CDS: Machine Learning

Dirren van Vlijmen
Gerwin de Kruijf
Olivier Brahma

2.1 Perceptron

1. a)

$P \le n$, sum is limited by $P-1$

change $n-1$ to $P-1$

$$C(P,n) = 2\sum_{i=0}^{P-1} \binom{P-1}{i} = 2 \cdot \sum_{i=0}^{P-1} \binom{P-1}{i} 1^i$$

use formula =

$$2 \cdot (1+1)^{P-1} = 2^1 \cdot 2^{P-1} = 2^P$$

1. b) $P = 2n$, use $2n$ in sum

$$C(2n,n) = 2\sum_{i=0}^{n-1} \binom{2n-1}{i}$$

$$= 2 \cdot \frac{1}{2} (1+1)^{2n-1}$$

$$= 2^{2n-1} = 2^{P-1}$$

Perceptron 4

a.) $\delta = 4m(2p)\exp\left(\frac{-\epsilon^2 p}{8}\right) = 0.01$

we have two cases:

① $p > N \rightarrow m(2p) = C(N, 2p) \leq \left(\frac{2ep}{N}\right)^N$

② $p \leq N \rightarrow m(2p) = C(N, 2p) = 2^{2p}$

①: $0.01 = 4\left(\frac{2ep}{N}\right)^N \exp\left(\frac{-\epsilon^2 p}{8}\right)$

$\rightarrow \ln(0.01) = \ln(4) + N\ln\left(\frac{2ep}{N}\right) - \frac{\epsilon^2 p}{8}$

$\rightarrow \frac{\epsilon^2 p}{8} = N\left(\ln\left(\frac{2p}{N}\right) + 1\right) - \ln\left(\frac{0.01}{4}\right)$

$$\rightarrow \epsilon = \sqrt{\frac{8\left(N\left(\ln\left(\frac{2p}{N}\right) + 1\right) - \ln(\delta/4)\right)}{p}}$$

②: $0.01 = 2^{2p+2}\exp\left(\frac{-\epsilon^2 p}{8}\right)$

$\rightarrow \frac{\epsilon^2 p}{8} = (2p+2)\ln(2) - \ln(0.01)$

$$\rightarrow \epsilon = \sqrt{\frac{8\left((2p+2)\ln(2) - \ln(0.01)\right)}{p}}$$