Dynamic Seat Assignment with Social Distancing

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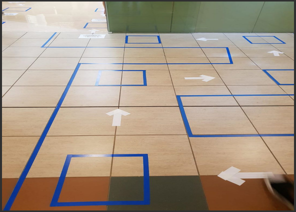
- Social distancing measures.



Social Distancing under Pandemic

- Social distancing in seating areas.

Literature Review

Seat Planning with Social Distancing

- Allocation of seats on airplanes (Ghorbani et.al 2020), classroom layout planning (Bortolete et al. 2022), seat planning in long-distancing trains (Haque & Hamid 2022).

- Social distancing can be enforced in different groups (Moore et al. 2021).

- Seating planning for known groups in amphitheaters (Haque & Hamid 2022), airplanes (Salari et al. 2022), theater (Blom et al. 2022).

Dynamic Seat Assignment

- Related to multiple knapsack problem (Pisinger et al. 1999) and dynamic knapsack problem (Kleywegt et al. 1998).
- Dynamic seat assignment on airplane (Hamdouch et al. 2011), train (Berge et al. 1993).
- Assign-to-seat: dynamic capacity control for selling high-speed train tickets (Zhu et al. 2023).

Problem Definition

Seat Planning with Social Distancing

- Group type $\mathcal{M} = \{1, \ldots, M\}$.
- Row $\mathcal{N} = \{1, \ldots, N\}$.
- The social distancing: δ seat(s).
- $n_i = i + \delta$: the new size of group type i for each $i \in \mathcal{M}$.
- The number of seats in row j: $S_j, j \in \mathcal{N}$.
- $L_j = S_j + \delta$: the length of row j for each $j \in \mathcal{N}$.

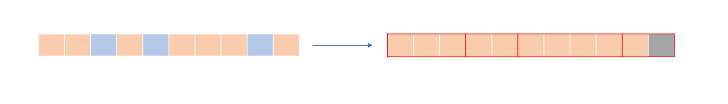


Figure: Problem Conversion with One Seat as Social Distancing

Basic Concepts

- $P_k = (t_1, \ldots, t_M)$, pattern, the seat planning for each group type in one row.
  - For each pattern k, a_k, b_k indicate the number of groups and the unused seats, respectively.
  - Loss for pattern k: $a_k\delta + b_k - \delta$.
- Largest patterns: Minimal loss.
- Full patterns: $b_k = 0$.

- Example:
  $\delta = 1,\ M = 4,\ n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5,\ L = 21.$
  Largest patterns: $(0, 0, 0, 4), (0, 0, 4, 1), (0, 2, 0, 3)$.
  Largest may not be full: $(0, 0, 0, 4)$.
  Full may not be largest: $(1, 1, 4, 0)$.

Dynamic Seat Assignment Problem

- There is one and only one group arrival at each period, $t = 1, \dots, T+1$.
- The probability of an arrival of group type i: p_i.
- $\mathbf{L} = (l_1, l_2, \dots, l_N)$, where $l_j = 0, \dots, L_j, j \in \mathcal{N}$: Remaining capacity.
- $u_{i,j}$: Decision. Assign group type i to row j, $u_{i,j} = 1$.
- $U(\mathbf{L}) = \{u_{i,j} \in \{0,1\}, \forall i, j | \sum_{j=1}^{N} u_{i,j} \leq 1, \forall i, n_i u_{i,j} \mathbf{e}_j^\top \leq \mathbf{L}, \forall i, j\}$.
- $\mathbf{e}_j^\top$: Unit row vector with j-th element being 1.
- $V_t(\mathbf{L})$: Value function at period t, given remaining capacity, $\mathbf{L}$.

$$V_t(\mathbf{L}) = \max_{u \in U(\mathbf{L})} \{\sum_{i=1}^{M} p_i (\sum_{j=1}^{N} i u_{i,j} + V_{t+1}(\mathbf{L} - \sum_{j=1}^{N} n_i u_{i,j} \mathbf{e}_j^\top))\}$$

