$r_a = r_i \, e^{\frac{\;}{c'}} = \underline{\underline{31{,}28\,\text{mm}}}$

(4.2)

$f = 5{,}8\,\text{GHz}, \quad P_s = 17\,\text{dBm}, \quad G_s = 31\,\text{dB}, \quad d = 8\,\text{km}, \quad G_e = 16\,\text{dBi}$

4.1)

$h = 25\,\text{m}$

$\lambda = \frac{c_0}{f} = 51{,}724\,\text{mm}$

$r = \sqrt{\frac{d\lambda}{4}} = 10{,}17\,\text{m}$

$h_{min} = r + h = 35{,}17\,\text{m}$

4.2)

$P_{e,dBm} = P_{s,dBm} - L_{iso} + G_s + G_e = -61{,}72\,\text{dBm}$

$L_{iso} = -20 \cdot \log\left(\frac{\lambda}{4\pi d}\right) = 125{,}77\,\text{dB}$

4.3)

$$f(\vartheta,\varphi) = \begin{cases} \sqrt{4\pi}\, A \cos^m(\vartheta) \sin^{32}(\varphi) \sqrt{\cos\varphi} & \text{für } 0 \le \varphi \le \pi/2 \\ 0 & \text{sonst} \end{cases}$$

$\cancel{\Omega = \int_0^{\pi}\int_0^{\pi} \sqrt{4\pi}\, A \cos^m(\vartheta) \sin(\vartheta) \sin^{32}(\varphi) \sqrt{\cos\varphi}\, d\vartheta\, d\varphi}$

4.4

4.3)

$$f(\vartheta,\varphi) = \begin{cases} \sqrt{4\pi}\, A \cos^{m}(\vartheta) \sin^{32}\varphi \sqrt{\cos\varphi} & 0 \le \varphi \le \frac{\pi}{2} \\ 0 \end{cases}$$

$$\Rightarrow \quad G_s = \frac{4\pi}{\Omega_ä}$$

$$\Omega_ä = 4\pi A^2 \int_0^{\frac{\pi}{2}}\int_0^{\pi} \cos^{2m}(\vartheta) \sin\vartheta \sin^{64}\varphi \cos\varphi \, d\vartheta \, d\varphi$$

$u = \cos\vartheta$ $\qquad$ $v = \sin\varphi$

$du = -\sin\vartheta\, d\vartheta \Rightarrow d\vartheta = -\frac{1}{\sin\vartheta} du$ $\qquad$ $dv = \cos\varphi\, d\varphi \Rightarrow d\varphi = \frac{1}{\cos\varphi} dv$

$$= 4\pi A^2 \int_0^1 \int_1^{-1} -u^{2m} v^{64} \, du \, dv$$

immer ungerade => neg (-1)

$$= 4\pi A^2 \left(-\frac{u^{2m+1}}{2m+1}\right)\Bigg|_1^{-1} \frac{v^{65}}{65}\Bigg|_0^1 = 4\pi A^2 \underbrace{\left(-\frac{(-1)^{2m+1}}{2m+1} + \frac{1}{2m+1}\right)}_{\frac{2}{2m+1}} \frac{1}{65}$$

$$= 4\pi A^2 \frac{2}{2m+1} \frac{1}{65}$$

$G_{s,dB} = 31\,dBi \mathrel{\hat=} 10^{3,1} = D$

$$10^{3,1} = \frac{65 \cdot (2m+1)}{2A^2} \Rightarrow \frac{2A^2 \cdot 10^{3,1}}{65} = 2m+1$$

$$\Rightarrow m = \frac{\frac{2A^2 \cdot 10^{3,1}}{65} - 1}{} = 20{,}047 \Rightarrow \underline{\underline{20 = m}}$$

4.4)

~~... f(90;55)^2~~

~~... L f(132,45) = 21,44·10^{-3}~~

$$\Omega_ä = 4\pi A^2 \int_0^{\frac{\pi}{2}}\int_{-7,5}^{132,5} \cos^{40}\vartheta \sin\vartheta \sin^{64}\varphi \cos\varphi = 4\pi A^2 \left(-\frac{\cos^{41}\vartheta}{41}\right)\Bigg|_{-7,5}^{132,5} \frac{1}{65}$$

$$= \frac{4\pi A^2}{65} \, \frac{\cos(-7{,}5)^{41} - \cos(132{,}5)^{41}}{41} = 7{,}6344 \cdot 10^{-3}$$

$$D = G_s = \frac{4\pi}{\Omega_ä} = 3572{,}83 = 35{,}51\,dBi$$

$$\Rightarrow P_{e,dBm} = P_{s,dBm} - L + G_s + G_e = -60{,}26\,dBm$$

$$\underline{\underline{\Delta SNR}} = SNR - SNR' = P_{e,dBm} - P_{e,dBm}' = \underline{\underline{1{,}51\,dB}}$$

