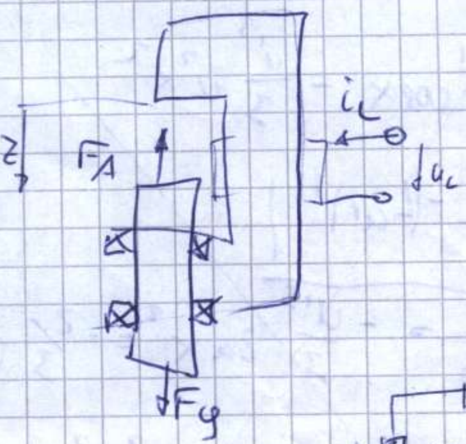
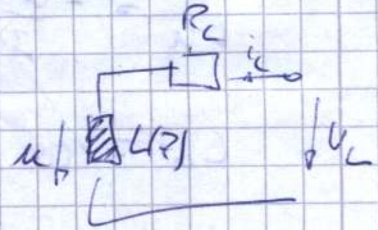
10.10.08

1) Magnetlager





$$F_g = m_B \cdot g$$

$$F_A = \frac{1}{2}\mu_0 A \frac{N i_L^2}{z}$$

$$L(z) = N^2 \frac{\mu_0 A}{z}$$

$$\Psi = L \cdot I$$

$$\frac{d\Psi}{dt} = U = \frac{d\Psi}{dt} = L \cdot \dot{i}_L + i_L N^2 \mu_0 A \frac{(-1)}{z^2}\dot{z} = u$$

$$u_L = u + R_L i_L = i_L R_L + N^2 \frac{\mu_0 A}{z}\left(\dot{i}_L - i_L \frac{\dot{z}}{z}\right)$$

$$X = \begin{bmatrix} i_L \\ z \\ v \end{bmatrix}$$

$\cancel{F_G + F_A}$ $\quad \sum F = \frac{d}{dt}(m \cdot \dot{z})$

$$F_G - F_A = m \cdot \dot{v}$$

$$\dot{v} = -\frac{\mu_0}{2m} A N \frac{i_L^2}{z} + g$$

$$\dot{z} = v$$

$$\dot{i}_L = \frac{u_L z}{N^2 \mu_0 A} + i_L \left[\frac{R_L z}{N^2 \mu_0 A} + \frac{v}{z}\right]$$

$$Y = z$$

b) $N = \mu_0 = A = R_L = g = m = 1$

$z_0 = 1$

$0 = -\frac{1}{2} i_L^2 \overset{+}{\cancel{-}} 1$

$i_L^2 = +2 \rightarrow i_{L,R} = \pm\sqrt{2}$

$v_R = 0$

$0 = u_{L,R} - i_{L,R}[1]$

$u_{L,R} = \pm\sqrt{2}$

$\dot{v} = -\frac{1}{2} \frac{i_L^2}{z^2} \overset{+}{\cancel{-}} 1$

$\dot{z} = v$

$\dot{i}_L = u_L z - i_L \left[z \overset{+}{\cancel{-}} \frac{v}{z} \right]$

c) $c^T = [0 \quad 1 \quad 0]$, $d = 0$

$$b = \frac{df(x_R, u_R)}{du} = \begin{bmatrix} \frac{z_R}{N^2 \mu_0 A} \\ 0 \\ 0 \end{bmatrix}$$

$$A = \frac{df(x_R, u_R)}{dx_1} = \begin{bmatrix} -\frac{R_L z_R}{N^2 \mu_0 A} & \frac{u_{LR}}{N^2 \mu_0 A} \overset{+}{\cancel{-}} \frac{i_{LR} R_L}{N^2 \mu_0 A} & \frac{i_{L,R}}{z_R} \\ 0 & 0 & 1 \\ -\frac{\cancel{2} i_{L,R}}{z_R} \frac{A N \mu_0}{2m} & +\frac{\mu_0}{2m} A N \left(\frac{i_{L,R}}{z_R}\right)^2 & 0 \end{bmatrix}$$

Ruhelage aus b

$c^T = [0 \quad 1 \quad 0]$, $d = 0$

$b^T = [1 \quad 0 \quad 0]$

$$A = \begin{bmatrix} -1 & \overset{0}{\cancel{\pm\sqrt{2} \mp \sqrt{2}}} & \pm\sqrt{2} \\ 0 & 0 & 1 \\ \mp\sqrt{2} & 2 & 0 \end{bmatrix}$$

2a) $x_{k+1} = \phi x_k + \Gamma u_k \quad \hat{x}(0) = \hat{x}_0$

$y_k = c^T x_k$

~~Luenberger~~ triviale $\overbrace{}^{\text{Luenberger}}$

$\hat{x}_{k+1} = \phi \hat{x}_k + \Gamma u_k + \hat{k}(\hat{y}_k - y_k)$

$\hat{y}_k = c^T \hat{x}_k$

---

können beliebig platziert werden, wenn System vollst. beobachtbar

b) Beobachtbar?

