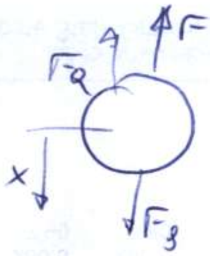
6.2.09

$$V = \frac{\pi x^2}{3}(3R - x) \quad , F_A = \rho V g$$



$$F_g = g m$$

$$m \cdot \dot{v} = F_A + F_g - F - F_a$$

$$X = \begin{bmatrix} x \\ v \end{bmatrix}, \quad u = F$$

$$\dot{x} = v$$

$$y = x$$

$$\dot{v} = g - \frac{\rho g \pi x^2}{3m}(3R - x) - \frac{F}{m}$$

$$y = [1 \quad 0] x$$

b) $x_S = R/3$, $v_R = 0$, $\dot{v} = 0 = g - \frac{\rho g \pi x_S^2}{3m}(3R - x_S) - F_R/m$

$$F_R = mg - \frac{\rho g \pi}{3}\frac{R^2}{9}(3R - R/3)$$

$$= mg - \frac{\rho g \pi}{3} R^3 \left(\frac{8}{27}\right)$$

c) $c^T = [1 \quad 0]$ $\quad d = 0$

$$b^T = [0 \quad -1/m]$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{\rho g \pi 2 x_R 3R}{3m} + \frac{\rho g \pi x_R^2}{3m} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{\rho g R^2}{9m} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ +\frac{\rho g \pi}{m}\left(\frac{R^2}{9} - \frac{6R^2}{9}\right) & 0 \end{bmatrix}$$

$$= -\frac{\rho g \pi}{m} 5\frac{R^2}{9}$$

d) char. Polynom $\quad |\lambda E - A| = \lambda^2 + \frac{\rho g \pi}{9m} 5R^2 = 0$

$$= \lambda^2 - \cancel{\frac{\rho g R^2}{9m}} = \cancel{\left(\lambda + \frac{\rho g R^2}{9m}\right)\left(\lambda - \frac{\rho g R^2}{9m}\right)} = 0$$

$$\lambda = \pm\sqrt{-\frac{\rho g \pi}{9m} 5R^2}$$

geschwindigkeitsprop. Dämpfung $F = d \cdot v$

$$|\lambda E - A| = \lambda(\lambda + d) + \frac{\rho g \pi}{9m} 5R^2 = 0$$

$$\lambda_{1,2} = -\frac{d}{2} \pm \sqrt{\frac{d^2}{4} - \frac{\rho g \pi}{9m} 5R^2}$$

$\rightarrow$ große Dämpfung, $\Rightarrow$ [illegible]

$$Re(\lambda) < 0$$

e) $O = \begin{bmatrix} c^T \\ c^T A \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ Rang $O = 2$ $\rightarrow$ beobachtbar

$p_{\beta,soll} = (\lambda+1)^2 = \lambda^2 + 2\lambda + 1$

$\hat{v}_1 = e_n O^{-1} = e_n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} e_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\hat{k} = -1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\rho g \pi 5 R^2}{4} \end{bmatrix}$

$\hat{k} = \begin{bmatrix} -2 \\ \frac{\rho g \pi 5 R^2}{4} - 1 \end{bmatrix}$

$$\begin{array}{cc|c|c} & & 0 & 1 \\ & & 1 & 0 \\ \hline 0 & 1 & 1 & 0 \\ -\frac{\rho g \pi 5 R^2}{4} & 0 & 0 & -\frac{\rho g \pi 5 R^2}{4} \end{array}$$

$A = \begin{bmatrix} 0 & 1 \\ -5 & 0 \end{bmatrix}$

$$\begin{array}{cc|c|c} & & 0 & 1 \\ & & 1 & 0 \\ \hline 0 & 1 & 1 & 0 \\ -5 & 0 & 0 & -5 \end{array}$$

$\hat{k} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -5 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

2008/02/06/

2a) → skrip'

2b) → ϕ .. nilpotent

$$\phi^3 = 0 \qquad \phi = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = \phi \cdot x_0 \,,\; x_2 = \phi^2 x_0 ,\; x_3 = \phi^3 x_0$$

c)