Method Overview

- Obtain seat planning composed of full or largest patterns.
  - Linear seat planning from stochastic programming
  - Integral seat planning from deterministic model
  - Construct largest or full patterns.
- Stochastic planning policy.
  - Group type control
  - Value of stochastic programming

Seat Planning by Stochastic Programming

Method Flow

- The formulation of scenario-based stochastic programming(SSP).
- Reformulate SSP to the benders master problem(BMP) and subproblem.
- The optimal solution can be obtained by solving BMP iteratively.
- To avoid solving IP directly, we consider the linear relaxation form.
- Obtain integral seat planning composed of full or largest patterns by deterministic model.

Scenario-based Stochastic Programming

$$
\begin{aligned}
(SSP)\ \max \quad & E_\omega \left[\sum_{i=1}^{M-1} (n_i - \delta)(\sum_{j=1}^{N} x_{ij} + y_{i+1,\omega}^{+} - y_{i\omega}^{+}) + (n_M - \delta)(\sum_{j=1}^{N} x_{Mj} - y_{M\omega}^{+}) \right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y_{i\omega}^{+} + y_{i+1,\omega}^{+} + y_{i\omega}^{-} = d_{i\omega}, \quad i = 1, \ldots, M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y_{i\omega}^{+} + y_{i\omega}^{-} = d_{i\omega}, \quad i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& y_{i\omega}^{+}, y_{i\omega}^{-} \in \mathbb{Z}_{+}, \quad i \in \mathcal{M}, \omega \in \Omega \\
& x_{ij} \in \mathbb{Z}_{+}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned} \tag{1}
$$

Reformulation

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} + z(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{n}\mathbf{x} \leq \mathbf{L} \\ & \mathbf{x} \in \mathbb{Z}_{+}^{M \times N}, \end{aligned} \tag{2}$$

where $z(\mathbf{x})$ is defined as

$$z(\mathbf{x}) := E(z_\omega(\mathbf{x})) = \sum_{\omega \in \Omega} p_\omega z_\omega(\mathbf{x}),$$

and for each scenario $\omega \in \Omega$,

$$\begin{aligned} z_\omega(\mathbf{x}) := \max \quad & \mathbf{f}^\intercal \mathbf{y} \\ \text{s.t.} \quad & \mathbf{x}\mathbf{1} + \mathbf{V}\mathbf{y} = \mathbf{d}_\omega \\ & \mathbf{y} \geq 0. \end{aligned} \tag{3}$$

Solution to Subproblem

Problem (3) is easy to solve with a given $\mathbf{x}$ which can be seen by the dual problem:

$$\begin{aligned} \min \quad & \alpha_\omega^\intercal(\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \\ \text{s.t.} \quad & \alpha_\omega^\intercal \mathbf{V} \geq \mathbf{f}^\intercal \end{aligned} \tag{4}$$

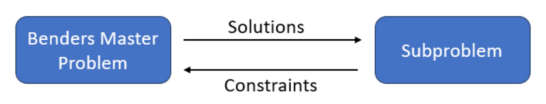
- The feasible region of problem (4), $P = \{\alpha | \alpha^\intercal V \geq \mathbf{f}^\intercal\}$, is bounded. In addition, all the extreme points of P are integral.
- The optimal solution to this problem can be obtained directly according to the complementary slackness property.

Benders Decomposition Procedure

Let z_ω be the lower bound of problem (4), SSP can be obtained by solving following restricted benders master problem:

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} + \sum_{\omega \in \Omega} p_\omega z_\omega \\ \text{s.t.} \quad & \mathbf{n}\mathbf{x} \leq \mathbf{L} \\ & (\alpha^k)^\intercal (\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \geq z_\omega, \alpha^k \in \mathcal{O}, \forall \omega \\ & \mathbf{x} \in \mathbb{Z}_+ \end{aligned} \tag{5}$$

Constraints will be generated from problem (4) until an optimal solution is found.



To avoid solving IP directly, we consider the linear relaxation of Problem (5).

