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ORIGINAL ARTICLE

A methodology for error characterization and quantification in rotary joints of multi-axis machine tools

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Abstract A novel methodology was developed and implemented for error characterization, evaluation, and its quantification in a rotary joint of multi-axis machine tools by using a calibrated double ball bar (DBB) system as a working standard. This methodology greatly simplified the measurement setup and accelerated the error quantification. Moreover, the methodology is observed to be highly economical by reducing the tooling and instrumentation. However, considerable time and little more efforts are required in measuring and computing the errors. The developed methodology is capable of evaluating the five degrees of freedom (DOF) errors including two radial errors, one axial error, and two tilt errors out of six DOF error components of rotary joints. A DBB system with point accessory was used for direct measurement by change of length at multi-points and various locations, whereas mathematical modeling technique was implemented, and equation solvers were exercised for error evaluation and its quantification. Validation and authentication of the methodology and results obtained were carried out by simulation process prior to implementing the methodology on rotary joints of a five-axis turbine blade grinding machine. The results obtained from "A" and "B" rotary joints of the turbine blade grinding machine were presented by applying cubic spline technique for error modeling and its further utilization. The methodology was found extremely pragmatic, quite simple, efficient, and easy to use for error characterization and its quantification in rotary joints of multi-axis machine tools.

Keywords Rotary joints · Error characterization · Machine tools · Six DOF · DBB

1 Introduction

Multi-axis machine tools are considered as the backbone of production industries because of its flexibility and ability to process geometrically intricate shapes and surfaces by combating the increasing demand of emerging technologies. High accuracy of the machine tools is the essential requirement to meet the challenges of manufacturing precision components and consistency of increasing quality level. Machine tools are composed of links and joints, and practically most of them are based on prismatic and rotary joints chained in parallel or serial manners [1]. Rotary joints such as a rotary indexing table, dividing head, and spindle are relevant to a great deal of angle displacements and transmit the rotational motion. These rotary joints possibly contain six degrees of freedom (DOF) errors, i.e., three linear (two radial and one axial) and three angular (one angular position and two tilt) errors as per theory of rigid body motion in three-dimensional (3D) space [2] due to erroneous motions influenced by various affecting factors [3]. These errors contribute a significant inaccuracy in overall performance of a machine tool [4, 5], which is one of the most important prerequisites for quality assurance of products. Error characterization and quantification predicts the accuracy and intended usage of a machine tool, which is also helpful for purchase, diagnostic, and maintenance purposes.

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Although a comprehensive and considerable amount of work was available on error characterization of prismatic joints like Nawara et al. [6], Soons et al. [7], Suh and Lee [8], Ferreira and Liu [9], Mou and Liu [10], Zhang et al. [11], and Chen et al. [12], work on rotary joint was sparse, as most researcher focused on prismatic joints only. To quantify the errors in rotary joints, most researchers used conventional methodologies and techniques to detect the six DOF errors partially and only in those rotary joints which have the mounting capability of standards or artifacts, so that it becomes a big question mark for the researchers to investigate a proper solution to measure the errors in quick and efficient manners for all types of rotary joints. The authors of this paper concentrated on this lacking field and developed a novel methodology which is capable of measuring the five DOF out of six DOF errors by just measuring the change of length between two points. Double ball bar (DBB), a metrology instrument, was used on a similar setup at multi-points and three various locations for five DOF error characterization and quantification efficiently and effectively in all types of rotary joints whether they have the mounting facility or not.

2 Literature survey on rotary joints

During literature review on the subject topic, it was revealed that a little work is available on rotary joints of the machine tool in comparison with prismatic joints, and no evidence was observed about a methodology which characterizes and quantifies the six DOF errors of a rotary joint in a single setup. Most methodologies are limited up to characterization and quantification of radial and axial errors of a machine tool. Although, the metrology of machine tools and their components have roots back to early twentieth century, and Dr. Schlesinger [13] was the first person who developed the earliest method for a rotation accuracy test for rotary joints and established specific techniques for performance of machine tools, which are still widely used in the industry. The development in error characterization and error mapping in rotary joint is hereby presented through a time line survey keeping in view the methodologies used by the researchers and giving an overview how the research revolutionized with the passage of time in the rotary joints.

2.1 Research on rotary joints up to year 1980

In year 1901, as aforementioned, Schlesinger started the work of establishing acceptance standards for machine tools, and in 1927, he published for the first time a comprehensive series of acceptance test specification [13]. Schlesinger used a mechanical displacement indicator for axial and radial error quantification through superposition of the form error of the measured surface and the errors in rotary joints. In 1959, Tlusty [14], firstly, used the capacitance-based non-contact displacement transducers for radial error quantification. Displacement transducers were mounted at 90° from each other for a two-axis pickup system, and this arrangement made it possible for the first time to show the radial error motion of an axis dynamically on a polar plot. In 1969, Bryan et al. [3] introduced a specific terminology and described the tests for error motion. They proposed a method of measuring rotary joints error with a master ball, while machining and error quantification was carried out by error separation method. On the basis of their work, Tlusty and Bryan are considered the founders of modern spindle metrology. They developed complete methods for accurately quantifying rotary joints performance and presenting the results in the form of polar plots. Bryan's work provided the basis for a new document published by the International Institution for Production Engineering Research (CIRP) in 1975 and again in 1976 on unifying the terminology of axis of rotation [15, 16]. These papers became the basis of the standard document B89.3.4M that was eventually published in 1985 and was considered the earliest cited standard; ANSI/ASME B89.3.4 axes of rotation methods of specifying and testing, issued in 1985 [17] and was revised in 1995 [18], defines the language of rotary joints metrology, and the reader is referred to the standard for concise and comprehensive definitions of the relevant terms. In 1972, Donaldson's contributed to a robust error separation (reversal technique) methodology, which made possible the measurements of errors in rotary joints [19]. The same year, Goddard et al. [20] published their testing techniques for quantifying the rotation accuracy. In 1973, digital processing of rotary joint measurements was introduced by Vanherck and Peters, and it became widespread in the 1980s as rotary encoders and resolvers, which appeared more frequently on precision machine tools [21]. In later 1970s and early 1980s, extensive work has been done on hardware development to facilitate the measurement. Optical and digital methods were used for error characterization and quantification. In 1977, Arora et al. used resolver mounted on a spindle to monitor the phase match or artifact eccentricity and efficiently removed it from the measurement data by cancelation of eccentricity of a master sphere, which was possible due to accurate angular triggering to properly synchronize data from successive revolutions [22]. In 1978, Murthy et al. used a method in which analog and digital notch filters synchronized to the rotary joint rotation rate to remove artifact eccentricity from measurement data [23].

2.2 Research on rotary joints year 1981–1990

By the late 1980s, inventors developed nanometer-resolution sensors that could be applied to the challenge of measuring precision of rotary joints. In 1982, Mitsui determined the radial error motion by the three-point method [24]. Chapman designed and built a spindle analyzer capable of measuring radial and axial error motions using analog and digital methods on a master cylinder with a resolution of 5 nm [25]. In 1988, Hansen introduced a general purpose digital instrument with double sphere artifact for measuring radial and axial error motion simultaneously [26]. Some optical methods have been developed for angular indexing measurement [27–31].

