
The Theory and Practice of Revenue Management

Recent titles in the
INTERNATIONAL SERIES IN
OPERATIONS RESEARCH & MANAGEMENT SCIENCE
Frederick S. Hillier, Series Editor, Stanford University

- Ramík, J. & Vlach, M. / *GENERALIZED CONCAVITY IN FUZZY OPTIMIZATION AND DECISION ANALYSIS*
- Song, J. & Yao, D. / *SUPPLY CHAIN STRUCTURES: Coordination, Information and Optimization*
- Kozan, E. & Ohuchi, A. / *OPERATIONS RESEARCH/ MANAGEMENT SCIENCE AT WORK*
- Bouyssou et al. / *AIDING DECISIONS WITH MULTIPLE CRITERIA: Essays in Honor of Bernard Roy*
- Cox, Louis Anthony, Jr. / *RISK ANALYSIS: Foundations, Models and Methods*
- Dror, M., L'Ecuyer, P. & Szidarovszky, F. / *MODELING UNCERTAINTY: An Examination of Stochastic Theory, Methods, and Applications*
- Dokuchaev, N. / *DYNAMIC PORTFOLIO STRATEGIES: Quantitative Methods and Empirical Rules for Incomplete Information*
- Sarker, R., Mohammadian, M. & Yao, X. / *EVOLUTIONARY OPTIMIZATION*
- Demeulemeester, R. & Herroelen, W. / *PROJECT SCHEDULING: A Research Handbook*
- Gazis, D.C. / *TRAFFIC THEORY*
- Zhu, J. / *QUANTITATIVE MODELS FOR PERFORMANCE EVALUATION AND BENCHMARKING*
- Ehrgott, M. & Gandibleux, X. / *MULTIPLE CRITERIA OPTIMIZATION: State of the Art Annotated Bibliographical Surveys*
- Bienstock, D. / *Potential Function Methods for Approx. Solving Linear Programming Problems*
- Matsatsinis, N.F. & Siskos, Y. / *INTELLIGENT SUPPORT SYSTEMS FOR MARKETING DECISIONS*
- Alpern, S. & Gal, S. / *THE THEORY OF SEARCH GAMES AND RENDEZVOUS*
- Hall, R.W. / *HANDBOOK OF TRANSPORTATION SCIENCE - 2nd Ed.*
- Glover, F. & Kochenberger, G.A. / *HANDBOOK OF METAHEURISTICS*
- Graves, S.B. & Ringuest, J.L. / *MODELS AND METHODS FOR PROJECT SELECTION: Concepts from Management Science, Finance and Information Technology*
- Hassin, R. & Haviv, M. / *TO QUEUE OR NOT TO QUEUE: Equilibrium Behavior in Queueing Systems*
- Gershwin, S.B. et al / *ANALYSIS & MODELING OF MANUFACTURING SYSTEMS*
- Maros, I. / *COMPUTATIONAL TECHNIQUES OF THE SIMPLEX METHOD*
- Harrison, T., Lee, H. & Neale, J. / *THE PRACTICE OF SUPPLY CHAIN MANAGEMENT: Where Theory And Application Converge*
- Shanthikumar, J.G., Yao, D. & Zijm, W.H. / *STOCHASTIC MODELING AND OPTIMIZATION OF MANUFACTURING SYSTEMS AND SUPPLY CHAINS*
- Nabrzyski, J., Schopf, J.M., Węglarz, J. / *GRID RESOURCE MANAGEMENT: State of the Art and Future Trends*
- Thissen, W.A.H. & Herder, P.M. / *CRITICAL INFRASTRUCTURES: State of the Art in Research and Application*
- Carlsson, C., Fedrizzi, M., & Fullér, R. / *FUZZY LOGIC IN MANAGEMENT*
- Soyer, R., Mazzuchi, T.A., & Singpurwalla, N.D. / *MATHEMATICAL RELIABILITY: An Expository Perspective*

*** A list of the early publications in the series is at the end of the book ***

THE THEORY AND PRACTICE OF REVENUE MANAGEMENT

KALYAN T. TALLURI

Department of Economics and Business
Universitat Pompeu Fabra
Barcelona

GARRETT J. VAN RYZIN

Graduate School of Business
Columbia University
New York

Library of Congress Cataloging-in-Publication Data

A C.I.P. Catalogue record for this book is available from the Library of Congress

Tallury & Van Ryzin/ *THE THEORY AND PRACTICE OF REVENUE MANAGEMENT*

ISBN 978-0-387-24376-4

ISBN 978-0-387-27391-4 (eBook)

DOI 10.1007/978-0-387-27391-4

First paperback printing © 2005 Springer Science+Business Media, Inc.

© 2004 Springer Science+Business Media, Inc.

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, Inc., 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

9 8 7 6 5 4 3 2 1

SPIN 11374756

springeronline.com

To Cristina and Uma
for the love and joy,
K.T.

