Roy Model

Kuan-Ming Chen* Ming-Jen Lin†

Spring 2023

1 Goal

- Be able to draw the Roy model graphically
- Algebraically derive the Roy model and discusses all cases.
- Talk a bit about identification

2 Original Example: Roy

The question: what would be your earning if I become an accountant instead of economist?

Have a chart of average salary pulled up.

One proposal is to look at $\bar{y}_A$ - $\bar{y}_E$ Is this correct?

Roy's model graphically: a spread distribution and a degenerated distribution.

Note that the treatment effect language.

*National Taiwan University. Email: kuanmingchen@ntu.edu.tw
†National Taiwan University. Email: mjlin@ntu.edu.tw

3 Greek Letters

αA	`\alpha A`	νN	`\nu N`
βB	`\beta B`	$\xi\Xi$	`\xi \Xi`
$\gamma\Gamma$	`\gamma \Gamma`	$o O$	`o O`
$\delta\Delta$	`\delta \Delta`	$\pi\Pi$	`\pi \Pi`
$\epsilon\varepsilon E$	`\epsilon \varepsilon E`	$\rho\varrho P$	`\rho \varrho P`
ζZ	`\zeta Z`	$\sigma\Sigma$	`\sigma \Sigma`
ηH	`\eta H`	τT	`\tau T`
$\theta\vartheta\Theta$	`\theta \vartheta \Theta`	$\upsilon\Upsilon$	`\upsilon \Upsilon`
ιI	`\iota I`	$\phi\varphi\Phi$	`\phi \varphi \Phi`
κK	`\kappa K`	χX	`\chi X`
$\lambda\Lambda$	`\lambda \Lambda`	$\psi\Psi$	`\psi \Psi`
μM	`\mu M`	$\omega\Omega$	`\omega \Omega`

4 Review of Some Statistic Properties

Truncated normal formula: Let $X \sim N(\mu, \sigma^2)$, then $E[X|X > a] = \mu + \sigma \frac{\phi(a)}{1-\Phi(a)}$

What is joint normality? Suppose X and Y are joint normal, then $E[Y|X = x] = \frac{\sigma_{XY}}{\sigma_X^2} x$, which happens to be the regression coefficient.

Now that we understand these properties, we can change the notation to emoji.

5 Review of Roy Model

Recall: The Roy model question: If you should become an accountant or economist? Is $\bar{y}_A - \bar{y}_E$ correct.

Case 1: One degenerate, and the other has a distribution Case 2: Either is de-generated. (but then show that this is misleading, because it's the joint distribution that matters.)

6 Example of Migration

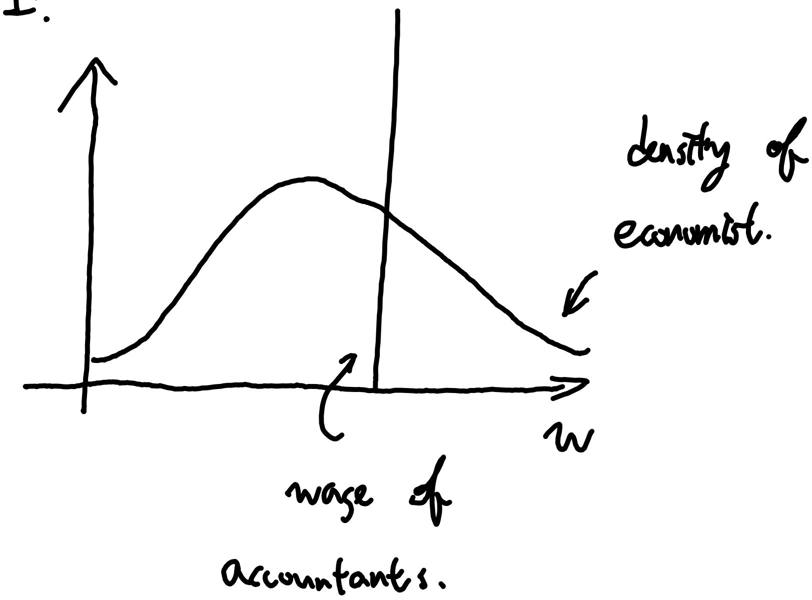
6.1 Who migrates?

