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Social classroom seating assignment problems

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ABSTRACT

University students benefit academically, personally and professionally from an expansion of their in-class social network. To facilitate this, we present a novel and broadly-applicable optimization approach that exposes individuals to as many as possible peers that they do not know. This novel class of "social seating assignment problems" is parameterized by the social network, the physical seating structure and "tie potentials" representing the likelihood for two neighbors to connect. The resulting problem is NP-hard and belongs to assignment problems with an elaborate objective that depends on the relation of both seats and individuals assigned to them. We develop compact integer programming formulations and strengthen them with valid inequalities to improve performance. In parallel, we suggest fast heuristics that are guided by network centrality measures. Finally, we present the necessary modeling techniques to integrate practically relevant instructor preferences and special student needs. Combining the above, we experiment on a set of 320 realistic instances with up to 200 students forming both sparse and dense social networks and for both rectangular and circular classrooms. For sparse or small instances we present optimality gaps under 1.3% within a few minutes, whereas in larger or denser cases the algorithmic performance decreases. We also evaluate our approach from both a quantitative and a qualitative perspective on three actual classes with more than 70 students. A network-analysis-based comparison of a priori and a posteriori networks shows that 40% of opportunities led to new connections, while feedback from both instructor and student is particularly favorable for our method.

1 | INTRODUCTION

Our research examines seating arrangements in classes in a manner that maximizes the potential for new ties among students. Put more simply, our goal is to seat students who do not already know one another closer together in the classroom. Although intuition alone designates the importance of the problem we introduce, let us recall some indicative arguments. A strong social network (SN) is an important factor for students' integration and persistence [44]. Moreover, increased social interaction leads to enhanced knowledge, critical thinking, problem-solving skills [17], and innovation [29]. Factors that facilitate the creation of new ties in class include instructor teaching style, mode of instruction (face-to-face or virtual), student body, and course content. It is also known that physical proximity remains a fundamental way to establish connections [42].

This brings us back to intuition. Though people can work in a variety of positions, most humans these days sit in offices, classrooms, or other types of workplaces. Seating patterns can influence social interactions even outside of the workplace. It is therefore surprising that existing literature, to the best of our

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knowledge, has overlooked the synergies between social and student life while sitting for long hours in a classroom. In fact, the literature remains limited for similar synergies outside the classroom, for example, in professional workshops, team-building events, therapeutic sessions, and social gatherings. As a result, one could argue that our work could be utilized in social seating settings outside of student environments and traditional classrooms.

In more technical terms, we consider two graphs. The first one represents the social network, having one node per student and an edge per pair of students who are already socially acquainted. The second graph represents the seating structure of a classroom, having one node per seat and one edge per pair of neighboring seats, accompanied by a positive weight that resembles the social potential of the seat pair. We associate with these two graphs an integer programming formulation, whose structure is novel, in order to determine the seating plan that maximizes the weighted sum of edges in the classroom graph whose endpoints are assigned to students unknown to each other. This is an assignment problem whose objective is determined by both seats and the students assigned to them. Different formulations of varying strength, accompanied by heuristics, exhibit a remarkable computational performance on instances of realistic size. Our evidence shows that a-priori knowledge of an existing SN can be exploited to create neighborhoods of students who do not know each other yet but can effectively interact. This is supported by a case study including three real classrooms, where students provided qualitative feedback that supports the quantitative results. The transparent nature of the deterministic approach makes the method explainable and trustworthy which we think is of particular importance in an educational setting.

Our work offers an optimization perspective to the literature on education that investigates the impact of seating arrangements on students' performance [35] or engagement [39]. In that sense, it bridges the research gap identified by Hill and Peuker [15]. A parallel gap shows up in the optimization community, which has been studying seating arrangement mostly from a location perspective and without exploiting or enhancing social aspects. Indicatively, Vangerven et al. [48] examine seating in a parliament as an assignment problem that uses a binary seat neighborhood relation to indicate whether communication between two seats is conveniently possible; here, the sole social aspect is whether two members of parliament belong to the same party.

Diverse subset selection is an appealing and impactful optimization problem. Specifically, the "maximum dispersion/diversity problem" (MDP) [20, 21] aims at maximizing the sum of the Euclidean distances among the selected seats. If distances were determined by a social network, that sum could become highly correlated with the objective function we consider. However, MDP overlooks the conditional benefits of an assignment coming from the existing social network. What is encouraging is that the MDP can be handled by exact optimization methods [40], hence its broader applicability in realistic conditions appears more plausible. Our presented model can also be used to solve "social team formation problems" [15, 34] in which students are assigned to teams such that the overall number of potential new ties is maximized, that is, the classroom graph is not taken into account. We could represent teams of given sizes in our model by assigning a uniform pairwise tie potential to all associated seats.

Our contribution is threefold: we advance education research by integrating seating with social networking; we introduce a relevant assignment problem for which integer programming is a valuable modeling and solution method; and we prove its applicability via a comprehensive case study. The details of our contribution are as follows: related literature is explored more thoroughly in Section 2 to both motivate our study and explain its significance. We describe our model along with the notation we used in Section 3, together with an illustrative example. The mathematical formulation(s) are presented in Section 4, while some variations and extensions arising from educational practices are discussed in Section 5. Section 6 describes our heuristics that complement the efficiency of the mathematical models, as documented by the experimentation presented in Section 7. Our case study, which is included in Section 8, provides additional insight into the efficacy of our approach.

2 | RELATED WORK

Finding seating plans that conform to the—potentially strange—particularities of social events has recently gained attention. Indicatively, Lewis and Carroll [24] and Tomić and Urošević [45] propose heuristics for finding seating arrangements for dinners where given individuals or groups (e.g., families) should sit at the same table, with the former also introducing a compact integer programming formulation. They do so by solving an associated graph partitioning problem where the sum of edges within each partition is to be minimized. Although [24] caters for certain seating requirements (i.e., groups seating on the same or on different tables), its focus is on groups rather than individuals and on finding solutions of good quality through a meta-heuristic.

Class seating has motivated two further interesting problems. The primary goal of Berriaud et al. [4] is envy-freeness and stability if individuals have cardinal preferences over where they would sit, that is, people should sit such that no pair wishes to switch their seats and no individual envies the seat of another. Social distancing in classrooms [12] also resembles the need for certain pairs of people not to sit too close. Similarly, exam hall seating problems address strategies to minimize cheating by maximizing the distance between students. Relevant side constraints include ensuring that students enrolled in the same course are not seated next to each other. Metaheuristic approaches have been suggested to solve this problem in the literature (e.g., [1, 9]).

As mentioned in the introduction, our objective is to strengthen social connections by seating individuals who are not acquainted with each other in close proximity. As this could be interesting in non-classroom arrangements, we should mention that seating on both professional and social occasions appears to be a non-trivial task. This explains why an optimization approach has been employed for social distancing in cinemas [41], airplanes [37] and buses [18]. Parliament seating is another relevant problem, as already discussed, that has been tackled—similar to our work—by integer programming and heuristic methods [48].

Our combinatorial problem is a special case of the well-known and notoriously hard quadratic assignment problem (QAP), which has numerous applications including facility location, material handling, and manufacturing (e.g., see [26]). The QAP focuses on the minimization of two types of costs: an assignment cost per pair selected and a "combined" cost occurs only when selected pairs have been assigned simultaneously. In our setting, there are no costs (negative benefits) for individual assignments and we aim at maximizing overall benefit. An early application that results in a QAP variant that is similar to ours is office-employee assignment [14], which minimizes communication efforts between employees assigned to different offices. Our problem focuses on the fact (which it also exploits) that only selected pairs of both students and seats (i.e., employees and offices, respectively) can contribute to enhanced social interaction.

Teachers themselves desire to organize classroom seating for a variety of reasons, including tightening social ties [16]. There is a breadth of papers on the multiple factors influencing how and whether the seating arrangement affects student performance. However, to the best of our knowledge, the impact of seating arrangements in higher education classrooms on the development of students' social networks has not been explored.

The concept of sociograms [25] has been extensively utilized in educational contexts to analyze the social relationships within a classroom via appropriate graphical representations. Visualizations map out social dynamics such as friendships, interactions, social isolates, and group structures, allowing educators to address social and emotional needs effectively. Sociograms are also used to capture changes in social relationships over time, providing a dynamic view of the classroom's social landscape which can be used to understand complex peer relations during childhood [2]. In our study, we complement the diagnostic nature of sociograms, offering a prescriptive approach to enhance classroom social networks.

In the field of STEM education, the application of social network analysis has been used in understanding and improving student interactions and social structures. Pearson et al. [32] have demonstrated how network analysis can uncover patterns of inclusion and exclusion within student cohorts. Additionally, studies have explored various aspects of student relationship networks (see, for example, [6, 7, 11, 27, 38]). The topic also considered important in educational psychology [36]. Similar to our work, all these studies focus on fostering bridging and bonding social capital among students.

In more detail, Levine et al. [23] present a two-phase study examining the effects of classroom seating on test scores and participation. They report that, when students selected their seats, those in the front performed better, but there was no effect on participation. When randomly assigned to seats, students in the front participated more than those in the rear. These results suggest that the relationship between seating position and grades is mediated by self-selection processes, while participation can be influenced by seat location.

As intuitively expected, students who prefer to sit near the front of the room have a higher probability of receiving better grades [3]. Along these lines, Perkins and Wieman [33] investigated a large introductory physics course in which students were randomly assigned to seats and observed that seat location had a noticeable impact on student success in the course, particularly in the top and bottom parts of the grade distribution. Furthermore, Tagliacollo et al. [43] conclude that students' motivation for learning determines their seat choice and this effect explains why seat position is associated with school performance. They suggest that displacing students to a frontal seat position in the classroom to improve learning performance does not necessarily change a student's performance if a lack of motivation persists. Analogous observations are made in [47]. The relationship between students' classroom seating location and their learning outcomes in five marketing classes ($N = 232$) over five years is reported by Pichierri and Guidoin [35]. They examine the moderating role of shyness and nonconformity; results indicate that classroom seating does affect learning performance, however shyness but not nonconformity moderates this association. Less surprisingly, Lacroix and Lacroix [22] conclude that seat location affects grades in larger classrooms, but not in smaller ones, according to their review of previous literature and analysis of student performance in six small economics classes.

Learning is typically associated with engagement and the overall class experience, hence seating could meaningfully be associated not just with performance. Indeed, Shernoff et al. [39] examined the influence of seating location on student engagement, attention, classroom learning experience, and course performance. Results showed lower engagement, attention, and quality of classroom experience when sitting in the back of the classroom. Multilevel models revealed both within-student and between-student effects. Social interaction is no longer neglected: indicatively, Nehyba et al. [30] investigated university students' interaction intensity in pre-service teachers' groups during reflective practice. Results showed more intensive interaction in rows with moderation influenced by the field of study and facilitator involvement. The study highlights the importance of seating arrangements in student interaction.

In a recent study, Bluteau et al. [5] compared the well-being and mental health of 10–12 years old students in fixed and flexible classrooms. The research, based on a quasi-experimental, quantitative design, found that flexible classroom seating positively impacted girls' well-being and mental health, while fixed classroom seating was most beneficial for boys. The impact of classroom seating on student engagement is examined in [13] via wearable physiological sensors. The study, conducted at a high school, found that individual and group seating experiences are associated with perceived engagement and physiologically-based engagement. Students who sit close together are more likely to have similar learning engagement and high physiological synchrony. The findings suggest that flexible seating arrangements can maximize student engagement and suggest intelligent seating choices in the future, something that our work could easily facilitate.

Last, Van den Berg and Cillessen [46] explored the relationship between classroom seating arrangements and peer status using the social relations model. Results show that children who sit closer to the center of the classroom are more liked and popular. Additionally, placing liked peers closer to themselves leads to better likeability and popularity. The findings suggest further research on classroom seating arrangements and peer relationships.

As this literature review shows, further research is needed to examine the potential link between seating arrangements and social network development among higher education students. This could provide valuable insights into how classroom design can promote not only academic success but also social integration and support among students. Moreover, it could possibly be intertwined with the growing interest in optimizing over group centrality metrics [8].

3 | THE SOCIAL SEAT ASSIGNMENT PROBLEM

3.1 | Definition

The social seat assignment problem (SSAP) can be defined as follows. Let $N = (V, E)$ be an undirected graph that represents the student social network. That is, every student is represented by a node $i \in V$ and an edge $\{i, i'\}$ is contained in E if the students $i, i' \in V$ know each other. The physical seating structure is represented by the undirected edge-weighted neighborhood graph $M = (W, F)$: a node $j \in W$ represents a seat and an edge $\{j, j'\} \in F$ is associated with two seats $j, j' \in W$ that are considered to allow direct student interaction. Such seats are typically neighboring seats—horizontal, vertical, or diagonal. For two seats that are connected through an edge $\{j, j'\} \in F$, we are given a tie potential value $p_{j,j'} > 0$, which indicates how likely a new connection is created given that the students assigned to seats j and j' do not know each other. A higher value would be chosen for seats that are close to each other, whereas seats that are too far apart will not even be connected through an edge in M. We assume that a sufficient number of seats is always available to accommodate all students; that is, $|W| \geq |V|$.

