Dynamic Seat Assignment

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Abstract

We consider the critical challenge of seat planning and assignment in the context of social distancing measures, which have become increasingly vital in today's environment. Ensuring that individuals maintain the required social distances while optimizing seating management requires careful consideration of several factors, including group sizes, venue layouts, and fluctuating demand patterns. We introduce critical concepts and analyze seat planning with deterministic requests. Subsequently, we introduce a scenario-based stochastic programming approach to address seat planning with stochastic requests. The seat planning can serve as the foundation for the seat assignment. Furthermore, we explore a dynamic situation where groups arrive sequentially. We propose a seat-plan-based assignment policy for either accommodating or rejecting incoming groups. This policy outperforms traditional bid-price and booking-limit strategies. Our study evaluates social distancing measures through the lens of revenue optimization, delivering actionable insights for both venue operators and policymakers. For venue managers, we develop an operational framework to optimize seat allocations under distancing constraints, and quantitative tools to assess occupancy rate tradeoffs (considering the layout configurations). For policymakers, we provide analytical methods to evaluate the economic impact of distancing mandates, and guidelines for setting policy parameters (including achievable occupancy rate, maximum group size, physical distance requirements).

Keywords: seating management, social distancing, scenario-based stochastic programming, dynamic seat assignment.

1 Introduction

Social distancing has proven effective in containing the spread of infectious diseases. For instance, during the recent COVID-19 pandemic, the fundamental requirement of social distancing involved establishing a minimum physical distance between individuals in public spaces. In fact, the principles of social distancing can also be applied to various industries beyond public health. For example, restaurants may adopt social distancing practices to enhance guest experience and satisfaction while fostering a sense of privacy. In event management, particularly for large gatherings, social distancing can improve comfort and safety, even in non-health-related contexts, by providing attendees with more personal space.

As a general principle, social distancing measures can be defined from different dimensions. The basic requirement of social distancing is the specification of a minimum physical distance between individuals in public areas. For example, the World Health Organization (WHO) suggests to"keep physical distance of at least 1 meter from others" (WHO, 2020). In the US, the Centers for Disease Control and Prevention (CDC) describes social distancing as "keeping a safe space between yourself and other people who are not from your household" (CDC, 2020). It's important to note that this requirement is typically applied with respect to groups of people. For instance, in Hong Kong, the government has implemented social distancing measures during the Covid-19 pandemic by limiting the size of groups in public gatherings to two, four, and six people per group over time. Moreover, the Hong Kong government has also established an upper limit on the total number of people in a venue; for example, restaurants were allowed to operate at 50% or 75% of their normal seating capacity (GovHK, 2020).

From a company's perspective, social distancing can disrupt normal operations in certain sectors. For example, a restaurant needs to change or redesign the layout of its tables to comply with social distancing requirements. Such change often results in reduced capacity, fewer customers, and consequently, less revenue. In this context, affected firms face the challenge of optimizing its operational flow when adhering to social distancing policies. From a government perspective, the impact of enforcing social distancing measures on economic activities is a critical consideration in decision-making. Facing an outbreak of an infectious disease, a government must implement social distancing policy based on a holistic analysis. This analysis should take into account not only the severity of the outbreak but also the potential impact on all stakeholders. What is particularly important is the evaluation of business losses suffered by the industries that are directly affected.

We will address the above issues of social distancing in the context of seating management. Consider a venue, such as a cinema or a conference hall, which is used to host an event. The venue is equipped with seats of multiple rows. During the event, requests for seats arrive in groups, each containing a limited number of individuals. Each group can be either accepted or rejected, and those that are accepted will be seated consecutively in one row. Each row can accommodate multiple groups as long as any two adjacent groups in the same row are separated by one or multiple empty seats to comply with social distancing requirements. The objective is to maximize the number of individuals accepted for seating.

Seat management is critically dependent on demand forms, leading us to examine three distinct problems: the Seat Planning with Deterministic Requests (SPDR) problem, the Seat Planning with Stochastic Requests (SPSR) problem, and the Seat Assignment with Dynamic Requests (SADR) problem. In the SPDR problem, complete information about seating requests in groups is known. This applies to scenarios where the participants and their groups are identified, such as family members attending a church gathering or staff from the same office at a company meeting. In the SPSR problem, the requests are unknown but follow a probabilistic distribution. This problem is relevant in situations where a new seating layout must accommodate multiple events with varying seating requests. For example, during the COVID-19 outbreak, some theaters physically removed some seats and used the remaining ones to create a seating plan that accommodates stochastic requests. In the SADR problem, groups of seating requests arrive dynamically. The problem is to decide, upon the arrival of each group of request, whether to accept or reject the group, and assign seats for each accepted group. Such seat assignment is applicable in commercial settings where requests arrive as a stochastic process, such as ticket sales in movie theaters.

We develop models and derive optimal solutions for each of these three problems. Specifically, we formulate the SPDR problem using Integer Programming and discuss the key characteristics of the optimal seating plan. For the SPSR problem, we utilize scenario-based optimization and develop solution approaches based on Benders decomposition. In addressing the SADR problem, we implement a two-stage seat-plan-based assignment approach. In the first decision phase, a relaxed dynamic programming evaluates each incoming request to determine its acceptance. The accepted requests then proceed to the second assignment phase, where the group-type control allocation is performed. This seat-plan-based assignment policy outperforms traditional bid-price and booking-limit policies. Although each of these models represents a standalone problem tailored to specific situations, they are closely interconnected in terms of problem-solving methods and managerial insights. In the seat planning with deterministic requests (SPDR) problem, we identify important concepts such as the full pattern and the largest pattern, which play a crucial role in developing solutions for the other two problems. Additionally, the SPDR problem serves as a useful offline benchmark for evaluating the performance of policies in the SADR problem. Furthermore, the solution to the SPSR problem can serve as a reference seat plan for dynamic seat assignment in the SADR problem.

We investigate the impact of social distancing from the perspective of revenue loss. To facilitate this analysis, we introduce the concept of the threshold of request-volume, which represents the upper limit on the number of requests an event can accommodate without being affected by social distancing measures. Specifically, if an event receives fewer requests than the threshold of request-volume, it will experience virtually no revenue loss due to social distancing. Our computational experiments demonstrate that the threshold of request-volume primarily depends on the mean group size and is relatively insensitive to the specific distribution of group sizes. This finding provides a straightforward method for estimating the threshold of request-volume and evaluating the impact of social distancing. In some instances, the government imposes a maximum allowable occupancy rate to enforce stricter social distancing requirements. To assess this effect, we introduce the concept of the threshold of occupancy rate, defined as the occupancy rate at the threshold of request-volume. The maximum allowable occupancy rate is effective for an event only if it is lower than the event's threshold of occupancy rate. Moreover, it becomes redundant if it exceeds the maximum achievable occupancy rate for all events.

