Seat Planning and Seat Assignment with Social Distancing

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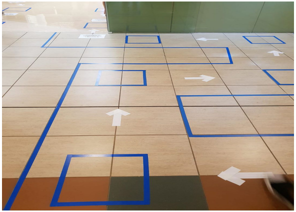
Social Distancing under Pandemic

- Social distancing measures



Social Distancing under Pandemic

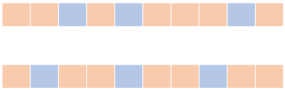
- Social distancing in seating areas

Seat Planning and Seat Assignment

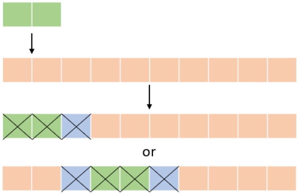
Social distancing requirement:

- The size of a group is confined.
- People in the same group sit together.
- Different groups should keep distance.

Seat planning:



Seat assignment:



Overview

- Seat planning
  - Deterministic demand: make the seat planning with the known specific demand for groups.
  - **Stochastic demand**: make the seat planning with the known demand distribution before the realization of demand.
- Seat assignment with dynamic demand
  - Assign seats to each group under flexible seat planning.
  - **Real-time seat assignment**
  - Accept or reject each group after its realization, but assign them later.
  - Assign seats to each group under fixed seat planning.

Contributions

Seat Planning

- New model and technique for stochastic demand
- Provide seat planning as a basis for seat assignment

Guidance →

Seat Assignment

- New model for seat assignment problem
- Provide practical policies and insights

Literature Review

Seat Planning with Social Distancing

- Applications:

  Airplanes seat planning (Ghorbani et.al 2020)
  Classroom layout planning (Bortolete et al. 2022)
  Trains seat planning (Haque & Hamid 2022).

- Group-based seat planning for known groups:

  Amphitheaters (Haque & Hamid 2022)
  Airplanes (Salari et al. 2022)
  Theaters (Blom et al. 2022).

  Our work considers seat planning with stochastic demand.

Dynamic Seat Assignment

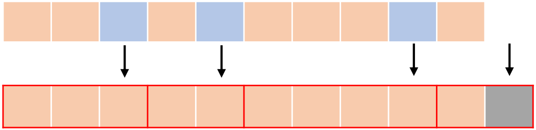
- Dynamic multiple knapsack problem:
  - Multiple knapsack problem (Pisinger et al. 1999)
  - Dynamic knapsack problem (Kleywegt et al. 1998)
- Network revenue management: Group-based (Talluri et al. 2006).
- Dynamic capacity control: Assign-to-seat for selling high-speed train tickets (Zhu et al. 2023)

Our work considers the group-based seat assignment.

Problem Description

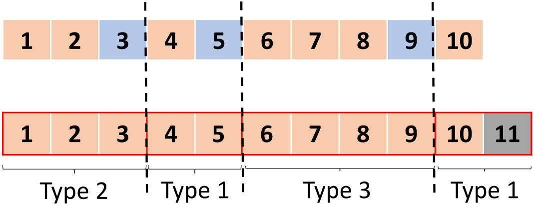
Seat Planning with Social Distancing

- Group type $\mathcal{M} = \{1, \ldots, M\}$.
- Row $\mathcal{N} = \{1, \ldots, N\}$.
- Number of seats in row j: $L_j^0, j \in \mathcal{N}$.
- Social distancing: δ seat(s).
  - $n_i = i + \delta$: the size of type i group for each $i \in \mathcal{M}$.
  - $L_j = L_j^0 + \delta$: the size of row j for each $j \in \mathcal{N}$.



Pattern

- Pattern: $\boldsymbol{h} = (h_1, \ldots, h_M)$, the seat planning for one row, where h_i is the number of type i groups.
  - Feasible pattern: $\sum_{i=1}^{M} n_i h_i \leq L$.
  - Maximum number of people accommodated: $|\boldsymbol{h}| = \sum_{i=1}^{M} i h_i$.



$$L = 11,\ \delta = 1,\ M = 4.$$
$$\boldsymbol{h} = (2, 1, 1, 0),\ |\boldsymbol{h}| = 7.$$

Largest and Full Patterns

- Largest pattern:
  $\boldsymbol{h}$ is a largest pattern if $|\boldsymbol{h}| \geq |\boldsymbol{h}'|$ for any feasible $\boldsymbol{h}'$.
  $|\boldsymbol{h}| = qM + \max\{r - \delta, 0\}$, where $q = \lfloor L/n_M \rfloor$, $r = L - q \cdot n_M$.
- Full pattern:
  $\boldsymbol{h}$ is a full pattern if $\sum_{i=1}^{M} n_i h_i = L$.

