12.12.08 / 1) $q_1 = A_s\sqrt{2gh}$ $\quad u = \begin{bmatrix} q_{in} \\ F \end{bmatrix}$

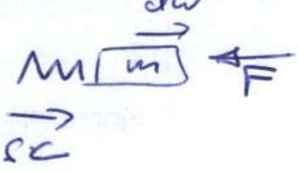
$A_s = s\cdot b$

$y = q_{out}$

Totzeit $T = \frac{l}{v} = \frac{2l}{\omega_B\cdot d_B}$ $\quad x = \begin{bmatrix} s \\ \omega \\ h \end{bmatrix}$

$\dot s = \omega$

$m\cdot\dot\omega = F - sc - d\omega$



$\dot\omega = \frac{F}{m} - s\frac{c}{m} - \omega\frac{d}{m}$

$[q] = \frac{m^3}{s}$

$\dot V = \dot h\cdot A = q_{in} - q_1$

$\dot h = \frac{q_{in}}{A} - \frac{q_1}{A} = \frac{q_{in}}{A} - \frac{b\sqrt{2g}}{A}\cdot s\cdot\sqrt{h}$

$y = q_{out} = q_1(t-T) = \sqrt{2g}\,b\;s(t-T)\cdot\sqrt{h(t-T)}$

b) $F_R = konst$, $q_{in,R} = konst$

$\omega = 0$

$0 = \frac{F_R}{m} - s_R\cdot\frac{c}{m} \rightarrow s_R = \frac{F_R}{c}$

$0 = \frac{q_{in,R}}{A} - \frac{b\sqrt{2g}}{A}\cdot\frac{F_R}{c}\cdot\sqrt{h_R} \rightarrow h_R = \left(\frac{q_{in,R}\,c}{F_R\cdot b\sqrt{2g}}\right)^2$

$y = \cancel{\sqrt{2g}\,b}\;\cancel{\frac{F_R}{c}}\;\frac{q_{in,R}\,c}{F_R\,b\sqrt{2g}} = q_{in,R}$

linearisier

$$B = \frac{d f(x, n, u_n)}{du} = \begin{bmatrix} 0 & 0 \\ 0 & 1/m \\ 1/A & 0 \end{bmatrix} \qquad d = 0$$

$$C^T = \frac{d g}{dx} = \cancel{\left[\sqrt{2g}\, b \sqrt{h_R} \,\middle|\, 0 \,\middle|\, \frac{\frac{1}{2}\sqrt{2g}\, b s_R}{\sqrt{h_R}} \right]} \quad \sqrt{2g}\, b \sqrt{h(t-T)}\, s(t-T)$$

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ -c/m & -d/m & 0 \\ -\frac{b\sqrt{2g}}{A}\sqrt{h_R} & 0 & -\frac{b\sqrt{2g}\, s_R}{2A\sqrt{h_R}} \end{bmatrix}$$

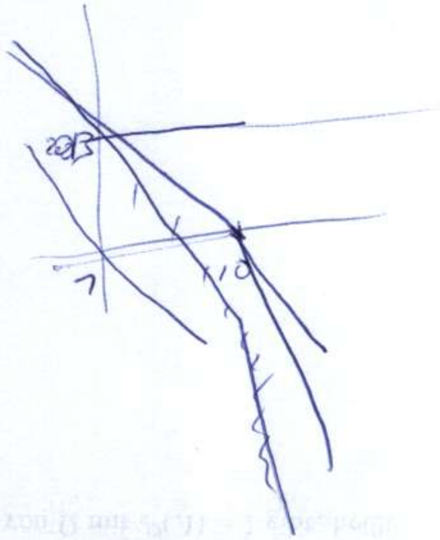
c) Totzeit

$$G(s) = e^{-sT}$$

e) ~~d_i~~ 12.12.08

~~$\phi = e^{AT_a}$~~

1) $G(s) = \dfrac{1}{\frac{s^2}{100} + \frac{s}{10}} = \dfrac{10}{s(\frac{s}{10}+1)}$



b) $t_r = 0{,}15\,s$

$\omega_c t_r = 1{,}5 \quad \omega_c = 10$

$PR = 45 = \frac{\pi}{4}$

$|\arg(G(s))|_{\omega_c = 1} = 0 - \frac{\pi}{2} - \frac{\pi}{4} = -\frac{3\pi}{4} \rightarrow PR = \frac{\pi}{4}$ ☺

