Exploration of chaos in shock waves

Jithin D. George

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Outline

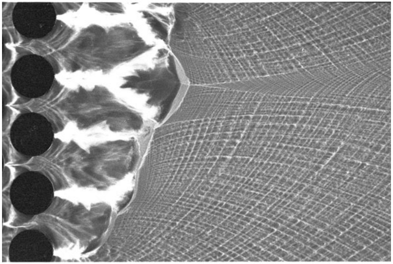
Background: Reactive Euler equations and detonation

$$\frac{D\rho}{Dt} = -\rho\nabla.u$$

$$\rho\frac{Du}{Dt} = -\nabla P$$

$$\rho\frac{D}{Dt}\left(h + \frac{|u|^2}{2}\right) = \frac{\partial P}{\partial t}$$

$$\frac{DY_i}{Dt} = \dot{\omega}_i$$



Model Equation

$$u_t + \frac{1}{2}(u^2 - uu_s)_x = f(x, u_s)$$

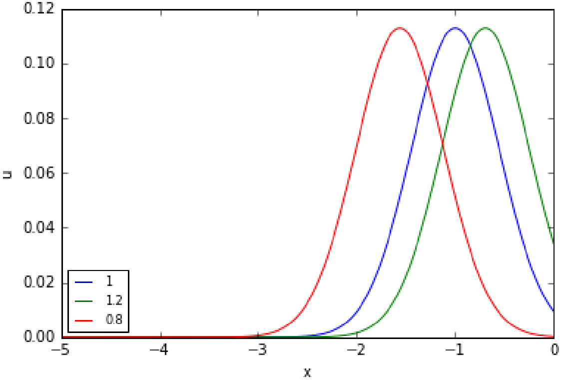
$$x \in (-\infty, 0)$$

$$\textbf{Characteristic speed } = u - \frac{u_s}{2}$$

The nature of the source term

$$f(x, u_s) = \frac{q}{2\sqrt{4\pi\beta}} e^{-\frac{[x - x_f(u_s)]^2}{4\beta}}$$

$$x_f(u_s) = \left(\frac{u_{0s}}{u_s}\right)^{\alpha}$$



The steady state solution (The fixed point of a pde)

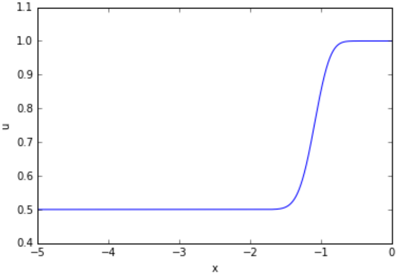
$$\frac{1}{2}(u_0^2 - u_0 u_{0s})' = f(x, u_{0s})$$

$$u_0(x) = \frac{u_{0s}}{2} + \sqrt{2\int_{-\infty}^{x} f(y, u_{0s})dy}$$

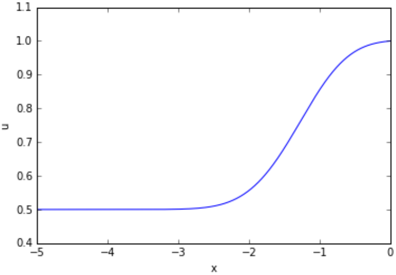
$$f = \frac{q}{2\sqrt{4\pi\beta}} e^{-\frac{[x - x_f(u_s)]^2}{4\beta}}$$

$$u_0(x) = \frac{1}{2}\left[1 + \sqrt{\frac{1 + erf((x+1)/2\sqrt{\beta})}{1 + erf(1/2\sqrt{\beta})}}\right]$$

Effects of β



(a) β =0.01



(b) β =0.1

Figure 1: u_0 for various values of β

Movie

Numerical setup

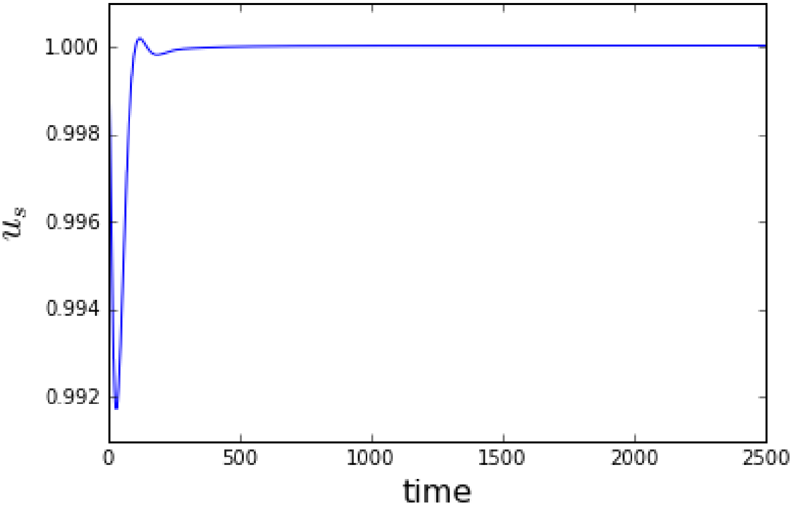
- $$u_t + \frac{1}{2}(u^2 - uu_s)_x = ae^{-\frac{(x+us^{-\alpha})^2}{4\beta}}$$
- $$x \in (-10, 0)$$
- Outflow boundary conditions
- Godunov splitting
- Lax-Wendroff fluxes with MC limiters

