Seat Planning and Seat Assignment with Social Distancing

Examinee: Zikang LI

Supervisors: Prof. Xiangtong QI
Prof. Qian LIU
Chairperson: Prof. Zhenyang LIN
Members: Prof. Richard SO
Prof. Yi CHEN
Prof. Mitch LI
Prof. Yimin YU

Department of Industrial Engineering and Decision Analytics
The Hong Kong University of Science and Technology

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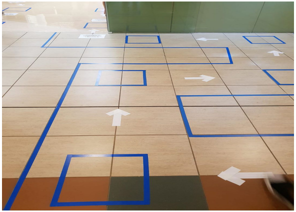
- Social distancing measures



Social Distancing under Pandemic

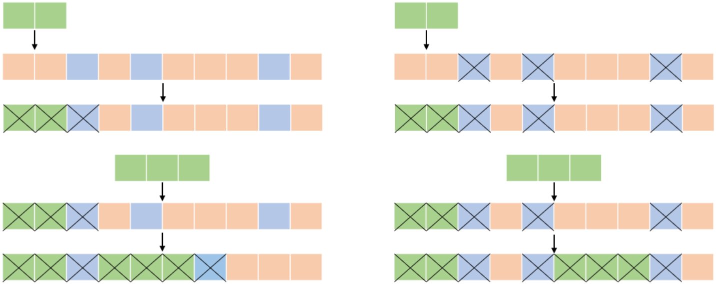
- Social distancing in seating areas

Seat Planning and Seat Assignment

Social distancing requirement:

- The size of a group is confined.
- People in the same group sit together.
- Different groups should keep distance.

Seat assignment under flexible seat planning and fixed seat planning:



Overview

- Seat planning
  - Deterministic demand: make the seat planning with the known specific demand for each group.
  - Stochastic demand: make the seat planning with the known demand distribution before the realization of demand.
- Seat assignment with dynamic demand
  - Assign seats to each group under flexible seat planning.
    - Assign seats to each group after its realization.
    - Accept or reject each group after its realization, but assign them later.
  - Assign seats to each group under fixed seat planning.

Literature Review

Seat Planning with Social Distancing

- Static seat allocation on airplanes (Ghorbani et.al 2020), classroom layout planning (Bortolete et al. 2022), seat planning in long-distancing trains (Haque & Hamid 2022).

- People in the same group can sit together:

  Seat planning for known groups in amphitheaters (Haque & Hamid 2022), airplanes (Salari et al. 2022), theater (Blom et al. 2022).

Dynamic Seat Assignment

- Static form: multiple knapsack problem (Pisinger et al. 1999)
  One row: dynamic knapsack problem (Kleywegt et al. 1998)

  Our work considers the dynamic form and multiple rows.

- Group-based network revenue management is the real complication (Talluri et al. 2006).

- Assign-to-seat: dynamic capacity control for selling high-speed train tickets (Zhu et al. 2023)

Problem Description

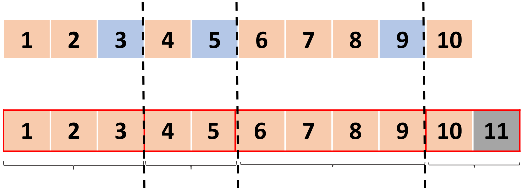
Seat Planning with Social Distancing

- Group type $\mathcal{M} = \{1, \ldots, M\}$.
- Row $\mathcal{N} = \{1, \ldots, N\}$.
- The number of seats in row j: $L_j^0, j \in \mathcal{N}$.
- The social distancing: δ seat(s).
- $n_i = i + \delta$: the size of group type i for each $i \in \mathcal{M}$.
- $L_j = L_j^0 + \delta$: the size of row j for each $j \in \mathcal{N}$.

