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1) 1) $\dot{V}_{0B} = \dot{H} \cdot A_{0B} = Q_i - Q$

1. Teilsystem

2) $p_1 = p_{atm} + \rho g H$

3) $m = \rho A L$

4) $Q = v A$

2. Teilsystem

5) $\rho A L \dot{v} = \rho A L g \cdot \sin\alpha + (p_1 - p_2) A$

6) $Q = C \sqrt{p_2 - p_{atm}}$

7) $T_m = 2\rho R Q\left(\frac{Q}{A_j} - \omega R\right)$

8) $T_L = k\omega^2$

9) $T_d = d\omega$

Drallsatz

10) $\Theta \dot{\omega} = T_m - T_L - T_d$

$= 2\rho R Q\left(\frac{Q}{A_j} - \omega R\right) - k\omega^2 - d\omega$

DGL 1. Ordnung

$\dot{x} = f(x,u) \qquad u = Q_i$

$y = g(x,u) \qquad y = \omega$

$x = [H, v, \omega]$

„trivialster Impulssatz ever"

$m\ddot{x} = F$

$x_2 = \dot{x}$

$m\dot{x}_2 = F$

$x_1 = x$

$\dot{x}_1 = x_2$

$1) \Rightarrow \dot{x}_1 = \frac{1}{A_{OB}} u - \frac{A}{A_{OB}} x_2$

$5) \Rightarrow \dot{x}_2 = g \cdot \sin(\alpha) + \frac{g}{L} x_1 - \frac{A^2}{C^2 g L} x_2^2$

$2) \Rightarrow p_1 = p_{atm} + \rho g H$

$6) \Rightarrow p_2 = \frac{(VA)^2}{C^2} + p_{atm}$

$p_1 - p_2 = \rho g H + p_{atm} - \left(\frac{(VA)^2}{C^2} + p_{atm}\right)$

$10) \Rightarrow \dot{x}_3 = \frac{1}{\Theta} \cdot \left(2\rho R A x_2 \left(\frac{A \cancel{x_2}}{A_j} - R x_3\right) - K x_3^2 - d x_3\right)$

$= \frac{2\rho R A^2}{\Theta A_j} x_2^2 - \frac{2\rho R^2 A}{\Theta} x_2 x_3 - \frac{K}{\Theta} x_3^2 - \frac{d}{\Theta} x_3$

$$\dot{x} = \begin{bmatrix} f_{11} u - f_{12} x_2 \\ f_{21} x_1 - f_{22} x_2^2 + f_{23} \\ f_{31} x_2^2 - f_{32} x_2 x_3 - f_{33} x_3^2 - f_{34} x_3 \end{bmatrix}$$

RL $u_R > 0$ $\quad \omega_R > 0$ $\quad$ alle Ruhelagen $\overset{!}{\nabla}$

$x_{2R} = \frac{f_{11}}{f_{12}} u_R = \frac{1}{A} u_R$

$x_{1R} = \frac{1}{f_{21}} \left(f_{22} x_{2R}^2 - f_{23}\right) = \frac{1}{g \rho L C^2} u_R^2 - L \sin\alpha$

$x_{3R1/2} = -\frac{1}{2K} \left(d + 2\rho R^2 u_R\right) \overset{+}{(-)} \left[\frac{1}{4K^2} \left(d + 2\rho R u_R\right)^2 + \frac{2\rho R}{A_j K} u_R\right]^{1/2}$

durch $\omega_R > 0$

e) lin. $\Delta \dot{x} = A \Delta x + B \Delta u \quad \Delta y = C \Delta x + D \Delta u$

