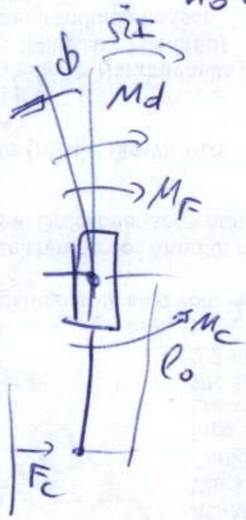
2009/06/09

$$C_c(x) = \varepsilon \frac{A}{h_0+x} \qquad F_c = \frac{1}{2}\frac{\partial C(x)}{\partial x} u_c^2 \qquad x = \begin{pmatrix} u_c \\ \phi \\ \Omega \end{pmatrix}$$



$$u = U, \quad y = \phi \qquad \boxed{\dot\phi = \Omega}$$

$$M_d = d\cdot\dot\phi, \quad M_F = k\phi$$

$$\frac{1}{I}\sum M = \dot\Omega = \frac{-d\cdot\dot\phi \mp k\phi + F_c\cdot l_0}{I}$$

$$M_c = F_c\cdot l_0$$

$$x = \sin\phi\cdot l_0 = \cancel{l_0\cos\phi}\cdot l_0\cdot\phi$$

$$F_c = \frac{A\varepsilon}{2}u_c^2\frac{\partial}{\partial x}\frac{1}{h_0+x} = \frac{A\varepsilon u_c^2}{2}\cdot(-1)\frac{1}{(h_0+x)^2}$$

$$u_c = U - iR \qquad Q = C\cdot u_c$$

$$\frac{dQ}{dt} = \frac{\partial C\cdot u_c}{\partial t} = C\cdot\dot u_c + \frac{\partial C}{\partial t}u_c$$

$$u_c = U - RC\dot u_c + \frac{R\varepsilon A\,\dot x u_c}{(h_0+x)^2} \qquad = C\cdot\dot u_c + \frac{\partial C}{\partial x}\dot x u_c$$

$$\dot u_c = \frac{U}{RC} + \left(\frac{\varepsilon A}{C}\frac{l_0\Omega}{(h_0+x)^2} - \frac{1}{RC}\right)u_c \qquad i_c = C\cdot\dot u_c - \frac{\varepsilon A}{(h_0+x)^2}\dot x u_c$$

$$\boxed{\dot u_c = \frac{U}{RC} + \left(\frac{l_0\Omega}{(h_0+x)} - \frac{1}{RC}\right)u_c} = \boxed{\frac{U h_0}{R\varepsilon A} + \left(\frac{l_0\Omega}{(h_0+l_0\phi)} - \frac{1}{R\varepsilon A}\right)u_c}$$

$$= \frac{U(h_0+l_0\phi)}{R\varepsilon A}$$

$$\boxed{\dot\phi = \Omega}$$

$$\boxed{\dot\Omega = -\frac{d\Omega}{I} - \frac{k\phi}{I} - \frac{l_0 A\varepsilon u_c^2}{2I}\frac{1}{(h_0+l_0\phi)^2}}$$

$$= \frac{U(h_0+l_0\phi)}{R\varepsilon A} + \left(\frac{l_0\Omega}{h_0+l_0\phi} - \frac{(h_0+l_0\phi)}{R\varepsilon A}\right)u_c$$

b) $u_R = 0 \quad \Omega = 0$

$0 = \cancel{k\omega} - u_C \rightarrow u_C = 0$

$0 = \frac{-k\phi_R}{I} \cancel{\ell\omega} = 0 \rightarrow \phi_R = 0$

c) $c^T = (0 \;\; 1 \;\; 0) \;, \; d = 0$

$b^T = \left(\frac{h_0}{R_E A} \quad 0 \quad 0 \right)$

$$A = \begin{pmatrix} \frac{-h_0}{R_E A} & 0 & \frac{h_0}{h_0} \\ 0 & 0 & 1 \\ 0 & -\frac{k}{I} & -\frac{d}{I} \end{pmatrix}$$

