7.3.2008) 1) $C_2(u_{C2}) = C_{20} + C_{21} u_{C2}^2$

$$Q = C \cdot U \rightarrow i = \frac{dQ}{dt} = \frac{dQ}{du} \cdot \frac{du}{dt} = \left(C + \frac{dC}{du} \cdot u\right) \dot{u}$$

$$i_{C2} = \left(C_{20} + C_{21} u_{C2}^2 + 2 C_{21} u_{C2}^2\right) \dot{u} = \underline{C_{20} \dot{u}_{C2} + 3 C_{21} u_{C2}^2 \dot{u}_{C2}}$$

$$x = \begin{bmatrix} u_{C1} \\ u_{C2} \end{bmatrix} \qquad i_{C1} = C_1 \frac{du_{C1}}{dt}$$

$$i_{C1} + i_{C2} = -u/R_0$$

$$u_{C1} = u - i_{C1} \cdot R_1 \rightarrow i_{C1} = \frac{u}{R_1} - \frac{u_{C1}}{R_1} \cancel{- \frac{\dot{u}_{C2} C_{20}}{R_1} - \frac{3 C_{21} u_{C2}^2 \dot{u}_{C2}}{R_1}}$$

$$y = u_{C2} + i_{C2} \cdot R_2 = u_{C2} + \dot{u}_{C2} C_{20} R_2 + 3 C_{21} R_2 u_{C2}^2 \dot{u}_{C2}$$

$$\cancel{-u/R_0 = \left(C_{20} \dot{u}_{C2} + 3 C_{21} u_{C2}^2 \dot{u}_{C2} + u/R_1 - \dot{u}_{C2} \frac{C_{20}}{R_1} - \frac{3 C_{21} u_{C2}^2 \dot{u}_{C2}}{R_1}\right)}$$

$$\cancel{\dot{u}_{C2} = \left(-u\left(\frac{1}{R_0} + \frac{1}{R_1}\right) \frac{u}{R_1}\right.}$$

$$-u/R_0 = \frac{u}{R_1} - \frac{u_{C1}}{R_1} + C_{20} \dot{u}_{C2} + 3 C_{21} u_{C2}^2 \dot{u}_{C2}$$

$$\dot{u}_{C2} = \frac{u_{C1}}{R_1 (C_{20} + 3 C_{21} u_{C2}^2)} - u \frac{1/R_0 + 1/R_1}{C_{20} + 3 C_{21} u_{C2}^2}$$

$$\dot{u}_{C1} = -\frac{u_{C1}}{R_1 C_1} + \frac{u}{R_1 C_1}$$

$$y = u_{C2} + R_2 \left(\frac{u_{C1}}{R_1} - u \left(1/R_0 + 1/R_1\right)\right)$$

$$y = \begin{bmatrix} R_2/R_1 & 1 \end{bmatrix} x + \left[-\left(\frac{R_2}{R_0} + \frac{R_2}{R_1}\right)\right] u$$

$$\cancel{\dot{x} = \frac{u_{C1}}{R_1 (C_{20} + 3 C_{21} u_{C2}^2)} - u \frac{1/R_0 + 1/R_1}{C_{20}}}$$

b) $u = 0$

$$\dot{u}_{C2} = 0 = \frac{u_{C1}}{R_1(C_{20} + 3 \cdots u_{C2}^2)} \rightarrow u_{C1} = 0$$

$u_{C2} \xrightarrow{\text{beliebig}}$

$$\dot{u}_{C1} = 0 = -\frac{u_{C1}}{R_1 C_1} \rightarrow u_{C1} = 0$$

u_{C2} nicht bestimmt -> unendlich viele Ruhelage

$$d = \frac{d\,g(x_R, u_R)}{d u} = -\frac{R_2}{R_0} + \frac{R_2}{R_1}$$

$$c^T = \frac{d g(x_R, u_R)}{d x} = \begin{bmatrix} \frac{R_2}{R_1} & 1 \end{bmatrix}$$

$$b = \frac{d f(x_R, u_R)}{d u} = \begin{bmatrix} 1/R_1 C_1 \\ -\frac{1/R_0 + 1/R_1}{C_{20} + 3C_{21} u_{C2,R}^2} \end{bmatrix}$$

