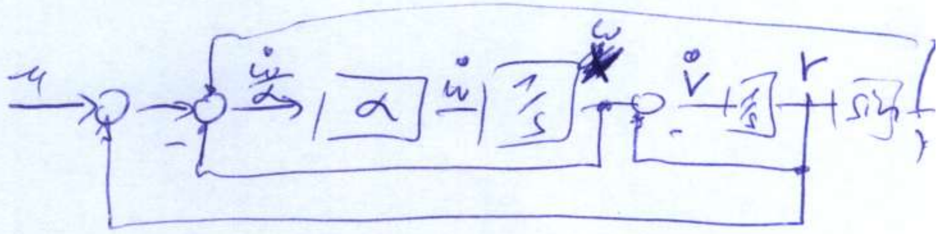
4. 4. 2008

1a) $y = \sin v$



$x = \begin{bmatrix} \omega \\ v \end{bmatrix}$

$u = u$

$\dot{v} = w - v$

$\alpha\dot{\omega} = \alpha(u + v + y - w) = \alpha(-\omega + v + \sin v + u)$

$\dot{\omega} = \frac{1}{\alpha}(-\omega + v + \sin v) + \frac{1}{\alpha} u$

ii) Ruhelage $\dot{\omega} = 0,\ \dot{v} = 0,\ u = 0$

$\rightarrow w = v$

$0 = \frac{\sin v}{\alpha} \rightarrow \sin v = 0 \rightarrow v = \pi \cdot k = w \quad , k \in \mathbb{Z}$

$\rightarrow y = 0$

$c^T = \frac{\partial g}{\partial x}(x_r, u_r)$

$c^T = [0 \quad 1]$

$b = \frac{\partial f}{\partial u}(x_r, u_r) = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} -\alpha & \alpha + \alpha\cos v_R \\ +1 & -1 \end{bmatrix} = \begin{bmatrix} -\alpha & 2\alpha \\ 1 & -1 \end{bmatrix}$

$\det(\lambda E - A) = 0$

$\det \begin{pmatrix} \lambda + \alpha & -2\alpha \\ -1 & \lambda + 1 \end{pmatrix} = 0$

$(\lambda + \alpha)(\lambda + 1) - 2\alpha) = 0$

$\lambda^2 + \lambda(\alpha + 1) + \alpha^2 = 0 \quad , da\ \alpha \neq 0$

$\lambda^2 + \lambda(\alpha + 1) + \alpha = 0$

$\lambda_{12} = \frac{-\alpha + 1}{2} \pm \sqrt{\frac{(\alpha + 1)^2}{4} - \alpha} = \frac{-\alpha + 1}{2} \pm \sqrt{\frac{\alpha^2 + 2\alpha + 1 - 4\alpha}{4}}$

~~ad 4aii)~~ $\cancel{\lambda_{12} = -\frac{\alpha+1}{2} \pm \frac{\sqrt{(\alpha+1)(\alpha-1)}}{2}}$

$$(\lambda+\alpha)(\lambda+1) - 2\alpha = 0$$

$$\lambda^2 + \lambda(\alpha+1) + \alpha - 2\alpha = 0$$

$$\lambda^2 + \lambda(\alpha+1) - \alpha = 0$$

$$\lambda_{12} = -\frac{\alpha+1}{2} \pm \sqrt{\frac{\alpha^2+2\alpha+1+4\alpha}{4}} = -\frac{\alpha+1}{2} \pm \frac{\sqrt{\alpha^2+6\alpha+1}}{2}$$

WC: $+$

$\operatorname{Re} \lambda < 0 \;=\; 0 > -\alpha - 1 + \sqrt{\alpha^2+6\alpha+1}$ $\quad |:(\alpha+1)$

$\alpha \neq -1$, weil sonst $> \to <$

$$1 > \sqrt{\frac{\alpha^2+6\alpha+1}{\alpha^2+2\alpha+1}}$$

$$1 > \frac{\alpha^2+6\alpha+1}{\alpha^2+2\alpha+1}$$

$$\alpha^2+2\alpha+1 > \alpha^2+6\alpha+1$$

$$-\alpha > 0$$

$\underline{\underline{\alpha < 0}}$ $\underline{\underline{\text{und } \alpha \neq -1}}$

b) $\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -\frac{1}{4} & 0 \end{bmatrix} \begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix} + \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix} u_1$