Outline

$\alpha = 2$

Movie

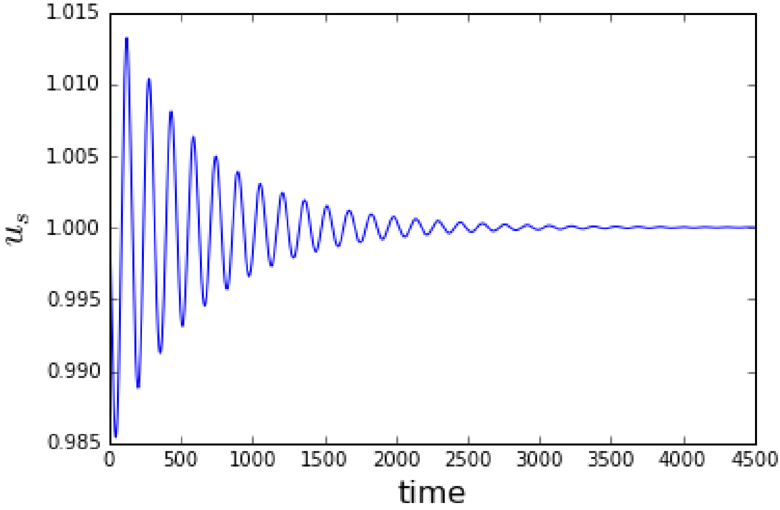
Behaviour of u_s



$\alpha = 3$

Movie

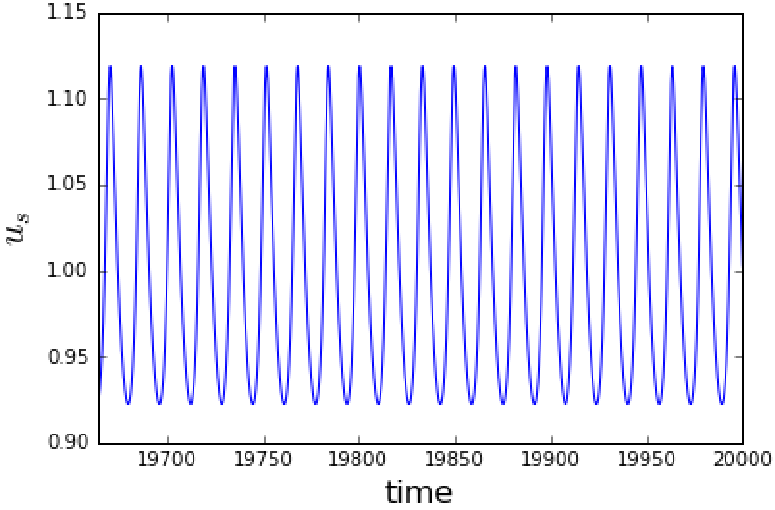
Behaviour of u_s



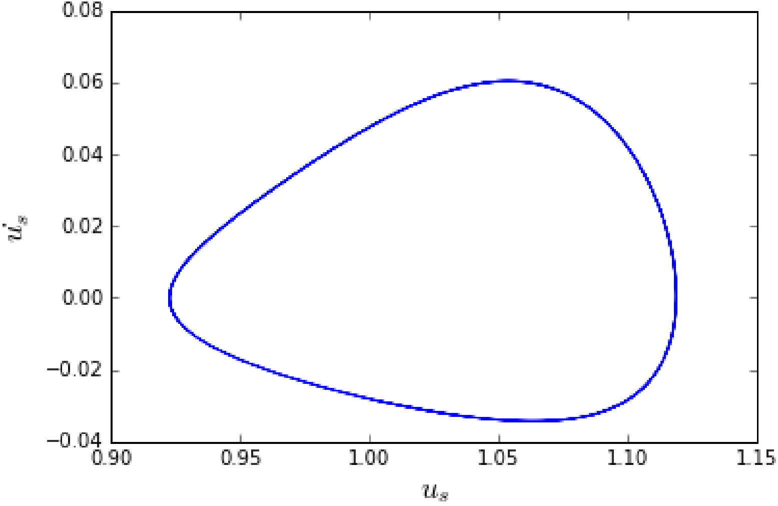
$\alpha = 4.6$

Movie

Asymptotic behaviour of u_s



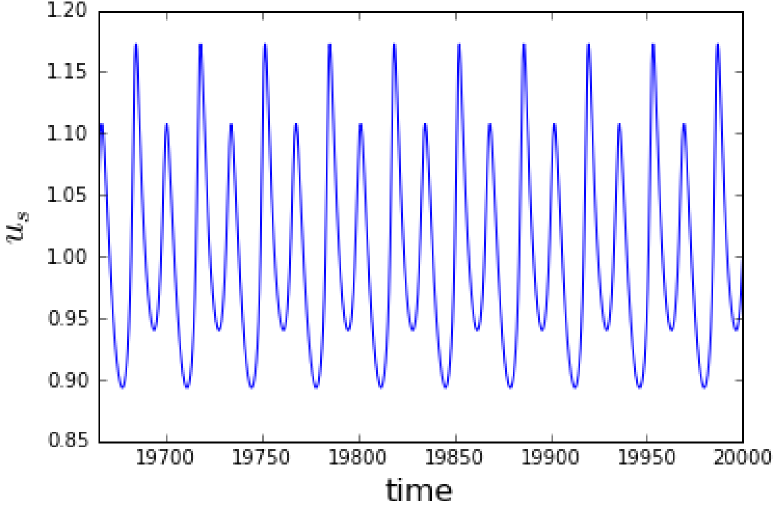
Limit cycle



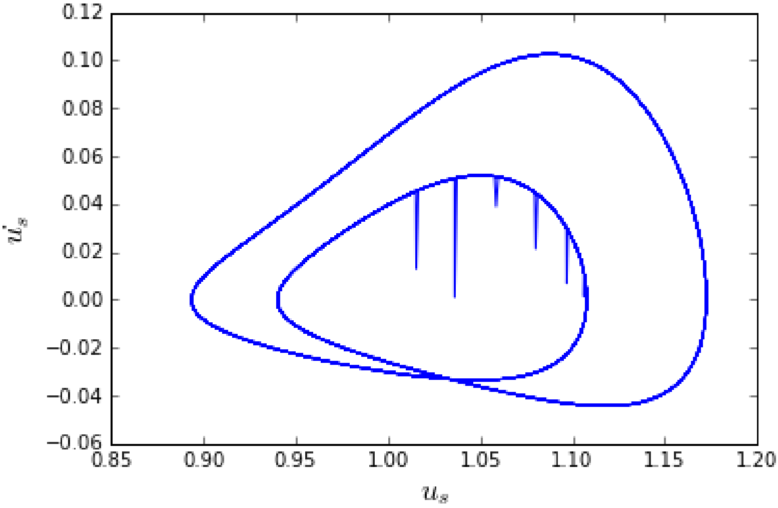
$\alpha = 4.85$

Movie

Asymptotic behaviour of u_s



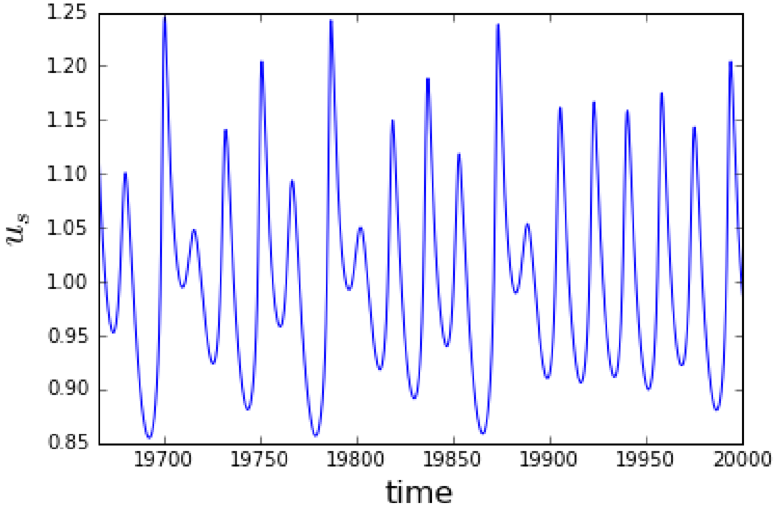
Limit cycle


0.12
