34.09

$u = M$

$y = Y$

$J_Q = m \cdot L^2$

Φ

$y = L \sin\beta$

$J_{ges_2} = J_Q \; J_b + J_2$

Drehimpulserhaltung

$\frac{d}{dt} I \dot\varphi = \Sigma M$

$\dot\omega = \frac{1}{I} \Sigma M$

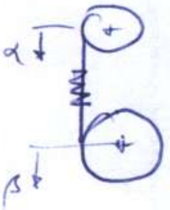
$x = \begin{array}{l} \beta \\ \dot\beta = \Omega \\ \alpha \\ \dot\alpha = \omega \end{array}$

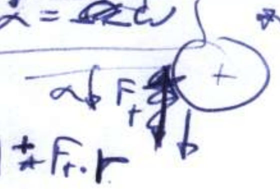
$\alpha r = \beta \cdot R + \sqrt[3]{\frac{F}{K}}$

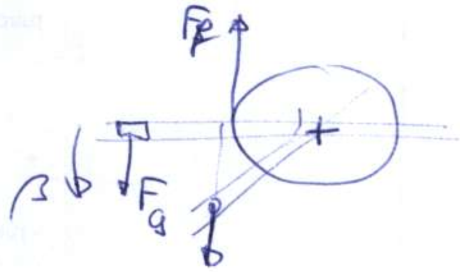
$J_1 \dot\omega = M + F_r \cdot r$

$\dot\Omega = \frac{1}{J_{ges}} (m \cdot g L + F_r \cdot R)$

$F_p = K(-\alpha r + \beta R)^3$







$$\boxed{\begin{aligned} \dot\omega &= \frac{M}{J_1} + K \cdot (\alpha r + \beta R)^3 \cdot r \frac{1}{J_1} \\ \dot\Omega &= \frac{1}{J_{ges}} \left(m \cdot g L \cos\beta - \frac{}{(\alpha r + \beta R)^3} KR\right) \\ y &= L \cdot \sin\beta \end{aligned}}$$

a) $\beta_R = \beta_0, \; \dot\omega = 0, \; \dot\Omega = 0$

$0 = \frac{M_R}{} + K(\beta R - \alpha r)^3 \cdot r$

$0 = m g L \cos\beta_0 - KR(\beta R - \alpha r)^3 \rightarrow \alpha_R = \frac{\sqrt[3]{\frac{m g L \cos\beta_0}{KR}} - \beta_0 R}{r}$

$$\boxed{M_R = -K\left(\beta_0 R - \beta_0 R - \frac{m g L \cos\beta}{KR}\right) r = \frac{-m g L \cos\beta_0 r}{R}}$$

c) $\beta_0 = 0 \rightarrow M_R = \frac{-m g L r}{R}$

$\alpha_R = \sqrt[3]{\frac{m g L}{K R r^3}}$

d) c) $\cancel{Y = KB_1}$

$d = 0, \quad c = \frac{dg}{dx} = \begin{vmatrix} L \cdot \cos\beta_0 \\ 0 \\ 0 \\ 0 \end{vmatrix} = [L \; 0 \; 0 \; 0]^T$

$b = \frac{df}{du} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1/J_1 \end{vmatrix}$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{KR}{J_{ges}} 3R(\alpha_R r)^2 & 0 & +\frac{KR}{J_{ges}} 3(\alpha_R r)^2 r & 0 \\ 0 & 0 & 0 & 1 \\ \frac{Kr}{J_1} 3(\alpha_R r)^2 R & 0 & -\frac{Kr}{J_1} 3(\alpha_R r)^2 r & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{3KR^2 (\alpha_R r)^2}{J_{ges}} & 0 & \frac{3KR}{J_{ges}} \alpha_R^2 r^3 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{3KR}{J_1} \alpha_R^2 r^3 & 0 & -\frac{3K r^4 \alpha_R^2}{J_1} & 0 \end{bmatrix}$$

d) Ähnlichkeit mit reeller Jord. Normalform

$\alpha_1 = \alpha_2 = 0 \quad \rightarrow \quad Re(\lambda_i) = 0$, nicht asymptotisch stabil → nicht BIBO-stabil

