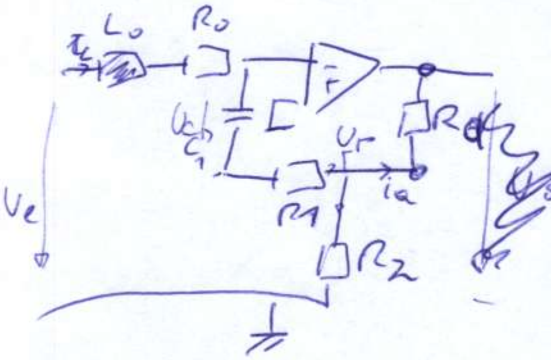
1) 26.2.2010

$L_0(\delta) = \frac{d_1}{d_0+\delta}$ $\quad \delta \geq 0$, $d_0, d_1$ ... konst $\quad \underline{\frac{\partial L}{\partial \delta}\frac{\partial \delta}{dt}}$

$\psi = L_0(\delta) i_L$, $u_L = \frac{d\psi}{dt} = L_0(\delta)\frac{di_L}{dt} - d_1(d_0+\delta)^{-2} i_L \cdot \frac{\partial\delta}{dt} = V_e - i_L R_0$




$V_e = u_L + i_L R_0 \rightarrow i_L = \frac{V_e - U_L}{R_0}$

$i_C = C \frac{du_C}{dt} - i_L$

$i_a = i_L - i_2 = i_L - \frac{U_P}{R_2}$

$U_a = U_P - i_a R_a = -U_C - i_L R_1 - i_L R_a$

$- \frac{U_C + i_L R_1}{R_2} \cdot R_a$

$U_a = -U_C\left(\frac{R_a}{R_2}+1\right) - i_L\left(\frac{R_1 R_a}{R_2} + R_1 + R_a\right)$

$i_a = u_a / R_0$

$i_a = +U_C \underbrace{\left(\frac{R_a}{R_2}+1\right)\frac{1}{R_a}}_{+K_C} + \frac{U_e}{R_0}\underbrace{\left(\frac{R_1 R_a}{R_2} + \frac{R_1}{R_a} + \frac{1}{R_a}\right)}_{+K_{ue}} - \frac{U_L}{R_0}\underbrace{\left(\frac{R_1 R_a}{R_2} + \frac{R_1}{R_a} + R_a\right)}_{-K_L}$

$i_a = U_C K_C + U_e K_{ue}$

$+ K_L \frac{d_1}{d_0+\delta}\frac{di_L}{dt} + d_1 \ln(d_0+\delta)\frac{d\delta}{dt} i_L K_L$

$u = \begin{bmatrix}\delta \\ u_e\end{bmatrix}$, $x = \begin{bmatrix}\delta \\ i_L \\ u_C\end{bmatrix}$ $\quad y = i_a$

$\dot{x} = f(x,u) =$

$U_P = -U_C - i_L R_1$

$i_a = i_L - \frac{U_P}{R_2} = i_L + \frac{U_C}{R_2} + i_L \frac{R_1}{R_2} =$

$u = \begin{bmatrix}\dot\delta \\ u_e\end{bmatrix}$

$x = \begin{bmatrix}\delta \\ i_L \\ u_C\end{bmatrix}$

$= U_e\left(\frac{1}{R_0} + \frac{R_1}{R_2 R_0}\right) + U_C/R_2 - U_L\left(\frac{1}{R_0} + \frac{R_1}{R_2 R_0}\right)$

$$\left|\begin{aligned}
y &= i_a = U_C \frac{1}{R_2} + i_L\left(\frac{R_1}{R_2}+1\right) \\
\dot{u}_C &= \frac{1}{C} i_L \\
\dot{i}_L &= \frac{U_e}{L_0} - i_L\frac{R_0}{L_0} + \frac{d_1}{(d_0+\delta)^2} i_L \frac{V}{L_0} \\
\dot{\delta} &= V
\end{aligned}\right| = \frac{U_e(d_0+\delta)}{d_1} - \frac{i_L R_0 (d_0+\delta)}{d_1} + \frac{i_L V}{(d_0+\delta)}$$

