

FLIGHT TRANSPORTATION LABORATORY
REPORT R91-5

**MODELING AIRLINE GROUP
PASSENGER DEMAND FOR
REVENUE OPTIMIZATION**

Tom Svrcek

May 1991

MODELING AIRLINE GROUP PASSENGER DEMAND
FOR REVENUE OPTIMIZATION

by
TOM SVRCEK

Submitted to the Department of Civil Engineering
on May 10, 1991 in partial fulfillment
of the requirements for the Degree of
Master of Science in Operations Research

ABSTRACT

Many airlines currently use a variety of analytic techniques for seat inventory control as part of a larger revenue or "yield" management system. However, much of this effort has emphasized decisions based on the individual passenger, and has neglected a significant segment of total airline passenger demand, namely that of group passengers. Group passenger demand differs in several important respects from individual passenger demand, and these differences have motivated the need for separate attention in booking procedures and future demand forecasting.

In this thesis we begin by discussing the issues involved with trying to characterize the stochastic nature of airline group passenger demand, and identify the primary elements of variability associated with it. Later, we use these primary elements of demand to develop a mathematical model for the distribution of group passengers on a given flight(s). Armed with a well-defined distribution for group passenger demand, we enhance current mathematical programming approaches for solving the seat inventory control problem to include the control of group seat inventories. We then present a model for determining the minimum per passenger fare an airline should charge an ad hoc group request based on the displacement of individual passengers. Finally, we discuss the issues involved with overbooking in the group demand context, and suggest areas for further research.

Thesis supervisor: Peter P. Belobaba
Title: Assistant Professor of Aeronautics and Astronautics

ACKNOWLEDGEMENTS

There are many whom I would like to thank for helping me to complete this thesis. First, I wish to thank the Office of Naval Research for their continued financial assistance, which allows me to concentrate my efforts on "doing my own thing". Also, I would like to thank the people at Delta Air Lines for providing their motivation and guidance in helping me to focus my efforts. Thanks also to my friends at the Flight Transportation Laboratory, including my advisor, Professor Peter Belobaba, who had to read several embryonic versions of this my final effort. And, of course, I would like to thank and apologize to my two families -- my mother and sister, for putting up with me trying to write my thesis during Christmas dinner, and the students of MacGregor's H-Entry, who sometimes didn't know they *had* a graduate tutor.

Contents

1	Introduction	9
1.1	Motivation for Thesis	9
1.2	Goal of Thesis	13
1.3	Structure of Thesis	14
2	Seat Inventory Control	17
2.1	The Problem Defined	17
2.2	The Realities of Seat Inventory Control	20
2.3	Planning vs. Decision Making Models	22
2.4	Problem Environments	24
2.4.1	The Deterministic Environment	25
2.4.2	The Probabilistic Environment	25
2.5	Mathematical Problem Formulations	28
2.5.1	Leg-Based Nested Formulations	29
2.5.2	Mathematical Programming Approaches	31
2.6	Including Group Passenger Demand	33
3	Review of Past Research	34
4	The Stochastic Nature of Group Demand	45
4.1	The Group Booking Process	45
4.2	Characterizing Group Demand	49
4.2.1	The Number of Group Requests	50
4.2.2	The Size of the Group Request	52
4.2.2.1	The Continuous Case	54
4.2.2.2	The Discrete Case	55
4.2.3	The Utilization Ratio Components	57
4.3	Ranking the Costs Associated with the Utilization Ratio	60
4.4	Independence of Parameters	63
4.4.1	Independence between N and S	64
4.4.2	Independence between N and U	64
4.4.3	Independence between S and U	65
4.5	Modeling Group Demand	65

5	Group Seat Inventory Control Implementation	70
5.0	Introduction	70
5.1	Mathematical Programming Approaches to the Planning Problem	71
5.1.1	The Deterministic Environment	71
5.1.2	The Probabilistic Environment	76
5.2	The Displacement Cost Model for Ad Hoc Group Requests	83
5.2.1	The Leg-Based Displacement Cost Model	84
5.2.2	A Network Based Displacement Cost Model	88
5.3	The N-Split Minimum Group Fare Model	89
5.4	Overbooking in the Group Passenger Context	91
6	Conclusions	97
6.1	Summary	97
6.2	Further Research Topics	99
Bibliography		103

List of Tables

2.1	Nested vs. Non-Nested Inventory Levels	21
5.1	Displacement Cost Model Setup	87
5.2	Displacement Cost Model Results	87

List of Figures

2.1	Three Airport Route Network	18
2.2	Four Airport Route Network	19
4.1	Probability Mass Function $p_n(n_0)$	51
4.2	Portion of Probability Mass Function $p_s(s_0)$	53
4.3	Continuous Approximation for $p_s(s_0)$	54
4.4	"Probability Buckets" Approximation of $p_s(s_0)$	56
4.5	Continuous Representation of $f_u(u_0)$	59
4.6	Discrete Representation of $p_u(u_0)$	60

Chapter 1

Introduction

1.1 Motivation for Thesis

The competitive nature of today's commercial airline industry has motivated the development of a variety of quantitative techniques aimed at increasing revenues and/or reducing costs. One such group of techniques, known as seat inventory control or revenue/yield management, has been the focus of much recent research. Since the deregulation of the U.S. airline industry in 1978, the elimination of a fixed fare structure by the now defunct Civil Aeronautics Board and the resulting increased competition brought about the need for new strategies to maximize revenue under an aggressive open market consisting of many new upstart, low-cost carriers. These carriers were given the opportunity to serve routes previously restricted to the large, major carriers, and due to low cost structures they could charge lower fares than were offered previously. The large major carriers that once enjoyed the benefits of fixed pricing and essentially no competition had to compete aggressively for passenger demand. Recognizing that the marginal cost of carrying an additional passenger on a flight is rather small, the airlines realized they could sell a number of seats at discounted prices in order to stimulate

otherwise dormant, price sensitive demand, and in doing so they could increase the total revenue for the flight. This practice is known as differential pricing.

The concept of differential pricing is straightforward -- more "valuable" variations of a similar product (those providing higher utility to the consumer) are offered at higher prices. The difficulty with applying the notion of differential pricing to the air transportation industry is that the product itself is rather homogeneous -- travel between a specified origin and destination. The idea of varying utility, then, had to come not from the travel itself but rather the flexibility with which one can purchase that travel. The airlines thus created distinct fare "products", placing varying degrees of restrictions on each product, affecting its utility to the consumer. The more deeply discounted the fare product, the more severe were the restrictions made on its purchase.

A good example of a common restriction placed on deeply discounted fares is the advance purchase deadline -- typically 7 or 14 days prior to the date of departure. The advance purchase restriction is generally inconvenient for the business traveler, who often cannot plan his trip very far in advance. Thus, the business traveler must purchase a more expensive, less restricted fare product -- one which allows him to book his ticket up to the time of departure. By creating effective barriers which prohibit high fare customers from purchasing lower fare products, the airlines can dramatically increase total revenue on each flight, without allowing the high fare customers to purchase the lower priced fare products.

While the benefits of a differential price structure are obvious to the passenger qualified to purchase the deeply discounted fare products, there are, in theory, benefits for passengers who must purchase the higher priced fare products as well. The incremental revenue generated by the price-sensitive traveler keeps the average ticket price down for

all passengers, as the burden of covering the operating costs of the flight is now spread over more passengers.

The practical difficulty with differential pricing is that the new, price-sensitive demand can exceed the number of seats that would otherwise go empty on a given flight, raising the possibility of low revenue passengers displacing the higher revenue passengers. Thus, the airlines developed a technique to limit the number of seats available at each of the different fare product levels, in order to maximize the expected revenue for a given flight. This technique, known as seat inventory control, strives to achieve the delicate balance between minimizing the number of empty seats on a given flight and minimizing the displacement of high revenue passengers by low revenue passengers.

Most of the recent work done in the area of seat inventory control has concentrated on decisions regarding the marginal individual passenger, i.e. the number of seats allocated to a particular fare class is a function of the forecasted demand of individual passenger bookings and the revenue each individual passenger will generate in a particular fare class. However, there exists another source of bookings which can constitute a significant portion of total passenger demand, particularly in markets dominated by international and leisure travel. These markets are typically characterized by a considerable number of group passenger bookings -- bookings made in large blocks at once by a third party (such as a tour operator or travel agent), rather than by the individuals themselves.

Although groups can be treated, in many cases, as simply a collection of individuals, there exist important differences that warrant separate attention in the context of seat inventory control. For example, in accepting a group request, the airline at

once reduces the uncertainty of future demand by the size of the group, rather than by a single individual. This reduction in uncertainty is valuable to the airline, in essentially the same way the certainty of bulk orders is valuable to most suppliers. Recognizing the value of their request to the airlines, travel agents and tour operators will generally negotiate for a lower fare per group passenger than the comparable individual passenger fare. There exists, then, the potential of displacing higher revenue passengers with the lower revenue group passengers and thus reducing the total revenue for that flight. The revenue lost in displacing higher revenue passengers could eliminate any advantage initially gained by accepting the group at the negotiated fare.

Moreover, simply booking a group request does not guarantee revenue for an airline. There still exists a distinct possibility that all or part of the initial block of seats requested will ultimately not be purchased and/or used. To complicate matters further, tour operators may book the same group on several carriers in the process of searching for the best group fare quote. The unused seats from a returned group request may be absent from the carrier's seat inventory for months, and the revenue lost due to their absence may not be recovered in the (possibly short) period of time the seats are once again made available. An intuitive solution to this problem might be to introduce non-refundable deposits or cancellation penalties on seats requested by a travel agent or tour operator. However, the threat of lost goodwill and the number of other competitors able to fulfill any request has generally limited the effectiveness of any such penalty scheme in practice.

Another solution might be for the carrier to accept more group passenger bookings on a flight than the available capacity, fully anticipating that some groups will cancel prior to departure. The airline group analyst, then, must carefully weigh the costs of having a flight depart with a large block of empty seats versus the cost of possibly denying boarding to many group passengers at once when accepting or rejecting a group request.

Any effective group overbooking policy would have to consider this "multiplier effect" of many passengers being affected by a single accept/reject decision.

The above discussion suggests a need for characterizing the nature of group demand independent of individual passenger demand. Moreover, it motivates the need for separate consideration of group demand on the part of the airline when making seat allocation and overbooking decisions. Indeed, errors in accepting or rejecting a single large group may mean the difference between a profitable and an unprofitable flight.

1.2 Goal of Thesis

Increasingly sophisticated models are currently being developed to control seat inventories. An integral part of the input to these models are the forecasts associated with future passenger demand. In order to incorporate the idea of group bookings as a source of future passenger demand, we must develop a means to model the nature of the group booking process. One major objective of this thesis to provide the necessary framework to develop a mathematical model to characterize group demand for a given flight itinerary.

In order to achieve this goal, we must first explore the group booking process qualitatively, so that we may better understand the differences between group and individual passenger demand. In our comparison, we will discuss such issues as overbooking, cancellation and no-show rates, even sell-up, and show how each of these issues can be modified to fit the group booking context. Later, we use this information to construct our mathematical model of group demand itself.

Our mathematical characterization begins by presenting a general formulation for the distribution of the number of group passengers expected for a particular flight itinerary. The model we present is a probabilistic formulation based on a random sum of two random variables. We then add the dimension of variability associated with group cancellation and no-show rates, in order to provide a more realistic model of group passenger demand.

Armed with a model for the distribution of group passenger demand, we then demonstrate how to incorporate our model into current formulations designed to solve the seat inventory control problem. More specifically, we present explicit formulations for both the single leg and multi-leg, multi-class (MLMC) problems, in order to demonstrate how to implement the group model into current practices.

It is not the intent of this thesis to provide an exhaustive set of empirical results to be used in comparing one inventory control technique with another in hopes of establishing a 'best' method. The empirical results included in this thesis are meant to serve two purposes. First, they are intended to show that the theoretical constructs presented are rather simple to implement in practice, and they are provided as a guide to this end. Second, they are meant to serve as incentives to implement these models and draw more relevant, case-specific conclusions as to which method will work best under a variety of circumstances.

1.3 Structure of Thesis

The remainder of this thesis is organized in six chapters, each addressing a separate issue of the group booking control problem. Chapter 2 introduces the seat inventory control problem. We begin by discussing the single leg problem, and work

toward the more complex multi-class, multi-leg problem. Along with a general description, we introduce a simple network which we will use as the foundation for our explicit formulations of optimization methods to solve the problem. Next, we discuss the difficulties associated with actually implementing the solutions in a real world environment. We then differentiate between two types of problem solving techniques, known as planning models and decision making models, and discuss the differences between them. Finally, we describe and state the necessary assumptions behind each of the different problem formulations, and clarify what is meant by the word "optimal" when discussing problem solutions. We define the set of assumptions used when formulating the seat inventory control problem as the specific "problem environment", which, like the problem formulation, can be either deterministic or probabilistic.

In Chapter 3 we discuss the current body of literature on the seat inventory problem, and discuss further, through examples, the differences between planning and decision making models for solving the problem. We also introduce a hybrid method, which utilizes advantages from two distinct approaches to the inventory control problem, in order to solve the problem in yet another "optimal" way.

Chapter 4 discusses, at length, the group booking process, and introduces crucial distinctions between group and individual passenger booking procedures. We describe a typical group booking scenario, including a discussion of the negotiation phase, which does not ordinarily take place in the context of individual passenger travel. We also discuss modified notions of overbooking, cancellation rates, and sell-up as they pertain to the group booking process. Also in the chapter, we discuss the major sources of variation associated with group travel demand, namely the number of group requests, the size of each individual group request, and the utilization rates associated with a group request. The remainder of Chapter 4 involves the development of the mathematical model for the

distribution of group passenger demand for a particular flight itinerary. We also discuss specific implementation issues pertinent to using our model, including simplifying assumptions which can be made to aid the ease of use.

Following the development of the model for group demand, we present in Chapter 5 specific formulations for both the single leg and MLMC seat inventory control problems. These formulations are extensions of previously developed models, modified to take into consideration the group demand model developed in Chapter 4. We consider, in each of our formulations of the planning model approach, the possibility of both deterministic and probabilistic demand. We then present the Displacement Cost Model for negotiating ad hoc group requests, both from a leg-based and a network based perspective. Included in our discussion of the Displacement Cost Model is a simple numerical example in order to address implementation issues. Finally, we discuss the practice of overbooking in the group passenger context. Although we do not present an explicit model for overbooking group passenger demand, we discuss the important issues related to developing such a model.

Finally, in Chapter 6 we summarize the findings of the thesis, and we suggest directions for further research. Since the area of group passenger seat inventory control is relatively new, there are many such opportunities which can have dramatic revenue impacts for even small sized carriers.

Chapter 2

Seat Inventory Control

2.1 The Problem Defined

Consider the problem of allocating seats to several different fare classes on a single leg flight, where all passenger itineraries are restricted to one origin-destination pair. The solution as to how many seats to allocate to each fare class depends almost exclusively on the average revenue associated with each fare class and the corresponding forecasted demand. The intuitive solution suggests that we begin by allocating as many seats as there exists demand for the high revenue passengers, and continue to allocate the remaining seats successively in a "top down" fashion to the lower revenue passengers. We assume that the necessary fare restrictions are in place so that higher revenue passengers are not able to purchase tickets in the lower fare classes.

Extending the above case to multiple leg flights is non-trivial. In the multiple flight leg case, we must account for differences in the revenues associated with accepting a booking request from a passenger travelling on only one leg and a passenger traversing both legs of a two-leg flight. In addition, we must consider these revenue differences among each of the different fare classes offered. This problem is referred to as the

multiple leg, multiple class, or MLMC seat inventory control problem. Consider the simple route network shown in Figure 2.1.

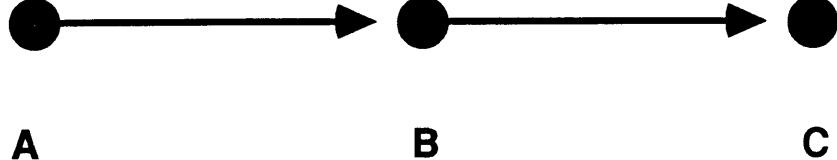


Figure 2.1: Three Airport Route Network

Passengers can fly any one of three distinct itineraries:

- i) A - B non-stop
- ii) B - C non-stop
- iii) A - C one-stop

Consider the case where there is strong demand for all three itineraries. If we employ our intuitive single leg methodology to just the A-B leg, the considerable revenue generated by the A-C passengers relative to the A-B passengers would cause our greedy intuitive approach to always give preference to A-C itineraries. The "network" optimal solution, however, might favor the sum of the non-stop revenues generated by A-B and B-C passengers, whose total revenue is generally higher than for the single one-stop A-C passenger (in the same booking class). In this instance, for the A-B leg inventory, our seat inventory control methodology should prohibit excessive A-C bookings in favor of the non-stop A-B bookings in order to maximize revenues.

