The Electric Dipole Moment of a Moving Magnetic Dipole*

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The fact that a magnetic dipole $\boldsymbol{\mu}$ *moving with velocity* $\boldsymbol{\beta}c$ *has an electric dipole moment* $\mathbf{p}=\boldsymbol{\beta}\times\boldsymbol{\mu}/c$ *has made periodic appearance in the literature but the importance of this fact and its general utility have not been given sufficient expression. It is the purpose of this paper to show how to derive the equation* $\mathbf{p}=\boldsymbol{\beta}\times\boldsymbol{\mu}/c$ *and then to use it for a simple description of the atomic spin–orbit interaction.*

I. INTRODUCTION

While preparing to teach an undergraduate course on relativity and atomic physics, the author was appalled by the poor and sometimes incorrect explanation given to the spin–orbit interaction in texts on atomic physics, atomic spectra, and quantum mechanics.

One of the keys to a simple understanding of the spin–orbit interaction is to realize that a *moving* magnetic dipole has a real physical electric dipole moment. Although this idea is not new,[1] nor is it very surprising, it is usually either avoided or not known to many people who could use it.

For example, this electric dipole moment must be taken into account when measuring the intrinsic (in the particle rest frame) electric dipole moment of particles such as neutrons.

This paper is intended to show the advanced undergraduate physics student how to derive the expression for the electric dipole moment of an object in terms of its intrinsic magnetic dipole moment and the velocity of the object. This expression will then be used in describing first, the spin–orbit interaction in atoms, second, the necessity of taking the velocity dependent electric dipole moment into account when measuring the intrinsic electric dipole moment of a particle such as the neutron, and finally in describing one type of spin-dependent scattering.

II. ELECTRIC DIPOLE MOMENT OF MOVING MAGNETIC DIPOLE

Let us take a classical current loop to describe a pure magnetic dipole moment $\boldsymbol{\mu}$ in the dipole rest frame:

$$\rho=0,$$
$$\mathbf{j}=(\boldsymbol{\mu}\times\mathbf{r}/\pi R^4)\delta(\theta-\tfrac{1}{2}\pi)\delta(r-R). \tag{1}$$

We are using spherical polar coordinates (r, θ, φ) with μ in the (polar) z direction,

$$\boldsymbol{\mu}\equiv\int\tfrac{1}{2}\mathbf{r}\times\mathbf{j}d^3x. \tag{2}$$

Rationalized mks units will be used throughout.

Now j_μ is a Lorentz four-vector $(\mathbf{j}, c\rho)$ and the value of j_μ' in another system is

$$\rho'=\gamma\rho+\gamma\boldsymbol{\beta}\cdot\mathbf{j}/c,$$
$$\mathbf{j}'=\mathbf{j}+\{[(\gamma-1)/\beta^2](\boldsymbol{\beta}\cdot\mathbf{j})+\gamma c\rho\}\boldsymbol{\beta},$$
$$\gamma\equiv(1-\beta^2)^{-1/2}. \tag{3}$$

This is a Lorentz transformation without rotation where the unprimed system (dipole rest system) moves with velocity $c\boldsymbol{\beta}$ relative to the primed (lab) system. Therefore, there is a nonzero charge distribution in the lab system:

$$\rho'=(\gamma\boldsymbol{\beta}\cdot\boldsymbol{\mu}\times\mathbf{r}/\pi R^4c)\delta(\theta-\tfrac{1}{2}\pi)\delta(r-R).$$

The electric dipole moment is defined by $\mathbf{p}'\equiv\int\rho'\mathbf{r}'d^3x'$ in the lab (prime) system.

It is easiest now if we evaluate the integral in the unprimed system. (The answer is still $\mathbf{p}'$ in the lab system.):

$$\mathbf{r}'=\mathbf{r}+\{[(\gamma-1)/\beta^2](\boldsymbol{\beta}\cdot\mathbf{r})+\gamma ct\}\boldsymbol{\beta},$$
$$t'=\gamma t+\gamma\boldsymbol{\beta}\cdot\mathbf{r}/c.$$

