Assignment 1.

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1. There are $2^{100} \approx 10^{30}$ configurations possible. I have an i3 processor which has roughly 3 GHz. This means it can do $3x10^9$ calculations in a second. So, it would take around 10^{20} seconds to do calculations of all the configurations. That's 10^{13} years of calculations!.

2. (a) $\uparrow\ \uparrow, \downarrow\ \uparrow, \uparrow\ \downarrow, \downarrow\ \downarrow$

   (b)
   $$Z = \sum_i e^{-\beta\epsilon U_i} = 2e^{\beta\epsilon} + 2e^{-\beta\epsilon} = 4cosh(\beta\epsilon)$$

   (c) There are 4 states. So, the mean energy is
   $$< E >= \frac{2\epsilon e^{-\beta\epsilon} - 2\epsilon e^{\beta\epsilon}}{Z} = -\frac{2\epsilon e^{\beta\epsilon} - 2\epsilon e^{-\beta\epsilon}}{2e^{\beta\epsilon} + 2e^{-\beta\epsilon}} = -\epsilon \tanh(\beta\epsilon)$$

   (d) Mean energy is close to $-\epsilon$ when $\tanh(\beta\epsilon)$ close to 1, which happens for $\beta\epsilon$ sufficiently greater than 0. This happens because the parallel spin terms dominate then giving the mean energy equivalent to that of a parallel spin.

   (e) One.
   $$\lim_{T\to 0} \frac{2e^{\beta\epsilon}}{2e^{\beta\epsilon} + 2e^{-\beta\epsilon}} = \lim_{\beta\to\infty} \frac{2e^{\beta\epsilon}}{2e^{\beta\epsilon} + 2e^{-\beta\epsilon}} = 1$$

   (f) Zero.
   $$\lim_{T\to 0} \frac{2e^{-\beta\epsilon}}{2e^{\beta\epsilon} + 2e^{-\beta\epsilon}} = \lim_{\beta\to\infty} \frac{2e^{-\beta\epsilon}}{2e^{\beta\epsilon} + 2e^{-\beta\epsilon}} = 0$$

   (g) Zero.
   $$\lim_{T\to\infty} < E >= \lim_{\beta\to 0} -\epsilon \tanh(\beta\epsilon) = 0$$

   (h) Ratio of probability for the all up state to the all down state is 1 regardless of temperature because they have the same probability. Similarly, the ratio of the probablities of the two anti-parallel states are 1.

   Ratio of the probability of a parallel state to an anti-parallel one is
   $$\lim_{\beta\to 0} \frac{e^{\beta\epsilon}}{e^{-\beta\epsilon}} = 1$$

(i) For the probabilities to be equal, we need

$$e^{\beta\epsilon} = 2e^{-\beta\epsilon}$$

$$2\beta\epsilon = \log(2)$$

$$T = \frac{2\epsilon}{k_B \log 2}$$

(j) It seems to be $\beta\epsilon$ that determines the probability of states and thus, $\frac{\epsilon}{k_B T}$ is the parameter that controls the nature of the system.

3. (a) Each state is influenced only by its two neighbours. So, it's a closed system.

   (b) The minimum prossible energy is -4ϵ.

   (c) $\begin{matrix} \uparrow & \uparrow \\ \uparrow & \uparrow \end{matrix}$ and $\begin{matrix} \downarrow & \downarrow \\ \downarrow & \downarrow \end{matrix}$

4. (a) idivide(a,b) returns the closest integer to a/b which is either lesser than or greater than or just the closest one to a/b depending on the option. Mod returns the remainder in a/b

   (b) It seems to be West, South, East, North.

   (c) From the Matlab output nbr, it seems to be periodic boundaries.

   (d)

$$[-1,-1,-1,-1] \rightarrow_{k=1} [1,-1,-1,-1] \rightarrow_{k=2} [1,1,-1,-1] \rightarrow_{k=1} [-1,1,-1,-1]$$

5. Tried running it for a 10x10 system. This is the result.