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Alternative techniques for forecasting mineral commodity prices

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ABSTRACT

Forecasting mineral commodity (MC) prices has been an important and difficult task traditionally addressed by econometric, stochastic-Gaussian and time series techniques. None of these techniques has proved suitable to represent the dynamic behavior and time related nature of MC markets. Chaos theory (CT) and machine learning (ML) techniques are able to represent the temporal relationships of variables and their evolution has been used separately to better understand and represent MC markets. CT can determine a system's dynamics in the form of time delay and embedding dimension. However, this information has often been solely used to describe the system's behavior and not for forecasting. Compared to traditional techniques, ML has better performance for forecasting MC prices, due to its capacity for finding patterns governing the system's dynamics. However, the rational nature of economic problems increases concerns regarding the use of hidden patterns for forecasting. Therefore, it is uncertain if variables selected and hidden patterns found by ML can represent the economic rationality. Despite their refined features for representing system dynamics, the separate use of either CT or ML does not provide the expected realistic accuracy. By itself, neither CT nor ML are able to identify the main variables affecting systems, recognize the relation and influence of variables though time, and discover hidden patterns governing systems evolution simultaneously. This paper discusses the necessity to adapt and combine CT and ML to obtain a more realistic representation of MC market behavior to forecast long-term price trends.

1. Introduction

Mineral commodities (MCs) have been fundamental for human development and exploited for more than 7000 years. Gold, copper, silver and iron ore are the oldest MCs being mainly used for construction, fabrication of domestic goods and also as a reservoir of value [1–7]. Mineral activities provide income to companies and investors, promoting investments and technological developments. Governments receive taxes and royalties from mineral exploitation and take advantage of technological developments through the improvement of workforce skills. Mining industry generates jobs, increases incomes and develops infrastructure [8–13]. The significance of this industry to economic, technological and social development is quite obvious [6]. Therefore, understanding the future behavior of MC prices is vital for all agents of the economy, so-called governments, companies and society.

Econometric, qualitative, survey, stochastic and time series methods have been mainly used to forecast MC prices and trends [14]. Due to the close relation between the global economy and MC prices, econometric models are the oldest and the most intensively used methods [15,16]. However, historical data do not guarantee accurate predictions, as there is no certainty that past events will be repeated in the future at the same intervals and intensity. The common assumption of random behavior of MC markets has encouraged the use of stochastic-Gaussian models working within pre-established and well known boundaries for forecasting prices [17]. Nonetheless, each MC market has its own features regarding processing, trading, transportation and application that result in particular configurations. These differences have significant implications for pricing not only for a particular MC, but also for the complementary and substitute commodities. Therefore, there is ongoing debate as to whether MC prices do exhibit random behavior. Furthermore, despite the long-term balance asserted by microeconomic theory, prices and costs do not fluctuate randomly [18].

The random behavior of systems evolving through time has been questioned in nature and the stock market since 1963, then

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CT characterized the concept of sensitivity to initial conditions and self-similarity patterns of variables in dynamic systems [19–26]. CT and ML arose in the 1960s as alternative methods able to represent dynamic systems by describing the causal relation between variables. They have been used to represent and predict the behavior of complex dynamic systems in the fields of medicine, neuroscience, meteorology, stock markets and also in MC markets [17,21,27–33].

The temporal and learning properties of CT and ML techniques are appropriate for capturing MC markets behavior. CT enables an accurate determination of two important properties to reconstruct chaotic systems dynamic so-called embedding dimension and time delay. The embedding dimension corresponds to the number of variables governing the system. The time delay corresponds to the temporal influence of variables, meaning for how long changes of variables can affect the system [23,34–37]. ML can emulate human learning and find hidden patterns into large high dimensional data sets. Thus, ML has become one of the most powerful techniques to represent complex dynamic systems and for forecasting MC prices and trends. Despite their refined features to represent dynamic systems, neither CT nor ML can represent the properties of MC markets by themselves. They are unable to simultaneously detect the embedding dimension and the time delay, select the most significant variables affecting behavior and discover the patterns governing the system. However, a combination of CT and ML provides a better representation of MC markets and prices.

This paper investigates the main features of different MC markets, their dynamics and key features to enable better understanding of the principles behind the current techniques for forecasting MC prices. Several of these forecasting techniques are examined and their main advantages and drawbacks discussed. This paper further introduces two techniques to represent complex dynamic systems so-called CT and ML. Finally, the necessity and suitability to combining CT and ML to obtain a more realistic representation of MC markets to forecast long-term prices trends were discussed.

2. Mineral commodities

Mineral Commodities (MCs) are non-renewable resources classified as energy, metallic and non-metallic. Energy commodities refer to fluid and solid fossil fuels used for power generation. This group encompasses oil, gas, coal and uranium. Non-metallic commodities are defined as those minerals that do not contain recoverable metals. The group includes (among others) phosphate rocks, potash, salts, clays, sands, boron, and crushed and broken stones such as limestone and granite. Metallic commodities are defined as solid materials containing an appropriate composition of metal ores to be extracted and used as a metal precursor or as a direct raw material for manufacturing. They are categorized as either ferrous, light, precious or base metals [14,38,39]. However, this grouping can vary according to ultimate uses, market configuration and trading peculiarities (e.g. silver). MCs are traded worldwide in diverse futures and spot exchange markets (EXM). Table 1 provides the main markets for mineral commodities. The New York Mercantile Exchange (NYMEX) and the London Metal Exchange (LME) are the most important markets.

As with any other product traded in the markets, MC prices are fundamentally determined by supply and demand. Supply is driven by production costs and technology that reflect the competitiveness of the business. Demand is on the other hand driven by income and customer preferences that reflect the strength of the economy [9,10,12]. Economic, technological, political and psychological factors are the main variables affecting the balance between supply and demand in MC markets; therefore, their prices.

Table 1
Main exchange markets for mineral commodities (Adapted from [14,16,30,40–44]).

Market	Mineral commodities
London Metal Exchange (LME)	Aluminum, Aluminum alloy (NASAAC), copper, lead, nickel, silver, zinc, North American Special.
New York Mercantile Exchange (NYMEX)	Coal, natural gas, palladium, platinum, WTI
Shanghai Metal Exchange (SHME)	Aluminum, copper, lead, nickel, tin, zinc
Commodity Exchange of New York (COMEX)	Copper, gold, silver
Tokyo Commodity Exchange (TOCOM)	Aluminum, gold, silver, palladium, platinum
Chicago Board of Trade (CBOT)	Gold, silver
Kuala Lumpur Tin Exchange	Tin
Intercontinental Exchange (ICE)	Brent, coal, natural gas
European Energy Exchange (EEX)	Brent, coal, natural gas
Multi Commodity Exchange (MCX)	Coal, natural gas

The relationship between economic development and mineral industries is obvious. Increasing demand for goods and services has been historically driven by the economic expansion of developing countries. As a result, MC production grows to meet this increasing demand. Pei and Tilton claimed that in the short-term, the demand for MCs has elastic behavior to the Gross Domestic Product (GDP) and inelastic to the income [45]. Technological advances have reduced the adverse economic effects of mineral depletion. Development of more efficient and low cost technologies for mining and processing has increased ore reserves and mineral recovery performance [10,46]. Governments have an important influence on MC markets by introducing trading policies and market regulations generally when MC prices rise. Periods with high prices have encouraged the emergence of economically nationalist governments adopting regulations aimed at increased taxes and royalties [9,13,47]. Governments have gone further, manipulating not only taxation, but also the base for prices and company revenues [48]. Therefore, it human psychology must also be taken into account when considering systems exhibiting dynamic equilibrium through time, such as technological developments and economic growth [49], having significant impact on MC prices. In the past, Smith also claimed that market expectations are generated by the preferences of customers driven by psychological states [12]. In the risk premium theory, Keynes restated the significance of expectation of markets stating that under "normal" supply conditions, expected prices exceed current prices [50,51]. Human beings are complex systems fluctuating between disordered phases and complex stages of order. Influenced by the fluctuating environment and their own genetic inheritance, humans acquire experience over time. Experience can change their attitude which has a crucial influence for the decision-making process, especially during critical situations [52].

The evolution of MC prices and supply levels can provide important clues to understand the effects of economic, political, technological and psychological factors on MC markets. Fig. 1 shows the evolution of copper prices and production between 1850 and 2015, which also includes important geopolitical and economic events occurred in this period. The graph emphasizes the strong temporal relation between copper prices and economic, financial and technological factors.