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
$\dot{u}_s$
u_s

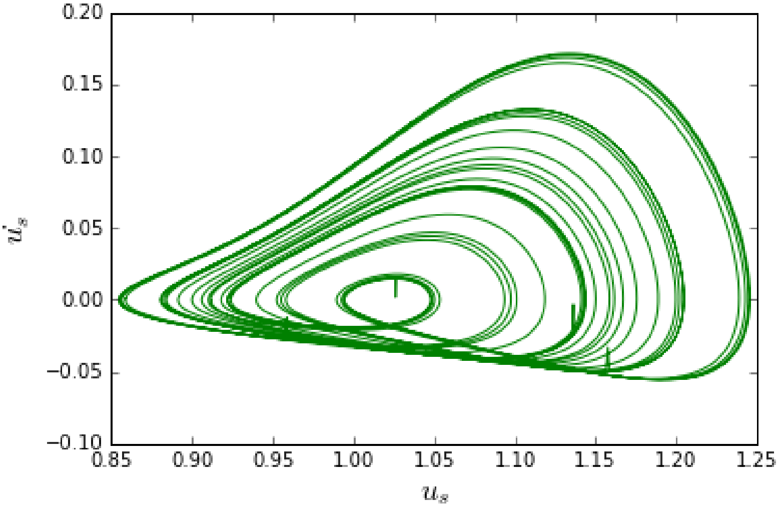
$\alpha = 5.1$

Movie

Asymptotic behaviour of u_s



Limit cycle



0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
1.25
$\dot{u}_s$
u_s

Outline

Bifurcation diagram

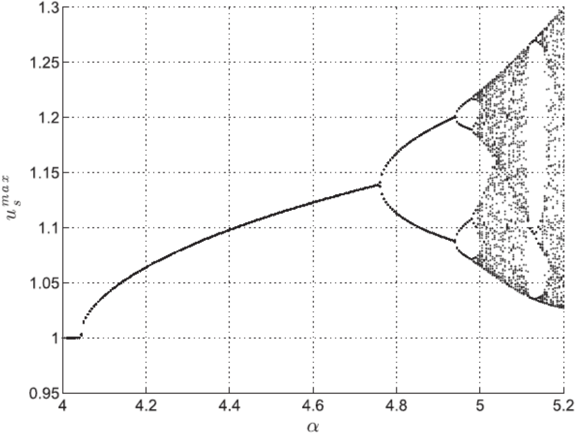


FIG. 3. The long-time values of the local maxima, $u_s^{\max}$, of the shock strength as a function of α.

Largest Lyapunov Exponents

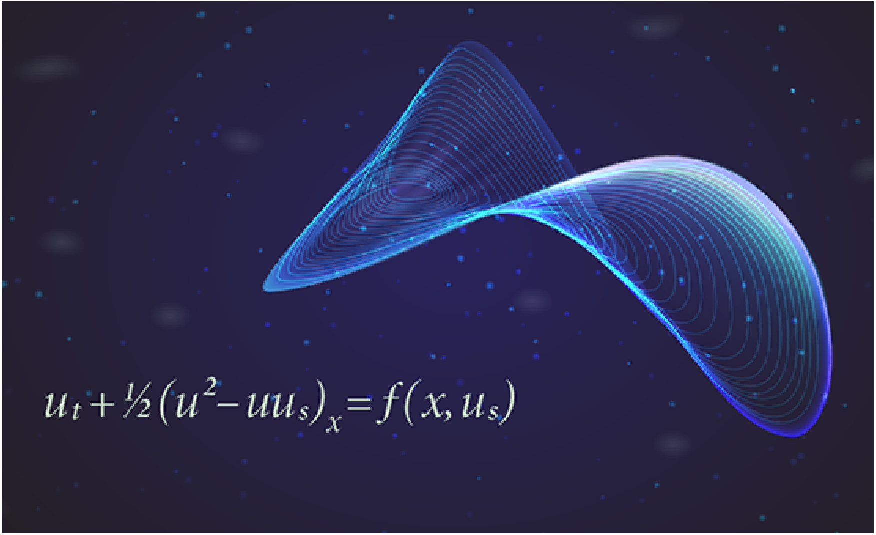
	4.85	4.96	4.97	5.1
LLE	0	0	0.0042	0.0315
D_C	1.0006	1.002	1.67	1.91

Weird pdes and insights

Movie

Resources

- Kasimov, Aslan R., Luiz M. Faria, and Rodolfo R. Rosales. "Model for shock wave chaos." Physical review letters 110.10 (2013): 104104.
- Faria, Luiz M., Aslan R. Kasimov, and Rodolfo R. Rosales. "Study of a model equation in detonation theory." SIAM Journal on Applied Mathematics 74.2 (2014): 547-570.
- `https://github.com/Dirivian/apps_detonation`


$u_t + ½(u^2 - uu_s)_x = f(x, u_s)$