2009/10/09

c) $\dot{\omega} = \frac{1}{m}(m \cdot g + f_e + f_m)$

$\Theta \Phi = N i$, $R_E = \rho_1$, $R_L(\delta) = \rho_2 \delta$

$L_G(\delta) = N^2 / R_G(\delta)$

$R_G(\delta) = ? = \rho_1 + \rho_2 \delta$

a) $L_G(\delta) = \frac{N^2}{\rho_1 + \rho_2 \delta}$

b) $f_m = \frac{1}{2} \frac{\partial L_G(\delta)}{\partial \delta} i^2 = -\frac{i^2}{2} \frac{N^2 \rho_2}{(\rho_1 + \rho_2 \delta)^2}$

d) $\frac{d\psi}{dt} = -R i + v$, $\psi = L_G(\delta) \cdot i$

$\frac{d\psi}{dt} = -\frac{R\psi}{\rho_1 + \rho_2 \delta} + v$

$-\frac{R(\rho_1 + \rho_2 \delta)}{N^2} \psi + v$

$\cancel{\frac{d\psi}{dt} = L_G \dot{i} + \frac{dL_G}{d\delta} \frac{d\delta}{dt} i}$

$\cancel{= L_G \dot{i} - \frac{N^2 \rho_2}{(\rho_1 + \rho_2 \delta)^2} \dot{\delta} i}$

$\cancel{= \frac{N^2}{\rho_1 + \rho_2 \delta} \left(\dot{i} - \frac{\rho_2}{\rho_1 + \rho_2 \delta} \omega i\right) = -R i + v}$

e) $x = \begin{bmatrix} \psi \\ \delta \\ \omega \end{bmatrix}$, $u = \begin{bmatrix} v \\ f_e \end{bmatrix}$ $y = i$

$\boxed{\dot{\delta} = \omega}$

$\Theta \Phi = N \cdot i$

$\cancel{\dot{\omega} = g + \frac{f_e}{g} - i}$

$\boxed{\dot{\psi} = -\frac{R\psi}{\rho_1 + \rho_2 \delta} + v}$

$\dot{\omega} = g + f_e/m - \frac{N^2 \rho_2 \psi^2 (\rho_1 + \rho_2 \delta)^2}{2 N^4 (\rho_1 + \rho_2 \delta)^2 m}$

$\boxed{\dot{\omega} = g + f_e/m - \frac{\rho_2 \psi^2}{N^2 2 m}}$

$\boxed{y = i = \frac{\psi(\rho_1 + \rho_2 \delta)}{N^2}}$

1)

$$C^* = \begin{bmatrix} \frac{p_1 + p_2 \delta_R}{N^2} \\ \psi_R \frac{p_2}{N^2} \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} \psi \\ \delta \\ \omega \end{bmatrix}$$

$$D = 0$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1/m \end{bmatrix} \qquad A = \frac{\partial f(x_R, u_R)}{\partial x_R} =$$

$$= \begin{bmatrix} -\frac{R(p_1 + p_2\delta_R)}{N^2} & -\frac{Rp_2}{N^2}\psi_R & 0 \\ 0 & 0 & 1 \\ -\frac{p_2}{N^2 m}\psi_R & 0 & 0 \end{bmatrix}$$

2)

$$G_i(s) = \frac{\hat{i}}{\hat{v}} = \frac{1}{L_\delta s + R} = \frac{1}{1 + \frac{\sqrt{3}\, s}{100}}$$

i) $\omega_c t_r = 1{,}5 \rightarrow \omega_c = 100$

$\ddot{u} = 10\% \rightarrow PR = 60° \rightarrow \varphi = -\frac{2\pi}{3}$

$$R = \frac{V(1 + sT_I)}{s}$$

ii) $\arg\left(\frac{1}{s} G_i(s)\right)\Big|_{s = I\omega_c} = -\pi/2 - \arctan \frac{\sqrt{3}}{1} = -\frac{\pi}{2} - \frac{\pi}{3} = -\frac{5\pi}{6}$