3. Background of forecasting of mineral commodity prices

The analysis of market data provides important guidelines for choosing the most suitable model to predict prices [30,53]. Qualitative, trend exploration, linear, econometric, stochastic and time

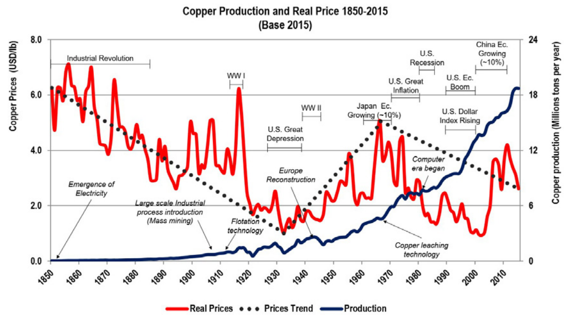


Fig. 1. Copper price and production. From electricity emergence to present times. Adapted from [2,7,60–64].

series models have been used separately or in combination to forecast metal prices [14,16,54–59].

3.1. Econometrics

The relationship between commodity markets and the economic environment has long been used in forecasting prices. It assumes that supply and demand are fundamental drivers of economic growth and prices balance. To forecast prices, historical supply and demand levels are correlated with prices by using statistical and mathematical tools [14].

Co-movement of commodity behavior has been asserted as a suitable approach to predict prices. Arezki et al. noted that the long-term behavior of a particular primary commodity (group formed by 25 commodities that includes 11 MC (Al, Cu, Pb, Zn, Sn, Au, Ag, pig iron, coal and oil)) can be predicted by using co-movements of another commodity which fits better compare to the random walk models [65]. The random walk models for forecasting the prices state that the price fluctuations have no memory, therefore, historical data and events cannot be used to predict time-series behavior [66]. The strong co-movement similarity between aluminum, copper, lead, iron ore, zinc, silver, platinum, tin, mercury and ferrous scrap prices was verified by Byrne et al. [67], Roberts [68] and Rossen [38].

Vector Auto Regression (VAR) and Vector Error Correction (VEC) techniques have been used to forecasting MC prices. VAR are simple multivariable models that use past values of all variables in the system to describe the behavior of each variable [69]. VEC models are special cases of VAR for stationarity variables. They are able to capture complex bi-directional movements without any assumption regarding the existence and direction of causality [40,70]. VAR and VEC techniques are capable of examining past information to understanding variables behavior and detecting their fluctuations. However, the unlimited search of past information to infer future behavior is a drawback for forecasting when not all past information is relevant to the future or loses significance over time.

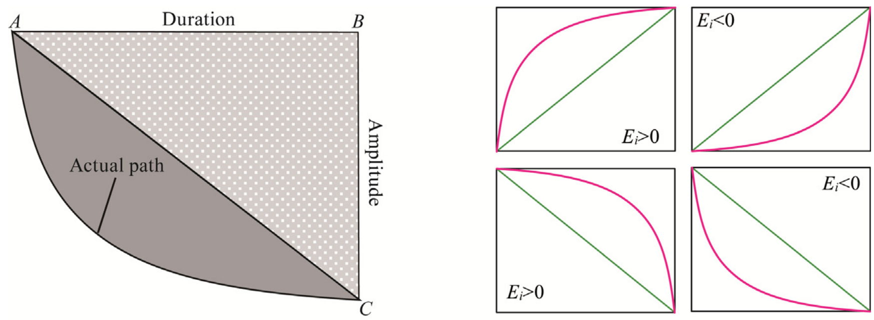
Haque et al. used VEC and VAR to examine the long-term relationship between iron ore prices and the U.S. Dollar (USD) versus Australian Dollar (AUD) exchange rate [71]. The USD represent the quote currency and the AUD the production currency. They concluded that the causality relationship occurs in a single way, where changes in the USD/AUD have been the results of price fluctuations. Kulshreshtha and Parikh used the VAR to demonstrate the relationship between coal demand and structural and technological changes in the Indian market [70]. It was claimed that coal demand in India is slightly elastic to prices but very elastic to GDP, which is a common pattern exhibited for developing economies at early stages. Walls used root Augmented Dickey-Fuller (ADF) and VAR techniques to determine future and expected spot prices of natural gas in the U.S. market [72]. The research assessed the relationship between natural gas prices and logistics, and distribution features of the market. At that time (in 1995) the U.S. market was characterized by many rigid regulations and the government intervention. Walls concluded that spot and future prices of natural gas are non-stationary, and that future prices are unbiased predictors of spot prices regardless the location of the nodes of the U.S. gas pipeline [72].

The research using VEC and VAR has demonstrated the temporal relation of variables. Haque et al. further demonstrated the temporal relationship between MC prices and economic variables describing an endless cyclical behavior of cause and effect [71]. The research provides evidence that MC prices fluctuations can affect the economy and the international trade performance of producer countries, due to exchange rate changes. Exchange rate fluctuations inevitably affect revenues, production costs and investments in the mineral extraction sector. Variations in the financial performances of mining companies promote new production and trading strategies to adjust costs. Production cost adjustments affect the competitiveness of the market resulting in unavoidable MC price fluctuations. Changes in MC prices affect exchange rates, becoming the starting point of a new adjustment period for producer economies, mining companies and the MC prices. The temporal relation and sensitivity of local economies to MC markets fluctuations are also stated by Kulshreshtha and Parikh [70]. They noted that at early stages of economic developments, MC price fluctuations have slight effects on demand. They further assert that MC demand elasticity to the price increases in direct relation to economic growth, which is pushed by the increasing demand for raw materials for infrastructure and goods production. Thus, the temporal relationship between MC prices and the evolution of economies is evident.

The dynamic and uncertain nature of the MC prices have also been stated for non-competitive markets through stationarity tests. Stationarity is a significant feature of dynamic time-related systems [33,34,73–76]. Haque et al. stated the stationarity of long term iron ore prices despite its oligopolistic configuration [71]. Walls also stated the stationarity of natural gas prices in the U.S. market which had many rigid regulations at time of the assessment [72].

Rossen introduced a novel econometric technique to assess the dynamics of metal prices by combining four methodologies to determine short and long-term price cycles, the amplitude and intensity of price fluctuations and trend turning points [38]. The dynamics of monthly prices of twenty MCs over the last hundred years was examined and grouped into five categories: non-ferrous metals (copper, zinc, tin, lead); precious (gold, silver, platinum, palladium); steel alloys (chromium, cobalt, manganese, molybdenum, nickel, tungsten); lights (aluminum, magnesium), and electricals (antimony, bismuth). In the model, short-term prices are driven by short-term cycles that normally evolve over two to eight years. The approximate regions of turning points were identified by using the maximum and minimum neighborhoods in the major cycles. Finally, the duration and intensity of price fluctuations were studied by the excess index (E_i) developed by Harding and Pagan that uses a "triangle approximation" (*TA*) [77], which is the main feature of the model.

In *TA* the amplitude (A_i) and duration (D_i) of price fluctuations are described by the leg (*BC*) and (*AB*) of the triangle *ABC* respectively as seen in Fig. 2a. The *TA* of "cumulative movements" (C_{Ti}) is defined by the area of the triangle, where $C_{Ti} = 0.5(D_i \times A_i)$. The "actual cumulative movements" (C_i) correspond to the sum of the rectangles that approximate to the triangle, where $C_i = (1^T r_i - 0.5A_i)$. However, as C_i may vary from the C_{Ti} the excess of cumulated movements must be removed, thus the rate of growth is defined by $E_i = ((C_{Ti} - C_i + 0.5A_i))/D_i$ as seen in Fig. 2b.



(a) Stylised cycle

(b) Shape of boom and slump phases and corresponding values of the excess index

Fig. 2. The excess index (E_i). Modified from [38,77].

