7.10.11

1) a) $R = r \cdot \frac{l}{\frac{d^2 \pi}{4}} = \frac{4 r l}{d^2 \pi}$

$P = u \cdot i = R \cdot i^2$

b) $x = \begin{bmatrix} T_g \\ T_{wi} \end{bmatrix} \qquad u = i$

$$\frac{d}{dt} U = P + Q_g$$

$$m c_v \dot{T}_g = R i^2 + A \sigma (T_{wi}^4 - T_g^4)$$

$$T_w(z,t) = \frac{d_w - z}{d_w} T_{wi}(t) + \frac{z}{d_w} T_{wo}$$

$$\dot{W} = A \rho_w c_w \int_0^{d_w} \frac{\partial T_w(t,z)}{\partial t} dz = A \rho_w c_w \int_0^{d_w} \frac{d_w - z}{d_w} \dot{T}_{wi} \, dz$$

$$= A \rho_w c_w \left(d_w - \frac{d_w^{\cancel{2}}}{2 \cancel{d_w}} \right) \dot{T}_{wi} = A \rho_w c_w \frac{d_w}{2} \dot{T}_{wi}$$

$$Q = -A \lambda_w \frac{\partial T_w(t,z)}{\partial z} \Big|_{z = d_w} - Q_w = -A \lambda_w \left(-\frac{T_{wi}}{d_w} + \frac{T_{wo}}{d_w} \right) + A\sigma (T_{wi}^4 - T_g^4)$$

$$\dot{W} = -Q$$

$$\cancel{A} \rho_w c_w \frac{d_w}{2} \dot{T}_{wi} = \cancel{A} \lambda_w \left(-\frac{T_{wi}}{d_w} + \frac{T_{wo}}{d_w} \right) - \cancel{A} \sigma (T_{wi}^4 - T_g^4)$$

$$\frac{d}{dt} \begin{bmatrix} T_g \\ T_{wi} \end{bmatrix} = \begin{bmatrix} \frac{1}{m c_v} \left(R i^2 + A \sigma (T_{wi}^4 - T_g^4) \right) \\ \frac{2}{\rho_w c_w d_w} \left[\lambda_w \left(\frac{T_{wo}}{d_w} - \frac{T_{wi}}{d_w} \right) - \sigma (T_{wi}^4 - T_g^4) \right] \end{bmatrix}$$

$$y = p = \frac{m \bar{R}}{V} T_g$$

c)

$$\frac{d}{dt}\begin{bmatrix}\alpha\\ \dot\alpha\\ \beta\\ \dot\beta\end{bmatrix} = \begin{bmatrix}\dot\alpha\\ \frac{k}{J_1}(\beta R-\alpha r)^3 r+\frac{M}{J_1}\\ \dot\beta\\ \frac{1}{J_2}\left(-k(\beta R-\alpha r)^3 R+G L\cos(\beta)\right)\end{bmatrix}$$

