Comment on Spin-Rotation-Gravity Coupling

The coupling of intrinsic spin to the angular velocity in a neutron interferometer was recently studied by Mashhoon [1] on the basis of a "tentative extension of the hypothesis of locality." I show that this extension is incorrect and that the predicted spin effect [1,2], contrary to what was implied [1], is a consequence of the usual treatment of interferometry [3,4] and no new hypothesis is needed. I propose a feasible experiment to observe this effect, which is 8 orders of magnitude more sensitive than the one proposed earlier [1], under present conditions. Also, I extend it to the gravitational field, and show that a recent extension [5] violates the principle of equivalence.

Physically, this effect is simply due to the neutron spin having a constant direction in a local inertial frame and therefore it rotates with respect to a rotating frame. This is seen from the neutron wave function between mirrors during free fall, which was obtained from a WKB approximation of the general relativistic Dirac's equation in a gravitational field to be [3]

$$\tilde{\Psi}(x)=\exp\left(-i\int_C k_\mu dx^\mu\right)P\exp\left(-i\int_C \Gamma_\mu dx^\mu\right)\tilde{\Psi}(x_0)\,, \tag{1}$$

where $\Gamma_\mu=\frac{1}{2}(e^a{}_\nu\nabla_\mu e^{b\nu})M_{ab}$, with M_{ab} generating the Lorentz group acting on spinor space, C is the integral curve of the curl free k^μ satisfying $k^\mu k_\mu=m^2c^2/\hbar^2$. Here, $\tilde{\Psi}(x)$ is the normalized value of the wave function at x that would be *observed* by a local inertial observer defined by the vierbein $e_a{}^\mu(x)$ $(a=0,1,2,3)$. Physically (1) implies that *the spin of the neutron during free fall is parallel transported as if it were a gyroscope* in this approximation. The effect of each mirror is to give the neutron a Lorentz boost and a Thomas precession in the laboratory frame [4] which does not commute with the operator in (1), in general. Hence, the effect of gravity alone cannot in general be written as an integral around a closed curve as done recently [5], unless second-order terms are neglected.

Suppose the interferometer has constant angular velocity $\mathbf{\Omega}$ in Minkowski space-time. Choose $e_a{}^\mu$ in (1) to rigidly rotate with the interferometer with $e_0{}^\mu$ as the four-velocity field of the apparatus. But the wave function Ψ of an inertial observer is obtained by choosing $e_a{}^\mu$ to be a Minkowski coordinate basis so that $\Gamma_\mu=0$. Hence, if x'^μ are the rotating coordinates labeling the same event as x^μ, then $\Psi'(x')\equiv\tilde{\Psi}(x)$ and $\Psi(x)$ are related by a *Lorentz boost* and a rotation, and not a pure rotation as assumed in the "generalization of the hypothesis of locality" [1].

To see the nonrelativistic analog, neglect the bending of the beams due to inertial fields, which does not contribute to the phase shift to lowest order. Then the velocity $\mathbf{u}$ of a given beam relative to the rotating frame is the same as if $\mathbf{\Omega}$ were $\mathbf{0}$. Hence, the two-component spinor field $\psi(\mathbf{x},t)$ in the *inertial frame* is related to the "unperturbed" $\psi_0(\mathbf{x}',t')$ by a *Galilei boost* corresponding to $\mathbf{V}=\mathbf{\Omega}\times\mathbf{x}$:

$$\psi(\mathbf{x},t)=\exp\left[\frac{im}{\hbar}\left(\int\mathbf{V}\cdot d\mathbf{x}'+\frac{\mathbf{V}^2t'}{2}\right)\right]\psi_0(\mathbf{x}',t')\,, \tag{2}$$

$(\mathbf{x},t)$ being related to $(\mathbf{x}',t')$ by the same *active* Galilei boost. (2) gives the "Sagnac" phase shift [6] $\Delta\phi=(m/\hbar)\oint\mathbf{V}\cdot d\mathbf{x}$.