A valid seating assignment assigns every student to exactly one seat, such that every seat has at most one student assigned. In such an assignment, the utilized tie potential for neighboring seats $\{j, j'\} \in F$ is $p_{j,j'}$ if two students $i, i' \in V$ are assigned to the seats and they do not know each other (i.e., $\{i, i'\} \notin E$). SSAP asks for a valid seating assignment such that the overall tie potential is maximized. It differs from classical assignment problems in two aspects. First, profit (here, the tie potential value) is achieved by assigning pairs of items (students) rather than by individual assignments. Second, the profit (tie potential) depends on where the two items (students) are assigned to. Note that the maximum number of new ties for a given social network N is given by the number of edges in its complement graph. Let us now examine the relation of SSAP to two problems discussed in the literature.

Recall the definition of QAP from [14]: we are given a set of employees I and a set of offices J, where $|I| = |J|$, together with a measure $c_{i,i'}$ reflecting the affinity between two employees $i, i' \in I$ and the distance $d_{j,j'}$ between two offices $j, j' \in J$. If $f(i)$ denotes the office assigned to employee i, the objective is to minimize the sum of $c_{i,i'} \cdot d_{f(i),f(i')}$. More intuitively, QAP asks to place in geographical proximity pairs of employees who feel close to each other.

Although SSAP serves the opposite purpose, it can be transformed to a special case of QAP as follows. (i) We introduce $|W| - |V|$ isolated "dummy" student nodes in N in order to obtain $|V| = |W|$ and set $I = V$, $J = W$; (ii) we set $c_{i,i'} = -1$ if $\{i, i'\} \in E$ and $c_{i,i'} = 0$ otherwise; (iii) last, we set $d_{j,j'} = p_{j,j'}$ if $\{j, j'\} \in F$ and $d_{j,j'} = 0$ otherwise. One can then verify that minimizing the sum of $c_{i,i'} \cdot d_{f(i),f(i')}$ over all pairs $i, i' \in I$ becomes equivalent to maximizing the sum of $p_{j,j'}$ for all $\{j, j'\} \in F$ such that $f(i) = j$, $f(i') = j'$ (or vice-versa) and $\{i, i'\} \in E$.

The optimal student assignment problem (OSAP) [15] is defined on the graph $N = (V, E)$ used by SSAP to represent the students' social network plus a set of teams T, each having a minimum size and a maximum size; let l_t and u_t denote such size limitations per $t \in T$. OSAP asks for an assignment of students, that is, nodes in V, to the T teams in a way that respects that allowable size per team and minimizes the pairs of neighboring nodes placed in the same team.

OSAP is the special case of SSAP where the neighborhood graph M can be partitioned into a set of cliques T, that is, we have one "clique of seats" per potential student team. Clique $t \in T$ has u_t nodes, while $l_t = 0$. If we set the same tie-potential for all pairs of neighboring seats, that is, $p_{j',j} = 1$ for all $\{j', j\} \in F$, maximizing the sum of tie potentials for SSAP becomes equivalent to minimizing the number of existing intra-team ties (as OSAP aims at). As OSAP is NP-hard [15], so is SSAP. However, for completeness of our exposition, let us provide a formal reduction.

Lemma 1. *SSAP is* NP*-hard.*

Proof. We show a reduction from the variant of the maximum clique problem (MCP) that asks whether a given undirected graph has a complete subgraph (i.e., a clique) of a given size. Formally, an instance of MCP receives as input an undirected graph $G' = (V', E')$ and a positive integer $k \leq |V'|$, asking whether G' has a clique of k nodes.

Given an arbitrary MCP instance, we construct an instance of SSAP as follows. We set the social network N to be the complement graph of MCP's input graph, that is, $N = \overline{G'}$. Also, we set the classroom network M as the union of the complete graph on k nodes and $|V'| - k$ isolated nodes, and set a tie potential $p_{j,j'} = 1$ for each edge $\{j, j'\}$ of M. Observe that G' has a clique of size k if and only if its complement $\overline{G'}$ has an independent set of size k, which is equivalent to SSAP having k students whose seating in the clique of M yields a solution with value $k \cdot (k - 1)/2$.

That is, MCP has a clique of size k if and only if k students can sit together in a "clique" classroom whereas all remaining students are assigned to isolated seats. As MCP is NP-complete [19], so does the version of SSAP asking whether a given instance admits a solution with a given value. The result follows. □

3.2 | An illustrative example

Consider a class with six students ($|V| = 6$) and a classroom with ten seats ($|W| = 10$). Figure 1 illustrates the room layout (left) and its neighborhood network representation (M) including tie potential values from 1 to 3 (right), respectively. Note that M is

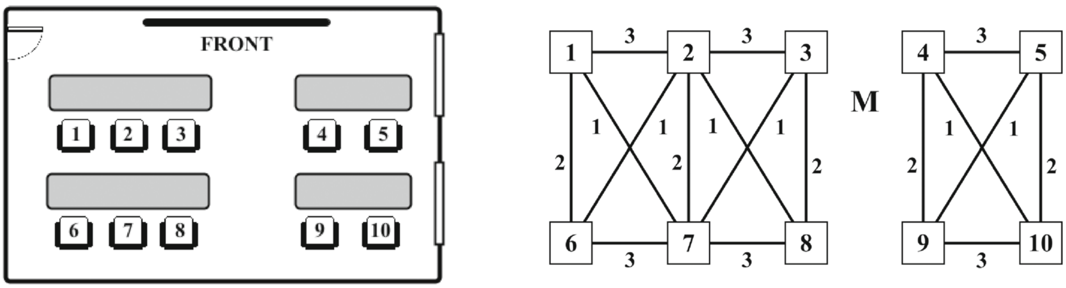

FIGURE 1 | An example classrooms with 10 seats (left) and its neighborhood network representation M with 17 edges ($|F| = 17$) including edge weights ($p_{i,i'} \in \{1, 2, 3\}$) representing tie potentials (right).

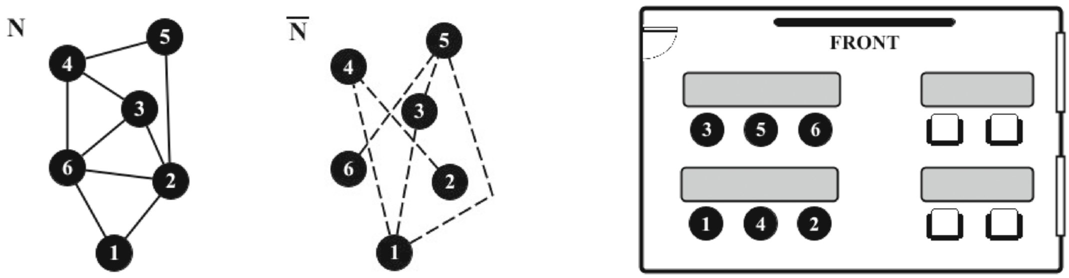

FIGURE 2 | An example social network N (left) with 6 students ($|V| = 6$) and 9 ties ($|E| = 9$), the complement network $\overline{N}$ representing possible new ties (center), and an optimal social seating assignment of the students to seats with an overall potential of 15 (right).

disconnected since across-aisle seats are not considered neighbors in this example. The given tie potentials favor students to be horizontal neighbors (3), followed by vertical neighborhood (2), and diagonally seated neighbors (1). We denote the corresponding sets of edges as F_h, F_v, and F_d, respectively.

The student social network ($N = (\{1, 2, 3, 4, 5, 6\}, \{\{1, 2\}, \{1, 6\}, \{2, 3\}, \{2, 5\}, \{2, 6\}, \{3, 4\}, \{3, 6\}, \{4, 5\}, \{4, 6\}\})$) is depicted in Figure 2 (left). The complement graph of N, that is, $\overline{N} = (V, \{\{i, i'\} \subseteq V : i \neq i' \wedge \{i, i'\} \notin E\})$, has an edge for two students that do not know each other (see Figure 2, center). An optimal seat assignment is shown in Figure 2 (right). Students are only seated at tables in the left part of the classroom to make best use of direct (horizontal) neighborhoods which have the highest tie potential (3). Nevertheless, each student has one vertical neighbor and at least one diagonal neighbor. Using the tie potentials assigned to edges in M, the overall potential is $4 \cdot 3 + 1 \cdot 2 + 1 \cdot 1 = 15$. Note that an equally good assignment can be obtained by swapping assignments in row one and row two. The maximum number of possible new ties for this optimal assignment is 6, namely $\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 4\}, \{3, 5\}$ and $\{5, 6\}$ which corresponds to the theoretical maximum represented by $\overline{N}$.

4 | MATHEMATICAL FORMULATION

Our integer programming (IP) formulations for SSAP use two types of variables. A binary assignment variable $x_{i,j}$ indicates whether student $i \in V$ is assigned to seat $j \in W$ ($x_{i,j} = 1$) or not ($x_{i,j} = 0$). Additionally, we introduce a binary tie potential variable $z_{j,j'}$ for every edge $\{j, j'\} \in F$ that becomes 1 if seats j and j' are assigned to two students that do not know each other, that is, if there is potential for a new tie.

Let us introduce three IP models, in increasing order regarding their tightness.

$$(F_0) \quad \text{Maximize} \sum_{\{j,j'\} \in F} p_{j,j'} \cdot z_{j,j'} \tag{1}$$

$$\text{Subject to} \quad \sum_{j \in W} x_{i,j} = 1 \qquad \forall i \in V, \tag{2}$$

$$\sum_{i \in V} x_{i,j} \leq 1 \qquad \forall j \in W, \tag{3}$$

$$z_{j,j'} + x_{i,j} + x_{i',j'} \leq 2, \quad z_{j,j'} + x_{i,j'} + x_{i',j} \leq 2 \quad \forall \{i, i'\} \in E, \{j, j'\} \in F, \tag{4}$$

$$2 \cdot z_{j,j'} \leq \sum_{i \in V} \left(x_{i,j} + x_{i,j'}\right) \qquad \forall \{j, j'\} \in F, \tag{5}$$

$$x_{i,j} \in \{0, 1\} \qquad \forall i \in V, j \in W, \tag{6}$$

$$z_{j,j'} \in \{0, 1\} \qquad \forall \{j, j'\} \in F. \tag{7}$$

The objective function (1) maximizes the total weight of potential new ties, which is measured by summing over the tie potential variables, multiplied by the pre-specified tie potential values. The assignment constraints (2) assure that every student is assigned to precisely one seat. Inequalities (3) forbid assigning more than one student to each seat. Binary variables $x_{i,j}$ and $z_{j,j'}$ are linked through constraints (4). For two neighboring seats $j, j' \in W$ and two students $i, i' \in V$ who know each other, variable $z_{j,j'}$ is forced to zero if students i and i' are both assigned to seats j and j' in one of the two possible ways. This only covers the case that exactly two connected students are assigned to a pair of neighboring seats. To prevent tie potentials in the case of one or two empty seats, we add inequalities (5). They ensure that the tie variable for two neighboring seats is bounded from above by 0.5 if one seats remains empty and forces it to zero in the case of both seats being empty. The variables are defined in (6) and (7). The overall number of potential new connections can be calculated as $\sum_{\{j,j'\}\in F} z_{j,j'}$.

A stronger version of Inequalities (4) leads to our second formulation.

$$(F_1)\ \text{Maximize} \sum_{\{j,j'\}\in F} p_{j,j'} \cdot z_{j,j'}$$

$$\text{Subject to } (2),(3),(5)\text{–}(7)$$

$$z_{j,j'} + x_{i,j} + x_{i',j'} + x_{i,j'} + x_{i',j} \leq 2 \quad \forall\{i,i'\} \in E, \{j,j'\} \in F. \tag{8}$$

Formulation F_1 can be strengthened further by a dis-aggregated version of Inequalities (5), which also address the case of no student assigned, but for one seat at a time:

$$(F_2)\ \text{Maximize} \sum_{\{j,j'\}\in F} p_{j,j'} \cdot z_{j,j'}$$

$$\text{Subject to } (2),(3),(6)\text{–}(8)$$

$$z_{j,j'} \leq \sum_{i\in V} x_{i,j} \qquad \forall\{j,j'\} \in F. \tag{9}$$

To obtain our last formulation, we propose an alternative version for Equation (5), derived after observing that the sum of all tie potential variables incident with one seat has to be zero if no student has been assigned to the seat:

$$(F_3)\ \text{Maximize} \sum_{\{j,j'\}\in F} p_{j,j'} \cdot z_{j,j'}$$

$$\text{Subject to } (2),(3),(6)\text{–}(8)$$

$$\sum_{\{j,j'\}\in F} z_{j,j'} \leq |\{\{j,j'\} \in F\}| \cdot \sum_{i\in V} x_{i,j} \qquad \forall j \in W. \tag{10}$$

To formally establish a relation among the above formulations, let, for $i \in \{0,1,2,3\}$, $P(F_i)$ denote the polytope defined as the convex hull of integer vectors satisfying formulation F_i.