$$A = \frac{d f(x_R, u_R)}{d x} = \begin{bmatrix} -\frac{1}{R_1 C_1} & 0 \\ \frac{1}{R_1(C_{20} + 3C_{21} u_{C2,R}^2)} & 0 \end{bmatrix}$$

c) $G(s) = c^T (sE - A)^{-1} b + d$

$$(sE - A)^{-1} = \begin{pmatrix} s + \frac{1}{R_1 C_1} & 0 \\ -\frac{1}{R_1 C_{20}} & s \end{pmatrix}^{-1} = \frac{1}{s(s + \frac{1}{R_1 C_1})} \begin{bmatrix} s & 0 \\ + \frac{1}{R_1 C_{20}} & s + \frac{1}{R_1 C_1} \end{bmatrix}$$

$$G(s) = \frac{s R_2}{R_1^2 C_1} - s\left(\frac{1}{R_0 C_{20}} + \frac{1}{R_1 C_{20}}\right) - \frac{1}{R_0 R_1 C_1 C_{20}} - \frac{R_2}{R_0} + \frac{R_2}{R_1} + \frac{2}{R_1^2 C_{20} C_1}$$

$$(sE-A)^{-1} b = \begin{bmatrix} \frac{1}{s+1/R_1C_1} & 0 \\ \frac{1}{R_1C_{20}\, s(s+1/R_1C_1)} & \cancel{\frac{s+\frac{1}{R_1C_1}}{s(s+1/R_1C_1)}} \; 1 \end{bmatrix}$$

$$\frac{1}{R_1C_1} \quad -\frac{1}{R_0C_{20}} - \frac{1}{R_1C_{20}}$$

$$\frac{1}{1+R_1C_1 s}$$

$$\cancel{\frac{1}{R_1^2C_1C_{20}\, s(s+\frac{1}{R_1C_1})}}$$

$$c^T(sE-A)^{-1} b + d \qquad + -\frac{1}{sR_0C_{20}} - \frac{1}{sR_1C_{20}}$$

$$G(s) = \frac{R_2}{R_1}\frac{1}{1+sR_1C_1} - \frac{1}{s}\left(\frac{1}{R_0C_{20}} + \frac{1}{R_1C_{20}}\right) - \frac{R_2}{R_0} - \frac{R_2}{R_1}$$

$$= \frac{R_2}{R_1}\left(\frac{-1-sR_1C_1+1}{1+sR_1C_1} - \frac{R_2}{R_0} - s\frac{R_2}{R_0}R_1C_1\right) - \frac{1}{s}\left(\frac{1}{R_0C_{20}} + \frac{1}{R_1C_{20}}\right)$$

$$= \frac{-\left(sR_2C_1 - \frac{R_2^2}{R_1R_0} - s\frac{R_2^2C_1}{R_0}\right)\cdot s - \left(\frac{1}{R_0C_{20}} + \frac{1}{R_1C_{20}}\right)(1+sR_1C_1)}{s(1+sR_1C_1)}$$

$$= \frac{s^2\left(\frac{R_2^2C_1}{R_0} - R_2C_1\right) + \frac{R_2^2}{R_1R_0}s - sR_1C_1\left(\frac{1}{R_0C_{20}} + \frac{1}{R_1C_{20}}\right) - \frac{1}{R_0C_{20}} + \frac{1}{R_1C_{20}}}{s(1+sR_1C_1)}$$

$$= \frac{\underbrace{-\left[\frac{1}{R_0C_{20}} + \frac{1}{R_1C_{20}}\right]}_{V_P} \cdot 1 + s\overbrace{\left[R_1C_1 - \frac{R_2^2}{R_1R_0\left(\frac{1}{R_0C_{20}} + \frac{1}{R_1C_{20}}\right)}\right]}^{T_D+T_I} + s^2\underbrace{\left(\frac{R_2C_1 - \frac{R_2^2C_1}{R_0}}{\frac{1}{R_0C_{20}} + \frac{1}{R_1C_{20}}}\right)}_{T_I T_D}}{s(1+s\underbrace{R_1C_1}_{T_R})}$$

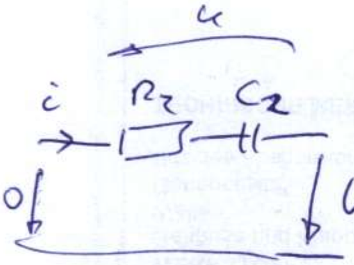
d) $R = [b, Ab] = \begin{bmatrix} \frac{1}{R_1C_1} & -\frac{1}{R_1^2C_1^2} \\ -\frac{1}{R_0C_{20}} - \frac{1}{R_1C_{20}} & -\frac{1}{RC_1}\left(\frac{1}{R_0C_{20}} + \frac{1}{R_0R_1}\right) \end{bmatrix}$

