usual variations in auroral luminosity. The fluctuations on October 29 are, indeed, unique in the Sacramento Peak records.

I thank Dr. C. T. Lewis, who sent me a pre-publication copy of his letter, and Prof. C. Gartlein for the data on the visual aurora in the northern hemisphere.

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* In this communication[1] the legend for the ordinate of Fig. 1, which appeared in the original tracing, was inadvertently left out in the published figure. The intensities for that figure are in Rayleighs, uncorrected for absorption or scattering.

[1] Silverman, S. M., *Nature*, **195**, 481 (1962).
[2] Lewis, C. T., *Nature*, **197**, 992 (1963).
[3] De Witt, R. N., *J. Geophys. Res.*, **67**, 1347 (1962).
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PHYSICS

Group Motions in Space-time and Doppler Effects

THE discovery of the Mössbauer effect has given fresh interest to the discussion of relativistic Doppler effects[1]. The purpose of this communication is to establish certain exact results in a very simple way.

Consider a space-time which admits a time-like group of motions, given infinitesimally by:

$$dx^r = X^r \, dw \tag{1}$$

where X^r is a vector field and dw an infinitesimal constant (Latin suffixes range 1–4 and the summation convention are used here). Let C' and C be two paths of this group (Fig. 1). Take events A' and A on these world lines and draw the geodesic $A'A$. On application of (1), A' is pushed to B' and A to B; the geodesic $A'A$ becomes the geodesic $B'B$, and these two geodesics have the same absolute measure (space-time 'length'). The measures (or proper time elements) of $A'B'$ and AB are respectively:

$$ds' = X' \, dw, \; ds = X \, dw \tag{2}$$

where:

$$X'^2 = (-g_{rs}X^rX^s)_{A'}, \; X^2 = (-g_{rs}X^rX^s)_A \tag{3}$$

g_{rs} being the metric tensor of space-time.

Suppose now that $A'A$ is a null geodesic. Then all its transforms under (1) are null geodesics; in particular,

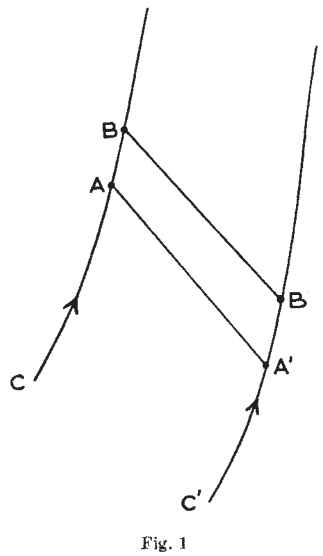

Fig. 1

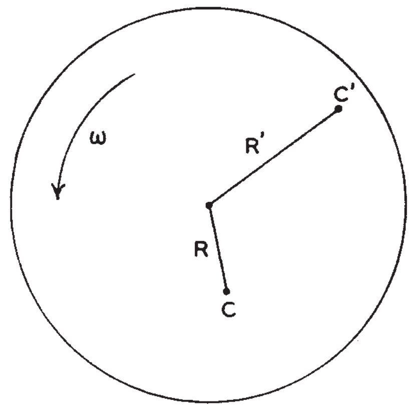

Fig. 2

$B'B$ is a null geodesic. If C' and C are the world-lines of a source of light (or other electromagnetic disturbance) and a receiver, we may regard $A'A$ and $B'B$ as the histories of two adjacent wave crests. Then the absolute (or proper) periods of emission and reception are respectively $\tau' = ds'$ and $\tau = ds$, and so the ratio of the frequencies of reception and emission is:

$$\nu/\nu' = \tau'/\tau = ds'/ds = X'/X \tag{4}$$

these frequencies being measured in the instantaneous rest frames of the receiver and the emitter. To calculate the Doppler effect, all we need is the ratio X'/X.

Consider now, in flat space-time with $g_{rs} = \text{diag}(1, 1, 1, -1)$, a disk rotating with constant angular velocity ω about the x^3-axis (Fig. 2). The world-lines of the particles of the disk are paths for the group of motions given by (1) with:

$$X^1 = -\omega x^2, \; X^2 = \omega x^1, \; X^3 = 0, \; X^4 = 1 \tag{5}$$

Let a source C' and a receiver C be carried round with the disk, at distances R' and R from its centre. Then by (3) and (5) we have:

$$X' = (1 - R'^2\omega^2)^{1/2}, \; X = (1 - R^2\omega^2)^{1/2} \tag{6}$$

and so, by (4), the ratio of the frequencies of reception and emission is:

$$\frac{\nu}{\nu'} = \left(\frac{1 - R'^2\omega^2}{1 - R^2\omega^2}\right)^{1/2} \tag{7}$$

This formula is exact. Note that the Doppler effect depends only on the distances R' and R, and not at all on the relative angular positions of source and receiver.

If the source and receiver are equidistant from the centre of the disk, there is no Doppler effect ($\nu = \nu'$).

Since the space-time of a stationary gravitational field admits a group of motions ($X^1 = X^2 = X^3 = 0$, $X^4 = 1$), the foregoing method gives at once the Doppler effect for any source and receiver which have paths of the group for world-lines. We have then $X' = (-g'_{44})^{1/2}$ and $X = (-g_{44})^{1/2}$.

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[1] Frauenfelder, H., *The Mössbauer Effect*, 62 (Benjamin, New York, 1962).

Longitudinal Sound Velocity in Columns of Granular Ammonium Perchlorate of Low Loading Density

THE longitudinal sound velocity in columns of granular ammonium perchlorate at an average loading density of $0{\cdot}93 \pm 0{\cdot}07$ g/c.c. was measured. The particle size distribution of the salt is shown in Fig. 1. The 50 per cent point (by weight) was 14μ.