4.2.2011

1) a) $V_L(s) = A \cdot s$

$$m_p \dot{w} = F_p - d\,w = (p_L - (a_0 h + a_1))A - d\,w$$

b) $$m \cdot \ddot{h} = m \cdot g - F_A = m \cdot g - \rho_W (V_B - V_W(s))\, g$$

$$m = \rho_W V_W(s) + m_K$$

$$\ddot{h} = g - \frac{\rho_W (V_B - V_W(s))\, g}{\rho_W V_W(s) + m_K} \qquad V_W(s) = A(l - s)$$

$$\ddot{h} = g - \frac{\rho_W (V_B - A(l-s))\, g}{\rho_W A(l-s) + m_K}$$

c) $$\frac{d}{dt}\begin{bmatrix} s \\ w \\ h \\ \dot{h} \end{bmatrix} = \begin{bmatrix} w \\ (p_L - (a_0 h + a_1))\frac{A}{m_p} - \frac{d}{m_p} w \\ \dot{h} \\ g - \frac{\rho_W (V_B - A(l-s))\, g}{\rho_W A(l-s) + m_K} \end{bmatrix} \qquad u = p_L$$

$$y = h$$

d) $w = 0$

$\dot{h} = 0$

$p_{LR} = a_0 h_R + a_1 \quad \Rightarrow \quad h_R = \frac{p_{LR} - a_1}{a_0}$ bei p_{LR} gegeben

$$1 = \frac{\rho_W (V_B - A(l - s_R))}{\rho_W A(l - s_R) + m_K}$$

$$\rho_W A(l - s_R) + m_K = \rho_W V_B - \rho_W A(l - s_R)$$

$$2\rho_w A(l-s_R) = \rho_w V_B - m_K$$

$$l - s_R = \frac{\rho_w V_B - m_K}{2\rho_w A}$$

$$s_R = l - \frac{\rho_w V_B - m_K}{2\rho_w A}$$

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{d}{m_P} & -\frac{A\,d_0}{m_P} & 0 \\ 0 & 0 & 0 & 1 \\ A_{41} & 0 & 0 & 0 \end{bmatrix}$$

$$A_{41} = -\frac{\rho_w A g(\rho_w A(l-s_R)+m_K) - \rho_w(V_B - A(l-s_R))g\cdot(-\rho_w A)}{(\rho_w A(l-s_R)+m_K)^2}$$

$$= -\frac{\cancel{\rho_w^2 A^2 g l} - \cancel{\rho_w^2 A^2 g s_R} + \rho_w A g m_K + \rho_w^2 A g V_B - \cancel{\rho_w^2 A^2 g l} + \cancel{\rho_w^2 A^2 g s_R}}{(\rho_w A(l-s_R)+m_K)^2}$$

$$= -\frac{\rho_w A g(m_K + \rho_w V_B)}{(\rho_w A(l-s_R)+m_K)^2}$$

$$b = \frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ \frac{A}{m_P} \\ 0 \\ 0 \end{bmatrix} \qquad c^T = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \qquad d = 0$$

2) a) $\lambda \cdot (\lambda - 2) + (1 + a^2) = 0$

$$\lambda^2 - 2\lambda + 1 + a^2 = 0$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot (1+a^2)}}{2} = \frac{2 \pm 2\sqrt{-a^2}}{2} =$$

$$= 1 \pm j a$$

$$(A - \lambda_1 E) v_1 = \begin{bmatrix} -1-ja & 1 \\ -(1+a^2) & 1-ja \end{bmatrix} v_1 = 0$$

$$v_{12} = (1+ja) v_{11}$$

$$v_{11} = 1$$
$$v_{12} = (1+ja)$$

$$v_1 = \begin{bmatrix} 1 \\ 1+ja \end{bmatrix}$$

$$\Rightarrow V = \begin{bmatrix} 1 & 0 \\ 1 & a \end{bmatrix}$$

b) $$\dot{z} = \underbrace{V^{-1} A V}_{\tilde{A}} z + \underbrace{V^{-1} b}_{\tilde{b}} u$$
$$y = \underbrace{c^T V}_{\tilde{c}^T} z$$

