

# Appointment Scheduling with Restricted People

## 1 Model

The service time for patient  $i$ ,  $\xi_i$ , stochastic with a mean of  $\mu_i$  and a standard deviation of  $\sigma_i$ . The service times are mutually independent. For each patient  $i = 1, \dots, n$ , we use  $A_i$  to denote the appointment time,  $S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\}$  denote the actual starting time of service. We assume that the patients will arrive at the appointed time. Especially,  $A_1 = S_1 = 0$ .

The waiting time for patient  $i$  is  $S_i - A_i$ , the total waiting time is  $\sum_{i=2}^n \alpha_i (S_i - A_i)$ , where  $\alpha_i$  is the weight for patient  $i$ . The overtime is  $(S_n + \xi_n - T)^+$  and the total idle time is  $\sum_{i=1}^{n-1} [S_{i+1} - (S_i + \xi_i)] = S_n - \sum_{i=1}^{n-1} \xi_i$ .

In the scenario with at least 2 patients overlapping in the waiting room, we can calculate the overlapping time. Let  $t_{ij}$  denote the overlapping time between two patients  $i$  and  $j$ . Then,  $t_{i,j} = (S_i - A_j)^+$ .

The duration when there are only  $(j - i + 1)$  people for patients  $i$  and  $j$  are waiting is  $t_{i,j} - t_{i,j+1}$ ,  $i = 2, \dots, n-1, j \geq i$ .

Total overlapping time:  $\sum_{i=2}^{n-1} \sum_{j=i}^{n-1} \gamma_{i,j} (t_{i,j} - t_{i,j+1})$

Problem to minimize the total time:

$$\begin{aligned} \min_{\mathbf{A}} \quad & E_{\xi} \left[ \left( S_n - \sum_{i=1}^{n-1} \xi_i \right) + \sum_{i=2}^{n-1} \sum_{j=i}^{n-1} \gamma_{i,j} (t_{i,j} - t_{i,j+1}) + \beta (S_n + \xi_n - T)^+ \right] \\ \text{s.t.} \quad & S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\} \\ & S_1 = 0 \end{aligned} \tag{1}$$

1. Possible traits: heterogeneous patients, no-show, lateness, walk-in

Different models: objective: minimize the total cost, minimize the makespan (the departure time of the last customer).

Two possible options: the time of several people waiting, what if it is not

Traditional Appointment Scheduling Model.

1. with overbooking and no-shows (partial punctuality)

- discrete n time slots.

- minimize the waiting cost, idle time and overtime costs.

2.

- minimize

3. Under a service-level constraint (waiting time threshold)
  - makespan