

AMATH 582 Problem Set 1: Eigenfaces

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Abstract

This homework explores the implementation of the singular value decomposition on a dataset of image files. An exploration of the application of the SVD to various kinds of images like cropped images, uncropped images and those of multiple people, yields new perspectives on the underlying features across different people.

1 Introduction and Overview

The Singular Value Decomposition is a result from linear algebra with powerful applications. The SVD enables the decomposition of many things in a matrix format into orthogonal basis elements. Just like it is often possible to decompose a complex function into underlying eigenfunctions, we explore in a similar way, the inherent features in facial images, the eigenfaces.

2 Theoretical Background

The SVD of a matrix decomposes it into three matrices.

$$A = USV^*$$

Here, U and V represent matrices of basis elements. S is the matrix of singular values. V could be considered as the original basis matrix. U would then be the principal component basis. Since the basis elements are orthogonal and the singular value decomposition can be applied to complex matrices, both U and V are unitary matrices. With that in mind, from basic matrix arithmetic, we can get that U is an eigenvector of A^*A and V is an eigenvector of AA^* . Thus, the SVD is similar to the eigenvalue decomposition. However, since it is between two basis, it is not unique and exists for any matrix A .

The SVD gives a geometric interpretation of a matrix A acting on a vector V stretching it by the singular values in S along the directions of U . Finding the largest singular values with the right U and V thus essentially gives us the most important "modes" of the matrix A . Information about the weights of the modes resides in the S matrix.

3 Implementation and Development

The resource available to us was a vast amount of image files from UCSD's computer vision group containing both original and cropped images. The cropped images were smaller and easier to work with.

Most of the code was on extracting the information from the image files into a matrix. Each image was reshaped into a column of the matrix. Three scenarios were implemented.

- Cropped images of a single person
- All the cropped images

- Uncropped images of a single person.

After the matrix was obtained, it was subjected to the singular value decomposition.

Then, the three most prominent modes were looked at. After gaining information from the statistics of the singular matrix, a low rank reconstruction is attempted.

4 Computational Results

4.1 Cropped images of a single person

The singular value decomposition of a matrix containing cropped images of a single person yields the following results.

4.1.1 The singular value matrix

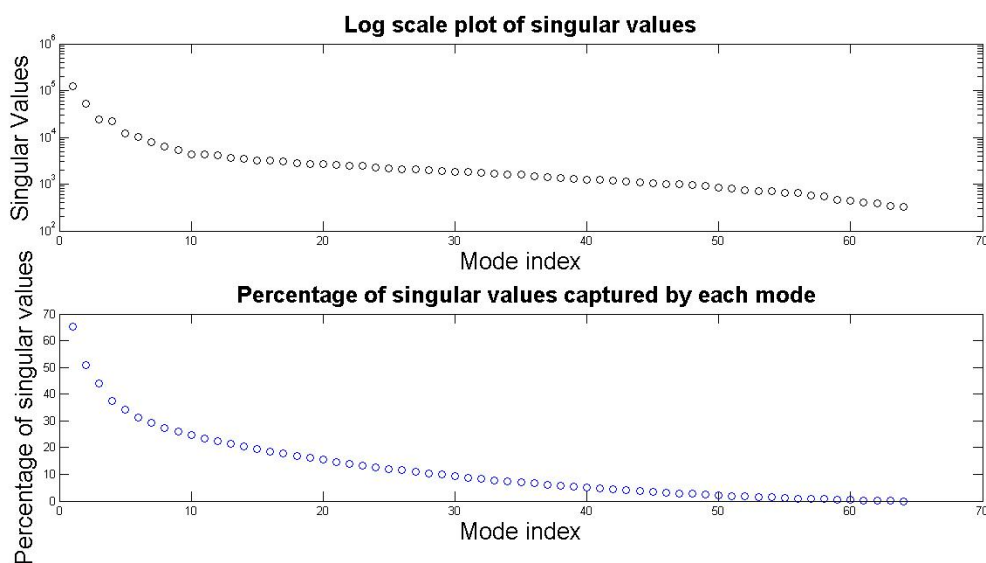


Figure 1: Statistics of the Singular Value Matrix

Since the singular values don't trail to the order of 10^{-15} , we can't ignore those values. Furthermore, to capture 90 percent of the variation, we would need around 30 modes.

4.1.2 The first three modes

The first mode seems to focus on the eyes and the nostrils. The second focuses on the cheeks and the shading and in the third mode, the nose seems to be highlighted. These features contribute the most to the images of the same person under different poses and lighting.