To Mary Beth,
Stephanie, Claire and
Andrea with love and
thanks, and to the
memory of my father
John R. van Ryzin,
G.V.

Contents

Dedication	v
List of Figures	xvii
List of Tables	xxi
Preface	xxv
Acknowledgments	xxxix
1. INTRODUCTION	1
1.1 What Is “RM”?	1
1.1.1 Demand-Management Decisions	2
1.1.2 What’s New About RM?	4
1.2 The Origins of RM	6
1.2.1 Airline History	6
1.2.2 Consequences of the Airline History	10
1.3 A Conceptual Framework for RM	11
1.3.1 The Multidimensional Nature of Demand	11
1.3.2 Linkages Among Demand-Management Decisions	12
1.3.3 Business Conditions Conducive to RM	13
1.3.4 Industry Adopters Beyond the Airlines	16
1.4 An Overview of a RM System	17
1.5 The State of the RM Profession	18
1.6 Chapter Organization and Reading Guide	20
1.6.1 Chapter Organization	20
1.6.2 Reading Guide	22
1.7 Notes and Sources	23

Part I Quantity-based RM

2. SINGLE-RESOURCE CAPACITY CONTROL	27
2.1 Introduction	27
2.1.1 Types of Controls	28
2.1.2 Displacement Cost	32
2.2 Static Models	33
2.2.1 Littlewood's Two-Class Model	35
2.2.2 n -Class Models	36
2.2.3 Computational Approaches	41
2.2.4 Heuristics	44
2.3 Adaptive Methods	50
2.3.1 Adaptive Algorithm	50
2.3.2 A Numerical Comparison with EMSR and Censored Forecasting	52
2.4 Group Arrivals	56
2.5 Dynamic Models	57
2.5.1 Formulation and Structural Properties	58
2.5.2 Optimal Policy	59
2.6 Customer-Choice Behavior	62
2.6.1 Buy-Up Factors	62
2.6.2 Discrete-Choice Models	64
2.7 Notes and Sources	75
3. NETWORK CAPACITY CONTROL	81
3.1 Introduction	81
3.1.1 The Promise and Challenge of Network Control	82
3.1.2 Types of Controls	83
3.2 The Theory of Optimal Network Controls	87
3.2.1 The Structure of Optimal Controls	88
3.2.2 Bid Price Controls	89
3.2.3 Nonoptimality of Bid-Price Controls	90
3.2.4 Evidence in Support of Bid Prices	91
3.2.5 Bid Prices and Opportunity Cost	91
3.3 Approximations Based on Network Models	92
3.3.1 The Deterministic Linear Programming Model	93
3.3.2 The Probabilistic Nonlinear Programming Model	95
3.3.3 The Randomized Linear Programming Model	98

3.4	Approximations Based on Decomposition	100
3.4.1	OD Factors Method	101
3.4.2	Prorated EMSR	102
3.4.3	Displacement-Adjusted Virtual Nesting (DAVN)	103
3.4.4	Dynamic Programming Decomposition	107
3.4.5	Iterative Decomposition Methods	108
3.5	Stochastic Gradient Methods	111
3.5.1	Continuous Model with Gradient Estimates	112
3.5.2	Discrete Model with First-Difference Estimates	116
3.6	Asymptotic Analysis of Network Problems	118
3.6.1	Asymptotic Optimality of Partitioned Controls	118
3.6.2	Asymptotic Optimality of Bid-Price Controls	120
3.6.3	Comments on Asymptotic Optimality	120
3.7	Decentralized Network Control: Airline Alliances	121
3.8	Notes and Sources	122
4.	OVERBOOKING	129
4.1	Business Context and Overview	130
4.1.1	A History of Legal Issues in Airline Overbooking	131
4.1.2	Managing Denied-Service Occurrences	135
4.1.3	Lessons Beyond the Airline Industry	137
4.2	Static Overbooking Models	138
4.2.1	The Binomial Model	139
4.2.2	Static-Model Approximations	147
4.2.3	Customer Class Mix	149
4.2.4	Group Cancellations	150
4.3	Dynamic Overbooking Models	152
4.3.1	Exact Approaches	152
4.3.2	Heuristic Approaches Based on Net Bookings	154
4.4	Combined Capacity-Control and Overbooking Models	155
4.4.1	Exact Methods for No-Shows	156
4.4.2	Class-Dependent No-Show Refunds	158
4.4.3	Exact Methods for Cancellations	159
4.4.4	Class-Dependent Cancellation Refunds	160
4.5	Substitutable Capacity	161
4.5.1	Model and Formulation	162
4.5.2	Joint Optimal Overbooking Levels	164
4.6	Network Overbooking	166

