10.12.2010 / 1)

$$\rho_u(s) = \rho_0 \cdot e^{-s/s_0}$$

$$x^T = \begin{bmatrix} T \\ s \\ w \end{bmatrix}$$

$$T_u = T_0 - K_2 s$$

$$\boxed{\dot{s} = w}$$

$$\rho(T,s) = \rho_u(s) - K_1(T - T_0)$$
$$= \rho_0 e^{-s/s_0} - K_1(T - T_0)$$

$$\frac{d}{dt}\left(c_v V \rho(T,s) T\right) = P_H - P_v$$

$$u = P_H$$

$$y = s$$

$$F_{auf} = \left(\rho_u(s) - \rho(T,s)\right) V g$$

$$F_R = d w$$

$$F_g = m \cdot g \qquad P_v = A \cdot \alpha \cdot (T - T_u)$$

Impulserh. $F_{auf} + F_g + F_r = m\dot{w}$ $\quad = A\cdot\alpha(T_0 + K_2 s)$

$$V g(\rho_u(s) - \rho(T,s)) - m g - d w = m\dot{w}$$

$$c_v V\Big[\underbrace{\rho(T,s\frac{dT}{dt} + T \cdot \frac{d\rho(T,s)}{dt}}_{}\Big] = P_H - P_v$$

$$\rho \cdot \dot{T} = + T \cdot \left[-\frac{\rho_0}{s_0} e^{-s/s_0} \cdot \dot{s} - k_1 \dot{T}\right]$$

$$\dot{T}[\rho_0 \cdot e^{-s/s_0} - k_1(T - T_0))$$

$$\dot{T}(\rho_0 \cdot e^{-s/s_0} - K_1(T - T_0)) + \frac{w\rho_0}{s_0} e^{-s/s_0} \cdot T - K_1 T\dot{T} = \frac{P_H}{V c_v} - \frac{A\alpha(T - T_u)}{c_v V}$$

$$\dot{T} = \frac{P_H}{V c_v\Big[\rho_0 \cdot e^{-s/s_0} \underbrace{-K_1(T - T_0) - K_1 T}_{\substack{\cancel{+K_1 T_0} \\ -K_1(2T - T_0)}}\Big]} + \frac{-\frac{A\alpha(T - T_u)}{V c_v} + \frac{w\rho_0}{s_0} e^{-s/s_0} T}{\Big[\rho_0 \cdot e^{-s/s_0} \underbrace{-K_1(T - T_0) - K_1 T}_{-K_1(2T - T_0)}\Big]}$$

$$\dot{w} = \frac{V g}{m}\left(\rho_0 \cdot e^{-s/s_0} + K_1(T - T_0)\right) - g - \frac{d w}{m}$$

$$\frac{d}{dt}\left(\cancel{\#}\,\rho\cdot T\right) = \frac{d\rho}{ds}\underbrace{\frac{ds}{dt}}_{\omega}\cdot T + \frac{d\rho}{dT}\underbrace{\frac{dT}{dt}}_{\dot{T}} T + \underbrace{\frac{dT}{dt}}_{\dot{T}}\rho$$

$$\underbrace{\qquad}_{-k_1\dot{T}(2T-T_0)}$$

$$= -\frac{\rho_0}{s_0}e^{-s/s_0}\,\omega T \cancel{+} -k_1 T\dot{T} + \dot{T}\rho_0 e^{-s/s_0} - \dot{T}k_1(T-T_0)$$

$$\overset{!}{=} \frac{P_H - A\alpha\,(T-T_0+k_2 s)}{V c_V}$$

$$\dot{T} = \frac{P_H}{V c_V(\rho_0 e^{-s/s_0} - k_1\cancel{T}(2T-T_0))} + \frac{\frac{\rho_0}{s_0}e^{-s/s_0}\omega T - \frac{A\alpha(T-T_0+k_2 s)}{c_V V}}{\rho_0 e^{-s/s_0} - k_1(2T-T_0)}$$

$$= \cancel{P_H}$$

$$\dot{\omega} = \frac{V\rho}{m}(T-T_0)k_1 - g - \frac{d\omega}{m}$$

b) $x_R = 0$ $\quad u_R = P_{HR}$

$\dot{s} = \omega_R = 0$

$$0 = \frac{V\rho}{m}(T_R-T_0)k_1 - g \rightarrow T_R = \frac{m}{k_1 V} + T_0$$

$$0 = \frac{P_H}{V c_V} + \cancel{\frac{\rho_0}{s_0}e} - \frac{A\alpha}{V c_V}(T_R - T_0 + k_2 s_R)$$

