Stabilizing Grand Cooperation of Machine Scheduling Game via Setup Cost Pricing

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Outline

Preliminaries

Cooperative Game

A **cooperative game** is defined by a pair (V, C):

- A set $V = \{1, 2, \ldots, v\}$ of players, **grand colaition**;
- A **characteristic function** $C(S) =$ the minimum total cost achieved by the cooperation of members in coalition $S \in \mathbb{S} = 2^V \setminus \{\emptyset\}$.

The game requires:

- A **cost allocation** $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_v] \in \mathbb{R}^v$, where $\alpha_k =$ the cost allocated to each player $k \in V$.

Core

Define $\alpha(S) = \sum_{k \in S} \alpha_k$.

A cost allocation $\alpha \in \mathbb{R}^v$ is in the **core** if it satisfies:

- **Budget Balance** Constraint: $\alpha(V) = C(V)$;
- **Coalition Stability** Constraints: $\alpha(S) \leq C(S)$ for each $S \in \mathbb{S}$.

$$\mathrm{Core}(V, C) = \Big\{\alpha : \ \alpha(V) = C(V), \\ \alpha(S) \leq C(S), \ \forall S \in \mathbb{S} \setminus \{V\}, \ \alpha \in \mathbb{R}^v\Big\}.$$

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$$\left. \alpha(S) \leq C(S),\ \forall S \in \mathbb{S} \setminus \{V\},\ \alpha \in \mathbb{R}^V \right\}.$$

However, $\mathrm{Core}(V, c)$ can be empty.

Existing Instruments

$$\mathrm{Core}(V, C) = \left\{ \alpha : \ \alpha(V) = C(V), \ \alpha(S) \leq C(S), \ \forall S \in \mathbb{S} \setminus \{V\} \right\}$$

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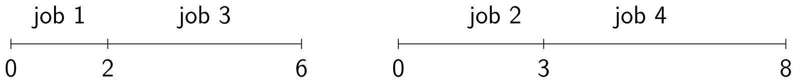
S.	Caprara and Letchford (2010, MP), Liu et al. (2016, IJOC)
P.	Faigle et al. (2001, IJGT), Schulz and Uhan (2010, OR)
P&S	Liu et al. (2018, OR)
Inv. Opt.	Liu et al. (2020, under review)

Illustrative Example

Example: Machine Scheduling Game (MSG)

Game of Parallel Machine Scheduling with Setup Cost:

- Grand coalition: $V = \{1, 2, 3, 4\}$;
- Processing times: $t_1 = 2$, $t_2 = 3$, $t_3 = 4$, $t_4 = 5$;
- Machine setup cost: $t_0 = 9.5$;
- $c(S)$ for $S \in \mathbb{S}$: minimizes the total completion time of jobs in S plus the machine setup cost;
- $\pi(N) = \pi(\{1, 3\}) + \pi(\{2, 4\}) = 38$ (SPT Rule).



Example: Empty Core

Coalitions	Cost
$\{1\}$	11.5
$\{2\}$	12.5
$\{3\}$	13.5
$\{4\}$	14.5
$\{1, 2\}$	16.5
$\{1, 3\}$	17.5
$\{1, 4\}$	18.5
$\{2, 3\}$	19.5
$\{2, 4\}$	20.5
$\{3, 4\}$	22.5
$\{1, 2, 3\}$	25.5
$\{1, 2, 4\}$	26.5
$\{1, 3, 4\}$	28.5
$\{2, 3, 4\}$	31.5
$\{1, 2, 3, 4\}$	38

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Optimal Cost Allocation Problem

$$\begin{aligned} \max \ & \left(\alpha_1+\alpha_2+\alpha_3+\alpha_4\right) = 37.25 < 38 \\ s.t. \ & \alpha_1 \le 11.5, \ \cdots, \ \alpha_4 \le 14.5, \\ & \alpha_1+\alpha_2 \le 16.5, \ \cdots, \ \alpha_3+\alpha_4 \le 22.5, \\ & \cdots, \\ & \alpha_1+\alpha_2+\alpha_3+\alpha_4 \le 38. \end{aligned}$$

$$\alpha^* = \left[6; 8.75; 10.75; 11.75\right]$$

Models & Analyses

Problem Definition and Formulation

Definition

An Identical Variable Parallel machine scheduling of Unweighted jobs game (IVPU):

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- Identical machine: $M = \{1, 2, \ldots, m\}$;
- Each machine Price: P and Each job processing time: t_k;
- Characteristic function: $c(S) = \min(\sum_{k \in S} C_k + Pm_S)$,

  where C_k is the completion time of job $k \in S$ and m_S is the number of using machine for the sub-coalition S.

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\begin{aligned}
c(S,P) = & \min \sum_{k\in V}\sum_{j\in O} c_{kj}x_{kj} + P\sum_{k\in s} x_{k1} \\
s.t. \quad & \sum_{j\in O} x_{kj} - y_k^s = 0, \forall k \in V, \\
& \sum_{k\in V} x_{kj} \le m, \forall j \in O, \\
& x_{kj} \in \{0,1\}, \forall k \in V, \forall j \in O, \\
& y_k^s = 1, k \in s \,; y_k^s = 0, k \notin s.
\end{aligned}
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Properties

Definition

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- For the grand coalition V, define $P_L(v,V) = 0, P_H(1,V) = P^*$. So the practical domain of the price is $[0, P^*]$ which is divided into v nonoverlapping subintervals by the number of using machines.
- Denote the right end of every subinterval as $P_i, 1 \leq i \leq v$, where $P_1 = P^*$.