4.7	Notes and Sources	168
Part II Price-based RM		
5.	DYNAMIC PRICING	175
5.1	Introduction and Overview	175
5.1.1	Price versus Quantity-Based RM	176
5.1.2	Industry Overview	177
5.1.3	Examples of Dynamic Pricing	179
5.1.4	Modeling Dynamic Price-Sensitive Demand	182
5.2	Single-Product Dynamic Pricing Without Replenishment	187
5.2.1	Deterministic Models	188
5.2.2	Stochastic Models	200
5.3	Single-Product Dynamic Pricing with Replenishment	209
5.3.1	Deterministic Models	209
5.3.2	Stochastic Models	212
5.4	Multiproduct, Multiresource Pricing	215
5.4.1	Deterministic Models Without Replenishment	216
5.4.2	Deterministic Models with Replenishment	218
5.4.3	Stochastic Models	219
5.4.4	Action-Space Reductions	220
5.5	Finite-Population Models and Price Skimming	223
5.5.1	Myopic Customers	223
5.5.2	Strategic Customers	226
5.6	Promotions Optimization	229
5.6.1	An Overview of Promotions	229
5.6.2	Retailer Promotions	232
5.6.3	Trade-Promotion Models	234
5.7	Notes and Sources	235
6.	AUCTIONS	241
6.1	Introduction and Industry Overview	241
6.1.1	An Overview of Auctions in Practice	242
6.1.2	Types of Auctions	245
6.2	Independent Private-Value Theory	247
6.2.1	Independent Private-Value Model and Assump- tions	247
6.2.2	An Informal Analysis of Sealed-Bid, First- and Second-Price Auctions	248

6.2.3	Formal Game-Theoretic Analysis	254
6.2.4	Revenue Equivalence	257
6.2.5	Optimal Auction Design	259
6.2.6	Relationship to List Pricing	262
6.2.7	Departures from the Independent Private-Value Model	266
6.3	Optimal Dynamic Single-Resource Capacity Auctions	272
6.3.1	Formulation	272
6.3.2	Optimal Dynamic Allocations and Mechanisms	274
6.3.3	Comparisons with Traditional RM	278
6.4	Optimal Dynamic Auctions with Replenishment	280
6.4.1	Dynamic Programming Formulation	281
6.4.2	Optimal Auction and Replenishment Policy	282
6.4.3	Average-Profit Criterion	284
6.4.4	Comparison with a List-Price Mechanism	285
6.5	Network Auctions	288
6.5.1	Problem Definition and Mechanism	289
6.5.2	Equilibrium Analysis	290
6.5.3	Relationship to Traditional Auctions	291
6.5.4	Relationship to Traditional Network RM	291
6.5.5	Revenue Maximization and Reserve Prices	293
6.6	Notes and Sources	294

Part III Common Elements

7.	CUSTOMER-BEHAVIOR AND MARKET-RESPONSE MODELS	301
7.1	The Independent-Demand Model	301
7.2	Models of Individual Customer Choice	303
7.2.1	Reservation-Price Models	303
7.2.2	Random-Utility Models	304
7.2.3	Customer Heterogeneity and Segmentation	308
7.3	Models of Aggregate Demand	310
7.3.1	Demand Functions and Their Properties	311
7.3.2	Multiproduct-Demand Functions	320
7.3.3	Common Demand Functions	321
7.3.4	Stochastic-Demand Functions	327
7.3.5	Rationing Rules	330
7.4	Notes and Sources	330

8. THE ECONOMICS OF RM	333
8.1 Introduction	333
8.2 Perfect Competition	336
8.2.1 Perfectly Competitive Markets	336
8.2.2 Firm-Level Decisions Under Perfect Competition	338
8.2.3 Precommitment and Demand Uncertainty	338
8.2.4 Peak-Load Pricing Under Perfect Competition	341
8.2.5 Identifiable Peak Periods	341
8.2.6 Uncertainty over the Timing of Peak Loads	343
8.2.7 Advance Purchases in Competitive Markets	345
8.3 Monopoly Pricing	349
8.3.1 Single-Price Monopoly	350
8.3.2 Monopoly with Capacity Constraints	351
8.3.3 Multiple-Price Monopoly and Price Discrimination	352
8.3.4 Strategic Customer Behavior	363
8.3.5 Optimal Mechanism Design for a Monopolist	369
8.3.6 Advance Purchases and Peak-Load Pricing Under Monopoly	372
8.4 Price and Capacity Competition in an Oligopoly	375
8.4.1 Static Models	376
8.4.2 Dynamic Models	388
8.4.3 Product Differentiation	395
8.5 Notes and Sources	402
9. ESTIMATION AND FORECASTING	407
9.1 Introduction	407
9.1.1 The Forecasting Module of RM Systems	408
9.1.2 What Forecasts Are Required?	410
9.1.3 Data Sources	412
9.1.4 Design Decisions	415
9.2 Estimation Methods	419
9.2.1 Estimators and Their Properties	420
9.2.2 MSE Estimators	422
9.2.3 Maximum-Likelihood (ML) Estimators	425
9.2.4 Method of Moments and Quantile Estimators	427
9.2.5 Endogeneity, Heterogeneity, and Competition	428
9.3 Forecasting Methods	433
9.3.1 Ad-Hoc Forecasting Methods	434
9.3.2 Time-Series Forecasting Methods	439