Consider the simple model:

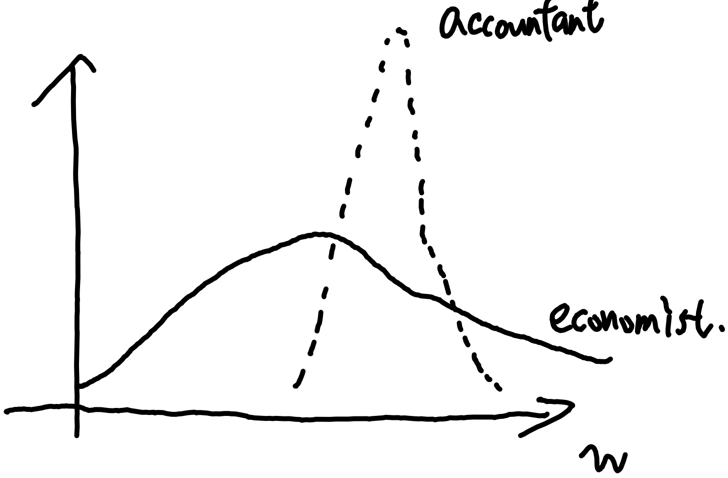
$$w_0 = \mu_0 + \epsilon_0$$

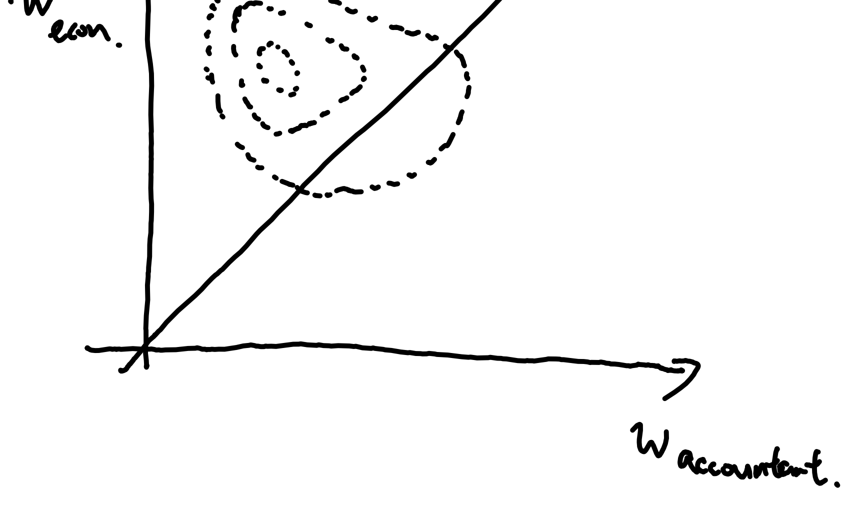
$$w_1 = \mu_1 + \epsilon_1$$

Case 1:


density of economist.
w
wage of accountants.

Case 2:


accountant
economist.
w


45°
$w_{econ.}$
$w_{accountant.}$

where migrant is 1, non-migrant is 0 Assume that $\epsilon_0 \sim N(0, \sigma_0^2)$ and $\epsilon_1 \sim N(0, \sigma_1^2)$ Assume the migrant cost C, and the correlation coefficient is $\rho = \frac{\sigma_{01}}{\sigma_0 \sigma_1}$

One will migrate if $w_1 > w_0 + C$. Let $\nu = \epsilon_1 - \epsilon_0$, then we have the probability of migration is $Pr(\nu > \mu_0 - \mu_1 + C) = Pr(\frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu})$. Call $z = \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}$, then we have the probability being $1 - \Phi(z)$.

6.2 Earning of migration?

What would be the staying income for those who immigrated?

$$E[w_0 | \text{Immigrate}] = \mu_0 + E[\epsilon_0 | \frac{\nu}{\sigma_\nu} > z]$$

then we do the algebra exercise to write RHS as a function of $E[\frac{\nu}{\sigma_\nu} | \frac{\nu}{\sigma_\nu} > z]$, and then derive the Inverse Mill Ratio.

The below derivation is directly drawn from David Autor's lecture note from his course "MIT 14.661 Spring 2003".