Evaluate $\mathbf{p}'$ at $t'=0$ everywhere:

$$ct=-\boldsymbol{\beta}\cdot\mathbf{r},$$
$$\mathbf{r}'=\mathbf{r}-[\gamma/(\gamma+1)](\mathbf{r}\cdot\boldsymbol{\beta})\boldsymbol{\beta},$$
$$d^3x=dx_1'dx_2'(\gamma dx_3'-\gamma\beta c dt'),$$

but $dt'=0$; so $d^3x'=d^3x/\gamma$;

$$\mathbf{p}'=\int(\gamma\boldsymbol{\beta}\cdot\boldsymbol{\mu}\times\mathbf{r}/\pi R^4c)\delta(\theta-\tfrac{1}{2}\pi)\delta(r-R)\times\{\mathbf{r}-[\gamma/(\gamma+1)](\mathbf{r}\cdot\boldsymbol{\beta})\boldsymbol{\beta}\}(d^3x/\gamma).$$

If for convenience we choose a coordinate system,

$$\boldsymbol{\beta}=\beta_x\hat{e}_x+\beta_z\hat{e}_z,$$
$$\mathrm{R}=R\cos\varphi\hat{e}_x+R\sin\varphi\hat{e}_y,$$
$$\boldsymbol{\mu}=\mu\hat{e}_z,$$

the integral can be evaluated easily, with the result

$$\mathbf{p}'=\boldsymbol{\beta}\times\boldsymbol{\mu}/c; \tag{4}$$

$\mathbf{p}'$ is the electric dipole moment (measured in the lab) of the magnetic dipole $\boldsymbol{\mu}$, where the right hand side is evaluated in the dipole rest frame. It will be shown that both sides may be evaluated in the lab with the same result to first order in β.

III. OTHER MOMENTS

Let us now examine the more general problem of a particle moving with velocity $\boldsymbol{\beta}c$ and having charge q and intrinsic electric and magnetic dipole moments $\mathbf{p}$ and $\boldsymbol{\mu}$. In the particle rest frame,

$$\rho=q\delta^3(\mathbf{r})+(p/2b)[\delta^3(\mathbf{r}-\mathbf{b})-\delta^3(\mathbf{r}+\mathbf{b})],$$
$$\mathbf{j}=(\boldsymbol{\mu}\times\mathbf{r}/\pi R^4)\delta(\theta-\tfrac{1}{2}\pi)\delta(r-R),$$
$$\mathbf{p}=p\hat{b}. \tag{5}$$

Now define the electric and magnetic moments in the laboratory:

$$q'\equiv\int\rho' d^3x',$$
$$\mathbf{p}'\equiv\int\rho'\mathbf{a}'d^3x',$$
$$\boldsymbol{\mu}'\equiv\int\tfrac{1}{2}\mathbf{a}'\times\mathbf{j}'d^3x',$$
$$\mathbf{a}'\equiv\mathbf{r}'-\boldsymbol{\beta}ct=\mathbf{r}-[(\gamma-1)/\gamma\beta^2](\boldsymbol{\beta}\cdot\mathbf{r})\boldsymbol{\beta}. \tag{6}$$

To be meaningful in the laboratory Eq. (6) must be evaluated at $t'=0$. The transformations for $\mathbf{j}'$ and ρ' are given in Eq. (3). Assume that q, $\mathbf{p}$, and $\boldsymbol{\mu}$ do not change with time t so that we can remove some complications when carrying out the integrals at fixed t'. The results are as follows:

$$q'=q,$$
$$\mathbf{p}'=\mathbf{p}+\boldsymbol{\beta}\times\boldsymbol{\mu}/c-[(\gamma-1)/\gamma\beta^2](\boldsymbol{\beta}\cdot\mathbf{p})\boldsymbol{\beta},$$
$$\boldsymbol{\mu}'=(\boldsymbol{\mu}/\gamma)-[(\gamma-1)/2(\gamma+1)]\times\boldsymbol{\beta}\times(\boldsymbol{\beta}\times\boldsymbol{\mu})-\tfrac{1}{2}c\boldsymbol{\beta}\times\mathbf{p}. \tag{7}$$

This shows that to first order in β, $\mathbf{p}'=\mathbf{p}+\beta\times\boldsymbol{\mu}/c$ and $\boldsymbol{\mu}'=\boldsymbol{\mu}-\tfrac{1}{2}c\boldsymbol{\beta}\times\mathbf{p}$.

