Homework 4 (Question 2)

In [1]:
```
import sys
!{sys.executable} -m pip install pulp
```

```
Requirement already satisfied: pulp in c:\users\muntakim\appdata\local\programs\python\p
ython38-32\lib\site-packages (2.4)
Requirement already satisfied: amply>=0.1.2 in c:\users\muntakim\appdata\local\programs
\python\python38-32\lib\site-packages (from pulp) (0.1.4)
Requirement already satisfied: docutils>=0.3 in c:\users\muntakim\appdata\local\programs
\python\python38-32\lib\site-packages (from amply>=0.1.2->pulp) (0.17.1)
Requirement already satisfied: pyparsing in c:\users\muntakim\appdata\local\programs\pyt
hon\python38-32\lib\site-packages (from amply>=0.1.2->pulp) (2.4.7)
```

In [2]:
```
import pulp
from pulp import *
```

In [3]:
```
"""
Constraint Problem

Author : Muntakim Rahman 2021
"""
```

Out[3]:
```
'\nConstraint Problem\n\nAuthor : Muntakim Rahman 2021\n'
```

Homework Problem

Consider the $m \times n$-matrix A and the vector $\vec{b} \in R^m$ that are given by

$$A = [a_{ij}], \qquad \vec{b} = [b_i]$$

where

$$a_{ij} = (-2)^{i+j}(i^2 - j^2), \qquad b_i = (-2)^i \quad \text{for } i = 1, \cdots m \text{ and } j = 1, \cdots, n.$$

Note that a_{ij} is the entry of A at the i-th row and the j-th column.

Consider the following condition on vectors $\vec{x} \in R^n$:

$$(\text{Condition1}) \qquad A\vec{x} \leq \vec{b} \quad \& \quad \vec{x} \geq \vec{0}.$$

Check whether there is a vector $\vec{x} \in R^n$ that satisfies (Condition1), when $m = n = 10$, and when $m = n = 20$. If the vector exists, solve for it.

Constraint Problem

$$\begin{aligned} \text{Maximize} \quad & -w_o \\ \text{Subject to} \quad & A\vec{x} + \vec{w}_o \leq \vec{b} \\ & \vec{x} \geq \vec{0} \\ & \vec{w}_o \geq 0. \end{aligned}$$

$$\begin{aligned} \text{Where} \quad & A = [a_{ij}], \qquad \vec{b} = [b_i], \\ & a_{ij} = (-2)^{i+j}(i^2 - j^2), \\ & b_i = (-2)^i, \\ & i = 1, \cdots, m \qquad j = 1, \cdots, n. \end{aligned}$$

$\vec{w}_o = [w_{oi}]$ *represents auxiliary variable.*

Check if Constraint Satisfied When M = N = 10

In [4]:
```
M = 10
N = 10
```

In [5]:
```
A = [[(-2)**(i+j) * (i**2 - j**2) for j in range(1, N + 1)] for i in range(1, M + 1)]
b = [(-2)**i for i in range(1, M + 1)]
```

In [6]:
```
## Print Condition Variables -> Mainly for Debugging Purposes.
print('Matrix A')
for i in range(M) :
    print(A[i])

print('\n')

print('Vector b')
print(b)
```

```
Matrix A
[0, 24, -128, 480, -1536, 4480, -12288, 32256, -81920, 202752]
[-24, 0, 160, -768, 2688, -8192, 23040, -61440, 157696, -393216]
[128, -160, 0, 896, -4096, 13824, -40960, 112640, -294912, 745472]
[-480, 768, -896, 0, 4608, -20480, 67584, -196608, 532480, -1376256]
[1536, -2688, 4096, -4608, 0, 22528, -98304, 319488, -917504, 2457600]
[-4480, 8192, -13824, 20480, -22528, 0, 106496, -458752, 1474560, -4194304]
[12288, -23040, 40960, -67584, 98304, -106496, 0, 491520, -2097152, 6684672]
[-32256, 61440, -112640, 196608, -319488, 458752, -491520, 0, 2228224, -9437184]
[81920, -157696, 294912, -532480, 917504, -1474560, 2097152, -2228224, 0, 9961472]
[-202752, 393216, -745472, 1376256, -2457600, 4194304, -6684672, 9437184, -9961472, 0]


Vector b
[-2, 4, -8, 16, -32, 64, -128, 256, -512, 1024]
```

