Elongated Gaussian

In this exercise you study the effect of correlations between variables on the efficiency of MCMC. Consider the distribution $p(x_1, x_2) \propto \exp(-E(x_1, x_2))$, with $E = \frac{1}{2}x'Ax$ and $x = (x_1, x_2)$ and

$$A = \begin{pmatrix} 250.25 & -249.75 \\ -249.75 & 250.25 \end{pmatrix}$$

which describes the distribution of two strongly correlated Gaussian variables.

1. Write a computer program to sample from this distribution using the Metropolis Hasting algorithm. Study the acceptance ratio and how well the sampler covers the entire distribution by varying the width σ of the proposal distribution. Report the optimal values.

2. Write a computer program to sample from this distribution using the Hamilton Monte Carlo algorithm. Study the acceptance ratio and how well the sampler covers the entire distribution by varying the step size ϵ, the number of leap frog steps τ. Report the optimal values.

3. Compute the mean of this Gaussian distribution using both methods and compare the accuracy as a function of the computation time.

Bayesian inference for Perceptron learning with MCMC

In this exercise you are asked to sample from the posterior of learning problem. The learning task is the perceptron/logistic regression classification problem as explained in Mackay chapter 39 and 41. The sampling methods are the Metropolis Hasting method and the Hamilton Monte Carlo method as described in MacKay chapter 29 and 30. The data are given by the files `x.ext` (input patterns) and `t.ext` (output label).

Write a computer program to sample from the distribution $p(w|D, \alpha)$ as given by MacKay Eqs. 41.8-10 using the Metropolis Hasting algorithm. Do the same using the Hamilton Monte Carlo method. For both methods, reproduce plots similar to fig. 41.5 and estimate the burn in time that is required before the sampler reaches the equilibrium distribution. Investigate the acceptance ratio for both methods and try to optimize this by varying the proposal distribution, the step size ϵ in HMC and the number of leap frog steps τ.