Phasenanhebung $\varphi - \arg(L_1) = \pi/6$

iii) $\arg(1 + sT_i)_{s = I\omega_c} = \arctan 100 T_i \overset{!}{=} \pi/6$

$$\boxed{T_i = \frac{\sqrt{3}}{3 \cdot 100}}$$

$$\left|\frac{(1 + sT_i)}{s} G_i(s)\right|\Bigg|_{s = I\omega_c} = \frac{\sqrt{1 + \frac{3}{3^2}}}{100} \cdot \frac{1}{\sqrt{3+1}} = \frac{2}{\sqrt{3}} \frac{1}{100} \cdot \frac{1}{2} \overset{!}{=} \frac{1}{V}$$

$$\boxed{V = \sqrt{3} \cdot 100}$$

2 iv) $e_{\infty}|_{\delta_{ref}(t)-\delta(t)} = \lim_{t\to\infty} (r(t) - y(t)) = \lim r(t)\left(1 - \frac{L}{1+L}\right)$

$= \lim r(t) \frac{1}{1+L}$

$\cancel{\lim_{s\to 0} (s R(s) G(s) \frac{1}{s})}$

$e_{\infty} = \lim_{s\to 0} s\hat{e}(s) = \lim s \frac{1}{1+L} \cdot \frac{1}{s}$

$$= \lim_{s\to 0} \frac{1}{1 + \frac{\sqrt{2}\cdot 100 (1+sT_I)}{s\left(1+\frac{\sqrt{2}s}{100}\right)}} = 0$$

b) $G_\delta(s) = \frac{1}{ms^2}$ $\quad V = \text{konst}$

$R_\delta(s) = V_\delta (1 + T_\delta s)$

i) $G_{p_e, e_\delta}(s) = \frac{\hat{e}_\delta}{p_e} = \frac{\delta_{ref} - \hat{\delta}}{\hat{p}_e}$

$\delta = \frac{p_e \cdot G_\delta}{1 + R_\delta V G_\delta}$

$\cancel{(\hat{e}_\delta \cdot R_\delta \cdot V + p_e) \cdot G_\delta = \hat{\delta}}$

$\cancel{(\delta_{ref} - \delta) R_\delta \cdot V + p_e = \frac{\delta}{G_\delta}}$

$$= \frac{\delta_{ref} - \frac{p_e G_\delta}{1 + R_\delta V G_\delta}}{p_e}$$

$$z = \frac{1 + \frac{T_a}{2} q}{1 - \frac{T_a}{2} q}$$

$\lim_{z\to 1} (z-1) \cdot G(z)\, q_0$

$\lim_{q\to 0} \left(\frac{1 + \frac{T_a}{2} q - 1 + \frac{T_a}{2} q}{1 - \frac{T_a}{2} q} G^{\#}(q) \right)$

$$\boxed{\lim_{q\to 0} \left(\frac{T_a q}{1 - \frac{T_a}{2} q} G^{\#}(q) \right)}$$

$$\frac{z}{z-1} = \frac{\frac{1 + \frac{T_a}{2} q}{1 - \frac{T_a}{2} q}}{\frac{T_a q}{1 - \frac{T_a}{2} q}} = \frac{1 + \frac{T_a}{2} q}{T_a q}$$

ad 3bii

$(A - \lambda E)\, v_{i+1} = v_i$

$$\begin{pmatrix} \alpha - \frac{\alpha+\beta}{2} & \frac{\beta-\alpha}{2} \\ \frac{\alpha-\beta}{2} & \beta - \frac{\alpha+\beta}{2} \end{pmatrix} \begin{matrix} v_1 \\ v_2 \end{matrix} = \begin{matrix} 1 \\ 1 \end{matrix}$$

$$\frac{\alpha-\beta}{2} v_1 = \frac{\alpha-\beta}{2} v_2 + 1$$

$$\frac{\alpha-\beta}{2} v_1 = \frac{\alpha-\beta}{2} v_2 + 1$$

$$v_2 = 0, \quad v_1 = \frac{2}{\alpha-\beta}$$

$$V = \begin{vmatrix} 1 & 2/\alpha-\beta \\ 1 & 0 \end{vmatrix}$$

$$V^{-1} = \begin{pmatrix} 0 & +2/\beta-\alpha \\ -1 & 1 \end{pmatrix} \cdot \frac{\alpha-\beta}{2} \qquad \tilde{A} = \begin{vmatrix} \frac{\alpha+\beta}{2} & 1 \\ 0 & \frac{\alpha+\beta}{2} \end{vmatrix}$$

iii $\quad x = Vz$

$$\begin{array}{cc|l} & & b_1 \\ & & b_2 \\ 0 & -1 & -b_2 \\ \frac{\beta-\alpha}{2} & \frac{\alpha-\beta}{2} & (b_1 - b_2) / \frac{\beta-\alpha}{2} \end{array}$$

$$V\dot{z} = AVz + bu$$

$$\dot{z} = \underbrace{V^{-1}AV}_{\tilde{A}} z + \underbrace{V^{-1}b}_{\tilde{b}} u$$

wenn $\quad \dot{z} = \tilde{A} z + \tilde{b} u$

iv $\quad b_1 = b$, hängt $\dot{z}_2$ nur von z_2 ab, $\dot{z}_1 = \frac{\alpha+\beta}{2} z_2$