In [7]:
```
decision_variables = []

for i in range(1, M + 1) :
    current_variable = LpVariable(name = 'x_' + str(i), lowBound = 0, cat = LpContinuou
    decision_variables.append(current_variable)
```

```
## Print Decision Variables -> Mainly for Debugging Purposes.
print(decision_variables)

aux_variable = LpVariable(name = 'Auxiliary_Variable', lowBound = 0)
```

```
[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_10]
```

In [8]:
```
# Check if there is a feasible solution to the LP Problem by using Phase One of the Two
LP_Prob_Aux = LpProblem(name = 'Constraint_Problem', sense = LpMaximize)

# The Objective Function is Added to 'LP_Prob_Aux' First.
LP_Prob_Aux += - aux_variable, 'Auxiliary_Problem'
```

In [9]:
```
# The Constraints are Added to 'LP_Prob_Aux'
for i in range(M) :
    LP_Prob_Aux += lpSum([A[i][j] * decision_variables[j] for j in range(N)]) + aux_var
```

In [10]:
```
print(LP_Prob_Aux)
```

```
Constraint_Problem:
MAXIMIZE
-1*Auxiliary_Variable + 0
SUBJECT TO
Constraint_1: Auxiliary_Variable + 202752 x_10 + 24 x_2 - 128 x_3 + 480 x_4
 - 1536 x_5 + 4480 x_6 - 12288 x_7 + 32256 x_8 - 81920 x_9 <= -2

Constraint_2: Auxiliary_Variable - 24 x_1 - 393216 x_10 + 160 x_3 - 768 x_4
 + 2688 x_5 - 8192 x_6 + 23040 x_7 - 61440 x_8 + 157696 x_9 <= 4

Constraint_3: Auxiliary_Variable + 128 x_1 + 745472 x_10 - 160 x_2 + 896 x_4
 - 4096 x_5 + 13824 x_6 - 40960 x_7 + 112640 x_8 - 294912 x_9 <= -8

Constraint_4: Auxiliary_Variable - 480 x_1 - 1376256 x_10 + 768 x_2 - 896 x_3
 + 4608 x_5 - 20480 x_6 + 67584 x_7 - 196608 x_8 + 532480 x_9 <= 16

Constraint_5: Auxiliary_Variable + 1536 x_1 + 2457600 x_10 - 2688 x_2
 + 4096 x_3 - 4608 x_4 + 22528 x_6 - 98304 x_7 + 319488 x_8 - 917504 x_9
 <= -32

Constraint_6: Auxiliary_Variable - 4480 x_1 - 4194304 x_10 + 8192 x_2
 - 13824 x_3 + 20480 x_4 - 22528 x_5 + 106496 x_7 - 458752 x_8 + 1474560 x_9
 <= 64

Constraint_7: Auxiliary_Variable + 12288 x_1 + 6684672 x_10 - 23040 x_2
 + 40960 x_3 - 67584 x_4 + 98304 x_5 - 106496 x_6 + 491520 x_8 - 2097152 x_9
 <= -128

Constraint_8: Auxiliary_Variable - 32256 x_1 - 9437184 x_10 + 61440 x_2
 - 112640 x_3 + 196608 x_4 - 319488 x_5 + 458752 x_6 - 491520 x_7
 + 2228224 x_9 <= 256

Constraint_9: Auxiliary_Variable + 81920 x_1 + 9961472 x_10 - 157696 x_2
 + 294912 x_3 - 532480 x_4 + 917504 x_5 - 1474560 x_6 + 2097152 x_7
 - 2228224 x_8 <= -512

Constraint_10: Auxiliary_Variable - 202752 x_1 + 393216 x_2 - 745472 x_3
 + 1376256 x_4 - 2457600 x_5 + 4194304 x_6 - 6684672 x_7 + 9437184 x_8
 - 9961472 x_9 <= 1024
```

```
VARIABLES
Auxiliary_Variable Continuous
x_1 Continuous
x_10 Continuous
x_2 Continuous
x_3 Continuous
x_4 Continuous
x_5 Continuous
x_6 Continuous
x_7 Continuous
x_8 Continuous
x_9 Continuous
```

