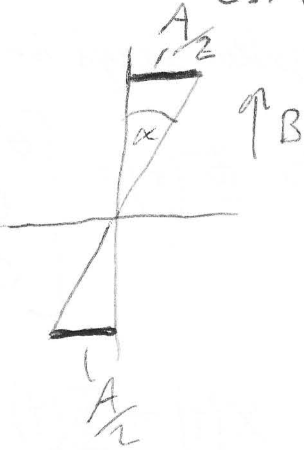
29.4.2011

1) a) $\Phi = BA + Li = B \cdot N \cdot \sin(\alpha) \cdot b \cdot a + Li$

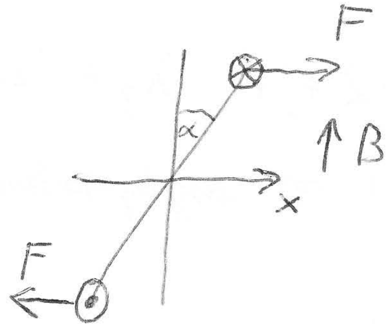


$$\frac{d}{dt}\Phi = BNab\cos(\alpha)\cdot\omega + L\frac{di}{dt}$$

$$= -Ri + u_L$$

b) $F = l i e_i \times B$

$= a \cdot i \cdot N \cdot B$



$$M_{el} = 2 \cdot F \cdot \frac{b}{2} \cdot \cos\alpha = a i B N b \cos\alpha$$

$$J\dot{\omega} = M_{el} - d\omega - c\alpha$$

c)

$$\frac{d}{dt}\begin{bmatrix}\alpha\\ \omega\\ i\end{bmatrix} = \begin{bmatrix}\omega\\ \frac{1}{J}(a i B b N \cos\alpha - d\omega - c\alpha)\\ \frac{1}{L}(u_L - BNab\cos(\alpha)\,\omega - Ri)\end{bmatrix}$$

$$u = u_L \qquad y = \alpha$$

d)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{a i_R B b N(-\sin(\alpha_R)) - c}{J} & -\frac{d}{J} & \frac{a\,BbN\cos\alpha_R}{J} \\ \frac{1}{L}(BNab\sin(\alpha_R)\omega_R) & -\frac{BNab\cos(\alpha_R)}{L} & -\frac{R}{L}\end{bmatrix} \qquad b = \begin{bmatrix}0\\0\\ \frac{1}{L}\end{bmatrix}$$

$$c^T = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

2) a) i) System liegt in Jordanscher Normalform vor

$$\Rightarrow \underline{\Phi}(t) = \begin{bmatrix} e^{-3t} & 0 & 0 \\ 0 & e^{-2t}\cos(4t) & e^{-2t}\sin(4t) \\ 0 & -e^{-2t}\sin(4t) & e^{-2t}\cos(4t) \end{bmatrix}$$

$$x(t) = \underline{\Phi}(t)\, x_0$$

ii) $\lambda_1 = -3 \quad \lambda_2 = -2 + j4 \quad \lambda_3 = -2 - j4$

$Re(\lambda_i) < 0 \ \forall i \Rightarrow$ asymptotisch stabil

iii)

$$\begin{array}{c|c|c} & \begin{matrix} -3 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & -4 & -2 \end{matrix} & \begin{matrix} -3 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & -4 & -2 \end{matrix} \\ \hline 0 \ 1 \ 1 & 0 \ \ -6 \ \ 2 & 0 \ \ 4 \ \ -28 \end{array}$$

$$\theta = \begin{bmatrix} 0 & 1 & 1 \\ 0 & -6 & 2 \\ 0 & 4 & -28 \end{bmatrix}$$

nicht vollst. beobachtbar, x_1 nicht beobachtbar

$$\tilde{c}^T = [1 \quad 0 \quad 1]$$

b) i)

$$\begin{array}{cc|cc} & A^2 & 1 & -1 \\ & & 1 & -1 \\ \hline 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{array}$$

$$\Rightarrow \underline{\Phi}(t) = E + A\cdot t + A^2 \frac{t^2}{2!} \dots$$

$$= E + A\cdot t = \begin{bmatrix} 1+t & -t \\ t & 1-t \end{bmatrix}$$

$$\underline{\Phi}(T_a) = \begin{bmatrix} 1+T_a & -T_a \\ T_a & 1-T_a \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\Gamma = \int_0^{T_a} \underline{\Phi}(\tau)\cdot b\, d\tau = \int_0^{T_a} \begin{bmatrix} -t \\ 1-t \end{bmatrix} d\tau = \begin{bmatrix} -\frac{T_a^2}{2} \\ T_a - \frac{T_a^2}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$c^T = [1 \quad 0]$$

