# Seat Planning and Seat Assignment with Social Distancing

April 10, 2024

LI, Zikang

IEDA
The Hong Kong University of Science and Technology

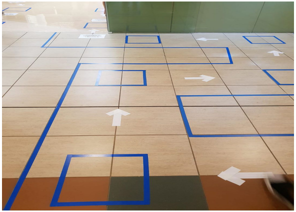
Supervisors: QI, Xiangtong; LIU, Qian

# Table of Contents

# Introduction

# Social Distancing under Pandemic

- Social distancing measures

# Social Distancing under Pandemic

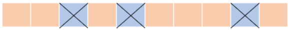
- Social distancing in seating areas

# Seat Planning and Seat Assignment

Social distancing requirement:

- The size of a group is confined.
- People in the same group sit together.
- Different groups should keep distance.

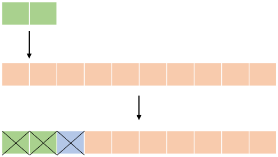
Seat planning:



Fixed seat planning

Flexible seat planning

Seat assignment:



# Situations

- Seat planning
  - Deterministic demand: make the seat planning with the known specific demand for each group.
  - Stochastic demand: make the seat planning with the known demand distribution before the realization of demand.
- Seat assignment with dynamic demand
  - Assign seats to each group under fixed seat planning.
  - Assign seats to each group under flexible seat planning.
  - Assign seats to each group after its realization.
  - Accept or reject each group after its realization, but assign them later.

# Literature Review

# Seat Planning with Social Distancing

- Allocation of seats on airplanes (Ghorbani et.al 2020), classroom layout planning (Bortolete et al. 2022), seat planning in long-distancing trains (Haque & Hamid 2022).
  - Social distancing can be enforced in different groups (Moore et al. 2021).
  - Seat planning for known groups in amphitheaters (Haque & Hamid 2022), airplanes (Salari et al. 2022), theater (Blom et al. 2022).

# Dynamic Seat Assignment

- Related to multiple knapsack problem (Pisinger et al. 1999) and dynamic knapsack problem (Kleywegt et al. 1998).
- Dynamic seat assignment on airplane (Notomista et al. 2016), train (Hamdouch et al. 2011).
- Assign-to-seat: dynamic capacity control for selling high-speed train tickets (Zhu et al. 2023).

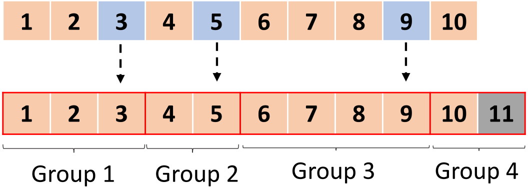
# Seat Planning with Social Distancing

- Group type $\mathcal{M} = \{1, \ldots, M\}$.
- Row $\mathcal{N} = \{1, \ldots, N\}$.
- The number of seats in row $j$: $L_j^0, j \in \mathcal{N}$.
- The social distancing: $\delta$ seat(s).
  - $n_i = i + \delta$: the size of group type $i$ for each $i \in \mathcal{M}$.
  - $L_j = L_j^0 + \delta$: the length of row $j$ for each $j \in \mathcal{N}$.

Figure: Problem Conversion with One Seat as Social Distancing

# Pattern

- Pattern: $\boldsymbol{h} = (h_1, \dots, h_M)$, the seat planning for one row, where $h_i$ is the number of group type $i$.
  - The maximum number of people accommodated: $|\boldsymbol{h}| = \sum_{i=1}^{M} i h_i$.




$$L = 11,\ \delta = 1,\ M = 4,\ n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5.$$
$$\boldsymbol{h} = (2, 1, 1, 0),\ |\boldsymbol{h}| = 7.$$

# Largest and Full Patterns

Suppose the length of the row is $L$.

- $\boldsymbol{h}$ is a feasible pattern if $\sum_{i=1}^{M} n_i h_i \leq L$.
- $\boldsymbol{h}$ is a largest pattern if $|\boldsymbol{h}| \geq |\boldsymbol{h}'|$ for any feasible $\boldsymbol{h}'$.
  $|\boldsymbol{h}| = qM + \max\{r - \delta, 0\}$, where $q = \lfloor L/n_M \rfloor$, $r \equiv L \bmod n_M$.
- $\boldsymbol{h}$ is a full pattern if $\sum_{i=1}^{M} n_i h_i = L$.