2.3 Research on rotary joints year 1991–2000

In 1992, Parks measured the radial, axial, and angular motion errors of rotary joints with a very simple method by using five linear variable differential transducer. This is achieved by a master ball and a master plate, which are employed as a reference [32]. In year 1993, Zhang et al. developed a four-point method for rotary joint's error measurement and error separation method [33]. In 1994, Park and Kim [34] developed an optical moiré technique to evaluate the radial motion of a rotary joint without using mechanical master artifacts. To measure the angular indexing of rotary axis, the most common conventional optical technique is autocollimator and multi-facet polygon prism, in which a 12-sided prism gives a limited angle measurement (at every 30°) through comparison method, whereas 72-sided polygons have been used but are not now available commercially due to higher cost of its manufacturing. To overcome this problem, in 1994, Lin developed a measurement technique. High-precision serrated table with a plane reflector was incorporated whose measuring step is 0.25° with an accuracy and repeatability of 0.1 and 0.05 s [31]. For calibration of higher accuracy serrated rotary table, a reference table and laser system was developed by A. G. Davis and named as Davis rotary calibrator for the calibration of rotary indexing error [35]. In 1995, Noguchi et al. used vector indication method to assess the radial errors of rotary axis [36]. In 1996, Gee's developed a measuring system based on Fizeau interferometer [37] for assessing the axial and angular error motion of an ultra-precision air spindle. The same year, Estler [38] proposed reversal techniques to separate the master axis error motion from that of the spindle under test. This reversal is achieved by using a rotary table and a reversal chuck, which eliminates the need for relocating the capacitance probe. Zhang et al. developed a multi-point methodology in 1997, which able to separate the radial error motion of the rotary joint from the roundness error of a precision sphere [39]. It permits the assessment of the radial, axial, and tilts error motions of the rotary joint. In 1998, another technique, three capacitance-type sensors, was employed by Lee et al. to measure the 3D positions of a master ball. This technique [40] traces the centre of the rotating master ball in 3D space using a polar plot. In 2000, Marsh and Robert [41] proposed a master axis method for machine tool rotary joint's measurement, which is capable to measure the errors under dynamic load conditions. Summarizing the developed rotary accuracy measurement system, a precision artifact (i.e., a precision sphere or cylinder) and multiple probes (i.e., capacitance probe or eddy current probe) were used to inspect the spindle axis error motion of the rotating sensitive direction.

2.4 Research on rotary joints year 2001 to up to date

Gao et al. experimented in 2002 an angular three-point method [42]; for this, three two-dimensional (2D) surface slope sensors were employed, which are capable of measuring simultaneously the workpiece out-of-roundness and the rotary joint radial and angular error motion. James et al. presented the Estler face motion reversal method in year 2003, which permits the separation of tilt and axial error motions from the circular flatness of a reference part [43]. Liu et al. [44] developed a measuring system in year 2004 for evaluating the radial and tilt error motions of the rotary joint without using a master sphere or cylinder. This system uses a rotational fixture with a built-in laser diode, which is mounted on the rotary joint. Two measuring devices with two positions sensitive detectors are fixed on the machine table in order to measure the laser point position from the laser diode. In year 2005, Grejda et al. [45] implemented a portable master axis of rotation (ultra-high-precision air-bearing spindle) and a capacitance probe for assessing radial and axial error motions of a rotary joint at the nanometer level, whereas Castro [46] presented a methodology in 2008 based on laser in conjunction with a convergent lens, which is capable of assessing the radial and axial error motions of a rotary joint with high accuracy, but it is unable to measure the tilt errors of the rotary joint.

Over the years, research was presented through a literature review, and it was concluded that a little research which is available relevant to this topic has shown that researchers worked on error characterization and quantification in rotary joints of machine tools in different perspectives and aspects. Most of them used a number of displacement sensors, either optical, digital, or laser-based sensors, and error separation techniques to solve the purpose. Most work was carried out limited to radial and axial motion error of rotary joint of machine tools, whereas in high- and ultra-high-precision accuracy requirement, the tilt errors contributed to the uncertainty significantly, and proper quantification to maintain the accuracy is needed. The use of laser interferometer was observed more complicated for measurement of radial and axial errors like Fizeau laser interferometer, which is quite expensive and very convoluted in its usage. The use of 2D slope sensors is cumbersome in the aspect that it is difficult to align the optical components like laser diodes, beam splitters, mirrors, autocollimators lenses, quadrant, photodiodes, etc. with the revolving rotary joints or mounted artifact on rotary joints. The rotation of joints creates hindrance in measurements, as most optical devices need to be mounted and well aligned. The alignment process itself is time consuming and quite cumbersome resulting in large machine downtime. A technique which needs a highly precise reference table for comparison is infeasible in the sense that most rotary joints can't be compared as they have already been fixed on the machine tools and are heavier in weight besides having a larger volume, so this technique is limited up to very restricted cases, for example, testing a small size spindle or its laboratory testing and metrology purposes. The major problem in measurements through artifacts is the manufacturing of high-precision artifacts, which are quite expensive, and it is difficult to maintain their accuracy for a long time and for its repetitive usage during measurements. When the spindle error motion is of the same order of magnitude as the accuracy of the reference artifact, then it is impossible to separate the superposition of the form error of the measured surface and the rotary joints' errors.

Several error quantification methodologies are based on error separation techniques and are quite reliable for accurate measurement of profile error to error motion, such as the Donaldson's reversal method [19, 47], the multi-step method, or multi-position method in which the measurement is recorded at multiple angular locations [48], and the multi-probe method in which measurements are recorded with multiple sensors [49, 50]. Estler face motion reversal methods, etc., Donaldson's and Estler reversal methods, are quite popular because of their authenticity. The accuracy of error separation method is adversely affected by imperfectly repositioning the gauges or by using multiple gauges of unequal sensitivity. In order to overcome some problems associated with the methods and their limitation, a method based on a DBB, a working standard is proposed in this paper, which is capable of characterization and quantification of five DOF out of six DOF errors of rotary joints.

3 Possible error sources and their characterization in rotary joints

3.1 Possible error sources in rotary joints

One of the major error sources of rotary joints is due to limited accuracy of the individual components such as shafts, bearing transmission system, indexing devices, etc. caused by dimensional and form inaccuracy of the elemental components and their subassemblies and assemblies, which introduces the systematic and kinematic inaccuracy. This inaccuracy is primarily based on manufacturing and assembling errors of the joints. Kinematic errors due to motion of the joints are actually dependent on the geometric errors and assembly defects combined with gravitational, vibrational, and forces effects on the rotary joints. The driving system and the controller also contribute to the errors, which is the function of motion direction. Wear of the elements in rotary joints also introduces errors, and it is due to aging, excessive usage, or mishandling. Assembly of joints and their misalignments also influence a lot. Stiffness error in rotary joint originates from imperfect stiffness, weight, gravitational force, and joints configuration, and basically, this error depends on designing factors or improper loading. All these errors result in lack of concentricity between mounting centers and the axis of rotation, lack of concentricity between the machine transmission mounting system and the axis of rotation, wobble in the instantaneous center of the axis of rotation, and errors in the graduations of the rotary scale. Temperature is also a major source of error and is considered as the largest single source of dimensional errors, which significantly affects the repeatability and stability of the rotary joints.