$\beta_1 = 0, 9; \; \beta_2 = 3$

2) a) i) $\underline{k^T} = -p_0 v_1^T - p_1 v_1^T \phi - v_1^T \phi^2$

$v_1^T = e_n \cdot R^{-1}$

$R = [\Gamma, \phi\Gamma] = \begin{pmatrix} 3b_1 & 3b_1+1 \\ 1/b_1 & 1/b_1 \end{pmatrix}$

$v_1^T = (1, -3b_1^2)$

$R^{-1} = \frac{1}{3-3-1/b_1} \cdot \begin{pmatrix} 1/b_1 & -3b_1-1 \\ -1/b_1 & 3b_1 \end{pmatrix} = +\begin{pmatrix} -1 & 3b_1^2+b_1 \\ 1 & -3b_1^2 \end{pmatrix}$

$$\boxed{\begin{aligned} k_1 &= -p_0 - p_1 - 1 \\ k_2 &= +3b_1^2 p_0 - p_1(b_1 - 3b_1^2) - (2b_1 - 3b_1^2) \end{aligned}}$$

$$\begin{array}{c|cc|cc} \alpha_n & 1 & b_1 & 1 & b_1 \\ & 0 & 1 & 0 & 1 \\ \hline & 1 & b_1-3b_1^2 & 1 & 2b_1-3b_1^2 \end{array}$$

ii) $p_0 = p_1 = 0$

$k_1 = -1$

$k_2 = 3b_1^2 - 2b_1$

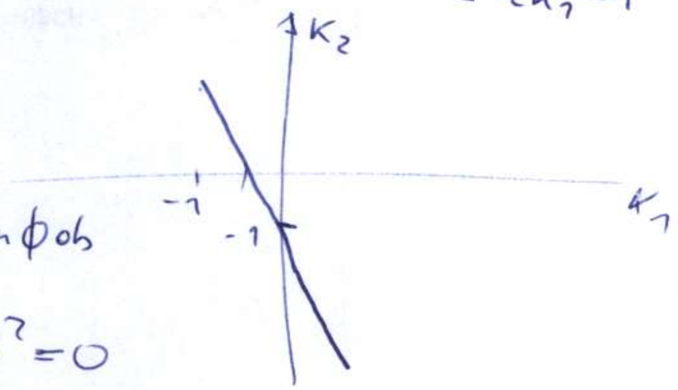
iii) $p = z(z+a) = z^2 + az$

$p_1 = a, p_0 = 0$

$k_1 = -a - 1 \to a = -k_1 - 1$

$k_2 = -a(1-3) - (2-3) = +2a + 1$

$= -2k_1 - 1$



iv) trivialer Beobachter

$e_{k+1} = \phi e_k = \underline{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}} e_k$

Stabilität hängt von ϕ ab

$\begin{vmatrix} z-1 & -1 \\ 0 & z-1 \end{vmatrix} = (z-1)^2 = 0$

↳ z am EHK → nicht BIBO-stabil

→ trivialer Beobachter instabil

b) $G(s) = \dfrac{(s+0{,}01)(s+10)}{s(s^2+6s+5)}$ $T_a = 4s$

$= s\cdot(s+5)(s+1)$

Pole $\cancel{s}_i \Leftrightarrow \cancel{z}_i = e^{+zT_a}$ poles: $0, -5, -1$

$\rho_i = \Omega_0 \frac{z_i-1}{z_i+1}$

$G(s)$ nicht sprungfähig $\rightarrow$ Nst bei Ω_0 im ρ bereich mit $\Omega_0 = \frac{2}{T_a} = 0{,}5$

