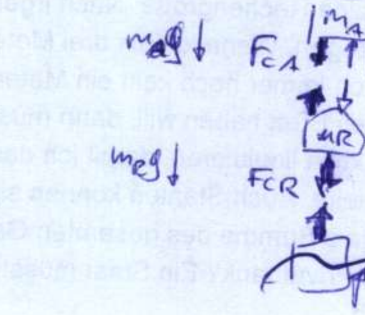
2.7.2010

$$x = \begin{bmatrix} z_R \\ z_A \\ v_R \\ v_A \end{bmatrix}, \quad y = z_A, \quad u = F, \quad d = z_s$$

$$\dot x = f(x,u,d) \qquad y = g(x,u,d)$$

$$F_{CA} = c_A(z_A - z_r - l_A)$$

$$F_{CR} = c_R(z_R - z_s - l_R)$$



$$\dot v_A = \frac{1}{m_A}\left(F - F_{CA} - m_A g\right) = \frac{F}{m_A} + c_A\frac{l_a + (z_R - z_a)}{m_A} - g$$

$$\dot v_R = \frac{1}{m_R}\left(-F + F_{CA} - F_{CR} - m_r g\right) = -\frac{F}{m_r} - \frac{c_R(z_R - z_s - l_R)}{m_r} - c_A\frac{l_a + z_R - z_a}{m_A} - g$$

$$\dot x = \begin{bmatrix} \dot z_R \\ \dot z_a \\ \dot v_R \\ \dot v_a \end{bmatrix} = \begin{bmatrix} v_R \\ v_A \\ -\frac{c_R(z_R - z_s - l_R)}{m_r} - \frac{c_a(l_a + z_R - z_a)}{m_r} - \frac{F}{m_r} - g \\ c_A\frac{(l_a + z_R - z_a)}{m_a} + \frac{F}{m_a} - g \end{bmatrix}$$

$$y = z_A$$

e) $v_{R,R} = 0,\ v_{A,R} = 0,\ u_R = 0,\ d_R = 0$

$$0 = c_R(z_{R,R} - l_r) + c_a(l_a + z_{R,R} - z_{a,R}) - m_r g$$

$$0 = c_A(l_a + z_{R,R} - z_a) - m_a g \Rightarrow z_{R,R} - z_{a,R} = \frac{m_a g}{c_A} - l_a$$

$$m_r g = c_R(z_{R,R} - l_r) + c_a\left(l_a + \frac{m_a g}{c_a} - l_a\right)$$

$$\boxed{z_{R,R} = \frac{m_r g - m_a g}{c_R} + l_R}$$

$$z_{A,R} = \frac{m_r g - m_a g}{c_R} + l_R + l_a - \frac{m_a g}{c_a}$$

c) $b_u = \left[0 \quad 0 \quad -\frac{1}{m_r} \quad +\frac{1}{m_a}\right]$

$b_d = \left[0 \quad 0 \quad -\frac{c_R}{m_r} \quad 0\right]$

$d_d = 0$

$d_u = 0$

$c^T = (0, 0 \; 0 \; 1)$

$$A = \frac{\partial f(x_R, u_R)}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\left(\frac{c_R}{m_r} + \frac{c_a}{m_r}\right) & +\frac{c_a}{m_r} & 0 \\ \frac{c_a}{m_a} & -\frac{c_a}{m_a} & 0 \end{bmatrix}$$

d) Da keine Dämpfung vorhanden, wird keine Energie abgegeben → schwingt ewig ⇒ nicht asympt. stabil

$$\begin{pmatrix} \dot z_R \\ 0 \\ \dot v_R \\ \dot Q \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ -\frac{c_a + c_h}{m_r} & \frac{c_a}{m_r} & 0 & 0 \\ \frac{c_a}{m_o} & -\frac{c_a}{m_o} & 0 & 0 \end{pmatrix} \begin{pmatrix} z_R \\ 0 \\ v_R \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{m_r} \\ \frac{1}{m_o} \end{pmatrix} \Delta u + \begin{pmatrix} 0 \\ 0 \\ \frac{c_R}{m_r} \\ 0 \end{pmatrix} \Delta z_s$$

$\dot z_R = v_R$

$\dot v_R = -\frac{z_R}{m_r}(c_a + c_h) - \frac{\Delta u}{m_r} + \frac{c_R \Delta z_s}{m_r}$

$0 = \frac{z_R c_a}{m_o} + \frac{\Delta u}{m_a} \rightarrow \boxed{\Delta u = -z_R c_a}$