$$x_{k+1} = \underline{\Phi}\, x_k + \Gamma u_k$$

$$y_k = c^T x_k$$

2) b) ii)

$$G(z) = c^T (zE - \Phi)^{-1} \Gamma$$

$$(zE - \Phi)^{-1} = \begin{bmatrix} z-2 & 1 \\ -1 & z \end{bmatrix}^{-1} = \frac{1}{(z-2)z+1} \begin{bmatrix} z & -1 \\ 1 & z-2 \end{bmatrix}$$

$$G(z) = \frac{-\frac{1}{2} - \frac{z}{2}}{(z^2 - 2z + 1)}$$

c) i)

$$(z + 0{,}5)(z + 0{,}9) + 0{,}85 = 0$$

$$z^2 + 1{,}4z + 1{,}3 = 0$$

$$\left| \frac{a_0}{a_n} \right| < 1$$

$$\left| \frac{1{,}3}{1} \right| > 1$$

notwendige Bedingung für Einheitskreis-polynom nicht erfüllt ⇒ nicht asymptotisch stabil

ii)

$$p_{g,soll} = z^2 + z + 0{,}24$$

$$R = \begin{bmatrix} 0 & 1 \\ 1 & -0{,}9 \end{bmatrix} \Rightarrow \text{vollst. erreichbar} \qquad R^{-1} = \frac{1}{-1} \begin{bmatrix} -0{,}9 & -1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{array}{cc|cc} & & 0{,}9 & 1 \\ & v_1^T & 1 & 0 \\ \hline 0 & 1 & 1 & 0 \end{array} \qquad \begin{array}{cc|cc} & & -0{,}5 & 1 \\ & \Phi^2 & -0{,}85 & -0{,}9 \\ \hline -0{,}5 & 1 & -0{,}6 & -1{,}4 \\ -0{,}85 & -0{,}9 & \times & \times \end{array}$$

$$p_{g,soll} = \Phi^2 + \Phi + 0{,}24E = \begin{bmatrix} -1{,}1 & -0{,}4 \\ \times & \times \end{bmatrix} + 0{,}24E = \begin{bmatrix} -0{,}86 & -0{,}4 \\ \times & \times \end{bmatrix}$$

$$k^T = -v_1^T p_{g,soll} = \begin{bmatrix} 0{,}86 & 0{,}4 \end{bmatrix}$$

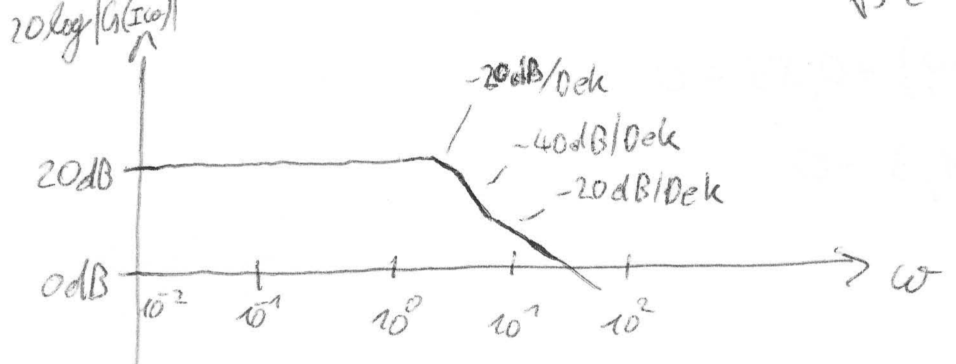
3) a) $T_{r_1,y_1} = \frac{R_1(s)\,G_1(s)}{1+R_1(s)\,G_1(s)} = \frac{2\cdot 3}{\sqrt{3}\,s+2\cdot 3} = \frac{6}{\sqrt{3}\,s+6} =$