$\boxed{R(s) = V = \sqrt{2}}$

$|L(s)|_{\omega_c} \overset{!}{=} 1 = \dfrac{V\,10}{10(\sqrt{2})} \rightarrow V = \sqrt{2}$

c) $T_{d,y} = \dfrac{G(s)}{1+RG} = \dfrac{\frac{10}{s(s/10+1)}}{1 + \frac{10\sqrt{2}}{s(s/10+1)}} = \dfrac{10}{10\sqrt{2} + s(s/10+1)}$

1) $d(s) = \dfrac{0{,}25}{s}$

$\lim_{s \to 0} \dfrac{0{,}25 \cdot 10}{\sqrt{2} + s(s/10+1)} = \dfrac{2{,}5}{\sqrt{2}}$

2) $|G(s)|_{s=i5} = \dfrac{10}{\sqrt{(10\sqrt{2} - \frac{25}{10})^2 + 5^2}} = K$

$\arg(s)|_{s=i5} = -\arctan \dfrac{5}{(10\sqrt{2} - \frac{25}{10})}$

2d) i) $R(s) = \frac{s-1}{s}$ $\quad G(s) = \frac{s+3}{s^2+s-2}$

$$T_{r,y} = \frac{RG}{1+RG} = \frac{\frac{s-1}{s}\frac{s+3}{s^2+s-2}}{1+\frac{s-1}{s}\frac{s+3}{s^2+s-2}} = \frac{(s-1)(s+3)}{(s-1)(s+3)+s(s^2+s-2)} =$$

$$= \frac{s^2-3+2s}{s^2-3+2s+s^3+s^2-2s} = \frac{s^2-3+2s}{\underbrace{s^3+2s^2+0s-3}}$$

kein Hurwitzpolynom → nicht BIBO

ii) nein, da G(s) nicht stabil ist

3)

b) $\dot{x} = t + 2x + u$
$y = x$ } nicht linear, ~~aber~~ und zeitvariant

Σ_2 $\dot{x} = 2tx + u$
$y = x$ } nicht linear, zeitvariant

c) $\det(\lambda E - A) = 0$

$$\begin{vmatrix} \lambda & 0 & 3 \\ 0 & \lambda-\alpha+2 & 0 \\ -\alpha & 0 & \lambda+\alpha+3 \end{vmatrix} = \lambda(\lambda-\alpha+2)(\lambda+\alpha+3) + \alpha 3(\lambda-\alpha+2) = 0$$

$$\rightarrow (\lambda-\alpha+2)(\lambda^2+\lambda(\alpha+3)+\alpha 3) = 0$$

$\lambda_1 = \alpha - 2$

$\lambda_2 = -\frac{\alpha+3}{2} \pm \sqrt{\frac{(\alpha+3)^2}{4} - \alpha 3}$

1:) $\alpha < 2$, $\alpha > 0$

$= \frac{-\alpha+3}{2} \pm \frac{(\alpha-3)}{2} < 0$

WC: $\frac{(\alpha+3)^2}{4} - \alpha 3 > 0$

$(-\alpha+3) \pm (\alpha-3) < 0$

$\alpha^2 + 6\alpha + 9 - 12\alpha > 0$ $\quad$ 1) $-\alpha-3+\alpha-3 < 0$ ✓

$\alpha^2 - 6\alpha + 9 > 0$ $\quad$ $-\alpha-3-\alpha+3 < 0$

~~$(\alpha-3)^2 > 0$ → gilt immer~~ $\alpha^2 - 6\alpha + 9 = (\alpha-3)^2 > 0$

4) a) $w_i^T A = \lambda_i w_i^T$

$A^T w = \lambda_i w$ $\qquad \begin{vmatrix} \lambda-2 & \cancel{0}-1 \\ 0 & \lambda-1 \end{vmatrix} = (\lambda-2)(\lambda-1) = 0$

$\cancel{\lambda^2 - 3\lambda + 2} = 0$

$\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} w = \lambda_i w$ $\qquad \lambda_1 = 2 \quad n = 1$

$\lambda_1 = 1 \quad n = 1$

$\lambda = 2$ $\quad \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 2w_1 \\ 2w_2 \end{pmatrix}$

w_1 beliebig $\qquad w_{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$w_1 + w_2 = 2w_2$

$w_1 = w_2$

$\lambda = 1$

$\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

$2w_1 \cancel{\ldots} = w_1 \qquad w_{(1)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$w_1 + w_2 = w_2$

