

## Retirement Finance Simulation Model

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Notes in red are my comments and explanations, not model output.

## Section 1. Simulation Parameters

The first section of this report identifies the parameters used for the simulation.

NOTE THAT ALL RESULTS AND INPUT PARAMETERS ARE EXPRESSED IN REAL (TODAY'S) DOLLARS. Assets with no inflation protection will show a decline in value over time based on simulated inflation rates.

Parameters for the simulations are as follows:

- Run 10,000 scenarios.
- Demographic data
  - Husband's current age is 63.
  - Wife's current age is 56.
- Social Security data
  - Husband claims at age 70
  - Wife claims at age 67
  - All claiming options:

	higherEarner	lowerEarner
Age62	21288	15480
FRA	28956	22752
Age70	37200	28656

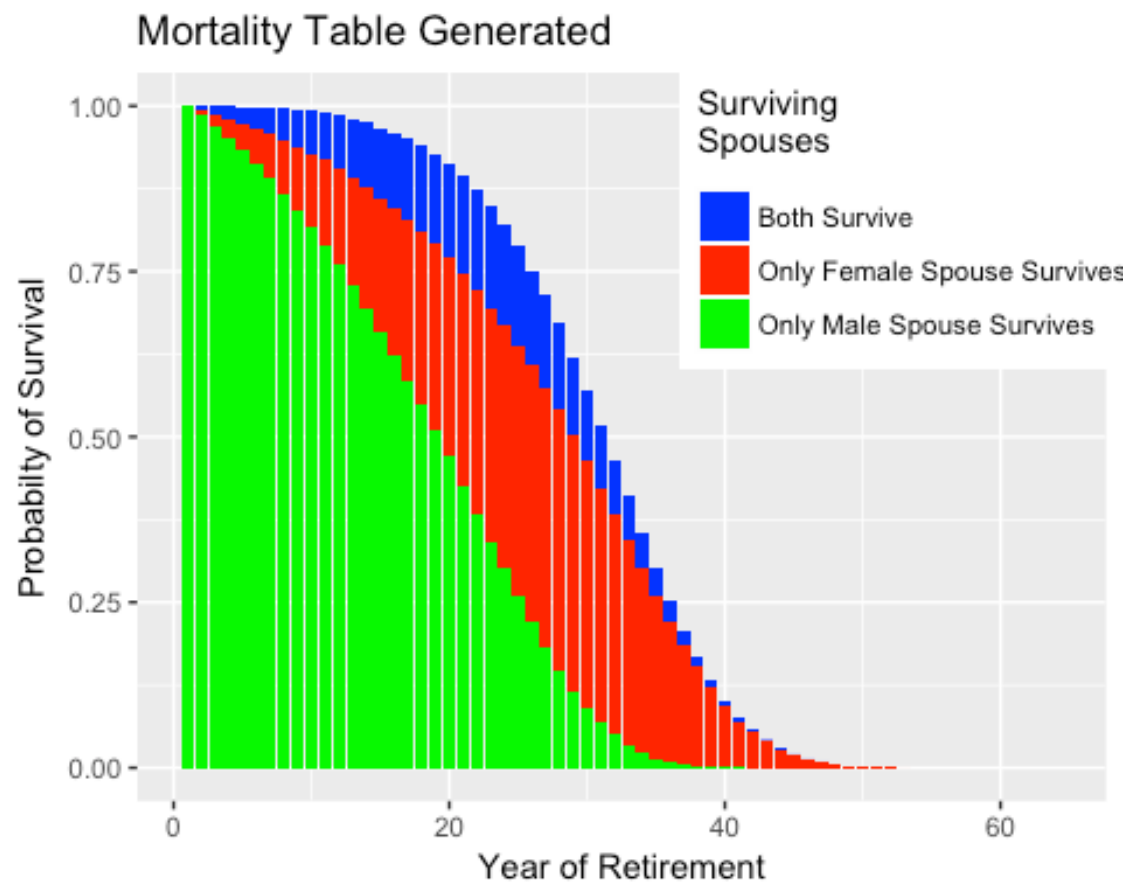
- Portfolio parameters
  - Initial portfolio balance before annuity purchase is \$ 2,615,616.
  - Equity allocations randomized from 0% to 100%.
- Market and inflation parameters
  - Annual average rate of inflation is 2%.
  - Inflation rate annual standard deviation is 1%.
  - Risk-free real return rate is 1%.
  - Equity risk premium is 4.25%.
  - Standard deviation of annual market returns is 12%.
- Immediate Annuity parameters
  - SPIA Purchase age is 65
  - Age of annuitant when payments will begin is 66.
  - Quote for annual payment before any deaths is \$ 0.
  - Annuity is inflation-protected is TRUE.
  - Husband owns Annuity is TRUE.
  - Percent of benefit that goes to survivor is 50%.

- SPIA payout rate is 5.36%.
  - Annuity allocation as percent of initial portfolio randomized from 0% to 100%.
- Deferred Annuity parameters
  - None simulated
- Spending parameters
  - Expected spending year one of retirement randomized from \$ 135,000 to \$ 150,000.
  - Percent expense decline after death of first spouse is 63%.
  - Expenses typically decline 0% annually throughout retirement.
- HECM Line of Credit
  - Home appreciation rate is 0% annually.
  - Initial HECM Line of Credit available is \$ 0
  - Initial Reverse Mortgage Balance is \$ 0
  - Home Market Value (no real annual growth assumed) is \$ 75,000
  - Mean long term return for 1-yr Libor= 2% with standard deviation= 1%
  - HECM line of credit's maximum lifetime interest rate cap 10.34%
  - HECM line of credit's margin added to Libor Index for variable rate loan 3%
  - HECM line of credit's Monthly Insurance Premium percentage 1.25%

## Section 2. Graphs of Simulation Parameters

Section 2 of this report graphically displays key simulation input parameters.

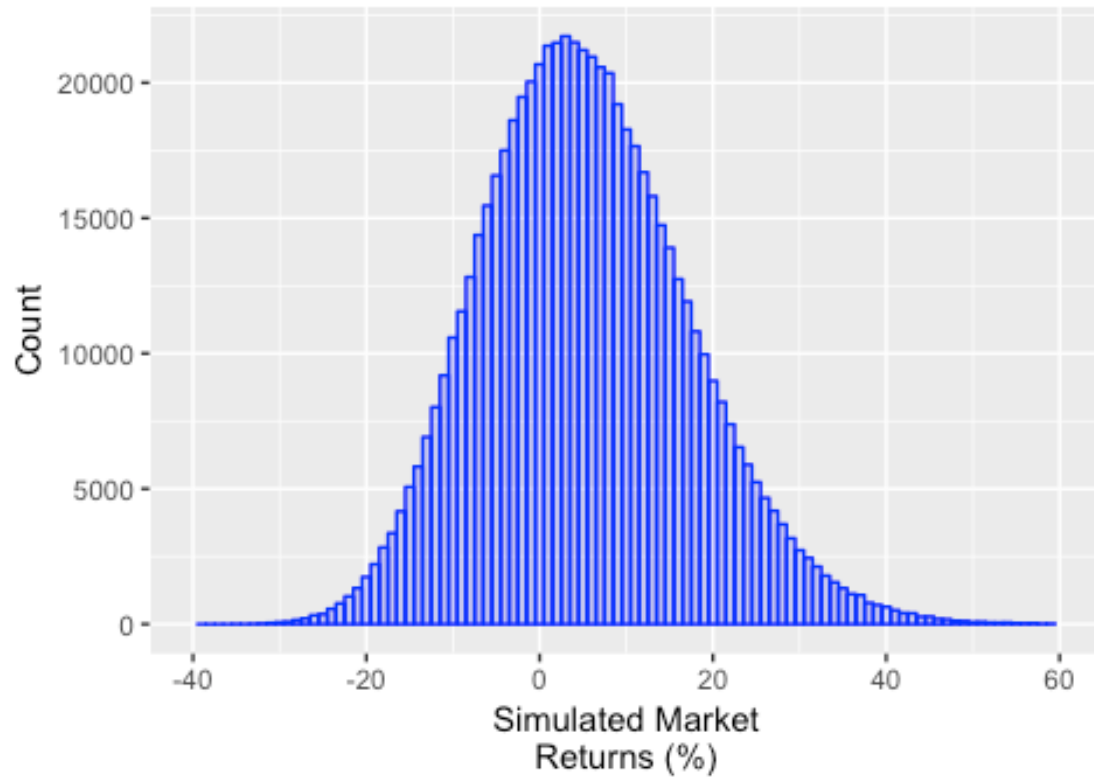
Life expectancies for husband and wife are plotted below.



