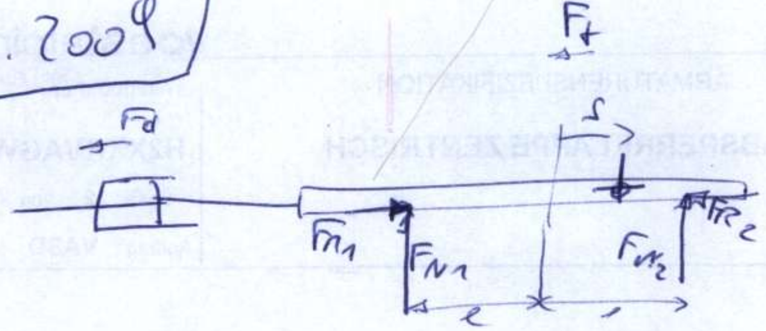
7.8.2009

1)




$X = \begin{bmatrix} s \\ \omega \end{bmatrix}$

$\dot{s} = \omega$

a)

$F_{R1} = \mu F_{N1} \qquad F_{R2} = \mu F_{N2} \qquad F_d = \omega \cdot d$

$$\frac{1}{m}\sum F = \dot{\omega} = \frac{1}{m}\left[F_{R1} - F_{R2} - F_d\right]$$

$$F_{N2} \cdot 2e = m \cdot g \cdot (e+s)$$

$$F_{N1} \cdot 2e = m \cdot g \cdot (e-s)$$

$$\dot{\omega} = \frac{\mu g}{2}(1 - s/e) - \frac{\mu g}{2}(1 + s/e) - \frac{\omega \cdot d}{m}$$

$$\boxed{\dot{\omega} = -\mu g \, s/e - \frac{\omega d}{m}}$$

b)

$\dot{\omega} = 0$

$\omega_R = 0$

$\boxed{s_R = 0}$ → 1 Ruhelage, da $\det A \neq 0$

$$A = \cancel{\begin{bmatrix} -\frac{\mu g}{e} & 0 \\ 0 & -\frac{d}{m} \end{bmatrix}} \begin{bmatrix} 0 & 1 \\ -\frac{\mu g}{e} & -\frac{d}{m} \end{bmatrix}$$

c)

$\cancel{\lambda_1 = -\frac{\mu g}{e} \quad \lambda_2 = -d/m}$ → ~~beide~~ beide Realteil $< 0$

→ asympt. stabil

$$\begin{vmatrix} \lambda & -1 \\ \mu g/e & \lambda + d/m \end{vmatrix} = \lambda(\lambda + d/m) + \mu g/e = 0$$

$$\lambda^2 + \lambda \, d/m + \mu g/e$$

$$\lambda_{1,2} = -\frac{d}{2m} \pm \sqrt{\frac{d^2}{4m^2} - \frac{\mu g}{e}}$$

$$= -\frac{d}{2m} \pm \frac{\sqrt{d^2 - 4m^2 \mu g}}{2m}$$

↳ Realteil kann nicht $> 0$ sein

wenn $d > 2m\sqrt{\mu g}$ nicht schwingfähig

2) $x_{k+1} = \begin{bmatrix} 1 & -3/4 \\ 1/2 & -1/4 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_k$

$y_k = \begin{bmatrix} c_1 & c_2 \end{bmatrix} x_k$

$\forall$ Rechtseigenvektor $v_i \neq 0$ $\quad c^T v_1 \neq 0$

$$|\lambda E - \Phi| = \begin{vmatrix} \lambda - 1 & 3/4 \\ -1/2 & \lambda + 1/4 \end{vmatrix} = (\lambda - 1)(\lambda + 1/4) + \frac{3}{8}$$

$$= \lambda^2 - \frac{3\lambda}{4} - \frac{1}{4} + \frac{3}{8} = \lambda^2 - \frac{3\lambda}{4} + \frac{1}{8}$$

$$\lambda_{1,2} = \frac{3}{8} \pm \sqrt{\left(\frac{3}{8}\right)^2 - \frac{1}{8}} = \frac{3}{8} \pm \frac{1}{8}$$

$$\lambda_1 = \frac{1}{2}, \quad \lambda_2 = \frac{1}{4}$$

$$\underline{A v_i = \lambda_i v_i}$$

1) $\begin{vmatrix} 1 & -3/4 \\ 1/2 & -1/4 \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} v_1 \\ v_2 \end{vmatrix}$

