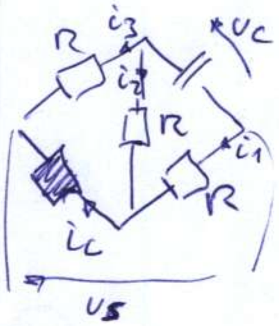
2016.08 $L(i_L) = L_0 + L_1 i_L^2 \qquad L_0, L_1 > 0$



$u = u_S \qquad x = \begin{bmatrix} u_C \\ i_L \end{bmatrix}$

$y = u_C$

$y = c^T x = [1\ 0]\, x$

$\cancel{u_L = L(i_L)\frac{di_L}{dt}}$

$\phi = LI$

$$u = \frac{d\phi}{dt} = \frac{d\phi}{di_L}\frac{di_L}{dt} = L\frac{di}{dt} + i_L\frac{di_L}{dt} \cdot L_1 2 i_L$$

$$= L_0 \dot i + L_1 \dot i\, i_L^2 + L_1 2\, i_L^2 \dot i$$

$$\boxed{u_L = L_0 \dot i_L + 3L_1 \dot i_L i_L^2}$$

$i_L = i_1 + i_2 \quad , \quad i_C = i_3 + i_2 = C\cdot\frac{du_C}{dt}$

$u_S = u_C + i_2 R + u_L$

$u_S = (i_1 - i_2 + i_3)R =$

$u_S = u_C + i_3 R$

$u_S = i_1 \cdot R + u_L$

$$\dot u_C = \frac{i_3}{C} + \frac{i_2}{C} = \frac{u_S - u_C}{RC} + \frac{u_S - u_C - u_L}{RC}$$

$$\dot u_C = \frac{2}{RC}u_S - \frac{2u_C}{RC} - \frac{L_0}{RC}\dot i_L - \frac{3L_1}{RC}\dot i_L i_L^2$$

$$i_L = \frac{u_S - u_L}{R} + \frac{u_S - u_C - u_L}{R}$$

$$i_L = \frac{2u_S}{R} - \frac{2u_L}{R} - \frac{u_C}{R} = 2\frac{u_S}{R} - \frac{2L_0}{R}\dot i_L - \frac{6L_1}{R}\dot i_L i_L^2 - \frac{u_C}{R}$$

$$\dot i_L\left(\frac{2L_0}{R} + \frac{6L_1}{R}i_L^2\right) = \frac{2u_S}{R} - i_L - \frac{u_C}{R}$$

$$\dot i_L = \frac{2u_S}{2L_0 + 6L_1 i_L^2} - \frac{R i_L}{2L_0 + 6L_1 i_L^2} - \frac{u_C}{2L_0 + 6L_1 i_L^2}$$

$$\dot u_C = \frac{2u_S}{RC} - \frac{2u_C}{RC} - \frac{\dot i_L (L_0 + 3L_1 i_L^2)}{RC} = \frac{2u_S}{RC} - \frac{2u_C}{RC} - \left(\frac{u_S}{RC} - \frac{i_L}{2C} - \frac{u_C}{2RC}\right)$$

$$\boxed{\dot u_C = \frac{u_S}{RC} - \frac{3u_C}{2RC} + \frac{i_L}{2C}}$$

b) $u = u_R = 0$

$\dot{x} = 0$

$$0 = -\frac{R i_{LR} + u_{CR}}{2L_0 + 6L_1 i_L^2} \rightarrow R i_{LR} = -u_{CR} \rightarrow i_{LR} = -\frac{u_{CR}}{R}$$

$$0 = -\frac{3u_{CR}}{2RC} + \frac{i_{LR}}{2C} \qquad \frac{3u_{CR}}{R} = i_{LR}$$