These qualitative insights are stable with respect to the government policy's strictness and the specific characteristics of various venues, such as minimum physical distance, allowable maximum group size, and venue layout. When the minimum physical distance increases, the threshold of request-volume, threshold of occupation rate and maximum achievable occupation rate decrease accordingly. Conversely, when the allowable maximum group size decreases, the number of accepted requests will increase; however, both the threshold of occupation rate and maximum achievable occupation rate decline. Although venue layouts may vary in shapes (rectangular or otherwise) and row lengths (long or short), the threshold of occupancy rate and maximum achievable occupancy rate do not exhibit significant variation.

The rest of this paper is structured as follows. We review the relevant literature in Section 2. Then we introduce the key concepts of seat planning with social distancing and formulate the seat planning with deterministic requests in Section 3. In Section ??, we establish the scenario-based stochastic programming for the seat planning with stochastic requests, then apply the Benders decomposition technique to obtain the solution. Section 4 presents the seat-plan-based assignment policy to assign seats for incoming requests. Section ?? presents the experimental results and provide insights gained from implementing social distancing. Conclusions are shown in Section 5.

2 Literature Review

Seating management is a practical problem that presents unique challenges in various applications, each with its own complexities, particularly when accommodating group-based seating requests. For instance, in passenger rail services, groups differ not only in size but also in their departure and arrival destinations, requiring them to be assigned consecutive seats (Clausen et al., 2010; Deplano et al., 2019). In social gatherings such as weddings or dinner galas, individuals often prefer to sit together at the same table while maintaining distance from other groups they may dislike (Lewis and Carroll, 2016). In parliamentary seating assignments, members of the same party are typically grouped in clusters to facilitate intra-party communication as much as possible (Vangerven et al., 2022). In e-sports gaming centers, customers arrive to play games in groups and require seating arrangements that allow them to sit together (Kwag et al., 2022).

Incorporating social distancing into seating management has introduced an additional layer of complexity, sparking a new stream of research. Some works focus on the layout design and determine seating positions to maximize physical distance between individuals, such as students in classrooms (Bortolete et al., 2022) or customers in restaurants and beach umbrella arrangements (Fischetti et al., 2023). Other works assume the seating layout is fixed, and assign seats to individuals while adhering to social distancing guidelines. For example, Salari et al. (2020) and Pavlik et al. (2021) consider the seat assignment in the airplanes. The above studies consider the seating management with social distancing for individual requests.

Our work relates to seating management with social distancing for group-based requests, which has found its applications in various areas, including single-destination public transits (Moore et al., 2021), airplanes (Ghorbani et al., 2020; Salari et al., 2022), trains (Haque and Hamid, 2022, 2023), and theaters (Blom et al., 2022). Due to the diversity of these applications, there are different issues to be addressed. For example, Salari et al. (2022) consider the distance between different groups and develop a seating assignment strategy that outperforms the simplistic airline policy of blocking all middle seats. In Haque and Hamid (2023), the design of seat allocation for groups with social distancing takes into account the transmission risk within the train and between different stops. Our work is closely related to Blom et al. (2022), who address the group-based seating problem in theaters. While they primarily focuses on scenarios with known groups (referred to as seat planning with deterministic requests in our work), we investigate a broader range of demand forms. In addtion to the deterministic requests, we also study group-based seat planning with stochastic requests and explore dynamic seat assignment, assuming that groups arrive sequentially according to a stochastic process.

From a technical perspective, when all requests are known, the seating planning with deterministic requests (SPDR) problem can be formulated as a multiple knapsack problem (Martello and Toth, 1990). While existing literature has primarily focused on deriving bounds or competitive ratios for general multiple knapsack problems (Khuri et al., 1994; Ferreira et al., 1996; Pisinger, 1999; Chekuri and Khanna, 2005), our work distinguishes itself by analyzing the specific structure and properties of solutions to the SPDR problem. This approach offers valuable insights for our investigation into situations involving dynamic demand.

While the dynamic stochastic knapsack problem (e.g., Kleywegt and Papastavrou (1998, 2001), Papastavrou et al. (1996)) has been extensively studied in the literature, these works primarily consider a single knapsack scenario where requests arrive sequentially and their resource requirements and rewards are unknown until they arrive. In contrast, the seat assignment with dynamic requests (SADR) problem extends this framework by incorporating multiple knapsacks, adding another layer of complexity to the decision-making process. Research on the dynamic or stochastic multiple knapsack problem is limited. Perry and Hartman (2009) employs multiple knapsacks to model multiple time periods for solving a multiperiod, single-resource capacity reservation problem. This essentially remains a dynamic knapsack problem but involves time-varying capacity. Tönissen et al. (2017) considers a two-stage stochastic multiple knapsack problem with a set of scenarios, wherein the capacity of the knapsacks may be subject to disturbances. This problem is similar to the SPSR problem in our work, where the number of items is stochastic.

Generally speaking, the SADR problem relates to the revenue management (RM) problem, which has been extensively studied in industries such as airlines, hotels, and car rentals, where perishable inventory must be allocated dynamically to maximize revenue (van Ryzin and Talluri, 2005). Network revenue management (NRM) extends traditional RM by considering multiple resources (e.g., flight legs, hotel nights) and interdependent demand (Williamson, 1992). The standard NRM problem is typically formulated as a dynamic programming (DP) model, where decisions involve accepting or rejecting requests based on their revenue contribution and remaining capacity (Talluri and van Ryzin, 1998). However, a significant challenge arises because the number of states grows exponentially with the problem size, rendering direct solutions computationally infeasible. To address this, various control policies have been proposed, such as bid-price (Adelman, 2007; Bertsimas and Popescu, 2003), booking limits (Gallego and van Ryzin, 1997), and dynamic programming decomposition (Talluri and van Ryzin, 2006; Liu and van Ryzin, 2008). These methods typically assume that demand arrives individually (e.g., one seat per booking). However, in our problem, customers often request multiple units simultaneously, requiring decisions that must be made on an all-or-none basis for each request. This requirement introduces significant complexity in managing group arrivals (Talluri and van Ryzin, 2006).