Figure: Problem Conversion with One Seat as Social Distancing

Pattern

- Pattern: $\boldsymbol{h} = (h_1, \ldots, h_M)$, the seat planning for one row, where h_i is the number of group type i.
  - Feasible pattern: $\sum_{i=1}^{M} n_i h_i \leq L$.
  - The maximum number of people accommodated: $|\boldsymbol{h}| = \sum_{i=1}^{M} i h_i$.



$$L = 11,\ \delta = 1,\ M = 4,\ n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5.$$
$$\boldsymbol{h} = (2, 1, 1, 0),\ |\boldsymbol{h}| = 7.$$

Largest and Full Patterns

- Largest pattern:
  $\boldsymbol{h}$ is a largest pattern if $|\boldsymbol{h}| \geq |\boldsymbol{h}'|$ for any feasible $\boldsymbol{h}'$.
  $|\boldsymbol{h}| = qM + \max\{r - \delta, 0\}$, where $q = \lfloor L/n_M \rfloor$, $r = L - q \cdot n_M$.
- Full pattern:
  $\boldsymbol{h}$ is a full pattern if $\sum_{i=1}^{M} n_i h_i = L$.

Example:

$\delta = 1,\ M = 4,\ L = 21.$

Largest patterns: $h_1 = (0,0,0,4)$, $h_2 = (0,0,4,1)$, $h_3 = (0,2,0,3)$.

Largest but not full: $h_1 = (0,0,0,4)$. $\sum_{i=1}^{M} n_i h_i \neq L$

Full but not largest: $h_4 = (1,1,4,0)$. $|h_4| = 15 < 16 = |h_1|$

Problem Formulation

Seat planning problem with given demand $\boldsymbol{d}$:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M} \sum_{j=1}^{N} (n_i - \delta) x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \leq d_i, \quad i \in \mathcal{M}, \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N}, \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned}
\tag{1}
$$

Objective: maximize the number of people accommodated.
x_{ij}: the number of group type i in row j.

Property

In the LP relaxation of problem (1), there is a threshold $\tilde{i} \in \{0, 1, \ldots, M\}$ such that the optimal solutions satisfy the following conditions:

- For $i = 1, \ldots, \tilde{i} - 1$ and all j, $x^*_{ij} = 0$.
- For $i = \tilde{i} + 1, \ldots, M$, $\sum_j x^*_{ij} = d_i$.
- For $i = \tilde{i}$, $\sum_j x^*_{ij} = \frac{L - \sum_{i=\tilde{i}+1}^{M} d_i n_i}{n_{\tilde{i}}}$

The seat planning obtained from problem (1) may not utilize all seats.

We aim to improve the seat planning utilizing all seats.

Generate the Full or Largest Patterns

Original seat planning: $\boldsymbol{H}$.
Desired feasible seat planning: $\boldsymbol{H}'$.
Meet the original group type requirements:
$\sum_{k=i}^{M} \sum_{j=1}^{N} H'_{ji} \geq \sum_{k=i}^{M} \sum_{j=1}^{N} H_{ji}, \forall i \in \mathcal{M}$.

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M} \sum_{j=1}^{N} (n_i - \delta) x_{ij} \\
s.t. \quad & \sum_{j=1}^{N} \sum_{k=i}^{M} x_{kj} \geq \sum_{k=i}^{M} \sum_{j=1}^{N} H_{ji}, i \in \mathcal{M} \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& x_{ij} \in \mathbb{N}, i \in \mathcal{M}, j \in \mathcal{N}
\end{aligned}
\tag{2}
$$

$\boldsymbol{H}'$ corresponding to the optimal solution is composed of full or largest patterns.

Seat Planning with Stochastic Demand

Method Flow

We aim to obtain a seat planning with known demand scenarios.

- Build the formulation of Scenario-based Stochastic Programming (SSP).
  - Consider the nested relation: a smaller group can take the seats planned for the larger group.
- Reformulate SSP to the Benders Master Problem (BMP) and subproblem.
- The optimal solution can be obtained by solving BMP iteratively.