Example:

$\delta = 1,\ M = 4,\ L = 21.$

Largest patterns: $h_1 = (0,0,0,4),\ h_2 = (0,0,4,1),\ h_3 = (0,2,0,3),$
$|h_1| = |h_2| = |h_3| = 16.$

Largest but not full: $h_1 = (0,0,0,4).$ $\sum_{i=1}^{M} n_i h_i \neq L$

Full but not largest: $h_4 = (1,1,4,0).$ $|h_4| = 15 < 16$

Problem Formulation

Goal: obtain the seat planning composed of full or largest patterns while maximizing the number of people accommodated.

Seat planning problem with given demand $\boldsymbol{d} = (d_1, \ldots, d_M)$:

$$\begin{aligned} \max \quad & \sum_{i=1}^{M} \sum_{j=1}^{N} (n_i - \delta) x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} \leq d_i, \quad i \in \mathcal{M}, \\ & \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N}, \\ & x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}. \end{aligned} \tag{1}$$

Objective: maximize the number of people accommodated.

x_{ij}: the number of type i groups in row j.

Property

In the LP relaxation of problem (1), there is a threshold $\tilde{i} \in \{0, 1, \ldots, M\}$ such that the optimal solutions satisfy the following conditions:

- For $i = 1, \ldots, \tilde{i} - 1$ and all j, $x^*_{ij} = 0$.
- For $i = \tilde{i} + 1, \ldots, M$, $\sum_j x^*_{ij} = d_i$.
- For $i = \tilde{i}$, $\sum_j x^*_{ij} = \frac{L - \sum_{i=\tilde{i}+1}^{M} d_i n_i}{n_{\tilde{i}}}$

The seat planning obtained from problem (1) may not utilize all seats.

We aim to obtain the seat planning composed of full or largest patterns.

Generate the Full or Largest Patterns

Original seat planning: $\boldsymbol{H}$.
Desired feasible seat planning: $\boldsymbol{H}'$.

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{M}\sum_{j=1}^{N}(n_i-\delta)x_{ij} \\
s.t. \quad & \sum_{j=1}^{N}\sum_{k=i}^{M} x_{kj} \geq \sum_{k=i}^{M}\sum_{j=1}^{N} H_{ji}, i \in \mathcal{M} \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& x_{ij} \in \mathbb{N}, i \in \mathcal{M}, j \in \mathcal{N}
\end{aligned} \tag{2}
$$

Group type requirement:

$\sum_{j=1}^{N} H'_{jM} \geq \sum_{j=1}^{N} H_{jM}$,

$\sum_{j=1}^{N}(H'_{jM} + H'_{j,M-1}) \geq \sum_{j=1}^{N}(H_{jM} + H'_{j,M-1})$, ...,

$\sum_{j=1}^{N}(H'_{jM} + \ldots + H'_{j,1}) \geq \sum_{j=1}^{N}(H_{jM} + \ldots + H'_{j,1})$.

Property: $\boldsymbol{H}'$ corresponding to the optimal solution to problem (2) is composed of full or largest patterns.

Seat Planning with Stochastic Demand

Method Flow

We aim to obtain a seat planning with known demand scenarios.

- Build the formulation of Scenario-based Stochastic Programming (SSP).
  - Consider the nested relation: a smaller group can take the seats planned for the larger group.
- Reformulate SSP to the Benders Master Problem (BMP) and subproblem.
- The optimal solution can be obtained by solving BMP iteratively.