Cayley Hamilton

~~K_3 ...~~ $\qquad \frac{K_0}{K_3} E + \frac{K_1}{K_3} A + \frac{K_2}{K_3} A^2 + A^3 = 0$

char. glg. $\qquad \rho_0 \qquad \rho_1 \qquad \rho_2$

$$|\lambda E - A| = \begin{vmatrix} \lambda + 2\alpha & 2\alpha - \beta - \gamma & 2\beta - 2\gamma \\ 0 & \lambda + \beta + \gamma & 2\gamma - 2\beta \\ 0 & \frac{1}{2}\gamma - \frac{1}{2}\beta & \lambda + \beta + \gamma \end{vmatrix}$$

$$= (\lambda + 2\alpha)(\cancel{\ldots})(\lambda + \beta + \gamma)^2 + \frac{\beta - \gamma}{2} 2(\gamma - \beta)(\lambda + 2\alpha)$$

$$= (\lambda + 2\alpha)(\lambda^2 + 2(\beta + \gamma)\lambda + (\beta + \gamma)^2) - (\gamma - \beta)^2 \lambda - 2\alpha(\gamma - \beta)^2$$

$$= \lambda^3 + 2\lambda^2(\beta + \gamma) + \lambda(\beta + \gamma)^2$$

$$+ 2\alpha\lambda^2 + 4\lambda\alpha(\beta + \gamma) + 2\alpha(\beta + \gamma)^2$$

$$- \lambda(\gamma - \beta)^2 - 2\alpha(\gamma - \beta)^2$$

$$= \lambda^3 + \lambda^2(2\alpha + 2\beta + 2\gamma) + \lambda((\beta + \gamma)^2 - (\gamma - \beta)^2 + 4\alpha(\beta + \gamma))$$

$$+ 2\alpha((\beta + \gamma)^2 - (\gamma - \beta)^2)$$

NR: $(\beta + \gamma)^2 - (\gamma - \beta)^2 = (\beta + \gamma + \gamma - \beta)(\beta + \gamma - \gamma + \beta) = 4\gamma\beta$

$$\text{pdy} = \lambda^3 + \lambda^2\, 2(\alpha + \beta + \gamma) + \lambda\, 4(\gamma\beta + \alpha(\beta + \gamma)) + 8\alpha\gamma\beta$$

$$\to \frac{K_0}{K_3} = 8\alpha\gamma\beta \,,\; \frac{K_1}{K_3} = 4[\gamma\beta + \alpha\beta + \alpha\gamma] \,,\; \frac{K_2}{K_3} = 2(\alpha + \beta + \gamma)$$

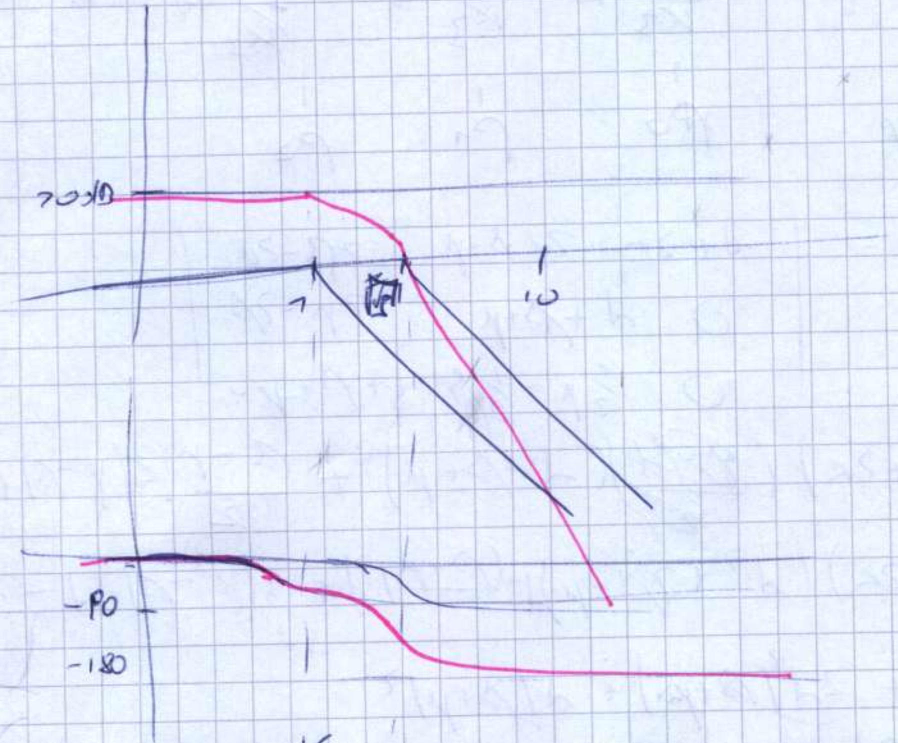
d) $\Delta \arg(r(j\omega)) = 0$

muss 4π sein -> (Michailov)

