Referee report on

Lagrangian Heuristic for Simultaneous Subsidization and Penalization: Implementations on Rooted Travelling Salesman Games

by

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The paper examines the problem of stabilizing the grand coalition of an unbalanced cooperative game under the concept of simultaneous subsidization and penalization (S&P). It develops Lagrangean relaxation techniques for computing subsidy-penalty pairs to stabilize the grand coalition of cooperative games. This technique is illustrated with an application to the rooted traveling salesman game.

The paper is well-written and the topic is sensible. It makes a minor contribution on some previous attempts by the same authors (see Li et al. 2016 and 2018) on stabilization of cooperative games using subsidies or penalizations in that in this new paper they combine both together rather than considering them individually. Nevertheless, the paper presents a framework based on upper and lower bounds on the characteristic function of the game that allows to approximate the solution of hard combinatorial games resorting to approximated games which may be polynomially solvable (at least theoretically speaking!).

The whole idea is to replace the combinatorial game for some Lagrangean relaxation having the property of being more suitable to be handle. Then, a standard column generation approach allows one to tackle the evaluation of the minimum penalization value for a given value of w.

Each one of the techniques used in the algorithm proposed by the authors is well-known and in this regard no major contribution is made. On the other hand, putting all the pieces together induces some novelty in the treatment of unbalanced cooperative games and the paper deserves some merit.

The computational experiments are only devoted to the rooted traveling salesman game. In this particular case this approach is competitive.

In the following there are listed a series of concerns to be taken into account:

Concerns:

1. Notation. The paper does not conform with the standard notation in TU-games. Usually, coalitions are referred with capital letters and its cardinal in lower case letters, i.e. $S \subset N$ and $|S| = s$. This paper uses a different notation with s for coalitions and then some inconsistencies appears when referring to v in some places. I would suggest to adapt the notation to the standard to ease the readability of potential readers.

2. Some confusion appears, here and there, when referring to LP or MIP. For instance, in page 4 line 3, it is mentioned that (2) is a combinatorial optimization problem. However, (2) is a LP since in its description c is given and thus all constraints and variables are linear and continuous. The same confusion can be found at other places of the paper. Please clarify!

3. Page 4 line -13: The authors must be more precise. The Lagrangean bound is more accurate than the linear relaxation whenever the problem does not fulfill the integrality property.

4. The statement of Theorem 1 should be modified since the value of the LP is one of the many possible upper bounds not the only one as stated there.

5. Remark 1. The meaning of v is unclear. One should guess that it refers to $|V|$ but this has to be made explicit.

6. Page 8, line -12. Note that (5) is not an LP but an ILP.

7. To better illustrate the proposed methodology, it would be advisable to apply it not only to the rooted traveling salesman problem. I would suggest to add another class of combinatorial games, for instance location games, to the computational study.

This referee finds the contribution of this paper to the field of computational cooperative games interesting and with potential. I would suggest to address a revision following the guidelines listed above before a positive recommendation can be achieved.