$\begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -\frac{1}{4} & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix} u_0$

$x_{1,1} = 4 + u_0$

$x_{2,1} = -\frac{1}{2} - \frac{1}{2} u_0$

I: $-\frac{1}{2} = 4 + u_0 - \frac{1}{2} - \frac{1}{2} u_0 + u_1 - \cancel{\frac{1}{2} u_1}$

II: $1 = \cancel{\ldots} - 1 - \frac{1}{4} u_0 - \frac{1}{2} u_1$

II: $u_0 = -8 - 2u_1$

I: $0 = 4 + \frac{1}{2} u_0 + u_1 = 4 - 4 - u_1 + u_1 = 0$?

oder II: $u_1 = -4 - \frac{1}{2} u_0$

I: $0 = 2 + \frac{1}{4} u_0 + \frac{1}{2} u_1 = 2 - 2 + \frac{1}{4} u_0 - \frac{1}{4} u_0 = 0$

$\boxed{\text{d.h. } u_0 = \alpha \quad u_1 = -4 - \frac{\alpha}{2}}$

ii) I: $\frac{1}{2} = 4 + U_0 - \frac{1}{2} - \frac{1}{2}U_0 + U_1$

II: $1 = -1 - \frac{1}{4}U_0 - \frac{1}{2}U_1$

$U_1 = -4 - \frac{1}{2}U_0$

$\frac{1}{2} = 3 + \frac{1}{2}U_0 + U_1 = 3 - 4 + \cancel{\frac{1}{2}U_0} - \cancel{\frac{1}{2}U_0} = -1$

Widerspruch, nicht erreichbar

iii) $R(A,B) = \begin{vmatrix} 1 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{4} \end{vmatrix}$

		1
		$-\frac{1}{2}$
1	1	$\frac{1}{2}$
$-\frac{1}{4}$	0	$-\frac{1}{4}$

$\text{Rang}(R) = 1 \neq 2$

2) G(s) aufstellen

$V_0: \; 26\,dB \mathrel{\hat=} 20$

$$G(s) = \frac{20\cdot(1+\frac{s}{2})}{(1+10s)} \text{ (vorzuhalten)}$$

$\omega_{c_1} = 0{,}1 \quad , \omega_{c2} = 2$

$$R(s) = \underbrace{\frac{V_I}{s}}_{R_1}(1+sT_i)$$

$\omega_c t_r = 1{,}5s$

$\omega_c = 1$

$\hat{u} + PR = 70$

$\hookrightarrow PR = 60$

$$L_1 = R_1 G \Big|_{I\omega_c} = \frac{20(1+\frac{s}{2})}{(1+10s)\,s} \Big|_{I\omega_c = I} = \frac{20+10I}{1+10I}$$

$\arg L_1(I\omega_c) =$

graphisch

$$\arg(L_1(I\omega_c)) = -60 - \underbrace{90}_{\text{Integrator}} = -150°$$

$\cancel{|L_1(I\omega_c)|} = 2\,(6\,dB)$

$\frac{\pi}{6} \overset{!}{=} \arctan T_i$

$T_i \overset{!}{=} \frac{\sqrt{3}}{3}$

$$|R_2(I\omega_c)| = \left|\frac{1+I\frac{\sqrt{3}}{3}}{I}\right| = \sqrt{1+\frac{3}{9}} = \frac{\sqrt{12}}{3} = \frac{\sqrt{3}\cdot 2}{3}$$

V_I fehlt

$$|L_2(I\omega_c)| = \underbrace{|G(I\omega_c)|}_{2\,(\hat=\,6\,dB)\text{ graphisch}} \cdot V\frac{\sqrt{3}\,2}{3} \overset{!}{=} 1$$

$$\to V = \frac{\sqrt{3}}{4}$$

b)

i) $e(t) = t\,\sigma(t)$ ○—● $1/s^2 = E(s)$

- $\lim_{\substack{(t\to\infty)\\ s\to 0}} \left(R(s)\cdot E(s)\cdot s\right) = \lim_{s\to 0}\left|\frac{\cancel{s}\cdot s}{\cancel{s^2}}\,\frac{1}{s^2+2s+1}\right| = 1$