Section2 just shows the inputs that were used for the simulation. Results come next.

The following annual market returns were simulated.

Figure 2. Histogram of  
Simulated Annual Real Market Returns

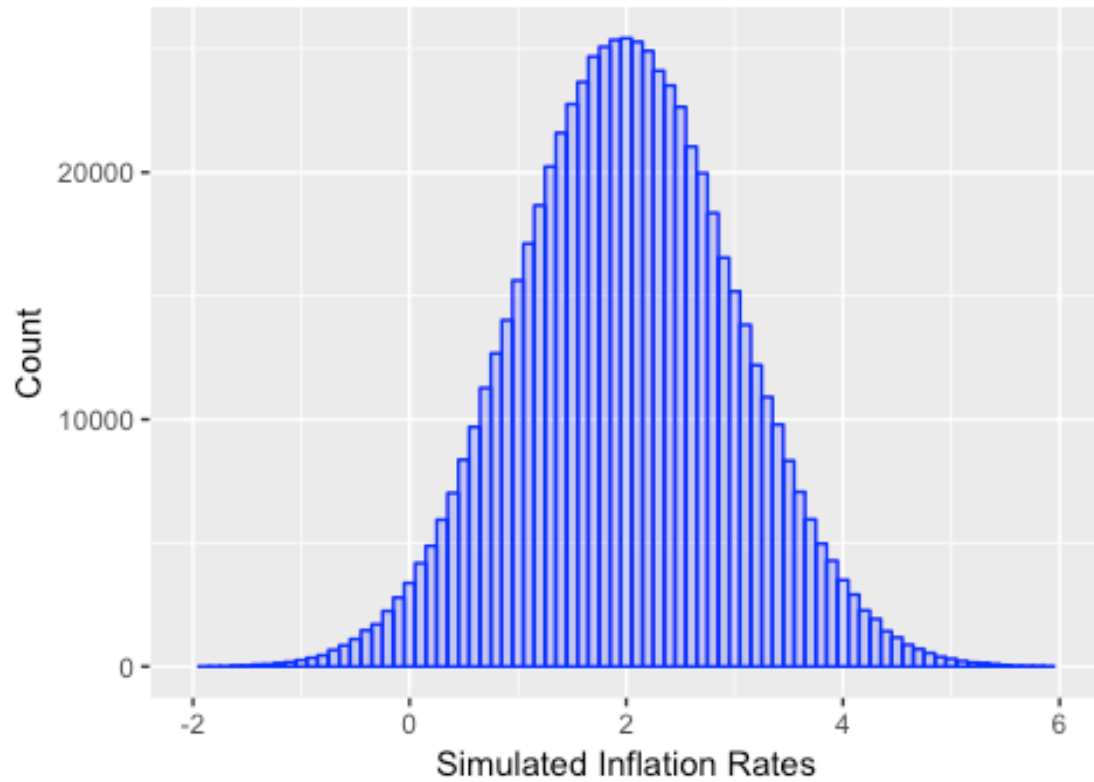


Arithmetic Mean of Simulated Annual Returns = 5.277%.

Standard Deviation of Simulated Annual Returns = 12.002%.

The following simulated annual inflation rates were simulated:

Figure 3. Histogram of  
Simulated Annual Inflation Rates

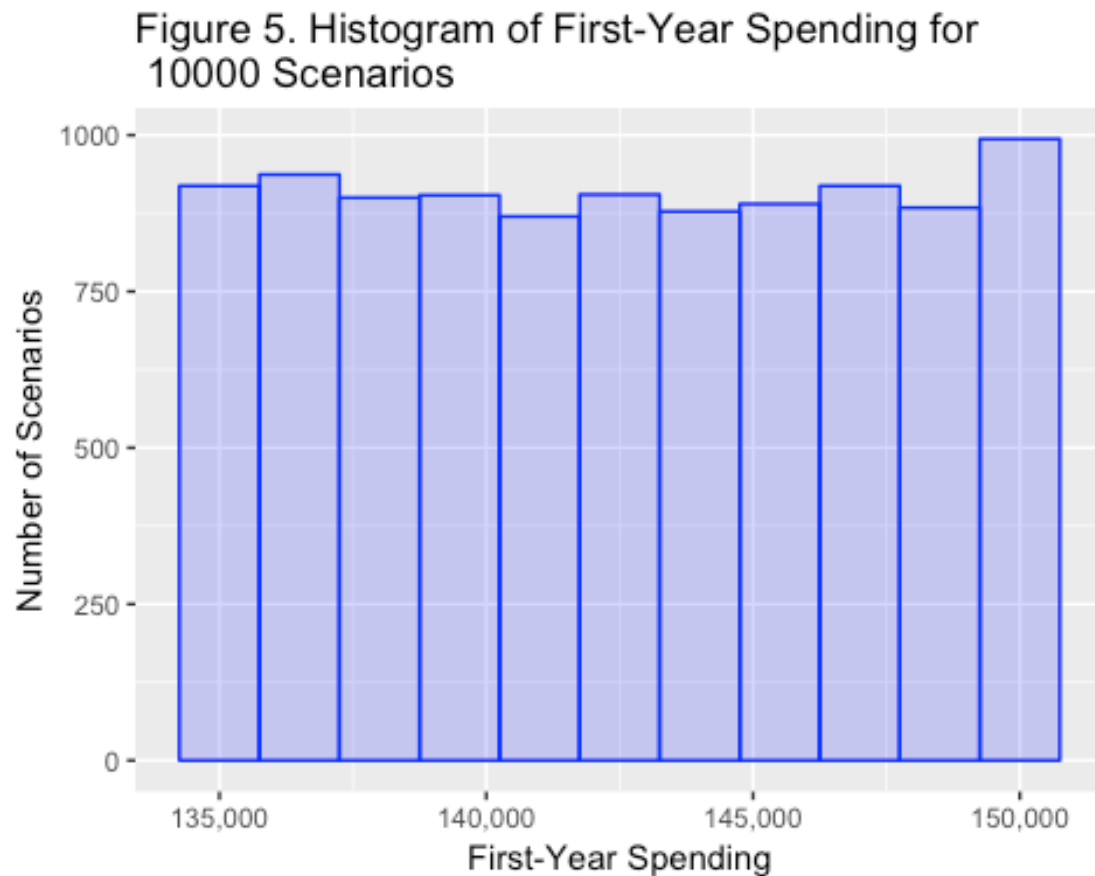


- Arithmetic Mean of Simulated Inflation Rates = 2%.
- Standard Deviation of Simulated Annual Inflation Rates = 0.999%.

The following graph shows the number of scenarios that were simulated at various first-year spending rates. The x-axis also displays the range of first-year spending that was simulated.

The minimum first-year spending simulated was \$ 135,000.

The maximum first-year spending simulated was \$ 150,000.