Scenario-based Stochastic Programming (SSP)

Objective: maximize the expected number of people
$y^{+}_{i\omega}$: excess supply for i, ω. $y^{-}_{i\omega}$: shortage of supply for i, ω.
$d_{i\omega}$: demand of group type i for scenario ω

$$
\begin{aligned}
\max \quad & E_{\omega}\left[\sum_{i=1}^{M-1}(n_i-\delta)(\sum_{j=1}^{N} x_{ij}+y^{+}_{i+1,\omega}-y^{+}_{i\omega})+(n_M-\delta)(\sum_{j=1}^{N} x_{Mj}-y^{+}_{M\omega})\right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij}-y^{+}_{i\omega}+y^{+}_{i+1,\omega}+y^{-}_{i\omega}=d_{i\omega}, \quad i=1,\ldots,M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij}-y^{+}_{i\omega}+y^{-}_{i\omega}=d_{i\omega}, \quad i=M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& y^{+}_{i\omega}, y^{-}_{i\omega} \in \mathbb{N}, \quad i \in \mathcal{M}, \omega \in \Omega \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned} \tag{3}
$$

$E_{\omega}\left[\sum_{i=1}^{M-1}(n_i-\delta)(\sum_{j=1}^{N} x_{ij}+y^{+}_{i+1,\omega}-y^{+}_{i\omega})+(n_M-\delta)(\sum_{j=1}^{N} x_{Mj}-y^{+}_{M\omega})\right] =$
$\sum_{j=1}^{N}\sum_{i=1}^{M} i \cdot x_{ij}-\sum_{\omega=1}^{|\Omega|} p_{\omega}\sum_{i=1}^{M} y^{+}_{i\omega}$

Reformulation

$$
\begin{aligned}
\max \quad & \mathbf{c}^\intercal \mathbf{x} + \sum_{\omega \in \Omega} p_\omega \mathbf{f}^\intercal \mathbf{y}_\omega \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y_{i\omega}^{+} + y_{i+1,\omega}^{+} + y_{i\omega}^{-} = d_{i\omega}, \\
& i = 1, \ldots, M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y_{i\omega}^{+} + y_{i\omega}^{-} = d_{i\omega}, \\
& i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& y_{i\omega}^{+}, y_{i\omega}^{-} \in \mathbb{N}, \quad i \in \mathcal{M}, \omega \in \Omega \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned}
$$

$\mathbf{c}^\intercal \mathbf{x} = \sum_{j=1}^{N} \sum_{i=1}^{M} i x_{ij}$, $\mathbf{f}^\intercal \mathbf{y}_\omega = -\sum_{i=1}^{M} y_{i\omega}^{+}$.

SSP is equivalent to the following master problem

$$
\begin{aligned}
\max \quad & \mathbf{c}^\intercal \mathbf{x} + z(\mathbf{x}) \\
\text{s.t.} \quad & \mathbf{x}^\intercal \mathbf{n} \leq \mathbf{L} \\
& \mathbf{x} \in \mathbb{N}^{M \times N, +},
\end{aligned}
\tag{4}
$$

where $z(\mathbf{x})$ is defined as

$$
z(\mathbf{x}) := E(z_\omega(\mathbf{x})) = \sum_{\omega \in \Omega} p_\omega z_\omega(\mathbf{x}),
$$

and for each scenario $\omega \in \Omega$, we have the subproblem

$$
\begin{aligned}
z_\omega(\mathbf{x}) := \max \quad & \mathbf{f}^\intercal \mathbf{y}_\omega \\
\text{s.t.} \quad & \mathbf{x}\mathbf{1} + \mathbf{V}\mathbf{y}_\omega = \mathbf{d}_\omega \\
& \mathbf{y} \geq 0.
\end{aligned}
\tag{5}
$$

Solution to Subproblem

Problem (5) is easy to solve with a given $\mathbf{x}$ from the perspective of the dual problem:

$$\begin{aligned} \min \quad & \alpha_\omega^\intercal(\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \\ \text{s.t.} \quad & \alpha_\omega^\intercal \mathbf{V} \geq \mathbf{f}^\intercal \end{aligned} \tag{6}$$

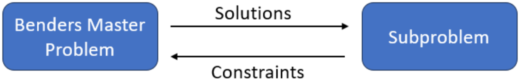
- The feasible region of problem (6), $P = \{\alpha | \alpha^\intercal \mathbf{V} \geq \mathbf{f}^\intercal\}$, is bounded. In addition, all the extreme points of P are integral.
- The optimal solution can be obtained directly according to the complementary slackness property.

