27.4.12

1a) $$\frac{d}{dt}\left(m_T(w_T - w_G)\right) = m_T g - F_A - c_T(s_T - l_0) - d_T w_T$$

b) $$\dot{m}_T = \rho_L q - \rho_W q = (\rho_L - \rho_W) q$$

c) $$x = \begin{bmatrix} s_T \\ w_T \\ m_T \end{bmatrix} \quad u = q \quad d = \begin{bmatrix} w_G \\ a_G \end{bmatrix} \quad y = s_T$$

$$\dot{m}_T(w_T - w_G) + m_T(\dot{w}_T - a_G) = m_T g - F_A - c_T(s_T - l_0) - d_T w_T$$

$$\frac{d}{dt}\begin{bmatrix} s_T \\ w_T \\ m_T \end{bmatrix} = \begin{bmatrix} w_T \\ \frac{1}{m_T}\left(m_T g - F_A - c_T(s_T - l_0) - d_T w_T - (\rho_L - \rho_W) q (w_T - w_G)\right) + a_G \\ (\rho_L - \rho_W) q \end{bmatrix}$$

$$y = s_T$$

d) $\dot{x} = 0$, $w_{G,R} = 0$, $a_{G,R} = 0$, $m_{T,R} = m_0 =$ konst.

$w_{T,R} = 0$, $q_R = 0$

$$0 = \frac{1}{m_0}\left(m_0 g - F_A - c_T(s_{TR} - l_0) - 0 - 0\right) + 0$$

$$s_{TR} = \frac{m_0 g - F_A}{c_T} + l_0$$

1 d) ff

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{c_T}{m_{T,R}} & -\frac{d_T + (\rho_L - \rho_W) q_R}{m_{T,R}} & A_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_{23} = \frac{d \dot{w}_T}{d m_T} = \frac{g \cdot m_{T,R} - (m_{T,R}\, g - F_A - c_T(s_{T,R} - l_0) - d_T w_{T,R} - (\rho_L - \rho_W) q_R (w_{TR} - w_G))}{m_{T,R}^2}$$

$$= \frac{F_A + c_T(s_{T,R} - l_0) + d_T w_{T,R} + (\rho_L - \rho_W) q_R (w_{TR} - w_G)}{m_{T,R}^2}$$

$$b = \frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ -\frac{(\rho_L - \rho_W)(w_{T,R} - w_G)}{m_{T,R}} \\ (\rho_L - \rho_W) \end{bmatrix} \qquad c^T = \frac{\partial h}{\partial x} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$b_d = \frac{\partial f}{\partial d} = \begin{bmatrix} 0 & 0 \\ \frac{(\rho_L - \rho_W) q_R}{m_{T,R}} & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Delta \dot{x} = A \Delta x + b \Delta u + b_d \Delta d$$

$$\Delta y = c^T \Delta x$$

2a) $G(z) = \frac{z-1}{z} \mathcal{Z}\left(\frac{G(s)}{s}\right) = \frac{\cancel{z-1}}{\cancel{z}} \frac{T_a \cancel{z}}{(z-1)^{\cancel{2}}} = \frac{T_a}{z-1}$

b) $T_{v,y}(z) = \frac{G(z)}{1+G(z)R(z)} = \frac{T_a}{z-1+T_a P}$

$T_{d,y}(z) = \frac{1}{1+G(z)R(z)} = \frac{z-1}{z-1+T_a P}$

c) $(1,1,1,\dots) \circ\!\!-\!\!\bullet \frac{z}{z-1} = \hat{v} = \hat{d}$

$\hat{y} = T_{v,y}\,\hat{v} + T_{d,y}\,\hat{d}$ $\qquad |T_a P - 1| < 1$ damit EWS angewendet werden darf

$\lim\limits_{k\to\infty} y_k = 0 = \lim\limits_{z\to 1} (z-1)\,\hat{y} = \lim\limits_{z\to 1} \frac{T_a}{z-1+T_a P} + \frac{z-1}{z-1+T_a P} =$

