7.12.11

1 a) $u_a = -u_{c2}$

$u_{c2} = u_D \qquad i_D = I_S e^{\frac{u_{c2}}{m u_T}}$

$i_{c2} = C_2 \dot{u}_{c2}$

$i_{c1} = C_1 \dot{u}_{c1} = \frac{u_e + u_0 - u_{c1}}{R_1}$

$i_{R2} = \frac{u_e + u_0}{R_2} \qquad i_{c1} + i_{R2} = i_D + i_{c2}$

$$\frac{u_e + u_0 - u_{c1}}{R_1} + \frac{u_e + u_0}{R_2} = I_S e^{\frac{u_{c2}}{m u_T}} + C_2 \dot{u}_{c2}$$

$$\frac{d}{dt}\begin{bmatrix} u_{c1} \\ u_{c2} \end{bmatrix} = \begin{bmatrix} \frac{u_e + u_0 - u_{c1}}{R_1 C_1} \\ \frac{1}{C_2}\left(\frac{u_e + u_0 - u_{c1}}{R_1} + \frac{u_e + u_0}{R_2} - I_S e^{\frac{u_{c2}}{m u_T}}\right) \end{bmatrix}$$

$y = -u_{c2}$

b) $\dot{x} = 0$, $u_{eR} = -u_0 + u_{c1R}$ bzw. $u_{c1R} = u_{eR} + u_0$

$$\overbrace{\frac{u_{eR} + u_0 - u_{c1}}{R_1}}^{0} + \frac{u_{eR} + u_0}{R_2} - I_S e^{\frac{u_{c2R}}{m u_T}}$$

$$\frac{u_{eR} + u_0}{I_S R_2} = e^{\frac{u_{c2R}}{m u_T}}$$

$$u_{c2R} = m\, u_T \ln\left(\frac{u_{eR} + u_0}{I_S R_2}\right)$$

1b) ff

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} -\frac{1}{R_1 C_1} & 0 \\ -\frac{1}{R_1 C_2} & -\frac{I_S e^{\frac{u_{C2R}}{mU_T}}}{C_2 m U_T} \end{bmatrix} \qquad b = \frac{\partial f}{\partial u} = \begin{bmatrix} \frac{1}{R_1 C_1} \\ \frac{1}{C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{bmatrix}$$

$$c^T = \frac{\partial h}{\partial x} = \begin{bmatrix} 0 & -1 \end{bmatrix}$$

c) $$\dot{x} = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} x + \begin{bmatrix} d \\ -c \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

$$\det(\lambda E - A) = (\lambda - a)(\lambda - c)$$

$$\lambda_1 = a \qquad \lambda_2 = c$$

λ_1:
$$(A - \lambda_1 E) v_1 = \begin{bmatrix} 0 & b \\ 0 & c-a \end{bmatrix} v_1 = 0$$

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$c^T v_1 = 0 \Rightarrow$ nicht vollst. beobachtbar

2) a) $x = Vz$

$\tilde{A} = V^{-1}AV \qquad \tilde{B} = V^{-1}B$

$$R = [\tilde{B}, \tilde{A}\tilde{B}, \tilde{A}^2\tilde{B}, \dots, \tilde{A}^{n-1}\tilde{B}]$$

$$= [V^{-1}B, V^{-1}AVV^{-1}B, V^{-1}AVV^{-1}AVV^{-1}B, \dots, V^{-1}AV \dots V^{-1}B]$$

$$= [V^{-1}B, V^{-1}AB, V^{-1}A^2B, \dots, V^{-1}A^{n-1}B]$$

wegen der Regularität von V^{-1} ändert sich der Rang von R nicht durch die reguläre Zustandstransformation

b) $G(s) = \dfrac{V}{1 + 2\xi(sT) + (sT)^2} \qquad |G(j\omega)| = \dfrac{V}{\sqrt{(1-(\omega T)^2)^2 + (2\xi\omega T)^2}}$

$$\frac{d|G(j\omega)|}{d\omega} = \frac{V}{[(1-(\omega T)^2)^2 + (2\xi\omega T)^2]^{\frac{3}{2}}} \left(-\frac{1}{2}\right) \cdot [2(1-(\omega T)^2) \cdot (-2\omega T^2) + 4 \cdot \xi^2 2\omega T^2] = 0$$

$$0 = \left(-\frac{1}{2}\right)(2\omega T^2)[-2(1-(\omega T)^2) + 4\xi^2]$$

$\omega = 0$

oder

$$0 = 4\xi^2 - 2 + 2(\omega T)^2$$

$$\frac{(\pm)\sqrt{1-2\xi^2}}{T} = \omega \qquad \text{für } \xi > 0{,}5 \text{ kein Maximum}$$