Scenario-based Stochastic Programming (SSP)

Objective: maximize the expected number of people
$y_{i\omega}^{+}$: excess supply for i, ω. $y_{i\omega}^{-}$: shortage of supply for i, ω.
$d_{i\omega}$: demand of type i groups for scenario ω

$$
\begin{aligned}
\max \quad & E_{\omega}\left[\sum_{i=1}^{M-1}(n_i-\delta)(\sum_{j=1}^{N} x_{ij}+y_{i+1,\omega}^{+}-y_{i\omega}^{+})+(n_M-\delta)(\sum_{j=1}^{N} x_{Mj}-y_{M\omega}^{+})\right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij}-y_{i\omega}^{+}+y_{i+1,\omega}^{+}+y_{i\omega}^{-}=d_{i\omega}, \quad i=1,\ldots,M-1, \omega\in\Omega \\
& \sum_{j=1}^{N} x_{ij}-y_{i\omega}^{+}+y_{i\omega}^{-}=d_{i\omega}, \quad i=M, \omega\in\Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j\in\mathcal{N} \\
& y_{i\omega}^{+}, y_{i\omega}^{-}\in\mathbb{N}, \quad i\in\mathcal{M}, \omega\in\Omega \\
& x_{ij}\in\mathbb{N}, \quad i\in\mathcal{M}, j\in\mathcal{N}.
\end{aligned} \tag{3}
$$

$E_{\omega}\left[\sum_{i=1}^{M-1}(n_i-\delta)(\sum_{j=1}^{N} x_{ij}+y_{i+1,\omega}^{+}-y_{i\omega}^{+})+(n_M-\delta)(\sum_{j=1}^{N} x_{Mj}-y_{M\omega}^{+})\right]=$
$\sum_{j=1}^{N}\sum_{i=1}^{M} i\cdot x_{ij}-\sum_{\omega=1}^{|\Omega|} p_{\omega}\sum_{i=1}^{M} y_{i\omega}^{+}$

Reformulation

$$
\begin{aligned}
\max \quad & \mathbf{c}^\intercal \mathbf{x} + \sum_{\omega \in \Omega} p_\omega \mathbf{f}^\intercal \mathbf{y}_\omega \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y_{i\omega}^{+} + y_{i+1,\omega}^{+} + y_{i\omega}^{-} = d_{i\omega}, \\
& i = 1, \ldots, M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y_{i\omega}^{+} + y_{i\omega}^{-} = d_{i\omega}, \\
& i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j, j \in \mathcal{N} \\
& y_{i\omega}^{+}, y_{i\omega}^{-} \in \mathbb{N}, \quad i \in \mathcal{M}, \omega \in \Omega \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}.
\end{aligned}
$$

$\mathbf{c}^\intercal \mathbf{x} = \sum_{j=1}^{N} \sum_{i=1}^{M} i x_{ij}$, $\mathbf{f}^\intercal \mathbf{y}_\omega = -\sum_{i=1}^{M} y_{i\omega}^{+}$.

SSP is equivalent to the following master problem

$$
\begin{aligned}
\max \quad & \mathbf{c}^\intercal \mathbf{x} + z(\mathbf{x}) \\
\text{s.t.} \quad & \mathbf{x}^\intercal \mathbf{n} \leq \mathbf{L} \\
& \mathbf{x} \in \mathbb{N}^{M \times N, +},
\end{aligned} \tag{4}
$$

where $z(\mathbf{x})$ is defined as

$$
z(\mathbf{x}) := E(z_\omega(\mathbf{x})) = \sum_{\omega \in \Omega} p_\omega z_\omega(\mathbf{x}),
$$

and for each scenario $\omega \in \Omega$, the subproblem

$$
\begin{aligned}
z_\omega(\mathbf{x}) := \max \quad & \mathbf{f}^\intercal \mathbf{y}_\omega \\
\text{s.t.} \quad & \mathbf{x}\mathbf{1} + \mathbf{V}\mathbf{y}_\omega = \mathbf{d}_\omega \\
& \mathbf{y} \geq 0.
\end{aligned} \tag{5}
$$

Solution to Subproblem

Problem (5) is easy to solve with a given $\mathbf{x}$ from the perspective of the dual problem:

$$\begin{aligned} \min \quad & \alpha_\omega^\intercal(\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \\ \text{s.t.} \quad & \alpha_\omega^\intercal \mathbf{V} \geq \mathbf{f}^\intercal \end{aligned} \tag{6}$$

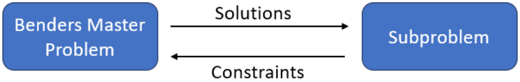
- The feasible region of problem (6), $P = \{\alpha | \alpha^\intercal \mathbf{V} \geq \mathbf{f}^\intercal\}$, is bounded. In addition, all the extreme points of P are integral.
- The optimal solution can be obtained directly according to the complementary slackness property.