In [11]:
```
LP_Prob_Aux.writeLP('ConstraintProblem_M_N_10.lp')
```

Out[11]: [Auxiliary_Variable, x_1, x_10, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9]

In [12]:
```
# The Problem is Solved Using PuLP's Choice of Solver.
LP_Prob_Aux.solve()
```

Out[12]: 1

In [13]:
```
print(f'Status: {LpStatus[LP_Prob_Aux.status]} \n')

for variable in LP_Prob_Aux.variables() :
    print(f'{variable.name} = {variable.varValue}')
print('\n')

if (LpStatus[LP_Prob_Aux.status] == 'Optimal') :
    print(f'Optimal Value : Z = {value(LP_Prob_Aux.objective)}')
    if (value(LP_Prob_Aux.objective) == 0) :
        print ('The Original LP Problem is feasible.')
    else :
        print ('The Original LP Problem is not feasible.')
else :
    print(f'No Optimal Value. Status Code : {value(LP_Prob_Aux.objective)}')
```

```
Status: Optimal

Auxiliary_Variable = 0.0
x_1 = 0.0
x_10 = 0.0
x_2 = 0.0032467532
x_3 = 0.0
x_4 = 0.0
x_5 = 0.0
x_6 = 0.0
x_7 = 0.0
x_8 = 0.0
x_9 = 2.536526e-05


Optimal Value : Z = 0.0
The Original LP Problem is feasible.
```

Check if Constraint Satisfied When M = N = 20

In [14]:
```
M = 20
N = 20
```

In [15]:
```
A = [[(-2)**(i+j) * (i**2 - j**2) for j in range(1, N + 1)] for i in range(1, M + 1)]
b = [(-2)**i for i in range(1, M + 1)]
```

In [16]:
```
decision_variables = []

for i in range(1, M + 1) :
    current_variable = LpVariable(name = 'x_' + str(i), lowBound = 0, cat = LpContinuou
    decision_variables.append(current_variable)

## Print Decision Variables -> Mainly for Debugging Purposes.
print(decision_variables)

aux_variable = LpVariable(name = 'Auxiliary_Variable', lowBound = 0)
```

```
[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_10, x_11, x_12, x_13, x_14, x_15, x_16,
x_17, x_18, x_19, x_20]
```

In [17]:
```
# Check if there is a feasible solution to the LP Problem by using Phase One of the Two
LP_Prob_Aux = LpProblem(name = 'Constraint_Problem', sense = LpMaximize)

# The Objective Function is Added to 'LP_Prob_Aux' First.
LP_Prob_Aux += - aux_variable, 'Auxiliary_Problem'
```

In [18]:
```
# The Constraints are Added to 'LP_Prob_Aux'
for i in range(M) :
    LP_Prob_Aux += lpSum([A[i][j] * decision_variables[j] for j in range(N)]) + aux_var
```

