Title

Discount

1 Lemma

For convenience of expression, we set the setup cost as $S_1, S_2, \ldots, S_n$ at interval point while the number of machine changes. And S_i denotes the setup cost when the machine number changes from i to $i-1$, especially, S_1 denotes the least setup cost when machine number is 1 and the corresponding subsidy is 0. We have the equation $S_1 = S_2 + \cdots + S_n = \sum_{i=2}^{n} S_i$.

At first by given job processing times and calculating the given program, we can get $n-1$ setup cost values at interval points, that is $S_2, S_3, \ldots, S_n$.

Now that $S_2, S_3, \ldots, S_n$ can be obtained by calculating, we can have a further result. $S_1 = (n-1, n-2, \ldots, 0) \cdot (t_1, t_2, \ldots, t_n)^T$, and $t_1 < t_2 < \cdots < t_n$.

Consider all the permutation and combination of processing jobs on two machines, we know that the following inequality must hold

$$S_0 + (n, n-1, \ldots, 1) \cdot (t_1, t_2, \ldots, t_n)^T \leq 2S_0 + \bigcirc \cdot (t_1, t_2, \ldots, t_n)^T.$$

Notice that the inequality holds under any circumstances, which means we should find the minimum of $\bigcirc$. Meanwhile, we know that $\bigcirc$ must contain jobs' processing time from t_1 to t_n once at least. That is $\bigcirc \geq (1, 1, \ldots, 1)$.

So when $\bigcirc = (1, 1, \ldots, 1)$, we get the equality

$$S_0 + (n, n-1, \ldots, 1) \cdot (t_1, t_2, \ldots, t_n)^T = 2S_0 + \sum_{i=1}^{n} t_i \tag{1}$$

Use S_1 to replace S_0, we can get

$$S_1 = (n-1, n-2, \dots, 0) \cdot (t_1, t_2, \dots, t_n)^T \tag{2}$$