A Novel Generative Model for Burst Error Characterization in Rayleigh Fading Channels

Cheng-Xiang Wang and Matthias Pätzold
Department of Information and Communication Technology
Faculty of Engineering and Science, Agder University College
Grooseveien 36, N-4876 Grimstad, Norway
E-mail: cheng.wang@hia.no

Abstract – **Generative models have significant applications in the design and performance evaluation of communication protocols as well as coding systems. In this paper, a novel deterministic process based generative model (DPBGM) is proposed for the characterization of burst errors in Rayleigh fading channels. A simple procedure has been developed for generating an error sequence with desired burst error statistics from a properly sampled and parameterized deterministic process followed by a threshold. The proposed generative model enables us to perfectly match any given error burst distribution (EBD) and error-free burst distribution (EFBD) of the underlying descriptive model. The gap distribution (GD) and the error-free run distribution (EFRD) obtained from the generated error sequence can also be fitted to those of the target error sequence with good accuracy.**

I. Introduction

In digital wireless communication systems, channel impairments will greatly influence the transmission process in such a way that the errors occur in bursts. Many researches have shown that the statistical properties of the error process have a significant impact on the performance of a communication system [1, 2]. A proper understanding and an accurate modeling of the error process are therefore necessary. Error models for describing the bursty error process can be classified as descriptive models [3] and generative models [4]. Descriptive models analyze burst error statistics of the error sequence obtained directly from experimental results. Generative models are parameterized mathematical models capable of generating a statistically similar error sequence as produced by the real channel. Error models are essentially required for the design and performance evaluation of communication protocols [5].

Up to now, most commonly used generative models are based on finite-state Markov chains (FSMCs) or hidden Markov chains. Gilbert [6] and Elliot [7] proposed simple two-state Markov models. The disadvantage of a two-state Markov model is its limited capability to reproduce the desired burst error statistics. One way to overcome this problem is to enlarge the number of states [8-11]. Frichman [8] investigated FSMC models with more than two states. These states are partitioned into error-free states and error states. Simplified Frichman's models (SFMs) with a single error state were further been applied to characterize HF channels [9], VHF channels [10], and UHF channels [11]. Another class of generative models are hidden Markov models [12, 13], which lack a direct intuition between the channel behavior and the underlying Markov chain. More recently, bipartite models [14] were proposed. The Markov chain used in a bipartite model forms a bipartite graph. Although a large number of states will almost surely provide a better representation of the channel, the complexity of the model makes the subsequent performance analysis of high layer protocols increasingly difficult.

Deterministic processes [15, 16], based on the principle of Rice's sum of sinusoids [17, 18], have so far exclusively been employed for emulating the fading behavior of the received signal, but not for the generation of error sequences. Deterministic processes have several advantages, e.g., the process parameters can easily be determined, the processes can efficiently be implemented on a computer, and their statistical properties can be varied in wide ranges. Despite these promising advantages, the development of generative models by using deterministic processes is still missing in the literature, due to the lack of a proper procedure for generating an error sequence from a deterministic process. The aim of the present paper is to fill this gap.

The rest of this paper is structured as follows. Some concepts and the relevant burst error statistics are reviewed in Section II. Section III presents a descriptive model, which is considered as the reference model for the development of the generative model in Section IV. Section V compares the burst error statistics of the underlying descriptive model, the proposed generative model, and a SFM. Finally, the conclusions are drawn in Section VI.

II. Burst Error Statistics

In this section, some concepts used to describe the burst error statistics are briefly reviewed. For convenience, an error sequence is in general represented by a binary sequence of ones and zeros, where "1" denotes an error bit and "0" denotes a correct bit.

Following [9, 10], a gap is defined as a string of consecutive zeros between two ones and has the length equal to the number of zeros. An error-free burst [14] is defined as an all-zero sequence with a length of at least η bits, where η is an positive integer. The difference between an error-free burst and a gap is that an error-free burst has the minimum length of η and it is not necessarily located between two errors. An error burst [14] is a sequence of zeros and ones beginning and ending with an error, and separated from neighboring error bursts by at least η error-free bits. It should be noted that the minimum length of an error burst is 1 and the number of consecutive error-free bits within an error burst is less than η. Hence, the local error density inside an error burst is greater than $\Delta = 1/\eta$.