$$\frac{P_{HR}}{k_2 A\alpha} - \frac{T_R + T_0}{k_2} = s_R \quad \text{um } P_{H,R} \text{ regeln}$$

$$P_{HR} = A\alpha(T_R - T_0 + k_2 s_R) \quad \text{um } s_R \text{ regeln}$$

Linearisieren

$c^T = (0 \; 1 \; 0)$, $d = 0$

$$b = \frac{\partial f(x_R, u_R)}{\partial u} = \begin{bmatrix} \frac{1}{V_{cv}(\rho_0 e^{-s/s_0} - k_1(2r - r_0))} \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{r} = \frac{\Gamma_{it} + V_v \frac{\rho_0}{s_0} e^{-s/s_0} \omega r - A\alpha(r - r_0 + k_2 s)}{V_{cv}(\rho_0 e^{-s/s_0} - k_1(2r - r_0))}$$

$$\frac{\partial \dot{r}(x_R, u_R)}{\partial r} = \frac{-A\alpha \cancel{V_{cv}}(\rho_0 e^{-s_R/s_0} - k_1(2r_R - r_0)) + A\alpha(r_R - r_0 + k_2 s) \cdot \cancel{V_{cv}}(-2k_1)}{(V_{cv})^{\cancel{2}}(\rho_0 e^{-s_R/s_0} - k_1(2r_R - r_0))^2}$$

$$= \frac{-A\alpha \rho_0 e^{-s_R/s_0} + A\alpha k_1(\cancel{2r_R} - r_0) - \cancel{2R} + 2r_0 + k_2 s)}{}$$

$$= \frac{-A\alpha(\rho_0 e^{-s_R/s_0} + \cancel{k_1} 2r_0 + k_1 k_2 s_R)}{V_{cv}(\rho_0 e^{-s_R/s_0} - k_1(2r_R - r_0))}$$

usw

2a) $\Omega_c \cdot t_r = 1{,}2 \rightarrow \Omega_c = \frac{1}{2}$ falls $\Omega_c \le \frac{0{,}2\;2}{T_a} = 1$ ✓

$\ddot{u} + \phi_R = 70$

$\phi = 60 \rightarrow \varphi = 120° \xrightarrow{-2\pi/3}$ [illegible] $= 0 \rightarrow$ I · 1. Lu

~~nicht~~ sprungfähig $\lim_{q \to \Omega_0} G^\#(q) \neq 0$ (kein Nst bei Ω_0)

$\rightarrow$ Nst bei $\Omega_0 = \frac{2}{T_a}$ benötigt! $\Omega_0 = 5$

$$R^\#(q) = V_R \frac{\left(1 - \frac{q}{\Omega_0}\right)\left(1 + 2 \cdot 0{,}75\, q + q^2\right)}{q(1 + qT)}$$

$$\left(1 - \frac{q^2}{5}\right) = \left(1 - \frac{\omega^2}{5}\right)$$

$$L_1 = R_1 \frac{(1 + 2q)\left(1 + \frac{q}{5}\right)\left(1 - \frac{q}{5}\right)}{q\left(1 + 2(2 - \sqrt{3})q\right)}$$

$$\arg L_1 \Big|_{q = j\Omega_c} = \frac{\pi}{4} + \cancel{\ldots} - \frac{\pi}{2} - \frac{\pi}{12} = -\frac{\pi}{3}$$

$$\arg\left(\overline{1 + qT}\right)\Big|_{\frac{\pi}{2}} \overset{!}{=} -\frac{\pi}{3} \rightarrow \arctan \frac{T}{2} = \frac{\pi}{3}$$

$T = 2\sqrt{3}$

$$\left|\frac{R^\#(q)}{V_R} G^\#(q)\right|_{\Omega_c} = \frac{\sqrt{2} \cdot \left(1 - \frac{1}{20}\right)}{\cancel{\frac{1}{2}} \sqrt{1 + (2 - \sqrt{3})^2} \cdot \cancel{2}} = \frac{19}{20} \cdot \sqrt{2} \frac{1}{\sqrt{5 + 3 - 4\sqrt{3}}}$$

$$= \frac{19}{20} \cdot \sqrt{2} \frac{1}{2\sqrt{2 - \sqrt{3}}} = \frac{1}{V_R}$$

b) $G^\#(q)$ realisierbar, wenn keine Pole bei ~~[illegible]~~ Ω_0 sind!

