$\boxed{2010/02/05}$

1a) $\dot{m}_{out} = \rho A_D \sqrt{2gh}$ $\qquad F_R = d \cdot w$ $\qquad x = \begin{vmatrix} h \\ s \\ w \end{vmatrix}$

$F_c = c \cdot s$

$w_{out} = \dfrac{\dot{m}_{out}}{\rho A_D} = \sqrt{2gh}$ $\qquad u = \dot{m}_{in}$

$y = s$

$p = m_{out} w_{out}$

$F_m = \dfrac{dp}{dt} = \dfrac{dm}{dt} w_{out} + \underbrace{\dfrac{dw_{out}}{dt}}_{\emptyset} m = \dfrac{\dot{m}_{out}}{\cancel{\rho A_D}} \cancel{dm} \rho A_D \sqrt{2gh}$

$\dot{h} A \cdot \rho = \dot{m}_{in} - \dot{m}_{out}$

$\dot{h} = \dfrac{\dot{m}_{in}}{A\rho} - \sqrt{2gh}$

$\dot{s} = w$

$\dot{w} = \dfrac{1}{m}\left(F_m - dw - cs\right) = \dfrac{\rho A_D 2gh}{m} - \dfrac{dw}{m} - \dfrac{cs}{m}$

$\boxed{y = s}$

b) $\begin{vmatrix} \dot{h} \\ \dot{s} \\ \dot{w} \end{vmatrix} = \begin{vmatrix} -\sqrt{2gh} + \dfrac{\dot{m}_{in}}{A\rho} \\ w \\ \rho \dfrac{A_D 2gh}{m} - \dfrac{dw}{m} - \dfrac{cs}{m} \end{vmatrix} = f(x, u)$

c) $\dot{m}_{in,R} = const$

$w = 0$

$0 = \rho \dfrac{A_D 2gh_R}{m} - \dfrac{cs_R}{m} \rightarrow \boxed{s_R = \dfrac{\rho A_D 2g h_R}{c}}$

$\sqrt{2gh_R} = \dfrac{m_{in,R}}{A\rho} \rightarrow \boxed{h_R = \left(\dfrac{m_{in,R}}{A\rho}\right)^2 \dfrac{1}{2g}}$

adc) $y = \underbrace{(0\ 1\ 0)}_{c^T} x$, $d=0$, $b = \begin{pmatrix} \frac{1}{A_S} \\ 0 \\ 0 \end{pmatrix}$

$$A = \frac{\partial f(x_R, u_R)}{\partial x} = \begin{bmatrix} -\frac{1}{2}\frac{2g}{\sqrt{2gh_R}} & 0 & 0 \\ 0 & 0 & 1 \\ \frac{2g\rho A_0}{m} & -c/m & -d/m \end{bmatrix}$$

evt h_R einsetzen

2a)
$$\phi(s) = (sE - A)^{-1} = \begin{vmatrix} s-1 & -3 \\ 1 & s-5 \end{vmatrix}^{-1} = \frac{1}{(s-1)(s-5)+3} \begin{vmatrix} s-5 & +3 \\ -1 & s-1 \end{vmatrix}$$

$$= \frac{1}{\underbrace{s^2 - 6s + 8}} \begin{bmatrix} s-5 & +3 \\ -1 & s-1 \end{bmatrix} = \begin{bmatrix} \frac{s-5}{s^2-6s+2} & \frac{+3}{s^2-6s+2} \\ \frac{-1}{s^2-6s+2} & \frac{s-1}{s^2-6s+2} \end{bmatrix}$$

$s_{1,2} = 3 \pm \sqrt{1} = 4, 2$

$\mathcal{L}^{-1}$ Rücktransformieren ? $\rightarrow \phi(t)$

e) $G(z) = c^T (zE - \phi)^{-1} \Gamma = (1\ \ -1) \begin{vmatrix} z+\frac{5}{2} & -5 \\ 1 & z+2 \end{vmatrix}^{-1} \begin{vmatrix} 1 \\ 0 \end{vmatrix}$

$$= (1\ \ -1) \frac{1}{(z+\frac{5}{2})(z+2)+5} \begin{vmatrix} z+2 & 5 \\ -1 & z+\frac{5}{2} \end{vmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$= \frac{\cancel{z+2+1}}{z^2 + \frac{1}{2}z \cancel{\ldots}} = \cancel{\frac{z \ldots -1}{z^2 + \frac{9}{2}z + 10}} \quad \frac{z-1}{z^2 + \frac{1}{2}z \cancel{\ldots}} = \frac{z-1}{z(z+\frac{1}{2})}$$

ii)
$$u(k) = \sqrt{10}\, 1^k + \frac{5}{2}\cos\left(\frac{\pi}{2}k + \frac{\pi}{10}\right)$$

