Exploration of chaos in shock waves

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Outline

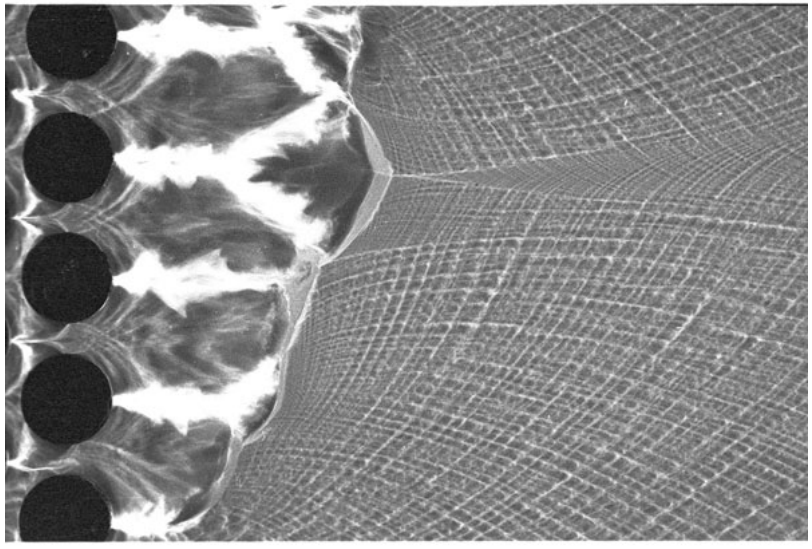
Background: Reactive Euler equations and detonation

$$\frac{D\rho}{Dt} = -\rho \nabla . u$$

$$\rho \frac{Du}{Dt} = -\nabla P$$

$$\rho \frac{D}{Dt}\left(h + \frac{|u|^2}{2}\right) = \frac{\partial P}{\partial t}$$

$$\frac{DY_i}{Dt} = \dot{\omega}_i$$



Model Equation

$$u_t + \frac{1}{2}(u^2 - uu_s)_x = f(x, u_s)$$

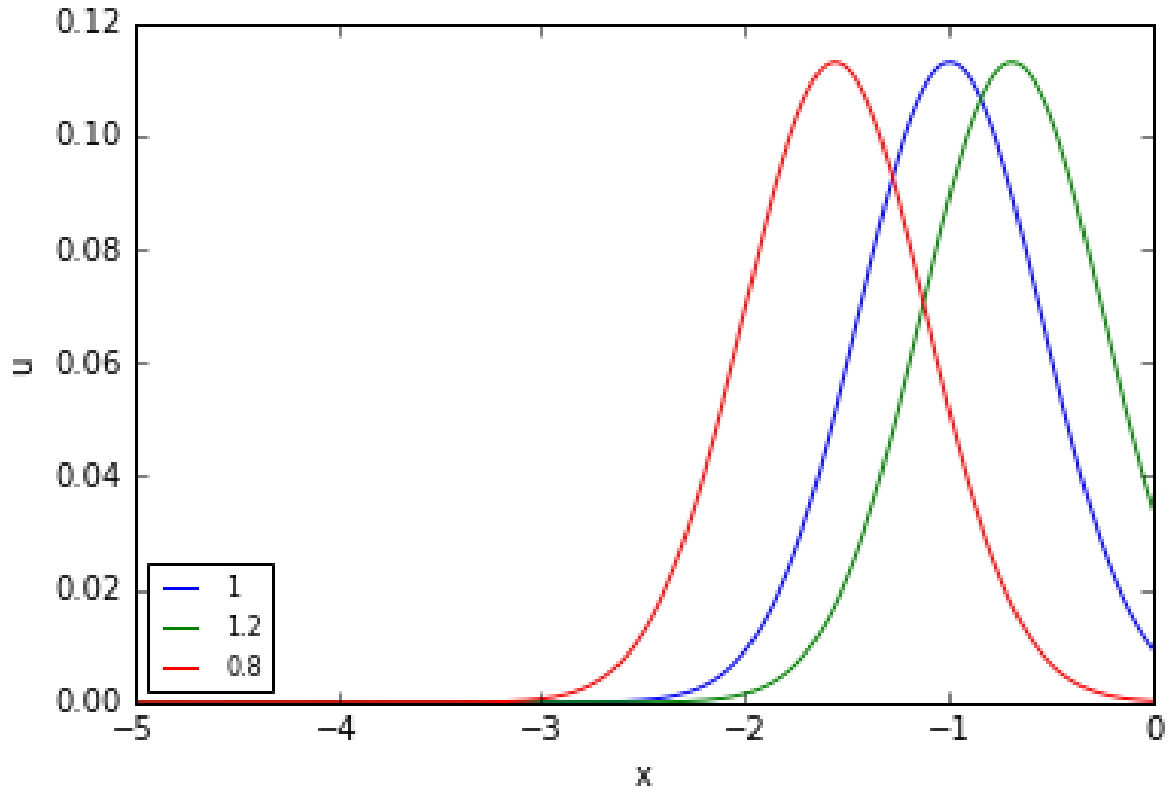
$$x \in (-\infty, 0)$$

$$\textbf{Characteristic speed } = u - \frac{u_s}{2}$$

The nature of the source term

$$f(x, u_s) = \frac{q}{2\sqrt{4\pi\beta}} e^{-\frac{[x - x_f(u_s)]^2}{4\beta}}$$

$$x_f(u_s) = \left(\frac{u_{0s}}{u_s} \right)^{\alpha}$$



The steady state solution

$$\frac{1}{2}(u_0^2 - u_0 u_{0s})' = f(x, u_{0s})$$

$$u_0(x) = \frac{u_{0s}}{2} + \sqrt{2\int_{-\infty}^{x} f(y, u_{0s})dy}$$

$$f = \frac{q}{2\sqrt{4\pi\beta}} e^{-\frac{[x-x_f(u_s)]^2}{4\beta}}$$

$$u_0(x) = \frac{1}{2}\left[1 + \sqrt{\frac{1 + erf((x+1)/2\sqrt{\beta})}{1 + erf(1/2\sqrt{\beta})}}\right]$$