The following notations for burst error statistics will be used throughout the rest of the paper:

1) $G(m_g)$: the GD, which is defined as the cumulative distribution of the gap length m_g.
2) $P(0^m/1)$: the EFRD, which is the probability that an error is followed by at least m error-free bits. The EFRD can be calculated from the GD [9]. It is clear that $P(0^m/1)$ is a monotonically decreasing function of m such that $P(0^0/1) = 1$ and $P(0^m/1) \to 0$ as $m \to \infty$.
3) $P_{EB}(m_e)$: the EBD, which is the cumulative distribution of the error burst length m_e.
4) $P_{EFB}(m_{\bar{e}})$: the EFBD, which is the cumulative distribution of the error-free burst length $m_{\bar{e}}$.

III. The Descriptive Model

A descriptive model analyzes the interested burst error statistics of the target error sequence obtained directly from experimental results. In this paper, the target error sequence is generated by a computer simulation of a coherent QPSK system with a Rayleigh fading channel. The underlying Rayleigh fading channel is modeled by the following deterministic process [16, 19]:

$$\tilde{\zeta}(t) = |\tilde{\mu}(t)| = |\tilde{\mu}_1(t) + j\tilde{\mu}_2(t)| \quad (1)$$

where

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}) \ , \ \ i = 1, 2 \ . \quad (2)$$

In (2), N_i defines the number of sinusoids, $c_{i,n}$, $f_{i,n}$, and $\theta_{i,n}$ are called the Doppler coefficients, the discrete Doppler frequencies, and the Doppler phases, respectively. By using the method of exact Doppler spread (MEDS) [16, 19], the Doppler phases $\theta_{i,n}$ are the realizations of a random generator uniformly distributed over $(0, 2\pi]$, while $c_{i,n}$ and $f_{i,n}$ are given by

$$c_{i,n} = \sigma_0 \sqrt{\frac{2}{N_i}} \quad (3)$$

$$f_{i,n} = f_{max} \sin\left[\frac{\pi}{2N_i}\left(n - \frac{1}{2}\right)\right] \quad (4)$$

respectively. Here, σ_0 is the square root of the mean power of $\tilde{\mu}_i(t)$ and f_{max} is the maximum Doppler frequency.

The average bit error probability (BEP) of the adopted transmission system is obtained by evaluating 8×10^6 transmission bits with a transmission rate of $F_s = 144$ kb/s, which is the same as specified for vehicular users in UMTS systems. The total transmission time is therefore $T_t \approx 55.6$ s. Fig. 1 depicts the simulated BEP together with the theoretical BEP given in [20] versus the average signal-to-noise ratio (SNR). The mean power was given by $\sigma_0^2 = 1/2$ and the maximum Doppler frequency was chosen as $f_{max} = 74$ Hz, which corresponds to a carrier frequency of 2 GHz and a vehicle velocity of 40 km/h. The numbers of sinusoids were chosen as $N_1 = 9$ and $N_2 = 10$. A SNR of 15 dB was selected for the generation of the target error sequence. The corresponding BEP is 7.5341×10^{-3}. Hence, the total number of errors is $N_e = 60273$ for the given length $N_t = 8 \times 10^6$ of the error sequence.

The relevant burst error statistics can be obtained from the resulting error sequence. To avoid a bit-by-bit processing of the error sequence, a sensible way of recording error data is to list the successive gap lengths. It is a means of compressing the error sequence and called gap recording [10]. Once the gap recording has been completed, the GD $G(m_g)$ and the EFRD $P(0^m/1)$ can easily be calculated. Figs. 2 and 3 show the resulting GD and EFRD, respectively. Note that the initial part of the GD has a relatively steep slope. This indicates that short gaps, say 1–300, occur with high probability. The last part of both curves illustrates that some long gaps, say 1000–29099, exist. The middle flat part demonstrates that very few gaps with lengths between 300 and 1000 are present.