A notable study addressing group-like arrivals in revenue management examines hotel multi-day stays (Bitran and Mondschein, 1995; Goldman et al., 2002; Aydin and Birbil, 2018). While these works focus on customer classification and room-type allocation, they do not prioritize real-time assignment. The work of Zhu et al. (2023), which addresses the high-speed train ticket allocation, is related to our

SADR problem in terms of real-time seat assignment. While Zhu et al. (2023) processes individual seat requests and implicitly accommodates group-like traits through multi-leg journeys (e.g., passengers retaining the same seat across connected segments), our SADR context explicitly treats groups as booking entities involving simultaneous multi-seat reservations (e.g., a family booking four seats in a single transaction).

3 Seat Planning Problem with Social Distancing

3.1 Concepts

Consider a seat layout comprising N rows, with each row j containing L_j^0 seats, for $j \in \mathcal{N} := \{1, 2, \ldots, N\}$. The venue will hold an event with multiple seat requests, where each request includes a group of multiple people. There are M distinct group types, where each group type i, $i \in \mathcal{M} := \{1, 2, \ldots, M\}$, consists of i individuals requiring i consecutive seats in one row. The request of each group type is represented by a demand vector $\mathbf{d} = (d_1, d_2, \ldots, d_M)^\intercal$, where d_i is the number of groups of type i.

We now formulate the seat planning with deterministic requests (SPDR) problem as an integer programming, where x_{ij} represents the number of groups of type i planned in row j.

$$
\begin{aligned}
\text{(SPDR)} \quad \max \quad & \sum_{i=1}^{M} \sum_{j=1}^{N} (n_i - \delta) x_{ij} && (1) \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \leq d_i, \quad i \in \mathcal{M}, && (2) \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N}, && (3) \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned}
$$

The objective function (1) is to maximize the number of individuals accommodated. Constraint (2) ensures the total number of accommodated groups does not exceed the number of requests for each group type. Constraint (3) stipulates that the number of seats allocated in each row does not exceed the size of the row.

The increasing nature of the ratio $\frac{i}{n_i}$ with respect to group size i leads to preferential inclusion of larger groups in the optimal fractional seat plan. This intuitive property is illustrated in Proposition 1.

Proposition 1. *For the LP relaxation of the* SPDR *problem, there exists an index $\tilde{i}$ such that the optimal solutions satisfy the following conditions: $x_{ij}^* = 0$ for all j, $i = 1, \ldots, \tilde{i} - 1$; $\sum_{j=1}^{N} x_{ij}^* = d_i$ for $i = \tilde{i} + 1, \ldots, M$; $\sum_{j=1}^{N} x_{ij}^* = \frac{L - \sum_{i=\tilde{i}+1}^{M} d_i n_i}{n_{\tilde{i}}}$ for $i = \tilde{i}$.*

4 Seat Assignment with Dynamic Requests

In many commercial situations, requests arrive sequentially over time, and the seller must immediately decide whether to accept or reject each request upon arrival while ensuring compliance with the required spacing constraints. If a request is accepted, the seller must also determine the specific seats to assign. Importantly, each request must be either fully accepted or entirely rejected; once seats are assigned to a group, they cannot be altered or reassigned to other requests.

To model this problem, we formulate it using dynamic programming approach in a discrete-time framework. Time is divided into T periods, indexed forward from 1 to T. We assume that in each period, at most one request arrives and the probability of an arrival for a group type i is denoted as p_i, where $i \in \mathcal{M}$. The probabilities satisfy the constraint $\sum_{i=1}^{M} p_i \leq 1$, indicating that the total probability of any group arriving in a single period does not exceed one. We introduce the probability $p_0 = 1 - \sum_{i=1}^{M} p_i$ to represent the probability of no arrival in each period. To simplify the analysis, we assume that the arrivals of different group types are independent and the arrival probabilities remain constant over time. This assumption can be extended to consider dependent arrival probabilities over time if necessary.

The remaining capacity in each row is represented by a vector $\mathbf{L} = (l_1, l_2, \ldots, l_N)$, where l_j denotes the number of remaining seats in row j. Upon the arrival of a group type i at time t, the seller needs to make a decision denoted by $u_{i,j}^t$, where $u_{i,j}^t = 1$ indicates acceptance of group type i in row j during period t, while $u_{i,j}^t = 0$ signifies rejection of that group type in row j. The feasible decision set is defined as

$$U^t(\mathbf{L}) = \left\{ u_{i,j}^t \in \{0,1\}, \forall i \in \mathcal{M}, \forall j \in \mathcal{N} \,\middle|\, \sum_{j=1}^{N} u_{i,j}^t \leq 1, \forall i \in \mathcal{M}; n_i u_{i,j}^t \mathbf{e}_j \leq \mathbf{L}, \forall i \in \mathcal{M}, \forall j \in \mathcal{N} \right\}.$$

Here, $\mathbf{e}_j$ represents an N-dimensional unit column vector with the j-th element being 1, i.e., $\mathbf{e}_j = (\underbrace{0, \cdots, 0}_{j-1}, 1, \underbrace{0, \cdots, 0}_{N-j})$. The decision set $U^t(\mathbf{L})$ consists of all possible combinations of acceptance and rejection decisions for each group type in each row, subject to the constraints that at most one group of each type can be accepted in any row, and the number of seats occupied by each accepted group must not exceed the remaining capacity of the row.

Let $V^t(\mathbf{L})$ denote the maximum expected revenue earned by the optimal decision regarding group seat assignments at the beginning of period t, given the remaining capacity $\mathbf{L}$. Then, the dynamic programming formulation for this problem can be expressed as:

$$V^t(\mathbf{L}) = \max_{u_{i,j}^t \in U^t(\mathbf{L})} \left\{ \sum_{i=1}^{M} p_i \Big(\sum_{j=1}^{N} i u_{i,j}^t + V^{t+1}(\mathbf{L} - \sum_{j=1}^{N} n_i u_{i,j}^t \mathbf{e}_j) \Big) + p_0 V^{t+1}(\mathbf{L}) \right\} \tag{4}$$

with the boundary conditions $V^{T+1}(\mathbf{L}) = 0, \forall \mathbf{L}$, which implies that the revenue at the last period is 0 under any capacity. The initial capacity is denoted as $\mathbf{L}_0 = (L_1, L_2, \ldots, L_N)$. Our objective is to determine group assignments that maximize the total expected revenue during the horizon from period 1 to T, represented by $V^1(\mathbf{L}_0)$.

Solving the dynamic programming problem in equation (4) presents computational challenges due to the curse of dimensionality that arises from the large state space.

