1a)

k	0	1	2	3	4	5	6	7	8
u_k	2	-1	1	0	0	0	0	0	0
y_k	0	2	-3	2	7	-4	4	0	0
$\Rightarrow$ $4\, u_{k-4}$	0	0	0	0	8	-4	4	0	0
$-\, u_{k-2}$	0	0	-2	1	7	-4	4	0	0
$+\, u_{k-1}$	0	2	-3	2	7	-4	4	0	0

$$\Rightarrow (g_k) = \delta_{k-1} - \delta_{k-2} + 4\delta_{k-4}$$

$$G(z) = \frac{1}{z} - \frac{1}{z^2} + \frac{4}{z^4} = \frac{z^3 - z^2 + 4}{z^4}$$

Hauptdiagonale von $\phi = 0$

ii) $$T = \begin{bmatrix} 2 \\ -1 \\ \beta \\ 1 \end{bmatrix} \qquad c^T = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

$$g_k = m_k = c^T \underline{\phi}^{k-1} T$$

$k=1$: $g_{k_1} = \delta_0 = c^T T = 2 - 1 + \beta + 1 = 2 + \beta = 1 \Rightarrow \beta = -1$

$k=2$: $g_{k_2} \overset{-\delta_0}{=} -1 = c^T \phi T = [1111] \begin{bmatrix} 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ -1 \\ 1 \end{bmatrix} = -a - b + c = -1$

$[0\, a\, b\, c]$

$k=4$: $g_4 = 4\delta_0 = 4 = c^T \phi^3 T = [1111] \begin{bmatrix} 0 & 0 & 0 & abc \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ -1 \\ 1 \end{bmatrix} = a \cdot b \cdot c = 4$

$[0\, 0\, 0\, abc]$

$k=3$: $g_3 = 0 = c^T \phi^2 T = [1111] \begin{bmatrix} 0 & 0 & ab & 0 \\ 0 & 0 & 0 & bc \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ -1 \\ 1 \end{bmatrix} = -ab + bc = 0$

$[0\, 0\, ab\, bc]$

$$\begin{aligned} -a - b + c &= -1 \\ a \cdot b \cdot c &= 4 \\ b \cdot (c - a) &= 0 \Rightarrow c = a \end{aligned}$$

$\cancel{-a} - \frac{4}{a^2} + \cancel{a} = -1 \Rightarrow a^2 = 4 \quad a = \pm 2 = c$

$a^2 b = 4 \Rightarrow b = \frac{4}{a^2} \qquad b = 1$

$\phi_{ij} \geq 0 \Rightarrow a = c = 2$

$$\Rightarrow \phi = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$\underline{\underline{\beta = -1}}$

$$c^T = [1\,1\,1\,1]$$

Kontrolle: $G(z) = c^T (zE - \phi)^{-1} T = \frac{4}{z^4} - \frac{1}{z^2} + \frac{1}{z}$ ✓

$R(\Phi, \Gamma) = [\Gamma, \Phi\Gamma, \Phi^2\Gamma, \Phi^3\Gamma]$

$$= \begin{bmatrix} 2 & -2 & -2 & 4 \\ -1 & -1 & 2 & 0 \\ -1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Rang} = 4 = n \Rightarrow \text{vollst. Erreichbar}$$

$\Rightarrow$ vollständig steuerbar

(es gilt nicht vollst. steuerbar $\Rightarrow$ vollst. Erreichbar.)

b) $A = \begin{bmatrix} -20 & 10 & 4 \\ 0 & -20 & 13 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \Phi = \begin{bmatrix} e^{-2} & e^{-2} & 0{,}5 \\ 0 & e^{-2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\Gamma = [1 0 0]$ $\quad y(k) = \underbrace{[1 \; 1 \; 0]}_{c^T} x(k)$

i) $e^{-20T_a} = e^{-2} \Rightarrow \underline{\underline{T_a = 0{,}1}}$

ii) $G(z) = c^T \underbrace{(zE - \Phi)^{-1}}_{L} \Gamma = [1 \; 1 \; 0] \begin{bmatrix} z-e^{-2} & -e^{-2} & -0{,}5 \\ 0 & z-e^{-2} & -1 \\ 0 & 0 & z-1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow G(z) = L_{11} + L_{12} = \dfrac{1}{(z-e^{-2})^2 \cdot (z-1)} \cdot ((z-e^{-2})(z-1) - 0)$

$$\underline{\underline{G(z) = \frac{1}{z-e^{-2}}}}$$

iii) Pole: $z = e^{-2} = \frac{1}{e^2} < 1 \Rightarrow$ $\underline{\underline{\text{Bibo stabil}}}$

~~$\not\Leftarrow$ asymp~~ $\not\Rightarrow$ asymptotische Stabilität. Jeder Pol (e^{-2}) ist Eigenwert, aber nicht jeder Eigenwert auch Pol