Benders Decomposition Procedure

Let z_ω be the lower bound of problem (6), SSP can be obtained by solving following benders master problem:

$$
\begin{aligned}
\max \quad & \mathbf{c}^\intercal \mathbf{x} + \sum_{\omega \in \Omega} p_\omega z_\omega \\
\text{s.t.} \quad & \mathbf{x}^\intercal \mathbf{n} \leq \mathbf{L} \\
& (\alpha^k)^\intercal (\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \geq z_\omega, \alpha^k \in \mathcal{O}, \forall \omega \\
& \mathbf{x} \in \mathbb{N}^{M \times N, +}
\end{aligned} \tag{7}
$$

Constraints will be generated from problem (6) until an optimal solution is found.



Obtain Seat Planning Composed of Full or Largest Patterns

In some cases, it is time-consuming to obtain the optimal solution to SSP.

Meanwhile, there exists an optimal solution to SSP such that the patterns associated with this optimal solution are composed of the full or largest patterns under any given scenarios.

We aim to obtain a seat planning composed of full or largest patterns by the optimal solution to the LP relaxation of SSP.

- Obtain the solution to the relaxed SSP, $\mathbf{x}^*$, by benders decomposition. Aggregate $\mathbf{x}^*$ to the number of each group type, $s_i = \sum_j x_{ij}^*, i \in \mathbf{M}$.
- Obtain the optimal solution, $\mathbf{x}^1$, by solving problem (1) with $d_i = s_i$.
- Construct the full or largest patterns with $\mathbf{x}^1$.

Seat Assignment with Dynamic Demand

Dynamic Demand

- There is at most one group arrival at each period, $t = 1, \dots, T$.
- The probability of an arrival of group type i: p_i
- Seat assignment under the seat planning

1. Assign the seats under the flexible seat planning
   Situation 1: the seller need to assign seats each time a group arrives.
   Situation 2: the seller only needs to accept or reject the group's request and then assign the seats after the selling period.
2. Assign the seats under the fixed seat planning
   The seats, which were arranged for social distancing purposes, need to be dismantled before people arrive to prevent them from occupying those seats.
   When each group arrives, we make decisions regarding whether to accept or reject them based on the predetermined seat planning.

Real-time Seat Assignment under Flexible Seat Planning

- There is one and only one group arrival at each period, $t = 1, \ldots, T+1$.
- The probability of an arrival of group type i: p_i.
- $\mathbf{L}^t = (l_1, l_2, \ldots, l_N)$, where $l_j = 0, \ldots, L_j, j \in \mathcal{N}$: Remaining capacity.
- $u_{i,j}^t$: Decision. Assign group type i to row j at period t, $u_{i,j}^t = 1$.
- $U^t(\mathbf{L}^t) = \{u_{i,j}^t \in \{0,1\}, \forall i, j \mid \sum_{j=1}^N u_{i,j}^t \leq 1, \forall i, n_i u_{i,j}^t \mathbf{e}_j \leq \mathbf{L}, \forall i, j\}$.
- $\mathbf{e}_j$: Unit column vector with j-th element being 1.
- $V^t(\mathbf{L}^t)$: Value function at period t, given remaining capacity, $\mathbf{L}^t$.

$$V^t(\mathbf{L}^t) = \max_{u_{i,j}^t \in U^t(\mathbf{L}^t)} \left\{ \sum_{i=1}^M p_i (\sum_{j=1}^N i u_{i,j}^t + V^{t+1}(\mathbf{L}^t - \sum_{j=1}^N n_i u_{i,j}^t \mathbf{e}_j)) + p_0 V^{t+1}(\mathbf{L}^t) \right\}$$

Proposed Methods

- Suppose the supply associated with the seat planning is $[X_1, \ldots, X_M]$. $(X_i = \sum_j x_{ij}, \forall i)$

  For the arriving group type i,

- if $X_i > 0$, accept the group, assign it by the tie-breaking rule;
- if $X_i = 0$, two methods:

  1. Based on the adjusted SSP.
  2. Based on the seat planning from the LP relaxation of SSP.
     - Determine the possible group type $\hat{i}^* > i$ by group-type control
     - Decision on assigning the group to a specific row