$v_1 - 3/4\, v_2 = 1/2\, v_1$

$\underline{1/2\, v_1 - v_2/4 = v_2/2}$

$v_1 = \frac{3 v_2}{2}$

$\frac{3v_2}{2} - 3/4\, v_2 = 3/4\, v_2$ ✓

$v_1 = \begin{vmatrix} 3/2 \\ 1 \end{vmatrix}$

2) $\begin{vmatrix} 1 & -3/4 \\ 1/2 & -1/4 \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \end{vmatrix} = \frac{1}{4} \begin{vmatrix} v_1 \\ v_2 \end{vmatrix}$

$v_1 - 3/4\, v_2 = 1/4\, v_1$

$\underline{v_1/2 - v_2/4 = 1/4\, v_2}$

$v_1 = v_2$

$v_2 = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$

$c^T v_i \neq 0 \rightarrow \boxed{\begin{matrix} c_1 + c_2 \neq 0 \\ \frac{3c_1}{2} + c_2 \neq 0 \end{matrix}}$

b) $d_{1,2} = 1/3$ $\qquad p = (d - \tfrac13)^2 = d^2 - \frac{2d}{3} + \frac19$

$\hat v_1 = O^{-1} e_n \qquad O = \begin{pmatrix} c^T \\ c^T\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & -3/4 \end{pmatrix}$

$$\begin{array}{cc|cc} & & 1 & -3/4 \\ & & 1/2 & -1/4 \\ 1 & 0 & 1 & -3/4 \end{array}$$

$O^{-1} = -\frac43 \begin{pmatrix} -3/4 & 0 \\ -1 & 1 \end{pmatrix}$

$\hat v_1 = \begin{bmatrix} 0 \\ -4/3 \end{bmatrix} \qquad = \begin{bmatrix} 1 & 0 \\ 4/3 & -4/3 \end{bmatrix} \quad \begin{matrix} 1 \\ 0 \\ -4/3 \end{matrix}$

$\hat k = -\frac19 \hat v_1 + \frac23 \phi \hat v_1 - \phi^2 \hat v_1$

$$\begin{array}{cc|c|c} & & 0 & 1 \\ & & -4/3 & 1/3 \\ \hline 1 & -3/4 & 1 & 3/4 \\ 1/2 & -1/4 & 1/3 & 5/12 \end{array}$$

$\hat k = -\frac19 \begin{pmatrix} 0 \\ -4/3 \end{pmatrix} + \frac23 \begin{pmatrix} 1 \\ 1/3 \end{pmatrix} - \begin{pmatrix} 3/4 \\ 5/12 \end{pmatrix}$

$\hat k = \begin{bmatrix} \frac23 - \frac34 \\ \frac{4}{27} + \frac29 - \frac{5}{12} \end{bmatrix} = \begin{pmatrix} -\frac{1}{12} \\ \frac{48-135}{27\cdot 12} + \frac29 \end{pmatrix} = \begin{pmatrix} -\frac{1}{12} \\ -\frac{\cancel{87}\,29}{4\,\cancel{12}\cdot 27} + \frac29 \end{pmatrix} \quad \begin{matrix} -\frac{1}{12} \\ -267 + \dots \end{matrix}$

c) $x_{k+1} = \phi x_k + \Gamma u_k \qquad e_{k+1} = \phi_e e_k = \phi_e(\hat x_k - x_k)$

$y_k = c^T x_k + \Delta y_k \qquad = (\hat x_{k+1} - x_{k+1})$

$\hat x_{k+1} = \phi \hat x_k + \Gamma u_k + \hat k (y_k - \hat y_k)$

$\hat y_k = c^T \hat x_k$

$\hat x_{k+1} = \phi \hat x_k + \cancel{\phi} x_{k+1} - \phi x_k + k(c^T \hat x_k - c^T x_k \mp \Delta y_k)$

$\hat x_{k+1} - x_{k+1} = \phi \hat x_k - \phi x_k + \hat k c^T \hat x_k - \hat k c^T x_k \mp \hat k \Delta y_k$

$= \underbrace{(\phi + \hat k c^T)}_{\phi_e}(\hat x_k - x_k) \mp \hat k \Delta y_k \qquad \boxed{-(E - \phi_e)^{-1} \hat k \Delta y_k}$

$e_{k+1} = \phi_e e_k \mp \hat k \Delta y_k$

$\lim_{k\to\infty} \cancel{e_{k+1} = \hat k \Delta y_k} \qquad e_k = \phi_e e_k + \hat k \Delta y_k \to e_k =$

3a) $V \overset{!}{=} 25$ BIBO-stabil, Ja

→ sprungfähig

→ nein, nicht phasenmin.