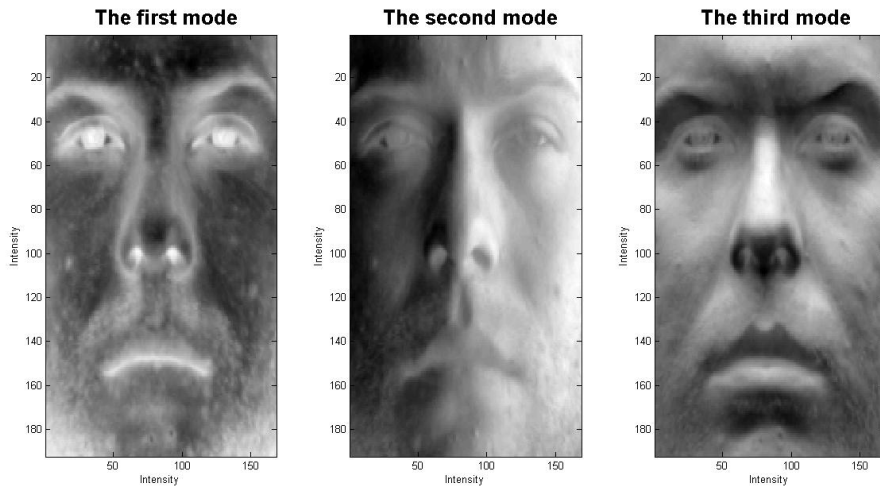


Figure 2: The first 3 modes

4.1.3 Low rank reconstruction

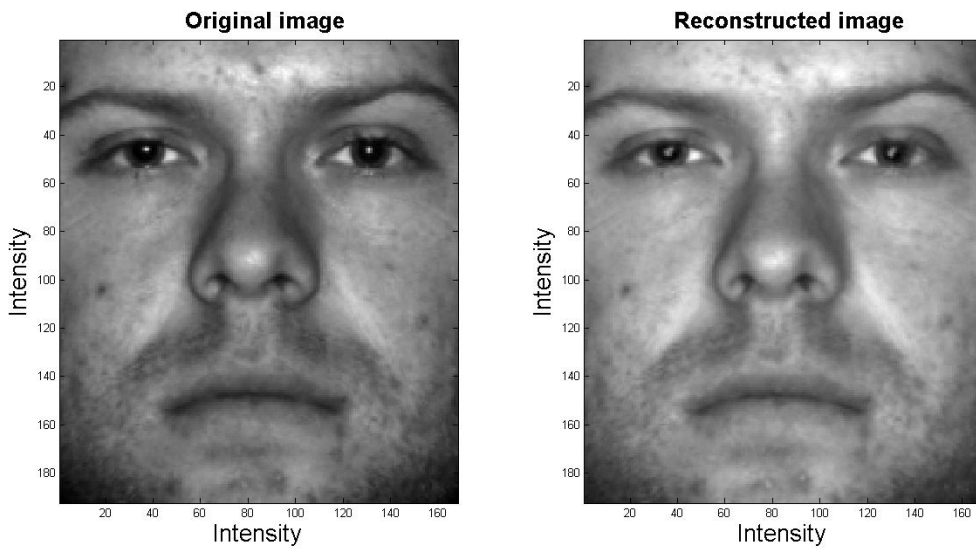


Figure 3: Reconstruction of the first image with 30 modes

The first image can be reconstructed to a good level using only 30 modes.

4.2 All cropped images

The singular value decomposition of a matrix containing cropped images of multiple people yields the following results.