93 TM_{010}, $a = r$, $d = h$

$E_z = E_0 J_0(k_r r)$ $\qquad J_0(k_r a) = 0$

$H_\varphi = \frac{j\omega\varepsilon}{k_r} E_0 J_1(k_r r)$ $\qquad k_r^2 + k_z^2 = \frac{\omega^2}{c_0^2} \Rightarrow k_r^2 = \frac{\omega^2}{c_0^2}$

5.1

$$\omega_{mnp} = c \sqrt{\left(\frac{j_{m,n}}{a}\right)^2 + \left(\frac{p\pi}{l}\right)^2}$$

$j_{m,n}$ … n-te Nullstelle der m-ten Besselfunktion

$\Rightarrow$ 1-te Nullstelle der 0-ten Bessel

$j_{01} = k_r \cdot r$

$$\Rightarrow \omega_{010} = c \cdot \frac{2{,}4048}{a} = c_0 \cdot \frac{2{,}4048}{a}$$

$$f_{010} = \frac{\omega_{010}}{2\pi}$$

5.2

$$Q_0 = \frac{2\pi W}{P} = \frac{\omega_{mnp} W}{P} \qquad W = \frac{1}{4}\int_V (\varepsilon E E^* + \mu H H^*)\, dV$$

$dA = r\,dr\,d\varphi$ $\qquad P = \frac{1}{2} R_H \int_A H \cdot H^*\, dA$

$dV = r\,dr\,d\varphi\,dz$

$$W = \frac{1}{4} E_0^2 \int_0^z \int_0^\varphi \int_0^r \left[\varepsilon_0 r J_0^2(k_r r) + \mu_0 \frac{\omega^2\varepsilon^2}{k_r^2} r J_1^2(k_r r)\right] dr\,d\varphi\,dz$$

$$= \varepsilon_0 \frac{E_0^2}{4} \cdot z \cdot \varphi \left[\frac{r^2}{2}\left(J_1^2(r k_r) + J_0^2(k_r r)\right) + \mu_0 \frac{\omega^2 \varepsilon_0 r^2}{k_r^2 2}\left(J_0^2(k_r r) - \frac{2}{k_r r} J_0(k_r r) J_1(k_r r) + J_1^2(k_r r)\right)\right]$$

$$P = \frac{1}{2} R_H \int_0^\varphi \int_0^r \frac{\omega^2 \varepsilon^2}{k_r^2} E_0^2 r J_1^2(k_r r)\, dr\,d\varphi$$

$$= \frac{1}{2} R_H \frac{\omega^2 \varepsilon^2}{k_r^2} E_0^2 \varphi \left[\frac{r^2}{2}\left(J_0^2(k_r r) - \frac{2}{k_r r} J_0(k_r r) J_1(k_r r) + J_1^2(k_r r)\right)\right]$$

5.3 $\qquad R_H = \sqrt{\frac{\omega\mu}{2\sigma}} = 34{,}49\,m\Omega$ $\qquad f_{010} = 17{,}18\,GHz$

mit $r = a \Rightarrow J_0(k_r a) = 0$, $z = d$ $\qquad k_r^2 = \frac{\omega^2}{c_0^2}$

$$W = \varepsilon_0 \frac{\cancel{E_0^2}}{4} d \cancel{\varphi} \cancel{\frac{a^2}{2}} \left[\cancel{J_1^2(k_r a)} + \mu_0 \frac{\omega^2 \varepsilon_0}{k_r^2} \cancel{J_1^2(k_r a)}\right]$$

$$P = \frac{1}{2} R_H \frac{\omega^2 \varepsilon^2}{k_r^2} \cancel{E_0^2} \cancel{\frac{a^2}{2}} \cancel{J_1^2(k_r a)} \cancel{\varphi}$$

$$\Rightarrow \underline{\underline{Q_0}} = \frac{\omega_{010} \cdot \frac{\varepsilon_0 d}{4}\left[1 + \cancel{\frac{\mu_0 \varepsilon_0 c_0^2}{1}}\right]}{\frac{1}{2} R_H \varepsilon_0^2 c_0^2} = \frac{\omega_{010} \cdot d}{R_H \cdot \varepsilon_0 \cdot c_0^2} = \underline{\underline{19{,}64 \cdot 10^3}}$$