$\Leftrightarrow$ ~~alle~~ sind erlaubt I, II, IV, V

r z ρ

~~$z_1 = 1$~~

0 1 $0{,}5 \cdot \frac{0}{2} = 0$

-5 e^{-20} $0{,}5\,\frac{e^{-20}-1}{e^{-20}+1}$

-1 e^{-4} $0{,}5\,\frac{e^{-4}-1}{e^{-4}+1}$ $\Big\}$ 2 Pole (aufspaltbar)

I: $50\rho\left(\frac{\rho^2}{0{,}5} + \frac{\rho}{0{,}5} + 1\right) = 100\rho\,(\underbrace{\rho^2 + \rho + 0{,}5})$

$-\frac{1}{2} \pm \sqrt{-\frac{1}{4} - 0{,}5}$ $\rightarrow$ nicht reell aufspaltbar

$\rightarrow$ I kommt nicht in Frage

stat. Verstärkung

$\lim\limits_{s\to 0} \frac{s}{s} G(s) = \frac{0{,}01 \cdot 10}{0} = \infty$

$\lim\limits_{\rho\to 0} \cancel{\frac{T_a\rho}{1-\frac{T_a}{2}\rho}}\, G^{\#}\, \cancel{\frac{1+\frac{T_a}{2}\rho}{T_a\rho}} = \lim\limits_{\rho\to 0} \frac{1+2\rho}{1-2\rho}\, G^{\#}$ $\rightarrow$ ~~III~~ V ausgeschlossen, da stat. Verstärkung endlich ist

Realisierbarkeit $G(s)$ ✓

Impulsantwort:

$\lim\limits_{s\to 0} s\,G(s) = \frac{0{,}01 \cdot 10}{5} = 0{,}02$

$\lim\limits_{\rho\to 0} G^{\#}(\rho)\, \frac{\rho}{1-\rho\frac{T_a}{2}} = 0{,}02$ für IV $\leftarrow$ Richtig

$0{,}2$ für II

c) $(g_k) = (0, 10, 4, \frac{8}{5}, \frac{16}{25}, \frac{32}{125}, \dots)$

$g_k = m_k = c^T \phi^{k-1} \Gamma$

$H(1,1) = \begin{bmatrix} m_1 & m_2 \\ m_2 & m_3 \end{bmatrix} = \begin{bmatrix} 10 & \cancel{5} \\ \cancel{10}\,4 & \cancel{\;}\frac{8}{5} \end{bmatrix}$