We propose our policy for assigning arriving requests in a dynamic context. First, we employ the traditional bid-price control policy. Then, we improve the bid-price control policy based on the seat plan (patterns).

For any policy π, let V^{π} denote the expected revenue collected under π. Among all policies, a special one is the dynamic programming (DP) policy as it achieves the maximal expected revenue. Apart from DP, another special policy is the hindsight optimum (HO), where the decision maker has full information on the demand realization for the entire time horizon and optimizes over the allocation schemes. Here, it is impossible for the HO to be attained because we can never know the full demand realization ahead. For ease of analysis, the hindsight optimum for the sample path is computed by solving a relaxed static problem and V^{HO} is the expected value over all sample paths. Then for any online policy π, we have

$$V^{\pi} \leq V^{DP} \leq V^{HO}.$$

It is well known that DP, despite its optimality, is computationally complex owing to the "curse of dimensionality." Thus we use HO as our benchmark to evaluate the performance of any policy π.

4.1 Traditional BPC Policy

Bid-price control is a classical approach discussed extensively in the literature on network revenue management. It involves setting bid prices for different group types, which determine the eligibility of groups to take the seats. Bid-prices refer to the opportunity costs of taking one seat. As usual, we estimate the bid price of a seat by the shadow price of the capacity constraint corresponding to some row. In this section, we will demonstrate the implementation of the bid-price control policy.

The dual of LP relaxation of the SPDR problem is:

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} d_i z_i + \sum_{j=1}^{N} L_j \beta_j \\ \text{s.t.} \quad & z_i + \beta_j n_i \geq (n_i - \delta), \quad i \in \mathcal{M}, j \in \mathcal{N} \\ & z_i \geq 0, i \in \mathcal{M}, \beta_j \geq 0, j \in \mathcal{N}. \end{aligned} \tag{5}$$

In (5), β_j can be interpreted as the bid-price for a seat in row j. A request is only accepted if the revenue it generates is no less than the sum of the bid prices of the seats it uses. Thus, if $i - \beta_j n_i \geq 0$, meanwhile, the capacity allows, we will accept the group type i. And choose $j^* = \arg\max_j \{i - \beta_j n_i\}$ as the row to allocate that group.

Lemma 1. *The optimal solution to problem* (5) *is given by* $z_1 = \ldots = z_{\tilde{i}} = 0$, $z_i = \frac{\delta(n_i - n_{\tilde{i}})}{n_{\tilde{i}}}$ *for* $i = \tilde{i}+1, \ldots, M$ *and* $\beta_j = \frac{n_{\tilde{i}} - \delta}{n_{\tilde{i}}}$ *for all* j.

Algorithm 1: Bid-Price Control

for $t = 1, \dots, T$ **do**

  Observe a request of group type i;

  Solve problem (5) with $\boldsymbol{d}^t = (T - t) \cdot \boldsymbol{p}$ and $\mathbf{L}^t$;

  Obtain $\tilde{i}$ such that the aggregate optimal solution is $xe_{\tilde{i}} + \sum_{i=\tilde{i}+1}^{M} d_i^t e_i$;

  if $i \geq \tilde{i}$ *and* $\max_{j \in \mathcal{N}} L_j^t \geq n_i$ **then**

    Set $k = \arg\min_{j \in \mathcal{N}} \{L_j^t | L_j^t \geq n_i\}$ and break ties;

    Assign the group to row k, let $L_k^{t+1} \leftarrow L_k^t - n_i$;

  else

    Reject the group;

  end

end

Let V(T-BPC), V(DLP) denote the optimal value of (5) and the LP relaxation of the SPDR problem with expected demand, respectively. Then we have $V(\text{T-BPC}) = V(\text{DLP})$.

However, traditional bid-price control (BPC) has two drawbacks. First, when capacity permits, the policy treats all rows as equally preferable, making no distinction between them. Second, the decision to accept a request may not be feasible if certain rows lack sufficient capacity.

4.2 BPC Policy Based on Patterns

To address these limitations, we consider an improved DP formulation. The key idea, as we explain next, is to represent row structures using patterns, not just track the capacity $\boldsymbol{L}$.

Suppose that $S(L_j)$ is the set of all patterns for row j. Let $v^t(\boldsymbol{L})$ indicate the maximal expected value to go at time t, given the capacity $\boldsymbol{L}$.

The dynamic programming can be expressed as follow:

$$\begin{aligned} & v^t(\boldsymbol{L}) = \mathbb{E}_{i \sim p^t}\left[\max \left\{ \max_{j: \boldsymbol{h} \in S(L_j), h_i \geqslant 1} \left\{ v^{t+1}(\boldsymbol{L} - e_j^T \cdot n_i) + i \right\}, v^{t+1}(\boldsymbol{L}) \right\} \right], && \forall t, \boldsymbol{L}, \\ & v^{T+1}(\boldsymbol{L}) = 0, && \forall \boldsymbol{L}. \end{aligned} \tag{6}$$

We can solve the following program to compute $v^1(\boldsymbol{L})$ for any given capacity $\boldsymbol{L}$:

$$\begin{aligned} \min \quad & v^1(\boldsymbol{L}) \\ \text{s.t.} \quad & v^t(\boldsymbol{L}) \geq \mathbb{E}_{i \sim p^t}\left[\max \left\{ \max_{j: \boldsymbol{h} \in S(L_j), h_i \geqslant 1} \left\{ v^{t+1}(\boldsymbol{L} - e_j^T \cdot n_i) + i \right\}, v^{t+1}(\boldsymbol{L}) \right\} \right], \\ & v^{T+1}(\boldsymbol{L}) \geq 0. \end{aligned} \tag{7}$$

Solving (7) remains computationally prohibitive. Following from the ADP approach, we approximate $v^t(\boldsymbol{L})$ as

$$\hat{v}^t(\boldsymbol{L}) = \theta^t + \sum_{j=1}^{N} \max_{\boldsymbol{h}\in S(L_j)} \{\sum_{i=1}^{M} \beta_{ij} h_i\}. \tag{8}$$

Unlike traditional linear approximations, our approach retains the linear term θ^t but introduces a nonlinear component for each row j. Specifically, we maximize the linear combination $\sum_{i=1}^{M} \beta_{ij} h_i$ over the feasible set $S(L_j)$.

Our approximation extends classical linear ADP by incorporating resource-specific nonlinear terms through constrained maximization over feasible allocations. While similar separable corrections appear in resource allocation ADP (e.g., Powell, 2007), our explicit use of $\max_{\boldsymbol{h}\in S(L_j)}$ captures local constraints more directly, akin to dual decomposition methods (Shapiro & Nemirovski, 2005).

