Stabilizing Grand Cooperation of Machine Scheduling Game via Setup Cost Pricing

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Outline

Preliminaries

Cooperative Game

A cooperative game is defined by a pair (V, C):

- A set $V = \{1, 2, \ldots, v\}$ of players, **grand colaition**;
- A **characteristic function** $C(S) =$ the minimum total cost achieved by the cooperation of members in coalition $S \in \mathbb{S} = 2^V \setminus \{\emptyset\}$.

The game requires:

- A cost allocation $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_v] \in \mathbb{R}^v$, where $\alpha_k =$ the cost allocated to each player $k \in V$.

Core

Define $\alpha(S) = \sum_{k\in S} \alpha_k$.

A cost allocation $\alpha \in \mathbb{R}^V$ is in the **core** if it satisfies:

- Budget Balance Constraint: $\alpha(V) = C(V)$;
- Coalition Stability Constraints: $\alpha(S) \leq C(S)$ for each $S \in \mathbb{S}$.

$$\mathrm{Core}(V, C) = \Big\{\alpha : \ \alpha(V) = C(V), \\ \alpha(S) \leq C(S), \ \forall S \in \mathbb{S} \setminus \{V\}, \ \alpha \in \mathbb{R}^V\Big\}.$$

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However, $\mathrm{Core}(V, c)$ can be empty.

Existing Instruments

$$\mathrm{Core}(V, C) = \left\{ \alpha : \ \alpha(V) = C(V), \ \alpha(S) \leq C(S), \ \forall S \in \mathbb{S} \setminus \{V\} \right\}$$

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- Penalization: $\alpha(S) \le C(S) + z$, **least core**;

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- Simul. S & P: $\alpha(V) = C(V) - \theta$ and $\alpha(S) \leq C(S) + z$, **PSF**;

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- Inv. Opt.: Changing c to d such that $\mathrm{Core}(V, D)$ is non-empty.

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S.	Caprara and Letchford (2010, MP), Liu et al. (2016, IJOC)
P.	Faigle et al. (2001, IJGT), Schulz and Uhan (2010, OR)
P&S	Liu et al. (2018, OR)
Inv. Opt.	Liu et al. (2020, under review)

Illustrative Example

Example of Single Machine Scheduling Game

Models & Analyses

Problem Definition and Formulation

Properties

Algorithms & Computations

... Algorithm

Computational Results

Extension & Generalization

Machine Scheduling Game with Weighted Jobs

Pricing in General IM Games

Conclusions

⋆ Cooperative Game Theory:

- New Instrument for Stabilization via Cost Adjustment.

⋆ Inverse Problem:

- Constrained Inverse Optimization Problem.

⋆ Models, Solution Methods and Applications:

- Several equivalent LP formulations;
- Feasibility analyses & How to handle infeasibility;
- Implementations on WMG and UFL games.

The End

Thank you!