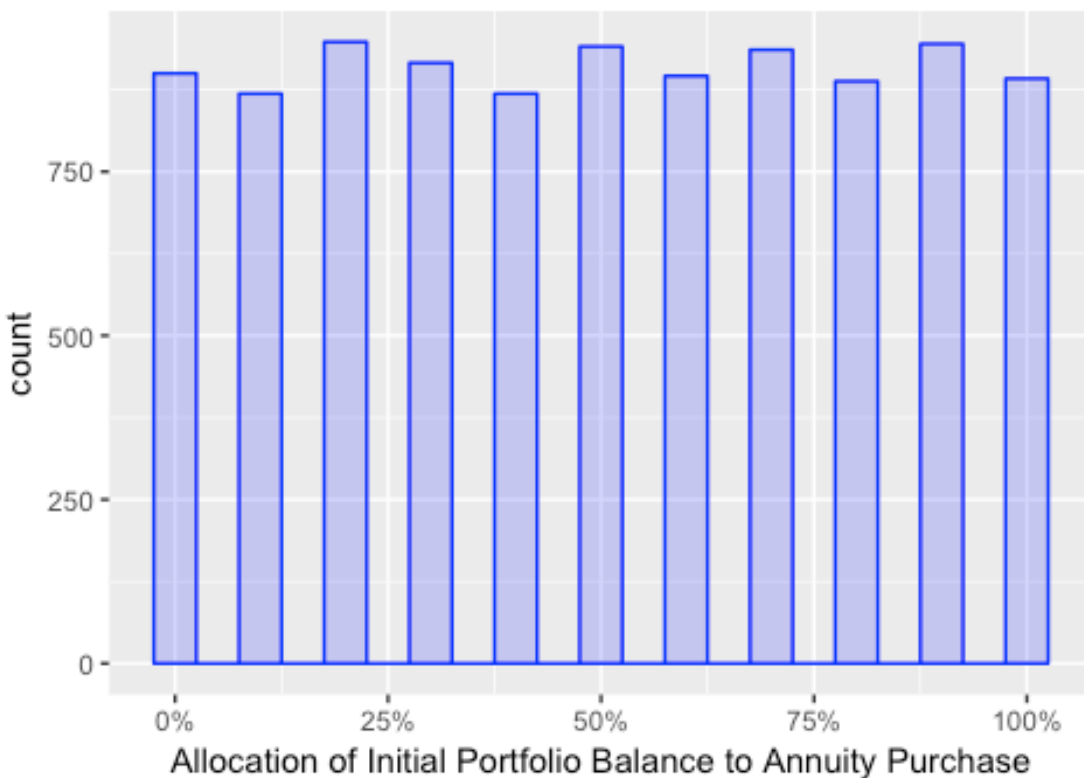


This chart shows that an equal number of scenarios were tested with first year of retirement spending ranging from \$135K to \$150K. In the first run, we try equal amounts of every important parameter.

The graphs that follow are intended to show the range of inputs used by the simulation model (for example, the range of spending tested) and the distributions of the input parameters (for example, life expectancy follows a Gompertz distribution, while spending parameters are randomized with a uniform distribution.)

The following chart shows the number of scenarios simulated at each level of annuity allocation. The x-axis shows the range of annuity allocations simulated.

**Figure 4. Histogram of Annuity Allocations for 10000 Scenarios**

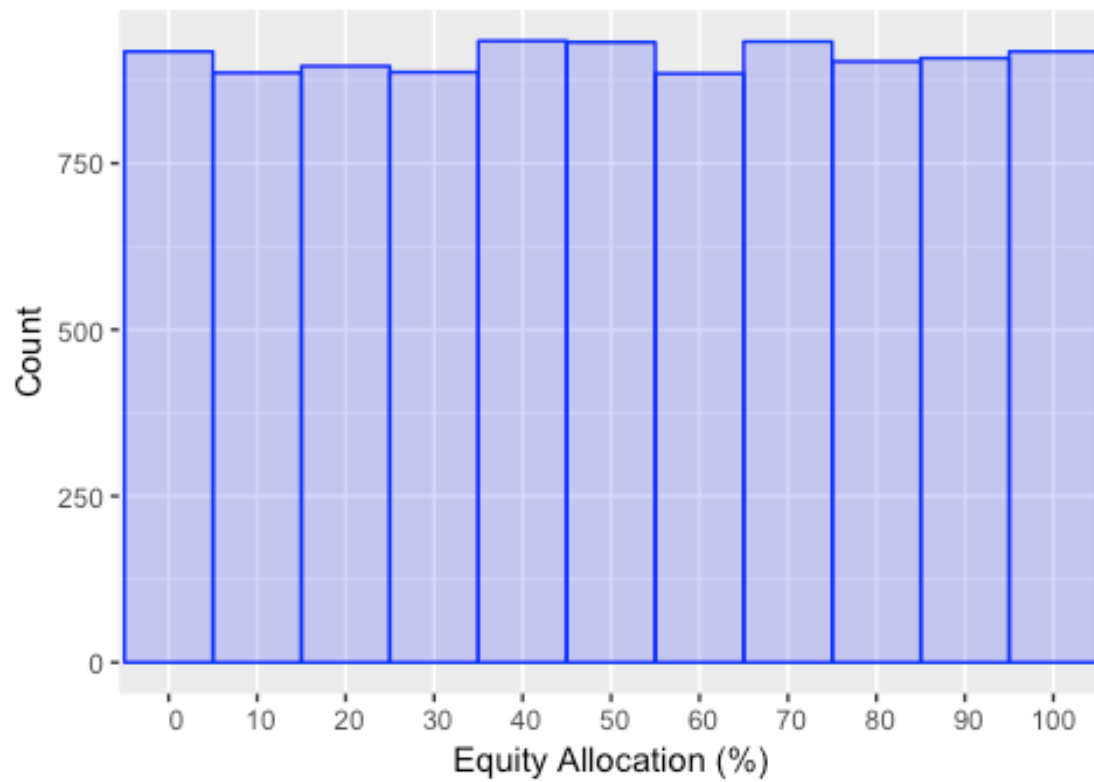


We tested allocating SPIA purchases from none to all of the savings portfolio.



The following chart shows the number of scenarios simulated at each level of equity allocation. The x-axis shows the range of equity allocations simulated.

**Figure 5. Histogram of Equity Allocations for 10000 Scenarios**



We simulated all equity allocations from 0% to 100% in 10% increments.

## Section 3. Results of Simulations

Section 3 provides the results of the simulation.

### Statistics For Underfunded Scenarios

- 5756 scenarios with unmet spending or 57.56%
- Some failed scenarios were almost completely funded, while some funded scenarios were just barely funded. These scenarios fall within the margin of error.
- Percent of scenarios that funded less than 95% of years 56.93%
- Number of years with unmet spending 130918
- Mean years with unmet spending when spending not met 23
- Depleted portfolios 57.56 %
- Scenarios depleting HECM Line of Credit 0 or 0%

