Modelling Electromagnetic Wave Scattering Using the Finite Difference Time Domain Technique

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Final Year Project Report

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Researchers' Declaration

I, Attique Dawood, do hereby solemnly declare that the work presented in this report is my own, and has not been presented previously to any other institution.

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(Researcher)

Signature

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(Supervisor)

Abstract

This report is divided into two sections. The first section is an introduction to the Finite Difference Time Domain (FDTD) technique, used to numerically implement Maxwell's equations. The second section is a detailed description of the problems solved using the FDTD technique. First, a transverse electromagnetic (TEM) wave is simulated in one dimension in the absence of any obstacle. Then, two problems involving scattering of a plane TEM wave by a dielectric cylinder and conducting plate are modelled. Finally, the characteristics of a monopole antenna are modelled using commercially available computational electromagnetics software.

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Introduction

Many numerical methods are used to solve Maxwell's equations. There are basically two approaches to formulate an electromagnetic scattering problem using Maxwell's equations; integral and differential. Examples of integral techniques are method of moments and finite elements. Finite Difference Time Domain (FDTD) is a differential technique used to numerically implement Maxwell's equations. The aim of this project was to get acquainted with this technique and to use it to solve "standard" electromagnetic scattering problems.

The FDTD method is conceptually simple, and does not need formulation of integral equations. It is simple to implement for complicated, inhomogeneous structures because constitutive parameters (σ, μ, ε) can be assigned to each lattice point. Also, its computer memory requirement is not prohibitive for many structures of interest. On the other hand, the method requires modelling object as well as its surroundings. Thus, the program execution time may be excessive. In addition, field quantities are only known at grid nodes.

This report analyzes simple electromagnetic scattering problems using FDTD. Starting point of the project was the simulation of a TEM wave propagation in one dimension. Afterwards, more complex electromagnetic problems are solved involving plane wave scattering by obstacles such as a dielectric cylinder and a conducting plate. Finally, a monopole antenna is modelled using a commercial electromagnitc software based on FDTD. The results obtained are matched with published results to verify the correctness of the approach.

Chapter 2

Yee's Finite Difference Algorithm

2.1 Introduction

The finite-difference time-domain (FDTD) formulation of Maxwell's equations is a convenient tool for solving electromagnetic scattering problems. The FDTD method, first introduced by Yee in 1966 [1], is a direct solution of Maxwell's time-dependent curl equations. The scheme treats the irradiation of the scatterer as an initial value problem. This chapter gives a description of the Yee's algorithm. This treatment follows the approach used by Sadiku [2].

The first two Maxwell's equations can be written as,

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \dots (2.1)$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad \dots (2.2)$$

This is all the information needed for linear isotropic materials to completely specify the field behaviour over time as long as the initial field distribution is given. Conveniently, the field and sources are set to zero at the initial time, taken as time $t = 0$. The starting point for the FDTD formulation is the curl equations. These can be recast into the form used for FDTD. Equations 2.1 and 2.2 represent a system of six scalar equations, which can be expressed in rectangular coordinate system as,

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right) \qquad \dots (2.3)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) \qquad \text{… (2.4)}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \qquad \text{… (2.5)}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right) \qquad \text{… (2.6)}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right) \qquad \text{… (2.7)}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right) \qquad \text{… (2.8)}$$

2.2 Finite Difference Representation of Maxwell's Equations

Following Yee's notation, we define a grid point in the solution region as,

$$(i, j, k) \equiv (i\Delta x, j\Delta y, k\Delta z) \qquad \text{… (2.9)}$$

A function of space and time is defined as,

$$F^n(i, j, k) \equiv F(i\delta, j\delta, k\delta, n\Delta t) \qquad \text{… (2.10)}$$

Where $\delta = \mathbf{\Delta}x = \mathbf{\Delta}y = \mathbf{\Delta}z$ is the space increment, $\mathbf{\Delta}t$ is the time increment, while i, j, k and n are integers. To discretize the derivatives we can use forward, backward or central difference schemes (Yee uses central difference scheme). Using central finite difference approximation for space and time derivatives that are second-order accurate we get,

$$\frac{\partial F^n(i, j, k)}{\partial x} = \frac{F^n(i + 1/2, j, k) - F^n(i - 1/2, j, k)}{\delta} + O(\delta^2)$$