From the gap recorder, the error burst recorder $\mathbf{EB}_{rec}$ and the error-free burst recorder $\mathbf{EFB}_{rec}$ can further be obtained. Here, $\mathbf{EB}_{rec}$ is a vector which keeps a record of successive error burst lengths, while $\mathbf{EFB}_{rec}$ records successive error-free burst lengths. Let us denote the minimum value and the maximum value in $\mathbf{EB}_{rec}$ as m_{B1} and m_{B2}, respectively. This means that the length m_e of error bursts satisfies $m_{B1} \leq m_e \leq m_{B2}$. By analogy, the minimum value and the maximum value in $\mathbf{EFB}_{rec}$ are denoted as $m_{\bar{B}1}$ and $m_{\bar{B}2}$, respectively. Moreover, for the derivation of the generative model, it is convenient to define the following parameters:

1) N_{EB}: the total number of error bursts, which is equal to the number of entries in $\mathbf{EB}_{rec}$.
2) N_{EFB}: the total number of error-free bursts, which is equal to the number of entries in $\mathbf{EFB}_{rec}$.
3) $N_{EB}(m_e)$: the number of error bursts of length m_e in $\mathbf{EB}_{rec}$.
4) $N_{EFB}(m_{\bar{e}})$: the number of error-free bursts of length $m_{\bar{e}}$ in $\mathbf{EFB}_{rec}$.
5) $\mathbf{E}_{EB}$: a vector which records the number of errors in each error burst of $\mathbf{EB}_{rec}$. The number of entries in $\mathbf{E}_{EB}$ is N_{EB}.

The EBD $P_{EB}(m_e)$ and the EFBD $P_{EFB}(m_{\bar{e}})$ can be calculated by

$$P_{EB}(m_e) = \frac{1}{N_{EB}} \sum_{k=m_{B1}}^{m_e} N_{EB}(k) \ , \ m_{B1} \leq m_e \leq m_{B2} \quad (5)$$

$$P_{EFB}(m_{\bar{e}}) = \frac{1}{N_{EFB}} \sum_{k=m_{\bar{B}1}}^{m_{\bar{e}}} N_{EFB}(k) \ , \ m_{\bar{B}1} \leq m_{\bar{e}} \leq m_{\bar{B}2} \quad (6)$$

respectively. By setting $\eta = 300$, altogether $N_{EB} = 1629$ error bursts and $N_{EFB} = 1630$ error-free bursts are obtained. The lengths of the error bursts range from $m_{B1} = 1$ to $m_{B2} = 6182$, while the lengths of the error-free bursts range from $m_{\bar{B}1} = 302$ to $m_{\bar{B}2} = 29099$. The resulting EBD and the EFBD are plotted in Fig. 4.

IV. The Generative Model

It is widely accepted that the second order statistics of the fading envelope process are closely related to the statistics of error bursts. This inspires us to develop a generative model by using a properly parameterized deterministic process followed by a threshold r_{th}. During the simulation, the level of the deterministic process will be from time to time below and above the given threshold depending on the chosen parameters. If the level of the deterministic process is below the threshold, then an error burst occurs at the model output. On the other hand, if the level of the deterministic process is above the threshold, then an error-free burst is generated at the model output. The level-crossing rate (LCR) of the deterministic process has to be fitted to the occurrence rate of error bursts by finding a proper threshold and proper values for the parameters describing the deterministic process.

A. The LCR Fitting

Let us consider the deterministic process $\tilde{\zeta}(t)$ in (1). Its parameters $c_{i,n}$, $f_{i,n}$, and $\theta_{i,n}$ are again determined by using the MEDS. The LCR of $\tilde{\zeta}(t)$ can approximately be expressed as [16]:

$$\tilde{N}_{\zeta}(r) = \sqrt{\frac{\tilde{\beta}}{2\pi}} p_{\zeta}(r) \ , \ r \geq 0 \quad (7)$$

where

$$\tilde{\beta} = 2\pi^2 \sum_{n=1}^{N_i} (c_{i,n} f_{i,n})^2 \quad (8)$$

and

$$p_{\zeta}(r) = \frac{r}{\sigma_0^2} \exp(-\frac{r^2}{2\sigma_0^2}) \ , \ r \geq 0 \quad (9)$$