Our goal is to determine the expected value of ϵ_0 when a specific value of ν is given. Since both ϵ_0 and ϵ_1 are normally distributed, this expectation can be calculated by the regression coefficient formula:

$$E(\epsilon_0 | \nu) = \frac{\sigma_{0\nu}}{\sigma_\nu^2} \nu. \tag{1}$$

We obtain

$$E(\frac{\epsilon_0}{\sigma_0} \mid \frac{\nu}{\sigma_\nu}) = \underbrace{\frac{1}{\sigma_0}}_{\text{since we divide by } \sigma_0} \underbrace{\frac{\sigma_{0\nu}}{\sigma_\nu^2} \nu \frac{1}{\sigma_\nu}}_{\text{origin eq1}} \underbrace{\frac{1}{\sigma_\nu^{-2}}}_{\text{denominator eq1 divided by } \sigma_\nu} \underbrace{\frac{1}{\sigma_\nu}}_{\text{from change of } \sigma_{0\nu}} = \frac{\sigma_{0\nu}}{\sigma_0 \sigma_\nu} \frac{\nu}{\sigma_\nu} = \rho_{0\nu} \frac{\nu}{\sigma_\nu}$$

. Note that due to normalization, the covariance $\text{cov}(\epsilon_0, \epsilon_1)$ is reduced by $\frac{1}{\sigma_0 \sigma_1}$, and the variance of $\frac{\nu}{\sigma_\nu}$ is 1. Hence, we can rewrite the equation as

$$E(w_0 \mid \text{Immigrate}) = \mu_0 + \sigma_0 E(\frac{\epsilon_0}{\sigma_0} \mid \frac{\nu}{\sigma_\nu} > z) = \mu_0 + \rho_{0\nu} \sigma_0 E(\frac{\nu}{\sigma_\nu} \mid \frac{\nu}{\sigma_\nu} > z) = \mu_0 + \rho_{0\nu} \sigma_0 \frac{\phi(z)}{1 - \Phi(z)}$$

where $\frac{\phi(z)}{1-\Phi(z)}$ is the Inverse Mills Ratio, which represents the conditional expectation of a standard normal random variable truncated from the left at point z. The IMR is sometimes referred to as a "hazard ratio" because a hazard function answers the question "what is the probability of an event given that the event has not already occurred?" Similarly, the IMR answers the question: "what is the expectation of epsilon given that epsilon is greater than or equal to z?" We can calculate the expected wage in the source country of workers who do migrate as: $E(w_1 \mid \text{Immigrate}) = \mu_1 + E(\epsilon_1 \mid \frac{\nu}{\sigma_\nu} > z) = \mu_1 + \rho_{1\nu} \sigma_1 \frac{\phi(z)}{\Phi(-z)}$.

6.3 Understand Who will Immigrate?

$$E[w_0|I] = \mu_0 + \frac{\sigma_0\sigma_1}{\sigma_v}(\rho - \frac{\sigma_0}{\sigma_1})(\frac{\phi(z)}{1-\Phi(z)}) \tag{2}$$

$$E[w_1|I] = \mu_1 + \frac{\sigma_0\sigma_1}{\sigma_v}(\frac{\sigma_1}{\sigma_0} - \rho)(\frac{\phi(z)}{1-\Phi(z)}) \tag{3}$$

Let $Q_0 = E[\epsilon_0|I]$ and $Q_1 = E[\epsilon_1|I]$

6.3.1 Case 1: Positive Selection

$Q_0 > 0, Q_1 > 0$. That means $\frac{\sigma_1}{\sigma_0} > \rho > \frac{\sigma_0}{\sigma_1}$ and $\sigma_1 > \sigma_0$.

People moving to the place that is more spread and there is sufficient correlation between both jobs ($\rho > \frac{\sigma_0}{\sigma_1}$). (bright people move to gain more opportunity)

6.3.2 Case 2: Negative Selection

$Q_0 < 0, Q_1 < 0$. That means $\frac{\sigma_1}{\sigma_0} < \rho < \frac{\sigma_0}{\sigma_1}$ and $\sigma_1 < \sigma_0$

People moving to the place that is less spread and there is sufficient correlation between both jobs ($\rho > \frac{\sigma_1}{\sigma_0}$). (worst people move to gain "insurance")

6.3.3 Case 3: Refugee, Sorting

$Q_0 < 0, Q_1 > 0$. $\rho < \frac{\sigma_1}{\sigma_0}$ and $\rho < \frac{\sigma_0}{\sigma_1}$.

The skills in the two countries are opposite, so people sort into where needed the most.

6.3.4 Case 4?

Is it possible that $Q_0 > 0, Q_1 < 0$? (PSet)

7 Back to Our Problem: What's Wrong by Subtracting Means?

$E[w_1|I] - E[w_0|!I]$

References