Lemma 2. *$P(F_2) \subset P(F_1) \subset P(F_0)$ and $P(F_2) \subset P(F_3)$.*

Proof. It becomes easy to see that each inequality (5) is the sum of two inequalities (9), since graph M is undirected. Hence any vector violating (5) must also violate one of the associated (9) and this shows that $P(F_2) \subset P(F_1)$.

Now observe that (8) dominates any of the inequalities (4) in the sense that it has more variables in its left-hand side (it is a "lifted" version of (4)). Therefore, any vector violating any of (4) must also violate (8) and this explains why $P(F_1) \subset P(F_0)$.

Last, (10) for a given $j \in W$ is the sum of (9) for all $j' \in W$ such that $j, j' \in F$. Hence, $P(F_2) \subset P(F_3)$. □

Simple arguments can show that all inequalities in Formulation (F_2) are maximally valid except for inequalities (8), that is, no such inequality can either get a larger coefficient or an additional variable in its left-hand side (written as "≤"), without increasing its right-hand side.

Inequalities (8) are in fact special cases of what could be called clique inequalities for SSAP:

$$z_{j,j'} + \sum_{i\in C} x_{i,j} + \sum_{i\in C} x_{i,j'} \leq 2 \qquad \forall C \in \mathcal{C}(N), \{j,j'\} \in F, \tag{11}$$

where $\mathcal{C}(N) \subseteq 2^V$ is the set of all maximal cliques in N. Note that it reduces to inequality (8) when we limit ourselves to cliques of cardinality 2. However, not all clique inequalities can be added without increasing unreasonably (or even exponentially) the size of the formulation. Even finding a clique whose corresponding inequality (11) is maximally violated by a given vector is NP-hard. Efficient separation is plausible through enumeration only for small instances and cliques of small cardinality.

Although there is symmetry in the problem, it becomes tough to incorporate as it requires detecting the automorphism groups of the underlying graphs. For example, two equally-sized front rows without any back rows are interchangeable. More formally, let $M_1, \ldots, M_k \subset M$ be a collection of maximal connected components of M that are pairwise isomorphic; that is, it is sufficient that $M_{k'} \simeq M_{k'+1} \forall k' \in \{1, \ldots, k-1\}$.

$$\sum_{i\in V} \sum_{j\in W(M_{k'})} x_{i,j} \leq \sum_{i\in V} \sum_{j\in W(M_{k'+1})} x_{i,j} \qquad \forall k' \in \{1, \ldots, k-1\}. \tag{12}$$

Similarly, we have symmetry breaking constraints regarding tie potentials (e.g., as in [28]):

$$\sum_{e\in F(M_{k'})} z_e \leq \sum_{e\in F(M_{k'+1})} z_e \qquad \forall k' \in \{1, \ldots, k-1\}. \tag{13}$$

For two maximally connected components of M, we can perform an isomorphism test considering edges weights as explained by Cordella et al. [10]. However, our experiments showed that the presented algorithms are not able to take notable advantage of these inequalities, possibly due to the powerful solver-internal symmetry-breaking techniques. Nevertheless, we mention them because they could be useful for related models and methods in future research.

5 | PRACTICAL MODEL EXTENSIONS

When implementing our model in a real-world classroom, it is very important to incorporate students and instructor preferences and needs. Additional side requirements can arise from the spatial table arrangement, course content, teaching style, and student special needs. They can notably complicate the problem introduced above. In the following, we provide model-based techniques that allow us to respect practical constraints and preferences in order to ensure a positive classroom experience.

5.1 | Seat, zone and neighbor pre-assignment

If a student $i \in V$ is required to sit on a seat, say $j \in W$, then we can enforce this assignment by adding the variable fixing constraint $x_{i,j} = 1$. In the opposite case that student i must not sit on seat j, we add $x_{i,j} = 0$ instead. Similarly, a seat $j \in W$ can be forced to be used (stay empty) by adding the constraint $\sum_{i \in V} x_{i,j} = 1$ ($\sum_{i \in V} x_{i,j} = 0$).

More generally, a student might need to sit in the front due to vision or hearing problems. Moreover, students might be restricted to certain seats due to physical accessibility limitations. Let $Z \subset W$ be a zone represented by an arbitrary subset of seats that student i has to be assigned to. Then adding the constraint $\sum_{j \in Z} x_{i,j} = 1$ ensures that no seat outside of Z is selected for student i. Note that if $|Z| = 1$, then this model extension corresponds to fixing a seat for the student, as described above.

A student may require support from a classmate who sits nearby. Enforcing two students $i, i' \in V$ to sit in the same neighborhood can be achieved by adding the following constraints to the formulation:

$$x_{i,j} \leq \sum_{j' \in N[j]} x_{i',j'} \qquad \forall j \in W.$$

Here, $N[j]$ denotes the set of neighbors of j in M and can be substituted by an arbitrary set of suitable instructor-defined seats.

5.2 | Student density

Similar to the seating preferences described above, instructors might want to ensure a certain minimum number of students assigned to a classroom area. Let $u \geq 1$ be the desired number of occupied seats for a specific subset of seats $W' \subset W$. Then we can ensure this minimum occupation in a seating assignment via

$$\sum_{i \in V} \sum_{j \in W'} x_{i,j} \geq u.$$

Note that these inequalities can be used to model lower bounds on the number of students in teams in the OSAP [15]. Finally, we can also limit the student density for subsets of seats from above through similar constraints.

5.3 | Neighbor aversion

When incompatibility between two students is known, then we want our model to avoid seating them together. Let $i, i' \in V$ be the two students that need to sit apart. Then our formulations can be extended by the following constraint to account for this requirement.

$$x_{i,j} + x_{i',j} + \sum_{j' \in N[j]} x_{i,j'} + \sum_{j' \in N[j]} x_{i',j'} \leq 1 \qquad \forall j \in W.$$

5.4 | Student isolation

A student might prefer to not having neighbors in order to minimize potential distraction or due to mental health reasons (e.g., ADHD). Our model can account for such a requirement for student i by adding the following constraints to any of the formulations.

$$\sum_{i' \in V} x_{i',j'} \leq |V| \cdot (1 - x_{i,j}) \qquad \forall j \in W, j' \in N[j].$$

Here, the neighboring seats of j are denoted by $N[j]$. Note that the addition of such a constraint tends to increase the number of seats that must be empty which can cause infeasibility to the model. However, we can guarantee feasibility if the following condition holds for a specific number of isolated students h: $|W| \geq (|V| - h) + h \cdot (1 + \overline{d_M}) = |V| + h \cdot \overline{d_M}$, where $\overline{d_M}$ is the maximum node degree in M.

5.5 | Neighborhood ties enforcement

Students might appreciate both having some neighbors that they already know and being exposed to peers that they do not know. Although this feature might come at the cost of losing some potential for new ties, it can address a student need, as revealed by our case study (Section 8, critical comment 7). To ensure that each student has at least k familiar neighbors, we can add the following constraints.

$$\sum_{\{j,j'\} \in F} (1 - z_{j,j'}) \geq k \cdot \sum_{i \in V} x_{i,j} \qquad \forall j \in W.$$

These additional requirements can also be adjusted to student-dependent needs; for example, use $k = 1$ for student $i \in V$ and $k = 2$ for student $i' \in V$. To achieve this, we simply exchange the right-hand side summation by corresponding student assignment variable $x_{i,j}$. Note that corresponding requirements may turn out to be infeasible; for example, when k exceeds the number of ties of student i. In a similar manner, we can also enforce a minimum number of unacquainted neighbors by omitting the constant on the left-hand side.

5.6 | Spatial preferences

Our model does not account for inconveniences that could come with certain seats. An optimal solution might place students in the back of the classroom although an equally good configuration exists that places them in the front. Preferred seats can be defined based on several aspects, including room lighting, board view, room acoustics and furniture. Let us assume that seats $W' \subset W$ are preferred seats. To model favor for seats in W', we substitute objective (1) by the following alternative objective.

$$\sum_{\{j,j'\} \in F} p_{j,j'} \cdot z_{j,j'} + \frac{\phi}{|W'| + 1} \cdot \sum_{i \in V} \sum_{j \in W'} x_{i,j}.$$

The parameter $\phi \geq 1$ is used to specify the importance of preferred seats. When using $\phi = 1$ then the preferences will not override the objective criterion of overall new tie potential, but they will be considered when multiple optimal solutions exist. An optimal assignment that uses the maximum number of seats in W' will be returned. The effect is illustrated for the example class with $W' = \{1, 2, 3, 4, 5\}$ and $\phi = 6$ in Figure 3 (left). Since $\phi > 1$, the overall potential reduces from 15 to 12.

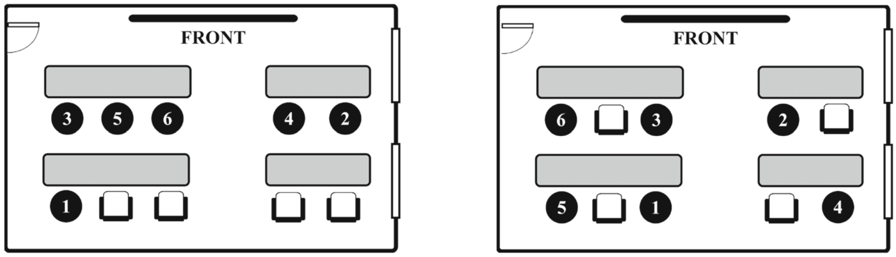

FIGURE 3 | Optimal seat assignments with preferred seating on seats 1–5 (left) and when avoiding conflicts (right).

5.7 | Avoiding conflicts

Our formulations aim at maximizing the opportunities for students to get to know their classmates. A related but generally weaker approach is to minimize the number of already connected students that sit together. This could be useful when, for example, seeking a seating plan for exams. Students who are familiar with each other are more likely to recognize those who are knowledgeable. The level of familiarity among students may make them tempted to "cheat" by viewing the answers of knowledgeable people during in-person exams. This strategy tends to create gaps between students and can be useful when the student social network is dense. Let $q_{j,j'}$ be a penalty value for students that know each other and sit on seats $j, j' \in W$. Then formulation (F') can be used to model this situation.

$$(F') \quad \text{Minimize} \sum_{\{j,j'\}\in F} q_{j,j'} \cdot z_{j,j'} \tag{14}$$

$$\text{Subject to} \quad (2), (3), (6), (7)$$

$$z_{j,j'} \geq x_{i,j} + x_{i',j'} + x_{i,j'} + x_{i',j} - 1 \qquad \forall \{i,i'\} \in E, \{j,j'\} \in F. \tag{15}$$

Inequality (15) will force the reinterpreted conflict variables $z_{j,j'}$ to take the value of 1 if connected students i and i' are both assigned to seats j and j' in any of the two possible ways. Note that the value of the right-hand side is at most 1. Figure 3 (right) shows an optimal assignment with an overall penalty of zero for the example class ($q_{j,j'} = p_{j,j'} \forall \{j,j'\} \in F$). The overall potential is 5 which is lower than the previously obtained value of 15.

6 | HEURISTIC ASSIGNMENT STRATEGIES

In addition to the exact methods presented in Section 4, we developed heuristics to find optimized seating arrangements quickly. Although problem specific heuristics have been developed for a wide range of assignment problems (e.g., [48]), the particular structure of SSAP requires special attention. Hence, we propose approaches that integrate information from both network layers, the social network and the classroom neighborhood network.

Our heuristic methods assign individuals to seats in an iterative fashion. They differ in the sequence in which the students are assigned using various dynamic priority rules. Finally, we apply an efficient local search that explores two polynomial neighborhoods to further optimize the assignment. For a (partial) assignment of students $V' \subseteq V$ to seats $W' \subseteq W$, let $W(i) \in W'$ denote the seat that student $i \in V'$ is assigned to, and $V(j) \in V'$ the student on seat $j \in W'$. During the assignment process, we calculate a tie potential score $\sigma(i,j)$ for an unseated student $i \notin V'$ and an empty seat $j \notin W'$ as

$$\sigma(i,j) = \sum_{\{j,j'\}\in F:\, j'\in W' \wedge \{i,V(j')\}\notin E} p_{j,j'}. \tag{16}$$

The σ-score takes into account potentials of neighboring seats that are occupied by unrelated students. There may be multiple assignment options with identical score. Therefore, we break ties by calculating the priority $\pi_{i,j}$ of assignment $(i,j) \in \tilde{A} = V \setminus V' \times W \setminus W'$ via one of the following rules.

(A) Lexicographical first student:	$\pi_{i,j} = -i$
(B) High-degree seat:	$\pi_{i,j} = d_M(j)$
(D) Low-degree student on high-degree seat:	$\pi_{i,j} = d_M(j)/d_N(i)$
(E) High-centrality seat:	$\pi_{i,j} = c_M(j)$
(F) High-degree student:	$\pi_{i,j} = d_N(i)$
(G) Low-degree seat:	$\pi_{i,j} = -d_M(j)$
(I) High-centrality seat and student:	$\pi_{i,j} = c_M(j) + c_N(i)$

Note that rules B-I are all based on network centrality, measured either by node degree centrality (d) or node closeness centrality (c) in the corresponding network. We recall that the latter, node closeness centrality (c), is calculated as the reciprocal of the average distance to all other nodes in the network, measured by the shortest path lengths. We also experimented with using closeness centrality instead of degree centrality in (F) but did not observe improved results although computation times were longer. We also experimented with inverse versions of these rules which led to inferior results.