$y_{k1}(1^{(k)}) = 0$

$$|G(z)|_{z = e^{j\pi/2} = j} = \frac{2\sqrt{2}}{\sqrt{5}}$$

$$\arg(z)_{z=j} = +\frac{3\pi}{4} - \frac{\pi}{2} - \operatorname{atan} 2$$

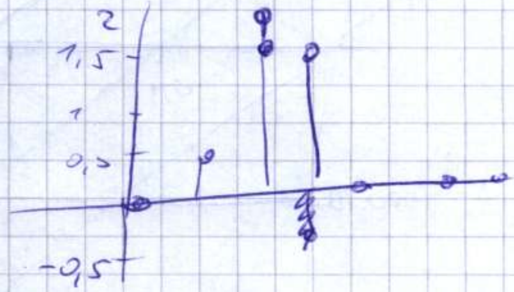
iii) nein $\rightarrow$ Shannon

2c) $|g_k| = |0, \frac{1}{2}, 1, 1, 1|$

$(u_k) = (1, 2, -3, 0, 0)$

$y_0 = 0, \; y_1 = \frac{1}{2}, \; y_2 = 1 + \cancel{\frac{1}{2}}, \; y_3 = \cancel{-\frac{3}{2} + 2 + 1} \; 1 + 2 - \frac{3}{2} = 1{,}5$

$y_4 = -3 + 2 + 1 = 0$



ii) nicht BIBO stabil, weil $\sum |g_k| \to \infty$

Hankelm.: $\begin{vmatrix} \frac{1}{2} & 1 \\ 1 & 1 \end{vmatrix} = \frac{1}{2} - 1 = -\frac{1}{2} \neq 0$

$\to$ voller Rang
erreichbar + beobachtbar

3)

$t_r = 1{,}5 s \to \omega_c = 1$

$\hat{u} = 10 \to \Phi = 60° \to \varphi = -\frac{2\pi}{3} = -\frac{8\pi}{12}$

$R_1 = \frac{P}{(s + \frac{1}{2-\sqrt{3}})}$

$\arg(G(s))\big|_{s=j1} = \cancel{1} - \pi/2 - \underbrace{\arctan(2-\sqrt{3})}_{-\pi/12} = -\frac{7\pi}{12}$

