Assignment 1.

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Due Oct 2

1.

$$x^2y' + (1+x^2)y = 0$$
$$\frac{y'}{y} = -1 - \frac{1}{x^2}$$
$$\log(y) = -x + \frac{1}{x} + C$$
$$y = Ae^{\frac{1}{x}-x}$$

2.

$$xy' + 3y = -x$$

Multiplying by the integrating factor,

$$x^3y' + 3x^2y = -x^3$$
$$(x^3y)' = -x^3$$
$$x^3y = -\frac{x^4}{4} + C$$
$$y = -\frac{x}{4} + \frac{C}{x^3}$$

3.

$$y'' + 6y' + 5y = 0$$

Using the ansatz e^{mx},

$$m^2e^{mx} + 6me^{mx} + 5e^{mx} = 0$$
$$m^2 + 6m + 5 = 0$$
$$m = -1, -5$$

Thus,

$$y = c_1e^{-x} + c_2e^{-5x}$$

4.

$$y'' + 6y' + 9y = 0$$

Using the ansatz e^{mx},

$$m^2e^{mx} + 6me^{mx} + 9e^{mx} = 0$$
$$m^2 + 6m + 9 = 0$$
$$m = -3, -3$$

Since the root is repeated, we can use reduction of order to find that the general solution is

$$y = c_1e^{-3x} + c_2xe^{-3x}$$

5.

$$y'' + 6y' + 13y = 0$$

Using the ansatz e^{mx},

$$m^2e^{mx} + 6me^{mx} + 13e^{mx} = 0$$
$$m^2 + 6m + 13 = 0$$
$$m = -3 + 2i, -3 - 2i$$

Since the root are complex, the general solution is

$$y = e^{-3x}(c_1sin(2x) + c_2cos(2x))$$

6.

$$y''' + y'' - y' - y = 0$$

Using the ansatz e^{mx},

$$m^3e^{mx} + m^2e^{mx} - me^{mx} - e^{mx} = 0$$
$$m^3 + m^2 - m - 1 = 0$$
$$(m^2 - 1)(m + 1) = 1$$
$$m = 1, -1, -1$$

Since e -1 is repeated twice, the general solution is

$$y = c_1e^x + c_2e^{-x} + c_3xe^{-x}$$

7.

$$xy'' + 3y' = x^2$$

Multiplying by the integrating factor,

$$x^3y'' + 3x^2y' = x^4$$
$$(x^3y')' = x^4$$

$$x^3y' = \frac{x^5}{5} + c_1$$
$$y' = \frac{x^2}{5} + \frac{C}{x^3}$$
$$y = \frac{x^3}{15} - \frac{c_1}{2x^2} + c_2$$

8.
$$(x-1)y'' - xy' + y = 0$$

Plugging in $y_2 = u(x)y_1$, we get

$$(x-1)(u''y_1 + 2u'y_1' + uy_1'') - x(u'y_1 + uy_1') + uy_1 = 0$$

$$(x-1)(u''y_1 + 2u'y_1') - x(u'y_1) = 0$$
$$(x-1)(u''e^x + 2u'e^x) - x(u'e^x) = 0$$
$$(x-1)u'' + (x-2)u' = 0$$
$$\frac{u''}{u'} = -\frac{x-2}{x-1} = -1 + \frac{1}{x-1}$$
$$\log(u') = c_1 - x + log(x-1)$$
$$u' = c_1(x-1)e^{-x}$$
$$u = c_1xe^{-x} + c_2$$
$$y = uy_1 = c_1x + c_2e^x$$