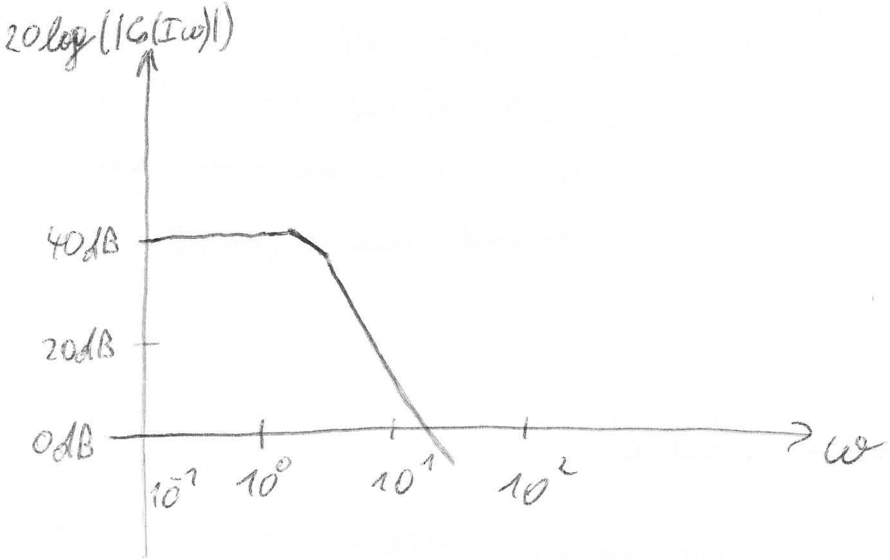
$u = M$

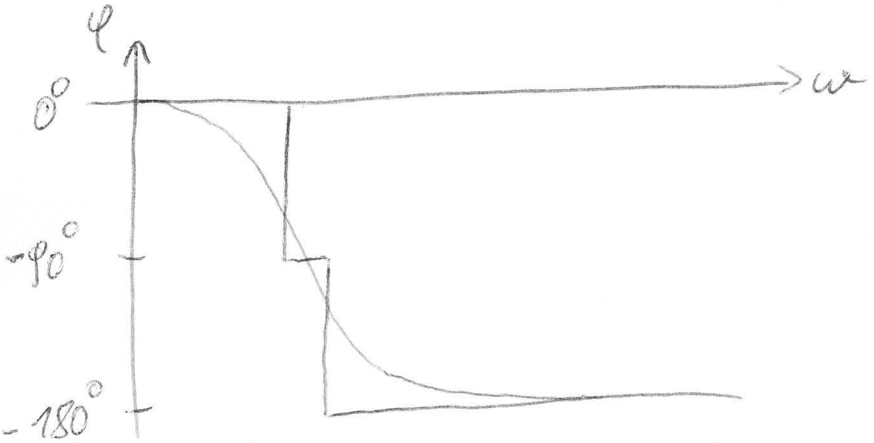
$y = L\sin\beta$

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{k}{J_1}3(\beta_R R-\alpha_R r)^2(-r^2) & 0 & 3\frac{k}{J_1}(\beta_R R-\alpha_R r)^2 Rr & 0\\ 0 & 0 & 0 & 1\\ -\frac{3k}{J_2}(\beta_R R-\alpha_R r)^2(-r)R & 0 & -\frac{3k}{J_2}(\beta_R R-\alpha_R r)^2 R^2-GL\sin(\beta_R) & 0\end{bmatrix}$$

$$b = \begin{bmatrix}0\\ \frac{1}{J_1}\\ 0\\ 0\end{bmatrix} \qquad c^T = \begin{bmatrix}0 & 0 & L\cos(\beta_R) & 0\end{bmatrix}$$

2) a) $G(s) = \frac{400\sqrt{3}}{(s+2\sqrt{3})(s+2)} = \frac{\cancel{400\sqrt{3}}^{\,100}}{\cancel{2\sqrt{3}}\cdot\cancel{2}} \frac{1}{\left(\frac{s}{2\sqrt{3}}+1\right)\left(\frac{s}{2}+1\right)} = \frac{100}{\left(\frac{s}{2\sqrt{3}}+1\right)\left(\frac{s}{2}+1\right)}$





b) $R(s) = V\frac{(1+sT_I)}{s}$

$\omega_c T_r = 1{,}5$ $\qquad \Phi + \ddot{u} = 70$ $\qquad e_\infty\big|_{r(t)=\sigma(t)} = 0$ erfüllt

$\omega_c = \frac{\cancel{3}\,\cancel{4}^{\,2}}{\cancel{2}\,\cancel{3}} = 2$ $\qquad \Phi = 60^\circ$ $\qquad$ durch $\frac{1}{s}$

$\arg(G(I)) = -\arctan\left(\frac{1}{\sqrt{3}}\right) - \arctan(1) = -30^\circ - 45^\circ = -75^\circ$

$\arg(L(I\omega)) = -75^\circ - 90^\circ + \arctan(2\cdot T_I) \overset{!}{=} -120^\circ$

