Assignment #1

UW-Madison MATH 421

GEOFF YOERGER

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Exercise One:

a)

```
If $f(x) = x^n$, then
$$
f^\prime(x) = n x^{n-1}.
$$
```

If $f(x) = x^n$, then

$$f'(x) = nx^{n-1}.$$

b)

```
If $n \neq -1$, then
$$
\int x^{n} dx = \frac{1}{n+1} x^{n+1} + C.
$$
```

If $n \neq -1$, then

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C.$$

c)

```
The derivative of a function $f$ at $x=a$ is
$$
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.
$$
```

The derivative of a function f at $x = a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Exercises Two:

a)

```
The number $e$ is defined by
$$
e = \lim_{n \to \inf} \left( 1 + \frac{1}{n} \right) ^ {n}.
$$
```

The number e is defined by

$$
e = \lim_{n \to \infty} \left(1 + \frac{1}{n} \right)^{n}.
$$

b)

```
If $f$ is a continuous function, then
$$
\frac{d}{dx} \left[ \int_{a}^{x} f(t) dt \right] = f(x).
$$
```

If f is a continuous function, then

$$
\frac{d}{dx} \left[\int_{a}^{x} f(t) dt \right] = f(x).
$$

Exercise Three:

```
\begin{center}
    \begin{tabular}{|rl|cc|}
    \hline
    First Name & Last Name & Ice Cream Flavor & Number of Scoops \\
    \hline
    Alexa & Leal & Vanilla & $4$ \\
    Julia & Maschi & Chocolate & $2$ \\
    Johnny & Tran & Strawberry & $18$ \\
    Geoff & Yoerger & Chocolate Malt & $\infty$ \\
    \hline
    \end{tabular}
\end{center}
```

First Name	Last Name	Ice Cream Flavor	Number of Scoops
Alexa	Leal	Vanilla	4
Julia	Maschi	Chocolate	2
Johnny	Tran	Strawberry	18
Geoff	Yoerger	Chocolate Malt	∞

Exercise Four:

```
$$
\det
\begin{pmatrix}
    a & b \\
    c & d \\
\end{pmatrix}
= ad - bc.
$$
```

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

Exercise Five:

1)

```
\newcommand{\Rb}{\mathbb{R}}
\newcommand{\Nb}{\mathbb{N}}
\newcommand{\Zb}{\mathbb{Z}}
\newcommand{\Qb}{\mathbb{Q}}
\newcommand{\Cb}{\mathbb{C}}

$$
\Nb \subset \Zb \subset \Qb \subset \Rb \subset \Cb
$$
```

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

2)

```
\newcommand{\pder}[2]{\frac{\partial #1}{\partial #2}}

If $z = x^2 + xy + y^2$, then
$$
\pder{z}{x} = 2x + y
$$
```

If $z = x^2 + xy + y^2$, then

$$\frac{\partial z}{\partial x} = 2x + y$$

Exercise Six:

```
If $f(x) = x^2$, then
\begin{align*}
    f'(a) & = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\
    & = \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h} \\
    & = \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - a^2}{h} \\
    & = \lim_{h \to 0} 2a + h = 2a. \\
\end{align*}
```

If $f(x) = x^2$, then

$$\begin{aligned} f'(a) & = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\ & = \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h} \\ & = \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - a^2}{h} \\ & = \lim_{h \to 0} 2a + h = 2a. \end{aligned}$$