Conversely, consider the case where there is little demand in the B-C local market, but continued strong demand in the A-B and A-C markets. If our seat inventory control methodology does not differentiate between the A-C and A-B passengers, we run the risk of filling the A-B leg with local A-B traffic, and subsequently flying the B-C leg virtually empty due to the absence of the A-C through passengers. Ideally, we would like to be able to set allocations for each O-D/fare class combination on the flight, in order to maximize our revenue potential. Network considerations are further complicated if we expand our simple network as shown in Figure 2.2.

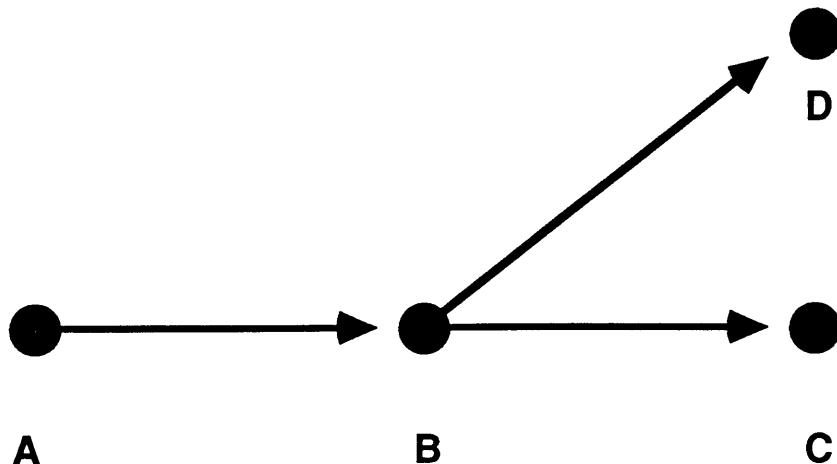


Figure 2.2: Four Airport Route Network

Passengers on the A-B leg may be part of three distinct itineraries now, and we must consider network revenue contributions of the A-D passengers (who will change aircraft for the B-D leg) versus the contributions we have discussed previously. The sophisticated hub and spoke networks operated by today's carriers are several orders of magnitude more complex than our simple network. In order to achieve system-wide revenue maximization objectives, many airlines are currently researching methodologies

for controlling seat inventories over multiple leg itineraries, which as we have discussed, is a more complex problem than simply controlling each leg individually.

2.2 The Realities of Seat Inventory Control

In order to be effective in generating incremental revenues for the airlines, the theories and methodologies developed for seat inventory control must be implemented through current computer reservations systems, each with its own set of constraints and capabilities. Unfortunately, these reservations systems are not standardized, nor do they currently support real-time updates of input data to the seat inventory control algorithms after each booking request is received. Therefore, the current practice is for the seat inventory control system to set booking limits for the reservations system based on off-line analysis of historical and/or current passenger data, and these limits are generally updated periodically throughout the booking period for each flight. The reservations systems maintain the booking limits, and accept or reject booking requests from passengers based on the established booking limits for each fare class. Some reservations systems maintain fare class inventories in discrete "buckets" -- each fare class is allocated a distinct number of seats which are used exclusively for bookings in that fare class. The difficulty with maintaining seat inventories in this manner is the potential for refusing a booking request for a high fare passenger while a lower fare bucket remains open.

Other reservations systems allow for fare class nesting, or sharing of inventories. Such systems order the fare classes by some measure of revenue/desirability, from highest to lowest, and allow higher fare classes to share the inventories of classes lower in value. Instead of non-nested, partitioned fare class allocations, the reservation

system maintains fare class booking limits, where the booking limit is defined as the sum of the seats available to the class in question and all lower ranked fare classes. Such a system prevents the refusal of a higher fare booking request while lower fare inventories are still available. Table 2.1 demonstrates the difference between nested and non-nested inventory control.

Table 2.1 - Nested vs. Non-Nested Inventory Levels

Fare	Non-Nested	Nested Booking
<u>Class</u>	<u>Allocation</u>	<u>Limit</u>
Y	20	120
B	30	100
M	15	70
H	25	55
Q	30	30

In the above example, if the number of bookings for, say, fare class M reaches 15, in the non-nested environment all further requests for fare class M will be refused (assuming no cancellations). In the nested system, an M-class request will not be refused until the total number of bookings for classes Q, H, and M reaches 70. The remaining 50 seats of the aircraft are similarly being protected for classes Y and B.

In current practice, inventories for most reservations systems are maintained strictly for each leg, rather than the entire origin-destination path for passengers traversing multiple legs. In other words, a connecting itinerary BOS-ORD-SFO will be subject to control separately as one BOS-ORD flight and one ORD-SFO flight. A Y-class request for BOS-SFO requires availability in both BOS-ORD and ORD-SFO Y-class inventories, and no distinction is made between a local BOS-ORD Y-class request

and a BOS-ORD-SFO request for the BOS-ORD leg. More advanced systems allow restricted O-D control for multiple leg flights maintaining the same flight number, thus exerting separate control for BOS-ORD-SFO and BOS-ORD passengers in the above example. However, the large hub and spoke networks operated by today's major carriers create possible multiple leg itineraries over numerous flight numbers. It is believed that the proper control of individual O-D/fare class itineraries can increase potential revenues dramatically.

The next three sections are included to familiarize the reader with the different aspects involved with developing a seat inventory control system. Section 2.3 describes two distinct problems which an inventory control system can address, either independently or in tandem. These two types of problems are known as planning models and decision making models. In Section 2.4, we define what we call "problem environments", which describe the simplifying assumptions made in order to characterize each problem mathematically. The more simplified the problem, the easier it is to solve, but at the expense of the potential usefulness of the results. In Section 2.5, we describe various problem formulations and solution algorithms used to solve these problems. Finally in Section 2.6, we discuss the implications of including group booking requests into any seat inventory control model.

2.3 Planning Models vs. Decision Making Models

We make a distinction here between two similar types of problems which can be modeled when addressing the seat allocation or seat inventory control problem -- planning and decision making models.

The first type of problem we will discuss is the planning model. Planning models seek to allocate the remaining seats on an aircraft to the optimal mix of fare classes (or fare class/itinerary combinations), given the forecasted demand for the remaining bookings to come in each fare class (or fare class/itinerary combination). Initially, before the airline receives any bookings for a flight, the solution to the planning model is a best attempt to allocate the authorized capacity of the aircraft to the anticipated demand in each fare class (or fare class/itinerary combination), in order to maximize the expected revenue. If the airline does not update its demand forecasts prior to departure, these allocations will be maintained throughout the booking period for that flight departure. If, on the other hand, the demand forecasts are updated periodically as actual passenger demand is realized, the planning model can be employed iteratively after each demand update, in order to find the optimal mix of allocations for the remaining available authorized seats. The airline can thus exploit more credible input information to better capture actual demand patterns, and potentially increase the revenue on each flight.

The second type of seat inventory control problem is solved by the decision making model. Whereas the output of the planning model is a set of seat allocations for each booking class, the output of the decision making model is an "accept" or "reject" response to a request for the next available seat at a specified ticket price. In essence, the decision making model evaluates the expected revenue of the next available seat based on demand forecasts, and accepts the request only if the price offered meets or exceeds that expected revenue. The decision making model can, in some sense, be thought of as a "real-time" tool, designed to make yes/no decisions for each individual request, based on the expected revenue of the next seat to be sold.

The difficulty with such models is that demand forecasts are traditionally expressed in terms of future "bookings to come". Thus, by accepting a request for a seat,

we by definition alter the distribution of the forecast for the remaining bookings to come for that fare class. The decision making model, in order to make optimal seat allocation decisions, would at the extreme require updates of demand forecasts after each request is accepted. This method is obviously computationally prohibitive given the enormous number of booking requests received by an airline, though approximations may elicit perfectly usable results.

The two models are not disjoint, although the time frame over which each is implemented is often different. The planning model, as mentioned above, is typically implemented prior to the booking period, as well as after each demand forecast update, if demand forecasts are indeed updated periodically throughout the booking period. The decision making model would in theory be implemented as each individual booking request is received. In theory, then, the planning model converges to the decision making model as passenger demand forecasts are updated more and more frequently. If passenger forecasts are updated after each request, then the planning model and the decision making model are essentially the same. Although grossly unrealistic for individual passenger requests, the decision making methodology is directly applicable for group booking requests, as we will discuss later.

2.4 Problem Environments

As discussed earlier, the term "problem environment" is simply a way of describing the set of assumptions we make about the problem we wish to solve in trying to characterize it mathematically. Such assumptions include the distributions of demand, the order in which booking requests are received for different fare classes, and the

relationships of demand between fare classes. The two primary environments we will discuss are the deterministic environment and the probabilistic environment.

2.4.1 The Deterministic Environment

The assumption of a deterministic environment for the seat inventory control problem, although somewhat unrealistic, is fundamental to the understanding of more complex environments involving less restrictive assumptions and perhaps more accurately depicting the actual booking process. As the name implies, everything in the deterministic environment is known with certainty. Demand levels for each booking class are known, and can be forecasted well in advance of the time frame over which demand is realized. Periodic adjustment of demand forecasts is unnecessary -- if our initial forecast for the demand of Y-class passengers is 10, we know we will receive exactly 10 bookings in the Y fare class. Note that in the deterministic environment there is also no need to make any assumption regarding the relative timing of when passengers book in the different fare classes. We do, however, make the assumption that bookings between fare classes are independent. The independence assumption states that the number of bookings received in one fare class does not in any way influence the number of bookings received in any other fare class -- an assumption which greatly simplifies the analysis for our seat inventory control models.

2.4.2 The Probabilistic Environment

The uncertain nature of the true airline booking process suggests that the deterministic environment may grossly oversimplify the problem in modeling passenger demand behavior. Indeed, seat inventory control would be quite straightforward if demand levels were always known with certainty.

A more realistic model which captures the inexact behavior of future passenger demand, at least in part, is described by the probabilistic environment. In it we recognize the forecasted demand as being an expected value with some degree of uncertainty. Indeed, it is unlikely that the realized demand will consistently emerge as identical to the forecasted value; there will be times when the actual demand will exceed our forecast and times when it will fall short.

Nevertheless, if we have a model of the distribution of demand rather than simply an expected value, it is possible when formulating the problem to account for situations when demand levels are different from our forecasts. We account for such disparities through the idea of probability distributions and expected marginal revenues, which we briefly describe in the following section. The theory behind such models is not new, and credit is given to the original investigators in our review of past research in Chapter 3.

The Expected Marginal Revenue Approach

Quite simply, the expected marginal revenue (EMR) approach captures the uncertain nature of passenger demand, and evaluates the expected revenue generated by the next available seat if it were assigned to a particular fare class. To calculate the EMR, we are given a well-defined probability density for the distribution of demand for that fare class, which we will call $f_X(x_i)$ for fare class i.

The first seat assigned to a fare class with any appreciable demand will almost certainly be sold, and thus the expected revenue generated by that seat (if we allocate it to that fare class) is equal to the revenue associated with that fare class. The next seat, however, is theoretically not as likely to be sold, and we may or may not receive the

ticket price if we allocate it to that same fare class. The EMR of the second (nth) seat, then, can be expressed as the ticket price multiplied by the probability that the demand will exceed one (n-1) seat(s).

The probability of selling the nth seat is reasonably assumed to be a non-increasing function in n, and can be expressed as the integral

$$P(n_i) = \int_{n_i}^{\infty} f_x(x_0) dx_0 \quad (2.1)$$

or simply

$$P(n_i) = 1 - F_X(n_i) \quad (2.2)$$

where n_i is the current allocation of fare class i, and $F_X(n_i)$ is the cumulative probability of $f_x(x_i)$, evaluated at n_i . The expected marginal revenue of each additional seat allocated to fare class i ($P(n_i)$) scaled by the average revenue of fare class i) is thus also a non-increasing function in n.

It follows, then, that each available seat of an aircraft can potentially contribute one of k (where k is the number of fare classes offered) different expected revenues to the total flight revenue, depending on which fare class the seat is allocated to. The expected revenue may be significantly lower than the actual ticket price, reflecting the uncertainty of actually selling that seat in that fare class. Unlike the deterministic environment in which the expected revenue of a seat is known to be the revenue of that fare class (provided sufficient demand), it is not assumed, a priori, that a seat will be sold once allocated in the probabilistic environment. We do, however, maintain the

independence assumption for demand between fare classes. In the next section, we describe the problem formulations associated with solving the seat inventory control problem.

2.5 Mathematical Problem Formulations

Having discussed the two types of problem models and problem environments, we are ready to describe the mathematical formulations used to solve seat inventory control problems. Each formulation necessarily depends upon the type of problem to be solved and the set of underlying assumptions made to solve it. The exhaustive set of all the combinations of problem types, environments and formulations includes many cases which are not considered by the airlines either because of they over simplify or over complicate the problem. Therefore, we discuss in the remainder of this chapter a subset of problems we believe represents a realistic collection of the cases most useful in solving many seat inventory control problems.

We begin by discussing leg-based approaches, as most computer reservations systems maintain flight inventories on a flight leg basis. Since the single leg, partitioned inventory problem is unrealistically simple, we discuss only the nested case, where seat inventories among different fare classes can be shared. We then discuss methods to adapt traditional leg-based approaches to include the "network effects" discussed in Section 2.1.

Finally, we discuss network-based approaches to the seat inventory control problem, which are formulated using mathematical programming techniques. These formulations assume partitioned inventories, as the fully nested, multiple leg, multiple fare

class problem is very difficult formulate, let alone to solve on even dramatically simplified networks. We can, however, discuss network problems in terms of both the deterministic and probabilistic environments for partitioned O-D allocations, both of which can be solved for moderately large networks.

2.5.1 Leg-Based Nested Formulations

Our common sense, intuitive approach to allocating seats on a single flight leg turns out to be a rather sophisticated problem mathematically when seat inventories are shared. Due to fare class nesting, the j th seat of an aircraft cannot be classified as uniquely belonging to the Y-class inventory. It can, in fact, depending on demand distributions, belong to all fare class inventories, with different, non-zero probabilities of being sold to each one. Our objective of maximizing revenue is well-defined if we myopically consider only the current individual flight leg, though this set of booking limits may not be optimal for the entire network. To account for "network effects", leg-based revenue values must be altered somehow to reflect total revenue contributions. One method that allows airlines to partially account for such network effects through reservations inventory structures is known as virtual nesting.

Virtual Nesting

As the name implies, virtual nesting establishes a fare class hierarchy in artificial, or "virtual" booking classes, based on some measure of system contribution or value associated with each specific O-D/fare class combination. Combinations with similar values are grouped into the same virtual class, and leg-based control is exerted on virtual classes rather than the original fare classes. One method of ranking different O-D/fare class combinations is to use the corresponding average revenue dollar values.

Returning to our previous example, say the BOS-ORD-SFO discounted M-class revenue is higher in dollar value than the BOS-ORD full coach Y-class fare. The two-leg M-class itinerary would then be put into a virtual class X_1 , and the single leg Y-class itinerary might be placed in a lower valued virtual class X_2 . Booking limits based on total itinerary revenues would then favor refusing the BOS-ORD Y-class passenger for the BOS-ORD-SFO M-class passenger in such a version of the the virtual fare class structure.

There are difficulties with a virtual fare class hierarchy based strictly on total itinerary dollar values. Consider a situation with strong BOS-ORD and ORD-SFO Y-class demand, where the sum of the two single leg Y-class revenues exceeds the revenue contribution of the through M-class itinerary. In this instance, we would prefer not to favor the virtual class X_1 over the virtual class X_2 .

There are other methods used to create virtual fare class hierarchies which avoid such "greedy" solutions. By defining fare class values differently, an airline can order its O-D/fare class combinations in a variety of ways, which can potentially change throughout the booking process. Thus, in allocating seats to different virtual inventories, we are no longer restricted to the explicit set of originally designated fare classes. We continue our discussion of mathematical problem formulations to the seat inventory control problem by describing mathematical programming approaches.

2.5.2 Mathematical Programming Approaches

Deterministic Formulation

Similar to the description of its environment, the deterministic formulation to the seat inventory control problem is the more straightforward of the two math programming approaches. The following section describes the foundation for formulating the deterministic seat inventory control problem as an integer linear program.

