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# Scenario-based Stochastic Programming

$$V_t(\mathbf{L}) = E_i\left[\max_{k\in N:L_k\geq i+s}\{[V_{t-1}(\mathbf{L}-U_{ik})+i], V_{t-1}(\mathbf{L})\}\right], \mathbf{L}\geq\mathbf{0}$$

$$V_{T+1}(\mathbf{L}) = 0,$$

- $\mathbf{L}$, remaining capacity.
- $n_i$.
- $p_i$: the probability of an arrival of group type $i$.

# Scenario-based Stochastic Programming

$$
\begin{aligned}
\max \quad & E_\omega\left[\sum_{i=1}^{M-1}(n_i - s)(\sum_{j=1}^{N} x_{ij} + y^+_{i+1,\omega} - y^+_{i\omega}) + (n_M - s)(\sum_{j=1}^{N} x_{Mj} - y^+_{M\omega})\right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y^+_{i\omega} + y^+_{i+1,\omega} + y^-_{i\omega} = d_{i\omega}, \quad i \in [M-1], \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y^+_{i\omega} + y^-_{i\omega} = d_{i\omega}, \quad i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \le L_j, j \in [N] \\
& y^+_{i\omega}, y^-_{i\omega} \in \mathbb{Z}_+, \quad i \in [M], \omega \in \Omega \\
& x_{ij} \in \mathbb{Z}_+, \quad i \in [M], j \in [N].
\end{aligned}
\tag{1}
$$

# Two-stage

$$
\begin{aligned}
\max \quad & c'\mathbf{x} + z(\mathbf{x}) \\
\text{s.t.} \quad & \mathbf{n}\mathbf{x} \leq \mathbf{L} \\
& \mathbf{x} \in \mathbb{Z}_+^{M \times N},
\end{aligned}
\tag{2}
$$

where $z(\mathbf{x})$ is the recourse function defined as

$$
z(\mathbf{x}) := E(z_\omega(\mathbf{x})) = \sum_{\omega \in \Omega} p_\omega z_\omega(\mathbf{x}),
$$

and for each scenario $\omega \in \Omega$,

$$
\begin{aligned}
z_\omega(\mathbf{x}) := \max \quad & \mathbf{f}'\mathbf{y}_\omega \\
\text{s.t.} \quad & \mathbf{x}\mathbf{1} + \mathbf{V}\mathbf{y}_\omega = \mathbf{d}_\omega \\
& \mathbf{y}_\omega \geq 0.
\end{aligned}
\tag{3}
$$

# Solve the Second Stage Problem

$$\begin{aligned} \min \quad & {\alpha'}_\omega(\mathbf{d}_\omega - \mathbf{x1}) \\ \text{s.t.} \quad & {\alpha'}_\omega \mathbf{V} \geq \mathbf{f}' \end{aligned} \tag{4}$$

Let $P = \{\alpha | \alpha' V \geq \mathbf{f}'\}$. The feasible region of problem (4), $P$, is bounded. In addition, all the extreme points of $P$ are integral.

# Delayed Constraint Generation

Restricted Benders Master Problem

$$
\begin{aligned}
\max \quad & c'x + \sum_{\omega \in \Omega} p_\omega z_\omega \\
\text{s.t.} \quad & \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in [N] \\
& (\alpha^k)'(\mathbf{d}_\omega - \mathbf{x1}) \geq z_\omega, \alpha^k \in \mathcal{O}^t, \forall \omega \\
& \mathbf{x} \geq 0
\end{aligned}
\tag{5}
$$

# Obtain the Feasible Seat Planning

Step 1. Obtain the solution, $\mathbf{x}^*$, from stochatic linear programming by benders decomposition.

Step 2. Aggregate the solution to the supply, $s_i^0 = \sum_j x_{ij}^*$.

Step 3. Obtain the optimal solution, $\mathbf{x}^1$, from problem (7) by setting the supply $\mathbf{s}^0$ as the upper bound.