$\Delta\varphi = -\pi/12$

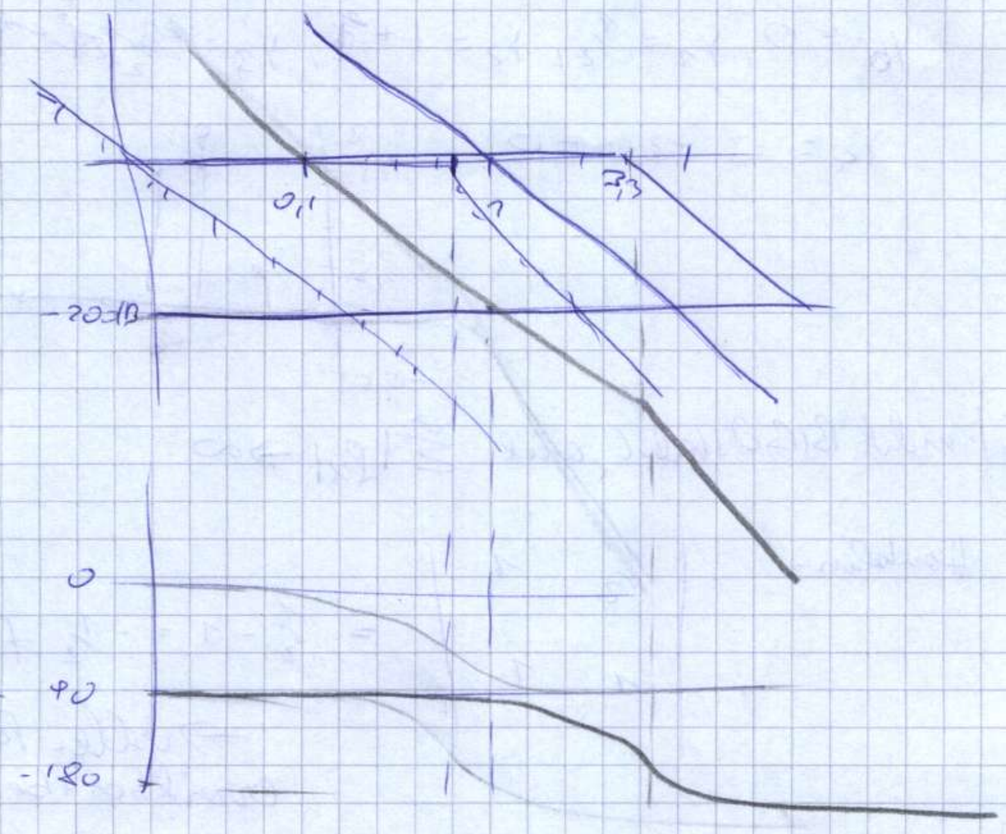
$|L_1|_{\omega_c} = \left|\frac{1}{3s(1 + \frac{1}{2-\sqrt{3}})^2}\right| = \frac{1}{3(1 + \frac{1}{(2-\sqrt{3})^2})} = \frac{1}{P}$

$P = 3\left(1 + \frac{1}{(2-\sqrt{3})^2}\right)$

$R = \frac{3\left(1 + \frac{1}{4 - 4\sqrt{3} + 3}\right)}{\left(s + \frac{1}{2-\sqrt{3}}\right)} = \frac{3\left(2-\sqrt{3} + \frac{1}{(2-\sqrt{3})}\right)}{1 + s(2-\sqrt{3})}$

a) $G(s) = \frac{2-\sqrt{3}}{3} \cdot \frac{1}{s} \cdot \frac{1}{1+s(2-\sqrt{3})}$

$20 \cdot \log \frac{2-\sqrt{3}}{3} = -20\,dB$, $\omega_c = \frac{1}{0{,}3} = 3{,}3$



b) → Regler 2 ~~(Siehe)~~ (siehe oben)

c) $- 1 \cdot$ ———

d)

$$\lim_{s \to 0} s \cdot \left(\frac{1}{s} \cdot \frac{G(s)}{1+R(s)G(s)} - \frac{1}{s} \right) = \frac{\frac{1}{s(1+s)}}{1+\frac{1}{s(1+s)\cdot(1+s)}} - 1 = \frac{1+s}{1+s(1+s)^2} \cdot 1 = 0$$

4) $R = [\Gamma, \phi\Gamma, \phi^2\Gamma] = \begin{bmatrix} 0 & 0 & 2 & 0 & 8 & 0 \\ 1 & 0 & 3 & 0 & 13 & 0 \\ 0 & 1 & 0 & 4 & 0 & 16 \end{bmatrix}$

$$\begin{array}{cccc|cc|cc} & & & & 0 & 0 & 2 & 0 \\ & & & & 1 & 0 & 3 & 0 \\ & & & & 0 & 1 & 0 & 4 \\ \hline 0 & 1 & 2 & 0 & 2 & 0 & 8 & 0 \\ 1 & 2 & 3 & 0 & 3 & 0 & 13 & 0 \\ 0 & 0 & 0 & 4 & 0 & 4 & 0 & 16 \end{array}$$

→ voller Rang

$$x_{k+1} = \phi x_k + \Gamma K x_k = \underbrace{[\phi + \Gamma K]}_{\phi_e} x_k$$

$$\begin{array}{cc|ccc} & & k_{11} & k_{12} & k_{13} \\ & & k_{21} & k_{22} & k_{23} \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & k_{11} & k_{12} & k_{13} \\ 0 & 1 & k_{21} & k_{22} & k_{23} \end{array}$$

$$\phi_e = \begin{vmatrix} 1 & 2 & 0 \\ 2+k_{11} & 3+k_{12} & k_{13} \\ k_{21} & k_{22} & 4+k_{23} \end{vmatrix} \qquad k_{13}, k_{21}, k_{22} = 0$$