We should note that $\boldsymbol{\mu}$ and $\mathbf{p}$ also give rise to nonvanishing higher multipoles in the lab system.

IV. OTHER METHODS OF OBTAINING $\mathbf{p}'$

A. J. D. Jackson[2] suggests obtaining Eq. (4) by taking a vector potential for a magnetic dipole,

$$\mathbf{A}=\mu_0\boldsymbol{\mu}\times\mathbf{r}/4\pi r^3, \qquad \varphi=0,$$

and transforming the four-vector potential from a moving frame to the laboratory.

B. Panofsky and Phillips[3] obtain Eq. (4) by applying Maxwell's equations to a current carrying rectangular wire. They discuss the relationship of this problem with the classic problem of unipolar induction.

C. A. O. Barut,[4] Panofsky and Phillips,[3] among others have made Lorentz transforms of the electromagnetic field equations in a material medium and the partial fields corresponding to the tensor for the dipole moments. Just as **E** and **B** fields mix when transformed so do **D** and **H** fields as well as electric and magnetic dipole moments.

V. SPIN–ORBIT INTERACTION ENERGY

A. The atomic fine structure is due to the spin–orbit interaction. Unfortunately this interaction term consists of two very different physical phenomena that just happen to have almost identical mathematical form. I shall call these phenomena the electric dipole and the Thomas precession terms.

The electric dipole term is simply the potential energy (U) of the electric dipole moment of the orbiting electron ($\mathbf{p}$) coupled to the electric field ($\mathbf{E}$) from the rest of the atom:

$$U = -\mathbf{p}\cdot\mathbf{E}. \tag{8}$$

Using Eq. (4), we obtain

$$U = -\boldsymbol{\beta}\times\boldsymbol{\mu}\cdot\mathbf{E}/c.$$

Express $\boldsymbol{\mu}$ in terms of the electron spin $\mathbf{s}$ having magnitude ($\hbar/2$), the electron charge q, mass m, and "g factor" g:

$$\boldsymbol{\mu} = (gq/2m)\mathbf{s}.$$

After permuting the triple product,

$$U = -(gq/2mc)\mathbf{E}\times\boldsymbol{\beta}\cdot\mathbf{s}. \tag{9}$$

For a central field

$$\mathbf{E} = -(d\varphi/dr)\hat{r}.$$

The orbital angular momentum of the electron may be written

$$\mathbf{L} = m\mathbf{r}\times\boldsymbol{\beta}c.$$

Putting these together now yields the usual result in atomic physics texts:

$$U = -(gq/2m^2c^2r)(d\varphi/dr)(\mathbf{L}\cdot\mathbf{s}). \tag{10}$$

B. The Thomas precession term is the change in rotational kinetic energy of the electron due to the precession of the electron as seen by the laboratory observer. According to a slightly extended theory of special relativity, it can be shown that to a stationary observer any object which accelerates will precess with angular velocity[5]

$$\boldsymbol{\omega}_T = -(2c)^{-1}\boldsymbol{\beta}\times\mathbf{a} \qquad (\beta \ll 1); \tag{11}$$

$\boldsymbol{\beta}c$ is the velocity and $\mathbf{a}$ the acceleration of the object as measured by the laboratory observer who we assume is at rest.

The change in kinetic energy of a gyroscope due to a slow precession is

$$U(\text{Thomas}) = \mathbf{s}\cdot\boldsymbol{\omega}_T, \tag{12}$$

where $\mathbf{s}$ is the angular momentum of the object about its axis and $\boldsymbol{\omega}_T$ is the slow precession angular velocity. Substitute Eq. (11) into Eq. (12):

$$U(\text{Thomas}) = -(2c)^{-1}\mathbf{s}\cdot\boldsymbol{\beta}\times\mathbf{a}. \tag{13}$$

Permute the triple product to

$$U(\text{Thomas}) = +(2c)^{-1}\mathbf{a}\times\boldsymbol{\beta}\cdot\mathbf{s}. \tag{14}$$

We have not said what caused the acceleration and as far as the Thomas term is concerned it does not matter. Inglis[6] applied Eq. (12) to the acceleration of neutrons by nuclear forces.