3) a) $G_1(s) = \frac{1}{\sqrt{3}s}$, $G_2(s) = \frac{10}{1+s}$, $R_1(s) = 3$

$$T_{r1,y1} = \frac{R_1 G_1}{1 + R_1 G_1} = \frac{\frac{3}{\sqrt{3}s}}{1 + \frac{3}{\sqrt{3}s}} = \frac{3}{\sqrt{3}s + 3} = \frac{1}{1 + s/\sqrt{3}}$$

b) $\overline{G_3(s)} = T_{r1,y1} G_2(s) = \frac{10}{(1+s)(1+s/\sqrt{3})}$



c) $R_1 = \frac{V}{s}$ $\quad t_r \cdot \omega_c = 1{,}5 \rightarrow \omega_c = 1$

$\ddot{u} = 10\% \rightarrow PR = 60 \Rightarrow \arg L = -\frac{2\pi}{3}$

$$\arg\left(G_3 \cdot \frac{1}{s}\right) = -\frac{\pi}{2} - \frac{\pi}{4} - \frac{\pi}{6} = -\frac{\cancel{10}\pi}{12} = -\frac{\cancel{11}\pi}{\cancel{12}}$$

$$R_2 = \frac{V}{s}(1 + sT)$$

$$\arg(G_3 R_2) = -\frac{11\pi}{12} + \arctan T \overset{!}{=} -\frac{2\pi}{3} = -\frac{8\pi}{12}$$

$$T = \tan\frac{3\pi}{12} = \cancel{\tfrac{1}{\sqrt{3}}}\ 1$$

$$\left|G_3 \cdot \frac{R_2}{V}\right|_{\omega_c} = \frac{10 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{\frac{1}{3}+1}} = \frac{10\sqrt{3}}{\cancel{2}\,2} \rightarrow V = \cancel{\tfrac{2}{10\sqrt{3}}}\ \frac{1}{5\sqrt{3}}$$

d) $T_{d,y} = \frac{1}{1+R_2 G_3} = \frac{1}{1+\frac{1}{5\sqrt{3}}\frac{10}{(\frac{s}{\sqrt{3}}+1)s}} = \frac{5\sqrt{3}\, s\cdot(1+\frac{s}{\sqrt{3}})}{10+5\sqrt{3}\, s(\frac{s}{\sqrt{3}}+1)}$

$d(t) = a\cdot\sigma(t)$

$d(s) = \frac{a}{s}$

$\lim\limits_{s\to 0} s\cdot d(s)\cdot T_{d,y}(s) = \lim\limits_{s\to 0} \frac{5\sqrt{3}\, a(s(1+\frac{s}{\sqrt{3}}))}{10+5\sqrt{3}\, s(\frac{s}{\sqrt{3}}+1)} = 0$

4) $(g_k) = \delta_k + \underbrace{\left(\frac{1}{2}\right)^{k-2}\sigma[k-2]}_{\text{Dämpfung + Zeitverschiebung}}$

$g(z) = 1 + z^{-2}\frac{2z}{2z-1} = 1 + \frac{2}{2z^2-z} = \frac{2z^2-z+2}{z(2z-1)}$

$\cancel{(2z^2-z+2):(2z-1) = z}$

$= \frac{2z^2-z+2}{z(2z-1)} = \frac{z^2-\frac{z}{2}+1}{z(z-\frac{1}{2})}$

e) BIBO-stabil? Ja, da alle Pole im EK

c) $(u_k) = -1^k \circ\!\!-\!\!\bullet\; -\frac{z}{z-1}$

$y_1\quad \lim\limits_{z\to 1}(z-1)\frac{-z}{(z-1)}\frac{z^2-\frac{z}{2}+1}{z(z-\frac{1}{2})} = -\frac{1-\frac{1}{2}+1}{1-\frac{1}{2}} = -3$

$|G(z)|_{z=e^{j\frac{\pi}{2}}=j} = \left|\frac{-j/2}{j(j-\frac{1}{2})}\right| = \frac{+\frac{1}{2}}{\sqrt{1+\frac{1}{4}}} = +\frac{1}{\sqrt{5}}$

$(y_k) = -3 + \frac{4}{\sqrt{5}}\left(k\frac{\pi}{2}+\frac{\pi}{3} - \operatorname{atan}(-2)\right)$

$\arg(G(z))|_{z=j} = -\frac{\pi}{2} - \frac{\pi}{2} - (\operatorname{atan}(-2)+\pi)$

$= -2\pi - \operatorname{atan}(-2) = -\operatorname{atan}(-2)$

4d) $x_{k+3} - x_{k+2} + 5x_{k+1} - 7x_k = u_k$

$y_k = x_{k+2} - 10x_k$

$z_{1,k} = x_k$

$z_{2,k} = x_{k+1}$

$z_{3,k} = x_{k+2}$

$z_{1,k+1} = z_{2,k}$

$z_{2,k+1} = z_{1,k}$

$z_{3,k+1} = u_k + z_{3,k} - 5z_{2,k} + 7z_{1,k}$

$y_k = z_{3,k} - 10z_{1,k}$