$O(c^T, \phi) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix}$ $\quad \det O = 8 + \cancel{12} - \cancel{12} + 4 = 4 \neq 0$

$\to$ rang $= n = 3$

	1 0 2 / 0 2 1 / 1 0 0	1 0 2 / 0 2 1 / 1 0 0
1 1 0	1 2 3	4 4 4

$\det O =$

Dead-Beat Beobachter: $P_{g,soll} = z^3$

$\hat{v}_1 = O^{-1} \cdot e_n$

$\hat{v}_1 = O^{-1} \cdot e_n = \begin{bmatrix} x & x & 3 \\ x & x & -3 \\ x & x & 1 \end{bmatrix} \frac{\cdot e_n}{\det O} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix} \frac{1}{\det O} = \frac{1}{4} \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$

$\hat{k} = -\phi^3 \hat{v}_1 = -\frac{1}{4} \begin{bmatrix} 21 \\ -9 \\ 11 \end{bmatrix}$

	1 0 2 / 0 2 1 / 1 0 0	1 0 2 / 0 2 1 / 1 0 0	3 / -3 / 1
1 0 2 / 0 2 1 / 1 0 0	3 0 2 / 1 4 2 / 1 0 2	5 0 6 / 3 8 6 / 3 0 2	21 / -9 / 11

$\phi_e = [\phi + \hat{k} c^T] = \begin{bmatrix} 1 - \frac{21}{4} & -\frac{21}{4} & 2 \\ \frac{9}{4} & 2 + \frac{9}{4} & 1 \\ 1 - \frac{11}{4} & -\frac{11}{4} & 0 \end{bmatrix}$

$e_{k+1} = \phi_e e_k$

	1	1	0
21	21	21	0
-9	-9	-9	0
11	11	11	0

3) a) $\to 2$

$\xi = \frac{1}{\sqrt{1}}$

b) $\to$ ~~4~~

d) $\to 3$

c) $\to 1$

3b) I-Anteil $G_I(s) = \frac{10}{s}$

PT$_2$: $G_2(s) = \frac{1}{1 + 2\xi\frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2}$ mit $\omega_c = 10\,\frac{1}{s}$

PD-Glied $G_D(s) = \left(1 + \frac{s}{\omega_D}\right)$ mit $\omega_D = 100\,\frac{1}{s}$

$$|G_2(s)|_{\omega_c = 10} \overset{!}{=} 12\,dB = 10^{\frac{12}{20}} = \frac{1}{\sqrt{\left(1 - \frac{1}{100}\right)^2 + (2\xi)^2}} = \frac{1}{2\xi}$$

$$\xi = \frac{1}{2} 10^{-\frac{12}{20}}$$

4 c) Dualitätsprinzip

$$\underbrace{\begin{aligned} x_{p,k+1} &= \Phi x_{p,k} + \Gamma u_k \\ y_{p,k} &= c^T x_{p,k} + d u_k \end{aligned}}_{\text{Primales System}} \qquad \underbrace{\begin{aligned} x_{d,k+1} &= \Phi^T x_{d,k} + c u_k \\ y_{d,k} &= \Gamma^T x_{d,k} + d u_k \end{aligned}}_{\text{duales System}}$$

besitzen die selbe Übertragungsfunktion

da $G(z) = c^T (zE - A)^{-1} \Gamma + d = G(z)^T$

$= \Gamma^T (zE - A^T)^{-1} c + d$

$(A \cdot B \cdot C)^T = C^T (A \cdot B)^T$

$= C^T B^T A^T$

$(A^{-1})^T = (A^T)^{-1}$

d)

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi & 0 \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} c^T & 0 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix}$$

A_{22} ... beliebig

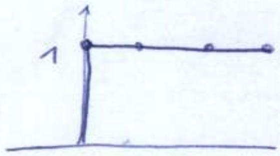
$\det \begin{vmatrix} \Phi & 0 \\ 0 & A_{22} \end{vmatrix} = 0 \quad \det(A_{22}) = 0$

$\det \begin{bmatrix} c_1^T & 0 \\ c_1^T \Phi & 0 \\ c_1^T \Phi^2 & 0 \\ c_1^T \Phi^3 & 0 \end{bmatrix}$

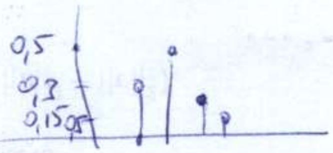
$\begin{matrix} \Phi & 0 \\ 0 & A_{22} \end{matrix} \Bigg| \begin{matrix} 0 & A_{21} \\ 0 & 0 \\ 0 & A_{21} \end{matrix}$

$[c_1^T \cdot 0] \begin{pmatrix} \Phi & 0 \\ 0 & A_{21} \end{pmatrix}$

4a) $T_a = 2s$



$\sum h(t)\,\sigma(t-$



$g(k) = \sum_{n=0}^{\infty} h(k-n)\,\sigma(n)$

$\int_{-\infty}^{\infty} h(\tau-\tau')\,\sigma(\tau')\,d\tau'$

b) $G(z) = \dfrac{(z+1)\,V}{(z-0{,}2-0{,}2j)(z-0{,}2+0{,}2j)} = \dfrac{z+1}{(z-0{,}2)^2+0{,}2^2} = \dfrac{(z+1)V}{z^2-0{,}4z+0{,}08}$