In [19]:
```
print(LP_Prob_Aux)
```

```
Constraint_Problem:
MAXIMIZE
-1*Auxiliary_Variable + 0
SUBJECT TO
Constraint_1: Auxiliary_Variable + 202752 x_10 - 491520 x_11 + 1171456 x_12
 - 2752512 x_13 + 6389760 x_14 - 14680064 x_15 + 33423360 x_16 - 75497472 x_17
 + 169345024 x_18 - 377487360 x_19 + 24 x_2 + 836763648 x_20 - 128 x_3
 + 480 x_4 - 1536 x_5 + 4480 x_6 - 12288 x_7 + 32256 x_8 - 81920 x_9 <= -2

Constraint_2: Auxiliary_Variable - 24 x_1 - 393216 x_10 + 958464 x_11
 - 2293760 x_12 + 5406720 x_13 - 12582912 x_14 + 28966912 x_15 - 66060288 x_16
 + 149422080 x_17 - 335544320 x_18 + 748683264 x_19 - 1660944384 x_20
 + 160 x_3 - 768 x_4 + 2688 x_5 - 8192 x_6 + 23040 x_7 - 61440 x_8
 + 157696 x_9 <= 4

Constraint_3: Auxiliary_Variable + 128 x_1 + 745472 x_10 - 1835008 x_11
 + 4423680 x_12 - 10485760 x_13 + 24510464 x_14 - 56623104 x_15
 + 129499136 x_16 - 293601280 x_17 + 660602880 x_18 - 1476395008 x_19
 - 160 x_2 + 3279945728 x_20 + 896 x_4 - 4096 x_5 + 13824 x_6 - 40960 x_7
 + 112640 x_8 - 294912 x_9 <= -8

Constraint_4: Auxiliary_Variable - 480 x_1 - 1376256 x_10 + 3440640 x_11
 - 8388608 x_12 + 20054016 x_13 - 47185920 x_14 + 109576192 x_15
 - 251658240 x_16 + 572522496 x_17 - 1291845632 x_18 + 2894069760 x_19
```

```
 + 768 x_2 - 6442450944 x_20 - 896 x_3 + 4608 x_5 - 20480 x_6 + 67584 x_7
 - 196608 x_8 + 532480 x_9 <= 16

Constraint_5: Auxiliary_Variable + 1536 x_1 + 2457600 x_10 - 6291456 x_11
 + 15597568 x_12 - 37748736 x_13 + 89653248 x_14 - 209715200 x_15
 + 484442112 x_16 - 1107296256 x_17 + 2508193792 x_18 - 5637144576 x_19
 - 2688 x_2 + 12582912000 x_20 + 4096 x_3 - 4608 x_4 + 22528 x_6 - 98304 x_7
 + 319488 x_8 - 917504 x_9 <= -32

Constraint_6: Auxiliary_Variable - 4480 x_1 - 4194304 x_10 + 11141120 x_11
 - 28311552 x_12 + 69730304 x_13 - 167772160 x_14 + 396361728 x_15
 - 922746880 x_16 + 2122317824 x_17 - 4831838208 x_18 + 10905190400 x_19
 + 8192 x_2 - 24427626496 x_20 - 13824 x_3 + 20480 x_4 - 22528 x_5
 + 106496 x_7 - 458752 x_8 + 1474560 x_9 <= 64

Constraint_7: Auxiliary_Variable + 12288 x_1 + 6684672 x_10 - 18874368 x_11
 + 49807360 x_12 - 125829120 x_13 + 308281344 x_14 - 738197504 x_15
 + 1736441856 x_16 - 4026531840 x_17 + 9227468800 x_18 - 20937965568 x_19
 - 23040 x_2 + 47110422528 x_20 + 40960 x_3 - 67584 x_4 + 98304 x_5
 - 106496 x_6 + 491520 x_8 - 2097152 x_9 <= -128

Constraint_8: Auxiliary_Variable - 32256 x_1 - 9437184 x_10 + 29884416 x_11
 - 83886080 x_12 + 220200960 x_13 - 553648128 x_14 + 1350565888 x_15
 - 3221225472 x_16 + 7549747200 x_17 - 17448304640 x_18 + 39862665216 x_19
 + 61440 x_2 - 90194313216 x_20 - 112640 x_3 + 196608 x_4 - 319488 x_5
 + 458752 x_6 - 491520 x_7 + 2228224 x_9 <= 256

Constraint_9: Auxiliary_Variable + 81920 x_1 + 9961472 x_10 - 41943040 x_11
 + 132120576 x_12 - 369098752 x_13 + 964689920 x_14 - 2415919104 x_15
 + 5872025600 x_16 - 13958643712 x_17 + 32614907904 x_18 - 75161927680 x_19
 - 157696 x_2 + 171261820928 x_20 + 294912 x_3 - 532480 x_4 + 917504 x_5
 - 1474560 x_6 + 2097152 x_7 - 2228224 x_8 <= -512

Constraint_10: Auxiliary_Variable - 202752 x_1 + 44040192 x_11
 - 184549376 x_12 + 578813952 x_13 - 1610612736 x_14 + 4194304000 x_15
 - 10468982784 x_16 + 25367150592 x_17 - 60129542144 x_18 + 140123308032 x_19
 + 393216 x_2 - 322122547200 x_20 - 745472 x_3 + 1376256 x_4 - 2457600 x_5
 + 4194304 x_6 - 6684672 x_7 + 9437184 x_8 - 9961472 x_9 <= 1024

Constraint_11: Auxiliary_Variable + 491520 x_1 - 44040192 x_10
 + 192937984 x_12 - 805306368 x_13 + 2516582400 x_14 - 6979321856 x_15
 + 18119393280 x_16 - 45097156608 x_17 + 108984795136 x_18 - 257698037760 x_19
 - 958464 x_2 + 599147937792 x_20 + 1835008 x_3 - 3440640 x_4 + 6291456 x_5
 - 11141120 x_6 + 18874368 x_7 - 29884416 x_8 + 41943040 x_9 <= -2048

Constraint_12: Auxiliary_Variable - 1171456 x_1 + 184549376 x_10
 - 192937984 x_11 + 838860800 x_13 - 3489660928 x_14 + 10871635968 x_15