Rossen also argued for the asymmetry of price cycles where slumping phases are significantly longer than the booming phases [38]. However, insignificant correlation between the amplitude and duration of price fluctuations was found. It was also observed that pattern similarity is not a necessary characteristic of metal prices, but precious and non-ferrous metals shared a common pattern. As most metals are complementary or substitute products for industrial use and are traded in the same EXM is likely that pattern similarities be characteristic in metals prices. Despite the econometric nature of the model, the use of geometrical approximations to establish patterns and trend similarities mimic fractal geometry, which describes self-similarities, which is a characteristic of chaotic behavior. In addition, the introduction of a time delay to capture the dynamics of metal prices is an approximation of the Takens' Theorem which affirms the dynamic of the system is preserved. The Takens' Theorem states that the phase space (p) of the original system can be reconstructed by using the evolution of previous observations framed in a specific dimension (m) and time delay (τ) as seen in Eq. (1) [23]. Thus, self-similarities and temporal relation of economic variables was recognized, suggesting the dynamic time-related behavior of the MC prices in the long-term, or at least during the last century. Therefore, the widely-held belief in stochastic behavior in the MC market is questioned ones again.

$$p(i) = (x_i, x_{i+\tau}, x_{i+2\tau}, \ldots x_{i+(m-1)\tau}) \tag{1}$$

Watkins and McAleer examined the adequacy of econometric and return models for forecasting spot and long-term prices of aluminum, copper, lead, nickel, tin and zinc [16]. One hundred and seven models contained in 45 publications between 1980 and 2002 were analyzed. The research questioned the statistical adequacy of econometric models to forecast the prices of non-ferrous metals and concluded that existing models have a lack of technical, economic and temporary assumptions.

Econometric models can deal with a large number of variables to represent the intricacy of the market. However, their complexity, relatively high cost and static nature are significant drawbacks. They are complex because a deep technical knowledge is required to develop detailed market analysis. They are costly because a number of specialists and large database analyzes are needed to carry out the study. They are static because if market conditions change suddenly or new variables are added or subtracted the model is no longer valid and a new one should be developed to represent the new scenario. Despite the use of formal mathematical and statistical tools to predict demand and supply levels, modelling all variables involved is not feasible by traditional static techniques taking into account the time-related nature of variables and the uncertain effects of unexpected events regarding geopolitics, customer patterns and desires, and technological developments. Moreover, although industrial production and investment can be used to predict long-term trends [15], predict MCs demand is much harder because it is also linked to consumption, which depend of economic growing becoming MCs demand even more volatile than economic activity [9].

As an example, even though the favorable conditions of the copper market during the last 80 years, prices have fluctuated significantly, far more than the forecast. Fig. 3 shows real copper prices evolution between 1935 and 2014 and the forecasts of the most prestigious financial institutions worldwide. The graph reveals significant differences between real and predicted copper prices. It shows that the copper market dynamics are not well represented by the static and flat trend forecasting made by financial institutions. Changes in copper prices trend are only exhibited by The World Bank (TWB) forecasting, but with five-year delay. The delay between real copper prices between 2003 and 2013 and TWB forecasted prices for the period 2008–2014 demonstrates the short-term demand volatility and long-term supply rigidity of the copper market [13].

3.2. Time-series

Drawbacks of econometric models have encouraged the introduction of time-series modelling techniques to analyze and extrapolate future trends [14]. Time-series models are commonly used for forecasting metal prices [81]. Linear models such as Autoregressive Conditional Heteroscedastic (ARCH), the generalized version of ARCH (GARCH), Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) have been intensively used to forecast the commodity prices [56,57,82]. Such time-series models have become competitive alternatives, because they do not require expensive software packages and the analysis can be conducted in-house reducing the need of external assistance for economists, statisticians or other specialists.

The ARMA model combines Autoregressive (AR) and Moving Average (MA) models. It is based on the assumption that a

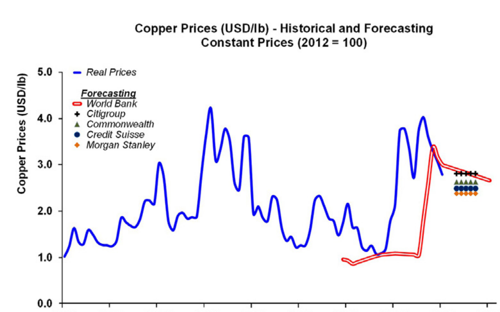


Fig. 3. Copper price evolution and forecasting. Adapted from [60,62,78–80].

sequence of random variables depends on time and fluctuates in a regular pattern [82]. The ARIMA model is the generalized version of the ARMA model that uses a linear function to describe past behavior and random errors. ARIMA models are based on the assumption of lagged random changes of the dependent variables represented by an algebraic equation with fixed coefficients estimated from past data. The flexibility to represent different time series, their statistical properties and the use of the Box–Jenkins methodology are the main advantages of ARIMA models. However, the use of pre-established linear correlation between variables, the assumption of uniform changes in the long-term and no causal relationship between variables are significant disadvantages that have resulted in an unsatisfactory approximation of real world problems [14,56].

Ru and Ren developed an ARMA model to capture aluminum market dynamics to predict short-term prices [82]. It was claimed that regular changes of variables over time fits short-term price fluctuations behavior and efficiently predict prices. The significant impact of economics, politics and technological uncertainties on prices was noted by Ru and Ren stressing that the model requires continuous updating to maintain performance [82]. Although the model can identify short-term price fluctuations using previous data the reliability of the technique is debatable. The necessity for continuous updating proves the static nature of the model that in addition to its high sensitivity to errors demonstrate that ARIMA models cannot properly represents aluminum market dynamics in the short term.

Kriechbaumer et al. claimed that traditional ARIMA models are unsuitable for predicting base metals prices [59]. However, they can reach high levels of predictive accuracy if wavelet techniques are introduced to generate asymmetric price behavior by changing the extension and location of cycles. Kriechbaumer et al. forecasted monthly prices of aluminum, copper, lead and zinc by an improved combined Wavelet-ARIMA model [59]. It was found that using a wavelet-based multiresolution analysis before the ARIMA model can separate prices in several cycles, increasing predictability. Fig. 4 shows the results obtained by Kriechbaumer et al. [59] demonstrating that the Wavelet-ARIMA configuration is more accurate than the traditional ARIMA models. The technique exhibits good performance in tracking trends. However, its inability to predict sudden fluctuations is noticeable and is a significant drawback, because not only predicting trends, but also sudden fluctuations are important inputs for decision making, especially during "stable" market seasons. The low performance of this technique in detecting sudden fluctuations can be explained by the use of a linear model.

ARCH and GARCH techniques are commonly used for modelling the effects that economic, financial and trading variables have on market prices volatility. These models have been largely used to measure volatility in derivative pricing and risk analysis [83]. For forecasting, ARCH and GARCH models examine volatility of variables searching for unusual variances of the error also called "heteroscedasticity". The heteroscedasticity phenomenon occurs when variances of the error are not equal or reasonably larger than expected values. Thus, the relationship between current and previous errors are used to setting the trend of volatility in clusters [84].

Tully and Lucey argued for the suitability of the Asymmetric Power GARCH (APGARCH) for modelling the effects of macroeconomic variables on gold prices [83]. The influence of Financial Times Stock Exchange index, the U.S. Dollar (USD), British Pound, the U.S. interest rate and the U.K. consumer price index on the gold prices was investigated through an APGARCH model. It was confirmed that the USD is the main variable affecting gold prices. Ma used the Exponential GARCH (EGARCH) technique to examine the volatility of iron ore spot prices resulting from pricing mechanism changes [1]. The EGARCH model can capture long memory patterns and asymmetric movements. Ma suggested that the new pricing mechanism has alleviated spot prices volatility that are mainly the result of negative shocks in the market [1]. It was found that the volatility of prices does not follow market changes instantly, but there is a time lag between market changes and price fluctuations that should be further studied.

Figuerola-Ferretti and Gilbert used the Bivariate Fractionally Integrated GARCH (FIGARCH) technique to represent the short-term dynamics of aluminum and copper prices volatility in the LME [85]. According to Baillie et al. [86], the FIGARCH model is more capable for representing the temporal dependence of financial volatility by assuming a slow hyperbolic rate of decay to account for the influence of the time lag on the conditional variance of the process. Figuerola-Ferretti and Gilbert suggested that spot and three-month prices of aluminum and copper have a long memory and symmetric process [85]. Common patterns in prices variability of both commodities were also found and attributed to the common LME trading process. Adrangi et al. examined the non-linear behavior of daily prices of crude oil, heating oil and unleaded gasoline traded on the NYMEX [87]. Robust evidence for the non-linearity of oil and derivative market prices was found, arguing that ARCH processes drive those three commodities. Morana noted that GARCH models are appropriate not only for forecasting Brent oil prices in the short-term, but also at different horizons without a need to include structural changes [88]. McKenzie et al. used the Asymmetric Power GARCH (APGARCH) model to analyze the conventional features of futures prices' contract volatility at different horizons of seven commodities traded in the LME; so-called aluminum alloy, aluminum, copper, nickel, lead, zinc and tin [89]. The research concluded that data of future contract prices, generally, do not present asymmetry, because the so-called "leveraging effect" leads in different market response. McKenzie et al. also stated that APGARCH model could not achieve sufficient level of performance to adequately predict the future behavior of MC prices [89].