1b) $v \equiv 0$, $v_c = v_{cR}$ $\Rightarrow$ $\dot{v}_c = 0,\ \dot{i}_L = 0,\ \dot{\delta} = 0$

$0 = \frac{u_{cR}}{L_0}$ $\cancel{\quad}$ $\dot{i}_L = 0$

$v_{cR} = 0$ $\qquad 0 = \frac{v_c(d_0+\delta)}{d_1}$ entweder $u_{cR} = 0$ oder $\delta_R = -d_0$

$u_{cR}$ beliebig $\to$ unendlich viele Ruhelagen

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1c) $D = 0$

$$C = \frac{\partial}{\partial x} h(x_R, u_R) = \left[0,\ \frac{R_1}{R_2}+1,\ \frac{1}{R_2}\right]$$

$$B = \frac{\partial}{\partial u} f(x_R, u_R) = \left.\begin{bmatrix} 1 & 0 \\ 0 & \frac{(d_0+\delta)}{d_1} \\ 0 & 0 \end{bmatrix}\right|_{x_R, u_R}$$

$$A = \frac{\partial}{\partial x} f(x_R, u_R) = \begin{matrix} \dot{\delta} & 0 & 0 \\ \cancel{+\dot{\delta}\frac{i_L}{R_0(d_0+\delta)}\frac{d_1}{} - \frac{\dot{\delta}\, d_1 i_L}{(d_0+\delta)R}} & -\frac{R_0(d_0+\delta)}{d_1} & 0 \\ 0 & 1/C & 0 \end{matrix}$$

$$\frac{\partial}{\partial \delta} f(x_R, u_R) = \frac{d_0 u_{cR}}{d_1}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ \frac{u_{cR}}{d_1} & -\frac{R_0(d_0+\delta)}{d_1} & 0 \\ 0 & 1/C & 0 \end{bmatrix}$$

2 a) I) linear, zeitvariant

II) nichtlinear, zeitvariant

III) nichtlinear, zeitvariant

IV) ~~linear,~~ ze LTI

b) $y_k = u_k - \frac{b}{2} u_{k-1} - \frac{b}{5} y_{k-1}$

$$Y(z) = U(z) - \frac{b}{2} z^{-1} U(z) - \frac{b}{5} z^{-1} Y(z)$$

$$G(z) = \frac{Y(z)}{U(z)} = \frac{1 - \frac{b}{2} z^{-1}}{1 + \frac{b}{5} z^{-1}} = \frac{z - \frac{b}{2}}{z + \frac{b}{5}} = \frac{z}{z+\frac{b}{5}} - z^{-1} \frac{\frac{b}{2} z}{z + \frac{b}{5}}$$

$$g_k = G(z) = -\frac{5}{b}\left(-\frac{b}{5}\right)^k 1^k + \frac{5}{2}\left(-\frac{b}{5}\right)^{k-1} 1^{k-1}$$

$$= -\frac{5}{b} \frac{z}{\frac{z}{(-\frac{b}{5})} - 1} + \frac{5}{2} z^{-1} \frac{z}{\frac{z}{(-\frac{b}{5})} - 1}$$

c) BIBO stabil, wenn Pole in EK

$|b| < 5$

$b := -4$, Pole bei $+\frac{4}{5}$

Nst bei $0 = z + 2$, $z = -2$

-> nicht Phasenminimal

d) $u_k = \left(\sqrt{2} \sin\left(k \frac{5\pi}{4} - \frac{5\pi}{\sqrt{12}}\right)\right) \sigma_k + \sigma_{k-5}$

$$|G(e^{j5\pi/4})| = \left| \frac{e^{j5\pi/4} - b/2}{e^{j5\pi/4} + b/5} \right|$$

arg)

d) $T_{u,y} = \frac{1}{1+R_S G_S} = \frac{1}{1+\frac{1}{\sqrt{3}\,\cancel{8}}\frac{\cancel{10}\;2}{s\,(s\frac{\sqrt{3}}{3}+1)}} = \frac{s\,(s\frac{\sqrt{3}}{3}+1)}{\frac{2}{\sqrt{3}}+s\,(s\frac{\sqrt{3}}{3}+1)}$

$d(s) = 0{,}5/s \qquad Y(s) = \frac{0{,}5(s\frac{\sqrt{3}}{3}+1)}{\frac{2}{\sqrt{3}}+s\,(s\frac{\sqrt{3}}{3}+1)}$