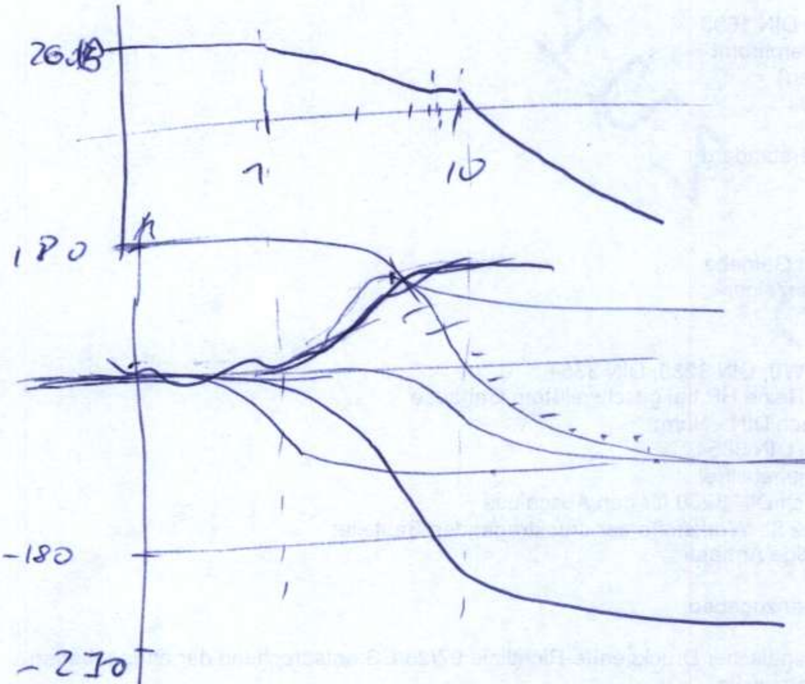
$$G(s) = \frac{\cancel{1648}(s+4)(s-5)}{(s+8)(s+1)} \cdot V$$

$$\lim_{s \to 0} s \cdot G(s) \frac{1}{s} = \lim V \cdot \frac{-20}{8} \overset{!}{=} 25$$

$$\boxed{V = -10}$$

b) $G_1$ ist gleich, $G_3$ gleich, genau

$G_2$ gleich




$\arg(G) = -\pi + \operatorname{atan}\frac{\omega}{-5} - \operatorname{atan}\frac{\omega}{10} - \operatorname{atan}\omega$

$\arg(s-5) = -\pi + \operatorname{atan}\frac{\omega}{5}$

| | |
|---|---|
| 0 | $\cancel{\pi}$ |
| 1 | |
| 5 | $-\frac{\pi}{4} + \pi = \frac{3\pi}{4}$ |
| 10 | |
| $\infty$ | $-\pi - \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} \cancel{= -\frac{5\pi}{2}}$ $+ \frac{\pi}{2}$ |

c) $\dot{x}(t) = \begin{bmatrix} -4 & 2 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t-2)$

$y(t) = [0 \;\; 1]\, x(t)$

linear, zeitinvariant

$X(s) \cdot s - X(0) = A\, X(s) + b\, U(s) \cdot e^{-2s}$

$x(0) = 0!$ $\quad X(s) = (sE - A)^{-1} b\, e^{-2s} U(s)$

$$G(s) = \frac{Y(s)}{U(s)} = c^T X(s) = c^T (sE-A)^{-1} b\, e^{-2s}$$

$$(sE-A)^{-1} = \begin{vmatrix} s+4 & -2 \\ 0 & s+2 \end{vmatrix}^{-1} = \frac{1}{(s+4)(s+2)} \begin{vmatrix} s+2 & 2 \\ 0 & s+4 \end{vmatrix}$$

$$= \begin{pmatrix} \frac{1}{s+4} & \frac{2}{(s+4)(s+2)} \\ 0 & \frac{1}{s+2} \end{pmatrix} \quad \begin{matrix} 2 \\ 1 \end{matrix}$$

$$\begin{matrix} 0 & 1 \end{matrix} \quad \begin{matrix} 0 & \frac{1}{s+2} \end{matrix} \quad \frac{1}{s+2}$$

$$\boxed{G(s) = \frac{e^{-2s}}{s+2}}$$

$z = e^{+sT_a} \rightarrow \cancel{\ln z = -sT_a}$

$\hookrightarrow z^{1/T_a} = e^s$ $\qquad \cancel{s = \frac{-\ln z}{T_a}}$

$\cancel{G(z) = \frac{e^{+\frac{2}{T_a}\ln z}}{2 - \frac{\ln z}{T_a}}}$

Vgl. mit $e^{-2s} \leftrightarrow z^{-2/T_a}$

$$G(z) = \frac{z}{z - e^{-2/10}} \cdot z^{-2/T_a} = \frac{1}{z^{19}(z - e^{-2/10})}$$