The ARCH models have demonstrated the ability to detect and replicate commodity price fluctuations and volatility in the short-term. At this horizon, volatility and fluctuations are far more

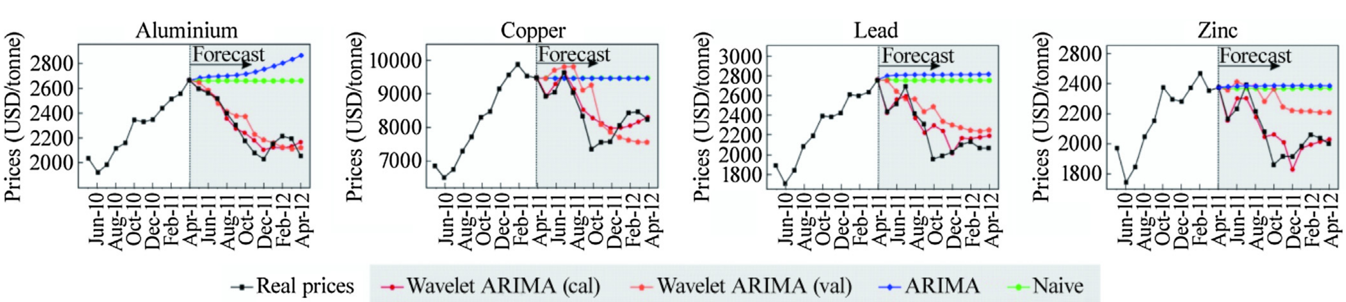


Fig. 4. Price forecasting May 2011 to April 2012. cal and val stand for calibration and validation, respectively (Modified from [59]).

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intensive compared to long-term annual base quotation. The ARCH models have shown their suitability for examining not only single, but also multiple commodities to find common volatility features. They are capable for capturing features and similarities between different commodities and horizons, and provide valuable information regarding their past behavior. However, their capacity to properly simulate future prices is debatable, because the behavior of a time series is solely based on symmetries or asymmetries emphasized by seeking not equal variances of the error between clusters of samples. Except for FIGARCH models, ARCH techniques disregard the temporal dependence of variables volatility. While it is true that "unusual deviation of errors" can describe pattern changes and provide important clues of time-series behavior. Their causes are unknown and unable to be tracked through time. Also, it is not possible to determine if detected "unusual deviation of errors" are the result of sudden unexpected events, the effect of an evolutionary process or both. Therefore, the cause and effect nature present in economic time-series is disregarded. FIGARCH models have proven their capacity to represent temporal dependency of variables solely in the short-term, where effects are almost "immediate". The capacity of FIGARCH models to capture effects of a long lags or time delays have not been proved. Thus, there is no evidence for the ability of the technique to predict long-term prices.

3.3. Stochastic-Gaussian models

Forecasting market prices using stochastic-Gaussian models built on the base of Monte Carlo Simulation has been mainly the result of the widely held conviction that market fluctuations have random sources [17,90]. Monte Carlo Simulation can solve problems of continuous dynamic systems by discrete-time approximation. To avoid discretization, Monte Carlo Simulation manipulates stochastic differential equations such as Geometric Brownian Motion that is commonly used to represent the belief of random behavior of stock prices [91].

Miller and Ni examined the relationship between GDP growth and oil price forecasting by using a state-space model [92]. They concluded that trends in economic activity and real oil prices are stochastic. However, similar work conducted by Abdullah and Zeng [30], Alquist and Kilian [93], Alquist et al. [94] and Miller and Ni [92] noted that historical prices and economic activities also contain relevant and useful information to predict prices which contradicts the belief of random behavior. Dong et al. suggested the mean-reverting shifting behavior of coal prices in the Chinese market using a shifting trend model able to capturing technological developments [95]. They argued that Brownian motion or other random-walks methods are also appropriate for capturing the dynamics of the Chinese coal market.

Based on several assumptions regarding the markets behavior Geometric Brownian Motion (GBM), Stochastic Brownian Motion (SBM) and Mean Reversion (MR) models have been used for forecasting metallic commodity prices. Baldursson claimed that GBM method can represent random asymmetric fluctuations (peak and trough) of long-term aluminum prices at different supply and demand balance levels [97]. Due to the long planning and building stages of mineral industry, Baldursson assumed demand elasticity and supply levels remain constants over time which is the weakness of the method taking into account the sensitivity of aluminum demand to technological development and other substitute mineral commodities [97]. Shafiee and Topal claimed that SBM and MR models are unable to detect and forecast sudden price fluctuations [53]. They stated that modelling prices through the random walk phenomenon resulted in a lack of rationality between prices and historical data. To overcame this drawback, Shafiee and Topal introduced an econometric model which is capable to identify the long-term relationship between prices and historical data in non-stationary time-series and able to measure random shocks [53]. Using annual gold prices as a case study, they concluded that the model has high predictability for forecasting long-term gold prices, capable to anticipate spot price trends by replicating past fluctuations (jump and dips) and has better performance than ARIMA models. Nonetheless, the model also has two main drawbacks. First, the assumptions that reversion of prices occurs in the same manner as in the past is not realistic. As noted before, the use of historical information may provide clues to understanding market behavior, but there is no guarantee that the past events will be repeated in the future, at the same time intervals and with the same intensity. Second, assuming the increase of the prices over time does not match with the real behavior of gold prices. Even though nominal gold prices in the examined period exhibited a growing trend, real prices have a fluctuating pattern as seen in Fig. 5. Price trend fluctuations reflecting the evolution of market prices had up and down trends following a logical development linking to economic, financial and geopolitical factors and their temporal relation. Thus, the ability to measure or control shocks inside boundaries is debatable.

Monte Carlo Simulation has been largely used in the mining industry to quantify a large number of geological uncertainties and their variability in an ore body through conditional simulation techniques also called sequential Gaussian [98]. However, the suitability of Gaussian Monte Carlo Simulation models to represent market commodity price variations has been controversial, since economic boundaries have not concise limits. Gaussian Monte Carlo Simulation models can assess geological uncertainties, because the distribution and limits of ore grade fluctuations can be established accurately by geological, mineralogical and petrological characteristics governed by well-known science laws. On the other hand, the economic environment and the commodity price fluctuations are influenced not only by economic and financial variables embedded in a set of rules and theories, but also by psychological, geopolitical and technological variables who fluctuate inside of the unestablished boundaries. Furthermore, a statistical analysis conducted by [83] argued that financial variables are characterized by non-normal distribution stressing that assuming a normal distribution of the prices is inappropriate, at least in the case of gold.

Mandelbrot used wool market prices to discuss the suitability to representing the economic behavior through random variables generated by Gaussian (normal) GBM models, and he concluded that traditional stochastic-Gaussian distribution models are not suitable to represent the large amount of information provided by economists and are unable to describe the relationships between the variables [21]. He further demonstrated that changes in the prices are not stochastic, but exhibit a self-similarity evolution pattern only interrupted by the Civil War between 1861 and

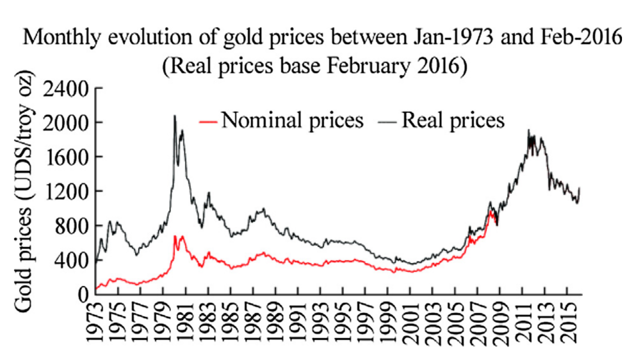

Fig. 5. Monthly Real and Current Gold prices evolution (Adapted from [64,96]).

