1.10.10

1) a) $c(u_c) = C_0 + C_1 u_c^2$

$u = \begin{bmatrix} u_{e1} \\ u_{e2} \end{bmatrix}$ $\quad y = u_a = u_{c2} - u_{c1}$

$x = \begin{bmatrix} u_{c1} \\ u_{c2} \end{bmatrix}$

$u_{e2} = R \cdot i_{c2} + u_{c2}$

$\Rightarrow i_{c2} = \frac{u_{e2} - u_{c2}}{R}$

$i_{c2} = \frac{d}{dt}(C u_{c2}) = \dot{C} u_{c2} + C \dot{u}_{c2}$

$= C_1 \cdot 2 u_{c2} \dot{u}_{c2} \cdot u_{c2} + (C_0 + C_1 u_{c2}^2) \dot{u}_{c2}$

$\frac{u_{e2} - u_{c2}}{R} = (3 C_1 u_{c2}^2 + C_0) \dot{u}_{c2}$

$\dot{u}_{c2} = \frac{u_{e2}}{R(3C_1 u_{c2}^2 + C_0)} - \frac{u_{c2}}{R(3C_1 u_{c2}^2 + C_0)}$

$u_{e1} = R \cdot i_{c1} + u_{c2}$

$i_{c1} = \frac{u_{e1} - u_{c2}}{R}$

$i_{c1} = \frac{d}{dt}(c \cdot u_{c1}) = (3 C_1 u_{c1}^2 + C_0) \dot{u}_{c1}$

$\dot{u}_{c1} = \frac{u_{e1}}{R(3C_1 u_{c1}^2 + C_0)} - \frac{u_{c2}}{R(3C_1 u_{c1}^2 + C_0)}$

$$\frac{d}{dt}\begin{bmatrix} u_{c1} \\ u_{c2} \end{bmatrix} = \begin{bmatrix} \frac{u_{e1}}{R(3C_1 u_{c1}^2 + C_0)} - \frac{u_{c2}}{R(3C_1 u_{c1}^2 + C_0)} \\ \frac{u_{e2}}{R(3C_1 u_{c2}^2 + C_0)} - \frac{u_{c2}}{R(3C_1 u_{c2}^2 + C_0)} \end{bmatrix}$$

$y = u_{c2} - u_{c1}$

1b) •) $u_{e1} = u_{e2} = 0$

$$\dot{u}_{c1} = 0 = -\frac{u_{c2R}}{R(3C_1 u_{c1R}^2 + C_0} \Rightarrow u_{c2R} = 0$$

$$\dot{u}_{c2} = 0 = -\frac{u_{c2R}}{R(3C_1 u_{c2}^2 + C_0)}$$

u_{c1} beliebig

•) $u_{e1} = u_{e2} = u_{c1R} = u_{c2R} = 0$

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & -\frac{1 \cdot R(3C_1 u_{c1R}^2 + C_0) - u_{c2R} \cdot R \cdot 0}{(R(3C_1 u_{c1R}^2 + C_0))^2} \\ 0 & -\frac{1 \cdot R(3C_1 u_{c2R}^2 + C_0) - 0}{(R(3C_1 u_{c2R}^2 + C_0))^2} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -\frac{RC_0}{R^2 C_0^2} \\ 0 & -\frac{1}{RC_0} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{RC_0} & 0 \\ 0 & \frac{1}{RC_0} \end{bmatrix}$$

$$c^T = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$\begin{array}{cc|cc} & & \frac{1}{RC_0} & 0 \\ & & 0 & \frac{1}{RC_0} \\ \hline 0 & -\frac{1}{RC_0} & 0 & -\frac{1}{R^2C_0^2} \\ 0 & -\frac{1}{RC_0} & 0 & -\frac{1}{R^2C_0^2} \end{array}$$

c) $$R = \begin{bmatrix} \frac{1}{RC_0} & 0 & 0 & -\frac{1}{R^2C_0^2} \\ 0 & \frac{1}{RC_0} & 0 & -\frac{1}{R^2C_0^2} \end{bmatrix}$$

voller Rang $\Rightarrow$ vollständig erreichbar d)