But ψ cannot be transformed to the rotating frame by a *passive* transformation of the form (2) because then it would not be single valued, in general, because of the Sagnac phase. So, I *incorporate the Galilei boost into the new (kinetic) momentum and energy operators:* $\hat{\mathbf{p}}'=-i\hbar\nabla'-m\mathbf{V}$ and $\hat{p}_0'=i\hbar\,\partial/\partial t'+\frac{1}{2}m\mathbf{V}^2+\mathbf{\Omega}\cdot\mathbf{S}$, while ψ transforms as a scalar [7]. Schrödinger's equation then transforms to the rotating frame as

$$i\hbar\frac{\partial}{\partial t'}\psi'=-\frac{\hbar^2}{2m}\left(\nabla'-i\frac{m}{\hbar}\mathbf{V}\right)^2\psi'-\mathbf{\Omega}\cdot\mathbf{S}\psi'-\tfrac{1}{2}m\mathbf{V}^2\psi'\,. \tag{3}$$

The "minimal coupling" of $m\mathbf{V}/\hbar$ yields the Sagnac phase shift. Equation (3) and $\hat{\mathbf{p}}_\mu'$ may also be obtained as a nonrelativistic limit of the generally covariant Dirac's equation and relativistic energy-momentum operators $\hat{P}_\mu'=i\hbar\,\partial/\partial x^{\mu\prime}+i\Gamma_\mu$. The (energy) eigenvalues of $\hat{P}_0'$ which include mc^2 are positive. Thus there is no difficulty with quantum theory when applied to rotating frames as implied at the end of Ref. [1]. Indeed, this effect can even be understood classically: The spin state as viewed in the rotating frame must have an angular velocity $-\mathbf{\Omega}$ relative to its state as viewed in the inertial frame, because of the purely kinematical transformation between the two frames. This explains why, according to (3), the "g factor" for the neutron in the Coriolis and centrifugal fields is 1, instead of 2 as it is for a Dirac or Pauli electron in an electromagnetic field. To see this, rewrite the Hamiltonian in (3) as $H=-(\hbar^2/2m)\nabla'^2-\mathbf{\Omega}\cdot(\mathbf{L}+\mathbf{S})$, where $\mathbf{L}=-i\hbar\mathbf{x}\times\nabla'$ is the orbital angular momentum.

Consider now an interferometric experiment in which the spin state of one of the two interfering polarized beams is flipped and flipped back [1] by rotating it by 180° and −180° about axes A_1 and A_2, respectively, which are parallelly attached to the rotating frame and perpendicular to $\mathbf{\Omega}$. This can be done by suitable magnetic fields fixed with respect to the interferometer along A_1 and A_2. If $\mathbf{\Omega}$ is constant then (3) has a solution $e^{-i\omega t}\psi'(\mathbf{x})$. The spin components here have the same *total* canonical energy $\hbar\omega$ in the rotating frame, but different wave numbers k_+ and k_- for the quantization axis along $\mathbf{\Omega}$. Using (3), the phase shift due to the spin flips is $\Delta\theta=(k_+-k_-)l=\Omega l/u$, to first order, where l is the distance between A_1 and A_2 and u is the speed of the incoming neutron beam relative to the rotating frame. This differs from the treatments of $\Delta\theta$ in Refs. [1] and

[5], where it is obtained from different frequencies of the two spin components, because, here, the Hamiltonian in the rotating frame is stationary and therefore does not change the frequency.

But in the inertial frame the spin does not rotate except during the spin flips. And $\Delta\theta$ may be explained as due to A_2 having rotated with respect to A_1 by the angle $\mathbf{\Omega} l/u$ during the time that the neutron travels between the spin flips. The composition of the two rotations of the spin by π and $-\pi$ rad about A_1 and A_2 then is easily shown to be a total rotation of $2\mathbf{\Omega} l/u$. The action of this rotation on the neutron spinor gives the phase shift $\mathbf{\Omega} l/u$, as before. Hence this effect in the inertial frame is entirely due to the rotation of A_1 and A_2 which removes all the mystery from this effect. It would be necessary to eliminate the much larger Sagnac phase shift in this experiment by having a "figure-8" interferometer [1,8].