In the deterministic environment, demand levels are known with certainty. Therefore, the contribution of each seat allocated to a particular fare class will be equal to the revenue value associated with that fare class, until the number of seats allocated reaches the demand forecast. Any additional seat allocated in excess of demand will have no contribution. Thus for each fare class, we can define a decision variable which represents the optimal (integer) number of seats to allocate. The coefficient for each of the decision variables used in the objective function is simply the revenue value associated with that fare class.

The non-trivial constraints for the linear program can be grouped into two sets. The first set of constraints prevents the number of seats allocated to each fare class from exceeding the corresponding demand forecast for that fare class. Otherwise, all seats would be allocated to the fare class with the highest revenue value. The second set of constraints prevents the total number of seats allocated to all fare classes on a particular flight leg from exceeding the capacity of the aircraft. A more explicit description of the deterministic linear programming formulation is provided in Chapter 5.

Probabilistic Formulation

The probabilistic formulation to the seat inventory control problem differs from the deterministic formulation in that we acknowledge the forecast of demand for a fare class as being uncertain, and thus the revenue contribution for additional seats allocated to a fare class is characterized by a non-increasing function.

The decision variables in the seat-by-seat probabilistic formulation differ from those in the deterministic formulation in that we now have a separate, binary decision variable for each seat being allocated to each fare class. Thus, the number of decision variables is increased by a factor equal to the capacity of the aircraft. For example, if there are four fare classes and 100 seats to allocate, the probabilistic formulation can have as many as 400 decision variables. Judicious selection of the full set of decision variables, however, can significantly reduce the actual number used in the formulation.

The coefficients of the decision variables become the expected marginal revenues (EMR's) associated with allocating each additional seat to a particular fare class. Thus, the first seat to be allocated to a fare class will have a revenue contribution essentially equal to the full revenue value, but subsequent seats will have smaller contributions due to the decreasing likelihood of selling seats in that fare class.

The non-trivial constraints in the probabilistic formulation consist of similar capacity restrictions, in that the total number of seats allocated cannot exceed the capacity of the aircraft. However, the set of demand constraints are no longer necessary. Given that demand forecasts are given in terms of probability distributions, there may exist a non-zero probability of selling a very large number of seats in a particular fare

class, although the expected marginal revenue associated with such large numbers will be very low.

2.6 Including Group Passenger Demand

In Chapter 4, we provide a general description of the group booking process, as well as a discussion of differences between individual and group passenger demand behavior. From the discussions of the previous sections in this chapter, it should be clear that in order to accurately include group passenger demand into the general seat inventory control model, certain issues must first be addressed.

Logically, issues such as demand independence and the time frame over which group demand is realized will need to be included before we can characterize the problem mathematically. More importantly, we must define the general mathematical model we choose to employ -- whether it is a leg-based approach or a full network math programming formulation. Once an approach has been defined, we must address more specific concerns such as nesting and the problem environment we choose to use in our formulation. If a probabilistic environment is chosen, then a model for the distribution of group passenger demand is necessary. The remainder of this thesis is concerned with mathematically characterizing the distribution of group passenger demand and using the model to enhance current seat inventory control models.

Chapter 3

Review of Past Research

The problem of allocating seats to different booking classes or inventories has been studied and documented since the early 1970's. Once considered only by the large, major carriers, computerized yield or "revenue" management systems are currently being used even by smaller carriers that have recognized the incremental revenue potential from various forms of seat inventory control.

The body of literature dedicated to the MLMC problem has concentrated on mathematical programming formulations seeking a system-wide "optimal" solution to a greatly restricted version of the real problem. These original formulations, though theoretically correct for the specifically defined problem they assume, are often time consuming to solve and can produce "sub-optimal" revenue impacts when implemented in a real world environment. Thus, recent research has concentrated on developing methods of modifying the original formulations to generate solutions which are adaptable to current airline reservations systems. These modified solutions have been shown empirically to outperform the solutions from the strictly defined, original formulations. The following is a discussion of past and current research in the area of seat inventory control. We start by discussing the classic, single leg, two class example and conclude with a discussion of

the more general multiple leg, multiple class problem, including various techniques to resolve implementation issues.

One of the first approaches for controlling the number of seats allocated to lower fare passenger demand was proposed by Kenneth Littlewood [1] of BOAC in 1972. Littlewood recognizes that lower fare passengers tend to book earlier than higher revenue passengers, and he motivates the seat allocation problem in terms of service standards. He argues that if reservations were accepted on a strictly "first come, first served" basis, then the earlier booking low fare passengers would, in fact, be getting a higher standard of service than the later booking higher fare passengers. If in accepting a reservation for a low fare passenger a subsequent reservation for a higher fare passenger must be rejected, the airline loses money, which exacerbates the problem.

Recognizing that the stochastic nature of demand can be modeled as a probability distribution, Littlewood suggests, for a two fare class example, that reservations be accepted for lower revenue passengers (paying a mean revenue r) provided:

$$r \geq P * R \quad (3.1)$$

where R is the revenue associated with a higher yield passenger, and P is the maximum risk of losing a higher yield passenger, assuming that in accepting a lower fare reservation for that seat, the airline will have to reject a later booking request from a higher yield passenger. Thus, in order to maximize revenue for a given flight, reservations for lower yield passengers should continue to be accepted until P is equal to the ratio of the mean revenues from the lower and higher yield passengers. Littlewood suggests that this procedure can be generalized to many fare classes but does not present any such procedure in his paper.

Equivalent expressions were derived by Bhatia and Parekh [2] at TWA in 1973 and by Helmut Richter [3] at Lufthansa in 1982, though their approaches are quite different. Using differentiation and transform analysis, Bhatia and Parekh develop a formula which relates an integral of the distribution for higher fare class demand, $f_2(y)$, to the ratio of the lower to the higher fare class revenues, F_1/F_2 . The formula they develop is

$$\frac{F_1}{F_2} = \int_{C-T}^{\infty} f_2(y) dy \quad (3.2)$$

where C is the capacity of the aircraft and T is the optimal allocation for the lower fare class passengers. The integral represents the probability that the higher fare class demand will exceed its current allocation. Thus, the optimal allocation occurs when the value of this integral is equal F_1/F_2 .

Richter presents what he calls a differential revenue method, which examines the impact on the total revenue of a flight if one more seat is offered to the lower revenue fare class. When the differential revenue is equal to zero, we reach a condition of indifference, and thus the optimal seat allocation is found. Richter also makes the important observation that the optimal seat allocation itself is influenced only by the distribution of the higher yield and not the lower yield passengers, though the low yield demand distribution does influence the total expected revenue for that flight.

The extension of the optimal booking limit problem to more than two fare classes is a non-trivial one. Interactions between multiple probability distributions make the Littlewood result hard to extend to the general case of N fare classes. Approaches have

been proposed by Curry [4], Brumelle and McGill [5], and Wollmer [6]. Curry describes the differences among the various methods, and provides a summary of his own method using convolution integrals. Belobaba [7] is responsible for the EMSR heuristic of setting booking limits in the general case, and these limits have been found to produce expected revenues consistently within 0.5% of those obtained using the optimal limits under a variety of circumstances.

Jorn Buhr [8] at Lufthansa provides an excellent introduction to the multi-leg seat inventory control problem, for the single fare class case. In his paper he presents the 2-leg, 3-airport (A,B,C) network we described in Chapter 2, and defines sales limits, SL_{AB} , SL_{BC} , and SL_{AC} , and revenue per passenger R_{AB} , R_{BC} , R_{AC} , corresponding to the respective segments. Using the probabilistic nature of demand, he defines the expected residual revenue (E) on a flight segment as the probability of realizing additional passenger demand multiplied by the revenue per passenger, or, for example,

$$E_{AC}(x) = P_{AC}(x) \cdot R_{AC} \quad (\text{for segment AC}) \quad (3.3)$$

where $E_{AC}(x)$ is mutually exclusive of $[E_{AB}(y) + E_{BC}(y)]$. He then presents the optimality condition for his partition as

$$\text{Min } \Delta E_z = |E_{AC}(x) - (E_{AB}(y) + E_{BC}(y))| \quad (3.4)$$

Using successive adjustments of x and y depending on the sign of ΔE_z , he finds the optimal sales limits for each segment once a minimum value is found. Buhr presents a description of an iterative technique to find the minimum ΔE_z , and provides results from case specific examples using actual flight data from Lufthansa.

He concludes by discussing the input requirements of his model, as well as issues regarding sensitivity analysis. In this discussion, he suggests a technique for treating the multiple class case. Flight segment sales limits are determined first, using the average revenue values for all booking classes as initial input data. Once the O-D allocations are determined for each segment, each is then broken down further by fare class. Buhr notes, however, that the average revenue values will change for the new passenger mix problem per O-D itinerary, due to interrelations in the first step.

Ken Wang [9] at Cathay Pacific discussed the problem of setting optimal seat allocations based on O-D itineraries for a multi-class multi-leg environment. He describes a four step model, which essentially outlines the fundamental seat inventory control problem as it is approached even today. The third step of his model deals with the actual seat allocation algorithm, once estimates for unconstrained demand levels are determined using historical statistical data. Wang defines the expected revenue of each O-D itinerary i as

$$E(R_i) = \sum \Pr(x_{jk} \geq s_{jk}) \cdot Y_{jk} \quad (3.5)$$

This expression is equivalent to Buhr's, only it is applied to individual O-D/fare class combinations rather than to an entire flight segment. In the above expression, Y_{jk} is the revenue associated with the j th O-D itinerary for the k th fare class. The method of allocation he describes is essentially a greedy algorithm which assigns the first seat of the aircraft to the O-D/fare class combination with the highest $E(R_i)$ value. Once that seat is allocated, the expected revenue values are updated for the next seat, and the process is repeated until all seats on the aircraft are allocated. Unfortunately this method becomes computationally overwhelming when large networks with many O-D/fare class

combinations are used. Wang does not discuss the notion of nesting seat inventories once his "optimal" allocations have been determined.

Glover, Glover, Lorenzo, and McMillan [10] discuss what they define as the passenger mix problem, and present a method of solving the MLMC problem using a minimum cost (maximum profit) network flow formulation with side constraints. Nodes of the network represent the origin/destination points on the route structure, where a physical location may be separated into two or more nodes to reflect different arrival and departure times of actual flights. Forward arcs connecting the temporal location nodes represent the number of passengers on a flight segment, and are constrained (using arc capacities) by the authorized capacity of the aircraft assigned to that segment. Backward or reverse arcs represent specific passenger itineraries, or PI's, and the capacity constraints on these arcs are the demand levels of each corresponding PI.

Arc costs on the backward arcs represent the price of the ticket of that passenger itinerary. Thus the maximum value of the objective function is achieved when the largest flow is realized on the arcs with the highest priced PI's, subject to the demand and capacity constraints imposed by the arc capacities described above. The authors describe a network of 600 flights and 30,000 PI's, with up to five fare classes allowed for each PI, for which their network flow formulation was applied at Frontier Airlines. Finally, they discuss significant improvements in running times over a more traditional LP formulation of the problem using 200,000 variables and 3,000 non-trivial constraints.

There are several difficulties with the authors' formulation of the problem, however. Most importantly, the authors assume a deterministic demand environment, i.e. demand levels used in the formulation are constant and are known with certainty. This assumption alone makes the strict use of the presented model ineffective in a practical

application. Moreover, the solution of the network formulation gives seat allocations in "discrete buckets", which do not allow the sharing of lower fare class inventories with those allocated to the higher revenue fare classes. Glover et al. do not discuss the possibility of nesting, which would eliminate this concern , but may produce varying degrees of "sub-optimal" solutions depending on how they choose to nest the distinct inventories.

Richard Wollmer [11] of the McDonnell Corporation presents what is essentially a probabilistic linear programming formulation of the seat allocation or planning model approach to the single leg, discrete inventory problem. Using the notion of probabilistic demand, Wollmer's method begins by calculating the expected revenue associated with allocating the first of the remaining seats of the aircraft to each individual fare class i . He defines this quantity as V_i ,

$$V_i = r_i \cdot [1 - F_i(0)] \quad (3.6)$$

where r_i is the revenue for fare class i , and $F_i(x)$ is the cumulative probability distribution of the forecasted demand for fare class i , evaluated at the x th seat. The first of the remaining seats of the aircraft is allocated to the fare class with the highest V_i . We increase the allocation of that fare class by one, and recalculate the V_i given that we have decreased the demand of future bookings to come by one, or

$$V_i^- = V_i - r_i^- \cdot [x_i^- = \xi] \quad (3.7)$$

We then repeat the allocation procedure successively, each time increasing the allocation of the fare class with the highest V_i by one, until all remaining authorized seats on the aircraft have been allocated. Wollmer notes that this procedure can be applied not only to

an empty aircraft, but also to partially booked flights, where the allocation algorithm is repeated for the number of unsold seats, rather than the full authorized capacity of the aircraft. An interesting and important output of this algorithm is the sum of all the V_i 's found in the course of the allocations. The sum, which we denote as $Z(n)$, where n is the number of seats allocated using the algorithm, represents the total revenue expected for that particular flight leg.

Wollmer extends his single leg planning model to the two-leg, multi-class case. This transition is fairly straightforward; instead of r_i he defines r_{1i} , r_{2i} , for the revenues associated with allocating seats to passengers on respective individual legs, and r_{12i} for the revenues associated with through passengers using both legs. Again, he calculates a series of (now double subscripted) V_{1i} 's, V_{2i} 's, and V_{12i} 's, and using the same "highest contribution" technique to allocate all the remaining seats of the aircraft. This algorithm extends cleanly to the general MLMC problem, though the number of combinations becomes prohibitively large beyond even a few legs and a few fare classes.

Wollmer finishes his paper with a rigorous proof that the allocation produced by the two leg seat allocation algorithm is optimal. One difficulty with his proof is the operational definition of the word "optimal". The allocations produced by Wollmer's algorithm are optimal only for discrete fare class buckets, and only if the remaining capacities of the aircraft on both legs are equal. The algorithm assumes that we are constrained, through our reservations system, to allocate seats exclusively to each fare class/itinerary and forbid any further bookings in a fare class once the number of bookings reaches its allocation limit. Under these conditions, Wollmer's allocations would indeed be optimal. However, as we discussed in Chapter 2, the idea of refusing a reservation for a full coach fare passenger even though a deeply discounted fare class remains open is counter-intuitive to a revenue maximizing objective. The practice of fare class nesting,

with booking limits replacing booking allocations, provides a means of always accepting a booking request for a higher revenue passenger, provided there are seats remaining on the aircraft.

Wollmer acknowledges that nesting should take place, but his solution is simply to allow any seat allocated to fare level j , to also be available to a fare level i , if $r_i > r_j$, essentially creating booking limits based simply on the allocations derived by his algorithm. This form of nesting, however, has been shown to be sub-optimal to other forms of nesting. Thus his proof of optimality is valid for an environment in which fare classes do not share inventories, regardless of revenue, though in practice there exist methods which provide solutions with greater revenue potential.

Wollmer [12] employs the techniques presented in his first paper to formally develop a mathematical programming formulation of the single leg seat allocation problem, which he later extends to the general case of N legs. He then introduces a technique which is the foundation for a general decision making type algorithm. Simply stated, the algorithm optimally allocates the N remaining seats of the aircraft, and calculates the value of $Z(N)$ as defined earlier. He then repeats the allocation algorithm for $N-1$ seats, and finds the difference $\bar{r} = Z(N) - Z(N-1)$. This quantity \bar{r} , represents the expected revenue value of the next seat to be allocated. Wollmer argues, if $r_i < \bar{r}$ for any fare class i , then fare class i should be closed to further bookings. Put another way, we will accept a booking request only if the value of the request exceeds the expected revenue value associated with the next seat to be allocated.

Although it is not the intention of this thesis to discuss the means of nesting the discrete seat allocations derived from the mathematical programming approaches to the seat allocation problem, the reader is encouraged to read Williamson's thesis [13] on the

comparison of optimization techniques for origin-destination seat inventory control. In it, she provides a comprehensive description of possible methods to improve solutions which result from a strict mathematical programming approach (both deterministic and probabilistic) to the problem, as well as methods of increasing potential revenues by modifying the average fare class revenue inputs to leg based approaches such as Belobaba's EMSR heuristic. Much work is currently being done in this research area, due to the sizeable revenue implications.