$|H| \overset{\text{nicht}}{\neq} 0 \Rightarrow$ regulär

$\overset{\text{nicht}}{\Rightarrow}$ vollständig erreichbar + beobachtbar

3a) I) n.l., zeitvariant

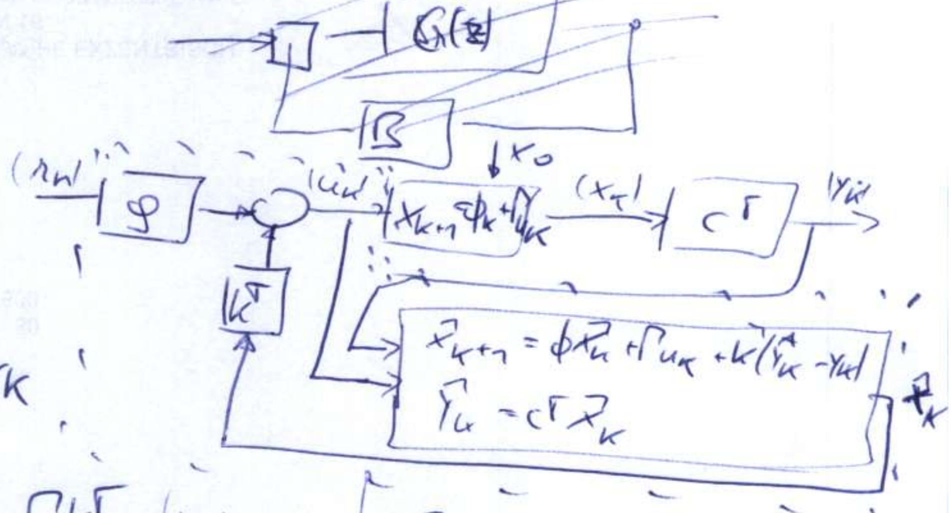
II) linear zeitvariant

III n.l. zeitinvariant

IV linear, zeitinvariant

b) i) $\phi = \begin{bmatrix} -1 & \frac{1}{6} \\ \frac{1}{3} & -2 \end{bmatrix}$ $\Gamma = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $c^T = [0\ 1]$

ii)



$x_{k+1} = \phi x_k + \Gamma u_k$

mit $u_k = k^T \hat{x}_k + g r_k$

$e = \hat{x}_k - x_k$

$x_{k+1} = \phi x_k + \Gamma k^T (e + x_k) + g r_k$

$$\begin{bmatrix} x_{k+1} \\ e_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi + \Gamma k^T & \Gamma k^T \\ 0 & \phi + \hat{k} c^T \end{bmatrix}}_{\phi_{ges}} \begin{bmatrix} x_k \\ e_k \end{bmatrix} + \begin{bmatrix} \Gamma g \\ 0 \end{bmatrix} u_k$$

$$Y_k = [c^T \ 0^T] \begin{bmatrix} x_k \\ e_k \end{bmatrix}$$

iii) $d_i(E\Phi_\varphi) = \det\left|zE-(\Phi+\Gamma k^T)\right| \cdot \det\left|zE-(\Phi+\bar{k}c^T)\right|$

$k^T = \left(-\frac{11}{6} \quad 2\right) \qquad \bar{k}^T = \left(-\frac{29}{6} \quad \frac{7}{2}\right)$

$$\begin{array}{c|cc} & -\frac{11}{6} & 2 \\ \hline 0 & 0 & 0 \\ 1 & -\frac{11}{6} & 2 \end{array} \qquad \begin{array}{c|cc} & 0 & 1 \\ \hline -\frac{29}{6} & 0 & -\frac{29}{6} \\ \frac{7}{2} & 0 & \frac{7}{2} \end{array}$$

$$\begin{vmatrix} z+1 & -\frac{1}{6} \\ -\frac{1}{3}+\frac{11}{6} & z+2-2 \end{vmatrix} = (z+1)(z) + \frac{1}{6}\cdot\frac{9}{6}\cdot\frac{3}{2} = z^2+z+\frac{1}{4} \; \boxed{=\left(z+\frac{1}{2}\right)^2}$$

$$\cancel{= z^2 + \frac{20}{3}z + \frac{17}{3} + \frac{1}{12} = z^2 + \frac{20}{3}z + \frac{76}{12}}$$

$$\begin{vmatrix} z+1 & -\frac{1}{6}+\frac{29}{6} \\ -\frac{1}{3} & z+2-\frac{7}{2} \end{vmatrix} = (z+1)\left(z-\frac{3}{2}\right) + \frac{1}{3}\left(\frac{28}{6}\right) =$$

$$= z^2 - \frac{z}{2} - \frac{3}{2} + \frac{14}{9} = z^2 - \frac{z}{2} + \frac{1}{18} =$$

$$z_{12} = \frac{1}{4} \pm \sqrt{\frac{1}{16}-\frac{1}{18}} = \frac{1}{4} \pm \frac{1}{\sqrt{2}}\sqrt{\frac{9-8}{9\cdot 8}}$$

$$= \frac{1}{4} \pm \frac{1}{\sqrt{2}\,3\cdot 2\sqrt{2}} = \frac{1}{4} \pm \frac{1}{12}$$

$$\hookrightarrow \boxed{\left(z - \frac{1}{3}\right)\left(z - \frac{1}{6}\right)}$$

$$\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{6}\right)$$

4a) $G(s) = \dfrac{300}{(s^2+s+15^2)(s^2+6s+9^2)} = \dfrac{4}{3 \cdot 81} \cdot \dfrac{1}{\left(\left(\frac{s}{15}\right)^2 + \frac{s}{15}\cdot\frac{2}{30}+1\right)\left(\left(\frac{s}{9}\right)^2 + \frac{s}{9}\cdot\frac{2\cdot 3}{9\cdot 3}+1\right)}$

