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11.12.09

1a) $\psi = B\, A(s) + Li$ $\qquad A(s) = s^2 \tan\alpha$

$\frac{d\psi}{dt} = -Ri + v$ $\qquad i = \frac{\psi}{L} - \frac{B}{L} s^2 \tan\alpha$

$\frac{di}{dt} = \frac{d\psi}{dt}\frac{1}{L} - \frac{B}{L}\tan\alpha\,\frac{ds^2}{ds}\dot{s}$

$\dot{i} = -\frac{R}{L} i + \frac{v}{L} - \frac{B}{L}\tan\alpha\, 2s\dot{s}$

$\boxed{\dot{i} = -\frac{R}{L} i + \frac{v}{L} - \frac{B}{L}\tan\alpha\, 2s\omega}$

b) $F_m = \frac{\partial \bar{W}_m}{\partial s} = \frac{\partial}{\partial s}\int \psi\, di = \frac{\partial}{\partial s}\left(s^2 \tan\alpha\, B i + \frac{L i^2}{2} + C\right)$

$F_m = 2s\tan\alpha\, B i + \cancel{L i^2}$

$\dot{\omega} = \frac{F_{ext}}{m} + \frac{F_m}{m} - \frac{s \cdot c}{m} - \frac{\omega \cdot d}{m}$

$\boxed{\dot{\omega} = \frac{F_{ext}}{m} + \frac{2s}{m} B\tan\alpha\, i - \frac{sc}{m} - \frac{\omega d}{m}}$

$\boxed{y = 2s\tan\alpha\, B i}$ $\qquad \boxed{\dot{s} = \omega}$

c) $\omega = 0$ $\qquad s_R = \frac{F_{ext\,R}}{c - 2B\tan\alpha\, i_R}$

$0 = \frac{F_{ext,R}}{m} + \frac{2s_R}{m} B\tan\alpha\, i_R - \frac{s_R c}{m}$

$0 = -\frac{R}{L} i_R + \frac{v_R}{L}$ $\rightarrow \boxed{v_R = i_R R}$

$\rightarrow F_{ext,R} = 0$ $\qquad 2B\tan\alpha\, i_R = \boxed{c = 2B\tan\alpha\,\frac{v_R}{R}}$

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e) $x = \begin{vmatrix} s \\ \omega \\ i \end{vmatrix}, u = \begin{vmatrix} V \\ F_{ext} \end{vmatrix} \quad d = 0 \qquad c^T = \frac{\partial h(x_R, u_R)}{\partial x} = \begin{vmatrix} 2\tan\alpha \, B \, i_R \\ 0 \\ 2\tan\alpha \, B \, s_R \end{vmatrix}^T$

$B = \frac{\partial f(x_R, u_R)}{\partial u} = -\begin{vmatrix} 0 & 0 \\ 0 & 1/m \\ +1/L & 0 \end{vmatrix}$

$A = \frac{\partial f(x_R, u_R)}{\partial x} = \begin{vmatrix} 0 & 1 & 0 \\ -\frac{c}{m} + i_R \tan\alpha \frac{2B}{m} & -d/m & \frac{2 s_R B \tan\alpha}{m} \\ 0 & -\frac{2 s_R B}{L}\tan\alpha & -R/L \end{vmatrix}$

2) i.) $r(t) = t\sigma(t) \quad \circ\!\!-\!\!\bullet \quad = \frac{1}{s^2}$

$u(s) = G(s) \cdot \frac{1}{s^2} = \frac{1}{s(s^2+2s+100)}$

$\lim_{s \to 0} s\,u(s) = 1/100$

ii.) $u_\infty = 1/100$

$|G_2(0)|_{s=0} = 1$

$\boxed{y_\infty = 1/100}$

iii) $|G_1(s)|_{s=10I} = \frac{10}{\sqrt{20^2 + (100-10^2)^2}} = 1/2$

$(\arg G_1(s))_{s=10I} = \pi/2 - \operatorname{atan}\frac{20}{0} = 0$

$u(t) = 5/2 \sin(10t)$

$90dB, -135 = -\frac{3\pi}{4}$

$y(t) = 2 \cdot \frac{\sqrt{2}}{2} 5 \sin(10t - 3\pi/4) = 5\sqrt{2}\sin(10t - 3\pi/4)$

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2b) $\to c$ (Betrag bei $\infty = 0$, bei $0 = 1$)

c) nli. zeitv.

linear, zeitva.

d)

i) $z_1 = y$

$z_2 = \dot{z}_1 = \dot{y}$

$\dot{z}_2 = \ddot{y} = 5 + u + \frac{2}{25} z_2 z_1$

$$\dot{z} = \begin{bmatrix} z_2 \\ \frac{2}{25} z_2 z_1 + 5 + u \end{bmatrix}$$

ii) $\cancel{10\ddot{y} - 2\frac{\dot{y}(1+t) - y}{(1+t)^2} = \sqrt{2}\, u(t)}$

$$\dot{y} = \frac{1}{10}\frac{y}{1+t} + \int_0^t \frac{\sqrt{2}}{10} u(\tau)\, d\tau$$