3.2 Error characterization in rotary joints

Rotary joints can be categorized as a rigid body system which has possibly six DOF in 3D space and kinematically constrained. When a rotary joint moves, it may have a limited six DOF due to aforesaid causes and unwanted movements, which introduce six DOF [12] errors and result to a decrease in the accuracy of rotary joints of multi-axis machine tools. The nominal axis of rotation is *Z*-axis, which depends on different indicating conventions on the shop floor or topology of a machine and can be called *A*-axis, *B*-axis, or *C*-axis, which is described in detail in a standard document EIA-267-B [51]. A reference coordinate's axis is introduced where the axis of rotation is coincident with the main rotation axis. Rotation axis rotates in relation to this reference axis. Due to imperfection of manufacturing and assembly errors and erroneous motions, these errors magnify and can be distinguished by linear error motion and angular error motion of a rotary axis. Rotary axis may exhibit unintended linear motions as they rotate, which can be mentioned as rotary axis error in linear direction XYZ as $\delta_x(\theta)$, $\delta_y(\theta)$, and $\delta_z(\theta)$ named as axial and radial errors, respectively. Rotary axis may exhibit about these errors either of the two axes perpendicular to the axis of rotation and one along the axis of rotation. Angular errors are two tilt errors and one drive-related error, which is the difference between the commanded and the realized angular position. Angular error motions and scale motions are described as a tilt along the axis of rotation and perpendicular to the axis of rotation, which can be notated as $\varepsilon_x(\theta)$, $\varepsilon_y(\theta)$, and $\varepsilon_z(\theta)$, respectively. These errors are elaborated through Fig. 1 and clarified as mentioned below when X-axis is considered as the main rotating axis.

1. Linear displacement or axial error along X coordinates $\delta_x(\theta)$
2. Linear displacement or radial error along Y coordinates $\delta_y(\theta)$
3. Linear displacement or radial error along Z coordinates $\delta_z(\theta)$
4. Angular displacement or angular position error about X coordinates $\varepsilon_x(\theta)$
5. Angular displacement or tilt around the Y coordinates $\varepsilon_y(\theta)$
6. Angular displacement or tilt around the Z coordinates $\varepsilon_z(\theta)$

where "δ" and "ε" are the linear error and angular errors, respectively, subscript is the error direction and the position coordinate is inside the parenthesis.

The rotation about the axis line error $\varepsilon_x(\theta)$ can be defined by comparing the actual and angular displacement with the nominal rotation and calculated as

$$\varepsilon_x(\theta) = \theta(\text{actual}) - \theta(\text{nominal}) \tag{1}$$

The methodology presented in this research paper can measure five DOF errors out of six DOF, and this sixth error, which is rotation about the axis line error $\varepsilon_x(\theta)$ incase of "A" axis and $\varepsilon_y(\theta)$ incase of "B" axis, is unable to measure with this novel methodology, so for determining these errors, some sort of rotation measuring instruments can be used.

4 Methodology for errors evaluation in a rotary joint

The authors developed this novel methodology on the basis of rotation displacement, in which two metrological

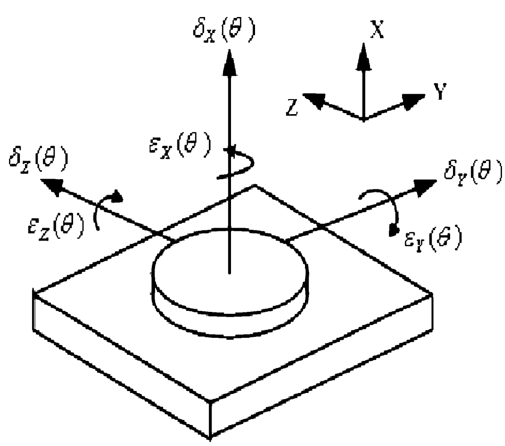


Fig. 1 Six degrees of freedom errors in rotary joints

coordinates placed on the center position of a rotary axis named "A" rotary joint as mentioned in Fig. 2. One metrological coordinate, i.e., O_S is considered as a stationary coordinate, whereas the second coordinate is considered dynamic or moveable coordinates whose rotation is relative to the motion of fixed coordinate and is described as O_d. Initially, they are in superposition and coincide to each other. A point "m" is taken on moveable metrological coordinate whose position in O_S metrological coordinate is (x_m, y_m, z_m). When "A" rotary joint rotates at an angle "θ" and there is no error in movement (by considering an ideal motion), the point "m" moves, and its new position is (x_{m1}, y_{m1}, z_{m1}). Then, the ideal movement of point "m" can be measured in relation to the metrological coordinate O_S through a homogeneous transformation matrix called T_{sd}. So, the new position of the point can be calculated as per below mentioned formulae.

$$\begin{vmatrix} x_{m1} \\ y_{m1} \\ z_{m1} \\ 1 \end{vmatrix} = T_{sd} \begin{vmatrix} x_m \\ y_m \\ z_m \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_m \\ y_m \\ z_m \\ 1 \end{vmatrix} \tag{2}$$

As it is impossible that rotation axis rotates with ideal movement due to imperfection and inaccuracies due to the aforesaid six DOF errors, therefore, it has erroneous movement. The actual position of point "m" is considered as "m′" and can be calculated by introducing a homogeneous error transformation matrix named as T_{sde}. Now, the actual coordinates position in relevance to O_S metrological coordinate is $(x_{m1'}, y_{m1'}, z_{m1'})$, whereas the T_{sde} is as mentioned below.

$$T_{sde} = \begin{vmatrix} 1 & -\varepsilon_z(\theta) & \varepsilon_y(\theta) & \delta_x(\theta) \\ \varepsilon_z(\theta) & 1 & -\varepsilon_x(\theta) & \delta_y(\theta) \\ -\varepsilon_y(\theta) & \varepsilon_x(\theta) & 1 & \delta_z(\theta) \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{3}$$

$$\begin{vmatrix} x'_{m1} \\ y'_{m1} \\ z'_{m1} \\ 1 \end{vmatrix} = T_{sde} T_{sd} \begin{vmatrix} x_m \\ y_m \\ z_m \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & -\varepsilon_z(\theta) & \varepsilon_y(\theta) & \delta_x(\theta) \\ \varepsilon_z(\theta) & 1 & -\varepsilon_x(\theta) & \delta_y(\theta) \\ -\varepsilon_y(\theta) & \varepsilon_x(\theta) & 1 & \delta_z(\theta) \\ 0 & 0 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_m \\ y_m \\ z_m \\ 1 \end{vmatrix} \tag{4}$$