Example:

$\delta = 1$, $M = 4$, $L = 21$.

Largest patterns: $h_1 = (0,0,0,4)$, $h_2 = (0,0,4,1)$, $h_3 = (0,2,0,3)$.

Largest may not be full: $h_1 = (0,0,0,4)$. $\sum_{i=1}^{M} n_i h_i \neq L$

Full may not be largest: $h_4 = (1,1,4,0)$. $|h_4| = 15 < 16 = |h_1|$

# Seat Planning with Deterministic Demand

# Deterministic Formulation

Seat planning problem with given demand $\boldsymbol{d}$:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M} \sum_{j=1}^{N} (n_i - \delta) x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \leq d_i, \quad i \in \mathcal{M}, \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N}, \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned} \tag{1}
$$

Objective: maximize the number of people accommodated.

$x_{ij}$: the number of group type $i$ in row $j$.

# Property

In the LP relaxation of problem (1), there is a threshold $\tilde{i} \in \{0, 1, \ldots, M\}$ such that the optimal solutions satisfy the following conditions:

- For $i = 1, \ldots, \tilde{i} - 1$ and all $j$, $x_{ij}^* = 0$.
- For $i = \tilde{i} + 1, \ldots, M$, $\sum_j x_{ij}^* = d_i$.
- For $i = \tilde{i}$, $\sum_j x_{ij}^* = \frac{L - \sum_{i=\tilde{i}+1}^{M} d_i n_i}{n_{\tilde{i}}}$

The seat planning obtained from problem (1) may not utilize all seats.

We aim to improve the seat planning utilizing all seats.

# Generate the Full or Largest Patterns

Original seat planning: $\boldsymbol{H}$.
Desired feasible seat planning: $\boldsymbol{H}'$.
Meet the original group type requirements:
$\sum_{k=i}^{M}\sum_{j=1}^{N} H_{ji}^{'} \geq \sum_{k=i}^{M}\sum_{j=1}^{N} H_{ji}, \forall i \in \mathcal{M}$.

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(n_i-\delta)x_{ij} \\
s.t. \quad & \sum_{j=1}^{N}\sum_{k=i}^{M} x_{kj} \geq \sum_{k=i}^{M}\sum_{j=1}^{N} H_{ji}, i \in \mathcal{M} \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& x_{ij} \in \mathbb{N}, i \in \mathcal{M}, j \in \mathcal{N}
\end{aligned}
\tag{2}
$$

$\boldsymbol{H}'$ is composed of full or largest patterns.

# Seat Planning with Stochastic Demand

# Method Flow

We aim to obtain a seat planning with known demand scenarios.

- Build the formulation of scenario-based stochastic programming (SSP).
  - Consider the nested relation: a smaller group can take the seats planned for the larger group.
- Reformulate SSP to the benders master problem (BMP) and subproblem.
- The optimal solution can be obtained by solving BMP iteratively.

# Scenario-based Stochastic Programming (SSP)

Objective: maximize the expected number of people
$y^{+}_{i\omega}$: excess supply for $i, \omega$. $y^{-}_{i\omega}$: shortage of supply for $i, \omega$.
$d_{i\omega}$: demand of group type $i$ for scenario $\omega$

$$
\begin{aligned}
\max \quad & E_{\omega}\left[\sum_{i=1}^{M-1}(n_i-\delta)(\sum_{j=1}^{N} x_{ij} + y^{+}_{i+1,\omega} - y^{+}_{i\omega}) + (n_M-\delta)(\sum_{j=1}^{N} x_{Mj} - y^{+}_{M\omega})\right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y^{+}_{i\omega} + y^{+}_{i+1,\omega} + y^{-}_{i\omega} = d_{i\omega}, \quad i = 1,\ldots,M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y^{+}_{i\omega} + y^{-}_{i\omega} = d_{i\omega}, \quad i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& y^{+}_{i\omega}, y^{-}_{i\omega} \in \mathbb{N}, \quad i \in \mathcal{M}, \omega \in \Omega \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned} \tag{3}
$$

