27.1.12

1a) $x = \begin{bmatrix} i_m \\ \omega \\ p \end{bmatrix}$ $u = \begin{bmatrix} u_m \\ u_p \end{bmatrix}$ $y = p$

d ... Störung

$$q_p = k_p \omega u_p^2$$

$$q_l = k_l \sqrt{p}\, d$$

$$\frac{d}{dt} p = \frac{\beta}{V}(q_p - q_l) = \frac{\beta}{V}(k_p \omega u_p^2 - k_l \sqrt{p}\, d)$$

$$q_p \cdot p = M_p \omega$$

$$I_m \dot{\omega} = -M_p + M_m = -k_p u_p^2 \cdot p + k_m i_m$$

$$M_m = k_m i_m$$

$$u_{ind} = k_m \omega$$

$$u_m = i_m R_m + L_m \frac{d i_m}{dt} + u_{ind}$$

$$\frac{d}{dt}\begin{bmatrix} i_m \\ \omega \\ p \end{bmatrix} = \begin{bmatrix} \frac{1}{L_m}(u_m - i_m R_m - k_m \omega) \\ \frac{1}{I_m}(-k_p u_p^2 p + k_m i_m) \\ \frac{\beta}{V}(k_p \omega u_p^2 - k_l \sqrt{p}\, d) \end{bmatrix}$$

$$y = p$$

1b)

$$\dot{x} = 0$$

$$u_{m,r} = i_{m,r} R_m + k_m \omega_r$$

$$i_{m,r} = \frac{k_p}{k_m} u_{p,r}^2 p_r$$

$$\omega_r = \left(u_{m,r} - \frac{k_p}{k_m} u_{p,r}^2 p_r \cdot R_m\right) \frac{1}{k_m}$$

$$d_r = \frac{1}{k_l \sqrt{p_r}} k_p \omega_r u_{p,r}^2$$

c)

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} -\frac{R_m}{L_m} & -\frac{k_m}{L_m} & 0 \\ \frac{k_m}{I_m} & 0 & -\frac{k_p u_{p,r}^2}{I_m} \\ 0 & \frac{\beta}{V} k_p u_{p,r}^2 & -\frac{\beta k_l d_r}{V 2 \sqrt{p_r}} \end{bmatrix}$$

$$B_u = \frac{\partial f}{\partial u} = \begin{bmatrix} \frac{1}{L_m} & 0 \\ 0 & -\frac{2 k_p u_{p,r} p_r}{I_m} \\ 0 & \frac{2 \beta k_p \omega_r u_{p,r}}{V} \end{bmatrix}$$

$$b_d = \frac{\partial f}{\partial d} = \begin{bmatrix} 0 \\ 0 \\ -\frac{\beta k_l \sqrt{p_r}}{V} \end{bmatrix}$$

$$c^T = \frac{\partial h}{\partial x} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

2a) $\exists \xi \in (a,b): f'(\xi) = \frac{f(b)-f(a)}{b-a}$

$\dot{x} = f(x) \qquad f(x) = \sin(x)$

Lipschitz-Bedingung: $\| f(x,t) - f(y,t) \| \leq L \| x - y \| \qquad 0 < L < \infty$

$$f(b) - f(a) = \frac{df}{dx}\Big|_{x\in(a,b)} (b-a) \qquad \Big| \, |\cdot|$$

$$|f(b) - f(a)| \leq \left| \frac{df}{dx}\Big|_{x\in(a,b)} (b-a) \right|$$

$$|f(b) - f(a)| \leq \left| \frac{df}{dx}\Big|_{x\in(a,b)} \right| \cdot |b-a| \qquad \frac{df}{dx} = \cos(x)$$

$$|\cos(x)| \leq 1 \quad \forall x \in \mathbb{R}$$

$$|\sin(b) - \sin(a)| \leq 1 \cdot |b-a|$$

$$L = 1$$

Lipschitz-Bedingung erfüllt $\Rightarrow$ Existenz und Eindeutigkeit

b) allg. Lsg. für autonomes System: $x(t) = \sum_{k=0}^{\infty} A^k \frac{t^k}{k!} x_0$

