

Exploration of chaos in shock waves

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Outline

Introduction

Background

Model Equation

The nature of the source term

The Steady state solution

Effects of β

Numerical setup

Behaviour

$\alpha = 2$

$\alpha = 3$

$\alpha = 4.6$

$\alpha = 4.85$

$\alpha = 5.1$

Conclusions

Bifurcation diagram

LLE

Weird pdes and insights

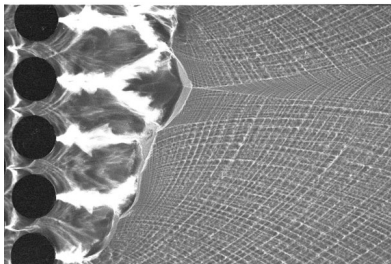
Background: Reactive Euler equations and detonation

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot u$$

$$\rho \frac{Du}{Dt} = -\nabla P$$

$$\rho \frac{D}{Dt} \left(h + \frac{|u|^2}{2} \right) = \frac{\partial P}{\partial t}$$

$$\frac{DY_i}{Dt} = \dot{\omega}_i$$



Model Equation

$$u_t + \frac{1}{2}(u^2 - uu_s)_x = f(x, u_s)$$

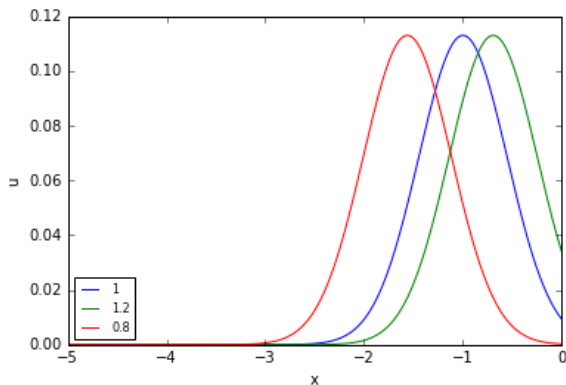
$$x \in (-\infty, 0)$$

$$\textbf{Characteristic speed} = u - \frac{u_s}{2}$$

The nature of the source term

$$f(x, u_s) = \frac{q}{2\sqrt{4\pi\beta}} e^{-\frac{[x-x_f(u_s)]^2}{4\beta}}$$

$$x_f(u_s) = \left(\frac{u_{0s}}{u_s}\right)^\alpha$$



The steady state solution

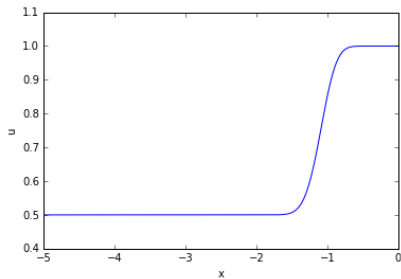
$$\frac{1}{2}(u_0^2 - u_0 u_{0s})' = f(x, u_{0s})$$

$$u_0(x) = \frac{u_{0s}}{2} + \sqrt{2 \int_{-\infty}^x f(y, u_{0s}) dy}$$

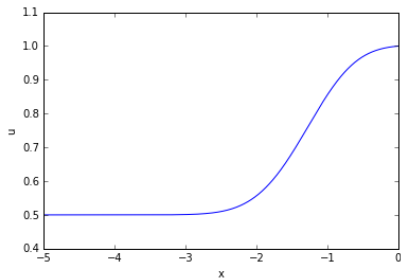
$$f = \frac{q}{2\sqrt{4\pi\beta}} e^{-\frac{[x-x_f(u_s)]^2}{4\beta}}$$

$$u_0(x) = \frac{1}{2} \left[1 + \sqrt{\frac{1 + \operatorname{erf}((x+1)/2\sqrt{\beta})}{1 + \operatorname{erf}(1/2\sqrt{\beta})}} \right]$$