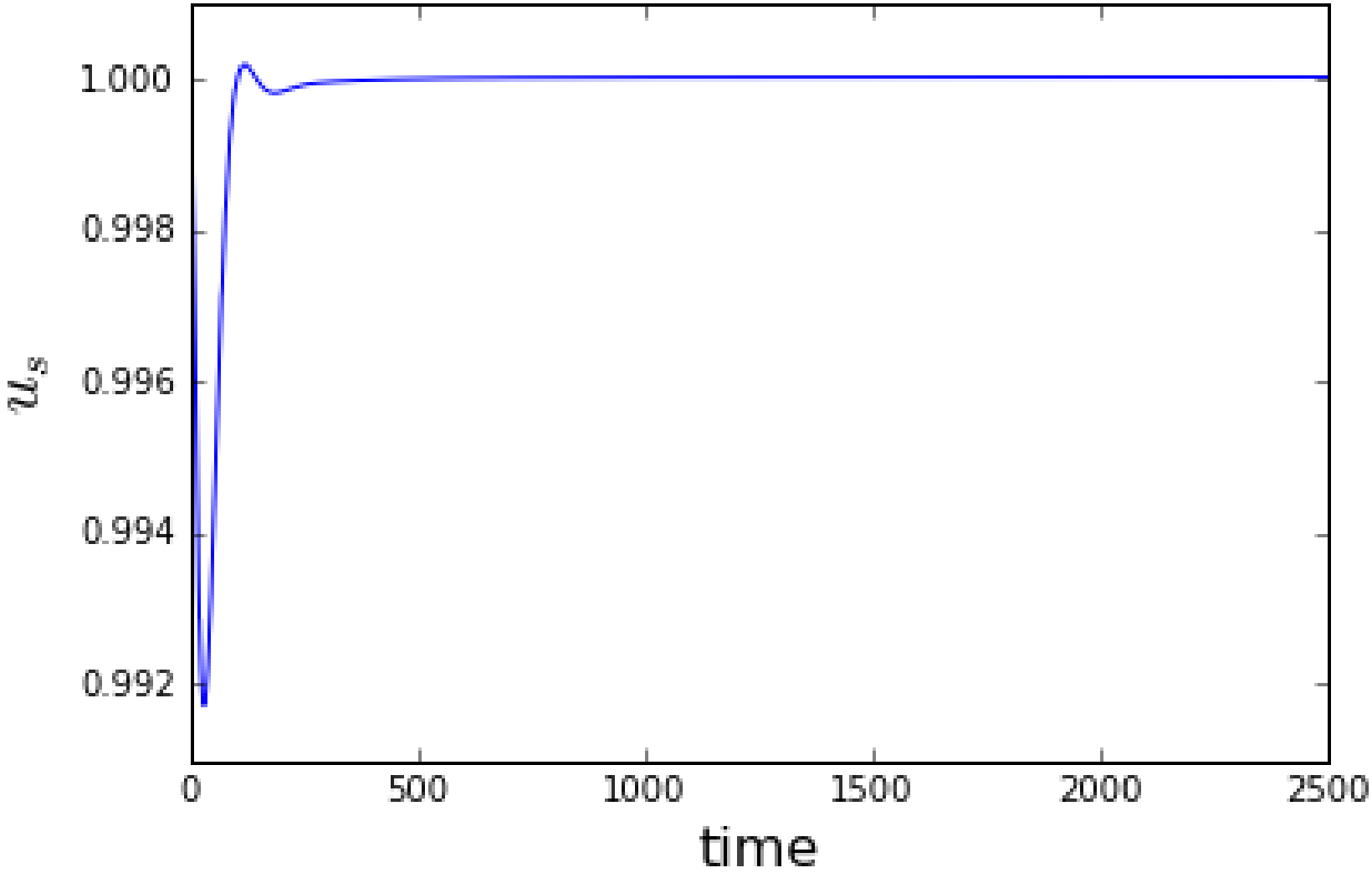
Effects of β

Numerical setup

- $$u_t + \frac{1}{2}(u^2 - uu_s)_x = ae^{-\frac{(x+us-\alpha)^2}{4\beta}}$$
- $$x \in (-10, 0)$$
- Outflow boundary conditions
- Godunov splitting
- L-W with MC limiters