Benders Decomposition Procedure

Let z_ω be the lower bound of problem (6), SSP can be obtained by solving following benders master problem:

$$
\begin{aligned}
\max \quad & \mathbf{c}^\intercal \mathbf{x} + \sum_{\omega \in \Omega} p_\omega z_\omega \\
\text{s.t.} \quad & \mathbf{x}^\intercal \mathbf{n} \leq \mathbf{L} \\
& (\alpha^k)^\intercal (\mathbf{d}_\omega - \mathbf{x}\mathbf{1}) \geq z_\omega, \alpha^k \in \mathcal{O}, \forall \omega \\
& \mathbf{x} \in \mathbb{N}^{M \times N, +}
\end{aligned}
\tag{7}
$$

Constraints will be generated from problem (6) until an optimal solution is found.



Obtain Seat Planning Composed of Full or Largest Patterns

Property: there exists an optimal solution to SSP such that the patterns associated with this optimal solution are composed of the full or largest patterns under any given scenarios.

- Obtain the solution to the LP relaxation of SSP, $\mathbf{x}^*$.
- Obtain a fesible seat planning, $\boldsymbol{H}$, by solving problem (1) with $d_i = \sum_j x^*_{ij}$.
- Obtain full or largest patterns by solving problem (2) with $\boldsymbol{H}$.

Seat Assignment with Dynamic Demand

Real-time Seat Assignment

- There is at most one group arrival at each period, $t = 1, \ldots, T$.
- The probability of an arrival of type i: p_i.
- $\mathbf{L} = (l_1, l_2, \ldots, l_N)$, where $l_j = 0, \ldots, L_j, j \in \mathcal{N}$: Remaining capacity.
- $u_{i,j}^t$: Decision. Assign type i to row j at period t, $u_{i,j}^t = 1$.
- $U^t(\mathbf{L}) = \{u_{i,j}^t \in \{0,1\}, \forall i, j \,|\, \sum_{j=1}^N u_{i,j}^t \leq 1, \forall i, n_i u_{i,j}^t \mathbf{e}_j \leq \mathbf{L}, \forall i, j\}$.
- $\mathbf{e}_j$: Unit column vector with j-th element being 1.
- $V^t(\mathbf{L})$: Value function at period t, given remaining capacity, $\mathbf{L}$.

$$V^t(\mathbf{L}) = \max_{u_{i,j}^t \in U^t(\mathbf{L})} \left\{ \sum_{i=1}^M p_i (\sum_{j=1}^N i u_{i,j}^t + V^{t+1}(\mathbf{L} - \sum_{j=1}^N n_i u_{i,j}^t \mathbf{e}_j)) + p_0 V^{t+1}(\mathbf{L}) \right\}$$

Seat Assignment under The Seat Planning

- Assign the seats under the flexible seat planning

  Situation 1: **real-time seat assignment**

  Situation 2: accept or reject each arriving group, then assign the seats after all groups arrive.

- Assign the seats under the fixed seat planning

  The seats will be dismantled before people arrive to prevent them from occupying those seats.

Proposed Methods to Real-time Seat Assignment

- Suppose the supply associated with the seat planning is $[X_1, \ldots, X_M]$. ($X_i = \sum_j x_{ij}, \forall i$)

  For the arriving type i group,

  - if $X_i > 0$, accept the group, assign it by the tie-breaking rule (full pattern first);
  - if $X_i = 0$, two methods:

  1. Based on the modified SSP.
  2. Based on the seat planning from the LP relaxation of SSP.
     - Determine the possible type $\hat{i}^* > i$ by group-type control
     - Decision on assigning the group to a specific row

Method 1: Modified SSP

Introduce the decision variables $I_j, j \in \mathcal{N}$ indicating whether we accept the arriving type i' in row j.

