Introduction to

Optimization

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School of Management University of Science and Technology of China

Sept 1, 2020

Summary

- Backpack problem
- Playing Around With Our New Toy
- 3 Fourier's Physics Playground
 - Maxwell's Electrodynamics
 - Heisenberg's Uncertainty Principle

Dis (USTC)

Backpack problem



Dis (USTC)

Ordinary Differential Equations

$$\frac{\mathrm{d}}{\mathrm{d}x}y(x) + \frac{1}{CR}y(x) = 0$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}y(x) + \gamma \frac{\mathrm{d}}{\mathrm{d}x}y(x) + \omega_0^2 y(x) = f(x)$$

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Ordinary Differential Equations

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$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}y(x) + \gamma \frac{\mathrm{d}}{\mathrm{d}x}y(x) + \omega_0^2 y(x) = f(x)$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \gamma \frac{\mathrm{d}}{\mathrm{d}x} + \omega_0^2\right] y(x) = f(x)$$

$$y(x) = \frac{f(x)}{\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \gamma \frac{\mathrm{d}}{\mathrm{d}x} + \omega_0^2}$$

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$$f(x) \longrightarrow \hat{f}(\xi)$$

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$$(f + \alpha g)(x) \longrightarrow \hat{f}(\xi) + \alpha \hat{g}(\xi)$$

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$$\frac{\mathrm{d}}{\mathrm{d}x}f(x) \longrightarrow \mathcal{F} \qquad \longrightarrow i\xi \hat{f}(\xi)$$



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$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \gamma \frac{\mathrm{d}}{\mathrm{d}x} + \omega_0^2\right] y(x) = f(x)$$

$$\downarrow$$

$$\mathcal{F}$$

$$\left[-\xi^2 + i\gamma\xi + \omega_0^2\right] \hat{y}(\xi) = \hat{f}(\xi)$$

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Box Proposal

$$\mathcal{F}[f](\xi) = \hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)e^{-ix\xi} dx$$

$$\mathcal{F}^{-1}[\hat{f}](x) = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \hat{f}(\xi) e^{ix\xi} d\xi$$

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Box Proposal

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$$(\widehat{f+\alpha g})(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (f(x) + \alpha g(x)) e^{-ix\xi} dx$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad$$

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$$\widehat{(f+\alpha g)}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (f(x) + \alpha g(x)) e^{-ix\xi} dx$$

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$$(\widehat{f+\alpha g})(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)e^{-ix\xi} dx + \frac{\alpha}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(x)e^{-ix\xi} dx$$

$$(\widehat{f + \alpha g})(\xi) = \widehat{f}(\xi) + \alpha \widehat{g}(\xi)$$

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$$(\widehat{f} + \alpha g)(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (f(x) + \alpha g(x)) e^{-ix\xi} dx$$

$$\downarrow \downarrow$$

$$(\widehat{f} + \alpha g)(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ix\xi} dx + \frac{\alpha}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(x) e^{-ix\xi} dx$$

$$\downarrow \downarrow$$

$$(\widehat{f} + \alpha g)(\xi) = \widehat{f}(\xi) + \alpha \widehat{g}(\xi)$$

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$$\widehat{f}'(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f'(x)e^{-ix\xi} dx$$

$$\downarrow \downarrow$$

$$\widehat{f}'(\xi) = \frac{f(x)e^{-ix\xi}}{\sqrt{2\pi}} \Big|_{-\infty}^{+\infty} + i\xi \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)e^{-ix\xi} dx$$

$$\downarrow \downarrow$$

$$\widehat{f}'(\xi) = i\xi \widehat{f}(\xi)$$

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$$\downarrow \downarrow$$

$$\widehat{f}'(\xi) = i\xi \widehat{f}(\xi)$$

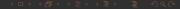
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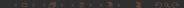
The inverse does work

for appropriate functions

and, sometimes, the Fourier Transform of a function is not in the same set as the original function, but let's forget about this since we do not know a decent theory of integration