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F_s = \lim_{s\to 0}\frac{0{,}5\,s\,(s\frac{\sqrt{3}}{3}+1)}{\frac{2}{\sqrt{3}}+s\,(s\frac{\sqrt{3}}{3}+1)} = \frac{\cancel{0{,}5}}{\frac{2}{\sqrt{3}}} = \cancel{0}$

$= \cancel{\frac{0{,}5\sqrt{3}}{2}} = \cancel{0{,}5}$

e)

$G_{S,neu} = \frac{10}{(s(\frac{\sqrt{3}}{3}+a)+1)\,(s+1)} = \frac{\cancel{30/\sqrt{3}}}{s+}$

$$\frac{\dfrac{\cancel{10}\;2}{\cancel{(s+1)}(s[\frac{\sqrt{3}}{3}+a]+1)}\;\dfrac{1}{\sqrt{3}\,\cancel{8}}\;\dfrac{\cancel{(1+s)}}{s}}{1+\dfrac{2}{\sqrt{3}}\dfrac{1}{s\,(1+s(\frac{\sqrt{3}}{3}+a))}} = \frac{1}{\frac{s\,(1+s(\frac{\sqrt{3}}{3}+a))\sqrt{3}}{2}+1}$$

$$= \boxed{\frac{1}{1+\frac{\sqrt{3}}{2}s+\frac{\sqrt{3}}{2}s^2\,(\frac{\sqrt{3}}{3}+a)}}$$

$= \cancel{\frac{7}{4a}}$

$\cancel{s^2+\frac{s}{(\frac{\sqrt{3}}{3}+a)}+\frac{2}{\sqrt{3}}\frac{1}{\frac{\sqrt{3}}{3}+a}=0}$

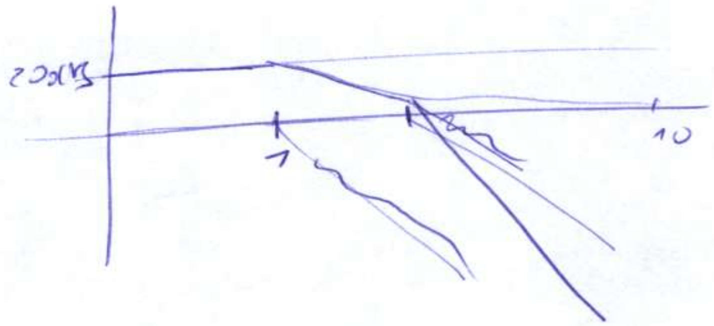
$\cancel{s_{12} = -\frac{1}{(\frac{\sqrt{3}}{3}+a)2} \pm \sqrt{\frac{1}{4(\frac{\sqrt{3}}{3}+a)^2}-\frac{2}{\sqrt{3}}\frac{1}{\frac{\sqrt{3}}{3}+a}}}$

$\frac{\sqrt{1-\frac{8}{\sqrt{3}}(\frac{\sqrt{3}}{3}+a)}}{2(\frac{\sqrt{3}}{3}+a)}$

3) $\frac{\sqrt{3}}{3}\frac{d^2y(t)}{dt^2} + \left(1+\frac{\sqrt{3}}{3}\right)\frac{dy}{dt} + y = 10u$

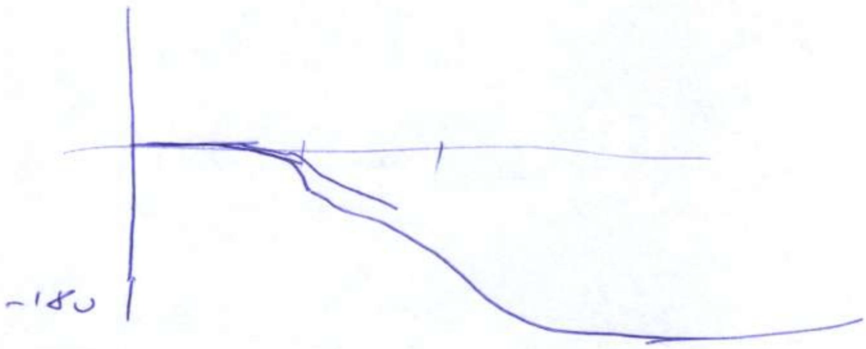
$$G(s) = \frac{10}{1+\left(1+\frac{\sqrt{3}}{3}\right)s + s^2\frac{\sqrt{3}}{3}} = \frac{10}{\left(s\frac{\sqrt{3}}{3}+1\right)(s+1)} = \frac{\frac{30}{\sqrt{3}}}{\left(s+\frac{3}{\sqrt{3}}\right)(s+1)}$$

