① Prüfung 5.2.2016

a)

i) $f(x_R, u_R) = \emptyset$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 \cdot x_2 + \sqrt{2} \sin u \\ x_1 \cdot x_2^2 + \sqrt{2} \cos u \end{bmatrix} \qquad y = \arctan \frac{x_2}{x_1} + 2u^2$$

$$0 = x_{1R}^2 \, x_{2R} + \sqrt{2} \sin u_R \rightarrow x_{2R} = \frac{-\sqrt{2} \sin u_R}{x_{1R}^2} = -\sqrt{2} \frac{\cos u_R^2}{\sin u_R} = \underline{-1}$$

$$0 = x_{1R} \cdot x_{2R}^2 + \sqrt{2} \cos u_R \rightarrow \cancel{x_{1R}} \cdot \frac{-\sqrt{2} \sin u_R}{x_{1R}^{\cancel{2}}} = -\sqrt{2} \cos u_R$$

$$x_{1R} = \frac{\sin u_R}{\cos u_R} = \tan(u_R) = \tan\left(\frac{\pi}{4}\right) = \underline{1}$$

ii) $$A = \frac{\partial f(x_R, u_R)}{\partial x_i} = \begin{bmatrix} x_{2R} & x_{1R} \\ x_{2R}^2 & x_{1R} \cdot 2x_{2R} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$B = \frac{\partial f(x_R, u_R)}{\partial u} = \begin{bmatrix} \sqrt{2} \cos(u_R) \\ -\sqrt{2} \sin(u_R) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C = \frac{\partial h(x_R, u_R)}{\partial x_i} = \begin{bmatrix} -\frac{x_{2R}}{1 + \left(\frac{x_{2R}}{x_{1R}}\right)^2} & \frac{1}{x_{1R}\left(1 + \left(\frac{x_{2R}}{x_{1R}}\right)^2\right)} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$D = \frac{\partial h(x_R, u_R)}{\partial u} = \begin{bmatrix} 4 u_R \end{bmatrix} = \begin{bmatrix} \pi \end{bmatrix}$$

b) Zeige durch Laplace:

$$\hat{\Phi}(s) = (sE - J)^{-1} = \begin{bmatrix} s-\lambda & -1 \\ 0 & s-\lambda \end{bmatrix}^{-1} = \frac{1}{(s-\lambda)^2} \begin{bmatrix} (s-\lambda) & +1 \\ 0 & (s-\lambda) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-\lambda} & \frac{1}{(s-\lambda)^2} \\ 0 & \frac{1}{s-\lambda} \end{bmatrix}.$$

$$\Phi(t) = \mathcal{L}^{-1}\{\hat{\Phi}(s)\} = \begin{bmatrix} e^{t\lambda} & t \cdot e^{\lambda t} \\ 0 & e^{t\lambda} \end{bmatrix} \checkmark$$

P.B.Z.:

$$\frac{1}{(s-\lambda)^2} = \frac{A}{(s-\lambda)} + \frac{B}{(s-\lambda)^2} \quad \bullet\!\!-\!\!\circ \quad \emptyset + t \cdot e^{\lambda t}$$

$$1 = A(s-\lambda) + B$$

$$\hookrightarrow A = \emptyset, \; B = 1$$

c)

i) $\det(A-\lambda E) = \det\left(\begin{bmatrix} -2-\lambda & 1 & 0 \\ -1 & -\lambda & 0 \\ -7 & -4 & 2-\lambda \end{bmatrix}\right)$, $x = V\cdot z$, $z = V^{-1}x$

$= (2-\lambda)\cdot\left[(-2-\lambda)(-\lambda)+1\right] = (2-\lambda)\cdot\left[\underbrace{\lambda^2+2\lambda+1}_{\lambda_{2,3} = -1 \pm \sqrt{1-1}}\right]$

Eigenwerte: $\lambda_1 = 2$ $a.V = 1$; $g.V = 1$

$\lambda_{2,3} = -1$ $a.V = 2$; $g.V = 1$ → Hauptvektor berechnen

$(A-\lambda_1 E)\vec{v_1} = \vec{0}$

$\begin{bmatrix} -4 & 1 & 0 \\ -1 & -2 & 0 \\ 7 & -4 & 0 \end{bmatrix}\vec{v_1} = \vec{0}$