… (2.11)

$$\frac{\partial F^n(i,j,k)}{\partial y} = \frac{F^n(i,j+1/2,k) - F^n(i,j-1/2,k)}{\delta} + O(\delta^2)$$

… (2.12)

$$\frac{\partial F^n(i,j,k)}{\partial z} = \frac{F^n(i,j,k+1/2) - F^n(i,j,k-1/2)}{\delta} + O(\delta^2)$$

… (2.13)

$$\frac{\partial F^n(i,j,k)}{\partial t} = \frac{F^{n+1/2}(i,j,k) - F^{n-1/2}(i,j,k)}{\Delta t} + O(\Delta t^2)$$

… (2.14)

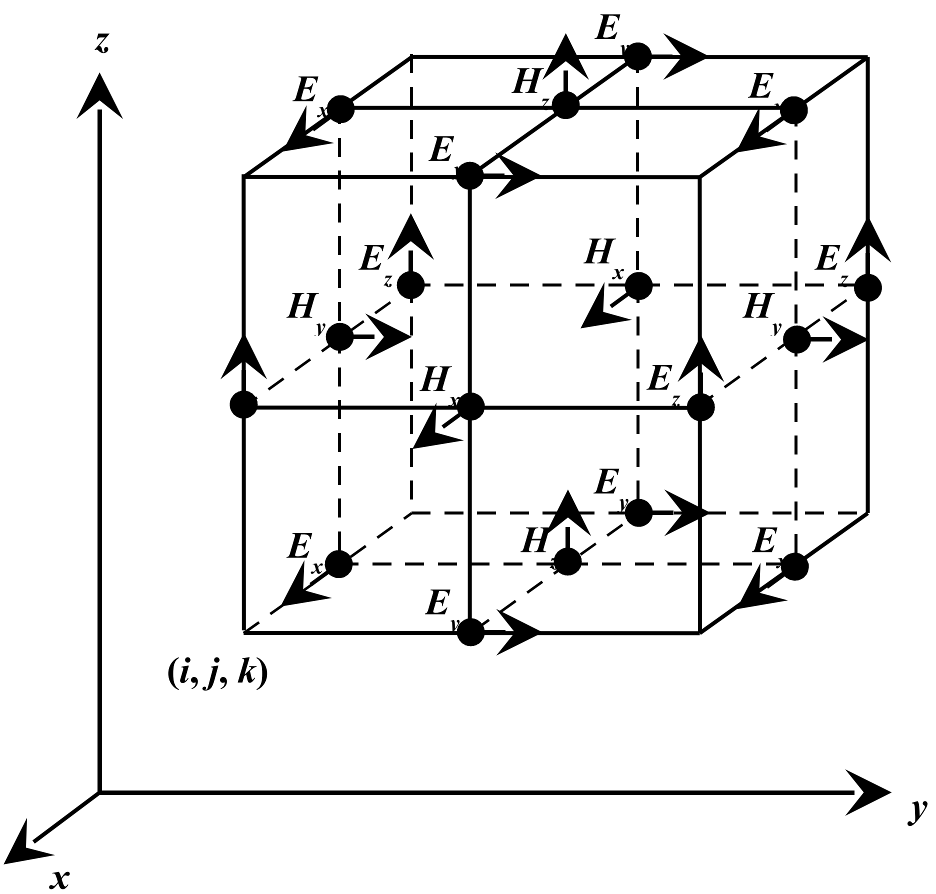

Figure 2.1: Position of field components in a unit cell of Yee's lattice

The central difference is simpler as far as practical implementation is concerned. In applying the above equations to space derivatives, Yee positions the components of electric and magnetic fields about a unit cell of the lattice shown in figure 2.1. The components of electric and magnetic field are evaluated at alternate half-time steps. All the field components are present in a quarter of a unit cell as shown in figure 2.2.