Deterministic Formulations

To obtain an integral seat planning, we consider the following two deterministic formulations.

$$\begin{aligned} \max \quad & \sum_{i=1}^{M} \sum_{j=1}^{N} (n_i - \delta) x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \leq s_i^0, \quad i \in \mathcal{M}, \\ & \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\ & x_{ij} \in \mathbb{Z}_+, \quad i \in \mathcal{M}, j \in \mathcal{N}. \end{aligned} \tag{6}$$

$$\begin{aligned} \max \quad & \sum_{i=1}^{M} \sum_{j=1}^{N} (n_i - \delta) x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \geq s_i^1, \quad i \in \mathcal{M}, \\ & \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\ & x_{ij} \in \mathbb{Z}_+, \quad i \in \mathcal{M}, j \in \mathcal{N}. \end{aligned} \tag{7}$$

Problem (6) can generate a feasible seat planning.
Problem (7) can generate a seat planning no inferior than a given feasible seat planning.

Obtain Seat Planning Composed of Full or Largest Patterns

Step 1. Obtain the solution, $\mathbf{x}^*$, by benders decomposition. Aggregate $\mathbf{x}^*$ to the number of each group type, $s_i^0 = \sum_j x_{ij}^*, i \in \mathbf{M}$.

Step 2. Solve problem (6) to obtain the optimal solution, $\mathbf{x}^1$. Aggregate $\mathbf{x}^1$ to the number of each group type, $s_i^1 = \sum_j x_{ij}^1, i \in \mathbf{M}$.

Step 3. Solve problem (7) to obtain the optimal solution, $\mathbf{x}^2$. Aggregate $\mathbf{x}^2$ to the number of each group type, $s_i^2 = \sum_j x_{ij}^2, i \in \mathbf{M}$.

Step 4. For each row, construct a full or largest pattern.

Dynamic Seat Assignment for Each Group Arrival

Policies

- Stochastic planning policy
- Bid-price control
- Dynamic programming based heuristic
- Booking limit control
- First come first served

Stochastic Planning Policy(SPP)

- Group-type control
  - Seat planning from stochastic programming.
  - When there is no small group, decide which group type to be assigned.

$$d(i,j;t) = \underbrace{i + (j-i-\delta)P(D_{j-i-\delta} \geq x_{j-i-\delta}+1;t)}_{\text{acceptance}} - \underbrace{jP(D_j \geq x_j;t)}_{\text{rejection}},\ j > i.$$

If $d(i,j^*;t) > 0$, we will place the group of i in $(j^*+\delta)$-size seats. Otherwise, reject the group.

Stochastic Planning Policy(SPP)

- Use the value of stochastic programming as the approximation of value function in DP.
  - Value of Acceptance(VoA): approximation of $V_t(\mathbf{L} - n_i \mathbf{e}_j^\top) + i$. (Find a pattern containing group type j^*)
  - Value of Rejection(VoR): approximation of $V_t(\mathbf{L})$.
  - If VoA is no less than VoR, accept group type i, otherwise, reject it.

Bid-price Control

The dual problem of LP relaxation of problem (6) is:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{M} d_i z_i + \sum_{j=1}^{N} L_j \beta_j \\
\text{s.t.} \quad & z_i + \beta_j n_i \geq (n_i - \delta), \quad i \in \mathcal{M}, j \in \mathcal{N} \\
& z_i \geq 0, i \in \mathcal{M}, \beta_j \geq 0, j \in \mathcal{N}.
\end{aligned}
\tag{8}
$$

There exists h such that the aggregate optimal solution to relaxation of problem (6) takes the form $xe_h + \sum_{i=h+1}^{M} d_i e_i$, $x = (L - \sum_{i=h+1}^{M} d_i n_i)/n_h$.