$\Rightarrow$ A ist nilpotent mit Ordnung n

$\Rightarrow$ alle Eigenwerte von A sind null (algebraische Vielfachheit $= n$)

charakteristisches Polynom: $p = \lambda^n$

2c) $\hat{\Phi}(s) = \begin{bmatrix} \frac{1}{s-1} & \frac{1}{s^2+3s-4} \\ 0 & \frac{1}{s+4} \end{bmatrix}$

$$s_{1,2} = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm 5}{2}$$

$$s_1 = -4 \quad s_2 = 1$$

$$\frac{1}{(s-1)(s+4)} = \frac{\frac{1}{5}}{s-1} + \frac{-\frac{1}{5}}{s+4}$$

$$\Phi(t) = \begin{bmatrix} e^t & \frac{1}{5}e^t - \frac{1}{5}e^{-4t} \\ 0 & e^{-4t} \end{bmatrix}$$

$$\frac{d}{dt}\Phi(t)\Big|_{t=0} = A = \begin{bmatrix} 1 & 1 \\ 0 & -4 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 1 & 1 \\ 0 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ \beta \end{bmatrix} u$$

A hat obere Dreiecksstruktur $\Rightarrow$ EW: $\lambda_1 = 1 \quad \lambda_2 = -4$

$Re(\lambda_1) > 0 \Rightarrow$ nicht asymptotisch stabil

$$R = \begin{bmatrix} 1 & 1+\beta \\ \beta & -4\beta \end{bmatrix}$$

$$\begin{array}{cc|c} & & 1 \\ & & \beta \\ \hline 1 & 1 & 1+\beta \\ 0 & -4 & -4\beta \end{array}$$

$$\det(R) = -4\beta - \beta - \beta^2 = -\beta(5+\beta) = 0$$

$\Rightarrow \beta = 0 \vee \beta = -5$ dann hat R keinen vollen Rang $\Rightarrow$ nicht vollständig erreichbar

3a) $p = \left(\lambda - \frac{3}{4}\right)\cdot \lambda + \frac{1}{8} = \lambda^2 - \frac{3}{4}\lambda + \frac{1}{8}$

$$\lambda_{1,2} = \frac{+\frac{3}{4} \pm \sqrt{\frac{9}{16} - \frac{1}{2}}}{2} = \frac{\frac{3}{4} \pm \frac{1}{4}}{2} \Rightarrow \lambda_1 = \frac{1}{2} \quad \lambda_2 = \frac{1}{4}$$

$$\lambda_1 : \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{2} \end{bmatrix} = 0 \qquad w_1^T = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\lambda_2 : \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{4} \end{bmatrix} = 0 \qquad w_2^T = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$w_1^T \Gamma = 1 \neq 0 \qquad w_2^T \Gamma = 2 \neq 0$$

$\Rightarrow$ vollst erreichbar

b)

$$\begin{bmatrix} x_{k+1} \\ x_{I,k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi & 0 \\ -c^T & 1 \end{bmatrix}}_{\Phi_i} \begin{bmatrix} x_k \\ x_{I,k} \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma_i} u_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_k$$

$$u_k = \begin{bmatrix} k_1^T & k_2 \end{bmatrix} \begin{bmatrix} x_k \\ x_{I,k} \end{bmatrix} \qquad y_k = \underbrace{\begin{bmatrix} c^T & 0 \end{bmatrix}}_{c^T_i} x_k$$

$$\Phi_i = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} & 0 \\ -\frac{1}{4} & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} \qquad \Gamma_i = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\Phi_i^2 \Gamma$			
			$\frac{1}{2}$
			0
			-1
$\frac{3}{4}$	$\frac{1}{2}$	0	$\frac{3}{8}$
$-\frac{1}{4}$	0	0	$-\frac{1}{8}$
0	-1	1	-1

$$R = \begin{bmatrix} 0 & \frac{1}{2} & \frac{3}{8} \\ 1 & 0 & -\frac{1}{8} \\ 0 & -1 & -1 \end{bmatrix} \qquad \det(R) \neq 0$$