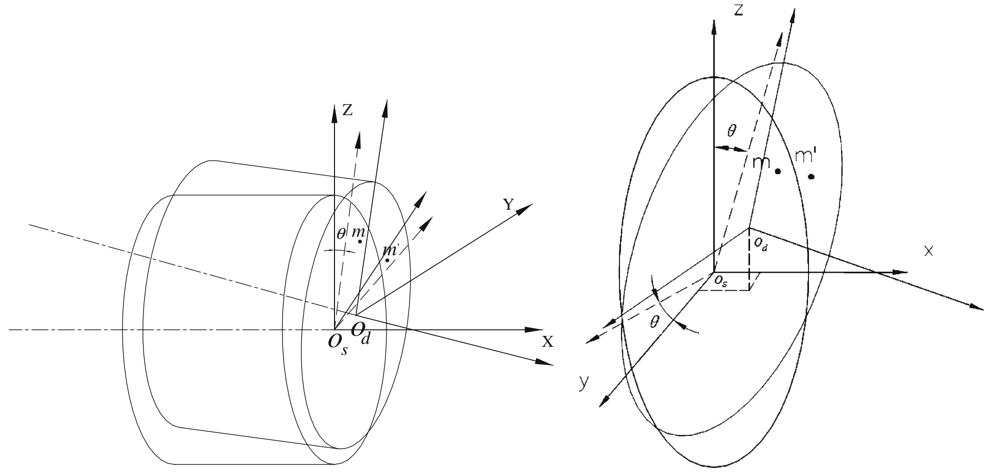


Fig. 2 Schematic diagram of rotary joint by placing metrological coordinates

If ΔM can be used to express the error between actual and nominal position at the point "m" between coordinates O_S and O_d, then it can be written as

$$\Delta \mathrm{M} = \text{actual position} - \text{nominal position} \tag{5}$$

By substituting the values of actual and nominal position in Eq. 5, it becomes

$$\Delta \mathrm{M} = \begin{vmatrix} \Delta x_m(\theta) \\ \Delta y_m(\theta) \\ \Delta z_m(\theta) \\ 0 \end{vmatrix} = \begin{vmatrix} x_{m1'} \\ y_{m1'} \\ z_{m1'} \\ 1 \end{vmatrix} - \begin{vmatrix} x_{m1} \\ y_{m1} \\ z_{m1} \\ 1 \end{vmatrix} \tag{6}$$

$$\begin{aligned} \Delta \mathrm{M} = & \begin{vmatrix} 1 & -\varepsilon_z(\theta) & \varepsilon_y(\theta) & \delta_x(\theta) \\ \varepsilon_z(\theta) & 1 & -\varepsilon_x(\theta) & \delta_y(\theta) \\ -\varepsilon_y(\theta) & \varepsilon_x(\theta) & 1 & \delta_z(\theta) \\ 0 & 0 & 0 & 1 \end{vmatrix} \\ & \times \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_m \\ y_m \\ z_m \\ 1 \end{vmatrix} \\ & - \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_m \\ y_m \\ z_m \\ 1 \end{vmatrix} \end{aligned} \tag{7}$$

To measure the actual position of the moving point, a displacement-measuring instrument DBB was used; first, it was calibrated, and calibration procedure was presented through a paper [52]. The measurement arrangement was explained through Fig. 3 in which a DBB and a fixture were mounted on "A" rotary joint. The DBB consists of two high-precision balls whose sphericity is controlled under a specified limit. Between these balls, a sensor is placed to measure the change in length. This sensor is highly precise and sensitive with a high resolution. However, for experimentation, a calibrated DBB was used to authenticate the results. The placement of the DBB is very simple, and for placement, balls are distinguished as ball "0" and ball "1." Ball "0" is mounted through a point fixture or with special arrangement at "A" rotary joint at a point "m," and this ball is rotated with "A" rotary joint. Position of ball "0" at coordinate O_S is $S(x, y, z)$. When "A" rotary joint rotates equal to an angle say "θ," then ball "0" position at coordinate O_S is $S(x(\theta), y(\theta), z(\theta))$. Ball "1" is placed at the center of "A" rotary joint, and its position at coordinate O_S is $S_1(x_1, 0, 0)$, whereas the measuring distance between two balls can be taken as "d." So, from this, an equation can be made for distance measurement between these two positions where the balls are placed and can be written as

$$d^2 = (x(\theta) - x_1)^2 + y^2(\theta) + z^2(\theta) \tag{8}$$

If there is no error while "A" rotary joint rotates, then bar's length will be unchanged. If the "A" rotary joint has errors during rotation due to six DOF errors, then the length will be changed in relation to the errors' magnitude, and this error magnitude will represent the six DOF errors. Change of the length phenomenon is explained through Fig. 4, in which dotted lines exhibit the ideal position, and confirmed lines show the actual position. In Fig. 4(a), the pitch error and, in Fig. 4(b), the axial displacement error were explained in combination with change of the length phenomenon in DBB. The similar can happen at opposite side point position.

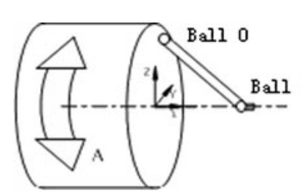


Fig. 3 Ball location method

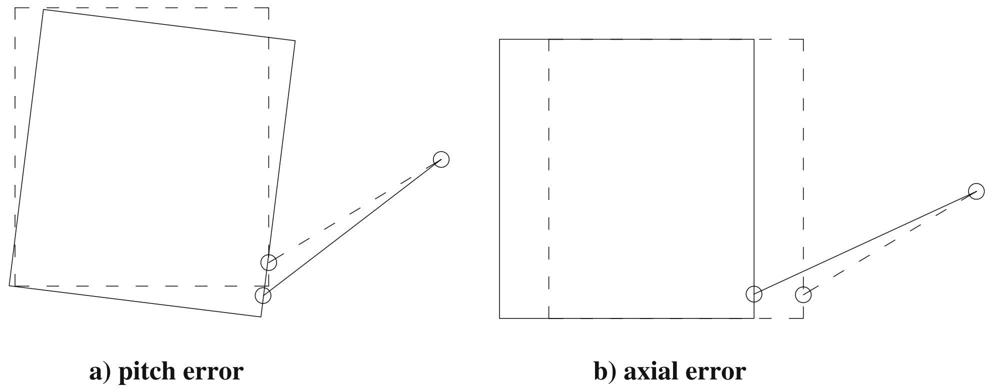


Fig. 4 Length change phenomenon of DBB in rotary joints (**a**) Pitch error (**b**) Axial error

The measure of the output of a very small input or increment introduced is calculated by taking the partial derivative of Eq. 8, and the new equation will become as

$$d\Delta d(\theta) = (x(\theta) - x_1)\Delta x(\theta) + y(\theta)\Delta y(\theta) + z(\theta)\Delta z(\theta) \tag{9}$$

whereas in Eq. 9, $\Delta x(\theta)$, $\Delta y(\theta)$, and $\Delta x(\theta)$ are the difference between the actual and nominal position value at point "0" as mentioned in Eq. 7. For a point measurement, place the ball "1" at rotation axis center, ball "0" at point "m," and rotate the "A" rotary joint to measure the length change through DBB sensor. Substitute Eq. 7 into Eq. 9 and get a new equation in which by plugging the measured change length obtained from sensor for solving the equations of six DOF errors for further calculation.