$$G(s) = c^T (sE - A)^{-1} B$$

$$(sE-A)^{-1} = \begin{bmatrix} s & \frac{1}{RC_0} \\ 0 & s+\frac{1}{RC_0} \end{bmatrix}^{-1} = \frac{1}{s\left(s+\frac{1}{RC_0}\right)} \begin{bmatrix} s+\frac{1}{RC_0} & -\frac{1}{RC_0} \\ 0 & s \end{bmatrix}$$

$$\begin{array}{cc|cc|cc}
 & & s+\frac{1}{RC_0} & -\frac{1}{RC_0} & \frac{1}{RC_0} & 0 \\
 & & 0 & s & 0 & \frac{1}{RC_0} \\
\hline
-1 & 1 & -s-\frac{1}{RC_0} & s+\frac{1}{RC_0} & -\frac{s}{RC_0}-\frac{1}{R^2C_0^2} & \frac{s}{RC_0}+\frac{1}{RC_0}
\end{array}$$

$$G(s) = \frac{1}{s\left(s+\frac{1}{RC_0}\right)} \begin{bmatrix} -\frac{s}{RC_0}-\frac{1}{R^2C_0^2} & \frac{s}{RC_0}+\frac{1}{RC_0} \end{bmatrix}$$

$$= \frac{1 \cdot \cancel{\left(s+\frac{1}{RC_0}\right)}}{RC_0 s \cancel{\left(s+\frac{1}{RC_0}\right)}} \begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{\hat{u}_a(s)}{\hat{u}_{e1}(s)} = -\frac{1}{RC_0 s} = -\frac{\hat{u}_a(s)}{\hat{u}_{e2}(s)}$$

2) a) $y_k = 1 \cdot (1^{k-2}) + 4 \cdot (1^{k-3}) + 5 \cdot (1^{k-4})$

$$Y_z = z^{-2} \frac{z}{z-1} + 4 z^{-3} \frac{z}{z-1} + 5 z^{-4} \frac{z}{z-1}$$

$$G_z = (z^{-2} + 4z^{-3} + 5z^{-4}) \frac{z}{z-1} = \frac{(z^2+4z+5) \cdot \cancel{z}}{z^{\cancel{4}\,3} (z-1)}$$

$$= \frac{(z^2+4z+5)}{z^3 (z-1)}$$

$$g_1[k] \circ\!\!-\!\!\bullet\; G_{1z} = 1 + 4z^{-1} + 5z^{-2} = \frac{z^2+4z+5}{z^2}$$

$$G_{2z} = \frac{G_z}{G_{1z}} = \frac{1}{z(z-1)} = \frac{-1}{z} + \frac{1}{z-1}$$

$$= -z^{-1} + z^{-1} \frac{z}{z-1}$$

$$g_2[k] = -\delta_{k-1} + (1^{k-1})$$

oder besser: $G_{2z} = \frac{1}{z^2} \frac{z}{z-1} \;\bullet\!\!-\!\!\circ\; g_2[k] = (1^{k-2})$

b) G ist nicht BIBO-stabil, da es einen Pol am Einheitskreis besitzt

Ja, Impulsantwort ist nicht absolut summierbar

c) $G_z = \dfrac{z^2+4z+5}{z^3(z-1)}$

$$G(e^{I\frac{\pi}{4}}) = \frac{e^{I\frac{\pi}{2}} + e^{I\frac{\pi}{4}} \cdot 4 + 5}{e^{I\frac{3\pi}{4}}\left(e^{I\frac{\pi}{4}} - 1\right)} = \frac{I + 4\left(\frac{\sqrt{2}}{2} + I\frac{\sqrt{2}}{2}\right) + 5}{e^{I\frac{3\pi}{4}}\left(\frac{\sqrt{2}}{2} - 1 + I\frac{\sqrt{2}}{2}\right)}$$