An assignment in $\tilde{A}$ with the highest priority is implemented in each iteration. Note that all rules are based on the structure of the dynamic social and classroom networks. Algorithm 1 formally describes the constructive technique.

To potentially improve the obtained assignments, we developed an iterated local search procedure that is applied to the constructed assignments. We explore two types of neighborhoods. First, we consider single-reassignment (MOVE) in a best-first fashion. That is, for each student i and each open seat j, we evaluate the benefit of reassigning i to j. To quantify the latter, we calculate the objective function value of the modified seating assignment (after assigning student j to available seat j) and subtract the initial objective function value (j remains assigned to its original seat). If the resulting value is greater than zero then we identified a potential improvement and we discard the (non-beneficial) student relocation otherwise. Among all potential improvements, we choose the move with the highest

ALGORITHM 1 | Heuristic seating assignment.

Input: Social network N, classroom network M, priority strategy X$\in$ {A-I}

Output: Seating assignment

$V' \leftarrow \emptyset, W' \leftarrow \emptyset, A^* \leftarrow \emptyset$
while $|V'|<|V|$ **do** ▷ Iteratively assign students to seats
  $A \leftarrow (V \setminus V') \times (W \setminus W')$
  for all $(i,j) \in A$ **do** ▷ Update scores for potential assignments
    $\sigma(i,j) \leftarrow \sum_{\{j,j'\}\in F} p_{j,j'}$
  $\tilde{A} \leftarrow \{(i,j) \in A : \sigma_{i,j} = \max_{(i',j')\in A} \sigma_{i',j'}\}$ ▷ Select max-score assignments
  $(i^*, j^*) \leftarrow \mathrm{argmax}_{(i,j)\in\tilde{A}} \pi_{i,j}$ ▷ Select next testing opportunity
  $A^* \leftarrow A^* \cup \{(i^*, j^*)\}$ ▷ Add assignment
  $V' \leftarrow V' \cup \{i^*\}$ ▷ Update set of assigned students
  $W' \leftarrow W' \cup \{j^*\}$ ▷ Update set of occupied seats
return A^* ▷ Return seating assignment

benefit and repeat this MOVE search until no improvement can be identified. Afterwards, we analogously search for a best way to swap two seated individuals (SWAP). We consider each pair of two students i and i' that are assigned to seats, say j and j', respectively. A potential improvement is the reassignment of student i to seat j' and student i' to seat j such that the overall objective function value increases. Through experiments, we found out that the best result can be obtained when applying MOVE first, followed by SWAP. In the case where an improvement was found, we repeat these two search steps.

7 | COMPUTATIONAL ANALYSIS

In the following, we provide an empirical evaluation of our model and the developed methods. We implemented all procedures using Python (v3.12) on a win-x64 machine with an Intel i7-1365U processor (10 cores) and 32 GB of RAM. Linear programs and integer linear programs were solved using Gurobi[i] (Version 11.0.1) using the Python interface gurobipy[ii].

7.1 | Test instances

We evaluate our exact and heuristic methods on a set of realistic test instances. Therefore, we generate social networks of various sizes by randomly extracting subnetworks from the real social network of the industrial engineering program at Cal Poly. This data was collected using an online survey system that students could access via computer and mobile phone [31]. Students were presented with the names of peers from each year in the program where they were asked to indicate "who they know." We did incentivize participation in the social network surveys through coffee shop gift card raffles that took place in class after the assigned survey class time. From our experience, the engagement of the instructors themselves in person are extremely important for high response rates [15]. In addition, instructors sent email reminders to students who did not participate. Finally, we symmetrized the network assuming that one indication for a tie is sufficient. The class social networks used in our case study (Section 8) are subnetworks of this network. The overall network with 272 student nodes and 3315 ties is depicted in Figure 4. We select student sets of six cardinalities (10, 25, 50, 100, 150, 200) using two different random seeds. Moreover, we consider two density scenarios: the original set of induced connections between selected individuals and an augmented network in which we randomly insert four times the number of ties of the sparse case. The detailed social network data can be found in Table 1.

We use two types of classroom layouts: rectangular and circular. Rectangular classrooms are parameterized by the row block sizes $R = [R_1, \dots]$ and the column block sizes $C = [C_1, \dots]$. Circular layouts use parameters for the number of rows (R), the number of circular fragments (F), and the number of seats on the innermost ring (I). Circular rows past the innermost ring get one additional seat per new row. For example, if the first row has 2 seats, then the second row has 3 seats, the third row has 4 seats and so on. The schemes are illustrated in Figure 5 for a rectangular $[R, C] = [[1,3],[2,4,2]]$ configuration and a circular $[R, F, I] = [3,3,2]$ configuration. Tie potentials are as follows: 3 (horizontal), 2 (vertical), 1 (diagonal). Note that circular classrooms do not have edges of weight 2.

The detailed classroom networks are described in Table 2. Here, F_h, F_v and F_d denote the sets of horizontal, vertical and diagonal neighbor edges, respectively (as explained in Section 3.2).

For our experiments, we pair each of the 24 random social networks with each of the 44 classrooms if the following two properties are satisfied: the number of students must not exceed the number of seats ($|V| \leq |W|$) and the student to seat ratio is at least 1/3 ($|V|/|W| \geq 1/3$). This results in 320 test instances[iii].

7.2 | Formulation strengths

We conduct a computational evaluation of formulations (F_0)–(F_3). To this end, we compare the strengths of the LP relaxations of all formulations. The aggregated results for all 320 test instances are given in Table 3. We consider average solution time in seconds ($\bar{t}$), maximal solution time ($t_{\max}$), average optimality gap ($\bar{\Delta}$), and maximal optimality gap ($\Delta_{\max}$). Note that we were able to solve all root node LPs with any formulation. The optimality gap is calculated as $100 \cdot (ub - lb)/lb$ where ub is the objective value of the LP solution and lb is the objective value of the best assignment that we found; that is, the best lower bound found by our exact method (see Section 7.4). We also report average gaps for circular and rectangular classroom layouts ($\overline{\Delta_{\bigcirc}}$ and $\overline{\Delta_{\square}}$), showing a very similar trend. Sparse social networks seem to lead to easier problems than dense social networks ($\overline{\Delta_S}$ and $\overline{\Delta_D}$). Note that we could not observe any impact of Inequalities 11 for our instances when solving the root node LP, and we therefore decided to omit their separation.

We observe that formulation (F_2) clearly outperforms the other formulations. This significant empirical benefit supports our theoretical relationships shown in Section 4. The average gap (6.1%) is significantly lower than for the other formulations. However, the LP relaxation of formulation (F_1) can be solved slightly faster than the one for formulation (F_2). On average, solving an LP instance takes 4.7 s, compared to 5.1, 6.3, and 7.0 respectively.

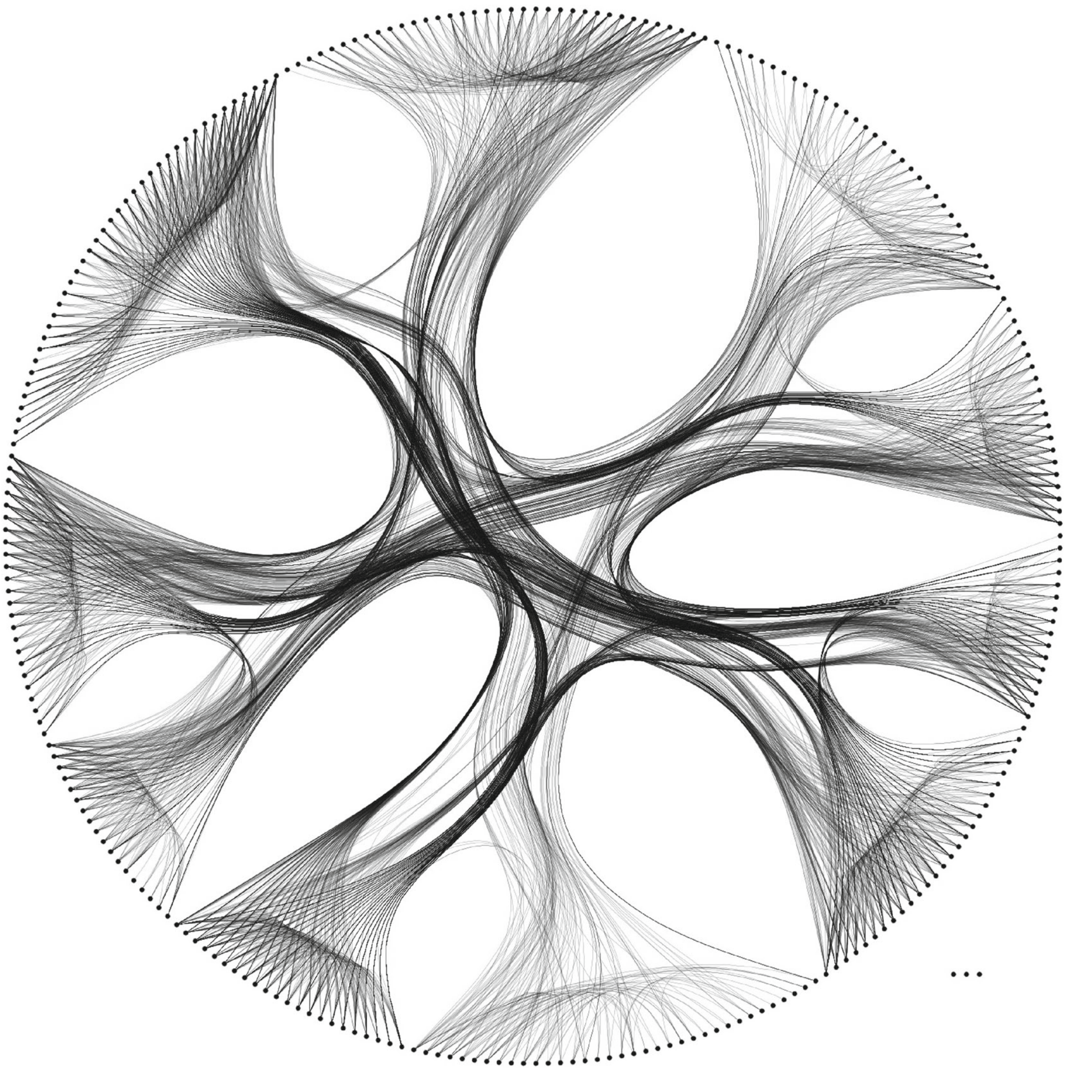

FIGURE 4 | The social network of the 272 students in the industrial engineering program connected by 3315 ties.

TABLE 1 | The 24 realistic social networks with random seed s, density scenario d (S = sparse, D = dense), number of student nodes $|V|$, and number of ties $|E|$.

#	s	d	$\|V\|$	$\|E\|$	#	s	d	$\|V\|$	$\|E\|$	#	s	d	$\|V\|$	$\|E\|$
1	1	S	10	6	9	5	S	50	100	17	9	S	150	945
2	1	D	10	36	10	5	D	50	600	18	9	D	150	5670
3	2	S	10	4	11	6	S	50	142	19	10	S	150	998
4	2	D	10	24	12	6	D	50	852	20	10	D	150	5988
5	3	S	25	24	13	7	S	100	446	21	11	S	200	1728
6	3	D	25	144	14	7	D	100	2676	22	11	D	200	10368
7	4	S	25	39	15	8	S	100	433	23	12	S	200	1846
8	4	D	25	234	16	8	D	100	2598	24	12	D	200	11076

These observations hold for both classes of neighborhood networks, circular and rectangular. Moreover, the social network density does not seem to impact these relationships between the formulations. Nevertheless, it can be seen that dense social networks lead to an average optimality gap that is almost four times the one for sparse instances.

7.3 | Heuristic performance

We evaluate the different heuristic strategies based on their performance in all test instances. To better understand the quality of the best solutions found, we illustrate the achieved average optimality gaps in Figure 6 (left). Clearly, the inclusion of

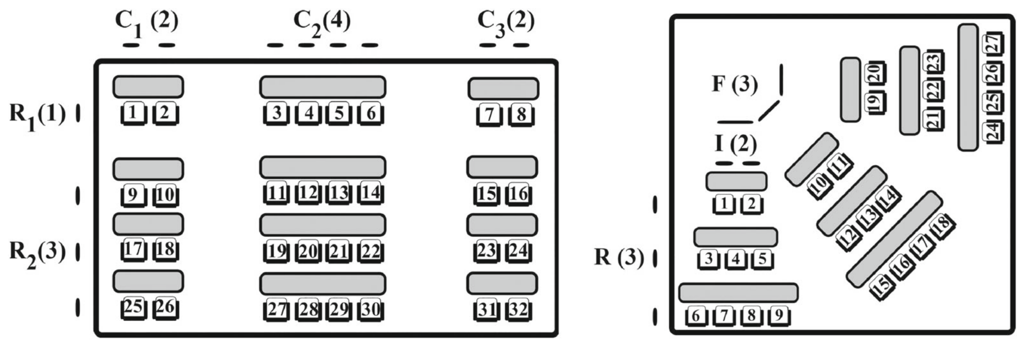

FIGURE 5 | Illustration of the rectangular (left) and the circular classroom (right) scheme with their parameters using classrooms with 32 seats and 27 seats, respectively.