$\rightarrow$ nicht steuerbar

3a) $\dot{x} = f(x)$ mit $f(x) = \lambda x$

i) $\lambda = \alpha,\ \alpha < 0$

$$x_{k+1} = x_k + T_a \alpha x_k = x_k \underbrace{(1 + T_a \alpha)}_{\Phi}$$

asympt. stabil, wenn Eigw. von Φ im EK

$$|1 + T_a \alpha| < 1$$

weil $\alpha < 0$ $\quad T_A \cancel{*} > 0,\ T_a < \left|\frac{2}{\alpha}\right|$

ii) $x_{k+1} = x_k + I\alpha x_k T_a = x_k \underbrace{(1 + I\alpha T_a)}_{\Phi}$

$$|1 + I\alpha T_a| < 1$$

$$\sqrt{1 + (\alpha T_a)^2} < 1$$

$$1 + (\alpha T_a)^2 < 1$$

$$(\alpha T_a)^2 < 0$$

$$\alpha^2 T_a^2 < 0 \quad \text{nie erfüllt.}$$

-> funktioniert für ungedämpfte Schwingungs-/systeme nicht

b) $R \doteq [b, Ab]$

erreichbar $\Leftrightarrow$ steuerbar

$$\begin{array}{cc|c}
 & & b_1 \\
 & & b_2 \\
\hline
\alpha & \frac{\beta-\alpha}{2} & \alpha b_1 + \left(\frac{\beta-\alpha}{2}\right) b_2 \\
\frac{\alpha-\beta}{2} & \beta & b_1 \frac{\alpha-\beta}{2} \quad \beta b_2
\end{array}$$

$$R = \begin{bmatrix} b_1 & ; & \alpha b_1 + \left(\frac{\beta-\alpha}{2}\right) b_2 \\ b_2 & ; & b_1 \left(\frac{\alpha-\beta}{2}\right) + \beta b_2 \end{bmatrix}$$

$$b_1^2 \left(\frac{\alpha-\beta}{2}\right) + \beta b_1 b_2 - \alpha b_1 b_2 \cancel{+} \left(\frac{\beta-\alpha}{2}\right) b_2^2 \neq 0$$

$$\left(b_1 \sqrt{\frac{\alpha-\beta}{2}}\right)^2 - 2 \frac{(\alpha-\beta)}{2} b_1 b_2 + \left(b_2 \sqrt{\frac{\alpha-\beta}{2}}\right)^2$$

$$\left(b_1 \sqrt{\frac{\alpha-\beta}{2}} - b_2 \sqrt{\frac{\alpha-\beta}{2}}\right)^2 \neq 0 \quad \boxed{b_1 = 1,\ b_2 = 2}$$

$$b_1 \neq b_2$$

$\frac{\alpha-\beta}{2} + \cancel{2\beta} - \cancel{2\alpha} - \cancel{2(\beta} + \cancel{2\alpha} \neq 0$

i)

$$\begin{vmatrix} \lambda-\alpha & \frac{\alpha-\beta}{2} \\ \frac{\beta-\alpha}{2} & \lambda-\beta \end{vmatrix} = (\lambda-\alpha)(\lambda-\beta) + \frac{(\alpha-\beta)^2}{4} =$$

$$= \lambda^2 + \alpha\beta - \lambda(\alpha+\beta) + \frac{(\alpha-\beta)^2}{4}$$

$$\lambda_{1,2} = \frac{\alpha+\beta}{2} \pm \sqrt{\frac{(\alpha+\beta)^2}{4} - \frac{(\alpha-\beta)^2}{4} - \alpha\beta}$$

$$= \frac{\alpha+\beta}{2} \pm \frac{\sqrt{\cancel{\alpha^2} + \cancel{2\alpha\beta} + \cancel{\beta^2} - \cancel{\alpha^2} + \cancel{2\alpha\beta} - \cancel{\beta^2} - \cancel{4\alpha\beta}}}{2}$$