One problem during placing is that it is quite difficult to place ball "1" at the exact center. If it is not at the center position, an offset error is introduced, which influences the length change. It means Δd contains offset information, which needs to be eliminated. For elimination of these offset errors, assume ball "1" offset is only possible at Y and Z direction which are "a" and "b," respectively. From this, ball "1" coordinate is $S_1'(x_1, a, b)$ at metrological coordinate O_S, so Eq. 8 can be rewritten in the form of the given equation below.

$$d^2 = (x(\theta) - x_1)^2 + (y(\theta) - a)^2 + (z(\theta) - b)^2 \tag{10}$$

Taking partial derivative of the equation

$$d\Delta d(\theta) = (x(\theta) - x_1)\Delta x(\theta) + (y(\theta) - a)\Delta y(\theta) + (z(\theta) - b)\Delta z(\theta) \tag{11}$$

by the addition of two additional measuring points, it needs to add two more equations to solve the six DOF errors and the offset errors. Bar length change is a combination of length change due to six DOF errors and the errors due to offset. So, it needs to remove the offset contribution error for getting only the six DOF errors. The offset errors $\Delta d(\theta)''$ are mentioned in Eq. 12.

$$\Delta d(\theta)'' = \sqrt{(x - x_1)^2 + (y - a)^2 + (z - b)^2} - \sqrt{(x(\theta) - x_1)^2 + (y(\theta) - a)^2 + (z(\theta) - b)^2} \tag{12}$$

where x, y, and z are the position coordinates of the point, then the bar length change has only the six DOF effect which can be calculated from the equations given below.

$$\Delta d(\theta) = \Delta d(\theta)' - \Delta d(\theta)'' \tag{13}$$

$$\Delta d(\theta) = \Delta d(\theta)' - \left(\sqrt{(x - x_1)^2 + (y - a)^2 + (z - b)^2} - \sqrt{(x(\theta) - x_1)^2 + (y(\theta) - a)^2 + (z(\theta) - b)^2}\right. \tag{14}$$

In the above equation, $\Delta d\ (\theta)'$ represents the bar length change, $\Delta d\ (\theta)''$ represent the offset, and $\Delta d\ (\theta)$ represents the bar length change due to six DOF errors. During method verification and validation, it was revealed that this method exhibited being capable of measuring the six DOF, but actually, it was only capable of measuring the five DOF errors, which were checked by plugging the offset value and $\delta_x(\theta)$, $\delta_y(\theta)$, $\delta_z(\theta)$, $\varepsilon_y(\theta)$, and $\varepsilon_z(\theta)$ as zero. The change in length in $\varepsilon_x(\theta)$ was not observed through DBB though it did exist. When the offset was less than 0.5 mm, $\varepsilon_x(\theta)$ only produce very small change in bar length, which was less than the resolution of the DBB. So, this method is not feasible to measure the angular position error $\varepsilon_x(\theta)$ which is the only limitation of this novel methodology. Point placement position values at ball point “0” and bar length are the coefficient, and their location errors are very small in relation to their own value, so it is concluded that measurement errors by using this method is not influenced on error determination of rotary axis, even if they are in millimeter but not greater than 2 mm, which depicts that very accurate positioning and accurate length of bar are not required for location of point “0.” The main purpose is to get the length change, which is important information.

5 Simulation to validate the methodology

Simulation was carried out to validate the newly developed method. Simulation was processed in UG software in which the same measurement environment was generated by placing points on metrological coordinates. A sensor was placed on the designated points in which one end was placed at point “0” and other at point “1,” whereas the length of the sensor was initially fixed. A motion error was introduced in linear displacement of X, Y, and Z direction within the range of 10 µm, and angular errors were introduced in Y and Z direction within the range of 0.1 rad. An offset was given to the point “1” in up to 0.2 mm range, as eccentricity error in X and Y direction is considered as “a” and “b” offset. The effect of rotation errors and offset errors were measured through the change of length of the measuring sensor. So, by offsetting the point “1” and the effect of rotation errors, the change of length (Δd) at different rotational displacement interval (θ) was measured accurately between point “0” and point “1” in whole range of 0~360°. Errors observed were substituted into Eq. 14 to formulate the measurement at different interval points, and further from this information, a function was generated, and equations were made. In the same way, it was repeated eight times at multi-point position to get a minimum of eight equations, which is the requirement of solving the equations in the equation solver. Then, these equations were solved through developed equation solving software and for determination of the errors due to offset effects. Results obtained from this solution were compared to the initial setting data, which were induced prior to measure the change of length in the sensor for its correctness. The simulation method in UG is elucidated through Fig. 5, and obtained results are mentioned through Figs. 6, 7, 8, 9, and 10.

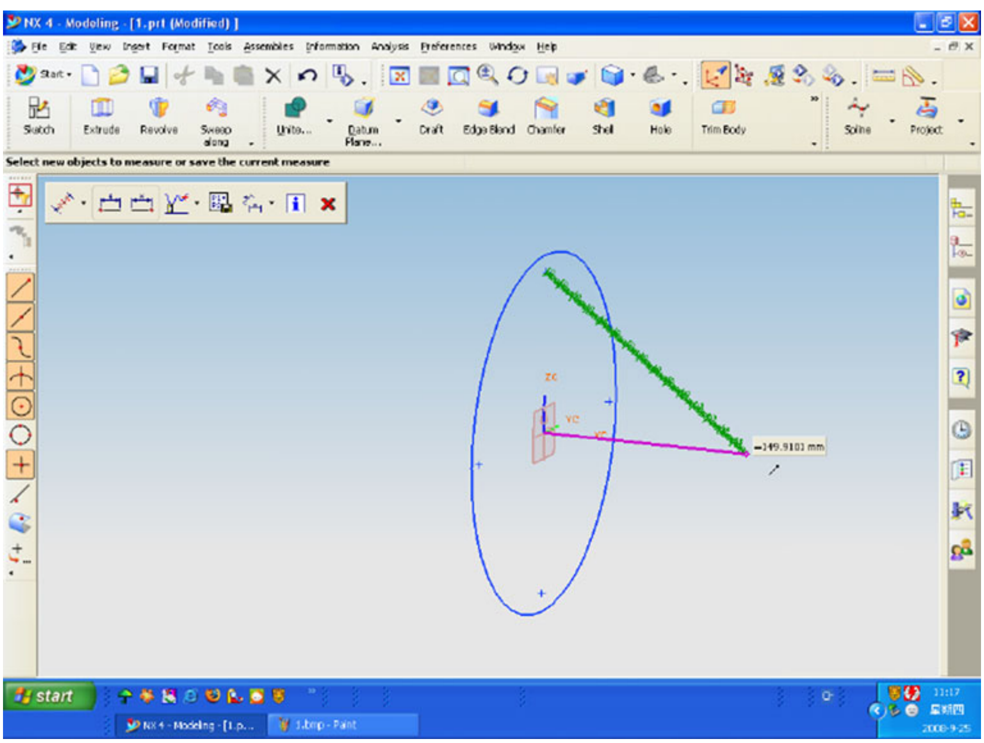


Fig. 5 Measuring the distance between points “0” and “1”