9.3.3	Stationary Time-Series Models	442
9.3.4	Nonstationary Time-Series Models	447
9.3.5	Box-Jenkins Identification Process	449
9.3.6	Bayesian Forecasting Methods	450
9.3.7	State-Space Models and Kalman Filtering	458
9.3.8	Machine-Learning (Neural-Network) Methods	464
9.3.9	Pick-up Forecasting Methods	470
9.3.10	Other Methods	472
9.3.11	Combining Forecast Methods	472
9.4	Data Incompleteness and Unconstraining	473
9.4.1	Expectation-Maximization (EM) Method	474
9.4.2	Gibbs Sampling	481
9.4.3	Kaplan-Meier Product-Limit Estimator	483
9.4.4	Plotting Procedures	484
9.4.5	Projection-Detruncation Method	485
9.5	Error Tracking and System Control	486
9.5.1	Estimation Errors	487
9.5.2	Forecasting Errors and System Control	496
9.6	Industry Models of RM Estimation and Forecasting	499
9.6.1	Airline No-Show and Cancellations Forecasting	499
9.6.2	Groups Demand and Utilization Forecasting	502
9.6.3	Sell-Up and Recapture Forecasting	504
9.6.4	Retail Sales Forecasting	505
9.6.5	Media Forecasting	508
9.6.6	Gas-Load Forecasting	510
9.7	Notes and Sources	511
10.	INDUSTRY PROFILES	515
10.1	Airlines	515
10.1.1	History	515
10.1.2	Customers, Products, and Pricing	516
10.1.3	RM Practice	521
10.2	Hotels	524
10.2.1	Customers, Products, and Pricing	524
10.2.2	RM Practice	526
10.3	Rental Car	531
10.3.1	Customers, Products, and Pricing	531
10.3.2	RM Practice	532

10.4	Retailing	533
10.4.1	Customers, Products, and Pricing	534
10.4.2	RM Practice	541
10.5	Media and Broadcasting	542
10.5.1	Customers, Products, and Pricing	543
10.5.2	RM Practice	545
10.6	Natural-Gas Storage and Transmission	546
10.6.1	Customers, Products, and Pricing	547
10.6.2	RM Practice	550
10.7	Electricity Generation and Transmission	551
10.7.1	Industry Structure	552
10.7.2	Customers, Products, and Pricing	554
10.7.3	RM Practice	554
10.8	Tour Operators	555
10.8.1	Customers, Products, and Pricing	556
10.8.2	Capacity Management and Base-Price Setting	556
10.8.3	RM Practice	558
10.9	Casinos	559
10.9.1	Customers, Products, and Pricing	559
10.9.2	RM Practice	559
10.10	Cruise Ships and Ferry Lines	560
10.10.1	Customers, Products, and Prices	560
10.10.2	RM Practice	561
10.11	Passenger Railways	561
10.11.1	Customers, Products, and Pricing	561
10.11.2	RM Practice	563
10.12	Air Cargo	563
10.12.1	Customers, Products, and Pricing	563
10.12.2	RM Practice	563
10.13	Freight	564
10.13.1	Customers, Products, and Pricing	565
10.13.2	RM Practice	566
10.14	Theaters and Sporting Events	567
10.14.1	Customers, Products, and Pricing	567
10.14.2	Ticket Scalping and Distribution	567
10.14.3	RM Practice	571
10.15	Manufacturing	574

10.15.1 Customers, Products, and Pricing	574
10.15.2 RM Practice	575
10.16 Notes and Sources	576
11. IMPLEMENTATION	579
11.1 Segmentation and Product Design	579
11.1.1 Segmentation	580
11.1.2 Product Design	585
11.2 System Architecture, Hardware, Software, and Interfaces	594
11.2.1 Hardware Requirements	594
11.2.2 User-Interface Design	594
11.2.3 GDS, CRS, and PMS Interfaces	598
11.2.4 Retail Management Systems	605
11.3 Revenue-Opportunity Assessment and Revenue-Benefits Measurement	608
11.3.1 Revenue-Opportunity Assessment	608
11.3.2 Revenue-Benefits Measurement	610
11.4 RM Simulation	611
11.4.1 Generating Aggregate Number of Customers	613
11.4.2 Generating the Customer-Arrival Pattern	613
11.5 Customer Perceptions and Reactions	614
11.5.1 RM Perception Problems	614
11.5.2 Managing Perceptions	618
11.5.3 Overbooking Perceptions	619
11.6 Cultural, Organizational, and Training Issues	620
11.6.1 Changes in Responsibility by Function	620
11.6.2 Project and Organizational Structure	623
11.6.3 Training	627
11.7 Notes and Sources	628
Appendices	631
A Notation	631
B Probability	635
C Convexity and Optimization	643
D Dynamic Programming	651
E The Theory of Choice	657
F Game Theory	667