Our approximation adopts a separable nonlinear structure, but unlike classical concave/quadratic approximations, the nonlinearity is implicitly defined by constrained maximization over feasible allocations. This captures problem-specific constraints without requiring explicit parametric forms for .

The term β_{ij} can be regarded as the approximated value for each group in row j.

Substituting (8) into (7), we have:

$$\theta^t - \theta^{t+1} = \hat{v}^t(\boldsymbol{L}) - \hat{v}^{t+1}(\boldsymbol{L}) \geq \sum_i p_i \max\left\{ \max_{j:\boldsymbol{h}\in S(L_j), h_i \geqslant 1} \{v^{t+1}(\boldsymbol{L} - e_j^T n_i) - v^{t+1}(\boldsymbol{L}) + i\}, 0 \right\} \tag{9}$$

For each j, $v^{t+1}(\boldsymbol{L} - e_j^T n_i) - v^{t+1}(\boldsymbol{L}) = \max_{\boldsymbol{h}\in S(L_j - n_i)}\{\sum_i \beta_{ij} h_i\} - \max_{\boldsymbol{h}\in S(L_j)}\{\sum_i \beta_{ij} h_i\} = -\beta_{ij}$.
Let $\alpha_i = \max\{\max_j \{i - \beta_{ij}\}, 0\}$ and $\gamma_j = \max_{\boldsymbol{h}\in S(L_j)}\{\sum_i \beta_{ij} h_i\}$.
Then, we obtian $\theta^1 = \sum_{t=1}^{T}(\theta^t - \theta^{t+1}) \geq \sum_t \sum_i \alpha_i p_i = \sum_i d_i \alpha_i$ and $\hat{v}^1(\boldsymbol{L}) = \sum_i d_i \alpha_i + \sum_j \gamma_j$.
Since $\hat{v}^1(\boldsymbol{L})$ is a feasible solution to (7), then $V(\text{I-BPC}) \geq V^{DP}$.

The corresponding bid-price problem can be expressed as:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{M} \alpha_i d_i + \sum_{j=1}^{N} \gamma_j \\
\text{s.t.} \quad & \alpha_i + \beta_{ij} \geq i, \quad \forall i, j, \\
& \sum_{i=1}^{M} \beta_{ij} h_i \leq \gamma_j, \quad \forall j, \boldsymbol{h} \in S(L_j), \\
& \alpha_i \geq 0, \quad \forall i, \\
& \gamma_j \geq 0, \quad \forall j.
\end{aligned} \tag{10}$$

α_i represents marginal revenue for group i. β_{ij} represents the cost for group i assigned in row j. γ_j represents the capacity cost associated with row j.

Let $V(\text{T-BPC})$ and $V(\text{I-BPC})$ denote the expected optimal value of (5) and (10), respectively.

Lemma 2. *For any optimal bid-prices, β_j, in (5), there exist optimal bid-prices, β_{ij}, in (10) such that we have the relation: $\beta_{ij} \leq n_i \beta_j, \forall i$. Furthermore, $V(\text{T-BPC}) \geq V(\text{I-BPC})$.*

Both bid-price approaches give upper bounds on the value function at any state, meanwhile it follows that I-BPC provides a tighter approximation to the value function more accurately than T-BPC does.

Algorithm 2: Improved Bid-Price Control

for $t = 1, \dots, T$ **do**
  Observe a request of group type i;
  Solve problem (5) with $\boldsymbol{d}^t = (T - t) \cdot \boldsymbol{p}$ and $\mathbf{L}^t$;
  if $i - \beta_{ij} \geq 0$ **then**
    Set $k = \arg\max_{j \in \mathcal{N}} \{i - \beta_{ij}\}$ and break ties;
    Assign the group to row k, let $L_k^{t+1} \leftarrow L_k^t - n_i$;
  else
    Reject the group type;
  end
end

Lemma 2 shows that the I-BPC policy does not uniformly reject all small groups. Instead, it selectively accepts them based on pattern-based allocation, enabling more flexible decision-making.

Unlike the traditional BPC, the I-BPC policy accounts for row-specific characteristics in allocation decisions. Meanwhile, it guarantees feasible placement. Once a request is accepted, the policy ensures it can be assigned to a suitable row without additional feasibility checks. Therefore, the I-BPC policy better captures the structure of the capacity and features greater flexibility than the T-BPC policy does.

However, the bid-price policies are established via a "dual" formulation, which lose the information contained in the primal problem. Hence, we consider the dynamic primal formulation.

4.3 Dynmaic Primal Based on Patterns

Let $y_{j\boldsymbol{h}}$ denote the proportion of pattern $\boldsymbol{h}$ used in row j. The primal problem can be formulated as:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M} \sum_{j=1}^{N} i x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} + x_{i0} = d_i, \quad i \in \mathcal{M}, \\
& x_{ij} = \sum_{\boldsymbol{h} \in S(L_j)} h_i y_{j\boldsymbol{h}}, \quad i \in \mathcal{M}, j \in \mathcal{N}, \\
& \sum_{\boldsymbol{h} \in S(L_j)} y_{j\boldsymbol{h}} \leq 1, \quad j \in \mathcal{N}.
\end{aligned}
\tag{11}
$$

Here, x_{i0} is the number of unassigned groups of type i. The first set of constraints demonstrate that for each group type i, the sum of assigned groups and unassigned groups equals the total demand. The second set of constraints shows that the number of groups of type i assigned to row j equals the sum of h_i (the count of type i groups in pattern $\boldsymbol{h}$) weighted by the pattern proportions $y_{i\boldsymbol{h}}$. The total proportion of patterns uesd in each row j cannot exceed 1.

Let $\phi(\boldsymbol{d})$ indicate the optimal value of the linear relaxation of the SPDR problem.

$V^{\text{HO}} = E[\phi(\boldsymbol{d})] \le \phi(E[\boldsymbol{d}]) = V(\text{DLP})$.

Then, the following hierarchy holds: $V(\text{DLP}) = V(\text{T-BPC}) \ge V(\text{I-BPC}) \ge V^{\text{DP}}$.

Any feasible solution to 11 yields a feasible solution to linear relaxation of SPDR having the same objective value.