One method of combining the optimal booking limits of a nested inventory approach and the distinct fare class bucket results of an origin-destination math programming formulation is presented by Renwick Curry [14]. In his paper, he outlines a hybrid approach which borrows from both seat allocation methodologies. After a thorough narrative on the differences between the two approaches and the real world constraints posed by computer reservation systems, Curry introduces his new method, which is formulated as a mathematical program:

$$\text{maximize } \sum_k R^k (A^k) \quad (3.8)$$

$$\text{subject to } \sum_{k \in j} A^k \leq C^j \quad (3.9)$$

The variable R^k is defined as the expected revenue from the i th fare class nest; similarly, A^k is the allocation given to the i th fare class nest. A fare class nest is a collection of fare classes, all from the same O-D pair, which are ordered hierarchically according to revenue. Inventories within a fare class nest may be shared, but inventories between different nests may not. The allocation of a fare class nest, A^k , is the booking limit associated with the highest revenue booking class in the nest. Booking limits within each fare class nest are determined simultaneously using the optimal booking limit equations discussed by Brumelle and McGill, Curry, and Wollmer.

In essence, Curry's method exploits some of the shared inventory benefits possible through CRS nesting, while determining separate, distinct nest allocations through an O-D mathematical programming formulation which aims for a "network optimal" rather than a "leg-based" optimal solution. The fact that the inventories between individual O-D nests cannot be shared, however, makes the revenue impacts of Curry's solution "sub-optimal", just like the discrete fare class inventories from any mathematical programming formulation of the problem. Nevertheless, his approach does strive to improve previous efforts, albeit theoretically. Curry concludes his discussion with several special considerations in airline management which can be addressed with the fare class nest approach. He mentions overlapping flights, point-of-sale control, and group acceptance.

The methodologies for seat inventory control just reviewed do not explicitly derive formulations to handle the problems associated with group booking control. Possible extensions are mentioned, but an examination of the group booking process and how it differs from that of individual passenger demand is noticeably absent from the recent literature. We hope to address, at least in part, those issues most crucial to the inclusion of group booking control into future generations of revenue management systems.

Chapter 4

The Stochastic Nature of Group Demand

4.1 The Group Booking Process

The actual booking process for group travel differs from that of individual passengers in several important respects. We begin our analysis of group demand with a description of a typical group booking scenario. It is provided to highlight some of the key differences, though the steps described are by no means standard procedure for all airlines. We include them simply to familiarize the reader with the differences between the group and individual booking processes and to establish a group booking taxonomy to which we will refer in later sections. Before continuing our description, we make a distinction between the two primary types of group requests -- ad hoc and contract.

The more common of the two types of requests are ad hoc requests, which are generally made for one specific group flying on a specific itinerary. Typically, travel agents or tour operators will make an ad hoc request to an airline after receiving a request from a third party, or they may request a group of seats on speculation, based on anticipated demand. Since ad hoc group requests are made on the spot, accept/reject decisions are required for the specific requested itinerary, using current data maintained by both the

seat inventory control and reservations systems. The second type of request is known as a contract request. A contract is generally long term in nature, involving many groups travelling on many dates. Usually there is some common element to the itineraries, such as origin-destination pairs, and/or the days of the week. Typical contract requests are made by cruise lines, the military, and tour wholesalers who book similar itineraries over several weeks rather than individually. As we will discuss, if contract group demand is consistent enough, it can be forecasted and subsequently included in seat inventory control algorithms. The following is a description of a typical ad hoc group request, though many of the elements discussed are applicable to contract groups as well.

An Ad Hoc Group Request

The first major difference between the group and individual booking processes is the time frame over which they are received. Although the majority of individual passenger demand in domestic O-D markets is generally not realized until some time within the eight weeks preceding the day of departure, group requests are routinely received as many as eight or nine months in advance, possibly more. Special events such as college football bowl games, high school band competitions, and Mardi Gras can lead to significant group demand well over a year in advance due in large part to speculation by third party customers such as tour operators and travel agents.

Whereas individuals may not be willing, initially, to commit to a specific itinerary, say thirteen months prior to departure, a tour operator who knows she can get a large discount on a block of tickets may reserve seats in anticipation of selling the space, at least partially, within a specified amount of time. Because the tour operator can offer individual tickets at a discounted fare, there is now an incentive for the individual passenger to reserve a ticket through the tour operator.

In order to receive a discounted fare on a block of seats, the tour operator or travel agent who has established sufficient group demand must go through the phase of the group booking process known as negotiation. During negotiation, the tour operator contacts the airline and requests a specified number of seats for a given itinerary, typically asking for a quote on the lowest possible fare. The analyst at the airline checks the availability of the specified itinerary, and she will usually quote as a base price, the ticket price of the individual fare associated with the lowest open booking class large enough to accommodate the size of the group requested. As an example, consider a group request for twenty seats on a two leg flight. If the lowest open booking class on both legs is the M-class, the analyst will quote, as a base group price, the M-class fare for an individual passenger traveling on that two leg flight. If the tour operator accepts the fare, the analyst would then book those seats on the flight(s) -- although acceptance by the tour operator of the base quote is exceedingly rare. Far more common is negotiation between the analyst and the tour operator, which ultimately results in a reduction of the base quote.

The logic behind this reduction in the group fare is, at first, a typical example of bulk pricing. The tour operator, in requesting a block of seats at once, is reducing the uncertainty of passenger demand, which is worth something to the airline. The savings is in some sense being passed on to the tour operator through the reduced fare. However, there is a key difference between ordinary bulk pricing and the pricing of group requests. In ordinary bulk pricing, we generally assume that the marginal cost of each additional unit of goods is constant, or even decreasing due perhaps to some economies of scale. The margin of profit per item is decreased by the reduction in price, but it is justified by the guarantee of the large size of the purchase.

The marginal cost of each additional seat of a group request, however, is characterized by a non-decreasing function rather than a non-increasing one. Since group requests can ultimately displace passengers paying higher fares, each additional group passenger may occupy a seat with a higher expected marginal revenue value than the previous one. For example, in a request of size twenty, if the lowest open booking class with 20 seats available is a deeply discounted Q-class, the average individual passenger fare displaced by accepting the group cannot exceed the average Q-class fare. For a request of size fifty, however, the lowest open, common booking class may be the full coach Y-class. If the group is accepted, a group passenger may now displace a full fare Y-class passenger, rather than a deeply discounted Q-class passenger. The average per passenger group fare for the group of size fifty, then, should reflect the possibility of higher fare passenger displacement, rather than a reduction in price over the group request of size twenty.

After the negotiated price has been agreed upon, the analyst will typically book the seats in the booking class with the average fare closest to the negotiated fare, unless special provisions for group bookings exist in the reservations system. The tour operator will then have a specified amount of time, typically a few weeks, to actually commit to the seats, at which time a non-refundable deposit on each seat requested must be placed. Some time prior to departure, the tour operator must actually purchase the requested seats she still wants, for the ticket price agreed upon during the negotiation phase. Seats not purchased are still charged the non-refundable deposit, and any unpurchased seats are then returned to the inventory for individual passenger demand. Any cancellation by a passenger holding a purchased group ticket may now be subject to the same penalties associated with individual passenger bookings. Typically, specific cancellation penalties and refundability restrictions are also determined during negotiation, and may vary significantly among groups.

As mentioned above, this description of the group booking process is by no means standard to the airline industry. In fact, there are likely to be as many variations in procedure as there are airlines. The important points to keep in mind as we try to characterize group demand are:

- 1) Group requests are often received as much as a year in advance.
- 2) Nominal deposits are all that is required to hold space until a short time prior to departure.
- 3) The marginal cost of each additional seat of a group request is non-decreasing.

4.2 Characterizing Group Demand

Our discussion in the previous section suggests that the nature of the group booking process differs enough from that of the individual passenger to warrant separate consideration when modeling passenger demand. Indeed, we feel that the differences are significant enough to motivate an entirely separate model to characterize the distribution of group demand. The problem therefore becomes how to quantify the parameters associated with the group booking process in order to accurately model group passenger demand.

Before continuing with our model development, it is important to clarify precisely what we are trying to model when we discuss group passenger demand. Unless otherwise stated, a group request is assumed to be "genuine", in that it reflects a request made on behalf of an identified single group of passengers wishing to travel on a particular

origin-destination itinerary, and who are at least somewhat flexible as to changes in departure times and perhaps even departure dates. The size of the group request can be based on speculation of anticipated demand by a travel agent or tour operator, though it is assumed that the same group does not make multiple requests on different carriers. Thus, partial or entire cancellation of the group request is based on inaccurate speculation on behalf of the travel agent or tour operator, and/or the cancellation behavior of the individual passengers. We present our model in terms of group passenger demand for a specific flight leg, though entirely equivalent analysis can be used to model group passenger demand for a specific flight or flight itinerary. Ultimately, the choice of precisely what group passenger demand segment we model depends on the specific application of the results.

The process we are trying to characterize is by nature a stochastic one; therefore, we begin the model building process by briefly discussing the major sources of variation associated with group demand, and later we use these variables to derive a probability distribution for the number of group passengers on a specific flight leg. The three major sources of variation we have identified are: the number of group requests received, the size of each individual group request, and the utilization rate, a measure of what fraction of the original group passenger bookings will ultimately be used.

4.2.1 The Number of Group Requests

The first dimension of variability associated with our model for group demand is the actual number of groups requests an airline receives for a given flight leg. This number, which we define as n , is bounded from below by zero and is theoretically unbounded from above. Since requests are countable and not fractional (we cannot receive a half request), n is a discrete random variable, and the probability distribution for

n , $p_n(n_0)$, can be expressed as a probability mass function, with impulses of probability corresponding to the likelihood of receiving any one of up to n group requests for a given flight leg. A typical distribution for $p_n(n_0)$ is shown in Figure 4.1.

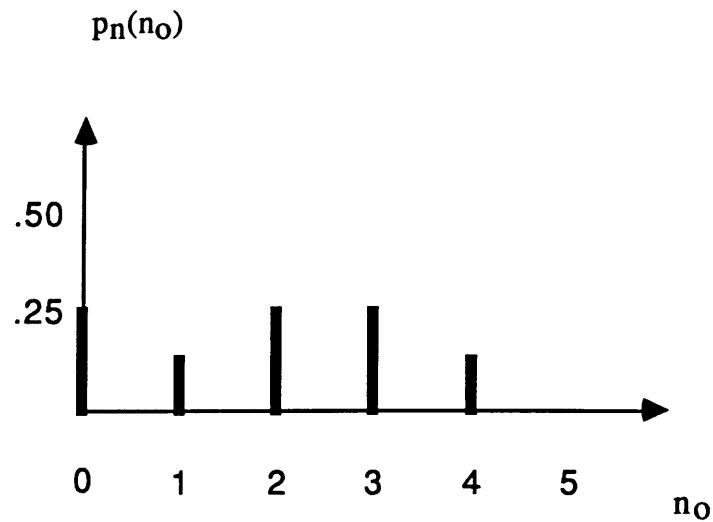


Figure 4.1: Probability Mass Function $p_n(n_0)$

Though theoretically unbounded from above, we recognize that the number of group requests for a given flight leg will have some reasonable upper bound, which is likely to be rather small (in the order of four or five) in most instances. We do not need to make any assumptions regarding an underlying distribution for $p_n(n_0)$. Presently, it suffices to assume that for an individual flight leg, the distribution of the number of group requests can be estimated using historical data.

The distribution shown in Figure 4.1, for instance, suggests that the probability of receiving two group requests on some flight, say Flight 1234, is approximately 25%, or

stated another way, historical data indicates 25% of the departures of Flight 1234 received two requests for group bookings. The distribution of $p_n(n_0)$ for flight 1234 may be similar to that of another flight, say flight 78, and in fact it may be reasonable to group flights with similar distributions of $p_n(n_0)$ to make our estimates more robust. The tradeoffs involved with grouping flights are well known -- gains in an indicator's robustness due to aggregation may translate to losses in the ability to infer trends on an individual flight basis.

4.2.2 The Size of the Group Request

The second dimension of variability in the group booking process is the actual size of each group request received. The tour operator or travel agent who requests a block of seats will generally request a specified number of seats, which we define as the variable s . Less frequently, the tour operator will quote a maximum ticket price she is willing to pay for each seat in a block of seats, and will then ask for the greatest number of seats she can reserve at that price. For our analysis, however, we consider the first case, where the the actual size of the group request is the variable.

Like the variable n , the number of seats s in a group request is bounded from below by zero, is theoretically unbounded from above, and can take only discrete values. Unlike n , however, the practical upper bound on s is not often a small number less than four or five. In fact, many airlines do not even consider a request for multiple seats a "group" request unless the block size is greater than ten seats. Though no upper bound exists on the theoretical demand for an individual flight leg, the number of passengers in a group request cannot reasonably exceed the authorized capacity of the aircraft, as is commonly done in a charter flight situation. Thus, the practical bounds on the size of a group request virtually span the capacity of the aircraft -- for a wide body jet, this upper

bound can easily be in the order of 400 seats. Due to such a large number of possible values for s , it seems impractical to maintain the complete probability mass function for $p_s(s_0)$. In fact, with 400 possible values of s , we may falsely attribute differences in consecutive values of s to a fundamental difference in group demand, when we may be dealing only with small sample bias. There is, for example, more reason to believe the portion of the PMF shown in Figure 4.2 is the result of small sample bias than an inherent lack of demand for groups of size twenty.

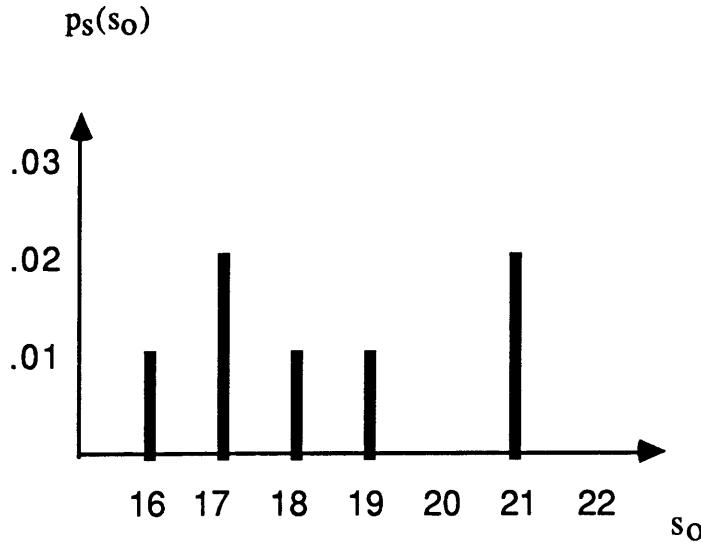


Figure 4.2: Portion of Probability Mass Function $p_s(s_0)$

We are thus left with the objective of how to characterize the distribution of the size of a single group request. We present arguments for two separate models, both of which provide more tractable and more easily maintained distributions than complete enumeration.

4.2.2.1 The Continuous Case

A well known means of approximating a probability mass function with an impractical number of discrete points would be to use a continuous function to describe the distribution of the possible values of s . One possible continuous approximation for the PMF of s is shown in Figure 4.3.

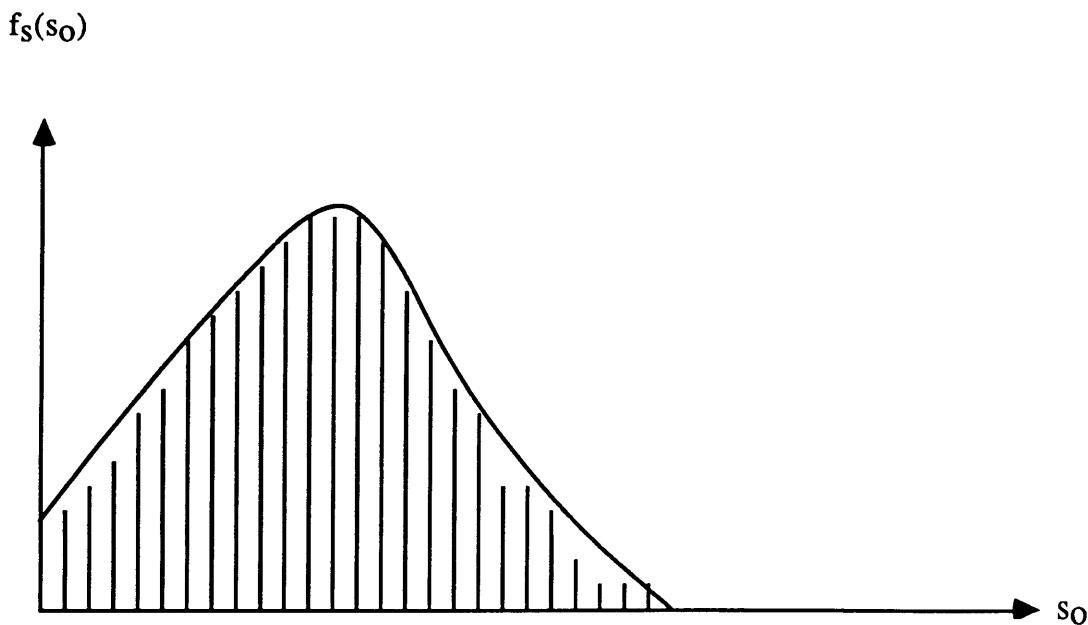


Figure 4.3: Continuous Approximation for $p_s(s_o)$

To determine the probability that s will take on a specific discrete value, say S , we need to evaluate the integral of the new PDF $f_s(s_o)$ over the interval $[S - 0.5, S + 0.5]$, or

$$P[s = S] = \int_{S - 0.5}^{S + 0.5} f_s(s_o) ds_o \quad (4.1)$$

There are several difficulties with the continuous approximation approach. Computationally, the integral may be time consuming to evaluate, given no prior assumptions on the distribution of s and the curve fitting methods used to determine the function $f_s(s_0)$. Moreover, the resulting value may still be, at best, an approximation of an approximation, with two degrees of unwanted uncertainty. To simplify the calculations somewhat, we can start to make assumptions about the underlying distribution of group demand, perhaps the normal distribution, which has well documented tables of integration values. The obvious drawback of this approach is that we may or may not have the empirical evidence necessary to support our assumptions. Of the two methods, however, we feel the distribution assumption has the most promise.