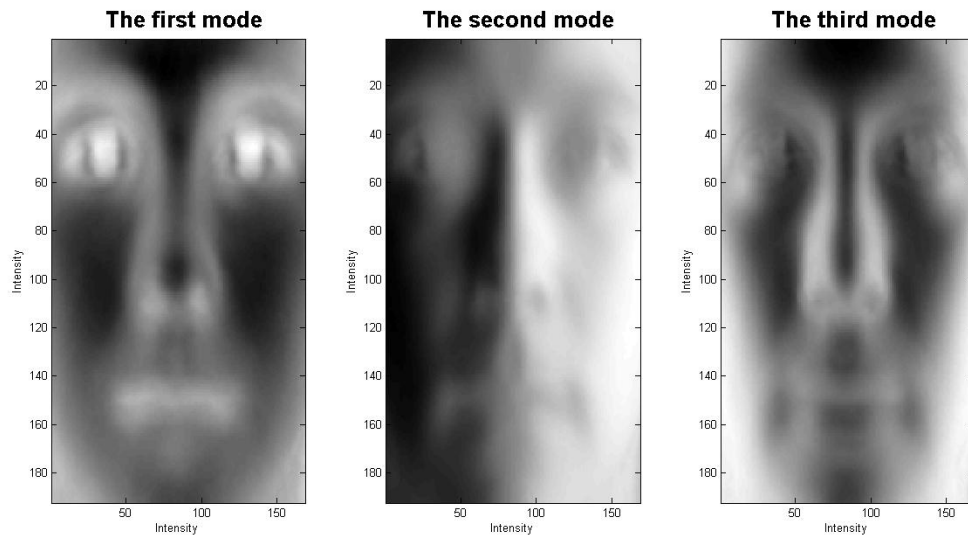


Figure 4: The first 3 modes

4.2.1 The first three modes

Unlike the previous case, these modes look less human because they are for multiple people. Still, the eyes and the cheeks remain important features in the first two modes.

4.2.2 Low rank reconstruction

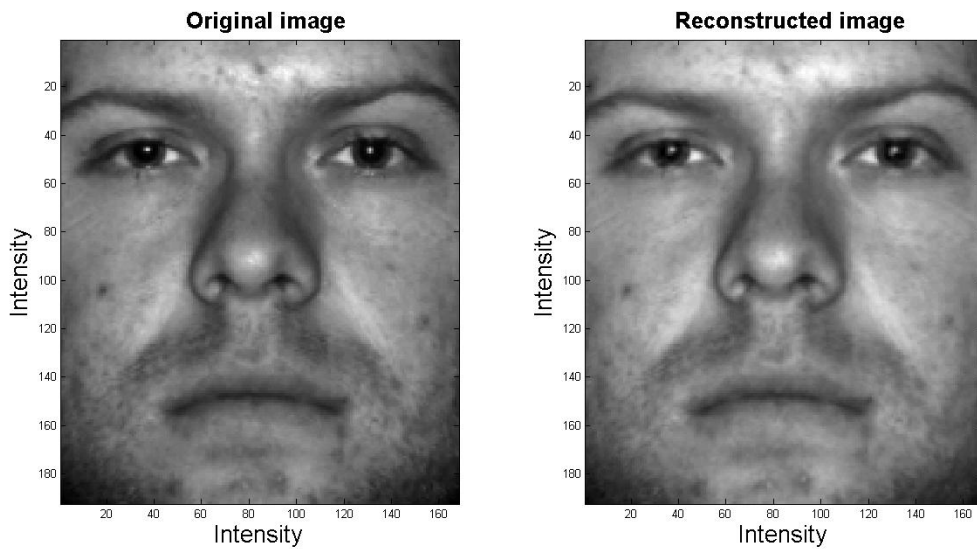


Figure 5: Reconstruction of the first image with 800 modes

It takes 800 modes to reconstruct the same image which only needed 30 last time. That is because these modes cater to multiple people. As a result, the following reconstruction of another image is possible with the same basis.

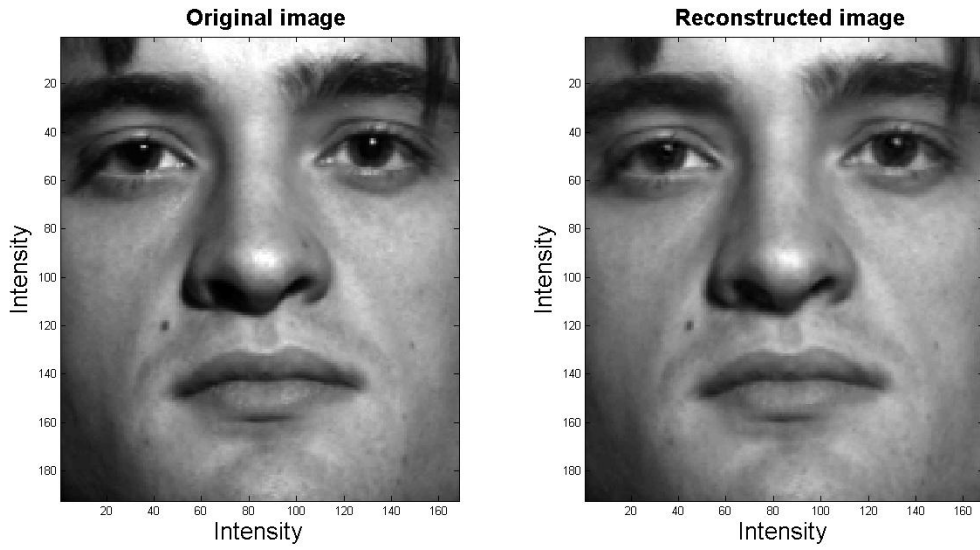


Figure 6: Reconstruction of the 1000th image with 800 modes

4.3 Uncropped images of a single person

The singular value decomposition of the uncropped images of a single person yields the following results.