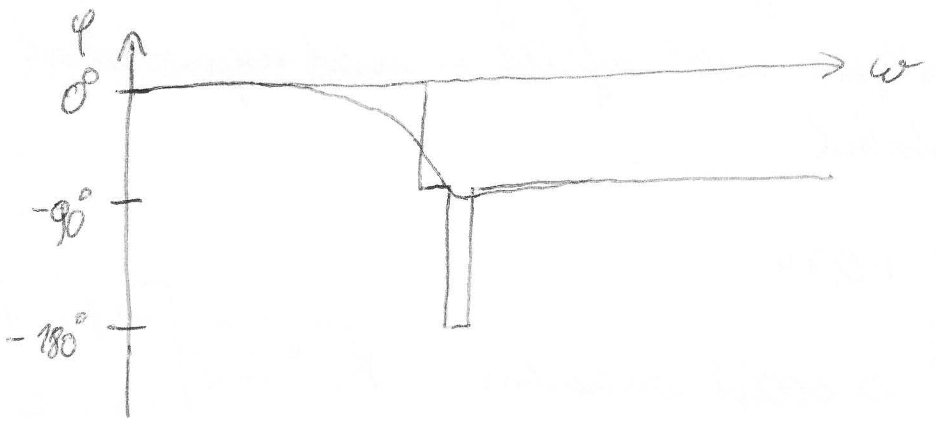
$= \frac{1}{\frac{s}{\sqrt{3}\,2}+1}$

b) $G_3 = \frac{10\left(1+\frac{2-\sqrt{3}}{2}s\right)}{\left(1+\frac{1}{2}s\right)\left(1+\frac{s}{\sqrt{3}\,2}\right)}$

$\left(\frac{2-\sqrt{3}}{2}\right)^{-1} \approx \frac{2}{2-\frac{7}{4}} = \frac{2}{\frac{1}{4}} = 8$

$\sqrt{3}\,2 \approx \frac{7}{2} = 3{,}5$





c) $\omega_c t_r = 1{,}5$ $\qquad \Phi + ü = 70$ $\qquad e_\infty\big|_{r_2(t)=\sigma(t)} = 0 \Rightarrow \frac{1}{s}$ in $R_2(s)$

$\omega_c = \frac{3\cdot 4}{2\cdot 3} = 2$ $\qquad \Phi = 60°$

$\arg(G_3(I2)) = \frac{\pi}{12} - \frac{\pi}{4} - \frac{\pi}{6} = -\frac{\pi}{3} \mathrel{\hat{=}} -60°$ $\qquad R_2 = V\,\frac{1+sT}{s}$

$\arg(R_2(I2)) \stackrel{!}{=} -60°$

$\Rightarrow \arctan(2T) - 90° = -60° \Rightarrow T = \frac{\tan(30°)}{2} = \frac{\sqrt{3}}{6}$

3 c) ff

$$1 \overset{!}{=} |L(I2)| = V \cdot \frac{\sqrt{1+(\frac{\sqrt{3}}{6}2)^2}\ \sqrt{1+(2-\sqrt{3})^2} \cdot 10}{2 \cdot \sqrt{2}\ \sqrt{1+\frac{1}{3}}} =$$

$$\Rightarrow V = \frac{\cancel{2}\sqrt{2}\ \cancel{\sqrt{1+\frac{1}{3}}}}{\cancel{10}_5 \cdot \cancel{\sqrt{1+\frac{1}{3}}}\ \sqrt{1+4-4\sqrt{3}+3}} = \frac{\sqrt{2}}{5\ \sqrt{8-4\sqrt{3}}} = \frac{1}{5\sqrt{4-2\sqrt{3}}}$$

d) $T_{d,Y} \overset{!}{=} 0$

$$T_{d,Y} = G_d \frac{1}{1+R_2G_3} - R_d \cdot \frac{G_3}{1+R_2G_3} \overset{!}{=} 0$$

$$G_d - R_d\, G_3 = 0$$

$$R_d = \frac{G_d}{G_3} = \frac{1}{50} \frac{(1+\frac{1}{2}s)(1+\frac{s}{\sqrt{3}2})}{(1+\frac{\sqrt{3}}{20}s)(1+\frac{\sqrt{3}}{30}s)(1+\frac{2-\sqrt{3}}{2}s)}$$