4.2.2.2 The Discrete Case

The second approach to making the distribution of the size of a group request more tractable is motivated by the idea of a histogram. Rather than making a continuous approximation out of the discrete probability mass function $p_s(s_0)$, we instead make even larger, discrete buckets of width w . Thus, instead of having C (where $C =$ the capacity of the aircraft) individual buckets of width 1, we have C/w buckets, each having width w . Figure 4.4 shows a plausible PMF for s before and after aggregating the data into probability "buckets". Note that the method just described addresses our earlier concern that consecutive values of s may have misleadingly large differences in $p_s(s_0)$ due to small sample bias. Under the new method, consecutive values of s will be placed, in general, into a single probability bucket, and thus the aggregation of the individual values into these buckets should provide a more robust model for the true distribution of the size of a group request.

$p_s(s_0)$

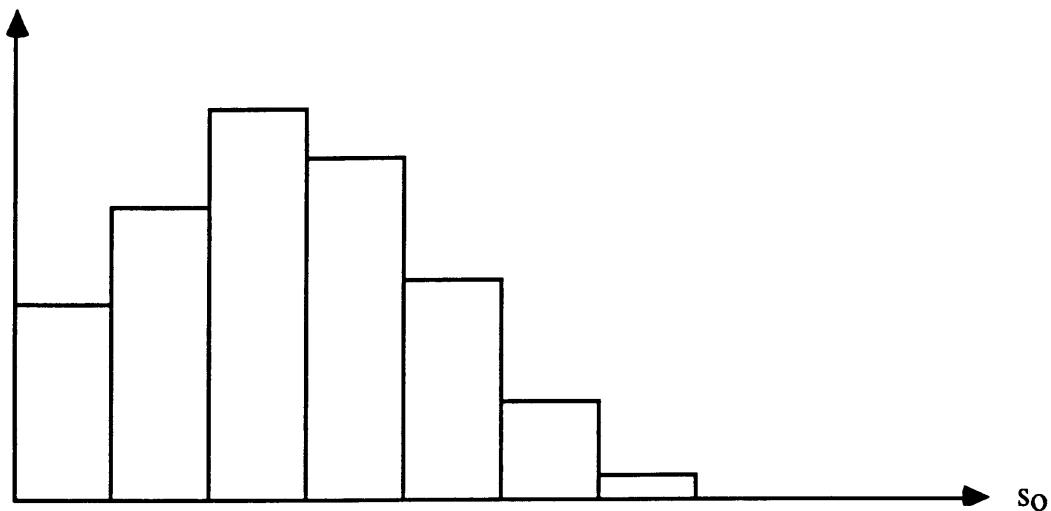


Figure 4.4: "Probability Buckets" Approximation of $p_s(s_0)$

In addition to being more robust, the probability bucket concept has a rather attractive intuitive appeal from the perspective of the tour operator. If we assume that a tour operator's request for a group of seats is based initially on a speculative estimate of the number of seats she will be able to sell for a particular itinerary, then it is somewhat more plausible to think of her request being in multiples of five or ten, rather than some large prime number. Even if we relax the assumption that her request is made entirely on speculation, it may be reasonable to assume that she will, in some sense, "round up" to cover some unanticipated demand over the period of time between her request and the day of departure.

Thus in the absence of strong empirical evidence to support the assumption of an underlying distribution governing the size of an individual group request, we believe the method of creating "probability buckets" provides an intuitive and easily tractable means of characterizing the desired distribution of $p_s(s_0)$.

4.2.3 The Utilization Ratio Components

The third and perhaps most problematic dimension of variability when trying to characterize group demand is known as the utilization ratio or utilization rate. Simply put, the utilization ratio expresses the fraction of seats in the original group request that will ultimately be used by that group. The idea of the utilization ratio is sometimes expressed in terms of its complement, commonly known as the cancellation rate. For individual passenger demand, the determination of a utilization rate is straightforward -- for a given flight, simply divide the number of passengers who actually fly by the number of reservations made for the same flight.

From our previous description of the group booking process, it is obvious that such a singular calculation is not possible for a group booking. There are in fact three possible instances where separate components of the total utilization ratio can be calculated. The first occurs when the tour operator must commit the non-refundable deposit for each seat she still wishes to reserve. As the analyst has already booked a specified number of seats according to the original negotiation, seats no longer wanted by the tour operator (those for which no deposit is made) can be returned to the flight's seat inventory, and can again be used for additional group or individual passenger bookings. Nevertheless, the unused seats have incurred an opportunity cost to the airline during the time they were unavailable for further bookings. We will denote this cost as c_1 , and the fraction of originally booked seats for which a deposit is placed as p_1 .

Once a deposit is placed for the desired seats, there is a second opportunity for the tour operator or travel agent to renege on all or part of the booking agreement. The tour operator has the option of purchasing only a fraction of the remaining booked seats, and

losing the non-refundable deposit on those returned. Again, the airline can return the unpurchased seats to the seat inventory. However, the time frame over which the seats can be resold has been significantly reduced, as the purchase deadline is typically quite close to the departure date. Although they may and often do collect the non-refundable deposits on the unpurchased seats, the opportunity costs associated with potential spoilage (seats that depart empty when there exists demand for those seats) may far outweigh the benefits gained by collecting those deposits. We denote the cost associated with not having the seats available for further bookings to be c_2 , and the fraction of tickets actually purchased by the tour operator as p_2 .

There is, in fact, a third utilization ratio associated with group travel which expresses the fraction of the individual group passengers who actually use the tickets they have purchased. In some sense, this rate is similar to the utilization rate of an individual passenger (depending on restrictions), as the airline has already received payment for the seat, and whether or not the passenger chooses to use the ticket is not necessarily affected by the actions of other passengers in the group. There is, again, a cost to the airline if the passenger does not use his ticket, albeit an opportunity cost. The unused seats are generally not recognized until the day of departure, thus they cannot be put back into the seat inventory for further bookings. We denote the cost of losing potential further bookings as c_3 , and the fraction of purchased group tickets actually used on the given flight(s) as p_3 .

Having separated the notion of utilization into three distinct components, we note that for analysis we may wish to express just a single rate, which nevertheless incorporates vital information associated with all three constituents. In order to accomplish this aggregation, it is necessary to reflect the potentially high variation of such a parameter. Thus, we choose to model the utilization ratio, u , as a random variable

rather than the more traditional characterization of a deterministic value. The distribution for u can be expressed either as a continuous variable, with PDF $f_u(u_0)$, taking on decimal values between zero and one, or equivalently as a discrete variable with PMF $p_u(u_0)$, taking on nonnegative integer values between zero and one hundred. Examples of each are shown in Figures 4.5 and 4.6. For the purposes of further analysis, we will use the discrete representation.

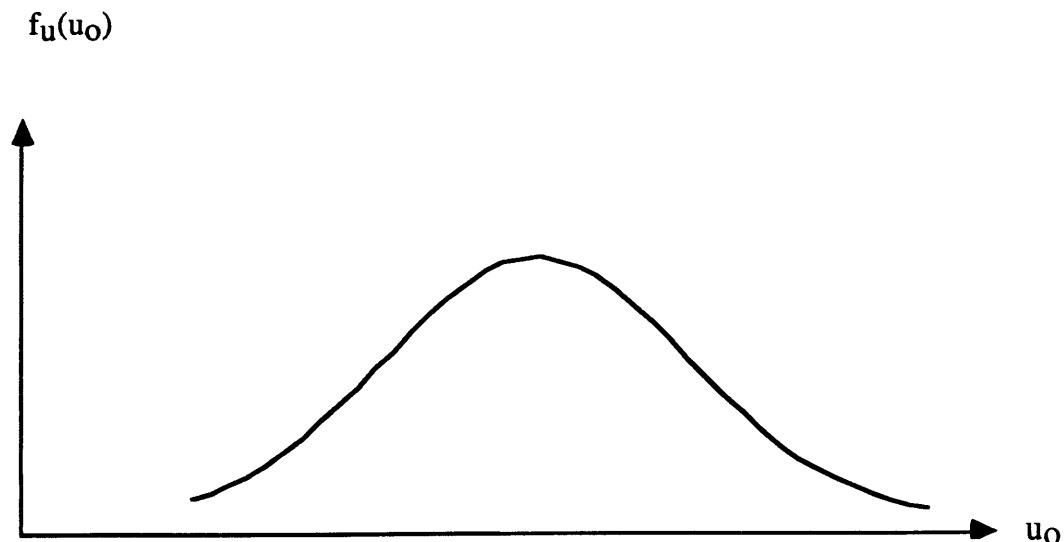


Figure 4.5: Continuous Representation of $f_u(u_0)$

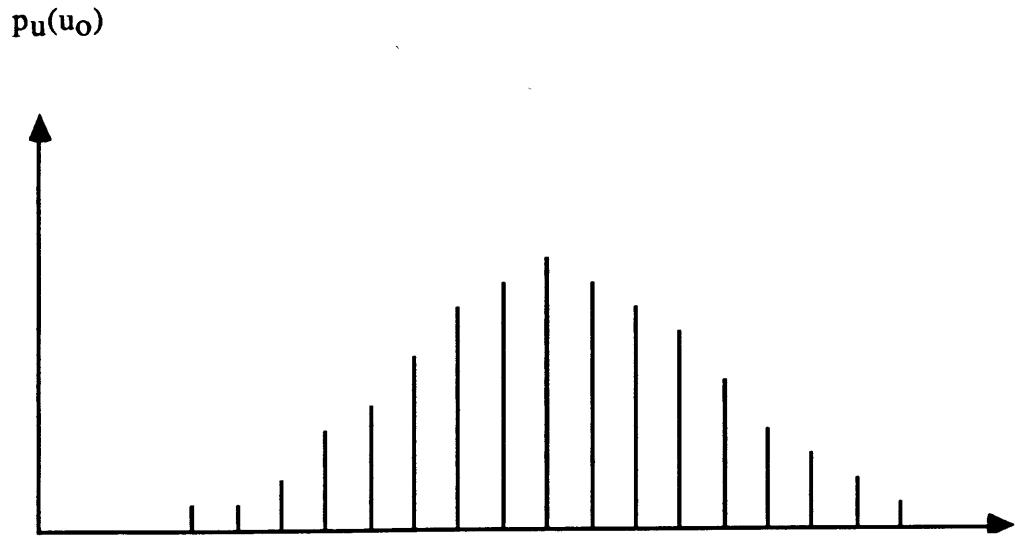


Figure 4.6: Discrete Representation of $p_u(u_0)$

4.3 Ranking the Costs Associated with the Utilization Ratio

In the previous section, we described the three potential costs associated with denying further bookings due to reservations that are ultimately not used. Briefly, these costs are incurred at the following times:

- i) Time between negotiation phase and placement of non-refundable deposit.
- ii) Time between placement of non-refundable deposit and actual purchase of tickets.
- iii) Time between actual ticket purchase and day of departure.

Although ideally we would like to track all three of the utilization parameters independently, data restrictions may prevent us from maintaining separate records, or even allowing us to estimate one or more of the individual parameters. Thus, if we are able to estimate only one or two of the parameters, or perhaps simply the aggregate, it would be helpful to establish a hierarchy of the relative importance of the three components -- a precedence list, in some sense. The list can then be used in assessing how much effort should be placed in tracking each of the three components. After discussing the three costs, c_1 , c_2 , and c_3 , we present what we believe is the proper precedence list for ranking these costs.

Of the three, the cost associated with the first time period, c_1 , seems to be the lowest to the airline. Consider the time during which this cost is incurred. In general, group negotiations are performed several months in advance of the flight departure, and the time between negotiation and placement of the non-refundable deposit typically does not exceed more than a few weeks. Thus, in many cases, there are still months until the day of departure when the seats are unavailable to further bookings. Since individual passenger demand is largely realized within eight weeks prior to departure, the opportunity cost of not selling the unused seat to an individual passenger can be considered negligible. The unused seats, however, do impact further group bookings. Since the base quote during the negotiation phase of another group depends upon the ticket price of the lowest fare class large enough to accommodate the size of the group request, superfluous bookings may drive up the base quote for the next group artificially. This situation is detrimental to the airline only in the extreme case when the extra bookings on hand are numerous enough to artificially close a booking class. Even in this case, the final ticket price is still decided upon by an analyst, and the negotiated price may be well below the ticket price of the higher open fare class.

In situations less extreme than the one described above, we can in fact discuss the possibility of a slight benefit of extra bookings on hand during the first time period, in terms of a potential "sell-up" effect. In the context of individual passengers, sell-up occurs when the passenger's initial choice for a fare class is closed, and the next higher fare class is sufficiently close in price that the passenger will purchase the higher priced ticket rather than seeking a different flight or carrier. In the group context, recall that the marginal cost of each additional seat is an increasing function. Thus a tour operator negotiating for a group of size twenty on a flight with 50 bookings on hand is, in some sense, trying to purchase more valuable seats than for a similar flight with no bookings on hand. Provided the increase in marginal seat costs due to the extra bookings already on hand is not excessive enough to drive the ticket price over the tour operator's negotiating limit, the final negotiated price may indeed be slightly higher for the first case (with 50 bookings on hand), thus paralleling the notion of sell-up.

We believe the next smallest cost associated with the three utilization ratios is c_3 , the cost during the period just prior to departure. As described in Section 4.2.3, c_3 is essentially the opportunity cost of a further booking (depending on restrictions), given that a group traveler does not show up on the day of departure. The unused seat can be sold to another passenger, group or individual, and thus the airline can receive another ticket's worth of revenue for that flight.

By far the largest of the three costs is c_2 , the cost of the unused booking being out of the seat inventory between the time of deposit placement and the actual purchase of the desired seats. Though the notion of sell-up associated with c_1 still applies, the " c_1 " seats are generally out of the seat inventory for only a few weeks, and those few weeks are likely to take place months before individual passenger demand begins to materialize. The seats associated with c_2 , on the other hand, may be out of the seat inventory for

several months, beyond the time when most group demand is realized. Thus any possible benefits are quickly diminished by the opportunity costs of displacing individual passengers, particularly during the time frame over which individual passenger demand is realized. In addition to potential individual passenger displacement, unused bookings during the second time frame may displace other group passengers as well, thus compounding the opportunity cost we have described.

In summary, then, the relative ranking of the costs associated with the three utilization ratios, from lowest to highest, is

$$c_1 < c_3 \ll c_2$$

As mentioned previously, the above expression can be used as an indicator of precedence for determining the order in which estimates of the utilization ratio components should be determined when modeling group demand.

4.4 Independence of Parameters

In the previous discussion, we characterized the three dimensions of variability associated with group demand: the number of group requests, the size of any individual request, and the corresponding utilization ratios. Before continuing with our mathematical characterization, some assumptions must be made about the interrelationships between these variables. Namely, our model assumes all three variables to be statistically independent of each other.

4.4.1 Independence between N and S

At first, it might seem there is a rather strong relationship between the number of group requests and the size of any individual request. Clearly, if we know that there are twenty separate groups on an aircraft with a capacity of 100 seats, it is logical to infer that the average group size is quite small. However, the distributions we are modeling are not intended to characterize passengers flown but rather passenger demand, which is realized far in advance of flight departure. The distribution of the number of group requests is not in any way constrained by physical aircraft limitations; for example, the number of group requests for a flight to Mardi Gras or the Rose Bowl may far exceed the number an airline is able to carry. Similarly, the size of any one group is also theoretically unbounded. The independence assumption of these variables follows from this unboundedness argument. Why, for instance, would the fact that an airline received two group requests earlier in the day impact the size of the next group request? Thus, for our model, we will assume that the number of group requests and the size of any individual request are independent.