$= -\frac{R_1+R_0}{R_1^2C_1C_{20}R_0}$

$\mathrm{Rang}(R) = 2 \rightarrow$ vollst. erreichbar

c) Methode 2

$Y_{a12} = -\frac{1}{R_0} \cdot \frac{1+s(R_0+R_1)C_1}{1+s\frac{(R_0+R_1)C_1R_2}{R_1+R_2}}$



$Y_{b12} = \frac{i}{u} = -\frac{sC_2}{1+sR_2C_2}$

$i \cdot R + i \cdot \frac{1}{sC} = u$

$i = -\frac{u}{R+1/sC} = \frac{sC_2u}{1+sR_2C_2}$

$G(s) = -\frac{1}{R_0sC_2} \frac{(1+sR_2C_2)(1+s(R_0+R_1)C_1)}{\left(1+s\frac{(R_0+R_1)C_1R_2}{R_2+R_1}\right)R}$

$V = -\frac{1}{C_2R_0}$

$T_R = \frac{(R_0+R_1)}{R_1+R_2} \cdot C_1R$

$T_I = R_2C_2$

$T_D = (R_0+R_1)C_1$

2a) $\dot{x} = f(x,u,t)$

$y = h(x,u,t)$

lineares System lässt sich in die Form

$\dot{x} = Ax + Bu$

$y = Cx + Du$

1) $f(\alpha x_1 + \beta x_2, 0, t) = \alpha f(x_1, 0, t) + \beta f(x_2, 0, t)$

$f(0, \alpha u_1 + \beta u_2, t) = \alpha f(0, u_1, t) + \beta f(0, u_2, t)$

$f(x,u,t) = f(x,0,t) + f(0,u,t)$

schreiben, invariant, wenn A, B, C, D nicht von t abhängen

oder, $y(t)$, $x(t_0) = x_0$, $u(\tau)$ (für $t_0 \le \tau \le t$), dann ist $y(t-T)$ für $x(t_0+T) = x_0$ und $u(\tau - T)$ $t_0 + T \le \tau \le t + T$

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b) wenn $K(s)$ und $G(s)$ BIBO-stabil

c) $K = \cancel{f_r}\, y - f$

$f = K \cdot \frac{1}{s} \frac{1}{s^2+2s+2} = (\cancel{f_r} y) \frac{1}{s} \frac{1}{s^2+2s+2}$

$y = (r - f) \cdot \frac{K}{s^2+1}$ $\qquad f = \dfrac{+\frac{y}{s} \frac{1}{s^2+2s+2}}{1 + \frac{1}{s(s^2+2s+2)}} = \dfrac{+y}{1+s(s^2+2s+2)}$

$y = \left(r - \frac{y}{1+s(s^2+2s+2)}\right) \cdot \frac{K}{s^2+1} = \frac{rK}{s^2+1} - \frac{yK}{(1+s(s^2+2s+2))(s^2+1)}$

$G(s) = \frac{y}{r} = \dfrac{K}{(s^2+1)\left(1 + \frac{K}{(1+s(s^2+2s+2))(s^2+1)}\right)} =$

$= \dfrac{K}{(s^2+1) + \frac{K}{1+s(s^2+2s+2)}} = \dfrac{K(1+s(s^2+2s+2))}{K + (s^2+1)(1+s(s^2+2s+2))} =$

$= \dfrac{K(1+2s+2s^2+s^3)}{K+1+2s+2s^2+s^3+s^2+s^5+2s^4+2s^3} = \dfrac{K(1+2s+2s^2+s^3)}{s^5+2s^4+3s^3+3s^2+2s+(1+K)}$

ad 2c)

$$
\begin{array}{c|ccc}
s^5 & 1 & 3 & 2 \\
s^4 & 2 & 3 & (1+K) \\
s^3 & \frac{3}{2} & \frac{4-(1+k)}{2} = \frac{3-k}{2} & 0 \\
s^2 & \frac{2}{3}\left(\frac{9}{2} - \cancel{\ldots}\right) & (1+k) & \\
s^1 & \frac{\left(1+\frac{2k}{3}\right)\frac{(3-k)}{2} - \frac{3}{2}(1+k)}{\left(1+\frac{2k}{3}\right)} & & \\
s^0 & (1+k) & &
\end{array}
$$