$-4x_1 + x_2 = 0$

$-x_1 - 2x_2 = 0$ $x_3 = t$ → $\vec{v_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$7x_1 - 4x_2 = 0$

$(A-\lambda_2 E)\vec{v_2} = \vec{0}$

$\begin{bmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 7 & -4 & 3 \end{bmatrix}\vec{v_2} = \vec{0}$

$-x_1 + x_2 = 0 \to x_1 = x_2 \to x_1 = t$

$-x_2 + x_2 = 0 \to x_2 = t$ $\vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$7x_1 - 4x_2 + 3x_3 = 0$

$\to 7-4+3x_3 = 0 \to x_3 = -1$

Hauptvektor:

$(A-\lambda_3 E)\vec{v_3} = \vec{v_2}$

$\begin{bmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 7 & -4 & 3 \end{bmatrix}\vec{v_3} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$-x_1 + x_2 = 1 \to x_1 = x_2 - 1 = 0$

$-x_1 + x_2 = 1 \to -x_2 + 1 + x_2 = 1 \to x_2 = t$

$-4 + 3x_3 = -1 \to \underline{\underline{x_3 = 1}}$

$\to \vec{v_3} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\underline{\underline{V = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}}}$

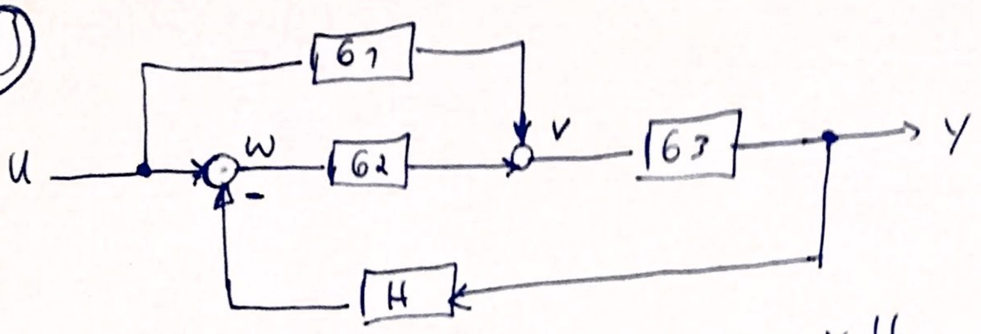
$\underline{\underline{\tilde{A} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}}}$ ... weil $\underline{a.V = g.V}$, und $\underline{aV = 2\ \ gV = 1}$

ii) $z(t) = \tilde{\Phi}(t)\, z_0 = \tilde{\Phi}(t)\cdot V^{-1}x_0 = \underline{\underline{\begin{bmatrix} e^{2t} \\ -t\cdot e^{-t} \\ -e^{-t} \end{bmatrix}}}$

$V^{-1} = (-1)\cdot\begin{bmatrix} x & x & x \\ -1 & 0 & +1 \\ x & x & x \end{bmatrix}^T = \begin{bmatrix} x & 1 & x \\ x & 0 & x \\ x & -1 & x \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\tilde{\Phi}(t) = \begin{bmatrix} e^{2t} & 0 & 0 \\ 0 & e^{-t} & t\cdot e^{-t} \\ 0 & 0 & e^{-t} \end{bmatrix}$

2)

a) 

i)

$y = G_3 \cdot v$; $v = G_1 \cdot u + w \cdot G_2$; $w = u - y \cdot H$

$y = G_3 G_1 \cdot u + G_3 \cdot G_2 \cdot w = G_3 \cdot G_1 \cdot u + G_3 \cdot G_2 u - G_3 \cdot G_2 \cdot H y$