Properties

$$\begin{aligned}\omega(P) \quad &= \min_\alpha\{c(V, m(V,P)) - \alpha(V) : \\ &\alpha(s) \le c(s, m(s,P)), \forall s \in S, \alpha \in \mathbb{R}^v\};\end{aligned}$$

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- $P_i, 2 \leq i \leq v$ can be obtained by SPT rules.
- $P_1 = P_2 + \cdots + P_n = \sum_{i=2}^{n} P_i$.

Properties

- When the number of using machines is 1 for the grand coalition, the range of slopes of the line segments in the interval is $\left(-1, -\frac{1}{n-1}\right]$, and the number of break-points is $O(v^2)$;

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- When the number of using machines is 1 for the grand coalition, the range of slopes of the line segments in the interval is $\left(-1, -\frac{1}{n-1}\right]$, and the number of break-points is $O(v^2)$;
- Define that

$$\begin{aligned} \omega_1(P) = \quad & \min_{\alpha}\{c(V, m(V,P)) - \alpha(V) : \\ & \alpha(s) \leq c(s) + P, \forall s \in S, \alpha \in \mathbb{R}^v\} \end{aligned}$$

Then the original problem $\omega(P)$ is equivalent to $\omega_1(P)$ which means that all sub-coalitions only use one machine.

Algorithms & Computations

IPC Algorithm

The Intersection Points Computation(IPC) Algorithm to Construct $\omega(P)$ Function.

Step 1. Initially, set $I^* = \{P_L, P_H\}$ and $\mathbb{I} = \{[P_L, P_H]\}$.

Step 2. When $\mathbb{I}$ is not empty: Sort values in I^* by $P_0 < P_1 < \cdots < P_q$, where $P_0 = P_L, P_q = P_H$ and $q = |I^*| - 1$.

Step 3. Obtain an LPB cost allocation α_{LP}^{GF} with $\alpha_{LP}^{GF}(N) = \pi_{\mathrm{LP}}(N)$.

Computational Results

Instrument 1: LRB Cost Allocation

Step 1. Find an LP $\min_x \left\{cx : Gx \geq F\gamma^s\right\}$ giving a lower bound $\pi_{\mathrm{LP}}(S)$ to $\pi(S)$;

Step 2. Compute $\left(\alpha_{LP}^{GF}\right)_j = \left(\mu^*\right)^T F_{\cdot j}$, where μ^*: dual solution; $F_{\cdot j}$: j-th column.

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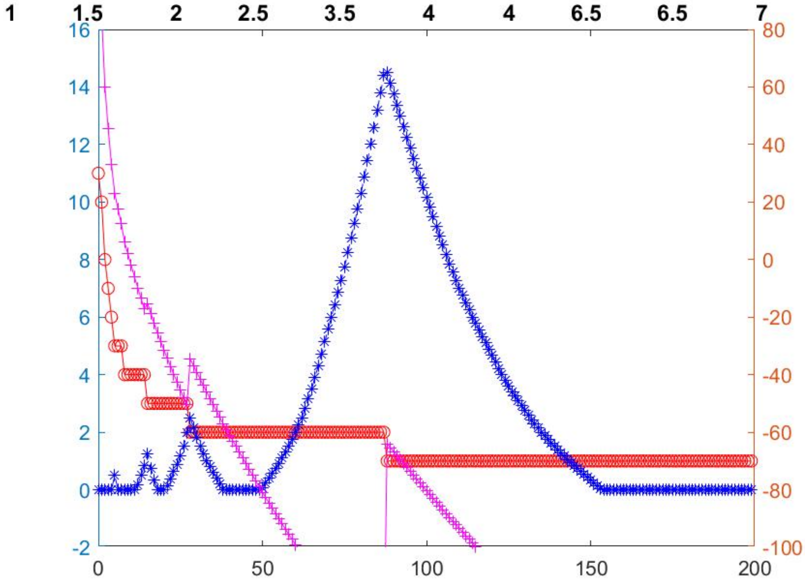
Image



1
1.5
2
2.5
3.5
4
4
6.5
6.5
7
16
14
12
10
8
6
4
2
0
-2
80
60
40
20
0
-20
-40
-60
-80
-100
0
50
100
150
200

Extension & Generalization

Machine Scheduling Game with Weighted Jobs

Pricing in General IM Games

Conclusions

- ⋆ Cooperative Game Theory:
  - New Instrument for Stabilization via Cost Adjustment.
- ⋆ Inverse Problem:
  - Constrained Inverse Optimization Problem.
- ⋆ Models, Solution Methods and Applications:
  - Several equivalent LP formulations;
  - Feasibility analyses & How to handle infeasibility;
  - Implementations on WMG and UFL games.

The End

Thank you!