Method 1

Introduce the decision variables $I_j, j \in \mathcal{N}$ indicating whether we accept the arriving group type i' in row j.

$$
\begin{aligned}
\max \quad & \sum_j i' I_j + E_\omega \left[\sum_{i=1}^{M-1} (n_i - \delta)(\sum_{j=1}^{N} x_{ij} + y^+_{i+1,\omega} - y^+_{i\omega}) + (n_M - \delta)(\sum_{j=1}^{N} x_{Mj} - y^+_{M\omega}) \right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y^+_{i\omega} + y^+_{i+1,\omega} + y^-_{i\omega} = d_{i\omega}, \quad i = 1, \ldots, M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y^+_{i\omega} + y^-_{i\omega} = d_{i\omega}, \quad i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j - n_{i'} I_j, j \in \mathcal{N} \\
& \sum_{j=1}^{N} I_j \leq 1 \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}, y^+_{i\omega}, y^-_{i\omega} \in \mathbb{N}, \quad i \in \mathcal{M}, \omega \in \Omega, a_j \in \{0, 1\}, j \in \mathcal{N}.
\end{aligned} \tag{8}
$$

- If $X_i = 0$, solve problem (8), make the decision and update the seat planning.

Method 2: Dynamic Seat Assignment (DSA)

Group-type control: determine the group type $\hat{i}^*$ to place arriving group type i.

Suppose the corresponding supply is $[X_1, \ldots, X_M]$.

$P(D_i^{T-t} \geq X_i)$ is the probability that the demand of group type i in $(T-t)$ periods is no less than X_i.

D_j^t is a random variable indicating the number of group type j in t periods.

$$d^t(i, \hat{i}) = \underbrace{i + (\hat{i} - i - \delta)P(D_{\hat{i}-i-\delta}^{T-t} \geq X_{\hat{i}-i-\delta} + 1)}_{\text{acceptance}} - \underbrace{\hat{i}P(D_{\hat{i}}^{T-t} \geq X_{\hat{i}})}_{\text{rejection}}$$

For all $\hat{i} > i$, find the maximum value denoted as $d^t(i, \hat{i}^*)$.
If $d^t(i, \hat{i}^*) \geq 0$, we place the group of i in $(\hat{i}^* + \delta)$-size seats. Otherwise, reject the group.

Method 2: Dynamic Seat Assignment (DSA)

1. Determine the group type $\hat{i}^*$ by the group-type control.
2. Make the decision on assigning the group to a specific row.
   - Determine a specific row including group type $\hat{i}^*$ by tie-breaking rule.
   - Decision on assigning the group
   - Value of Acceptance (VoA): value of LP relaxation of SSP with $(\mathbf{L} - n_i \mathbf{e}_{\hat{i}})$ plus i.
   - Value of Rejection (VoR): value of LP relaxation of SSP with $\mathbf{L}$.
   - If VoA is no less than VoR, accept group type i; otherwise, reject it.

Regenerate the seat planning

- When $X_M = 0$
- When comparing VoA and VoR

Compared with Other Policies

We compared DSA with the following policies

- Bid-price control
- Dynamic programming based heuristic
- Booking limit control
- First come first served

– Offline optimal solution: full knowledge of demands before decision

Bid-price Control

The dual problem of LP relaxation of problem (1) is:

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} d_i z_i + \sum_{j=1}^{N} L_j \beta_j \\ \text{s.t.} \quad & z_i + \beta_j n_i \geq (n_i - \delta), \quad i \in \mathcal{M}, j \in \mathcal{N} \\ & z_i \geq 0, i \in \mathcal{M}, \beta_j \geq 0, j \in \mathcal{N}. \end{aligned} \tag{9}$$

The optimal solution to problem (9) is given by $z_1, \ldots, z_{\tilde{i}} = 0, z_i = \frac{\delta(n_i - n_{\tilde{i}})}{n_{\tilde{i}}}$, for $i = \tilde{i} + 1, \ldots, M$, $\beta_j = \frac{n_{\tilde{i}} - \delta}{n_{\tilde{i}}}$ for all j.