$\arctan(2T_I) \overset{!}{=} 45^\circ$

$T_I = \frac{1}{2}$

$1 = |L(I2)| = V\cdot \frac{\cancel{\sqrt{2}}\cdot 100}{2\sqrt{1+\frac{1}{3}}\,\cancel{\sqrt{2}}} \Rightarrow V = \frac{2\cdot 2}{\sqrt{3}\cdot 100} = \frac{4}{\sqrt{3}\,100}$

c) i) nicht intern stabil wegen Pol-Nullstellen-Kürzung eines instabilen Pols

ii) $1 + L(s) = 0$

$1 + L(s) \neq 0$ für $Re(s) > 0$

$\Rightarrow$ intern stabil wenn keine Pol-Nullstellen-Kürzung von instabilen Polen auftritt

$$1 + \frac{s+10}{s-3} = 0$$

$$2s + 7 = 0$$

$$s = -\frac{7}{2} < 0 \Rightarrow \text{intern stabil}$$

Regler ist jedoch nicht realisierbar!

iii) $1 + L(s) = 0$

$$1 + \frac{7s+4}{s(s-3)} = 0$$

$$s^2 - 3s + 7s + 4 = 0$$

$s^2 + 4s + 4 = 0$ alle Koeff. > 0 und grad $= 2$

$\Rightarrow$ Hurwitzpolynom $\Rightarrow$ intern stabil

3) a) i) $\dot{x} = -x^2$

$$\frac{dx}{dt} = -x^2$$

$$-\frac{1}{x^2}\,dx = dt \quad \Big| \int$$

$$\frac{1}{x} + c = t$$

$$x = \frac{1}{t-c}$$

$$x(0) = x_0$$

$$x_0 = \frac{1}{-c}$$

$$x = \frac{1}{t + \frac{1}{x_0}}$$

$$x_{k+1} = \frac{1}{T_a + \frac{1}{x_k}}$$

ii) $x_{k+1} = \dfrac{1}{1+\frac{1}{x_k}} = \dfrac{x_k}{x_k+1}$

IA: $x_1 = \dfrac{x_0}{x_0+1} = \dfrac{1}{2} \overset{!}{=} \dfrac{1}{1+k} = \dfrac{1}{1+1} = \dfrac{1}{2}$ ✓

IS:

$$x_{k+1} = \frac{1}{1+\frac{1}{x_k}}$$

$$\frac{1}{1+(k+1)} = \frac{1}{1+(1+k)} \quad ✓$$

3b) $u(t) = \frac{1}{T_a}\left(t\,\sigma(t) - 2(t-T_a)\,\sigma(t-T_a) + (t-2T_a)\,\sigma(t-2T_a)\right)$

$$\hat{u}(s) = \frac{1}{T_a s^2}\left(1 - 2e^{-sT_a} + e^{-2sT_a}\right)$$

$$\hat{y}(s) = \frac{1}{T_a s^2}\left(1 - 2e^{-sT_a} + e^{-2sT_a}\right) G(s)$$

$$y_{k-1} \circ\!\!-\!\!\bullet\; z^{-1} y_z = Z\left(\frac{1}{T_a s^2}\left(1 - 2e^{-sT_a} + e^{-2sT_a}\right) G(s)\right)$$

$$= \frac{1}{T_a}\left(1 - 2z^{-1} + z^{-2}\right) Z\left(\frac{G(s)}{s^2}\right)$$

$$y_z = \frac{z^2 - 2z + 1}{T_a z}\, Z\left(\frac{G(s)}{s^2}\right)$$

$$= \frac{(z-1)^2}{T_a z}\, Z\left(\frac{G(s)}{s^2}\right)$$

4a)

$$\mathcal{O} = \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{array}{cc|cc} & & 1 & \frac{1}{2} \\ & & -\frac{1}{2} & 0 \\ \hline 1 & 1 & \frac{1}{2} & \frac{1}{2} \end{array}$$

$\det(\mathcal{O}) = 0 \Rightarrow$ nicht voller Rang $\Rightarrow$ nicht vollständig beobachtbar