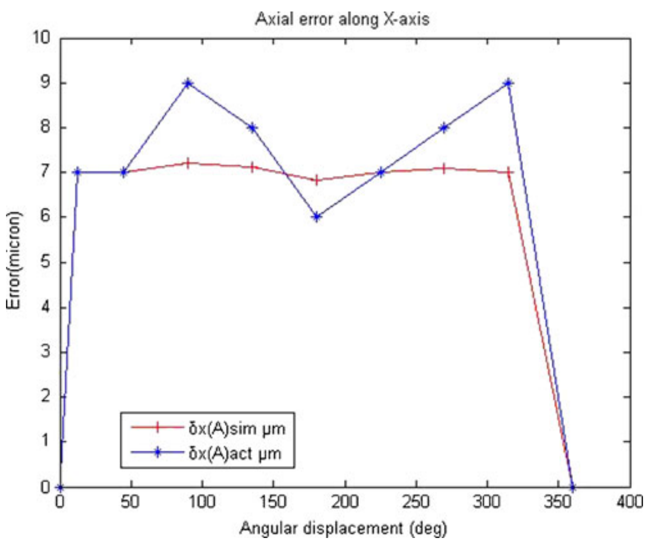


Fig. 6 Axial error along *X*-axis

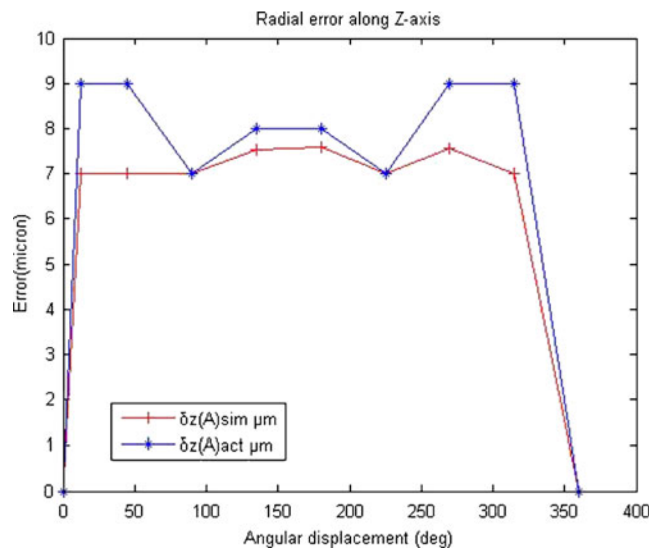


Fig. 8 Radial error along *Z*-axis

6 Implementation of the methodology on "A" and "B" rotary joints of a five-axis turbine blade grinding machine

In this methodology, the errors in rotary joints can be measured through the change in length of a displacement sensor during rotational displacement at specified intervals and specific locations at static manner and continuous as dynamic manner. To distinguish and characterize the error more precisely, the measurements were taken at multi-points and multi-locations as mentioned through Figs. 11 and 12. However, care was taken during sign convention of recording the data. The data was collected at different equally spaced intervals "n" and at various locations. The collected data were needed in a special treatment in which this change of length observed was substituted into Eq. 14. So, the number of equations has been obtained for measuring at different point position and various locations. These equations can be expressed in matrix form as **M**=**AX**, where **M** is a column vector of measured data, **X** is the column vector of the unknown, and **A** is the Jacobean. These equations can be solved directly through some good equation solving software. In this case, specialized equation solver software was