The Sagnac and centrifugal force effects can be eliminated much more easily by having a *single* polarized beam. Then the spin would precess by the angle $-\mathbf{\Omega} l/u$ in the rotating frame when the beam travels a distance l. This is measurable by nuclear or molecular beam magnetic resonance methods [9]. This effect due to the Earth's rotation may be varied by changing the orientation of the apparatus relative to the Earth, by placing the apparatus, shielded from external fields, on a table that is turned about an axis. This experiment is sensitive to a precession frequency of 4.7×10^{-7} Hz for the neutron whereas the magnitude of the Earth's angular velocity is 1.16×10^{-5} Hz. Also, it has the advantage that beams of atoms or molecules with much bigger spins and intensities than the neutron beam can be used.

Consider now the gravitational field due to a rotating object, assumed to be stationary for simplicity. Then, according to general relativity, if a nearby platform is nonrotating relative to the distant stars, which may be ensured by telescopes rigidly attached to it and focused on the stars, the local inertial frames would rotate with respect to the platform. Therefore, the momentum and the spin of a freely falling particle would rotate relative to the platform with the *same* angular velocity as the inertial frames, neglecting any interaction with the local space-time curvature. Hence there will be no change in the helicity of the particle, whether the particle is massive or massless. This is unlike a neutrino with a magnetic moment in a magnetic field $\mathbf{B}$, which rotates the spin but not the momentum and therefore changes the helicity. Because the helicity operator $\mathbf{S}\cdot\mathbf{p}/|\mathbf{p}|$, which commutes with $\mathbf{\Omega}\cdot(\mathbf{L}+\mathbf{S})$, does not commute with $\mathbf{B}\cdot\mathbf{S}$.

More generally in an arbitrary space-time, the principle of equivalence, according to which space-time is locally Minkowskian, implies that the helicity cannot change: In Eq. (1), the geodesic [3] C is the world line of the particle in the classical limit. In this limit, the energy-momentum $p^\mu=\hbar k^\mu$ and the Pauli-Lubanski spin vector W^μ, satisfying $p_\mu W^\mu=0$, are parallel transported along C. If the particle is massless, the helicity s is defined by $W^\mu=sp^\mu$. For a massive particle, suppose v^μ is the observer field obtained by parallel transporting along C the four-velocity of the source when it emitted the particle. Then

$$n^\mu\equiv\{m^2c^2(p_\rho v^\rho)^2-m^4c^4\}^{-1/2}(p_\rho v^\rho p^\mu-m^2c^2v^\mu)\,,$$

which satisfies $p_\mu n^\mu=0$ and $n^\mu n_\mu=-1$, is also paralle transported. Define the helicity with respect to the observer v^μ to be $s=(mc)^{-1}W^\mu n_\mu$. Hence the helicity does not change along C, whether the particle is massless or massive. If the particle is a neutrino emitted from a star and detected on the Earth, transforming from the observer v^μ at the detector to the detector four-velocity by a Lorentz boost would, in general, result in a small change in helicity for a massive neutrino but no change for a massless neutrino [10].

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[1] B. Mashhoon, Phys. Rev. Lett. **61**, 2639 (1988).
[2] A. Gorbatsevich, Exp. Tech. Phys. **27**, 529 (1979); V. G. Baryshevskii and S. V. Cherepitsa, Classical Quantum Gravity **3**, 713 (1986).
[3] J. Anandan, Nuovo Cimento **53A**, 221 (1979).
[4] J. Anandan, Phys. Rev. D **24**, 338 (1981); J. Anandan and B. Lesche, Lett. Nuovo Cimento **37**, 391 (1983); in Eq. (16) of the latter reference, $A_0^i=-\Omega^i/2c$, $A_j^i=0$, for a rotating vierbein field, which gives the present spin effect.
[5] Y. Q. Cai and G. Papini, Phys. Rev. Lett. **66**, 1259 (1991).
[6] J. Anandan, Phys. Rev. D **15**, 1448 (1977).
[7] This gauge transformation of the operators justifies the transformation of the wave function of Ref. [1], but only in the nonrelativistic case.
[8] J. Anandan, in *Proceedings of the Third Marcel Grossmann Meeting*, edited by Hu Ning (North-Holland, Amsterdam, 1983), p. 1187.
[9] N. F. Ramsey, *Molecular Beams* (Oxford Univ. Press, Oxford, 1956); K. F. Smith *et al.*, Phys. Lett. B **234**, 191 (1990).
[10] This disagrees with Ref. [5] which predicts a change in the helicity of a massless neutrino due to gravity.