$\lim\limits_{z\to 1} zG(z) = \dfrac{V(z+1)}{(z-0{,}2-0{,}2j)(z-0{,}2+0{,}2j)} = \dfrac{2V}{0{,}8^2+0{,}04} = \dfrac{2V}{0{,}68} \overset{!}{=} 1$

$V = \dfrac{0{,}68}{2} = 0{,}34$

c) $e^{j\frac{\pi}{4}T_a}$, $j+1/2$ $\quad |{-1}+2j|\cdot 2+1) = \sqrt{17}$

$|G(z)|_{z=j+1/2} = \dfrac{0{,}34\cdot\sqrt{5}}{0{,}92^2+0{,}4^2} \quad | 1+2j-1-0{,}4-0{,}4j+0{,}081$

$= \dfrac{0{,}34\sqrt{5}}{(1{,}6j-0{,}32)} \quad \dfrac{0{,}34\sqrt{5}}{}$

$u(t) = 5\sin(\frac{\pi}{4}t) + \frac{1}{2}\sigma(t) \qquad t = k\cdot T_a$

$u_k = 5\sin(\frac{\pi}{2}k) + \frac{1}{2}(1^k)$

$e^{j\frac{\pi}{2}} = j \qquad y_{k_2} = \frac{1}{2}$

$y_k = \frac{1}{2} + \dfrac{5\cdot 0{,}34\sqrt{2}}{\sqrt{\dots}}\cdot$

$|G(z)|_{z=j} = \dfrac{0{,}34\sqrt{2}}{0{,}92^2+0{,}4^2} = \dfrac{0{,}34\sqrt{2}}{\sqrt{0{,}23^2+0{,}1^2}}$ $\quad \sin(\frac{\pi}{2}k + \frac{\pi}{4} + \arctan\frac{10}{23})$

$\arg(G(z)_{z=j}) = \dfrac{\pi/4}{\arctan\frac{-0{,}4}{0{,}92}} - \arctan\dfrac{+0{,}4}{-0{,}92} = \frac{\pi}{4} \mp \arctan(+\frac{10}{23}) / -\pi$

c.)

$$G(z) = \frac{z-1}{z} \mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

$$\frac{4}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \underbrace{\frac{4}{s} - \frac{4}{s+1}}_{\circ\!\!-\!\!\bullet\ \mathcal{Z}}$$

$$A = \frac{4}{s+1} = 4$$

$$B = -4 \qquad \frac{4z}{z-1} - \frac{4z}{z-\frac{1}{2}}$$

$$G(z) = \frac{z-1}{z} \cdot \;=\; 4 - \frac{4(z-1)}{z-\frac{1}{2}} = \frac{2}{z-\frac{1}{2}}$$

$$T_{u,y} = \frac{R\cdot G}{1+RG} = \frac{\frac{z}{z-1+k}\cdot\frac{2}{z-\frac{1}{2}}}{1+\frac{z}{z-1+k}\cdot\frac{2}{z-\frac{1}{2}}} = \frac{2z}{2z+(z-\frac{1}{2})(z-1+k)}$$

$$= \frac{2z}{z^2+z^2+z(-\frac{1}{2}-1+k)+\frac{1-k}{2}} = \frac{2z}{z^2+z(\frac{1}{2}+k)+\frac{1-k}{2}}$$

Jury

z^2 | 1 | $\frac{1}{2}+k$ | $\frac{1}{2}-\frac{k}{2}$

$\frac{1}{2}-\frac{k}{2}$ | $\frac{1}{2}+k$ | 1

z | $-\frac{1}{4}+\frac{k}{2}-\frac{k^2}{4}$ | $\frac{1}{4}+\frac{3k}{4}+\frac{k^2}{2}$

1 | 0

$$d_2 = \frac{a_{2,0}}{a_{2,2}} = \frac{1}{2}-\frac{k}{2}$$

$$d_1 = \frac{\frac{1}{4}+\frac{3k}{4}+\frac{k^2}{2}}{-\frac{1}{4}+\frac{k}{2}-\frac{k^2}{4}}$$

$$\frac{1}{2}+k-\left(\frac{1}{2}-\frac{k}{2}\right)\left(\frac{1}{2}+k\right)$$

$$= \frac{1}{2}+k-\frac{1}{4}-\frac{k}{2}+\frac{k}{4}+\frac{k^2}{2}$$

$$-\frac{1}{4}+\frac{k}{4}\frac{k}{2}-\left(\frac{1}{4}+\frac{3k}{4}+\frac{k^2}{2}\right)^2 \Big/ \left(-\frac{1}{4}+\frac{k}{2}-\frac{k^2}{4}\right)$$

$$= \frac{1}{4}+\frac{3k}{4}+\frac{k^2}{2}$$

$$k > 1$$

$$-\frac{1}{4}+\frac{k}{2}-\frac{k^2}{4} > 0$$

$$k^2-2k+1 < 0$$

$$(k+1)(k-1) < 0$$