INTRODUCTION

"It's just that I have a reasonable doubt—that's all—so I think that we oughta talk about it."

– Juror #8 in *Twelve Angry Men*[1]

A powerful and informative way to teach climate change is to focus on the controversy itself. Structure a curriculum around the points of contention—the accusations of bias—and focus class-time on the skills that help students address and remove these biases from analyses. Students are regularly exposed to discussions of climate change; many of them including claims of bias and misconception. Clarity and literacy may be fostered in the students if the controversies are highlighted and used as a teaching tool to show how mathematics is frequently misapplied.

A number of studies have shown that students and even teachers hold erroneous notions of the science and mathematics behind climate change.[2] Often these misconceptions are a result of improper (or altogether absent) applications of uncertainty to data. While high school math standards include studies of uncertainty, they are often limited to problems of probability. Without taking uncertainty one step further—one step beyond the trivial calculations of the probability of pulling the same color sock out of a drawer in the dark—numbers that are unqualified by a probability *but are still uncertain* are taken to be certain and thus misapplied.[3]

Much of the literature on uncertainty in climate change is deeply technical and difficult to access for classroom use. This should not prevent educators, however, from granting the mathematics of uncertainty a prominent place in the classroom. This paper highlights recent claims of bias, miscalculation and misconception in the climate change debate and proposes connections between these and high school education standards. This paper also asserts that current math standards might more adequately address uncertainty.

The value of teaching clear mathematical approaches to uncertainty is well-documented. The National Council of Teachers of Mathematics claims that "Students must develop enough confidence in their reasoning abilities to question others' mathematical arguments as well as their own." Professor David Orr (1992) writes, "Ecological literacy... presumes an ability to use numbers, and the ability to know what is countable and what is not." Professor Hardy Grant (2000) summarizes Plato's claim that the tools of math can cut through deception, "The *Republic* stressed the study of mathematics as the best way to lead the mind upward from the deceptive and perishable world of the senses to the contemplation of eternal reality and absolute truth." A mathematical study of biases pervasive in the climate change debate can improve students' ability to quantitatively address uncertainty, thereby sharpening their critical eye and ecological literacy.

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[1] The character played by Henry Fonda (1957) and Jack Lemmon (1997).

[2] See Boyes (1993), Koulaidis and Christidou (1999), Meadows and Wiesenmayer (1999), Groves and Pugh (1999), and Summers et al (2001).

[3] This is true even for official statistics released by the US Labor Department. In early March 2002, the government reported that corporate payrolls had increased in February. In the April report, however, the government siad that the previous report was incorrect—jobs had in fact *decreased* in February—but that the new report showed an increase in payrolls by 58,000 through March. In the May report, the government repeated its poor estimation: it revised the previous month's report to show a decrease (-21,000 jobs) but reported an increase in the current month (43,000). (NPR, Market Place, May 3, 2002, TNT, 2002))

THE TREATMENT OF UNCERTAINTY IN MATH EDUCATION:
HEAVY ON PROBABILITY

As students in grades 9-12 extend their knowledge, what they know and are able to do includes... using... probability to represent and solve problems involving uncertainty.
– Colorado Mathematics Standards

Gail Burrill (2002), a former president of the National Council of Teachers of Mathematics, credited a Japanese colleague with telling her that "teaching math is 80 percent certainty and 20 percent doubt." A glance through three sets of mathematics standards (see Table 1), however, shows that uncertainty is addressed only indirectly in official documents through related concepts such as probability, estimation, and results communication. Probability is discussed at length in each of the standards and is the concept that most completely addresses uncertainty. The standards also include varying requirements for proficiency in estimation and communication—two concepts that are related to mathematical uncertainty.

Estimation is of course inherently uncertain—the goal of estimation is implicitly determining an acceptable level of uncertainty and choosing a result that one feels is within that range. Communication is less clearly, but just as importantly, related to uncertainty. The inherent uncertainty in much quantitative analysis is often left out of results summaries. An understanding of how to communicate results—and a critical eye in interpreting others' results—is necessary in order to incorporate uncertainty in analysis.

The standards indicate, however, that explicit treatment of uncertainty in high school classrooms is minimal. Harvard education professor Kay Merseth (1993) writes about her observations of math classrooms, "Important topics such as probability and statistics or mathematical modeling and data analysis are either buried in the final chapters of the textbook or given no consideration at all." The focus instead is on "mastery of the computational skills which would be needed by a shopkeeper in the year 1940." (Mathematical, 1987, in Merseth, 1993)

In the three documents examined in Table 1, the word "uncertainty" only appears once. The final column of Table 1 lists selected points that seem to attempt to capture the need for uncertainty analysis (points that are, in themselves, very important), but do a poor job of capturing the extent of involvement that uncertainty encompasses in quantitative analysis. Washington's concern about imprecision and tool choice, Colorado's weak equation of probability and uncertainty, the NCTM's desire for graphical expression of summary statistics—none of these give students the ability to handle uncertainty in data analysis.

