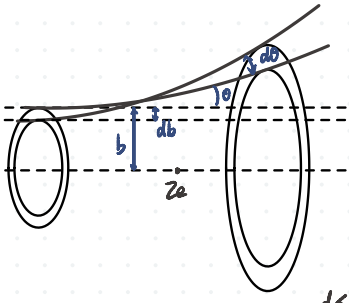
2. 卢瑟福散射公式



$$b=\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{E_\alpha}\cot\frac{\theta}{2}$$

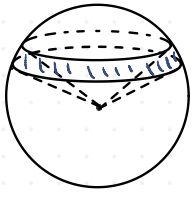
$$(\cot\chi)'=-1-\frac{\cos^2\chi}{\sin^2\chi}=-\frac{1}{\sin^2\chi}$$

$$db=\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{E_\alpha}\left(-\frac{1}{\sin^2\frac{\theta}{2}}\right)\frac{1}{2}d\theta$$

$b\sim b-db$

$$-2\pi b\,db=2\pi\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{E_\alpha}\cot\frac{\theta}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{E_\alpha}\left(-\frac{1}{\sin^2\frac{\theta}{2}}\right)\frac{1}{2}d\theta$$

$$d\sigma=2\pi b\,db=\pi\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Ze^2}{E_\alpha}\right)^2\frac{\cos\frac{\theta}{2}}{\sin^3\frac{\theta}{2}}d\theta$$



$$d\Omega=\frac{dS}{r^2}=\sin\theta\,d\theta\,d\varphi$$

$$d\Omega'=\int_0^{2\pi}\sin\theta\,d\theta\,d\varphi=2\pi\sin\theta\,d\theta=4\pi\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,d\theta$$

$$\frac{d\sigma}{d\Omega'}=\frac{\pi\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Ze^2}{E_\alpha}\right)^2\frac{\cos\frac{\theta}{2}}{\sin^3\frac{\theta}{2}}d\theta}{4\pi\sin\frac{\theta}{2}\cos\frac{\theta}{2}d\theta}=\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Ze^2}{E_\alpha}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}$$

1个α粒子被原子核散射到θ方向单位立体角的概率

$$\frac{nAt\,d\sigma}{A}=\frac{dN}{N}$$

$$d\sigma=\frac{dN}{ntN}$$

$$\frac{dN}{d\Omega'}=ntN\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Ze^2}{E_\alpha}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}$$