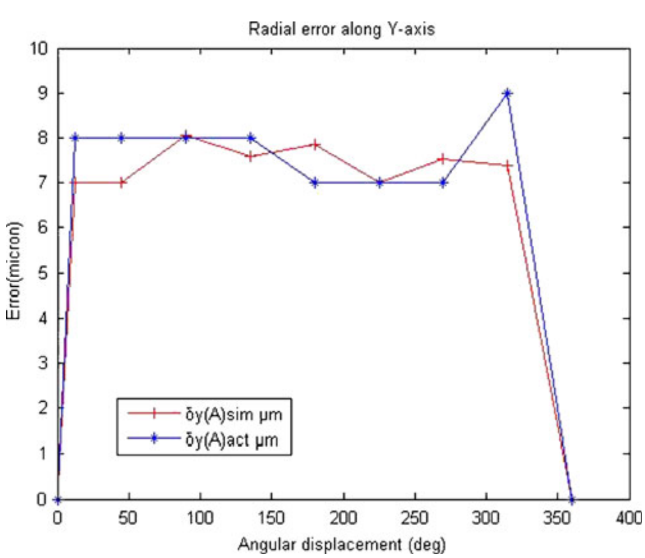


Fig. 7 Radial error along *Y*-axis

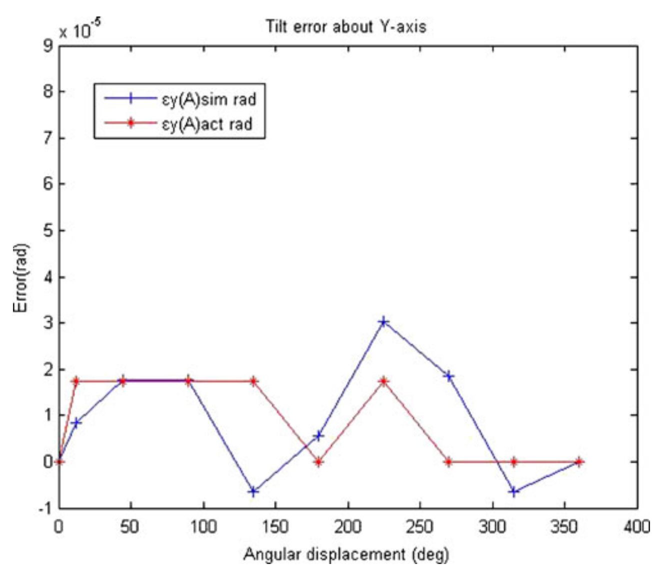


Fig. 9 Tilt error about *Y*-axis

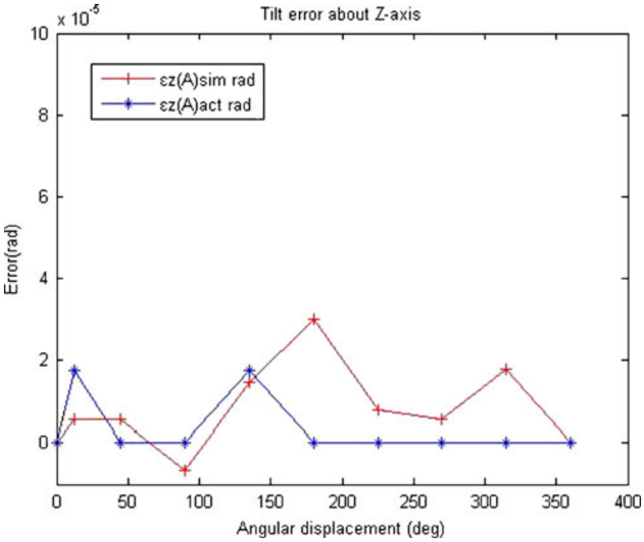


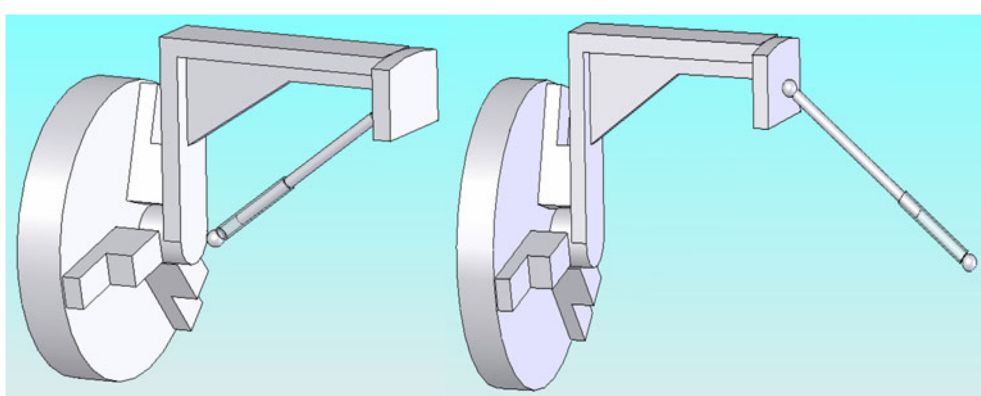
Fig. 10 Tilt error about *Z*-axis

developed in Matlab in combination of genetic algorithm technique to solve the equations efficiently for error separation. For measurement and data collection, a DBB and an L-shaped fixture was used. The L-shaped fixture was mounted on the "A" and "B" rotary joint simultaneously, and the DBB ball "0" was placed on the first point of fixture while the location was right sided, whereas ball "1" was mounted on the axis of rotation of "A" or "B" rotary joint or close to the rotation axis of "A" or "B" rotary joint. The "A" rotary joint rotated 0~360° at predefined intervals of 10°, whereas "B" rotary joint was rotated ±30° with a specified interval of 5° and recorded the change of length of DBB bar against the rotational displacement in one complete circle. In the second repetition, the position of the point was changed, and DBB ball "0" was placed by rotating the fixture at the next interval and taking this as a second point, but for data collection, this point was considered as initial point and, in same manner, measured the length change in DBB for a complete circle. By using this method, measurement was taken around four circular point displacement intervals. Then, the location was changed, and the DBB point "0" was placed at the middle and left location of the fixture (as explained in Fig. 12) and, in the same way, the DBB point "1" was placed on or near the "A" or "B" rotary joint rotation axis. The same process was repeated four times at each location, and the data was recorded at specific intervals of "A" and "B" rotary joints as already discussed. Obtained data was used to make the equations through some fitness method. These equations consist of known and unknown parameters, so unknown parameters were calculated through special equation solver software made in Matlab in combination of genetic algorithm for separating the six DOF errors efficiently. Although from this method, the six DOF errors can be resolved, but only five DOF are authentic and effective, which was revealed and proved from the simulation process. The measuring method is explained through Figs. 11, 12, 13, and 14, as given below. The obtained results were shown in Figs. 15, 16, 17, and 18 by using the function of cubic spline for error modeling and further usage.

7 Measurement results and discussion

Developed methodology was validated through simulation and further through experimentation. Simulation results are displayed in linear and angular error graphs, and it was concluded that results obtained were found in good conformance of the required accuracy and quite satisfactory to solve the purpose in a specified accuracy requirement. However, it was also noted that if the size of data obtained through measurement was increased in both manners, i.e., interval size and the rotational cycle numbers, the results obtained will be closer to the true value, which is a clear indication that the accuracy can be approached up to a super high accuracy level by increasing the size of obtained

Fig. 11 Measurement on point position through calibrated double ball bar and the L-shaped fixture



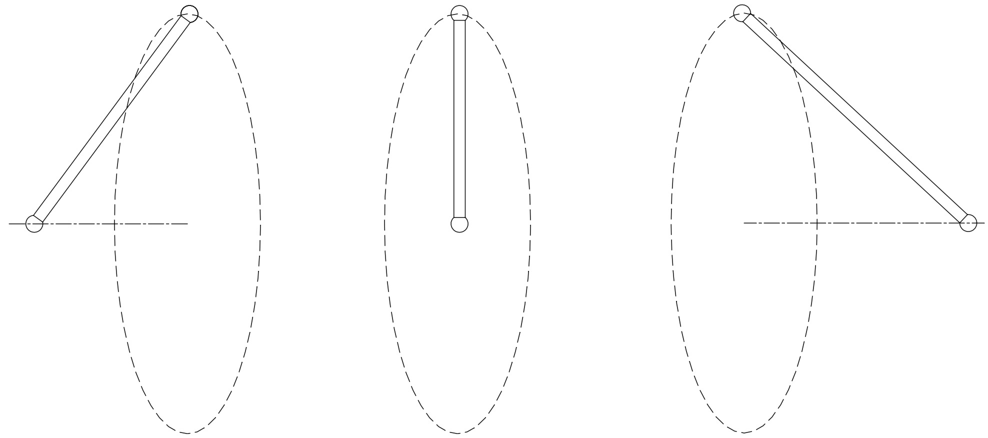


Fig. 12 Schematic diagrams of various locations of measurements

data. To approach higher accuracy, more data means more time, more efforts, and need of good equation solvers, so it depends on the user for accuracy choice. This method is capable of fulfilling all specified accuracy requirements, but for detection of errors with sufficient accuracy requirement, minimum of eight circular movements, four at each side of the fixture, is required to serve the purpose. The error characterization and its quantification were carried out in “A” and “B” rotary joint of five-axis turbine blade grinding machine. Measuring method and results obtained are explained through Figs. 13, 14, 15, 16, 17, and 18. The “A” rotary joint was measured in the range of 0~360° with interval of 10°, and “B” rotary joint was measured in the range of ±30° with interval of 5°. The measured results were substituted into the equations, which were further processed in developed equation solver, and at each point, the results were calculated five times, averaging them at point results, and were presented. In “A” and “B” rotary joints, the linear errors were observed within range of ±9 μm; similarly, the tilt errors were observed within range of ±0.00004 rad, which varied at different intervals. The rotary joints are newly made, so their results are observed well in conformance to the tolerance limits of the specifications provided by the manufacturer.

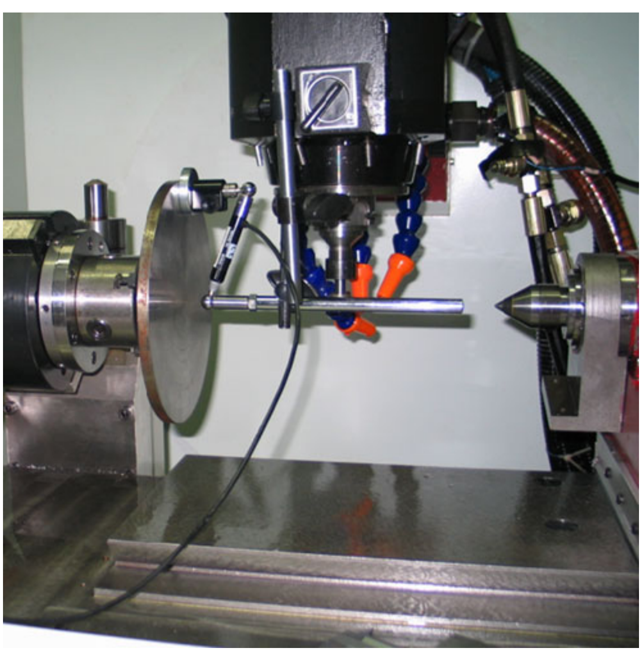


Fig. 13 Measurement of A-rotary joint in five-axis turbine blade grinding machine