References	671
------------	-----

Index	709
-------	-----

List of Figures

1.1	A firm's demand landscape	11
1.2	RM process flow	19
2.1	Booking limits b_j , protection levels y_j , and bid prices $\pi(x)$	29
2.2	Optimal protection level y_j^* in the static model	39
2.3	Monte Carlo estimates of optimal protection levels for 50 simulated data points	44
2.4	Adaptive-method example 1	54
2.5	Adaptive-method example 2	55
2.6	Optimal protection level $y_j^*(t)$ in the dynamic model	61
2.7	Scatter plot of $Q(S)$ and $R(S)$	69
3.1	Network examples	82
3.2	Comparison of DLP, RLP, and dynamic programming decomposition	100
3.3	A network example	115
3.A.1	Stochastic gradient calculation for nested booking limits: gradient equal to zero	127
3.A.2	Stochastic gradient calculation for nested booking limits: gradient nonzero	127
4.1	Overbooking notification statement	134
4.2	Overbooking limits over time	140
4.3	Overbooking for the multiclass and binomial models	165
4.4	Network overbooking	168
5.1	Sample price path at a discount air carrier	181

5.2	The maximum concave envelope produced by discrete prices	196
5.3	Sales volume example	198
5.4	Effect of markdowns	200
5.5	An example of the optimal price path in the stochastic case	204
5.6	A six-node, two-hub airline network	218
5.7	Optimal price-skimming solution for myopic customers	225
6.1	Perturbing the bid v_i in a second-price auction	249
6.2	Illustration of the direct-revelation mechanism	257
6.3	Optimal allocations in the dynamic auction model	275
6.4	Optimal allocations in the dynamic-auction model with replenishment	284
7.1	Individual demand and the aggregate demand	311
7.2	Revenue and marginal-revenue curves	319
7.3	Some common aggregate price-response functions	323
8.1	Revenue from selling a product at multiple prices	355
8.2	The equilibrium bipartite graph	382
8.3	Response functions for a duopoly RM game	383
8.4	The two cases for the function $g(\cdot)$	400
8.5	Best response cycle example	400
8.6	No-purchase probabilities causing a best-response cycle	401
9.1	Forecasting module in a RM system	409
9.2	Wedge-shaped bookings data	416
9.3	Time series components	435
9.4	Exponential smoothing with different smoothing parameters	437
9.5	Sample ACF and PACF functions	443
9.6	ACF and PACF examples	446
9.7	The hierarchical Bayes model	454
9.8	Kalman filter smoothing	463
9.9	Neural network example	465
9.10	Neural network activation functions	467
9.11	Incremental booking data	471
9.12	Over-fitting example	495
9.13	Booking curve with cancellations and no-shows	500
9.14	Cancellation probabilities as a function of booking time	501

9.15	Induction tree on cancellations data	503
9.16	Neural network for gas-load forecasting	510
10.1	Pricing an air travel itinerary	519
10.2	Revenue sources and revenue drivers for a hotel	526
10.3	A rental car RM system implementation	533
10.4	Store type breakdown for the top 200 retailers	535
10.5	Growth of department store markdowns	537
10.6	Gas pipeline network	551
10.7	Electricity industry structures	553
10.8	Capacity planning at a tour operator	557
10.9	Purchase plan and price setting at a tour operator	557
10.10	RM process for tour-operators	558
10.11	Washington Opera Kennedy Center layout	573
11.1	Advance-purchase and max-stay restrictions for an airline trip represented by a grid	590
11.2	Products and customers utility reduction modeling	591
11.3	Nightly batch processing	595
11.4	Forecasting under the independent-class model	596
11.5	RM process flow	597
11.6	Quantity-based RM user interface	598
11.7	Quantity-based RM user interface	599
11.8	GDS reservation processing	600
11.9	Seamless availability	604
11.10	Generating arrivals over time	613
11.11	RM organization charts	625

List of Tables

2.1	Static single-resource model and protection levels	48
2.2	Revenue performance for Example 2.3	49
2.3	Static single-resource model data for Example 2.4	49
2.4	Revenue performance for Example 2.4	50
2.5	Fares, demand statistics, and protection levels for adaptive-method numerical examples	53
2.6	Starting values of protection levels for adaptive-method numerical examples	53
2.7	Fare-product revenues and restrictions for Example 2.5	65
2.8	Segments and their characteristics for Example 2.5	65
2.9	Choice probabilities $P_j(S)$, probability of purchase $Q(S)$, and expected revenue $R(S)$ for Example 2.5	66
2.10	Illustration of nested policy for Example 2.5	72
2.11	The different segment choices in Example 2.5 if all classes are open and resulting demand for a population size of 20	74
2.12	Inputs to the EMSR-b model	75
2.13	Protections for the EMSR-b model without and with buy-up factors	75
2.14	Simulation results comparison between choice dynamic program and EMSR-b with buy-up	75
3.1	Example of a bid price table for a single resource based on remaining time and remaining capacity	90
3.2	Problem data for the bid price counterexample	90
3.3	Data for the iterative proration-method example	111