$\Rightarrow$ vollst erreichbar

3 b) ff

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = v_1^T \begin{bmatrix} 0 & \frac{1}{2} & \frac{3}{8} \\ 1 & 0 & -\frac{1}{8} \\ 0 & -1 & -1 \end{bmatrix}$$

$$v_{11} \cdot \frac{1}{2} = v_{13}$$

$$\Rightarrow v_1^T = \begin{bmatrix} -8 & 0 & -4 \end{bmatrix}$$

$$\frac{3}{8} \cdot 2 v_{13} - v_{13} = 1$$

$$v_{13} = -4$$

$$v_{11} = -8$$

$$p_{\varphi soll} = \left(\lambda - \frac{1}{2}\right)^3$$

$$\Phi_i - \frac{1}{2} E = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & 0 \\ -\frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & -1 & \frac{1}{2} \end{bmatrix} = \Phi'$$

$$p_{\varphi soll}(\Phi) = \Phi'^3$$

			$\frac{1}{4}$	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$	0
	Φ'^3		$-\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$-\frac{1}{2}$	0
			0	-1	$\frac{1}{2}$	0	-1	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{2}$	0	$-\frac{1}{16}$	$-\frac{1}{8}$	0	$\frac{1}{64}$	$\frac{1}{32}$	0
$-\frac{1}{4}$	$-\frac{1}{2}$	0	$\frac{1}{16}$	$\frac{1}{8}$	0	X	X	X
0	-1	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{16}$	$-\frac{1}{8}$	$\frac{1}{8}$

$$k^T = -v_1^T \Phi'^3 = \begin{bmatrix} \frac{\cancel{8} \cdot 1}{\cancel{64}_{8}} + \frac{\cancel{4}}{\cancel{16}_{4}} & \frac{1}{4} - \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

3c)

$$\text{fixpunkt: } x_k = \Phi x_k + \Gamma k_p r$$

$$x_k = (E-\Phi)^{-1} \Gamma k_p r$$

$$y_s = c^T x_k = r$$

$$\Rightarrow k_p = \frac{1}{c^T (E-\Phi)^{-1} \Gamma}$$

$$(E-\Phi)^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{4} & 1 \end{bmatrix}^{-1} = \frac{1}{\frac{1}{4}+\frac{1}{8}} \begin{bmatrix} \times & \times \\ \times & \frac{1}{4} \end{bmatrix} = \frac{8}{3} \begin{bmatrix} \times & \times \\ \times & \frac{1}{4} \end{bmatrix}$$

$$k_p = \frac{1}{\frac{\cancel{8}\,2}{3} \cdot \frac{1}{\cancel{4}}} = \frac{3}{2}$$

$$k_x = k_1^T + c^T k_p = \begin{bmatrix} \frac{3}{8} & -\frac{1}{4} \end{bmatrix} + \begin{bmatrix} 0 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & \frac{5}{4} \end{bmatrix}$$

$$4a) \text{-}) e_\infty = \lim_{s \to 0} s\,\hat{e}(s) = \lim_{s \to 0} s(\hat{r} - \hat{y}) = \lim_{s \to 0} s\left(\frac{1}{s^2} - \frac{RG}{1+RG}\frac{1}{s^2}\right) =$$

$$= \lim_{s \to 0} \frac{1}{1+R\cdot G}\cdot\frac{1}{s} = \lim_{s \to 0} \frac{1}{1+\frac{V_R V}{s(1+sT)}}\cdot\frac{1}{s} = \lim_{s \to 0} \frac{\cancel{s}(1+sT)}{(s(1+sT)+V_R V)\cancel{s}}$$