 - 30064771072 x_16 + 77846282240 x_17 - 193273528320 x_18 + 466003951616 x_19
 + 2293760 x_2 - 1.09951162778e+12 x_20 - 4423680 x_3 + 8388608 x_4
 - 15597568 x_5 + 28311552 x_6 - 49807360 x_7 + 83886080 x_8 - 132120576 x_9
 <= 4096

Constraint_13: Auxiliary_Variable + 2752512 x_1 - 578813952 x_10
 + 805306368 x_11 - 838860800 x_12 + 3623878656 x_14 - 15032385536 x_15
 + 46707769344 x_16 - 128849018880 x_17 + 332859965440 x_18
 - 824633720832 x_19 - 5406720 x_2 + 1.98427489075e+12 x_20 + 10485760 x_3
 - 20054016 x_4 + 37748736 x_5 - 69730304 x_6 + 125829120 x_7 - 220200960 x_8
 + 369098752 x_9 <= -8192

Constraint_14: Auxiliary_Variable - 6389760 x_1 + 1610612736 x_10
 - 2516582400 x_11 + 3489660928 x_12 - 3623878656 x_13 + 15569256448 x_15
 - 64424509440 x_16 + 199715979264 x_17 - 549755813888 x_18
 + 1.41733920768e+12 x_19 + 12582912 x_2 - 3.50469331354e+12 x_20
 - 24510464 x_3 + 47185920 x_4 - 89653248 x_5 + 167772160 x_6 - 308281344 x_7
 + 553648128 x_8 - 964689920 x_9 <= 16384
```

```
Constraint_15: Auxiliary_Variable + 14680064 x_1 - 4194304000 x_10
 + 6979321856 x_11 - 10871635968 x_12 + 15032385536 x_13 - 15569256448 x_14
 + 66571993088 x_16 - 274877906944 x_17 + 850403524608 x_18
 - 2.33646220902e+12 x_19 - 28966912 x_2 + 6.0129542144e+12 x_20
 + 56623104 x_3 - 109576192 x_4 + 209715200 x_5 - 396361728 x_6
 + 738197504 x_7 - 1350565888 x_8 + 2415919104 x_9 <= -32768

Constraint_16: Auxiliary_Variable - 33423360 x_1 + 10468982784 x_10
 - 18119393280 x_11 + 30064771072 x_12 - 46707769344 x_13 + 64424509440 x_14
 - 66571993088 x_15 + 283467841536 x_17 - 1.16823110451e+12 x_18
 + 3.60777252864e+12 x_19 + 66060288 x_2 - 9.89560464998e+12 x_20
 - 129499136 x_3 + 251658240 x_4 - 484442112 x_5 + 922746880 x_6
 - 1736441856 x_7 + 3221225472 x_8 - 5872025600 x_9 <= 65536

Constraint_17: Auxiliary_Variable + 75497472 x_1 - 25367150592 x_10
 + 45097156608 x_11 - 77846282240 x_12 + 128849018880 x_13 - 199715979264 x_14
 + 274877906944 x_15 - 283467841536 x_16 + 1.20259084288e+12 x_18
 - 4.94780232499e+12 x_19 - 149422080 x_2 + 1.52557238354e+13 x_20
 + 293601280 x_3 - 572522496 x_4 + 1107296256 x_5 - 2122317824 x_6
 + 4026531840 x_7 - 7549747200 x_8 + 13958643712 x_9 <= -131072

Constraint_18: Auxiliary_Variable - 169345024 x_1 + 60129542144 x_10
 - 108984795136 x_11 + 193273528320 x_12 - 332859965440 x_13
 + 549755813888 x_14 - 850403524608 x_15 + 1.16823110451e+12 x_16
 - 1.20259084288e+12 x_17 + 5.08524127846e+12 x_19 + 335544320 x_2
 - 2.08907209277e+13 x_20 - 660602880 x_3 + 1291845632 x_4 - 2508193792 x_5
 + 4831838208 x_6 - 9227468800 x_7 + 17448304640 x_8 - 32614907904 x_9
 <= 262144

Constraint_19: Auxiliary_Variable + 377487360 x_1 - 140123308032 x_10
 + 257698037760 x_11 - 466003951616 x_12 + 824633720832 x_13
 - 1.41733920768e+12 x_14 + 2.33646220902e+12 x_15 - 3.60777252864e+12 x_16
 + 4.94780232499e+12 x_17 - 5.08524127846e+12 x_18 - 748683264 x_2
 + 2.14404767416e+13 x_20 + 1476395008 x_3 - 2894069760 x_4 + 5637144576 x_5
 - 10905190400 x_6 + 20937965568 x_7 - 39862665216 x_8 + 75161927680 x_9
 <= -524288

Constraint_20: Auxiliary_Variable - 836763648 x_1 + 322122547200 x_10
 - 599147937792 x_11 + 1.09951162778e+12 x_12 - 1.98427489075e+12 x_13
 + 3.50469331354e+12 x_14 - 6.0129542144e+12 x_15 + 9.89560464998e+12 x_16
 - 1.52557238354e+13 x_17 + 2.08907209277e+13 x_18 - 2.14404767416e+13 x_19
 + 1660944384 x_2 - 3279945728 x_3 + 6442450944 x_4 - 12582912000 x_5
 + 24427626496 x_6 - 47110422528 x_7 + 90194313216 x_8 - 171261820928 x_9
 <= 1048576

VARIABLES
Auxiliary_Variable Continuous
x_1 Continuous
x_10 Continuous
x_11 Continuous
x_12 Continuous
x_13 Continuous
x_14 Continuous
x_15 Continuous
x_16 Continuous
x_17 Continuous
x_18 Continuous
x_19 Continuous
x_2 Continuous
x_20 Continuous
x_3 Continuous
x_4 Continuous
x_5 Continuous
x_6 Continuous
```

```
x_7 Continuous
x_8 Continuous
x_9 Continuous
```