Playing Around With Our New Toy



$$f(t) = \cos(\omega_0 t) e^{-\pi t^2}$$

$$\widehat{f}(\omega) = \frac{e^{-\frac{(\omega - \omega_0)^2}{4\pi}} + e^{-\frac{(\omega + \omega_0)^2}{4\pi}}}{2\sqrt{2\pi}}$$

$$\omega = 2\pi\nu$$

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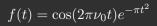
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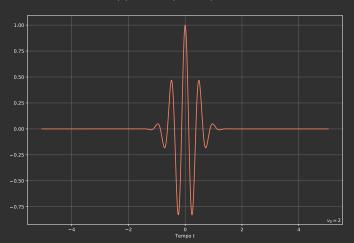
Dis (USTC) Introduction to

$$f(t) = \cos(2\pi\nu_0 t)e^{-\pi t^2}$$
$$\hat{f}(\nu) = \frac{e^{-\pi(\nu - \nu_0)^2} + e^{-\pi(\nu + \nu_0)^2}}{2\sqrt{2\pi}}$$

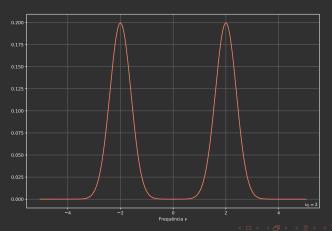
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$$\widehat{f}(\nu) = \frac{e^{-\pi(\nu - \nu_0)^2} + e^{-\pi(\nu + \nu_0)^2}}{2\sqrt{2\pi}}$$



A Harder Example

$$f(t) = e^{i\omega_0 t} = \cos(\omega_0 t) + i\sin(\omega_0 t)$$

$$\widehat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\omega_0 t} e^{-i\omega t} dt$$

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A Harder Example

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The Mathematical Moonwalk

$$f(t) = e^{i\omega_0 t}$$
 $e^{i\omega_0 t} = rac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \widehat{f}(\omega) e^{i\omega t} d\omega$ $\widehat{f}(\omega) = \sqrt{2\pi} \delta\left(\omega - \omega_0
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The Mathematical Moonwalk

$$f(t) = e^{i\omega_0 t}$$
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The Mathematical Moonwalk

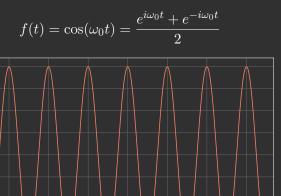
$$f(t) = e^{i\omega_0 t}$$

$$e^{i\omega_0 t} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \widehat{f}(\omega) e^{i\omega t} d\omega$$

$$\widehat{f}(\omega) = \sqrt{2\pi} \delta(\omega - \omega_0)$$

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Cosines



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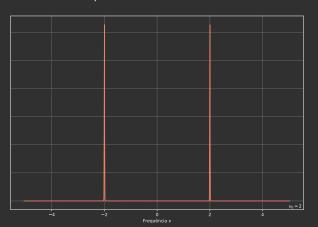
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Cosines

$$\widehat{f}(\omega) = \sqrt{\frac{\pi}{2}} \left(\delta \left(\omega - \omega_0 \right) + \delta \left(\omega + \omega_0 \right) \right)$$



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Fourier's Physics Playground

In the beggining, God said:

$$\begin{cases} \mathbf{\nabla \cdot E} = \frac{\rho}{\epsilon_0} \\ \mathbf{\nabla \cdot B} = 0 \\ \mathbf{\nabla \times E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \mathbf{\nabla \times B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

and there was light!

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and there was light!