- ~~stat~~ $u(t) = 1$ (stationär) $\rightarrow$ $\underline{\underline{y(t) = 20}}$

ii) $e(t) = 5\sin(t)$ $\qquad R_s = \frac{s}{(s+1)(s+1)}$

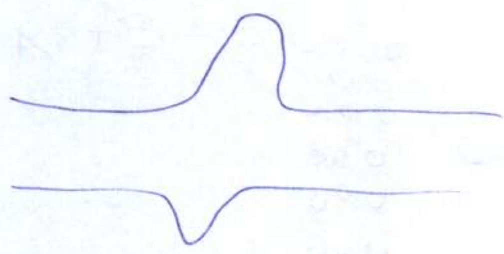
$|R(\cancel{j}\omega = I)| = \frac{1}{|\cancel{I} - 1 + 1 + 2I|} = 1/2$

$\arg(R(I\omega = I)) = \pi/2 - \pi/4 - \pi/4 = 0$

$u(t) = 5/2\,\sin(t)$

$y(t) = 5\cdot\sin(t - \underbrace{60°}_{-\pi/3})$

c) Lead/Lag
+ Allpass



3a) Q: $G^{\#}(q)$ darf keine Polstelle bei $\Omega_0 = \frac{2}{T_a}$ aufweisen

Z: $\text{grad}(a(z)) \leq \text{grad}(b(z))$

b) i) nicht gl. asympt. stabil

$p = \lambda^3$, alle Eigenwerte auf Achse

(d) ii) Eigenwert $\det(A - \lambda E) = p(A)$

$$\det(V^{-1}AV - \lambda E) = \det(V^{-1}AV - \lambda \underbrace{V^{-1}EV}_{E})$$

$$= \det(V^{-1}(A - \lambda E)V) =$$

$$= \cancel{\det V^{-1} \cdot \det \ldots} \; p(A) \cdot \det V$$

iii) Nein $(2 \neq 1)$

iv) Nilpotent, A

~~exp~~ $\phi(t) = \exp(At) = E + At + A^2\frac{t^2}{2} + \underbrace{A^3 \ldots}_{0}$

$$= E + At + A^2\frac{t^2}{2} = \begin{pmatrix} 1 & 2t & 2t^2 \\ 0 & 1 & 2t \\ 0 & 0 & 1 \end{pmatrix}$$

			0	2	0
			0	0	2
			0	0	0
0	2	0	0	0	4
0	0	2	0	0	0
0	0	0	0	0	0

...

zu vi) $0 = x(3) = \phi(3) x_0 + \int_0^3 \phi(3-\tau) b u(\tau) d\tau$

$$+ \int_1^2 \phi(3-\tau) b \, d\tau$$

$$\phi(3) x_0 + \int_1^2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} d\tau = \begin{pmatrix} 1 & 6 & 18 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{pmatrix} x_0 + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \vec{0}$$

$x_{0,3} = 0, \quad x_{0,2} = 0, \quad x_{0,1} = 1$

4) $u_k = x_k k_1 + \xi_k k_2$

$\xi_{k+1} = a \xi_k + 2 e_k$

$\xi z - a = 2 e_k$

$\boxed{T_2(z) = \frac{\xi_k}{e_k} = \frac{2}{z-a} = \frac{\xi_k}{(r-y)}}$

$x_{k+1} = -\frac{1}{2} x_k + u_k$

$z x = -\frac{1}{2} x + u$

$T_{1,1} = \frac{x}{u} = \frac{1}{z + 1/2}$

$\longrightarrow u = x \cdot k + \xi k_2$

$u = \frac{k \cdot u}{z + 1/2} + \xi k_2 = \frac{k_1 u}{z + 1/2} + \frac{2 k_2}{z-a} (r-y)$

$T_{1,2} = \frac{3}{z + 1/2}$

$y = T_{1,2} \cdot u = \cancel{G_{1,2}}$

$u \left(1 - \frac{k}{z + 1/2} \right) = \frac{2 k_2}{z-a} (r-y)$

$= \cancel{G_{1,2} \cdot \frac{2 k_2}{(z-a)(1 - \frac{k_1}{z+1/2})} (r-y) = \frac{3 \cdot 2 k_2 \cdot (r-y)}{(z+1/2)(z-a)(1 - \frac{k_1}{z+1/2})}}$