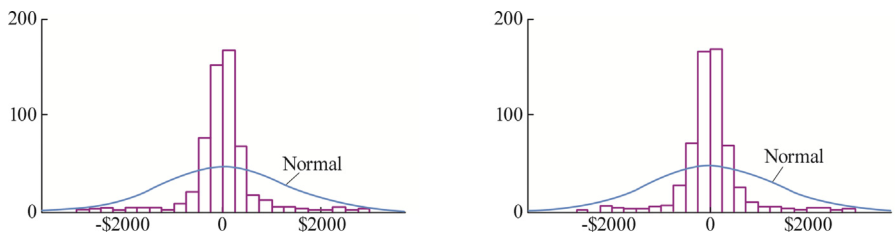

Fig. 6. Two histograms illustrating departure from normality of the fifth and tenth difference of monthly wool prices, 1890–1937 [21].

1865, and by periods of price control. Mandelbrot emphasized that, even though historical data seem to have a "Gaussian Bell" shape, they contain several deviations and price volatility [21]. These fluctuations are not an isolated phenomenon, but resulting from several previous price changes that may exceed the last observed changes. Hence, sudden fluctuation periods should not be assumed as random events and cannot be eliminated from the analysis without a close examination.

Fig. 6 shows the histograms of fifth and tenth differences[1] of monthly wool prices between 1890 and 1937. It demonstrated that the characteristic "bell" curve of Gaussian models becomes flatter and lower as a result of the large number of outliers that determine price changes over time [21]. Fig. 6a and b show monthly wool price curves departing from normality into a flat shape, for the fifth and tenth difference respectively. Despite the fact that both histograms have different price difference distributions, they exhibit a self-similarity pattern, which is characteristic of fractals.

In addition, Ahrens and Sharma indicated that aluminum, bituminous coal, lead, nickel, petroleum, and zinc prices have a stationary trend process, which is a characteristic of chaotic systems [73]. Ahrens and Sharma and Withagen also noted that copper, iron, natural gas, silver and tin prices have a different stationary trend [73,99]. Lee et al. [17] affirmed that the prices of the eleven commodities assessed by Ahrens and Sharma [73] are "stationary around deterministic trends with structural breaks" supporting the conclusion that the behavior of the prices is non-random.

Stochastic-Gaussian distribution models have attempted to simplify market uncertainties for forecasting MC prices handling a substantial amount of data fluctuating in a stochastic way. However, ignore causal relationship over the time is unrealistic and a significant drawback. There is a temporal relation between MC price fluctuations and the economic, technological, and social development, all of them driven by diverse and extensive past and current events. The effects of these variables evolution may be accumulated over time or, at least, create several evolving patterns. Thus, it can be emphasized that behavior of mineral commodity markets and their uncertainties exceed the capacity of Random-Gaussian Monte Carlos Simulation methods.

4. Dynamic systems as an alternative technique for MC prices forecasting

Econometric, stochastic-Gaussian, time-series and forecasting techniques can deal with economic and financial variables with a relative efficiency. However, they are unable to capture the dynamic and time-related behavior of psychological factors affecting MC markets. Their static, linear and random nature also reduce their capacity to properly represent the economic environment. Hence, alternative technologies capable to manage a large number of variables in dynamic systems and recognize their temporal relationship are needed.

Dynamic systems involve the evolution and dependency between variables through time and have been used to understand and conceptualize those systems governed by intense changes in levels of organization and time scales. They are mathematically represented by a fundamental equation that describes the evolution of the state of variables through time. It is the transformation of an initial state, x, at time t to a new state x at time $t+1$ which depends on the previous states $x(t)$ as seen in Eq. (2) [100–102].

$$x(t+1) = f(x(t)) \tag{2}$$

In dynamic systems, all variables are interrelated, which is called "Complete Interconnectedness". The change of any variable at any time will inevitably affect the dynamics of the other variables and the system, spreading out in its future states. Dynamic systems have a nested structure which means that each system is part of another system sharing the same dynamic principles at any level in which it occurs evolving over time to fit to a particular state so-called attractor states that are "preferred, but not necessarily predictable" [101–103,107–111].

4.1. Chaos theory

Chaos theory (CT) was devised by Lorenz while attempting to predict the weather by calculating uncontrolled approximations [19]. Lorenz noted that the similarities of two patterns starting from almost the same point are diluted through time to disappear completely. CT has popularized the self-organization of heterogeneous conditions in dynamic systems that exhibit ordered patterns in the absence of codes or rules [102].

Strange attractors, which determine self-organization, are significant for the detection of chaotic behavior. Ruelle and Takens discovered that strange attractors follow continuous curve orbits [24]. However, they are not concentric and never cross between each other [104]. Takens discovered that the dynamics of the system can be reconstructed by reorganizing the sequence of time-series observations, and Takens also claimed that changes in its shapes provide significant information hidden inside the dynamics of the system [23]. These findings are known as the Takens' Theorem which is one of the most well-known and used theorem to recognize chaotic behavior in time-series. McCullough et al. studied the effects that health information releasing has on costumer behavior in the U.S. beef market using the Takens' Theorem [32]. The research suggested that health information has direct and long-term effects on customers' preferences where consumption behavior changes according to the currently available information and to previously released information. Fig. 7 shows the phase space reconstruction technique used in the research. It graphically represents consumers' reaction to health information demonstrating the causal relation and memory of the beef market. It can be shown two states fluctuating through a well-defined transition period.

[1] Refers to the interpolation between two values of a function: y_a and y_b. If differences are well known in the form: $(y_b–y_a)/(b–a)$. the divided difference is given by the linear case of Newton's fundamental formula: $y_x = y_a + (x–a)y(a, b)$. For non-symmetrical interpolation of data namely: y_0, y_1, y_2, y_3..., the proportional interpolation is denoted by: $y_1^{(1)}(x) = y_1^{(1)} = (1-x)y_0 + xy_1$; $y_2^{(1)}(x) = y_2^{(1)} = [(2-x)y_0 + xy_2]/2$ and so on [161].

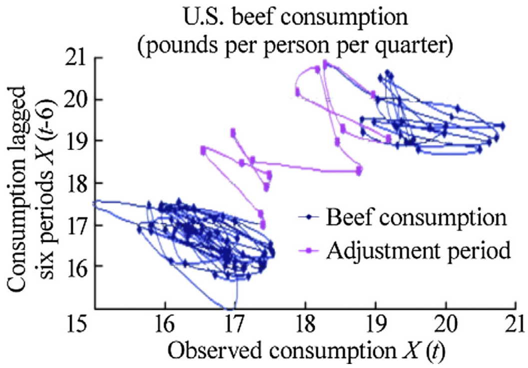

Fig. 7. The effects of human cholesterol information on U.S. Beef consumption (Modified from [32]).

4.2. Machine learning

Machine learning (ML) is currently one of the most active topics in Artificial Intelligence (AI) [105,106]. Flach defines ML as "the systematic study of algorithms and systems that improve their knowledge or performance with experience" [107]. ML can construct predictive rules and learn from examples to recreate a large number of dynamic systems promoting its use to studying discontinuous phenomenon [108,109]. ML has been used for recreating and forecasting complex dynamics in the fields of biology, medicine, ecology, astronomy, automation, banking, stock market, the oil industry, robotics, weather predictions, and image tracking among others [31,106,108–112].

The defeat of the world chess champion Gary Kasparov by Deep Blue Program in 1997 and the recent triumph of Google DeepMind's AlphaGo over the master of the board game Go Lee Se-dol are significant milestone that demonstrate the capacity of ML algorithms to find complex patterns in large amounts of data [113,114].

Artificial Neural Networks (ANN), Support Vector Machines (SVM) and Decision Trees (DT) are some of the most used techniques to predict patterns, classes, trends and values from a data set. ANNs were inspired by the biology of the nervous system and human brain function, aiming to mimic the neural interconnections in the human brain by using artificial neurones as seen in Fig. 8a. ANNs are adaptive in nature providing pattern recognition characteristics that may be used to simulate a wide range of complex systems [31,56,112,115].

A SVM is a machine learning technique used for pattern recognition, classification and regression analysis which uses a number of hyperplanes embedded in a high dimensional space as decision boundaries to separate classes [107,110,116]. SVM can be used for classification of non-linear data due to its capacity to learn in high-dimensional space. To solve non-linear separable problems, SVM must be combined with Kernel methods. Kernel methods are mathematical tricks that map non-linear input space to a high-dimensional linear features space where linear separation is achievable [107,110,116–119]. Kernels are based on functions that calculate the inner product of features directly from the vectors. Common choices for Kernel functions include polynomial of degree[2] *d* and *p*, radial basis, sigmoid and Gaussian [107,110,116,118–120]. Real world non-linear problems solved by SVM include face identification, handwritten digit recognition, text classification, and prediction of protein secondary structure and microarray classification in bioinformatics [116,118,120]. The application of SVM to classify gene-gene interaction in genetic epidemiology where classes of inputs are separated by hyperplanes is shown in Fig. 8b.

