Mackay 3.12

Options

$W_1 W_2$

$W_1 B_2$

$B_1 W_2$

~~B B~~ ← impossible

$$P(W_2 | W_1) = \frac{P(W_1, W_2)}{P(W_1)} = \frac{1/3}{2/3} = \frac{1}{2}$$

Extra opgave 3.1

a $N = 2$ options $S = \{hh, ht, th, tt\}$

$H_0: p_h = 0.5$ $H_1: p_h \neq 0,5$

assume equal probabilities

$P(H_0) = 0.5$ $P(H_1) = 0.5$

$N_H = 0$ $S = tt$

$P(S = tt | H_0) = 0.5^2 = 0,25$

$P(S = tt | H_1) = 1^2 = 1$

$$P(S = tt) = P(S=tt | H_0) P(H_0) + P(S=tt | H_1) P(H_1)$$
$$= 0,25 \cdot 0,5 + 1 \cdot 0,5$$
$$= 0,625$$

$$P(H_0 \mid s=tt) = \frac{p(s=tt \mid H_0)\, p(H_0)}{p(s=tt)}$$

$$= \frac{0{,}25 \cdot 0{,}5}{0{,}625}$$

$$= 0{,}2$$

$$p(H_1 \mid s=tt) = \frac{p(s=tt \mid H_1)\, p(H_1)}{p(s=tt)}$$

$$= \frac{1 \cdot 0{,}5}{0{,}625}$$

$$= 0{,}8$$

$N_H = 2$

$s = hh$

~~exactly~~ equal to $NH = 0$

$p(H_0 \mid s = hh) = 0{,}2$

$p(H_1 \mid s = hh) = 0{,}8$

$N_H = 1$

$s = \{ht, th\}$

$$p(s = \{ht, th\} \mid H_0) = 0{,}5^2 + 0{,}5^2$$

$$= 0.5$$

$p(s = \{ht, th\} \mid H_1) < 0{,}5$ for any $H_1: p_h \neq 0{,}5$

$$P(s=\{ht,th\}) = P(s=\{ht,th\}|H_0)P(H_0) + P(s=\{ht,th\}|H_1)P(H_1)$$

$$= 0{,}5 \cdot 0{,}5 + P(s=\{ht,th\}|H_1) \cdot 0{,}5$$

$$< 0{,}25 + 0{,}25$$

$$< 0{,}5$$

$$P(H_0 | s=\{ht,th\}) = \frac{P(s=\{th,ht\}|H_0)P(H_0)}{P(s=\{ht,th\})}$$

$$> \frac{0{,}5 \cdot 0{,}5}{0{,}5}$$

$$> \frac{0{,}25}{0{,}5}$$

$$> 0{,}5$$

$$P(H_1 | s=\{ht,th\}) = \frac{P(s=\{th,ht\}|H_0)P(H_0)}{P(s=\{ht,th\})}$$

$$= \frac{P(s=\{th,ht\}|H_0)P(H_1)}{(P(s=\{ht,th\}|H_0)P(H_0) + P(s=\{ht,th\}|H_1)P(H_1))}$$

$$P(H_1 \mid s=\{ht, th\}) = \frac{P(s=\{ht, th\} \mid H_1) \cdot 0{,}5}{(0{,}5 \cdot 0{,}5 + P(s=\{ht, th\} \mid H_1) \cdot 0{,}5)}$$

$$= \frac{P(s=\{ht, th\} \mid H_1)}{0{,}5 + P(s=\{ht, th\} \mid H_1)}$$

$$< \frac{0{,}5}{0{,}5+0{,}5}$$

$$< 0{,}5$$

b An unfair coin will prefer one result (side) over the other. Heads-Heads or Tails Tails will more likely be unfair. A fair coin will prefer a split result, such as Heads-tails or reverse. In the end, It is important to realise that $N=2$, and the coin should be tossed more to draw meaningful conclusions.