$$= \frac{1}{V_R V}$$

$$-) e_\infty = \lim_{s \to 0} s\,\hat{e}(s) = \lim_{s \to 0} s(-\hat{y}) = \lim_{s \to 0} s\left(-\frac{G}{1+RG}\frac{1}{s}\right) =$$

$$= \lim_{s \to 0} -\frac{G}{1+RG} = \lim_{s \to 0} \frac{\frac{V}{s(1+sT)}}{1+\frac{V_R V}{s(1+sT)}} = \lim_{s \to 0} \frac{V}{s(1+sT)+V_R V} = \frac{1}{V_R}$$

b) $$G(s) = \frac{(s+4)}{(s+7)(s+3)}$$

$$G_z = \frac{z-1}{z} Z\left(\frac{G(s)}{s}\right) \qquad \frac{s+4}{s(s+7)(s+3)} = \frac{\frac{4}{21}}{s} + \frac{\frac{-3}{28}}{s+7} + \frac{\frac{1}{-12}}{s+3}$$

$$G_z = \frac{z-1}{z}\left(\frac{4}{21}\frac{z}{z-1} - \frac{3}{28}\frac{z}{z-e^{-7T_a}} - \frac{1}{12}\frac{z}{z-e^{-3T_a}}\right)$$

$$= \frac{4}{21} - \frac{3}{28}\frac{z-1}{z-e^{-7T_a}} - \frac{1}{12}\frac{z-1}{z-e^{-3T_a}}$$

G_z ist BIBO stabil, da durch Abtasten ein stabiles System nicht instabil werden kann und $G(s)$ stabil ist (alle Pole in der linken offenen s-Halbebene)

c) $\omega T_a = \frac{\pi}{3}$

$$G(e^{I\frac{\pi}{3}}) = \frac{e^{I\pi/3}+1}{e^{I\pi/3}-\frac{1}{2}} = \frac{\frac{1}{2}+I\frac{\sqrt{3}}{2}+1}{\frac{1}{2}+I\frac{\sqrt{3}}{2}-\frac{1}{2}}$$

$$|G(e^{I\pi/3})| = \frac{\sqrt{(\frac{3}{2})^2+\frac{3}{4}}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{\cancel{\frac{12}{4}}\,4}}{\cancel{\frac{\sqrt{3}}{2}}} = 2$$

$$\arg(G(e^{I\pi/3})) = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right)-\frac{\pi}{2} = \arctan\left(\frac{\sqrt{3}}{3}\right)-\frac{\pi}{2} = -\frac{\pi}{3}$$

$$y_k = 3\cdot 2\sin\left(\frac{\pi}{3}k+\frac{\pi}{12}-\frac{\pi}{3}\right) = 6\sin\left(\frac{\pi}{3}k-\frac{\pi}{4}\right)$$

d) -) $G_{23} = \frac{G_2}{1-G_2G_3}$

$$T_{r,y} = \frac{G_1\cdot G_{23}}{1+G_1\cdot G_{23}\cdot G_4} = \frac{G_1\cdot\frac{G_2}{1-G_2G_3}}{1+G_1\frac{G_2}{1-G_2G_3}G_4} = \frac{G_1G_2}{1-G_2G_3+G_1G_2G_4}$$

-) $$T_{r,y} = \frac{\frac{1}{s}\cdot\frac{1}{1+sT}}{1-\frac{V}{1+sT}+\frac{1}{s}\cdot\frac{1}{1+sT}} = \frac{1}{(1+sT)\cdot s-Vs+1} = \frac{1}{s^2T+s(1-V)+1}$$

Polynom 2. Grades: alle Koeffizienten müssen >0 sein damit notwendiges und hinreichendes Kriterium für ein Hurwitzpolynom erfüllt sind

$\Rightarrow T>0,\ V<1$ damit BIBO-stabil