Data Mining applies machine learning and statistical techniques to search for patterns in large data sets, including human patterns [121]. To detect cybercrimes, Quah and Sriganesh proposed a data mining model using ANN for the early detection of customer profiles and recognition of buyers and sellers patterns in e-commerce [31]. ANNs have also been used to forecast stock price and currency quotes [31]. As a methodology, data mining uses machine learning and statistical methods to explore data sets to discover patterns. The methodology can be viewed as using these techniques to expand a human data analyst's ability to find patterns in large datasets.

5. Application of dynamic systems to understand and predict MC prices behavior

Over the last two decades strong evidence has appeared suggesting that the economy has a non-linear and chaotic structure that generates complex time paths by a deterministic process [87,122]. Chaos theory and Machine learning have become powerful techniques to study dynamic systems in a large range of fields such as natural phenomenon, ecosystems, economics, commodity prices, and the stock market [19,22,27,32,33,122,123]. The concept of sensitivity to initial conditions stated by the CT has been highly criticized in the market environment. However, over the last decade several researchers have provided significant evidence of its effectiveness in determining chaotic behavior and the self-similarities of market prices including mineral commodity prices [21,27,32,33,122,124]. ML has been used far more to assess market prices behavior mainly for Energy Commodities [30,87,125–127].

5.1. Evidence of chaotic behavior

Identifying the type of trend of MC prices (stochastic or deterministic) might only be assessed empirically and is fundamental for any attempt to explain or predict long-run behavior. Despite its significance, the study of trend types in MC markets is relatively new. To our knowledge, the first research into trend types in MC was conducted by Ahrens and Sharma [73].

Panas and Ninni examined the behavior of eight crude oil products in the two main European markets, Rotterdam and Mediterranean [122]. Daily prices were assessed to determine the chaotic structure of markets. The evidence provided by correlation dimension, entropies and the Lyapunov exponent tests, demonstrated that five oil products exhibited chaotic behavior. The research emphasizes the deterministic nature of the oil market driven by political variables that also control its structure. The Six Days War in 1967, the crisis in the Persian Gulf in 1991 and the re-introduction of Iraqi oil into the market in 1996 are some deterministic events quoted that have affected oil market structure of spot and future oil prices [122]. Panas investigated the chaotic behavior of daily prices of aluminum, copper, lead, tin, nickel and zinc in the LME [33]. Market dependencies was examined by ARCH, long memory and chaotic non-linear models. It was found that the price of the six MCs examined have dynamic nature and cumulative long-term dependence and the chaotic behavior of zinc was asserted. The research further claimed that stochastic models are inappropriate for describing copper and aluminum prices that exhibit long-term memory described by self-similarity pattern, a distinctive characteristic of chaos systems.

[2] Considering two data points $X_1 = (x_1, y_1)$ and $X_2 = (x_2, y_2)$, and consider X'_1 and X'_2 as their respective mapping. Assuming the mapping to a three dimensional feature space. For a kernel of degree *d*: $(X'_1 \cdot X'_2) = (X_1 \cdot X_2)^d$ is the inner product of feature vectors, and $(x_i, y_i) \rightarrow (x_i^2, y_i^2, \sqrt{2x_i, y_i})$ the mapping points in features space. For a kernel of degree *p*: $(X'_1 \cdot X'_2) = (X_1 \cdot X_2 + 1)^p$ is the inner product of feature vectors, and $(x_i, y_i) \rightarrow (x_i^2, y_i^2, \sqrt{2x_i, y_i}, \sqrt{2x_i}, \sqrt{2y_i}, 1)$ the mapping points in features space [107,118,119].

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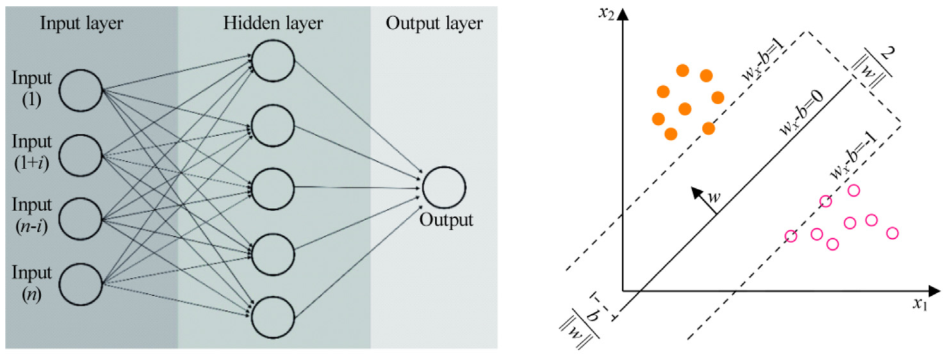

(a) An example of an ANN. Modified from[112]

(b) Linear Support Vector Machine (SVM) with maximum-margin hyperplane. Modified from[110]

Fig. 8. Machine learning.

He et al. used the Curvelet Based Multiscale Forecasting methodology in the Heterogeneous Market Hypothesis to empirically analyze the chaotic and multiscale features of metal price movements [27]. The Curvelet-based transform allows the identification of geometrical singularities (orientation) of multiscale objects that exhibit spatial–temporal organization and non-equilibrium dynamics [128]. The Heterogeneous Market Hypothesis assumes that investor transactions are held based on diverse preferences at different time horizons and frequency describing different investment patterns. He et al. modelled the dynamics of lead and zinc daily prices through phase space reconstruction, using the time delay methods [27]. The heterogeneous structure of metal market cycles was observed suggesting that lead and zinc prices have chaotic multi-scale characteristics. He et al. described their behavior as complex, exhibiting significant and sudden non-linear and chaotic fluctuations [27].

Svedberg and Tilton noted that the copper market behavior suffered many changes between 1870 and 2000 because unexpected changes of the variables driven the market or the introduction of new variables [129]. They stated that new uses of copper, decreasing production costs, availability of substitutive products and structural changes in global economy resulted changes in copper price trends. This statement supports the assertion of small changes in the initial condition of the copper market have effects on behavior of future prices, which can be considered as characteristic of chaos systems. As copper is traded in one of the most competitive MC markets, it is likely that other MC markets may have the similar behavior that facing changes on initial conditions.

The findings of the chaotic behavior in several MC prices time-series shows that those systems exhibit sensitivity to initial conditions, self-organization and are governed by ordered patterns in the absence of codes. CT has demonstrated its capacity to identify the number of variables affecting the system (dimension) as well as their temporal relationship (time delay). Nevertheless, the significant information provided by CT (time delay and dimension) has been disregarded for forecasting and solely chaotic behavior of MC prices has been asserted in the literature. Moreover, the assertion of the chaotic behavior has been limited to short-term horizons (daily and monthly), mainly due to the small number of samples available in annual base. The lack of chaotic assessments for long-term MC prices in an annual base is an significant gap taking into account the number of existing techniques to assess chaotic behavior by using small data sets.

5.2. Current machine learning techniques for forecasting MC prices

Abdullah and Zeng proposed an approach to predicting short and long-term oil prices by using Hierarchical Conceptual and Artificial Neural Networks-Quantitative (ANNQ) models [30]. The methodology determines oil price volatility by integrating the quantitative and qualitative data affecting the market. While quantitative information is related to those external agents such as supply and demand, qualitative information refers to the particular point of view and interpretation of market experts and news releases regarding predicted oil prices. The Hierarchical Conceptual model detected the most important events and data affecting oil price fluctuations. Then the ANNQ model learns from these events and uses them to predict prices. The methodology demonstrated that using Hierarchical Conceptual models to select the key factors affecting oil price volatility improves ANNQ predictions [30].

Sánchez Lasheras et al. assessed the performance of ANN, Elman Neural Network (ENN) and ARIMA models for forecasting spot copper prices [130]. ENN is a particular case of Recurrent Neural Network (RNN) that uses constant feedforward networks. In this network, a layer called "context layer" contain neurones that hold the outputs from the hidden layer and use them as new inputs in the next computing step. It was asserted that ANN and ENN models could reproduce actual prices with high detail demonstrating better performance than ARIMA models. The best configuration settled for ENN occurs with 17 neurones in the hidden layer. Dubey claimed that Support Vector Regression (SVR) model exceeds the performance of Fuzzy techniques for forecasting daily gold prices [131]. SVR is the combination of SVM and regression techniques and ANFIS combines fuzzy inference system within the adaptive neural network.

