the first and second factors (Figure 10 and Figure 11 respectively), indicating the need to consider these trends in the estimation of values of the coefficients when making a forecast of consumption. This is indicated in Equation (16) by the time functions, $f_0(t)$ $f_1(t)$ and $f_2(t)$:

$$D_t = 1.0488 f_0(t) + 0.9787 f_1(t) F_1(t) + 1.0180 f_2(t) F_2(t) \tag{16}$$

Figure 12 shows the fit of the model of (16) to the actual data on aluminium consumption.

A forecast of aluminium consumption for year 2000 was made by projecting time trends of the coefficients and by projecting future factor scores by Equation (13).

Evaluation of forecasting methods

The evaluation, selection and use of a forecasting method must consider several factors, such as the degree of accuracy desirable, the time period (*ex post* or *ex ante*) to be forecast, availability of comprehensive data, the time available to make all projections, the historical context and information about the commodity and the cost–benefit of the forecast.

Techniques vary in their costs, as well as in scope and accuracy. Measures of accuracy can help to assess costs and benefits in a particular situation on forecasting.

In this work the accuracy of the respective forecasting models is evaluated by average absolute error (AAE), which is defined as follows:

$$AAE = \sum_{i=1}^{n} \left(\frac{| \, Ac - Pr \, |}{Pc} \right)_i / n$$

where

Ac = actual value
Pr = predicted value
n = number of periods over which the model is simulated

Evaluation of the accuracy of forecasting models is based on *ex post* performance on a test period (1981–87). For this purpose the last few years were deliberately withheld in the estimation of the models. Subsequent to evaluation of *ex post* forecasting accuracy, the models were reestimated on the entire data set (1950–87) and these models were used to forecast aluminium consumption in the year 2000. Table 3 shows the accuracy of results for the different models.

The simple learning and time varying coefficients models have the highest *ex post* (1981–87) forecasting accuracies (0.07); the extensive translog accuracy of 0.09 is only slightly lower. The least accurate of the models in *ex post* forecasting is the linear IU model, with the lognormal IU and extensive learning models only slightly less inaccurate.

The lognormal IU, extensive translog consumption, logistic trend and time varying coefficient consumption models presented the best fits for the entire data set (1950–87). One of the reasons for the good fit of the

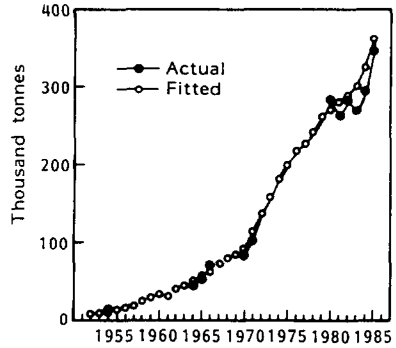

Figure 12. Actual and estimated aluminium consumption: based upon time varying coefficient factor model.

two former models is the key role played by GDP in shaping consumption in the study period, as pointed out by Suslick and Harris.[24]

Table 4 summarizes the forecasts of aluminium consumption in the year 2000 based on the entire data set (1950–87). Rather than forecasting price in the extended translog and extensive learning models, demand forecasts were developed under an assumption that the prices of aluminium and copper will remain constant at their average values for the data period. GDP estimates are based upon World Bank projections of two scenarios of 3.6% and 6.1% annual average rates of growth;[25] these give the low and high forecasts respectively.

Considering the *ex post* forecasting accuracies of the various models and their forecasts for year 2000, it seems reasonable to forecast Brazilian consumption of aluminium in year 2000 to be in the range of 800 to 1 300 thousand tonnes. This range excludes both the high and low forecasts but includes the extensive translog and time varying coefficient models' forecasts of consumption; both of these models demonstrated small *ex post* forecast errors. This range does not include the high bound of the simple learning, a model which also demonstrated very small *ex post* forecast errors.

The midpoint of this range agrees approximately with the results found by World Bank.[26] Interestingly, the selected range conforms closely to that of the extensive translog model of consumption, not IU. Moreover, this is the only model which included along with price and GDP a proxy for technical change. Thus, since the constructed price variable includes aluminium and copper prices, this model includes the major determinants of across time consumption: price induced materials substitution, level of economic activity and changes in aluminium using products and in product technologies.

[24]*Op cit*, Ref 4.

[25]World Bank, *World Development Report 1988*, Oxford University Press, New York, 1988.

[26]World Bank, *Worldwide Investment Analysis: The Case of Aluminum*, World Bank Staff Working Paper, No 603, 1983.

Table 3. Average absolute errors (AAE) of the estimated methods for aluminium consumption in Brazil.

Method	Observed period	AAE	*Ex post* period	AAE
Logistic IU trend[a]	1962–87	0.07		
Linear IU trend[a]	1950–80	0.12	1981–87	0.10
Linear IU model[b]	1950–80	0.22	1981–87	0.13
Linear IU model[b]	1950–87	0.21		
Lognormal IU model	1950–80	0.12	1981–87	0.12
Lognormal IU model	1950–87	0.12		
Extensive translog consumption	1950–80	0.13	1981–87	0.09
Extensive translog consumption	1950–87	0.11		
Learning IU model	1950–80	0.15	1981–87	0.07
Extensive learning IU model	1950–80	0.14	1981–87	0.11
Extensive learning IU model	1950–87	0.14		
Time varying coefficient	1952–80	0.10	1981–85	0.07
Time varying coefficient	1952–85	0.09		

[a] Models based upon time trend;
[b] models based upon GDP.

Table 4. The year 2000 Brazil demand forecasts for aluminium by different methods.[a]

Method	Time	Low	High
Logistic IU trend	1 068.7	na	na
Linear IU model	1 511.8	na	na
Linear IU model	na	983.8	1 752.8
Lognormal IU model	na	513.0	604.5
Extensive translog consumption	na	829.1	1 249.8
Learning model for IU	na	1 093.9	1 588.7
Extensive learning model for IU	na	862.0	977.7
Time varying coefficient for consumption	na	1 009.1	1 379.4

[a] Values expressed in thousand tonnes.
Time = forecasting method based only on time-trend projections; GDP and GDP per capita forecast from World Bank, *Worldwide Investment Analysis: The Case of Aluminum*, World Bank Staff Working Paper, No 603, 1983; low = average annual rate of 3.6% over the period 1987–2000; high = average annual rate of 6.1% over the period 1987–2000; na = not applicable.

Conclusions

The purpose of this study was to forecast Brazil's aluminium consumption for year 2000 using economics, quantitative methods, and models especially selected for their long-range capability. These methods and models differ from those commonly used in several ways: formal specification of the intensity of use measure as a quantitative economic model, application of formal statistical methods, use of learning patterns and incorporating relations with variable parameters.

Evaluation of accuracy of *ex post* forecasts for the 1981–87 test period indicates that the simple learning model and the time varying coefficients consumption model, followed by the extensive translog consumption model, showed the highest accuracies. The least accurate of the models in this period is the linear IU model, with the lognormal IU and extensive learning models only slightly less inaccurate.

Based upon the accuracies of the *ex post* forecasts and the patterns of the forecasts, Brazil's aluminium consumption for year 2000 is forecast to be within the range of 800 to 1 300 tonnes. This range covers the forecasts made by the extensive translog consumption model and the time varying coefficients model, both of which gave high accuracies in the *ex post* forecast period.

The lower bound of this range is associated with a low rate of growth in Brazil's GDP of about 3% and the upper bound is associated with a rate of a growth of about 6%. Even at the low rate Brazil's aluminium consumption is expected to approximately double that of 1986, while the high growth rate would require a tripling of consumption.