Designing Push Plans for Disk-Shaped Robots

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TU/e Technische Universiteit Eindhoven University of Technology

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- Context & problem description
- Our new algorithm
- New contributions & future research

Path planning

A fundamental problem in robotics.

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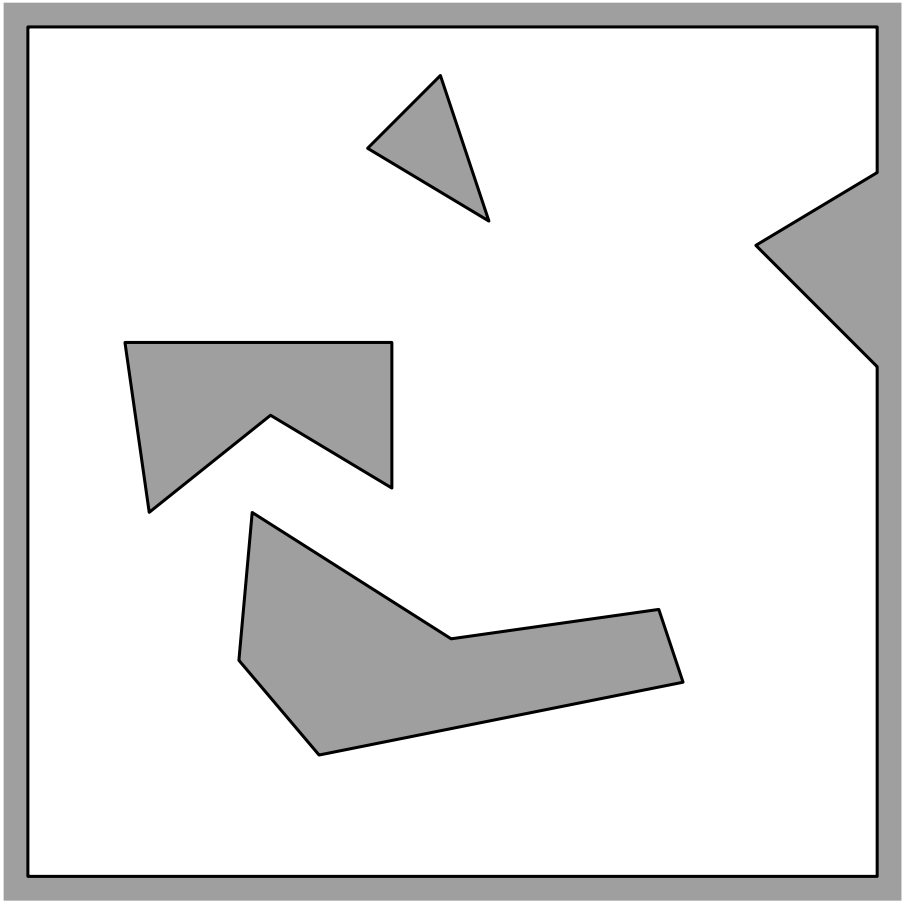

Given:

- A collection of obstacles.

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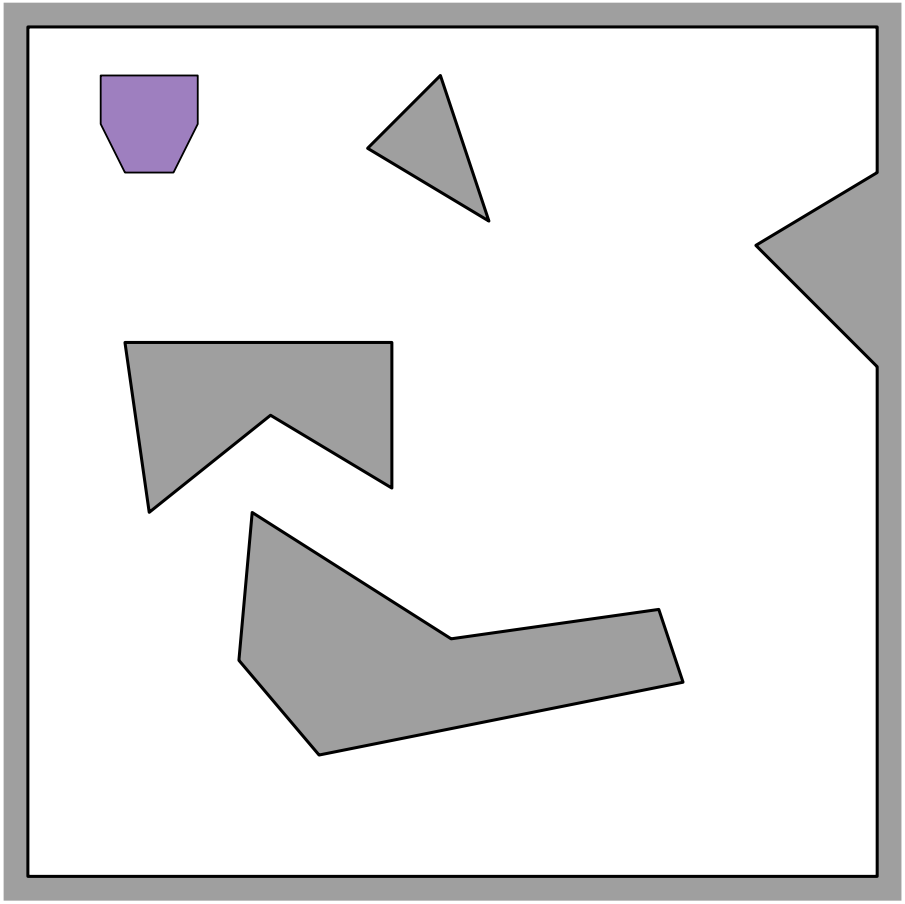

Given:

- A collection of obstacles.
- A robot

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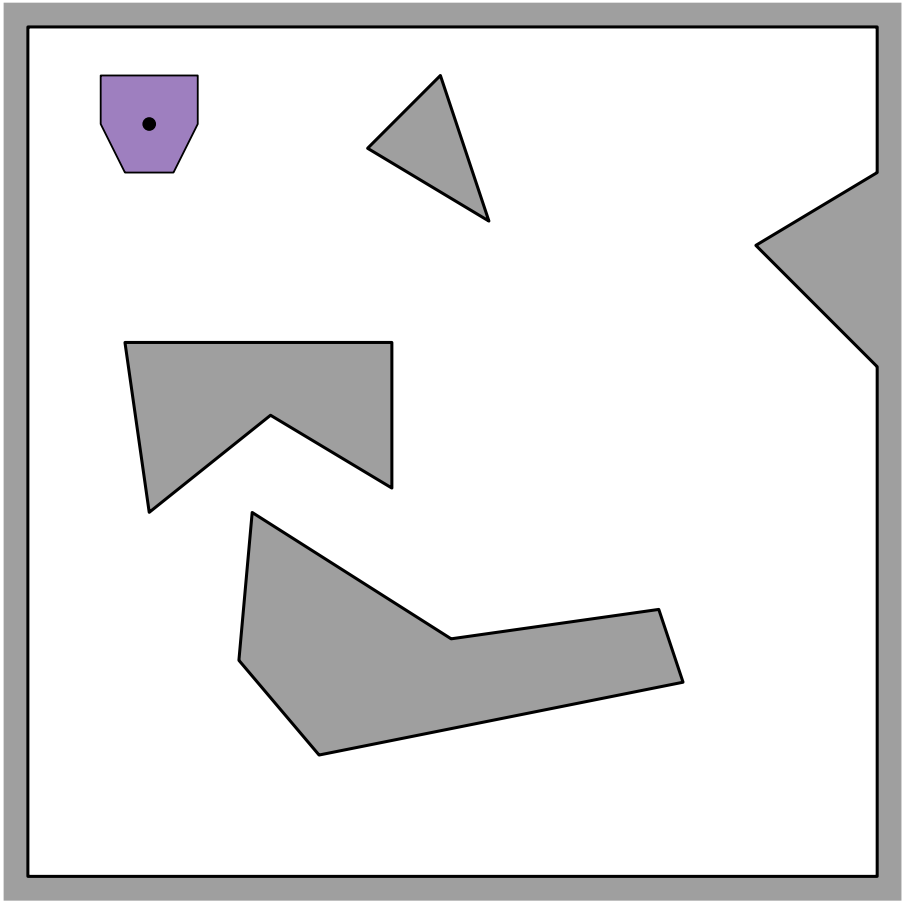

Given:

- A collection of obstacles.
- A robot at initial configuration.

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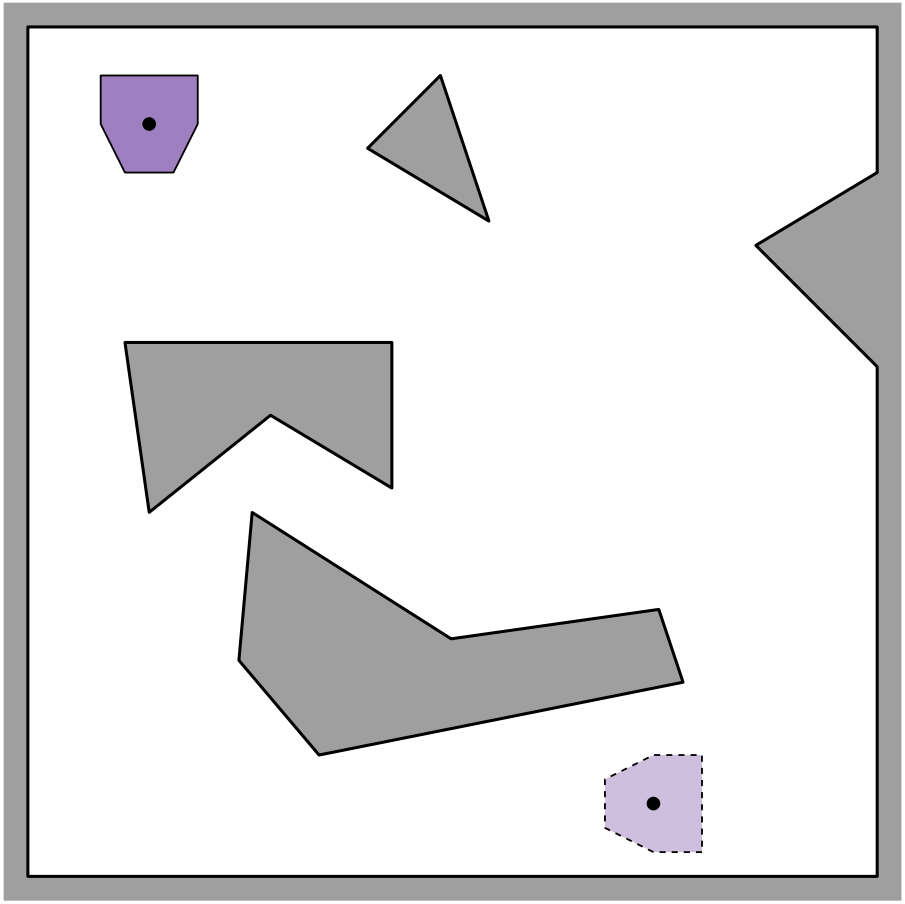

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- A destination configuration.

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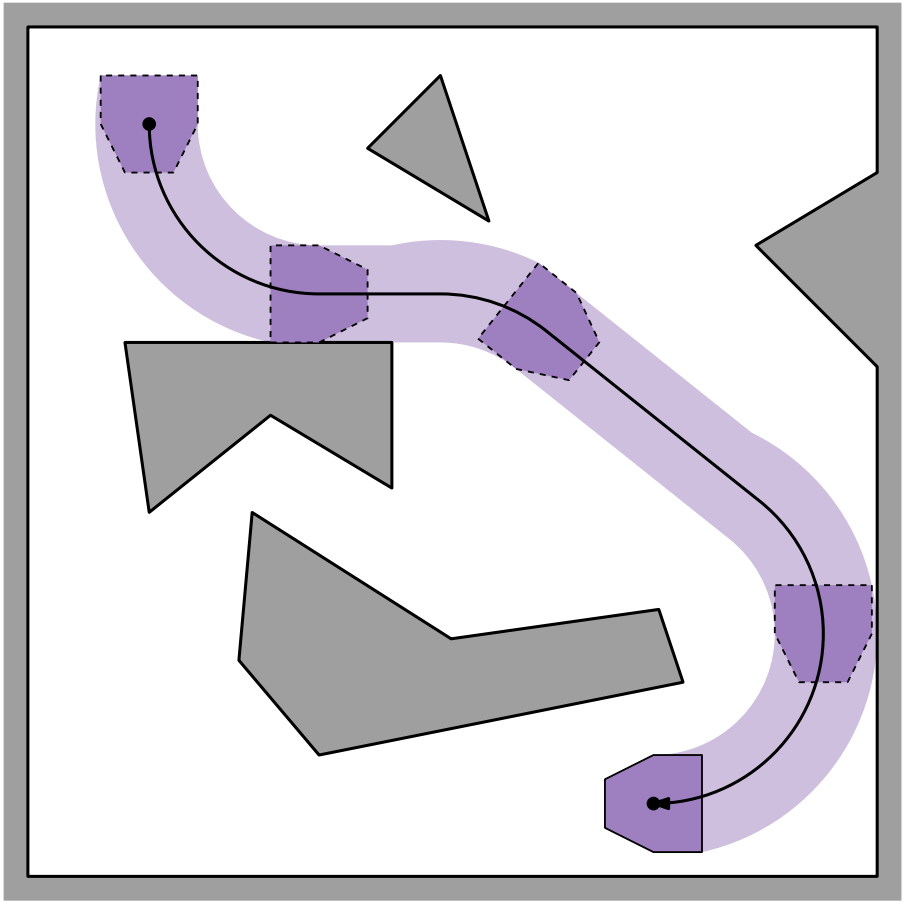

Given:

- A collection of obstacles.
- A robot at initial configuration.
- A destination configuration.

Find:

- A collision-free path from the initial configuration to the destination configuration. (Or report that none exists.)

Manipulation path planning

Now it's not the (active) robot that needs to reach a destination, but a (passive) object.

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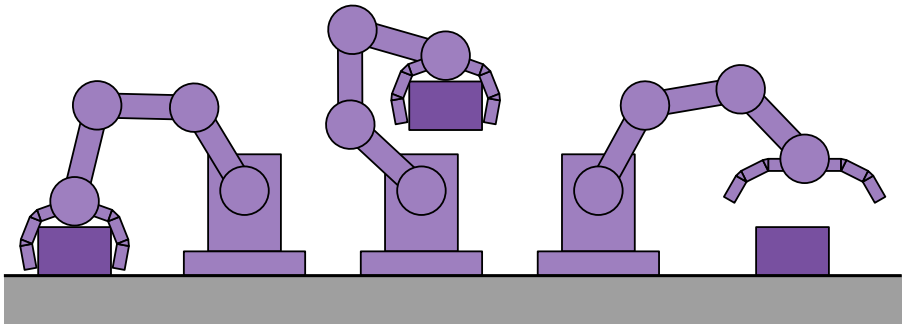

- Grasping

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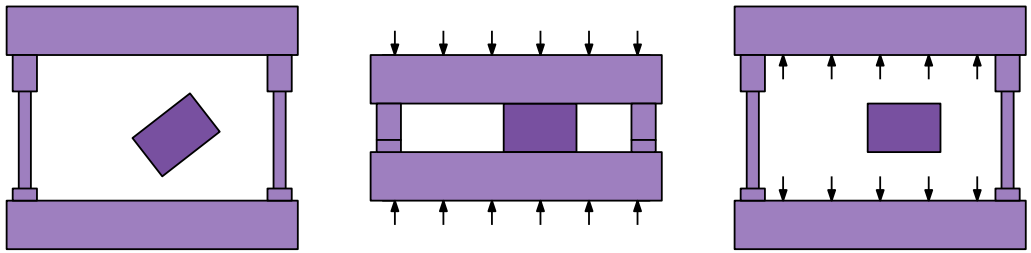
Now it's not the (active) robot that needs to reach a destination, but a (passive) object. Many forms of manipulation have been studied:

- Grasping
- Squeezing



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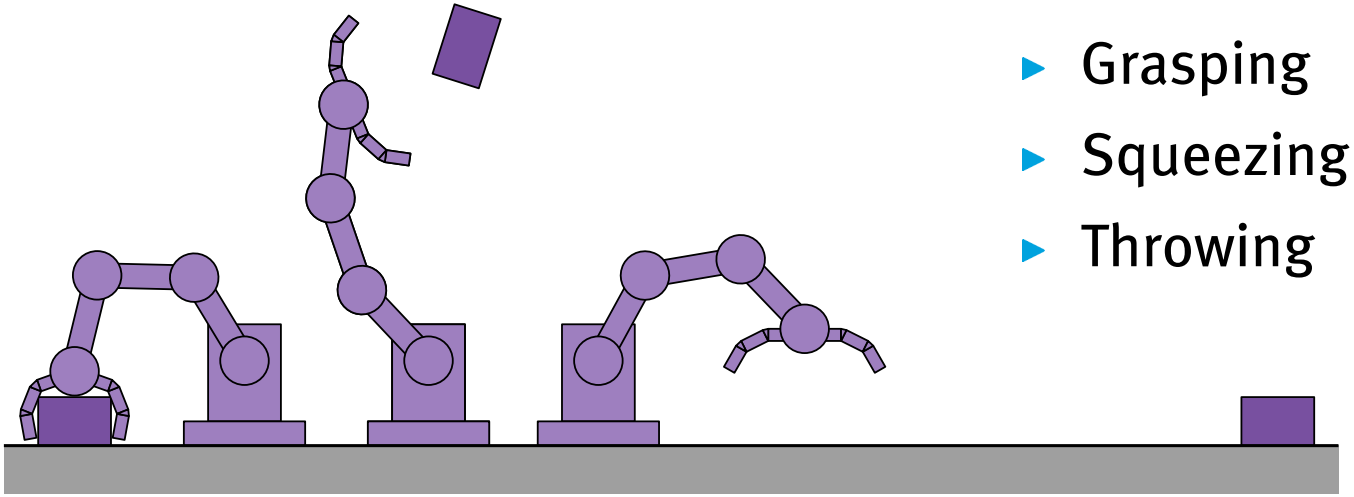

- Grasping
- Squeezing
- Throwing

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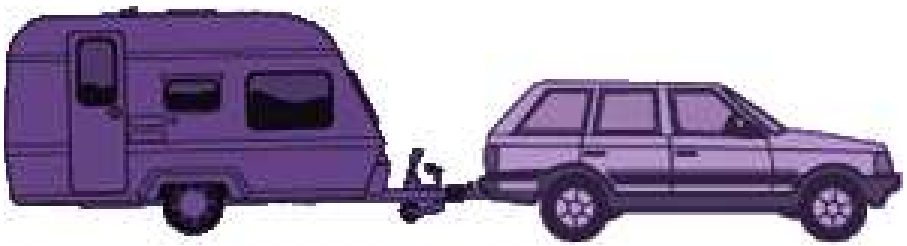

- Grasping
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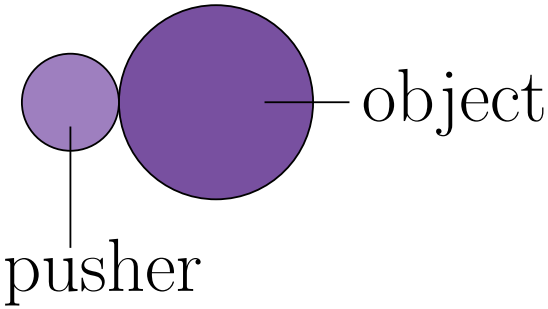
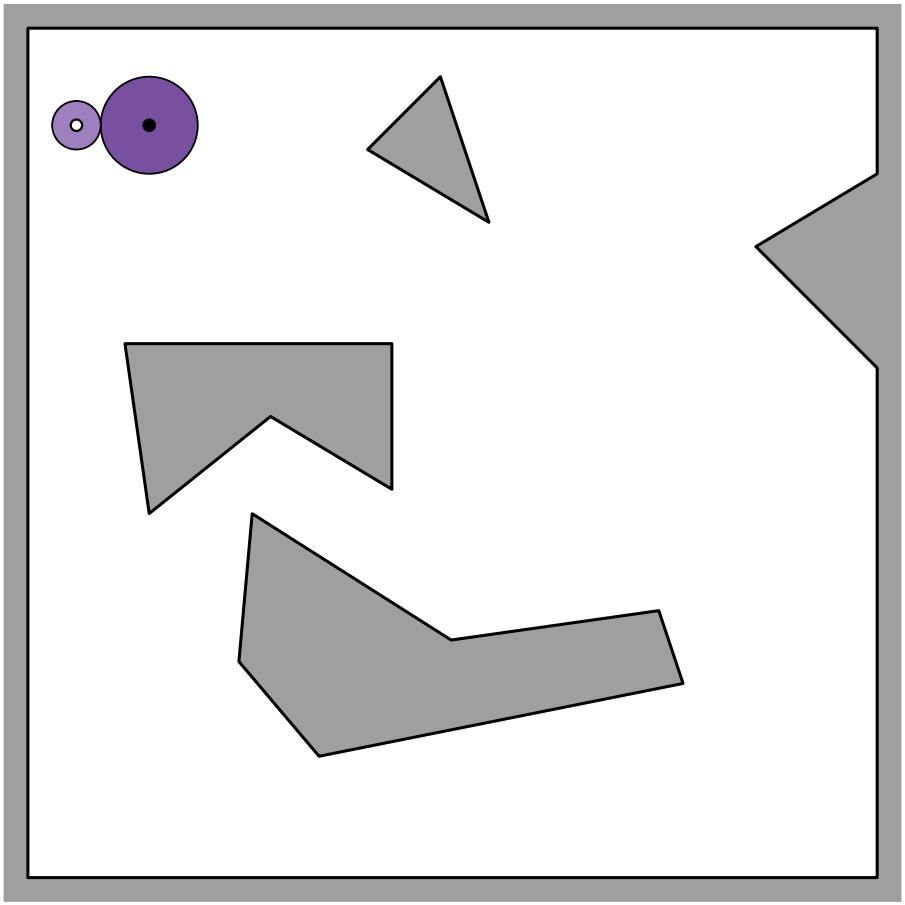
The global problem



Given:

- Two circular disks in the plane: a smaller pusher and a larger object.

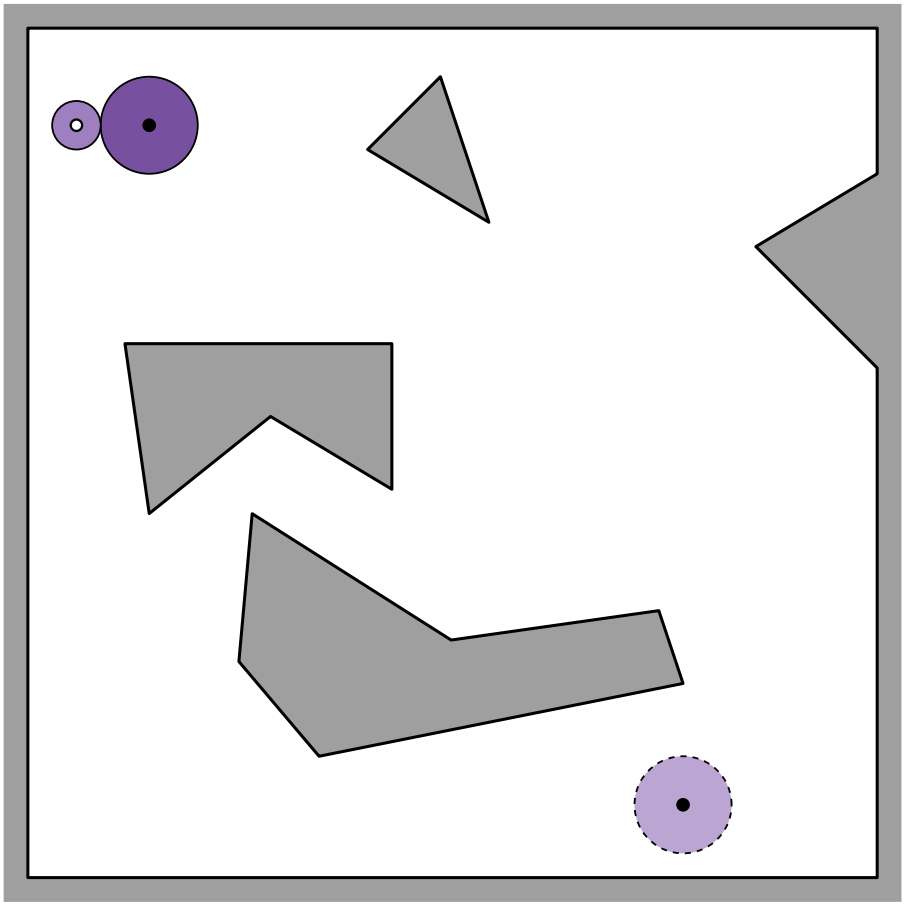
The global problem



Given:

- Two circular disks in the plane: a smaller pusher and a larger object.
- A collection of line-segment obstacles.

The global problem



Given:

- Two circular disks in the plane: a smaller pusher and a larger object.
- A collection of line-segment obstacles.
- A destination position for the object.

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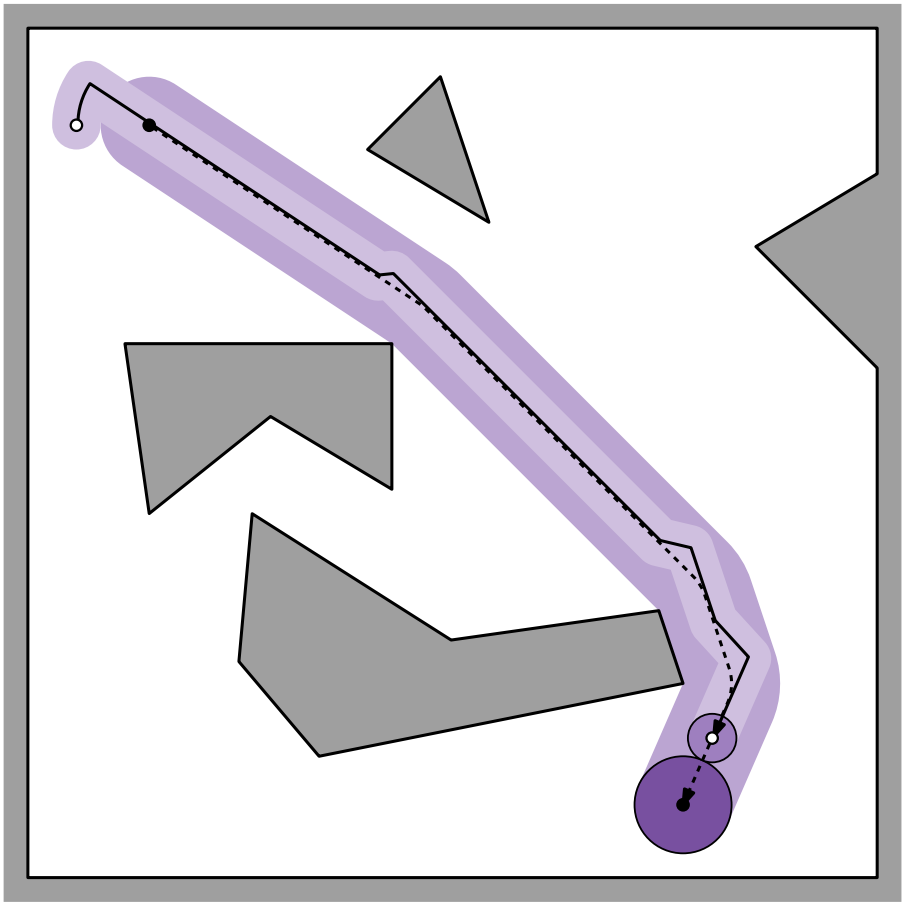


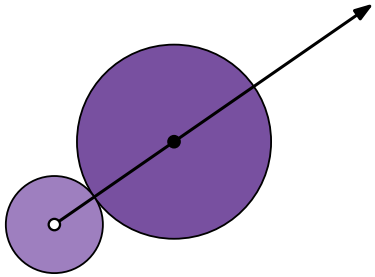
Given:

- Two circular disks in the plane: a smaller pusher and a larger object.
- A collection of line-segment obstacles.
- A destination position for the object.

Find:

- A path for the pusher to follow (a push plan) that makes it push the object to its destination.
  (Or report that none exists.)