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3a) $x_{k+1} = \begin{bmatrix} 1 & 3 \\ -5 & -7 \end{bmatrix} x_k + \begin{bmatrix} 2 \\ -2 \end{bmatrix} u_k$

$y_k = [4 \ 2] x_k$

$\begin{vmatrix} 1-\lambda & 3 \\ -5 & -7-\lambda \end{vmatrix} = \underbrace{-(1-\lambda)}_{(\lambda-1)}(\lambda+7) + 15 = \lambda^2 + 6\lambda + 8 = 0$

$\lambda_{1,2} = -3 \pm \sqrt{9-8} = \boxed{\begin{matrix} -4 \\ -2 \end{matrix}}$

$w_i^T \phi = \lambda_1 w_i^T$

$\phi^T w_1 = \lambda_1 w_i \rightarrow (\phi^T - \lambda_i E) w_i = 0$

$\lambda_1 = -2$

$\begin{vmatrix} 3 & -5 \\ 3 & -5 \end{vmatrix} \begin{matrix} w_1 \\ w_2 \end{matrix} = 0 \qquad 3w_1 = 5w_2 \quad \rightarrow \vec{w}_1 = \begin{vmatrix} 1 \\ 3/5 \end{vmatrix}$

$\lambda_1 = -4$

$\begin{vmatrix} 5 & -5 \\ 3 & -3 \end{vmatrix} \begin{matrix} w_1 \\ w_2 \end{matrix} = 0 \qquad w_1 = w_2 \qquad \vec{w}_2 = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$

$w_2^T \cdot \Gamma = 0 \rightarrow$ nicht erreichbar

<u>duales System:</u>

$z_{k+1} = \phi^T z_k + c\, v_k$

$y_k = b^T z_k$

$G(z) = c^T (zE - \phi)^{-1} \Gamma$

$G(z)^T = \Gamma^T (zE - \phi^T)^{-1} c$

↳ da hier Dynamikmatrix $\phi' = \phi^T$ ist, vertauschen sich krit. bei PBH-Test, da einmal $\phi^T w_i = \lambda_i w_i$ → erreichbar, rechtseigenw. und $\phi v_i = \lambda_i v_i$ → beobachtbar, linkseigenw. benötigt wird.

und $b^T \rightarrow c$ vertauscht wird

3 c)

$$u = \cancel{k} k_x^T x + (r - y) + (r - y) \cancel{k_i} x_I k_i$$

$$x_I = r - y \quad \dot{x}_I = r - c^T x$$

$$y = c^T x$$

$$\dot{x} = Ax + b\, k_x^T x + br - bc^T x + b x_I k_i$$

$$= (A + bk_x^T - bc^T)x + bk_i x_I + br$$

$$x_g = \begin{bmatrix} A + b(k_x^T - c^T) & bk_i \\ -c^T & 0 \end{bmatrix} x + \begin{bmatrix} b \\ 1 \end{bmatrix} r$$

$b(k_x^T - c^T)$

	$k_1 - 1$	k_2
0	0	0
1	$k_1 - 1$	k_2

$$\boxed{\dot{x}_g = \begin{bmatrix} -1 & 1 & 0 \\ k_1 - 1 & k_2 - 2 & k_i \\ -1 & 0 & 0 \end{bmatrix} x_g + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} r}$$

$$\begin{vmatrix} -1-\lambda & 1 & 0 \\ k_1 - 1 & k_2 - 2 - \lambda & k_I \\ -1 & 0 & -\lambda \end{vmatrix} = \lambda(\lambda+1)(k_2 - 2 - \lambda) + k_i + \lambda(k_1 - 1)$$

$$= \lambda(\lambda(k_2 - 2 - 1) - \lambda^2 + k_2 - 2) + k_I + \lambda(k_1 \ldots$$

$$= -\lambda^3 + \lambda^2(k_2 - 3) + \lambda(k_2 + k_1 - 3) + k_I$$

$$= \lambda^3 + \lambda^2(3 - k_2) + \lambda(3 - k_2 - k_1) - k_I = 0$$

gewünscht: $(\lambda+1)(\lambda+1)(\lambda+2) =$

$$(\lambda^2 + 2\lambda + 1)(\lambda + 2) = \lambda^3 + 2\lambda^2 + \lambda + 2\lambda^2 + 4\lambda + 2 = \lambda^3 + 4\lambda^2 + 5\lambda + 2$$

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ad 3c)

$$\boxed{\begin{aligned} k_I &= -2 \\ k_2 &= -1 \\ k_1 &= -1 \end{aligned}}$$

d) Separationsprinzip (wenn vollst. beob. u. erreichbar)

$P_{ges}(z) = \hat{P}_{Beob}(z) \, P_{Zustandsreg.}(z)$

4a) $\phi(0) = E$

$\phi(t+e) = \phi(t) \cdot \phi(e)$

$\phi^{-1}(t) = \phi(-t)$

$\frac{d\phi}{dt}(t) = A \cdot \phi(t)$

b) Plan: $\frac{d\phi}{dt} \cdot \phi^{-1}(t) = A$

e^{t}	0	0
$-2e^{t}+2e^{2t}$	e^{2t}	0
$\frac{21}{2}e^{3t}+\frac{3}{2}e^{t}-12e^{2t}$	$-6e^{2t}+6e^{2t}$	e^{3t}

