Assignment #3

UW-Madison MATH 421

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Exercise #1: Sketch the set of all points (x, y) in the plane satisfying

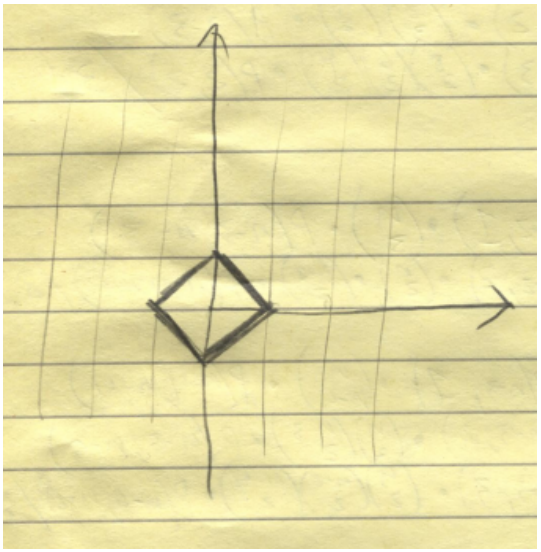

Figure 1: $|x| + |y| = 1$: A diamond

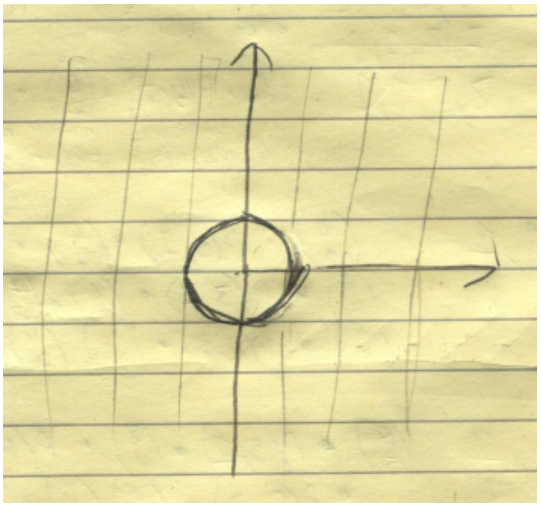

Figure 2: $x^2 + y^2 = 1$: A circle

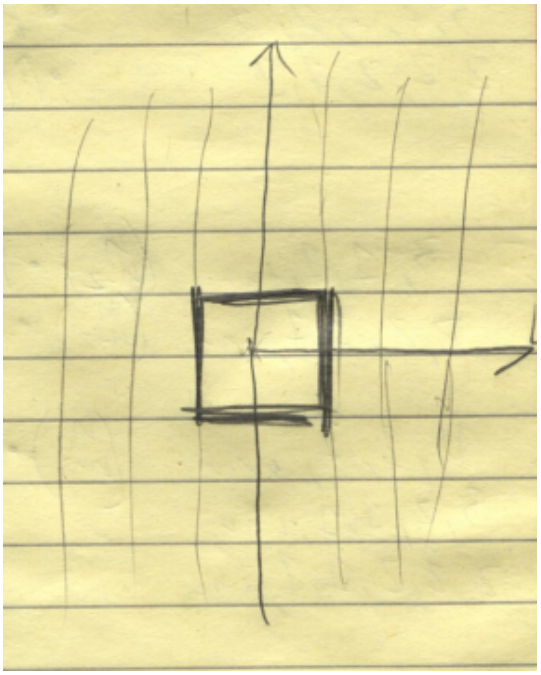

Figure 3: $\max\{|x|, |y|\} = 1$: A square

Exercise #2: Following the instructions on the previous problem: Spivak, Chapter 4, Problem 17 (i) and (ii).

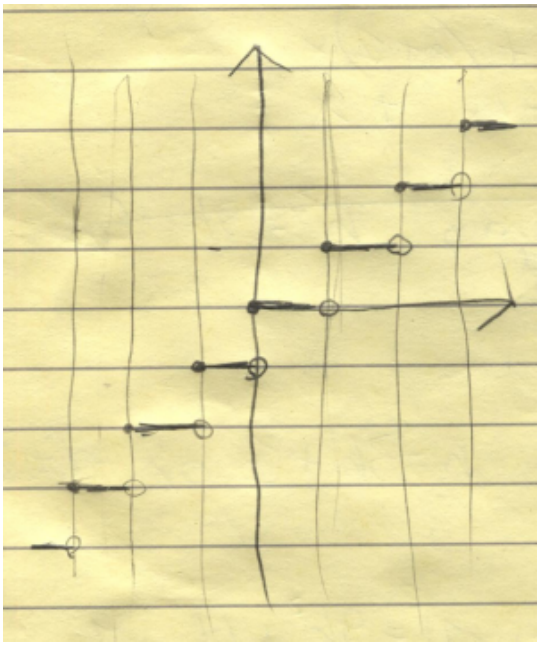

Figure 4: $f(x) = \lfloor x \rfloor$: Steps

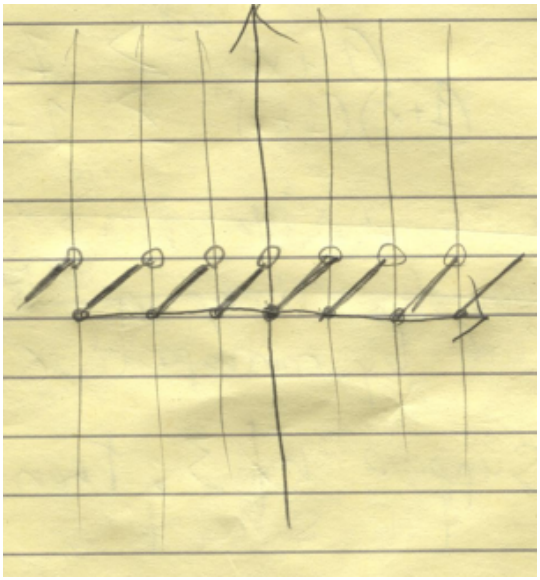

Figure 5: $f(x) = x - \lfloor x \rfloor$: Sawteeth

Exercise #3: Prove, using only the definition, that $\lim_{x \to 3} 5x = 15$.

Proof. □

Exercise #4: Prove, using only the definition, that $\lim_{x \to 2} x^2 + 2x = 8$.

Proof. □

Exercise #5: Prove the following theorem:

Theorem. *If x and y are numbers, then $||x| - |y|| \leq |x - y|$.*

Proof. (Hint: use a previous HW problem)

□

Exercise #6: Spivak, Chapter 5, 16 (a)

Proof. □

Exercise #7: Spivak, Chapter 5, Problem 12 (a)

Proof. (Hint: a proof by contradiction can work)

□

Exercise #8: Spivak, Chapter 5, Problem 37 (a)

Proof. □