Compliance & path sections (1/2)



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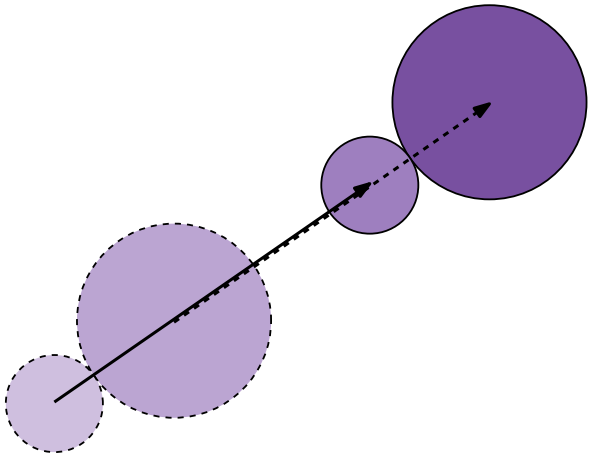

straight line

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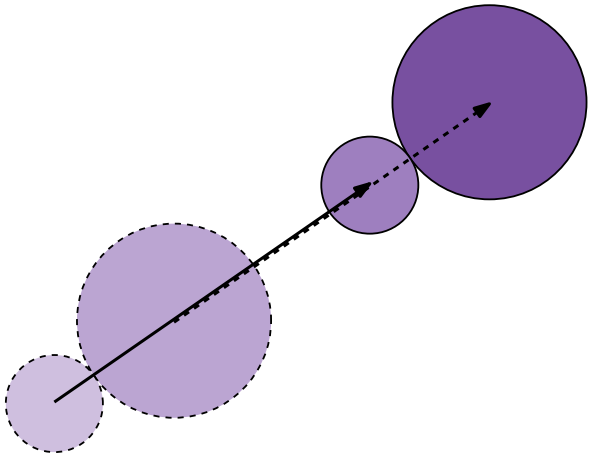

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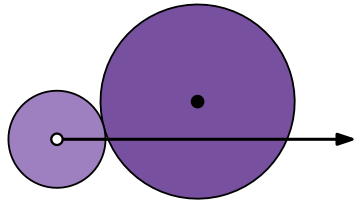

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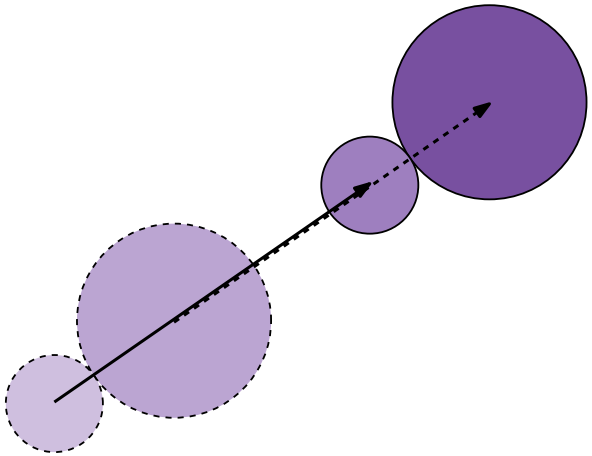

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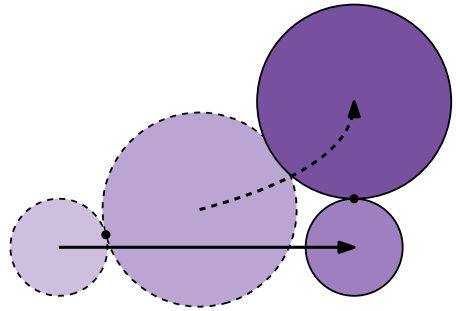

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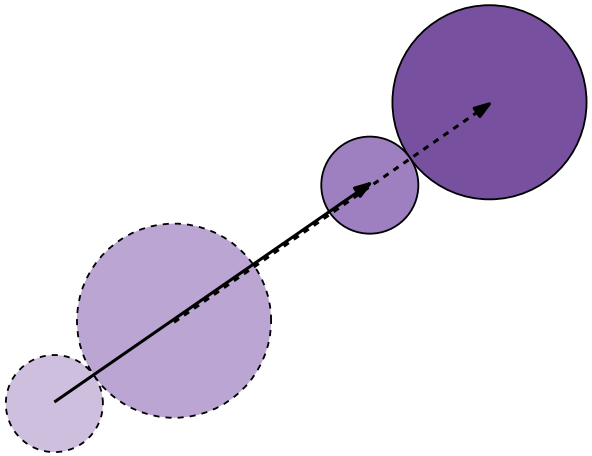

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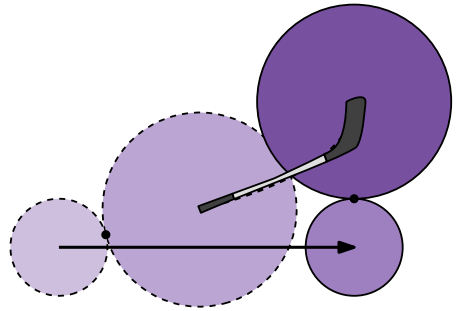

hockey stick

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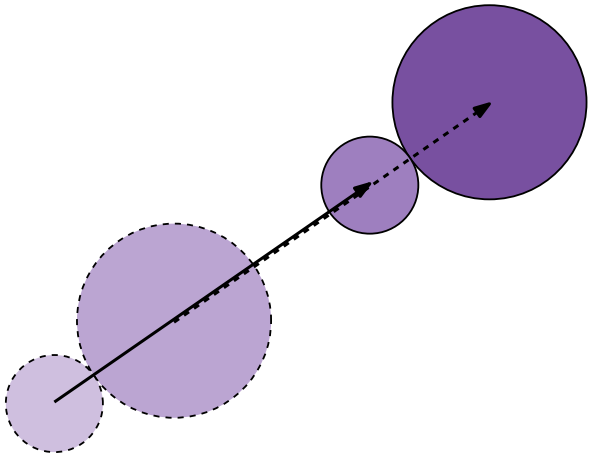

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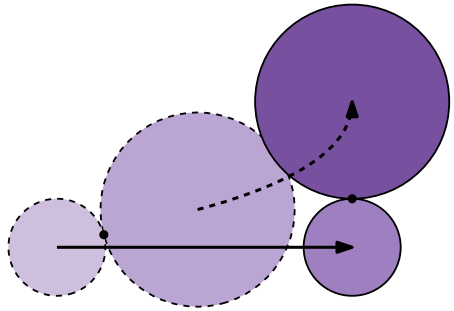

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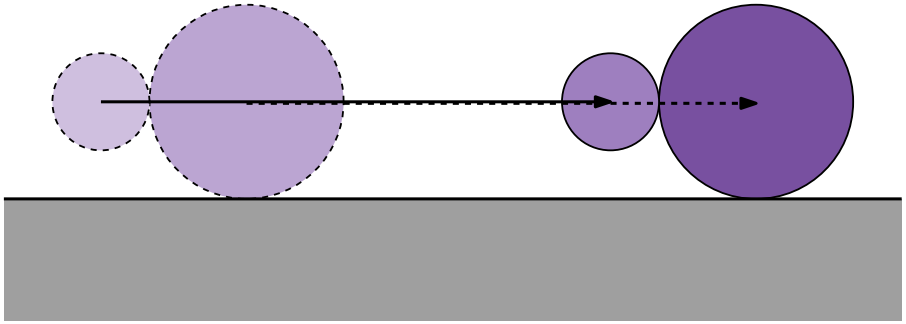

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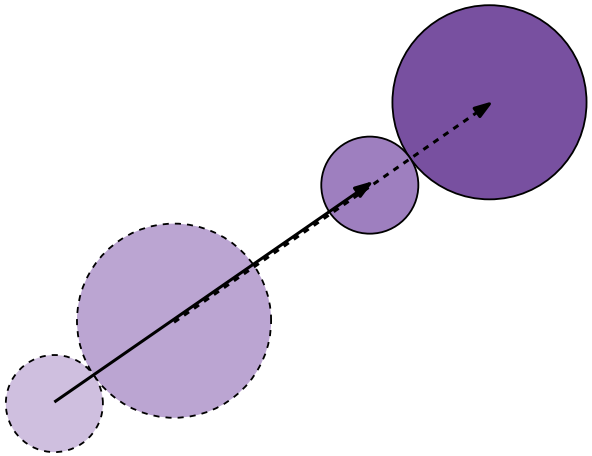

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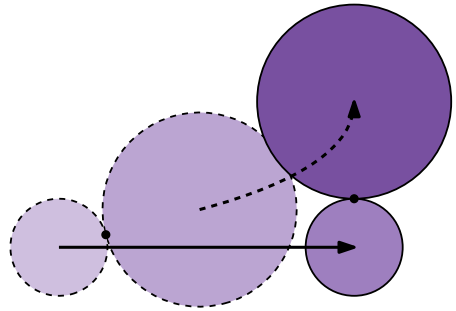

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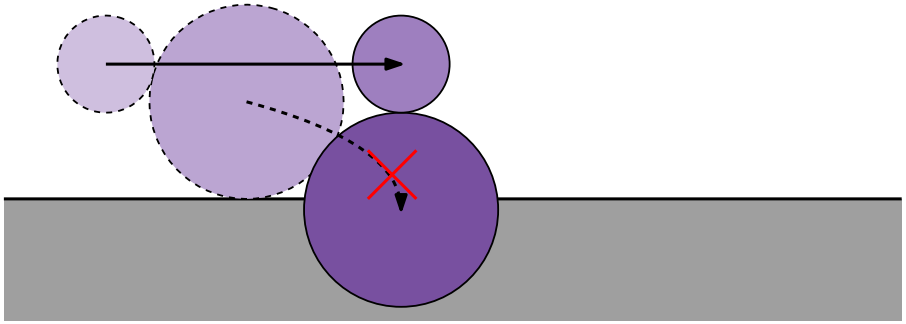

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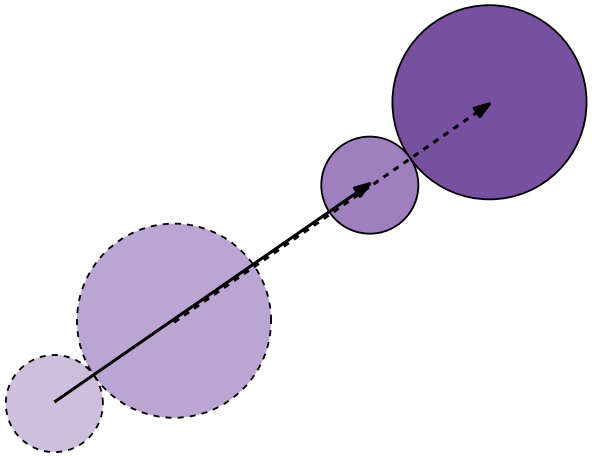

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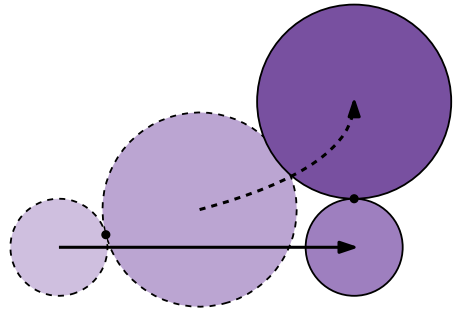

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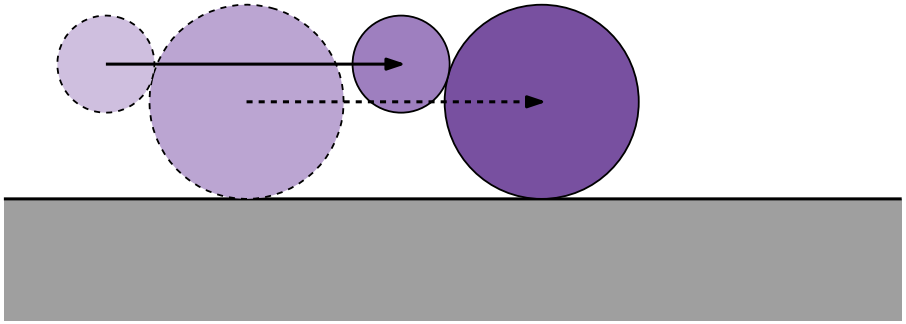

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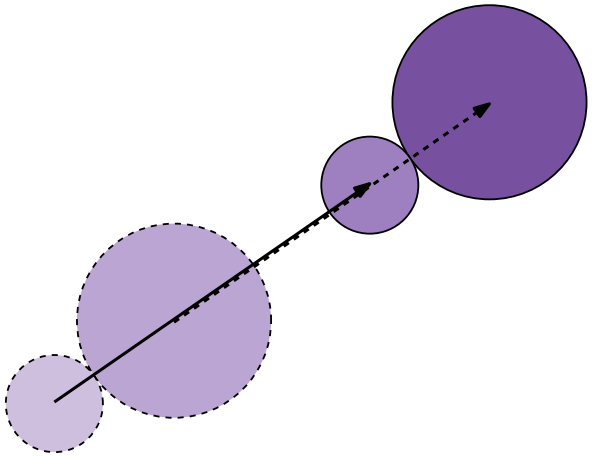

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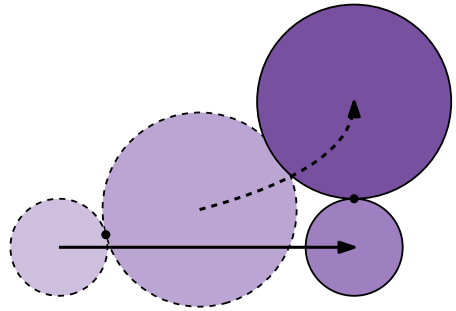

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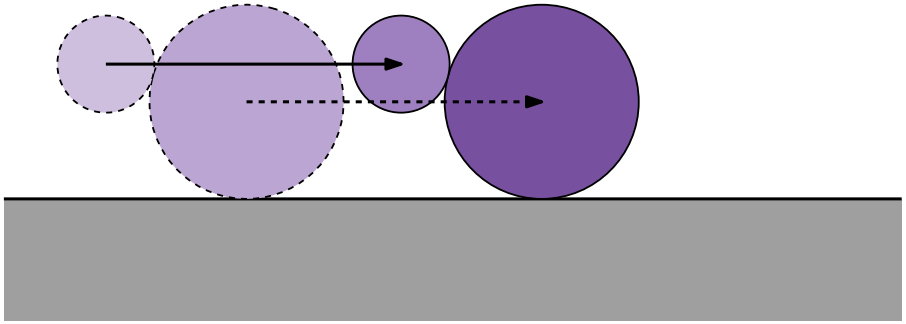

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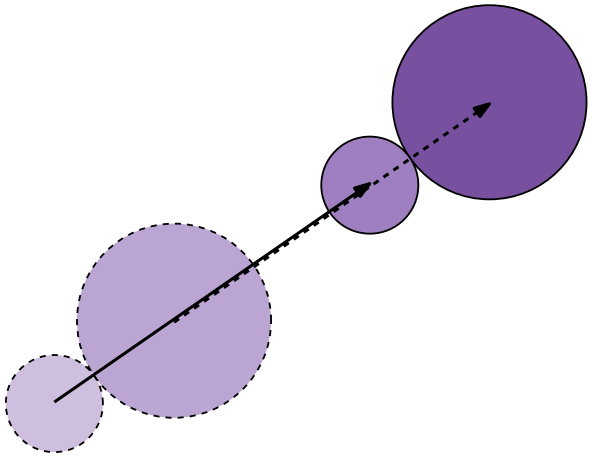

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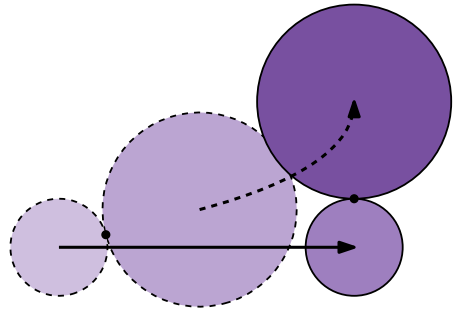

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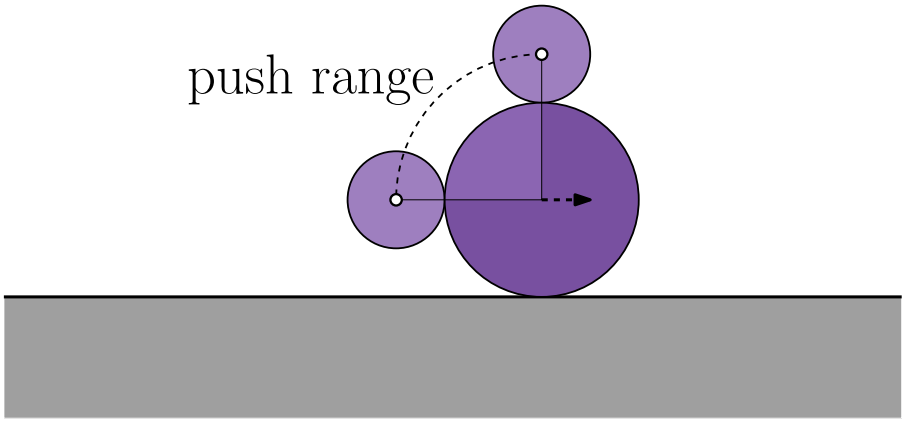

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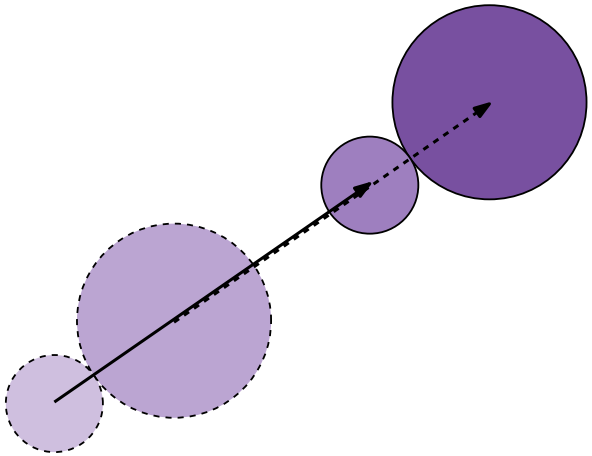

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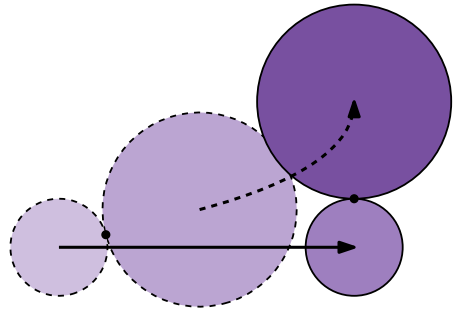

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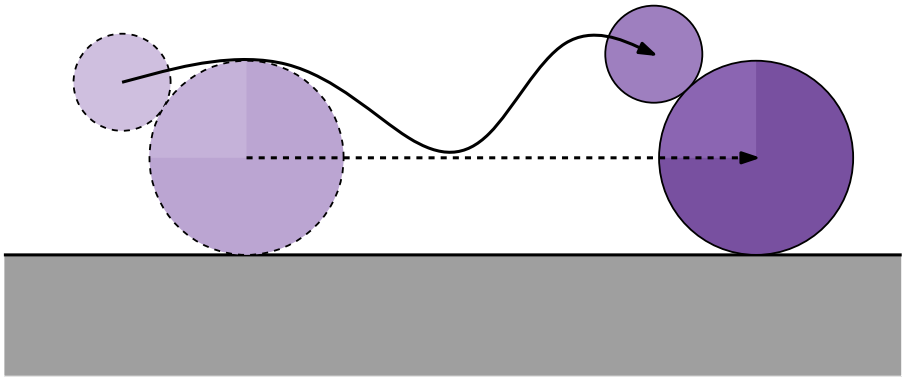

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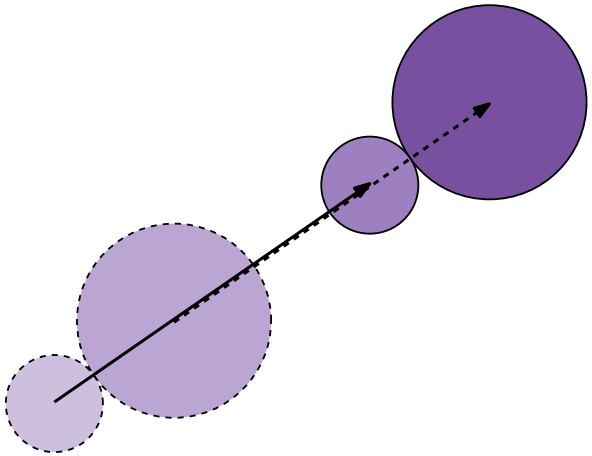

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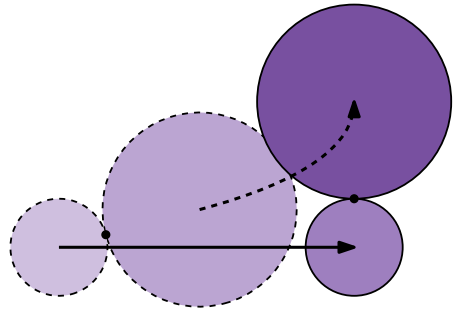

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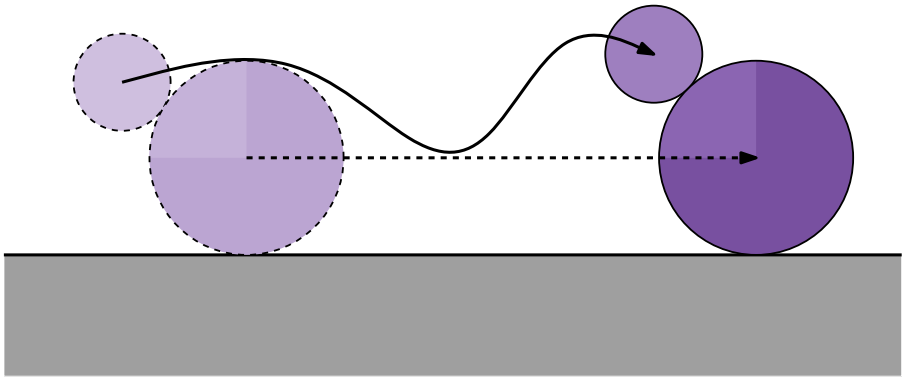

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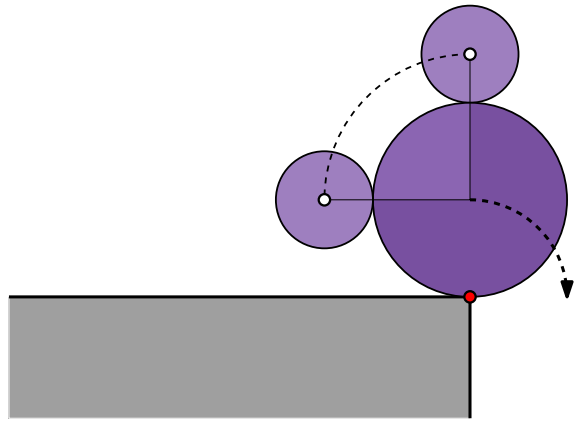
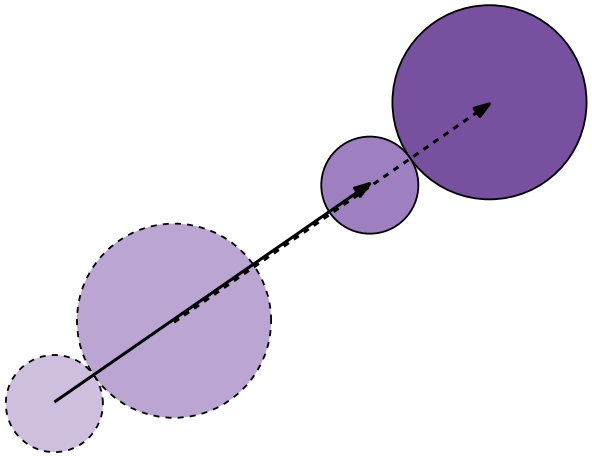


circular compliant

Compliance & path sections (1/2)



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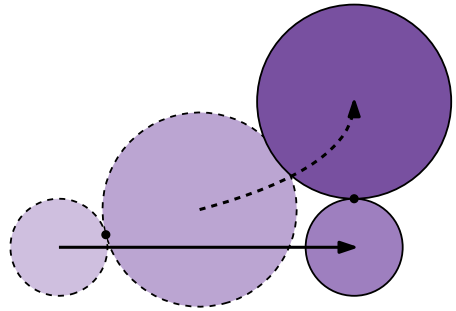

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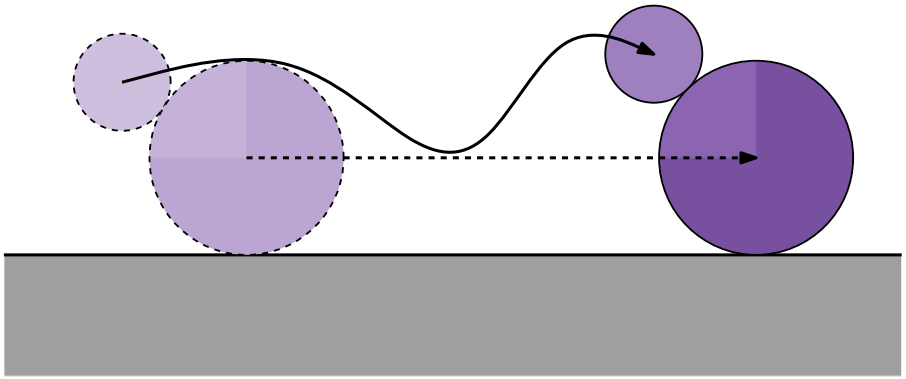

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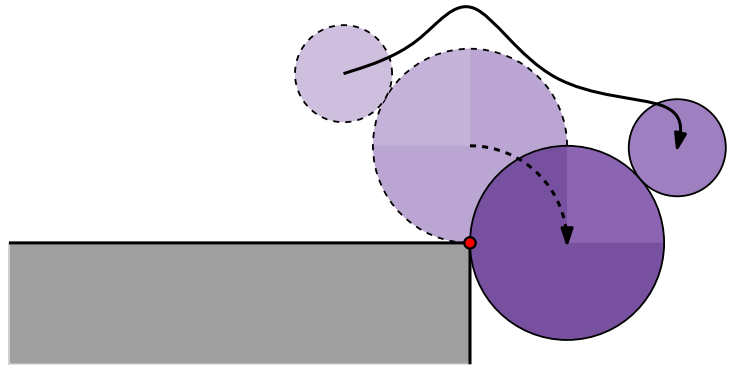

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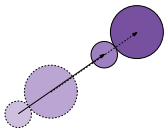
Compliance & path sections (2/2)



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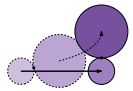
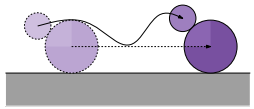
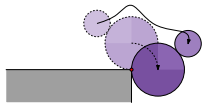


hockey-stick non-compliant



straight-line compliant

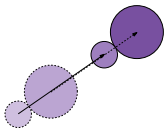


circular compliant

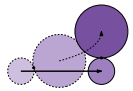
path sections

(object moves along curve, pusher stays within associated push range)

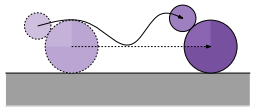
Compliance & path sections (2/2)



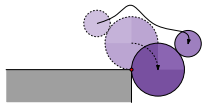
straight-line non-compliant



hockey-stick non-compliant



straight-line compliant



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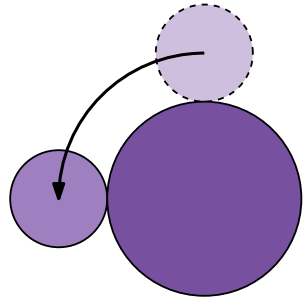

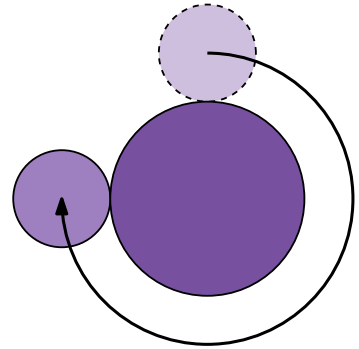

contact transits

(object is stationary, pusher turns around it)

Nieuwenhuisen's algorithm

Dennis Nieuwenhuisen designed an algorithm for this problem based on the Rapidly-exploring Random Trees path-planning algorithm.

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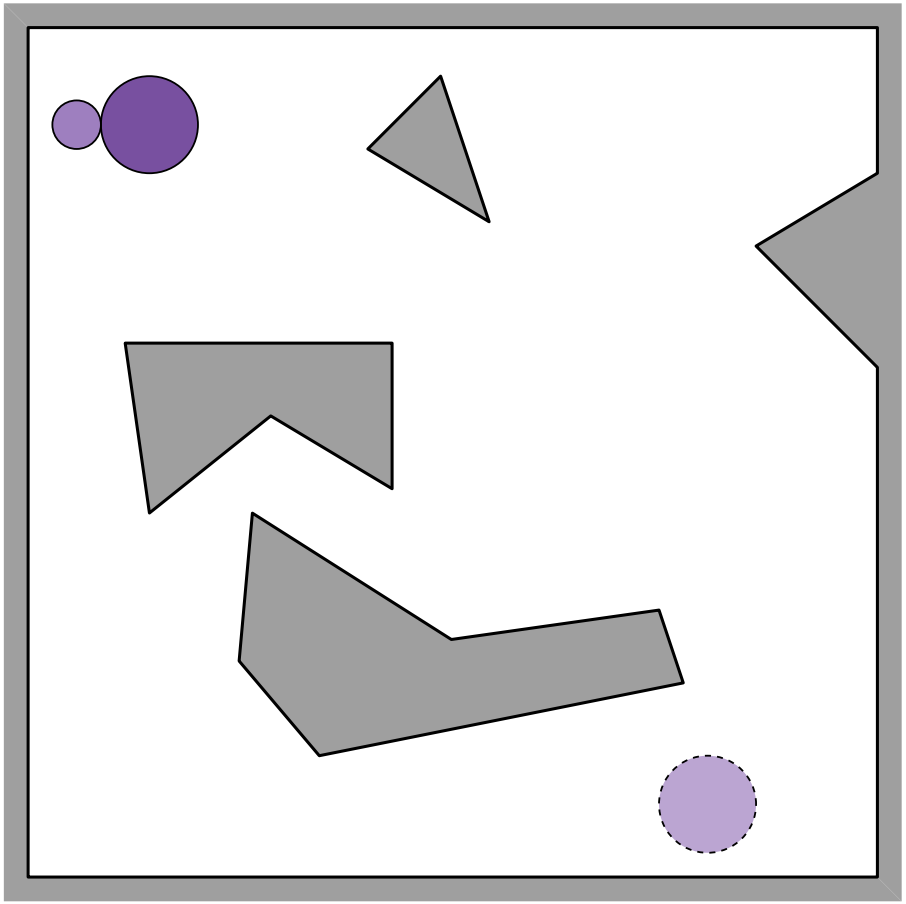

Create a tree with the initial configuration as the root:

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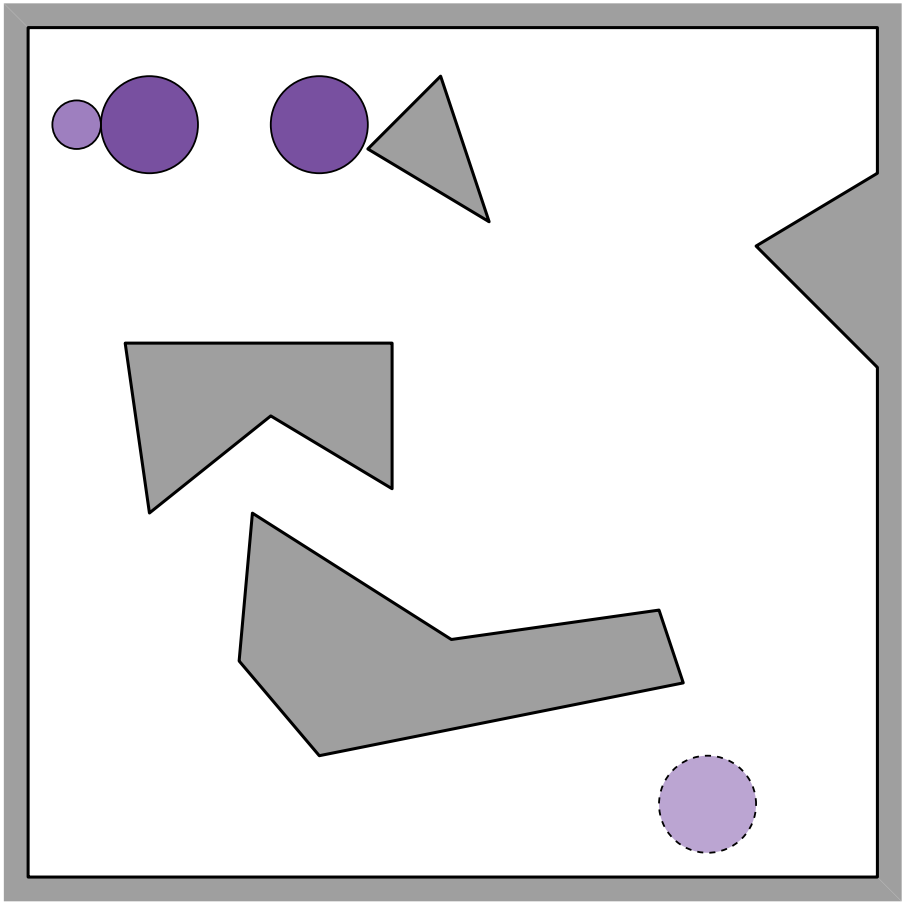

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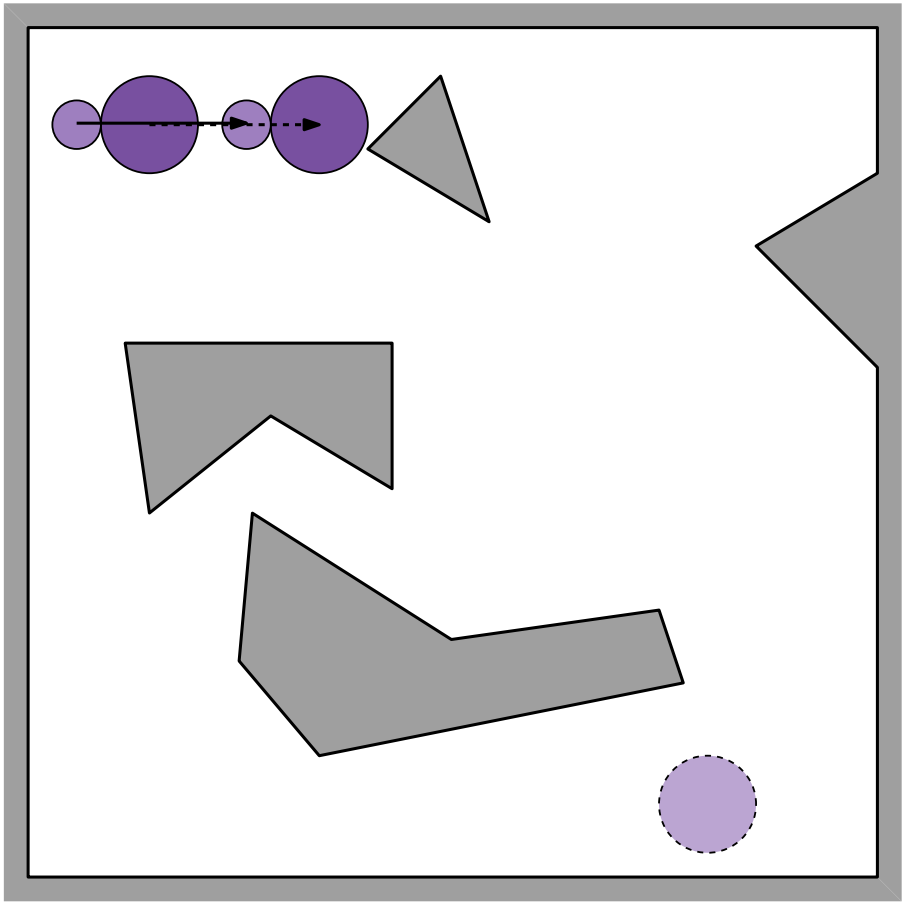