Compared to econometrics, linear and time-series models, ML techniques have demonstrated better performance for forecasting MC prices in different horizons. Using historical and related information for learning from past events, reasoning and discovering hidden patterns in large high dimensional data sets have been crucial to reach high forecasting performance. However, is important to notes that as long as the number of variables increase (the dimension) the complexity of the problem also increases. This problem becomes more complex for forecasting MC prices once supervised, unsupervised or semi-supervised ML algorithms seek for hidden patterns that, in theory, are unknown for human reasoning. Nonetheless, economic time series forecasting, that includes MC prices, are based and explained by the economic theory which is driven by rational decisions of all agents that participate in the market [18]. Therefore, uncontrolled or unknown hidden patterns found by ML may become a significant threat and hard to detect.

5.3. Combining chaos theory and machine learning

Traditional techniques are unable to represent all the dynamic features of MC markets to forecast prices. CT and ML are alternative techniques that have proven superior for understanding and describing the behavior of the complex dynamic systems. CT can detect the number of features that govern the system and their influence over time. It improves understanding of the system, which is crucial before any attempt to predict behavior. ML, on the other hand, can discover patterns to describe the dynamics of the system that are hidden within large datasets which are often updated. ML models can reduce complexity as well as acquire more experience and learn to increase accuracy.

However, despite the advantages of CT and ML for understanding and representing dynamic systems, their individual use is not adequate for forecasting MC prices. Similarly for traditional forecasting techniques, the sole use of CT or ML has several drawbacks that cause models to be incomplete. The advantages and disadvantages to using a single technique for forecasting MC prices in the long term are shown in Table 2.

In chaotic systems, strange attractors describe a set of periodic orbits that have ergodic dynamic. It means that through temporal evolution of the systems the orbit of every neighbor variable are also embedded within the dynamic of the chaotic attractors of the system [132,133]. As the dynamic properties of the chaotic systems stay invariant regardless the observation period and its resolution (distance between observations), the number of variables (or embedding dimension) and time delay driving the system's behavior which will also remain invariant. Thus, a deep insight of the time delay and embedding dimension allows the reconstruction of the dynamic of chaotic systems.

Understanding the periodic properties of the infinite number of unstable periodic orbits (UPO) described by strange attractors allows chaos control. Controlling chaotic systems aims to reduce its chaotic behavior by controlling UPOs trajectory or converging them to a point. Chaos control is also able to reconstruct complex periodic oscillators (or attractors). It is obvious that controlling the behavior of MC markets is impossible because fluctuations resulting from changes in exogenous and endogenous variables cannot be suppressed either controlled. However, its behavior can be well known via geometric observation of the dense number of UPOs embedded within strange attractors governing the system [134–136]. Pyragas developed a method based on a delayed feedback process for controlling chaotic systems. Pyragas proved the suitability of UPOs to describe the behavior of strange attractors of chaotic systems. He also noted that strange attractors can also be stabilized by using delayed feedback perturbations or external periodic oscillator [136].

Time delay has a major role for understanding and modelling chaotic systems, including chaos control and chaos synchronization. In chaotic systems, synchronization occurs when trajectories of attractors of different systems will converge to the same values and will stay together. This means that a proper definition of the temporal characteristics of two or more coupled systems can evolve sharing similar dynamic [137,138]. This is a remarkable property for economic time series as the correlation between variables affecting MC markets has been documented in the literature. On MC markets that have chaotic dynamics, chaos synchronization can help to reduce the complexity of forecasting problems by reducing data processing and analysis time, and narrowing markets research. Despite the important benefits that time delay has for studying and improving the knowledge of chaotic systems, there is a lack of research describing what the conditions of time delay that makes dynamic systems chaotic are [134].

CT gives an insight to the number of variables governing the system and how they are related over time. It can accurately find the time delay in the system. However, it is unable to identify, by itself, which are the relevant variables affecting the system. This is a significant drawback for modelling economic time series where phenomena are linked to hundred thousand of variables. Not having a proper identification of variables affecting the system for forecasting takes out the rationality of economic problems. Therefore, the use of CT alone, for forecasting MC prices is not enough because the main principle for forecasting economic time series, knowledge of the system, is incomplete due to the lack of variable selection. To obtain models to represent the behavior of MC markets for forecasting prices, CT needs the help of another technique capable of identifying relevant variables based on embedding dimension guidelines and to model their behavior based on time delay patterns.

A chaotic system with unknown features is a black-box corresponding to several non-linear classes, and therefore, suitable to be modelled using non-linear methods. Chaotic activity can make an efficient association of memories through autocorrelation learning [139,140]. ML techniques are designed to learning from informative features. However, irrelevant information usually

Table 2
Advantages and disadvantages to use a single technique for forecasting MC prices in the long term.

Technique/models	Advantages	Drawbacks
Econometrics	Used for forecasting long-term price trends Can deal with a large number of variables Can recognize the relationship between variables	Historical data relation does not guarantee accurate predictions. Complex, high cost and statics
Time series	May capture fluctuations Capable to examine multiple commodities and seek common volatility features	Fluctuations based on symmetries or asymmetries obtained from the unusual variance of errors Time evolution is not considered Mainly used for forecasting short-term prices
Stochastic	Used for forecasting long and short-term prices Able to solve problems of continuous dynamic systems by a discrete-time approximation	Normalization assumption needs boundaries Cannot represent the information provided by economists Unable to represent the temporal relationships between variables
Chaos theory	Able to find the features of dynamic systems so-called time delay and embedding dimension. Able to represent the causal relationship between variables and memory	Time delay and embedding dimension have been used solely to describe systems' behavior and not for forecasting MC prices
Machine learning	Used for forecasting long and short-term prices Better performance Able to find hidden patterns and learn Can deal with a large number of variables	High dimensionality increases complexity Hidden patterns, in theory, are unknown for human reasoning The evolution of economic time-series is based on the economic theory driven by market rationality

confuses algorithms [141,142]. A selection of relevant information enables the ML algorithm to use the fewer variables to obtain more information for modelling of dynamic of systems [143]. Avoiding the use of irrelevant, noisy or redundant information reduces overfitting [144–146]. Thus, to obtain reasonable and realistic outputs from ML forecasting models, a careful selection of variables is fundamental. The accuracy of ML models may be improved if guidelines are used for seeking the most relevant variables within large datasets.

There are several statistical and ML techniques to select relevant information by either reconstructing or reducing datasets. Based on correlation and relevance of variables, datasets can be reconstructed by grouping the original features to create new ones [143]. Using an iterative process usually automated in some feature selection algorithms, datasets can be reduced by removing irrelevant and redundant information as much as possible [147]. Variables are selected by measuring the accuracy of ML forecasting models. If the tested ML forecasting model does not reach the required accuracy, main variables are updated and tested until reaching the required level. Although selecting variables solely through ML techniques provides accurate statistical and mathematical evidence, which they are not sufficient for forecasting MC prices. The autonomous selection of variables takes out the rationality of economic problems that in addition of the unknown modelling of the temporal relation makes models incomplete. Thus, the sole use of statistical and ML techniques for variables selection generates three main concerns: First, it is uncertain, if these techniques can select informative variables affecting the system dynamics through time or are solely capable of selecting variables that generate the largest fluctuations of outputs. Secondly, it is also uncertain, if the number of variables selected properly representing the dimension of the system. Thirdly, temporal relation and effects propagations are not considered to determine variables driven by the system.

Previous research have proven that chaos theory helps and improves the convergence of orbits and the solution of the systems that have an associative memory created from interconnected parametric elements. The approximation properties of Neural Networks (NN) and its capacity to predict and control non-linear systems allows seeking for certain chaotic behavior, recognize its patterns and reconstruct chaotic dynamics by modelling the information storage in the system [133,148–152]. Based on a fully-connected NN, the Hopfield network can store patterns inside an associative memory able to recover exact patterns from memory given a partial or distorted version of original patterns [153,154]. Tan & Ali claimed that hidden information regarding patterns of systems dynamic are contained in the trajectory of strange attractors that can be extracted by using NN [155].

