Dynamic Seat Assignment with Social Distancing

Dis· count

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Abstract

Keywords: Social Distancing, Seat Assignment, Dynamic Arrival.

1 Introduction

1. define a different social distancing rule.

use the expected demand as the group portfolio to obtain the seat planning.

2. consider ‘departure’

Each arrival has an arrive time and leave time.

3. different price regions in cinemas.

4. How to assign the high-speed train tickets fairly?

2 Literature Review

3 Model

There are some rules: time Distancing: The time gap of each used room. Space Distancing: Enlarge the space distancing of each room as much as possible. That is, if we have a room contains q_k seat numbers, and distance ratio is r, then the number of customers who can be served p_i is less than $q_k \cdot r$. This condition can be set as a constraint.

Virables: Room numbers $k \in K = \{1, \ldots, |K|\}$. Room k contains seat number q_k. Number of customers in session i is p_i for each $i \in N = \{1, \ldots, n\}$. w_{ik} is the session i's start time in the room k. s_i is the service time for session i.(Given, in our case, we set all the service times are 2 hours.)

Feasibility:

Time window constraints: Time window $[a_i, b_i]$ for each group, but it satisfies the time constraints during opening time $[E, L]$ for the room.

Capacity constraint: The largest number of customers in a session p_i^* cannot exceed the product of the largest room capacity and ratio r, that is $p_i^* \leq q_k^* \cdot r$.

Solution:

Define the time distance t_i for session i. It can be variable or the constant. In our case, we set the time interval as the variable and it should be larger than half an hour.

Define a binary variable x_{ijk} for each room. If the room k is used by (i, j) and i followed by j, then $x_{ijk} = 1$, else $x_{ijk} = 0$.

Define w_{ik} is the session i's start time in the room k.

s_i is the service time for each session.(Given) Set it as a time window VRP problem and add the distance constraints.

Analysis:

Add two virtual nodes (0,n+1) for each room. One is the start node, its time window can be a time point E meaning the room is open; the other is the end node, its time window is also a time point L meaning the room is closed.

Expected result: Show the specific assignment for the coming people.

Give the sequence of each room, and the corresponding service start time.

Benchmark: First in First out. Manual work.

Question: How to determine the objective function?

How to determine the distance for the only one group in a room?

How to compare the result with the benchmark?

MODEL:

$$\min_{i,j,k} \quad \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} x_{ijk} \tag{1}$$

$$s.t. \quad \sum_{k\in K} \sum_{j\in\delta^+(i)} x_{ijk} = 1 \qquad \forall i \in N \tag{2}$$

$$\sum_{j\in\delta^+(0)} x_{0jk} = 1 \qquad \forall k \in K \tag{3}$$

$$\sum_{i\in\delta^-(n+1)} x_{i,n+1,k} = 1 \qquad \forall k \in K \tag{4}$$

$$\sum_{i\in\delta^-(j)} x_{ijk} - \sum_{i\in\delta^+(j)} x_{ijk} = 0 \qquad \forall k \in K, j \in N \tag{5}$$

$$w_{ik} + s_i + t_i - w_{jk} \le (1 - x_{ijk}) M_{ij} \qquad \forall k \in K, (i,j) \in A \tag{6}$$

$$a_i \sum_{j\in\delta^+(i)} x_{ijk} \le w_{ik} \le b_i \sum_{j\in\delta^+(i)} x_{ijk} \qquad \forall k \in K, i \in N \tag{7}$$

$$w_{0k} = E, w_{n+1,k} = L \qquad \forall k \in K \tag{8}$$

$$t_i \ge 0.5 \sum_{j\in\delta^+(i)} x_{ijk} \qquad \forall k \in K, i \in N \tag{9}$$

$$p_i \sum_{j\in\delta^+(i)} x_{ijk} \le 0.3 q_k \qquad \forall k \in K, i \in N \tag{10}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall k \in K, (i,j) \in A \tag{11}$$

The constraint (1) is to minimize the cost resulted by opening sessions.

The constraint (2) Every session i which is followed by session j is only served once by one room k.