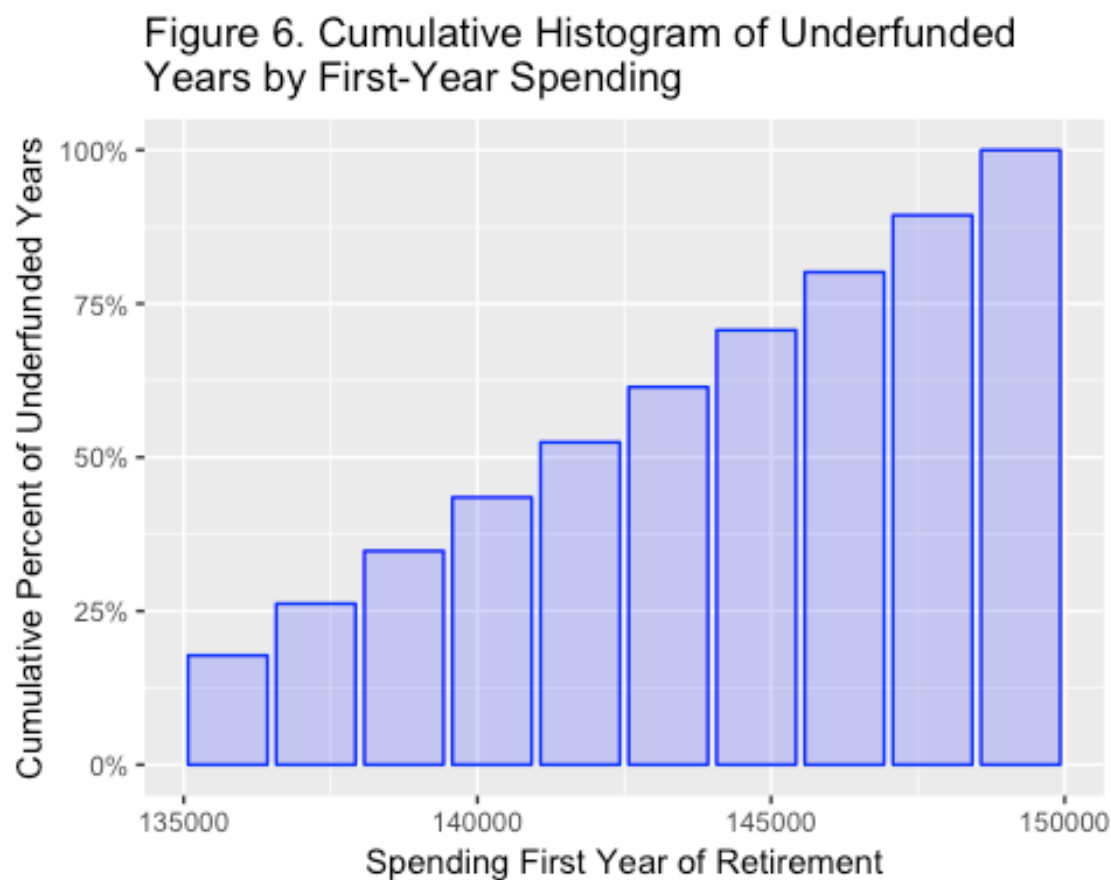
Here's where the results start and this first run, where we try everything in equal proportions, is awful: a 57.56% failure rate. So, we look for the primary causes.

Following is a histogram showing spending amounts (and the range of spending along the x-axis) for the first year of each retirement scenario.

This histogram shows the cumulative ratio of underfunded simulated years by the amount of spending for the first year of retirement. The right-most column, for example, shows that 100% of 5756 unfunded years had spending in the first year of retirement of \$ 0.3 or less. The column to its left shows that about 80% of all 5756 unfunded years had spending in the first year of retirement of \$ 0.2 or less.

About half of the 5756 underfunded years in this simulation could have been funded by spending less than \$ 0.2 from the beginning of retirement.

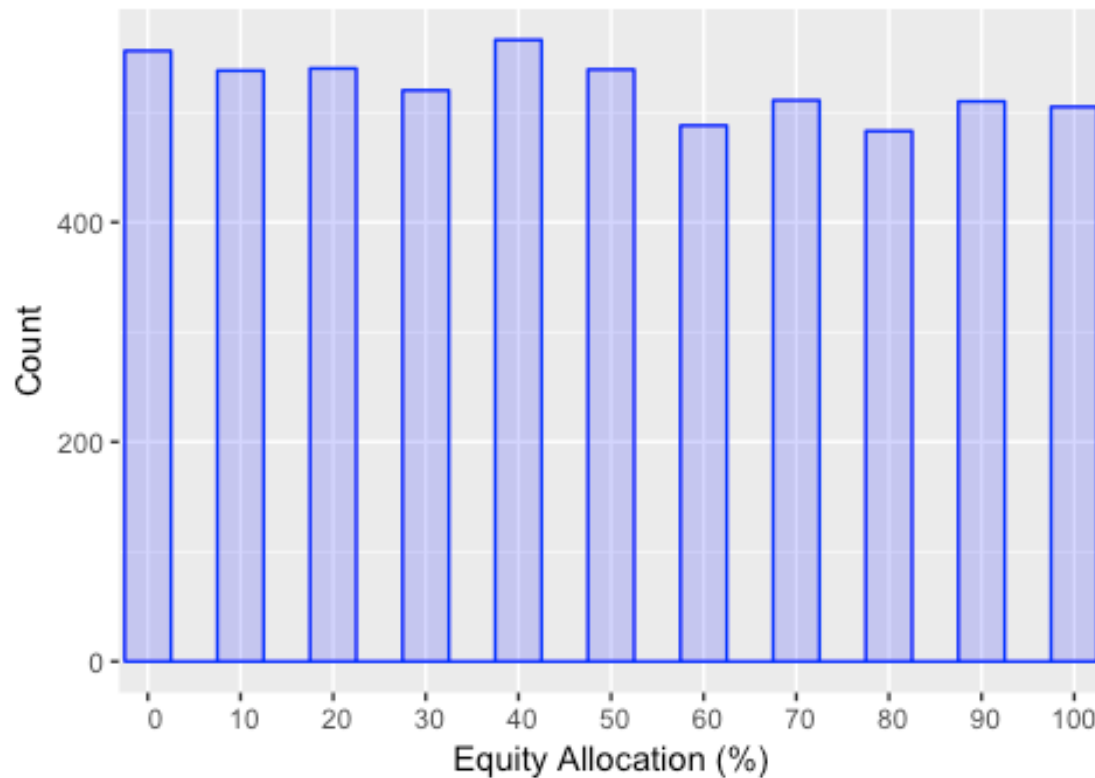
If an empty chart appears below, then there were no underfunded years.



This chart shows the first-year spending for scenarios that failed. We could lower the failure rate from 58% to about 15% by spending \$135,000 instead of \$150,000, suggesting that the failures are caused by spending too much. In a minute, we will see that is not necessary.

Following is a histogram showing the equity allocation for 5756 underfunded scenarios (an underfunded scenario had at least one underfunded year). If no chart appears below, then there were no underfunded scenarios.

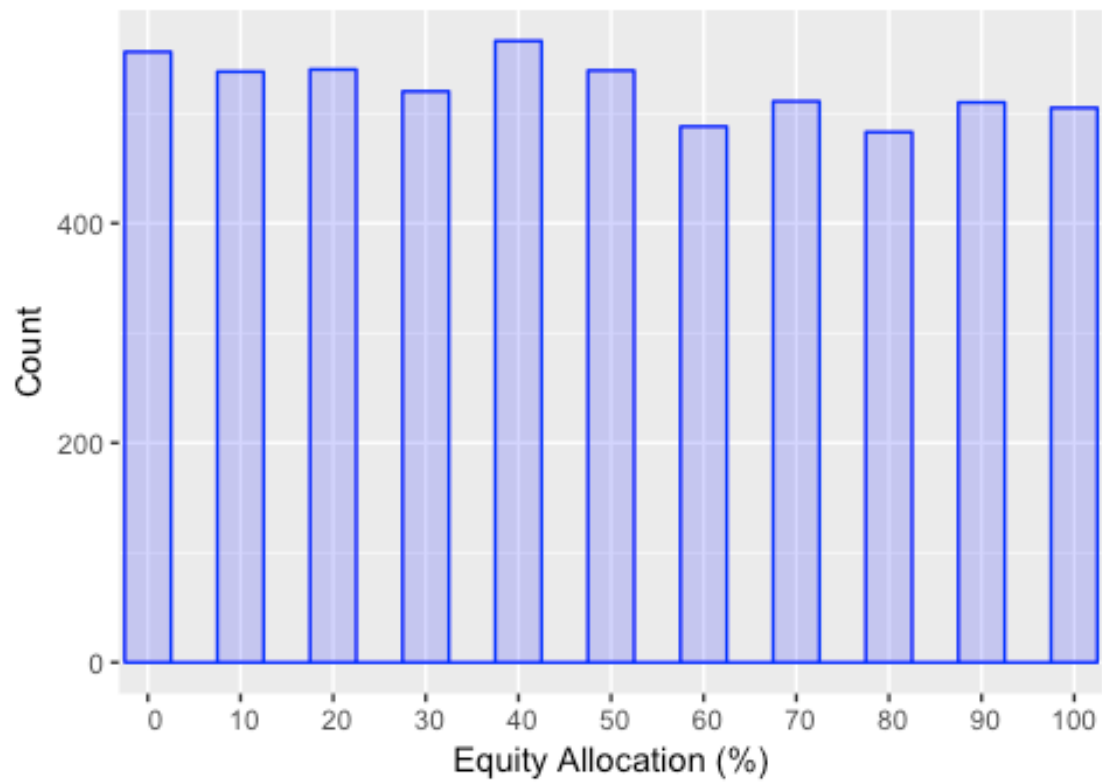
**Figure 7. Histogram of Equity Allocations for 5756 Underfunded Scenarios**