In the atomic situation, the acceleration is due to Coulomb forces between the electron charge (q) and the electric field ($\mathbf{E}$) at the electron due to the rest of the atom:

$$\mathbf{a} = q\mathbf{E}/m. \tag{15}$$

Put this in Eq. (14) to get

$$U(\text{Thomas}) = (q/2mc)\mathbf{E}\times\boldsymbol{\beta}\cdot\mathbf{s}. \tag{16}$$

Now compare this with the electric dipole term [Eq. (9)] and we see at once that U(dipole) equals $-gU$(Thomas). Since g for an electron is 2, we get the well-known factor of $-\frac{1}{2}$.

$$U(\text{Thomas}) = -\tfrac{1}{2}U(\text{dipole}).$$

When we compute the total spin–orbit energy we get

$$\begin{aligned} U(\text{spin–orbit}) &= U(\text{dipole}) + U(\text{Thomas}) \\ &= -(g-1)(q/2mc)\mathbf{E}\times\boldsymbol{\beta}\cdot\mathbf{s}. \end{aligned} \tag{17}$$

The origin and application of the Thomas precession will be discussed in greater detail in a forthcoming paper by this author.

C. The usual method for obtaining the dipole term is to make a Lorentz transformation of the electromagnetic field from the lab to the instantaneous rest system of the electron using the transformation

$$\mathbf{E}=\gamma\mathbf{E}'-[(\gamma-1)/\beta^2](\boldsymbol{\beta}\cdot\mathbf{E}')\boldsymbol{\beta}+\gamma c\boldsymbol{\beta}\times\mathbf{B}',$$

$$\mathbf{B}=\gamma\mathbf{B}'-[(\gamma-1)/\beta^2](\boldsymbol{\beta}\cdot\mathbf{B}')\boldsymbol{\beta}-\gamma\boldsymbol{\beta}\times\mathbf{E}'/c. \quad (18)$$

In our example,

$$\mathbf{B}'=0, \qquad \mathbf{E}'=-(d\varphi/dr)\hat{r},$$

so

$$\mathbf{B}=-\gamma\boldsymbol{\beta}\times\mathbf{E}'/c.$$

In this frame of reference there is no electric dipole moment so the coupling is between the magnetic field $\mathbf{B}$ and the magnetic dipole moment $\boldsymbol{\mu}$:

$$U=-\boldsymbol{\mu}\cdot\mathbf{B}=-\boldsymbol{\mu}\cdot(-\gamma\boldsymbol{\beta}\times\mathbf{E}'/c). \quad (19)$$

Now transform this energy to the lab system:

$$U'=\gamma U=-\gamma^2\boldsymbol{\beta}\times\boldsymbol{\mu}\cdot\mathbf{E}'/c. \quad (20)$$

So $U'=-\boldsymbol{\beta}\times\boldsymbol{\mu}'\cdot\mathbf{E}'/c$ to first order in β, which is Eq. (8).

By computing the energy in the electron rest frame and then transforming to the lab frame we include all of the higher multipoles which were neglected in Sec. V.*A*. But remember that this is only the dipole term of the spin–orbit interaction.

The author objects to this method of approaching the spin–orbit interaction because of the insinuation that there is no interaction in the lab system.

D. The Dirac equation of an electron in an arbitrary electromagnetic field contains both effects in the spin–orbit interaction among others and does so in a very elegant way.[7] The customary way of showing the presence of the spin–orbit energy terms in the Dirac equation is to multiply by the conjugate Hamiltonian and then to solve for $E'=E-mc^2$ in the nonrelativistic limit, yielding,[7,8] in mks units,

$$E'\psi=[(2m)^{-1}(\mathbf{p}-e\mathbf{A})^2+e\varphi$$
$$-(e\hbar/2m)\boldsymbol{\sigma}'\cdot\mathbf{B}-(ie\hbar/2mc)\boldsymbol{\alpha}\cdot\mathbf{E}]\psi.$$

The term in $\mathbf{B}$ predicts the electron magnetic moment of $(e\hbar/2m)\boldsymbol{\sigma}'$, which has the correct g factor of 2.

Of interest to us is the last term which is the spin–orbit term.