$$\begin{aligned}
\max \quad & \sum_j i' I_j + E_\omega \left[\sum_{i=1}^{M-1} (n_i - \delta)(\sum_{j=1}^{N} x_{ij} + y^+_{i+1,\omega} - y^+_{i\omega}) + (n_M - \delta)(\sum_{j=1}^{N} x_{Mj} - y^+_{M\omega}) \right] \\
\text{s.t.} \quad & \sum_{j=1}^{N} x_{ij} - y^+_{i\omega} + y^+_{i+1,\omega} + y^-_{i\omega} = d_{i\omega}, \quad i = 1, \ldots, M-1, \omega \in \Omega \\
& \sum_{j=1}^{N} x_{ij} - y^+_{i\omega} + y^-_{i\omega} = d_{i\omega}, \quad i = M, \omega \in \Omega \\
& \sum_{i=1}^{M} n_i x_{ij} \leq L_j - n_{i'} I_j, j \in \mathcal{N} \\
& \sum_{j=1}^{N} I_j \leq 1 \\
& x_{ij} \in \mathbb{N}, \quad i \in \mathcal{M}, j \in \mathcal{N}, y^+_{i\omega}, y^-_{i\omega} \in \mathbb{N}, \quad i \in \mathcal{M}, \omega \in \Omega, I_j \in \{0,1\}, j \in \mathcal{N}.
\end{aligned} \tag{8}$$

Intuitive but not efficient.

Method 2: Dynamic Seat Assignment (DSA)

For the arriving type i, if $X_i = 0$:

Step 1. Determine the type $\hat{i}^* > i$ by the group-type control.

Step 2. Decision on assigning type i to a row containing type $\hat{i}^*$.

Method 2: Dynamic Seat Assignment (DSA)

1. Group-type control: determine the type $\hat{i}^*$ to place arriving type i.

Recall that the supply is $[X_1, \ldots, X_M]$.

$P(D_i^{T-t} \geq X_i)$ is the probability that the demand of group type i in $(T-t)$ periods is no less than X_i.

D_j^t is a random variable indicating the number of group type j in t periods.

$$d^t(i, \hat{i}) = \underbrace{i + (\hat{i} - i - \delta) P(D_{\hat{i}-i-\delta}^{T-t} \geq X_{\hat{i}-i-\delta} + 1)}_{\text{acceptance}} - \underbrace{\hat{i} P(D_{\hat{i}}^{T-t} \geq X_{\hat{i}})}_{\text{rejection}}$$

For all $\hat{i} > i$, find the maximum value denoted as $d^t(i, \hat{i}^*)$.

If $d^t(i, \hat{i}^*) \geq 0$, we place the type i in the seats planned for type $\hat{i}^*$. Otherwise, reject the group.

Method 2: Dynamic Seat Assignment (DSA)

2. Decision on assigning the group to a specific row.

- Determine the specific row $\hat{j}$ including type $\hat{i}^*$ by tie-breaking rule (Non-full pattern first).
- Whether to assign the group in row $\hat{j}$
  - Value of Acceptance (VoA): value of LP relaxation of SSP with $(\mathbf{L} - n_i \mathbf{e}_{\hat{j}})$ plus i.
  - Value of Rejection (VoR): value of LP relaxation of SSP with $\mathbf{L}$.
  - If VoA is no less than VoR, accept type i; otherwise, reject it.

Compared with Other Policies

We compare DSA with the following policies

- Bid-price control
- Dynamic programming based heuristic
- Booking limit control
- First come first served

- Benchmark:
  Offline optimal solution with knowing demands before decision

Bid-price Control

The dual problem of LP relaxation of problem (1) is:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{M} d_i z_i + \sum_{j=1}^{N} L_j \beta_j \\
\text{s.t.} \quad & z_i + \beta_j n_i \geq (n_i - \delta), \quad i \in \mathcal{M}, j \in \mathcal{N} \\
& z_i \geq 0, i \in \mathcal{M}, \beta_j \geq 0, j \in \mathcal{N}.
\end{aligned} \tag{9}
$$

The optimal solution to problem (9) is given by $z_1, \ldots, z_{\tilde{i}} = 0, z_i = \frac{\delta(n_i - n_{\tilde{i}})}{n_{\tilde{i}}}$, for $i = \tilde{i}+1, \ldots, M$, $\beta_j = \frac{n_{\tilde{i}} - \delta}{n_{\tilde{i}}}$ for all j.

For type i, if $i - \beta_j n_i \geq 0$, accept it; otherwise, reject it.

Dynamic Programming Based Heuristic

- Relax all rows to one row with the same capacity by $\tilde{L} = \sum_{j=1}^{N} L_j$.
  - Deterministic problem: $\{\max \sum_{i=1}^{M} (n_i - \delta) x_i : x_i \leq d_i, i \in \mathcal{M}, \sum_{i=1}^{M} n_i x_i \leq \tilde{L}, x_i \in \mathbb{Z}_0^+\}$.
- Decision: u^t. If we accept a request in period t, $u^t = 1$; otherwise, $u^t = 0$.
  - DP with one row can be expressed as:

$$V^t(l) = \max_{u^t \in \{0,1\}} \left\{ \sum_i p_i [V^{t+1}(l - n_i u^t) + i u^t] + p_0 V^{t+1}(l) \right\}$$

$$V^{T+1}(l) = 0, \forall l.$$

- After accepting one group, assign it in some row arbitrarily when the capacity of the row allows.