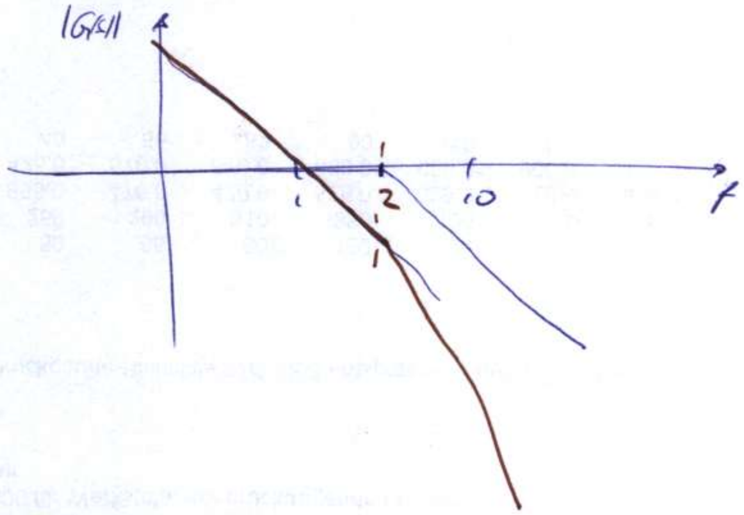
$$\boxed{\Leftrightarrow u_{CR} = 0,\ i_{LR} = 0}$$

c) $$b = \frac{df}{du} = \cancel{\begin{bmatrix} \frac{1}{RC} \\ \frac{1}{L_0} \end{bmatrix}} \begin{bmatrix} 1/RC \\ \frac{1}{L_0 + 3L_1 i_{LR}^2} \end{bmatrix}$$

$$A = \frac{df}{dx} = \begin{bmatrix} -\frac{3}{2RC} & \frac{1}{2C} \\ -\frac{1}{2L_0 + 6L_1 i_{LR}^2} & A_{22} \end{bmatrix}$$

$$A_{22} = \frac{d\dot{i}_L}{di_L} = \frac{2u_S - Ri_L - u_C}{2L_0 + 6L_1 i_L^2} = \frac{-R(2L_0 + 3L_1 i_{LR}^2) - (2u_{SR} - Ri_{LR} - u_{CR}) \cdot 12 L_1 i_{LR}}{(2L_0 + 6L_1 i_{LR}^2)^2}$$

2a) $G(s) = \dfrac{1}{s(0{,}5s+1)}$ $\qquad \omega_1 = 2$



b) $\omega_c t_R = 1{,}5 \rightarrow \omega_c = 1$

$\hat{u} = 30\%$ $\qquad \hat{u} + PR = 70$

$\hookrightarrow PR = 40^\circ$

$e_\infty = 0 \Rightarrow$ I-Anteil in R

$$R = \frac{K(1+sT_1)}{1+sT_1 n}$$

$$\arg(L)\big|_{\omega_c} = \arg(K G(j\omega))\big|_{\omega_c} = 0 - \pi/2 - 25^\circ = -115^\circ$$

$$\arg\left(\frac{1+sT_1}{1+snT_1}\right) = \cancel{-25^\circ + 10^\circ =} -15^\circ \Rightarrow \operatorname{atan} T_1 - \operatorname{atan} nT_1 = 15^\circ$$

$\cancel{\operatorname{atan} T_1 = 15^\circ = \pi/12}$

$\cancel{T_1 = 2 - \sqrt{3}}$

$\Delta\varphi_{Lead} = -15$

$\tan(-15) = -(2-\sqrt{3})$

$$n_{Lead} = 1 + 2(\sqrt{3}-2)\left(\sqrt{3}-2-\sqrt{(\sqrt{3}-2)^2+1}\right)$$

$$= \qquad \sqrt{3-4\sqrt{3}+4+1} \qquad -2\sqrt{2-\sqrt{3}}$$

$$T_{Lead} = \frac{1}{\sqrt{n_{Lead}}} = \ldots\ldots$$

3b) $\mathcal{O} = \begin{bmatrix} c^T \\ c^T A \\ \vdots \\ c^T A^{n-1} \end{bmatrix}$ $\qquad z = Tx$

$$\dot{x} = Ax + Bu = \cancel{T}TAT^{-1}z + TBu$$

$$y = c^T x + du = c^T T^{-1} z + du$$

$$\tilde{\mathcal{O}} = \begin{bmatrix} c^T T^{-1} \\ \cancel{c^T A T^{-1}}\ c^T T A T^{-1} \\ c^T T^{-1} TAT^{-1} \ldots ^2 \end{bmatrix} = \begin{bmatrix} c^T \\ c^T A \\ c^T A^2 \\ \vdots \\ c^T A^{n-1} \end{bmatrix} T^{-1}$$

