Dynamic Seat Assignment with Social Distancing

IEDA
The Hong Kong University of Science and Technology

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Introduction

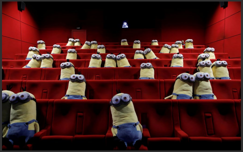
Social Distancing under Pandemic

- Governments issued the policy about social distancing constraint.

> As a result, the Hong Kong government announces a series of new measures that will come into effect from **July 29th**:
> - Gathering in public will be limited to only two people per group. Members of the same family are exempted.
> - Restaurants are unable to offer dine-in services for the whole day. Certain public establishments are exempted, such as eateries in public hospitals.
> - Masks are now required outdoors as well. There are no exemptions for exercising or smoking.

- Theater tickets booking: assign to seat.

Cinemas are required to strictly comply with the relevant anti-epidemic measures, e.g. only accepting patrons up to 50 per cent capacity limit allowing a maximum of four consecutive seats in the same row to be occupied, arranging cleansing and sterilisation work at regular intervals, not allowing live performance, etc. Patrons will also need to hold valid Vaccine Pass, use the "LeaveHomeSafe" mobile application, wear masks and take body temperature measurements, etc.



Literature Review

Seat Planning with Social Distancing

- Seat planning on airplanes, classrooms, trains.
  Allocation of seats on airplanes [4], classroom layout planning [3], seat planning in long-distancing trains [6].

- Group seat assignment in amphitheaters, airplanes, theater.
  - Group reservations can increase revenue without increasing the risk of infection [8].
  - Seating planning for known groups in amphitheaters [6], airplanes [10], theater [2].

Dynamic Seat Assignment

- Related to multiple knapsack problem [9] and dynamic knapsack problem [7].
- Dynamic seat assignment on airplain [5], train [1, 11].
- Assign-to-seat: dynamic capacity control for selling high-speed train tickets. [11]

Problem Definition

Seat Planning with Social Distancing

- Group type $\mathcal{M} = \{1, \ldots, M\}$.
- Row $\mathcal{N} = \{1, \ldots, N\}$.
- The social distancing: δ seat(s).
- $n_i = i + \delta$: the new size of group type i for each $i \in \mathcal{M}$.
- The number of seats in row j: $S_j, j \in \mathcal{N}$.
- $L_j = S_j + \delta$: the length of row j for each $j \in \mathcal{N}$.

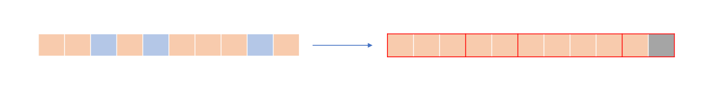


Figure: Problem Conversion with One Seat as Social Distancing

Basic Concepts

- Pattern: the seat planning for each group type in one row.
  Denoted by $P_k = (t_1, \ldots, t_M)$, where t_i is the number of group type i.
- For each pattern k, α_k, β_k indicate the number of groups and the left seats, respectively.
- Loss for pattern k: $\alpha_k \delta + \beta_k - \delta$.
- The largest patterns: patterns with the minimal loss.
- Full patterns: $\beta_k = 0$.

- Example:
  $\delta = 1$, $M = 4$, $n_i = i + 1, i \in \mathcal{M}$, $L = 21$.
  Largest patterns: $(0,0,0,4), (0,0,4,1), (0,2,0,3)$.
  Not a Full pattern: $(0,0,0,4)$.

Loss of The Largest Patterns

- Find a largest pattern: consider $L = n_M \cdot q + r, 0 \leq r < n_M$, where q is the qutionet and r is remainder. If the remainder r is greater than δ, the seats can be occupied by a group of size $(r - \delta)$. However, if r is less than or equal to δ, the seats should be left empty.
- Example: $\delta = 1$, $M = 4$, $L = 21$, $n_M = 5$, $q = 4$, $(0, 0, 0, 4)$ is a largest pattern.