3.4	Example of convergence of the iterative proration method ($t = 1$)	111
3.5	Data for the Williamson [566] network example	115
3.6	Initial protection levels produced by DAVN	116
3.7	Improved protection levels produced by the stochastic gradient algorithm	116
4.1	U.S. major airline denied-boarding rates, 1990-2000	133
4.2	Binomial and normal approximation overbooking probabilities	143
4.3	Comparison of normal and Gram-Charlier (G-C) approximations	149
4.4	Empirical distribution of group sizes	150
5.1	Allocations of capacity between periods 1 and 2 and the marginal values and total revenue	190
5.2	Example of the marginal-allocation algorithm	192
5.3	Example of discrete prices and revenues	195
5.4	Solution of a linear program for the discrete-price example	195
5.5	Results of different markdown policies on 60 markdown styles	200
5.6	Example performance of the deterministic price heuristic	206
5.7	Demand-function parameters, itineraries, and optimal solution for Example 5.6	219
5.8	Empirical generalizations on promotions	231
6.1	Dynamic auction revenues for different concentrations of customers	280
6.2	DLPCC suboptimality gaps relative to a dynamic auction for different demand to capacity ratios	280
6.3	Dynamic auction and replenishment profits for different numbers of customers	287
6.4	Dynamic auction and replenishment profits for different holding costs	288
6.5	Network auction simulation results: average revenues as a function of reserve price	294
7.1	Attribute weights x_m^j for attributes $m = 1, 2$ in alternative $j = 1, 2, 3$	311
7.2	Estimated elasticities (absolute values) for common products	314

7.3	Common demand functions	322
8.1	Prices and capacities for Example 8.2 without an advance-purchase market	347
8.2	Prices and capacities for Example 8.3 with an advance-purchase market	349
8.3	Revenue and variance calculations for Example 8.9 with a single-price policy	361
8.4	Revenue and variance calculations for Example 8.9 with a multiple-price policy	361
9.1	Assumptions of ordinary least-squares (OLS) estimation	424
9.2	Means and covariances of some stationary time-series processes	445
9.3	Results of the AR(2) forecasting example	448
9.4	EM algorithm iterations on constrained data	478
10.1	An example of airline fare codes, classes, and their restrictions	522
10.2	Features of a hotel property management system (PMS)	528
10.3	World's top 10 retailers, store types, and their revenues for the year 2002	534
10.4	U.S. Apparel sales by channel	537
10.5	U.S. apparel sales by category	538
10.6	Inventory definitions in television, radio, and print media	544
10.7	A sample advertising purchase plan	544
10.8	An example of a pipeline delivery contract	547
10.9	Sample pipeline tariffs	548
10.10	Sample natural gas transportation and storage products	549
10.11	Amtrak accommodation and fare types	562
10.12	Sample freight product differentiation	565
10.13	An example of ticket categories for a Broadway show	568
10.13	(continued) An example of ticket categories for a Broadway show	569
10.14	Washington Opera Kennedy Center pricing (2003–2004 season)	573
10.15	The Mets four-tier pricing plan (year 2002)	574
11.1	Customer segments and subsegments by industry	581
11.2	Classification of segment bases	582

11.3	Some common segment bases used in RM	583
11.3	(continued) Some common segment bases used in RM	584
11.4	Attributes and their levels for a hotel application	586
11.5	Major global distribution systems (GDSs) as of 1998	601
11.6	An availability request message as software code and the same request as a message	602
11.7	Typical data tables provided by a hotel PMS	602
11.8	Table BIDPRICE	603
11.9	RMS pricing and inventory functions	607
11.10	Functionality of EDI for the travel and tourism industries	609
11.11	Commonly tracked RM system performance measures	612
11.12	Task list for a RM implementation	624

Preface

Revenue management (RM) has gained attention recently as one of the most successful application areas of operations research (OR). The practice has grown from its origins as a relatively obscure practice among a handful of major airlines in the post-deregulation era in the U.S. (circa 1978) to its status today as a mainstream business practice with a growing list of industry users from Walt Disney Resorts to National Car Rental and a supporting industry of software and consulting firms. Major airlines, hotel chains, and car rental companies have large staffs of developers and analysts working on RM, and major consulting and software firms also employ large numbers of RM professionals.

There are now several major industry RM conferences each year: The Airline Group of the International Federation of Operational Research Societies (AGIFORS) sponsors an annual reservation and yield management conference that attracts up to 200 professionals, and The International Air Travel Association (IATA) hosts an annual RM conference that has drawn up to 800 attendees in recent years. The Professional Pricing Society also hosts professional conferences that address science-based pricing methods and technologies and general pricing strategy.