$$\boxed{\lambda_{1,2} = \frac{\alpha+\beta}{2}}$$

$$\begin{bmatrix} \alpha & \frac{\beta-\alpha}{2} \\ \frac{\alpha-\beta}{2} & \beta \end{bmatrix} \quad \begin{matrix} v_1 = \frac{\alpha+\beta}{2} v_1 \\ v_2 = \frac{\alpha+\beta}{2} v_2 \end{matrix} \rightarrow \begin{matrix} \alpha v_1 + \frac{\beta-\alpha}{2} v_2 = \frac{\alpha+\beta}{2} v_1 \\ \frac{\alpha-\beta}{2} v_1 + \beta v_2 = \frac{\alpha+\beta}{2} v_2 \end{matrix}$$

α

$$\Rightarrow \begin{cases} v_1\left(\alpha - \frac{\alpha}{2} - \frac{\beta}{2}\right) + \frac{\beta-\alpha}{2} v_2 = 0 \\ v_1\left(\frac{\alpha}{2} - \frac{\beta}{2}\right) + v_2\left(\beta - \frac{\alpha}{2} - \frac{\beta}{2}\right) = 0 \end{cases}$$

$$v_1 \frac{\alpha-\beta}{\cancel{2}} = v_2 \frac{\alpha-\beta}{\cancel{2}} \qquad v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\cancel{\frac{1}{s} \circ\!-\!\bullet \frac{1-e^{-sT_a}}{s}}$$

$$\cancel{s\,G(s) \circ\!-\!\bullet (z-1)\,G(z)}$$

$$\cancel{s \circ\!-\!\bullet\, Z\left(\mathcal{L}^{-1}\left(1-e^{-sT_a}\right)\Big|_{t=kT_a}\right)}$$

$$\cancel{Z\left(\delta(t) - \delta(t-T_a)\right)} \qquad \cancel{t=kT_a}$$

$$\cancel{1 - 1\cdot z^{-1}} \qquad \cancel{\delta(kT_a - T_a)}$$

$$\cancel{\frac{z-1}{z}} \qquad \cancel{\delta(T_a(k-1))}$$

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4a) $\phi = \begin{bmatrix} 1 & 0 \\ 1 & a \end{bmatrix}$ $\quad x_{k+1} = \phi^k x_k$

$\phi^k = \begin{bmatrix} 1 & 0 \\ \frac{a^k - 1}{a-1} & a^k \end{bmatrix}$

VI (Ind)

$\phi^1 = \phi$

$\phi^{k+1} = \phi \cdot \phi^k$

$$\begin{array}{cc|cc} & \phi\cdot\phi^k & 1 & 0 \\ & & \frac{a^k-1}{a-1} & a^k \\ \hline 1 & 0 & 1 & 0 \\ 1 & a & 1+\frac{(a^k-1)a}{a-1} & a^{k+1} \end{array}$$

$\Big\} \begin{bmatrix} 1 & 0 \\ \frac{\cancel{a}-1+a^{k+1}-\cancel{a}}{a-1} & a^{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{a^{k+1}-1}{a-1} & a^{k+1} \end{bmatrix}$

b) $G(z) = c^T (zE - \phi)^{-1} \Gamma$

$\begin{bmatrix} z-1 & 0 \\ -1 & z-a \end{bmatrix}^{-1} \Rightarrow \begin{bmatrix} z-a & 0 \\ 1 & z-1 \end{bmatrix} \frac{1}{(z-1)(z-a)} = \begin{bmatrix} \frac{1}{z-1} & 0 \\ \frac{1}{(z-1)(z-a)} & \frac{1}{z-a} \end{bmatrix}$

$$\begin{array}{cc|cc|c} & & \frac{1}{z-1} & 0 & 0 \\ & & \frac{1}{(z-1)(z-a)} & \frac{1}{z-a} & 1 \\ \hline 0 & 1 & \frac{1}{(z-1)(z-a)} & \frac{1}{(z-a)} & \frac{1}{z-a} \end{array}$$