→ kann nicht nilpotent werden

Ansatz: 2 Teilsysteme

$$x_{k+1} = \begin{pmatrix} \boxed{\begin{matrix} 1 & 2 \\ 2 & 3 \end{matrix}} & 0 \\ 0 \quad 0 & \boxed{4} \end{pmatrix} x_k + \begin{pmatrix} \boxed{\begin{matrix} 0 \\ 1 \end{matrix}} & 0 \\ 0 & \boxed{4} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\boxed{K = \begin{bmatrix} -5/2 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}}$$

2) $1 = v \cdot 4 \rightarrow v = 1/4$

$k_2 = -\frac{1}{4} \cdot 4 = -1 = k_{23}$

1) $\begin{matrix} 0 \\ 1 \end{matrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}^T \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix}$ $\quad v_2 = 0$

$1 = 2v_1 + 3v_2 \rightarrow v_1 = 1/2$ $\qquad v = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$

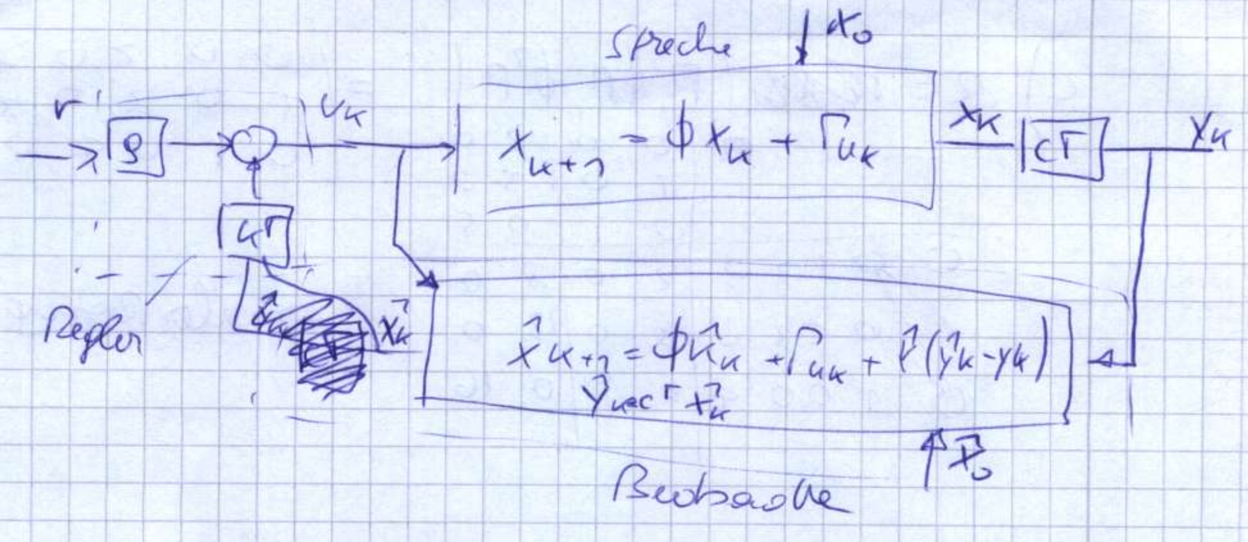
$$k_1 = -\begin{pmatrix} 1/2 \\ 0 \end{pmatrix}^T \cdot \begin{vmatrix} 5 & 8 \\ 8 & 13 \end{vmatrix}$$

$$\begin{array}{cc|cc} & & 1 & 2 \\ & & 2 & 3 \\ \hline 1 & 2 & 5 & 8 \\ 2 & 3 & 8 & 13 \end{array}$$

$k_{11} = -5/2 \rightarrow k_1^T = \begin{vmatrix} -5/2 \\ -4 \end{vmatrix}$

$k_{22} = -4$

d)



e) siehe Skript