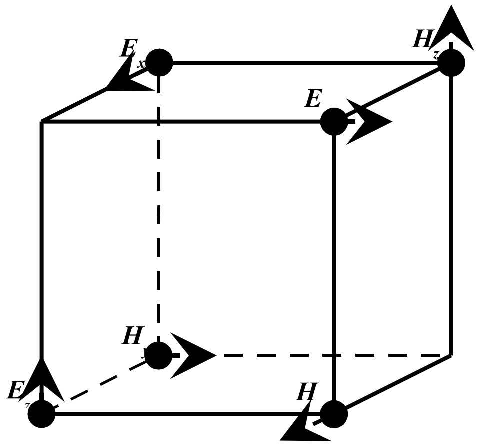

Figure 2.2: Relation between field components within unit quarter of a cell

Figure 2.3 illustrates typical relations between field components on a plane. This is useful when incorporating the boundary conditions. In translating these hyperbolic systems of equations into a computer code it must be insured that within the same time loop, one type of field components is calculated first and the results obtained are used in calculating the second.

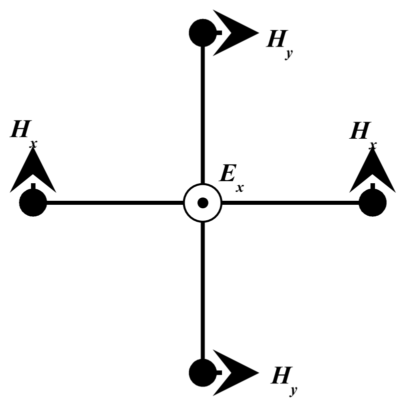

Figure 2.3: Relation between field components in a plane

2.3 The Yee Algorithm

The algorithm can be summarized as follows:

1. Replace all the derivatives in Ampere's and Faraday's laws with finite differences. Discretize space and time so that the electric and magnetic fields are staggered in both space and time.

2. At $t = n\Delta t$ solve for magnetic field at time $(n+1/2)\Delta t$ in terms of electric and magnetic fields at times $n\Delta t$ and $(n-1/2)\Delta t$ respectively. That is to say, solve the resulting difference equations for "future" fields in terms of "past" fields. Effectively these future fields become past fields when the time step is incremented.

3. At next half time step (i.e., $t = (n+1/2)\Delta t$) solve for electric field at time $(n+1)\Delta t$ in terms of magnetic and electric fields at times $(n+1/2)\Delta t$ and $n\Delta t$ respectively.

4. Repeat steps 2 and 3 until the desired solution is obtained.

2.4 Update Equations in One Dimension

Following the treatment of Sullivan [3], we consider a one-dimensional space where there are only variations in the x direction. The electric field is assumed to only have a z-component. In this case Faraday's law can be written,

$$-\mu \frac{\partial \mathbf{H}}{\partial t} = \nabla \times \mathbf{E} = -\hat{a}_y \frac{\partial E_z}{\partial x} \qquad \dots (2.15)$$

Here H_y is the only non-zero component of the magnetic field. Now, Ampere's law can be written as,

$$\varepsilon \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} = \hat{a}_z \frac{\partial H_y}{\partial x} \qquad \dots (2.16)$$

We can obtain the two scalar equations from the above two equations as,

$$\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x} \qquad \dots (2.17)$$

$$\varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} \qquad \ldots (2.18)$$

Notice, that equations 2.17 and 2.18 can be obtained by setting E_x, E_y and all the partial derivatives with respect to y and z equal to zero (equations 2.3 to 2.8). Equation 2.17 gives the temporal derivative of the magnetic field in terms of the spatial derivative of the electric field. Conversely, equation 2.18 gives the temporal derivative of the electric field in terms of the spatial derivative of the magnetic field. As will be shown, the first equation will be used to advance the magnetic field in time while the second will be used to advance the electric field. Advancing one field, then the other, and then repeating the process is known as a leap-frog scheme [4]. The next step is to replace these equations with finite differences. To do this, space and time need to be discretized. The following notation will be used in the following example:

$$E_z(x,t) = E_z(m\Delta x, n\Delta t) = E_z^n[m] \qquad \ldots (2.19)$$

$$H_y(x,t) = H_y(m\Delta x, n\Delta t) = H_y^n[m] \qquad \ldots (2.20)$$

Here, n denotes temporal step and m denotes the spatial step. When written as a superscript n still represents the temporal step and is not a power.