Over this same period, academic and industry research on the methodology of RM has also grown rapidly. The number of published papers on RM has increased dramatically in the last ten years. INFORMS, the leading professional society of OR, has started a Pricing and RM Section, which has now hosted several annual conferences on RM, each drawing in excess of 100 researchers and professionals. And several universities now offer specialized RM courses, at both the M.B.A and Ph.D. levels.

Despite this explosion of both professional and scholarly activity, no book has comprehensively covered the field of RM. For any area in such a mature state of development and with such widespread industry usage,

such a reference is desirable. However, for RM the need is particularly acute for several reasons:

- RM is very much a professional practice and as such there is a considerable amount of “institutional” knowledge surrounding it that is relatively inaccessible to those outside the profession.
- Many of the early and even some more recent seminal ideas do not appear in published journals. Even those that have been published sometimes appear in relatively obscure sources such as AGIFORS proceedings, industry newsletters, and standard industry practice.
- The terminology, concepts, and notation have not been standardized to date, so it is often confusing for an outsider to reconcile the various contributions of the extant literature.
- There is often a considerable gap between practitioners and academics in the field. Academics are often not aware of the real world complexities faced by practitioners of RM, and industry practitioners are often not aware of the more recent advances in the academic literature.

Our aim in writing this book is to meet this need. The book seeks—as its title indicates—to cover both the *theory* and the *practice* of RM. Fundamentally, RM is an applied discipline; its value and significance ultimately derive from the business results it achieves. At the same time it has strong elements of an applied science, and the technical elements of the subject deserve rigorous treatment. Both these practical and theoretical elements of the field reinforce each other, and to a large extent this is what makes the topic exciting. It is this constructive interplay of theory and practice that we have strived to capture in this book.

Audience

We have two primary audiences in mind for this book—(1) analytically trained (or at least “analytically tolerant”) practitioners in industry and (2) academic researchers and teachers. We view our core reader as someone who has the equivalent of a master’s degree or higher in a technical subject such as engineering, operations research, statistics, or economics. However, significant portions of the text are accessible to general or business readers, particularly the introduction, Chapter 10 on industry profiles, and Chapter 11 on implementation issues. In addition, the introductions to the technical chapters provide high-level overviews of

each chapter, which are designed to provide a qualitative understanding of the main topics covered and their business context, and give the reader a sense of the essence—if not the details—of the material.

For experienced practitioners this book serves as a single-source reference for the major theory and application issues involved in RM. The key technical results in the field are organized and presented precisely and in consistent notation, so that practitioners can easily refer to relevant models, formulas, and algorithms as needed. For new employees in the RM industry our book also serves as a useful primer on the subject, allowing them to “get up to speed” on the details of the field quickly through a consistent presentation of the material. For the technically oriented user it serves as an unbiased, noncommercial source for understanding the competing methodologies available for RM and their relative strengths and weaknesses.

We view the academic audience for the book as consisting of the many researchers now working on various RM-related topics, as well as those who work in related areas (such as supply-chain management), who may want a single-source, accessible overview of the main theory and practice components of the field. Academics who teach management science or operations management courses may also find the book useful, either directly as a supplementary text or simply for the instructor’s personal use as a reference on the subject. Our experiences with colleagues outside the field has suggested that most are curious about RM but perhaps not confident enough about the theory and practice to introduce the subject in their classes. This book should help “demystify” the subject for them.

Finally, a growing number of courses have specifically focused on RM. Though not designed particularly as a textbook, the book should serve as a useful reading and reference in such courses. While we have not put in homework exercises, we did include many small, technically uncluttered examples throughout the book that illustrate the core concepts being discussed.

We forewarn the reader that the material in some places in the book has an airline bias. This is as it should be in our opinion; airline RM practice remains an important topic in its own right. In addition, a large number—indeed the vast majority—of RM practitioners and researchers working in the field today are involved directly in airline RM practices. So airline RM is deserving of rigorous and careful coverage, which is one of our goals in writing this book.

At the same time, not every industry is like the airline industry and “airlinelike” conditions are not, in our view, that necessary to apply RM ideas. Therefore, we have attempted to present RM in as generic terms as possible and included several topics and chapters that generalize

beyond the airline industry. We have tried to be somewhat forward looking in this regard, while at the same time not venturing too far into the realm of pure speculation.

Content and Style

As for the choices of material, we have aimed for an applied technical (engineering) level in our treatment of the subject. For example, we have chosen to present all problems in discrete time. This eliminates several technical complications, while still allowing us to address a wide range of problems in a simple, yet rigorous way. Moreover, continuous-time models and methods are not frequently used in practice, so the focus on discrete-time methods is well justified from a practical standpoint.

Similarly, we have not included a large number of proofs. This is both consistent with the applied orientation of the field and reflects our view that RM models and theory do not share enough in common to justify a highly formalistic, deductive approach to the subject. In a few cases we provide proofs of the theoretical results, but even these are relegated to appendices. When proofs are omitted, we provide references to the original sources and if possible give either informal arguments or intuition about the results.