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Too hard, let's try something different

$$\begin{cases} \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} = \nabla \times \mathbf{A} \end{cases}$$

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Wave Equations

$$\begin{cases} \nabla^2 V - \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0} \\ \nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} \end{cases}$$

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All Wave Equations In One

$$\nabla^2 \psi(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = -g(\mathbf{r}, t)$$

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Fourier's Opinion

$$\widehat{g}(\mathbf{r},\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(\mathbf{r},t) e^{-i\omega t} dt$$

$$g(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \widehat{g}(\mathbf{r},\omega) e^{i\omega t} d\omega$$

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Fourier's Opinion

$$\widehat{\psi}(\mathbf{r},\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(\mathbf{r},t) e^{-i\omega t} dt$$

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Fourier's Opinion

$$\nabla^2 \psi(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = -g(\mathbf{r}, t)$$

$$\nabla^2 \widehat{\psi}(\mathbf{r}, \omega) + \frac{\omega^2}{c^2} \widehat{\psi}(\mathbf{r}, \omega) = -\widehat{g}(\mathbf{r}, \omega)$$

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$$L\phi(\mathbf{r}) = -s(\mathbf{r})$$

$$LG(\mathbf{r} - \mathbf{r}') = -\delta \left(\mathbf{r} - \mathbf{r}'\right)$$

$$\phi(\mathbf{r}) = \int G(\mathbf{r} - \mathbf{r}') s(\mathbf{r}') d\tau'$$

$$L\phi(\mathbf{r}) = \int LG(\mathbf{r} - \mathbf{r}')s(\mathbf{r}') d\tau' = -\int \delta(\mathbf{r} - \mathbf{r}') s(\mathbf{r}') d\tau' = -s(\mathbf{r})$$

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One At a Time

$$\nabla^2 \widehat{\psi}(\mathbf{r}, \omega) + \frac{\omega^2}{c^2} \widehat{\psi}(\mathbf{r}, \omega) = -\widehat{g}(\mathbf{r}, \omega)$$

$$\nabla^2 G(\mathbf{r} - \mathbf{r}') + \frac{\omega^2}{c^2} G(\mathbf{r} - \mathbf{r}') = -\delta \left(\mathbf{r} - \mathbf{r}'\right)$$

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One At a Time

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Solution for $\mathbf{r}-\mathbf{r}' eq \mathbf{0}$

$$\frac{1}{r}\frac{\mathrm{d}^2(rG)}{\mathrm{d}r^2} + k^2G = 0$$

$$G(r) = \frac{A}{r}e^{\pm ikr}$$

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Recovering 0 Psychological Trauma

$$\nabla^2 G(\mathbf{r} - \mathbf{r}') + \frac{\omega^2}{c^2} G(\mathbf{r} - \mathbf{r}') = -\delta \left(\mathbf{r} - \mathbf{r}' \right)$$

$$A \int \nabla^2 \frac{1}{r} d\tau' + 4\pi A \frac{\omega^2}{c^2} \int \frac{r^2}{r} dr = -\int \delta \left(\mathbf{r} - \mathbf{r}' \right) d\tau'$$

$$-4\pi A = -1$$

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Recovering 0 Psychological Trauma

$$\nabla^2 G(\mathbf{r} - \mathbf{r}') + \frac{\omega^2}{c^2} G(\mathbf{r} - \mathbf{r}') = -\delta \left(\mathbf{r} - \mathbf{r}' \right)$$

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Recovering 0 Psychological Trauma

$$\nabla^2 G(\mathbf{r} - \mathbf{r}') + \frac{\omega^2}{c^2} G(\mathbf{r} - \mathbf{r}') = -\delta \left(\mathbf{r} - \mathbf{r}' \right)$$

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$$-4\pi A = -1$$

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Back To Our Problem

$$\widehat{\psi}(\mathbf{r},\omega) = \int G(\boldsymbol{z})\widehat{g}(\mathbf{r}',\omega) \,\mathrm{d}\tau'$$

$$G(\mathbf{r}) = \frac{1}{4\pi\,\mathbf{r}}e^{\pm ik\,\mathbf{r}}$$

$$\widehat{\psi}(\mathbf{r},\omega) = \frac{1}{4\pi} \int \frac{\widehat{g}(\mathbf{r}',\omega)e^{\pm ikz}}{z} d\tau'$$

