Dynamic Seat Assignment with Social Distancing

Dis· count

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Abstract

Keywords: Social Distancing, Seat Assignment, Dynamic Arrival.

1 Introduction

1. define a different social distancing rule.

use the expected demand as the group portfolio to obtain the seat planning.

2. consider 'departure'

Each arrival has an arrive time and leave time.

3. different price regions in cinemas.

4. How to assign the high-speed train tickets fairly?

2 Literature Review

3 Model

Virables:

Room numbers $k \in K = \{1, \dots, |K|\}$.

Room k contains seat number q_k.

Number of customers in session i is p_i for each $i \in N = \{1, \dots, n\}$. w_{ik} is the session i's start time in the room k. s_i is the service time for session i.(Given, in our case, we set all the service times are 2 hours.)

Feasibility:

Capacity constraint: The number of customers in a session p_i cannot exceed the product of the largest room capacity and ratio r, that is $p_i \leq q_k \cdot r$.

Solution:

Define the time interval t_i for session i. It can be variable or the constant. In our case, we set the time interval as the variable and it should be larger than half an hour.

Define a binary variable x_{ijk} for each room. If the room k is used by (i, j) and i followed by j, then $x_{ijk} = 1$, else $x_{ijk} = 0$.

Define w_{ik} is the session i's start time in the room k.

s_i is the service time for each session.(Given)

Analysis:

Add two virtual nodes (0,n+1) for each room. One is the start node, its time window can be a time point E meaning the room is open; the other is the end node, its time window is also a time point L meaning the room is closed.

Expected result: Show the specific scheduling for the sessions.

Give the corresponding service start time.

Benchmark: Manual work.

Question: How to determine the objective function?

How to compare the result with the benchmark?

MODEL:

$$
\begin{aligned}
\min_{i,j,k} \quad & \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} x_{ijk} && && (1)\\
s.t. \quad & \sum_{k\in K} \sum_{j\in\delta^+(i)} x_{ijk} = 1 && \forall i \in N && (2)\\
& \sum_{j\in\delta^+(0)} x_{0jk} = 1 && \forall k \in K && (3)\\
& \sum_{i\in\delta^-(n+1)} x_{i,n+1,k} = 1 && \forall k \in K && (4)\\
& \sum_{i\in\delta^-(j)} x_{ijk} - \sum_{i\in\delta^+(j)} x_{ijk} = 0 && \forall k \in K, j \in N && (5)\\
& w_{ik} + s_i + t_i - w_{jk} \le (1 - x_{ijk}) M_{ij} && \forall k \in K, (i,j) \in A && (6)\\
& w_{0k} = E, w_{n+1,k} = L && \forall k \in K && (7)\\
& t_i \ge 0.5 \sum_{j\in\delta^+(i)} x_{ijk} && \forall k \in K, i \in N && (8)\\
& p_i \sum_{j\in\delta^+(i)} x_{ijk} \le 0.3 q_k && \forall k \in K, i \in N && (9)\\
& x_{ijk} \in \{0,1\} && \forall k \in K, (i,j) \in A && (10)
\end{aligned}
$$

The constraint (1) is to minimize the cost resulted by opening sessions.

The constraint (2) Every session i which is followed by session j is only served once by one room k.

The constraint (3) For every room k, start from session 0.

The constraint (4) For every room k, end at session (n+1).

The constraint (5) For every room k, session j will leave when it is served.

The constraint (6) session i start time + service time + interval(required) less than next session j start time. M for linearization.

The constraint (7) Add two node indicate the start node and end node.

The constraint (8) Time interval constraint.

The constraint (9) Space distance constraint.

3.1 M0

Maximize the distance.

Input: Service time s_i for each session instead of the time window [a,b].

Add $p_0 = 0, s_0 = 0, E = 0/8, L = 24$.