$\rightarrow w_1 = 0 \quad w_2$ beliebig

PBH: $w_i^T b \neq 0 \quad \forall w_i$

erreichbar

$(1\ 1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \neq 0$ ✓

$(0\ 1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \neq 0$ ✓

b) $v_1^T = e_2^T R^{-1} \qquad R = (\Gamma, \Phi\Gamma) = \begin{pmatrix} 1 & 4 \\ 2 & 2 \end{pmatrix}$

$v_1^T = [-2 \quad 1] \cdot -\frac{1}{6} \qquad R^{-1} = \begin{pmatrix} 2 & -4 \\ -2 & 1 \end{pmatrix}\left(-\frac{1}{6}\right)$

$k^T = \cancel{\ldots} \qquad (\lambda - \frac{1}{2})\cdot(\lambda - \frac{2}{3}) = \lambda^2 - \frac{7}{6}\lambda + \frac{1}{3}$

$-\frac{1}{3} v_1^T + \frac{7}{6} v_1^T \phi - v_1^T \phi^2$

$k^T = +\frac{1}{18}(-2\ 1) - \frac{7}{36}[-4\ \ -1] + \frac{1}{6}[-8\ \ -5] \qquad \begin{array}{cc|cc|cc} -2 & 1 & 2 & 1 & 2 & 1 \\ & & 0 & 1 & 0 & 1 \\ & & -4 & -1 & -8 & -5 \end{array}$

$k = \begin{bmatrix} -\frac{2}{18} + \frac{7}{9} - \frac{8}{6} \\ \frac{1}{18} + \frac{7}{36} - \frac{5}{6} \end{bmatrix} = \frac{1}{18}\begin{bmatrix} -12 \\ -\frac{21}{2} \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -4 \\ -\frac{7}{2} \end{bmatrix}$

ad 4b) $\ell = \dfrac{1}{c^T (E - \phi - \Gamma k^T)^{-1} \Gamma}$

$E - \phi - \Gamma k^T =$

$$\begin{bmatrix} -6/18 & -15/36 \\ 12/9 & 21/18 \end{bmatrix}$$

$$\begin{array}{c|cc} & -\frac{12}{18} & -\frac{21}{36} \\ \hline 1 & -12/18 & -21/36 \\ 2 & -12/9 & -21/18 \end{array}$$

$$-\text{"}-^{-1} = \frac{1}{-\frac{\cancel{6}\cdot \cancel{21}}{18 \cdot \cancel{18}} + \frac{\cancel{15}\cdot \cancel{12}}{3\cancel{9}\cdot \cancel{36}}} \begin{bmatrix} 21/18 & 15/36 \\ -12/9 & -6/18 \end{bmatrix}$$

$$= \frac{18}{3} \begin{bmatrix} 21/18 & 15/36 \\ -12/9 & -6/18 \end{bmatrix} = \begin{bmatrix} 7 & 5/2 \\ -8 & -2 \end{bmatrix}$$

$c^T \cdot (-\text{"}-) = (7 \quad 5/2)$

$-\text{"}- \cdot \Gamma = 7 + 5 = 12$

$$\boxed{\ell = 1/12}$$

c) $G(z) = c^T \underbrace{(zE - \phi)^{-1}} \Gamma$

$$\begin{bmatrix} z-2 & -1 \\ 0 & z-1 \end{bmatrix}^{-1} = \begin{bmatrix} z-1 & 1 \\ 0 & z-2 \end{bmatrix} \frac{1}{(z-1)(z-2)} = \begin{bmatrix} \frac{1}{z-2} & \frac{1}{(z-1)(z-2)} \\ 0 & \frac{1}{(z-1)} \end{bmatrix}$$

$$G(z) \quad c^T \cdot -\text{"}- \Gamma = \begin{bmatrix} \frac{1}{z-2} & \frac{1}{(z-1)(z-2)} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{z-2} + \frac{2}{(z-1)(z-2)}$$

$$= \frac{(z-1)+2}{(z-1)(z-2)} = \boxed{\frac{(z+1)}{(z-1)(z-2)}}$$

d) $x_{k+2} + 2x_{k+1} + x_k = -u_k$

$$y_k = x_{k+1} + 3x_k$$

$$z_{k,1} = x_k$$

$$z_{k,2} = x_{k+1}$$

$$z_{k+1,1} = z_{k,2}$$

$$z_{k+1,2} = x_{k+2} = -u_k - x_k - 2x_{k+1} =$$

$$= -u_k - z_{k,1} - 2z_{k,2}$$

$$z_{k+1} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} z_k + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 3 & 1 \end{bmatrix} z_k$$