Dis (USTC) Introduction to

Back To Our Problem

$$\widehat{\psi}(\mathbf{r},\omega) = \int G(\mathbf{z})\widehat{g}(\mathbf{r}',\omega) d\tau'$$

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Back To Our Problem

$$\widehat{\psi}(\mathbf{r},\omega) = \int G(\boldsymbol{z})\widehat{g}(\mathbf{r}',\omega) \,\mathrm{d}\tau'$$

$$G(\mathbf{z}) = \frac{1}{4\pi \, \mathbf{z}} e^{\pm ik \, \mathbf{z}}$$

$$\widehat{\psi}(\mathbf{r},\omega) = \frac{1}{4\pi} \int \frac{\widehat{g}(\mathbf{r}',\omega)e^{\pm ik\,\boldsymbol{\imath}}}{\boldsymbol{\imath}} \,\mathrm{d}\tau'$$

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$$\psi(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \widehat{\psi}(\mathbf{r},\omega) e^{i\omega t} d\omega$$

$$\psi(\mathbf{r},t) = \frac{1}{4\pi\sqrt{2\pi}} \iint \frac{\widehat{g}(\mathbf{r}',\omega)e^{i\omega t \pm i\omega\frac{c}{c}}}{\mathbf{z}} d\omega d\tau$$

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$$\psi(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \widehat{\psi}(\mathbf{r},\omega) e^{i\omega t} d\omega$$

$$\psi(\mathbf{r},t) = \frac{1}{4\pi\sqrt{2\pi}} \iint \widehat{g}(\mathbf{r}',\omega) e^{i\omega t \pm i\omega \frac{\epsilon}{c}} d\omega d\tau'$$

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$$\psi(\mathbf{r},t) = \frac{1}{4\pi\sqrt{2\pi}} \iint \frac{\widehat{g}(\mathbf{r}',\omega)e^{i\omega\left(t \pm \frac{\imath}{c}\right)}}{\imath} \,\mathrm{d}\omega \,\mathrm{d}\tau'$$

$$\psi(\mathbf{r},t) = \frac{1}{4\pi} \int \frac{g(\mathbf{r}',t \pm \frac{\mathbf{z}}{c})}{\mathbf{z}} d\tau'$$

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$$\psi(\mathbf{r},t) = \frac{1}{4\pi\sqrt{2\pi}} \iint \frac{\widehat{g}(\mathbf{r}',\omega)e^{i\omega\left(t\pm\frac{\imath}{c}\right)}}{\imath} d\omega d\tau'$$

$$\psi(\mathbf{r},t) = \frac{1}{4\pi} \int \frac{g(\mathbf{r}',t \pm \frac{\mathbf{z}}{c})}{\mathbf{z}} d\tau'$$

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$$\psi(\mathbf{r},t) = \frac{1}{4\pi\sqrt{2\pi}} \iint \frac{\widehat{g}(\mathbf{r}',\omega)e^{i\omega\left(t\pm\frac{\imath}{c}\right)}}{\imath} d\omega d\tau'$$

$$\psi(\mathbf{r},t) = \frac{1}{4\pi} \int \frac{g(\mathbf{r}',t-\frac{\mathbf{z}}{c})}{\mathbf{z}} d\tau'$$

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Back at Maxwell's

$$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}',t-\frac{\mathbf{z}}{c})}{\mathbf{z}} \, \mathrm{d}\tau'$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}',t-\frac{\mathbf{z}}{c})}{\mathbf{z}} \, \mathrm{d}\tau'$$

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One Last Step

$$\begin{cases} \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} = \nabla \times \mathbf{A} \end{cases}$$