# Reformulation

Problem (3) is equivalent to the following master problem

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} + z(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{n}\mathbf{x} \leq \mathbf{L} \\ & \mathbf{x} \in \mathbb{N}^{M \times N, +}, \end{aligned} \tag{4}$$

where $z(\mathbf{x})$ is defined as

$$z(\mathbf{x}) := E(z_\omega(\mathbf{x})) = \sum_{\omega \in \Omega} p_\omega z_\omega(\mathbf{x}),$$

and for each scenario $\omega \in \Omega$, we have the subproblem

$$\begin{aligned} z_\omega(\mathbf{x}) := \max \quad & \mathbf{f}^\intercal \mathbf{y} \\ \text{s.t.} \quad & \mathbf{x}\mathbf{1} + \mathbf{V}\mathbf{y} = \mathbf{d}_\omega \\ & \mathbf{y} \geq 0. \end{aligned} \tag{5}$$

$\mathbf{c}^\intercal \mathbf{x} = \sum_{j=1}^{N} \sum_{i=1}^{M} i x_{ij}$, $\mathbf{f}^\intercal \mathbf{y} = -\sum_{i=1}^{M} y_{i\omega}^{+}$, $\mathbf{V}$ is the system matrix for $\mathbf{y}$.

# Solution to Subproblem

Problem (5) is easy to solve with a given $\mathbf{x}$ from the perspective of the dual problem:

$$\begin{aligned} \min \quad & \alpha_\omega^\intercal(\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \\ \text{s.t.} \quad & \alpha_\omega^\intercal \mathbf{V} \geq \mathbf{f}^\intercal \end{aligned} \tag{6}$$

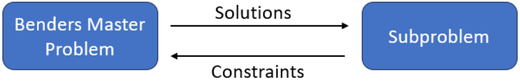
- The feasible region of problem (6), $P = \{\alpha | \alpha^\intercal \mathbf{V} \geq \mathbf{f}^\intercal\}$, is bounded. In addition, all the extreme points of $P$ are integral.
- The optimal solution to this problem can be obtained directly according to the complementary slackness property.

# Benders Decomposition Procedure

Let $z_\omega$ be the lower bound of problem (6), SSP can be obtained by solving following benders master problem:

$$\begin{aligned}
\max \quad & \mathbf{c}^\intercal \mathbf{x} + \sum_{\omega \in \Omega} p_\omega z_\omega \\
\text{s.t.} \quad & \mathbf{x}^\intercal \mathbf{n} \leq \mathbf{L} \\
& (\alpha^k)^\intercal (\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \geq z_\omega, \alpha^k \in \mathcal{O}, \forall \omega \\
& \mathbf{x} \in \mathbb{N}^{M \times N, +}
\end{aligned} \tag{7}$$

Constraints will be generated from problem (6) until an optimal solution is found.



# Running Time of Benders Decomposition and IP

Parameters:
(150, 350): the demand of each group type is randomly sampled from (150, 350).
(21, 50): the number of seats of each row is randomly sampled from (21, 50).

| # of scenarios | Demands | # of rows | # of groups | # of seats | Running time of IP(s) | Benders (s) |
|---|---|---|---|---|---|---|
| 1000 | (150, 350) | 30 | 8 | (21, 50) | 4.12 | 0.13 |
| 5000 | | | | | 28.73 | 0.29 |
| 10000 | | | | | 66.81 | 0.54 |
| 50000 | | | | | 925.17 | 2.46 |
| 1000 | (1000, 2000) | 200 | 8 | (21, 50) | 5.88 | 0.18 |
| 5000 | | | | | 30.0 | 0.42 |
| 10000 | | | | | 64.41 | 0.62 |
| 50000 | | | | | 365.57 | 2.51 |
| 1000 | (150, 250) | 30 | 16 | (41, 60) | 17.15 | 0.18 |
| 5000 | | | | | 105.2 | 0.37 |
| 10000 | | | | | 260.88 | 0.65 |
| 50000 | | | | | 3873.16 | 2.95 |

# Obtain Seat Planning Composed of Full or Largest Patterns

In some cases, it is time-consuming to obtain the optimal solution to SSP.

Meanwhile, there exists an optimal solution to SSP such that the patterns associated with this optimal solution are composed of the full or largest patterns under any given scenarios.