Dynamic Programming Based Heuristic

- Relax all rows to one row with the same capacity by $L = \sum_{j=1}^{N} L_j$.
  - Deterministic problem: $\{\max \sum_{i=1}^{M} (n_i - \delta) x_i : x_i \leq d_i, i \in \mathcal{M}, \sum_{i=1}^{M} n_i x_i \leq L, x_i \in \mathbb{Z}_+\}$.
- Decision: u. If we accept a request in period t, $u(t) = 1$; otherwise, $u(t) = 0$.
  - DP with one row can be expressed as:

$$V_t(L) = \mathbb{E}_{i \sim p}[\max_{u \in \{0,1\}} \{[V_{t+1}(L - n_i u) + iu]\}], L \geq 0$$

$$V_{T+1}(x) = 0, \forall x.$$

- After accepting one group, assign it in some row arbitrarily when the capacity of the row allows.

Booking limit Control

Basic idea: for every type of requests, we only allocate a fixed amount according to the static solution and reject all other exceeding requests.

1. Observe the arrival group type i.
2. Solve problem (6) using the expected demand.
3. Obtain the optimal solution, x_{ij}^* and the aggregate optimal solution, $\mathbf{X}$.
4. If $X_i > 0$, accept the arrival and assign the group to row k where $x_{ik} > 0$, update $\mathbf{L}^{t+1} = \mathbf{L}^t - n_i \mathbf{e}_k^\top$; otherwise, reject it, let $\mathbf{L}^{t+1} = \mathbf{L}^t$.

Numerical Results

Running time of Benders Decomposition and IP

# of scenarios	demands	running time of IP(s)	Benders (s)	# of rows	# of groups	# of seats
1000	(150, 350)	5.1	0.13	30	8	(21, 50)
5000		28.73	0.47	30	8	
10000		66.81	0.91	30	8	
50000		925.17	4.3	30	8	
1000	(1000, 2000)	5.88	0.29	200	8	(21, 50)
5000		30.0	0.62	200	8	
10000		64.41	1.09	200	8	
50000		365.57	4.56	200	8	
1000	(150, 250)	17.15	0.18	30	16	(41, 60)
5000		105.2	0.67	30	16	
10000		260.88	1.28	30	16	
50000		3873.16	6.18	30	16	

Feasible Seat Planning versus IP Solution

# samples	T	probabilities	# rows	people served by FSP	IP
1000	45	[0.4,0.4,0.1,0.1]	8	85.30	85.3
1000	50	[0.4,0.4,0.1,0.1]	8	97.32	97.32
1000	55	[0.4,0.4,0.1,0.1]	8	102.40	102.40
1000	60	[0.4,0.4,0.1,0.1]	8	106.70	NA
1000	65	[0.4,0.4,0.1,0.1]	8	108.84	108.84
1000	35	[0.25,0.25,0.25,0.25]	8	87.16	87.08
1000	40	[0.25,0.25,0.25,0.25]	8	101.32	101.24
1000	45	[0.25,0.25,0.25,0.25]	8	110.62	110.52
1000	50	[0.25,0.25,0.25,0.25]	8	115.46	NA
1000	55	[0.25,0.25,0.25,0.25]	8	117.06	117.26
5000	300	[0.25,0.25,0.25,0.25]	30	749.76	749.76
5000	350	[0.25,0.25,0.25,0.25]	30	866.02	866.42
5000	400	[0.25,0.25,0.25,0.25]	30	889.02	889.44
5000	450	[0.25,0.25,0.25,0.25]	30	916.16	916.66

Each entry of people served is the average of 50 instances. IP will spend more than 2 hours in some instances, as 'NA' showed in the table.