The constraint (3) For every room k, start from session 0.

The constraint (4) For every room k, end at session (n+1).

The constraint (5) For every room k, session j will leave when it is served.

The constraint (6) session i start time + service time + interval(required) less than next session j start time. M for linearization.

The constraint (7) Time window constraints for every session.?

The constraint (8) Add two node indicate the start node and end node.

The constraint (9) Time distance constraint.?

The constraint (10) Space distance constraint.

3.1 M0

Maximize the distance.

Input: Service time s_i for each session instead of the time window [a,b].

Add $p_0 = 0, s_0 = 0, E = 0/8, L = 24$.

MODEL:

$$
\begin{aligned}
max_{i,j,k} \quad & \sum_{(i,j)\in A}\sum_{k\in K} \frac{p_i}{q_k} x_{ijk} + \frac{1}{24} T_{ijk} && && (12)\\
s.t. \quad & \sum_{k\in K}\sum_{j\in\delta^+(i)} x_{ijk} = 1 && \forall i \in N && (13)\\
& \sum_{j\in\delta^+(0)} x_{0jk} = 1 && \forall k \in K && (14)\\
& \sum_{i\in\delta^-(n+1)} x_{i,n+1,k} = 1 && \forall k \in K && (15)\\
& \sum_{i\in\delta^-(j)} x_{ijk} - \sum_{i\in\delta^+(j)} x_{ijk} = 0 && \forall k \in K, j \in N && (16)\\
& y_{ijk} \geq (x_{ijk} - 1)M_{ij} && \forall k \in K, (i,j) \in A && (17)\\
& w_{0k} = E, w_{n+1,k} = L && \forall k \in K && (18)\\
& x_{ijk} \in \{0,1\} && \forall k \in K, (i,j) \in A && (19)
\end{aligned}
$$

How to change the quadratic terms to the linear terms(linearization)

Note that the $y = x_1 x_2$ where $x_1 \in \{0,1\}, x_2 \in [l,u] \rightarrow$

$$
\begin{aligned}
& y \leq x_2\\
& y \geq x_2 - u(1 - x_1)\\
& l x_1 \leq y \leq u x_1
\end{aligned}
$$

Let $(w_{jk} - w_{ik} - s_i) = y_{ijk}$ and $T_{ijk} = x_{ijk} y_{ijk}$

$$
\begin{aligned}
& T_{ijk} \leq y_{ijk}\\
& T_{ijk} \geq y_{ijk} - u(1 - x_{ijk})\\
& T_{ijk} \leq u x_{ijk}
\end{aligned}
$$

The constraint (1) Maximize the distance.

The constraint (2) Every session i which is followed by session j is only served once by one room k.

The constraint (3) For every room k, start from group 0.

The constraint (4) For every room k, end at group (n+1).

The constraint (5) For every room k, group j will leave when it is served.

The constraint (6) i start time + service time + interval(required) ¡ next j start time. M for linearization.

The constraint (7) Time window constraints for every group.

The constraint (8) Add two node which indicate the start node and end node.

4 Other Models

4.1 M1

q_k capacity.

$k \in K$ The Number of room

s_i service time for each group.

p_i demand number of people. $i \in N$

- Length for time. 24 for K. s_i for N. - Width for the capacity. q_k for K. p_i for N. - variable x_{ik} indicates group i served by room k.

Now we change the objective function q_k to a concave function $f(q_k)$. How to influence the result?

Search for the minimization makespan problem.

To be specific, how to deal/handle with minimax format?

$$\begin{aligned}
\min \quad & (\max(\sum_i x_{ik} s_i p_i)/(24 * f(q_k)), \quad \forall k \in K) \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}$$

To:

$$\begin{aligned}
(M1) = max \quad & t \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& t \leq \sum_i x_{ik} s_i p_i/(24 * q_k), \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}$$

$$\begin{aligned}
(M2) = min \quad & t \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& t \geq \sum_i x_{ik} s_i p_i/(24 * q_k), \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}$$

At first, it is clear that M1(max min) less than M2(min max). Thus, the true value will be between M1 and M2.