Step 4. Aggregate the solution to the supply, $s_i^1 = \sum_j x_{ij}^1$.

Step 5. Obtain the optimal solution, $\mathbf{x}^2$, from problem (**??**) by setting the supply $\mathbf{s}^1$ as the lower bound.

Step 6. Aggregate the solution to the supply, $s_i^2 = \sum_j x_{ij}^2$, which is the feasible seat planning.

# Dynamic Seat Assignment

# Results

# Deterministic Model

# Seat Planning with Social Distancing

Group type $[M] = \{1, \ldots, M\}$ Row $[N] = \{1, \ldots, N\}$ Let $n_i = i + s$ denote the new size of group type $i$ for each $i \in [M]$. Let $L_j = S_j + s$ denote the length of row $j$ for each $j \in [N]$, where $S_j$ represents the number of seats in row $j$.

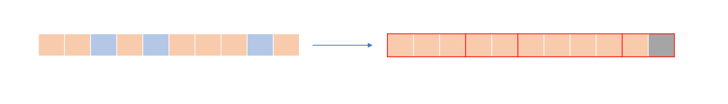

Figure: Problem Conversion

# Formulation

When $|\Omega| = 1$ in problem (1), the stochastic programming will be

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(n_i - s)x_{ij} - \sum_{i=1}^{M} y_i^+ \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y_i^+ + y_{i+1}^+ + y_i^- = d_i, \quad i \in [M-1], \\
& \sum_{j=1}^{N} x_{ij} - y_i^+ + y_i^- = d_i, \quad i = M, \\
& \sum_{i=1}^{M} n_i x_{ij} \le L_j, j \in [N] \\
& y_i^+, y_i^- \in \mathbb{Z}_+, \quad i \in [M] \\
& x_{ij} \in \mathbb{Z}_+, \quad i \in [M], j \in [N].
\end{aligned} \tag{6}
$$

# Formulation

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(n_i - s)x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \le d_i, \quad i \in [M], \\
& \sum_{i=1}^{M} n_i x_{ij} \le L_j, j \in [N] \\
& x_{ij} \in \mathbb{Z}_+, \quad i \in [M], j \in [N].
\end{aligned} \tag{7}
$$

# Analysis

# Example

Suppose the social distancing is one seat, then the new sizes of groups are $2, 3, 4, 5$, respectively. The length of one row is $L = 21$ and the demand is $[10, 12, 9, 8]_d$. Then these patterns, $(5, 5, 5, 5, 1), (5, 4, 4, 4, 4), (5, 5, 5, 3, 3)$, belong to $I_1$. For pattern 1, $(5, 5, 5, 5, 1)$, $P_1 = \{5\}$, thus a group with a size smaller than 5 cannot be put in this pattern.

# Properties

- $\alpha_k$ indicates the number of items for pattern $k$. $\beta_k$ indicates the left space for maximal pattern $k$. Notice that the left space is the true loss.
- Denote $\alpha_k + \beta_k - 1$ as the loss for pattern k, $l(k)$. When $l(k)$ reaches minimum, the corresponding pattern $k$ is the optimal solution for a single row.
- If the group sizes are consecutive integers starting from 2, $\{2, 3, \ldots, u\}$, then a greedy-based pattern is optimal, i.e., select the maximal group size,$u$, as many as possible and the left space is occupied by the group with the corresponding size. The loss is $k + 1$, where $k$ is the number of times $u$ selected. Let $S = u \cdot k + r$.

- Let $I_1$ be the set of patterns with the minimal loss.
- For a seat layout, $\{S_1, S_2, \ldots, S_N\}$, the total loss is $\sum_j(\lfloor\frac{S_j+1}{u}\rfloor - f((S_j+1) \mod u))$. The maximal number of people assigned is $\sum_j(S_j - \lfloor\frac{S_j+1}{u}\rfloor + f((S_j+1) \mod u))$.

# The End