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad D = 0$$

$$A = \partial_x f = \frac{\partial}{\partial x} f$$

$$C = \partial_x g \Big|_{x = x_R,\, u = u_R} \quad D = \partial_u g$$

$$A = \left[\begin{array}{c|c|c} 0 & -f_{12} & 0 \\ f_{21} & -2 f_{22} x_{2RL} & 0 \\ 0 & 2 f_{31} x_{2RL} - f_{32} x_{3RL} & -f_{32} x_{2RL} - 2 f_{33} x_{3RL} - f_{34} \end{array}\right]$$

$$B = \partial_u f = \begin{bmatrix} f_u & 0 & 0 \end{bmatrix}$$

2)a) $$T_{yr} = \frac{L}{1+L} = \frac{z_L}{z_L + n_L} \qquad L = R\,G = \frac{z_L}{n_L} \qquad V = 1 \qquad \frac{\text{Vorwärtszweig}}{1 + \text{Schleifenverstärkung}}$$

$$z_L + n_L = (a_0 + a_1 s)(s + 0{,}5) + (b_0 + b_1 s)(s^2 - s - 1) \overset{!}{=} (s+1)^3$$

Koeffizientenvergleich

$$= b_1 s^3 + (b_0 - b_1) s^2 + (-b_0 - b_1) s - b_0 = s^3 + 3s^2 + 3s + 1$$

$$(a_1) s^2 \quad (a_0 + 0{,}5 a_1) s + 0{,}5 a_0$$

s^3: $b_1 = 1$

s^2: $(b_0 - 1 + a_1) = 3$

s^1: $-b_0 - 1 + a_0 + 0{,}5 a_1 = 3$

s^0: $-b_0 + 0{,}5 a_0 = 1$

$$R = \frac{2 + 4s}{s}$$

b) $z_L + n_L = 2\left(s^3 + (4V-1)s^2 + (4V-1)s + V\right) = n(T_{yr})$

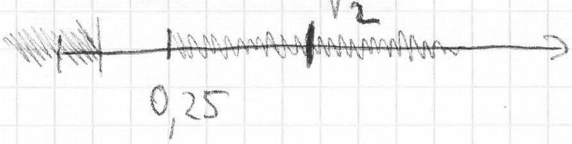
Routh-Hurwitz ab 3. Ordnung; bei Polynomen 2. Ordnung das notwendige und hinreichende Kriterium anwenden! nicht rechnen!

$s^3 \quad a_{01} = a_3 = 1 > 0$

$s^2 \quad a_{11} = a_2 = 4V-1 > 0 \longrightarrow V > 0{,}25$

$s^1 \quad a_{21} = \dfrac{a_{11}a_{12} - a_{01}a_{12}}{a_{11}} = a_1 - \dfrac{a_3 a_0}{a_2} = (4V-1) - \dfrac{V}{4V-1} > 0$

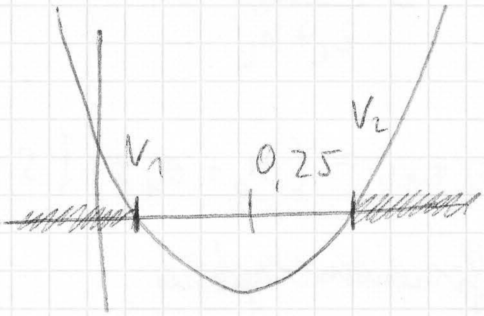
$s^0 \quad a_{31} = a_0 = V > 0$



$(4V-1)^2 - V > 0$

$V_{12} = \dfrac{9 \pm \sqrt{17}}{32}$

$V > \dfrac{9 + \sqrt{17}}{32}$



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2c) $R = \frac{\hat{y}}{\hat{u}} = \frac{2+4s}{s}$ muss in normierter Form vorliegen

$$\hat{y} = \left(\frac{2}{s} + 4\right)\hat{u}$$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$\dot{x} = 2u \qquad A = 0 \quad B = 2$$
$$y = x + 4u \qquad C = 1 \quad D = 4$$