Effects of β



(a) $\beta = 0.01$



(b) $\beta = 0.1$

Figure 1: u_0 for various values of β

Movie

Numerical setup



$$u_t + \frac{1}{2}(u^2 - uu_s)_x = ae^{-\frac{(x+us-\alpha)^2}{4\beta}}$$



$$x \in (-10, 0)$$

- ▶ Outflow boundary conditions
- ▶ Godunov splitting
- ▶ L-W with MC limiters

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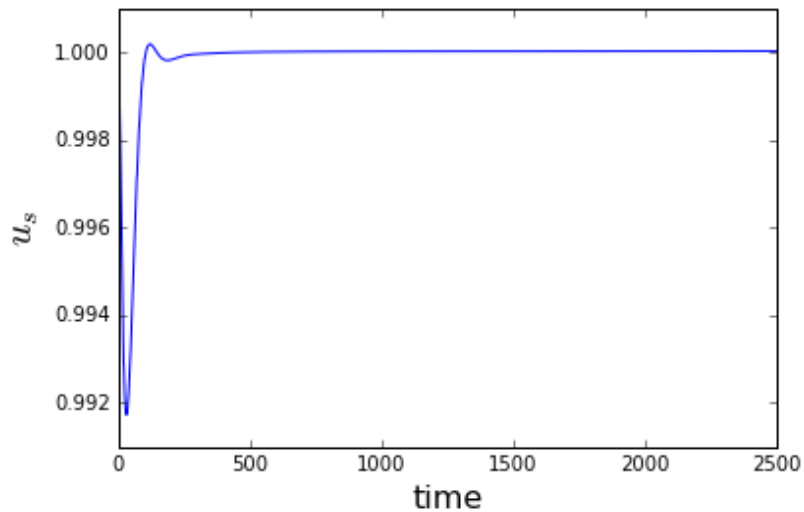
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$$\alpha = 2$$