denotes the Rayleigh distribution. The problem at hand is to find a proper parameter vector $\mathbf{\Psi} = (\sigma_0, r_{th}, f_{max})$ in order to fit the LCR $\tilde{N}_{\zeta}(r)$ of the deterministic process at $r = r_{th}$ to the given occurrence rate $R_{EB} = N_{EB}/T_t$ of error bursts. For this purpose, we first fix σ_0 and r_{th}, e.g., by choosing $\sigma_0 = 0.5$ and $r_{th} = 0.2$. Then, we calculate f_{max} according to the following expression:

$$f_{max} = \frac{N_{EB}}{T_t p_{\zeta}(r_{th}) \sqrt{\frac{\tilde{\beta}'}{2\pi}}} \quad (10)$$

where

$$\tilde{\beta}' = 2\pi^2 \sum_{n=1}^{N_i} \left\{ c_{i,n} \sin\left[\frac{\pi}{2N_i}(n - \frac{1}{2})\right] \right\}^2 . \quad (11)$$

The chosen parameter vector is $\mathbf{\Psi} = (0.5, 0.2, 44.8025)$, which leads to the perfect LCR fitting.

B. The Properly Sampled Deterministic Process

By using the obtained parameters, a deterministic process is generated in the time interval $t \in [0, T_t]$. It should be mentioned that error bursts and error-free bursts correspond to fading intervals and inter-fade intervals, respectively, of the deterministic process with the specified threshold level. However, the lengths of the generated error bursts and error-free bursts depend on the sampling interval T_A of the deterministic process. In order to adapt the EBD and the EFBD of the generative model to any given EBD and EFBD, T_A should be sufficiently small with respect to the symbol duration $T_s = 1/F_s$ of the reference transmission system. In this paper, we have selected $T_A = T_s/6$ empirically. Consequently, $\tilde{N}_t = 48 \times 10^6$ samples of the deterministic process are produced. An error burst recorder $\widetilde{\mathbf{EB}}_{rec}$ and an error-free burst recorder $\widetilde{\mathbf{EFB}}_{rec}$ are then obtained, which record the numbers of samples in successive fading intervals and inter-fade intervals, respectively. Due to the perfect LCR fitting, $\widetilde{\mathbf{EB}}_{rec}$ has the same number of entries as in $\mathbf{EB}_{rec}$. Also, $\widetilde{\mathbf{EFB}}_{rec}$ and $\mathbf{EFB}_{rec}$ have the same number of entries. Let us define the similar parameters to those in Section III by simply putting the tilde sign on all affected symbols, i.e., we write $\tilde{m}_{B1}$, $\tilde{m}_{B2}$, $\tilde{N}_{EB}(m_e)$, etc. The idea is to modify $\widetilde{\mathbf{EB}}_{rec}$ and $\widetilde{\mathbf{EFB}}_{rec}$ in such a way that one can find for all entries of these vectors corresponding entries in $\mathbf{EB}_{rec}$ and $\mathbf{EFB}_{rec}$, respectively. This implies that $\tilde{N}_{EB}(m_e) = N_{EB}(m_e)$ and $\tilde{N}_{EFB}(m_{\bar{e}}) = N_{EFB}(m_{\bar{e}})$ hold. It follows that the resulting EBD $\tilde{P}_{EB}(m_e)$ and EFBD $\tilde{P}_{EFB}(m_{\bar{e}})$ are fitted perfectly to $P_{EB}(m_e)$ and $P_{EFB}(m_{\bar{e}})$, respectively.

For the purpose of properly modifying $\widetilde{\mathbf{EB}}_{rec}$, we first find the error burst length values $\ell^1_{m_e}$ and $\ell^2_{m_e}$ ($\tilde{m}_{B1} \leq \ell^1_{m_e}, \ell^2_{m_e} \leq \tilde{m}_{B2}$) to satisfy the following conditions:

$$\sum_{k=\ell^1_{m_e}}^{\ell^2_{m_e}-1} \tilde{N}_{EB}(k) < N_{EB}(m_e) \text{ and } \sum_{k=\ell^1_{m_e}}^{\ell^2_{m_e}} \tilde{N}_{EB}(k) \geq N_{EB}(m_e)$$

$$\text{for } N_{EB}(m_e) \neq 0 \text{ and } m_{B1} \leq m_e \leq m_{B2} . \quad (12)$$