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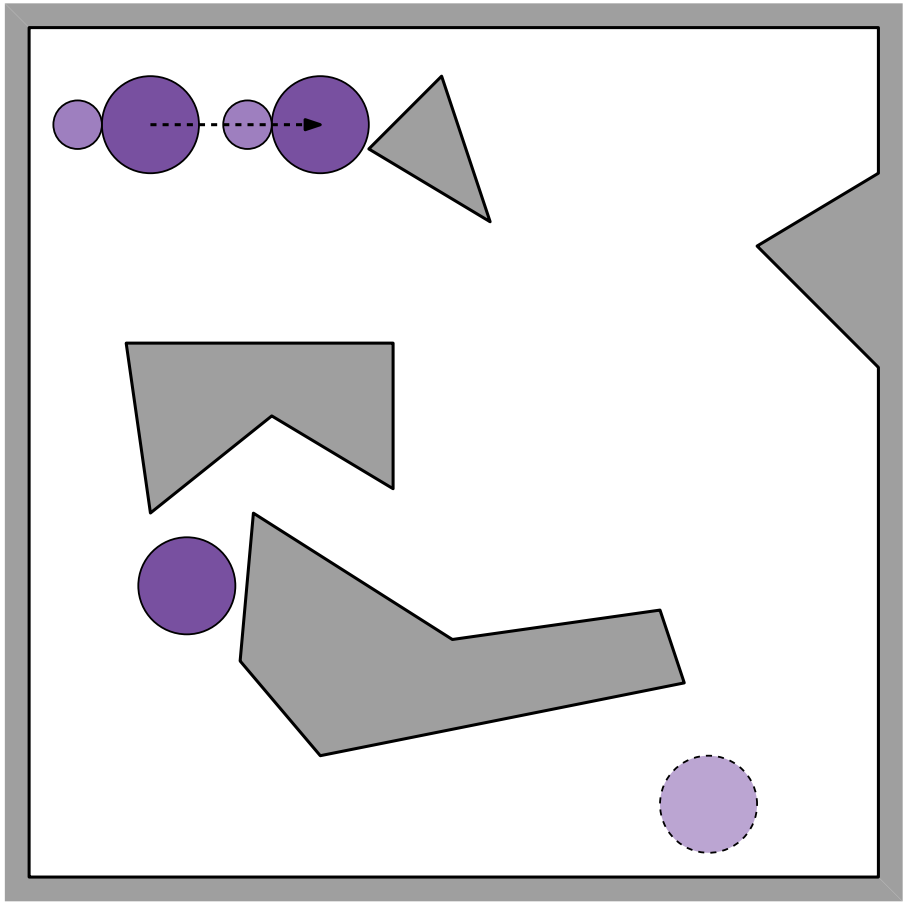

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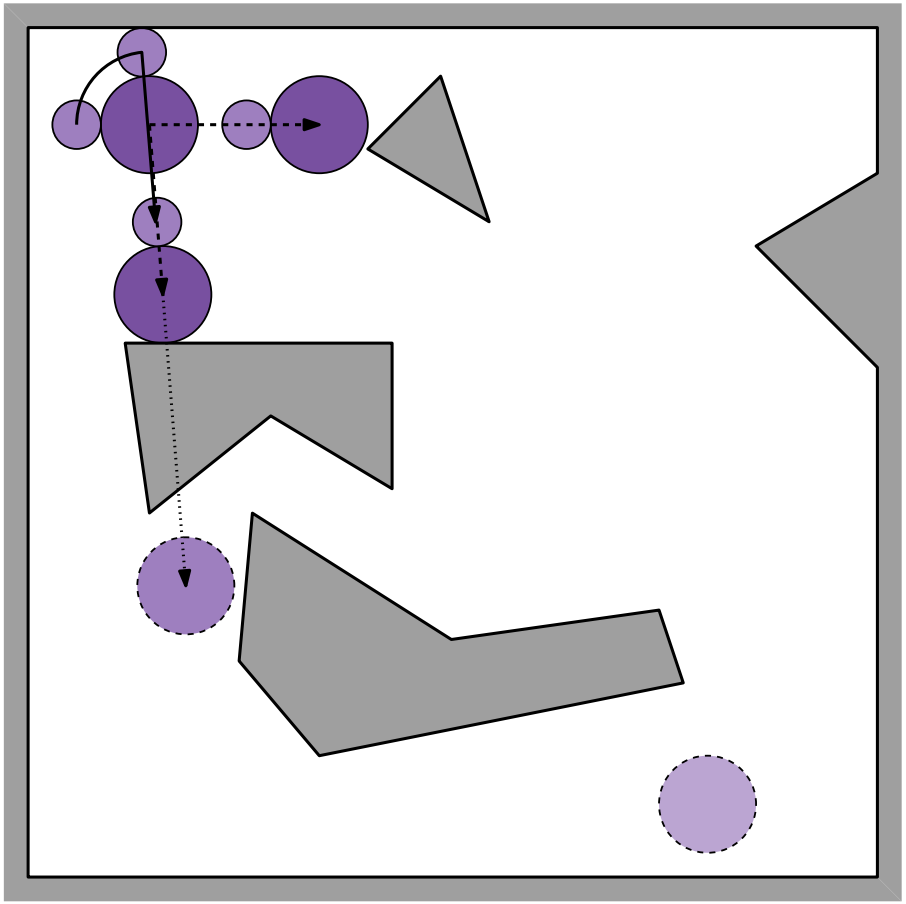

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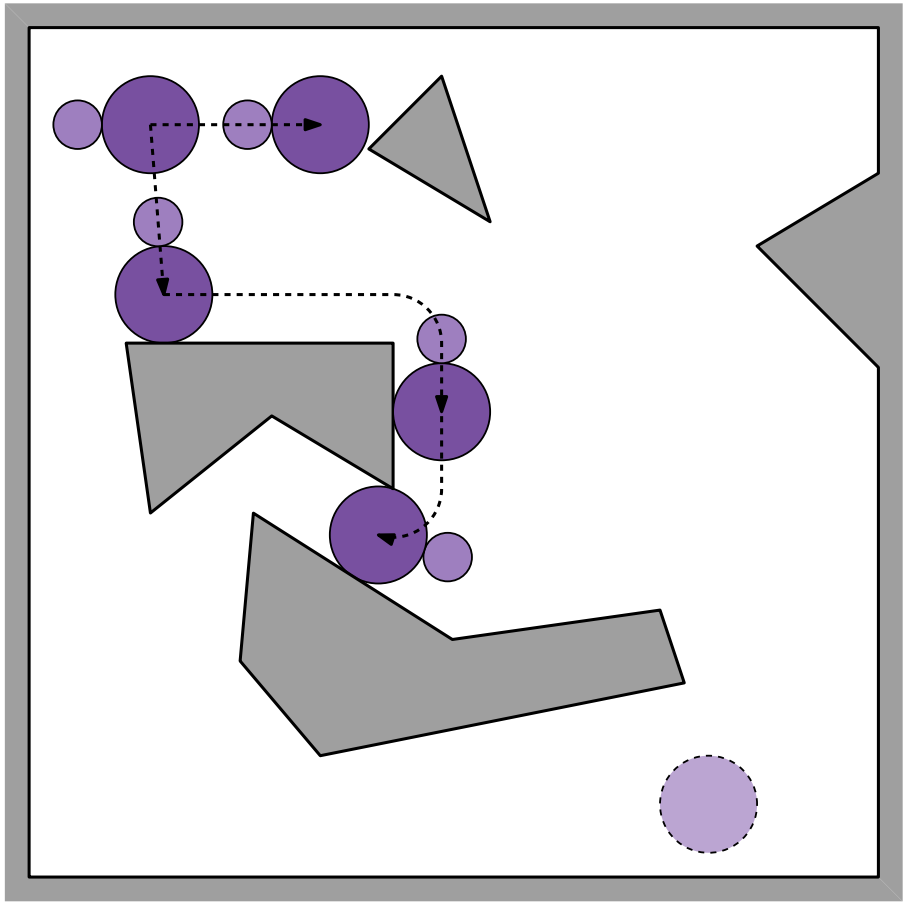

Create a tree with the initial configuration as the root:

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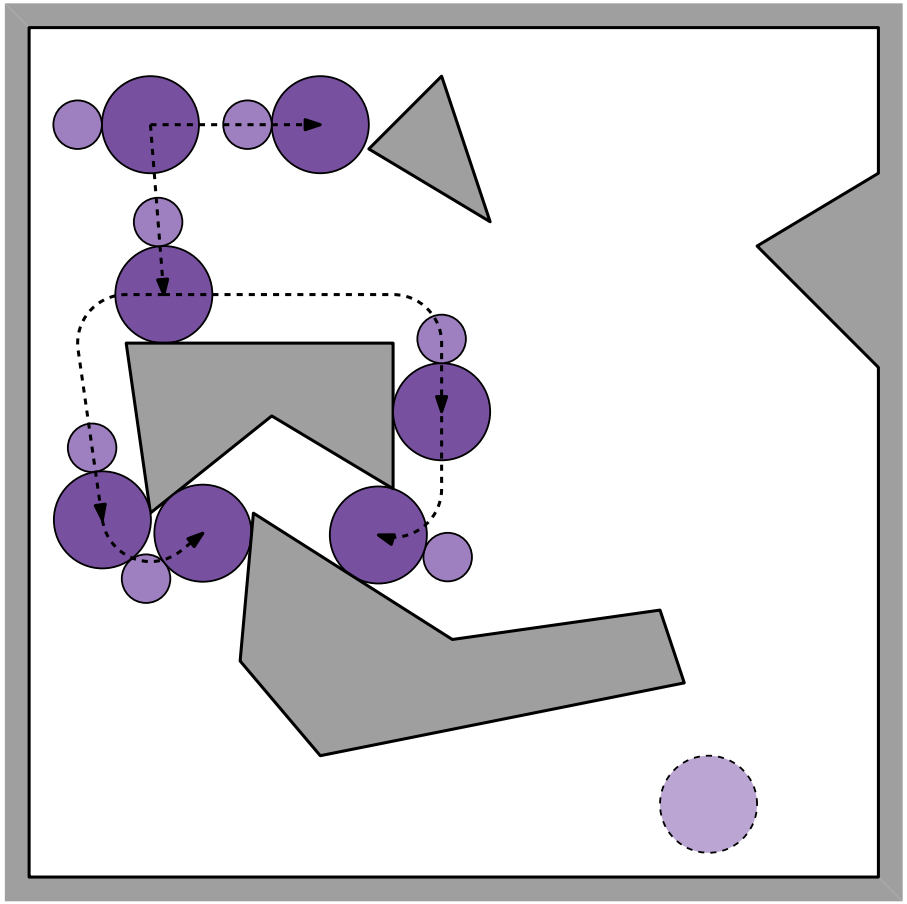

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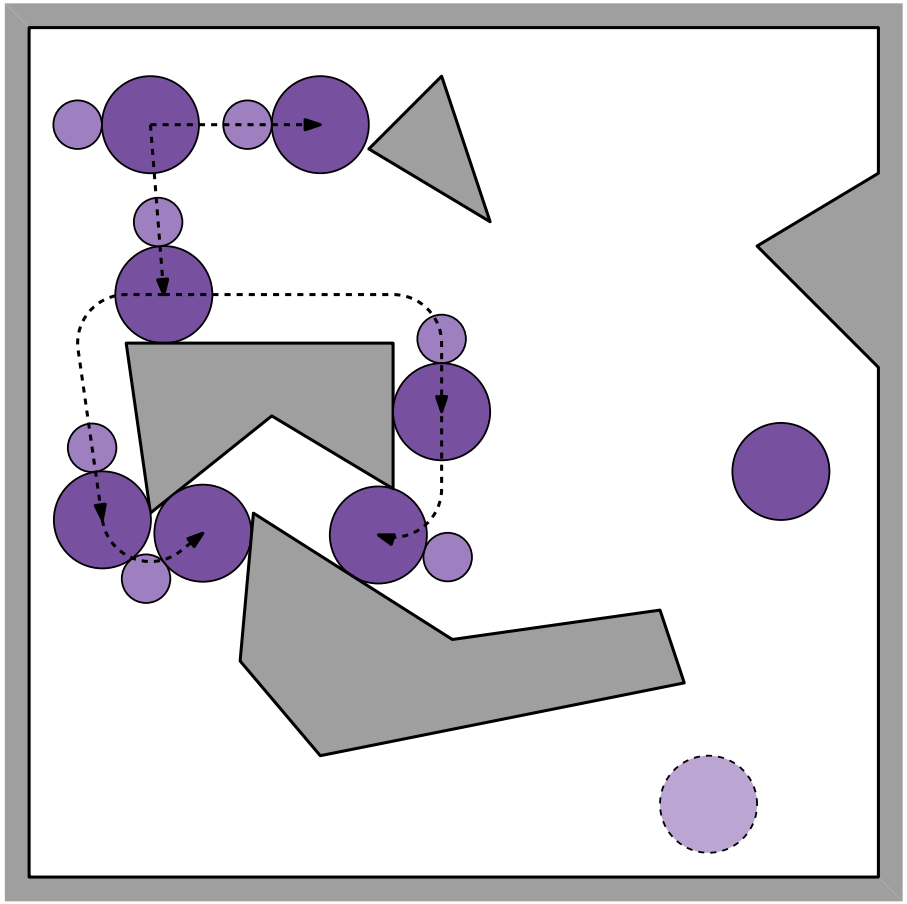

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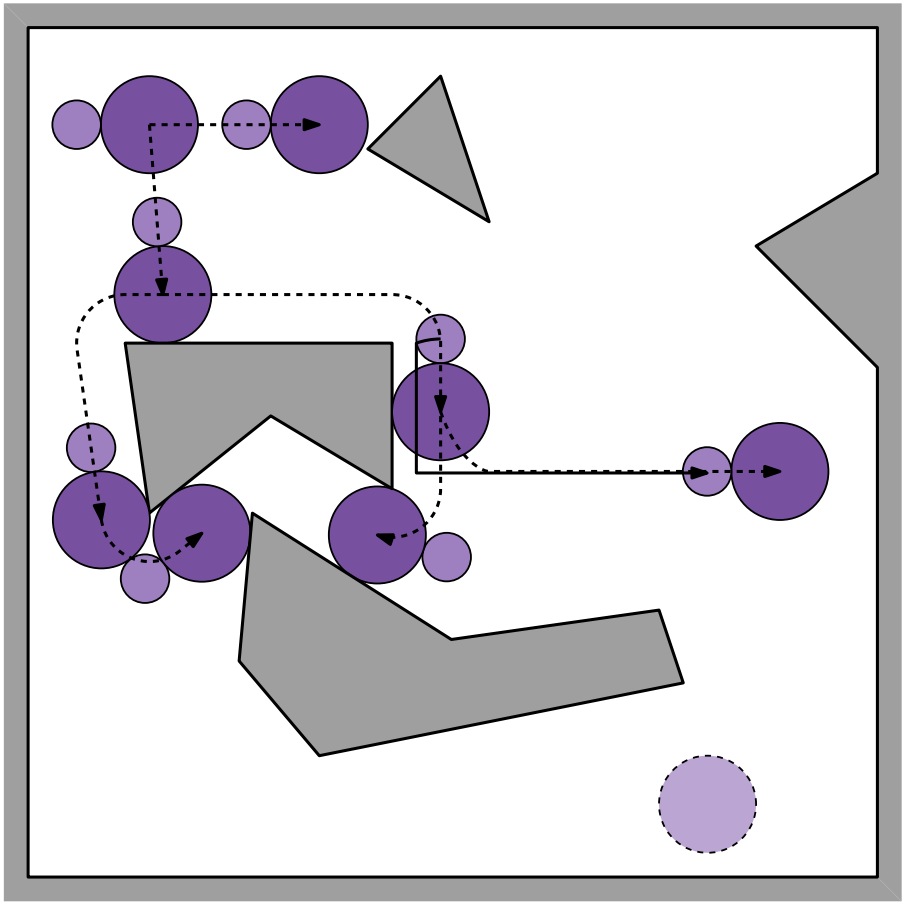

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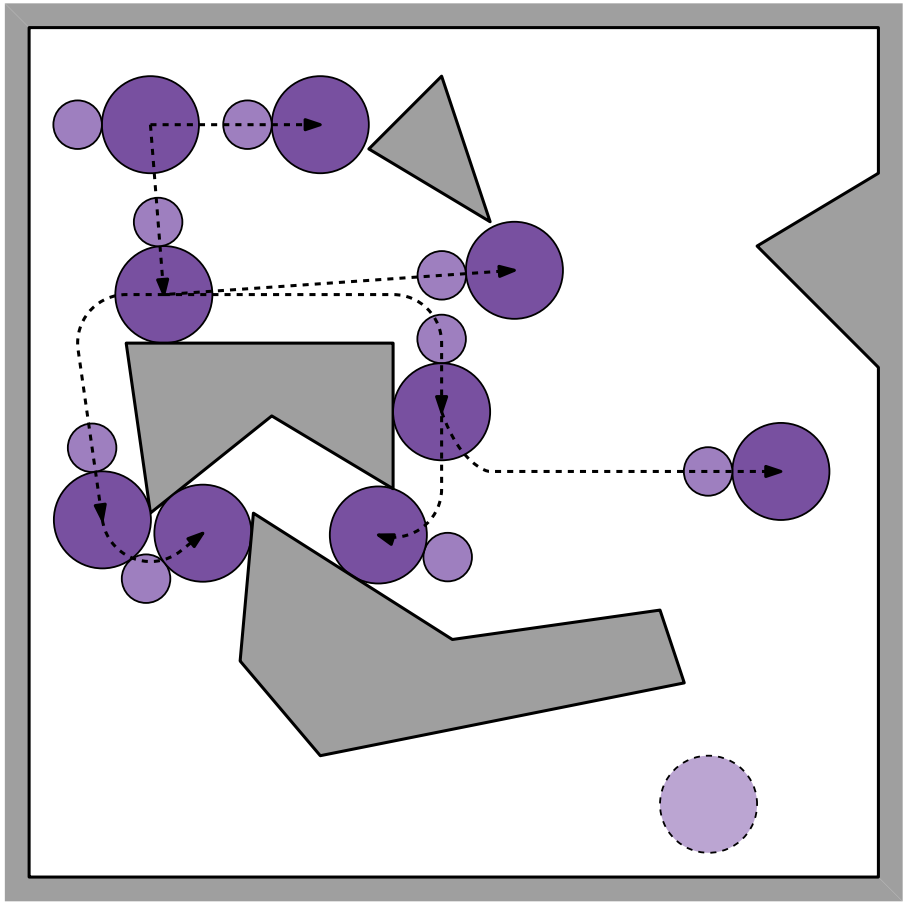

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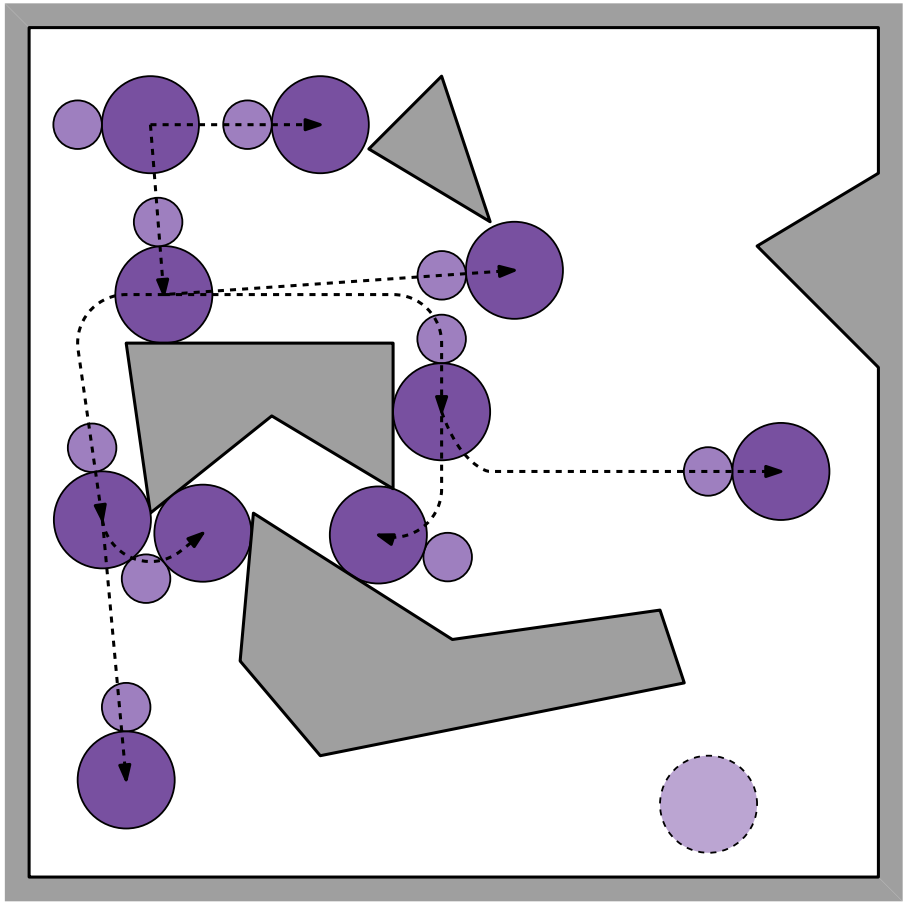

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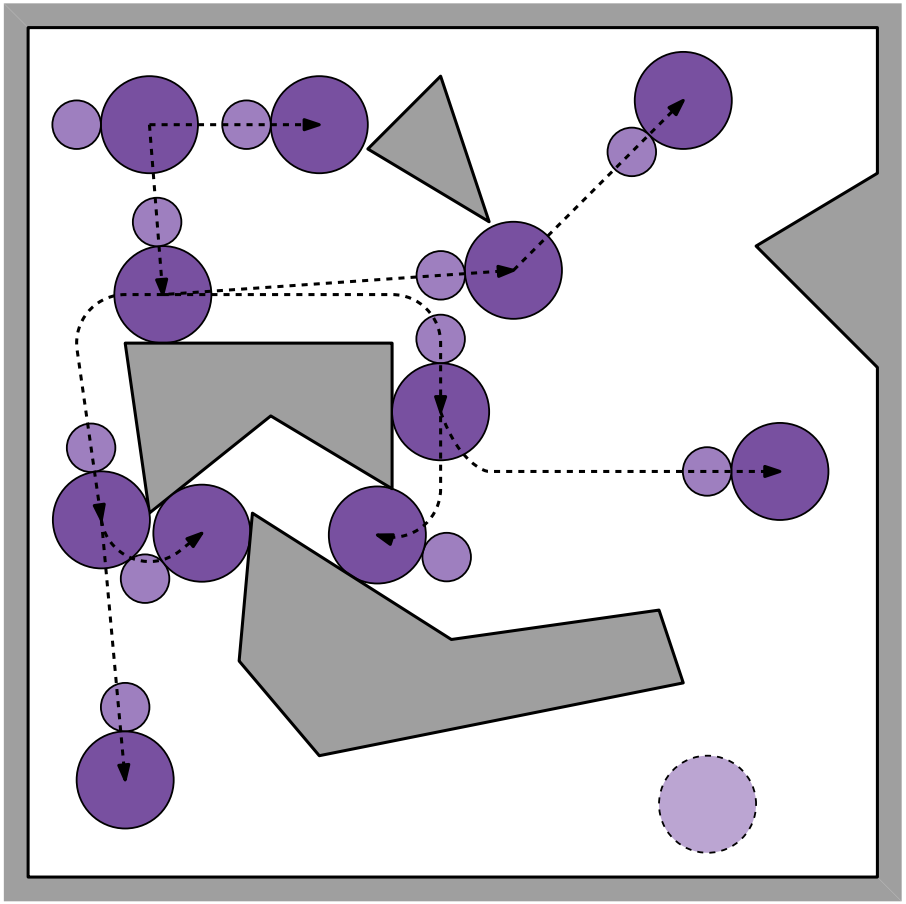

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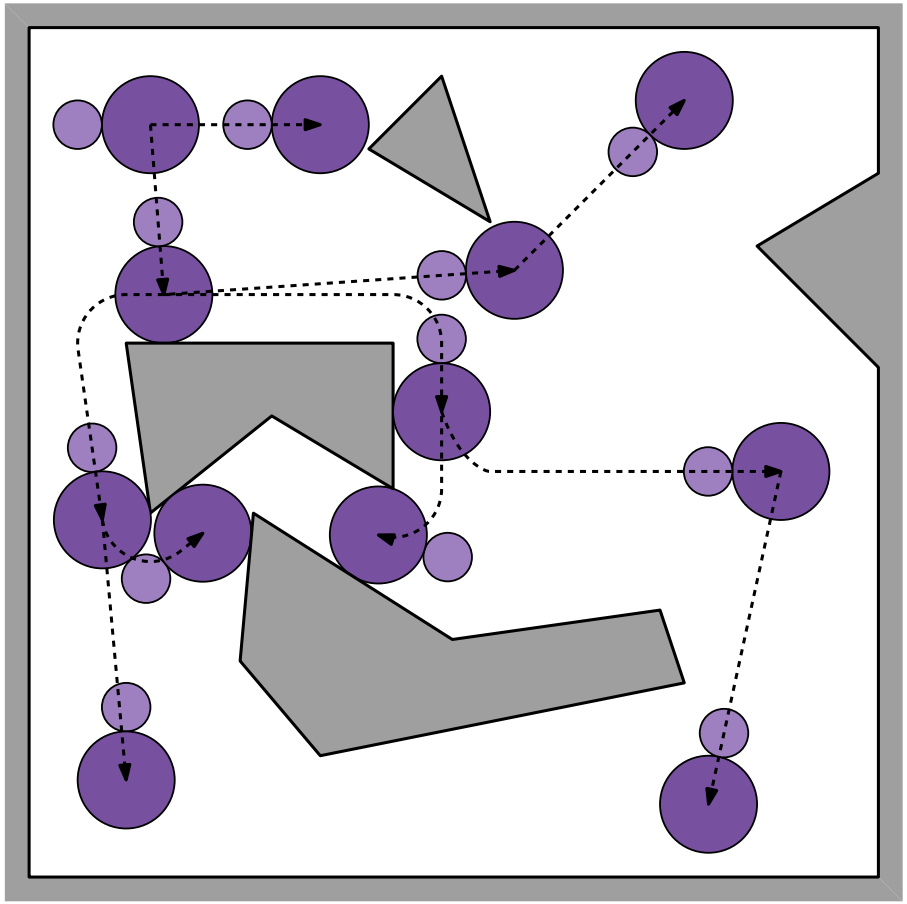

Create a tree with the initial configuration as the root:

- Take a random position.
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- Repeat until destination gets added to the tree (or other stopping criterion).

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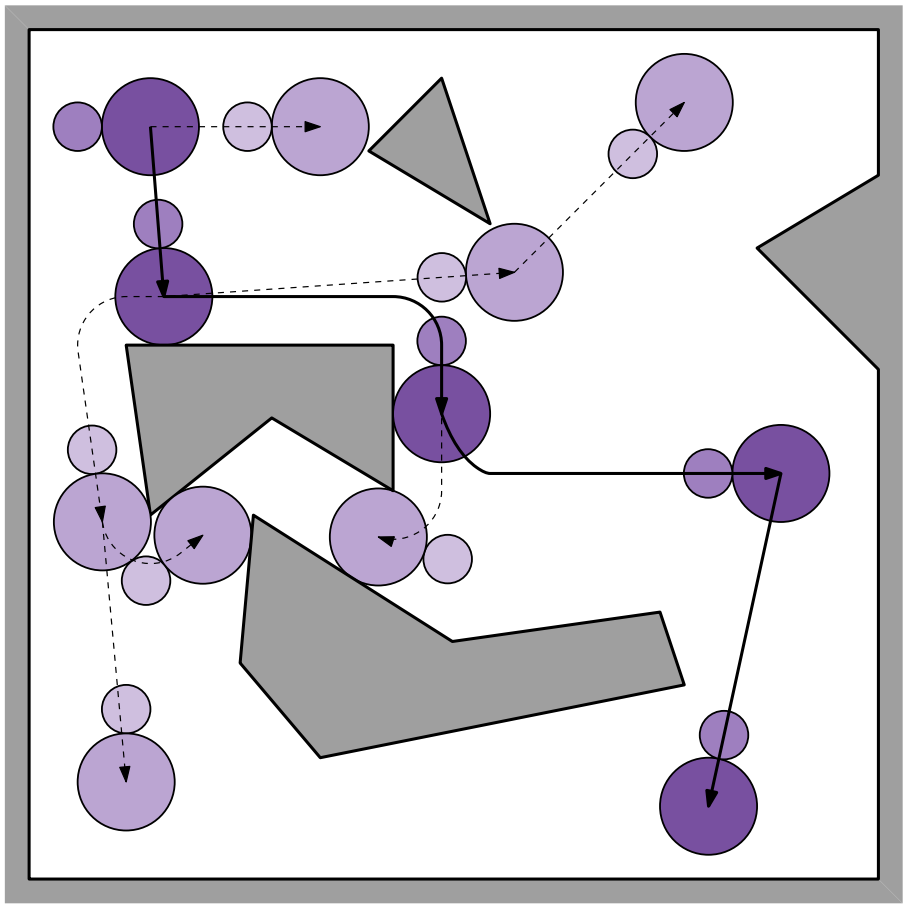

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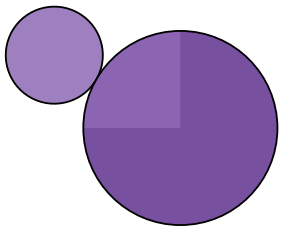
The studied subproblem

Nieuwenhuisen's algorithm needs a subroutine for this subproblem:



Given:

- Two circular disks in the plane: a smaller pusher and a larger object.

The studied subproblem

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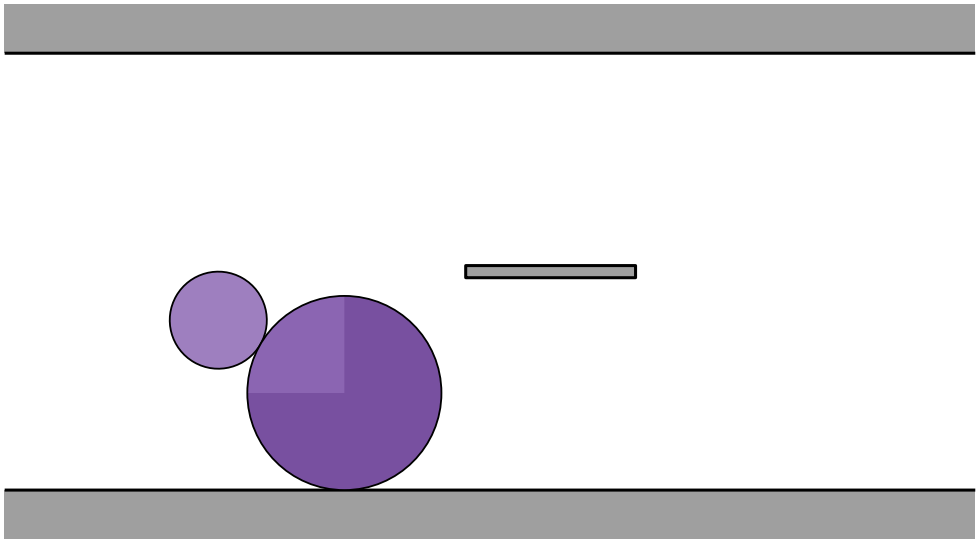

Given:

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- A collection of line-segment obstacles.