$V(\text{T-BPC}) \ge V^{\text{HO}} \ge V^{\text{DP}}$

Lemma 3. *$V(DLP) \ge V(HO)$ results from the concave property.*

Consider the standard linear program: $\{\max c^T\boldsymbol{x} : A\boldsymbol{x} \le \boldsymbol{d}, \boldsymbol{x} \ge \boldsymbol{0}\}$. Suppose that $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ are two demand vectors, the optimal solution is $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. For any $\lambda \in [0, 1]$, $\boldsymbol{d}_\lambda = \lambda\boldsymbol{d}_1 + (1-\lambda)\boldsymbol{d}_2$. Let $\boldsymbol{x}_\lambda = \lambda\boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2$, then $A\boldsymbol{x}_\lambda = A(\lambda\boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2) \le \lambda\boldsymbol{d}_1 + (1-\lambda)\boldsymbol{d}_2 = \boldsymbol{d}_\lambda$. Thus, $\boldsymbol{x}_\lambda$ is a feasible solution for $\boldsymbol{d}_\lambda$. Then, $\phi(\boldsymbol{d}_\lambda) \ge \boldsymbol{c}^T\boldsymbol{x}_\lambda = \lambda\boldsymbol{c}^T\boldsymbol{x}_1 + (1-\lambda)\boldsymbol{c}^T\boldsymbol{x}_2 = \lambda\phi(\boldsymbol{d}_1) + (1 = \lambda)\phi(\boldsymbol{d}_2)$, which indicates $\phi(\boldsymbol{d})$ is concave. Substitute $\boldsymbol{x}$ with $y_{j\boldsymbol{h}}$ and view $y_{j\boldsymbol{h}}$ as the decision variables, then the concave property still holds for (11).

Algorithm 3: Dynamic Primal

```
for t = 1, ..., T do
    Observe a request of group type i;
    Solve problem (11) with d^t = (T − t) · p ;
    Obtain an optimal solution x_ij ;
    Set k = arg max_j {x_ij} and break ties;
    Assign the group to row k (k = 0 means that the request is rejected), let L_k^{t+1} ← L_k^t − n_i;
end
```

4.3.1 Solve the dynamic primal

The pattern $\boldsymbol{h}$ is efficient for row j if and only if, for some $(\alpha_1, \ldots, \alpha_M, \gamma_j)$ (except that $\alpha_i = i, \forall i$), $\boldsymbol{h}$ is the optimal solution to

$$\max_{\boldsymbol{h}} \sum_{i=1}^{M} (i - \alpha_i)h_i - \gamma_j$$

To generate all efficient patterns, we need to solve the subproblem for each row j:

$$\begin{aligned} \max \quad & \sum_{i=1}^{M} (i - \alpha_i)h_i - \gamma_j \\ \text{s.t.} \quad & \sum_{i=1}^{M} n_i h_i \le L_j, \\ & h_i \in \mathbb{N}, \quad i \in \mathcal{M}. \end{aligned} \tag{12}$$

If the optimal value of (12) is larger than 0, the primal (11) reaches the optimal. Otherwise, a new pattern can be generated.

One important fact is that only efficient sets are used in the solution to (12). Specifically,

Lemma 4. *If $y^*_{j\boldsymbol{h}} > 0$ is the optimal solution to (11), then $\boldsymbol{h}$ is an efficient pattern.*

A pattern $\boldsymbol{h}$ is dominant if there is no distinct pattern $\boldsymbol{h}'$ where every component of $\boldsymbol{h}'$ is greater than or equal to the corresponding component of $\boldsymbol{h}$. The efficient pattern is a dominating pattern. (If $\alpha_i = i$, (11) reaches the optimal and no pattern will be generated.)

The relation between the capacity and the demand shows the different structure of the optimal solution.

Lemma 5. *When $\sum_{i=1}^{M} d_i n_i < \sum_{j=1}^{N} L_j$, we have $\gamma_j^* = 0, \forall j$ and $\alpha_i^* = i, \forall i$. There exists at least one row j such that $\sum_{\boldsymbol{h} \in S(L_j)} y_{j\boldsymbol{h}}^* < 1$.*

When $\sum_{i=1}^{M} d_i n_i \geq \sum_{j=1}^{N} L_j$, we have $\sum_{\boldsymbol{h} \in S(L_j)} y_{j\boldsymbol{h}}^* = 1, \forall j$.

It is obvious that the dominating patterns include the full and largest patterns.

The dominating pattern can be regarded as the feasible patterns when the number of occupied seats is in $[L - \delta, L]$.

Let $DP(L, M)$ denote the number of dominating patterns. Let $DP_t(L, M)$ denote the number of all feasible patterns with length L.

$DP(L, M) = DP_t(L, M) - DP_t(L - \delta - 1, M)$.

$$DP_{\mathrm{t}}(L, M) = \sum_{k=0}^{\lfloor L/(M+\delta) \rfloor} DP_{\mathrm{t}}(L - k(M + \delta), M - 1) \tag{13}$$

polynomial

4.4 Static BLC Policy

Booking limit control policy:

$$\begin{aligned}
\max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(n_i-\delta)x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \le d_i, \quad i \in \mathcal{M}, \\
& \sum_{i=1}^{M} n_i x_{ij} \le L_j, j \in \mathcal{N},
\end{aligned} \tag{14}$$

Let $d_i^* = \sum_j x_{ij}^*$, x_{ij}^* is an integral optimal solution to (14) with $d_i = \sum_t p_i^t$ (Expected demand).

Let d_i indicate the number of group type i during time T. $d_i = \sum_t \mathbf{1}_{i_t=i}$. Let $val(I;\{d_i\})$ denote the optimal objective value of (14).

$V^{BL}(I) = E_{\{d_i\}}[\sum_i (n_i-\delta)\min\{d_i^*, d_i\}]$, $V^{OPT}(I) = E_{\{d_i\}}[val(I;\{d_i\})] \le val(I;\{E[d_i]\})$.