Outline

Behaviour of u_s



movie

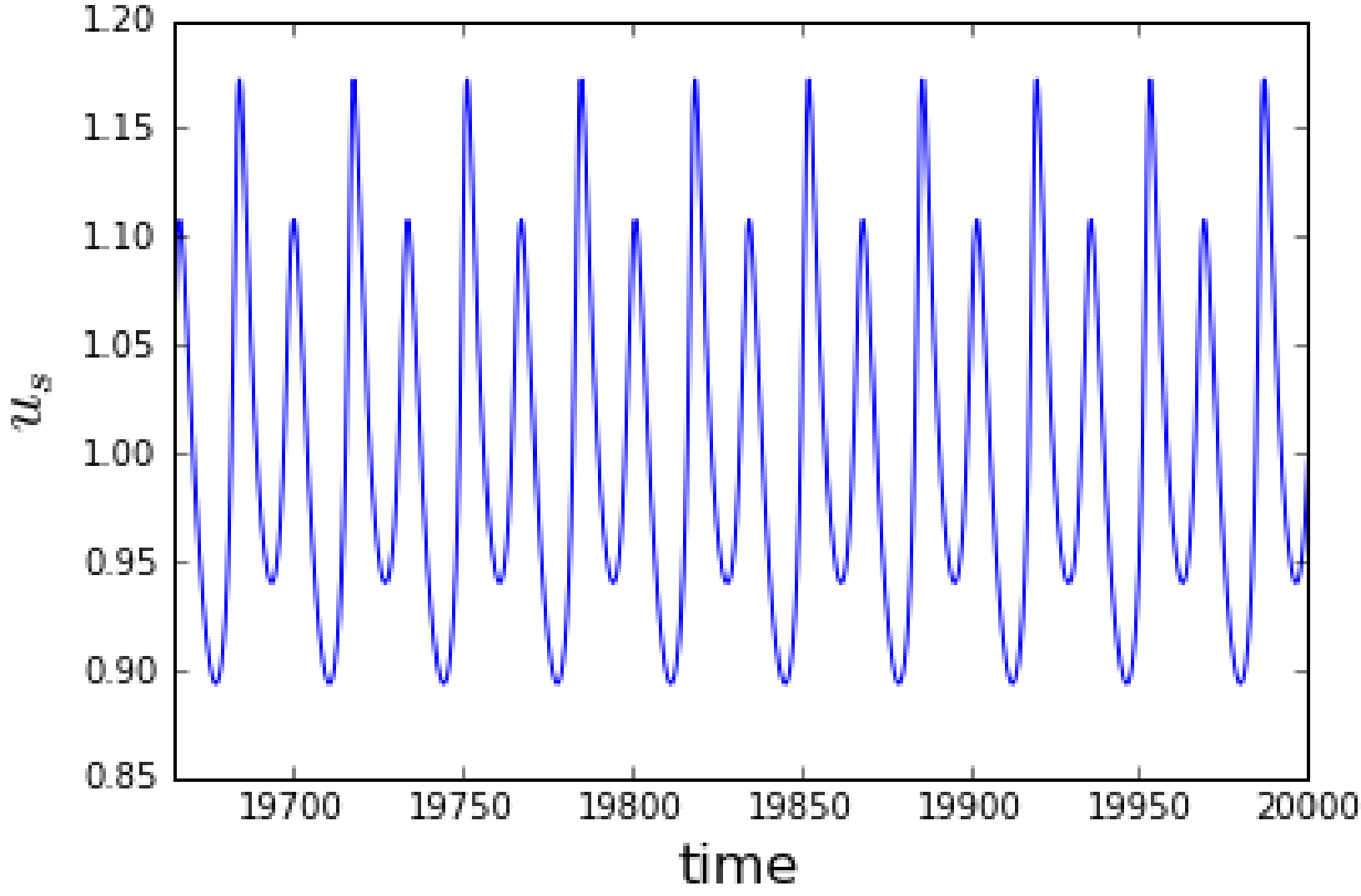
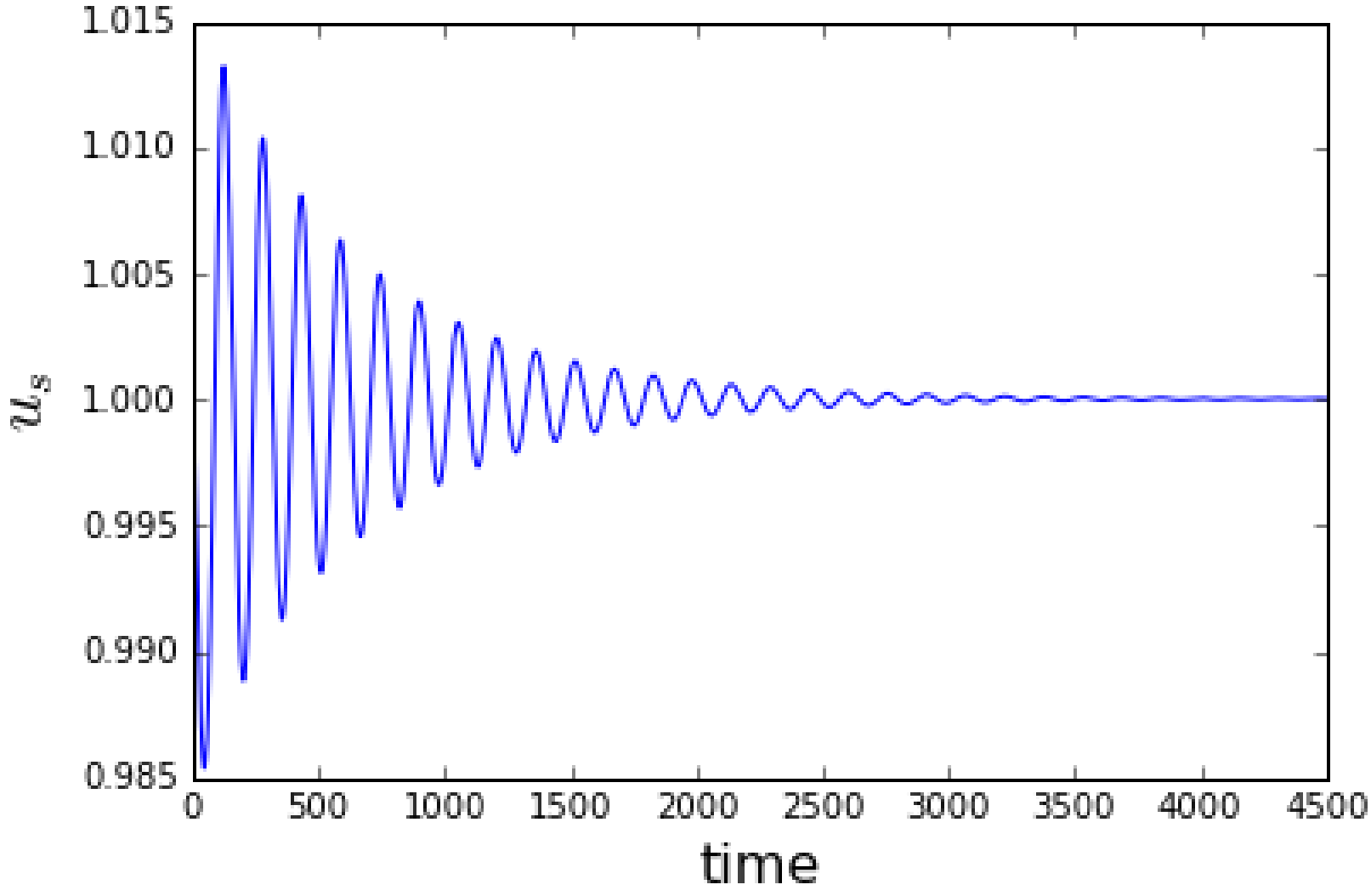