$R^\#(q) \rightarrow$ realisierbar

c) $q = \frac{2}{T_a} \frac{z-1}{z+1}$

---

3) $T_{u,y}(z)$, $T_{d,y}(z)$

$$(u_k - y_k) \cdot G_2 \cdot \frac{G_3}{1+G_3} = y_k \qquad T_{u,y}(z) = \frac{\frac{G_2 G_3}{1+G_3}}{1 + \frac{G_2 G_3}{1+G_3}} =$$

$$= \frac{G_2 G_3}{1 + G_3 + G_2 G_3} = *$$

$$* = \frac{\frac{\cancel{k}}{z-1} \frac{\frac{1}{2}-k}{\cancel{-k}}}{1 + \frac{k}{z-1} + \frac{\frac{1}{2}-k}{z-1}} =$$

$$= \frac{\frac{1}{2}-k}{\frac{1}{2}\cancel{-k}\cancel{+k}+z-1} = \frac{\frac{1}{2}-k}{z-\frac{1}{2}}$$

$$T_{d,y} \, \| \, ((\cancel{u_k} - y_k) G_2 + d_k G_1) \cdot \frac{G_3}{1+G_3} = y_k$$

$$T_{d,y} = \frac{\frac{G_1 G_3}{1+G_3}}{1 + \frac{G_2 G_3}{1+G_3}} = \frac{G_1 G_3}{1 + G_3 + G_2 G_3} = \frac{\frac{5}{z-1} \frac{k}{z-1}}{1 + \frac{k}{z-1} + \frac{\frac{1}{2}-k}{z-1}} =$$

$$= \frac{5k}{(z-1)^2 + \cancel{k(z-1)} + (\frac{1}{2} - \cancel{k})(z-1)} = \frac{5k}{z^2 - 2z + 1 + \frac{1}{2}z - \frac{1}{2}}$$

b) innen stabil, wenn alle Teilübertragungsfkt. BIBO stabil

→ nein, da G_1, G_3 nicht stabil

$T_{u,y}$ ist BIBO-stabil

$$z^2 - \frac{3}{2}z + \frac{1}{2} \qquad z_{12} = \frac{3}{4} \pm \sqrt{\frac{9}{16} - \frac{8}{16}} = \frac{2}{4} / 1$$

$$T_{d,y} = \frac{5k}{(z-\frac{1}{2})(z-1)}$$ → nicht BIBO stabil

c) $(d_k) = (2^{-k}\, k) =$ Dämpfungssatz

$p_z\, k \circ\!\!-\!\!\bullet \frac{z}{(z-1)^2}$

$p_z\left(\frac{z}{1/2} = 2z\right) = \frac{2z}{(2z-1)^2}$

$Y_z = G(z)\, p_z = \frac{10kz}{(z-\frac{1}{2})(z-1)(2z-1)^2} = \frac{20kz}{(z-1)(2z-1)^3}$

$y_{k\to\infty} = \lim_{z\to1} \frac{\cancel{(z-1)}\,20kz}{\cancel{(z-1)}(2z-1)^3} = 20k$

d) linear, zeitvariant

$x_1 = y$

$x_2 = \dot{y} = \dot{x}_1$

$x_3 = \ddot{y} = \dot{x}_2$

$\dot{x}_3 = \dddot{y} = \frac{u(t)}{\cosh(t)} - \frac{3\cos t}{\cosh(t)} x_3 + \frac{(1-e^{-3t})}{\cosh(t)} x_2$