Let us assume that this last term is $\frac{1}{2}$ the dipole term (assuming it already contains the Thomas term). Then the electric dipole moment operator is

$$\mathbf{p}=(ie\hbar/mc)\boldsymbol{\alpha}. \quad (21)$$

It is a straightforward matter to show that

$$2i\boldsymbol{\alpha}=\boldsymbol{\sigma}'\times\boldsymbol{\alpha}, \quad (22)$$

$$\therefore\ \mathbf{p}=(e\hbar/2mc)\boldsymbol{\sigma}'\times\boldsymbol{\alpha}. \quad (23)$$

According to Dirac theory the operator $\boldsymbol{\alpha}=-\boldsymbol{\beta}$ and the magnetic dipole moment is $\boldsymbol{\mu}=(e\hbar/2m)\boldsymbol{\sigma}'$; therefore

$$\mathbf{p}=\boldsymbol{\mu}\times(-\boldsymbol{\beta})/c,$$

$$\therefore\ \mathbf{p}=\boldsymbol{\beta}\times\boldsymbol{\mu}/c,$$

which is the classical Eq. (4).

VI. MEASUREMENT OF ELECTRIC DIPOLE MOMENT

Measurements have been made on the intrinsic electric dipole moment (p) of the neutron,[9–11] proton,[12] and electron.[13] The intrinsic electric dipole moment of the neutron has been measured with the greatest accuracy with the present upper limit being[9] $p\leq 5\times 10^{-23}$ e cm. Since the magnetic dipole moment of a neutron is 1.91 nuclear magnetons, the electric dipole moment (p'') due to the neutron motion is $p''=2.0\times 10^{-14}\beta$ e cm. If p'' is to be less than p (the upper limit on the intrinsic moment) then βc must be less than 0.75 m/sec. Some investigators are using ultracold neutrons with speeds as low as

6 m/sec.[14] Since this is still much larger than 0.75 m/sec, all of these experiments must either be insensitive to the effect of p'', the velocity dependent electric dipole moment, or make an appropriate correction for it. By making the magnetic and electric fields very close to parallel ($\theta < 1.5 \times 10^{-3}$ rad) the detection system of Ramsey *et al.*[9] is insensitive to p''. Their experiment essentially measures the interaction energy of the magnetic and electric dipole moments in parallel magnetic and electric fields:

$$U = -\boldsymbol{\mu}' \cdot \mathbf{B} - \mathbf{p}' \cdot \mathbf{E}.$$

To first order in β this becomes

$$U = -\boldsymbol{\mu} \cdot \mathbf{B} - \mathbf{p} \cdot \mathbf{E} - \boldsymbol{\beta} \times \boldsymbol{\mu} \cdot \mathbf{E}/c,$$

where the velocity dependent part of the electric dipole moment is $p'' = c^{-1}\boldsymbol{\beta} \times \boldsymbol{\mu}$. If the magnetic field is strong enough to orient $\boldsymbol{\mu}$ along the $\mathbf{B}$ direction that is parallel to the $\mathbf{E}$ direction and perpendicular to $\boldsymbol{\beta}$, then $\mathbf{p}'' \cdot \mathbf{E} = c^{-1}\boldsymbol{\beta} \times \boldsymbol{\mu} \cdot \mathbf{E} = 0$, and p'' does not contribute to the energy.

VII. SPIN–ORBIT SCATTERING BY ELECTRIC FIELD

Since an electric dipole experiences a force when placed in a nonuniform electric field, we are lead to expect a small, possibly significant, spin-dependent scattering of beams of magnetic dipoles passing through electric field gradients. For example one could do a Stern–Gerlach experiment with a nonuniform electric field rather than a nonuniform magnetic field. Of particular interest here is the production and analysis of polarized neutron beams. This was first discussed by Schwinger[15] in 1948 and again recently by Handel,[16] and Shull and Nathans.[17]

Schwinger[15] showed how to compute the spin-dependent scattering cross section and polarization of fast (1 MeV) neutrons in the Coulomb field of materials such as lead. Handel's paper,[16] entitled "Asymmetric Spin–Orbit Effects Calculated for the Total Reflection of Polarized Cold Neutrons," helps explain effects observed in neutron guides with ferromagnetic walls.