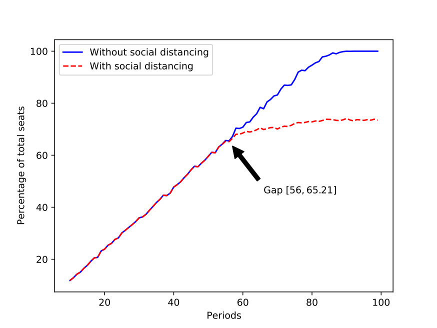
Performances of Different Policies

T	Probabilities	SPP(%)	DP1(%)	Bid(%)	Booking(%)	FCFS(%)
60	[0.25, 0.25, 0.25, 0.25]	99.12	98.42	98.38	96.74	98.17
70	[0.25, 0.25, 0.25, 0.25]	98.34	96.87	96.24	97.18	94.75
80	[0.25, 0.25, 0.25, 0.25]	98.61	95.69	96.02	98.00	93.18
90	[0.25, 0.25, 0.25, 0.25]	99.10	96.05	96.41	98.31	92.48
100	[0.25, 0.25, 0.25, 0.25]	99.58	95.09	96.88	98.70	92.54
60	[0.25, 0.35, 0.05, 0.35]	98.94	98.26	98.25	96.74	98.62
70	[0.25, 0.35, 0.05, 0.35]	98.05	96.62	96.06	96.90	93.96
80	[0.25, 0.35, 0.05, 0.35]	98.37	96.01	95.89	97.75	92.88
90	[0.25, 0.35, 0.05, 0.35]	99.01	96.77	96.62	98.42	92.46
100	[0.25, 0.35, 0.05, 0.35]	99.23	97.04	97.14	98.67	92.00
60	[0.15, 0.25, 0.55, 0.05]	99.14	98.72	98.74	96.61	98.07
70	[0.15, 0.25, 0.55, 0.05]	99.30	96.38	96.90	97.88	96.25
80	[0.15, 0.25, 0.55, 0.05]	99.59	97.75	97.87	98.55	95.81
90	[0.15, 0.25, 0.55, 0.05]	99.53	98.45	98.69	98.81	95.50
100	[0.15, 0.25, 0.55, 0.05]	99.47	98.62	98.94	98.90	95.25

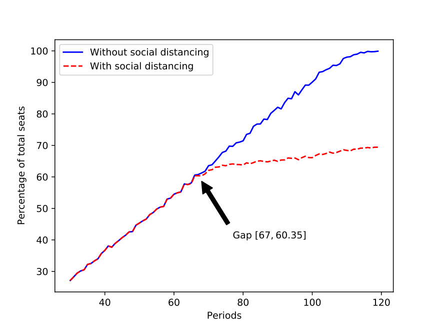
SPP has better performance than other policies under different demands.

Impact of Social Distancing as Demand Increases

$\gamma = p_1 * 1 + p_2 * 2 + p_3 * 3 + p_4 * 4$: the expected number of people at each period.



(a) When $\gamma = 2.5$



(b) When $\gamma = 1.9$

The gap point represents the first period where the number of people without social distancing is larger than that with social distancing and the gap percentage is the corresponding percentage of total seats.

Estimation of Gap Point

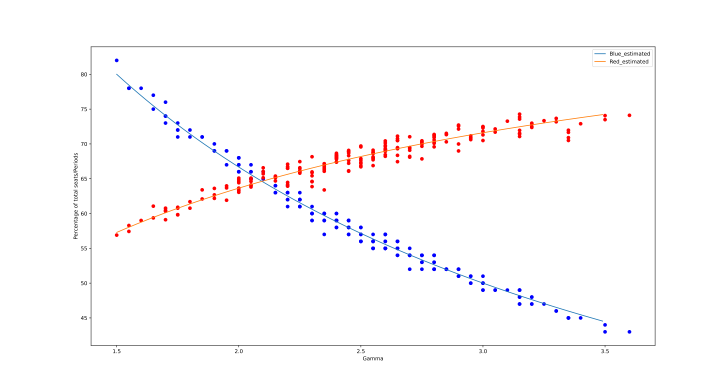


Figure: Gap points under 200 probabilities

Blue points: period of the gap point. Red points: occupancy rate of the gap point. Gap points can be estimated.

Conclusion

Conclusion

- We address the problem of dynamic seat assignment with social distancing.
- Our approach, stochastic planning policy, provides a comprehensive solution for optimizing seat assignments while ensuring the safety of customers under dynamic situation.
- We can estimate the occupancy rate when applying SPP according to γ.

The End