$= \frac{1}{P} \Rightarrow$ es existiert kein P damit $\lim\limits_{k\to\infty} (y_k) = 0$

d) $R(z) = K_P \frac{z-0{,}5}{z-a}$

$\lim\limits_{k\to\infty} y(k) = \lim\limits_{z\to 1} (z-1)\,\hat{y} = \lim\limits_{z\to 1} \cancel{(z-1)} \frac{G(z)}{1+G(z)R(z)} \cdot \frac{z}{\cancel{z-1}} =$

$= \lim\limits_{z\to 1} \frac{T_a(z-a)}{(z-1)(z-a)+T_a \cdot K_P(z-0{,}5)} \stackrel{!}{=} 0$

$a = 1$

K_P muss so gewählt sein dass alle Pole von $T_{v,y}(z)$ innerhalb des Einheitskreises liegen

2e) $n_L = z^2 - z - z\alpha + \alpha + T_\alpha \cdot K_p z - T_\alpha K_p \cdot 0{,}5$

$= z^2 + z(-1 - 1 + 1) + (1 - 0{,}5) = z^2 - z + 0{,}5$

$z_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 0{,}5}}{2} = \frac{1 \pm j}{2}$

$|z_{1,2}| = \frac{\sqrt{1+1}}{2} = \frac{\sqrt{2}}{2} < 1 \Rightarrow 1 + L(z) \neq z$ für $|z| \geq 1$

es treten keine Pol-Nullstellen-Kürzungen von instabilen Pole auf $\Rightarrow$ intern stabil

f) $z = \frac{1 + \frac{T_\alpha}{2} q}{1 - \frac{T_\alpha}{2} q} = \frac{1+q}{1-q}$

$R(z) = K_p \frac{z-b}{z-1}$ $\quad R^{\#}(q) = K_p \frac{1+q - b(1-q)}{1+q-1+q} = K_p \frac{1-b+q(1+b)}{2q}$

$G(z) =$

???

3 a) $z_k = V x_k$

$$V^{-1} z_{k+1} = \Phi V^{-1} z_k + \Gamma u_k$$

$$z_{k+1} = \underbrace{V \Phi V^{-1}}_{\Phi_s} z_k + \underbrace{V \Gamma}_{\Gamma_s} u_k$$

$$y_k = \underbrace{c^T V^{-1}}_{c_s^T} z_k + \underbrace{d}_{d_s} u_k$$

b) $$\Phi_s V = V \Phi$$

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_{n-1}^T \\ v_n^T \end{bmatrix} = \begin{bmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_{n-1}^T \\ v_n^T \end{bmatrix} \Phi$$

$$v_2^T = v_1^T \Phi$$

$$v_{i+1}^T = v_i^T \Phi \quad \text{für } i = 1 \ldots n-1$$

$$-a_0 v_1^T - a_1 v_2^T \ldots - a_{n-1} v_n^T = v_n^T \Phi$$

$$\Gamma_s = V \Gamma$$

$$v_1^T \Gamma = \ldots = v_{n-1}^T \Gamma = 0$$

$$v_n^T \Gamma = 1$$

3c) $v_2^T = v_1^T \Phi \qquad v_3^T = v_2^T \Phi$

$$
\begin{array}{ccc|ccc|ccc}
 & & & 2 & 1 & 0 & 2 & 1 & 0 \\
 & & & 1 & 0 & 0 & 1 & 0 & 0 \\
 & & & 0 & 1 & 1 & 0 & 1 & 1 \\
\hline
-\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} & -2 & -\frac{1}{3} & \frac{1}{3} & -\frac{13}{3} & -\frac{5}{3} & \frac{1}{3}
\end{array}
$$

$$\underbrace{\qquad\qquad}_{v_2^T} \qquad \underbrace{\qquad\qquad}_{v_3^T}$$

$$V = \begin{bmatrix} -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -2 & -\frac{1}{3} & \frac{1}{3} \\ -\frac{13}{3} & -\frac{5}{3} & \frac{1}{3} \end{bmatrix}$$