We aim to obtain a seat planning composed of full or largest patterns by the optimal solution to the LP relaxation of SSP.

- Obtain the solution to the relaxed SSP, $\mathbf{x}^*$, by benders decomposition. Aggregate $\mathbf{x}^*$ to the number of each group type, $s_i = \sum_j x_{ij}^*, i \in \mathbf{M}$.
- Obtain the optimal solution, $\mathbf{x}^1$, by solving problem (1) with $d_i = s_i$.
- Construct the full or largest patterns with $\mathbf{x}^1$.

# Seat Assignment with Dynamic Demand

# Dynamic Demand

- There is at most one group arrival at each period, $t = 1, \ldots, T$.
- The probability of an arrival of group type $i$: $p_i$.

1. Assign the seats under the fixed seat planning

The seats, which were arranged for social distancing purposes, need to be dismantled before people arrive to prevent them from occupying those seats. When each group arrives, we make decisions regarding whether to accept or reject them based on the predetermined seat planning.

2. Assign the seats under the flexible seat planning

Scenario 1: the seller need to assign seats each time a group arrives.

Scenario 2: the seller only needs to accept or reject the group's request and then assign the seats after the selling period.

# Seat Assignment under Fixed Seat Planning

- Group-type Control

- Obtain the seat planning from stochastic programming. Suppose the corresponding supply is $[X_1, \ldots, X_M]$. $(X_i = \sum_j x_{ij}, \forall i)$

  For the arrival of group type $i$,

- if $X_i > 0$, accept it directly, assign it the seats planned for group type $i$;

- if $X_i = 0$, determine which group type to accept it.

$P(D_i^{T-t} \geq X_i)$ is the probability that the demand of group type $i$ in $(T-t)$ periods is no less than $X_i$.

$D_j^t$ is a random variable indicating the number of group type $j$ in $t$ periods.

$$d^t(i, \hat{i}) = \underbrace{i + (\hat{i} - i - \delta) P(D_{\hat{i}-i-\delta}^{T-t} \geq X_{\hat{i}-i-\delta} + 1)}_{\text{acceptance}} - \underbrace{\hat{i} P(D_{\hat{i}}^{T-t} \geq X_{\hat{i}})}_{\text{rejection}}$$

For all $\hat{i} > i$, find the maximum value denoted as $d^t(i, \hat{i}^*)$.
If $d^t(i, \hat{i}^*) \geq 0$, we place the group of $i$ in $(\hat{i}^* + \delta)$-size seats. Otherwise, reject the group.

# Performance

The assignment is based on the fixed seat planning and we use the group-type control to make the decision.
$M = 4,\ \delta = 1,\ L_j = 21, j \in \mathcal{N},\ p_0 = 0,\ |\Omega| = 1000.$

| T | Probabilities | # of rows | SSP (%) | Expected demand (%) |
|---|---|---|---|---|
| 70 | [0.25, 0.25, 0.25, 0.25] | 10 | 94.97 | 94.71 |
| 80 | | | 96.48 | 96.16 |
| 90 | | | 97.94 | 97.36 |
| 100 | | | 98.91 | 96.27 |
| 70 | [0.25, 0.35, 0.05, 0.35] | 10 | 95.90 | 95.60 |
| 80 | | | 97.06 | 96.69 |
| 90 | | | 98.58 | 98.58 |
| 100 | | | 99.47 | 95.97 |
| 70 | [0.15, 0.25, 0.55, 0.05] | 10 | 97.41 | 96.70 |
| 80 | | | 98.85 | 96.06 |
| 90 | | | 98.73 | 97.63 |
| 100 | | | 98.46 | 98.19 |
| 140 | [0.25, 0.25, 0.25, 0.25] | 20 | 95.83 | 95.78 |
| 160 | | | 97.46 | 96.89 |
| 180 | | | 99.05 | 96.42 |
| 200 | | | 99.74 | 97.57 |