TABLE 2 | The 20 rectangular and 24 circular classroom networks with scheme (row block sizes R and column block sizes C); number of seats and neighbors.

Rectangular layout

#	R	C	$\|W\|$	$\|F\|$	$\|F_h\|$	$\|F_v\|$	$\|F_d\|$
1	2, 2	2, 2	16	24	8	8	8
2	2, 2, 2	2, 2, 2	36	54	18	18	18
3	2, 2, 2, 2	2, 2, 2, 2	64	96	32	32	32
4	2, 2, 2, 2, 2	2, 2, 2, 2, 2	100	150	50	50	50
5	2, 2, 2, 2, 2, 2	2, 2, 2, 2, 2, 2	144	216	72	72	72
6	2, 2	5	20	42	16	10	16
7	3, 3	5	30	76	24	20	32
8	4, 4	4, 4	64	168	48	48	72
9	2, 4, 2	2, 4, 2	64	130	40	40	50
10	3, 6, 3	3, 6, 3	144	378	108	108	162
11	4, 7, 4	4, 7, 4	225	648	180	180	288
12	5, 5, 5, 5	5, 5	200	576	160	160	256
13	8	8	64	210	56	56	98
14	10	10	100	342	90	90	162
15	12	12	144	506	132	132	242
16	14	14	196	702	182	182	338
17	6	2, 2, 2	36	78	18	30	30
18	10	2, 2, 2, 2, 2	100	230	50	90	90
19	14	2, 2, 2, 2, 2, 2, 2	196	462	98	182	182
20	18	2, 2, 2, 2, 2, 2, 2, 2, 2	324	774	162	306	306

Circular layout

#	R	F	I	$\|W\|$	$\|F\|$	$\|F_h\|$	$\|F_d\|$
21	1	5	3	18	37	15	22
22	1	15	3	48	107	45	62
23	1	30	3	93	212	90	122
24	2	2	3	18	32	12	20
25	3	3	3	36	69	27	42
26	4	1	3	24	36	12	24
27	4	5	3	72	148	60	88
28	4	10	3	132	288	120	168
29	1	5	5	35	82	30	52
30	1	15	5	85	212	80	132
31	1	30	5	160	407	155	252
32	2	2	5	40	86	30	56
33	3	3	5	75	168	60	108
34	4	1	5	60	120	40	80
35	4	5	5	140	328	120	208
36	4	10	5	240	588	220	368
37	1	5	7	56	139	49	90
38	1	15	7	126	329	119	210
39	1	30	7	231	614	224	390
40	2	2	7	70	164	56	108
41	3	3	7	126	303	105	198
42	4	1	7	112	252	84	168
43	4	5	7	224	556	196	360
44	4	10	7	364	936	336	600

local search procedures improves the results. It can be seen that the various strategies collectively help to obtain an average optimality gap of 4.2% versus 5.4% without local search. We note that the maximal gap is still 85.1% and that for 36 instances (11.3%) the gap is over 10.0%. Overall, both local search techniques (MOVE and SWAP) were effective. Improvements were mostly found during the first two iterations of the two local searches. Figure 6 (right) shows the relative number of best assignments found by a strategy and the optimal solutions found (including local search). We see that strategies F and I find more best solutions than the other strategies (58.8% and 55.6%). We could not detect any dominance, that is, none of the strategies consistently produced equally good or better solutions than any other single strategy. Finally, we suggest running all strategies to obtain optimal solutions in 34.1% of the instances.

TABLE 3 | Comparison of LP relaxation strengths and solution times (in seconds) for all formulations ((F_0)–(F_3)) using the 320 sparse/dense and rectangular/circular test instances.

Formulation	$\bar{t}$	t_{max}	$\overline{\Delta_S}$	$\overline{\Delta_D}$	$\overline{\Delta_\bigcirc}$	$\overline{\Delta_\square}$	$\overline{\Delta}$	Δ_{max}
(F_0)	7.0	89.1	14.4	22.7	20.6	16.3	18.6	169.6
(F_1)	4.7	75.4	14.4	22.7	20.6	16.3	18.6	169.6
(F_2)	5.1	70.9	2.5	9.6	5.3	6.9	6.1	108.7
(F_3)	6.3	170.7	35.0	45.1	43.4	36.4	40.1	227.7

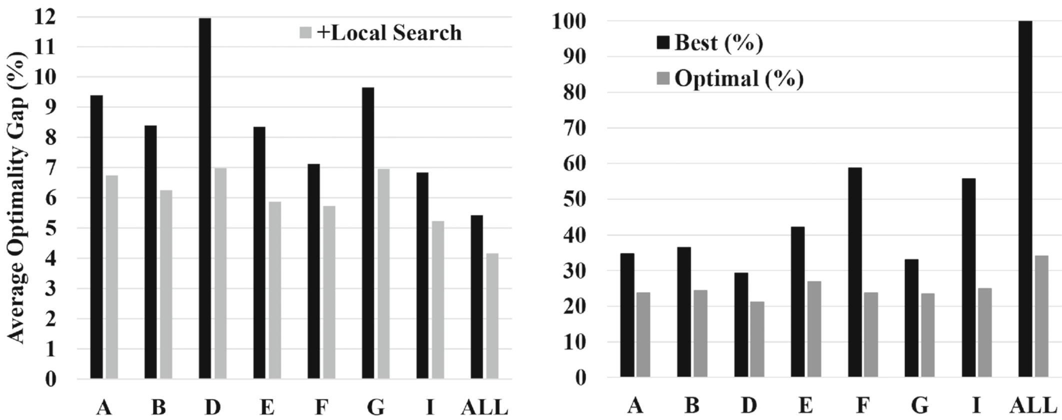

FIGURE 6 | The achieved optimality gaps by the heuristic construction strategies and after the local search procedures (left); the relative number of best and optimal solutions found including local search (right).

TABLE 4 | Aggregated results for instances with sparse and dense social networks, circular and rectangular classroom layouts.

M	N	#	#opt	$\overline{\Delta}$	$\overline{\Delta'}$	Δ_{max}	$\bar{t}$
CIRCULAR	SPARSE	84	77	0.07	0.80	1.26	267.3
CIRCULAR	DENSE	84	30	6.22	9.67	85.04	2008.2
RECTANGULAR	SPARSE	76	71	0.04	0.60	0.78	373.5
RECTANGULAR	DENSE	76	28	5.62	8.90	62.88	1986.9
*	SPARSE	160	148	0.05	0.72	1.26	317.7
*	DENSE	160	58	5.93	9.31	85.04	1998.1
CIRCULAR	*	168	107	3.14	8.65	85.04	1137.7
RECTANGULAR	*	152	99	2.83	8.12	62.88	1180.2
*	*	320	206	2.99	8.40	85.04	1157.9

7.4 | Overall results

In the following, we report results on the efficacy of our exact solution method. We ran a branch-and-cut method based on formulation (F_2) with a time limit of 3600 s. Gurobi cuts and heuristics were enabled. The aggregated results for various instance categories are shown in Table 4. We differentiate between instances with sparse and dense social networks, and classrooms with circular (CIRCULAR) and rectangular (RECTANGULAR) layout. The number of instances is given in column #, followed by the number of instances that we could solve to optimality (#opt). The overall average optimality gap and the average optimality gap for unsolved instances are given in columns $\overline{\Delta}$ and $\overline{\Delta'}$. Finally, we report the maximal optimality gap Δ_{max} and the average runtime in seconds ($\bar{t}$). For the detailed instance results we refer to Appendix B (Tables B1 and B2).

We were able to solve 206 (64%) of the 320 instances to optimality. The average optimality gap over all instances was 2.99% and 8.40% for unsolved ones. The maximum gaps of 85.04% and 63.88% were outliers. The average solution time for solved instances was 416.6 s. In experiments with larger time limits (10800 s) for some of the open instances, we observed that we were not able to close optimality gaps. We already saw notable tailing off with respect to the number of optimally solved instances when approaching the 3600 s time limit. Only 17 gaps could be closed within 600 and 3600 s. For 74 of the open instances, we could not prove optimality within 3600 s. Moreover, the solver ran out of memory during the branch-and-bound procedure for 48 of the instances with 100 or more students.

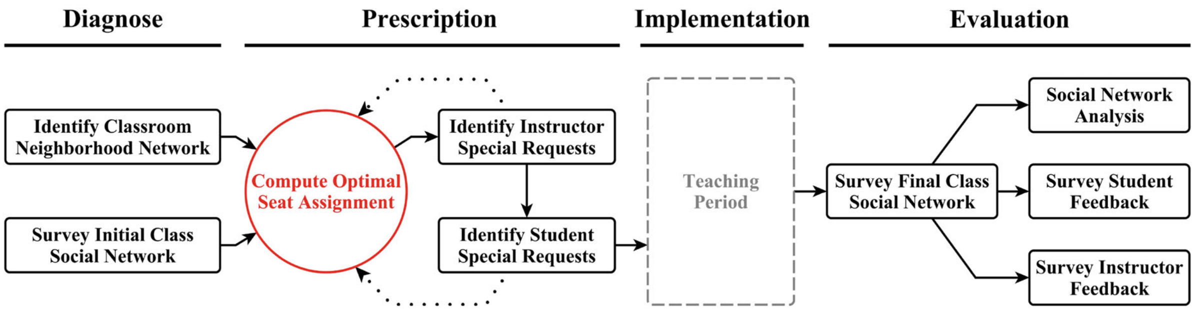

FIGURE 7 | The overall process for the in-class seating assignment experiment.

Our methods were able to solve all 20 small-classroom problems ($|W| < 50$) to optimality within an average of 15 seconds. We could solve 55 of 72 (76.4%) medium-sized classroom instances ($50 \leq |W| \leq 150$), leaving an average optimality gap of 16.5% for unsolved problems. 131 of 228 (57.5%) large-classroom instances ($|W| > 150$) were solved, and optimized to an average optimality gap of 7.0%, otherwise. Finally, we consider spatial ratio as a more important factor for instance hardness compared to classroom size, at least for our test scenarios. There was no significant difference in resolution performance between circular and rectangular instances. The density of the SN did have an impact on the instance hardness. Most sparse problems could be solved (92.5%) but the majority of dense problems remain unsolved (63.8%). This is also reflected in the average gaps for unsolved instances where the dense case (9.31%) is about 13 times higher than the sparse case (0.72%).

The SN density does not only have an impact on the hardness of the instances. Obviously, it can also affect the optimal overall tie potential for a given classroom. To better understand this influence, we also solved all instances to optimality using empty social networks. It turned out that the potential decreased by 5.2% for the 160 dense social network instances whereas the 160 realistic sparse networks did not hinder achieving full potential. Overall, adding ties caused a potential reduction of 2.6%.

Another metric that is directly related to the solvability of an instance is the spatial ratio ρ; that is, the number of students per seat, that is, $\rho = |N|/|M|$. We observe that our formulations suffer from an increased seat availability. The optimality gap tends to increase for both root node relaxation and the final result as ρ decreases. We could solve 12 out of 16 instances (75.0%) with $\rho = 1$ without branching whereas from 228 instances with $\rho < 0.75$, this occurred only 25 times (11.0%). A similar effect could be observed when considering the final optimality gaps.

8 | CASE STUDY

We conducted a multi-week case study in which we assigned seats optimally. Our goal for this real-world experiment is to demonstrate the practicability and effectiveness of our approach. In the following, we describe the experimentation process and setup, followed by a comprehensive quantitative, qualitative and visual analysis of both input data and results.

8.1 | Experiment process and setup

We designed a process that can be applied to periods of various lengths during an academic term (e.g., a semester or trimester), as illustrated in Figure 7. Prior to the actual seating experiment phase, instructors need to identify both, the initial student social network (N) and the classroom network (M). These diagnostic tasks can be performed in parallel. We suggest reserving at least one week for the social network survey. Details for how to conduct the latter can, for example, be found in [15]. In the prescriptive phase, an optimal seating assignment is computed using the integer program suggested in Section 4. The first session in which students sit in their assigned seats, instructors should identify special needs and integrate them into the optimization model using the techniques described in Section 5. An updated seating plan is then used in subsequent class meetings during the teaching period. After the experiment, the class social network is surveyed again, followed by an analysis. At the same time, the instructor's feedback and the students' comments should be collected to detect potential improvements.

