Homework 2

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5-7

(a) Show that $F_{ts}(x) = \sum_j p_{sj}^{\delta_t(s)} F_{t+1,j}\left(\frac{x-r(s,\delta_t(s))}{\alpha}\right)$.

According to the definition of X_{ts}, the next stage of the returns x' satisfies $\alpha x' + r(s,\delta_t(s) = x$.

(b) Show that $\hat{v}_{ts} = \alpha^2 \sum_j p_{sj}^{\delta_l(s)} \hat{v}_{t+1,j} + \sum_j p_{sj}^{\delta_t(s)} \left[r\left(s,\delta_t(s)\right) + \alpha v_{t+1,j}\right]^2 - (v_{ts})^2$

$\hat{v}_{ts} := E\left[(X_{ts} - v_{ts})^2\right] = E(X_{ts}^2) - 2E(X_{ts})v_{ts} + (v_{ts})^2$

$E(X_{ts}) = (v_{ts} - 1/2\alpha^2 \sum_j p_{sj}^{\delta_l(s)} \hat{v}_{t+1,j})$

According to the definition of the $E(X_{ts}^2)$, we can obtain that

$E(X_{ts}^2) = F_{ts}X_{ts}^2 = \sum_j p_{sj}^{\delta_t(s)} \left[r\left(s,\delta_t(s)\right)\right.$

6-4

The generalization of the inventory: Demand is negative. (section 6.3)

The original expression is that:

$\gamma(y) := cy + L(y) + \alpha \int_0^y \left[-c\beta_{\mathbf{R}}(y-\xi)\right]\phi(\xi)d\xi + \alpha \int_y^\infty \left[c\beta_{\mathbf{B}}(\xi - y)\right]\phi(\xi)d\xi$
In this case, the demand can be negative, when extending the demand to the whole real space, the above expression still holds. Because the stock could be returned to the supplier with the initial cost c.

To sum up, when the equation $\gamma(y)$ holds, it is still optimal to order up to S.

6-9

(a) Generalize (6.5) for this case. Period t. Unit order cost is c_t. unit holding cost is c_t^H. unit shortage cost is c_t^P demand distribution is Φ_t

$\Phi_t(S) = \frac{c_t^P - c_t(1-\alpha\beta_B)}{c_t^P + c_t^H + \alpha c_t(\beta_B - \beta_R)}$

(b) Show the increasing. When t increases, the critical fraction $\Phi_t(S)$ also increases.

7-7

(a) Verify that the optimality equations can be written as: When $y \geq x$, we discuss the result of $f_t(x)$: $f_t(x) = \min -cx + \min_{y \geq x} G_t(y)$. When $y \leq x$, replace the cost c with d, we can obtain that $f_t(x) = \min -dx + \min_{y \leq x} g_t(y)$. So comprehensively considering both situations, the optimality equations satisfy

$f_t(x) = \min[-cx + \min_{y \geq x} G_t(y), -dx + \min_{y \leq x} g_t(y)]$.

(b) Define $f_t^1 = -cx + \min_{y \geq x} G_t(y)$, $f_t^2 = -dx + \min_{y \leq x} g_t(y)$. According to the conclusion of Exercise 7.6, if $f_1' \leq f_2'$, their corresponding minimizers satisfy that $S_1 \geq S_2$.

Because of $d < c$, it is obvious that $f_t'^1 < f_t'^2$, so the minimizers $S_t^1 > S_t^2$. Reminding the initial model without the disposal, S_t^1 corresponds S. Define $S_t^1 = \overline{S_t}$ and $S_t^2 = \underline{S_t}$, so there exists an optimal target interval policy.

8-4

Let $g(x) = x^2$, it is obvious that $g(x)$ is a convex function. In this case, $f(x, y, z) = g(x + y - z) = x^2 + y^2 + z^2 + 2xy - 2xz - 2yz$. Its cross partials are not all negative(corresponds to submodularity) and not all positive(corresponds to supermodularity) either. So f is neither submodular nor supermodular.