# Real-time Seat Assignment

## Dynamic Seat Assignment Problem

- There is one and only one group arrival at each period, $t = 1, \ldots, T+1$.
- The probability of an arrival of group type $i$: $p_i$.
- $\mathbf{L}^t = (l_1, l_2, \ldots, l_N)$, where $l_j = 0, \ldots, L_j, j \in \mathcal{N}$: Remaining capacity.
- $u_{i,j}^t$: Decision. Assign group type $i$ to row $j$ at period $t$, $u_{i,j}^t = 1$.
- $U^t(\mathbf{L}^t) = \{u_{i,j}^t \in \{0,1\}, \forall i, j \,|\, \sum_{j=1}^{N} u_{i,j}^t \leq 1, \forall i, n_i u_{i,j}^t \mathbf{e}_j \leq \mathbf{L}, \forall i, j\}$.
- $\mathbf{e}_j$: Unit column vector with $j$-th element being 1.
- $V^t(\mathbf{L}^t)$: Value function at period $t$, given remaining capacity, $\mathbf{L}^t$.

$$V^t(\mathbf{L}^t) = \max_{u_{i,j}^t \in U^t(\mathbf{L}^t)} \left\{ \sum_{i=1}^{M} p_i \Big( \sum_{j=1}^{N} i u_{i,j}^t + V^{t+1}\big(\mathbf{L}^t - \sum_{j=1}^{N} n_i u_{i,j}^t \mathbf{e}_j\big)\Big) + p_0 V^{t+1}(\mathbf{L}^t) \right\}$$

# Proposed Methods

We make the decision under the flexible seat planning.

- Suppose the supply associated with the seat planning is $[X_1, \ldots, X_M]$.

  For the arriving group type $i$,

- if $X_i > 0$, accept the group, assign it according to the break tie rule;
- if $X_i = 0$, two approaches:

  1. Based on the adjusted SSP.
  2. Based on the seat planning from the LP relaxation of SSP.

# Method 1

To determine the appropriate row assignment for the current group type $i'$, we introduce the decision variables $I_j, j \in \mathcal{N}$ indicating whether we accept it in row $j$.

$$
\begin{aligned}
\max \quad & \sum_j i' I_j + E_\omega \left[ \sum_{i=1}^{M-1} (n_i - \delta)(\sum_{j=1}^{N} x_{ij} + y^+_{i+1,\omega} - y^+_{i\omega}) + (n_M - \delta)(\sum_{j=1}^{N} x_{Mj} - y^+_{M\omega}) \right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y^+_{i\omega} + y^+_{i+1,\omega} + y^-_{i\omega} = d_{i\omega}, \quad i = 1, \ldots, M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y^+_{i\omega} + y^-_{i\omega} = d_{i\omega}, \quad i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j - n_{i'} I_j, j \in \mathcal{N} \\
& \sum_{j=1}^{N} I_j \leq 1 \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}, y^+_{i\omega}, y^-_{i\omega} \in \mathbb{N}, \quad i \in \mathcal{M}, \omega \in \Omega, a_j \in \{0,1\}, j \in \mathcal{N}.
\end{aligned} \tag{8}
$$

- If $X_i = 0$, solve problem (8), make the decision and update the seat planning.

# Method 2: Dynamic Seat Assignment (DSA)

If $X_i = 0$,

1. Determine the group type $\hat{i}^*$ by the group-type control.
2. Make the decision on assigning the group to a specific row.
   - Break tie for determining a specific row
   - Decision on assigning the group
   - Value of Acceptance (VoA): value of LP relaxation of SSP with $(\mathbf{L} - n_i \mathbf{e}_{\hat{i}^*})$ plus $i$.
   - Value of Rejection (VoR): value of LP relaxation of SSP with $\mathbf{L}$.
   - If VoA is no less than VoR, accept group type $i$; otherwise, reject it.

Regenerate the seat planning

- When $X_M = 0$
- When comparing VoA and VoR

# Compared with Other Policies

We have the following policies compared with DSA when we make the instant allocation.

