Assignment 5.

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1. (a)
$$y'' - \lambda y = 0, y'(0) = y(1) = 0$$
Let y be an eigenfunction of form e^{nx}.
$$n^2 - \lambda = 0$$
If $\lambda > 0$,
$$y = ae^{\sqrt{\lambda}x} + be^{-\sqrt{\lambda}x}$$
With the initial conditions, we get
$$y = 0$$
If $\lambda = 0$,
$$y = c_1 x + c_2$$
With the initial conditions, we get
$$y = 0$$
If $\lambda < 0$,
$$\lambda = -\mu^2$$
$$y = a\sin(\mu x) + b\cos(\mu x)$$
With the initial conditions, we get
$$y = b\cos(\mu x)$$
where
$$\mu = (2n+1)\frac{\pi}{2}, n = 0, 1, 2, ...$$
(b)
$$y'' - 2y' + (1+\lambda)y = 0, y(0) = y(1) = 0$$
Let y be an eigenfunction of form e^{nx}.
$$n^2 - 2n + 1 + \lambda = 0$$
$$n = 1 \pm \sqrt{-\lambda}$$

If $\lambda > 0$,

$$\lambda = -\mu^2$$

$$y = ae^x \sin(\mu x) + be^x \cos(\mu x)$$

With the initial conditions, we get

$$y = ae^x \sin(\mu x)$$

where

$$\mu = n\pi, n = 1, 2, ..$$

If $\lambda = 0$,

$$y = c_1 x e^x + c_2 e^x$$

With the initial conditions, we get

$$y = 0$$

If $\lambda < 0$,

$$\lambda = -\mu^2$$

$$y = ae^{(1+\mu)x} + be^{(1-\mu)x}$$

With the initial conditions, we get

$$y = 0$$

2. (a)

$$x^4 y'' + \lambda y = 0$$

From the boundary conditions, $\alpha = 0, \beta = 0$, so by the S-L theory,

$$\lambda \geq 0$$

If $\lambda = 0$,

$$y = c_1 x + c_2$$

With the boundary conditions,

$$y = 0$$

If $\lambda > 0$,

$$\lambda = -\mu^2$$

$$y = c_1 x \sin(\frac{\sqrt{\lambda}}{x}) + c_2 x \cos(\frac{\sqrt{\lambda}}{x})$$

With the boundary conditions,

$$c_1 a \sin(\frac{\sqrt{\lambda}}{a}) + c_2 a \cos(\frac{\sqrt{\lambda}}{a}) = 0$$

$$c_2 b\sin(\frac{\sqrt{\lambda}}{b}) + c_2 b\cos(\frac{\sqrt{\lambda}}{b}) = 0$$

$$\frac{c_2}{c_1} = -\tan(\frac{\sqrt{\lambda}}{a}) = -\tan(\frac{\sqrt{\lambda}}{b})$$

$$\frac{\sqrt{\lambda}}{a} = \frac{\sqrt{\lambda}}{b} + 2n\pi$$

$$\lambda = \frac{4n^2\pi^2}{(\frac{1}{a} - \frac{1}{b})^2}$$

(b) Thus, the eigenfunctions would be

$$y_n = c_1(a\sin(\frac{\sqrt{\lambda}}{x}) - a\tan(\frac{\sqrt{\lambda}}{b})\cos(\frac{\sqrt{\lambda}}{x}))$$