* Loss of the largest patterns: $l(L) = \lfloor \frac{L}{n_M} \rfloor \delta - \delta + f((L \mod n_M))$, where $f(r) = 0$ if $r > \delta$; $f(r) = r$ if $r \leq \delta$.
* For a original seat layout, $\{S_1, S_2, \ldots, S_N\}$, the minimal total loss: $\sum_j l(S_j + \delta)$.

Dynamic Seat Assignment Problem

- $T+1$ periods in total.
- There is one and only one group arrival at each period.
- The probability of an arrival of group type i: p_i.
- Remaining capacity: $\mathbf{L} = (l_1, l_2, \ldots, l_N)$, where $l_j = 0, \ldots, L_j, j \in \mathcal{N}$.
- $\mathbf{e}_j^\intercal$: decision variable whether to assign the current group to row j.
- Value function: $V_t(\mathbf{L})$.

$$V_t(\mathbf{L}) = \sum_i p_i \left[\max_{\substack{j \in \mathcal{N}: \\ L_j \geqslant n_i}} \left\{ V_{t+1}\left(\mathbf{L} - n_i \mathbf{e}_j^\intercal\right) + i, V_{t+1}(\mathbf{L}) \right\} \right], V_{T+1}(\mathbf{L}) = 0.$$

- DP is computationally complex caused by the curse of dimensionality.
- We give a seat planning by stochastic programming firstly, then apply stochastic planning policy to make the decision.

Seat Planning by Stochastic Programming

Method Flow

- The formulation of scenario-based stochastic programming(SSP).
- Reformulate (SSP) to the benders master problem(BMP) and subproblem.
- The optimal solution can be obtained by solving (BMP) iteratively.
- To avoid solving IP directly, we consider the LP relaxation form.
- Obtain integral seat planning by deterministic model.

Scenario-based Stochastic Programming

$$
\begin{aligned}
(SSP)\ \max \quad & E_\omega \left[\sum_{i=1}^{M-1} (n_i - \delta)(\sum_{j=1}^{N} x_{ij} + y_{i+1,\omega}^{+} - y_{i\omega}^{+}) + (n_M - \delta)(\sum_{j=1}^{N} x_{Mj} - y_{M\omega}^{+}) \right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y_{i\omega}^{+} + y_{i+1,\omega}^{+} + y_{i\omega}^{-} = d_{i\omega}, \quad i = 1, \ldots, M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y_{i\omega}^{+} + y_{i\omega}^{-} = d_{i\omega}, \quad i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& y_{i\omega}^{+}, y_{i\omega}^{-} \in \mathbb{Z}_{+}, \quad i \in \mathcal{M}, \omega \in \Omega \\
& x_{ij} \in \mathbb{Z}_{+}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned} \tag{1}
$$

Reformulation

$$
\begin{aligned}
\max \quad & \mathbf{c}^{\intercal}\mathbf{x} + z(\mathbf{x}) \\
\text{s.t.} \quad & \mathbf{n}\mathbf{x} \leq \mathbf{L} \\
& \mathbf{x} \in \mathbb{Z}_{+}^{M \times N},
\end{aligned} \tag{2}
$$

where $z(\mathbf{x})$ is defined as

$$
z(\mathbf{x}) := E(z_{\omega}(\mathbf{x})) = \sum_{\omega \in \Omega} p_{\omega} z_{\omega}(\mathbf{x}),
$$

and for each scenario $\omega \in \Omega$,

$$
\begin{aligned}
z_{\omega}(\mathbf{x}) := \max \quad & \mathbf{f}^{\intercal}\mathbf{y} \\
\text{s.t.} \quad & \mathbf{x}\mathbf{1} + \mathbf{V}\mathbf{y} = \mathbf{d}_{\omega} \\
& \mathbf{y} \geq 0.
\end{aligned} \tag{3}
$$