$$|G(e^{I\frac{\pi}{4}})| = \frac{\sqrt{(5+\sqrt{2}\,2)^2 + (1+\sqrt{2}\,2)^2}}{1 \cdot \sqrt{\left(\frac{\sqrt{2}}{2} - 1\right)^2 + \frac{2}{4}}} = \frac{\sqrt{25 + 4\sqrt{2} + 8 + 1 + 4\sqrt{2} + 8}}{\sqrt{\frac{1}{2} - \sqrt{2} + 1 + \frac{1}{2}}}$$

$$= \frac{\sqrt{42 + 8\sqrt{2}}}{\sqrt{2 - \sqrt{2}}}$$

$$\arg\left(G(e^{I\frac{\pi}{4}})\right) = \arctan\left(\frac{1+\sqrt{2}\,2}{5+\sqrt{2}\,2}\right) - \frac{3\pi}{4} - \arctan\left(\frac{\frac{1}{2}}{\frac{\sqrt{2}}{2} - 1}\right)$$

$$(y_k) = \frac{1}{2} \cdot |G(e^{I\frac{\pi}{4}})| \cos\left(\frac{\pi}{4}k + \frac{\pi}{8} + \arg\left(G(e^{I\frac{\pi}{4}})\right)\right)$$

3) a) $\left(\frac{1}{4}-\lambda\right)\left(\frac{1}{6}-\lambda\right)\left(\frac{1}{2}-\lambda\right)=0$

$\lambda_1=\frac{1}{4} \quad \lambda_2=\frac{1}{6} \quad \lambda_3=\frac{1}{2}$

$$\lambda_1: \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{12} & \frac{1}{3} \\ 3 & 0 & \frac{1}{4} \end{bmatrix} V_1 = 0$$

$\frac{1}{12} v_{12} = \frac{1}{3} v_{13} \qquad -3 v_{11} = \frac{1}{4} v_{13}$

$v_{12} = 4 v_{13} \qquad -12 v_{11} = v_{13}$

$v_{11}=1 \quad v_{13}=-12$

$v_{12}=-48$

$$V_1 = \begin{bmatrix} 1 \\ -48 \\ -12 \end{bmatrix} \qquad c^T v_1 \neq 0$$

$$\lambda_2: \begin{bmatrix} \frac{1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{3} \\ 3 & 0 & \frac{1}{3} \end{bmatrix} V_2 = 0 \qquad V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$c^T V_2 = 0 \Rightarrow$ nicht vollst. beobachtbar

b)

$$\Theta = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix} \qquad \Theta^{-1} = 4 \cdot \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{8} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -\frac{1}{2} & 2 \end{bmatrix}$$

$\rightarrow$ vollst. beobachtbar

$$\hat{V}_1 = \Theta^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{array}{cc|c} & & 0 \\ & & 1 \\ \hline 2 & 0 & 0 \\ -\frac{1}{2} & 2 & 2 \end{array}$$

$$\hat{p}_{g,soll} = \left(z-\frac{1}{2}\right)^2 = z^2 - z + \frac{1}{4}$$

Φ^2:

$$\begin{array}{cc|cc} & & \frac{1}{4} & 1 \\ & & 0 & \frac{1}{8} \\ \hline \frac{1}{4} & 1 & \frac{1}{16} & \frac{3}{8} \\ 0 & \frac{1}{8} & 0 & \frac{1}{64} \end{array}$$

$$\hat{v}_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\hat{p}_{soll}(\Phi) = \begin{bmatrix} \frac{1}{16} & \frac{3}{8} \\ 0 & \frac{1}{64} \end{bmatrix} - \begin{bmatrix} \frac{1}{4} & 1 \\ 0 & \frac{1}{8} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} X & -\frac{5}{8} \\ X & \frac{9}{64} \end{bmatrix} \qquad \frac{1}{64} - \frac{8}{64} + \frac{16}{64} = \frac{9}{64}$$

$$\begin{array}{cc|c} \hat{k} = & & 0 \\ & & -2 \\ \hline X & -\frac{5}{8} & \frac{10}{8} \\ X & \frac{9}{64} & \frac{-18}{64} \end{array}$$