Let us define

$$N_{\ell^2_{m_e}} = \sum_{k=\ell^1_{m_e}}^{\ell^2_{m_e}-1} \tilde{N}_{EB}(k) - N_{EB}(m_e)\,. \qquad (13)$$

Note that $\ell^1_{m_{B1}} = \tilde{m}_{B1}$ and $\ell^2_{m_{B2}} = \tilde{m}_{B2}$ hold. Moreover, $\ell^1_{m_e+1} = \ell^2_{m_e}$ if $N_{\ell^2_{m_e}} > 0$, while $\ell^1_{m_e+1} = \ell^2_{m_e} + 1$ if $N_{\ell^2_{m_e}} = 0$. Once $T_A \ll T_s$ is ensured, $\ell^2_{m_e} \leq m_e$ is always fulfilled. The next step is to find the entries between $\ell^1_{m_e}$ and $\ell^2_{m_e} - 1$ in $\widetilde{\mathbf{EB}}_{rec}$, and then replace them by m_e. Also, find the entries with the value $\ell^2_{m_e}$ in $\widetilde{\mathbf{EB}}_{rec}$. Only $N_{\ell^2_{m_e}}$ of them are replaced by m_e in order to ensure that $\tilde{N}_{EB}(m_e) = N_{EB}(m_e)$ holds. Following this procedure, corresponding values can be found in possibly different positions of $\widetilde{\mathbf{EB}}_{rec}$ and $\mathbf{EB}_{rec}$. Then, proper numbers of samples in successive fading intervals of the deterministic process are chosen according to the modified values in $\widetilde{\mathbf{EB}}_{rec}$.

The above described procedure applies also to $\widetilde{\mathbf{EFB}}_{rec}$. Proper numbers of samples in successive inter-fade intervals of the deterministic process are chosen according to the modified values in $\widetilde{\mathbf{EFB}}_{rec}$. Consequently, from the resulting properly sampled deterministic process, the distributions $\tilde{P}_{EB}(m_e)$ and $\tilde{P}_{EFB}(m_{\bar{e}})$ of the generated error bursts and error-free bursts are fitted perfectly to the given EBD $P_{EB}(m_e)$ and the EFBD $P_{EFB}(m_{\bar{e}})$, respectively.

It is important to mention that the whole procedure described in this subsection is nothing else than a mapping system. For example, if ℓ_{m_e} ($\ell^1_{m_e} \leq \ell_{m_e} < \ell^2_{m_e}$) samples of the deterministic process are observed in a fading interval, a mapping $\ell_{m_e} \rightarrow m_e$ is first performed and then an error burst with length m_e is generated.

C. The Generation of Error Sequences

In this subsection, from the deterministic process followed by a specified threshold and a mapper, an approach is described to enable the generation of an error sequence.

For generating error bursts, it is convenient to construct a parameter vector $\tilde{\mathbf{E}}_{EB}$, which records the number of errors in each error burst of $\widetilde{\mathbf{EB}}_{rec}$. To reach this aim, we first have to find the numbers of errors in the error bursts with length m_e ($m_{B1} \leq m_e \leq m_{B2}$) in $\mathbf{EB}_{rec}$. Then, we assign randomly these corresponding numbers of errors to the error bursts with length m_e in $\widetilde{\mathbf{EB}}_{rec}$. The generation rule of an error burst with a certain number of errors is as follows: an error occurs at least at both ends, the remaining errors occur randomly, and the local error density inside an error burst is always greater than the given value Δ. For generating an error-free burst, the length is interpreted as the number of consecutive zeros.

V. Performance Evaluation

In order to evaluate the overall performance of the proposed DPBGM, the average BEP, the GD, the EFRD, the EBD, and the EFBD calculated from the generated error sequence will be compared to those of the target error sequence. Furthermore, the relevant results of a SFM will also be presented for comparison purposes. The parameters of a N-state SFM are obtained by fitting the weighted sum of $N-1$ exponentials to the EFRD $P(0^m/1)$ [8]. In this paper, a 6-state SFM is employed. Note that no better fitting can be obtained by increasing the number of states.