$val(I;\{d_i\})$ is concave in d_i.

$$\begin{aligned}
& V^{OPT}(I) - V^{BL}(I) \\
\le & val(I;\{E[d_i]\}) - V^{BL}(I) \\
= & val(I;\{E[d_i]\}) - val(I;\{\lfloor E[d_i]\rfloor\}) + val(I;\{\lfloor E[d_i]\rfloor\}) - E_{\{d_i\}}[\sum_i (n_i-\delta)\min\{d_i^*, d_i\}] \\
\le & \sum_i (n_i-\delta) + N\sum_i i + E_{\{d_i\}}[\sum_i (n_i-\delta)(d_i^* - \min\{d_i^*, d_i\})] \\
= & \sum_i (n_i-\delta) + N\sum_i i + E_{\{d_i\}}[\sum_i \frac{1}{2}(n_i-\delta)(d_i^* - d_i + |d_i^* - d_i|)] \\
\overset{(a)}{\le} & \sum_i (n_i-\delta) + N\sum_i i + \frac{1}{2}\sum_i (n_i-\delta)(d_i^* - E[d_i] + |d_i^* - E[d_i]| + \sqrt{\mathrm{Var}[d_i]}) \\
\le & \sum_i (n_i-\delta) + N\sum_i i + \frac{1}{2}\sum_i (n_i-\delta)\sqrt{\mathrm{Var}[d_i]} \\
\le & \sum_i (n_i-\delta) + N\sum_i i + \frac{1}{2}\sum_i (n_i-\delta)\sqrt{Tp_i(1-p_i)} = O(\sqrt{T})
\end{aligned}$$

Thus, $\lim_{T\to\infty}(V^{OPT}(I) - V^{BL}(I))/T \to 0$.

$val(I;\{E[d_i]\}) - val(I;\{\lfloor E[d_i]\rfloor\}) \le val(I;\{\lceil E[d_i]\rceil\}) - val(I;\{\lfloor E[d_i]\rfloor\}) = \sum_i (n_i-\delta)$

$LP - IP \le \sum_i\sum_j (n_i-\delta)(x_{ij}^* - \lfloor x_{ij}^*\rfloor) \le N\sum_i i \Rightarrow val(I;\{\lfloor E[d_i]\rfloor\}) \le IP + N\sum_i i$.

$IP = \sum_i\sum_j (n_i-\delta)x_{ij}^* = \sum_i (n_i-\delta)d_i^*$

(a) results from the following inequalities: $|d_i^* - d_i| = |(d_i^* - E[d_i]) + (E[d_i] - d_i)| \le |d_i^* - E[d_i]| + |d_i - E[d_i]|$. Take the expectation, we have $E[|d_i^* - d_i|] \le |d_i^* - E[d_i]| + E[|d_i - E[d_i]|]$. $E[|d_i - E[d_i]|] \le \sqrt{\mathrm{Var}[d_i]}$(Since $E[|X|] \le \sqrt{E[X^2]}$). $d_i^* \le E[d_i]$.

Surrogate relaxation (0-1 single):

$$\begin{aligned}
\max \quad & \sum_{i=1}^{M}(n_i-\delta)x_i && (15)\\
\text{s.t.} \quad & x_i \le d_i, \quad i\in\mathcal{M}, && (16)\\
& \sum_{i=1}^{M} n_i x_i \le L. && (17)
\end{aligned}$$

LP optimal solution: $[0,\ldots,0,X_{\tilde{i}},d_{\tilde{i}+1},\ldots,d_M]$, $X_{\tilde{i}}=\frac{L-\sum_{i=\tilde{i}+1}^{M} d_i n_i}{n_{\tilde{i}}}$.

One feasible IP optimal solution: $[0,\ldots,0,\lfloor X_{\tilde{i}}\rfloor,d_{\tilde{i}+1},\ldots,d_M]$.

$LP-IP\le \tilde{i}(X_{\tilde{i}}-\lfloor X_{\tilde{i}}\rfloor)$

$$\begin{aligned}
& V^{OPT}(I)-V^{BL}(I)\\
\le & val(I;\{E[d_i]\})-V^{BL}(I)\\
= & val(I;\{E[d_i]\})-val(I;\{\lfloor E[d_i]\rfloor\})+val(I;\{\lfloor E[d_i]\rfloor\})-E_{\{d_i\}}[\sum_i(n_i-\delta)\min\{d_i^*,d_i\}]\\
\le & \sum_i(n_i-\delta)+\tilde{i}(X_{\tilde{i}}-\lfloor X_{\tilde{i}}\rfloor)+E_{\{d_i\}}[\sum_i(n_i-\delta)(d_i^*-\min\{d_i^*,d_i\})]\\
\le & \sum_i(n_i-\delta)+\tilde{i}(X_{\tilde{i}}-\lfloor X_{\tilde{i}}\rfloor)+\frac{1}{2}\sum_i(n_i-\delta)\sqrt{Tp_i(1-p_i)}
\end{aligned}$$

$$V^t(l)=\max_{u_i^t\in\{0,1\}}\left\{\sum_{i=1}^{M}p_i\left[V^{t+1}(l-n_iu_i^t)+iu_i^t\right]+p_0V^{t+1}(l)\right\} \tag{18}$$

Always accept the largest group unless the capacity is insufficient.

We consider the problem with one row (stochastic knapsack problem).

The DP (optimal online) policy: $V_t(l-n_i)-V_t(l)+i\ge 0$.

$E[\text{loss}]=V^{\text{off}}-V_\pi^{on}\ge V^{\text{opt}}-V_\pi^{on}$

One sample path. d^r realization of M types.

$V_t(l)=\sum_{i=\hat{i}+1}^{M} r_i d_i^r + r_{\hat{i}}(l-\sum_{i=\hat{i}+1}^{M} d_i^r)$

Static deterministic heuristic policy: accept $i\ge\hat{i}$ if $\bar{d}_{\hat{i}+1}+\ldots+\bar{d}_M<l\le\bar{d}_{\hat{i}}+\ldots+\bar{d}_M$.

Let $V^{\text{OPT}}(I)$ denote the expected value under offline optimal policy (relaxed) during T periods for instance I (capacity, probability distribution).

The revenue loss between the static deterministic heuristic and the optimal is bounded by $C\sqrt{T}$.

Let γ_i, γ_i^0 denote the number of type i accepted and rejected by some heuristic policy, respectively.

$$
\begin{aligned}
\mathrm{OPT}(L, \hat{d}, \gamma): \quad \max \quad & \sum_{i=1}^{M} (n_i - \delta) x_i \\
\text{s.t.} \quad & x_i^0 + x_i = \hat{d}_i, \quad i \in \mathcal{M}, \\
& x_i \geq \gamma_i, \quad i \in \mathcal{M}, \\
& x_i^0 \geq \gamma_i^0, \quad i \in \mathcal{M}, \\
& \sum_{i=1}^{M} n_i x_i \leq L.
\end{aligned}
$$

Heuristic policy: At time t, solve problem (15) with $d_i = d_i^t = (T-t) * p_i$, $L = L^t$. When $x_i \geq 1$ for the request of type i, accept the request.

$d^{[1,T]}$ is the demand realization during $[1, T]$. $\gamma^{[1,t)}$ represents the number of requests rejected and accpeted by some heuristic policy during $[1, t)$.