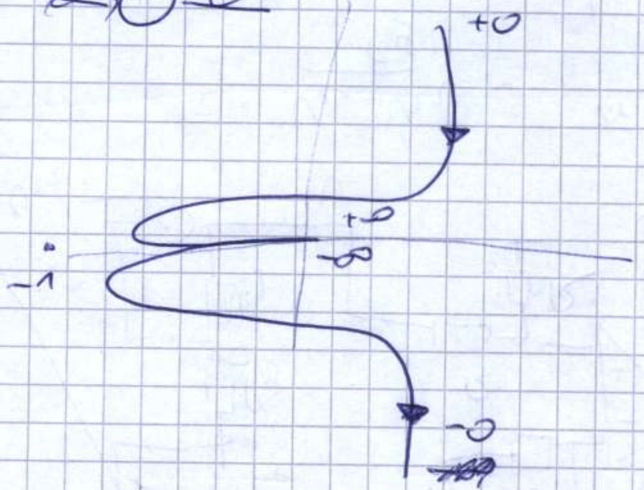
$y = x_1$

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \frac{(1-e^{-3t})}{\cosh t} & -\frac{3\cos t}{\cosh t} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\cosh(t)} \end{bmatrix} u$$

$y = [\,1 \quad 0 \quad 0\,]\, x$

4a)



$L(s) = \frac{5(2s+k)}{s(s^2+4s+3)} = \frac{5(2s+1)}{s^3+4s^2+3s}$

s	
$-\infty$	0^-
$+\infty$	0^+
$+0$	$+\infty$
-0	$-\infty$

$(s+1)\cdot(s+3)s$

$\Delta \arg\{L(j\omega)\} = \pi$

$\text{soll} = (\max(\text{grad } z, \text{grad } n) - N_-(n_L) + N_+(n_L))\pi$

$= (3 - 2 - 0)\pi = \pi$

→ BIBO stabil

b) $G(s) = \frac{L}{1+L} = \frac{5(2s+k)}{5(2s+k)+s^3+4s^2+3s}$

$= \frac{5(2s+K)}{s^3+4s^2+13s+5K}$

s^3	1	13
s^2	4	5K
s	$13 - \frac{5}{4}k$	
1	5K	

$\rightarrow K > 0$

$13 > \frac{5}{4}k$ $\qquad 0 < K < \frac{52}{5}$

$k < \frac{52}{5}$

c) $x_{k+1} = \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} x_k + \begin{pmatrix}0\\ 1\end{pmatrix} u_k$

$u_k = k^T x_k$

für $k = 2$ $\quad x_2 = (1\ \ 1)^T \quad (\cancel{x}_0 = [3\ \ 3])$

$x_{k+1} = \phi x_k + \Gamma k^T x_x = (\phi + \Gamma k^T) x_k$

$x_1 = (\phi + \Gamma k^T) x_0$

$x_2 = (\phi + \Gamma k^T)^2 x_0$

	k_1	k_2
0	0	0
1	k_1	k_2

$\begin{pmatrix}1\\ 1\end{pmatrix} = \cancel{\phi 2} \begin{pmatrix}1 & 1\\ k_1 & 1+k_1\end{pmatrix}^2 \begin{pmatrix}3\\ 3\end{pmatrix}$

$$\begin{array}{cc|cc} & & 1 & 1\\ & & k_1 & 1+k_2\\ \hline 1 & 1 & 1+k_1 & 2+k_2\\ k_1 & 1+k_2 & \cancel{k_1} & k_1+(1+k_2)^2 \end{array}$$

$\underbrace{k_1+k_1+k_1k_2}_{k_1(2+k_2)}$

I: $1 = 3+3k_1 + 6+3k_2$

II: $1 = 6k_1 + 3k_1k_2 + 3k_1 + 3(1+k_2)^2$

$k_1 = -k_2 - 8/3$

~~$1 = -18k_2 - 48 - \cancel{9k_2^2} - 24k_2 - 9k_2 - 24 + 3 + 6k_2 + \cancel{3k_2^2}$~~

~~$6k_2^2 + 45k_2 + 70 = 0$~~

~~$k_2^2 + \frac{45}{6}k_2 + \frac{70}{6} = 0$~~

~~$k_{1,2} = -\frac{45}{12} \pm \sqrt{\frac{45}{12}^2 - \frac{70}{6}}$~~

weiter auflösen

$1 = \cancel{-6k_2} - 16 \cancel{-3k_2^2} - 8k_2$
$-3k_2 - 8 + 3 \cancel{+6k_2} \cancel{+3k_2^2}$

$11k_2 = -22$

$k_2 = \cancel{-10}\ 2$

$k_1 = -2/3$