It should be pointed out that we are not considering either the strong-nuclear spin–orbit interaction (which was described by Inglis[6] as a Thomas precession effect due to acceleration by nuclear forces), nor the spin–spin interactions between the neutron and the scattering material. The following is a simple physical picture which is useful when trying to understand the processes described by Schwinger and Handel. Since the neutron has an electric dipole moment given by Eq. (4), the spin–orbit energy (U) of a neutron in an electric field ($\mathbf{E}$) is

$$U = -\mathbf{p} \cdot \mathbf{E} = -\boldsymbol{\beta} \times \boldsymbol{\mu} \cdot \mathbf{E}/c;$$

the force on the neutron is the negative gradient of the potential energy.

If the velocity is in the y direction and the magnetic moment is in the z direction, then the electric dipole moment is in the positive x direction. We are, therefore, only interested in the gradient of the x component of the electric field

$$\mathbf{F} = p \nabla E_x.$$

It is important to note that the x component of the gradient of E_x is always negative outside of the nucleus. Therefore, the force on the dipole will always be to the left (i.e., is the negative x half-plane). Since the neutron magnetic moment is in the direction opposite to its spin, we can restate the result in the following way: If we look from behind the neutron whose spin is up (down) the neutron will always have a force tending to scatter it to the right (left). Therefore, if an unpolarized beam of neutrons is scattered from the electric field of an atom the polarization of the neutrons coming out depends on the angle of scatter. Thus a spin-zero atom can act as a polarizer. Similarly we can use these atoms as polarization analyzers.

CONCLUSION

The Lorentz transformations for electric and magnetic dipoles are derived and presented in Eq. (7). To first order in the relative velocity a magnetic dipole transforms into a magnetic dipole as well as an electric dipole. As an example, a pure magnetic dipole of one Bohr magneton moving with velocity βc relative to the lab will have a lab electric dipole moment $p' = \frac{1}{2}\beta e\lambda$, where e is the electronic charge and λ is the electron Compton wavelength:

$$p' = 1.93 \times 10^{-11}\beta \text{ e cm}.$$

Small as this is it is shown that the electric dipole part of the atomic spin–orbit interaction energy can be simply interpreted as the coupling between the electric dipole moment of the moving electron and the local electric field.

Even when the magnetic dipole is of the order of a nuclear magneton and moving very slowly, as in the case for neutrons, the velocity dependent part of the electric dipole moment is large compared to present upper limits on the intrinsic electric dipole moment.

As a last example it has been shown how the spin–orbit forces can change the polarization of a beam of particles.

Making use of the fact that a moving magnetic dipole has an electric dipole moment makes the above examples straightforward. Perhaps this will be useful in making other problems better understood.

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[1] J. Frenkel, Z. Physik **37**, 243 (1926). C. G. Darwin, Proc. Roy. Soc. (London) **120**, 621 (1928). F. K. Richtmyer, E. H. Kennard, and T. Lauritsen, *Introduction to Modern Physics* (McGraw–Hill, New York, 1955), p. 252.

[2] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), Problem 11.9, p. 389.

[3] W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism* (Addison–Wesley, Reading, Mass., 1962), 2nd ed., p. 334–337.

[4] A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (MacMillan, New York, 1964), p. 73. Note the different definition for the moments.

[5] L. H. Thomas, Nature **117**, 514 (1926); Phil. Mag. **3**, 1 (1927).

[6] D. R. Inglis, Phys. Rev. **50**, 783 (1936).

[7] P. A. M. Dirac, Proc. Roy. Soc. (London) **117**, 610 (1928).

[8] L. I. Schiff, *Quantum Mechanics* (McGraw–Hill, New York, 1955), p. 330.

[9] J. K. Baird, P. D. Miller, W. B. Dress, and N. F. Ramsey, Phys. Rev. **179**, 1285 (1969).

[10] References contained in article by M. S. Rothenberg, Phys. Today **22**, 56 (November 1969).

[11] L. B. Okun, Comments Nucl. Particle Phys. **3**, 133 (1969).

[12] R. M. Sternheimer, Phys. Rev. **113**, 828 (1959).

[13] R. E. Rand, Phys. Rev. **140**, B1605 (1965).

[14] V. I. Lushchikov *et al.*, JETP Letters **9**, 23 (1969).

[15] J. Schwinger, Phys. Rev. **73**, 407 (1948).

[16] P. H. Handel, Phys. Rev. Letters **24**, 835 (1970).

[17] C. G. Shull and R. Nathans, Phys. Rev. Letters **19**, 384 (1967).