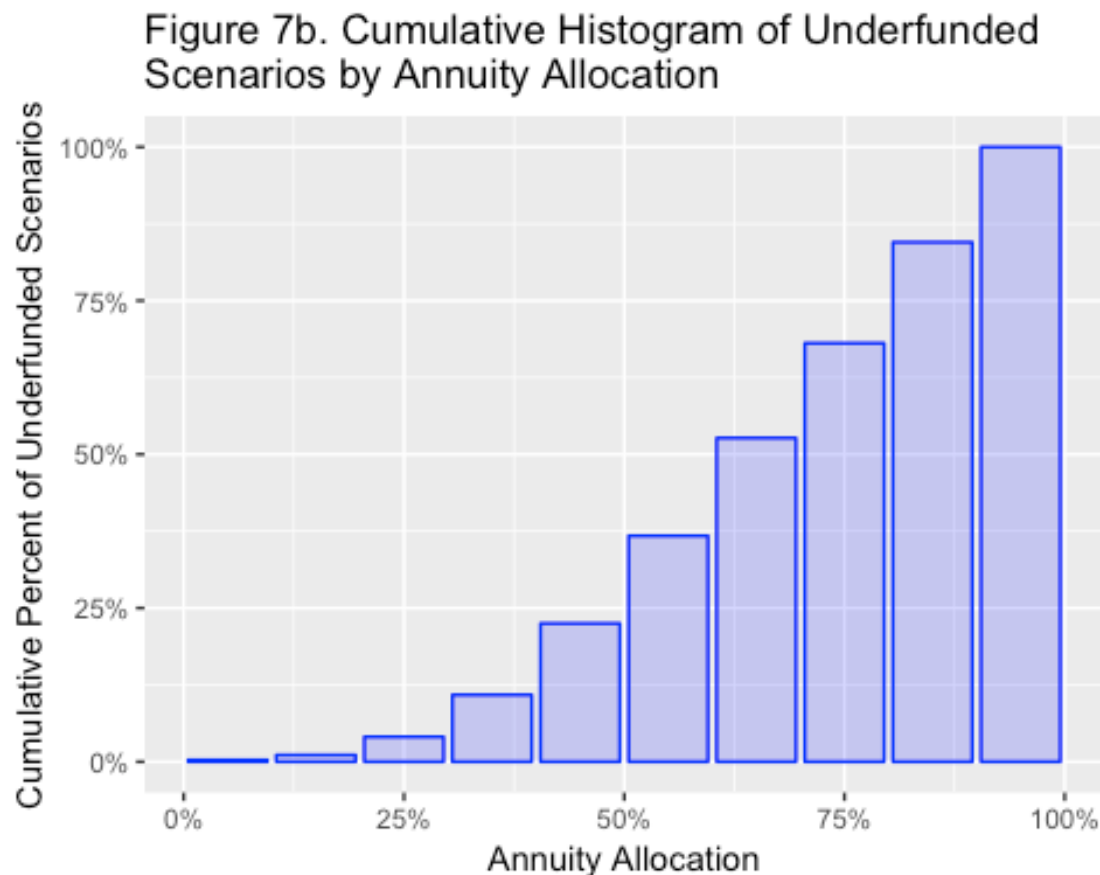
The equity allocations for failed scenarios is relatively uniform, so that doesn't seem to be the problem.

Following is a histogram showing the equity allocation for 53 underfunded *years*. If no chart appears below, then there were no underfunded years.

**Figure 7a. Histogram of Equity Allocations for 130918 Underfunded Years**



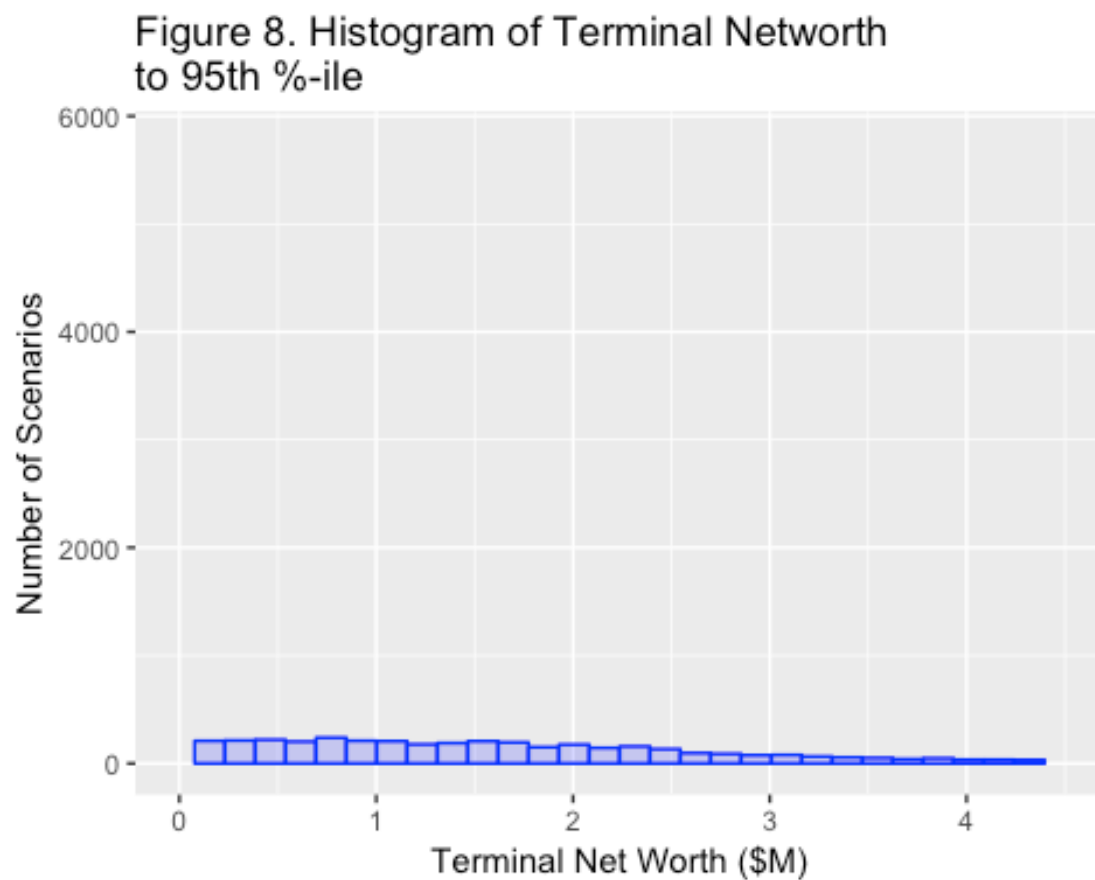
Following is a cumulative histogram of annuity allocations for the 5756 scenarios with unmet spending.



This chart shows the annuity allocations for the failed scenarios. We can decrease failed scenario percentage into single digits by restricting annuity purchases to 25% or less of the initial portfolio balance. This is the likely culprit, so our next simulation run won't allocate more than 25% to annuity purchase.

