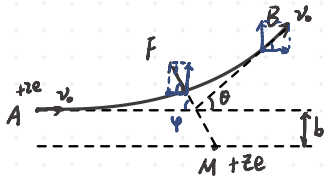
1. 库伦散射公式



受力分析：$F=\frac{Ze\cdot Ze}{4\pi\varepsilon_0 r^2}=\frac{2Ze^2}{4\pi\varepsilon_0 r^2}=\frac{Ze^2}{2\pi\varepsilon_0 r^2}$

垂直方向：$ma_\perp=F_\perp$

$$m\frac{dv_\perp}{dt}=F\sin\varphi=\frac{Ze^2}{2\pi\varepsilon_0 r^2}\sin\varphi$$

$$m\frac{dv_\perp}{dt}=\frac{Ze^2}{2\pi\varepsilon_0}\cdot\frac{1}{bv_0}\frac{d\varphi}{dt}\sin\varphi$$

$$\int_0^{v_0\sin\theta}m\,dv_\perp=\int_0^{\pi-\theta}\frac{Ze^2}{2\pi\varepsilon_0 bv_0}\sin\varphi\,d\varphi$$

$$mv_0\sin\theta=\frac{Ze^2}{2\pi\varepsilon_0 bv_0}(-\cos\varphi)\Big|_0^{\pi-\theta}$$

$$=\frac{Ze^2}{2\pi\varepsilon_0 bv_0}\left[-\cos(\pi-\theta)+1\right]$$

$$=\frac{Ze^2}{2\pi\varepsilon_0 bv_0}(\cos\theta+1)$$

角动量守恒：$b\cdot mv_0=mr^2\cdot\omega=mr^2\cdot\frac{d\varphi}{dt}$

$$bv_0=r^2\frac{d\varphi}{dt}$$

$$\frac{1}{r^2}=\frac{1}{bv_0}\frac{d\varphi}{dt}$$

能量守恒：$v_A=v_B$

$$2mv_0\sin\frac{\theta}{2}\cos\frac{\theta}{2}=\frac{Ze^2}{2\pi\varepsilon_0 bv_0}\left(2\cos^2\frac{\theta}{2}-1+1\right)$$

$$\frac{1}{2}mv_0\sin\frac{\theta}{2}\cos\frac{\theta}{2}=\frac{Ze^2}{2\pi\varepsilon_0 bv_0}\frac{1}{2}\cos^2\frac{\theta}{2}$$

$$mv_0\tan\frac{\theta}{2}=\frac{Ze^2}{2\pi\varepsilon_0 bv_0}$$

$$b=\frac{Ze^2}{2\pi\varepsilon_0 mv_0^2}\cot\frac{\theta}{2}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{E_{k\alpha}}\cot\frac{\theta}{2}$$