- Bid-price control
- Dynamic programming based heuristic
- Booking limit control
- First come first served
- Optimal: full knowledge of demands

# Bid-price Control

The dual problem of LP relaxation of problem (1) is:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{M} d_i z_i + \sum_{j=1}^{N} L_j \beta_j \\
\text{s.t.} \quad & z_i + \beta_j n_i \geq (n_i - \delta), \quad i \in \mathcal{M}, j \in \mathcal{N} \\
& z_i \geq 0, i \in \mathcal{M}, \beta_j \geq 0, j \in \mathcal{N}.
\end{aligned} \tag{9}
$$

The optimal solution to problem (9) is given by $z_1, \ldots, z_{\tilde{i}} = 0, z_i = \frac{\delta(n_i - n_{\tilde{i}})}{n_{\tilde{i}}}$, for $i = \tilde{i} + 1, \ldots, M$, $\beta_j = \frac{n_{\tilde{i}} - \delta}{n_{\tilde{i}}}$ for all $j$.

For the group type $i$, if $i - \beta_j n_i \geq 0$, accept it.

# Dynamic Programming Based Heuristic

- Relax all rows to one row with the same capacity by $L = \sum_{j=1}^{N} L_j$.
  - Deterministic problem: $\{\max \sum_{i=1}^{M} (n_i - \delta) x_i : x_i \leq d_i, i \in \mathcal{M}, \sum_{i=1}^{M} n_i x_i \leq L, x_i \in \mathbb{Z}_0^+\}$.
- Decision: $u^t$. If we accept a request in period $t$, $u^t = 1$; otherwise, $u^t = 0$.
  - DP with one row can be expressed as:

$$V^t(l) = \max_{u^t \in \{0,1\}} \left\{ \sum_i p_i [V^{t+1}(l - n_i u^t) + i u^t] + p_0 V^{t+1}(l) \right\}, l \geq 0$$

$$V^{T+1}(l) = 0, \forall l.$$

- After accepting one group, assign it in some row arbitrarily when the capacity of the row allows.

# Booking limit Control

Basic idea: for each type of requests, we only allocate a fixed amount according to the static solution and reject all other exceeding requests.

1. Observe the arrival group type $i$.
2. Solve problem (1) using the expected demand.
3. Obtain the optimal solution, $x_{ij}^*$ and the aggregate optimal solution, $\mathbf{X}$.
4. If $X_i > 0$, accept the arrival and assign the group to row $k$ where $x_{ik} > 0$, update $\mathbf{L}^{t+1} = \mathbf{L}^t - n_i \mathbf{e}_k$; otherwise, reject it, let $\mathbf{L}^{t+1} = \mathbf{L}^t$.

# Performances of Different Policies

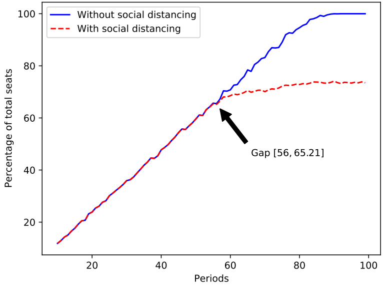
$M = 4,\ \delta = 1,\ N = 10,\ L_j = 21, j \in \mathcal{N},\ p_0 = 0,\ |\Omega| = 1000.$

| T | Probabilities | DSA (%) | DP1 (%) | Bid (%) | Booking (%) | FCFS (%) |
|---|---|---|---|---|---|---|
| 60 | [0.25, 0.25, 0.25, 0.25] | 99.12 | 98.42 | 98.38 | 96.74 | 98.17 |
| 70 | | 98.34 | 96.87 | 96.24 | 97.18 | 94.75 |
| 80 | | 98.61 | 95.69 | 96.02 | 98.00 | 93.18 |
| 90 | | 99.10 | 96.05 | 96.41 | 98.31 | 92.48 |
| 100 | | 99.58 | 95.09 | 96.88 | 98.70 | 92.54 |
| 60 | [0.25, 0.35, 0.05, 0.35] | 98.94 | 98.26 | 98.25 | 96.74 | 98.62 |
| 70 | | 98.05 | 96.62 | 96.06 | 96.90 | 93.96 |
| 80 | | 98.37 | 96.01 | 95.89 | 97.75 | 92.88 |
| 90 | | 99.01 | 96.77 | 96.62 | 98.42 | 92.46 |
| 100 | | 99.23 | 97.04 | 97.14 | 98.67 | 92.00 |
| 60 | [0.15, 0.25, 0.55, 0.05] | 99.14 | 98.72 | 98.74 | 96.61 | 98.07 |
| 70 | | 99.30 | 96.38 | 96.90 | 97.88 | 96.25 |
| 80 | | 99.59 | 97.75 | 97.87 | 98.55 | 95.81 |
| 90 | | 99.53 | 98.45 | 98.69 | 98.81 | 95.50 |
| 100 | | 99.47 | 98.62 | 98.94 | 98.90 | 95.25 |