2a)

i) $C = \frac{2\cdot(s+9)}{s^2-1} = \frac{2(s+9)}{(s+1)(s-1)}$

$\Delta \arg(1+L(j\omega)) = (2-1+1)\pi = 2\pi$

stetige Winkeländerung $= 2\pi \rightarrow$ BIBO-stabil

ii) $R(s) = \frac{s-1}{s+1}$ $\quad G(s) = \frac{s+9}{(s+1)(s-1)}$

$T_{r,y} = \frac{s+9}{s+9+(s+1)^2} = \frac{s+9}{s^2+3s+10} \rightarrow$ HW-Polynom $\rightarrow$ BIBO-stabil

$(s^2+3s+10):(s+9) = s-6$ .. Rest

$\underline{-s^2+9s}$

$-6s+0$

$T_{d,y}(s) = \frac{\frac{s+9}{(s+1)(s-1)}}{1+\frac{s+9}{(s+1)^2}} = \frac{(s+9)(s+1)}{(s+9)(s+1)+(s+1)^2(s-1)}$ $\frac{s^2+10s+9}{(s+1)(s+9+(s^2+8))} =$

$\hookrightarrow$ instabil

$= \frac{s^2+10s+9}{s^2+10s+9+s^3+2s^2+s-s^2-2s-1}$ $\cancel{= \frac{s^2+10s+9}{s^3+2s^2+9s+8}}$

$\cancel{s^3+2s^2+9s+8} = \cancel{\frac{s^2+10s+9}{(s+1)(s^2+s+8)}}$

iii) $T_{r,y} = \frac{\frac{(1+s)}{s}\frac{s+9}{(s^2-1)(s-1)}}{1+\frac{(1+s)(s+9)}{s(s^2-1)(s-1)}}$ $\frac{s^2+10s+9}{s^2+10s+9+(s^2-s)\,s^{3}\ldots -s}$ $= \frac{s^2+10s+9}{s^3+s^2+9s+9}$

$= \frac{s+9}{s+9+s^2-s} = \frac{s+9}{s^2+9} = \frac{s}{s^2+9} + \frac{9}{s^2+9}$

$Y_s = \cancel{\frac{1}{s^2+9}} + \cancel{\frac{9/s}{s^2+9}}$ $\frac{s+9}{s(s^2+9)} = \frac{As+B}{s^2+9} + \frac{C}{s} = \frac{1}{s} + \frac{1-s}{s^2+9}$

$C = \left.\frac{s+9}{s^2+9}\right|_0 = 1$

$\cancel{As+B} =$ $s+9 = As^2+Bs+s^2+9$

$A = -1$

$B = 1$

ad 2a iii) $Y(s) = \frac{1}{s} + \frac{1}{s^2+9} - \frac{s}{s^2+9}$

$= \frac{1}{s} + \frac{1}{3}\frac{3}{s^2+3^2} - \frac{s}{s^2+9} \multimap \sigma(t) + \frac{1}{3}\sin 3t - \cos 3t$

iv) $T_{r,y} = \dfrac{\frac{V(1+sT)}{s}\,\frac{s+9}{(s+1)(s-1)}}{1 + \frac{V(1+sT)(s+9)}{s(s+1)(s-1)}} = \dfrac{V(1+sT)(s+9)}{V(1+sT)(s+9) + s(s^2-1)}$

$N(s) = s^3 - s + V9 + s^2VT + sV(1+9T) =$

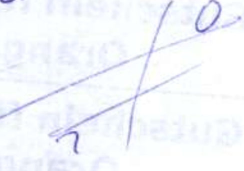
$s^3 + s^2VT + s(V(1+9T)-1) + V9$

s^3	1	$V+9VT-1$
s^2	VT	$V9$
s^1	$\frac{VT(V+9VT-1)-V9}{VT}$	0
s^0	$V9$	

$V+9VT-1-\frac{9}{T}>0$

3a) $R = \begin{bmatrix}0 & -2\\ 1 & -1\end{bmatrix}$ → vollständig erreichbar

b) $O = \begin{vmatrix}-1 & 1\\ -1 & 1\end{vmatrix}$ → nicht volls. erreichb. -1



$= (\lambda-1)(\lambda+1)+1 = \lambda^2 + 0 = 0$

$\lambda_{1,2} = 0$

$\begin{vmatrix}-1 & 1\\ -1 & 1\end{vmatrix} v = 0 \to v = \begin{vmatrix}1\\1\end{vmatrix}$