4 a) i) g_k ... Dead-Beat Verhalten $\Rightarrow G_4$ fällt weg

nach $k>3$ ist $g_k = 0 \Rightarrow G_1$ fällt weg

AWS: $f_0 = \lim_{z\to\infty} G_3(z) = -2$

$g_0 = 0 \Rightarrow G_3$ fällt weg

G_2 ist richtig

ii) Grenzwert existiert da alle Pole innerhalb des Einheitskreises

EWS: $\lim\limits_{k\to\infty} g_k = \lim\limits_{z\to 1} \cancel{(z-1)} \cdot \frac{z}{\cancel{(z-1)}} \quad \frac{-2z^2+\frac{1}{2}z+2}{z^3}$

$$= \frac{-2+\frac{1}{2}+2}{1} = \frac{1}{2}$$

iii) $\rho_1 = -2 = m_1$

$\rho_2 = \frac{1}{2} = m_2$

$\rho_3 = 2 = m_3$

$$H = \begin{bmatrix} -2 & \frac{1}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$$

voller Rang → vollst. beobachtbar und vollst. erreichbar

b) i) $G_z = \frac{z-1}{z} Z\left(\frac{G(s)}{s}\right) = \frac{z-1}{z} Z\left(\frac{4}{s^2}\right) = \frac{\cancel{z-1}}{\cancel{z}} \frac{\cancel{4}}{\cancel{4}} \frac{z}{(z-1)^2}$

$$= \frac{1}{z-1}$$

ii) $T_{r,y} = \frac{R(z)G(z)}{1+R(z)G(z)} = \frac{z-\frac{1}{2}}{(z-1)(z-\gamma)+z-\frac{1}{2}} = \frac{z-\frac{1}{2}}{z^2-\gamma z-z+\gamma+z-\frac{1}{2}}$

$$= \frac{z-\frac{1}{2}}{z^2-\gamma z-\frac{1}{2}+\gamma}$$

$p = z^2+a_1 z+a_0 \qquad a_1 = -\gamma \qquad a_0 = \gamma - \frac{1}{2}$

4b) (ii) ff

z^2	1	α_1	α_0	
	α_0	α_1	1	$\lambda_2 = \alpha_0$
z	$1-\alpha_0^2$	$\alpha_1 - \alpha_0\alpha_1$	0	
	$\alpha_1 - \alpha_0\alpha_1$	$1-\alpha_0^2$	0	$\lambda_1 = \frac{\alpha_1 - \alpha_0\alpha_1}{(1+\alpha_0)(1-\alpha_0)} = \frac{\alpha_1\cancel{(1-\alpha_0)}}{(1+\alpha_0)\cancel{(1-\alpha_0)}}$
	$1-\alpha_0^2 - \frac{\alpha_1^2(1-\alpha_0)}{1+\alpha_0}$			

$1 > 0$

$1-\alpha_0^2 > 0$

$(1+\alpha_0)(1-\alpha_0) > 0$

1. Fall: $1+\alpha_0 > 0 \Rightarrow \alpha_0 > -1$

$1-\alpha_0 > 0 \Rightarrow \alpha_0 < 1$

2. Fall $1+\alpha_0 < 0 \Rightarrow \alpha_0 < -1$

$1-\alpha_0 < 0 \Rightarrow \alpha_0 > 1$

2. Fall unsinnig $\Rightarrow -1 < \alpha_0 < 1$

$-1 < \gamma - \frac{1}{2} < 1$

$-\frac{1}{2} < \gamma < \frac{3}{2}$

$$1-\alpha_0^2 - \frac{\alpha_1^2(1-\alpha_0)}{1+\alpha_0} > 0$$

$$\frac{(1+\alpha_0)^2(1-\alpha_0) - \alpha_1^2(1-\alpha_0)}{1+\alpha_0} > 0$$

$$\frac{\overbrace{(1-\alpha_0)}^{>0 \text{ siehe oben}}\left((1+\alpha_0)^2 - \alpha_1^2\right)}{\underbrace{1+\alpha_0}_{>0 \text{ siehe oben}}} > 0 \Rightarrow \left((1+\alpha_0)^2 - \alpha_1^2\right) > 0$$

4b) iff $(1 + a_0 + a_1)(1 + a_0 - a_1) > 0$

$$\left(1 + \cancel{\gamma} - \frac{1}{2} + \cancel{(-\gamma)}\right)\left(1 + \gamma - \frac{1}{2} + \gamma\right) > 0$$

$$\underbrace{\left(\frac{1}{2}\right)}_{>0}\left(\frac{1}{2} + 2\gamma\right) > 0$$

$$\Rightarrow \left(\frac{1}{2} + 2\gamma\right) > 0$$

$$2\gamma > -\frac{1}{2}$$

$$\gamma > -\frac{1}{4}$$

$$\Rightarrow -\frac{1}{4} < \gamma < \frac{3}{2}$$