$\to$ BIBO-stabil $\to$ Ph. minimal




$\omega_1 = 1$

$\omega_2 = \frac{3}{\sqrt{3}} = \sqrt{3}$




$PR + \ddot{u} = 70° \to PR = 60$

$\omega_c t_r = 1{,}5 \to \omega_c$

$\to$ I glied

$$R = \frac{1(1+sT)}{s \cdot \cancel{(\ldots)}}$$

$$\boxed{R = \frac{1}{\sqrt{3}5}\ \frac{1+s}{s}}$$

$$L_1\Big|_{\omega_c} = R_1 G = \frac{10}{\sqrt{3}\left(\sqrt{3}\frac{\sqrt{3}}{3}+1\right)(\sqrt{3}+1)}$$

$-\frac{\pi}{2} - \frac{\pi}{6} - \frac{\pi}{4} = -\frac{11\pi}{12} \hat{=} PR\ \frac{\pi}{12}$

$\frac{\pi}{3} - \frac{\pi}{12}$

$$\arg(1+sT) = \cancel{-\frac{\pi}{12}} \cancel{+\ldots} = +\frac{3\pi}{12} = \frac{\pi}{4}, \to T = 1$$

$$L_2 = \frac{10(1+s)}{\left(s\frac{\sqrt{3}}{3}+1\right)\cancel{(1+s)}\cdot s} \qquad |L_2|_{\omega_c} = \left|\frac{10}{\left(\frac{\sqrt{3}}{3}+1\right)}\right| = \frac{10}{\sqrt{1+\frac{1}{3}}} = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$$

$V = \frac{1}{\sqrt{3}5}$

ad 2d)

$$1 - \frac{8}{\sqrt{3}}\left(\frac{\sqrt{3}}{3} + a\right) \leq 0$$

$$1 \leq \frac{8}{3} + \frac{8a}{\sqrt{3}}$$

$$-\frac{5}{3} \cdot \frac{\sqrt{3}}{8} \leq a \quad \rightarrow a \geq -\frac{5}{8\sqrt{3}}$$

$\hookrightarrow$ Realteil d. Polstelle $< 0$, $\quad -\frac{1}{\left(\frac{\sqrt{3}}{3} + a\right) \cdot 2} < 0$ ?

$$a > -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$$

Routh Hurwitz

| | | |
|---|---|---|
| $s^2$ | $\left(\frac{\sqrt{3}}{3} + a\right)\frac{\sqrt{3}}{2}$ | $1$ |
| $s^1$ | $\frac{\sqrt{3}}{2}$ | $0$ |
| $s^0$ | $1$ | |

$$a > -\frac{1}{\sqrt{3}}$$

4a) i) $\mathcal{O}(C,A) = \begin{matrix} C \\ C\Phi \\ C\Phi^2 \\ \vdots \\ C\Phi^{n-1} \end{matrix}$ $\quad n=3$

$$\mathcal{O} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & -1 \\ 13 & 0 & -6 \end{bmatrix}$$

<u>Rang < 3</u>

$A^2$:

| | | | | | |
|---|---|---|---|---|---|
| | | | 3 | 0 | -1 |
| | | | 3 | -2 | 0 |
| | | | -4 | 0 | 3 |
| 3 | 0 | -1 | 13 | 0 | -6 |
| 3 | -2 | 0 | 3 | 4 | -3 |
| -4 | 0 | 3 | -24 | 0 | 13 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 2 | 0 | -1 | 2 | 0 | -1 |
| | | | 3 | -2 | 0 | 3 | -2 | 0 |
| | | | -4 | 0 | 3 | -4 | 0 | 3 |
| 1 | 0 | 0 | 2 | 0 | -1 | 8 | 0 | -5 |