Booking limit Control

Basic idea: for each group type, we only allocate a fixed amount according to the static solution and reject all other exceeding requests.

1. Observe the arriving type i.
2. Solve problem (1) with $d_i = p_i \times T, i \in \mathcal{M}$.
3. Obtain the optimal solution, x^*_{ij} and the aggregate optimal solution, $\mathbf{X}$.
4. If $X_i > 0$, accept the arrival and assign the group to row k where $x_{ik} > 0$, update $\mathbf{L}^{t+1} = \mathbf{L}^t - n_i \mathbf{e}_k$; otherwise, reject it, let $\mathbf{L}^{t+1} = \mathbf{L}^t$.

Parameters Description

$M = 4$, $\delta = 1$, $|\Omega| = 1000$, $N = 10$, $L_j = 21, j \in \mathcal{N}$.

Consider three sets of probability distributions with the same expectation of demand each period:

D1: [0.25, 0.25, 0.25, 0.25]

D2: [0.25, 0.35, 0.05, 0.35]

D3: [0.15, 0.25, 0.55, 0.05]

Each entry in the result columns is the average of 100 instances.

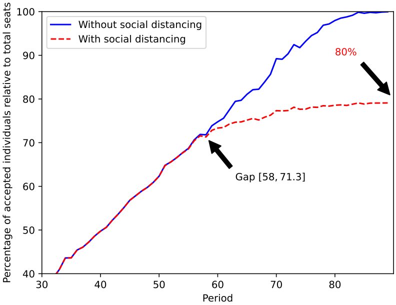
Performances of Different Policies

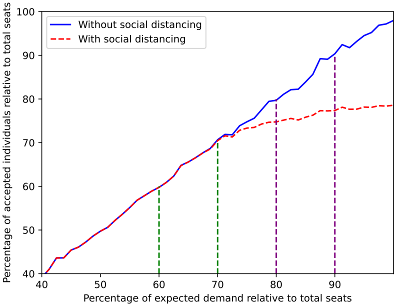
Distribution	T	DSA (%)	DP1 (%)	Bid (%)	Booking (%)	FCFS (%)
D1	60	99.12	98.42	98.38	96.74	98.17
	70	98.34	96.87	96.24	97.18	94.75
	80	98.61	95.69	96.02	98.00	93.18
	90	99.10	96.05	96.41	98.31	92.48
	100	99.58	95.09	96.88	98.70	92.54
D2	60	98.94	98.26	98.25	96.74	98.62
	70	98.05	96.62	96.06	96.90	93.96
	80	98.37	96.01	95.89	97.75	92.88
	90	99.01	96.77	96.62	98.42	92.46
	100	99.23	97.04	97.14	98.67	92.00
D3	60	99.14	98.72	98.74	96.61	98.07
	70	99.30	96.38	96.90	97.88	96.25
	80	99.59	97.75	97.87	98.55	95.81
	90	99.53	98.45	98.69	98.81	95.50
	100	99.47	98.62	98.94	98.90	95.25

DSA has better performance than other policies under different demands.

Impact of Social Distancing As Demand Increases

When the probability distribution is $[0.25, 0.25, 0.25, 0.25]$





- Gap point: the first period when there is a difference
- The largest occupancy rate is 80% by the property of the largest pattern.
- Demand less than 71.3% total seats: no difference
- Demand larger than 71.3% total seats: difference becomes larger

Gap Points and Occupancy Rates under Different Probability Distributions

$\gamma = p_1 * 1 + p_2 * 2 + p_3 * 3 + p_4 * 4$: the expected number of people at each period.