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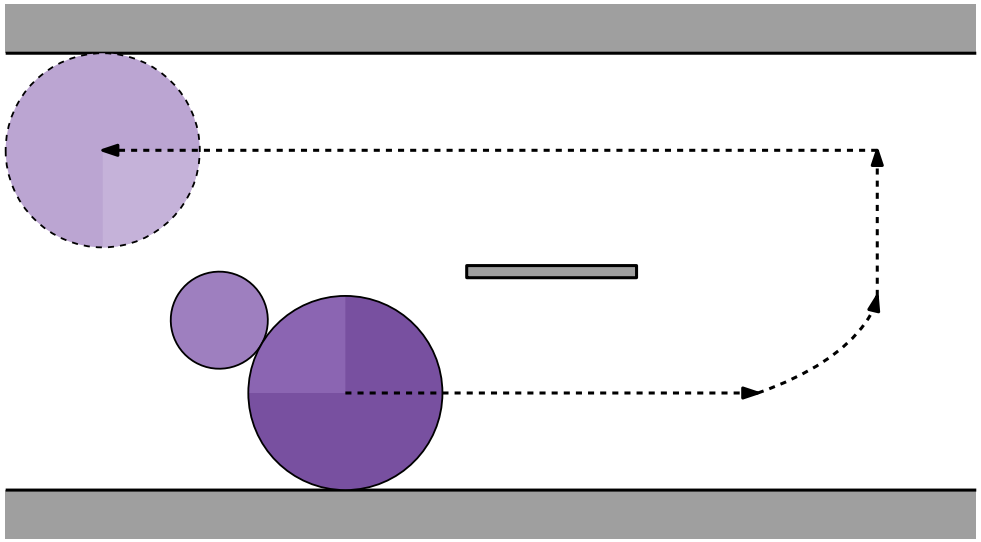

Given:

- Two circular disks in the plane: a smaller pusher and a larger object.
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- A path of obstacle-free path sections for the object.

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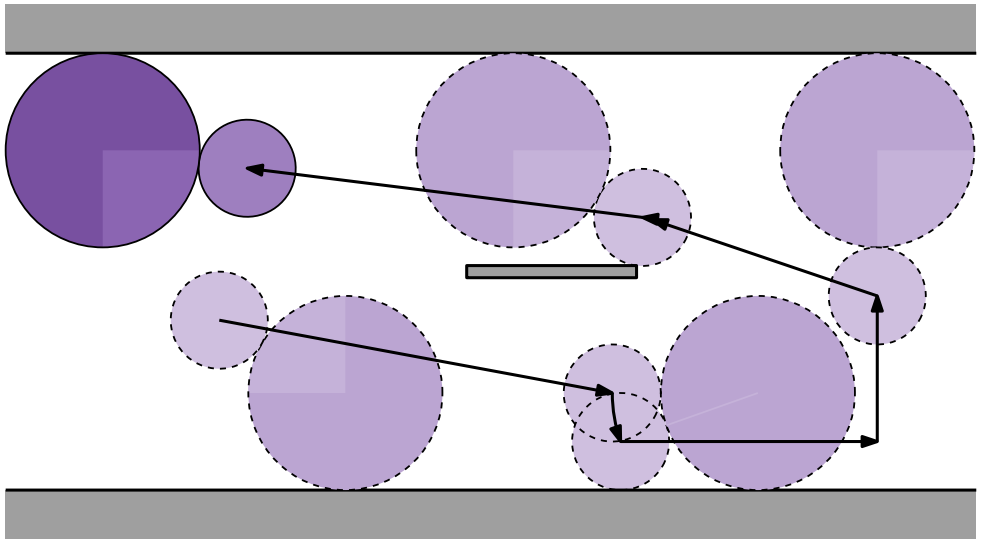


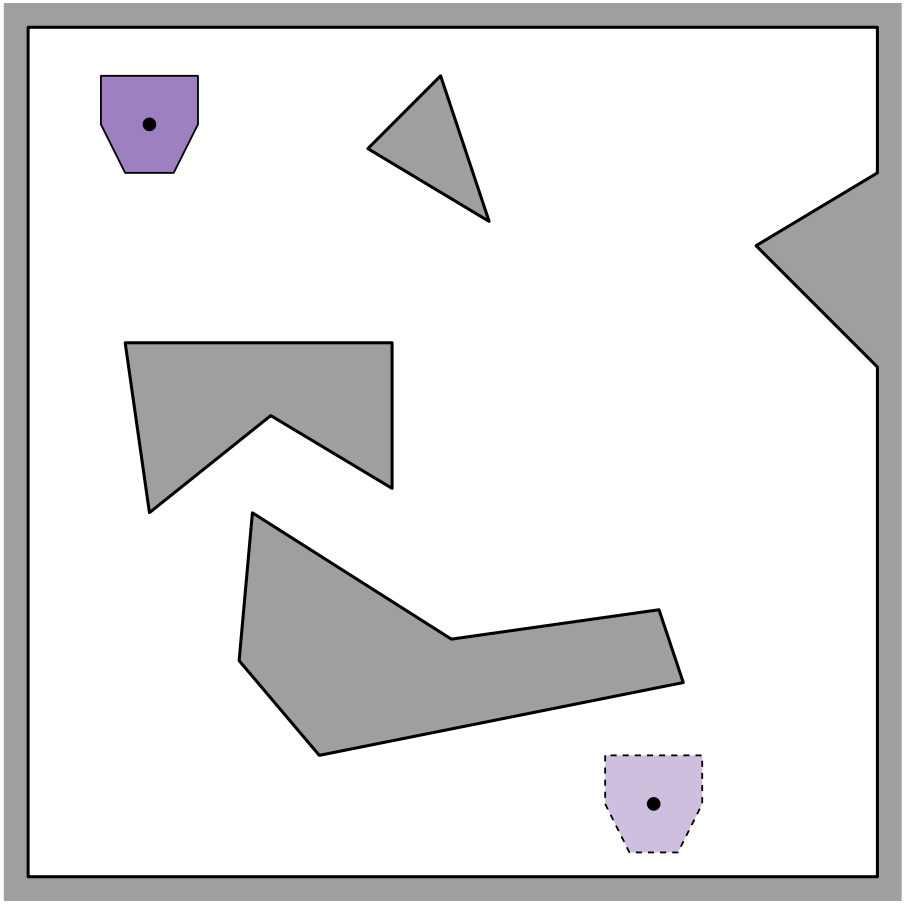
Given:

- Two circular disks in the plane: a smaller pusher and a larger object.
- A collection of line-segment obstacles.
- A path of obstacle-free path sections for the object.

Find:

- A push plan that makes the pusher push the object (as far as possible) along its path.

The configuration-space method



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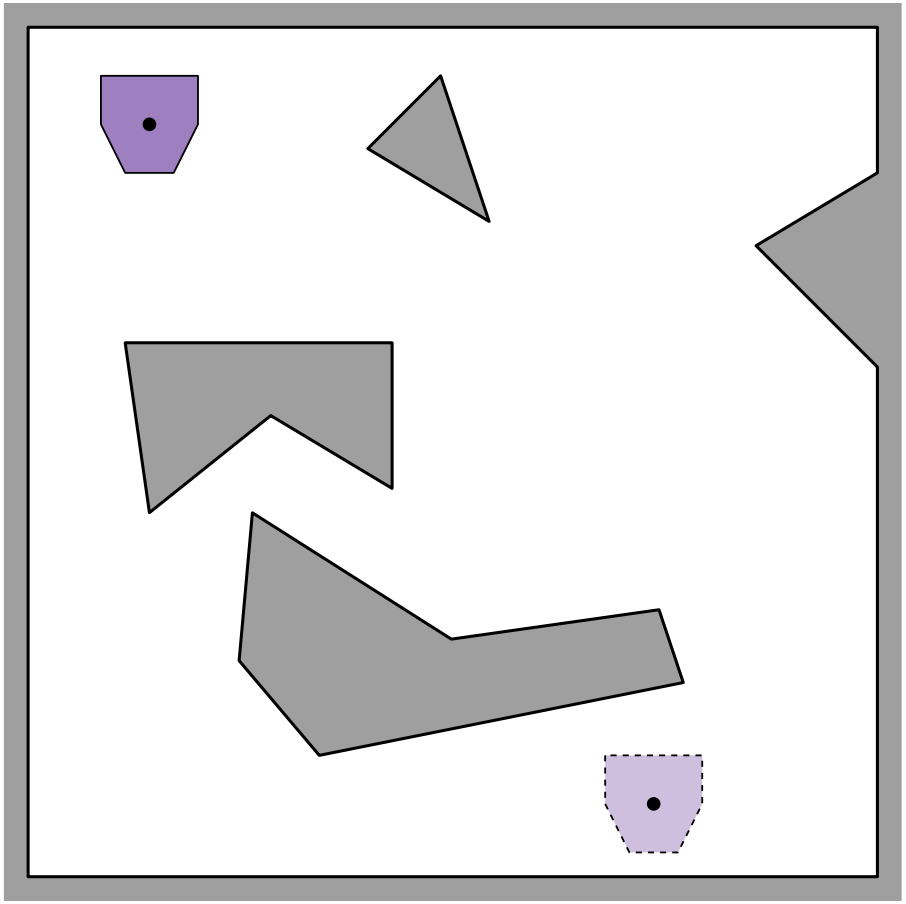

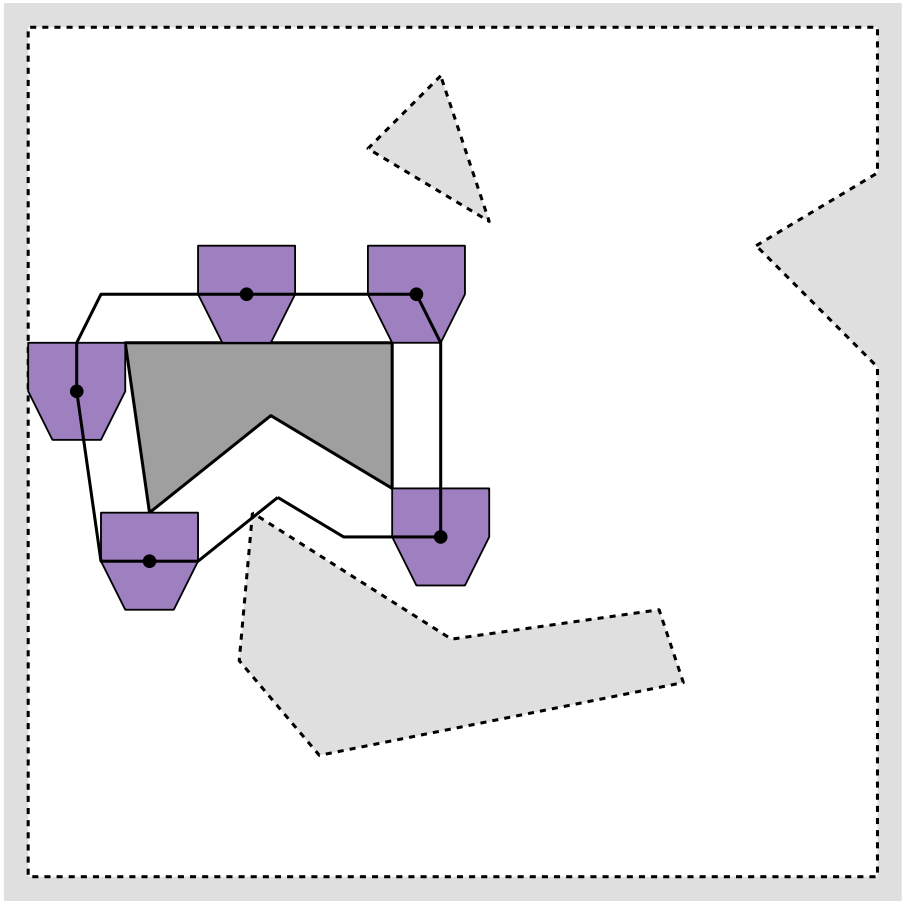

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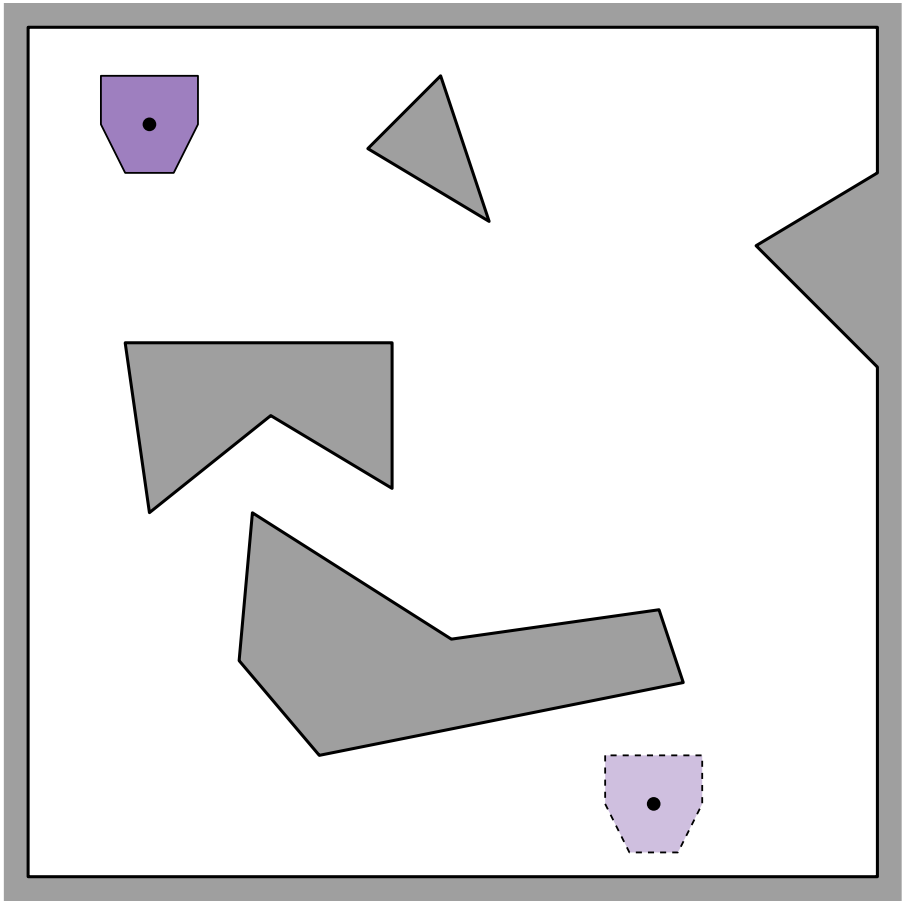

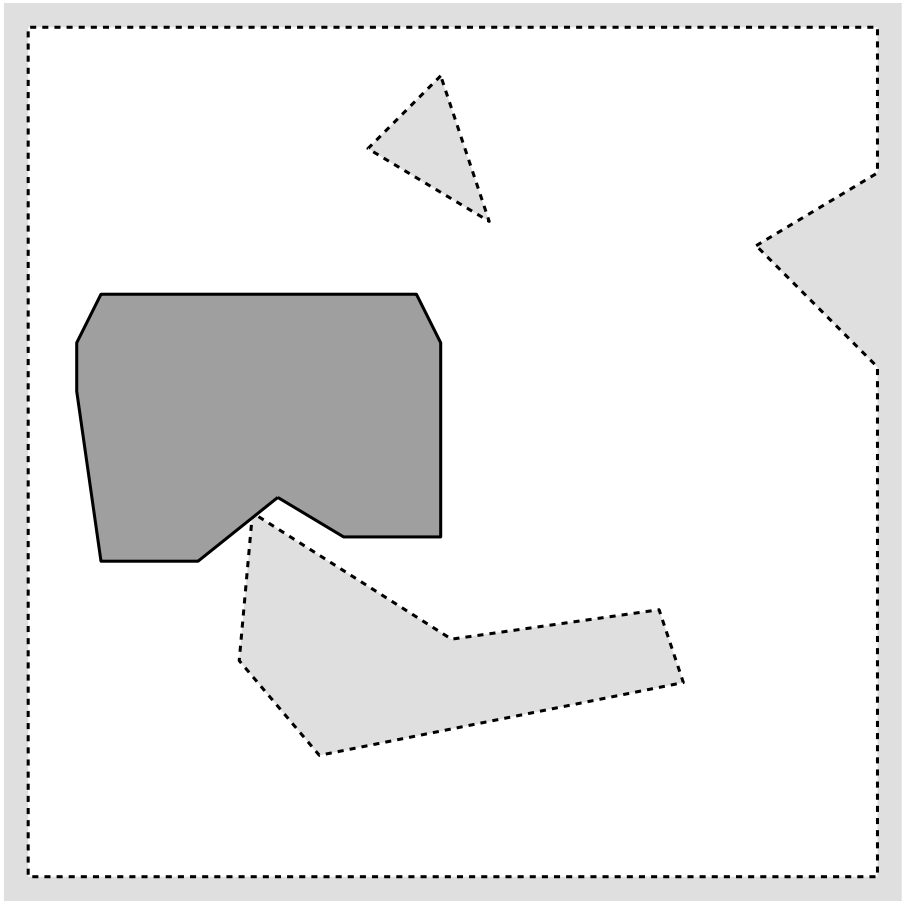

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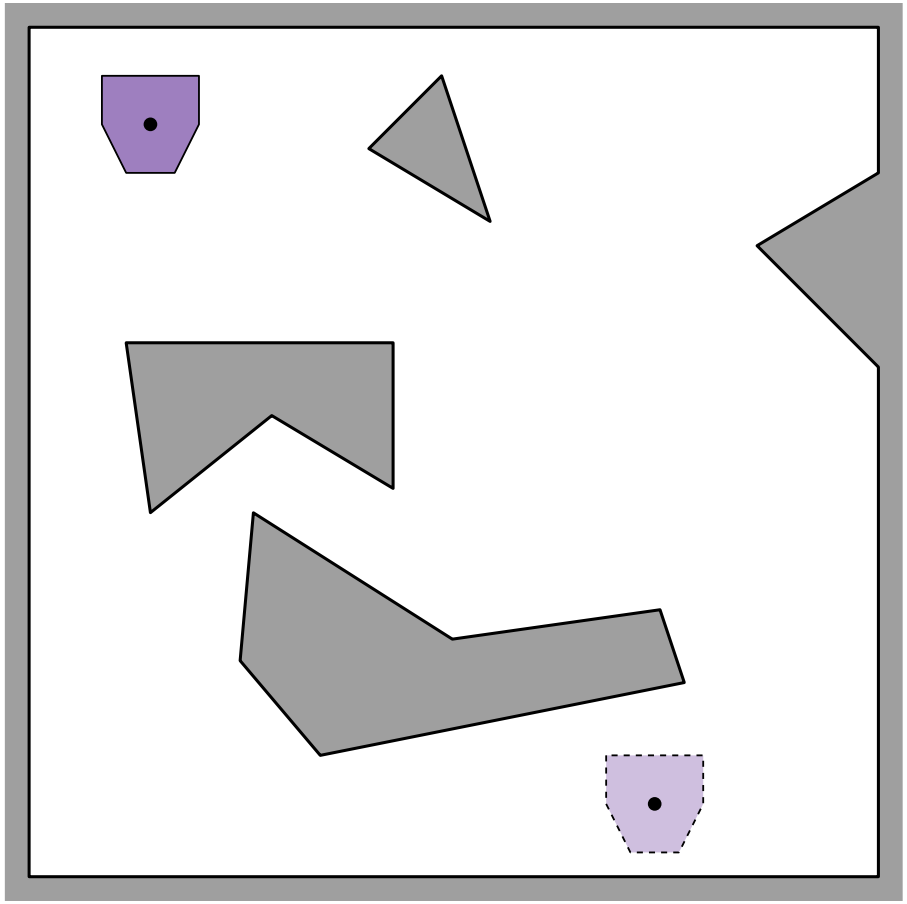
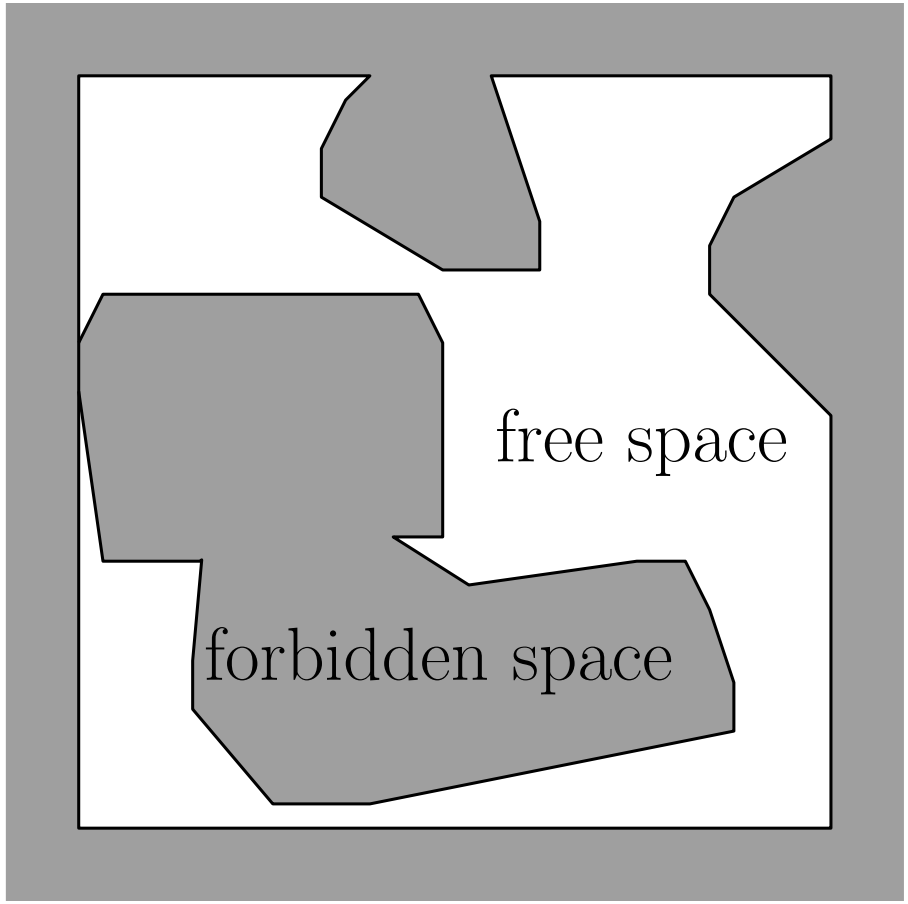
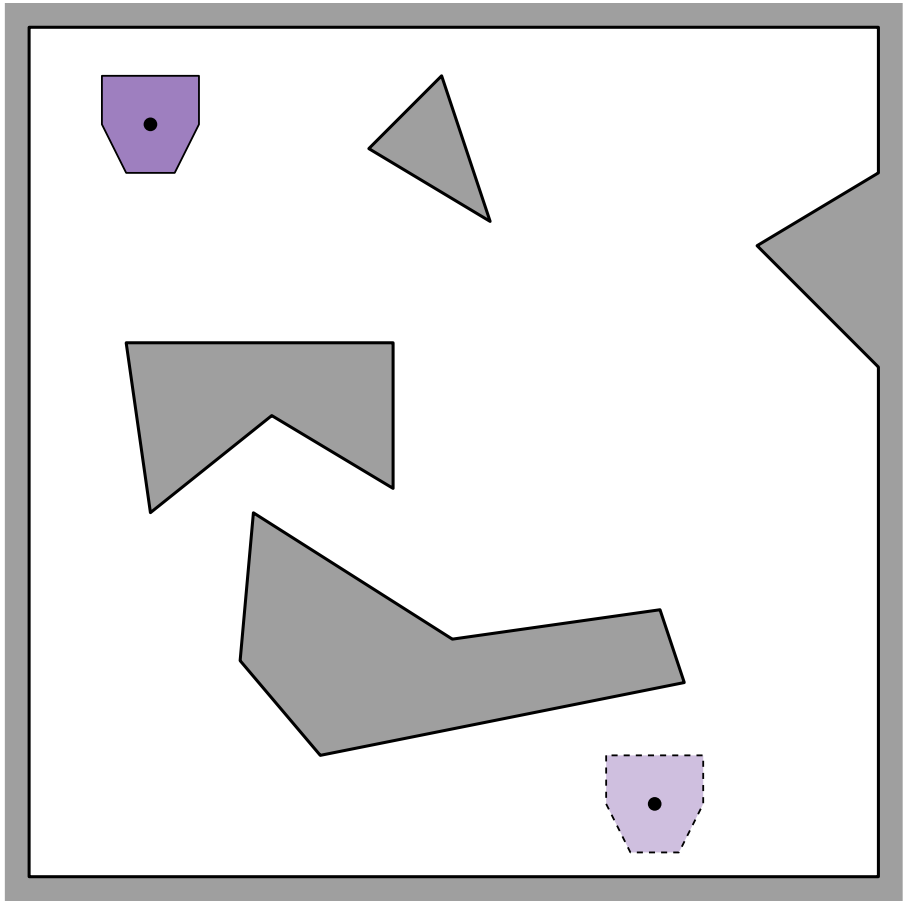


work space



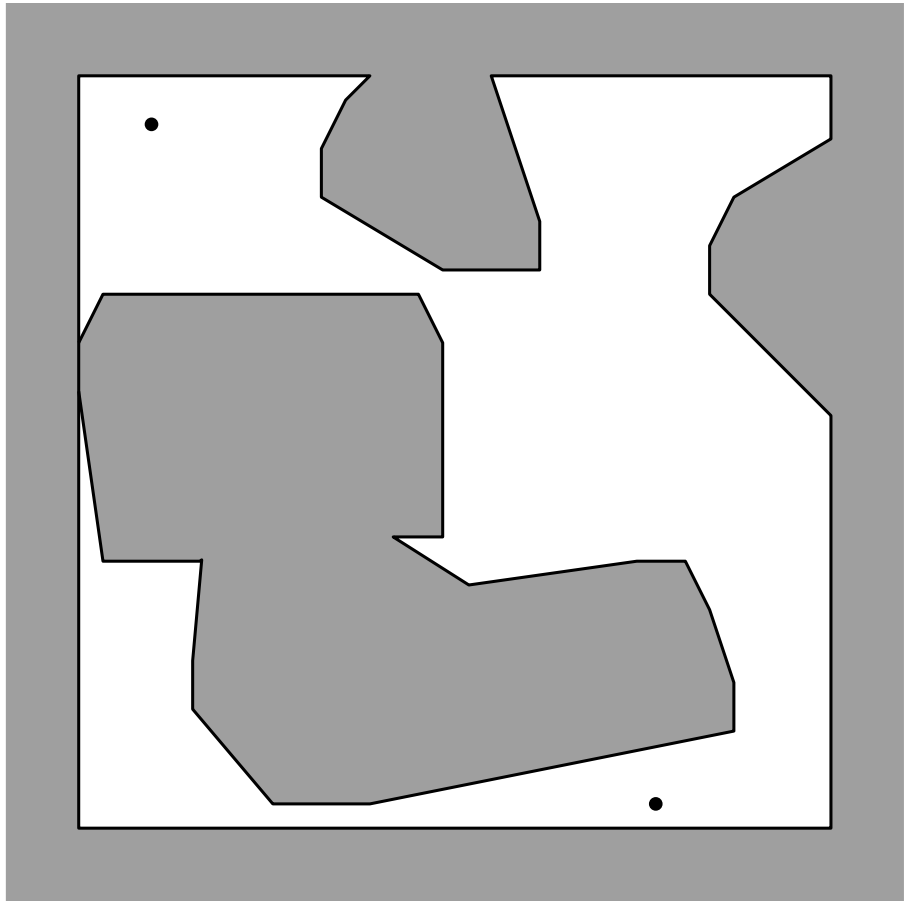
configuration space

The configuration-space method



work space

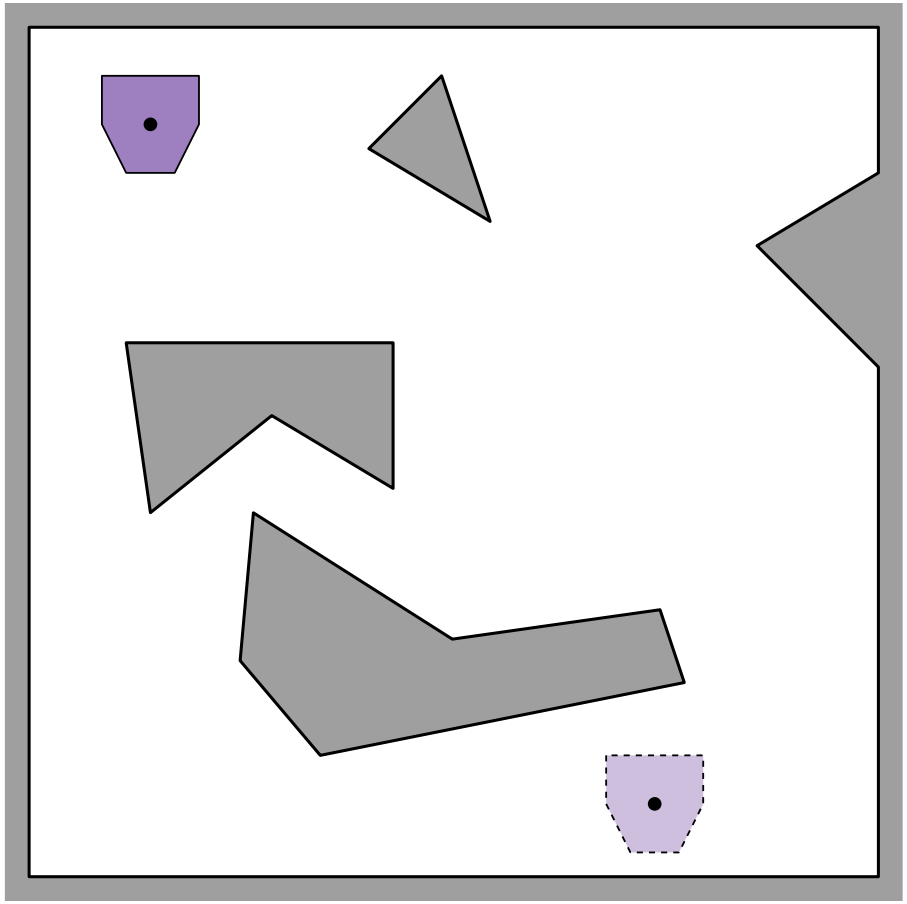
configuration = 2D position



configuration space

point in 2D

The configuration-space method



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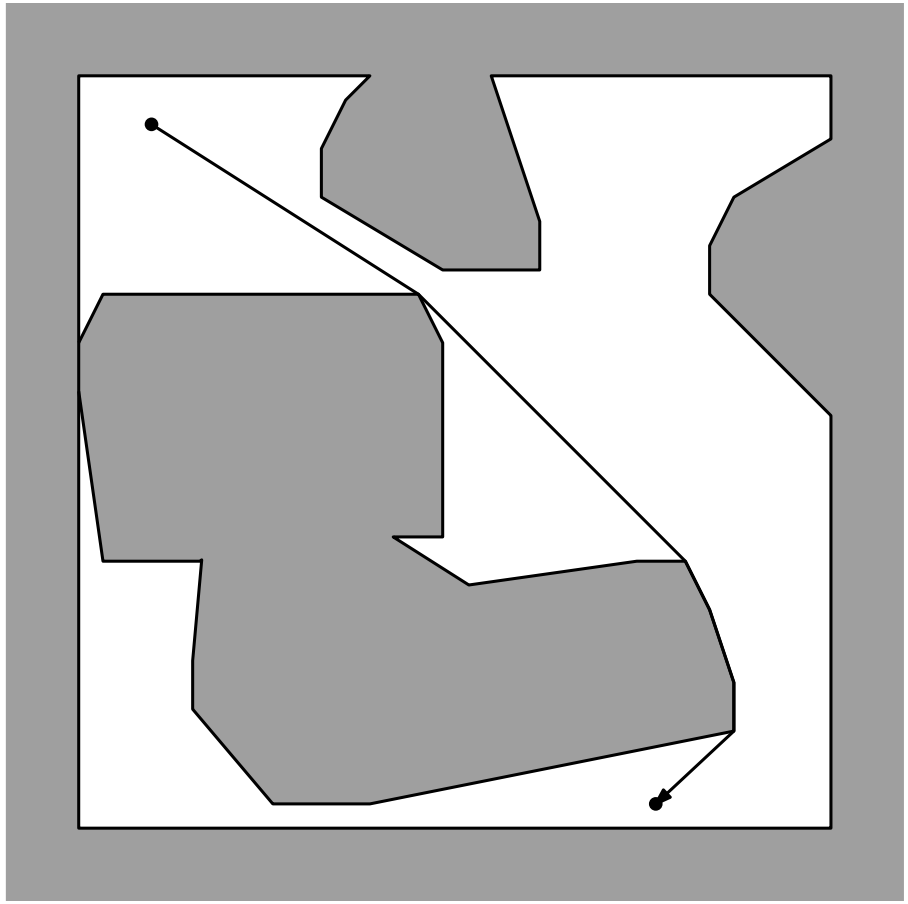

configuration space

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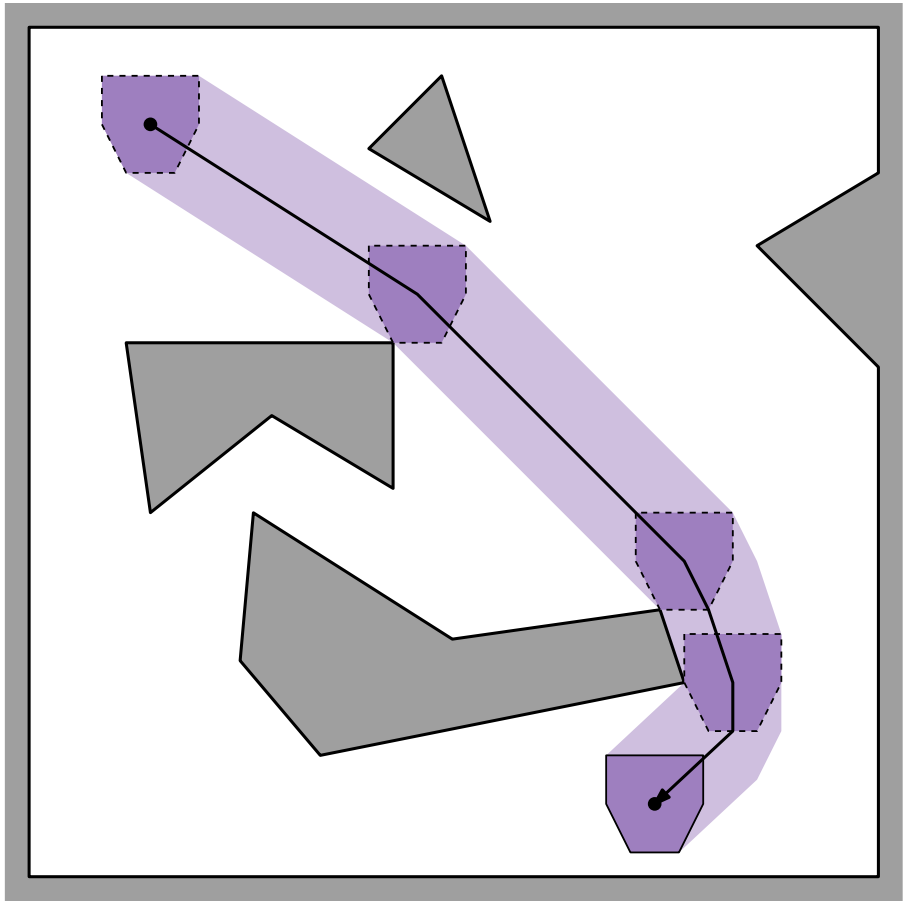