2c) $\lim\limits_{\omega \to 0} G_2(I\omega) = \dfrac{I\omega}{I\omega \cdot 10 + 1} = I0 \rightarrow G_2(s) = G_b(s)$

$\lim\limits_{\omega \to 0} G_3(I\omega) = \lim\limits_{\omega \to 0} \dfrac{0{,}02}{I\omega\left(I\omega + \frac{1}{10}\right)\left(I\omega + \frac{1}{5}\right)} = -I\infty \rightarrow G_3(s) = G_d(s)$

Stetige Winkeländerung:

$\Delta \arg(G_1(s)) = -3\pi$ $\qquad$ $\Delta \arg(G_c(s)) = -3\pi$

$\Delta \arg(G_4(s)) = -2\pi$ $\qquad$ $\Delta \arg(G_d(s)) = -2\pi$

$$G_1(s) = G_c(s)$$

$$G_4(s) = G_d(s)$$

3a) wenn das System vollständig beobachtbar und vollständig erreichbar ist dann ergibt sich das charakteristische Polynom des geschlossenen Kreises zu $p_g(z) = p_{g\,soll}(z)\,\hat{p}_{g\,soll}(z)$

b) $x_{k+1} = \Phi x_k + \Gamma u_k$ $\qquad$ $u_k = \overset{\text{Matrix!}}{k}\, \hat{x}_k + g r_k$

$y_k = C x_k$

$\hat{x}_{k+1} = \Phi \hat{x}_k + \Gamma u_k + \hat{k}(\hat{y}_k - y_k)$

$\hat{y}_k = C \hat{x}_k$

$e_{k+1} = \hat{x}_{k+1} - x_{k+1} = \Phi \hat{x}_k + \cancel{\Gamma u_k} + \hat{k} C(\hat{x}_k - x_k) - \Phi x_k - \cancel{\Gamma u_k}$

$\qquad = (\Phi + \hat{k} C) e_k$

$$\begin{bmatrix} x_{k+1} \\ e_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi + \Gamma k & \Gamma k \\ 0 & \Phi + \hat{k} C \end{bmatrix}}_{\Phi_{ges}} \begin{bmatrix} x_k \\ e_k \end{bmatrix} + \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} g \, r_k$$

$$y_k = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix}$$

Wegen der Block-Diagonalstruktur von Φ_{ges} ergibt sich das Separationsprinzip

c) $$y - 9u = \frac{1}{5}\left\{ \frac{d}{dt}(y - 2u) + \int \left[(16y - 29u) + \int (38u - 24y)\,dt \right] dt \right\}$$

$$\hat{y} - 9\hat{u} = \frac{1}{5}\left\{ s\hat{y} - 2s\hat{u} + \frac{1}{s}(16\hat{y} - 29\hat{u}) + \frac{1}{s^2}\left(38\hat{u} - 24\hat{y}\right) \right\}$$

$$5\hat{y} - s\hat{y} + \frac{1}{s}16\hat{y} + \frac{1}{s^2}(-24)\hat{y} = 9 \cdot 5\hat{u} - 2s\hat{u} - \frac{29}{s}\hat{u} + \frac{38}{s^2}\hat{u}$$

$$G(s) = \frac{5s^2 - s^3 + 16s - 24}{-2s^3 + 45s^2 - 29s + 38} = \frac{\frac{1}{2}s^3 - \frac{5}{2}s^2 - 8s + 12}{s^3 - \frac{45}{2}s^2 + \frac{29}{2}s - 19}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 19 & -\frac{29}{2} & \frac{45}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 12 + \frac{19}{2} & -8 - \frac{29}{4} & -\frac{5}{2} + \frac{45}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3d) $G(z) = \frac{z-1}{z} Z\left(\frac{G(s)}{s}\right)$

$$\frac{G(s)}{s} = \frac{2s^2+10s+8}{s^2(s^2+7s+10)} = \frac{2s^2+10s+8}{s^2(s+5)(s+2)} = \frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+5}+\frac{D}{s+2}$$

$$s_{1,2} = \frac{-7\pm\sqrt{49-40}}{2} = \frac{-7\pm 3}{2} = \begin{cases}-5\\-2\end{cases}$$