$$\frac{2}{3}\left(\frac{9}{2} - \left(\frac{3-k}{2}\right)\cdot 2\right) = 1 + \frac{2k}{3}$$

$$\cancel{\frac{2}{3}\left(\frac{9}{2} + \frac{k}{2}\right) \ldots}$$

$$= \frac{\frac{3}{2} - \frac{k}{2} + \frac{2k}{2} - \frac{2k^2}{6} - \frac{3}{2} - \frac{3}{2}k}{1 + \frac{2k}{3}} =$$

$$\cancel{\frac{9 + \ldots - 4k^2}{1 - \frac{2k}{3}} = \frac{9 - 3k - 4k^2}{6 + 4k}}$$

$$\frac{-k\left(1 + \frac{k}{3}\right)}{1 + \frac{2k}{3}} = \frac{-k(3+k)}{3+2k}$$

$(1+k) > 0 \rightarrow k > -1$

$1 + \frac{2k}{3} > 0 \rightarrow k > -\frac{3}{2}$

$$\cancel{\frac{9-3k-4k^2}{6+4k} > 0 \rightarrow 6+4k > 0}$$

$$\cancel{k > -\frac{6}{4} = -\frac{3}{2}}$$

$$\cancel{9 - 3k - 4k^2 > 0}$$

$$\cancel{k^2 + \frac{3}{4}k - \frac{9}{4} < 0}$$

$$\cancel{k_{1,2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{9}{4}} = -\frac{3}{2} \pm \frac{3\sqrt{2}}{2}} \quad \cancel{= k_1 \ldots \text{kein Einschränkung}}$$

$$\cancel{k_2 = \frac{-3 + 3\sqrt{2}}{2} = \frac{3(\sqrt{2} - 1)}{2}}$$

$$\frac{-k(3+k)}{3+2k} > 0 \quad \Big| \quad \text{Fall: } 3 + 2k > 0, \; k > -\frac{3}{2} \; \text{(keine Einschränk.)}$$

$-k(3+k) > 0 \Rightarrow k < 0$

$k \in [-1, 0]$

a+c)

$$G(s)\big|_{k=-0,5} = \frac{0,5\,(1+2s+2s^2+s^3)}{s^5+2s^2+3s^3+3s^2+2s+0,5}$$

$$\lim_{t\to\infty} = \lim_{s\to 0} s\cdot G(s)\cdot r(s) = \lim_{s\to 0} G(s) r_s = 1 r_s$$

$$\lim_{t\to\infty} r(s) = r_s$$

$$e_\infty = y_\infty - r_\infty = 0$$

---

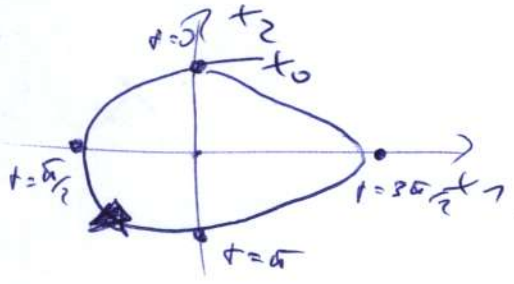
d) $\dot{x}_1 = -x_2$ $\quad x_1(0) = 0$

$\dot{x}_2 = x_1$ $\quad x_2(0) = 1$

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad x(t) = \phi(t)\cdot x_0$$

$$\phi(t) = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix} \qquad \boxed{\beta = -1}, \boxed{k = 0}$$

$$x(t) = \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix}$$



3) kein direkter DG $\quad g(0) \stackrel{!}{=} 0 \quad$ (iii) fällt aus

$g(1) = \frac{1,5}{3} \stackrel{!}{=} 0,5 \quad$ (ii) verbleibt

$$-0,5\cdot 1 + x\cdot 3 = 0,25$$

$$x = \frac{0,5}{6} + \frac{0,25}{3}$$

maximal 2 spätestens nach

3) $\cancel{g_k = 0{,}5\delta(t-1) - 0{,}25(t-2) - 0{,}25(t-3)}$

$G(z) \;\circ\!\!-\!\!\bullet\; 0{,}5\,\delta_{k-1} + 0{,}25\,\delta_{k-2}$

$G(z) = \cancel{\frac{0{,}5 z}{z-1}}\; z^{-1}\,0{,}5 + 0{,}25 \cdot z^{-2} = \frac{z\,0{,}5 + 0{,}25}{z^2}$

