1. min $f(x)=\frac{1}{2}x^TPx+q^Tx$

s.t. $Ax=b$

最优性条件 $\begin{cases}Ax^*=b\\ Px^*+q+A^Tv^*=0\end{cases}$

infeasible start 的牛顿法.

$$\begin{pmatrix}P & A^T\\ A & 0\end{pmatrix}\begin{pmatrix}\Delta x_{pd}\\ \Delta v_{pd}\end{pmatrix}=-\begin{pmatrix}Px+q+A^Tv\\ Ax-b\end{pmatrix}$$

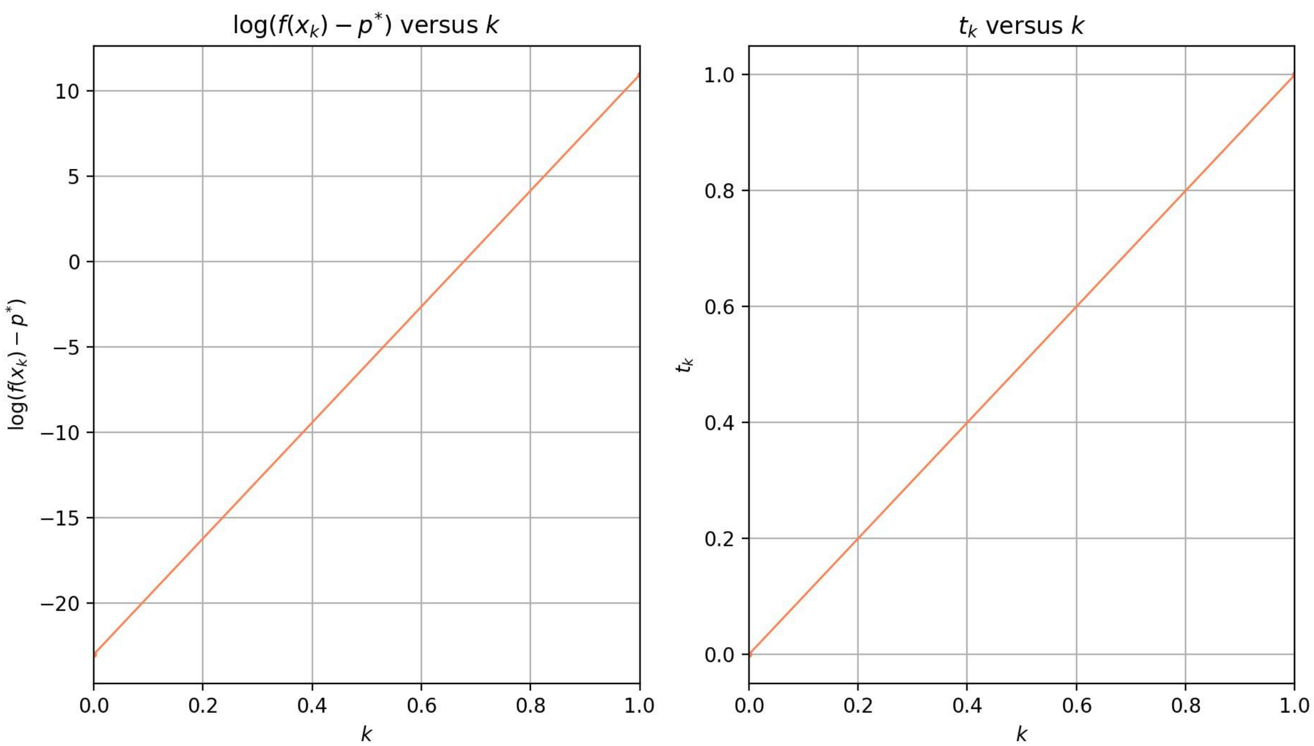
$$r(x,v)=\begin{pmatrix}Px+q+A^Tv\\ Ax-b\end{pmatrix}=\begin{pmatrix}r_{daul}\\ r_{pri}\end{pmatrix}$$

终止条件 $\lambda^2(x)=\Delta x_{nt}^T\nabla^2 f(x)\Delta x_{nt}<10^{-5}$

超参数 $\alpha=0.4$, $\beta=0.5$

初值 $x^{(0)}=\mathbb{1}^{200}$, $v^{(0)}=\mathbb{1}^{100}$.

图像如下:



原问题最优解x*、对偶问题最优解v*、函数最优值p*见 ./results/T1.txt

2. $\min\ f(x) = \frac{1}{2}x^T P x + q^T x$

s.t. $Ax = b$

$x \geq 0$.

①. 障碍函数法.

$\frac{n}{t}$ 次优的优化问题.

$$\min\quad tf(x) + \phi(x) = \frac{1}{2}t x^T P x + t q^T x - \sum_{i=1}^{n} \log(x_i)$$

s.t. $Ax = b$.