Figure 1: caption

Behaviour of u_s



movie

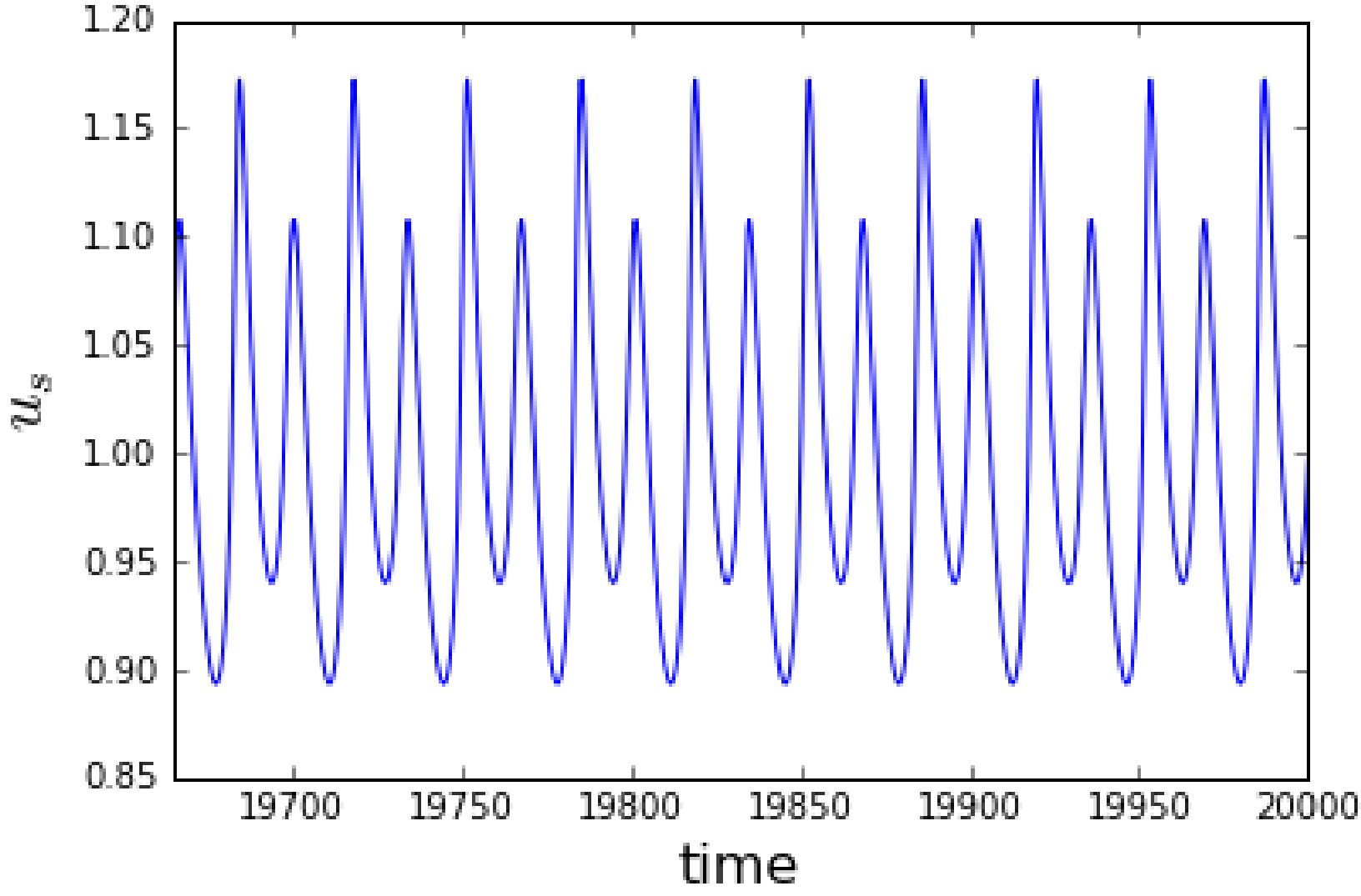


Figure 2: caption

movie