4.4.2 Independence between N and U

Using similar reasoning, we feel that the number of group requests for a particular itinerary and the utilization ratio for an individual group request are also independent. How many seats will be purchased and used from an initial request of 50 should not be influenced by the number of other group requests received. Additionally, we assume that passengers are not booked on multiple group requests.

4.4.3 Independence between S and U

Of the three interrelationships, the one for which dependence is most conceivable is between the size of an individual request and the utilization ratio. We might believe, for instance, that there is a greater chance of partial cancellation from a request of 200 seats than there is for one of only 20. The 200 seat request intuitively seems more of a speculation. Obviously, it is not necessarily the case, and there are arguments to the contrary as well. In order to resolve the question of possible dependence we make a rationality assumption that the actual number of seats requested for a group may be based on some proportion of speculation (or systematic variation), but that this proportion is uncorrelated to the request size. We do not claim the proportion of speculation itself is constant, simply uncorrelated to the size of a group request. For example, the proportion of speculation might vary uniformly between 5 and 10 percent of the group request. For a group of size 20, the number of speculated seats might therefore be between one and two seats. For the group of size 200, the number of speculated seats might be between ten and twenty. However, our rationality assumption claims there is no reason to believe the true number of speculated seats in the group request of size 200 is closer to 20 than the true number of speculated seats in the group of size twenty is to two. Having argued the independence among all three components of group demand, we continue with its mathematical characterization.

4.5 Modeling Group Demand

We can now formally develop our model for the distribution of group passengers on a particular flight. We are given as inputs to our model the probability distributions for the

three primary variables associated with group demand:

- i) $p_n(n_0)$, the number of group requests
- ii) $p_s(s_0)$, the size of any individual request
- iii) $p_u(u_0)$, the utilization ratio for any individual request

As discussed in previous sections, these distributions can all be expressed as discrete random variables with nonnegative integer experimental values, and we assume that all three are statistically independent. To combine these variables into a single distribution, we will use the techniques of discrete transform analysis and convolution.

Discrete Transforms of Probability Mass Functions

As a review [15], recall that if $p_x(x_0)$ is the PMF of a discrete random variable taking on only nonnegative integer experimental values, we can completely define its discrete or z-transform, $p_x^T(z)$, as

$$p_x^T(z) = E(z^x) = \sum_{x_0=0}^{\infty} z^{x_0} \cdot p_x(x_0) \quad (4.2)$$

Furthermore, it is possible to determine the individual terms of a PMF from $p_x^T(z)$ by the following formula,

$$p_x(x_0) = \frac{1}{x_0!} \left[\frac{\partial^{x_0}}{\partial z^{x_0}} p_x^T(z) \right]_{z=0} \quad x_0 = 0, 1, 2, \dots \quad (4.3)$$

Thus, we need only define the z-transform of a discrete random variable for its complete probability mass function to be well-defined.

Finally, we state an important result when finding the sum of a random number of independent, identically distributed random variables. We let n and x be independent random variables, where n is described by the PMF $p_n(n_0)$ and x by the PDF $f_x(x_0)$. Define r to be the sum of n independent experimental values of random variable x . The s transform for the PDF $f_r(r_0)$ is $f_r^T(s) = p_n^T[f_x^T(s)]$. [X]. We lose no generality by making random variable x discrete, and the distribution for x a PMF described by $p_x(x_0)$. The result simply becomes a z-transform for the PMF $p_r(r_0)$,

$$p_r^T(z) = p_n^T[p_x^T(z)] \quad (4.4)$$

Using the chain rule for differentiation, we can obtain expressions for the expectation and variance of r . These expressions are:

$$E(r) = E(n)E(x) \quad (4.5)$$

$$\sigma_r^2 = E(n) \cdot \sigma_x^2 + [E(x)]^2 \cdot \sigma_n^2 \quad (4.6)$$

We proceed with our analysis first by defining a distribution for the number of group passenger bookings for a flight, then later we add the notion of utilization to define the distribution for the actual number of group passengers.

The Distribution of Genuine Group Requests for a Flight Leg

Under the assumption that the number of group requests and the size of any individual request are statistically independent, the distribution for the number of group requests for a given flight leg, which we define as r , can be thought of a random sum of n group requests, where the size of any individual group request is itself a random variable, s . Thus, from the discussion above, the transform of the PMF for r , $p_r(r_0)$, is defined as:

$$p_r^T(z) = p_n^T[p_s^T(z)] \quad (4.7)$$

with expectation and variance

$$E(r) = E(n)E(s) \quad (4.8)$$

$$\sigma_r^2 = E(n) \cdot \sigma_s^2 + [E(s)]^2 \cdot \sigma_n^2 \quad (4.9)$$

A completely well-defined PMF can be determined from the resulting transform as discussed above.

We now add into our model the third dimension of variability, the utilization ratio, to completely define the distribution for the number of group passengers on a given flight leg. Recall that the number of seats ultimately used from our original, genuine group requests is reduced by travel agent or tour operator speculation as well as individual passenger cancellations.

The Distribution of Group Passengers for a Flight Leg

Adding the final dimension of variability into our model is simply one more random sum of random variables, if we consider the utilization ratio to be an integer number between zero and one hundred, as discussed previously. We define the number of group passengers on a given flight to be G , with a PMF defined by $p_g(g_o)$. The discrete transform for $p_g(g_o)$, $p_g^T(z)$, is defined as:

$$p_g^T(z) = p_u^T[p_f^T(z)] \quad (4.10)$$

Using with expectation and variance defined appropriately as:

$$E(g) = E(u)E(r) \quad (4.11)$$

$$\sigma_g^2 = E(u) \cdot \sigma_r^2 + [E(r)]^2 \cdot \sigma_u^2 \quad (4.12)$$

where $E(r)$ and $\text{Var}(r)$ can be determined using equations 4.8 and 4.9 from above. Again, the complete PMF for $p_g(g_o)$ is uniquely determined using equation 4.3.

In summary, we propose that the distribution for the number of group passengers on a given flight leg, g , can be modeled as a random sum of random variables, each with its own well-defined probability mass function. Using transform analysis, the expectation and variance of g are easily determined, and the complete PMF $p_g(g_o)$ can be evaluated using equation 4.3. Armed with a distribution for group passenger demand on a given flight, we proceed in Chapter 5 by trying to incorporate our group demand model into the traditional seat inventory control formulations discussed in Chapter 2.

Chapter 5

Group Seat Inventory Control Implementation

5.0 Introduction

Motivated by the need for separate attention to group passenger demand in the control of airline seat inventories, we present in this chapter explicit formulations of the planning and decision making models introduced in Chapter 2 which include the control of group passenger demand. Recall that planning models are typically formulated as mathematical programs which seek to allocate seats to the optimal mix of passengers, given future demand forecasts and average revenue values, and that decision making models elicit accept/reject responses to individual requests given the number of bookings on hand for the requested itinerary. We present both deterministic and probabilistic formulations for the planning model approach to seat allocation, and our approach to solving the decision making problem is presented in the form of the Displacement Cost Model for ad hoc group requests. The Displacement Cost Model is motivated by the expected marginal revenue approach of evaluating seat revenues first presented in Section 2.4.2.

5.1 Mathematical Programming Approaches To The Planning Problem

As previously discussed, the planning model approach to seat inventory control uses mathematical programming to allocate seats to discrete buckets, based on anticipated individual passenger demand. In extending these methods to include group passengers, we introduce the notion of a group demand "bucket" or "inventory", for which seats are actually allocated for anticipated group demand. Similar to individual passenger inventories, the group bucket should somehow be nested into the entire fare class hierarchy, though we do not discuss such issues here. We present the group demand inventory to motivate the introduction of group demand modeling and forecasting into current seat inventory control practice.

5.1.1 The Deterministic Environment

Recall that in the deterministic environment, individual passenger demand levels for all fare classes are known with certainty well in advance of the flight departure. Thus, the revenue contribution of each additional seat allocated to a particular fare class is the average revenue value associated with that fare class, until the number of seats allocated to that fare class reaches the forecasted demand. Seats allocated to a fare class in excess of the demand forecast will have no revenue contribution as there is zero probability of receiving requests above the forecasted number of bookings.

The objective function for a linear programming approach to solving the single leg deterministic seat allocation problem can be modeled as a revenue maximization problem. If we define x_i to be the (integer) number of seats allocated to fare class i , then the revenue contribution of all x_i seats to the total revenue is $F_i x_i$, where F_i is the average

revenue associated with fare class i . To complete the objective function, we simply want to maximize the sum of all $F_i x_i$ for $i = 1 \dots k$, where k is the number of fare classes, or

$$\text{Maximize} \quad \sum_{i=1}^k F_i \cdot x_i \quad (5.1)$$

There are, naturally, constraints to the objective function. In addition to the trivial non-zero constraints, there exist demand constraints and capacity constraints. The demand constraints place a limit on the number of seats we can allocate to any single fare class. Intuitively, for a deterministic demand forecast of μ_i , the variable x_i cannot exceed μ_i , or

$$x_i \leq \mu_i \quad \text{for } i = 1, \dots, k \quad (5.2)$$

Similarly, the capacity constraints put a limit on the total number of seats we can allocate -- these constraints are binding only when there is demand in excess of the capacity, C . The capacity we consider may not necessarily be the physical capacity of the aircraft. If we take into account no show behavior for overbooking purposes, then the capacity we use as a constraint may actually exceed the aircraft's physical capacity. The total number of seats an airline chooses to make available for a particular flight leg is known as the authorized capacity. Conversely, if we are allocating seats only within a subset of the coach cabin, we may wish to limit the sum of allocations to be some number below the physical capacity of the aircraft. For an authorized capacity of C on the single flight leg, the capacity constraint is simply

$$\sum_{i=1}^k x_i \leq C \quad (5.3)$$

The complete single leg integer linear programming formulation for the deterministic environment is thus,

$$\text{Maximize} \quad \sum_{i=1}^k F_i \cdot x_i$$

Subject to:

$$x_i \leq \mu_i \quad \text{for } i = 1, \dots, k$$

$$\sum_{i=1}^k x_i \leq C$$

$$x_i \geq 0 \text{ and integer} \quad \text{for } i = 1, \dots, k$$

For the multiple leg case, we simply expand the decision variables to two dimensions, where $x_{i,O-D}$ is the number of seats allocated to the i th fare class/O-D combination. The linear program can thus be written as:

$$\text{Maximize} \quad \sum_{O-D} \sum_{i=1}^k F_{i,O-D} \cdot x_{i,O-D}$$

Subject to:

$$x_{i,O-D} \leq \mu_{i,O-D} \quad \text{for all O-D itineraries and } i \text{ fare classes}$$

$$\sum_{O-D} \sum_{i=1}^k x_{i,O-D} \leq C_j \quad \text{for all O-D itineraries and } i \text{ fare classes on flight } j,$$

for all flights j

$$x_{i,O-D} \geq 0 \text{ and integer}$$

Including Group Demand

Under the assumptions of a deterministic environment, we expect to know, with certainty, the group size and the per passenger group fare of each group request well in advance of the flight departure. Without loss of generality, let us assume that the group demand for our single flight leg comes from one request, with size μ_g and average fare F_g . We define x_g to be a binary variable taking the value of one if we accept the group request and zero otherwise. The revenue contribution of an accepted group request is thus $F_g\mu_g$, and zero for a rejected request. Our objective function can thus be rewritten as

$$\text{Maximize } \sum_{i=1}^k F_i \cdot x_i + F_g \cdot \mu_g \cdot x_g \quad (5.4)$$

Since the group request is either accepted or rejected, there is no explicit constraint on the demand level. There is, nevertheless, an implied demand constraint, in that we cannot consider group requests larger than the authorized capacity of the aircraft. The capacity constraints, however, still apply. If we accept the group request, then the authorized capacity of the aircraft is in essence reduced by the size of the group, μ_g . We then must optimally allocate the remaining seats to individual passenger demand. Our expression for the capacity constraint thus becomes

$$\sum_{i=1}^k x_i \leq C - (\mu_g \cdot x_g) \quad (5.5)$$

or

$$\sum_{i=1}^k x_i + (\mu_g \cdot x_g) \leq C \quad (5.6)$$

The complete integer linear programming formulation for a single flight leg in the deterministic environment is thus:

$$\text{Maximize } \sum_{i=1}^k F_i \cdot x_i + F_g \cdot \mu_g \cdot x_g$$

Subject to

$$x_i \leq \mu_i \quad \text{for } i = 1, \dots, k$$

$$\sum_{i=1}^k x_i + (\mu_g \cdot x_g) \leq C$$

$$x_i \geq 0 \text{ and integer} \quad \text{for } i = 1, \dots, k$$

$$x_g = 0 \text{ or } 1$$

For the multiple flight leg case, we additionally define $x_{g,O-D}$ as a binary decision variable for a group request on a specific O-D itinerary. The mathematical programming formulation is entirely similar to the multiple leg case described previously, with the objective function modified to include the revenue of the group request, and the capacity constraints modified to reflect the reduction in capacity if the group request is accepted.

As discussed previously, the deterministic environment, though analytically straightforward, does not realistically characterize the true nature of airline passenger demand. To incorporate the uncertainty involved in passenger demand forecasts, we now discuss the probabilistic formulation of the planning model approach to seat inventory control.

5.1.2 The Probabilistic Environment

An accurate depiction of airline passenger demand should incorporate the uncertain nature of forecasted demand levels - indeed, it is unreasonable to think that forecasts will always be realized with certainty. To account for such uncertainty, the expected marginal revenue approach, as described in Chapter 2, evaluates the probability of selling a seat if it is allocated to a particular fare class, and determines the expected revenue contribution of that seat by scaling the average revenue for that fare class by that probability. We define $EMR_{i,j}$ to be the expected marginal revenue of the j th seat if it is allocated to the i th fare class, and $x_{i,j}$ to be a binary variable taking the value one if the j th seat is allocated to the i th fare class, and zero otherwise.

Consider the single flight leg case again. Since each of the C seats of the aircraft can be allocated to any one of the k fare classes, there can be as many as $k * C$ decision variables $x_{i,j}$ each with a corresponding $EMR_{i,j}$. The revenue maximizing objective function then becomes

$$\text{Maximize} \quad \sum_{i=1}^k \sum_{j=1}^C EMR_{i,j} \cdot x_{i,j} \quad (5.7)$$

In the probabilistic environment, the notion of strict demand constraints is not appropriate, since there exists a non-zero probability of selling a very large number of seats in any or all of the fare classes. There is an implicit demand constraint driven by the aircraft capacity, as we cannot allocate more seats than are authorized. Thus, in recognizing only $k * C$ decision variables we do not deny the possibility of demand in excess of capacity, we merely recognize that we cannot fulfill that demand given the physical limitations of the aircraft.

To ensure that we do not exceed the authorized capacity with our allocations, we again include a capacity constraint, which can be written as

$$\sum_{i=1}^k \sum_{j=1}^C x_{i,j} \leq C \quad (5.8)$$

The complete probabilistic linear program for the single leg case can be written formally as

$$\text{Maximize} \quad \sum_{i=1}^k \sum_{j=1}^C EMR_{i,j} \cdot x_{i,j}$$

Subject to

$$\sum_{i=1}^k \sum_{j=1}^C x_{i,j} \leq C$$

$$x_{i,j} = 0,1$$

Again, we can generalize the probabilistic linear program to the multiple leg, multiple fare class case by defining the binary decision variable $x_{i,j,O-D}$, which receives a value of one if the j th seat of the aircraft is allocated to the i th O-D/fare class combination, and zero otherwise. We now discuss the addition of group demand into the probabilistic environment. There are two cases which we will consider, deterministic group demand and probabilistic group demand.

Including Deterministic Group Demand

Although we are under the assumptions of probabilistic individual passenger demand, it is not unreasonable to imagine the negotiation of a group request which, if the origin-destination and price (but not the specific itinerary) were already accepted, would represent deterministic group demand. The decision as to whether or not to accept an already negotiated group request for a specific flight leg is thus equivalent to allocating deterministic group demand in a probabilistic individual passenger environment. The accept/reject decision would be determined by comparing the total expected flight revenues with and without the group. We can model such a situation with a probabilistic linear program, including the deterministic group additions presented in Section 5.1.1. More formally, the linear program for the single leg problem can be expressed as

$$\text{Maximize} \quad \sum_{i=1}^k \sum_{j=1}^C [EMR_{i,j} \cdot x_{i,j}] + F_g \cdot \mu_g \cdot x_g$$

Subject to

(5.9)

$$\sum_{i=1}^k \sum_{j=1}^C x_{i,j} + x_g \leq C$$

$$x_{i,j}, x(g) = 0,1$$

The solution set includes a value for $x_g = 1$ if the group should be accepted on the specific flight leg and zero otherwise, along with the optimal allocation of individual passenger demand over the remaining capacity. Similar analysis can be repeated for other flight legs, if necessary. The difficulty of this approach is that many flight leg combinations

may have to be searched before finding a feasible group itinerary -- one for which $x_g = 1$ for all flight legs.

