Summary

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1 Outline

This outline shows how to solve the IPU game with alternative machines.

Here, I paste some Lemmas and Theorems at the beginning.

Lemma 1. *The value at the extreme points of the sub-intervals I_i can be calculated with processing times t_i by comparing the costs of the grand coalitions where all the players use two adjacent numbers of machines.*

Theorem 1. *$\omega(P)$ is piecewise linear, and convex in price P at each sub-interval $[P_L(m, V), P_H(m, V)]$ in $[0, P^*]$.*

Theorem 2. *According to the foregoing description, we have the equation $P_L(V, 1) = P_L(V, 2) + \cdots + P_L(V, n) = \sum_{i=2}^{n} P_L(V, i)$.*

Theorem 3. *The subsidy is always zero when the number of using machines, m, is larger than $\frac{n}{2}$.*

Theorem 4. *The sum of absolute values of slopes at both sides of $P_L(V, i)$ is 1. When the number of machines used is 1, the range of slopes of the line segments in the interval is $\left(-1, -\frac{1}{n-1}\right]$, and the number of breakpoints is $O(v^2)$.*

Theorem 5. *The original problem is equivalent to using only one machine for all coalitions.*

Theorem 6. *The IPU game with alternative machines can be solved in polynomial time.*

As we all know, the cost arised from the partial players in the grand coalition can be calculated handily by the SPT rule.(The corresponding conclusion see Lemma 1) Meanwhile, inspired by the paper (Please refer to Liu et.al.2018), we have the following approach to solve this game.

There is an effective domain $[0, P^*]$, where the grand coalition may need a subsidy from the external to maintain the balance. (The corresponding conclusion see Theorem 1 and 2 where $P^* = P_L(V, 1)$).

For the effective domain, we just need to divide this interval into several parts and the points are denoted as $P_L(V, i), i = 1, 2, \ldots, n$. At first, we don't need to calculate the initial part because this part the corresponding subsidy is 0 always.(The corresponding conclusion see Theorem 3) Then we just need to focus on the latter part which shows some interesting properties we presents above.

As we've already known that $P_L(V,i), i = 1,2,\ldots,n$ can be obtained by Lemma 1 and Theorem 2, we just need to follow the CP approach(Algorithm 3) to calculate the weak derivative at each sub-interval $[P_L(m,V), P_H(m,V)]$ where the corresponding derivative is $m_V - \sum_{s\in S\subset\{V\}} \rho_s$. Then use Algorithm 1(IPC Algorithm) which will return all the breakpoints and the subsidies $\omega(P)$ during the $[0, P^*]$ to obtain the subsidy $\omega(P)$. Notice that we can calculate the characteristic function $c(s)$ easily according to Theorem 5, we can formulate Theorem 6. Until here, we've solved the IPU game with alternative machines.