We selected three core courses in the undergraduate industrial engineering program in which we assigned seats. All courses were taught by different instructors in face-to-face mode. We only assigned seats in lecture meetings and excluded lab sessions. The experiment period was from week 4 to week 8 (5 out of the overall 10 weeks). For each class, there were two 80-minute lecture meetings per week, resulting in a total of 800 minutes that the students sat in the optimized arrangement. These three classes have to be taken in sequence through prerequisite requirements (A→B→C). However, we experimented in parallel since they were all offered in the winter quarter of 2023. Herewith, we minimize interference between the experiment social networks. There was no student overlap between the experiment classes.

We extracted the in-class student social networks from the program SN survey data, as explained in Section 7.1. In our approach, we prioritize seating two students together when we are certain that they have not yet formed a connection, rather than relying on the uncertain potential of a new tie. Additionally, we have chosen to include ties that were surveyed at the start of the term in the final social network, as we do not anticipate a significant forgetting effect over the span of one quarter.

We posted the seating plan at each door of the experiment rooms. Student first names and last name initials were printed directly on the assigned table segment. An example (Class B, Room R_1) is shown in Appendix A (Figure A1).

Table 5 shows the classes (A–C) that we included in our case study, including the student population, survey response rate and room details. On average, social network survey response rates at the beginning of the term (RR_0) were 97.3%, and 92.9% at the end

TABLE 5 | The courses, social network (SN) survey, and classroom data for experiment classes (A–C).

Course				SN survey		Classroom					
#	Code	Name	$\|V\|$	RR_0	RR_1	#	$\|W\|$	$\|F_H\|$	$\|F_V\|$	$\|F_D\|$	Method
A	IME 301	Operations Research I	27	96.2	92.3	R_1	32	20	24	30	OPT
B	IME 305	Operations Research II	24	95.8	95.8	R_1	32	20	24	30	OPT
C	IME 420	Simulation	21	100.0	90.5	R_2	30	20	15	20	OPT
All			72	97.3	92.9						

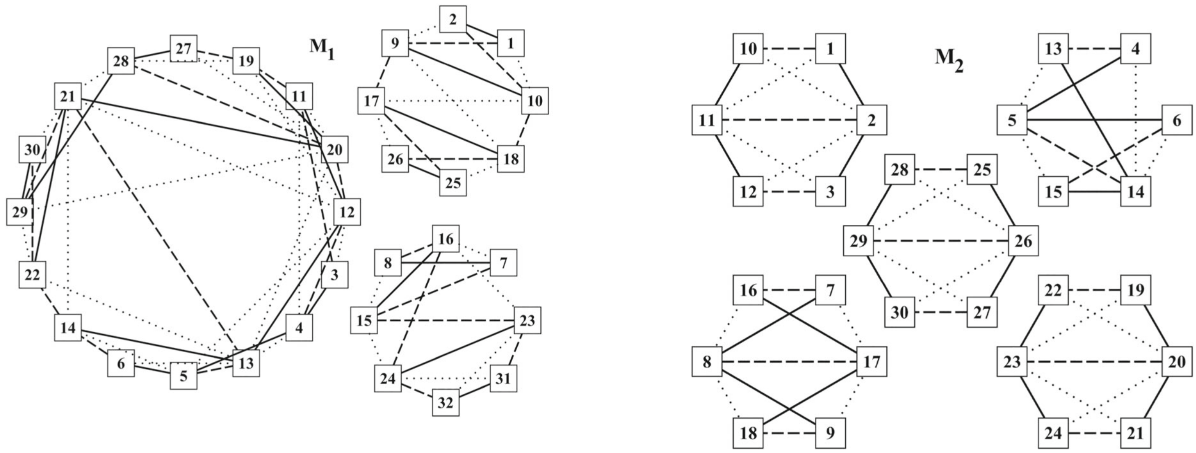

FIGURE 8 | The neighborhood networks M_1 and M_2 for classrooms R_1 and R_2 (Figure 9) where we experimented with optimum seating arrangements for three courses.

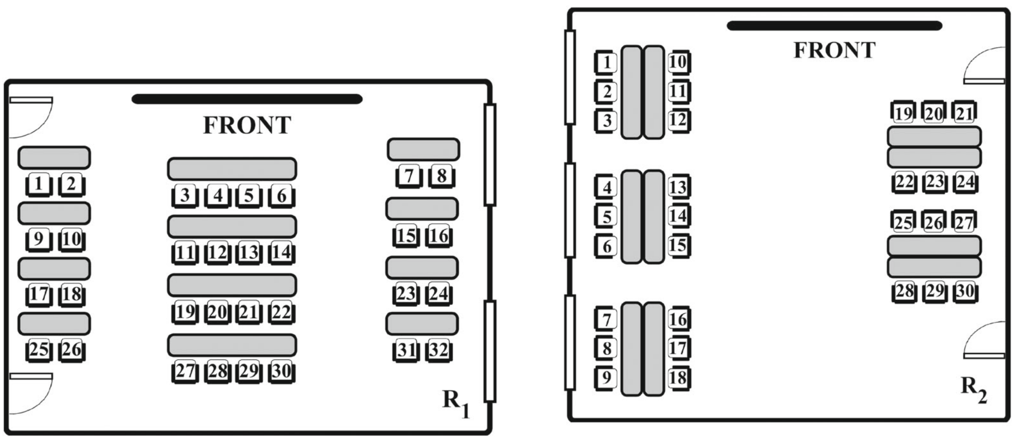

FIGURE 9 | The two classrooms, R_1 (left) and R_2 (right), where we experimented with optimum seating arrangements for three courses.

of the term (RR_1). The social networks were rather sparse with an average density of 0.24. Classes took place in two different rooms (R_1, R_2). The number of seats is given in column $|W|$. The classroom neighborhood structure is given in columns $|F_H|$, $|F_V|$ and $|F_D|$ by the overall number of horizontal, vertical and diagonal neighborships, respectively.

We used the same potential function as in the example in Section 3: direct neighbors (left/right) have potential 3; vertical neighbors (front/back) 2; diagonal neighbors 1. In Figure 8, we use solid, dashed and dotted edges to indicate this in the classroom networks. The networks consist of 3 and 5 connected components, respectively, which resembles their physical layout (Figure 9). Note that formulations (F_0)–(F_3) can be implemented in a spreadsheet for effective use by instructors in a template fashion. The model can be connected to selected external solvers via OpenSolver[iv].

8.2 | Results

In this section, we present the results obtained at the different stages of the experiment process (Section 8.1). The surveyed social networks and the classroom neighborhood networks are presented first. We then present the optimized seating

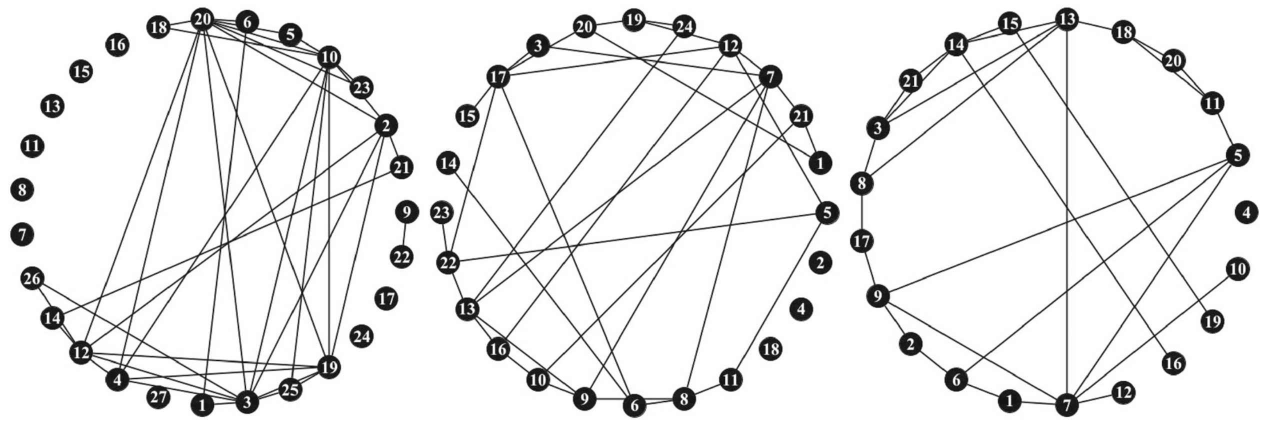

FIGURE 10 | The social networks N_A (left), N_B (center) and N_C (right) for the experiment classes A, B and C at the beginning of the academic term.

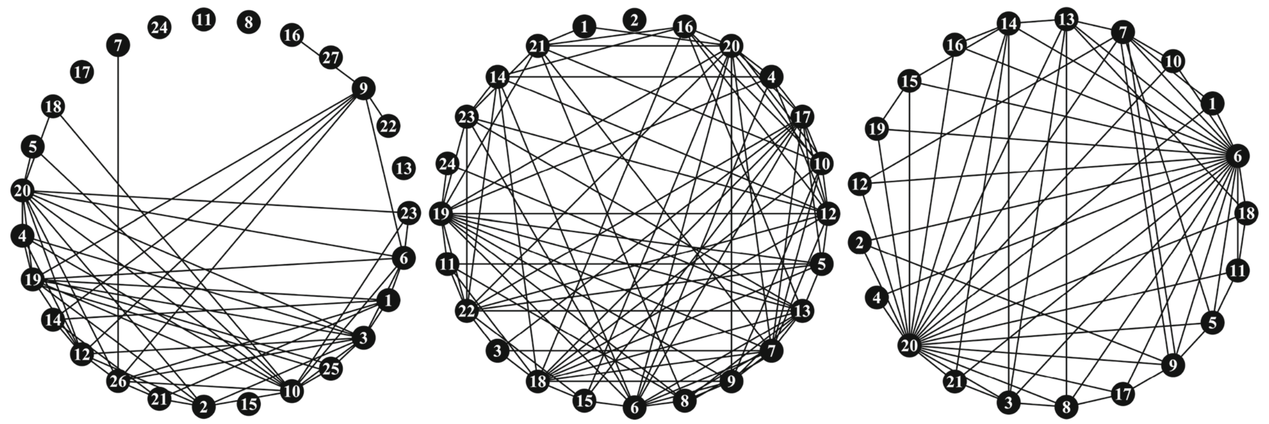

FIGURE 11 | The social networks N_A (left), N_B (center) and N_C (right) for the experiment classes A, B and C at the end of the academic term.

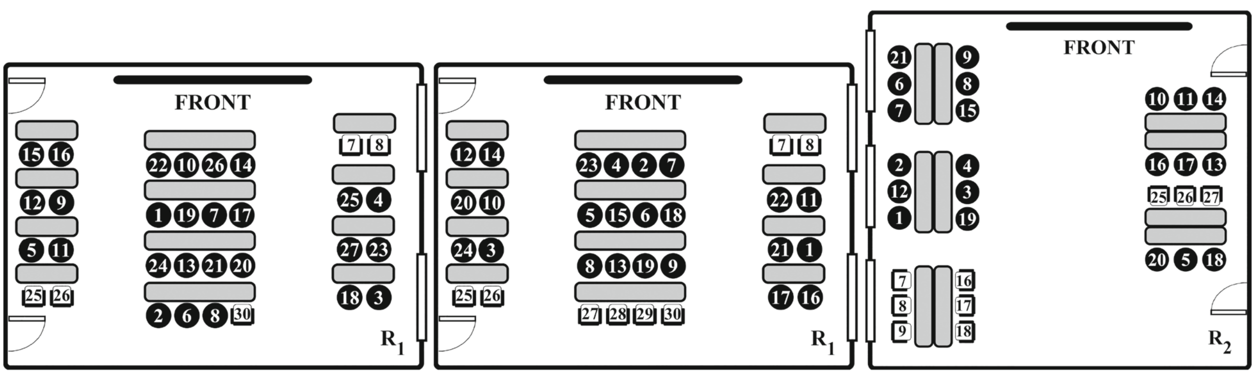

FIGURE 12 | The optimized seating assignments used for classes A (left), B (center) and C (right).

assignments and describe the special accommodations that were made. Finally, we present the final class social networks.

The physical layout of the classrooms is illustrated in Figure 9 and the corresponding neighbor networks are shown in Figure 8. Figure 10 shows the class social networks for the experiment classes as captured in the initial surveys. There were isolated students in class A (8), class B (3), and class C (1) This is common and can, for example, be due to students having transferred into the program, international exchange students, or students taking the class as an elective in a different program. The class social networks surveyed at the end of the term are illustrated in Figure 11. Note that they include all ties from the initial networks.

The optimized assignments used in experiment classes are shown in Figure 12. All models could be solved to optimality. Aligned with previous social network and classroom illustrations, black round nodes represent students allocated to tables. White square nodes are associated with empty seats. All classrooms had spare capacity such that empty seats were still available after assigning all students. No student sat in an isolated spot, and the model tended to fill tables rather than spread students over multiple tables that were only partially filled.