$y(1 + G_3 \cdot G_2 \cdot H) = u(G_3 \cdot G_1 + G_3 \cdot G_2)$

$\hookrightarrow G(s) = \dfrac{G_3 \cdot G_1 + G_3 \cdot G_2}{1 + G_3 \cdot G_2 \cdot H}$

ii) $n(s): s^3 + 4s^2 + s(K+1) + 2K - 6$

s^3	1	$(K+1)$	0
s^2	4	$2K-6$	0
s^1	$\frac{4K+4-2K+6}{4}$	0	
s^0	$2K-6$		

$2K - 6 > 0 \rightarrow K > 3$

$2K + 10 > 0 \quad K > -5$

iii)

$\sigma(t) \circ\!\!-\!\!\bullet \frac{1}{s}$

$-2e^{-2t} \rightarrow 0$; $\lim_{s \to 0} s \cdot \frac{1}{s} \cdot G(s) = \frac{2}{10} = \frac{1}{5}$ ... EWS

$|G(s=3j)| = \dfrac{-7}{-\cancel{j27} - 36 + \cancel{j27} + 10} = +\dfrac{7}{26}$,

$\arg(G(s=3j)) = 0$

$y(t) = \frac{1}{5} + \frac{14}{26} \sin(3t)$

② b)

i) $L(j\omega) = K \cdot \dfrac{4(j\omega+1)^2}{j\omega(j\omega-10)^2}$

$$\lim_{\omega\to 0_+} L(j\omega) = \lim_{\omega\to 0_+} \frac{K\cdot 4\left(\frac{1}{j\omega}+1\right)(j\omega+1)}{(j\omega-10)(j\omega-10)} = \lim_{\omega\to 0_+} -j\frac{4\cdot K}{\omega} = \underline{\underline{-\infty}}$$

$$\lim_{\omega\to 0_-} L(j\omega) = \lim_{\omega\to 0_-} -j\frac{K\cdot 4}{\omega} = \underline{\underline{+\infty}}$$

$$\lim_{\omega\to \infty_+} L(j\omega) = \lim_{\omega\to \infty_+} \frac{K\cdot\frac{4}{j\omega}\left(1+\frac{1}{j\omega}\right)\left(1+\frac{1}{j\omega}\right)}{1\left(1-\frac{10}{j\omega}\right)\left(1-\frac{10}{j\omega}\right)} = \lim_{\omega\to\infty_+} -j\frac{K\cdot 4}{\omega} = \underline{\underline{-0}}$$

$$\lim_{\omega\to \infty_-} L(j\omega) = \lim_{\omega\to \infty_-} -j\frac{4\cdot K}{\omega} = \underline{\underline{+0}}$$

ii) Nyquist kriterium: $\Delta \arg(1+L(j\omega)) = (3-0+2)\pi = \underline{\underline{5\pi}}$

Stetige Winkeländerung für $K=4$: $\underline{\underline{\pi}}$ -> NICHT STABIL!

Stetige Winkeländerung für $K=6$: $\underline{\underline{5\pi}}$ -> STABIL!

(3)

$$\begin{bmatrix} -\frac{1}{2} & -1 & -1 \\ -\frac{1}{2} & 0 & 0 \\ 1 & -1 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{2} & -1 & -1 \\ -\frac{1}{2} & 0 & 0 \\ 1 & -1 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{2} & -1 \\ -\frac{1}{4} & +\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

a)

i) Wenn nicht v. Erreichbar dann auch nicht v. Steuerbar!

$$R = [\Gamma, \Phi\Gamma \quad \Phi^2\Gamma] = \begin{bmatrix} 1 & \frac{3}{2} & \frac{3}{4} \\ 0 & -\frac{1}{2} & -\frac{3}{4} \\ -1 & \frac{1}{2} & \frac{4}{4} \end{bmatrix}$$

$\det(R) = \emptyset$
↳ Nicht regulär

$$\det(R) = -\frac{8}{8} + \frac{3}{8} - \left[\frac{4}{8} + \frac{3}{8}\right] = \emptyset \rightarrow \text{Nicht v. Erreichbar}$$

ii) 2
0

iii) 2
0

b) ii) $e_{k+1} = \hat{x}_{k+1} - x_{k+1} = \Phi \hat{x}_k + \cancel{\Gamma u_k} + k(\hat{y}_k - y_k) - \Phi x_k - \cancel{\Gamma u_k}$

