Assignment #2

UW-Madison MATH 421

YOUR NAME HERE
February 2, 2021

Exercise #1: Prove the following theorem by cases.

Theorem. *If x is an integer, then $x^2 + 3x - 9$ is odd.*

Proof. Add your proof here! $\square$

Exercise #2: Prove the following theorem in two ways: by contrapositive and by contradiction.

Theorem. *Suppose x is an integer. If x^2 is even, then x is even.*

Proof by contrapositive. Add your proof here! $\square$

Proof by contradiction. Add your proof here! $\square$

Exercise #3: Prove the following theorem.

Theorem. *If the name of a month has 5 or more characters, then a 4-letter word can be formed using those characters.*

Proof. Add your proof here! (Hint cases might work) $\square$

Exercise #4: Prove the following theorem.

Theorem. *For all numbers x and y, $(x+y)^2 = x^2 + y^2$ if and only if $x = 0$ or $y = 0$.*

Proof. ($\Rightarrow$):
($\Leftarrow$): $\square$

Exercise #5: Using only properties P1-P12 and noting every time you use one, prove the following theorem.

Theorem. *Suppose a and b are numbers. If $ab = 1$, then $b = a^{-1}$.*

Proof. Add your proof here! $\square$

Exercise #6: Using only properties P1-P12 and noting every time you use one, prove the following theorem.

Theorem. *Suppose a and b are numbers. If $a \neq 0$ and $b \neq 0$, then $(ab)^{-1} = a^{-1}b^{-1}$.*

Proof. Add your proof here! $\square$

Exercise #7: Using only properties P1-P12 and noting every time you use one, prove the following theorem.

Theorem. *Suppose a, b, and c are numbers. If $a < b$ and $0 < c$, then $ac < bc$.*

Proof. Add your proof here! $\square$

$\boxed{\textbf{Exercise \#8:}}$ Prove the following: if x and y are numbers, then

1. $|xy| = |x|\,|y|$,
2. $|x - y| \leq |x| + |y|$,
3. $|x| - |y| \leq |x - y|$.

Hint: you can give a short proof of (2) and (3) by reducing to the triangle inequality. You do not have to reference properties P1-P12.

Proof of (1). Add your proof here! $\square$

Proof of (2). Add your proof here! $\square$

Proof of (3). Add your proof here! $\square$