Appointment Scheduling

Discount

October 2024

1 Model

We assume that the patients will arrive at the appointed time.

The service time for patient i, ξ_i, stochastic with a mean of μ_i and a standard deviation of σ_i.

The service times are mutually independent.

For each patient $i = 1, \ldots, n$,

A_i: appointment time.

$S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\}$: actual starting time of service.

$A_1 = S_1 = 0$.

Patient i will arrive at A_i but start service at S_i.

Waiting time: $S_i - A_i$

Overtime: $(S_n + \xi_n - T)^+$

Total idle time: $\sum_{i=1}^{n-1} = [S_{i+1} - (S_i + \xi_i)] = S_n - \sum_{i=1}^{n-1} \xi_i$

Problem to minimize the total time:

$$
\begin{aligned}
\min_{\mathbf{A}} \quad & E_\xi \left[\left(S_n - \sum_{i=1}^{n-1} \xi_i \right) + \sum_{i=2}^{n} \alpha_i \left(S_i - A_i \right) + \beta (S_n + \xi_n - T)^+ \right] \\
\text{s.t.} \quad & S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\} \\
& S_1 = 0
\end{aligned} \tag{1}
$$

$$
S_i = \max\left\{A_i, S_{i-1} + \xi_{i-1}\right\}, \tag{2}
$$

$$
S_i - A_i = \max\left\{0, (A_{i-1} + W_{i-1}) + \xi_{i-1} - A_i\right\}, \tag{3}
$$

$$
W_i = \left(W_{i-1} + \xi_{i-1} - X_{i-1}\right)^+. \tag{4}
$$

Slot time: $X_i = A_{i+1} - A_i \rightarrow A_j = \sum_{i=1}^{j-1} X_i$ Waiting time: $W_i = S_i - A_i \rightarrow S_n = \sum_{i=1}^{n-1} X_i + W_n$

$$
\begin{aligned}
\min_{\mathbf{X}} \quad & E_\xi \left[\sum_{i=1}^{n-1} (X_i - \xi_i) + W_n + \sum_{i=2}^{n} \alpha_i W_i + \beta (\sum_{i=1}^{n-1} X_i + W_n + \xi_n - T)^+ \right] \\
\text{s.t.} \quad & W_i = \max\{0, W_{i-1} + \xi_{i-1} - X_{i-1}\} \\
& W_1 = 0.
\end{aligned} \tag{5}
$$

Suppose that σ are the same for all patients.
Let

$$x_i = (X_i - \mu_i)/\sigma, \tag{6}$$

$$\zeta_i = (\xi_i - \mu_i)/\sigma, \text{ and} \tag{7}$$

$$w_i = W_i/\sigma; \tag{8}$$

Take out $-\sum_{i=1}^{n-1} \mu_i$.

$$
\begin{aligned}
\sigma \cdot \min_{\mathbf{x}} \quad & \left\{ \sum_{i=1}^{n-1} x_i + E_\zeta w_n + \sum_{i=2}^{n} \alpha_i E_\zeta [w_i] \right\} \\
\text{s.t.} \quad & w_i = \max\{0, w_{i-1} + \zeta_{i-1} - x_{i-1}\} \\
& w_1 = 0.
\end{aligned} \tag{9}
$$

Traditional: idle time + waiting time + (overtime)
Social distance: Ilde time + Overlap + Overtime
Conclusion: dome-shaped
Graph:

$$
\begin{aligned}
\min_{\mathbf{A}} \quad & E_\xi \left[\left(S_n - \sum_{i=1}^{n-1} \xi_i \right) + \sum_{i=2}^{n-1} \alpha_i \max(S_i - A_{i+1}, 0) \right] \\
\text{s.t.} \quad & S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\} \\
& S_1 = 0
\end{aligned} \tag{10}
$$

Consider: three people

$$
\begin{aligned}
\min_{\mathbf{A}} \quad & E_\xi \left[\left(S_n - \sum_{i=1}^{n-1} \xi_i \right) + \sum_{i=2}^{n-1} \alpha_i \max(S_i - A_{i+1}, 0) + \sum_{i=2}^{n-2} \beta_i \max(S_i - A_{i+2}, 0) \right] \\
\text{s.t.} \quad & S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\} \\
& S_1 = 0 \\
& \\
& \text{j-i: } \max(\min(S_i - A_j, 0) + A_j - A_{j-1}, 0)
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\min_{\mathbf{A}} \quad & E_{\xi}\left[\left(S_n - \sum_{i=1}^{n-1} \xi_i\right) + \sum_i \sum_j w'_{ij}\right] \\
\text{s.t.} \quad & S_i = \max\{A_i, S_{i-1} + \xi_{i-1}\} \\
& A_1 = S_1 = 0 \\
& w_{ij} = \max\{0, S_i - A_j\} \\
& w_{ij} = \sum_{t \le i \le j \le k} w'_{tk}
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
& w_{ij} \geq S_i - A_j \\
& w_{ij} \geq 0 \\
& w_{ij} \leq M \cdot (1 - bin) \\
& w_{ij} \leq S_i - A_j + M \cdot bin
\end{aligned}
$$