oder mit Formel aus Formelsammlung

d) $\lim_{t \to \infty} g(t) \circ\!\!-\!\!\bullet \lim_{s \to 0} s\,\hat{u}(s)$

Grenzwert muss existieren

$$T_{yd} = \frac{G_d}{1+L} = \frac{\frac{1}{s}}{1 + \frac{z_L}{n_L}} \qquad L = R \cdot G$$

$$= \frac{n_L}{s(z_L + n_L)} = \frac{\cancel{s}\, n_G}{\cancel{s}(z_L + n_L)}$$

$$n(T_{yd}) = n(T_{yr}) \Rightarrow T_{yd} = \text{BIBO} \Rightarrow \text{GWS ok}$$

$$\lim_{s \to 0} s \, \frac{2\cdot(s^2 - s + 1)}{2\cdot(s^3 + 7s^2 + 7s + 2)} \cdot \frac{d_c}{s} = -\frac{d_c}{2} \qquad \sigma(t) \circ\!\!-\!\!\bullet \frac{1}{s}$$

$$T_{yd} \cdot \frac{d_c}{s}$$

e) $T_{yr} = \dfrac{G}{1+G} = \dfrac{2s+1}{s^2+s}$

$$\hat{Y} = T_{yr} \cdot \frac{1}{s} = \frac{2s+1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$s^2: A + C = 0 \Rightarrow C = -1$

$s^1: A + B = 2 \rightarrow A = 1$

$s^0: B = 1$

$$y(t) = \sigma(t) + t\,\sigma(t) - e^{-t}\,\sigma(t)$$

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3) a) $\dot{x}_1 = x_1(x_2-2)$

$\dot{x}_2 = \cos(x_1) + (x_2-2) + u$

a) I F1: $x_1 = 0 \Rightarrow 0 = 1 + (x_{2R}-2) \Rightarrow x_{2R} = 1 \quad x_R = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

F2: $x_{2R} = 2 \Rightarrow 0 = \cos(x_{1R}) \Rightarrow x_{1R} = \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$

$x_R = \begin{pmatrix} \frac{\pi}{2}+k\pi \\ 2 \end{pmatrix}$

II $z = x - x_R = \begin{pmatrix} x_1 \\ x_2 - 1 \end{pmatrix} \Rightarrow x_2 = z_2 + 1$

$\dot{x}_1 = \dot{z}_1 = z_1(z_2 - 1)$

$\dot{x}_2 = \dot{z}_2 = \cos(z_1) + (z_2 - 1) + u_R$

b) $\dot{x} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$

$\mathbb{H} = \underbrace{O}_{\text{Observability}}\ \underbrace{R}_{\text{Reachability}}$

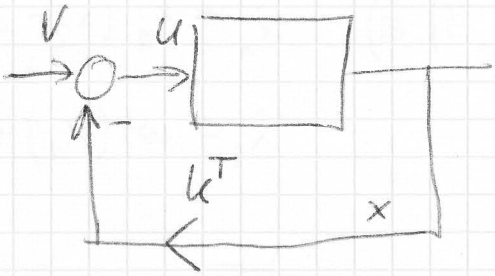
$\mathbb{H}_{11} = \begin{bmatrix} c^T b & c^T A b \\ c^T A b & c^T A^2 b \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 8 \end{bmatrix}$ $\Rightarrow \det(\mathbb{H}) \neq 0$

voller Rang

$\Rightarrow$ vollst. beobachtbar
erreichbar

$A \cdot b = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad A^2 b = \begin{bmatrix} 8 \\ * \end{bmatrix}$