Including Probabilistic Group Demand

Planning seat allocations for future group demand is not necessarily limited to deterministic group requests. Ideally, we can foresee a revenue management system which forecasts individual as well as group passenger demand based on historical booking patterns and optimally allocates the seats of an aircraft to both types of demand. We would thus introduce a group demand fare class, say G-class, where an availability for group demand is explicitly calculated. Since per passenger group fares are typically lower than individual passenger fares, the G-class allocation could be used to limit the number of seats sold to travel agents and tour operators. Not all group fares are lower than every individual passenger fare, and thus the G-class allocation could also be used to protect seats for group passengers away from the deeply discounted individual passengers. Such a system suggests that probabilistic group demand forecasts will be needed as inputs to the optimization models.

In order to obtain probabilistic forecasts of group demand, we need a mathematical model for the distribution of group passenger demand. Such a model is described in detail in Chapter 4. Section 4.5 explicitly characterizes a mathematical derivation of the distribution of group passenger demand, $p_g(g_0)$ and includes expressions for the expectation and variance of the distribution, along with a completely defined formula for obtaining the probability mass function itself.

With a completely defined probability mass function for the distribution of group passenger demand, it is possible to evaluate the expected marginal revenue values

(EMR's) associated with allocating seats to a separate group demand fare class. Provided with the average revenue paid by a group passenger on a flight segment, we can calculate the $EMR_{j,g}$ (the expected marginal revenue of allocating seat j to the G-class) by simply multiplying the average revenue per group passenger times the probability of selling that seat to a group passenger. The probability that group demand will exceed the j th seat is obtainable from the probability mass function $p_g(g_0)$. More specifically, it is the complement of the cumulative mass function evaluated at the j th seat. or

$$P(g,j) = 1 - P_g(j) \quad (5.10)$$

where $P(g,j)$ is the probability of selling the j th seat to a group passenger, and $P_g(j)$ is the cumulative mass function of $p_g(g_0)$ evaluated at j . The expected marginal revenue of the j th seat if allocated to the group demand fare class is thus

$$EMR_{j,g} = P(g,j) * F_g \quad (5.11)$$

where F_g is the average fare of a group passenger on that segment. Thus, the treatment of groups in the planning model is equivalent to a new fare class with average revenue F_g , in the probabilistic environment.

The single leg, probabilistic linear program which includes group demand can thus be written as

$$\text{Maximize } \sum_{i=1}^{k,g} \sum_{j=1}^C EMR_{i,j} \cdot x_{i,j}$$

Subject to (5.12)

$$\sum_{j=1}^C \sum_{i=1}^{k,g} x_{i,j} \leq C$$

$$x_{i,j}, x_{g,j} = 0,1$$

For the multiple leg, multiple fare class case, the probabilistic linear program can be written

$$\text{Maximize}_{O-D} \sum_{i=1}^{k,g} \sum_{j=1}^C EMR_{i,O-D,j} \cdot x_{i,O-D,j} \quad (5.13)$$

Subject to

$$\sum_{O-D} \sum_{i=1}^{k,g} \sum_{j=1}^{C_k} x_{i,O-D,j} \leq C_k \quad \text{for all O-D itineraries and } i \text{ classes on flight } k,$$

for all flights k

$$x_{i,O-D,j} = 0 \text{ or } 1$$

From the solution set, we can determine how many seats should be allocated to anticipated group demand by summing the non-zero $x_{g,O-D,j}$'s.

It is necessary again to emphasize that fares for group demand are almost always negotiated. Thus, it may seem inappropriate to use a seat allocation of say, 20 seats,

which is calculated based on some mean revenue value, to determine whether or not to accept a group request of size 30 with a per passenger group fare considerably higher than the mean revenue. The purpose of the group fare class allocation is not necessarily to prevent large groups from booking tickets on a particular flight. As discussed previously, through the sharing or nesting of inventories, we can keep economically sound requests from being rejected due to inadequate space in a single fare class. The group fare class allocation can serve a dual purpose. For lower than average group fare requests, the G-class allocation serves to limit the number of seats available to group demand, while for higher than average requests, the G-class can serve as a guideline to protect a certain number of seats from being sold to lower revenue individual passengers. This idea is similar to the protection levels used for higher individual passenger fare class inventories.

Control of group passenger demand does not necessarily require the forecasting of future demand based on historical data. Indeed, not all group passenger demand can be anticipated. For individual passengers, unanticipated demand is controlled by updating passenger demand forecasts and possibly redistributing seat allocations for the remaining capacity of the aircraft. For unanticipated group demand, there is increased flexibility in the absence of a fixed fare structure, and thus through negotiation of the group fare, the airline can make accept/reject decisions for each individual group request in the absence of a group demand fare class. Effective negotiation is aided by an analytic means of determining the costs and benefits of accepting a group request for any given fare. The Displacement Cost Model presented in the following section provides analytic support for the negotiation of ad hoc group requests, single requests made by groups, generally for a given origin-destination itinerary and departure date.

5.2 The Displacement Cost Model for Ad Hoc Group Requests

In the previous section, we detailed mathematical programming formulations of the planning model approach to the single and multiple leg, multiple class seat inventory control problems, including explicit consideration of the control of forecasted group demand through a separate "G-class" inventory. Future group demand, can be modeled as either deterministic or probabilistic. In the case of probabilistic demand, historical data regarding the likelihood of receiving requests (along with the request size and utilization ratios) can be maintained and used in much the same manner individual passenger data is used to forecast future passenger demand.

On the other hand, in the absence of an explicit group demand fare class for which future demand levels are forecasted, an airline can still make accept/reject decisions for group requests based on the future demand forecasts of individual passenger demand. We now present strategies for using the decision making model, which can be implemented in real-time for ad hoc group requests.

Our approach to solving the decision making problem for an ad hoc request is based on a displacement cost model. In accepting a group request, an airline potentially displaces up to S (where S = the size of the group request) individual passengers on each flight leg the group travels. Since group fares are often discounted below individual passenger fares in the same booking class, the decision of whether or not to accept a group request is directly dependent on the anticipated individual passenger demand for each flight leg in the group's itinerary. The group request should be accepted as long as the revenue generated from the group is greater than the expected revenue of the individual passengers the group displaces. Since ad hoc group requests are typically

subject to price negotiation rather than a decision based upon a single fare quote, the output of our displacement cost model will be a minimum acceptable group fare for the given itinerary. This minimum group fare represents the indifference point for the airline between accepting the group and rejecting the group in favor of anticipated individual passenger demand [16]. In equation form, the minimum group fare can be thought of as

$$\text{Minimum Group Fare} = \frac{\text{Total Expected Revenue of Displaced Passengers}}{\text{Group Request Size}}$$

Determining the total expected revenue of the displaced individual passengers can be accomplished using a variety of methods. In general, however, we recognize that individual passenger displacement costs can be thought of in two ways: in a leg-based scenario, and a network-based, (multiple-leg) scenario. Section 5.2.1 illustrates the leg-based approach to solving the displacement cost problem using the expected marginal revenue approach, while Section 5.2.2 describes how traditional MLMC seat inventory control formulations such as the planning models discussed in Section 5.1 can be employed as a decision making tool.

5.2.1 The Leg-Based Displacement Cost Model

The leg-based displacement cost model is based upon the expected marginal revenue approach described in Section 2.4.2. Recall that, due to the uncertain nature of passenger demand, forecasts are often characterized by an expected value and some measure of the variation of the demand. In allocating a seat to a particular booking class, we are not guaranteed to receive the revenue associated with that booking class for that seat, and, in fact, the more the number of seats allocated exceeds the forecast, the less likely we are to receive that revenue. Thus the expected marginal revenue function for

allocating seats to a fare class is a non-increasing function. Even in a fully nested environment where seat inventories among the different fare classes are shared to increase flight revenues, insufficient passenger demand may elicit infinitesimally small expected revenues for the last seats on an aircraft.

For each leg on which the group will travel, our displacement cost model explicitly calculates the expected marginal revenues associated with each seat of the aircraft, based on the individual passenger demand forecasts and the specific fare structures associated with each flight leg. Since group requests are typically made several months in advance of the actual flight departure, we will assume that passenger demand forecasts can be adjusted, if necessary, to reflect seasonal variation in demand levels. Also, since group requests are typically made for the lowest possible fare, we assume in our model that the group will be booked into the seats of the aircraft having the lowest expected marginal revenues, filling the remaining seats in a "bottom up" fashion. Thus for a request of size 20, the total individual passenger displacement cost is the sum of the 20 lowest expected marginal revenues of the remaining available seats on the aircraft. This calculation is repeated for each leg of the group request, and the sum of the displacement costs for all legs represents the total expected revenue of displaced individual passengers. The minimum group fare the airline can charge without losing revenue follows directly from this calculation.

A Simple Numerical Example

Consider the following 2-leg example for a one-way group request of size 15. The individual passenger demand forecasts and the fare structures for each leg are given in Table 5.1. The aircraft assigned to both flight legs has a capacity of 100 seats and we assume that there are no bookings on hand at the time the request is made. We also

assume that the demand forecasts shown reflect seasonal demand for the departure dates requested.

Table 5.2 shows the approximate expected marginal revenues associated with the last 15 seats of the aircraft calculated using Belobaba's EMSR heuristic, along with the calculations necessary to determine the minimum group fare for the specific request. The minimum group fare of \$190.31 per group passenger reflects an expected total displacement of \$2,854.66 in individual passenger revenues in accepting the group request on both legs.

As the name implies, the leg-based displacement cost model does not directly take into account the "network effects" discussed in the overview of the general MLMC seat inventory control problem. Clearly, an estimate of the true optimal minimum group fare based on individual passenger revenue displacement requires specific forecasts for individual passenger traffic on the origin-destination itinerary the group will travel, though such data are often unavailable or inaccurate. To partially account for such "network effects", we can modify the assumed revenue values of the different inventories on individual flight legs in order to reflect the specific O-D/itinerary composition of each fare class revenue, similar to what is done by some leg-based seat inventory control models. Nevertheless, the concepts behind the leg-based displacement cost model remain the same.

Table 5.1: Displacement Cost Model Setup

<u>Leg 1</u>	Fare	Demand	Demand	Mean
	<u>Class</u>	<u>Mean</u>	<u>Stdev</u>	<u>Fare</u>
	Y	36	10	180
	B	25	6	140
	M	16	5	115
	Q	42	13	86

<u>Leg 2</u>	Fare	Demand	Demand	Mean
	<u>Class</u>	<u>Mean</u>	<u>Stdev</u>	<u>Fare</u>
	Y	14	5	380
	B	12	4	320
	M	35	10	270
	Q	22	6	220

Table 5.2: Displacement Cost Model Results

Seat #	EMR1	EMR2	TOTAL
86	\$85.26	\$135.00	\$220.26
87	\$85.10	\$130.94	\$216.04
88	\$84.59	\$128.41	\$213.00
89	\$84.66	\$124.56	\$209.22
90	\$84.37	\$124.24	\$208.61
91	\$84.04	\$113.60	\$197.97
92	\$83.66	\$110.00	\$193.66
93	\$83.46	\$104.22	\$187.68
94	\$83.21	\$103.16	\$186.37
95	\$82.83	\$98.74	\$181.57
96	\$82.69	\$95.44	\$178.13
97	\$82.10	\$93.04	\$175.14
98	\$81.43	\$83.31	\$164.74
99	\$80.67	\$81.28	\$161.95
100	\$79.81	\$80.51	\$160.32

Total Displaced Revenue =	\$2,854.66
---------------------------	------------

Min Group Fare =	\$190.31 / pax
------------------	----------------

5.2.2 A Network Based Displacement Cost Model

As mentioned in the previous section, the (true) optimal solution to the minimum group fare problem for an ad hoc group request must take into account the impacts of multiple O-D/fare class itineraries on each flight leg the group travels. Our leg-based approach can only approximate such influences through a modification of the assumed revenue values of leg-based inventories. A somewhat more appropriate method is motivated by the planning models formulated in section 5.1.

Recall that one set of constraints in both mathematical programming formulations of the MLMC problem requires that the sum of all allocations to the multiple fare classes on each flight leg does not exceed the capacity of the corresponding aircraft. Also, recall that the value of the objective function in the optimal solution represents the maximum expected revenue over the entire route network under consideration. Let us define $Z(C)$ to be the optimal objective value of the linear programming formulation using the initial set of aircraft capacities, C . Now consider an ad hoc group request of size S . We define $Z(C-S)$ to be the optimal objective value of the same linear programming formulation, where the capacity constraints on each aircraft the group will travel is decreased by the size of the request S .

The value of $Z(C-S)$ represents the "best" solution to the problem given that we have accepted the group request and the S seats they requested are no longer available to further individual passenger bookings. We define the difference between the two objective values $Z(C)$ and $Z(C-S)$ to be $D(S)$. $D(S)$ represents the opportunity cost of accepting the group request in terms of expected individual passenger revenues. In other words, $D(S)$ represents the total expected revenue of displaced individual passengers

over the entire network. In algorithmic form, the planning model approach to solving the minimum group fare problem for an ad hoc group of size S over an entire network involves the following steps:

Step 1: Find $Z(C)$ using the appropriate mathematical programming formulation for the given network.

Step 2: Reformulate the mathematical program to find $Z(C-S)$, where the capacity constraints used in Step 1 are decreased by S on each flight the group will travel.

Step 3: Find $D(S) = Z(C) - Z(C-S)$.

Step 4: Minimum Group Fare = $D(S) / S$.

Note that our algorithm is completely general in that it does not make any assumptions regarding the actual mathematical programming formulation used to calculate $Z(C)$ and $Z(C-S)$.

5.3 The N-Split Minimum Group Fare Model

Thus far in our discussion of the minimum group fare problem, we have made the implicit assumption that the group request was somehow indivisible, such that the entire group must fly on the same itinerary. If, however, we relax this assumption and allow the same group request to be split up and carried over different itineraries, we may in theory

achieve an even lower minimum group fare for the same ad hoc request, or alternatively, a lower displacement cost at the same fare, translating to a greater profit for the airline.

The relaxation of the indivisible request assumption is quite reasonable given the large hub and spoke route networks operated by most of today's major carriers. It is not uncommon, for instance, for an airline to be able to service a specific origin-destination pair with connecting flights through more than one hub, while offering virtually the same departure and arrival times. Thus, an airline can better utilize excess capacity over more flight legs simply by dividing the group in such a way as to displace the minimum expected revenue from individual passengers, jointly over multiple itineraries, rather than just one.

The problem of finding the optimal division of the group request is in theory, no more difficult than the minimum group fare problem for a specific itinerary, only now we must specify a set of acceptable alternatives to the initial request. Acceptable alternatives, for example, may be those itineraries having departure and/or arrival times within some designated time window of the initial request. Once the set of alternatives has been specified, we must then decide upon the maximum number of different itineraries our group request will be allowed to travel. Note that this is not a rigid constraint, and in fact, the true upper bound is the number of acceptable alternatives; however, a large number of selections becomes cumbersome in practice.

We will continue to discuss the actual algorithm in terms of the leg-based model, for simplicity, though entirely equivalent procedures can be performed for the network model as well. In the leg-based formulation, the minimum group fare for a request of size S can be found from a set of A alternatives split over n different itineraries by creating $\binom{A}{n}$ itinerary n -tuples. The expected marginal revenues associated with the last S seats of each itinerary in an n -tuple are pooled, and our minimum group fare algorithm is applied to

the pooled set of expected marginal revenue values in exactly the same manner. We fill the group request from the bottom to the top of our pooled set of revenues, keeping track of which itinerary provided the minimum displaced revenue at each iteration, until the number of seats allocated to the group request reaches S . The n-split follows directly from the allocations.

Thus the displacement cost model for ad hoc group requests is not only limited to finding the minimum group fare for a given itinerary, but it can also be used to find the theoretically lowest possible fare if the group is willing to be divided over multiple itineraries. From a competitive perspective, an airline using the n-split displacement cost model can better utilize excess capacity, while simultaneously offering a variety of pricing alternatives for all ad hoc group requests. Ultimately, better utilization of resources lowers unit operating costs and provides incremental revenue on a larger scale.