$g(t) = e^{-2(t-2)}$

$g_k = e^{-2(kT_a - 2)}$

$= e^{-2(k - \frac{2}{T_a})T_a}$

$$\frac{z}{z - e^{-2T_a}}\, z^{-2/T_a} = \frac{1}{z^{19}(z - e^{-2T_a})}$$

4a) $u_k = (1^k) - (1^{k-1})$ $\cancel{G(z)}$ $(g_k) = \frac{1}{2}\delta_{k-1} + 1(1^{k-1})$

$y_k = \frac{1}{2}\delta_{k-1} + 1\delta_{k-2} + \frac{1}{2}\delta_{k-3}$ $g(z) = \frac{z^{-1}}{2} + \frac{z}{z-1}z^{-2}$

$= \frac{1}{2z} + \frac{1}{z(z-1)} = \frac{z+2}{2z(z-1)}$

$= \frac{1}{2}z^{-1} + 2z^{-2} + \frac{1}{2}z^{-3}$

$= \frac{z^2 + 2z + 1}{2z^{+3}}$

$u(z) = \frac{Y(z)}{G(z)} = \frac{(z^2+2z+1)\,2z(z-1)}{2z(z-1)(z+2)\,2z^{3}}$ $= \frac{z^3+2z^2+z-z^2-2z-1}{}$

$= \frac{z^3+z^2-z-1}{(z+2)z^3}$

$= \frac{z}{z+2} + \frac{z^{-1}}{z+2} - \frac{z^{-2}}{z+2} - \frac{z^{-3}}{(z+2)}$

$\frac{1}{2}^k(1^k) + \frac{1}{2}^{k-1}(1^{k-1}) - \frac{1}{2}^{k-2}1^{k-2} - \frac{1}{2}^{k-3}1^{k-3}$

$\cancel{\frac{1}{2}^k(1^k + 2\,1^{k-1})}$

0: 1

b)

$u_k \; \underbrace{\boxed{H} - \boxed{\frac{1}{s^2}} - \boxed{A}}\; 1 = (y_k)$

$-y_k \rightarrow \underbrace{\boxed{H} - \boxed{-\frac{1}{s}} - \boxed{A}} \rightarrow y_{k_2}$

$-\frac{z}{z-1}$

$Y_1 = \frac{T_a z}{(z-1)^2}u(z)$

$y(z) = y_1 + y_2$

$Y(z) = \frac{T_a z}{(z-1)^2}u(z) + \cancel{\frac{T_a z}{(z-1)^2}}\left(-\frac{z}{z-1}\right)Y(z)$

$G(z) = \frac{Y}{u} = \cancel{\frac{T_a z(z-1) - T_a z^2}{(z-1)^3}}$ $(T_a=1)$ $\frac{\frac{T_a z}{(z-1)^2}}{1 + \frac{z}{z-1}}$ $\overset{T_a=1}{=} \frac{z}{z(z-1)+(z-1)}$

$$\boxed{G(z) = \frac{z}{z^2 - z + z^2 - 2z + 1} = \frac{z}{2z^2 - 3z + 1}}$$

4c) $\Psi(k) = \begin{bmatrix} \left(\frac{1}{2}\right)^{k-\alpha} & \beta^{k-1}-\gamma \\ \varepsilon & \phi \end{bmatrix}$

$\Psi(0) = E$

$\Psi(k+l) = \Psi(k)\Psi(l)$

$\Psi^{-1}(k) = \Psi(-k)$

$\Psi(k+1) = \phi\Psi(k)$

$\phi(0) = E$

$\phi(t+l) = \phi(t)\phi(l)$

$\phi^{-1}(t) = \phi(-t)$

$\frac{d\phi}{dt}(t) = A\phi(t)$

(1)