$\cancel{y = \frac{\frac{6 k_2}{(z+1/2)(z-a)(1 - \frac{k_1}{z+1/2})}}{1 + \frac{6 k_2}{(z+1/2)(z-a)(1 - \frac{k_1}{z+1/2})}}}$

$\boxed{a=1}$

4b) $\varepsilon_k = \frac{z}{z-1}\cdot(r-y) = \frac{2r}{z-1}\left(1-\frac{6k_2}{6k_2+(z-1)(z+\frac{1}{2}-k_1)}\right)$

$(r_k) = (r_s) \circ\!\!-\!\!\bullet \ \frac{r_s z}{z-1}$

$\varepsilon_s = \lim_{z\to 1} \frac{r_s z}{\cancel{z-1}}\cdot\cancel{(z-1)}\frac{2}{\cancel{z-1}}\left(\frac{\cancel{6k_2}+\cancel{(z-1)}(z+\frac{1}{2}-k_1)\cancel{-6k_2}}{6k_2+(z-1)(z+\frac{1}{2}-k_1)}\right)$

$\varepsilon_s = \lim_{z\to 1}\frac{2r_s z(z+\frac{1}{2}-k_1)}{6k_2+(z-1)(z+\frac{1}{2}-k_1)} = \frac{2r_s(\frac{3}{2}-k_1)}{6k_2} = \boxed{\frac{(3-2k_1)r_s}{6k_2}}$

$x_k = y_k/3$

$x_k = \frac{2k_2 r}{6k_2+(z-1)(z+\frac{1}{2}-k_1)}$

$\boxed{x_s = \lim_{z\to 1}\frac{\cancel{(z-1)}2k_2 r_s z}{\cancel{(z-1)}(6k_2+(z-1)(z+\frac{1}{2}-k_1))} = \frac{2k_2 r_s}{6k_2} = r_s/3}$

ad 4a) $u = \frac{k_1 u}{z+1/2} + \frac{2k_2}{z-a} \cdot (r-y)$

$$u = \frac{\frac{2k_2}{z-a}(r-y)}{\left(1-\frac{k_1}{z+1/2}\right)}$$

$$y = G_{1,2} \cdot u = \frac{3}{z+1/2} \cdot \frac{\frac{2k_2}{z-a}(r-y)}{1-\frac{k_1}{z+1/2}} = \frac{6k_2(r-y)}{(z+1/2)(z-a)\left(1-\frac{k_1}{z+1/2}\right)}$$

$$= \frac{6k_2(r-y)}{(z-a)(z+1/2-k_1)}$$

$$\boxed{G(z) = \frac{y}{r} = \frac{\cancel{r}\cdot\frac{6k_2}{(z-a)(z+1/2-k_1)}}{\cancel{r}\left(1+\frac{6k_2}{(z-a)(z+1/2-k_1)}\right)} = \frac{6k_2}{6k_2+(z-a)(z+1/2-k_1)}}$$

$$\lim_{k\to\infty} y_k \overset{!}{=} r_s \overset{!}{=} \lim_{z\to 1} \cancel{(z-1)}\, G(z) \cdot r_s \cdot \frac{z}{\cancel{(z-1)}} = \lim_{z\to 1} \frac{6k_2 z r_s}{6k_2+(z-a)(z+1/2-k_1)} \overset{!}{=} r_s$$

$$1 = \frac{6k_2}{6k_2+(1-a)(3/2-k_1)}$$

$$\cancel{6k_2}+(1-a)(3/2-k_1) = \cancel{6k_2}$$

$$\boxed{a=1} \text{ oder } k_1 = 3/2$$

4c) $$G(z) = \frac{6k_2}{(6k_2 + \frac{a}{2} - ak_1) - az + z^2 + \frac{z}{2} - k_1 z}$$

$$(a=1) = \frac{6k_2}{(6k_2 + \frac{1}{2} - k_1) - \cancel{z} + z^2 + z(\frac{1}{2} - k_1 - a)}$$