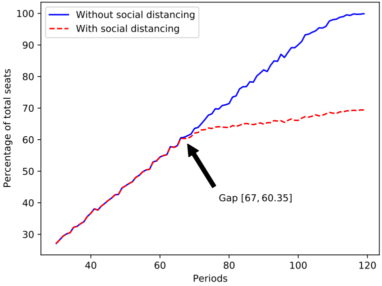
DSA has better performance than other policies under different demands.

# Impact of Social Distancing as Demand Increases

$\gamma = p_1 * 1 + p_2 * 2 + p_3 * 3 + p_4 * 4$: the expected number of people at each period.




(a) When $\gamma = 2.5$




(b) When $\gamma = 1.9$

The gap point represents the first period where the number of people without social distancing is larger than that with social distancing and the gap percentage is the corresponding percentage of total seats.

# Estimation of Gap Point

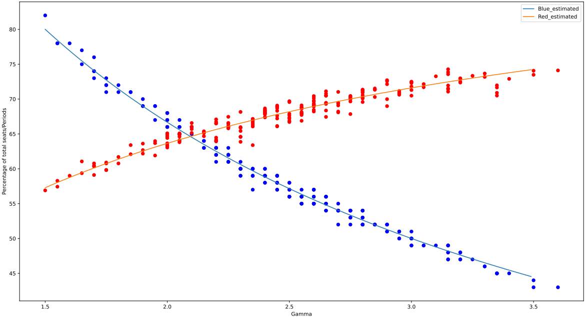



Figure: Gap points with 200 probabilities

Blue points: period of the gap point. Red points: occupancy rate of the gap point. Gap points can be estimated.

# Make A Later Allocation

This setting is particularly applicable to larger venues, such as stadiums, where an immediate decision is made when a group arrives, but the actual allocation of seats for that group is deferred to a later time.

The critical part is to make the decision, thus, we choose the following policies associated with relaxation forms.

Policies:

- Dynamic programming based heuristic
- Bid-price control

# Performances of Different Policies

| T | Probabilities | DP1-L (%) | Bid-L (%) | DP1 (%) | Bid (%) |
|---|---|---|---|---|---|
| 60 | [0.25, 0.25, 0.25, 0.25] | 99.52 | 99.44 | 98.42 | 98.38 |
| 70 | | 99.32 | 98.97 | 96.87 | 96.24 |
| 80 | | 99.34 | 99.30 | 95.69 | 96.02 |
| 90 | | 99.55 | 99.49 | 96.05 | 96.41 |
| 100 | | 99.78 | 99.66 | 95.09 | 96.88 |
| 60 | [0.25, 0.35, 0.05, 0.35] | 99.50 | 99.37 | 98.26 | 98.25 |
| 70 | | 99.40 | 98.97 | 96.62 | 96.06 |
| 80 | | 99.46 | 99.24 | 96.01 | 95.89 |
| 90 | | 99.59 | 99.35 | 96.77 | 96.62 |
| 100 | | 99.77 | 99.61 | 97.04 | 97.14 |
| 60 | [0.15, 0.25, 0.55, 0.05] | 99.57 | 99.54 | 98.72 | 98.74 |
| 70 | | 99.46 | 99.39 | 96.38 | 96.90 |
| 80 | | 99.50 | 99.30 | 97.75 | 97.87 |
| 90 | | 99.34 | 99.44 | 98.45 | 98.69 |
| 100 | | 99.34 | 99.55 | 98.62 | 98.94 |

# Conclusion and Future Work

# Conclusion and Future Work

Conclusion

- Address the problem of seat planning and assignment with social distancing.
- Provide a comprehensive solution for optimizing seat assignments under dynamic situation.
- Provide different methods in different scenarios and elaborate on the role of social distancing.

Future work

- Consider more flexible scenarios, people can choose the seats and leave randomly.
- Consider a scattered seat assignment when there are sufficient seats.

# Thank You!