In [20]:
```
LP_Prob_Aux.writeLP('ConstraintProblem_M_N_20.lp')
```

Out[20]:
```
[Auxiliary_Variable,
 x_1,
 x_10,
 x_11,
 x_12,
 x_13,
 x_14,
 x_15,
 x_16,
 x_17,
 x_18,
 x_19,
 x_2,
 x_20,
 x_3,
 x_4,
 x_5,
 x_6,
 x_7,
 x_8,
 x_9]
```

In [21]:
```
# The Problem is Solved Using PuLP's Choice of Solver.
LP_Prob_Aux.solve()
```

Out[21]:
```
1
```

In [22]:
```
print(f'Status: {LpStatus[LP_Prob_Aux.status]} \n')

for variable in LP_Prob_Aux.variables() :
    print(f'{variable.name} = {variable.varValue}')
print('\n')

if (LpStatus[LP_Prob_Aux.status] == 'Optimal') :
    print(f'Optimal Value : Z = {value(LP_Prob_Aux.objective)}')
    if (value(LP_Prob_Aux.objective) == 0) :
        print ('The Original LP Problem is feasible.')
    else :
        print ('The Original LP Problem is not feasible.')
else :
    print(f'No Optimal Value. Status Code : {value(LP_Prob_Aux.objective)}')
```

```
Status: Optimal

Auxiliary_Variable = 0.0
x_1 = 0.0
x_10 = 0.0
x_11 = 0.0
x_12 = 0.0
x_13 = 0.0
x_14 = 0.0
x_15 = 0.0
x_16 = 0.0
```

```
x_17 = 2.6769805e-08
x_18 = 0.0
x_19 = 0.0
x_2 = 0.00087719298
x_20 = 0.0
x_3 = 0.0
x_4 = 0.0
x_5 = 0.0
x_6 = 0.0
x_7 = 0.0
x_8 = 0.0
x_9 = 0.0


Optimal Value : Z = 0.0
The Original LP Problem is feasible.
```