The following special accommodations were made in the experiment classes using model variations (see Section 5). Class A (Room R_1): one student decided not to participate in the experiment. So the student was allowed to sit on any available (unassigned) seat. Class B (Room R_1): one student indicated difficulties seeing the board and the projector screen from the back of the classroom. We added a zone constraint to the model, making sure that the student was assigned to one of the seats in $\{1, \ldots, 16\}$. Class C (Room R_2): students were not able to properly see the front projector screen from seats 7 and 8. So we configured the model to keep these seats unassigned.

8.3 | Analysis

We assess the efficacy of our optimized seating strategy through a careful analysis of the student social network dynamics. To this end, we examine the initial social network, the potential new ties, the actual new ties, and the final social network as a whole. We are particularly interested in the absolute and relative growth of the social networks. Table 6 contains the corresponding detailed data that we use for our quantitative evaluation. For each class, we list the initial number of ties (column $|E_0|$) and the theoretically maximal number of new ties (column $\overline{E_0}$). Note that the latter corresponds to the number of edges in the complement network $\overline{N}$, as discussed in Section 3. The potential numbers of new neighborhood ties with respect to neighborhood networks in Figure 8 S (columns under Potentials) are given for each neighbor type: H (horizontal), V (vertical), D (diagonal), and their sum (Σ). Note that these are not the tie potential values used in the objective function of our formulations, but the neighbor counts. Additionally, we provide the percentage for maximal network growth with respect to $|E_0|$ (column %). The corresponding actual numbers of new ties that were observed at the end of the term are shown afterwards (columns under New). Columns under "Other" contains the absolute and relative increase of ties that were not established between neighbors as defined in Figure 8. The total network growth is given in columns under All, followed by the final number of ties at the end of the term (column Final). The results in Table 6 demonstrate a substantial increase in new social ties among students, on average by 94.5%, with Class B showing the highest relative growth at 184.8%. This increase in social connections suggests that our optimized seating arrangements effectively fostered new interactions. Additionally, the presence of other new ties indicates that interactions extended beyond immediate neighbors (54.2% on average), contributing to the overall social network growth. Note that, on average, about 30% of the potential neighborhood ties were established.

In Table 7, we provide a more detailed analysis of the network expansion, including connectivity, centrality, and subgroup metrics from social network analysis. The values of k and s, respectively, represent connected components and the quantity of isolated students (degree equals zero). The average degree of the student nodes, also called average degree centrality, is given in column $\overline{d}$. We also consider more sophisticated centrality measures: 2-step centrality (column $\overline{c_{s2}}$) and Katz-centrality (column $\overline{c_\kappa}$). Both metrics quantify the influence a node has in the network. The former is also called reach-centrality and focuses on the immediate neighbors and their neighbors, whereas the latter also takes into account nodes that can be reached indirectly. However, more distant nodes are taken into account by c_κ using

TABLE 6 | Analysis of the student social network growth for the three experiment classes (A–C).

Course	Ties																
			Potentials					New									
	Initial		Neighborhood					Neighborhood					Other		All		Final
#	$\|E_0\|$	$\|\overline{E_0}\|$	H	V	D	Σ	%	H	V	D	Σ	%	Σ	%	Σ	%	Σ
A	37	288	18	14	14	46	124.3	0	4	4	8	21.6	10	27.0	18	48.6	55
B	33	243	20	15	16	51	154.5	8	5	8	21	63.6	40	121.2	61	184.8	94
C	28	182	12	14	9	35	125.0	6	2	2	10	35.7	4	14.3	14	50.0	42
All	149	713	50	43	39	132	134.6	14	11	14	39	40.3	54	54.2	93	94.5	191

TABLE 7 | Detailed analysis of the initial and final student social networks for the three experiment classes (A–C).

Course	Social network																	
	Initial						Final						Δ %					
#	k	s	$\overline{d}$	$\overline{c_{s2}}$	$\overline{c_\kappa}$	ω	k	s	$\overline{d}$	$\overline{c_{s2}}$	$\overline{c_\kappa}$	ω	k	s	$\overline{d}$	$\overline{c_{s2}}$	$\overline{c_\kappa}$	ω
A	11	9	2.7	7.3	1.2	5	8	7	4.1	10.6	1.3	5	−27.3	−22.2	48.6	45.9	10.4	0.0
B	4	3	2.8	8.9	1.2	3	2	1	7.8	20.0	1.7	6	−50.0	−66.7	184.8	124.3	48.5	100.0
C	2	1	2.7	7.4	1.2	3	2	1	4.0	12.0	1.3	3	0.0	0.0	50.0	61.5	8.8	0.0
All	5.7	4.3	2.7	7.9	1.2	3.7	4.0	3.0	5.3	14.2	1.4	4.7	−25.8	−29.3	94.5	77.2	22.3	33.3

the attenuation factor α. In our experiment, we used $\alpha = 0.05$. To measure the impact of new ties on large groups of students that are fully connected to each other, we calculate the clique number ω. The relative change for each metric is presented in columns under Δ %.

We observe that for classes A and B, both the number of connected components and the number of isolated students decreased, whereas in class C, the initially low levels persisted. Most notable, the average degree almost doubled (+94.5%), and the other centralities increased in all classes. We observe an especially strong growth in class B which is also reflected in a duplication of the number of students in the largest clique ($3 \rightarrow 6$).

8.4 | Feedback

We surveyed both the instructors involved in the study and the students enrolled in the class. In the following subsections, we present an overview and analysis of the survey data.

8.4.1 | Instructors

All three experiment class instructors participated in our survey. According to the instructors, there were only a few occurrences of deviation from the seating plan (4 out of 5 on a Likert scale, with 5 being "never"). Attendance was reported as high in all classes (4.25 out of 5 on a Likert scale, with 5 being “no absences”). Collaboration time in the courses ranged from 15% to 33% per week, and the frequency of student collaboration activities ranged from 1 to 5 times per week. When asked if they observed any positive effects of the seating assignment on the students, all three instructors responded affirmatively and provided the comments shown in Table 8.

TABLE 8 | Positive instructor comments.

#	Comment
1	“Immediately introduced each other and collaborated. Good to see more diverse communication/collaboration.”
2	“Students met and socialized with some students they had never spoken with before.”
3	“There was a lot of interaction in the group assignments. I can’t be sure that it was more than not having the pre-arranged assignment, but it was definitely noticeable.”

When asked about any negative impacts on the students due to the seating assignment, the participating instructors provided the comments shown in Table 9. It is evident that there was no severe negative impact on the students. Comment 1 could be addressed through intervention and the addition of a single student to another group when a group consists of only one student due to absences. Comment 2 is about the classroom layout, while Comment 3 is about a special accommodation for a single student.

8.4.2 | Students

We surveyed the students in the three experiment courses, namely A, B, and C. Figure 13 shows the 26 responses that we received for four questions using a Likert scale. Students are, on average, neutral about the impact on learning experience (3.1) and classmate interactions (3.0). Consistent with the instructors’ observations, there were only a few seating plan deviations (4.3). Students tend to not prefer assigned seating in the future (2.2) which is probably a result of students being used to choosing their own seat.

The students’ “positive comments on the seating assignment” are listed in Table 10. Overall, the comments suggest that the seating assignment had a positive impact on the students’ social and academic experiences. Students reported increased comfort and familiarity with their neighbors, suggesting that the seating arrangement helped them “break the ice” and foster a sense of community. Many appreciated the opportunity for positive interactions with people they hadn’t spoken to before, including teaching assistants and peers from their department, hence indicating that the arrangement facilitated networking and academic support. The seating arrangement also encouraged diversity, prompting students to step out of their usual social circles and interact with a diverse group of peers, potentially leading to

TABLE 9 | Negative instructor comments.

#	Comment
1	“No communication for at least one table. No collaboration when non-shows caused isolated students.”
2	“In classroom 240 some students needed to move to the center tables to see the screen better.”
3	“Only one student - there was an external reason for the student desiring to sit separately, nothing to do with an individual in the class.”

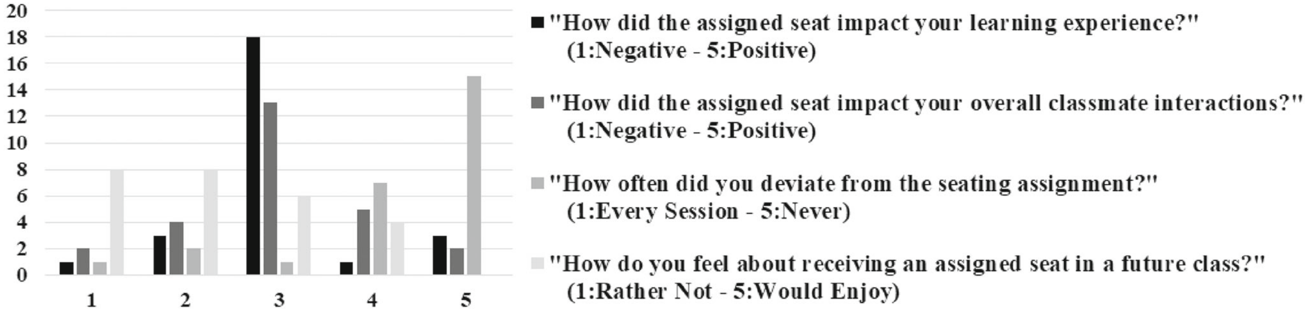

FIGURE 13 | Student survey results of the seating experiment.

TABLE 10 | All positive student comments.

#	Comment
1	"I knew my neighbors a little before, but I'm definitely more comfortable with them after this!"
2	"There weren't any glaringly obvious issues with it and it was fairly easy to follow."
3	"I sat next to my TA for 312 at the time so it was really cool for me to talk to him and get to know him more. Every time I see him we end up talking for a bit so that's really cool. I don't think that would've happened and I probably would've sat with people I already know if the seating chart didn't exist."
4	"I hadn't met them before and still talk to them now."
5	"It was nice being at the front of the class."
6	"I think that it was really good idea that could potentially have many positive impacts."
7	"I enjoyed being 'forced' to meet other people in my class as it brought me closer to other IME students."
8	"Was able to interact with people I have not talked to before."
9	"It was nice getting to know someone from my department and help each other out through the class! This would encourage students to interact out of your circle that you start to form as an upperclassmen."
10	"It was a good experience to interact with classmates I have not interacted."
11	"The seating assignment gave me a chance to talk to one of my neighbors that I had other classes with before but never really interacted with."
12	"Loved interacting with new students and building new friendships. It positively impacted school since interacting with these students helped me understand material better."
13	"I met more people that I didn't know."
14	"It was a change of pace from hanging with the same people."

a more inclusive and collaborative learning environment. Some students felt that interacting with new classmates enhanced their learning. Additionally, the creation of new friendships was expressed.

We asked students for "critical comments on the seating assignment" and obtained 14 responses, shown in Table 11. In summary, some individuals preferred sitting in the back of the classroom due to personal reasons, such as feeling nervous with people behind them in a work situation. However, others found it difficult to focus during lectures and discussions when not sitting in their preferred location. Students with vision impairments expressed the need to be seated closer to the board to avoid obstructions and distractions. Some students preferred sitting with people they knew and trusted, as it was more beneficial to their learning. Negative experiences were reported when seatmates did not engage or contribute to mutual learning. For some students with only a few connections in the class, being seated away from students they already know made it challenging to make connections and engage with peers.

The qualitative analysis of the critical comments highlights the importance of considering individual seating preferences, offering students ample opportunity to interact in class, ensuring visual and interactive accessibility, and instructor intervention when a student is non-participatory.

9 | CONCLUSIONS

The presented paper introduced a new model that optimizes student in-class seating assignments with respect to the maximum expansion of students' social networks. The optimization model is particular as it uses structural information from two networks, the pre-class student social network and the physical classroom neighborhood network, to evaluate student-to-seat assignments. The approach can be applied way beyond classrooms to settings such as networking events, social gatherings and professional development workshops.

After developing efficient mathematical formulations, techniques to model practically important extensions were presented. Additionally, we devised a set of fast heuristics that are guided by network centrality to complement our exact algorithms. We evaluated our model and methods on a comprehensive set of realistic test instances with up to 200 students and 364 seats. We used sparse and dense social networks from student surveys and coupled them to realistic classrooms of rectangular and circular shapes. We showed that all classrooms with up to 50 seats can be optimally arranged within a few seconds. For larger classrooms, we observe average optimality gaps of up to 15%. We identified both the density of the social network and the ratio of the number of seats to the number of students as important factors influencing problem hardness.

A multi-week case study including three engineering classes and over 70 students revealed the practical potential for social network growth. An average increase in ties of 40% could be observed. Interestingly, acquainted neighbors could be avoided in all classes. Furthermore, students and instructors reported that they enjoyed the increased interaction. Critical comments could be addressed through model readjustment to incorporate special needs.

We believe that future research could focus on better understanding student needs and how to best make use of practical model extensions to address them. It would also be interesting to isolate the seating impact from other relevant factors such as teaching methods. From a practical perspective, the exploration of real-time generation of seating assignments in-class with a focus on increasing student engagement and acceptance could be beneficial. A challenging model extension could be the repeated re-assignment of students to seats. Additionally, our model could be used to optimize classroom layouts from a social network perspective. Finally, the integration of diverse student relationships using weighted social network graphs could lead to interesting new insights.