ANN used on ML aims to model biological neurons by using artificial neurons which are threshold elements that generate non-linear output through a weighted sum transformation of inputs. From a neurophysiology point of view, this equivalence has been criticized because biological neurons are far more complicated that weighted threshold elements. The use of ANN for modelling human neurons is also criticized because its lack of chaotic behavior which is a common characteristic of biological neurons [152]. Although the complex dynamic of chaos systems and neurophysiologic characteristics have challenged the capacity of ANN for learning and emulating biological neurons, the analogy between biological and artificial neural networks can be still considered to investigate symmetric, asymmetric and nonlinear characteristics of chaos systems [156]. Many researches argued for the successful application of ANN for detecting and controlling chaotic systems and recommended that models must be as simple as possible. The introduction of time delay unavoidable increases the complexity of ANN models but can be effectively applied into chaotic delayed model such as the delayed Hopfield network [137,148,157]. Aihara et al. proposed a Chaotic Neural Network (CNN) to describe the chaotic behavior of a single neuron [152]. Based on feedback inputs via Hopfield networks and external input via back-propagation networks, the model introduces chaotic deterministic functions to ANN. The Transiently Chaotic Neural Networks (TCNN) with Chaotic Simulated Annealing (CSA) and the Discrete-time Recurrent Neural Networks (DRNN) have been successfully applied solving practical problems such as combinatorial optimization and associative memory [158]. It demonstrates that the memory and temporal properties of chaos system, and the learning and adaptive features of ML may be successfully combined and implemented.

In the context of MC markets, the ML technique described by Abdullah and Zeng has similarities with the chaotic behavior of the U.S. beef market described by McCullough et al., where information release has a significant impact on the market [30,32]. Although Abdullah and Zeng disregarded the temporal influence of information (time delay) [30], the similarity between both models argues for the psychological behavior of the oil market that evolves trough time. Based on this evidence it can be stated that MC markets are driven not only by statistical indices, and by the supply and demand, but also by decision-making processes based on the interpretation of information and analysis of past events. This statement describes sensitivity to initial condition in the oil market, which is a fundamental characteristic of chaotic behavior [34,37,101,104,132]. Due to the human decision-making nature of all MC markets, chaotic features of the oil market can also be found in other MC markets.

Chaotic dynamics are fundamental to cognitive and memory processes in the human brain. The chaotic behavior of the human brain has been reported by the observation of small sets of couple neurons embedded in low-dimensional strange attractors [151,156,159]. Markets behavior is fundamentally driven by the human nature, where human decision-making process is unavoidably linked to biological neural networks in the brain of all agents participating in the economy so-called governments, companies and consumers. According to [140], "a brain, is an ensemble of a large number of nonlinear and analog neurons, whose connections are asymmetric and highly structural. In such a complex system, one can naturally expect complex dynamics, including oscillations and chaotic activities, to occur". This simple, but remarkably relevant definition linked to the psychological state of human being supports the argument for the need to use and combine CT and ML in the field of mineral economics for forecasting MC prices.

Extensive researching argues for the suitability of the use of chaos theory for simulating the increasing complexity of dynamic systems. Thus, the need for linking chaos theory to mathematical models also increases [160]. In the case of mineral economics, the joint use CT and ML for forecasting MC prices is an efficient solution to overcome drawbacks generated by using a single technique (see Table 2). Systems dynamic information provided by CT in the form of time delay and embedding dimension are critical inputs for setting ML models for forecasting MC prices. The embedding dimension, which informs the exact number of variables governing the system, should be used as a guideline for a proper features selection. Time delay, which informs the limit of effects propagation over time, should be used to reconstruct datasets by including the delayed evolution of the effects that selected relevant variables have for the system. The joint use of CT and ML for modelling MC prices forecasting introduces the economic rationality and can also improve its accuracy.

Benefits of the combined used of CT and ML go beyond accuracy improvement of the models. Having a clear insight of the number of relevant variables for forecasting can helps forecasters to better identify particular variables of the market and focus on them.

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Instead of finding a large amount of information attempting to understand thousands of variables, the more precise information provided by CT and ML can reduce from thousands to couple the number of relevant variables to be monitoring. It has the potential to improve the knowledge of the systems, increases efficiency in data processing and reduces cost as only relevant variables should be monitored, updated and estimated.

The efficiency and efficacy resulting from the combination of both techniques can be explained by an example from a high precision military mission as an analogy in the form of sniper targets combines intelligence and field agents. This kind of mission aims to eliminate a target in a specific period and localization. As the combinations of time and places are infinite, and the field agent can seek and wait for the target only for a limited time, when supplies are available. Therefore, the intelligence is fundamental. Information regarding ‘‘localizations" of the target and the ‘‘time" when it moves from one point to the other can make the difference between accomplishing the mission or a mere attempt. In a similar way, predicting an MC price requires the intelligence information to set the dynamic of the target and field agents to execute predictions. To carry out the forecasting mission, the CT could act as an intelligence information source providing relevant information regarding ‘‘localization" of the target in the form of embedding dimension, and the ‘‘time" of target movements in the form of time delay. ML algorithm could act as a field agent modelling the information and reconstruct target's behavior to know when and how it moves.

6. Conclusions

Accurate and complex econometric techniques can correlate a large number of variables from historical data providing a good understanding of past behavior. However, they are incomplete to forecasting mineral commodity prices, due to their static nature based on the belief that past events may be repeated. Complexity is a significant drawback once a deep knowledge and extensive research is required to set a specific scenario. Stochastic-Gaussian models using Monte Carlo Simulation can incorporate uncertainties though a discrete time approximation of systems dynamics. However, the suitability of these models to represent the commodity price behavior is debatable, since long-term memory, self-similarity patterns, stationary and deterministic trends have been found in several prices time-series. Stochastic models fluctuating in pre-established boundaries are unable to manage the vast amount information required to forecast prices and represent the relationship between variables. As price fluctuations have temporal relation with the economic, technological and social factors the techniques appear to be unrealistic to represent MC markets and prices.

Time-series models are less complex than econometrics and able to replicate the behavior of MC prices in different horizons. ARIMA models may represent past trend with relative accuracy, however, they are unable to represent or predict sudden trend changes essentially, because of the use of linear models. ARCH models, on the other hand, have overcome non-linearity drawback by using the heteroscedasticity demonstrating the capacity to detect MC price fluctuations and volatility in different horizons. However, fluctuations are merely represented by unusual variance errors. Even though ‘‘unusual deviation of errors" provide significant information of fluctuations, their causes and effects are unknown and cannot be tracked through time.

Previous research indicates that CT and ML techniques can represent time relation between variables in complex dynamic systems. CT can recognize sensitivity to initial conditions and determine time delay and the embedding dimension governing the system. The chaotic behavior of several MC prices have been stated that suggesting the suitability of CT to represent MC market dynamics for forecasting prices. However, time delay and embedding dimension information have been not used for forecasting MC prices but only as descriptive information. The lack of chaotic assessment of long-term MC prices in an annual base was also found; although a number of techniques to assess small data sets are available.

ML finds patterns in the behavior of systems to recreate their behavior. ML has proved to have better performance for forecasting MC prices compared to all other techniques. Nonetheless, it was found that the ability of ML to deal with large high dimensional data sets and find hidden patterns to forecast MC prices might become a dangerous trap. An increase in variables in dataset (dimension) unavoidably increases the complexity of ML models that become hard to assess, train and adjust. Due to the rational nature of economic problems, dimensionality and selection of variables generate concerns regarding the suitability of the sole use of ML for forecasting MC prices. Using unknown hidden patterns from unknown variables may not properly represent the real MC market environment; therefore, prices.

None of the novel techniques proposed either CT or ML can be solely and separately used for forecasting long-term MC prices. To obtain a proper representation of MC markets CT and ML techniques need to work together. Information provided by CT in the form of embedding dimension and time delay is crucial to understand system dynamics and must be used as inputs for forecasting MC prices via ML models. Embedding dimension should be used to determine the main variables affecting the system and reduce dimensionality. Time delay should be used to reconstruct the dataset by including the evolution of those main variables to recreate system behavior.

Further research should be focused on three main steps. Firstly, assessing the chaotic dynamic of MC prices by using relatively small data sets on an annual base; secondly, investigating the most suitable techniques for features selection in high dimensional systems; and thirdly, integrating CT, features selection techniques and ML to forecasting MC long-term prices in an annual base.

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