$$G(z) = c^T (zE - \Phi)^{-1} \Gamma + d$$

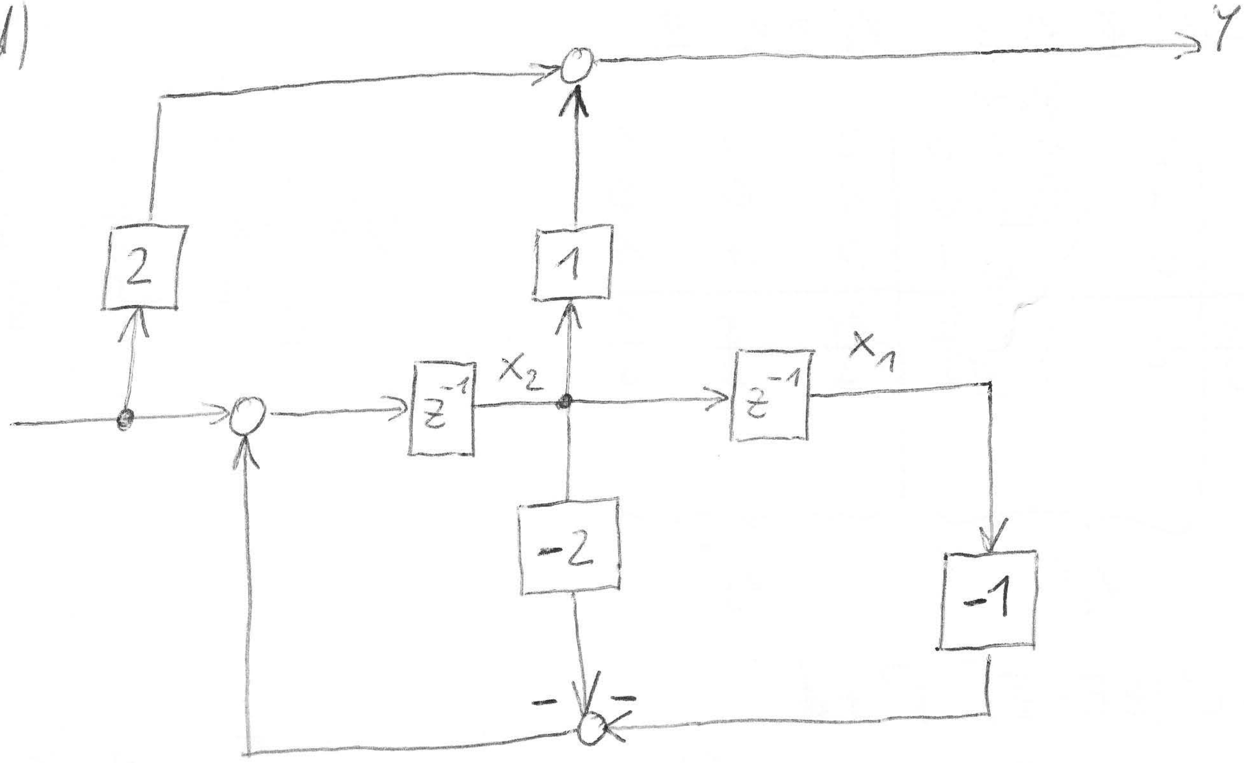
$$(zE - \Phi)^{-1} = \begin{bmatrix} z-2 & -1 & 0 \\ -1 & z & 0 \\ 0 & -1 & z-1 \end{bmatrix}^{-1} = \frac{1}{(z-2)z(z-1)-(z-1)} \begin{bmatrix} z\cdot(z-1) & X & 0 \\ X & X & X \\ X & X & X \end{bmatrix}$$

$$G(z) = \frac{z\cancel{(z-1)} + 0\cdot 2}{(z^2-2z)\cancel{(z-1)} - \cancel{(z-1)}} + 2 = \frac{z + 2z^2 - 4z - 2}{z^2 - 2z - 1} = \frac{2z^2 - 3z - 2}{z^2 - 2z - 1}$$

$$x_{k+1} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 0 & 1 \end{bmatrix} x_k + 2$$

