

For convenience of expression, we set the setup cost as S_1, S_2, \dots, S_n at interval point while the number of machine changes. And S_i denotes the setup cost when the machine number changes from i to $i - 1$, especially, S_1 denotes the least setup cost when machine number is 1 and the corresponding subsidy is 0. We have the equality

$$S_1 = S_2 + \dots + S_n = \sum_{i=2}^n S_i.$$

Notice that

$$(n-1) \sum_{s \in S \setminus \{V\}} \rho_s \geq \sum_{k \in V} \sum_{s \in S \setminus \{V\}: k \in s} \rho_s = n.$$

The left side of the inequality means for every ρ_s can appear at most $(n-1)$ times, so we should know that if and only if for every $\rho_s > 0$ appears $n-1$ times the quality holds. That is to say, the coalitions which contains $n-1$ players are all maximally unsatisfied coalitions. Then we have $\binom{n}{n-1}$ equalities.

$$\begin{cases} \alpha_1 + \alpha_2 + \dots + \alpha_{n-1} &= x_1 \\ \alpha_1 + \alpha_3 + \dots + \alpha_n &= x_2 \\ \vdots &\vdots \\ \alpha_2 + \alpha_3 + \dots + \alpha_n &= x_n. \end{cases}$$

Add these n equations together, and we can get

$$(n-1)(\alpha_1 + \alpha_2 + \dots + \alpha_n) = \sum_{i=1}^n x_i$$

As we know, x_1, x_2, \dots, x_n can be expressed as follows:

$$\begin{cases} x_1 = S_0 + (n-1)t_1 + (n-2)t_2 + \dots + t_{n-1} \\ x_2 = S_0 + (n-1)t_1 + (n-2)t_3 + \dots + t_{n-1} \\ \vdots \\ x_n = S_0 + (n-1)t_2 + (n-2)t_3 + \dots + t_n \end{cases}$$

According to SPT rule, we can obtain the equality $c(V) = \alpha_1 + \alpha_2 + \dots + \alpha_n = S_0 + nt_1 + (n-1)t_2 + \dots + t_n$. By replacing x_1, x_2, \dots, x_n together with the expression of $c(V)$, we can get a equality only with $S_0, x_1, x_2, \dots, x_n$. Finally, we can obtain $S_0 = \sum_{k=1}^n (n-k)t_k$.