TABLE 11 | All critical students comments on the seating assignment.

#	Comment
1	"My group was a group of 3, while the tables were organized to fit 6 each. Dispersing the empty seats among the class instead of having one half-empty table would help."
2	"I prefer to sit in the back of the classroom as it makes me nervous to have people behind me in a work situation. I found it incredibly difficult to focus during lecture and discussions when this was not the case. As well, I recently transferred into the IME dept so I was not sat next to the few people I had connected with and it was difficult to make connections in a 400 level lecture course."
3	"I feel like some people who typically sit at the front to have a good view/be more interactive with the professor were forced to sit somewhere in the back and that caused some issues. Other than that nothing really."
4	"Had to move seats because I have bad vision and was placed in the very back row."
5	"I did not enjoy being seated in the back of the class as I tend to not learn as much when I am not sitting in the front of the class."
6	"I prefer to be seated closer to the board so I can see without any large obstruction/distractions!"
7	"I sit with people I know I can trust and who are proven to be beneficial to my learning. I was friendly but the person I sat with (no one else at our two person table) hardly ever spoke to me. He was clearly behind and struggling in the class but refused to ask me questions/help himself. I would have been happy to help once in a while is it bc we were strangers, or I am a woman, or he just had too much or too little ego idk. So obviously since he was so far behind he couldn't help me if I had a question and it was def not a mutually beneficial arrangement. There is a reason why I sit by people I know and trust."
8	"I think the seating assignment might be less worthwhile if it's a class that doesn't lend itself well to discussions and/or group work."
9	"I did not get the chance to talk to the people that were assigned to nearby seats to me so it was overall a neutral experience."
10	"I am sure not everyone had the same experience, but my seatmates were very favorable."
11	"While I met more people most of the interactions were surface level."
12	"Some of the seats in the room are very hard to learn in because they face away from the lecture. I would not have chosen that seat because of that reason. I did not learn new names either because I was too busy learning in lecture and not talking with my neighbors beyond surface level."
13	"I got seated to the side of the room with only one classmate. I did not know him or interact with him at all throughout the quarter. I sat with people I knew before the assigned seating."
14	"I think the seating assignments results really just depends on the dynamic you have with the other person. For instance, I didn't feel comfortable with my partner because first off he didn't sit in the right seat. He sat on my seat even though he knew which one was his. Also he would always just leave his jacket laid out on the table making it uncomfortable. But if I was seated to someone who was friendly enough to make small talk, the experience would have been positive."

Acknowledgments

We thank both the instructors and the students in all classes for participating in the experiment. We are indebted to Gianna Nobili for her help in surveying the student social networks. We are grateful to the associate editor and the referees for their constructive input and to the journal editors for handling our manuscript.

Data Availability Statement

The data that support the findings of this study are openly available in https://github.com/ale-hill/SSAP.

ENDNOTES

[i] https://www.gurobi.com.

[ii] https://pypi.org/project/gurobipy.

[iii] Social network and classroom network data available at https://github.com/ale-hill/SSAP.

[iv] https://opensolver.org.

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APPENDIX A

EXAMPLE SEATING PLAN POST EXAMPLE

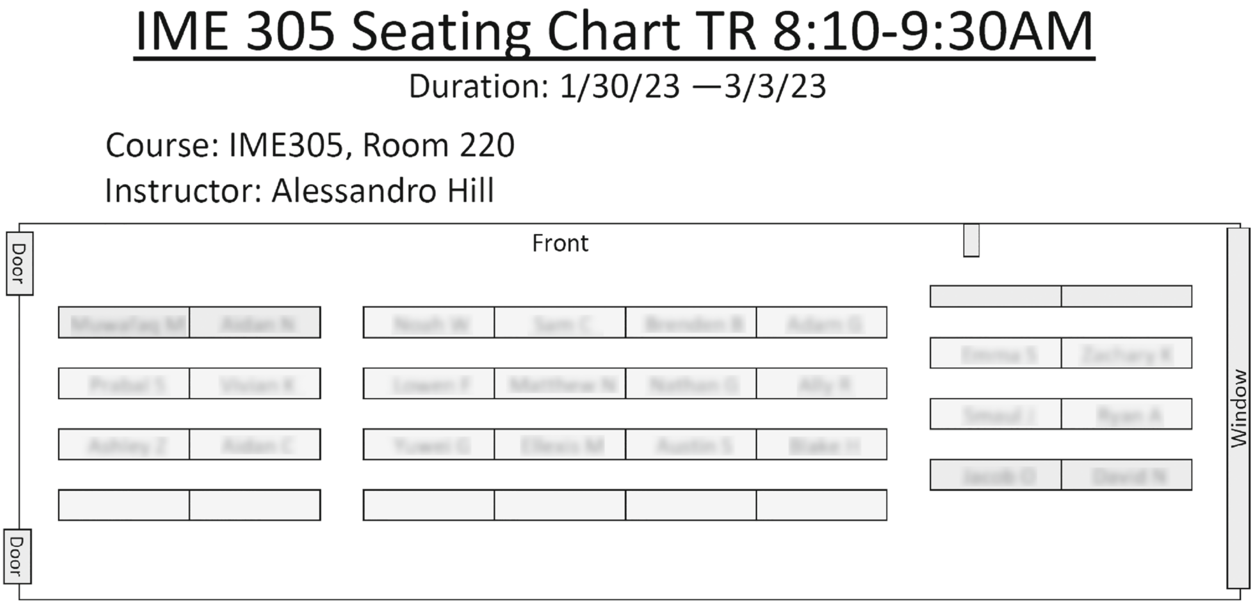

FIGURE A1 | The seating plan (here with 24 blurred student names) that we posted at each classroom door of Room R_1 for experiment class B during the 5-week experiment period.

APPENDIX B

FINAL RESULTS FOR ALL INSTANCES

TABLE B1 | Lower (lb) and upper (ub) bounds for all instances (social network id $\#(N)$, classroom network id $\#(W)$) obtained after one hour of branch-and-bound using formulation (F_2).

$\#(N)$	$\#(M)$	lb	ub	$\#(N)$	$\#(M)$	lb	ub	$\#(N)$	$\#(M)$	lb	ub	$\#(N)$	$\#(M)$	lb	ub	$\#(N)$	$\#(M)$	lb	ub
1	21	33	33	5	40	98	98	7	40	98	98	9	38	212	212	10	13	285	290
1	24	28	28	5	2	72	72	7	2	72	72	9	40	196	196	10	14	285	290
1	26	26	26	5	3	72	72	7	3	72	72	9	41	197	197	10	15	282	290
1	1	27	27	5	7	115	115	7	7	115	115	9	42	192	192	10	18	207	207
1	6	42	42	5	8	116	116	7	8	116	116	9	3	147	147	11	23	206	206
1	7	43	43	5	9	110	110	7	9	110	110	9	4	147	147	11	27	185	185
2	21	22	22	5	13	132	132	7	13	132	132	9	5	147	147	11	28	196	196
2	24	19	19	5	17	96	96	7	17	96	96	9	8	237	237	11	30	212	212
2	26	18	18	6	22	98	98	8	22	82	98	9	9	205	205	11	33	192	192
2	1	17	17	6	25	82	82	8	25	71	82	9	10	266	266	11	34	171	171
2	6	23	23	6	27	88	88	8	27	74	88	9	13	290	290	11	35	197	197
2	7	23	23	6	29	98	98	8	29	80	98	9	14	290	290	11	37	211	211
3	21	33	33	6	32	87	87	8	32	74	87	9	15	290	290	11	38	212	212
3	24	28	28	6	33	96	96	8	33	77	96	9	18	207	207	11	40	196	196
3	26	26	26	6	34	85	85	8	34	72	85	10	23	206	206	11	41	197	197
3	1	27	27	6	37	98	98	8	37	78	98	10	27	185	185	11	42	192	192
3	6	42	42	6	40	98	98	8	40	79	98	10	28	196	196	11	3	147	147
3	7	43	43	6	2	72	72	8	2	61	69	10	30	212	212	11	4	147	147
4	21	31	31	6	3	72	72	8	3	61	72	10	33	192	192	11	5	147	147
4	24	27	27	6	7	115	115	8	7	88	115	10	34	171	171	11	8	237	237
4	26	25	25	6	8	116	116	8	8	84	116	10	35	197	197	11	9	205	205
4	1	25	25	6	9	110	110	8	9	86	110	10	37	211	211	11	10	266	266
4	6	37	37	6	13	127	132	8	13	91	132	10	38	212	212	11	13	290	290
4	7	38	38	6	17	96	96	8	17	74	96	10	40	196	196	11	14	290	290
5	22	98	98	7	22	98	98	9	23	206	206	10	41	197	197	11	15	290	290
5	25	82	82	7	25	82	82	9	27	185	185	10	42	192	192	11	18	207	207
5	27	88	88	7	27	88	88	9	28	196	196	10	3	147	147	12	23	184	206
5	29	98	98	7	29	98	98	9	30	212	212	10	4	147	147	12	27	180	185
5	32	87	87	7	32	87	87	9	33	192	192	10	5	147	147	12	28	185	200
5	33	96	96	7	33	96	96	9	34	171	171	10	8	237	237	12	30	189	213
5	34	85	85	7	34	85	85	9	35	197	197	10	9	205	205	12	33	181	192
5	37	98	98	7	37	98	98	9	37	211	211	10	10	266	266	12	34	168	171

TABLE B2 | Lower (lb) and upper (ub) bounds for all instances (social network id #(N), classroom network id #(W)) obtained after 1 h of branch-and-bound using formulation (F_2).

#(N)	#(M)	lb	ub	#(N)	#(M)	lb	ub	#(N)	#(M)	lb	ub	#(N)	#(M)	lb	ub	#(N)	#(M)	lb	ub
12	35	183	202	13	16	615	615	15	5	300	300	17	44	658	661	20	12	790	792
12	37	195	211	13	18	420	420	15	10	506	506	17	11	812	812	20	16	934	953
12	38	189	216	13	19	426	426	15	11	552	552	17	12	792	792	20	19	639	640
12	40	179	196	14	28	395	399	15	12	528	528	17	16	946	950	20	20	645	650
12	41	182	201	14	31	423	444	15	14	612	612	17	19	639	639	21	36	850	856
12	42	172	192	14	35	406	406	15	15	615	615	17	20	645	650	21	39	918	918
12	3	144	148	14	36	419	427	15	16	615	615	18	31	672	672	21	43	845	845
12	4	145	150	14	38	444	447	15	18	420	420	18	36	633	641	21	44	873	883
12	5	145	150	14	39	423	458	15	19	426	426	18	39	666	689	21	11	1058	1058
12	8	212	239	14	41	400	407	16	28	396	399	18	43	631	634	21	12	1056	1056
12	9	191	205	14	42	382	384	16	31	426	443	18	44	654	664	21	20	864	864
12	10	234	266	14	43	422	422	16	35	405	406	18	11	808	818	22	36	844	858
12	13	247	290	14	4	300	300	16	36	423	427	18	12	792	792	22	39	911	920
12	14	249	293	14	5	300	300	16	38	441	448	18	16	939	953	22	43	841	848
12	15	244	298	14	10	504	510	16	39	426	459	18	19	639	642	22	44	870	1608
12	18	191	207	14	11	540	553	16	41	400	407	18	20	645	650	22	11	1055	1066
13	28	396	396	14	12	524	528	16	42	382	384	19	31	672	672	22	12	1053	1056
13	31	442	442	14	14	603	612	16	43	421	422	19	36	637	637	22	20	862	866
13	35	406	406	14	15	598	624	16	4	300	300	19	39	682	682	23	36	850	857
13	36	425	425	14	16	599	634	16	5	300	300	19	43	634	634	23	39	918	918
13	38	447	447	14	18	420	420	16	10	500	510	19	44	658	661	23	43	845	845
13	39	447	447	14	19	426	426	16	11	545	553	19	11	812	812	23	44	872	883
13	41	406	406	15	28	396	396	16	12	524	528	19	12	792	792	23	11	1058	1064
13	42	384	384	15	31	442	442	16	14	602	612	19	16	946	946	23	12	1056	1056
13	43	422	422	15	35	406	406	16	15	604	625	19	19	639	642	23	20	864	864
13	4	300	300	15	36	425	425	16	16	603	634	19	20	645	650	24	36	843	858
13	5	300	300	15	38	447	447	16	18	420	420	20	31	667	672	24	39	910	920
13	10	506	506	15	39	447	447	16	19	426	426	20	36	633	641	24	43	841	848
13	11	552	552	15	41	406	406	17	31	672	672	20	39	667	688	24	44	869	1608
13	12	528	528	15	42	384	384	17	36	637	642	20	43	630	634	24	11	1057	1066
13	14	612	612	15	43	422	422	17	39	682	682	20	44	654	664	24	12	1053	1056
13	15	615	615	15	4	300	300	17	43	634	634	20	11	804	818	24	20	862	1404