II) $\lambda_{12} = \{-1, -2\}$



$u = -k^T x + v$ $\qquad \tilde{A} = A - bk^T$

$$\tilde{A} = \begin{bmatrix} 1 & 2 \\ 2-k_1 & 3-k_2 \end{bmatrix}$$

$$|sE - \tilde{A}| = (s+1)(s+2) = (s-1)(s\cancel{4}-3+k_2) - (-2\cdot(-2+k_1))$$

$$s^2+3s+2 = s^2 + (-4+k_2)s + (1-k_2+2k_1)$$

$\Rightarrow k_2 = 7$

$k_1 = 5$

$k^T = \begin{bmatrix} 5 & 7 \end{bmatrix}$

3)c) 1) $\quad \Delta \arg(1+L) = \pi(\text{grad}(n_L) - N_-(n_L) + N_+(n_L))$

L_1: $\quad n_{1,2} = \pm\frac{1}{2} \qquad 3 \qquad -2 \quad +1$

$n_3 = -2$

$= 2\pi$ = SOLL

2π = IST

$\rightarrow$ BIBO-stabil

L_2: $(N_0 = 1 \quad s_1 = 0$

$s_{23} = 3/-2$

SOLL $= (3+1-1)\pi = 3\pi$

IST $= -\pi$ $\rightarrow$ nicht BIBO-stabil

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4) a) $x_{k+1} = \begin{bmatrix} 2 & 0 \\ 5 & 0{,}5 \end{bmatrix} x_k + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_k \quad c^T = [1, 0]$

$\hat{x}_{k+1} = \Phi x_k + \Gamma u_k$ ~~trivialen~~ trivialer Beobachter

$\hat{x}_{k+1} = \Phi x_k + \Gamma u_k - \hat{k}(\overset{c^T\hat{x}_k}{\hat{y}_k} - y_k)$ Luenberger Beobachter

$= (\Phi - \hat{k} c^T) x_k + \Gamma u_k + \hat{k}\, y_k$

$e_{k+1} = \hat{x}_{k+1} - x_{k+1} = (\Phi - \hat{k} c^T)\hat{x}_k + \cancel{\Gamma u_k} + \hat{k} c^T x_k$

$- \Phi x_k - \cancel{\Gamma u_k}$

$= (\Phi - \hat{k} c^T)\underbrace{(\hat{x}_k - x_k)}_{e_k}$ von u_k unabh.

steht auch im Skriptum

II $\quad \Theta = \begin{pmatrix} c^T \\ c^T \Phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$ nicht vollst. beobachtbar

III gilt im allg. nicht, aber hier schon
weil nicht beobachtbarer Eigenwert stabil ist

x_1 instabil aber beobachtbar, x_2 stabil, nicht
beobachtbar $\Rightarrow$ Fehlerdynamik stabilisierbar

$\tilde{\Phi} = \Phi - \hat{k} c^T = \begin{pmatrix} 2-\hat{k}_1 & 0 \\ 5-\hat{k}_2 & 0{,}5 \end{pmatrix}$ $\quad |zE - \tilde{\Phi}| = (z - 2 + \hat{k}_1)(z - 0{,}5)$

↳ so dass innerhalb des Einheitskreises

$= (z - 0{,}5)^2$

$\Rightarrow k_1 = 1{,}5;\ k_2 = 0$ bzw. beliebig

4) b) $x_{k+1} = \Phi x_k$ $\Phi = \begin{pmatrix} -7 & -4 \\ 8 & 5 \end{pmatrix}$ $\underline{x}_k = \underbrace{\varrho(k)}_{\uparrow\ \mathbb{R}} \underline{x}_0$

man bleibt auf derselben Richtung $\overset{!}{\nabla}$

$\Rightarrow Av = \lambda v$

$\Rightarrow (A - \lambda E) v = 0$

$|sE - \Phi| = (s+7)(s-5) + 32 = 0$

$s_{12} = -3 / 1$

$\begin{pmatrix} 8 & 4 \\ -8 & -4 \end{pmatrix} v_1 = 0 \rightarrow v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} = x_0$

$x_k = 1^k \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ Definition von Grenzstabil

klingt weder auf noch ab

Variation

$\dot{x} = A x$ $x_0 =$

$x(t) = \Phi x_0$

x_0 ist Kombination der Eigenvektoren

erspart Arbeit