$$\det(\lambda E - A) = \left(\lambda + \frac{h_0}{R_E A}\right) \cdot \lambda \left(\lambda + \frac{d}{I}\right) + \frac{k}{I}\left(\lambda + \frac{h_0}{R_E A}\right) = 0$$

$$= \lambda^3 + \lambda^2\left(\frac{h_0}{R_E A} + \frac{d}{I}\right) + \lambda\left(\cancel{\frac{h_0}{R_E A} + \frac{dk}{I}}\right) + \frac{d h_0}{R_E A I} + \frac{k h_0}{R_E A I} + \cancel{\frac{kd}{I}} = 0$$

$$\lambda\left(\frac{d h_0}{R_E A I} + \frac{k}{I}\right)$$

λ^3	1	$\frac{h_0}{R_E A I} + \frac{k}{I}$
λ^2	$\frac{h_0}{R_E A} + \frac{d}{I}$	$\frac{k h_0}{R_E A I}$
λ		
0	$\frac{k h_0}{R_E A I}$	

$$\frac{\left(\frac{h_0}{R_E A} + \frac{d}{I}\right)\left(\frac{h_0}{R_E A I} + \frac{k}{I}\right) - \frac{k h_0}{R_E A I}}{\frac{h_0}{R_E A} + \frac{d}{I}} \overset{!}{>} 0$$

$h_0 I + d$

2a) siehe Skript $u(t-T)$, $x_0(t-T) \to y(t-T)$

b) $\lim\limits_{t \to \infty} |g(t)| < \infty$, $Re(s_i) < 0$ ... s_i ... Pole der Übertragungsfunktion

c) i) $T_{ij} \Rightarrow$ BIBO stabil $i, j = 1, 2$

$T_{ij} = \frac{v_i}{w_j}$ $1 + RG \neq 0$ für $Re(s) \geq 0$

i.allg. keine Pol-Nst Kürzungen auftreten mit $Re(s_i) \geq 0$

d) c) $O = \begin{vmatrix} c^T \\ c^T A \\ \vdots \\ c^T A^{n-1} \end{vmatrix}$ muss $Rang\, O \overset{!}{=} n$

PBH Rangtest / Eigenvektortest

$\to$ Hankelmatrix muss regulär sein.

d) ~~PBH~~

e) Rang A = Ordnung von $n(s)$
$\hookrightarrow G(s)$

3a) $G(s) = \dfrac{Y(s)}{U(s)}$ $\qquad z = u + \cancel{2y + \ldots}$

$$= \frac{\frac{6}{1+s}}{1 + \frac{6}{1+s}\left(\frac{1}{2+s} - 1\right)} = \frac{6(2+s)}{(1+s)(2+s) + 6 - 12 - 6s} = \frac{6s+12}{s^2 - 3s - 4}$$

$$+\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}}$$

$$= \frac{3}{2} \pm \frac{5}{2} =$$

b) $\dfrac{6s+12}{(s-4)(s+1)} = \dfrac{A}{s-4} + \dfrac{B}{s+1}$

$$A = \frac{6s+12}{s+1}\bigg|_{s=4} = \frac{36}{5}$$

$$B = \frac{6s+12}{s-4}\bigg|_{s=-1} = -\frac{6}{5}$$

$$\cancel{= \frac{6}{5}\frac{s}{s-4} + \frac{12}{5}\frac{1}{s-4} - \frac{6}{5}\frac{s}{s+1} - \frac{12}{5}\frac{s}{s+1}}$$

$$G(s) = \frac{36}{5}\frac{1}{s-4} - \frac{6}{5}\frac{1}{s+1}$$

$$G(z) = \frac{36}{5}\frac{z}{z - e^{4T_a}} - \frac{6}{5}\frac{z}{z - e^{-T_a}}$$

c) $R(z) = c^T (zE - \Phi)^{-1} \Gamma + d$

$$= 1\,(z-1)^{-1} k_1 + k_2 = \frac{k_1}{z-1} + k_2$$

d) $\Delta \arg(1+L^{\#}(I\omega)) = (\max(\text{grad}(z_c), \text{grad}(n_c)) - N_-(n_c) + N_+(n_c))\pi$