$$B = \frac{8}{10} \qquad \frac{2+10+8}{1\cdot 6\cdot 3} = \frac{A}{1}+\frac{8}{10}-\frac{8}{75\cdot 6}+\frac{(-1)}{9}$$

$$C = \frac{2\cdot 25-50+8}{25(-3)} = -\frac{8}{75} \qquad \frac{20}{18}-\frac{8}{10}+\frac{8}{75\cdot 6}+\frac{1}{9} = A$$

$$D = \frac{2\cdot 4-20+8}{4\cdot 3} = -\frac{1}{3} \qquad \frac{10}{9}-\frac{4}{5}+\frac{4}{3\cdot 75}+\frac{1}{9} = A$$

$$\frac{250}{9\cdot 25}-\frac{4\cdot 45}{5\cdot 45}+\frac{4}{3\cdot 75}+\frac{25}{9\cdot 25} = A$$

$$\frac{G(s)}{s} = \frac{11}{25\cdot s}+\frac{8}{10s^2}-\frac{8}{75(s+5)}-\frac{1}{3(s+2)} \qquad \frac{\cancel{99}\ 11}{\cancel{9}\cdot 25} = A$$

$$A = \frac{11}{25}$$

$$G(z) = \frac{z-1}{z}\left(\frac{11}{25}\frac{z}{z-1}+\frac{8}{10}\frac{T_a z}{(z-1)^2}-\frac{8}{75}\frac{z}{z-e^{-5T_a}}-\frac{1}{3}\frac{z}{z-e^{-3T_a}}\right)$$

$$= \frac{11}{25}+\frac{8}{10}\frac{T_a}{z-1}-\frac{8}{75}\frac{z-1}{z-e^{-5T_a}}-\frac{1}{3}\frac{z-1}{z-e^{-3T_a}}$$

e) i) lineare Integro-Differentialgleichung → linear
Koeffizienten abhängig von t → zeitvariant

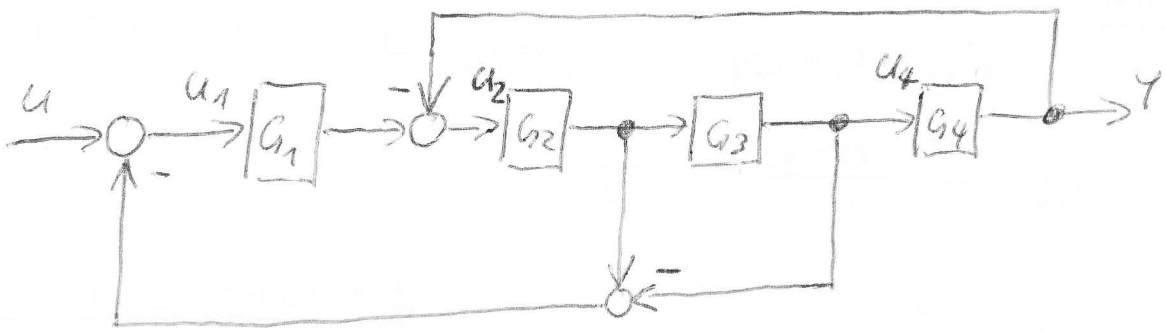
ii) nichtlineare Differentialgleichung → nichtlinear
konstante Koeffizienten → zeitinvariant

iii) lineare Gleichung → linear
konstante Koeffizienten → zeitinvariant

iV) nichtlineare Differentialgleichung → nichtlinear

konstante Koeffizienten → zeitinvariant

f)



$$\hat{u}_4 = \frac{\hat{y}}{G_4}$$

$$\hat{u}_2 = \frac{\hat{y}}{G_2 G_3 G_4}$$

$$\hat{u}_2 = \hat{u}_1 \cdot G_1 - \hat{y}$$

$$\hat{u}_1 = \frac{1}{G_1}\left(\hat{u}_2 + \hat{y}\right) = \left(\frac{1}{G_1 G_2 G_3 G_4} + \frac{1}{G_1}\right)\hat{y}$$

$$\hat{u}_1 = \hat{u} - \left(\hat{u}_2 G_2 - \hat{u}_4\right) = \hat{u} - \left(\frac{1}{G_3 G_4} - \frac{1}{G_4}\right)\hat{y}$$

$$\left(\frac{1}{G_1 G_2 G_3 G_4} + \frac{1}{G_1} + \frac{1}{G_3 G_4} - \frac{1}{G_4}\right)\hat{y} = \hat{u}$$

$$G(s) = \frac{1}{\frac{1}{G_1 G_2 G_3 G_4} + \frac{1}{G_1} + \frac{1}{G_3 G_4} - \frac{1}{G_4}} = \frac{G_1 G_2 G_3 G_4}{1 + G_2 G_3 G_4 + G_1 G_2 - G_1 G_2 G_3}$$