Rang n

$$\underbrace{c^T T^{-1}}\ \underbrace{(TAT^{-1})(TAT^{-1})} \cdots n-1 \text{ mal}$$

$$\text{Rang } \tilde{\mathcal{O}} = \min\{\text{Rang } \tilde{\mathcal{O}}, \text{Rang } T^{-1}\} =$$

c) $x_{k+1} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k$

$$\det \begin{bmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{bmatrix} = (2-\lambda)(4-\lambda) - 3 = 8 + \lambda^2 - 6\lambda - 3 = \lambda^2 - 6\lambda + 5$$

$$\lambda_{1,2} = 3 \pm \sqrt{4} = \underline{1, 5}$$

$$w_i^T A = \lambda_i w_i^T$$

$$A^T w_i = \lambda_i w_i$$

$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \Rightarrow \begin{matrix} 2w_1 + w_2 = w_1 \\ w_1 = -w_2 \\ -4w_1 = w_2 \end{matrix}$ $\qquad \boxed{\vec{w}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$ $\rightarrow$ nicht erreichbar da $w_1^T b = 0$

$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 5w_1 \\ 5w_2 \end{bmatrix}$ $\qquad \begin{matrix} 2w_1 + w_2 = 5w_1 \rightarrow w_2 = 3w_1 \\ 3w_1 + 4w_2 = 5w_2 \quad \text{da } 5w_1 - w_1 \end{matrix}$ $\qquad \boxed{\vec{w}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}}$

d) $w_i^T A = \lambda_i w^T$ $\qquad (A \cdot B)^T = B^T A^T$

$(w_i^T A)^T = \lambda_1 w$

$A^T w = \lambda_i w$

e) $y_{k+3} + 3\sin(y_{k+2}) + 5\sqrt{u_k} = \frac{\pi}{10} e^{y_{k+1}}$

~~$\sin y_{k+2} = z_1$~~

~~$e^{y_{k+1}} = z_2$~~

$y_k = z_{0,k}$

$y_{k+1} = z_{1,k}$

$y_{k+2} = z_{2,k}$

$z_{k+1} = f(z_k, u_k)$

$z_k \to y_k = g(z_k)$

$$\boxed{\begin{aligned} z_{0,k+1} &= y_{k+1} = z_1 \\ z_{1,k+1} &= y_{k+2} = z_2 \\ z_{2,k+1} &= y_{k+3} = -3\sin z_2 + \frac{\pi}{10} e^{z_1} - 5\sqrt{u_k} \\ y_k &= z_0 \end{aligned}}$$

f) Nyquist

A) ja, $\omega = 0$ liegt im positiv

B) ja, da für $\omega \to \infty$ geht Ok gegen 0

C) $\beta = 0$ und kurvit

D) ja

F) ja

$\phi = \arg(L(\mathrm{I}\omega_c)) + \pi = $ ~~positiv~~ negativ für L_1 $(\sim 45°)$

positiv für L_2 $(\sim + 90°)$

4) $x_{k+1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 5 & -2 & 1 \\ -8 & 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_k$

$y_k = (0 \; 0 \; 1) \, x_k$

$R = [\cancel{F}b, Ab, A^2 b] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 5 \\ 0 & 1 & -2 \end{bmatrix} \to \mathrm{Rang}(R) < 3$

$[\Gamma, \phi\Gamma, \phi^2\Gamma]$

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 5 & -2 & 1 \\ -8 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 5 & -2 & 1 \\ -8 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

$Ab = \phi\Gamma$: $\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

ϕ^2: $\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ -3\frac{1}{2} & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

$A^2b = \phi^2\Gamma$: $\begin{bmatrix} 0 \\ 5 \\ -2 \end{bmatrix}$

$\phi\phi\Gamma$: $\begin{bmatrix} 0 \\ 5 \\ -2 \end{bmatrix}$

b) b – d) ausgelassen

e) $x_{k+1} = \phi x_k + \Gamma u_k = \phi x_k + \Gamma \, [K_x^T \; K_I] \begin{bmatrix} z_{1,k} \\ z_{I,k} \end{bmatrix} + \Gamma k_p (r_k - y_k)$

$y_k = c^T \cdot x_k$