4.3.1 The first three modes

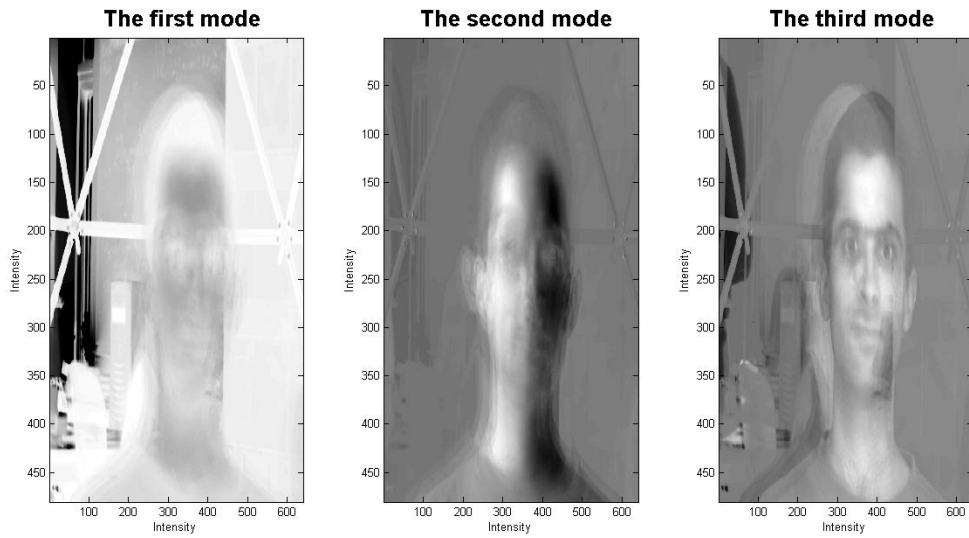


Figure 7: The first three modes

The biggest difference from the case of the cropped images is that the objects in the background come in the very first mode. The person and the lady in the background come in the third mode.

4.3.2 Low rank reconstruction

A nearly perfect reconstruction is possible with 250 modes.

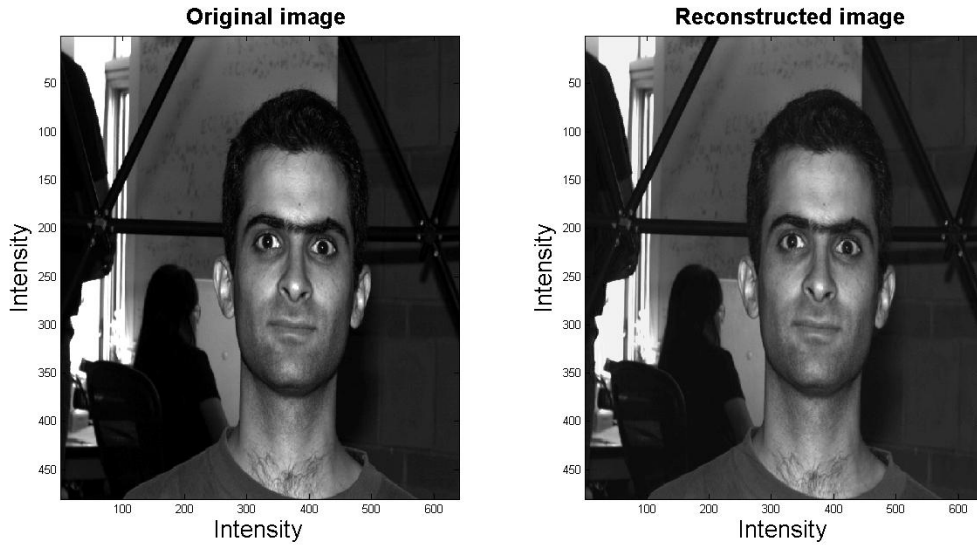


Figure 8: Reconstruction with 250 modes

5 Summary and Conclusions

The Singular Value Decomposition is capable of accurate low-rank approximations. It does so by obtaining the basis elements with the highest singular values. The basis differs from case to case. In the case of cropped images of multiple people, the SVD created a basis which was ideal for many faces rather than one particular face. Also, in the case of uncropped images, the background found its way into the prominent modes. Finally, using modes which cover around 90 to 95 percent of the variation yields a reconstruction almost indistinguishable to the original image.