Polstellen von $G(z)$ → Eigenwerte von ϕ

$$\cancel{p_{1,2} = +\frac{1}{2} \pm \sqrt{\frac{1}{2}}}$$

$$(6k_2 + \frac{1}{2} - k_1) \cancel{- z} + z^2 = 0 \quad \text{für } z = 0$$

$$\underbrace{(6k_2 + \frac{1}{2} - k_1)}_{=0} + z\underbrace{(\frac{1}{2} - k_1 - 1)}_{=0 \rightarrow \boxed{k_1 = -\frac{1}{2}}} + z^2 = z^2 \quad \cancel{\text{für } z=0}$$

$$6k_2 + \frac{1}{2} + \frac{1}{2} = 0 \rightarrow \boxed{k_2 = -\frac{1}{6}}$$

d) Analogie zum Dead-Beat-regler

→ 2 Schritte (wenn vollständig erreichbar)

weil ϕ Nilpotent, da ~~det~~ $p = z^n$

$$\cancel{x_{k+1} = \phi x_k + \Gamma u_k} \qquad \cancel{x_1 = 0 \cdot x_0}$$

$$y_k = c^T x_k \qquad y_1 = c^T \cdot x_0$$

$$x_{k+1} = \phi x_k \qquad x_1 = \phi \cdot x_0$$

$$x_2 = \phi x_1 = \phi^2 x_0 = 0$$

↳ da nilpotent nach $n=2$

$$y_k = c\, x_k$$

4e) Wenn das System vollständig erreichbar und beobachtbar ist, ergibt sich das char. Polynom des geschlossenen Kreises $P_{ges}(z) = \underbrace{P_{p,soll}(z)}_{\text{Zustands-regler}} \underbrace{\hat{P}_{p,soll}(z)}_{\text{Beobachter}}$

$$x_{k+1} = \phi x_k + \Gamma k^T \hat{x}_k + g r_k$$

$$y_k = c^T x_k$$

$$\hat{x}_{k+1} = \phi \hat{x}_k + \Gamma u_k + \hat{k}(\hat{y}_k - y_k)$$

$$\hat{y}_k = c^T \hat{x}_k$$

$$e_k = \hat{x}_k - x_k$$

$$u_k = k^T \hat{x}_k + g r_k$$

$$x_{k+1} = \phi x_k + \Gamma k^T (e_k + x_k) + g r_k$$

e_{k+1}

$$e_{k+1} = \hat{x}_{k+1} - x_{k+1} = \phi \hat{x}_k + \Gamma u_k + \hat{k}(\hat{y}_k - y_k) - \phi x_k - \underbrace{\Gamma k^T \hat{x}_k + g r_k}_{u_k}$$

$$= \phi e_k + \hat{k}(\hat{y}_k - y_k) =$$

$$= \phi e_k + \hat{k} c^T (\underbrace{\hat{x}_k - x_k}_{e_k}) = \underbrace{(\phi + \hat{k} c^T)}_{\phi_e} e_k$$

$$\begin{bmatrix} x_{k+1} \\ e_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi + \Gamma k^T & \Gamma k^T \\ 0 & \phi + \hat{k} c^T \end{bmatrix}}_{\phi_{ges}\ 2n \times n} \begin{bmatrix} x_k \\ e_k \end{bmatrix} + \begin{bmatrix} g \\ 0 \end{bmatrix} r_k$$

$$\det |z E_{2n \times 2n} - \phi_{ges}| = \det(z E_{n \times n} - (\phi + \Gamma k^T)) \det(z E_{n \times n} - (\phi + \hat{k} c^T))$$

ad 4e) Zustandsbeobachter

$$\hat{x}_{k+1} = \phi \hat{x}_k + \Gamma u_k + \hat{k} c^T x_k - \hat{k} y_k$$

$$= \underbrace{(\phi + \hat{k} c^T)}_{\phi_e} x_k + \Gamma u_k - \hat{k} y_k$$

$$\det \cancel{\phi_e}^{zE-\phi_e} = \det(zE_{n \times n} - \phi_e) = \det(zE_{n \times n} - (\phi + \hat{k} c^T))$$

Zustandsregler

$$x_{k+1} = \phi x_k + \Gamma u_k = \phi x_k + \Gamma k^T x_k + \Gamma g r_k =$$

$$\hookrightarrow u_k = k^T x_k + g r_k$$

$$= \underbrace{(\phi + \Gamma k^T)}_{\phi_g} x_k + \Gamma g r_k$$

char. Poly. $|zE_{n \times n} - \phi_g| = \det|zE_{n \times n} (\phi + \Gamma k^T)|$