Table 1. The treatment of various topics related to uncertainty in three sets of high school mathematics standards. ("The Essential...", 2001; "Colorado...", 2002; *Principles...*, 2001)

	PROBABILITY	ESTIMATION	COMMUNICATION	UNCERTAINTY
WASHINGTON: ESSENTIAL ACADEMIC LEARNING REQUIREMENTS IN MATHEMATICS	☑ "The student will... understand and apply concepts and procedures from probability and statistics (probability, statistics, and prediction and inference)." ☑ "The student will... use statistics to support different points of view." ☑ "The student will... understand and make inferences based on the analysis of experimental results."	☑ "The student will... use estimation to... determine the reasonableness of answers involving real numbers, for example, estimating the interest on a loan." ☑ "The student will... use estimation to obtain reasonable approximations."	☑ "The student will... represent and share information (share, explain, and defend mathematical ideas using terms, language, charts, and graphs that can be clearly understood by a variety of audiences)."	☒ "The student will... understand that the precision and accuracy of measurement is affected by the measurement tools and calculating procedures." ☒ "The student will... calculate and use the different measures of central tendency, variability, and range as appropriate to describe data." ☒ "The student will... understand and make inferences based on the analysis of experimental results."
COLORADO: MATHEMATICS STANDARDS	☑ Students will be "analyzing statistical claims for erroneous conclusions or distortions."	☑ "A student will... use number sense, including estimation and mental arithmetic, to determine the reasonableness of solutions."	☑ Multiple mentions of "... and communicate the reasoning used in solving these problems."	☒ Students will be "using experimental and theoretical probability to represent and solve problems involving uncertainty (for example, the chance of playing professional sports if a student is a successful high school athlete)."
NCTM: PRINCIPLES & STANDARDS FOR SCHOOL MATHEMATICS	☑ "All students should... understand how to compute the probability of a compound event." ☑ "All students should... evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of the conclusions."	☑ "Estimating is... a measuring technique that should be developed throughout the school years."	☑ "All students [should]... analyze and evaluate the mathematical thinking and strategies of others." ☑ "All students [should]... use the language of mathematics to express mathematical ideas precisely."	☒ "Students should be able to a variety of summary statis and graphical displays to analyze... center, spread, a shape... of both univariate bivariate data." ☒ "The shape of a distributio can be analyzed using grap displays such as histogram dotplots, stem-and-leaf plo box plots."

THE NATURE OF UNCERTAINTY:
THERE'S MORE TO IT THAN JUST PROBABILITY

Well, let's see, we have on the bags, Who's on first,
What's on second, I Don't Know is on third.
- Abbott & Costello

It is clear that conversations go nowhere when the parties involved are not talking about the same thing. So when someone asks, "Is the earth getting warmer?", the only good answer is that question is not specific enough. One person might be talking about the temperature increase between this morning (when there was frost on his windshield) and this afternoon (when we are playing soccer in the sun). Another person might be talking about the last 100 years. Or 50 years. Or 1000 years.

There are also other questions that cannot be ignored, such as "How much is the earth warming?" and "How quickly is the earth warming?". One would need to ask about the geographic extent of the question: "Where on the earth are we talking about?". The question of whether the earth is warming will yield different answers depending on whether you are considering only Boston, only North America or the whole earth?

The NOVA website ("Featured...", 2002) describes an activity based on their Temperature Trends section of the NOVA teacher guide for the Spring 2000 show "What's Up With the Weather?" by a teacher in Boston that provokes discussion about the importance of asking "what exactly are we talking about?" The teacher has students separately graph sections of monthly average temperatures from Boston in the last 10 years. When the graphs are put together, students are often surprised to see minimal warming, which leads to a discussion of whether 10 years of data from 1 location are enough to predictions about long-term climate change. The Union of Concerned Scientists ("Confronting...", 2000) also asks high school students to "understand that not every little year-to-year variation means that the overall climate is changing." One of the controversies in climate change science is about how much evidence we need in order to make sound conclusions and policy changes.