4 a) i) $\det(\lambda E - A) = (\lambda - 2)(\lambda - 5)(\lambda - 7) = 0$

$$\lambda_1 = 2 \quad \lambda_2 = 5 \quad \lambda_3 = 7$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 3 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} v_1 = 0 \qquad v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} v_2 = 0 \qquad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 0 & 1 \\ 3 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} v_3 = 0 \qquad \begin{aligned} v_{3,1} &= 1 \\ v_{3,2} &= \frac{3}{2} \\ v_{3,3} &= 5 \end{aligned} \qquad v_3 = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 5 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & \frac{3}{2} \\ 0 & 0 & 5 \end{bmatrix} \qquad V^{-1} = \frac{1}{5} \begin{bmatrix} 5 & -[0] & -1 \\ -[-5] & 5 & -[\frac{5}{2}] \\ 0 & -[0] & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -\frac{1}{5} \\ 1 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix} \qquad \tilde{b} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

$$\tilde{c}^T = \begin{bmatrix} 0 & 0 & 25 \end{bmatrix}$$

ii) $\tilde{\Phi} = \begin{bmatrix} e^{2t} & 0 & 0 \\ 0 & e^{5t} & 0 \\ 0 & 0 & e^{7t} \end{bmatrix}$

$$\begin{array}{ccc|ccc|ccc}
 & & & e^{2t} & 0 & 0 & 1 & 0 & 1 \\
 & & & 0 & e^{5t} & 0 & -1 & 1 & \frac{3}{2} \\
 & & & 0 & 0 & e^{7t} & 0 & 0 & 5 \\
\hline
1 & 0 & -\frac{1}{5} & e^{2t} & 0 & -\frac{1}{5}e^{7t} & e^{2t} & 0 & e^{2t}-e^{7t} \\
1 & 1 & -\frac{1}{2} & e^{2t} & e^{5t} & -\frac{1}{2}e^{7t} & e^{2t}-e^{5t} & e^{5t} & e^{2t}+\frac{3}{2}e^{5t}-\frac{5}{2}e^{7t} \\
0 & 0 & \frac{1}{5} & 0 & 0 & \frac{1}{5}e^{7t} & 0 & 0 & e^{7t}
\end{array}$$

$$\Phi(t) = \begin{bmatrix} e^{2t} & 0 & e^{2t}-e^{7t} \\ e^{2t}-e^{5t} & e^{5t} & e^{2t}+\frac{3}{2}e^{5t}-\frac{5}{2}e^{7t} \\ 0 & 0 & e^{7t} \end{bmatrix}$$

b) $p_{g,soll} = z^3$ $\qquad R = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

$$\begin{array}{ccc|c}
 & & & 0 \\
 & & & 1 \\
 & & & 2 \\
\hline
1 & 2 & 0 & 2 \\
2 & 0 & \frac{1}{2} & 1 \\
1 & 0 & 1 & 2
\end{array}$$

voller Rang $\rightarrow$ vollst erreichbar

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = v_1^T \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$v_1^T = \begin{bmatrix} \frac{1}{2} & 0 & 0 \end{bmatrix}$$

$$\begin{array}{ccc|ccc|ccc}
 & \Phi^3 & & 1 & 2 & 0 & 1 & 2 & 0 \\
 & & & 2 & 0 & \frac{1}{2} & 2 & 0 & \frac{1}{2} \\
 & & & 1 & 0 & 1 & 1 & 0 & 1 \\
\hline
1 & 2 & 0 & 5 & 2 & 1 & 10 & 10 & 2 \\
2 & 0 & \frac{1}{2} & \frac{5}{2} & 4 & \frac{1}{2} & 11 & 5 & \frac{5}{2} \\
1 & 0 & 1 & 2 & 2 & 1 & 7 & 4 & 2
\end{array}$$

$$k^T = -v_1^T \Phi^3 = \begin{bmatrix} -5 & -5 & -1 \end{bmatrix}$$