$\text{pole}(G(z)) = 0 \rightarrow$ liegt im EHK $\rightarrow$ BIBO-stabil

$\sum\limits_{0}^{\infty} |g_k| < \infty \rightarrow$ erfüllt $(0{,}75) \rightarrow$ BIBO-stabil

d) $|G(z)|_{z=e^{j\pi/2}=j} = \left| \frac{0{,}5j + 0{,}25}{-1} \right| = \sqrt{\frac{1}{16} + \frac{1}{4}} = \sqrt{5}/4$

$|G(z)|_{z=e^{j0}=1} = \left| \frac{0{,}5 + 0{,}25}{1} \right| = 3/4$

$\arg(G(z))_{z=j} = \arg\left(\frac{0{,}5j + 0{,}25}{-1} \right) = \operatorname{atan}\frac{0{,}5}{0{,}25} - \operatorname{atan}(-1)$

$= \operatorname{atan} 2 + \pi/4$

$y_k = \frac{\sqrt{5}}{2} \sin\left(k\frac{\pi}{2} + \operatorname{atan} 2 + \frac{\pi}{4}\right) - \frac{3}{4}\, 1^k$

e) Pole $z_i = e^{s_i T_a}$

Vielleicht keine aussage mgl.

4a) phasenminimal: wenn Pole + Nst in (abgeschlossener) linker HE ~~offene~~ liegen

c) i) $f_i = 1{,}5 \to \omega_c = 1{,}5$

$\alpha + \varphi_R = 70 \to \varphi_R = 60 = \frac{\pi}{3}$

$$\arg\left(\frac{(1+\frac{s}{2})\cdot 0{,}5}{s(\frac{s}{2}+1)}\right) = -\frac{\pi}{2}$$

$$\Delta\varphi = -\pi + \frac{\pi}{2} + \frac{\pi}{2} = -\frac{\pi}{2} + \frac{\pi}{3} = -\frac{\pi}{6}$$

$$\arg\left\{\frac{1}{1+sT_R}\right\}_{s=j1{,}5} = -\frac{\pi}{6} = -\arctan(1{,}5\,T_R)$$

$$T_R = \frac{\sqrt{3}}{3\cdot 1{,}5} \qquad T_R = \frac{2\sqrt{3}}{9}$$

$$\left|\frac{G(s)\cdot 1+\frac{s}{2}}{1+sT_R}\right| = \left|\frac{0{,}5}{s\,(1+s\frac{2\sqrt{3}}{9})}\right|_{s=j1{,}5} = \frac{0{,}5}{1{,}5\cdot\sqrt{\frac{3}{9}+1}} = \frac{0{,}5}{1{,}5\cdot\sqrt{12}/3} = \frac{1}{\sqrt{12}} = \frac{1}{2\cdot\sqrt{3}} \cdot \frac{1}{2\sqrt{3}}$$

$$V_R = 2\sqrt{3}$$

ii) $$L(s) = \frac{2\sqrt{3}\cdot 0{,}5}{s\cdot(1+sT_R)} = \frac{\sqrt{3}}{s^2T_R+s} = \frac{\sqrt{3}}{\cancel{j}\omega - \omega^2T_R} = \frac{(-\omega^2T_R - j\omega)\sqrt{3}}{\omega^4T_R^2+\omega^2}$$

$$= \frac{-\omega^2T_R\sqrt{3}}{\omega^4T_R^2+\omega^2} - \frac{\sqrt{3}\,\omega j}{\omega^2+\omega^4T_R^2}$$

	Re	Im
-0	$-T_R\sqrt{3}$	$+\infty$
$+0$	$-T_R\sqrt{3}$	$-\infty$
1	$-\frac{T_R\sqrt{3}}{1+T_R^2}$	$-\frac{\sqrt{3}}{1+T_R^2}$
$\sqrt{3}$	$-\frac{\sqrt{3}\,T_R\sqrt{3}}{1+3T_R^2}$	$-\frac{1}{1+3T_R^2}$
$-\infty$	$+0$	$+0$
$+\infty$	-0	-0