$$\tilde{A} = \begin{bmatrix} 1 & a \\ -a & 1 \end{bmatrix}$$

$$V^{-1} = \frac{1}{a} \begin{bmatrix} a & 0 \\ -1 & 1 \end{bmatrix}$$

$$\begin{array}{cc|c} \multicolumn{2}{c|}{\tilde{b}} & 1 \\ & & 1 \\ \hline 1 & 0 & 1 \\ -\frac{1}{a} & \frac{1}{a} & 0 \end{array} \qquad \begin{array}{cc|cc} & & 1 & 0 \\ \multicolumn{2}{c|}{\tilde{c}^T} & 1 & a \\ \hline -1 & 1 & 0 & a \end{array}$$

$$\tilde{\Phi} = \begin{bmatrix} e^t \cos(at) & e^t \sin(at) \\ -e^t \sin(at) & e^t \cos(at) \end{bmatrix}$$

c) $y(t) = \tilde{c}^T z$

$$z(t) = \tilde{\Phi}(t)\cdot z_0 + \int_0^t \tilde{\Phi}(t-\tau)\,\tilde{b}\,u(\tau)\,d\tau$$

$$z_0 = V^{-1}x_0 = 0 \qquad u(t) = \delta(t)$$

$$z(t) = \tilde{\Phi}(t)\tilde{b} = \begin{bmatrix} e^t\cos(\alpha t) \\ -e^t\sin(\alpha t) \end{bmatrix}$$

$$y(t) = \tilde{c}^T z(t) = -\alpha e^t \sin(\alpha t)$$

d) $u = \alpha\dot{y} = \alpha c^T \dot{x}$

$$\dot{x} = Ax + b\alpha c^T \dot{x}$$

$$(E - b\alpha c^T)\dot{x} = Ax$$

$$\dot{x} = (E - b\alpha c^T)^{-1} A x$$

$$(E - b\alpha c^T) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \alpha \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1+\alpha & -\alpha \\ \alpha & 1-\alpha \end{bmatrix}$$

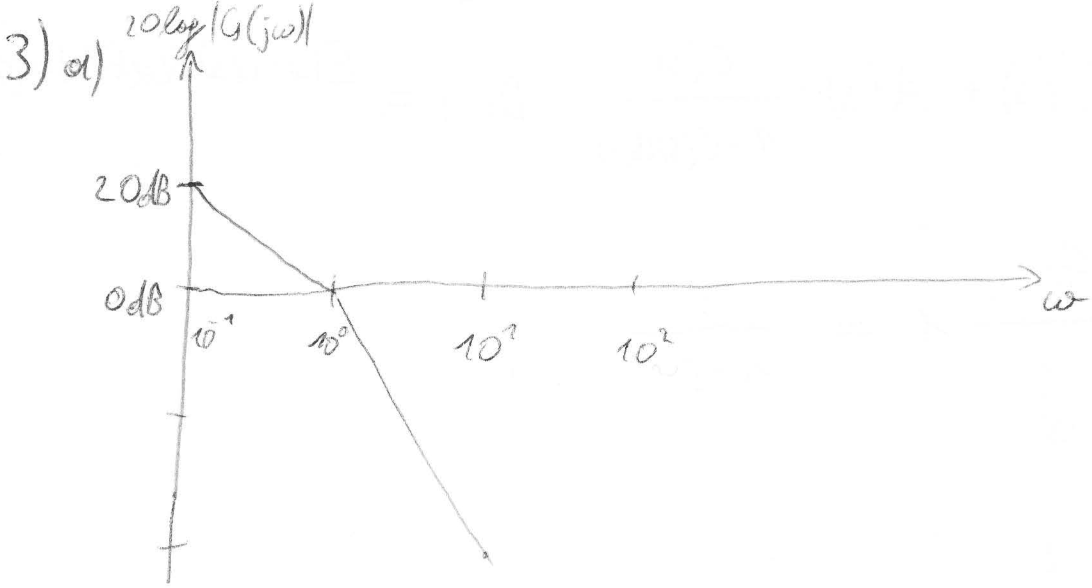
$$(E - b\alpha c^T)^{-1} = \frac{1}{(1-\alpha^2)+\alpha^2} \begin{bmatrix} 1-\alpha & \alpha \\ -\alpha & 1+\alpha \end{bmatrix}$$