$2\times \frac{3\pi}{2}$ $\quad = (3 - 1 + 1)\cdot\pi = 3\pi$

$\hookrightarrow$ BIBO-stabil

4a) i) $G(s)_{nom} = \frac{V_{nom}(1+sc)}{(1+sa)(1+sb)}$ $\quad$ mit $V_R \ldots \frac{1}{V_{nom}}$

$$R = \frac{V_R \quad (1+sa)(1+sb)}{\cancel{1+\ldots}(s+\frac{1}{e})^2(1+sc)}$$

$T_{nom} = R_s \cdot G_{nom}$

$\lim\limits_{s\to 0} |T_{nom}| \overset{!}{=} 1 = V_{nom}\, e^2 V_R \rightarrow V_R = \frac{1}{V_{nom}e^2}$

ii) $G_d = -R_d \cdot G_{nom}$

$$R_d = -\frac{G_d}{G_{nom}} = -\frac{\frac{1}{\cancel{1+sa}}}{\frac{V_{nom}(1+sc)}{\cancel{(1+sa)}(1+sb)}} = -\frac{(1+sb)}{V_{nom}(1+sc)}$$

iii) $S = \frac{T-T_{nom}}{T_{nom}}$ $\quad$ mit $T_{nom} = \frac{1}{e^2(s+\frac{1}{e})^2}$

$$T = \frac{1}{e^2(s+\frac{1}{e})^2} + \frac{\Delta V}{V_{nom}e^2(s+\frac{1}{e})^2}$$

$$S = \frac{\frac{\Delta V}{V_{nom}\cancel{e^2(s+\frac{1}{e})^2}}}{\frac{1}{\cancel{e^2(s+\frac{1}{e})^2}}} = \frac{\Delta V}{V_{nom}}$$

$\rightarrow$ Rückführung, kann von R nicht beeinflusst werden

4b) $G(s) = \frac{V_G}{s^2}$ $\quad V_G \geq 0 \quad R(s) = \frac{V_R(1+s\Gamma_1)}{(1+s\Gamma_2)}$

$$T = \frac{RG}{1+RG} = \frac{V_G V_R (1+s\Gamma_1)}{V_G V_R (1+s\Gamma_1) + s^2(1+s\Gamma_2)}$$

$$= \frac{V_G V_R (1+s\Gamma_1) / \cancel{\Gamma_2}}{\Gamma_1 s^3 + s^2 / \cancel{\Gamma_2} + s\,\Gamma_1 V_G V_R / \cancel{\Gamma_2} + V_G V_R / \cancel{\Gamma_2}}$$

$s^3 \quad \Gamma_2 \quad \Gamma_1 V_G V_R$

$s^2 \quad 1 \quad V_G V_R$

$s^1 \quad \Gamma_1 V_G V_R - V_G V_R \Gamma_2 \quad 0$

$s^0 \quad V_G V_R$

$\boxed{\Gamma_1 > \Gamma_2}$ $\quad \boxed{V_R > 0}$

$V_G > 0$

$$T = \frac{V_G\, 10\,(1+10s)}{5s^3 + s^2 + s V_G 100 + 10 V_G}$$

$\lim\limits_{t\to\infty} (r(t) - y(t))$

$e = \lim r - y$

$\lim\limits_{s\to 0} \left(s G(s) \frac{1}{s^2}\right) = \infty$

$$\left(\frac{s}{s^2} - sG(s)\frac{1}{s^2}\right) = \left(\frac{1}{s} - \frac{V_G 10(1+10s)}{s(5s^3 + s^2 + s V_G 100 + 10V_G)}\right) =$$

$$\lim_{s\to 0}\left(\frac{5s^{\cancel{3}\,2} + s^{2} + s V_G 100 + 10V_G - 10V_G - 100V_G s}{\cancel{s}(5s^4 + s^2 + s V_G 100 + 10V_G)}\right) = 0$$