4 b)

$$\Phi_e = (\Phi + \hat{k} c^T) = \begin{bmatrix} 1 & 1/2 \\ -\frac{1}{2} & 0 \end{bmatrix} + \begin{bmatrix} k_1 & k_1 \\ k_2 & k_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+k_1 & \frac{1}{2}+k_1 \\ k_2-\frac{1}{2} & k_2 \end{bmatrix}$$

$$\det(\lambda E - \Phi_e) = (\lambda - (1+k_1))(\lambda - k_2) - (k_2 - \tfrac{1}{2})(\tfrac{1}{2} + k_1) =$$

$$= \lambda^2 - \lambda k_2 - \lambda(1+k_1) + k_2 + \cancel{k_1 k_2} - (\cancel{k_2 k_1} + \tfrac{k_2}{2} - \tfrac{1}{4} - \tfrac{k_1}{2})$$

$$= \lambda^2 + \lambda(-1-k_1-k_2) + \frac{1}{4} + \frac{k_1}{2} + \frac{k_2}{2}$$

$$= \lambda^2 + a_1 \lambda + a_0$$

z^2	1	a_1	a_0	
	a_0	a_1	1	$\lambda_2 = a_0$
z	$1-a_0^2$	$a_1(1-a_0)$	0	
	$a_1(1-a_0)$	$1-a_0^2$	0	$\lambda_1 = \frac{a_1 \cancel{(1-a_0)}}{(1+a_0)\cancel{(1-a_0)}} = \frac{a_1}{1+a_0}$
	$1-a_0^2 - \frac{a_1^2(1-a_0)}{1+a_0}$			

4b) ff

$1 > 0$

$1 - a_0^2 > 0$

$(1+a_0)(1-a_0) > 0$

1. Fall:

$1 + a_0 > 0$

$a_0 > -1$

$1 - a_0 > 0$

$1 > a_0$

2. Fall

$1 + a_0 < 0$

$-1 \quad a_0$

$1 - a_0 < 0$

$1 < a_0$

unsinnig $\Rightarrow$ 1. Fall richtig

$$1 - a_0^2 - \frac{a_1^2(1-a_0)}{1+a_0} > 0$$

$$\frac{(1+a_0)^2(1-a_0) - a_1^2(1-a_0)}{\underbrace{1+a_0}_{>0}} > 0$$

$$\underbrace{(1-a_0)}_{>0}\left((1+a_0)^2 - a_1^2\right) > 0$$

$$(1+a_0+a_1)(1+a_0-a_1) > 0$$

1. Fall:

$1 + a_0 + a_1 > 0$

$a_1 > -1 - a_0$

$1 + a_0 - a_1 > 0$

$1 + a_0 > a_1$

bzw.

$a_0 + a_1 > -1$

$a_1 - a_0 < 1$

2. Fall:

$1 + a_0 + a_1 < 0$

$a_1 < -(1+a_0) < 0$

$1 + a_0 - a_1 < 0$

$0 < 1 + a_0 < a_1$

unsinnig $\Rightarrow$ Fall 1 richtig

$a_0 = \frac{1}{4} + \frac{k_1}{2} + \frac{k_2}{2}$

$-1 < \frac{1}{4} + \frac{k_1}{2} + \frac{k_2}{2} < 1$

$-\frac{5}{2} < k_1 + k_2 < \frac{3}{2}$

$-\frac{3}{4} - \frac{k_1}{2} - \frac{k_2}{2} > -1 \Rightarrow \frac{k_1}{2} + \frac{k_2}{2} < \frac{1}{4}$