In addition, the bodies of each chapter do not contain a large number of literature references. This is because we want the reader to “see the material for what it is” and not be sidetracked by a lot of discussion of the literature. Where ideas are strongly associated with specific papers and people, we, of course, point this out. Detailed references to the literature and a discussion of sources are collected in a Notes and Sources section provided at the end of each chapter. To further assist the reader, appendices containing basic results on probability theory, continuous optimization, dynamic programming, and game theory are provided to make the technical material in the book as self-contained as possible.

We tried to be comprehensive in our coverage of RM, covering both quantity- and price-based RM as well as the supporting topics of forecasting and economics. While we might have risked over-extending ourselves in this regard, we believe such a comprehensive approach is necessary to fully understand the subject. Indeed, a key contribution of the book is to unify the various forms of RM and to link them closely to each other and to the supporting fields of statistics and economics. The topics and coverage do, however, reflect our own personal choices about what is and is not important to understand RM. While we have tried to be as comprehensive, fair, and balanced as possible in arriving at these choices, undoubtedly our choices have resulted in some biases. However,

the benefit to the reader is that the text has a point of view and is not merely an uncritical inventory of all research results to date in the field.

Finally, we have also tried to come up with a notation that is generic and consistent across all the chapters. Much of this notation will not coincide with the notation found in the original papers in the field, which is by and large quite inconsistent anyway. A summary of notation is provided in Appendix A for reference. The consistency of notation and presentation, we believe, makes reading the book much easier than looking at the corresponding collection of original-source articles, and it also highlights the connections among topics.

Acknowledgments

This book has been a long time in the making, and writing it would not have been possible without strong support from our institutions, colleagues and family.

Jointly we would like to acknowledge the following colleagues for their time in reading, either sections, or significant parts of the book, and providing us with valuable feedback: Antonio Cabrales (UPF), James Dana (Northwestern), Srinivas Bollapragada (NBC), and the graduate OM class of UPF (2002); Gustavo Vulcano (NYU), Itir Karaesmen (U. of Maryland), Sanne de Boer (MIT), Michael Harrison (Stanford) (and doctoral students in Mike's 2002 Ph.D. seminar on RM), Costis Maglaras (Columbia), Serguei Netessine (Wharton), Qian Liu (Columbia), Yannis Paschalidis (Boston U.) and Andy Philpott (U. of Auckland) and the graduate students of the seminar taught at the Auckland University in 2002. The book has benefited greatly from their comments. Certainly, all remaining errors and obfuscations are our responsibility.

Interactions with many industry colleagues over the years, especially those with Surain Adyanthaya, Andy Boyd, Sebastian Ceria, Ren Curry, Mark Diamond, Kevin Geraghty, Craig Hopperstad, Bob Philips, Anand Rao, John Salch, Barry Smith, and Ben Vinod, have also greatly benefited the book. Dr. Rama Ramakrishnan of Profitlogic was kind enough to provide screenshots of markdown pricing software for use in the implementation chapter.

Kalyan Talluri would like to thank the Department of Economics and Business of the Universitat Pompeu Fabra for their support and healthy research environment, and the Deming Center of Columbia Business School for funding many trips to New York to work on the book. He also would like to acknowledge that his knowledge of RM, and his research, benefited from his long collaboration with the Pricing and RM department at Iberia airlines, specifically working for many years with

Fernando Castejon and Juan Magaz. On the personal side, the stress and labors of writing a long book like this, he would like to acknowledge, were vastly mitigated by the love and joy of companionship of Cristina Ferrer and Uma Talluri Ferrer, both of whom no doubt greet this book with a big sigh of relief.

Garrett van Ryzin would like to thank Columbia Business School for supporting this project over many years, and in particular the Deming Center and its director, Nelson Fraiman, who provided travel funds and research support which helped make writing this book possible. Significant portions of this book were written during a sabbatical visit to the University of Auckland in 2001-2002, and the support of the MSIS Department and especially its then head-of-department, Justo Diaz, is greatly acknowledged. A course taught at Auckland also helped improve early drafts of the book, as did input and discussions with Andy Philpott of the Engineering Science Department at Auckland. Much of the content of this book is the result of research collaborations with a number of colleagues, including Guillermo Gallego, Aliza Heching, Itir Karaesmen, Costis Maglaras, Siddharth Mahajan, Jeff McGill and Gustavo Vulcano. It has been a privilege to work with such a talented group of colleagues, and this book has benefited greatly from their collective contributions. Finally, writing this book would not have been possible without the patience, love and support of Mary Beth, Stephanie, Claire and Andrea—who generously (if not joyfully) tolerated Dad’s many long hours of isolation in his office. Like Cristina and Uma, they too very much deserve to celebrate the completion of this book.