Solution to Subproblem

Problem (3) is easy to solve with a given $\mathbf{x}$ which can be seen by the dual problem:

$$\begin{aligned} \min \quad & \alpha_\omega^\intercal(\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \\ \text{s.t.} \quad & \alpha_\omega^\intercal \mathbf{V} \geq \mathbf{f}^\intercal \end{aligned} \tag{4}$$

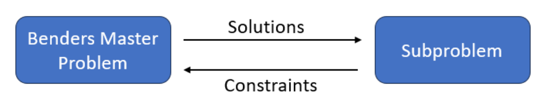
- The feasible region of problem (4), $P = \{\alpha | \alpha^\intercal V \geq \mathbf{f}^\intercal\}$, is bounded. In addition, all the extreme points of P are integral.
- The optimal solution to this problem can be obtained directly according to the complementary slackness property.

Bneders Decomposition Procedure

(SSP) can be obtained by solving following restricted benders master problem(BMP):

$$
\begin{aligned}
\max \quad & \mathbf{c}^\intercal \mathbf{x} + \sum_{\omega \in \Omega} p_\omega z_\omega \\
\text{s.t.} \quad & \mathbf{n}\mathbf{x} \leq \mathbf{L} \\
& (\alpha^k)^\intercal (\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \geq z_\omega, \alpha^k \in \mathcal{O}, \forall \omega \\
& \mathbf{x} \in \mathbb{Z}_+
\end{aligned}
\tag{5}
$$

Constraints will be generated from problem (4) until an optimal solution is found.



To avoid solving IP directly, we consider the LP relaxation of Problem (5).

Deterministic Formulation

To obtain an integral seat planning, we consider the following two deterministic formulations.

$$\begin{aligned} \max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(n_i - s)x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \leq s_i^0, \quad i \in \mathcal{M}, \\ & \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\ & x_{ij} \in \mathbb{Z}_+, \quad i \in \mathcal{M}, j \in \mathcal{N}. \end{aligned} \tag{6}$$

$$\begin{aligned} \max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(n_i - s)x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \geq s_i^1, \quad i \in \mathcal{M}, \\ & \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\ & x_{ij} \in \mathbb{Z}_+, \quad i \in \mathcal{M}, j \in \mathcal{N}. \end{aligned} \tag{7}$$

Problem (6) can generate a feasible seat planning.
Problem (7) can generate a seat planning no inferior than any given feasible seat planning.

Obtain The Feasible Seat Planning

Step 1. Obtain the solution, $\mathbf{x}^*$, by benders decomposition. Aggregate $\mathbf{x}^*$ to the number of each group type, $s_i^0 = \sum_j x_{ij}^*, i \in \mathbf{M}$.

Step 2. Solve problem (6) to obtain the optimal solution, $\mathbf{x}^1$. Aggregate $\mathbf{x}^1$ to the number of each group type, $s_i^1 = \sum_j x_{ij}^1, i \in \mathbf{M}$.

Step 3. Solve problem (7) to obtain the optimal solution, $\mathbf{x}^2$. Aggregate $\mathbf{x}^2$ to the number of each group type, $s_i^2 = \sum_j x_{ij}^2, i \in \mathbf{M}$.

Step 4. For each row, construct a full pattern.

Dynamic Seat Assignment for Each Group Arrival

Stochastic Planning Policy

- Group-type control
  - Feasible Seat planning from stochastic programming.
  - When there is no small group, decide which group type to be assigned.
- Value of Acceptance and Rejection
  - Compare the value of stochastic programming when assigning in the row versus not assigning.

Bid-price Control

The dual problem of linear relaxation of problem (6) is:

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} d_i z_i + \sum_{j=1}^{N} L_j \beta_j \\ \text{s.t.} \quad & z_i + \beta_j n_i \geq (n_i - \delta), \quad i \in \mathcal{M}, j \in \mathcal{N} \\ & z_i \geq 0, i \in \mathcal{M}, \beta_j \geq 0, j \in \mathcal{N}. \end{aligned} \tag{8}$$

There exists h such that the aggregate optimal solution to relaxation of problem (6) takes the form $xe_h + \sum_{i=h+1}^{M} d_i e_i$, $x = (L - \sum_{i=h+1}^{M} d_i n_i)/n_h$.