ii) $[1\ 1\ 0]$, weil in 2. Spalte nur die 2. Zeile $\neq 0$ ist

4.b)

$$
\begin{array}{c|ccc|ccc}
 & -3 & 0 & 1 & -3 & 0 & 1 \\
 & 1 & -2 & 1 & 1 & -2 & 1 \\
 & 3 & 0 & 0 & 3 & 0 & 0 \\
\hline
c^T:\ 1\ 1\ 0 & -2 & -2 & 2 & 10 & 4 & -4
\end{array}
$$

$$B = \begin{bmatrix} c^T \\ c^T \Phi \\ c^T \Phi^2 \end{bmatrix}$$

$$O = \begin{bmatrix} 1 & 1 & 0 \\ -2 & -2 & 2 \\ 10 & 4 & -4 \end{bmatrix}$$

$p_{gesoll}(z) = z^3$, weil minim. Zahl von Abtastschritten $\rightarrow$ 3 Schritte

$v_1 = O^{-1} e_n$

$$O^{-1} = \begin{bmatrix} x & x & 2/12 \\ x & x & 2/12 \\ x & x & 0/12 \end{bmatrix}$$

$\det O = 8 + 20 - 8 - 8 = 12$

$$v_1 = \frac{1}{6}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\hat{k} = (-\hat{p}_0 E - \hat{p}_1 \Phi - \hat{p}_2 \Phi^2 - \Phi^3) \cdot v_1 = -\Phi^3 v_1$$

$$
\begin{array}{ccc|ccc|ccc}
-3 & 0 & 1 & -3 & 0 & 1 & -3 & 0 & 1 \\
1 & -2 & 1 & 1 & -2 & 1 & 1 & -2 & 1 \\
3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 \\
\hline
-3 & 0 & 1 & 12 & 0 & -3 & -45 & 0 & 12 \\
1 & -2 & 1 & -2 & 4 & -1 & \cancel{-15}\,7 & -8 & +2 \\
3 & 0 & 0 & -9 & 0 & 3 & 36 & 0 & -9
\end{array}
$$

$$\hat{k} = -\frac{1}{6}\begin{bmatrix} -45 \\ 15 \\ 36 \end{bmatrix} = \boxed{\begin{bmatrix} 45/6 \\ -15/6 \\ -6 \end{bmatrix}}$$

$$e_{k+1} = (\Phi + \hat{k} c^T) e_k$$

$$= \left( \begin{bmatrix} -3 & 0 & 1 \\ 1 & -2 & 1 \\ 3 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 45/6 & 45/6 & 0 \\ -15/6 & -15/6 & 0 \\ -6 & -6 & 0 \end{bmatrix} \right) e_k$$

$$e_{k+1} = \begin{bmatrix} 4{,}5 & 7{,}5 & 1 \\ -1{,}5 & -4{,}5 & 1 \\ -3 & -6 & 0 \end{bmatrix} e_k$$

4c) $\dot{x}_1 - \frac{x_1}{2} = \pi(50x_2 + 2u)$

$$2\dot{x}_2 = -100\pi x_1 + x_2 - u/2$$

$$\dot{x} = \begin{bmatrix} 1/2 & 50\pi \\ -50\pi & 1/2 \end{bmatrix} x + \begin{bmatrix} 2 \\ -\cancel{1/4} \end{bmatrix} u$$

$$y = [1 \quad 1]\, x$$

ii)

$$\cancel{\phi(t) = \exp(At) = E + At + }$$

Vgl mit $A = \begin{pmatrix} \alpha_1 & \beta_1 \\ -\beta_1 & \alpha_1 \end{pmatrix}$

$$\phi(T_a) = \begin{pmatrix} e^{t/2}\cos 50\pi & e^{T/2}\sin 50\pi \\ -e^{t/2}\sin 50\pi & e^{t/2}\cos 50\pi \end{pmatrix} -$$

$$= \begin{bmatrix} \cos 50\pi & \sin 50\pi \\ -\sin 50\pi & \cos 50\pi \end{bmatrix} e^{t/2}$$

$$50\pi \neq \frac{\ell\pi}{T_a}$$

$T_a \neq \ell/50$ für $\ell = \pm 1, \pm 2, \ldots$