Although we only have one spatial dimension, time can be thought of as another dimension. Thus, this is really a sort of two-dimensional problem. Consider figure 2.4, where the electric field nodes are shown as circles and the magnetic field nodes as triangles. Assume that all the fields below the dashed line are known. They are considered in the past while the fields above the dashed line are future fields and hence unknown. The FDTD algorithm provides a way to obtain the future fields from the past fields.

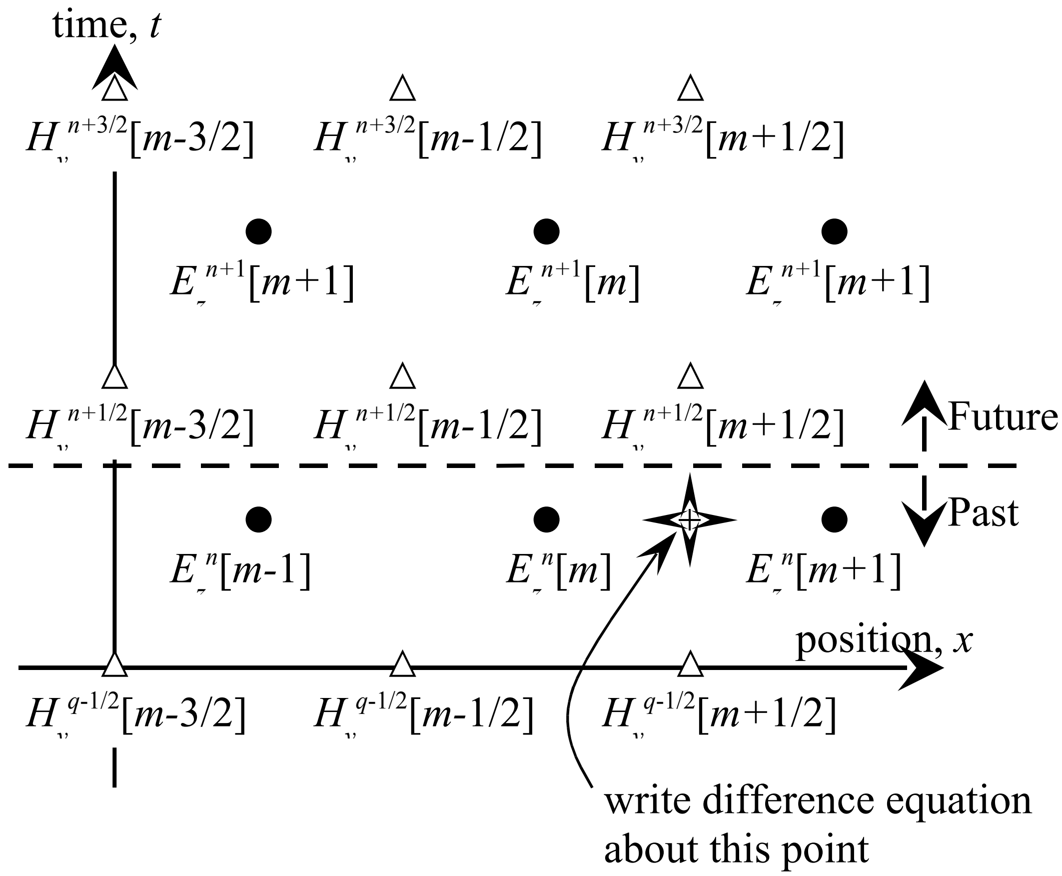

Figure 2.4: The arrangement of electric and magnetic field nodes in space and time. The electric field nodes are shown as circles and the magnetic field nodes as triangles. The indicated point where the difference equation is expanded to obtain an update equation for H_y

As indicated in figure 2.4, consider Faraday’s law at the space-time point $((m+1/2)\boldsymbol{\Delta}x\ ,n\boldsymbol{\Delta}t)$,

$$\mu \left.\frac{\partial H_y}{\partial t}\right|_{(m+1/2)\Delta x, n\Delta t} = \left.\frac{\partial E_z}{\partial x}\right|_{(m+1/2)\Delta x, n\Delta t} \qquad \ldots (2.21)$$