$\nabla f(x) = Px + q$, $\nabla^2 f(x) = P$.

$$\nabla \phi(x) = \begin{pmatrix} -1/x_1 \\ \vdots \\ -1/x_n \end{pmatrix}, \quad \nabla^2 \phi(x) = \begin{pmatrix} 1/x_1^2 & & & 0 \\ & 1/x_2^2 & & \\ & & \ddots & \\ 0 & & & 1/x_n^2 \end{pmatrix}$$

infeasible start 的牛顿法 (第一个循环).

$$\begin{pmatrix} t\nabla^2 f(x) + \nabla^2\phi(x) & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} \Delta x_{pd} \\ \Delta v_{pd} \end{pmatrix} = -\begin{pmatrix} t\nabla f(x) + \nabla\phi(x) + A^T v \\ Ax - b \end{pmatrix} = -r(x, v)$$

终止条件 $\|r(x,v)\|_2 < 10^{-8}$

feasible start 的牛顿法 (第二个及之后的循环

$$\begin{pmatrix} t\nabla^2 f(x) + \nabla^2\phi(x) & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} \Delta x_{nt} \\ w \end{pmatrix} = \begin{pmatrix} -t\nabla f(x) - \nabla\phi(x) \\ 0 \end{pmatrix}$$

终止条件: $\frac{1}{2}\lambda^2(x) < 10^{-8}$

$\frac{n}{t} < \varepsilon = 10^{-8}$ 后, 退出循环.

原问题最优解x*、对偶问题最优解λ*和v*、函数最优值p*见 ./results/T2_a.txt

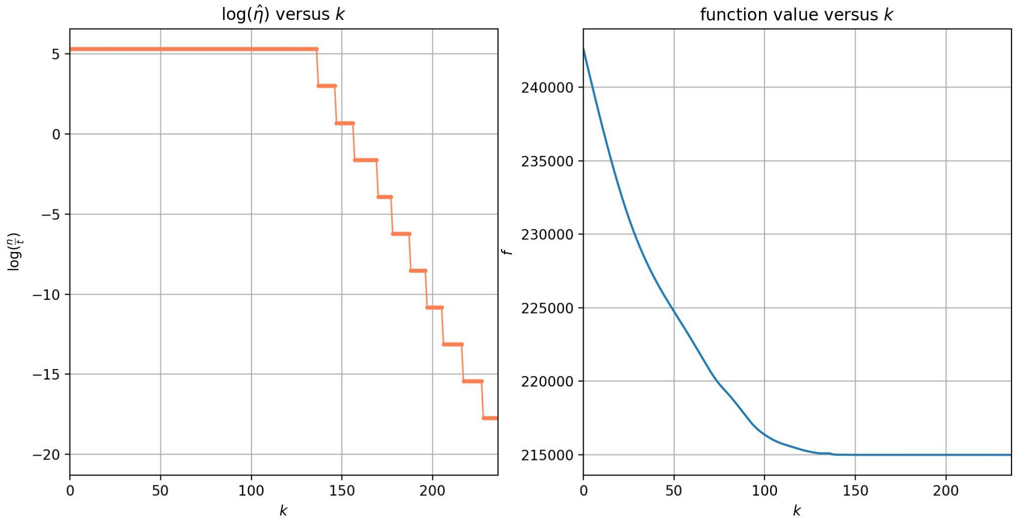
解出 x^* 后, $\lambda_i^* = -\frac{1}{t \cdot (-x_i)} = \frac{1}{t x_i}$

$$v_i^* = \frac{\hat{v}_i}{t} = \frac{w_i}{t}$$

回溯直线搜索 $\alpha = 0.1$, $\beta = 0.5$.

$\mu = 10$.

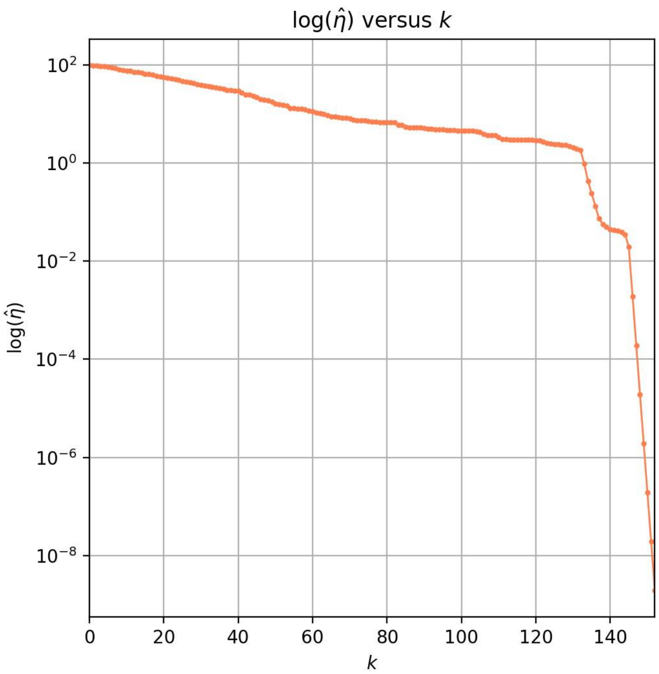
图像如下:

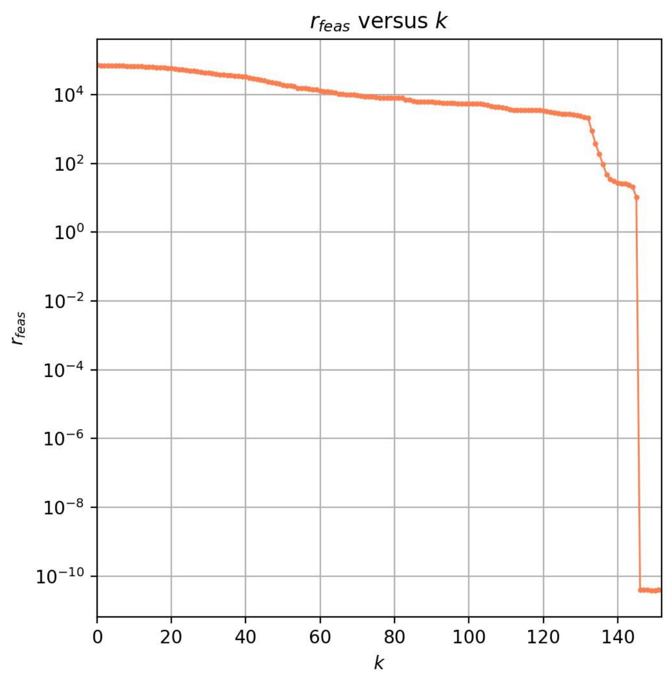


②. 原对偶内点法.

$$\begin{pmatrix} \nabla^2 f(x) & -I_{200} & A^T \\ \begin{pmatrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_{200}\end{pmatrix} & \begin{pmatrix} x_1 & & 0\\ & \ddots & \\ 0 & & x_{200}\end{pmatrix} & O_{200\times100} \\ A & O_{100\times200} & O_{100\times100}\end{pmatrix}\begin{pmatrix}\Delta x\\ \Delta\lambda \\ \Delta v\end{pmatrix} = -\begin{pmatrix} Px+q-\lambda+A^Tv \\ \mathrm{diag}(\lambda)x - \frac{1}{t}\mathbb{1}_{200} \\ Ax-b\end{pmatrix}$$

图像如下:





原问题最优解x*、对偶问题最优解λ*和v*、函数最优值p*见 ./results/T2_b.txt

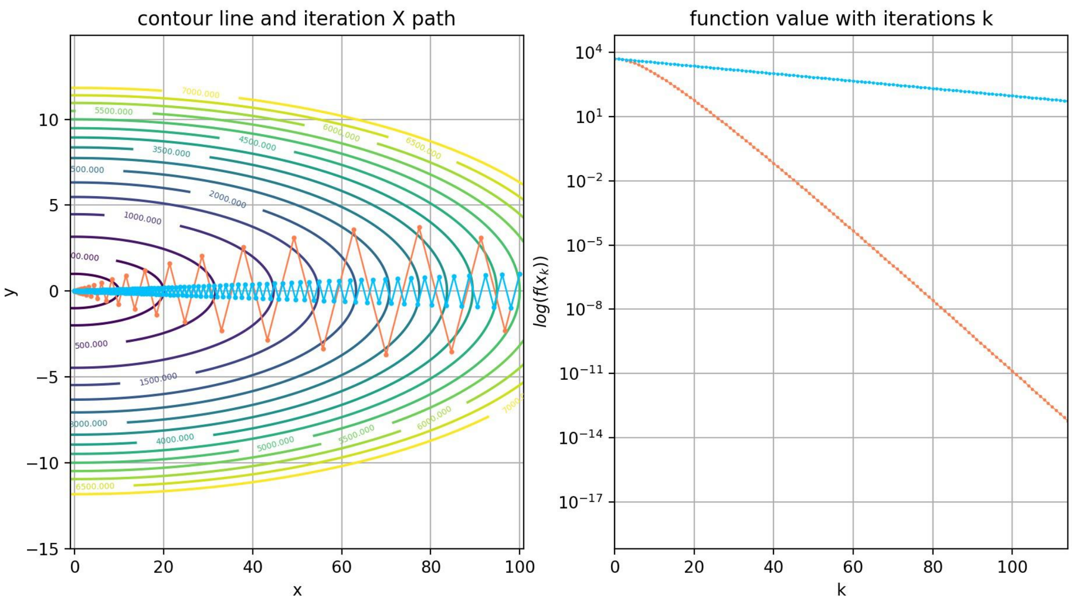
3. Heavy-Ball 算法.

$$x_{k+1} = x_k - \alpha \cdot A x_k + \beta (x_k - x_{k-1}).$$

Gradient Descent 算法.

$$\alpha = \frac{2}{101}.$$

图像:



蓝色曲线为梯度下降的结果；
红色曲线为Heavy-Ball加速的结果。