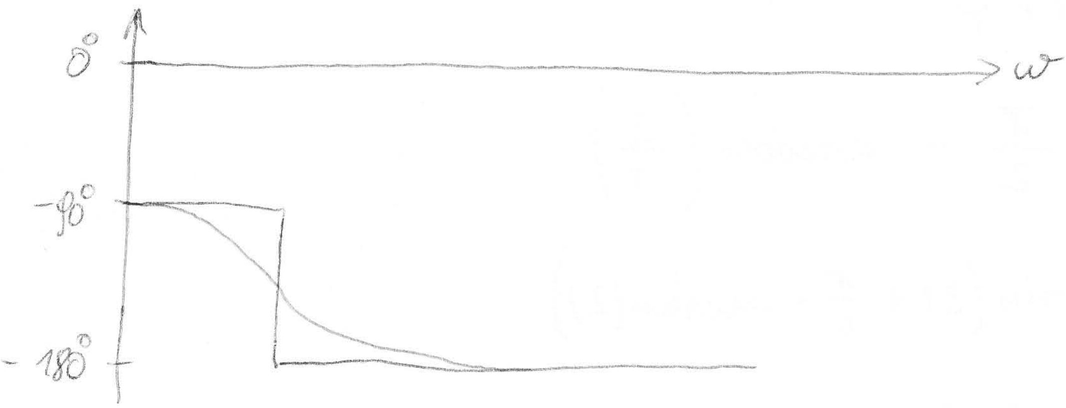
$$\dot{x} = \frac{1}{1} \underbrace{\begin{bmatrix} -2\alpha & 1+\alpha \\ -2-2\alpha & 2+\alpha \end{bmatrix}}_{A'}$$

$$\begin{array}{cc|cc} & & 0 & 1 \\ & & -2 & 2 \\ \hline 1-\alpha & \alpha & -2\alpha & 1+\alpha \\ -\alpha & 1+\alpha & -2-2\alpha & 2+\alpha \end{array}$$

$$\det(\lambda E - A') = (\lambda+2\alpha)(\lambda-2-\alpha) + (1+\alpha)(2+2\alpha)$$
$$= \lambda^2 - 2\lambda - \alpha\lambda + 2\alpha\lambda \cancel{-4\alpha} \cancel{-2\alpha^2} + 2(\cancel{\alpha^2} + \cancel{2\alpha} + 1)$$
$$= \lambda^2 + \lambda(-2+\alpha) + 2 \quad \Rightarrow \text{Hurwitzpolynom wenn } \alpha > 2$$

3) a)





b) $t_r = \frac{3}{2}\sqrt{3}$ $\quad \omega_c t_r = \frac{3}{2}$ $\quad \Phi + \ddot{u} = 70$

$\omega_c = \frac{1}{\sqrt{3}}$ $\quad \Phi = 55^\circ$

$$\arg\left(G(I\sqrt{3})\right) = -\frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{3}}\right) \hat{=} -120^\circ$$

$$R(s) = V\frac{(1+sT_I)}{s} \Rightarrow \arg\left(R(I\omega_c)\right) \stackrel{!}{=} -5^\circ$$

$$\arctan\left(\frac{T_I}{\sqrt{3}}\right) \stackrel{!}{=} 85^\circ$$

$$T_I = \tan(85^\circ)\cdot\sqrt{3}$$

$$1 = V\cdot\frac{\sqrt{1+(\tan(85^\circ))^2}}{\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\sqrt{1+\frac{1}{3}}} \Rightarrow V = \frac{\frac{2}{3\sqrt{3}}}{\sqrt{1+(\tan(85^\circ))^2}}$$

c) $F(s) = \frac{\hat{z}}{\hat{w}} = E(s) + A(s) \cdot \frac{C(s)}{1 - C(s)B(s)} \cdot D(s)$

d) $F(s) = s \cdot \frac{\frac{1}{s}}{1 + \frac{1}{s}} \, 1 = \frac{s}{1+s}$

$|F(j2)| = \frac{2}{\sqrt{1+4}} = \frac{2}{\sqrt{5}}$

$\arg(F(j2)) = \frac{\pi}{2} - \arctan\left(\frac{2}{1}\right)$

$z(t) = \frac{6}{\sqrt{5}} \sin\left(2t + \frac{\pi}{2} - \arctan(2)\right)$

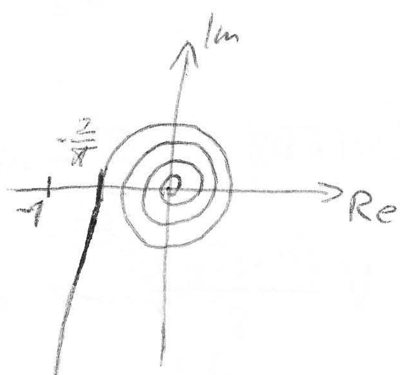
4 a) $G(j\omega) = j\frac{e^{-j\omega}}{\omega} = -j\frac{1}{\omega}\left(\cos(\omega) - j\sin(\omega)\right) = -\frac{\sin(\omega)}{\omega} - j\frac{\cos(\omega)}{\omega}$