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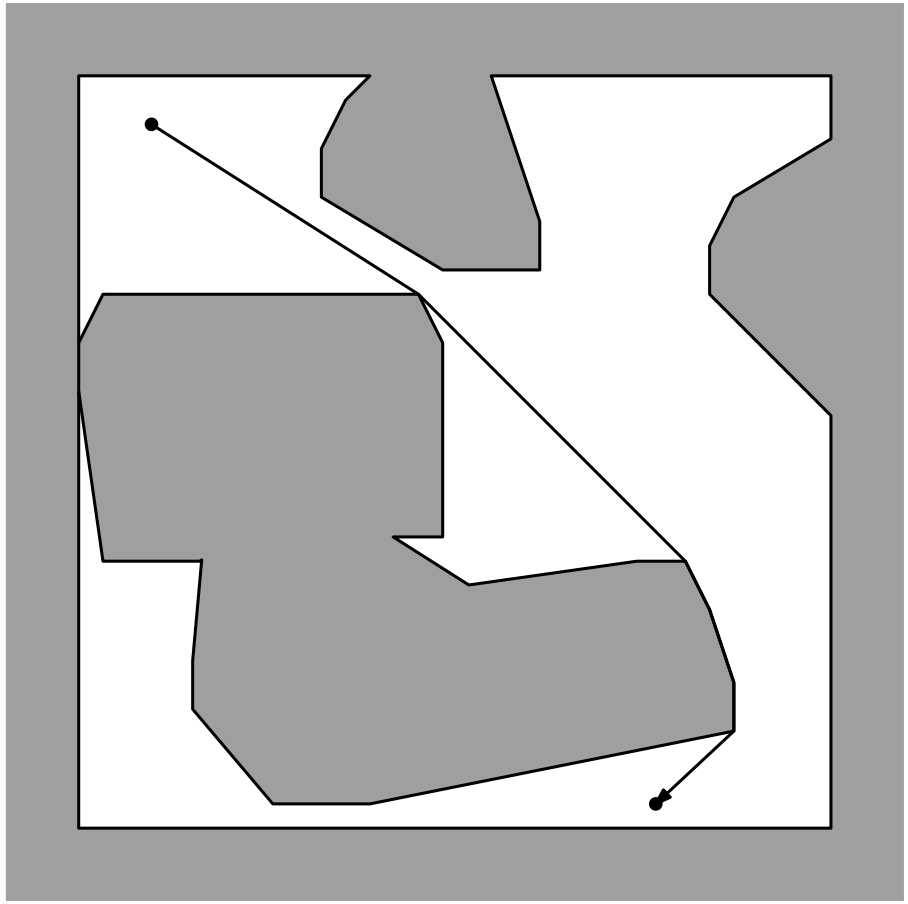

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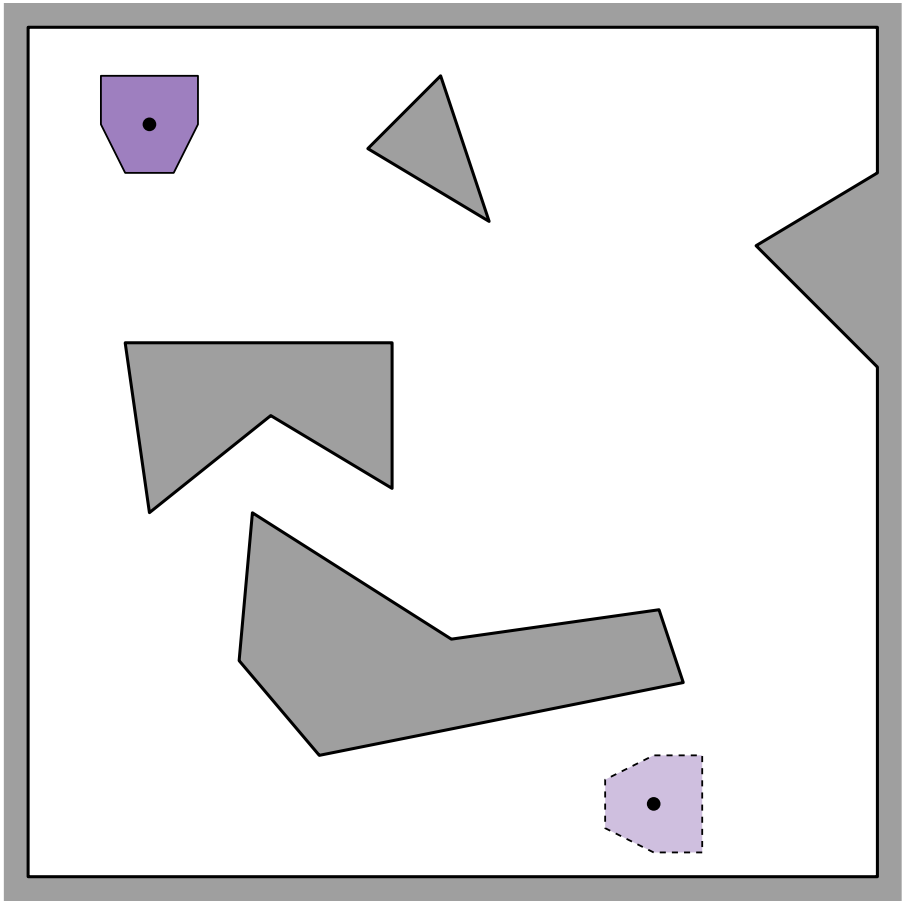

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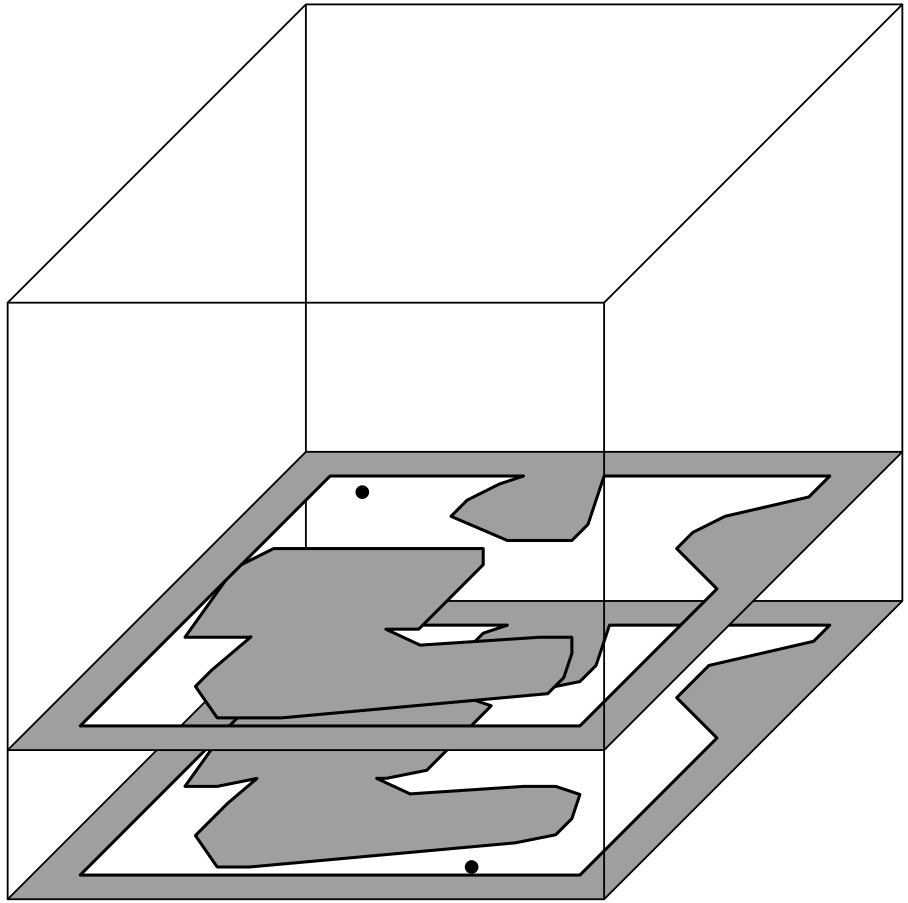
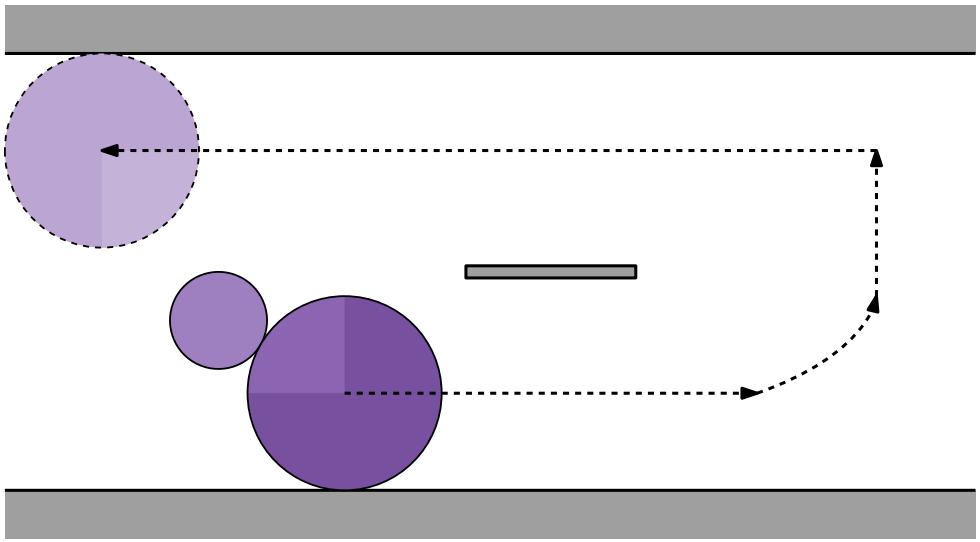


configuration space

configuration = 2D position +
1D orientation

point in 3D

The configuration-space method



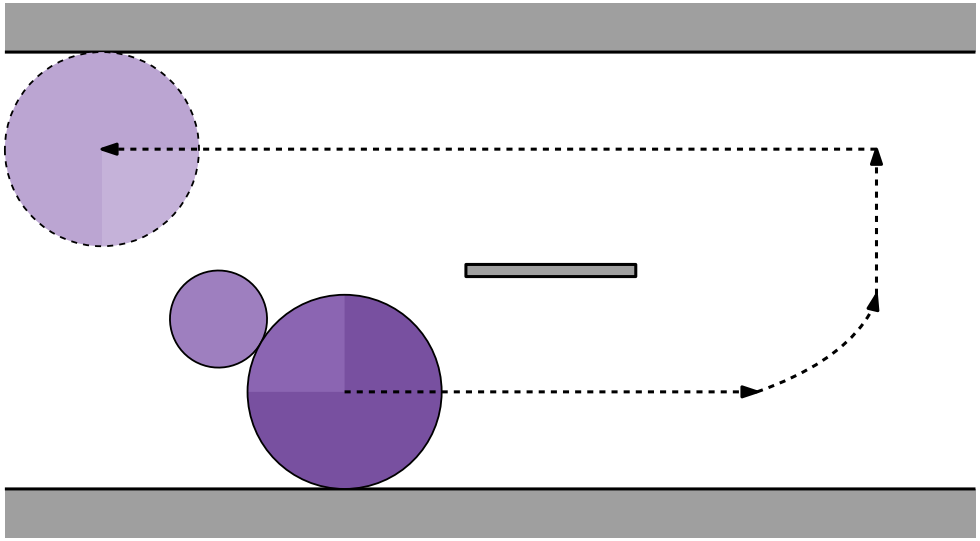
work space

configuration =
2D object position +
2D pusher position

configuration space

point in 4D ?!

The configuration-space method



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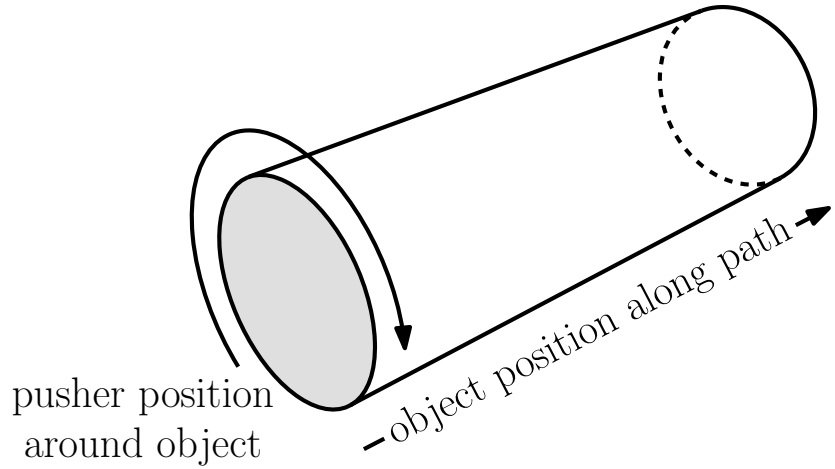
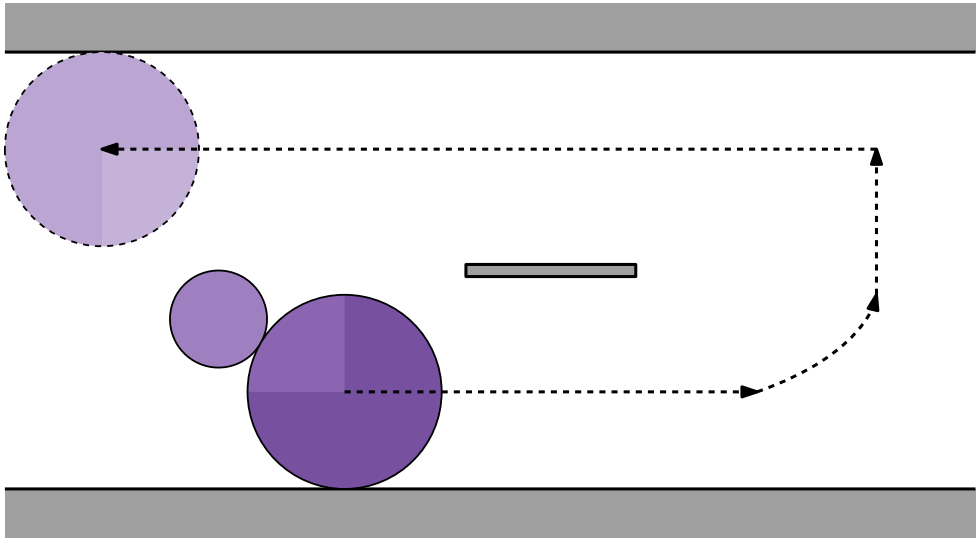


configuration space

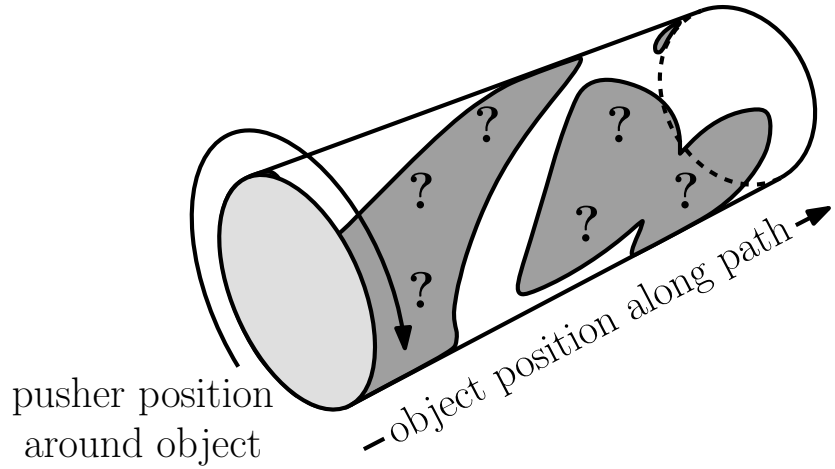
configuration =
1D object position along path +
1D pusher position around object

point in 2D (on cylinder)

The configuration-space method



work space

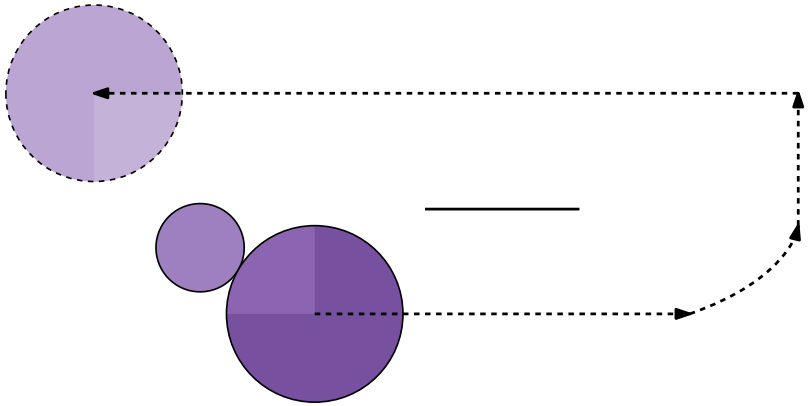


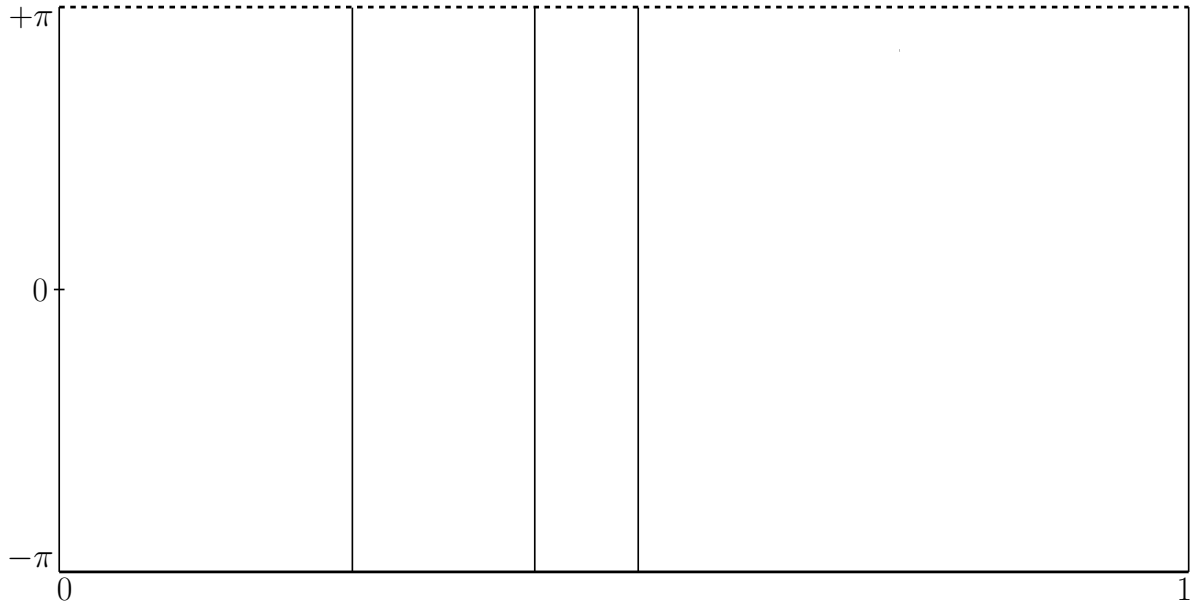
configuration space

configuration =
1D object position along path +
1D pusher position around object

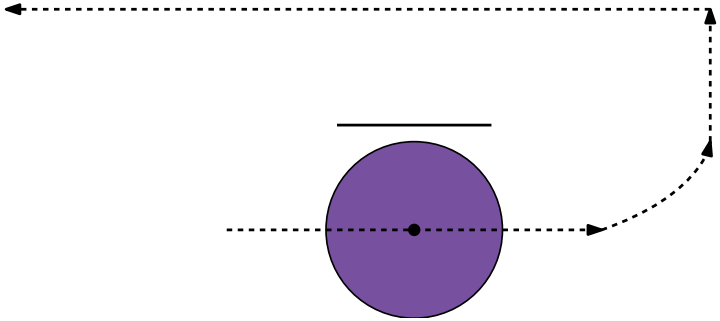
point in 2D (on cylinder)

