30.4.2010

$x = \begin{pmatrix} u_C \\ i_L \end{pmatrix}, \quad u = u_e$

a) $i_C = C\frac{du_C}{dt} = i_L - i_D$

$u_L = L\cdot\frac{di_L}{dt} = u_e - u_C - i_L \cdot R$

$$\boxed{\begin{aligned} \dot i_L &= \frac{u_e}{L} - \frac{u_C}{L} - i_L \frac{R}{L} \\ \dot u_C &= \frac{i_L}{C} - \frac{h(u_C)}{C} \end{aligned}}$$

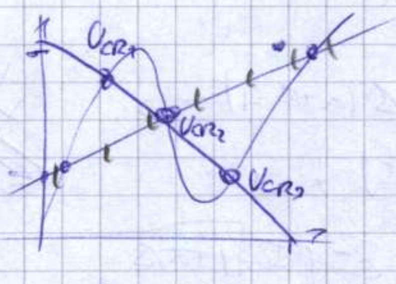
b) $0 = \frac{u_{eR}}{\cancel{L}} - \frac{u_{CR}}{\cancel{L}} - i_{LR}\frac{R}{\cancel{L}}$

$0 = i_{LR} - h(u_{CR}) \rightarrow i_{LR} = h(u_{CR})$

$u_{CR} = u_{eR} - R\cdot h(u_{CR})$

$h(u_{CR}) = \frac{u_{eR} - u_{CR}}{R} = i_{LR}$

→ 3 Ruhelage maximal



$h(u_{CR}) = 1mA - \frac{u_{CR}}{1000}$

c) $b = [0 \quad \frac{1}{L}]$

$A = \begin{bmatrix} -\frac{\partial h(u_C)}{\partial u_C}\frac{1}{C} & +\frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}$

ist dann ein HW-Poly, wenn $\dot h > -\frac{1}{R} = -10^{-3}$ und $\dot h > -\frac{RC}{L}$ → genau eine stabile Ruhelage $u_{CR1} + u_{CR3}$

d) $\cancel{\frac{R}{LC}\dot h(u_{CR}) + \frac{1}{LC} = 0}$

$(\lambda + \dot h/C)(\lambda + R/C) + \frac{1}{LC} = \lambda^2 + \lambda\left(\frac{\dot h}{C} + \frac{R}{L}\right) + \frac{\dot h R}{LC} + \frac{1}{LC} = 0$

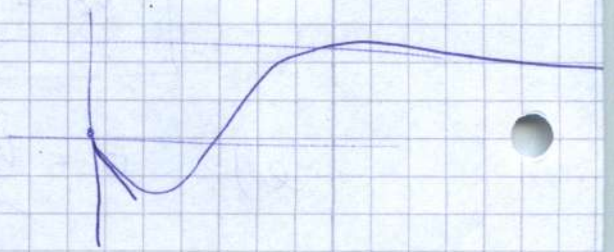
2d) nein, nicht p.m., da Nullstelle bei 1

e) $\lim_{t\to 0} h(t) = \lim_{s\to\infty} \frac{s^k(1-s)}{\cancel{s}(1+s)^2} = \lim \frac{-s^3+s^2}{s^2+2s+1} \cancel{=\infty} -1$

$\lim_{t\to 0} h(t) = \lim_{s\to\infty} \frac{\cancel{s}(1-s)}{(1+s)^2\cancel{s}} = \lim_{s\to\infty} \frac{-s^2+s}{s^2+2s+1} = \lim_{s\to\infty} \frac{-1 - \frac{1}{s}}{1+\frac{2}{s}+\frac{1}{s^2}} = -1$

$= \lim_{s\to\infty} \frac{(1-s)}{s^2+2s+1} = 0$

$\lim_{s\to 0} \frac{1-s}{(1+s)^2} = 1$



e) $|G(I\omega)| = \cancel{2} \frac{\sqrt{\omega^2+1}}{(\omega^2+1)}$

$\arg(I\omega) = \operatorname{atan}(-\omega) - 2\operatorname{atan}\omega + \operatorname{atan}\frac{\sin\omega T}{\cos\omega T}$

$= \operatorname{atan}-\omega - 2\operatorname{atan}\omega \mp \omega T = -3\operatorname{atan}\omega - \omega T$