$OPT(L, d^{[1,T]}, \gamma^{[1,t+1)})$ can be interpreted as the total reward obtained under a virtual policy where we first follow the heuristic policy during $[1, t+1)$ and then from time $t+1$ we follow the optimal solution assuming that we know the future demands.

For one sample path of the requests, the revenue loss can be decomposed into T increments.

$$
\begin{aligned}
& OPT(L, d^{[1,T]}, 0) - OPT(L, d^{[1,T]}, \gamma^{[1,T]}) \\
= & \sum_{t=1}^{T} [OPT(L, d^{[1,T]}, \gamma^{[1,t)}) - OPT(L, d^{[1,T]}, \gamma^{[1,t+1)})] \\
\leq & \sum_{t=1}^{T} (n_M - \delta)
\end{aligned}
$$

Let $L^t = L - \sum_i n_i \gamma_i^{[1,t)}$.

The expected revenue loss can be upper bounded:

$$
\begin{aligned}
& E[OPT(L, d^{[1,T]}, 0) - OPT(L, d^{[1,T]}, \gamma^{[1,T]})] \\
\leq & (n_M - \delta) \sum_{t=1}^{T} P(OPT(L, d^{[1,T]}, \gamma^{[1,t)}) - OPT(L, d^{[1,T]}, \gamma^{[1,t+1)}) > 0) \\
= & (n_M - \delta) \sum_{t=1}^{T} P(OPT(L^t, d^{[t,T]}, 0) - OPT(L^t, d^{[t,T]}, \gamma^{[t,t+1)}) > 0) \\
\leq & (n_M - \delta) \sum_{t=1}^{T} P(x_{it}^{*,t} < 1) \\
= & (n_M - \delta) \sum_{t=T_0}^{T} P(x_{it}^{*,t} < 1) \\
\leq & (n_M - \delta) \max_i \{ \frac{1}{p_i} \}
\end{aligned}
$$

Lemma 6. $OPT(L^1, \hat{d} + d^{[1,t_2)}, \gamma^{[1,t_2)}) = \sum_i (n_i - \delta)\gamma_i^{[1,t_1)} + OPT(L^t, \hat{d} + d^{[t_1,t_2)}, \gamma^{[t_1,t_2)})$

For any optimal solution x^* of $OPT(L^t, \hat{d} + d^{[t_1,t_2)}, \gamma^{[t_1,t_2)})$, $x^* + \gamma^{[1,t_1)}$ is a feasible solution of $OPT(L^1, \hat{d} + d^{[1,t_2)}, \gamma^{[1,t_2)})$. For any optimal solution x^* of $OPT(L^1, \hat{d} + d^{[1,t_2)}, \gamma^{[1,t_2)})$, $x^* - \gamma^{[1,t_1)}$ is a feasible solution of $OPT(L^t, \hat{d} + d^{[t_1,t_2)}, \gamma^{[t_1,t_2)})$ because $x^* - \gamma^{[1,t_1)} \geq \gamma^{[1,t_2)} - \gamma^{[1,t_1)} = \gamma^{[t_1,t_2)}$.

The first inequality results from $E[A] \leq r_M E[\mathbf{1}_{A>0}] = r_M P(A > 0)$.

The first equation follows from Lemma. (Let $t_1 = t_2 = t$, $\hat{d} = d^{[t,T]}$; let $t_1 = t, t_2 = t+1$, $\hat{d} = d^{[t+1,T]}$).

The second equation is as follows. If $x_{i^t}^{*,t} \geq 1$, then $x^{*,t}$ is still feasible for $OPT(L^t, d^{[t,T]}, \gamma^{[t,t+1)})$. (Because the optimal policy)

$x_{i^t}^{*,t}$ is the optimal solution for $\mathrm{OPT}(L^t, d^{[t,T]}, 0)$ at time t.

Let $T - T_0 = \max_i\{\frac{1}{p_i}\}$

For N rows,

$$
\begin{aligned}
OPT(\boldsymbol{L}, \hat{d}, \gamma): \quad \max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(n_i - \delta)x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} + x_{i0} = \hat{d}_i, \quad i \in \mathcal{M}, \\
& \sum_{j=1}^{N} x_{ij} \geq \gamma_i, \quad i \in \mathcal{M}, \\
& x_{i0} \geq \gamma_i^0, \quad i \in \mathcal{M}, \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, \quad j \in \mathcal{N}.
\end{aligned}
$$

5 Conclusion

We study the seating management problem under social distancing requirements. Specifically, we first consider the seat planning with deterministic requests problem. To utilize all seats, we introduce the full and largest patterns. Subsequently, we investigate the seat planning with stochastic requests problem. To tackle this problem, we propose a scenario-based stochastic programming model. Then, we utilize the Benders decomposition method to efficiently obtain a seat plan, which serves as a reference for dynamic seat assignment. Last but not least, to address the seat assignment with dynamic requests, we introduce the SPBA policy by integrating the relaxed dynamic programming and the group-type control allocation.

We conduct several numerical experiments to investigate various aspects of our approach. First, we compare SPBA with three benchmark policies: BPC, BLC, and RDPH. Our proposed policy demonstrates superior and more consistent performance relative to these benchmarks. All policies are assessed against the optimal policy derived from a deterministic model with perfect foresight of request arrivals.

Building upon our policies, we further evaluate the impact of implementing social distancing. By introducing the concept of the threshold of request-volume to characterize situations under which social distancing begins to cause loss to an event, our experiments show that the threshold of request-volume depends mainly on the mean of the group size. This leads us to estimate the threshold of request-volume by the mean of the group size.

Our models and analyses are developed for the social distancing requirement on the physical distance and group size, where we can determine a threshold of occupancy rate for any given event in a venue, and a maximum achievable occupancy rate for all events. Sometimes the government may impose a maximum allowable occupancy rate to tighten the social distancing requirement. This maximum allowable rate is effective for an event if it is lower than the threshold of occupancy rate of the event. Furthermore, the maximum allowable rate becomes redundant if it is higher than the maximum achievable rate for all events. These qualitative insights are stable concerning the tightness of the policy as well as the specific characteristics of various venues.

Future research can be pursued in several directions. First, when seating requests are predetermined, a scattered seat assignment approach can be explored to maximize the distance between adjacent groups when sufficient seating is available. Second, more flexible scenarios could be considered, such as allowing individuals to select seats based on their preferences. Third, research could also investigate scenarios where individuals arrive and leave at different times, adding an additional layer of complexity to the problem.

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