$k_1 + k_2 < \frac{1}{2}$

$-\frac{5}{4} - \frac{3}{2}k_1 - \frac{3}{2}k_2 < 1$

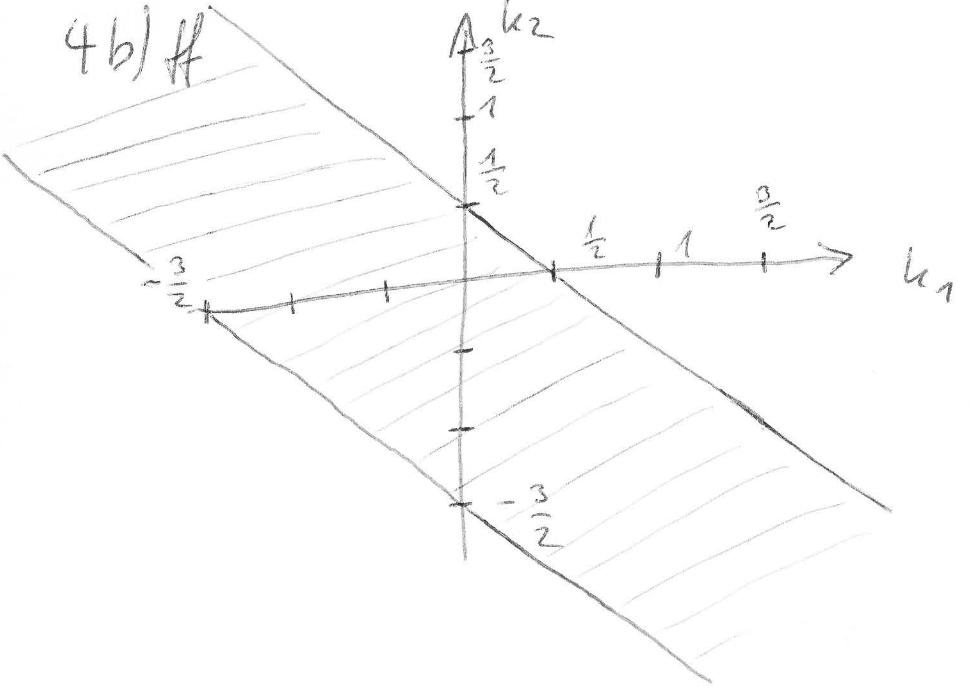
$\Rightarrow \frac{5}{4} + \frac{3}{2}k_1 + \frac{3}{2}k_2 > -1$

$\frac{3}{2}(k_1 + k_2) > -\frac{9}{4}$

$k_1 + k_2 > -\frac{\cancel{2}\cdot\cancel{9}^{3}}{\cancel{3}\cdot\cancel{4}_{2}} = -\frac{3}{2}$

$\Rightarrow -\frac{3}{2} < k_1 + k_2 < \frac{1}{2}$

4b) ff



Alternative: über Tustin-Transformation

$$z^2 + a_1 z + a_0 = 0 \qquad z = \frac{1+\frac{T_a}{2}q}{1-\frac{T_a}{2}q}$$

$$\frac{\left(1+\frac{T_a}{2}q\right)^2}{\left(1-\frac{T_a}{2}q\right)^2} + a_1 \frac{\left(1+\frac{T_a}{2}q\right)}{\left(1-\frac{T_a}{2}q\right)} + a_0 = 0$$

$$\left(1+\frac{T_a}{2}q\right)^2 + a_1\left(1+\frac{T_a}{2}q\right)\left(1-\frac{T_a}{2}q\right) + a_0\left(1-\frac{T_a}{2}q\right)^2 = 0$$

$$1 + 2\frac{T_a}{2}q + \left(\frac{T_a}{2}q\right)^2 + a_1\left(1-\left(\frac{T_a}{2}q\right)^2\right) + a_0\left(1 - 2\frac{T_a}{2}q + \left(\frac{T_a}{2}q\right)^2\right) = 0$$

$$\left(\frac{T_a}{2}q\right)^2\left(1-a_1+a_0\right) + \frac{T_a}{2}q \cdot 2\left(1-a_0\right) + \left(1+a_1+a_0\right) = 0$$

$\frac{T_a}{2} > 0$ für Hurwitz-Polynom müssen alle Koeffizienten > 0 sein

usw.

c) Nein, da das System nicht vollständig beobachtbar ist, können nicht alle Eigenwerte von Φ_e frei gewählt werden.