$\lambda = 1 \Rightarrow$ ~~ins~~ nicht asymptotisch stabiler Eigenwert von Φ

2) $\dot{x} = A x + b u$

$y = c^T x$

$\cancel{\dot{e} = A \cdot e}$

a) ? ? ? ? ? ?

b) $O(A, c^T) = \begin{bmatrix} c^T \\ c^T A \\ c^T A^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & -1 & 0 \\ 4 & 3 & -1 \end{bmatrix} \Rightarrow \text{rang } 3 = n$

$\Rightarrow$ vollst. Beobachtbar

c) $p(\lambda) = \lambda^3 + 9\lambda^2 + 26\lambda + 24$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & -1 & 0 \\ 4 & 3 & -1 \end{bmatrix} \vec{v}_1 \Rightarrow \vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\underline{\vec{k}} = -24\,\vec{v}_1 - 26\,A v_1 - 9\,A^2 v_1 - A^3 v_1 = \underline{\underline{\begin{bmatrix} -5 \\ 9 \\ -6 \end{bmatrix}}}$$

d) Skript S. 224 (1) und (2)

3) a)

$$R(s) = \frac{1+\beta s}{1-2s} \qquad G(s) = \frac{1}{s-\alpha}$$

i) $R(s)$ ist DT_1-Glied / Lag-Glied => kann keine instabile Strecke stabilisieren => $\alpha < 0$ damit $G(s)$ stabil ist. ??? Stimmt das ???

ii) $\cancel{L(s) = R(s) \cdot G(s)}$ $\frac{L(s)}{1+L(s)} = \frac{1+\beta s}{(1-2s)(s-\alpha) + \cancel{1+\beta s}} = \frac{1+\beta s}{-2s^2 + s(\beta + 1 + 2\alpha) + 1 - \alpha}$

intern stabil => $R(s)$ stabil => unabhängig von β ??

b) $p(\lambda) = \lambda^3 - \lambda$

$$b_R = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad A_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R(A,B) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ => rang $3 = n$ => vollst. erreichbar

iii) ????

4a) $\dot{x}_1 = x_1^2 \cos(x_2) - 10 + u_1^2$

$\dot{x}_2 = -x_2 + \frac{u_2}{1+u_1}$

$\dot{x}_3 = x_2^2 - x_3$

i) $\dot{x} = 0 \Rightarrow x_{1,R}^2 \cdot \cos(x_{2,R}) - 10 + u_{1R}^2 = 0$

$\Downarrow$

$x_{2,R} = \frac{u_{2,R}}{1+u_{1,R}}$ $\qquad x_{1,R} = \sqrt{\frac{10 - u_{1R}^2}{\cos\left(\frac{u_{2,R}}{1+u_{1,R}}\right)}}$

$x_{3,R} = \sqrt{\frac{u_{2,R}}{1+u_{1,R}}}$

ii) $y = \begin{bmatrix} x_1 + x_2 + x_3 \\ \cos(x_1) + x_2 u_1^2 \end{bmatrix}$ $\qquad \Delta\dot{x} = A\Delta x + B\Delta u$ $\qquad \Delta x(t_0) = \Delta x_0$

$\Delta y = C\Delta x + D\Delta u$ $\qquad = x_0 - \tilde{x}_0$

$\frac{\partial f}{\partial x} = \begin{bmatrix} 2\tilde{x}_1 \cos(\tilde{x}_2) & -\tilde{x}_1^2 \sin(\tilde{x}_2) & 0 \\ 0 & -1 & 0 \\ 0 & 2\tilde{x}_2 & -1 \end{bmatrix} = A$ $\qquad \frac{\partial f}{\partial u} = \begin{bmatrix} 2\tilde{u}_1 & 0 \\ -\frac{\tilde{u}_2}{(1+\tilde{u}_1)^2} & \frac{1}{1+\tilde{u}_1} \\ 0 & 0 \end{bmatrix} = B$

$\frac{\partial h}{\partial x} = \begin{bmatrix} \tilde{x}_2 & \tilde{x}_1 & 1 \\ -\sin(\tilde{x}_1) & \tilde{u}_1^2 & 0 \end{bmatrix} = C$ $\qquad \frac{\partial h}{\partial u} = \begin{bmatrix} 0 & 0 \\ 2\tilde{u}_1 x_2 & 0 \end{bmatrix} = D$

iii) A, B, C, D dürfen nicht von t abhängen

$\tilde{x} = \begin{bmatrix} 1 \\ 1 \\ t \end{bmatrix}$

4b) $G(s) = 10^3 \cdot \frac{\left(1 + \frac{s}{0{,}1}\right)\left(1 + \frac{s}{10^4}\right)}{\left(1 + \frac{s}{10}\right)^2}$