Introduction to Probability and Stochastic Processes – Exercises

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1 Exercises for Chapter 1: Review of Probability

Exercise 1.1. Let $\mathcal{F}$ be a σ-algebra of Ω. Suppose $B \in \mathcal{F}$. Show that $\mathcal{G} := \{A \cap B : A \in \mathcal{F}\}$ is a σ-algebra of B.

Exercise 1.2. Let $\mathcal{F}$ and $\mathcal{G}$ be σ-algebras of Ω. (a) Show that $\mathcal{F} \cap \mathcal{G}$ is a σ-algebra of Ω. (b) Show that $\mathcal{F} \cup \mathcal{G}$ is not necessarily a σ-algebra of Ω.

Exercise 1.3. Describe the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ for the following three experiments: (a) a biased coin is tossed three times; (b) two balls are drawn without replacement from an urn which originally contained two blue and two red balls; (c) a biased coin is tossed repeatedly until a head turns up.

Exercise 1.4. Suppose X is a continuous random variable with distribution F_X. Let g be a strictly increasing continuous function. Define $Y = g(X)$. (a) What if F_Y, the distribution of Y? (b) What is f_Y, the density of Y?

Exercise 1.5. Suppose X is a continuous random variable with distribution F_X. Find F_Y where Y is given by (a) X^2 (b) $\sqrt{|X|}$ (c) $\sin X$ (d) $F_X(X)$.

Exercise 1.6. Suppose X is a continuous random variable defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let f be the density of X under $\mathbb{P}$ and assume $f > 0$. Let g be the density function of a random variable. Define $Z := g(X)/f(X)$. (a) Show that $Z \equiv d\widetilde{\mathbb{P}}/d\mathbb{P}$ defines a Radon-Nikodym derivative. (b) What is the density of X under $\widetilde{\mathbb{P}}$?

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