We could also fix the problem by lowering annual spending -- it's just hard to generate \$150,000 a year if you purchase too much annuity -- but we won't need to do that.

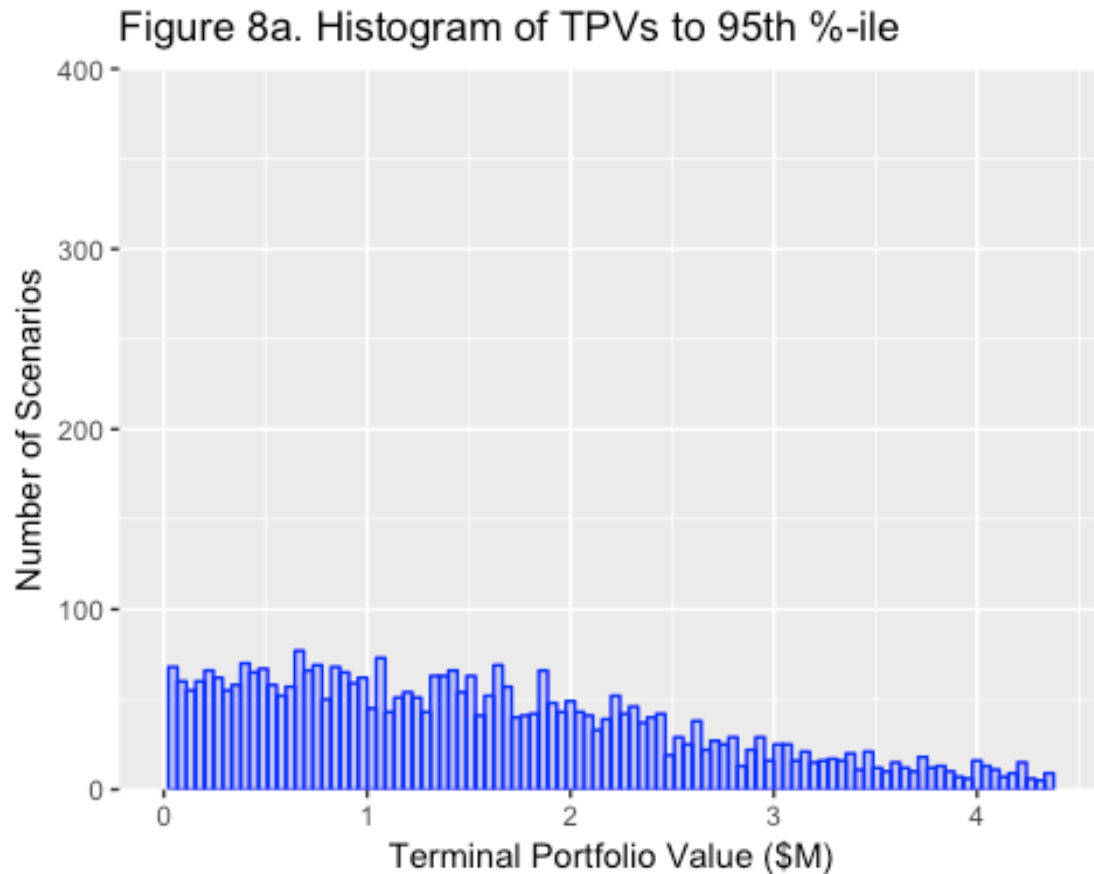
Following is a histogram of 95th percentile terminal net worth (portfolio value plus home equity at death of the second spouse). The largest 5% of terminal net worth values are excluded because they are highly unlikely and distort the graph.



- Mean Terminal Net Worth = \$ 1,059,766.
- 5% of Terminal Net Worth values fell below \$ 75,000.
- 10% of Terminal Net Worth values fell below \$ 75,000.
- 25% of Terminal Net Worth values fell below \$ 75,000.
- 50% of Terminal Net Worth values (the median) fell below \$ 75,000.

Note that the median terminal **net worth** is \$75,000. In your case, net worth = TPV plus home equity. That will improve when we limit annuities. Also notice that the median is much less than the average, telling us that the distribution of TPVs is left-skewed so the mean is not informative. The spreadsheet model you have only shows the average.

The following graph shows a histogram of terminal portfolio values (TPVs). The best 5% of TPVs are eliminated for this graph because a) they are highly improbable, b) they distort the graph and c) if they did occur, they would be excellent outcomes and not a downside risk.



- Mean Terminal Portfolio = \$ 984,766.
- 5% of Terminal Portfolio values fell below \$ 0.
- 10% of Terminal Portfolio values fell below \$ 0.
- 25% of Terminal Portfolio values fell below \$ 0.
- 50% of Terminal Portfolio values (the median) fell below \$ 0.

TPV excluding home equity. Though the mean TPV is nearly a million dollars, half the scenarios depleted the portfolio entirely.



Figure 8b.Ten Best and Worst TPV Paths

Excludes scenarios with 100% annuitization of portfolio.

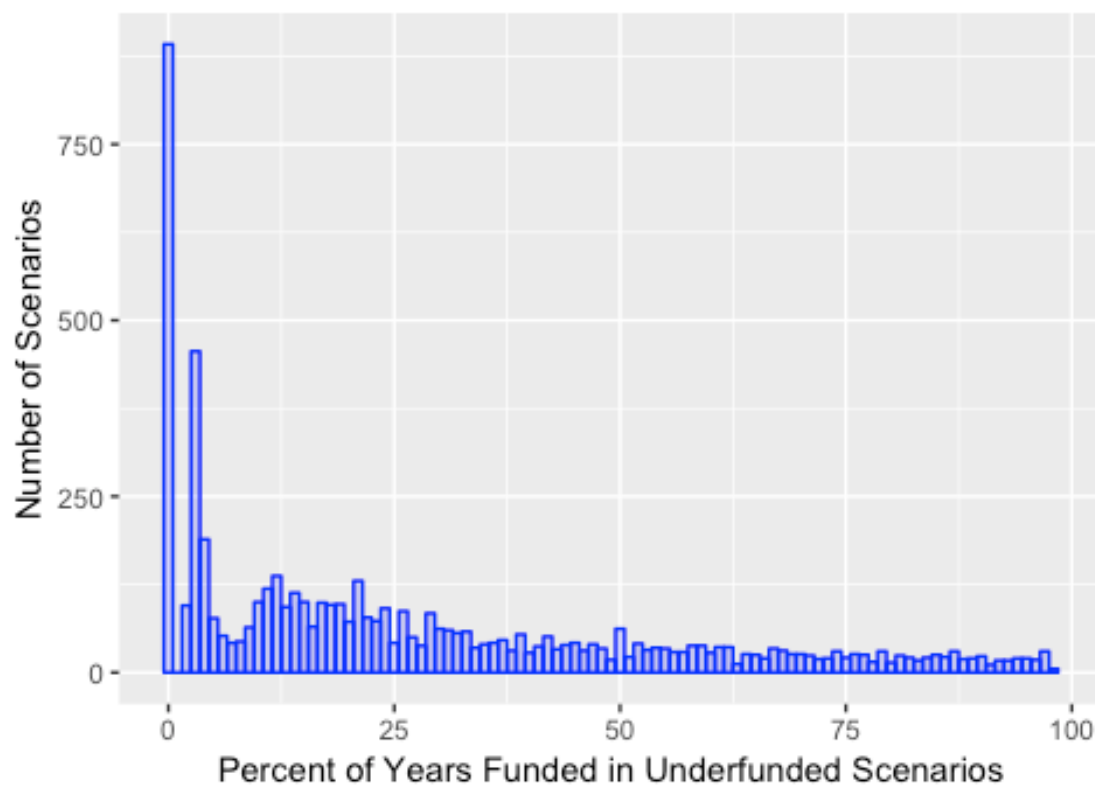


The chart above shows the ten portfolio balance paths with the highest TPV, the ten paths with the earliest portfolio depletion and the path with the median Terminal Portfolio Value.