The temporal derivative is replaced by a finite difference involving H_y at n+1/2 and n-1/2 (i.e., the magnetic field at a fixed location but two different times) while the spatial derivative can be replaced by a finite difference involving E_z at m+1 and m (i.e., the electric field at two different locations but one time). Using the finite difference scheme discussed earlier this yields:

$$H_y^{n+\frac{1}{2}}[m+\frac{1}{2}] = H_y^{n-\frac{1}{2}}[m+\frac{1}{2}] + \frac{\Delta t}{\mu \Delta x}(E_z^n[m+1] - E_z^n[m])$$

... (2.22)

Equation 2.22 is known as an update equation, specifically for the update equation for the H_y field. It is a generic equation which can be applied to any magnetic field node. After applying this equation to all the magnetic field nodes, the dividing line between future and past values has advanced a half time-step. The space-time grid thus appears as shown in figure 2.5 which is identical to figure 2.4 except for the advancement of the past/future dividing line.

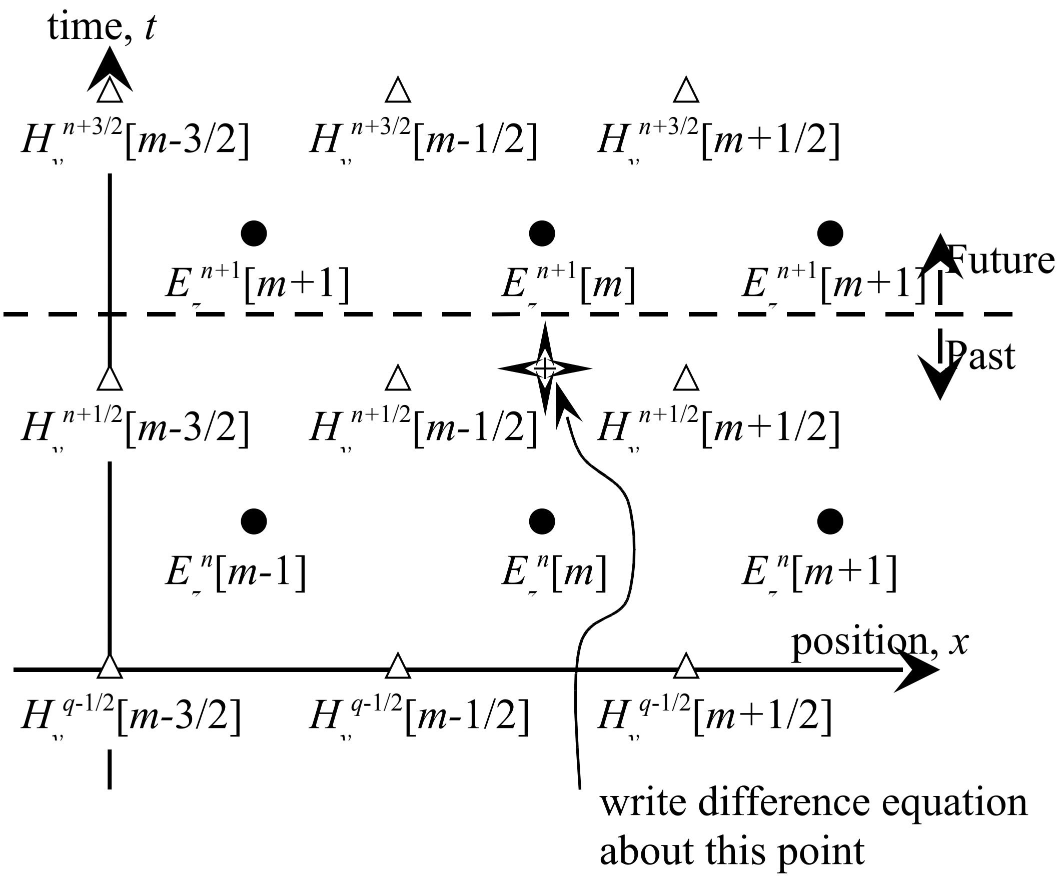

Figure 2.5: Space-time after updating the magnetic field. The dividing line between future and past values has moved forward a temporal step. The indicated point is where the difference equation is written to obtain an update equation for E_z.