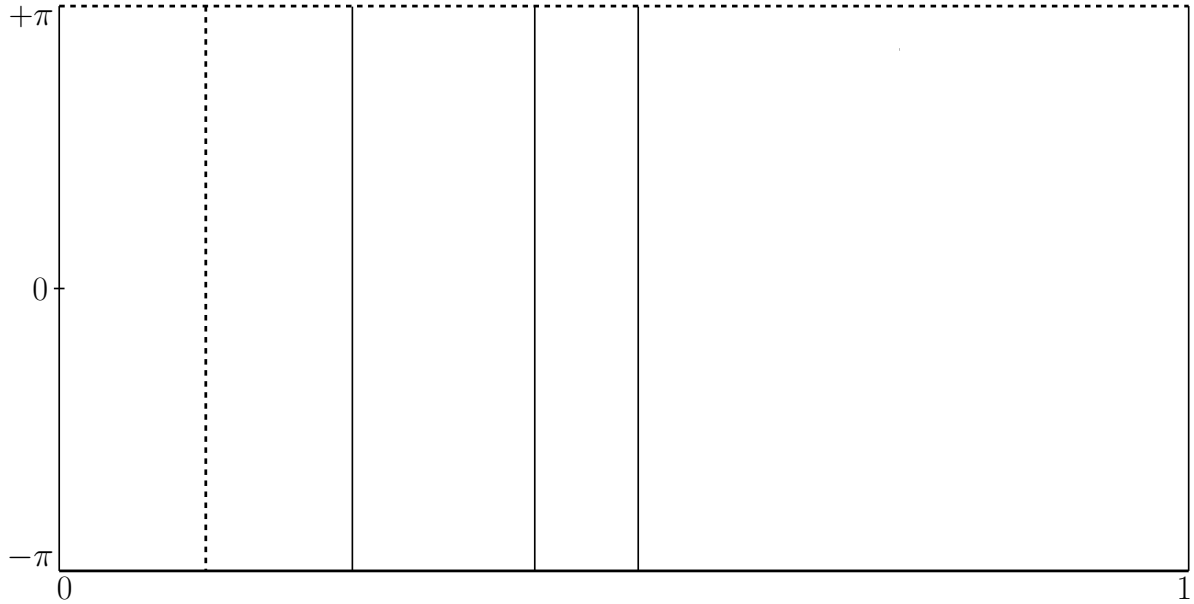
Configuration-space obstacles



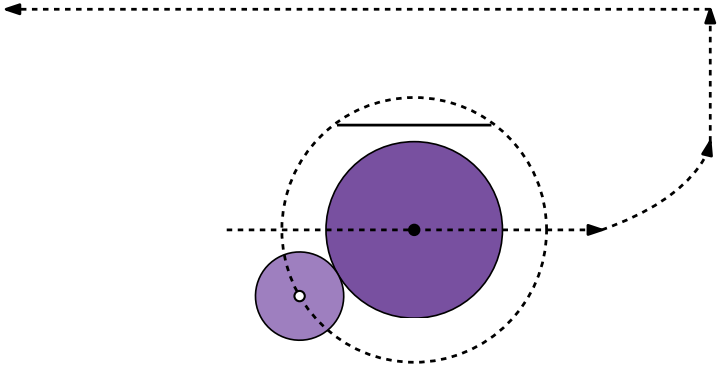


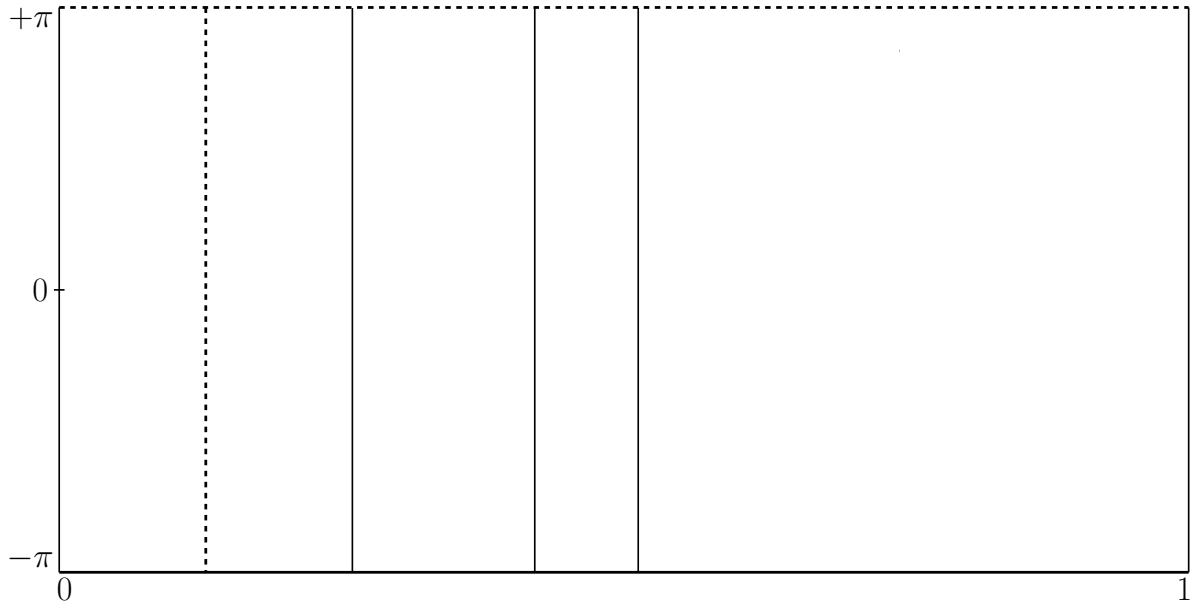
Configuration-space obstacles



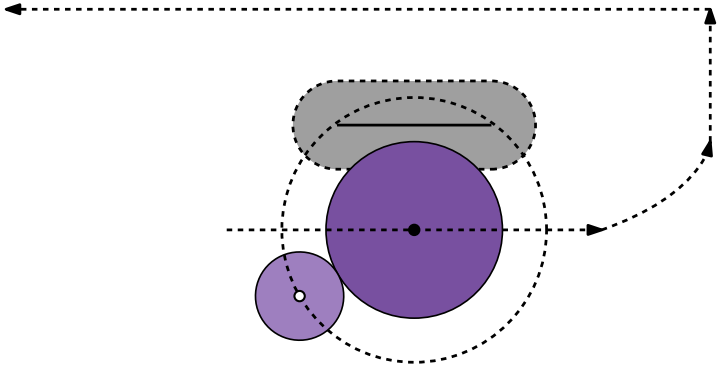


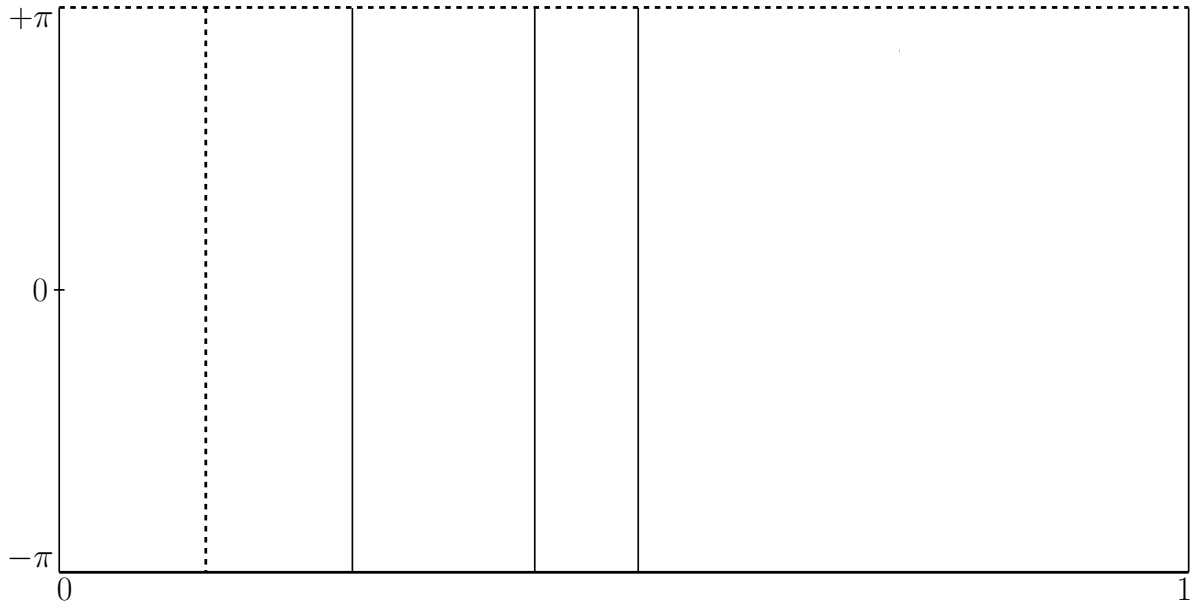
Configuration-space obstacles



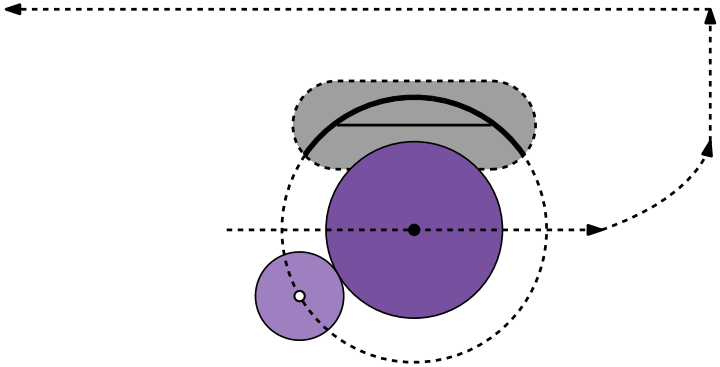


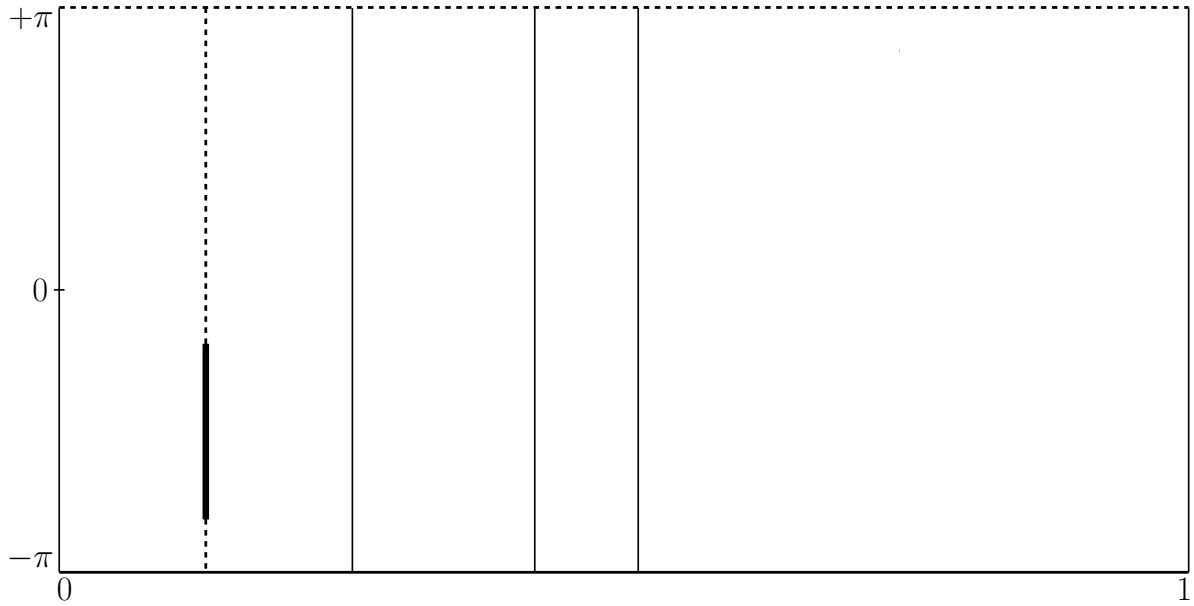
Configuration-space obstacles



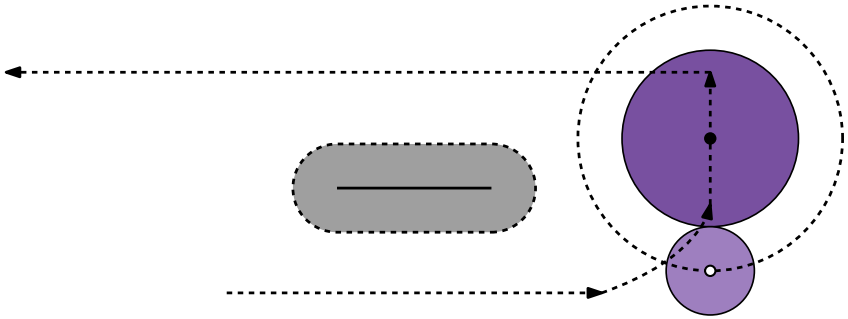


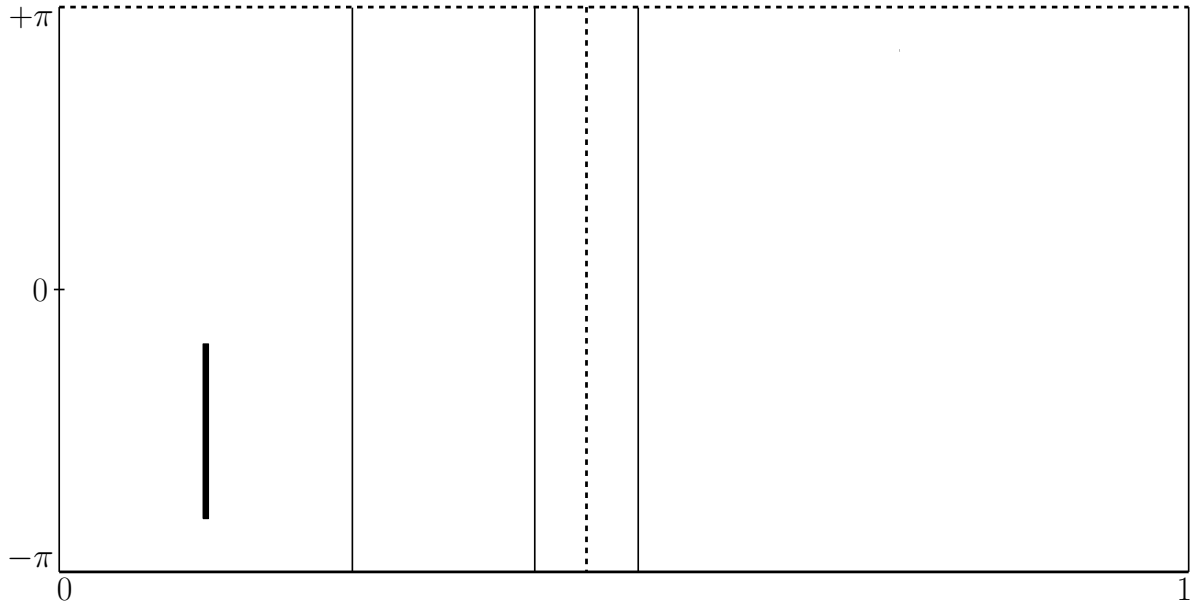
Configuration-space obstacles



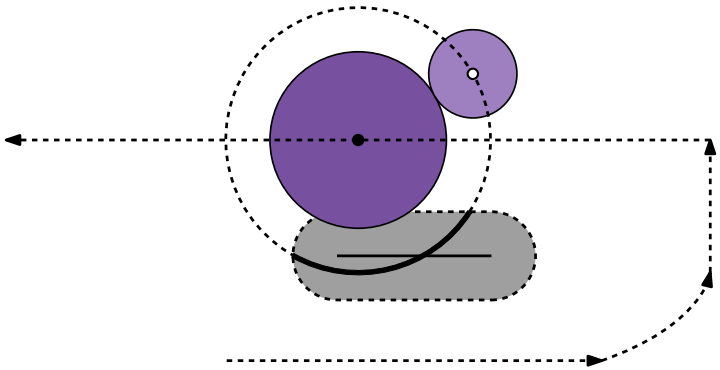


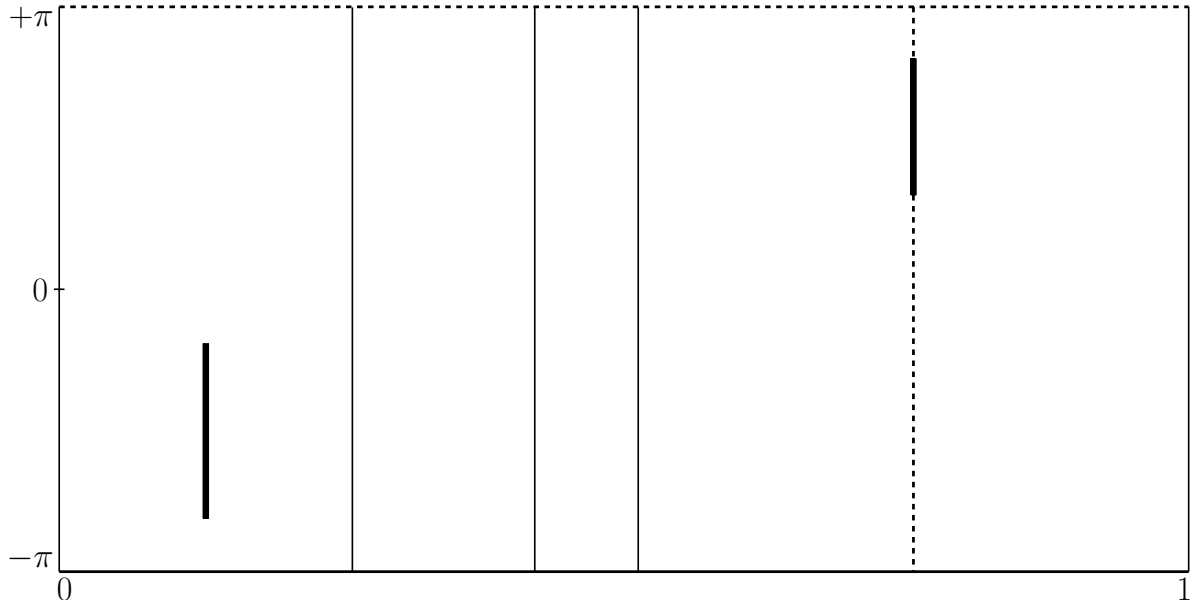
Configuration-space obstacles



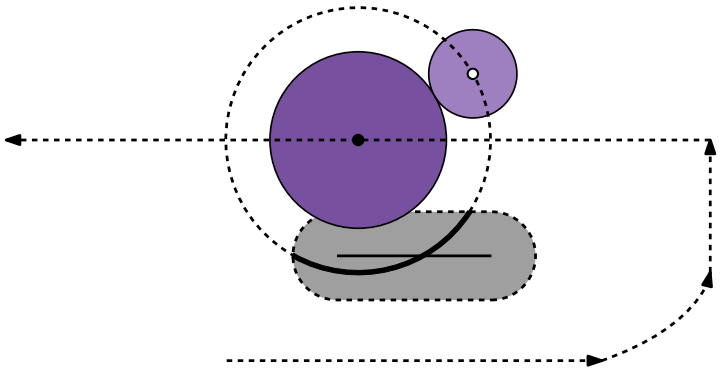


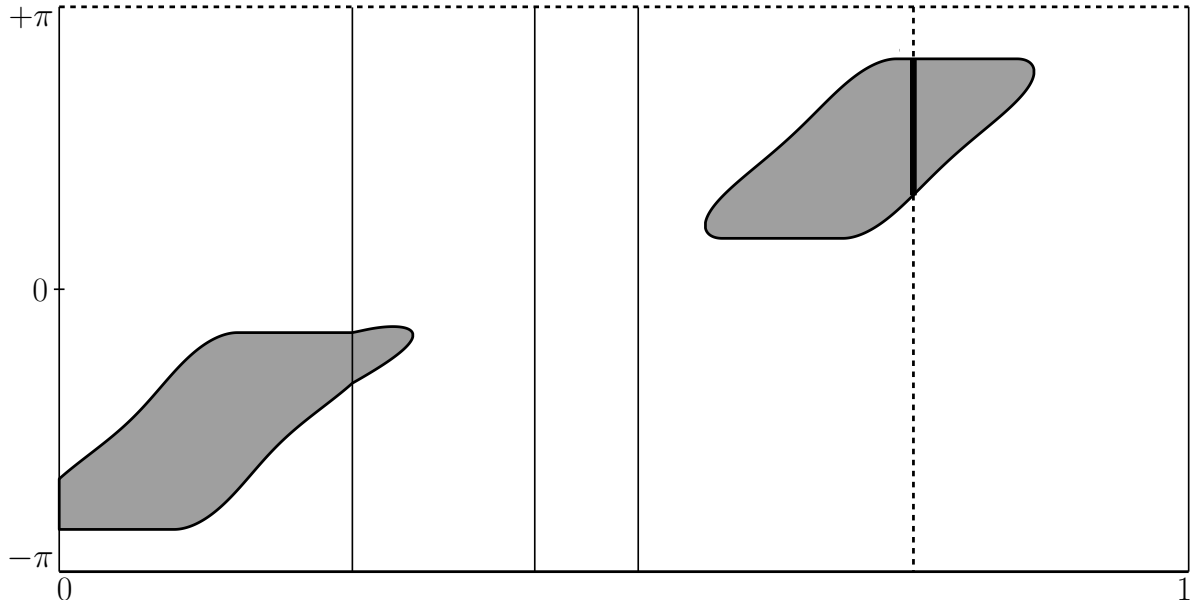
Configuration-space obstacles



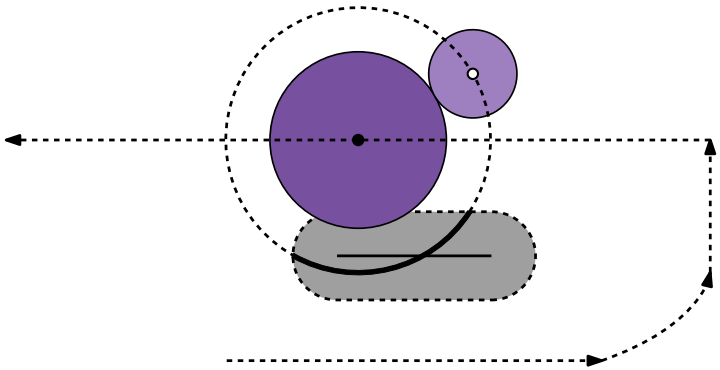


Configuration-space obstacles



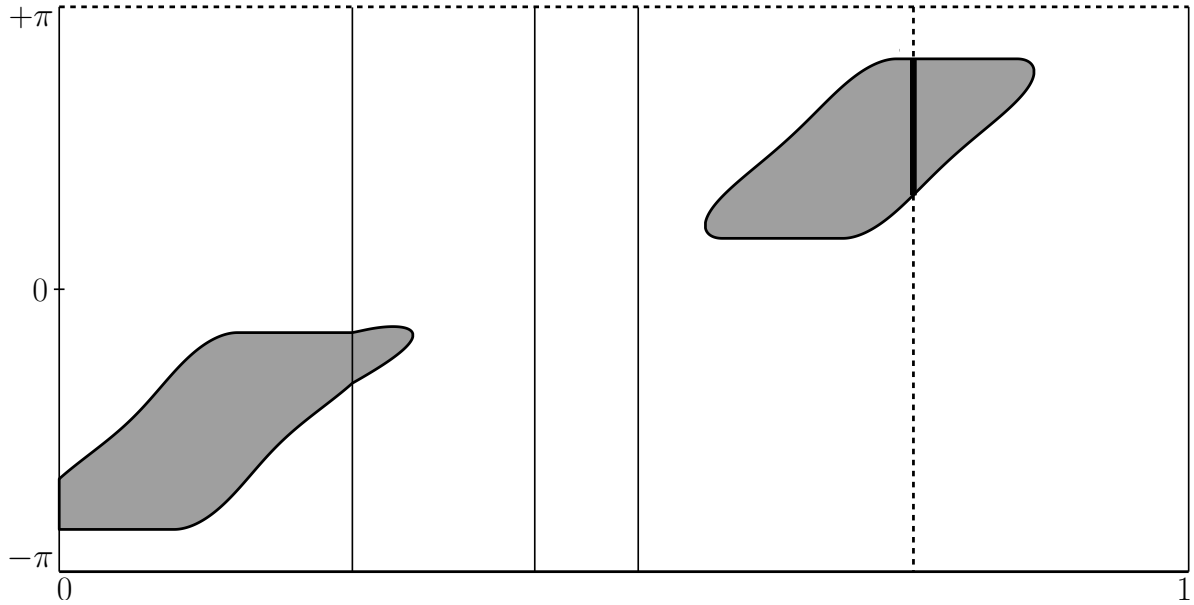


Configuration-space obstacles

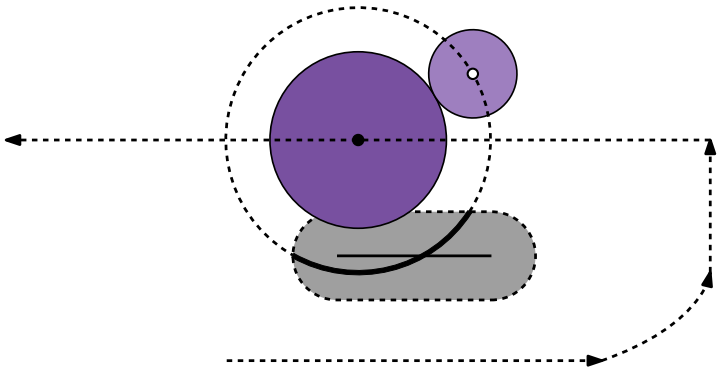


One $\mathcal{C}$-space obstacle:

- can have multiple components

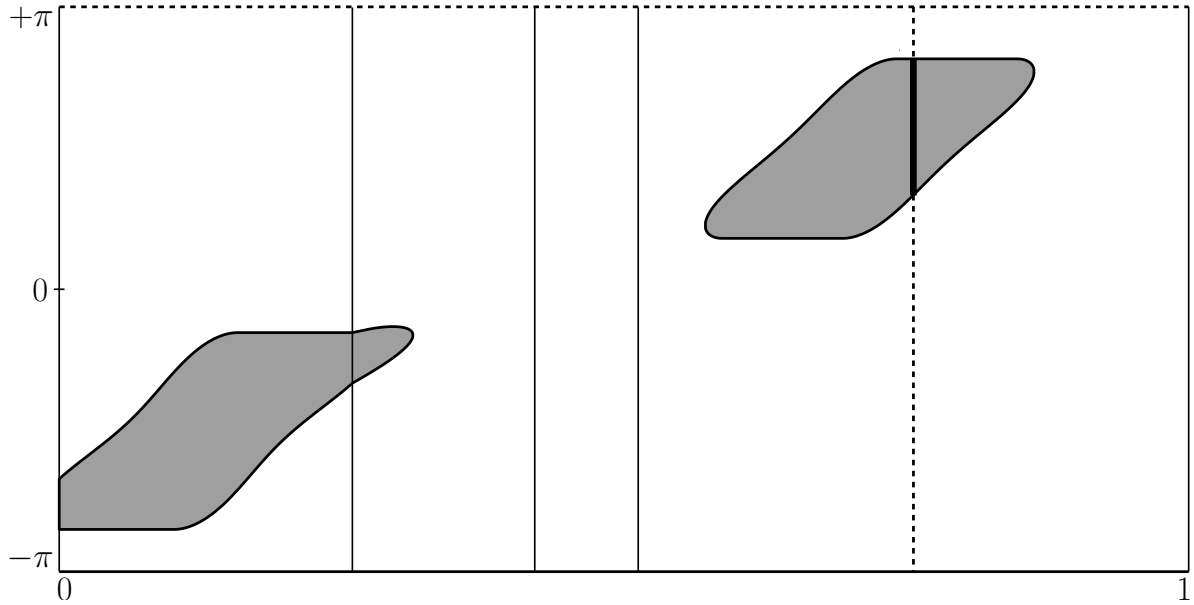


Configuration-space obstacles

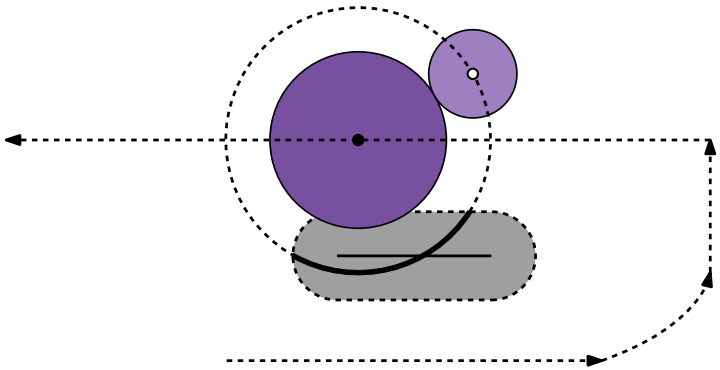


One $\mathcal{C}$-space obstacle:

- can have multiple components, but total complexity $O(\#\text{sections})$

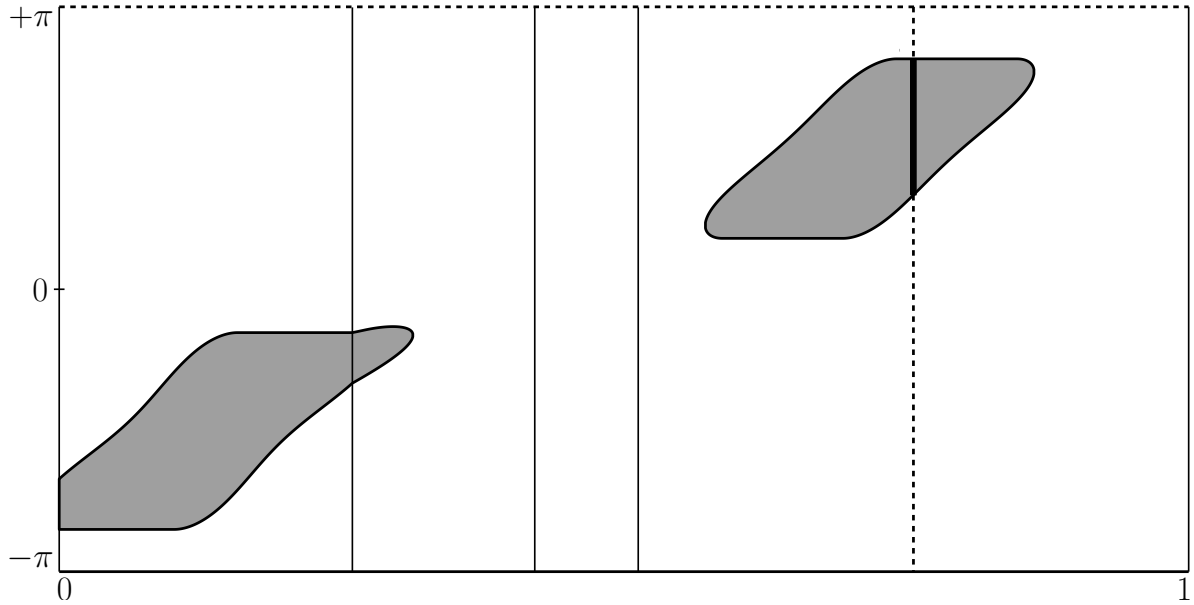


Configuration-space obstacles

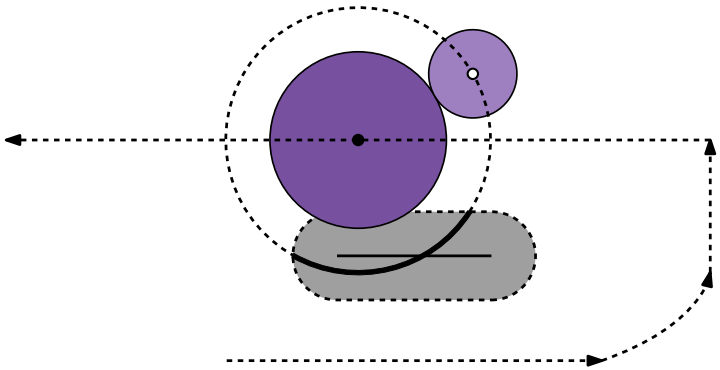


One $\mathcal{C}$-space obstacle:

- can have multiple components, but total complexity $O(\#\text{sections})$
- "weird" curves

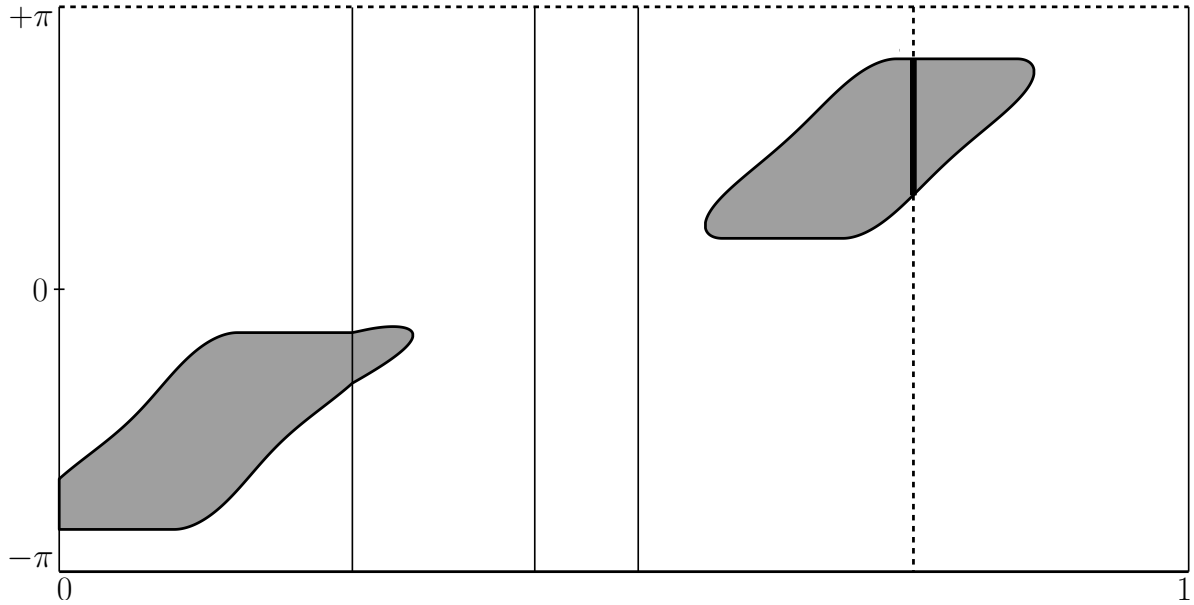


Configuration-space obstacles

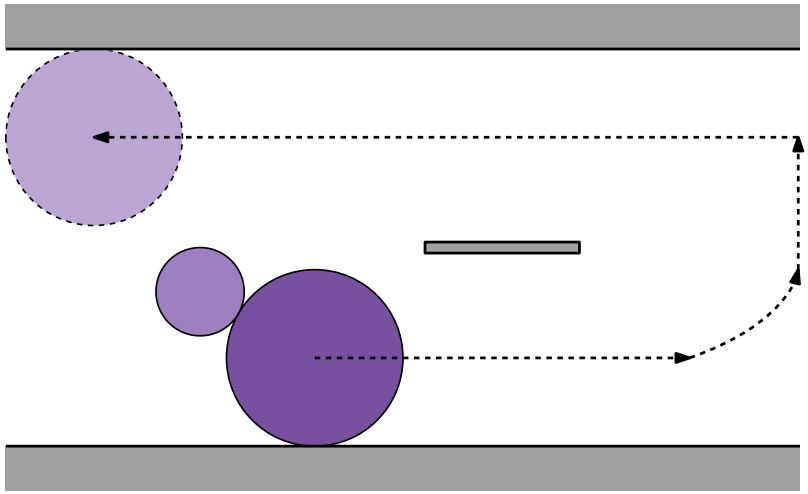


One $\mathcal{C}$-space obstacle:

- can have multiple components, but total complexity $O(\#\text{sections})$
- "weird" curves, but can be handled exactly with smart representation



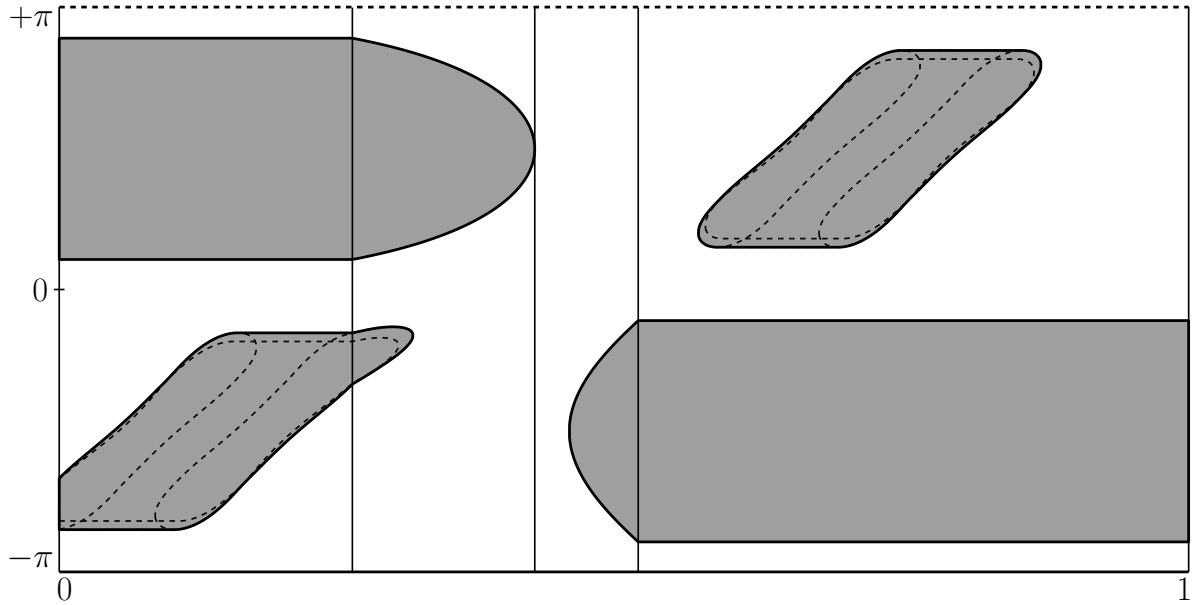
Configuration-space obstacles



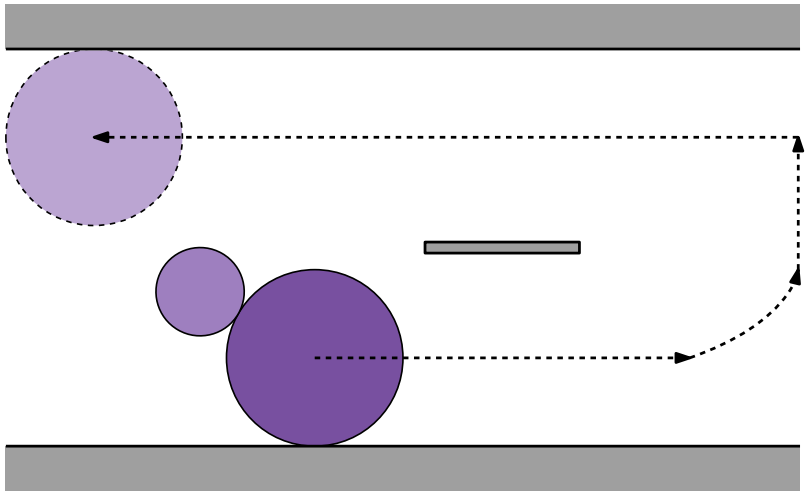
One $\mathcal{C}$-space obstacle:

- can have multiple components, but total complexity $O(\#\text{sections})$
- "weird" curves, but can be handled exactly with smart representation

Union of all $\mathcal{C}$-space obstacles



Configuration-space obstacles

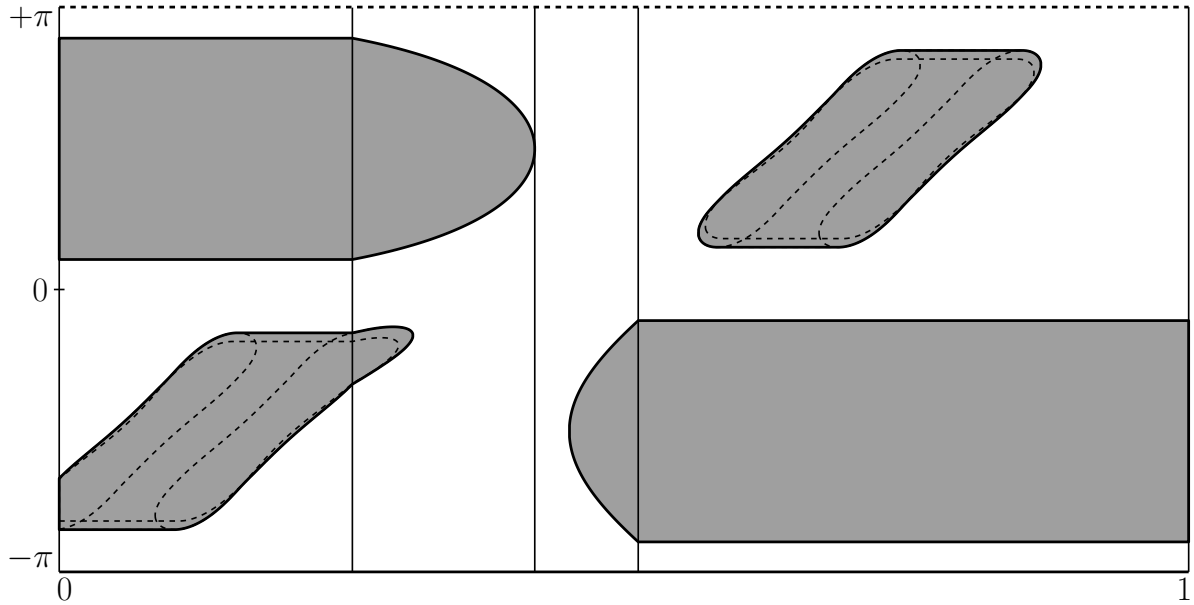


One $\mathcal{C}$-space obstacle:

- can have multiple components, but total complexity $O(\#\text{sections})$
- "weird" curves, but can be handled exactly with smart representation

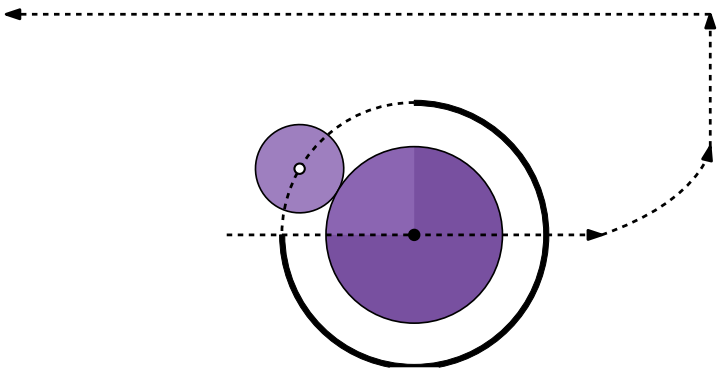
Union of all $\mathcal{C}$-space obstacles:

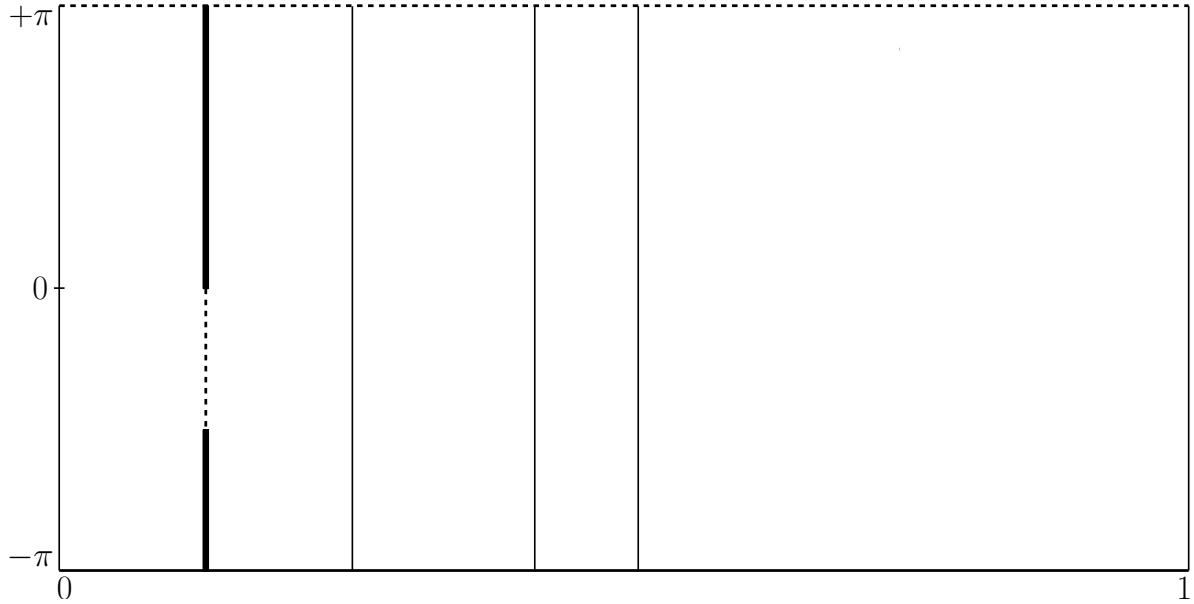
- linear complexity: $O(\#\text{obstacles} \times \#\text{sections})$



The forbidden push range

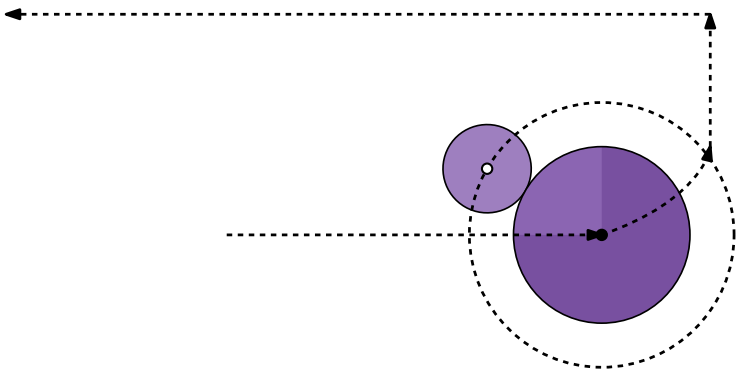
A configuration is also in the forbidden space if the pusher is outside the push range.

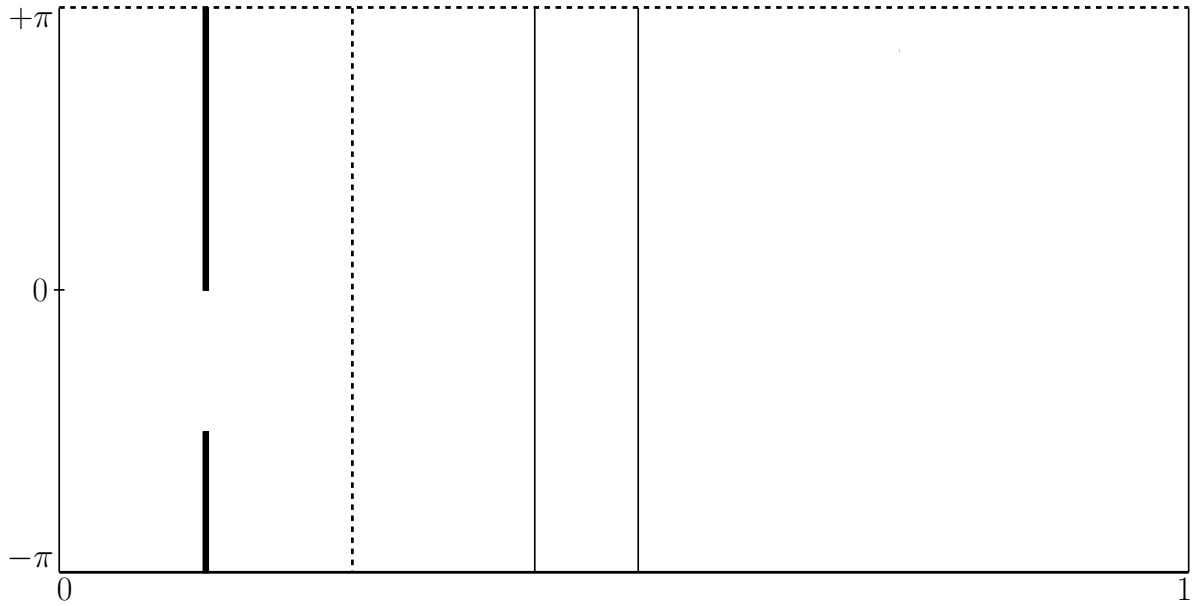




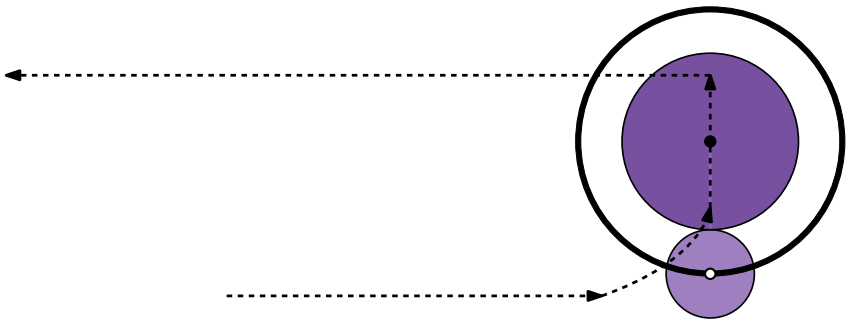
The forbidden push range

A configuration is also in the forbidden space if the pusher is outside the push range.





The forbidden push range



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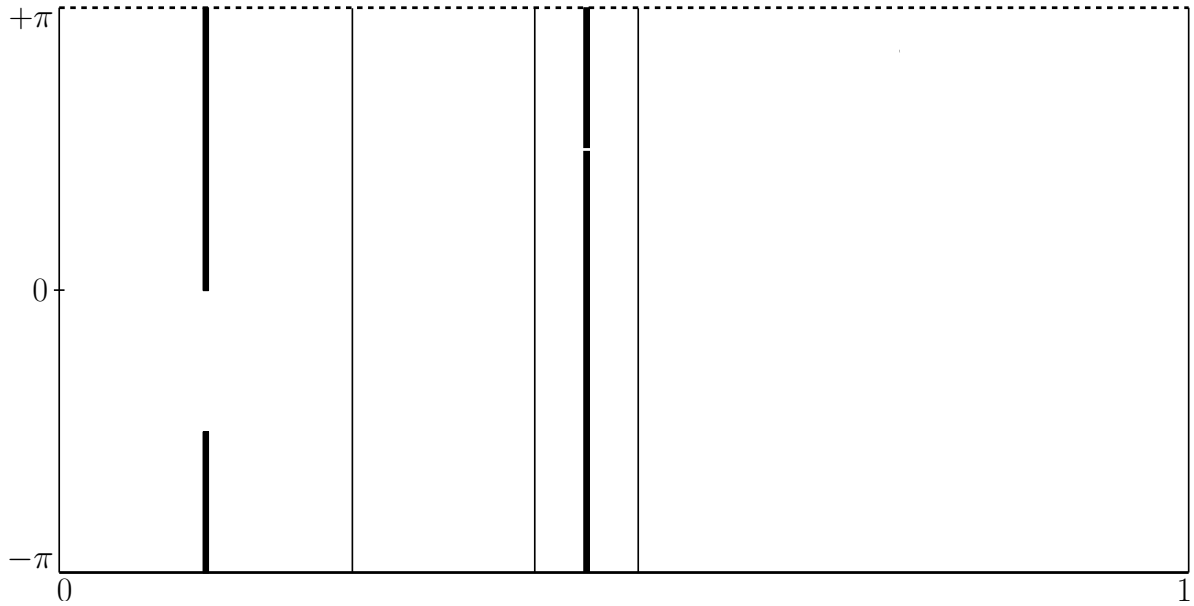
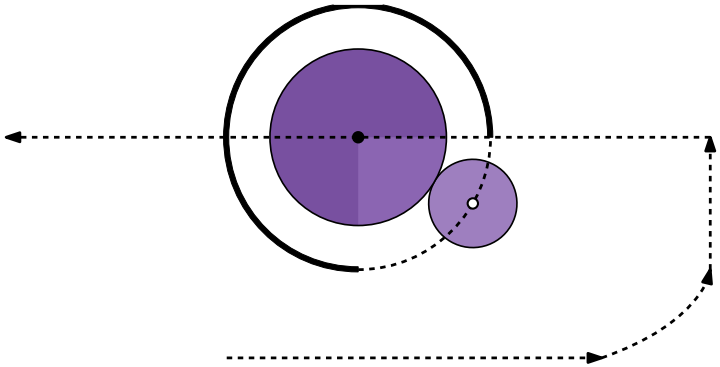
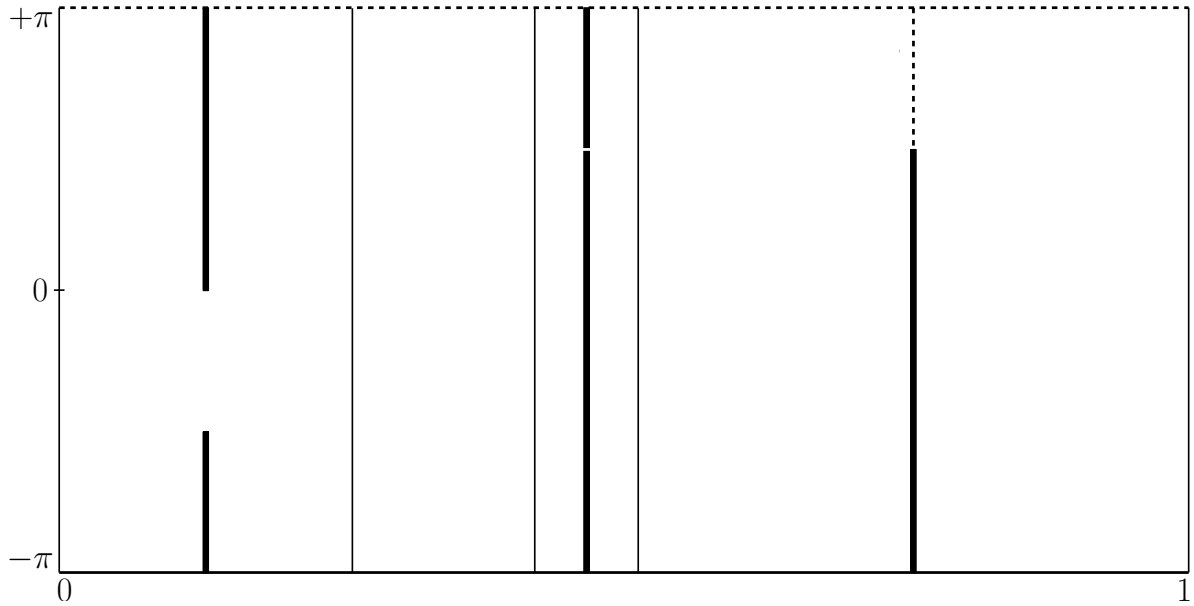


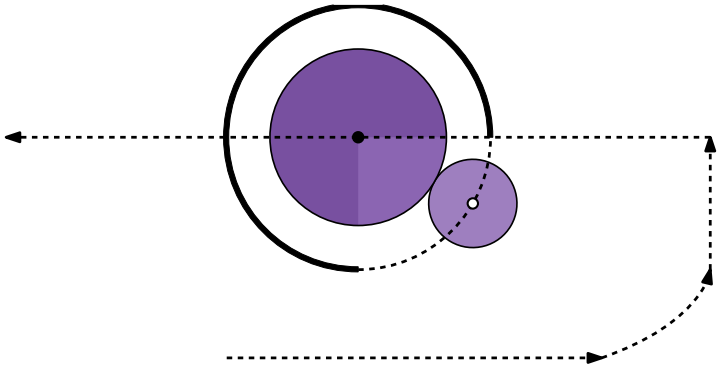
The forbidden push range



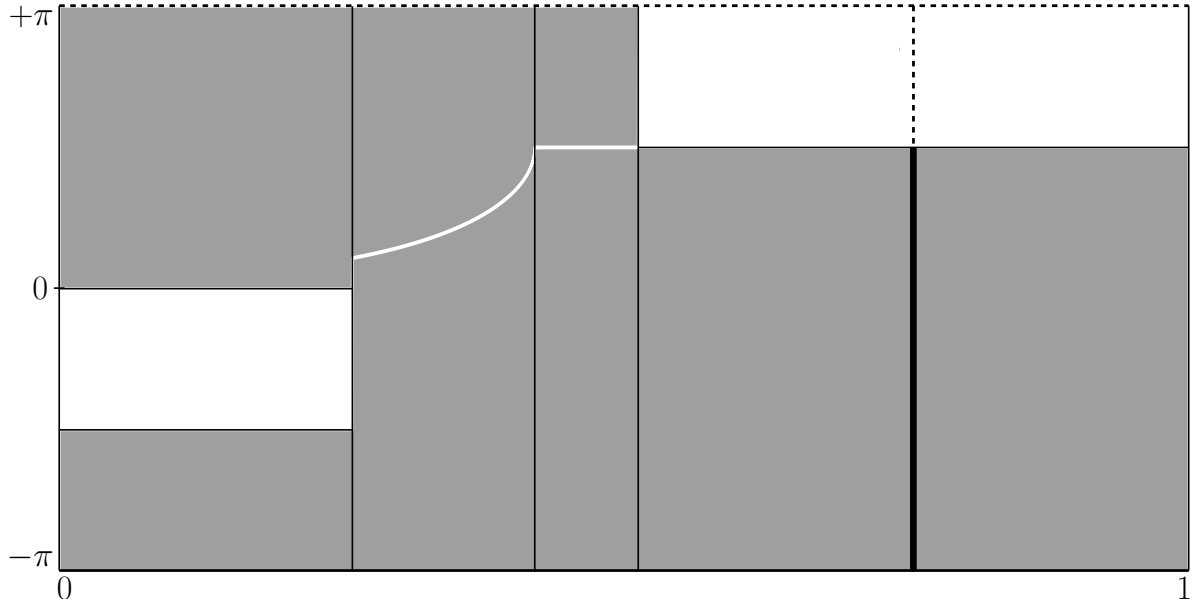
A configuration is also in the forbidden space if the pusher is outside the push range.



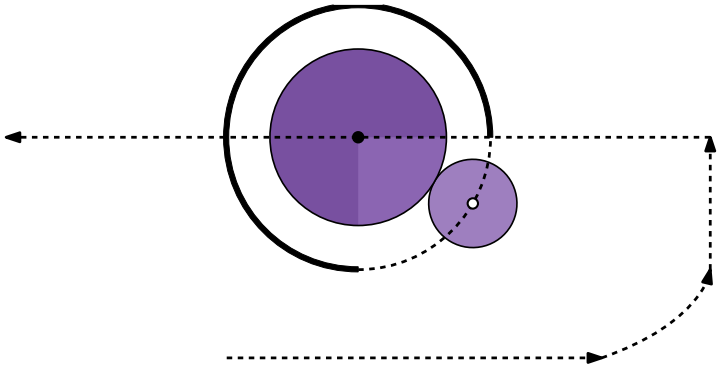
The forbidden push range



A configuration is also in the forbidden space if the pusher is outside the push range.

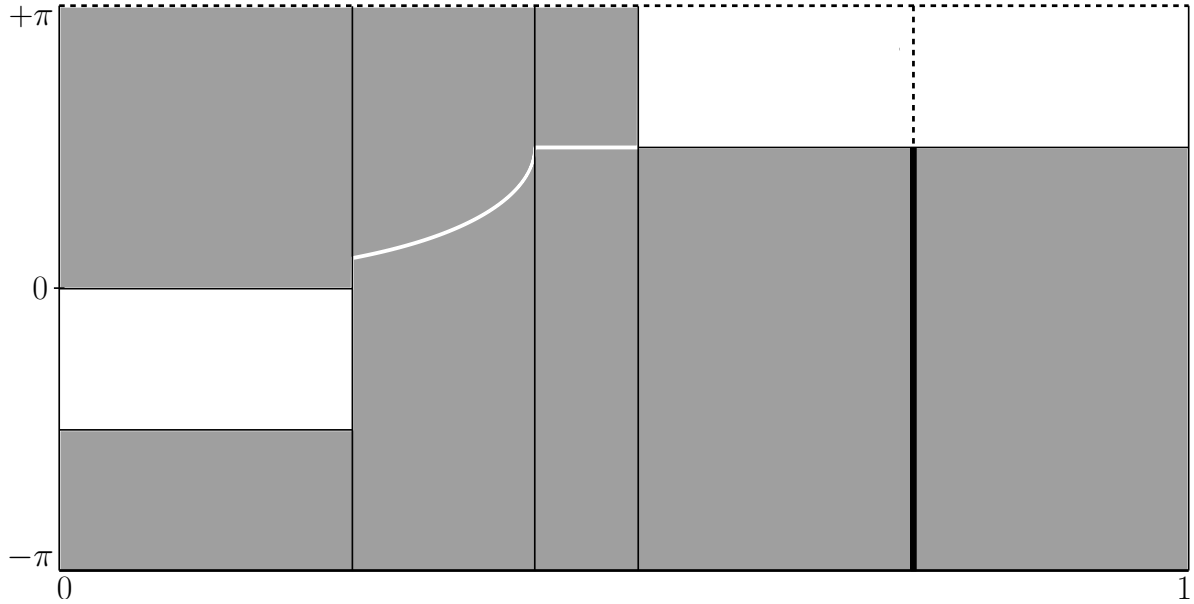


The forbidden push range

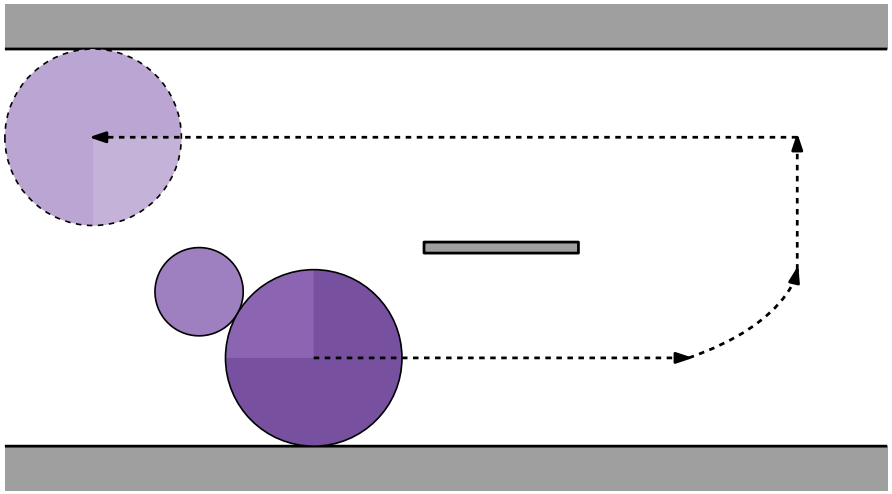


A configuration is also in the forbidden space if the pusher is outside the push range.

Forbidden push range also has $O(\#\text{sections})$ complexity.



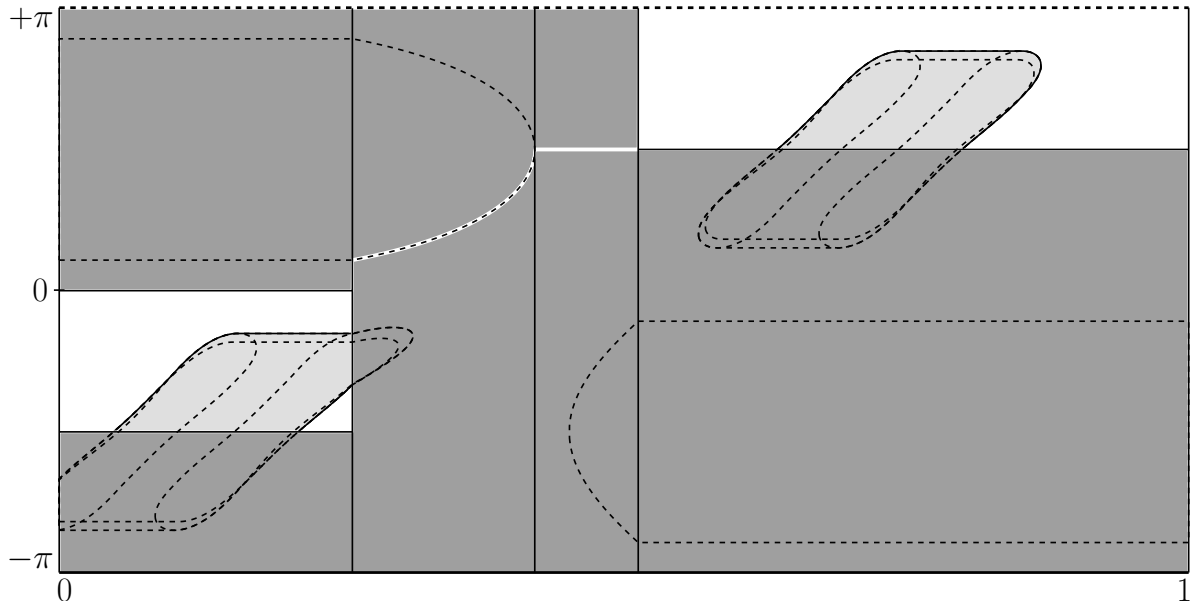
The forbidden push range



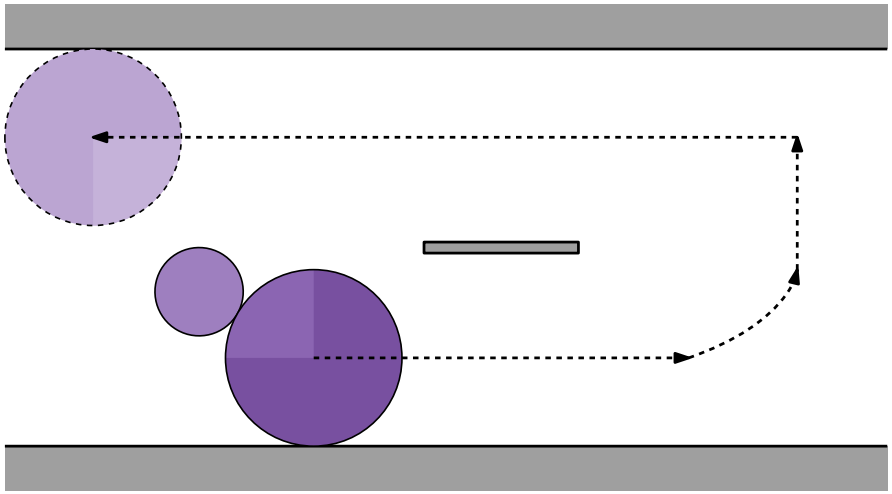
A configuration is also in the forbidden space if the pusher is outside the push range.

Forbidden push range also has $O(\#\text{sections})$ complexity.

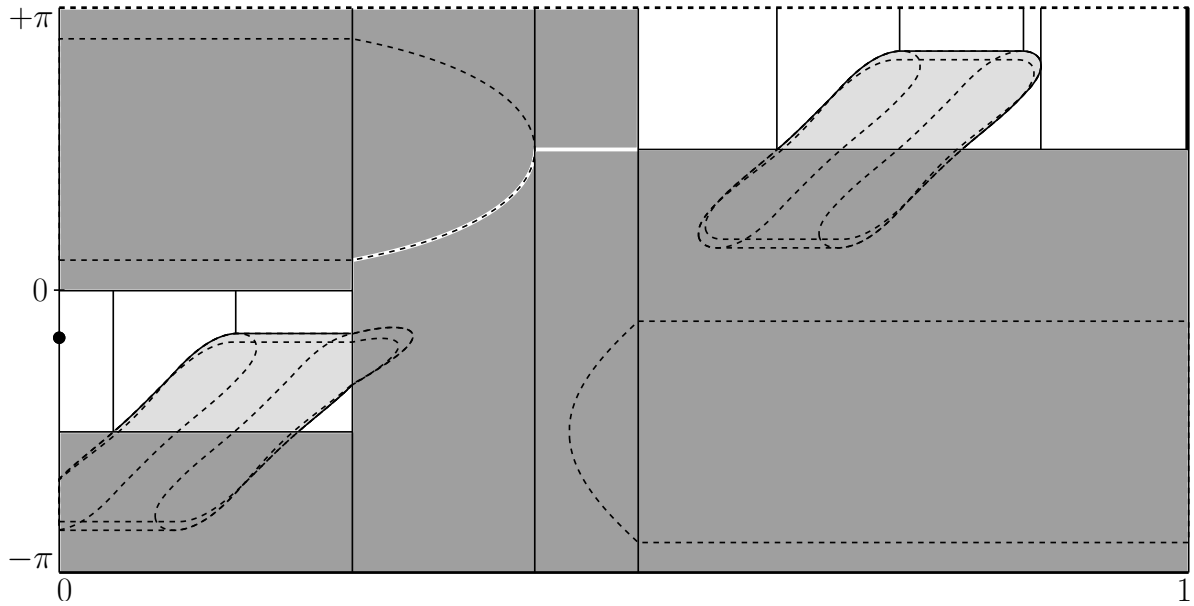
Total forbidden space: $O(\#\text{obstacles} \times \#\text{sections})$



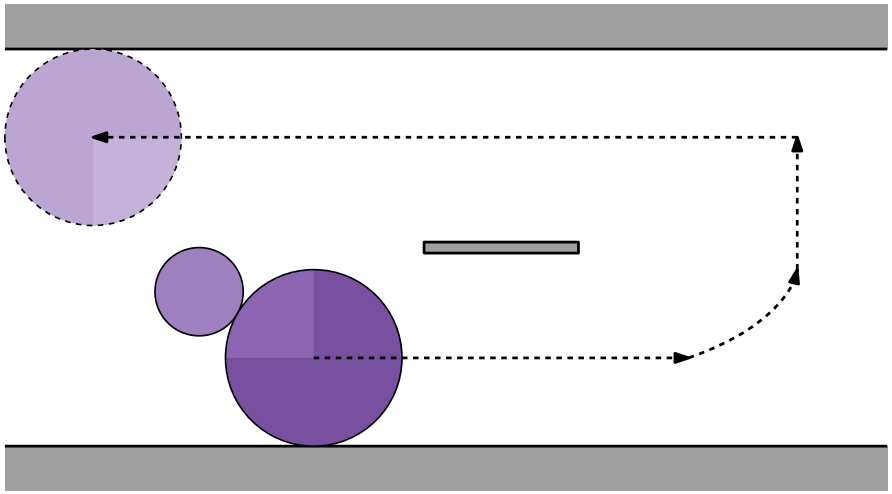
Finding a push plan



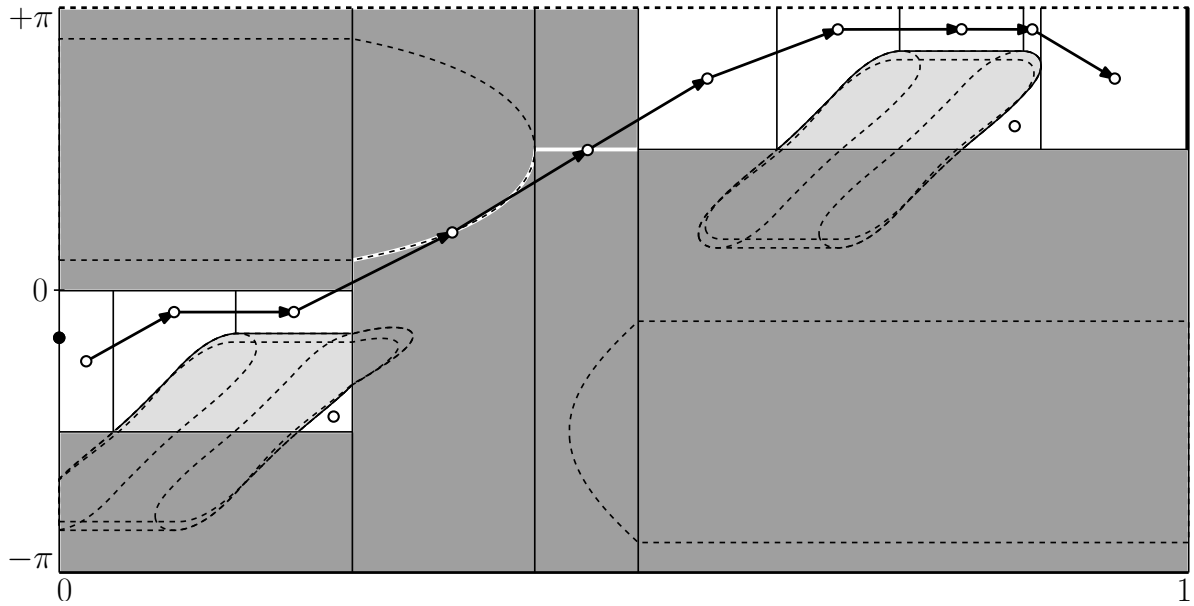
- Create vertical decomposition of the free space