For the group type i, if $i - \beta_j n_i \geq 0$, accept it.

Dynamic Programming Based Heuristic

- Relax all rows to one row with the same capacity by $L = \sum_{j=1}^{N} L_j$.
  - Deterministic problem: $\{\max \sum_{i=1}^{M} (n_i - \delta) x_i : x_i \leq d_i, i \in \mathcal{M}, \sum_{i=1}^{M} n_i x_i \leq L, x_i \in \mathbb{Z}_0^+\}$.
- Decision: u^t. If we accept a request in period t, $u^t = 1$; otherwise, $u^t = 0$.
  - DP with one row can be expressed as:

$$V^t(l^t) = \max_{u^t \in \{0,1\}} \left\{ \sum_i p_i [V^{t+1}(l^t - n_i u^t) + i u^t] + p_0 V^{t+1}(l^t) \right\}$$

$$V^{T+1}(l) = 0, \forall l.$$

- After accepting one group, assign it in some row arbitrarily when the capacity of the row allows.

Booking limit Control

Basic idea: for each type of requests, we only allocate a fixed amount according to the static solution and reject all other exceeding requests.

1. Observe the arrival group type i.
2. Solve problem (1) using the expected demand.
3. Obtain the optimal solution, x_{ij}^* and the aggregate optimal solution, $\mathbf{X}$.
4. If $X_i > 0$, accept the arrival and assign the group to row k where $x_{ik} > 0$, update $\mathbf{L}^{t+1} = \mathbf{L}^t - n_i \mathbf{e}_k$; otherwise, reject it, let $\mathbf{L}^{t+1} = \mathbf{L}^t$.

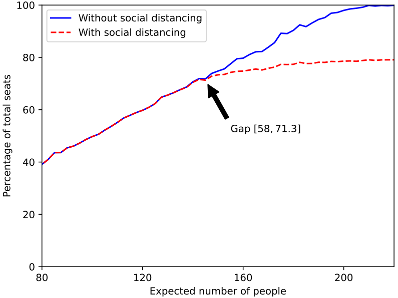
Performances of Different Policies

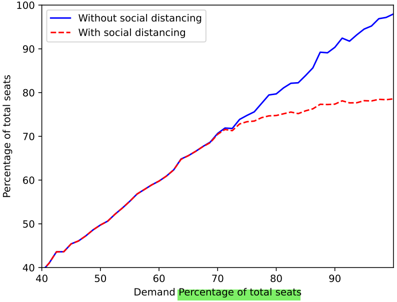
$M = 4,\ \delta = 1,\ N = 10,\ L_j = 21, j \in \mathcal{N},\ p_0 = 0,\ |\Omega| = 1000.$

T	Probabilities	DSA (%)	DP1 (%)	Bid (%)	Booking (%)	FCFS (%)
60	[0.25, 0.25, 0.25, 0.25]	99.12	98.42	98.38	96.74	98.17
70		98.34	96.87	96.24	97.18	94.75
80		98.61	95.69	96.02	98.00	93.18
90		99.10	96.05	96.41	98.31	92.48
100		99.58	95.09	96.88	98.70	92.54
60	[0.25, 0.35, 0.05, 0.35]	98.94	98.26	98.25	96.74	98.62
70		98.05	96.62	96.06	96.90	93.96
80		98.37	96.01	95.89	97.75	92.88
90		99.01	96.77	96.62	98.42	92.46
100		99.23	97.04	97.14	98.67	92.00
60	[0.15, 0.25, 0.55, 0.05]	99.14	98.72	98.74	96.61	98.07
70		99.30	96.38	96.90	97.88	96.25
80		99.59	97.75	97.87	98.55	95.81
90		99.53	98.45	98.69	98.81	95.50
100		99.47	98.62	98.94	98.90	95.25

DSA has better performance than other policies under different demands.

Impact of Social Distancing as Demand Increases





- Gap point: the first period when there is difference
- Demand less than 71.3% total seats: no difference
- Demand larger than 71.3% total seats: the difference becomes larger

Estimation of Gap Point

$\gamma = p_1 * 1 + p_2 * 2 + p_3 * 3 + p_4 * 4$: the expected number of people at each period.