These questions highlight one of the major roles that uncertainty plays in any analysis: uncertainty over what exactly is being discussed. Mathematicians regularly simplify problems so that the object being discussed is clear. Regarding the old calculus problem that begins "Consider a spherical cow...", Daniel Rockmore (2000) describes the clarifying value of such simplification:

> Having simplified the problem to a spherical cow, we can now proceed precisely and logically, deriving truth upon truth about this platonic beast. The facts may or may not say something about real cows, but they will be forever consistent with a simplified – spherical, non-four-footed, colorless, headless – and unchanging model of reality. Spherical cows allow for universal truths, real cows don't.

Heating arguments that regularly erupt in the popular and scientific literature are often quelled when a third party points out that the debaters are basing their arguments on different assumptions—they are considering two different, nearly-spherical cows.

Uncertainty means more than just clarity around assumptions, however. It also means that any estimate based on a sample needs to be presented in the form of a range. The surety of an answer often depends on how many times its question is asked. These three aspects of uncertainty—assumptions, confidence intervals, and sample size—are each important considerations in mathematical analysis, but none is related to probability.

Why risk confusing things with questions of uncertainty, as long as we're working with the best knowledge we have? Kammen and Hassenzahl (1999) assert that uncertainty is closely tied to risk, and levels of risk determine a number of decisions in fields as broad as public health, the environment and national security. In a consideration of the likely of a plane crash

Randomness is an important factor in understanding uncertainty. Almost every pollster claims that his subjects were chosen randomly, as if that claim gave the stamp of certainty to his data. The US Government, in the 1970 draft lottery dumped 366 numbered capsules – each with a day of the year on it –

into a large bin to determine who would be drafted for military duty. The numbers were placed into the bin in groups by month: first January went in, then February, all the way down to December. Apparently the bin was not mixed enough because a disproportionate number of the first dates selected were from December and dates from the early months of the year were selected near the end of the lottery significantly more than chance would dictate. In the 1971 draft lottery, the government abandoned the numbered balls and used a computer to randomly select the dates. (Paulos, 1988)

On other occasions, random sampling is applied when another method would be more appropriate. A geographically random set of local population densities would likely leave out cities that are dense but geographically small. A weighted random sample would be more appropriate in this case. Another example is the common digital model of land elevation[4], which samples on a regular grid. Some elevation models, however, are more appropriately sampled irregularly, such as one that includes desert and mountains. Close sampling in the desert is a waist of resources (time, money, memory), but close sampling in the mountains is imperative.

Another point of uncertainty in data analysis is the bias for many people to misunderstand the tendency for a data series to regress to its mean. People expect such regression in some circumstances. When George Brett batted .384 in 1980, few people expected him to do as well in 1981. When there are an unusually high number of car accidents on a stretch of highway on one day, few people expect there to be an unusually high number the next day as well.

There are, however, occasions when regression to the mean is inappropriately ignored. Kahneman and Tversky (1973, in Bazerman, 1998) describe a flight school in which pilots were praised for exceptionally good landings and harshly criticized for exceptionally poor ones. Instructors noticed that pilots who were praised tended to follow their great landings with average ones and criticized pilots tended to improve their landings after criticism. The fallacy is that the instructors concluded that punishments were more effective than rewards when in fact the only significant factor act work was regression to the mean.

Often these misconceptions are a result of improper (or altogether absent) application of notions of uncertainty to data.

THE NATURE OF THE CLIMATE CHANGE DEBATE:
HIGH STAKES AND HIGH UNCERTAINTY

The long-sustained political unwillingness to act decisively has been justified for a decade and more by emphasizing the uncertainties of climate change science, which have been significant.
– Glenn Willis, 2001

The debate over climate change is often fierce and like most debates, sometimes drops to a level of ungrounded accusations and personal attacks. These inappropriate means of argument are only distracting, however, and hide characteristics of the quieter, more real debate that has been dwelling in scientific and policy circles for years. One prominent characteristic in all forms of debate regarding climate change has been the acknowledgement (and sometimes accusation) of uncertainty. This makes it a particularly interesting debate to study, as well as an appropriate one to tackle in mathematics classes as a means to study quantitative uncertainty.

The nature of the climate change debate is unique and can be characterized by a number of features. Loosely, these features can be grouped into categories of those that are scientific in nature, those that are social in nature, and those that are related to time.

The science of climate change is, by nature, complicated because it is a science that attempts to study an open system over long time-scales. (Bechmann, 2000) The largeness of it all means that mathematical modeling is heavily relied on to recreate the past and make predictions about the future. This results in conclusions that are inherently uncertain and leads to frequent debate over the value and validity of climate models.[5]

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[4] These are often referred to as "D.E.M.'s", or digital elevation models.
[5] These are often referred to as "G.C.M.'s", or general circulation models.

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