0 für $\omega = \frac{\pi}{2}$

$\lim\limits_{\omega \to 0} \frac{-\sin(\omega)}{\omega} - j\frac{\cos(\omega)}{\omega} =$

$= -1 - j\infty$

$\lim\limits_{\omega \to \infty} G(j\omega) = 0$



4b) $f_k = 2^k k \left(e^{(k+1)T_a} - e^{kT_a}\right) = e^{T_a} f^* - f^*$

mit $f^* = 2^k k\, e^{kT_a} = \frac{1}{T_a} 2^k kT_a\, e^{kT_a}$

$$f_z^* = \frac{1}{T_a} \frac{\partial}{\partial a} \frac{z/2}{z/2 - e^{aT_a}} \Bigg|_{a=1} = \frac{1}{T_a} \frac{z/2}{\left(\frac{z}{2} - e^{aT_a}\right)^2} (-1)\cdot\left(-T_a e^{aT_a}\right) \Bigg|_{a=1}$$

$$= \frac{\frac{z}{2} e^{T_a}}{\left(\frac{z}{2} - e^{T_a}\right)^2}$$

$$f_z = \frac{\frac{z}{2} e^{2T_a} - \frac{z}{2} e^{T_a}}{\left(\frac{z}{2} - e^{T_a}\right)^2}$$

c) $G^{\#}(q) = \frac{2(q-1)(q^2-2)}{q^3+3q^2+3q+2}$ $\quad T_a = 1 \quad \Omega_0 = \frac{2}{T_a} = 2$

i) EWS: $\lim\limits_{s\to 0} s \cdot \frac{G(s)}{s} = \lim\limits_{s\to 0} V \cdot \frac{Z(s)}{N(s)} = V$ $\quad$ mit $Z(0) = N(0) = 1$

$$\lim_{q\to 0} G^{\#}(q) = \frac{2(-1)(-2)}{2} = 2$$

ii) BIBO-Stabilität

Routh-Hurwitz:

q^3	1	3
q^2	3	2
q^1	$\frac{2-9}{-3}$	0
q^0	2	

$\Rightarrow$ BIBO stabil

iii) $\lim\limits_{q \to \Omega_0} G^{\#}(q) \neq 0 \Rightarrow$ sprungfähig

d) $R = \begin{bmatrix} 1 & -\frac{1}{2} \\ -1 & -3 \end{bmatrix}$

$$\begin{array}{cc|c} & & 1 \\ & & -1 \\ \hline \frac{1}{2} & 1 & -\frac{1}{2} \\ -1 & 2 & -3 \end{array}$$

voller Rang → vollst. erreichbar

$v_1^T = \begin{bmatrix} 0 & 1 \end{bmatrix} R^{-1}$ $\qquad R^{-1} = \frac{1}{-3-\frac{1}{2}} \begin{bmatrix} -3 & \frac{1}{2} \\ 1 & 1 \end{bmatrix}$

$v_1^T = -\frac{2}{7} \begin{bmatrix} 1 & 1 \end{bmatrix}$

$P_{\varphi soll} = z^2 - \frac{1}{4}$

$$\begin{array}{cc|cc} & & \frac{1}{2} & 1 \\ \Phi^2 & & -1 & 2 \\ \hline \frac{1}{2} & 1 & -\frac{3}{4} & \frac{5}{2} \\ -1 & 2 & -\frac{5}{2} & 3 \end{array}$$

$P_{\varphi soll}(\Phi) = \begin{bmatrix} -1 & \frac{5}{2} \\ -\frac{5}{2} & \frac{11}{4} \end{bmatrix}$

$$\begin{array}{cc|cc} & & -1 & 5/2 \\ k^T & & -5/2 & 11/4 \\ \hline \frac{2}{7} & \frac{2}{7} & \frac{2}{7} - \frac{5}{7} & \frac{5}{7} + \frac{11}{14} \end{array}$$

$k^T = \begin{bmatrix} -1 & \frac{3}{2} \end{bmatrix}$