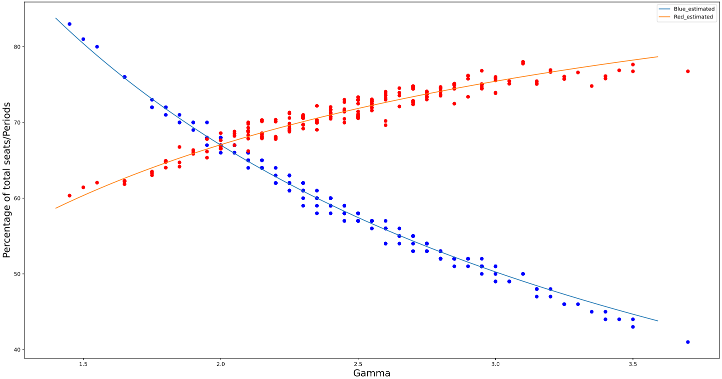


Figure: Gap points with 200 probabilities

Assumption: the groups before gap point will occupy all seats. c_1, c_2 can be seen as the discount factor compared to the ideal assumption.

$y_1 = \frac{c_1 \tilde{L}}{\gamma+\delta}$

$y_2 = c_2 \frac{\gamma}{\gamma+\delta} \frac{\tilde{L}}{\tilde{L}-N\delta}$

Blue points: period of the gap point. Red points: occupancy rate of the gap point. Gap points can be estimated with γ. Different seat layout affects c_1, c_2. The larger c_1, c_2, the closer to the ideal assumption.

Seat Assignment under Fixed Seat Planning

The assignment is based on the fixed seat planning and we use the group-type control to make the decision. $M = 4$, $\delta = 1$, $L_j = 21, j \in \mathcal{N}$, $p_0 = 0$, $|\Omega| = 1000$.

T	Probabilities	# of rows	Compared to the optimal (%)
70	[0.25, 0.25, 0.25, 0.25]	10	94.97
80			96.48
90			97.94
100			98.91
70	[0.25, 0.35, 0.05, 0.35]	10	95.90
80			97.06
90			98.58
100			99.47
70	[0.15, 0.25, 0.55, 0.05]	10	97.41
80			98.85
90			98.73
100			98.46
140	[0.25, 0.25, 0.25, 0.25]	20	95.83
160			97.46
180			99.05
200			99.74

Make A Later Allocation

This setting is particularly applicable to larger venues, such as stadiums, where an immediate decision is made when a group arrives, but the actual allocation of seats for that group is deferred to a later time.

The critical part is to make the decision, thus, we choose the following policies associated with relaxation forms.

Policies:

- Dynamic programming based heuristic
- Bid-price control

Performances of Different Policies

T	Probabilities	DP1-L (%)	Bid-L (%)	DP1 (%)	Bid (%)
60	[0.25, 0.25, 0.25, 0.25]	99.52	99.44	98.42	98.38
70		99.32	98.97	96.87	96.24
80		99.34	99.30	95.69	96.02
90		99.55	99.49	96.05	96.41
100		99.78	99.66	95.09	96.88
60	[0.25, 0.35, 0.05, 0.35]	99.50	99.37	98.26	98.25
70		99.40	98.97	96.62	96.06
80		99.46	99.24	96.01	95.89
90		99.59	99.35	96.77	96.62
100		99.77	99.61	97.04	97.14
60	[0.15, 0.25, 0.55, 0.05]	99.57	99.54	98.72	98.74
70		99.46	99.39	96.38	96.90
80		99.50	99.30	97.75	97.87
90		99.34	99.44	98.45	98.69
100		99.34	99.55	98.62	98.94

Conclusion and Future Work

Conclusion and Future Work

Conclusion

- Address the problem of seat planning and assignment with social distancing.
- Provide a comprehensive solution for optimizing seat assignments under dynamic situation.
- Provide different methods in different scenarios and elaborate on the role of social distancing.

Future work

- Consider more flexible scenarios, people can choose the seats and leave randomly.
- Consider a scattered seat assignment when there are sufficient seats.

Thank You!