$\begin{bmatrix} \frac{1}{2}^{-\alpha} & \beta^{-1}-\gamma \\ \varepsilon & \phi \end{bmatrix} \stackrel{!}{=} E$

$\varepsilon = 0, \ \phi = 1$

$\gamma = \frac{1}{\beta}, \quad \alpha = 0$

$\Psi_k = \begin{bmatrix} \left(\frac{1}{2}\right)^k & \beta^{k-1}-\gamma \\ 0 & 1 \end{bmatrix}$

(3) $\Psi^{-1}(k) = 2^k \begin{bmatrix} 1 & \gamma-\beta^{k-1} \\ 0 & \left(\frac{1}{2}\right)^k \end{bmatrix} = \begin{bmatrix} 2^k & 2^k(\gamma-\beta^{k-1}) \\ 0 & 1 \end{bmatrix}$

Vergleich $2^k = \left(\frac{1}{2}\right)^{-k}$ ✓

$\beta^{-k-1} - \gamma = 2^k(\gamma - \beta^{k-1}) \quad \Big| \cdot \beta = \frac{1}{\gamma}$

$\cancel{\beta^{-k}} \ \beta^{-k} - 1 = 2^k(1-\beta^k)$

$\beta^{-k} + \beta^{k}2^k = 2^k + 1 \quad | \beta^k$

$\cancel{\beta^{2k}}$

$1 + \beta^{2k}2^k = \beta^k 2^k + \beta^k$

subst $\beta^k = a$

$a^2 2^k - a(2^k+1) + 1 = 0$

$a^2 - a(1+2^{-k}) + 2^{-k} = 0$

$a_{1,2} = \frac{1+2^{-k}}{2} \pm \sqrt{\frac{(1+2^{-k})^2}{4} - 2^{-k}}$

$= \frac{1+2^{-k}}{2} \pm \sqrt{\frac{1+2\cdot 2^{-k}+2^{-2k}-4\cdot 2^{-k}}{4}} = \frac{1+2^{-k}}{2} \pm \frac{(2^{-k}-1)}{2} = \begin{cases} 2^{-k} \\ 1 \end{cases}$

RS: $\beta^k = 1$

$\beta^k = 2^{-k}$

$\boxed{\Rightarrow \beta = \begin{cases} 1 \\ \frac{1}{2} \end{cases}}$

$s = 1,\ \gamma = 1$

d) $\Psi(k) = \begin{vmatrix} \left(\frac{1}{2}\right)^k & 0 \\ 0 & 1 \end{vmatrix}$

$$\Psi(k+1) = \begin{vmatrix} \frac{1}{2}\left(\frac{1}{2}\right)^k & 0 \\ 0 & 1 \end{vmatrix} = \phi \cdot \begin{vmatrix} \frac{1}{2}^k & 0 \\ 0 & 1 \end{vmatrix}$$

$$\begin{matrix} & & 2^k & 0 \\ & & 0 & 1 \\ \frac{1}{2}\left(\frac{1}{2}\right)^k & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 1 \end{matrix}$$

$$\phi = \begin{vmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{vmatrix}$$

$\hookrightarrow$ keine Lsg, da Diagonalm.

$s = \frac{1}{2}$

$$\Psi(k) = \begin{vmatrix} \frac{1}{2}^k & \frac{1}{2}^{k-1} - 2 \\ 0 & 1 \end{vmatrix} \qquad \phi^{-1}(k)\quad \underbrace{\cancel{2 \cdot 2^k(1-2^k)}}_{2(2^k-1)}$$

$$\begin{matrix} 2^k & 2 \cdot 2^k - \underbrace{2^k \frac{1}{2}^{k-1}}_{2} \\ 0 & 1 \end{matrix}$$

$$\Psi(k+1) = \begin{vmatrix} \frac{1}{2}\left(\frac{1}{2}\right)^k & \left(\frac{1}{2}\right)^k - 2 \\ 0 & 1 \end{vmatrix} \qquad \boxed{\begin{matrix} \frac{1}{2} & \cancel{\frac{1}{2} \cdot (1-2^k)} \; 1-2 \\ 0 & 1 \end{matrix}}$$

$\phi$

$$\frac{1}{2}\left(\frac{1}{2}\right)^k \cdot 2(2^k - 1) + \left(\frac{1}{2}\right)^k - 2$$

$$= 1 - 2 = -1$$

$$\boxed{\phi = \begin{bmatrix} \frac{1}{2} & -1 \\ 0 & 1 \end{bmatrix}}$$