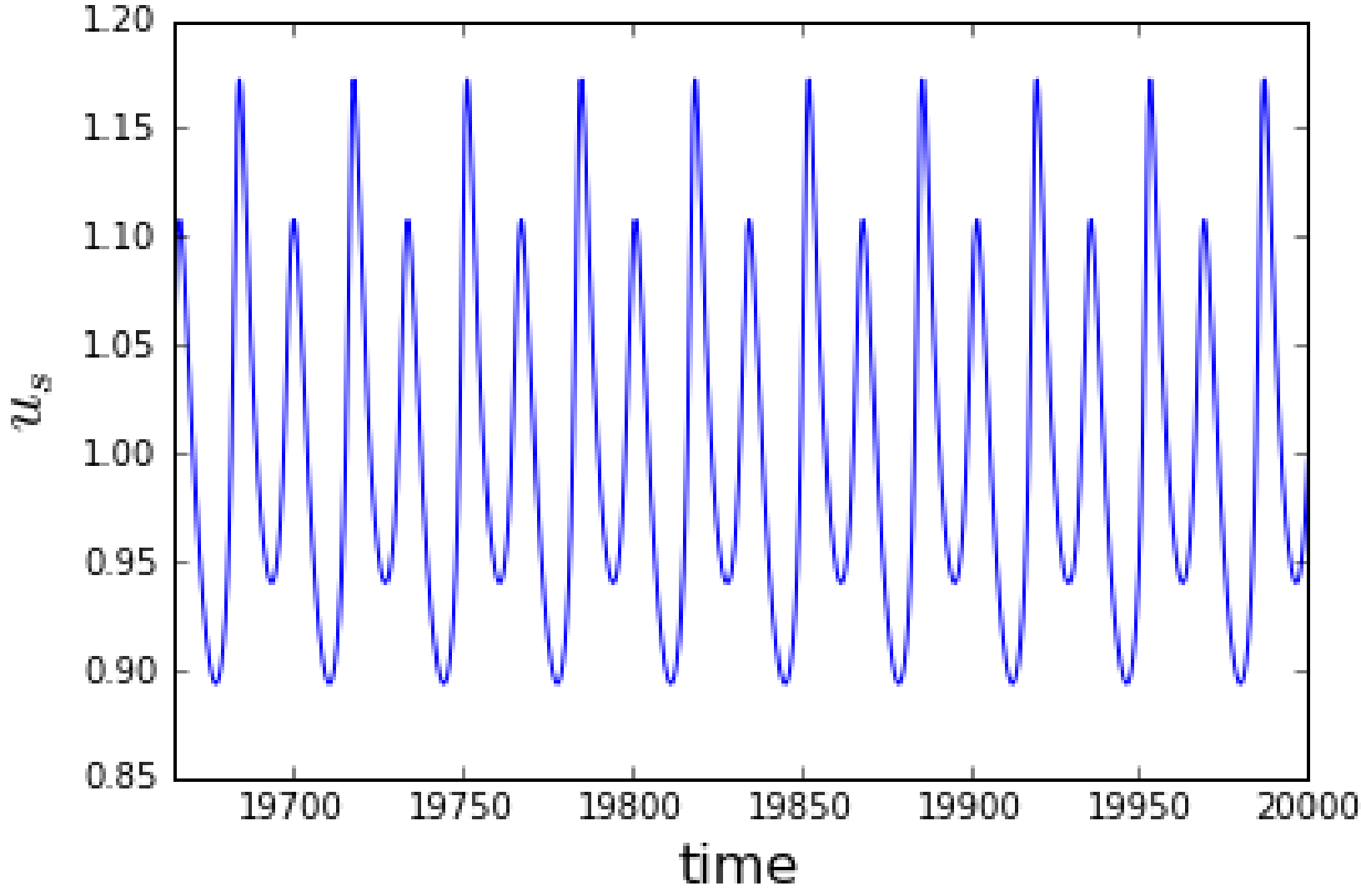
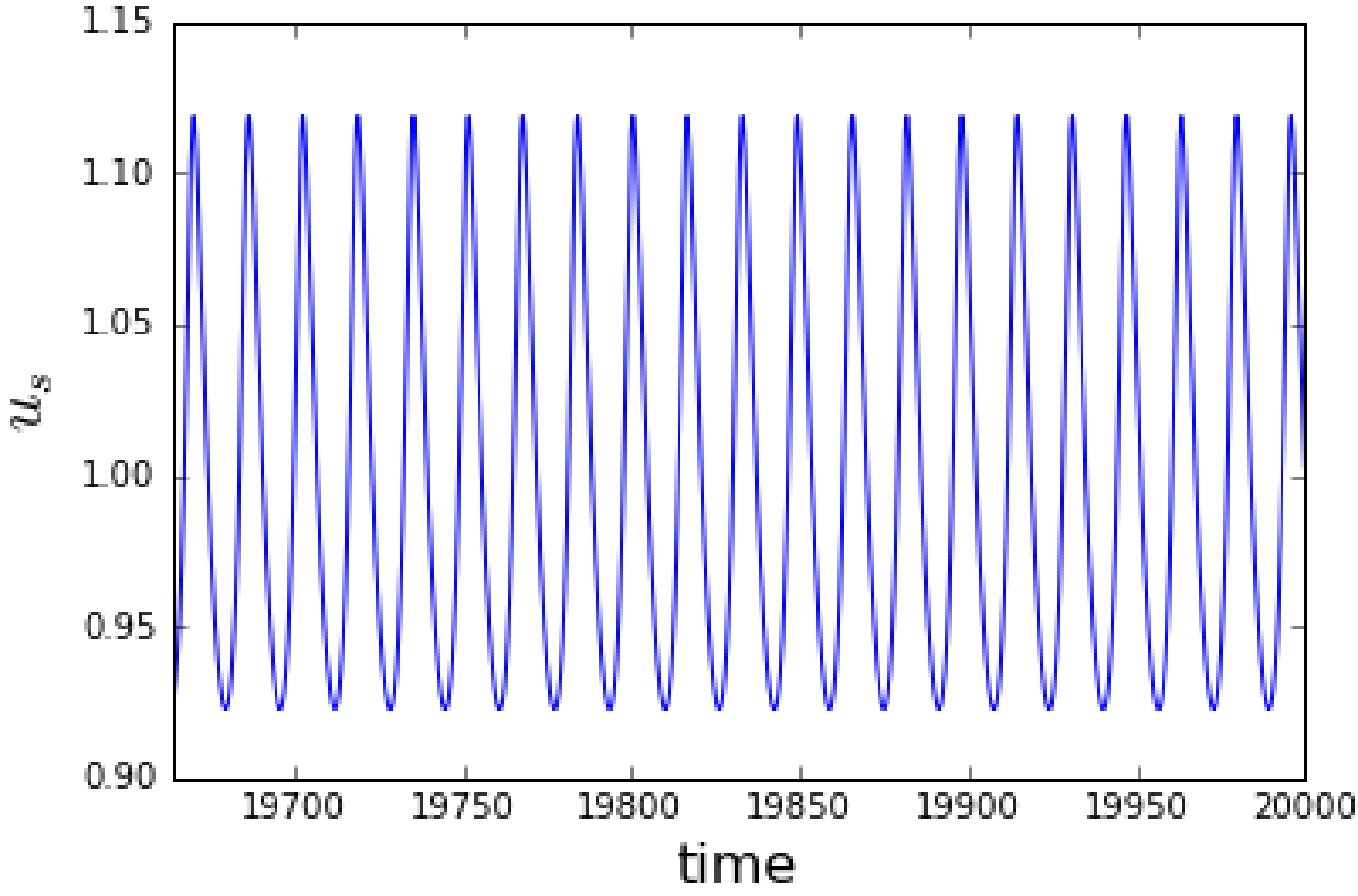
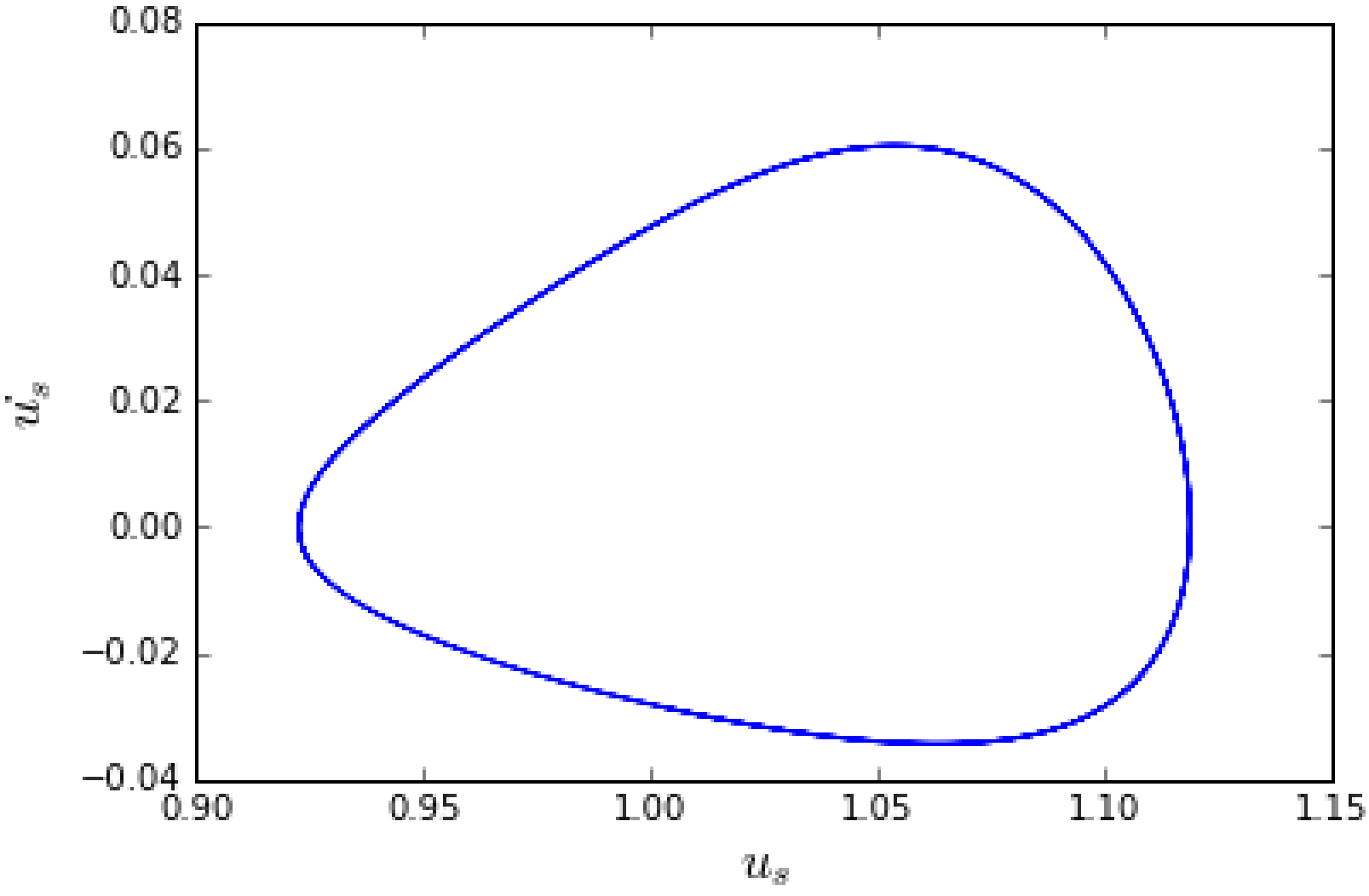