3a) i $e_{k+1} = A e_k$

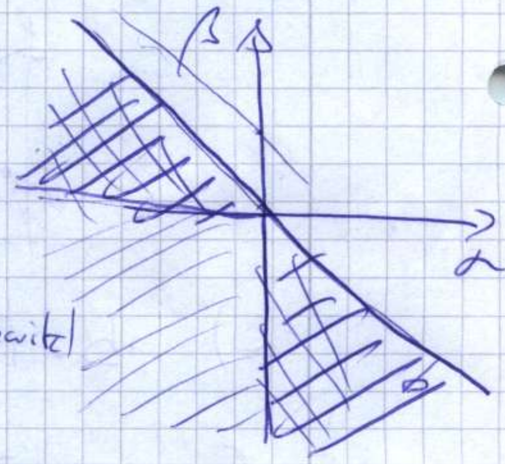
$(\alpha+\beta-\lambda)(-2-\lambda)(-\lambda) - \alpha\beta(\alpha+\beta-\lambda) = 0$

$(\alpha+\beta-\lambda)(\lambda^2+2\lambda-\alpha\beta) = 0$

$\lambda_1 = \alpha+\beta$, $\lambda_{2,3} \cancel{= -1 \pm \sqrt{1+\alpha\beta}}$

$\alpha+\beta < 0$ $\quad +\alpha\beta < 0$ (Hurwitz)

$\cancel{\alpha,\beta < 0}$



ii) $R = |b, Ab, A^2b| = \cancel{\ldots}$

$$\begin{matrix} 0 & 1 & \alpha+\beta \\ 0 & \beta & -2\beta \\ 1 & 0 & \alpha\beta \end{matrix}$$

				0	1
				0	β
				1	0
$\alpha+\beta$	0	1		1	$\alpha+\beta$
0	-2	β		β	-2β
0	α	0		0	$\alpha\beta$

$\det R \neq 0 = -2\beta - \beta(\alpha+\beta)$

$\boxed{\alpha+\beta \neq -2}$

iii) $O(c^T, A)$

$$\begin{array}{c|ccc|ccc} & \alpha+\beta & 0 & 1 & \alpha+\beta & 0 & 1 \\ & 0 & -2 & \beta & 0 & -2 & \beta \\ & 0 & \alpha & 0 & 0 & \alpha & 0 \\ \hline 1\ 0\ 0 & \alpha+\beta & 0 & 1 & (\alpha+\beta)^2 & \alpha & \alpha+\beta \end{array}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ \alpha+\beta & 0 & 1 \\ (\alpha+\beta)^2 & \alpha & (\alpha+\beta) \end{vmatrix}$$

$\alpha \neq 0$

iv)

$G(s) = c^T(sE-A)^{-1} b =$

$$= c^T \begin{pmatrix} s-(\alpha+\beta) & 0 & -1 \\ 0 & s+2 & -\beta \\ 0 & -\alpha & s \end{pmatrix}^{-1} b = \frac{\cancel{\alpha}\ -(s+2)}{(s+s)\cancel{2}(s+(\alpha+\beta)) - \alpha\beta(s-\alpha+\beta)}$$

$$= \frac{+(s+2)}{(s-(\alpha+\beta))(\cancel{2}s^2+\cancel{2s}-\alpha\beta)}$$

A) wenn $\alpha+\beta \neq 2$ ist, ist System stimmt 3. Ordnung -erreichbar

$(s^2+2s-\alpha\beta) : (s+2) = s$

$-\alpha\beta$

B) nur, wenn $\alpha \cdot \beta \neq 0$ stimmt

$\cancel{\mathcal{U}}$ A B

$v_i \Longleftrightarrow v_i$

$\lambda_i \Longleftrightarrow e^{+\lambda_i T_a}$

v) $\lambda_1 = \alpha_i + j\omega_i \rightarrow \omega_i \neq \frac{\ell\pi}{T_a}$ $\ell = \pm 1, \pm 2, \pm 3$

wenn $\alpha\beta < -1$ $\rightarrow \omega_i = \cancel{\sqrt{1+\alpha\beta}} \neq \frac{\ell\pi}{T_a}$