The best paths are highly improbable and not critical in the sense that if your portfolio did take one of those paths the outcomes would be wonderful. The worst paths are also highly improbable but should be given more consideration because, while best path outcomes would be wonderful, worst path outcomes would destroy the retirement plan. A good plan will eliminate these worst-case outcomes.

Though this graph shows 21 paths, the simulation actually generated 10000 hypothetical paths. Also note that the graph lines end when the last spouse dies.

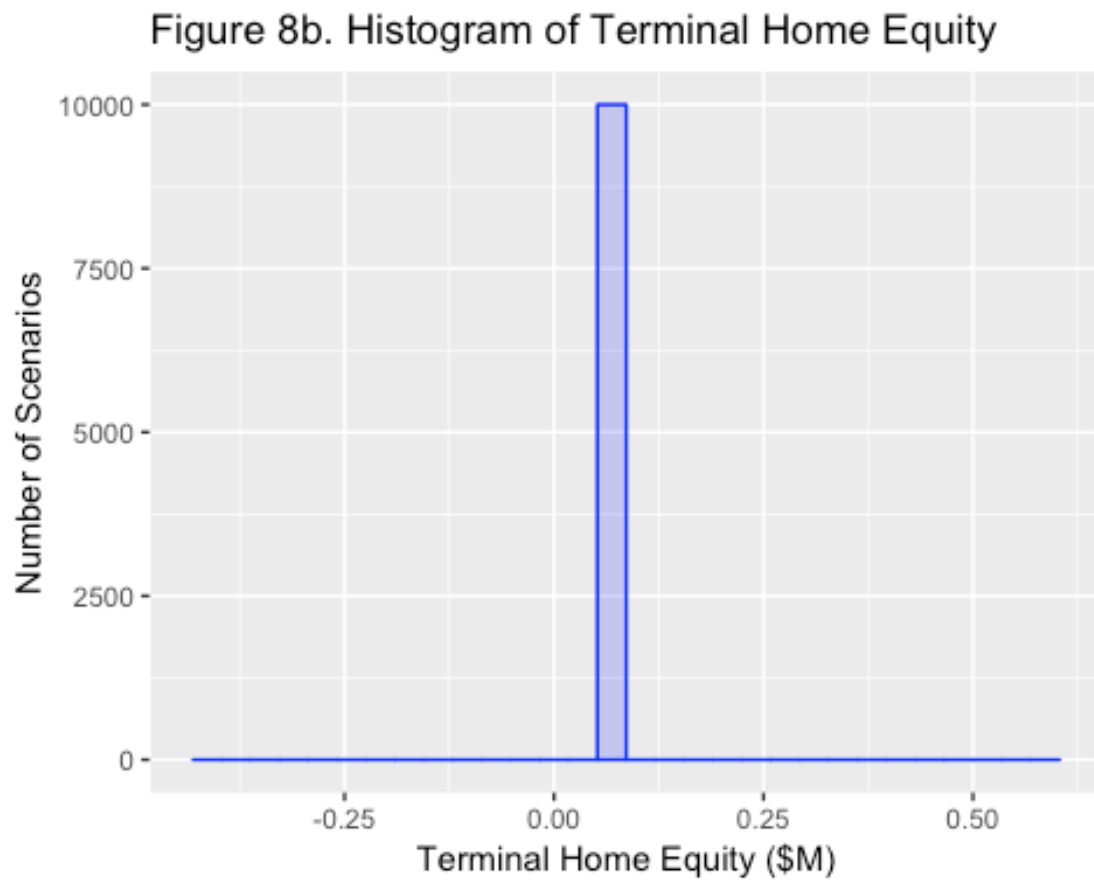
Figure 9. Histogram of Percent of Years Funded for Underfunded Scenarios



The graph above shows the 53 years of retirement for which spending demand (consumption) was not met. If an empty chart appears above, then there were no underfunded years.

The rightmost columns represent scenarios that were *mostly* funded. Leftmost columns represent scenarios that were *least* funded.

The following chart shows terminal home equity for all scenarios. If a HECM reverse mortgage is available, home equity can become negative but this is non-recourse debt and will not lower the household's terminal net worth.



Home appreciation rate is a real 0% per annum long term and you don't spend home equity.

The following chart shows safe (longevity-hedged) income (the "floor") by year of retirement. The dark line shows the mean safe floor income for all scenarios for each year of retirement, while the shaded area shows the range from the smallest floor income for any scenario for that year of retirement to the largest floor income for any scenario for that year of retirement. The black line shows mean annual expenses across all scenarios.

The floor does not include income from TIPS bond ladders because longevity risk is not hedged (you can outlive a ladder). HECM income is not included in the floor because it is contingent upon the continued ownership of your home and not on your lifetime.

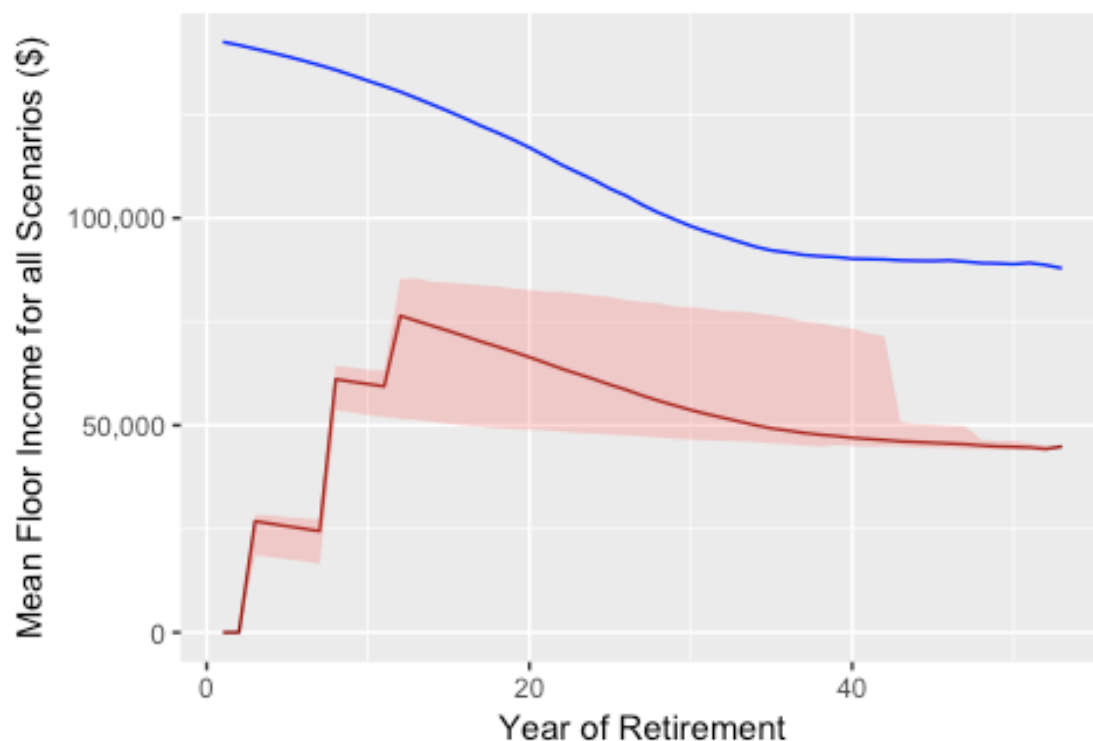
Note that annuities and pensions that are not inflation-protected will lose value to inflation each year, as simulation results are all calculated in real dollars.

Also note that for retired *couples*, the upper bound of floor income may decline sharply at the end of retirement. The upper bound of the pink area equals the maximum floor value of all scenarios at a given age. At some time, those scenarios will no longer include the probability that both spouses survive. When only one spouse survives, Social Security benefits may decline significantly and the maximum benefit for all scenarios at that age will also decline.

