

Assignment 1.

Jithin D. George

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1. There are $2^{100} \approx 10^{30}$ configurations possible. I have an i3 processor which has roughly 3 GHz. This means it can do 3×10^9 calculations in a second. So, it would take around 10^{20} seconds to do calculations of all the configurations. That's 10^{13} years of calculations!

2. (a) $\uparrow \uparrow, \downarrow \uparrow, \downarrow \downarrow, \downarrow \downarrow$
(b)

$$Z = \sum_i e^{-\beta \epsilon U_i} = 2e^{\beta \epsilon} + 2e^{-\beta \epsilon} = 4 \cosh(\beta \epsilon)$$

- (c) There are 4 states. So, the mean energy is

$$\langle E \rangle = \frac{2\epsilon e^{-\beta \epsilon} - 2\epsilon e^{\beta \epsilon}}{Z} = -\frac{2\epsilon e^{\beta \epsilon} - 2\epsilon e^{-\beta \epsilon}}{2e^{\beta \epsilon} + 2e^{-\beta \epsilon}} = -\epsilon \tanh(\beta \epsilon)$$

- (d) Mean energy is close to $-\epsilon$ when $\tanh(\beta \epsilon)$ close to 1, which happens for $\beta \epsilon$ sufficiently greater than 0. This happens because the parallel spin terms dominate then giving the mean energy equivalent to that of a parallel spin.

- (e) One.

$$\lim_{T \rightarrow 0} \frac{2e^{\beta \epsilon}}{2e^{\beta \epsilon} + 2e^{-\beta \epsilon}} = \lim_{\beta \rightarrow \infty} \frac{2e^{\beta \epsilon}}{2e^{\beta \epsilon} + 2e^{-\beta \epsilon}} = 1$$

- (f) Zero.

$$\lim_{T \rightarrow 0} \frac{2e^{-\beta \epsilon}}{2e^{\beta \epsilon} + 2e^{-\beta \epsilon}} = \lim_{\beta \rightarrow \infty} \frac{2e^{-\beta \epsilon}}{2e^{\beta \epsilon} + 2e^{-\beta \epsilon}} = 0$$

- (g) Zero.

$$\lim_{T \rightarrow \infty} \langle E \rangle = \lim_{\beta \rightarrow 0} -\epsilon \tanh(\beta \epsilon) = 0$$

- (h) Ratio of probability for the all up state to the all down state is 1 regardless of temperature because they have the same probability. Similarly, the ratio of the probabilities of the two anti-parallel states are 1.

Ratio of the probability of a parallel state to an anti-parallel one is

$$\lim_{\beta \rightarrow 0} \frac{e^{\beta \epsilon}}{e^{-\beta \epsilon}} = 1$$

(i) For the probabilities to be equal, we need

$$e^{\beta\epsilon} = 2e^{-\beta\epsilon}$$

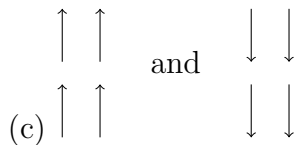
$$2\beta\epsilon = \log(2)$$

$$T = \frac{2\epsilon}{k_B \log 2}$$

(j) It seems to be $\beta\epsilon$ that determines the probability of states and thus, $\frac{\epsilon}{k_B T}$ is the parameter that controls the nature of the system.

3. (a) Each state is influenced only by its two neighbours. So, it's a closed system.

(b) The minimum possible energy is -4ϵ .



4. (a) `idivide(a,b)` returns the closest integer to a/b which is either lesser than or greater than or just the closest one to a/b depending on the option. `Mod` returns the remainder in a/b

(b) It seems to be West, South, East, North.

(c) From the Matlab output `nbr`, it seems to be periodic boundaries.

(d)

$$[-1, -1, -1, -1] \rightarrow_{k=1} [1, -1, -1, -1] \rightarrow_{k=2} [1, 1, -1, -1] \rightarrow_{k=1} [-1, 1, -1, -1]$$

5. Tried running it for a 10x10 system. This is the result.

