Appointment Scheduling

Discount

October 2024

1 Model

We assume that the patients will arrive at the appointed time.

The service time for patient i, ξ_i, stochastic with a mean of μ_i and a standard deviation of σ_i.

The service times are mutually independent.

For each patient $i = 1, \ldots, n$,

A_i: appointment time.

$S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\}$: actual starting time of service.

$A_1 = S_1 = 0$.

Patient i will arrive at A_i but start service at S_i.

Waiting time: $S_i - A_i$

Overtime: $(S_n + \xi_n - T)^+$

Total idle time: $\sum_{i=1}^{n-1} = [S_{i+1} - (S_i + \xi_i)] = S_n - \sum_{i=1}^{n-1} \xi_i$

Problem to minimize the total time:

$$\begin{aligned} \min_{\mathbf{A}} \quad & E_\xi \left[\left(S_n - \sum_{i=1}^{n-1} \xi_i \right) + \sum_{i=2}^{n} \alpha_i \left(S_i - A_i \right) + \beta (S_n + \xi_n - T)^+ \right] \\ \text{s.t.} \quad & S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\} \\ & S_1 = 0 \end{aligned} \tag{1}$$

$$S_i = \max\left\{A_i, S_{i-1} + \xi_{i-1}\right\}, \tag{2}$$

$$S_i - A_i = \max\left\{0, (A_{i-1} + W_{i-1}) + \xi_{i-1} - A_i\right\}, \tag{3}$$

$$W_i = \left(W_{i-1} + \xi_{i-1} - X_{i-1}\right)^+. \tag{4}$$

Slot time: $X_i = A_{i+1} - A_i \rightarrow A_j = \sum_{i=1}^{j-1} X_i$ Waiting time: $W_i = S_i - A_i \rightarrow S_n = \sum_{i=1}^{n-1} X_i + W_n$

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\begin{aligned}
\min_{\mathbf{X}} \quad & E_\xi \left[\sum_{i=1}^{n-1} (X_i - \xi_i) + W_n + \sum_{i=2}^{n} \alpha_i W_i + \beta(\sum_{i=1}^{n-1} X_i + W_n + \xi_n - T)^+ \right] \\
\text{s.t.} \quad & W_i = \max\{0, W_{i-1} + \xi_{i-1} - X_{i-1}\} \\
& W_1 = 0.
\end{aligned} \tag{5}
$$

Suppose that σ are the same for all patients.
Let

$$x_i = (X_i - \mu_i)/\sigma, \tag{6}$$

$$\zeta_i = (\xi_i - \mu_i)/\sigma, \text{ and} \tag{7}$$

$$w_i = W_i/\sigma; \tag{8}$$

Take out $-\sum_{i=1}^{n-1} \mu_i$.

$$
\begin{aligned}
\sigma \cdot \min_{\mathbf{x}} \quad & \left\{ \sum_{i=1}^{n-1} x_i + E_\zeta w_n + \sum_{i=2}^{n} \alpha_i E_\zeta [w_i] \right\} \\
\text{s.t.} \quad & w_i = \max\{0, w_{i-1} + \zeta_{i-1} - x_{i-1}\} \\
& w_1 = 0.
\end{aligned} \tag{9}
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Traditional: idle time + waiting time + (overtime)
Social distance: Ilde time + Overlap + Overtime
Conclusion: dome-shaped
Graph:

$$
\begin{aligned}
\min_{\mathbf{A}} \quad & E_\xi \left[\left(S_n - \sum_{i=1}^{n-1} \xi_i \right) + \sum_{i=2}^{n} \alpha_i \max(S_i - A_{i+1}, 0) \right] \\
\text{s.t.} \quad & S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\} \\
& S_1 = 0
\end{aligned} \tag{10}
$$

Let $O_i = S_i - A_{i+1}$, $X_i = A_{i+1} - A_i$.
$S_n = \sum_{k=1}^{n} X_k + O_n$, $S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\}$
$O_i = X_i + \max\{0, O_{i-1} + \xi_{i-1}\}$

$$
\begin{aligned}
\min_{\mathbf{A}} \quad & \left[\sum_{k=1}^{n} X_k + E_\xi O_n - E_\xi \sum_{i=1}^{n-1} \xi_i + E_\xi \sum_{i=2}^{n} \alpha_i O_i \right] \\
\text{s.t.} \quad & O_i \geq X_i + \max\{0, O_{i-1} + \xi_{i-1}\} \\
& O_i \geq 0
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\min_{\mathbf{A}} \quad & \left[\sum_{k=1}^{n} X_k + \sum_{k=1}^{M} \frac{1}{M} O_n - E_\xi \sum_{i=1}^{n-1} \xi_i + \sum_{i=2}^{n} \alpha_i \sum_{k=1}^{M} \frac{1}{M} O_i^k\right] \\
\text{s.t.} \quad & O_i^k \geq O_{i-1}^k + \xi_{i-1}^k \\
& O_i^k \geq X_i^k \\
& O_i^k \geq 0
\end{aligned} \tag{12}
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