$= \Phi \hat{x}_k - \Phi x_k + k c^T(\hat{x}_k - x_k) = \Phi(\hat{x}_k - x_k) + k c^T(\hat{x}_k - x_k)$

$= \Phi e_k + k c^T e_k = (\Phi + k c^T) e_k = \Phi_e e_k$

Deadbeat: $P_{y\,soll} = (\Phi_e)^n$ n-Schritte Notwendig Ordnung der Matrix.

i) $m_0 = c^T \Phi^{-1} \Gamma = [0\,2\,0] \begin{bmatrix} x & x & x \\ -1/3 & x & -2/3 \\ x & x & x \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = [-1/3 \;\; x \;\; -4/3] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \underline{\underline{1}}$

$\rightarrow \underline{\underline{m_0 = 1}}$

$m_k = c^T \Phi^{k-1} \Gamma$, $m_k = g_k$ d muss $\emptyset$ sein damit makrov erwerdbar. $\underline{\underline{d = \emptyset}}$

$m_1 = 0$, $m_2 = -1$, $m_3 = -\frac{3}{2}$

$0 = [c_1 \;\; c_2 \;\; 0] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \underline{c_1 = \emptyset}$ $\underline{\underline{c^T = [0\,2\,0]}}$

$-1 = [0 \;\; c_2 \;\; 0] \begin{bmatrix} -1/2 & -1 & -1 \\ -1/2 & 0 & 0 \\ 1 & -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \Rightarrow \underline{\underline{c_2 = 2}}$

(3) iii) $c^T = [2\ 0\ 0]$

Dead Beat-Regler: $p_{f\,soll} = \Phi^{-3}$

$$\theta = \begin{bmatrix} c^T \\ c^T\Phi \\ c^T\Phi^2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -2 & -2 \\ -\frac{1}{2} & 1 & -2 \end{bmatrix}, \quad \theta^{-1} = \frac{1}{12}\begin{bmatrix} x & x & x \\ x & x & x \\ 0 & +4 & -4 \end{bmatrix}^T$$

$$\hat{v}_1 = \theta^{-1}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x & x & 0 \\ x & x & \frac{1}{3} \\ x & x & -\frac{1}{3} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/3 \\ -1/3 \end{bmatrix}$$

$$\hat{k} = -p_{f\,soll}(\Phi)^3\,\hat{v}_1 = \underline{\underline{[-1/2\ \ 1/4\ \ -3/8]}}$$

$$\Phi^3 = \begin{bmatrix} -\frac{1}{4} & \frac{1}{2} & -1 \\ -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{2} & -\frac{3}{2} & -\frac{3}{4} \end{bmatrix}\cdot\begin{bmatrix} \frac{1}{2} & -1 & -1 \\ -\frac{1}{2} & 0 & 0 \\ 1 & -1 & 1/2 \end{bmatrix} = \begin{bmatrix} x & 5/4 & -1/4 \\ x & -1/4 & 1/2 \\ x & -3/4 & -15/8 \end{bmatrix}$$

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(4)

a) i) $G(s)$ → sprungfähig, → Realisierbar, → INSTABIL, → Reele Polstelle, $\omega_0 = \frac{2}{T_0} = 1$

- überprüfe Sprungfähigkeit:

$\lim_{\varrho \to 1} G_1^{\#}(\varrho) = \emptyset$; $\lim_{\varrho \to 1} G_4^{\#}(\varrho) = \emptyset$ } NICHT sprungfähig!

$G_1^{\#}(\varrho)$ und $G_4^{\#}(\varrho)$ fallen weg!

ii) über EWS!

$\lim_{\varrho \to \emptyset} s \cdot \frac{1}{s} G_3^{\#}(\varrho) = 1$, $\lim_{\varrho \to \emptyset} s \cdot \frac{1}{s} G_2^{\#}(\varrho) = 1$ Verstärkung muss 1 sein

b) 2 / 0

Ansatz: $x_k = \Phi^k x_0 = \psi(k)\,x_0$