A MATLAB Functions used

- **[U,S,V]=svd(A):**
This function performs the singular value decomposition of A and returns U,S and V.
- **imread:**
This function reads an image file into a matrix.
- **reshape:**
This function reshapes an mxn matrix into an ixj matrix such that m*n=i*j.
- **imagesc:**
This function plots a matrix as a scaled image.
- **cumsum(p):**
This function finds the cumulative sum of a vector p.
- **dir:**
This function list the items in a folder.

B MATLAB Code

B.1 imgextract.m

```
1 close all; clear all; clc;
2 wi=1;
3 for l = 1:9
4     l=num2str(l);
5     t=strcat('yaleB0',l);
6     cd(t);
7     for i=5:68
8         t=dir;
9         y=t(i);
10        x=y.name;
11        A = imread(x);
12        Z(:,wi)= reshape(A,[],1);
13        wi=wi+1;
14    end
15    cd ..
16 end
17 for l = 10:39
18     if l==14
19         continue
20     end
21     l=num2str(l);
22     t=strcat('yaleB',l);
23     cd(t);
24     for i=5:68
25         t=dir;
26         y=t(i);
27         x=y.name;
28         A = imread(x);
29         Z(:,wi)= reshape(A,[],1);
30         wi=wi+1;
31     end
32     cd ..
33 end
34 Z= double(Z);
35 [U,S,V]= svd(Z, 'econ');
36
```

B.2 imguncropped.m

```
1 close all; clear all; clc;
2 t= dir( '*.pgm');
3 wi=1;
4 for i=1:585
5     y=t(i);
6     x=y.name;
7     A = imread(x);
8     Z(:,wi)= reshape(A,[],1);
9     wi=wi+1;
10 end;
11 Z= double(Z);
12 [U,S,V]= svd(Z, 'econ');
13
```

B.3 plotsvd.m

```
1 %% Mode statistics
2 p= diag(S);
3 close all;
4 figure;
5 subplot(2,1,1);
6 semilogy(p, 'ko');
7 f=20;
8 xlabel('Mode index','FontSize', f) % x-axis label
9 ylabel('Singular Values','FontSize', f) % y-axis label
10 title('\bfLog scale plot of singular values','FontSize', f)
11 subplot(2,1,2);
12 plot((1-cumsum(p)/sum(p))*100, 'bo');
13 xlabel('Mode index','FontSize', f) % x-axis label
14 ylabel('Percentage of singular values','FontSize', f) % y-axis label
15 title('\bf Percentage of singular values captured by each mode','FontSize', f)
16
17 %% First three modes
18 figure;
19 m= 480;
20 n=640;
21 U1= reshape(U(:,1),m,n);
22 subplot(1,3,1);
23 colormap('gray'); imagesc(U1);
24 xlabel('Intensity') % x-axis label
25 ylabel('Intensity') % y-axis label
26 title('\bf The first mode','FontSize', f)
27 subplot(1,3,2);
28 U2= reshape(U(:,2),m,n);
29 colormap('gray'); imagesc(U2);
30 xlabel('Intensity') % x-axis label
31 ylabel('Intensity') % y-axis label
32 title('\bf The second mode','FontSize', f)
33 subplot(1,3,3);
34 U3= reshape(U(:,3),m,n);
35 colormap('gray'); imagesc(U3);
36 xlabel('Intensity') % x-axis label
37 ylabel('Intensity') % y-axis label
38 title('\bf The third mode','FontSize', f)
39
40 %% Reconstruction
41 j=300;
42 R= U(:,1:j)*S(1:j,1:j)*V(:,1:j)';
43
44 R1= reshape(R(:,1),m,n);
45 figure;
46 subplot(1,2,2);
```



```

47 colormap('gray'); imagesc(R1);
48 xlabel('Intensity','FontSize', f) % x-axis label
49 ylabel('Intensity','FontSize', f) % y-axis label
50 title('\bf Reconstructed image','FontSize', f)
51
52 subplot(1,2,1);
53 Z1= reshape(Z(:,1),m,n);
54 colormap('gray'); imagesc(Z1);
55 xlabel('Intensity','FontSize', f) % x-axis label
56 ylabel('Intensity','FontSize', f) % y-axis label
57 title('\bf Original image','FontSize', f)

```