Striving for a better utilization of resources is by no means a new concept to the airline industry. One of the most common and well-accepted means of attempting to maximize the utilization of fixed resources in the form of a scheduled flight departure is overbooking. The final section of this chapter discusses the implementation issues involved with incorporating the notion of group request utilization ratios into the general methodologies of passenger overbooking.

5.4 Overbooking in the Group Passenger Context

Just as the concepts of utilization ratios and sell-up can be modified to fit the framework of group demand as described in Chapter 4, so can the ideas and methodologies of passenger overbooking. In the case of individual passengers, the

privileges associated with fully-refundable tickets allow a traveler to make multiple reservations for a single trip, and not pay a penalty for reservations which go unused. By consistently and accurately estimating the proportion of bookings which ultimately are not used, an airline can book more seats than the physical capacity of the aircraft in order to depart with as few empty seats as possible, without denying boarding to an economically unsound number of passengers.

The amount by which an airline "overbooks" naturally depends on the estimate of the attrition rate of reservations. Prior to the day of departure, a confirmed reservation can be made and later cancelled, at which time the airline can still increase the availability of the seat inventory for the affected flight(s). The probability that a confirmed reservation will be cancelled prior to flight departure is known as the cancellation rate. The complement of the cancellation rate is often referred to as the utilization rate, which we discuss in detail in Chapter 2. In the weeks prior to departure, an airline can overbook a flight by taking into account the cancellation rate.

Passengers holding reservations for fully-refundable tickets, as we have discussed above, do not necessarily cancel unwanted reservations prior to flight departure and do not incur a penalty. Thus, airlines must account for so-called "no-shows" through overbooking in order to achieve maximum revenue potential. However, an overly aggressive overbooking policy or inaccurate estimates of no-show rates can lead to more passengers than available physical capacity at the time of departure. Such a situation leads to passengers being denied boarding on a flight. The costs of denying boarding to many passengers may exceed the incremental revenues associated with an aggressive overbooking policy, and should be considered before implementation.

A rather simplistic expression used for calculating the proportion above physical capacity a flight can profitably be overbooked is

$$\text{Overbooking Factor} = \frac{1}{1 - NS}$$

where NS is the average no-show rate for individual passengers and the overbooking factor is a scalar which, when multiplied by the physical capacity of the aircraft, gives the authorized number of seats which may be reserved on that flight. More complex expressions take into account the stochastic nature of the no-show rate and the costs associated with denied boardings, but the underlying principles are similar.

When dealing with group requests, however, such a singular calculation is not practical when trying to develop an analytic means of overbooking. In Chapter 2, we describe three distinct components to the utilization ratio associated with the group booking process, each with a corresponding cost. The third utilization rate, u_3 , is most closely analogous to the notion of a no-show rate, where each individual passenger of a group has a specific probability of using his purchased ticket. There is another "no-show" rate that we have not yet discussed, which can be thought of as a "no-show" rate for the group request as a whole. Recall that many months prior to departure, group requests can be booked and returned with little or no penalty. Thus, a large number of group bookings are made based on speculation, with a large degree of uncertainty. The initial opportunity cost to the airline of a single cancelled group booking is relatively small, but many such cancellations can have a serious negative impact on potential revenues, particularly if authorized capacities are based solely on the no-show behavior of individual passenger demand.

Ideally, we would like to be able to overbook a flight during the months prior to departure to reflect the number of groups that are likely to cancel their requests outright, in order to better accommodate genuine group passenger demand. Let us define the probability of a group request being canceled in its entirety prior to flight departure as p_1 . To determine the amount by which we should overbook the entire aircraft, let us separate the physical capacity of the aircraft into an "individual passenger" capacity and a "group passenger" capacity. This separation can be done using data based on the historical portions each segment of demand has occupied on previous flights. Returning to our simple overbooking expression, we might be tempted to overbook the "group passenger" capacity using a simple substitution of p_1 for NS, the no-show rate for an individual passenger. The danger in this substitution is the significant potential of denying boardings to many passengers at once.

For example, say our estimate for p_1 is .25, or the probability of an initial group request canceling outright is 25%. For an aircraft with a physical capacity of 360 and a "group passenger" capacity of 120, the authorized capacity for group demand based on our naive substitution of p_1 for NS is 160. Assume that we accept two group requests of size 50, and one of size 60 (if a group shows up, all passengers will show up) based on the authorization level calculated with our modified equation. From a purely expected value sense, the long run average number of passengers who will show up for the flight is $.75*160$, or 120. Assuming the three groups act independently, however, there is a $(.75)^3 = 14\%$ or a 9 in 64 chance that all three groups will show up on the day of departure, for a total of 160 passengers. This event translates to 40 extra group passengers than are accounted for from a strictly expected value sense, and is likely to be quite costly in terms of denied boarding costs. Thus, much greater attention to potential denied boarding risk is required when trying to establish a policy for overbooking groups on the basis of the no-show parameter, p_1 .

Returning to our earlier discussion of utilization ratios, it is clear that there is another opportunity to increase potential revenues through overbooking. Given that our group request will show up on the day of departure, there is still uncertainty associated with the fraction of the initial block of requested seats that will be purchased and used. In Chapter 2, we discussed the three separate components of the utilization rate associated with a genuine group request. If we consider the aggregate of all three utilization rates as the fraction of a genuine group request which will ultimately be used, then we can define the complement of that fraction as p_2 , the fraction of a genuine group request which will ultimately be returned to the seat inventory prior to the day of departure.

From our discussion of the group booking process, we recognize that the uncertainty or variability of p_2 is based, to a great extent, on the behavior of travel agents and tour operators who request blocks of seats many months in advance based on the speculation of fulfilling the request over the next few months. This uncertainty is in contrast to the underlying distribution of individual passenger cancellation rates, which are less likely to be motivated by such large degrees of future speculation. Due to the large degree of uncertainty behind the distribution of p_2 , similar caution should be used in substituting p_2 for NS in our simple expression for calculating overbooking levels for "group passenger" capacity.

The optimal overbooking policy for group passenger demand will naturally be influenced by existing policies already being used for individual passengers. The above discussion is intended merely to make readers aware that group passenger overbooking, like seat inventory control, deserves separate attention. Similarities do exist, however, to individual passenger methods, which make the possibilities of transfer and sharing improvements to either method encouraging.

The techniques presented in this chapter outline enhancements for current seat inventory control methodologies. Demand forecasting, seat allocation, and overbooking, in one form or another, are all currently being employed by even small sized carriers. The level of sophistication of these techniques depends on the available data as well as the costs and benefits associated with improving current practice. If a large portion of a carrier's total traffic can be attributed to group passenger demand, the enhancements presented can serve to improve the management of that demand. If separate group demand histories and even forecasts are currently maintained, the revenue improvements associated with implementing the planning and decision making models we have discussed could prove to be quite significant.

Chapter 6

Conclusions

6.1 Summary

The previous chapters of this thesis have discussed the general seat inventory control problem faced by airlines. The emergence of large hub and spoke networks has further complicated the problem due to the large number of potential origin-destination / fare class combinations which must be controlled on a single flight leg. Most seat inventory control mechanisms in use by airlines currently focus on making decisions regarding the individual passenger, but neglect a rather important component of airline passenger demand, namely group passenger demand. Group demand is prevalent in markets where there exists a large proportion of leisure traffic, as well as in many international markets. Decisions made regarding group requests require special attention due to several major differences in pricing and booking procedures, which we have detailed in Chapter 4. These differences include a flexible, negotiated price structure, multiple utilization ratios, and a longer time frame prior to departure over which demand is realized.

In order to consolidate individual and group passenger demand into a more general seat inventory control methodology, it was necessary to characterize mathematically the elements of variability unique to group passenger demand. These elements include the number of group requests, the size of any individual request, and the utilization ratios associated with each request. Armed with these elements, we developed a well-defined model for the distribution of group demand which considers these primary sources of variability inherent to the group booking process.

We then modified traditional seat inventory control techniques to include the control of group passenger demand, without changing any of the underlying assumptions made in the original models. These new formulations are presented in the first sections of Chapter 5, and can be employed to control the seat inventories on future flight departures based on forecasts for both individual and group passenger demand. Frequently, however, unanticipated group demand arises and decisions as to whether or not to accept a request or what fare to charge must be made on the spot. For these decisions, the individual passenger Displacement Cost Model was developed to determine the expected revenue of individual passenger demand displaced as a result of accepting a group request. The minimum per passenger group fare an airline should charge follows directly from the Displacement Cost Model. Issues such as overbooking groups and the opportunity costs associated with inaccurate estimates of utilization rates are also discussed.

To be an effective revenue generating tool, any model for improving seat inventory control procedures must be capable of being implemented. As our numerical examples demonstrate, the practical application of the theoretical models we have presented can be rather straightforward if the required data are available. Passenger demand forecasts and average fare class revenues are maintained by even the most basic seat inventory control systems, and are the primary pieces of information needed to implement the fundamental

displacement cost model for ad hoc group requests. Clearly though, the more sophisticated the available data, the more effective the actual implementation will be. For example, maintaining separate individual and group passenger demand forecasts makes implementing any of the group seat inventory control methodologies much simpler.

6.2 Further Research Topics

The area of group seat inventory control is relatively new and rich in opportunities for continued research. The two primary directions of such research are in group demand modeling and improved seat allocation methodologies.

Group Passenger Demand Modeling

A primary assumption of the methods discussed in this thesis is that demand forecasts for groups and individuals can be separated. Consider the Displacement Cost Model for ad hoc group requests. We base the calculations for the minimum per passenger group fare on the demand forecasts for individual passengers expected on each flight. These demand forecasts may include, in some sense, the group request we are negotiating if the forecasts are based on an aggregate booking history which does not differentiate between individuals and groups. The total displacement cost of strictly individual passenger demand may therefore be artificially high because our forecasts already include the anticipation of some group demand.

Methods to filter out historical group demand from individual passenger forecasts would undoubtedly make our model a more precise negotiating tool. Note, however, that even if group demand is not currently filtered out, the model errs on the proper side from

the carrier's perspective. By suggesting a slightly higher rather than a lower minimum fare, the analyst does not jeopardize a revenue maximization objective; rather, the model simply suggests a higher pricing floor for each group request. Such a problem can be remedied in the short term by simply educating analysts about how the model works.

The technique of characterizing group demand as a random sum of three random variables used to develop our model for the distribution of group passenger demand is not unique. Other models which incorporate other elements of variability can also be explored. Indeed, an empirical validation of our model is itself worthy of future research. Other approaches are certainly plausible, including an extensive examination of historical group booking patterns to identify trends in the group booking process not captured by our simple three variable model.

The notion of the multi-tiered utilization rate is another area which deserves attention from both a theoretical and a practical perspective. Theoretically, extending the ideas of non-refundable deposits and applying them to returned bookings in the earliest stages of the booking process promises to reduce the variability of utilization rates. Tour operators and travel agents who routinely book a single group on multiple carriers while trying to find the lowest quote would be discouraged to continue such a practice if non-refundable deposits were required many months prior to the date of departure. However, the threat of losing long-term goodwill may make this practice undesirable from a practical point of view. Instead, perhaps a policy of rewarding the travel agents and tour operators with more reliable group requests (from a utilization perspective) would do more to reduce the variability of utilization rates while maintaining, if not improving goodwill.

Improving Seat Inventory Control Techniques

Improvements to current seat inventory techniques are continually being researched due to the large potential revenue impacts. As mentioned previously, much research is being done in the area of O-D optimization, which directly accounts for network-wide revenue contributions rather than simply leg-based considerations. Certainly, improvements to individual passenger seat inventory control will directly impact group control, and as we have seen, techniques can be shared between the two methodologies with only minor adjustments. Empirical tests of various modified leg-based methods versus network-wide formulations are being performed for individual passengers, and an obvious extension of this research would be to explore the problem in terms of group demand.

In Chapter 5 we discussed the N-Split Minimum Group Fare Model, which is an extension of the leg-based Displacement Cost Model for determining the minimum per passenger fare for a group request. The concept of searching for the least "expensive" excess capacity over several fixed alternatives can be generalized to a complete network-wide search to elicit the "optimal" split of any group request over an appropriate subset of the route structure of an airline. A seat inventory control system capable of optimally distributing group requests over unused capacity could have dramatic revenue impacts. Moreover, the next logical step for such a system would be to distribute forecasted group requests over the capacity with the lowest displacement costs (determined from an expected marginal revenue perspective), thus managing seat inventories even more effectively.

The idea of better excess capacity management through seat allocation motivates the next step in the development of revenue management systems. Traditional seat

allocation techniques are currently employed under the assumption of fixed capacity based on the carrier's operating schedule. If we relax the fixed capacity assumption, we can begin to approach the problem from both the scheduling and the revenue management perspectives. A truly integrated system of scheduling and revenue management would approach the true "optimal" solution for revenue maximization, though implementation from an operations point of view might be prohibitively difficult.

Bibliography

- [1] Kenneth Littlewood, "Forecasting and Control of Passenger Bookings," AGIFORS Proceedings, October 1972, pp 95-117.
- [2] A. V. Bhatia and S. C. Parekh, "Optimal Allocation of Seats by Fare," AGIFORS Reservation Study Group, 1973.
- [3] Helmut Richter, "The Differential Revenue Method to Determine Optimal Seat Allotments by Fare Type," AGIFORS Symposium Proceedings, October 1982, pp 339-362.
- [4] R. E. Curry, "Optimum Seat Allocation with Fare Classes Nested on Segments and Legs," Tech. Note 88-1, Aeronomics Incorporated, Fayetteville, Ga., June, 1988.
- [5] S. L. Brumelle and J. I. McGill, "Airline Seat Allocation with Multiple Nested Fare Classes," ORSA/TIMS Conference, Denver, Colo., October, 1988.
- [6] R. D. Wollmer, "An Airline Seat Management Model for a Single Leg Route When Lower Fare Classes Book First," ORSA/TIMS Conference, Denver, Colo., October, 1988.
- [7] P.P. Belobaba, "Air Travel Demand and Airline Seat Inventory Management," Report R87-7, Flight Transportation Laboratory, MIT (May), 1987.
- [8] Jorn Buhr, "Optimal Sales Limits for 2-Sector Flights," AGIFORS Symposium Proceedings, October 1982, pp 291-303.
- [9] Ken Wang, "Optimum Seat Allocation for Multi-Leg Flights with Multiple Fare Types," AGIFORS Symposium Proceedings, October 1983, pp 225-237.
- [10] Fred Glover, Randy Glover, Joe Lorenzo and Claude McMillan, "The Passenger-Mix Problem in the Scheduled Airlines," *Interfaces*, Vol. 12, No. 3, June 1982, pp 73-79.
- [11] R. D. Wollmer, "Electronic Bulkhead Seat Allocation," Unpublished Internal Report, Douglas Aircraft Company, McDonnell-Douglas Corporation, Long Beach, Calif., 1984.
- [12] R. D. Wollmer, "An Airline Reservation Model for Opening and Closing Fare Classes," Unpublished Internal Report, Douglas Aircraft Company, McDonnell-Douglas Corporation, Long Beach, Calif., 1985.

- [13] E. L. Williamson, "Comparison of Optimization Techniques for Origin-Destination Seat Inventory Control," Report FIL-R88-2, Flight Transportation Laboratory, MIT, Cambridge, MA (1988).
- [14] R. E. Curry, "Optimal Airline Seat Allocation with Fare Classes Nested by Origins and Destinations," *Transportation Science*, Vol. 24, No. 3, August 1990, pp 193-204.
- [15] Alvin W. Drake, *Fundamentals of Applied Probability Theory*, McGraw-Hill Book Company, New York, 1967, pp 99-112.
- [16] Barry Smith, "Group Evaluation Using Break-even Analysis," AGIFORS Reservations and Yield Management Study Group Proceedings, April 1990, pp 177-192.

ACKNOWLEDGEMENTS

There are many whom I would like to thank for helping me to complete this thesis. First, I wish to thank the Office of Naval Research for their continued financial assistance, which allows me to concentrate my efforts on "doing my own thing". Also, I would like to thank the people at Delta Air Lines for providing their motivation and guidance in helping me to focus my efforts. Thanks also to my friends at the Flight Transportation Laboratory, including my advisor, Professor Peter Belobaba, who had to read several embryonic versions of this my final effort. And, of course, I would like to thank and apologize to my two families -- my mother and sister, for putting up with me trying to write my thesis during Christmas dinner, and the students of MacGregor's H-Entry, who sometimes didn't know they *had* a graduate tutor.