Dynamic Programming Based Heuristic

Relax all rows to one row with the same capacity by $L = \sum_{j=1}^{N} L_j$.
Deterministic problem is:
$\{\max \sum_{i=1}^{M} (n_i - \delta) x_i : x_i \leq d_i, i \in \mathcal{M}, \sum_{i=1}^{M} n_i x_i \leq L, x_i \in \mathbb{Z}_+\}$.
Let u denote the decision, where $u(t) = 1$ if we accept a request in period t, $u(t) = 0$ otherwise, the DP with one row can be expressed as:

$$V_t(L) = \mathbb{E}_{i \sim p}[\max_{u \in \{0,1\}} \{[V_{t+1}(L - n_i u) + iu]\}], L \geq 0$$

$$V_{T+1}(x) = 0, \forall x.$$

After accepting one group, assign it in some row arbitrarily when the capacity of the row allows.

Booking limit Control

Solve problem (6) using the expected demand. Then for every type of requests, we only allocate a fixed amount according to the static solution and reject all other exceeding requests.

When we solve the linear relaxation of problem (6), the aggregate optimal solution is the limits for each group type. Interestingly, the bid-price control policy is found to be equivalent to the booking limit control policy.

Numerical Results

Running time of Benders Decomposition and IP

# of scenarios	demands	running time of IP(s)	Benders (s)	# of rows	# of groups	# of seats
1000	(150, 350)	5.1	0.13	30	8	(21, 50)
5000		28.73	0.47	30	8	
10000		66.81	0.91	30	8	
50000		925.17	4.3	30	8	
1000	(1000, 2000)	5.88	0.29	200	8	(21, 50)
5000		30.0	0.62	200	8	
10000		64.41	1.09	200	8	
50000		365.57	4.56	200	8	
1000	(150, 250)	17.15	0.18	30	16	(41, 60)
5000		105.2	0.67	30	16	
10000		260.88	1.28	30	16	
50000		3873.16	6.18	30	16	

Feasible Seat Planning versus IP Solution

# samples	T	probabilities	# rows	people served by FSP	IP
1000	45	[0.4,0.4,0.1,0.1]	8	85.30	85.3
1000	50	[0.4,0.4,0.1,0.1]	8	97.32	97.32
1000	55	[0.4,0.4,0.1,0.1]	8	102.40	102.40
1000	60	[0.4,0.4,0.1,0.1]	8	106.70	NA
1000	65	[0.4,0.4,0.1,0.1]	8	108.84	108.84
1000	35	[0.25,0.25,0.25,0.25]	8	87.16	87.08
1000	40	[0.25,0.25,0.25,0.25]	8	101.32	101.24
1000	45	[0.25,0.25,0.25,0.25]	8	110.62	110.52
1000	50	[0.25,0.25,0.25,0.25]	8	115.46	NA
1000	55	[0.25,0.25,0.25,0.25]	8	117.06	117.26
5000	300	[0.25,0.25,0.25,0.25]	30	749.76	749.76
5000	350	[0.25,0.25,0.25,0.25]	30	866.02	866.42
5000	400	[0.25,0.25,0.25,0.25]	30	889.02	889.44
5000	450	[0.25,0.25,0.25,0.25]	30	916.16	916.66

Each entry of people served is the average of 50 instances. IP will spend more than 2 hours in some instances, as 'NA' showed in the table.