Probe mit MATLAB liefert korrekte Eigenwerte

c) $x = Vz \qquad V^{-1}AV = \tilde{A} \qquad V = \tilde{c}$

MIMO:

$$\Theta = \begin{bmatrix} \tilde{C} \\ \tilde{C}\tilde{A} \\ \vdots \\ \tilde{C}\tilde{A}^{n-1} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} CV \\ CVV^{-1}AV \\ \vdots \\ CVV^{-1}AV \cdots V^{-1}AV \end{bmatrix} = \begin{bmatrix} CV \\ CAV \\ \vdots \\ CA^{n-1}V \end{bmatrix}$$

Rang ändert sich nicht

$\Rightarrow$ Rang bleibt voll

$\Rightarrow$ vollst. beobachtbar

3d) alle Pole und NS. in linker offener s-Halbebene

$k-3>0$ $\qquad$ $h+1>0$

$k>3$ $\qquad$ $h>-1$

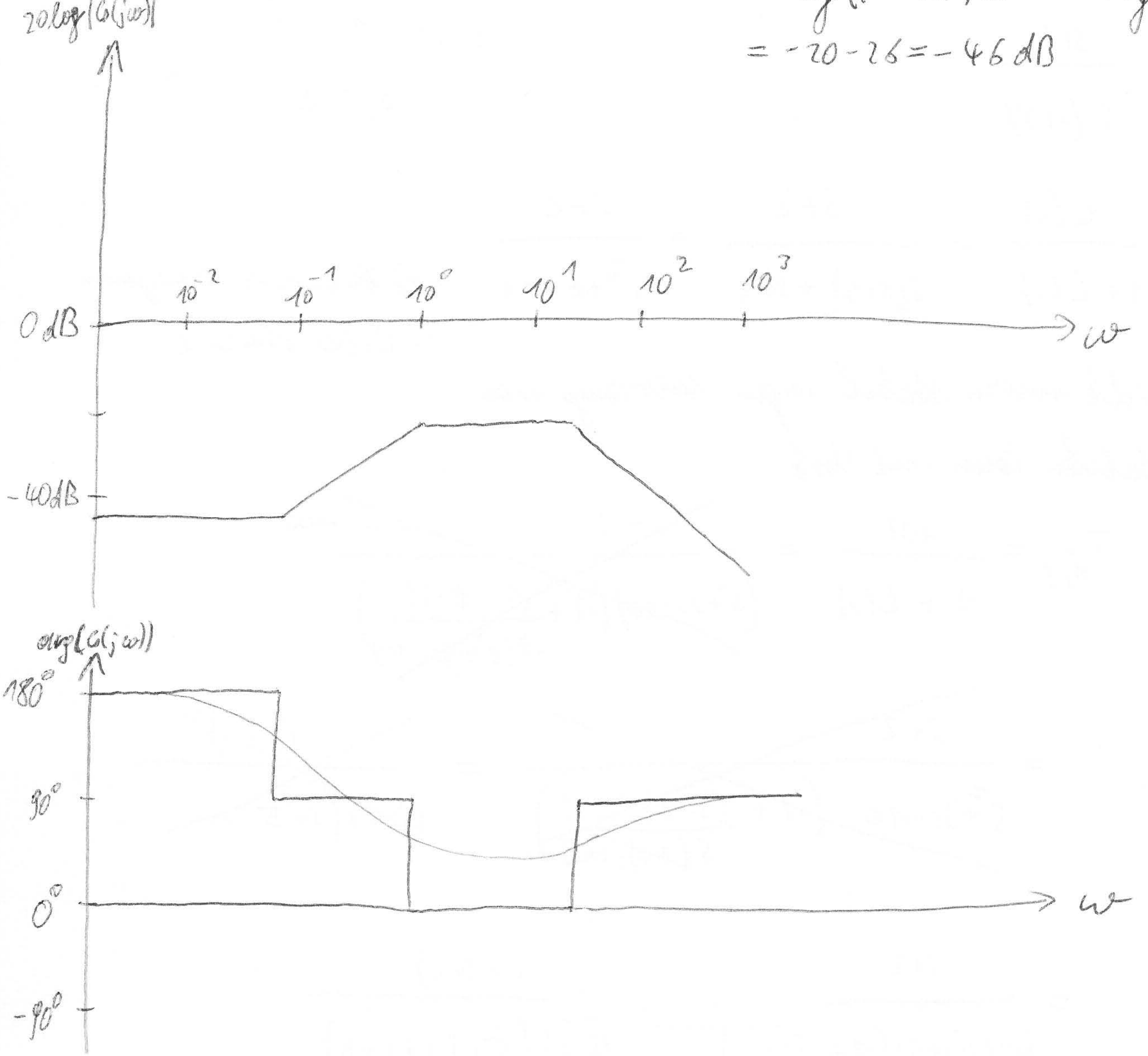
e) sprungfähig: $\lim_{s\to\infty} G(s) \neq 0$ $\qquad$ realisierbar: $\lim_{s\to\infty} G(s) < \infty$