$t_r = 0{,}5\,s \rightarrow \omega_c = 3$

$\hat{u} = 25 \rightarrow PR = 45 = \frac{\pi}{4}$

$\phi = -\frac{3\pi}{4}$

$\xi_1 = \frac{1}{30}$ $\qquad \xi_2 = \frac{1}{3}$

$R_1 = \dfrac{V \cdot \left[\left(\frac{s}{15}\right)^2 + \frac{s}{15}\cdot\frac{2}{30}+1\right]}{s \cdot (1+sT)^2}$

$L_1 = \dfrac{\frac{4}{3\cdot 81}}{s\cdot\left(\left(\frac{s}{9}\right)^2 + \frac{s}{9}\cdot\frac{2}{3}+1\right)} \rightarrow |L_1|_{\omega_c = 3} = \frac{1}{3}$

$\arg|L_1|_{\omega_c=3} = -\frac{\pi}{2} - \operatorname{atan}\left(\dfrac{\frac{2}{9}}{1-\frac{1}{9}}\right) = -\frac{\pi}{2} - \frac{\pi}{12} =$

$\frac{2/9}{8/9} = \frac{1}{4}$

$L_2 = \dfrac{R_1 L_1}{s \cdot (1+sT)^2}$

$\arg L_2 = -\frac{\pi}{2} - \frac{\pi}{12} - 2\cdot\operatorname{atan} 3T \stackrel{!}{=} -\frac{3\pi}{4}$

$2\left(-\frac{\pi}{2} - \frac{\pi}{12} + \frac{3\pi}{4}\right) \quad \frac{\pi}{12} = \operatorname{atan} 3T \rightarrow T = \frac{1}{3}\tan\frac{\pi}{12} = \frac{1}{12}$

$\left|\dfrac{\frac{4}{3\cdot 81}}{s\cdot(1+sT)^2\left(\left(\frac{s}{9}\right)^2 + \left(\frac{s}{9}\right)\cdot\frac{2}{3}+1\right)}\right|_{s=j3} = \dfrac{\frac{4}{3\cdot 81}}{3\cdot\left(1+\left(\frac{1}{4}\right)^2\right)\cdot\sqrt{\left(1-\frac{1}{9}\right)^2+\left(\frac{2}{9}\right)^2}} \stackrel{!}{=} \frac{1}{V}$

$\sqrt{\left(\frac{8}{9}\right)^2 + \left(\frac{2}{9}\right)^2} = \sqrt{\frac{68}{81}}$

$\frac{1}{V} = \dfrac{4 \cdot 4^2 \cdot 9}{9\cdot 81 \cdot 17 \cdot \sqrt{68}}$

$V = 17\cdot 81 \cdot \frac{17}{16} \cdot \frac{\sqrt{68}}{\ldots} \quad \frac{17^2\cdot 81}{16}$

$\frac{81}{4}\cdot\frac{16 4}{17}\cdot\sqrt{68} = \frac{81}{17}\,2\cdot\sqrt{17} = \frac{162}{\sqrt{17}}$

$$b)\ R(s) = \frac{1}{10} \cdot \frac{(1 + s/50)}{(1 + s/1000)}$$

$$\rightarrow G(s) = \frac{33}{1 + 2\xi s + s^2}$$

$$+ 40 \cdot \log \sqrt{2\xi} = 8\,dB$$

$$\sqrt{2\xi} = 10^{1/5}$$

$$\xi = \frac{10^{2/5}}{2}$$