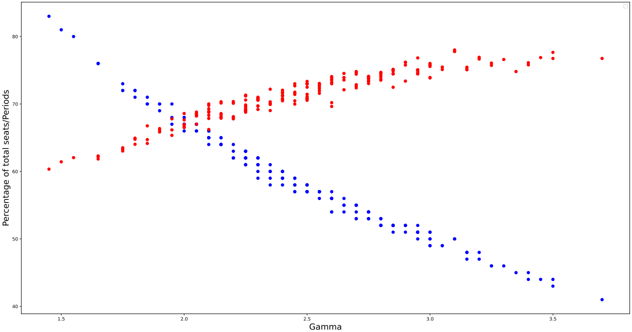


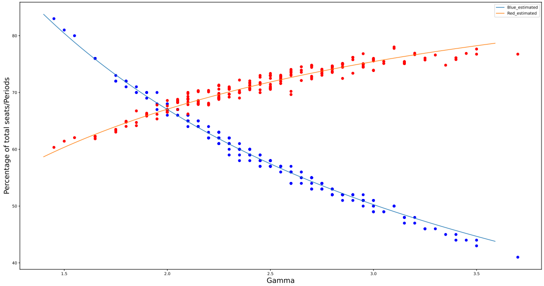
Figure: Gap points and occupancy rates with 200 probability distributions

Blue points: period of the gap point.

Red points: occupancy rate of the gap point.

Difference is small under the same gamma.

Estimation of Gap Points and Occupancy Rates



Ideal: the groups before gap point can occupy all seats.

Gap points can be estimated with γ.

$y_1 = \frac{c_1 \tilde{L}}{\gamma+\delta}$, $y_2 = c_2 \frac{\gamma}{\gamma+\delta} \frac{\tilde{L}}{\tilde{L}-N\delta}$

c_1, c_2 can be seen as the discount factor compared to the ideal situation.

The larger c_1, c_2 are, the closer to the ideal situation.

Seat layout(# of rows $\times$ # of seats)	Fitting Values of c_1	Fitting Values of c_2
10 $\times$ 11	0.909 $\pm$ 0.013	89.89 $\pm$ 1.436
10 $\times$ 16	0.948 $\pm$ 0.008	94.69 $\pm$ 0.802
10 $\times$ 21	0.955 $\pm$ 0.004	95.44 $\pm$ 0.571
10 $\times$ 26	0.966 $\pm$ 0.004	96.23 $\pm$ 0.386
10 $\times$ 31	0.965 $\pm$ 0.003	96.67 $\pm$ 0.434
10 $\times$ 36	0.968 $\pm$ 0.003	97.04 $\pm$ 0.289

The larger the size of row is, the larger the values of c_1, c_2 will be.

Seat Assignment After All Groups Arrive

DP1-A: Dynamic programming based heuristic policy after all groups arrive
Bid-A: Bid-price control policy after all groups arrive

Distribution	T	DP1-A (%)	Bid-A (%)	DP1 (%)	Bid (%)
D1	60	99.52	99.44	98.42	98.38
	70	99.32	98.97	96.87	96.24
	80	99.34	99.30	95.69	96.02
	90	99.55	99.49	96.05	96.41
	100	99.78	99.66	95.09	96.88
D2	60	99.50	99.37	98.26	98.25
	70	99.40	98.97	96.62	96.06
	80	99.46	99.24	96.01	95.89
	90	99.59	99.35	96.77	96.62
	100	99.77	99.61	97.04	97.14
D3	60	99.57	99.54	98.72	98.74
	70	99.46	99.39	96.38	96.90
	80	99.50	99.30	97.75	97.87
	90	99.34	99.44	98.45	98.69
	100	99.34	99.55	98.62	98.94

The performance is greatly improved without the assign-to-seat constraint.

Seat Assignment under Fixed Seat Planning

The assignment is based on the fixed seat planning and the group-type control.

Distribution	T	# of rows	Compared to the optimal (%)
D1	70	10	94.97
	80		96.48
	90		97.94
	100		98.91
D2	70	10	95.90
	80		97.06
	90		98.58
	100		99.47
D3	70	10	97.41
	80		98.85
	90		98.73
	100		98.46
D1	140	20	95.83
	160		97.46
	180		99.05
	200		99.74

The performance declines with the fixed seat planning constraint.

Conclusion and Future Work

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- Address the problem of seat planning and seat assignment with social distancing.
- Provide a comprehensive solution for optimizing seat assignments under dynamic situation.
- Provide different methods in different situations and elaborate on the role of social distancing.

Future work

- Consider more flexible situations, people can choose the seats and leave randomly.
- Consider a scattered seat assignment when there are sufficient seats.

Thank You!