$$\lim_{s\to\infty} \frac{s^2-2s}{s^2+2s+1} = 1 \Rightarrow \text{sprungfähig und realisierbar}$$

4a) $G(s) = -\dfrac{s - \frac{1}{10}}{(s+1)(s-20)}$

$G(j10^{-3}) \approx + \dfrac{\frac{1}{10}}{-20} = -\dfrac{1}{10 \cdot 20}$

$20 \log(10 \cdot 20^{-1})\,dB = -20\log(10) - 20\log(20)$
$= -20 - 26 = -46\,dB$



b) i) $R(s) = \frac{s-2}{s}$ $\quad G(s) = \frac{s+2}{s^2+3s-10}$ $\quad s_{1,2} = \frac{-3 \pm \sqrt{9+40}}{2} = \frac{-3 \pm 7}{2}$

$= \begin{cases} s_1 = -5 \\ s_2 = 2 \end{cases}$

$L(s) = \frac{s+2}{s(s+5)}$

$T_{r,y} = \frac{L(s)}{1+L(s)} = \frac{s+2}{s(s+5)+s+2} = \frac{s+2}{s^2+6s+2}$ ist Hurwitz-Polynom

$\Rightarrow$ BIBO stabil

ii) ist nicht intern stabil wegen Kürzung von instabilen Polen und Nst

zB: $T_{d,y} = \frac{G(s)}{1+L(s)} = \xcancel{\frac{s+2}{(s^2+3s-10)\left(1+\frac{(s-2)(s+2)}{s(s^2+3s-10)}\right)}} =$

$= \xcancel{\frac{s+2}{\left(1+\frac{(s-2)(s+2)}{s(s-2)(s+5)}\right)}} = \xcancel{\frac{s\cdot(s+2)}{(s-2)(s+5}}$

$= \frac{s+2}{(s-2)(s+5)\left(1+\frac{s+2}{s(s+5)}\right)} = \frac{s(s+2)}{(s-2)(s^2+5s+s+2)}$

$\uparrow$

instabiler Pol

c) $\hat{\Phi}(s) = (sE - A)^{-1} = \begin{bmatrix} s-2 & 1 \\ -4 & s-2 \end{bmatrix}^{-1} = \frac{1}{(s-2)^2+4} \begin{bmatrix} s-2 & -1 \\ 4 & s-2 \end{bmatrix}$

$$\Rightarrow \frac{1}{2} \frac{2}{(s-2)^2+4} \quad \bullet\!\!-\!\!\circ \quad \frac{1}{2} e^{2t} \sin(2t)$$

$$s\hat{f}(s) \circ\!\!-\!\!\circ \frac{d}{dt} f(t) = e^{2t} \sin(2t) + e^{2t} \cos(2t) = e^{2t}(\sin(2t) + \cos(2t))$$

$$\Phi(t) = \begin{bmatrix} e^{2t}(\sin(2t)+\cos(2t)) - e^{2t}\sin(2t) & -\frac{1}{2} e^{2t} \sin(2t) \\ 2e^{2t}\sin(2t) & e^{2t}\cos(2t) \end{bmatrix}$$

$$= e^{2t} \begin{bmatrix} \cos(2t) & -\frac{1}{2}\sin(2t) \\ 2\sin(2t) & \cos(2t) \end{bmatrix}$$