Assignment #3

UW-Madison MATH 421

YOUR NAME HERE
Due: February 16, 2020

Exercise #1: Prove the following theorem by induction.

Theorem. *If n is a natural number, then* $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

Proof. We argue by induction.
Base case:
Induction step:
$\square$

Exercise #2: Prove the following theorem by induction.

Theorem (Bernoulli's inequality)**.** *Suppose n is a natural number and x is a real number. If $x > -1$, then*

$$(1+x)^n \geq 1 + nx.$$

Proof. We argue by induction.
Base case:
Induction step:
$\square$

Exercise #3: For this problem we need the following definition:

Definition. An integer n is *divisible* by an integer k if the ratio n/k is an integer.

For example: -3, 0, 3, 6 are all divisible by 3 while 1, 2, 4, 5 are not divisible by 3. Prove the following:

Theorem. *Suppose n is an integer. If n^2 is divisible by 3, then n is divisible by 3.*

Proof. (Hint: if n is not divisible by 3, then $n = 3k+1$ or $n = 3k+2$ for some integer k.) $\square$

Exercise #4: Prove that $\sqrt{3}$ is irrational.

Proof. $\square$

Exercise #5: Prove that $\sqrt{2}+\sqrt{3}$ and $\sqrt{2}-\sqrt{3}$ are both irrational.

Proof. (Hint: $(a+b)(a-b) = a^2 - b^2$) $\square$

Exercise #6: Spivak, Chapter 3, Problem 14.

Solution. (justify your answer)
$\square$

Exercise #7: Spivak, Chapter 3, Problem 23.

Proof of (a). $\square$

Proof of (b). $\square$

Exercise #8: Spivak, Chapter 3, Problem 26.

Proof. $\square$

1 Extra Credit Questions

Each extra credit question is worth 1 extra point.

Exercise E.C.#1: Spivak, Chapter 2, Problem 17

Exercise E.C.#2: Spivak, Chapter 3, Problem 16

Exercise E.C.#3: Spivak, Chapter 3, Problem 17

Exercise E.C.#4: Spivak, Chapter 3, Problem 20 (b)