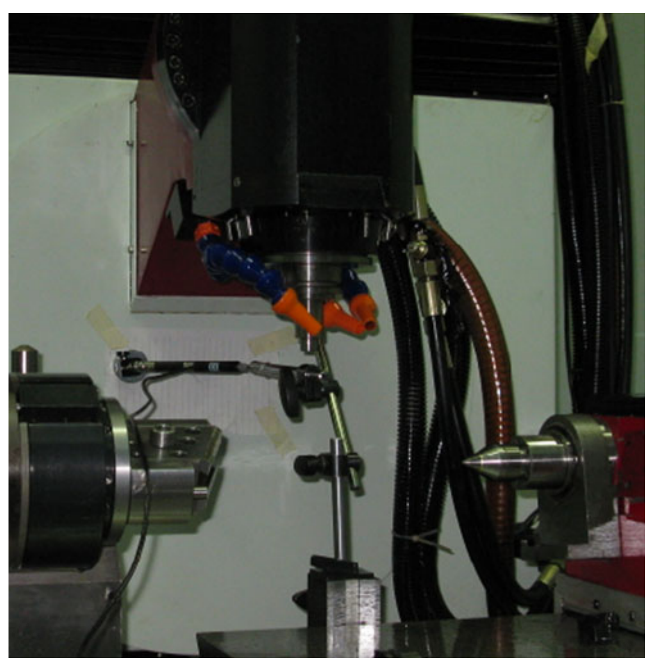


Fig. 14 Measurement of B-rotary joint in five-axis turbine blade grinding machine

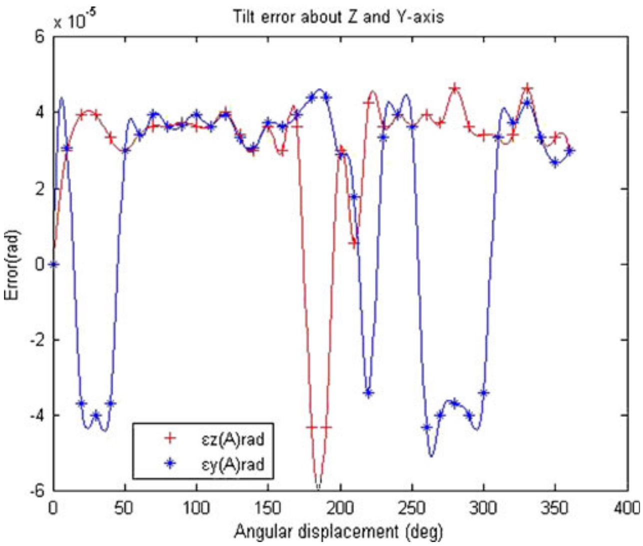


Fig. 15 Tilt errors about *Z* and *Y*-axis in A-rotary joint

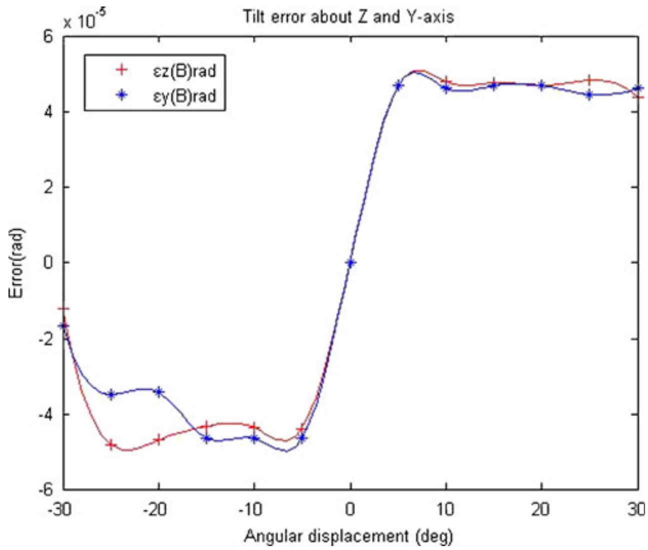


Fig. 17 Tilt error about *Z* and *Y*-axis in B-rotary joint

8 Summary and conclusion

In this paper, the authors developed and implemented a novel methodology to measure the six DOF errors through a calibrated DBB in a rotary joins of multi-axis machine tools. This method was observed as pragmatic, efficient, and easy to use. Five DOF errors out of six DOF errors can be calibrated in efficient manners by measuring the change of length of DBB in rotational displacement. It is a best substitute of already existing methods for error measurement in rotary joints. The main advantage of this method is that it measures all errors by using only a single instrument of DBB, which minimizes the usage of a lot of instruments, artifacts, accessories, and test devices, so only one calibrated DBB and a measuring fixture are enough to serve the purpose. Added feature of this methodology is capability to measure those rotary joints which have no mounting and centering facility. Experimentation was carried out on "A" and "B" rotary joints of a five-axis turbine blade grinding machine under controlled environmental conditions to minimize the random effects. Results exhibit that calibration of rotary joints is viable and accurate by implementing this technique. Calibration results are quite useful for the quantification and compensation of the rotary joints for maintaining its accuracy and extremely

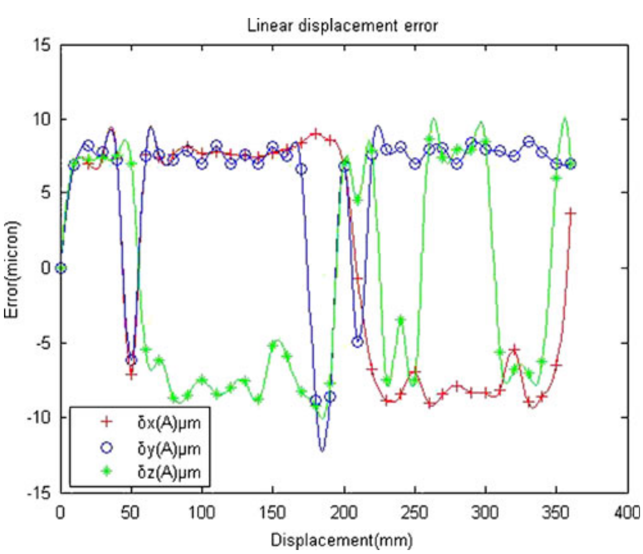


Fig. 16 Linear displacement error in A-rotary joint

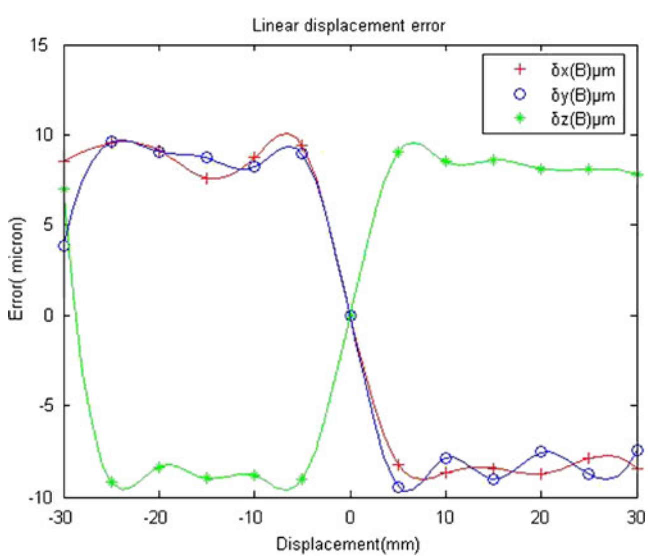


Fig. 18 Linear displacement error in B-rotary joint

helpful for the machine tool builders and multi-axis machine tool users.

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