3d)



4a) Matrix hat Dreiecksstruktur $\Rightarrow$ EW auf Hauptdiagonale

$\lambda_1 = 1,\ \lambda_2 = \frac{1}{4} \Rightarrow$ instabil da ein EW außerhalb des offenen Einheitskreises

b)

$$R = \begin{bmatrix} 0 & -1 \\ 2 & \frac{1}{2} \end{bmatrix} \Rightarrow \text{voller Rang} \Rightarrow \text{vollständig erreichbar}$$

$$R^{-1} = \frac{1}{2} \begin{bmatrix} \frac{1}{2} & 1 \\ -2 & 0 \end{bmatrix}$$

$$p_{g,soll} = \left(z - \frac{1}{2}\right)^2$$

$$p_{g,soll}(\Phi) = \left(\Phi - \frac{1}{2}E\right)^2$$

$$v_1^T = e_n^T R^{-1}$$

$$\begin{array}{cc|cc} & & \frac{1}{4} & \frac{1}{2} \\ & v_1^T & -1 & 0 \\ \hline 0 & 1 & -1 & 0 \end{array}$$

$$\begin{array}{cc|cc} & & \frac{1}{2} & -\frac{1}{2} \\ p_{g,soll} & & 0 & -\frac{1}{4} \\ \hline \frac{1}{2} & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{8} \\ 0 & -\frac{1}{4} & 0 & \frac{1}{16} \end{array}$$

$$k^T = -v_1^T p_{g,soll}(\Phi)$$

$$\begin{array}{cc|cc} & & \frac{1}{4} & -\frac{1}{8} \\ & k^T & 0 & \frac{1}{16} \\ \hline 1 & 0 & \frac{1}{4} & -\frac{1}{8} \end{array}$$

$$k^T = \begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \end{bmatrix}$$

c) $\lim\limits_{k\to\infty} y_k = \lim\limits_{k\to\infty} c^T x_k$

für $k \to \infty$ $\quad x_k = \Phi x_k + \Gamma u_k$

$$x_k = \Phi x_k + \Gamma k^T x_k + \Gamma g r_0$$

$$x_k = (E - \Phi - \Gamma k^T)^{-1} \Gamma g r_0$$

$$r_0 = c^T (E - \Phi - \Gamma k^T)^{-1} \Gamma g r_0$$

$$g = \frac{1}{c^T (E - \Phi - \Gamma k^T)^{-1} \Gamma}$$

$$E - \Phi - \Gamma k^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{1}{4} \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$(E - \Phi - \Gamma k^T)^{-1} = \frac{1}{\frac{1}{4}} \begin{bmatrix} x & -\frac{1}{2} \\ x & x \end{bmatrix} = \begin{bmatrix} x & -2 \\ x & x \end{bmatrix}$$

$$g = \frac{1}{-4} = -\frac{1}{4}$$

d) $\theta = \begin{bmatrix} 1 & 0 \\ 1 & -\frac{1}{2} \end{bmatrix}$ voller Rang $\Rightarrow$ vollständig beobachtbar

e) $\hat{v}_1 = \theta^{-1} e_n$ $\qquad \theta^{-1} = -2 \begin{bmatrix} x & 0 \\ x & 1 \end{bmatrix}$

$$\hat{v}_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\hat{p}_{g,soll} = z^2$$

$$\hat{p}_{g,soll}(\Phi) = \Phi^2$$

		1	$-\frac{1}{2}$
		0	$\frac{1}{4}$
1	$-\frac{1}{2}$	1	$-\frac{5}{8}$
0	$\frac{1}{4}$	0	$\frac{1}{16}$

$$\hat{k} = -\hat{p}_{g,soll}\, \hat{v}_1 = \begin{bmatrix} -\frac{5}{4} \\ \frac{1}{8} \end{bmatrix}$$

4 f)