Figure 3: caption

Asymptotic behaviour of u_s



Limit cycle



movie

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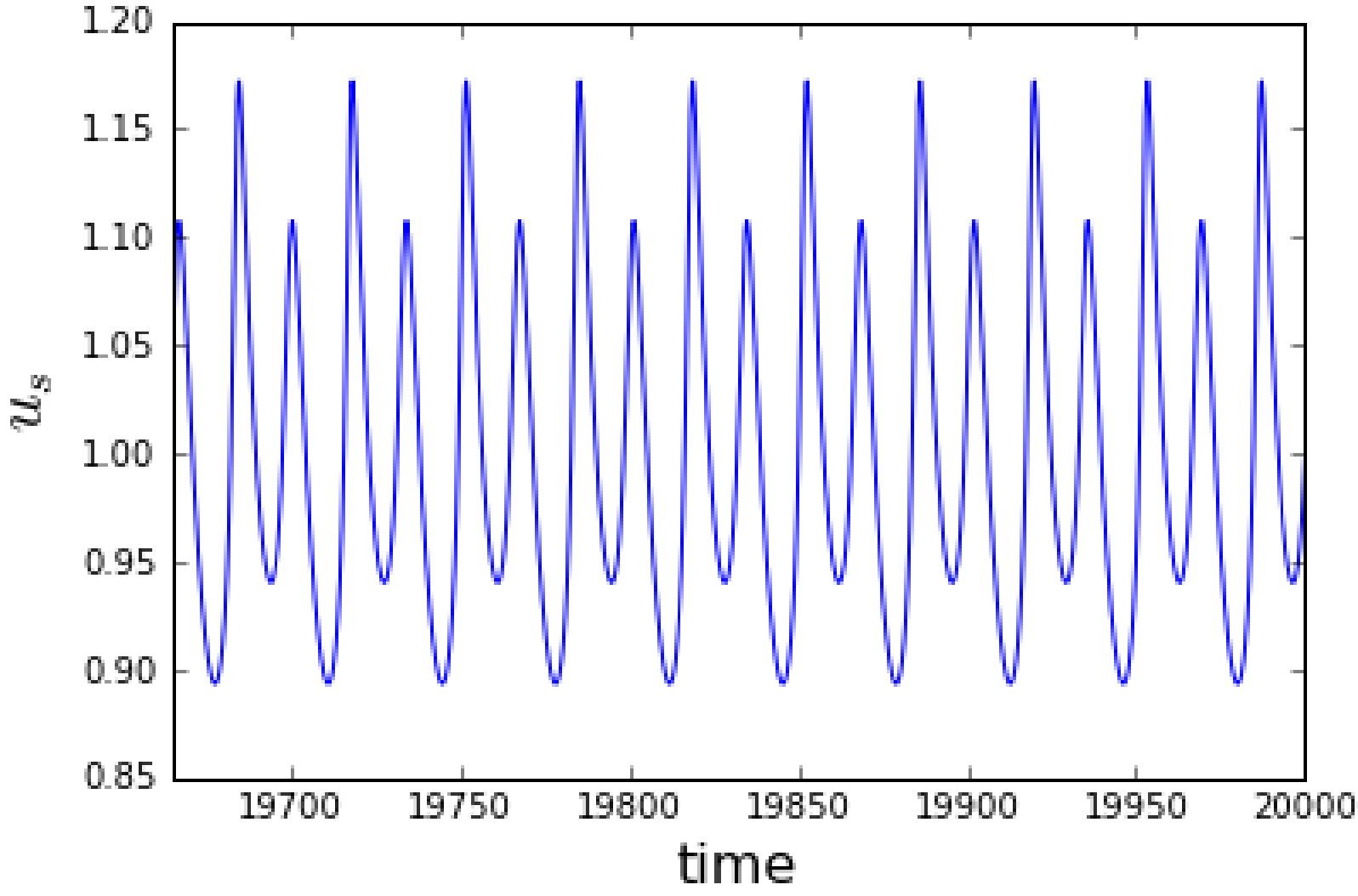
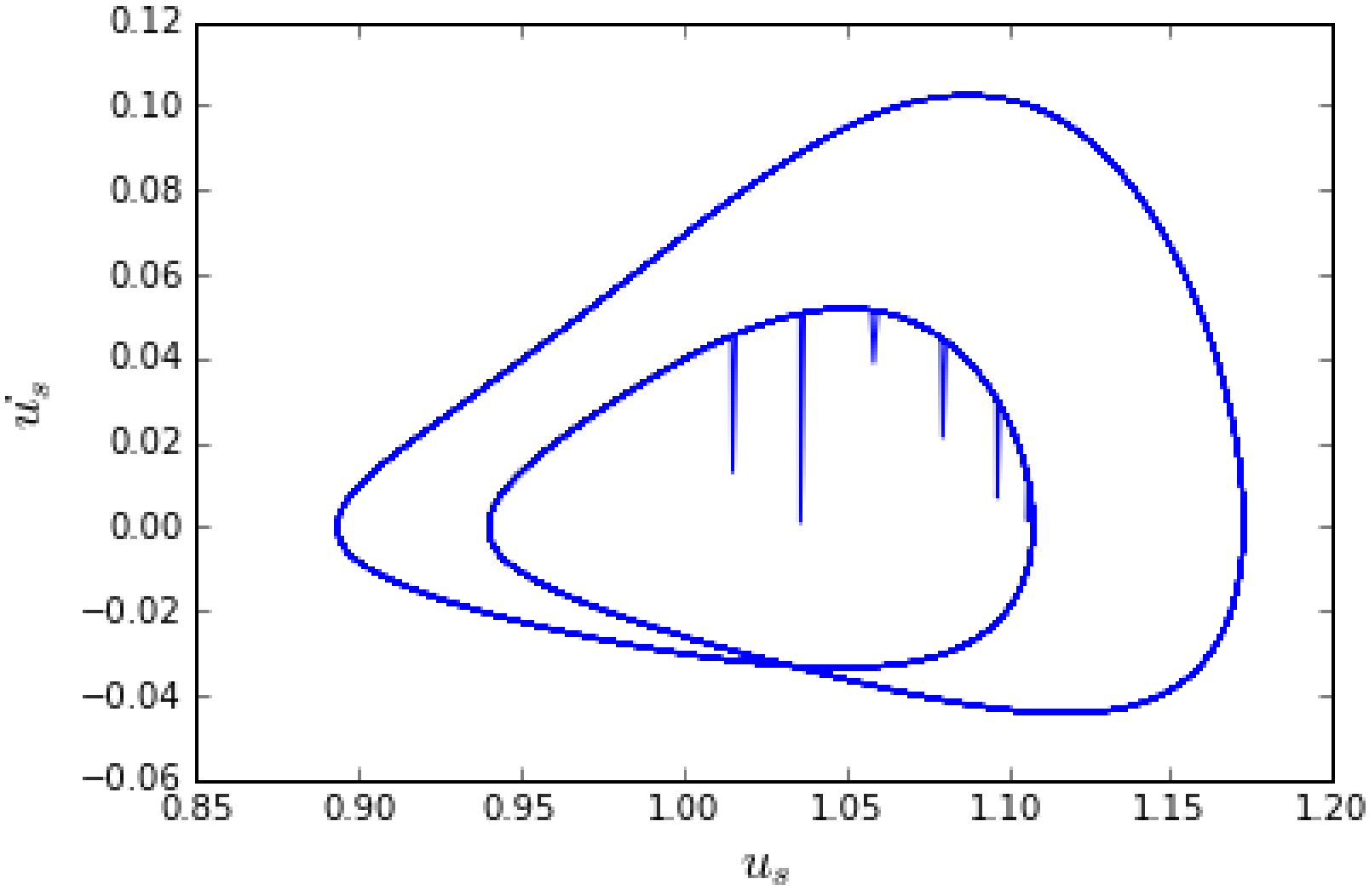
Asymptotic behaviour of u_s