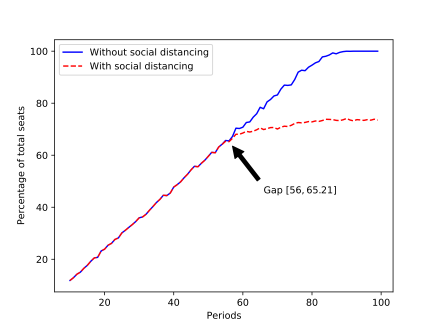
Performances of Different Policies

T	Probabilities	Sto(%)	DP1(%)	Bid(%)	Booking(%)	FCFS(%)
60	[0.25, 0.25, 0.25, 0.25]	99.12	98.42	98.38	96.74	98.17
70	[0.25, 0.25, 0.25, 0.25]	98.34	96.87	96.24	97.18	94.75
80	[0.25, 0.25, 0.25, 0.25]	98.61	95.69	96.02	98.00	93.18
90	[0.25, 0.25, 0.25, 0.25]	99.10	96.05	96.41	98.31	92.48
100	[0.25, 0.25, 0.25, 0.25]	99.58	95.09	96.88	98.70	92.54
60	[0.25, 0.35, 0.05, 0.35]	98.94	98.26	98.25	96.74	98.62
70	[0.25, 0.35, 0.05, 0.35]	98.05	96.62	96.06	96.90	93.96
80	[0.25, 0.35, 0.05, 0.35]	98.37	96.01	95.89	97.75	92.88
90	[0.25, 0.35, 0.05, 0.35]	99.01	96.77	96.62	98.42	92.46
100	[0.25, 0.35, 0.05, 0.35]	99.23	97.04	97.14	98.67	92.00
60	[0.15, 0.25, 0.55, 0.05]	99.14	98.72	98.74	96.61	98.07
70	[0.15, 0.25, 0.55, 0.05]	99.30	96.38	96.90	97.88	96.25
80	[0.15, 0.25, 0.55, 0.05]	99.59	97.75	97.87	98.55	95.81
90	[0.15, 0.25, 0.55, 0.05]	99.53	98.45	98.69	98.81	95.50
100	[0.15, 0.25, 0.55, 0.05]	99.47	98.62	98.94	98.90	95.25

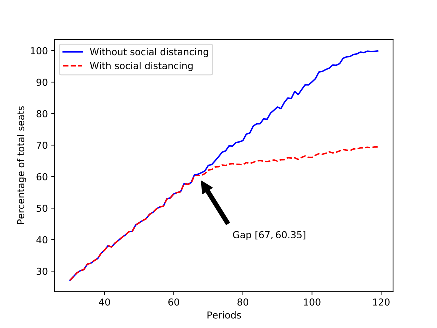
Sto has better performance than other policies under different demands.
The performance of FCFS drops as demand increases.

Impact of Social Distance as Demand Increases

Let $\gamma = p_1 * 1 + p_2 * 2 + p_3 * 3 + p_4 * 4$ denote the number of people at each period.



(a) When $\gamma = 2.5$



(b) When $\gamma = 1.9$

The gap point represents the first period where the number of people without social distancing is larger than that with social distancing and the gap percentage is the corresponding percentage of total seats.

When Supply and Demand Are Close

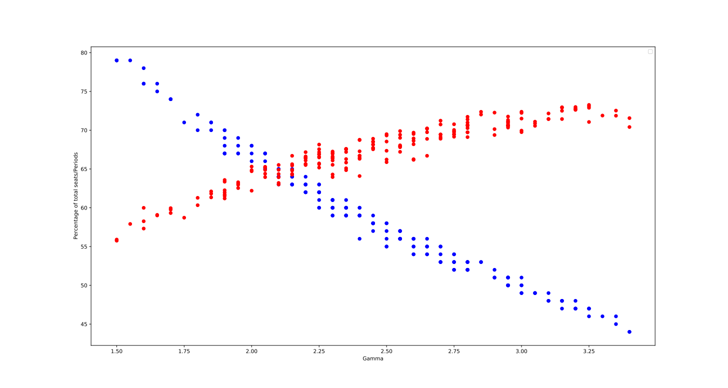


Figure: Gap points under 200 probabilities

Blue points: period of the gap point. Red points: occupancy rate of the gap point. Gap points can be estimated.

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