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Jefimenko Equations

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathbf{z}}}{\mathbf{z}^2} \left[\rho \right] + \frac{\hat{\mathbf{z}}}{c \, \mathbf{z}} \left[\frac{\partial \rho}{\partial t} \right] - \frac{1}{c^2 \, \mathbf{z}} \left[\frac{\partial \mathbf{J}}{\partial t} \right] d\tau'$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi} \int \left(\frac{1}{\mathbf{z}^2} \left[\mathbf{J} \right] + \frac{1}{c \, \mathbf{z}} \left[\frac{\partial \mathbf{J}}{\partial t} \right] \right) \times \hat{\mathbf{z}} d\tau'$$

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Fourier's Physics Playground Heisenberg's Uncertainty Principle

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Position and Momentum

$$\psi(x) = \langle x|\psi\rangle = \int \langle x|k\rangle \langle k|\psi\rangle dk = \frac{1}{\sqrt{2\pi}} \int e^{ikx} \psi(k) dk$$

$$\psi(k) = \langle k|\psi\rangle = \int \langle k|x\rangle \langle x|\psi\rangle dx = \frac{1}{\sqrt{2\pi}} \int e^{-ikx} \psi(x) dx$$

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Position and Momentum

$$\psi(x) = \langle x|\psi\rangle = \int \langle x|k\rangle \langle k|\psi\rangle dk = \frac{1}{\sqrt{2\pi}} \int e^{ikx} \psi(k) dk$$

$$\psi(k) = \langle k | \psi \rangle = \int \langle k | x \rangle \langle x | \psi \rangle dx = \frac{1}{\sqrt{2\pi}} \int e^{-ikx} \psi(x) dx$$

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Position and Momentum (but weirder)

$$\begin{cases} X |\psi\rangle = x\psi(x) \\ K |\psi\rangle = -i\frac{\partial \psi}{\partial x} \end{cases}$$

$$\begin{cases} X |\psi\rangle = -i\frac{\partial \psi}{\partial k} \\ K |\psi\rangle = k\psi(k) \end{cases}$$

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Position and Momentum (but weirder)

$$\begin{cases} X |\psi\rangle = x\psi(x) \\ K |\psi\rangle = -i\frac{\partial \psi}{\partial x} \end{cases}$$

$$\begin{cases} X |\psi\rangle = -i\frac{\partial \psi}{\partial k} \\ K |\psi\rangle = k\psi(k) \end{cases}$$

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Fourier Diplomacy

$$|x\rangle \stackrel{\mathcal{F}}{\longleftrightarrow} |k\rangle$$

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Fourier Uncertainty

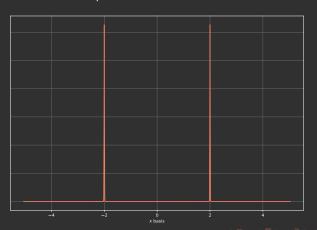
- $\psi(x)$: what is x?
- $\mathbf{Q} \ \psi(k)$: what is k?

Fourier Uncertainty

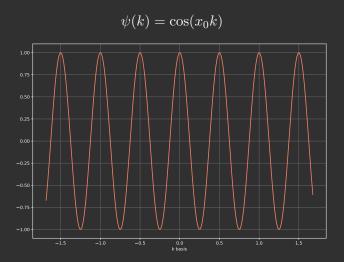
- $\psi(x)$: what is x?
- $\boxed{\mathbf{2} \; \psi(k) : \mathsf{what}} \; \mathsf{is} \; k?$

Definite Position

$$\psi(x) = \sqrt{\frac{\pi}{2}} \left(\delta \left(x - x_0 \right) + \delta \left(x + x_0 \right) \right)$$



Undefinite Momentum



Uncertainty Relation

$$\sigma_x \sigma_p \ge \frac{\hbar}{2}$$

The uncertainty relation is a consequence of the general fact that anything narrow in one space is wide in the transform space and vice versa. So if you are a 45 kg weakling and are taunted by a 270 kg bully, just ask him to step into momentum space!

Ramamurti Shankar

Acknowledgments

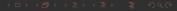
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