$G(z) = \frac{1}{z-a} = \frac{1}{a} \frac{z^{-1}}{(z/a)-1}$

$(g_k) = \frac{1}{a} \cdot a^k (1^{k-1})$

$= a^{k-1} (1^{k-1})$

$\sum^{\infty} |g_k| \overset{!}{<} \infty$, wenn $|a| < 1$

$-a + \frac{1}{1-a} < \infty$

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c) $\Gamma = \Gamma_v = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad c^T = (1 \; 1)$

$$\begin{array}{c|cc} & 1 & a \\ \phi & 1 & 2a \end{array}$$

Dead-Beat Beob. $P_{soll} = z^2$

$\hat{v}_1 = O^{-1} e_n \qquad O = \begin{bmatrix} c^T \\ c^T \phi \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & a \end{bmatrix} = \begin{matrix} 1 & 1 \\ 2 & 1/2 \end{matrix}$

$\cancel{\hat{v}_2 = \begin{bmatrix} +1 \\ 0 \end{bmatrix}}$

$\cancel{\hat{k} = -\phi^2 v_1 = -\begin{bmatrix} 1 \\ a+1 \end{bmatrix} = \begin{bmatrix} -1 \\ -3/2 \end{bmatrix}}$

$\cancel{e_{k+1} = (\phi + \hat{k} c^T) e_k = \begin{bmatrix} 0 & -3/2 \\ 1 & a \end{bmatrix} e_k}$

$\cancel{e_2 = \phi_e^2 e_0}$

$\cancel{e_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} e_0}$

$$\boxed{v_1 = O^{-1} e_n = \frac{1}{3}\begin{bmatrix} +2 \\ -2 \end{bmatrix}}$$

$$\boxed{\hat{k} = -\phi^2 v_1 = \frac{1}{3}\begin{bmatrix} -2 \\ -5/2 \end{bmatrix}}$$

$O^{-1} = -\frac{2}{3}\begin{bmatrix} 1/2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1/3 & +2/3 \\ +2/3 & -2/3 \end{bmatrix}$

$$\begin{array}{cc|c} & & 2 \\ & & -2 \\ \hline 1 & 0 & 2 \\ 3/2 & 1/4 & 5/2 \end{array} \qquad \begin{array}{c|cc} & 1 & 1 \\ \hline -2 & -2 & -2 \\ -5/2 & -5/2 & -5/2 \end{array}$$

$e_1 = (\phi + \hat{k} c^T) e_0$

$\cancel{= \begin{bmatrix} -1 & -2 \\ -1 & -a/4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/2 & \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ -5/2 & -5/2 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3/2 & -2 \end{bmatrix} e_0}$ $\quad \boxed{e_1 = 2e_1}$

$$\boxed{\phi_e = \begin{bmatrix} 1 & 0 \\ 1 & 1/2 \end{bmatrix} + \begin{bmatrix} -2/3 & -2/3 \\ -5/6 & -5/6 \end{bmatrix} = \begin{bmatrix} 1/3 & -2/3 \\ 1/6 & -1/3 \end{bmatrix}}$$

$\boxed{e_k = \vec{0} = \phi_e e_0}$

$\frac{1}{3} e_1 - \frac{2}{3} e_2 = 0$

$\frac{1}{6} e_1 - \frac{1}{3} e_2 = 0$

d) $e_k = \hat{x}_k - x_k \qquad e_{k+1} = \hat{x}_{k+1} - x_{k+1}$

$\hat{x}_{k+1} = \phi \hat{x}_k + x_{k+1} - \phi x_k - \Gamma_v v + \hat{k}(c^T \hat{x}_k - c^T x_k)$

$\hat{x}_{k+1} - x_{k+1} = e_{k+1} = (\phi + \hat{k} c^T)(\underbrace{x_{k+1} - x_k}_{e_k}) - \Gamma_v v$

$e_\infty \Rightarrow e_{k+1} = e_k \rightarrow \boxed{e_\infty} = - \cancel{\frac{\Gamma_v v}{B}} - (E - \underbrace{\phi - \hat{k} c^T}_{\phi_c})^{-1} \Gamma_v v$

$\cancel{e_\infty} \quad \cancel{123} \begin{vmatrix} 2/3 & +2/3 \\ -1/6 & 4/3 \end{vmatrix}^{-1} = \frac{1}{\underbrace{\frac{8}{9} + \frac{2}{18}}_{1}} \begin{bmatrix} 4/3 & -2/3 \\ +1/6 & 2/3 \end{bmatrix}$

$e_\infty = -\begin{bmatrix} 2 \\ 1 \end{bmatrix} v$