The Application of ML to Identifying Critical Constraints

in The Inverse Uncapacitated Facility Location Problem

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COMP 5212

Uncapacitated Facility Location Problem(UFLP)

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij} u_{ij} + \sum_{i=1}^{m} f_{i} v_{i}$$
s.t.
$$\sum_{i=1}^{m} u_{ij} = 1, \quad \forall j \in N$$

$$u_{ij} \leq v_{i}, \quad \forall i \in M, \forall j \in N$$

$$u_{ij}, v_{i} \in \{0, 1\}, \quad \forall i \in M, \forall j \in N$$
(2)

- (1) represents each customer $j \in N$ should be severd by one facility.
- (2) represents if customer j is served by faility i, then facility i must be open.

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IUFLP

$$\min_{f^0, r^0} \qquad \sum_{i=1}^m w_i^f |f_i^0 - f_i| + \sum_{i=1}^m \sum_{j=1}^n w_{ij}^r |r_{ij}^0 - r_{ij}|$$
s.t.
$$f_i^0 v_i^0 + r_{ij}^0 u_{ij}^0 \le f_i^0 v_i' + r_{ij}^0 u_{ij}', \quad \forall i \in M, \forall j \in N \qquad (3)$$

$$(v', u') \in \Upsilon \qquad (4)$$

 Υ is the set of all feasible solutions x' = (v', u').

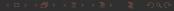
The number of solutions satisfying constraints (3) is m^n .

Only a few constraints will be critical to determine the feasible region of this problem. Once we can obtain the set of all critical constraints, we will get the optimal solution.

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Dataset Construction

- Set $w^f = 1, w^r = 1$ for simplicity.
- Data point (f, r, v^0, u^0, v', u') .
- Fixed cost: (f, r), the given optimal solution (v^0, u^0) , v', u' is any feasible solution to UFL.
- Use constraint (3) to label the data.
- If $f^0(v^0 v') + r^0(u^0 u') = 0$, then (v', u') is a critical constraint, the corresponding data will be labeled 1; otherwise, labeled 0.



Training Procedure

- The cost (f,r) can be very large while the solution always only contains 0,1. Thus, we need to normalize (f,r).
- The number of critical constraints will be way smaller than that of feasible solutions in each instance. Thus, we should avoid generating a feasible solution randomly as the data, otherwise, the dataset will be unbalanced.
- The batch size is 10; the number of epochs is 80; learning rate is 0.001; momentum is 0.9 for SGD-Momentum; the input size is (m+m*n)*3.

Results

(m,n)	Epoch	Logistic(SGD-M)	SVM(SGD-M)	Accuracy
(5,10)	20	0.2495	0.2669	
	40	0.2247	0.2669	
	60	0.2106	0.2670	
	80	0.2018	0.2670	100%, 99%
(10,20)	20	0.3935	0.7971	
	40	0.3799	0.7961	
	60	0.3741	0.7964	
	80	0.3715	0.7961	95%, 93%
(20,40)	20	0.3474	0.5028	
	40	0.3148	0.5028	
	60	0.2978	0.5024	
	80	0.2868	0.5030	99%, 98%

Challenges

- The results depend on the dataset. The different size of datasets and how the datasets are generated will give a very different result.
- Different sizes of m, n need different datasets. Constructing the dataset will be time-consuming with the large size of m, n.
- Finding the same feature behind the different size of this problem will be very crucial to solving the large-scale problem.

Thank You!