$-e^{-t}$	0	0
$+2e^{-t}-4e^{-2t}$	$-2e^{-2t}$	0
$-\frac{63}{2}e^{-3t}-\frac{3}{2}e^{-t}+24e^{-2t}$	$+12e^{-2t}-18e^{-3t}$	$-3e^{-3t}$

1	0	0
A_{12}	-2	0
A_{13}	A_{23}	-3

$A_{12} = 2 \cancel{-4e^{-t}} \cancel{+4e^{-t}} - 4 = -2$

$A_{23} = 12 \cancel{-18e^{-t}} \cancel{+18e^{-t}} - 18 = -6$

$A_{13} = \cancel{-\frac{63}{2}e^{-2t}} - \frac{3}{2} + \cancel{24e^{-t}} - \frac{63}{2} \cancel{-\frac{9}{2}e^{-2t}} + \cancel{36e^{-t}} + (12e^{-2t} - 18e^{-3t}) \cdot (-2e^{t} + 2e^{2t})$

$\underbrace{\quad}_{\cancel{-24e^{-t}} \cancel{+36e^{-2t}} + 24 \cancel{-36e^{t}}}$

$= -9$

$$\boxed{A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & -2 & 0 \\ -9 & -6 & -3 \end{pmatrix}}$$

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ad (4b). $\lim_{t \to \infty} x(t) = 0$

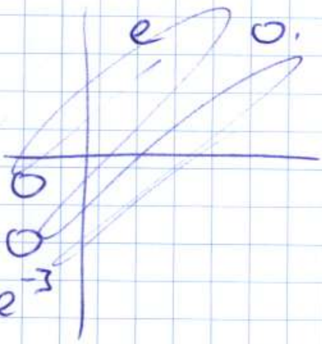
$\lim_{t \to \infty} \phi(t) \cdot x_0 = 0$ q.e.d.

Ruhelage $x = 0$
ist eindeutig, da $\det(A) \neq 0$

- $\begin{vmatrix} 0 \\ 0 \\ e^{-3} \end{vmatrix} = \phi(1) \cancel{x}_0$

$\phi(-1) x \; \cancel{\phi(-1)} = x_0$

$x_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



$$\begin{array}{cccc|c} & & & 0 & 0 \\ & & & 0 & 0 \\ & & & e^{-3} & e^{-3} \\ \hline x & + & 0 & 0 & \\ x & + & 0 & 0 & \\ x & x & e^{3} & 1 & \end{array}$$

c) $\begin{vmatrix} \lambda+4 & +6 \\ -1 & \lambda-1 \end{vmatrix} = (\lambda+4)(\lambda-1)+6 = \lambda^2+3\lambda-4+6 = \lambda^2+3\lambda+2$

$\lambda_{1,2} = -\frac{3}{2} \pm \sqrt{\frac{9-8}{4}} = \begin{cases} -2 \\ -1 \end{cases}$

$\phi = e^{At} = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-1t} \end{bmatrix}$

x_0 muss $[x\ 0]^T$ oder $[0\ x]^T$ sein,
denn $x_1(t) = e^{-2t}[x\ 0]^T$
oder $x_2(t) = e^{-t}[0\ x]^T$

4d) $G(z) = G_1(z) \cdot G_2(z) = c_1(z-a)^{-1} b_1 \cdot c_2 (z-a)^{-1} b_2$

$$= \frac{c_1 c_2 b_1 b_2}{(z-a)(z-a)}$$

$$G(z) = \frac{1}{(z-0,5)(z-1)} = \frac{1}{z^2 - 1,5z + 0,5}$$

Jury

$$T_{r,y} = \frac{P}{P + 0,5 - 1,5z + z^2}$$

$d_2 = 0,5$

z^2	1	$-1,5$	$0,5+P$
z	$P+0,5$	$-1,5$	1
z	$1-(P+0,5)^2$	$((P+0,5)+1)(-1,5)$	0
1	$-1,5(P+1,5)$	$1-(P+0,5)^2$	0
	$\boxed{1 - \frac{(1,5(P+1,5))^2}{1-(P+0,5)^2} - (P+0,5)^2}$	0	

$d_2 = P + 0,5$

$d_1 = \frac{-1,5(P+1,5)}{1-(P+0,5)^2}$

~~$P > 0,5$~~

$1 > (P+0,5)^2 \to 0,5 > P$

$(1-(P+0,5)^2)^2 - (1,5(P+1,5))^2 > 0 \to 1-(P+0,5)^2 > 1,5(P+0,5)^2$

$1 > 2,5(P+0,5)^2$

$\underbrace{\sqrt{4/10} - 0,5 > P}$

$\frac{2 - 0,5\sqrt{10}}{\sqrt{10}} > P$

$\cancel{P = \left(-0,5 ; \frac{2-0,5\sqrt{10}}{\sqrt{10}}\right)}$

$\boxed{P =]0,5 ; \infty]}$