MODEL:

$$
\begin{aligned}
max_{i,j,k} \quad & \sum_{(i,j)\in A}\sum_{k\in K}\frac{p_i}{q_k}x_{ijk} + \frac{1}{24}T_{ijk} & & (11)\\
s.t. \quad & \sum_{k\in K}\sum_{j\in\delta^+(i)} x_{ijk} = 1 & \forall i \in N & \quad (12)\\
& \sum_{j\in\delta^+(0)} x_{0jk} = 1 & \forall k \in K & \quad (13)\\
& \sum_{i\in\delta^-(n+1)} x_{i,n+1,k} = 1 & \forall k \in K & \quad (14)\\
& \sum_{i\in\delta^-(j)} x_{ijk} - \sum_{i\in\delta^+(j)} x_{ijk} = 0 & \forall k \in K, j \in N & \quad (15)\\
& y_{ijk} \geq (x_{ijk} - 1)M_{ij} & \forall k \in K, (i,j) \in A & \quad (16)\\
& w_{0k} = E, w_{n+1,k} = L & \forall k \in K & \quad (17)\\
& x_{ijk} \in \{0, 1\} & \forall k \in K, (i,j) \in A & \quad (18)
\end{aligned}
$$

How to change the quadratic terms to the linear terms(linearization)

Note that the $y = x_1x_2$ where $x_1 \in \{0, 1\}, x_2 \in [l, u] \rightarrow$

$$
\begin{aligned}
& y \leq x_2 \\
& y \geq x_2 - u(1 - x_1) \\
& lx_1 \leq y \leq ux_1
\end{aligned}
$$

Let $(w_{jk} - w_{ik} - s_i) = y_{ijk}$ and $T_{ijk} = x_{ijk}y_{ijk}$

$$
\begin{aligned}
& T_{ijk} \leq y_{ijk} \\
& T_{ijk} \geq y_{ijk} - u(1 - x_{ijk}) \\
& T_{ijk} \leq ux_{ijk}
\end{aligned}
$$

The constraint (1) Maximize the distance.

The constraint (2) Every session i which is followed by session j is only served once by one room k.

The constraint (3) For every room k, start from group 0.

The constraint (4) For every room k, end at group (n+1).

The constraint (5) For every room k, group j will leave when it is served.

The constraint (6) i start time + service time + interval(required) ¡ next j start time. M for linearization.

The constraint (7) Time window constraints for every group.

The constraint (8) Add two node which indicate the start node and end node.

——————————————————— 可行性：

影院有自主排片权

排片的重要性：

排片决定观影人次数；

决定收入：、票房、食品酒水、衍生品、广告场租；

排片是影院的核心竞争力；

Factors to consider:

Sequence/Assignment/Time/Price

1. 黄金时间段是上座率最高的时间段，两小时内每厅至少排一场，黄金时间段会随季节以及地域变化

2. 将票房最高的影片，在最黄金时间，排入最优最大厅

3. 把黄金时间段、黄金厅给黄金影片，从黄金向两边排起

4. 如果给一部电影相邻两个场次，最好两个影厅一大一小

5. 在同一个厅里，尽量避免插排两部不同的影片 A/B/A/B/A/B，只需安排同部电影 AAA，或 BBB 即可

6. 避免同时开场、同时散场。最短场间隔10分钟

7. 中文版、英文版都有的进口片，要根据观众的偏好排映

8. 进口动画影片，白天应更多安排中文版场次

9. 周六周日节假日应多排大片，票价可以考虑适当提高

10. 每日开场时间：节假日和有大片时可以适当提前，六一儿童节应提前早场开映时间

11. 影片每日结场时间：周五、周六、及各种节假日可以考虑推迟

12. 考虑成本控制。普通厅放映成本: 4张票. 巨幕厅：9张票.

Existing problems

1. 排片时缺少预估

2. 人流情况预估不够细化

3. 场次安排很少有影院个性化估算

4. 除最大厅和最小厅外，其他厅的安排随意

————————————————

self-study room (Reusable resources)

有social distance, 不考虑 group 比较简单，考虑 group 不符合实际。

4 Stochastic Situation

4.1 M1

q_k capacity.

$k \in K$ The Number of room

s_i service time for each group.

p_i demand number of people. $i \in N$

- Length for time. 24 for K. s_i for N. - Width for the capacity. q_k for K. p_i for N. - variable x_{ik} indicates group i served by room k.

Now we change the objective function q_k to a concave function $f(q_k)$. How to influence the result?

Search for the minimization makespan problem.