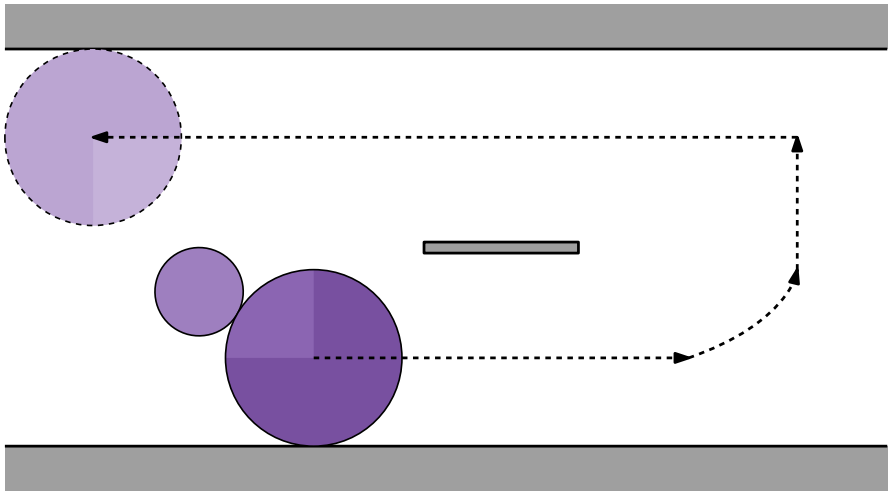
Finding a push plan



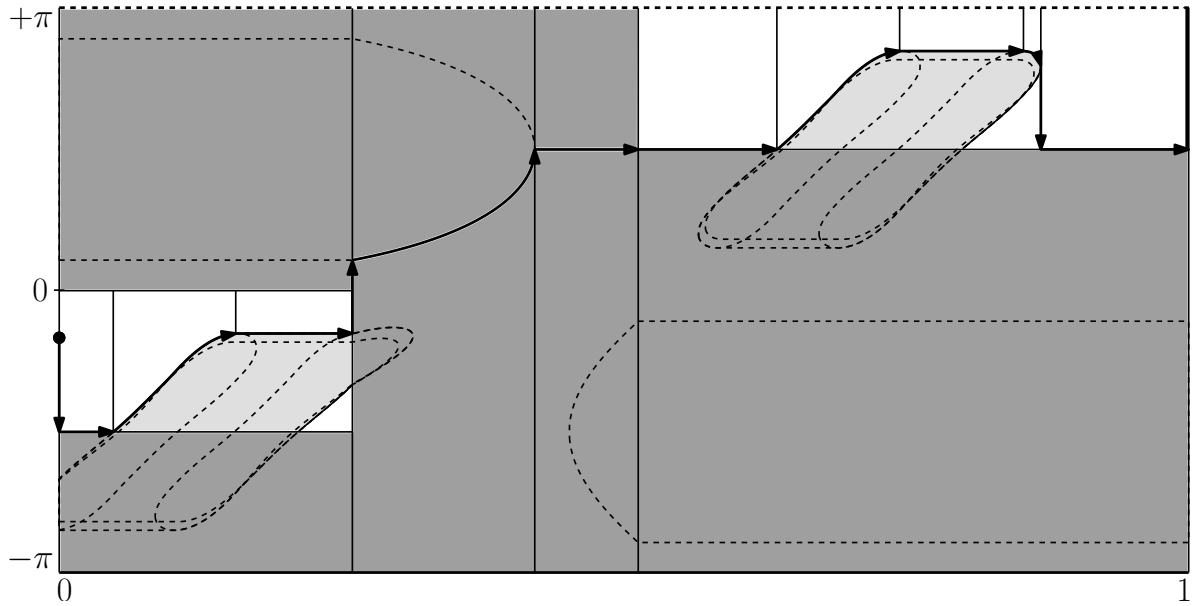
- Create vertical decomposition of the free space
- Find path through graph induced by cells



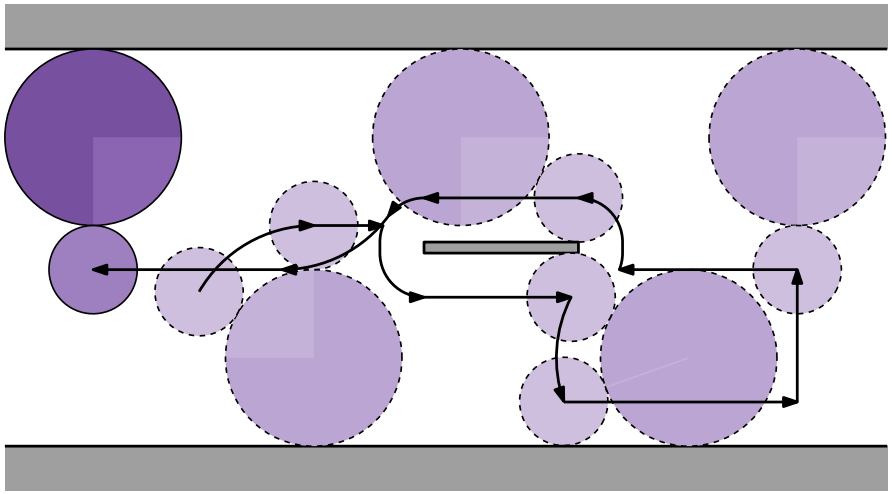
Finding a push plan



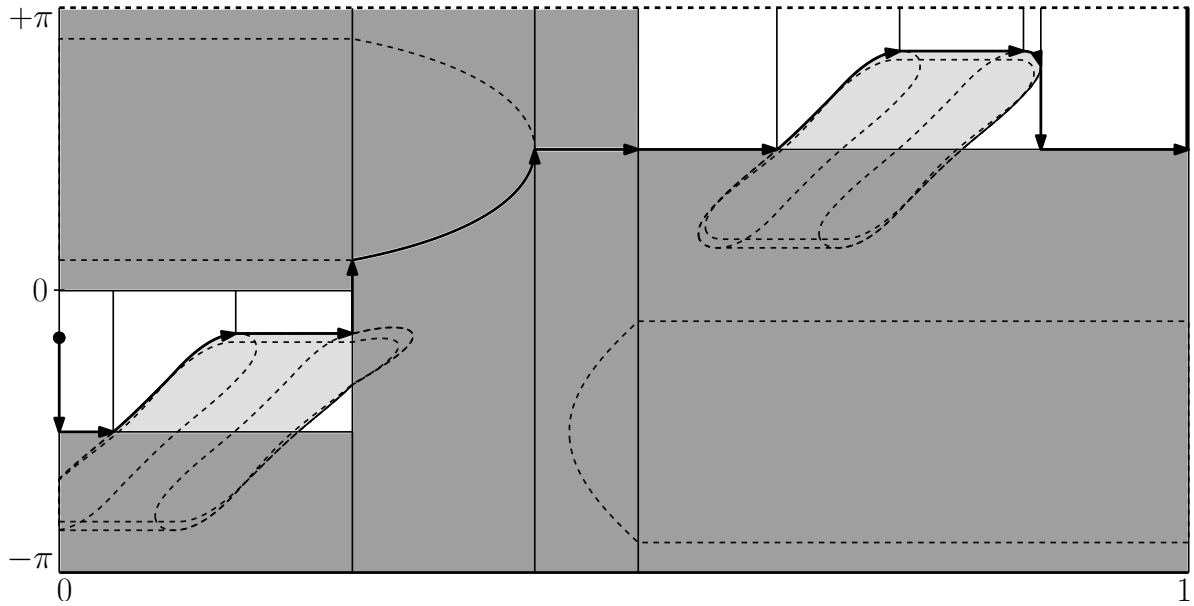
- Create vertical decomposition of the free space
- Find path through graph induced by cells
- Follow boundaries of those cells



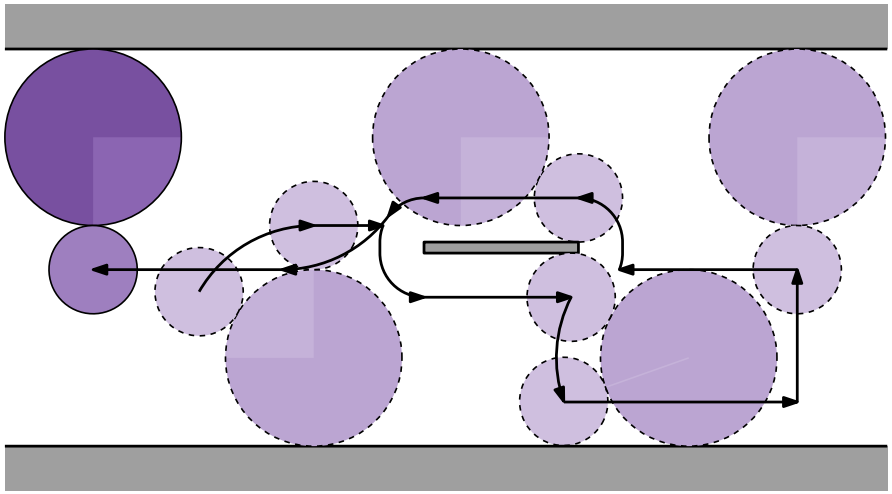
Finding a push plan



- Create vertical decomposition of the free space
- Find path through graph induced by cells
- Follow boundaries of those cells

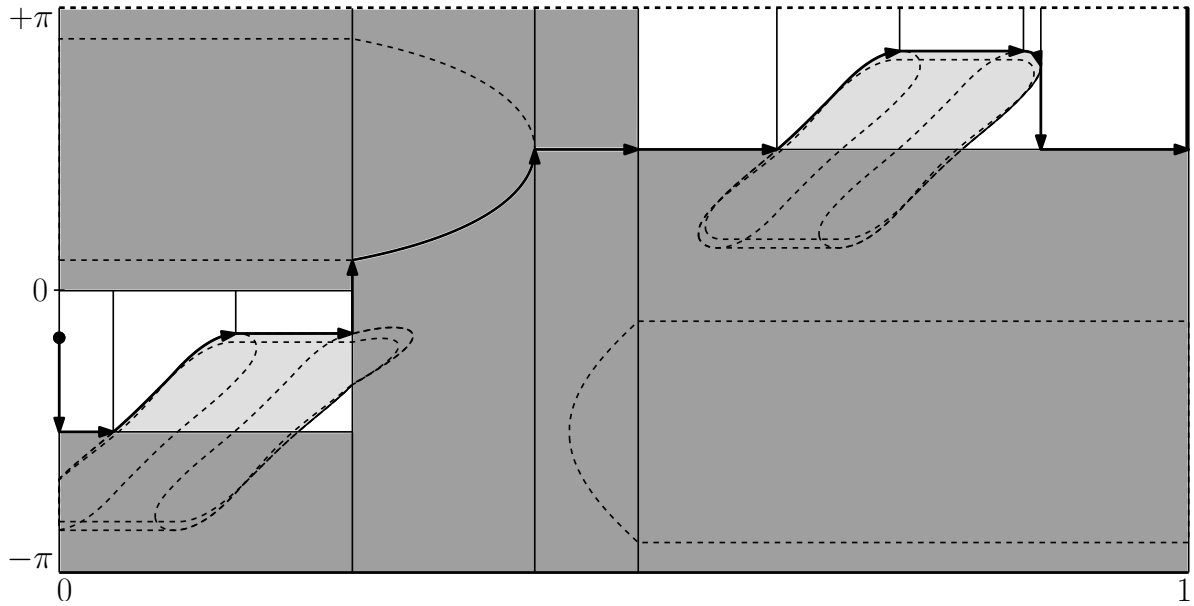


Finding a push plan

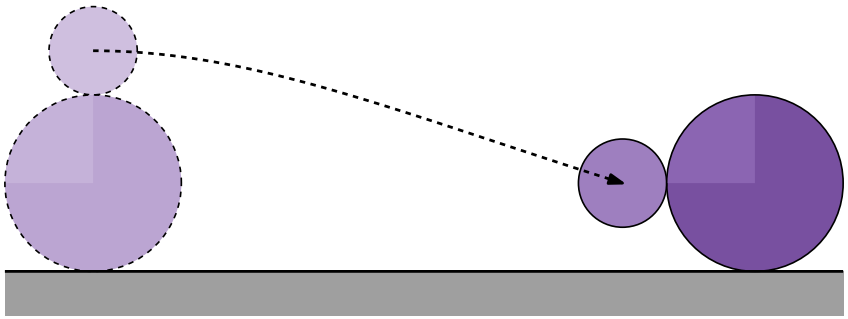


- Create vertical decomposition of the free space
- Find path through graph induced by cells
- Follow boundaries of those cells

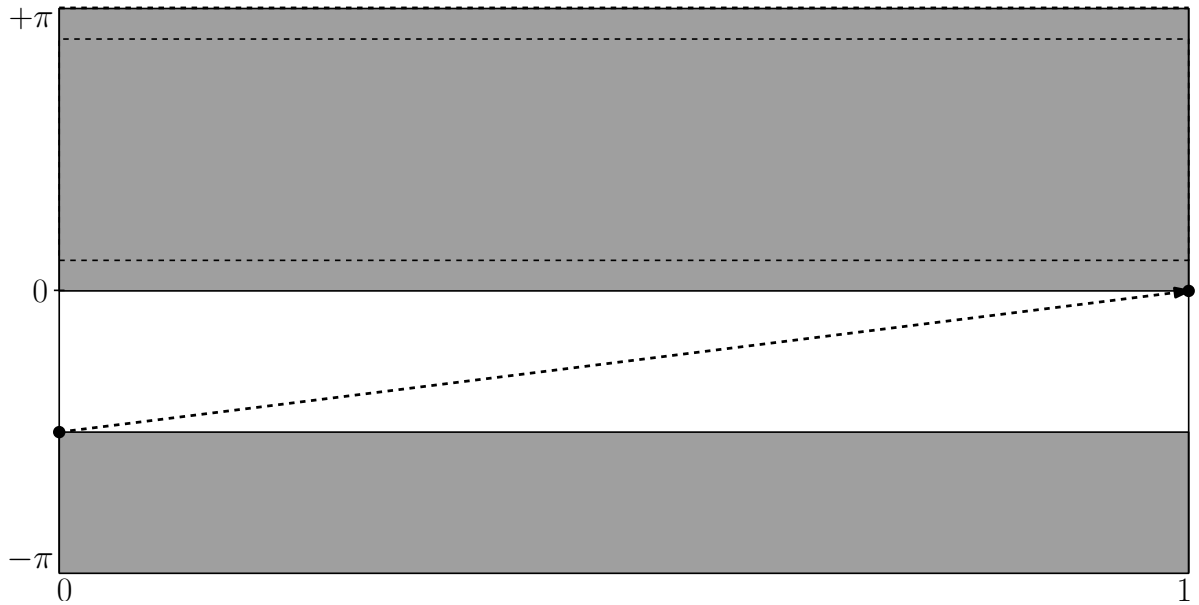
Fast method, but it gives a “bad” push plan. Can we find something shorter?



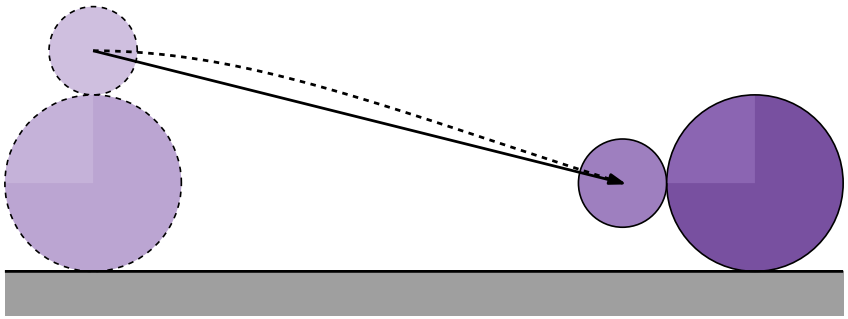
Finding a shortest push plan



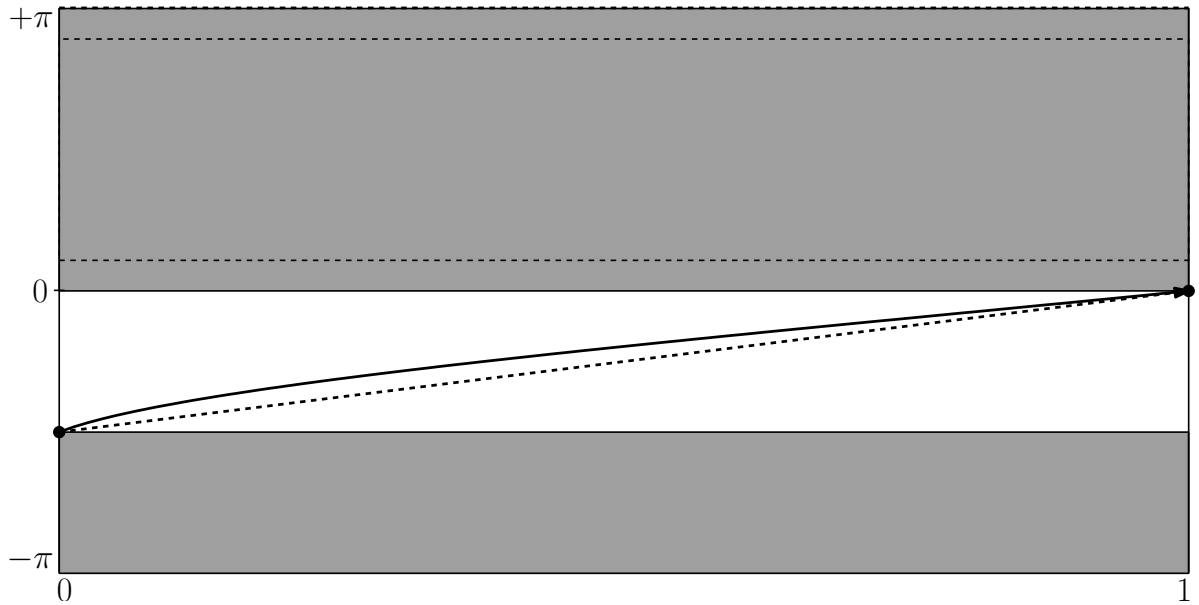
To minimize the distance traveled by the pusher, take the shortest path through free space?



Finding a shortest push plan



To minimize the distance traveled by the pusher, take the shortest path through free space? No!



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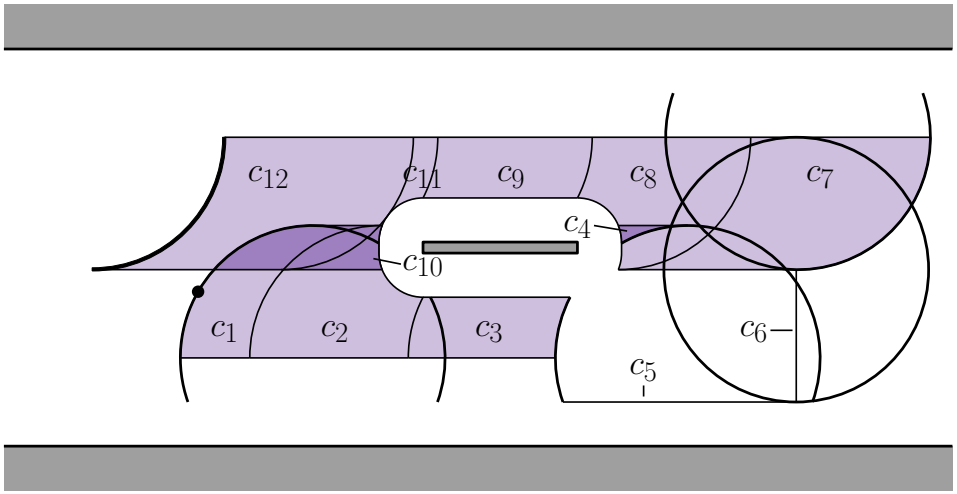
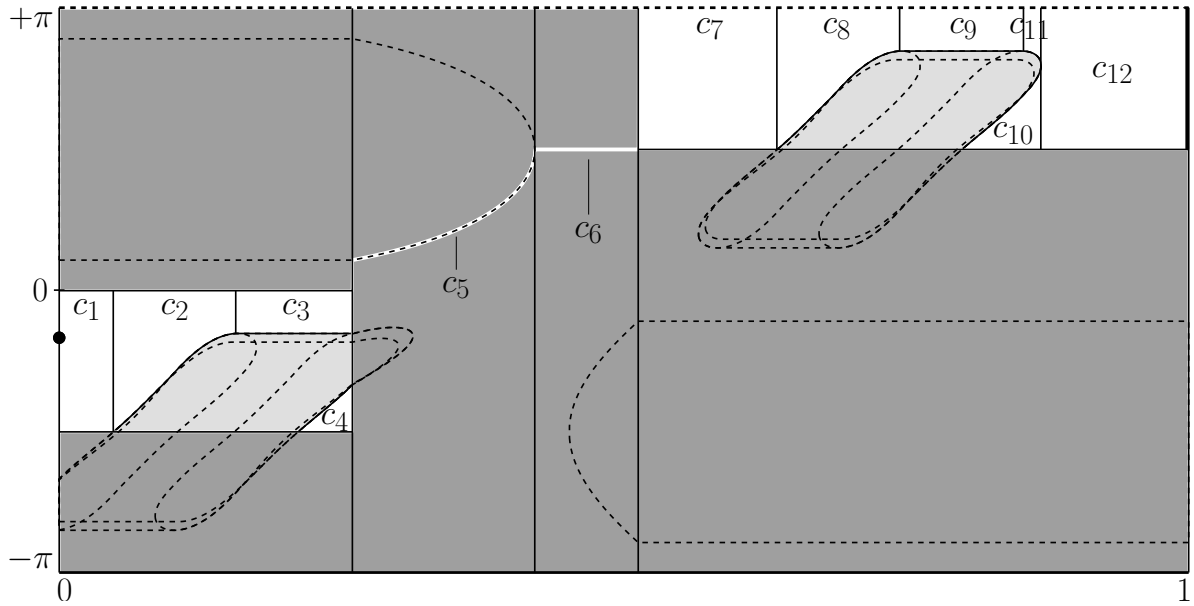
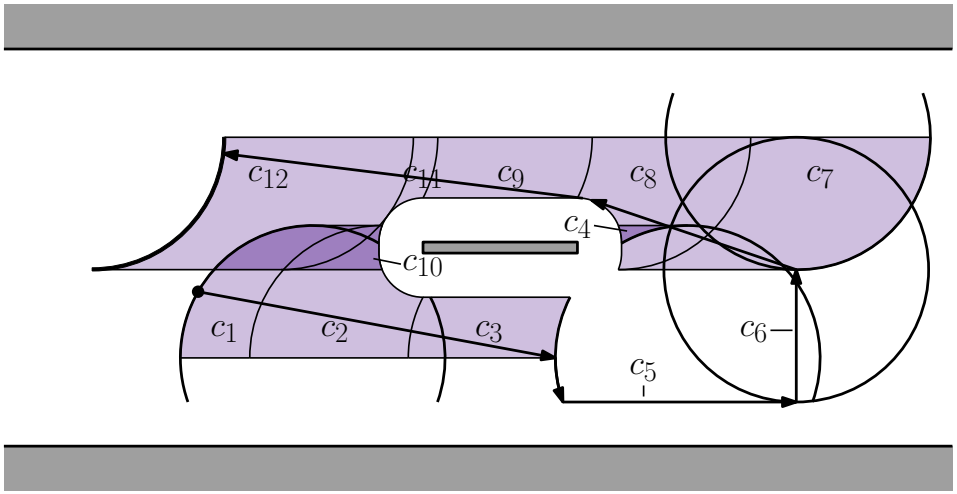


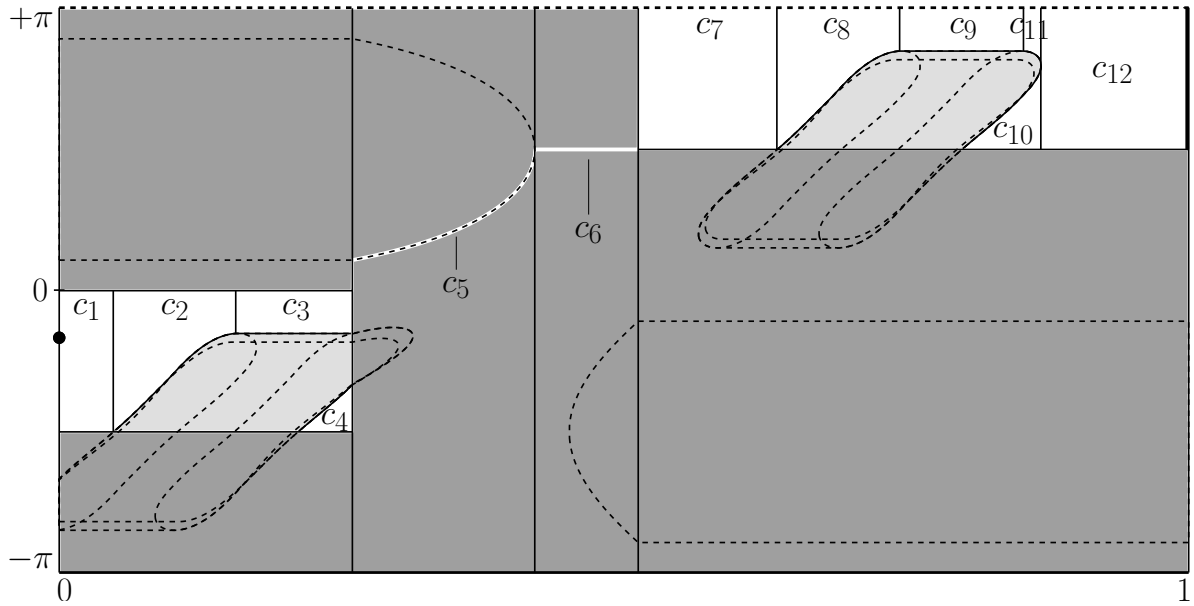
- Compute the area of allowed pusher positions for each cell



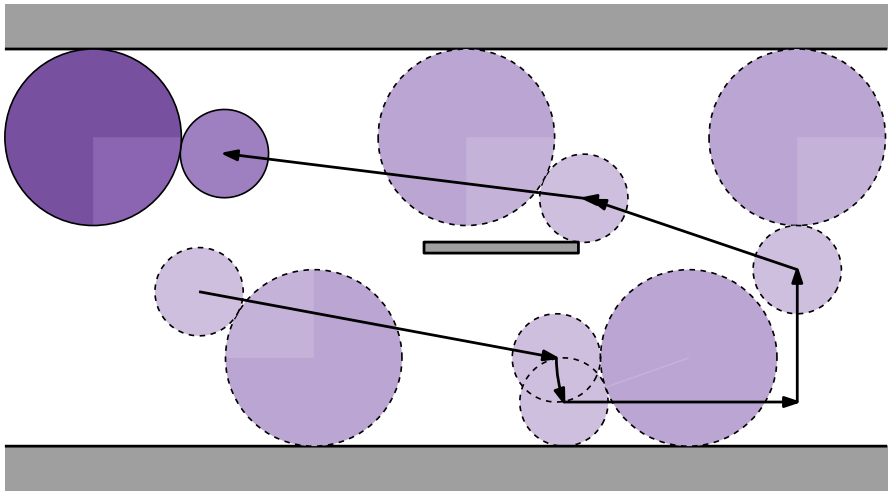
Finding a shortest push plan



- Compute the area of allowed pusher positions for each cell
- Find a shortest path through these work-space cells



Finding a shortest push plan



- Compute the area of allowed pusher positions for each cell
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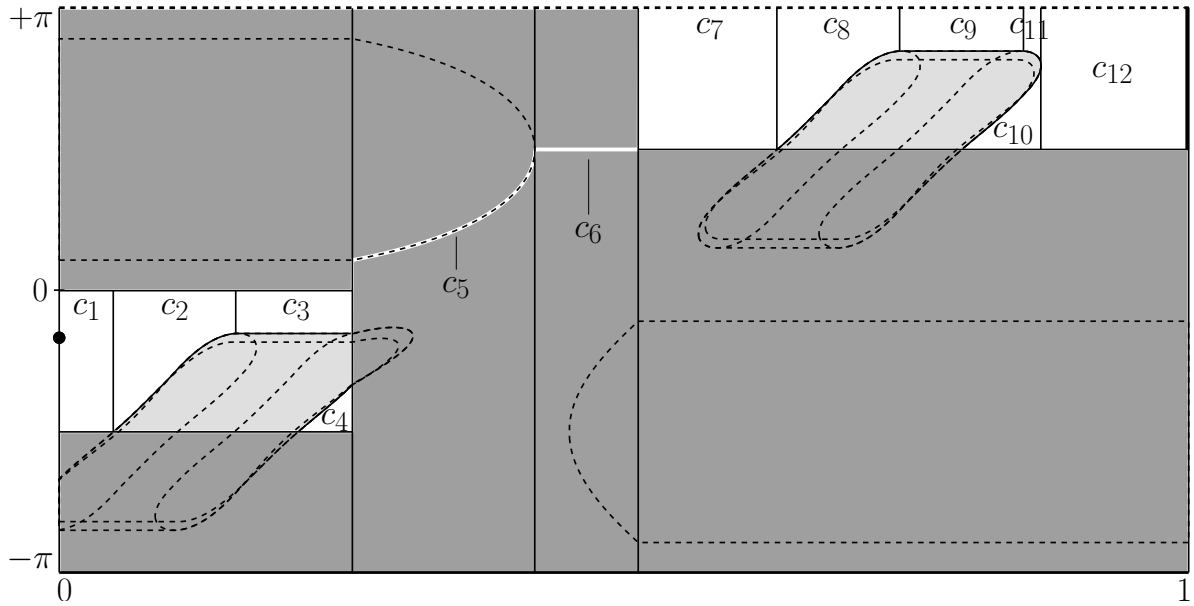

New contributions

- Our algorithm for contact-preserving push plans is more general:
  - it can solve the pulling problem

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  while having a better running time!

New contributions

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	High obstacle density		Low obstacle density	
	Nieuwenh.	Our method	Nieuwenhuisen	Our method
Preprocess	$n^2 \log n$	$n \log n$ (*)	$n^2 \log n$	$n \log n$ (*)
Any CPPP	$kn \log n$	$kn \log n$ (*)	$(k+n)\log(k+n)$	$(k+n)\log(k+n)$

(*) These entries are expected times. For the worst-case times, replace $\log n$ by $\log^2 n$.

New contributions

- Our algorithm for contact-preserving push plans is more general while having a better running time!
- Can also compute shortest contact-preserving push plans

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Preprocess	$n^2 \log n$	$n \log n$ (*)	$n^2 \log n$	$n \log n$ (*)
Any CPPP	$kn \log n$	$kn \log n$ (*)	$(k+n)\log(k+n)$	$(k+n)\log(k+n)$
A shortest CPPP	—	$k^2 n^2 \log(kn)$	—	$(k+n)\log(k+n) + k^2 \log k$ (**)

(*) These entries are expected times. For the worst-case times, replace $\log n$ by $\log^2 n$.

(**) This yields a "quasi-optimal" solution.

New contributions

- Our algorithm for contact-preserving push plans is more general while having a better running time!
- Can also compute shortest contact-preserving push plans
- Can also compute unrestricted push plans, which we proved is sometimes necessary

	High obstacle density		Low obstacle density	
	Nieuwenh.	Our method	Nieuwenhuisen	Our method
Preprocess	$n^2 \log n$	$n \log n$ (*)	$n^2 \log n$	$n \log n$ (*)
Any CPPP	$kn \log n$	$kn \log n$ (*)	$(k+n)\log(k+n)$	$(k+n)\log(k+n)$
A shortest CPPP	–	$k^2 n^2 \log(kn)$	–	$(k+n)\log(k+n) + k^2 \log k$ (**)
Any UPP	–	$kn \log(kn) + kn^2 \log n$	–	$(k+n)\log(k+n) + kn$

(*) These entries are expected times. For the worst-case times, replace $\log n$ by $\log^2 n$.

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Further research

- Non-disk-shaped object and/or pusher

Further research

- Non-disk-shaped object and/or pusher
- Finding shortest unrestricted push plans

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- Non-disk-shaped object and/or pusher
- Finding shortest unrestricted push plans
- Solving the global problem (i.e. no object path given) through the configuration-space method

Further research

- Non-disk-shaped object and/or pusher
- Finding shortest unrestricted push plans
- Solving the global problem (i.e. no object path given) through the configuration-space method
- Improved running times (especially for shortest push plans)

Questions?