The average BEP obtained from the DPBGM is exactly equal to the original BEP 7.5341×10^{-3}, while the SFM produces an error sequence with the average BEP of 7.5337×10^{-3}. Figs. 2 and 3 also include the GDs and the EFRDs of both generative models, respectively. These two curves of both the DPBGM and the SFM can approximate those of the descriptive model with good accuracy. Without any surprise, the SFM enables a better fitting to the desired GD and the EFRD compared with the DPBGM. The EBDs and the EFBDs of the generative models are illustrated in Fig. 4 together with the results already obtained for the descriptive model. As expected, the perfect match is observed in both curves for the DPBGM, while the SFM fails to capture these two statistics with high precision.

VI. Conclusion

This paper proposes a novel generative model, which is simply a deterministic process followed by a threshold and a mapping system. The design procedure runs as follows. In the first step, the parameters of the deterministic process and the specified threshold are determined by fitting the LCR to the occurrence rate of error bursts of the descriptive model. During the simulation, if the level of the deterministic process is below (above) the given threshold, an error burst (error-free burst) occurs at the model output. Then, by properly choosing the samples of the deterministic process, the distributions of the generated error bursts and error-free bursts can perfectly be adapted to any given EBD and EFBD, respectively. At last, a simple approach is presented to enable the generation of an error sequence from the properly sampled deterministic process. It is illustrated by various simulation results that the proposed DPBGM can accurately reproduce the desired burst error statistics.

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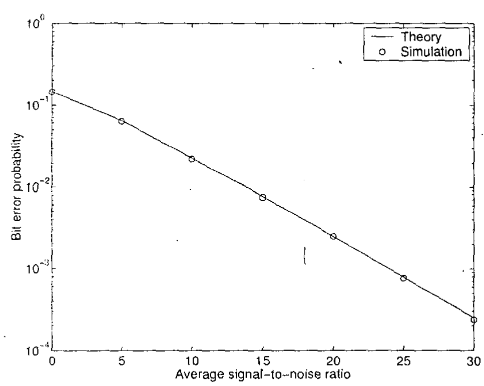

Fig. 1. The BEP for coherent QPSK systems by using the MEDS ($\sigma_0^2 = 1/2$, $f_{max} = 60$ Hz, $N_1 = 9$, $N_2 = 10$).

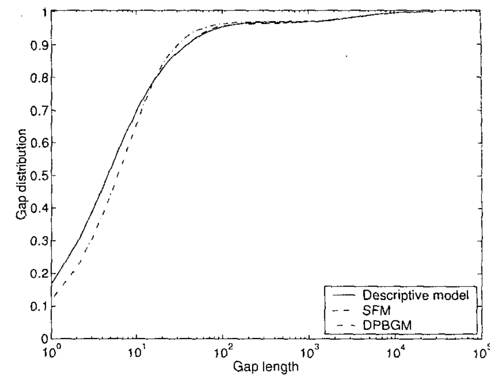

Fig. 2. The GDs of the generative models and the descriptive model.

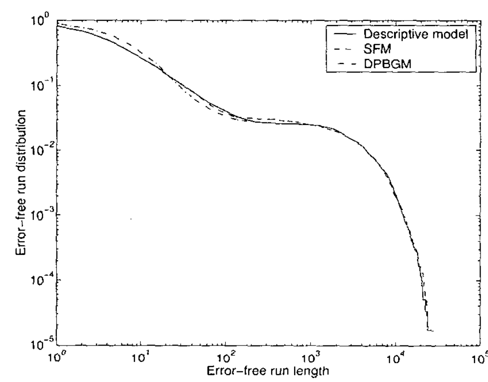

Fig. 3. The EFRDs of the generative models and the descriptive model.

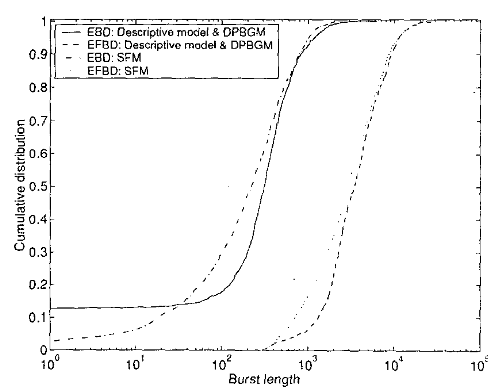

Fig. 4. The EBDs and the EFBDs of the generative models and the descriptive model.