For example, assume that in year 42, there are many scenarios in which the wife survives but only two scenarios in which the husband survives. Nonetheless, the maximum floor value for year 42 would be determined by the presence of those two scenarios. Assume further that in year 43, there are no simulated scenarios in which both spouses survive. The maximum floor value would then be determined only by the wife's benefit.

Figure 13. Safe Floor Income vs. Mean Expenses

Mean and Range: Pensions, Annuities, Social Security (red)  
Mean Expenses (blue)



The blue line shows mean annual spending. It declines over time due to inflation and due to the increasing probability that only one spouse has survived.

The dark red line shows mean safe floor income over time. It declines for the same two reasons. The pink ribbon shows maximum and minimum range of safe floor income across, say, all 10,000 scenarios simulated at age 40.

A TIPS ladder is not simulated but even if it were, it wouldn't be included in the floor. The floor should hedge longevity risk and you can outlive a ladder.

For example, at the 40<sup>th</sup> year of retirement, expenses across 10,000 scenarios averaged \$90,199. Floor income averaged \$46,963, with the best scenario having a floor of \$73,417 and the worst \$44,965.

The last two charts of this report provide details for a single randomly-selected scenario, scenario number

The following chart shows cash flow for scenario number 966. The black line shows consumption demanded. The colored areas indicate the source for funding that consumption.

Underfunded years will show that funding sources did not achieve demanded consumption -- the colored bars will not reach the black dots. When the colored bars exceed the black dots expenses for that year were exceeded by the safe floor income even without portfolio spending. The excess above the black dot is transferred to the savings portfolio.

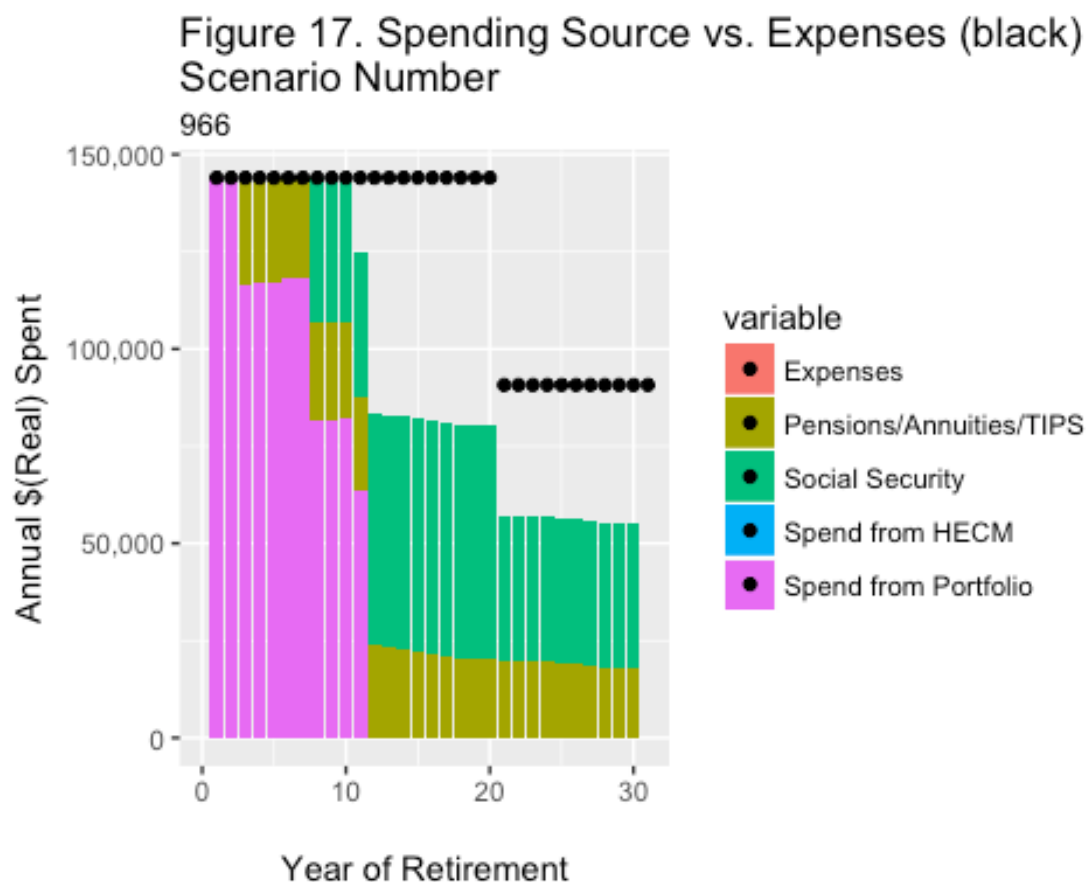
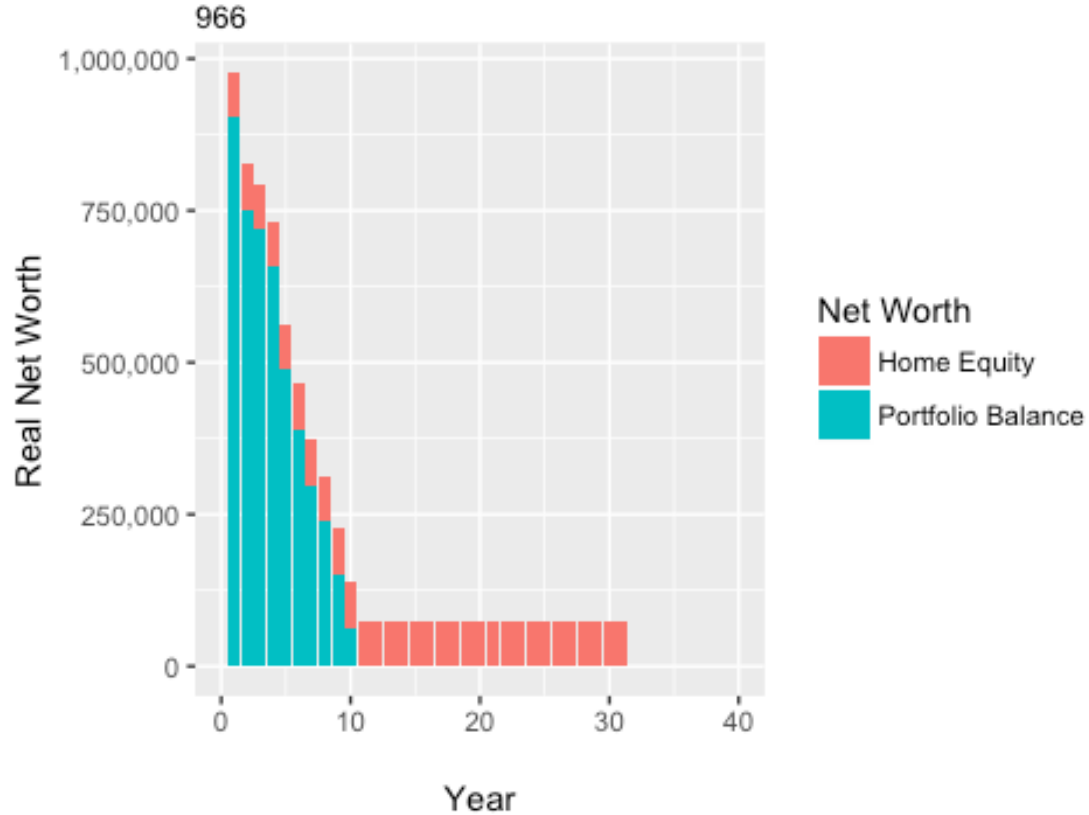


Figure 18. Net Worth Scenario Number



Scenario 966 was randomly selected as a demonstration. This is a failed scenario, but then more than half were.

**Bottom line: we need a new simulation run restricting annuity purchases to 25% of your savings and we should expect a single-digit failure rate.**