Movie

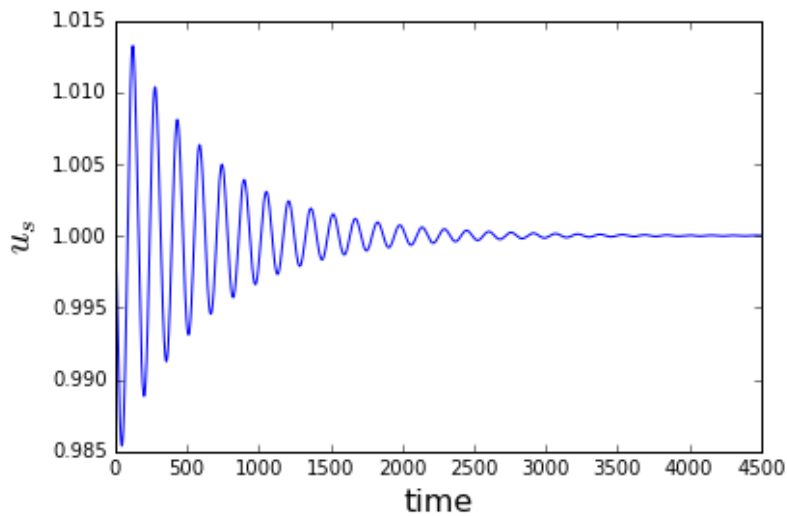
Behaviour of u_s



$$\alpha = 3$$

Movie

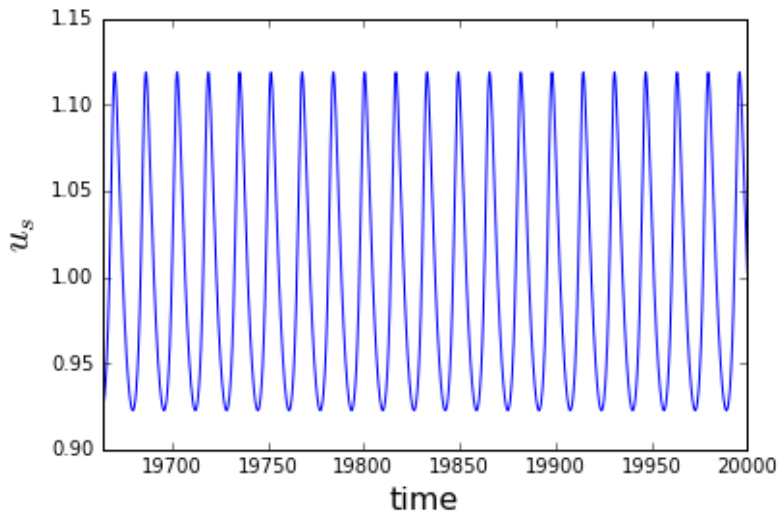
Behaviour of u_s



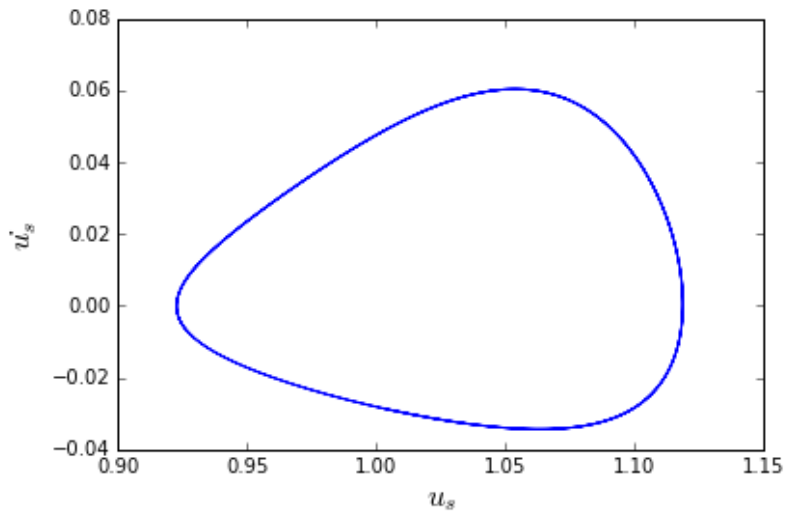
$$\alpha = 4.6$$

Movie

Asymptotic behaviour of u_s



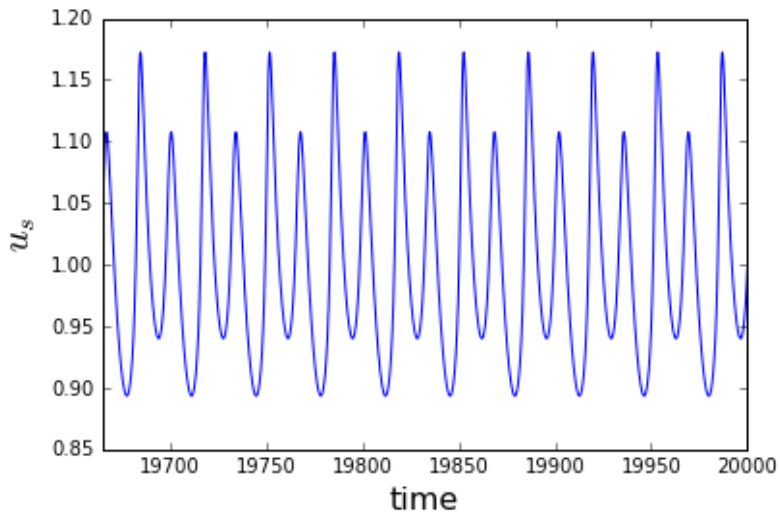
Limit cycle



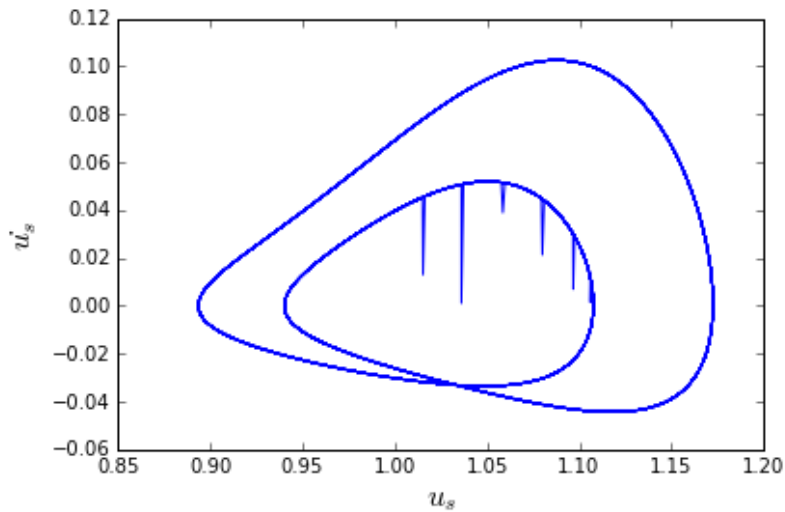
$$\alpha = 4.85$$

Movie

Asymptotic behaviour of u_s



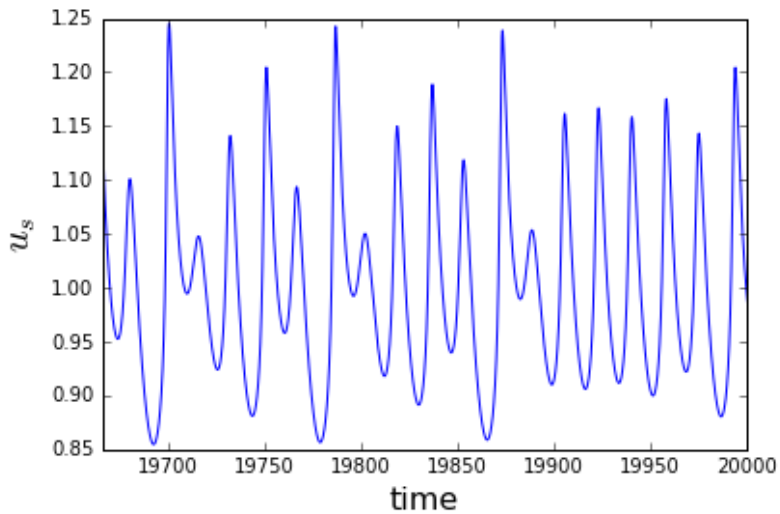
Limit cycle



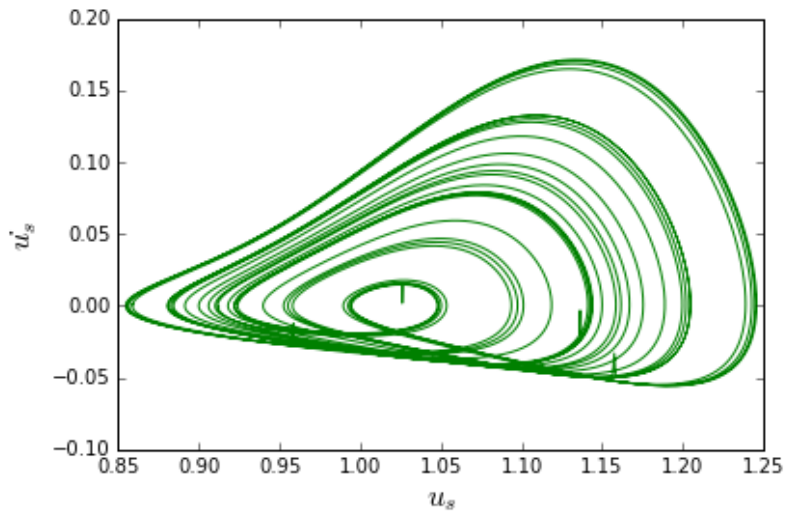
$$\alpha = 5.1$$

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Asymptotic behaviour of u_s



Limit cycle



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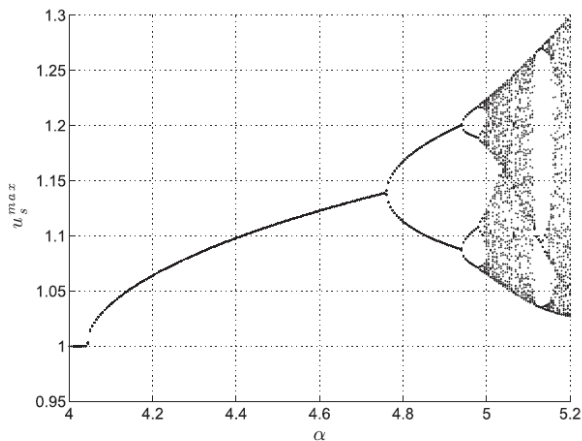


FIG. 3. The long-time values of the local maxima, u_s^{\max} , of the shock strength as a function of α .

Largest Lyapunov Exponents

	4.85	4.96	4.97	5.1
LLE	0	0	0.0042	0.0315
D_C	1.0006	1.002	1.67	1.91

Weird pdes and insights

Movie

Resources

- ▶ Kasimov, Aslan R., Luiz M. Faria, and Rodolfo R. Rosales. "Model for shock wave chaos." *Physical review letters* 110.10 (2013): 104104.
- ▶ Faria, Luiz M., Aslan R. Kasimov, and Rodolfo R. Rosales. "Study of a model equation in detonation theory." *SIAM Journal on Applied Mathematics* 74.2 (2014): 547-570.
- ▶ https://github.com/Dirivian/apps_detonation


$$u_t + \frac{1}{2}(u^2 - uu_s)_x = f(x, u_s)$$