$= \sqrt{-(1+\alpha\beta)}$

$T_a \neq \frac{\ell\pi}{\sqrt{-(1+\alpha\beta)}}$

a) $G^\#(q) = \frac{y}{u} = \frac{q^2-2}{q^2-8q+12}$

nicht sprungfähig, wenn Nst bei $\Omega_0 = 2/T_a$

$(q-\sqrt{2})(q+\sqrt{2}) \rightarrow \underline{T_a \neq \sqrt{2}}$

nicht realisierbar, wenn Pol bei $\Omega_0 = 2/T_a$

$q^2-8q+12 = (q-6)(q-2) \qquad T_a \neq 1, \frac{1}{3}$

b) $\Omega_0 = \frac{2}{T_a}\tanh\left(\frac{T_a}{2}\omega\right)$

in $|G^\#(I\Omega_0)|$ und $\arg(G^\#(I\Omega_0))$

4) a) $h_k \neq \Delta$

$g_k = (0, \frac{1}{2}, -1, \frac{3}{2}, 0 \ldots)$

$y_{max} = (0, 1, 0, 1, 0 \ldots) \rightarrow y_{max} = 2$

b) Eigenwerte von Φ $\rightarrow$ Eigenwerte von $\Psi \rightarrow e^{\lambda T_a}$

$x_{k+1} = \Phi x_k \qquad x_k = \Psi(k) \cdot x_0$

$x_k = \Phi^k x_0 \;\Big|\; x_1 = 2^k x_0 \qquad \cancel{x_1 = e^{2 T_a k} x_0}$

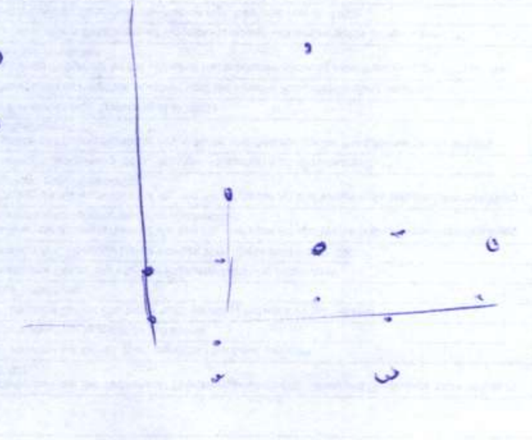
$x_1 = e^{k T_a} x_0$

$x_2 = x_0$

$x_3 = (-1)^k x_0$

$x_4 = (-0{,}5)^k x_0$

mit $x_0 = 1$



c) i) $-40 \log \sqrt{2\xi} = 14\,dB$

$= -20 \log 2\xi = \frac{-14}{20}$

$\frac{10^{-14/20}}{2} = \frac{1}{\cancel{5}} = 2\xi = 0{,}1$

i) $T_{yr}(s) = \frac{0{,}99}{1 + 2\xi \frac{s}{10} + \left(\frac{s}{10}\right)^2}$

ii) $e_\infty = 0{,}01 = \frac{1}{1+v} \Rightarrow v = 99$

$L = R(s)\,G(s) \Rightarrow R(s) = \frac{(s+1)\,v}{\cancel{1 + 2\xi \frac{s}{10}}}$

$T_{r,y} = \frac{\frac{v}{x}}{1 + \frac{v}{x}} = \frac{v}{v + x} = 0{,}99$

$T_{yr} = \frac{RG}{1+RG} \Rightarrow T_{yr} + RG\,T_{yr} = RG$

$R = \frac{T_{yr}}{G(1 - T_{yr})} = \frac{0{,}99(s+1)}{1 + 2\xi \frac{s}{10} + \frac{s^2}{10} - 0{,}99}$

$= \frac{99(s+1)}{1 + 2\xi s + s^2}$

mit $\xi = 1$

$= \frac{99(s+1)}{(s+1)^2} = \boxed{\frac{99}{(s+1)} = R(s)}$

$|G(j\omega_c)| = 1 = \frac{99}{1 + \omega_c^2} \Rightarrow \omega = \sqrt{98}$

iii) $T_R = \frac{1{,}5}{\omega_c} = \frac{1{,}5}{\sqrt{98}}$

$\hat{\omega} + \varphi_R = 70$ $\qquad \varphi = -2 \cdot \arctan \omega_c = -\cancel{245}$

$\hat{u} \cong 70\%$ $\qquad \cancel{\Leftrightarrow \varphi_R = 180^\circ}$