Limit cycle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−0.06
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0.85
0.90
0.95
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1.05
1.10
1.15
1.20
u_s

movie

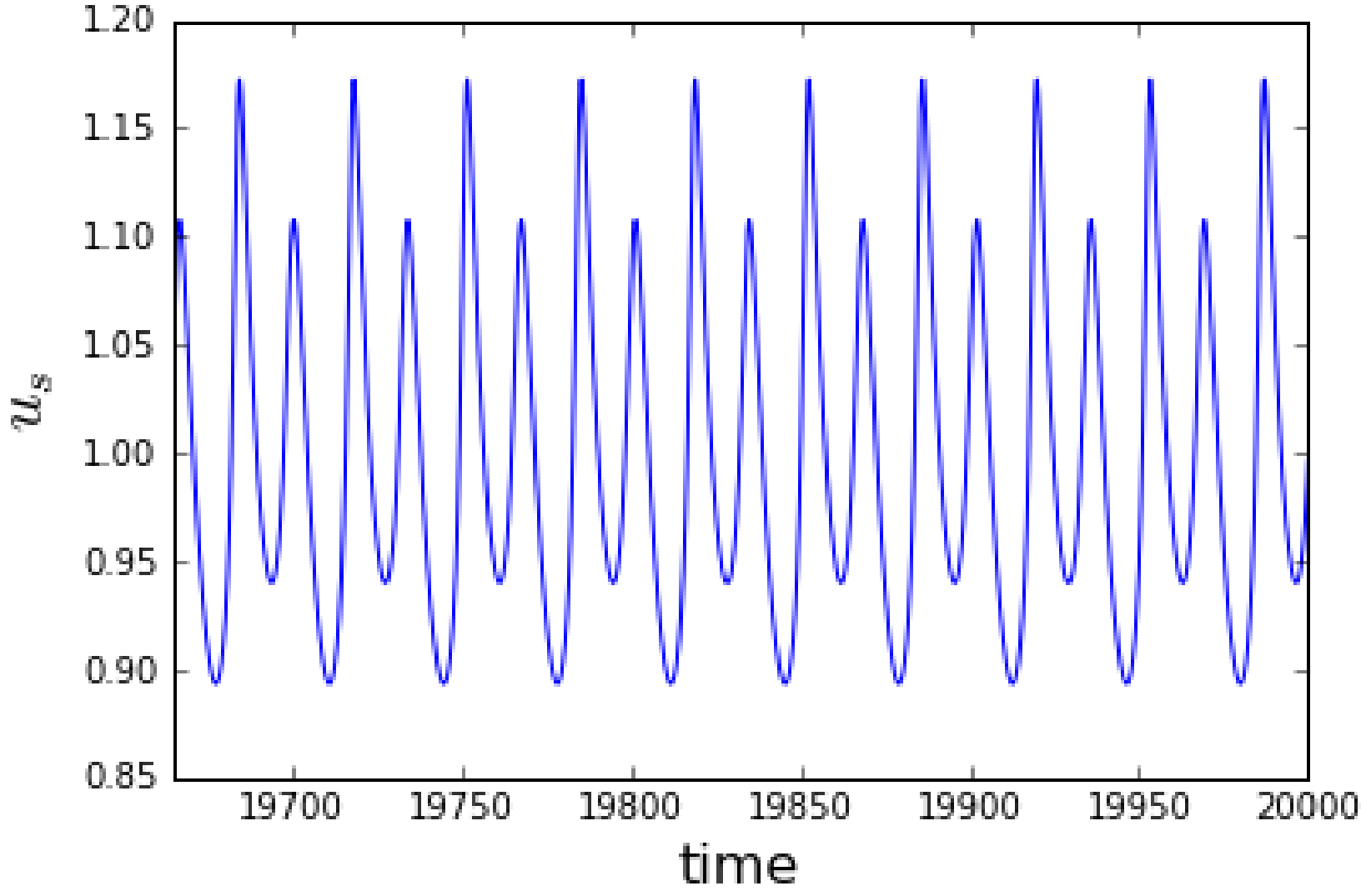
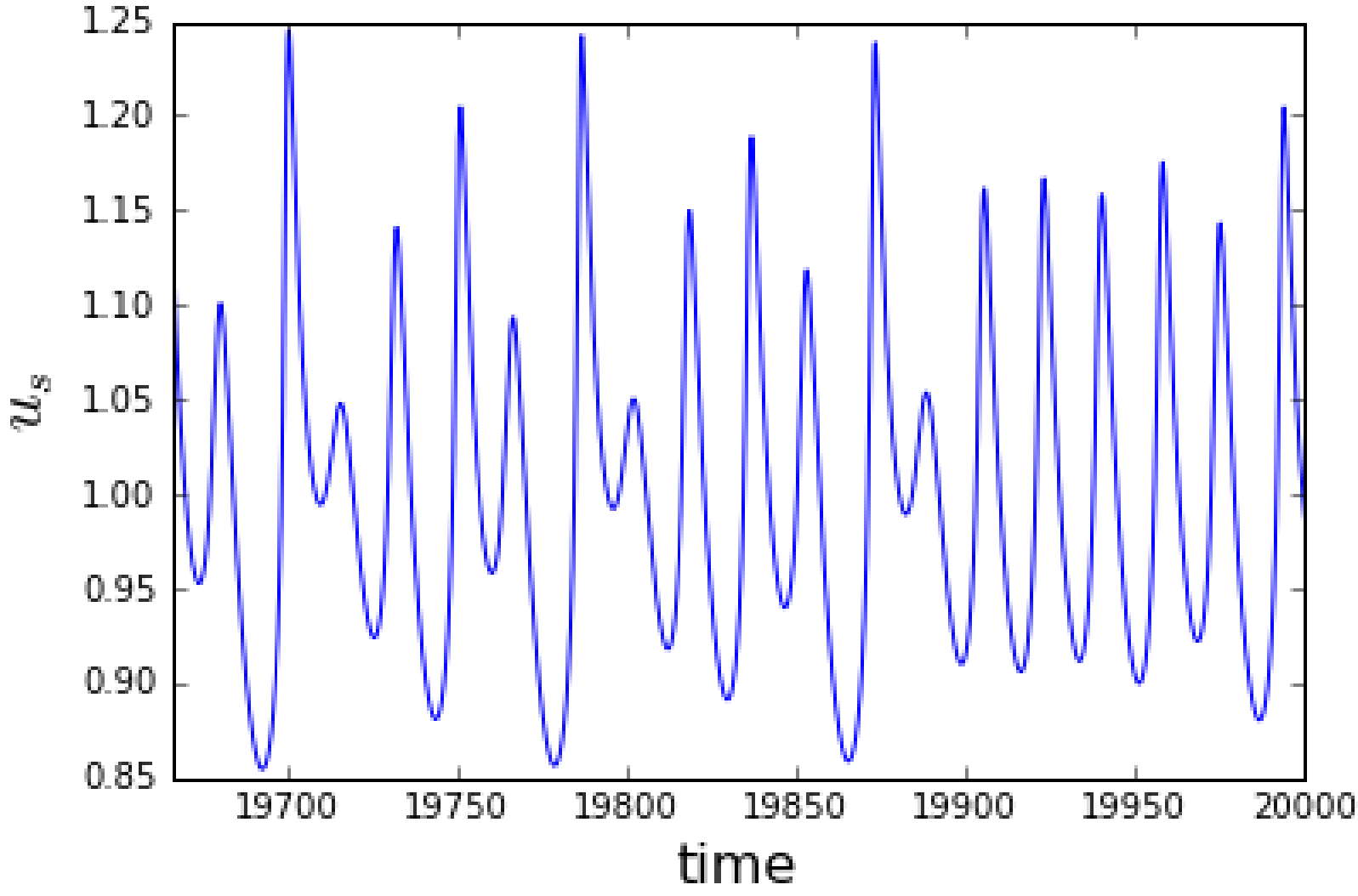
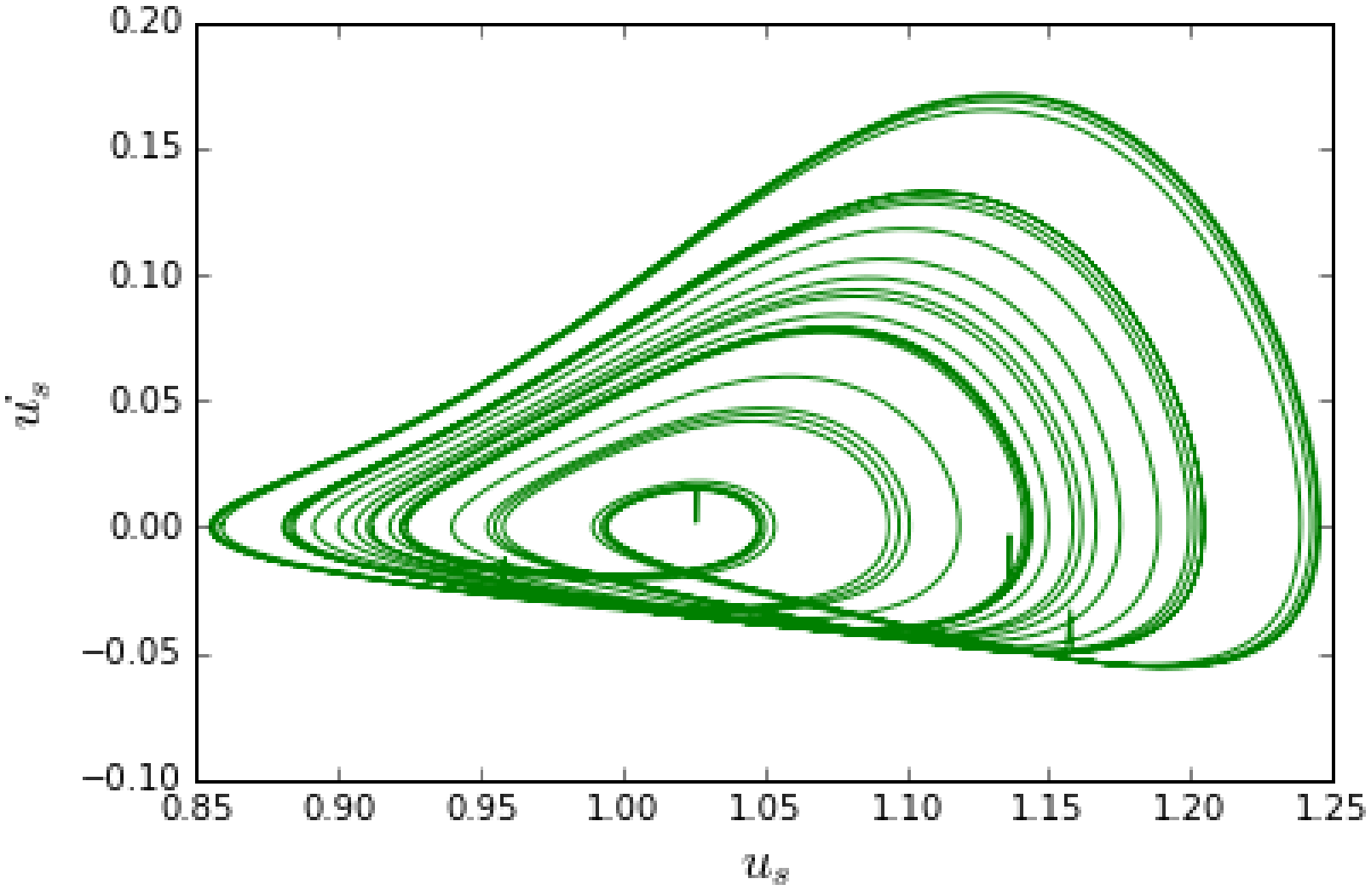


Figure 4: caption

Asymptotic behaviour of u_s



Limit cycle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