So what is the difference?

The constraint (1) Capacity ratio.

The constraint (2) Capacity constraints $|N| * |K|$.

The constraint (3) Time constraints $|K|$.

The constraint (4) Objective capacity ratio constraints $|K|$.

The constraint (5) Every group is served once $|N|$.

Virables: $|N| * |K| + 1$, refers to x_{ik}, t

Besides, when we convert the original problem into several sub-problems. Each sub-problem can be expressed as: Let $k = k_0$,

$$\begin{aligned}(Sub) = \min \quad & |\sum_i x_{ik_0} f(s_i, p_i) - r * f(24, q_{k_0})| \\ s.t. \quad & x_{ik_0} p_i \leq q_{k_0}, \quad \forall i \in N_0 \\ & \sum_{i \in N_0} x_{ik_0} s_i \leq 24\end{aligned}$$

Here, f(ServiceTime, Space) represents the area function. In fact, (12) is obviously satisfied because of the pretreatment which is used to get rid of the trouble of assignment constraints. Thus when we calculate the situation of under ratio, this sub-problem can be converted into

$$\begin{aligned}(Sub1) = max \quad & \sum_{i \in N_0} x_{ik_0} f(s_i, p_i) \\ s.t. \quad & \sum_{i \in N_0} x_{ik_0} f(s_i, p_i) \leq r * f(24, q_{k_0}) \\ & \sum_{i \in N_0} x_{ik_0} s_i \leq 24\end{aligned}$$

The final value equals to (-Sub1+$r * f(24, q_{k_0})$) This is a two-dimentional knapsack problem.

For any fixed $m \geq 2$, these problems do admit a pseudo-polynomial time algorithm (similar to the one for basic knapsack) and a PTAS.

Add one dimentional variable to the basic DP algorithm for knapsack.

Next time finish the code.

$$\begin{aligned}(Sub2) = min \quad & \sum_{i \in N_0} x_{ik_0} f(s_i, p_i) \\ s.t. \quad & \sum_{i \in N_0} x_{ik_0} f(s_i, p_i) \geq r * f(24, q_{k_0}) \\ & \sum_{i \in N_0} x_{ik_0} s_i \leq 24\end{aligned}$$

The final value equals to (Sub2 - $r * f(24, q_{k_0})$)

In fact, we do not need to calculate this form. We can obtain Sub1 firstly, then add a rest item with the minimum area.

4.2 M2

q_k capacity.

$k \in K$ The Number of room

s_i service time for each group.

p_i demand number of people. $i \in N$

- Length for time. 24 for K. s_i for N. - Width for the capacity. q_k for K. p_i for N. - variable x_{ik} indicates group i served by room k.

The Original model:

$$
\begin{aligned}
\min \quad & (\max(\sum_i x_{ik} s_i p_i)/(24 * q_k), \quad \forall k \in K) \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}
$$

To:

$$
\begin{aligned}
\max \quad & t \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& t \leq \sum_i x_{ik} s_i p_i/(24 * q_k), \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}
$$

$$
\begin{aligned}
\min \quad & t \\
s.t. \quad & x_{ik} p_i \leq q_k, \quad \forall i \in N, \forall k \in K \\
& \sum_{i \in N} x_{ik} s_i \leq T_k = 24 - (\sum_{i \in N} x_{ik} - 1) * 0.5, \quad \forall k \in K \\
& t \geq \sum_i x_{ik} s_i p_i/(24 * q_k), \quad \forall k \in K \\
& \sum_k x_{ik} = 1, \quad \forall i \in N
\end{aligned}
$$

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The constraint (2) Capacity constraints $|N| * |K|$.

The constraint (3) Time constraints $|K|$.

The constraint (4) Objective capacity ratio constraints $|K|$.

The constraint (5) Every group is served once $|N|$.

Virables: $|N| * |K| + 1$

5 Dynamic Situation

6 Results

7 Conclusion

References

Proof

(Theorem 1). ☐

(Lemma 1). ☐

(Lemma 2). ☐

(Theorem 2). ☐