To be specific, how to deal/handle with minimax format?

$$
\begin{aligned}
\min \quad & (\max(\sum_i x_{ik} s_i p_i)/(24 * f(q_k)), \quad \forall k \in K) \\
s.t. \quad & x_{ik} p_i \le q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \le T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}
$$

To:

$$
\begin{aligned}
(M1) = max \quad & t \\
s.t. \quad & x_{ik} p_i \le q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \le T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& t \le \sum_i x_{ik} s_i p_i/(24 * q_k), \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}
$$

$$
\begin{aligned}
(M2) = min \quad & t \\
s.t. \quad & x_{ik} p_i \le q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \le T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& t \ge \sum_i x_{ik} s_i p_i/(24 * q_k), \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}
$$

At first, it is clear that M1(max min) less than M2(min max). Thus, the true value will be between M1 and M2.

So what is the difference?

The constraint (1) Capacity ratio.

The constraint (2) Capacity constraints $|N| * |K|$.

The constraint (3) Time constraints $|K|$.

The constraint (4) Objective capacity ratio constraints $|K|$.

The constraint (5) Every group is served once $|N|$.

Virables: $|N| * |K| + 1$, refers to x_{ik}, t

Besides, when we convert the original problem into several sub-problems. Each sub-problem can be expressed as: Let $k = k_0$,

$$
\begin{aligned}
(Sub) = \min \quad & |\sum_i x_{ik_0} f(s_i, p_i) - r * f(24, q_{k_0})| \\
s.t. \quad & x_{ik_0} p_i \le q_{k_0}, \quad \forall i \in N_0 \\
& \sum_{i \in N_0} x_{ik_0} s_i \le 24
\end{aligned}
$$

Here, f(ServiceTime, Space) represents the area function. In fact, (12) is obviously satisfied because of the pretreatment which is used to get rid of the trouble of assignment constraints. Thus when we calculate the situation of under ratio, this sub-problem can be converted into

$$
\begin{aligned}
(Sub1) = max \quad & \sum_{i \in N_0} x_{ik_0} f(s_i, p_i) \\
s.t. \quad & \sum_{i \in N_0} x_{ik_0} f(s_i, p_i) \le r * f(24, q_{k_0}) \\
& \sum_{i \in N_0} x_{ik_0} s_i \le 24
\end{aligned}
$$

The final value equals to (-Sub1+$r * f(24, q_{k_0})$) This is a two-dimentional knapsack problem.

For any fixed $m \ge 2$, these problems do admit a pseudo-polynomial time algorithm (similar to the one for basic knapsack) and a PTAS.

Add one dimentional variable to the basic DP algorithm for knapsack.

Next time finish the code.

$$
\begin{aligned}
(Sub2) = min \quad & \sum_{i \in N_0} x_{ik_0} f(s_i, p_i) \\
s.t. \quad & \sum_{i \in N_0} x_{ik_0} f(s_i, p_i) \ge r * f(24, q_{k_0}) \\
& \sum_{i \in N_0} x_{ik_0} s_i \le 24
\end{aligned}
$$

The final value equals to (Sub2 - $r * f(24, q_{k_0})$)

In fact, we do not need to calculate this form. We can obtain Sub1 firstly, then add a rest item with the minimum area.

4.2 M2

q_k capacity.

$k \in K$ The Number of room

s_i service time for each group.

p_i demand number of people. $i \in N$

- Length for time. 24 for K. s_i for N. - Width for the capacity. q_k for K. p_i for N. - variable x_{ik} indicates group i served by room k.

The Original model:

$$
\begin{aligned}
\min \quad & (\max(\sum_i x_{ik} s_i p_i)/(24 * q_k), \quad \forall k \in K) \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
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$$

To:

$$
\begin{aligned}
\max \quad & t \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& t \leq \sum_i x_{ik} s_i p_i/(24 * q_k), \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}
$$

$$
\begin{aligned}
\min \quad & t \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& t \geq \sum_i x_{ik} s_i p_i/(24 * q_k), \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}
$$

So what is the difference?

The constraint (1) Capacity ratio.

The constraint (2) Capacity constraints $|N| * |K|$.

The constraint (3) Time constraints $|K|$.

The constraint (4) Objective capacity ratio constraints $|K|$.

The constraint (5) Every group is served once $|N|$.

Virables: $|N| * |K| + 1$

5 Dynamic Situation

6 Results

7 Conclusion

参考文献

Proof

(Theorem 1). $\square$

(Lemma 1). $\square$

(Lemma 2). $\square$

(Theorem 2). $\square$