Now consider Ampere's Law applied at the space-time point ($m\Delta x$, $(n+1/2)\Delta t$) which is indicated in figure 2.5:

$$E_z^{n+1}[m] = E_z^n[m] + \frac{\Delta t}{\varepsilon \Delta x}\left(H_y^{n+\frac{1}{2}}[m+\frac{1}{2}] - H_y^{n+\frac{1}{2}}[m-\frac{1}{2}]\right)$$

… (2.23)

This is the update equation for the E_z field. The indices in this equation are generic so that the same equation holds for any E_z node.

After applying the above equation to every electric field node in the grid, the dividing line between known and unknown field components moves forward another one-half temporal step. One is essentially back to the situation depicted in figure 2.4. The future fields closer to the dividing line between the future and past are magnetic fields. They would be updated again, and then the electric fields would be updated, and so on.

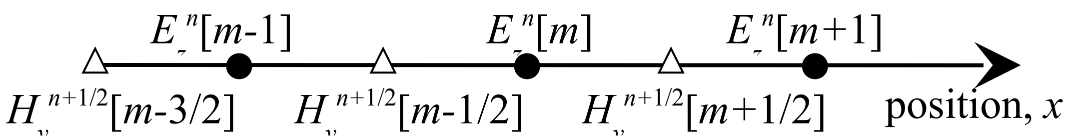

Figure 2.6: One-dimensional FDTD space showing the spatial offset between the magnetic and electric fields.

When first obtaining the update equations for the FDTD algorithm, it is helpful to think in terms of space-time. However, treating time as and additional dimension can be awkward. Thus, in most situations it is more convenient to think in terms of a single spatial dimension where the electric and magnetic fields are offset a half spatial step from each other. This is depicted in figure 2.6. The temporal offset between the electric and magnetic field is always understood whether explicitly shown or not.

The FDTD code for wave propagation in the form of a Gaussian pulse in one-dimension is given in appendix A.

2.5 Accuracy and Stability

In FDTD simulations there are restrictions on how large a temporal step can be. If it is too large, the algorithm produces unstable results (i.e., the numbers obtained are completely meaningless and generally tend quickly to infinity). Considering how fields propagate in an FDTD grid, it seems logical that energy should not be able to propagate any further than one spatial step for each temporal step. To ensure the accuracy of the computed results, the spatial step or increment δ must be small compared to the wavelength or minimum dimension of the scatterer. This amounts to having 10 or more cells per wavelength [5, 6]. To ensure the stability of the finite difference scheme, the time increment Δt must satisfy the following stability condition known as the Courant stability criterion [7, 8]:

$$u_{\max} \Delta t \leq \left[\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2} \right]^{-1/2} \qquad \dots (2.24)$$

u_{max} is the maximum wave phase velocity within the model. If we are using a cubic cell with $\delta = \Delta x = \Delta y = \Delta z$, the above condition becomes,

$$\frac{u_{\max} \Delta t}{\delta} \leq \left[\frac{1}{\sqrt{n}} \right] \qquad \dots (2.25)$$

Where n is the number of space dimensions. For practical reasons, it is best to choose the ratio of the time increment to spatial increment as large as possible.

2.6 Absorbing Boundary Conditions for FDTD

One of the major problems inherent in the standard FDTD, however, is the requirement for artificial mesh truncation (boundary) condition [9, 10]. The artificial termination truncates the solution region electrically close to the radiating/scattering object but effectively simulates the solution to infinity. These artificial termination conditions are known as absorbing boundary conditions (ABCs) [11, 12] as they theoretically absorb incident and scattered fields. The accuracy of the ABC dictates the accuracy of the FDTD method. The need for accurate ABCs has resulted in various types of ABCs. Here only Berenger's perfectly matched layer (PML) [13] type of ABC will be discussed, since PML has been the most widely accepted.

In the perfectly matched layer (PML) truncation technique, an artificial layer of absorbing material is placed around the outer boundary of the computational domain. The goal is to ensure that a plane wave that is incident from FDTD free space to the PML region at an arbitrary angle is completely absorbed there without reflection. There is complete transmission of the incident plane wave at the interface between free space and the PML region (figure 2.4). Thus, the FDTD and the PML region are said to be *perfectly matched*.

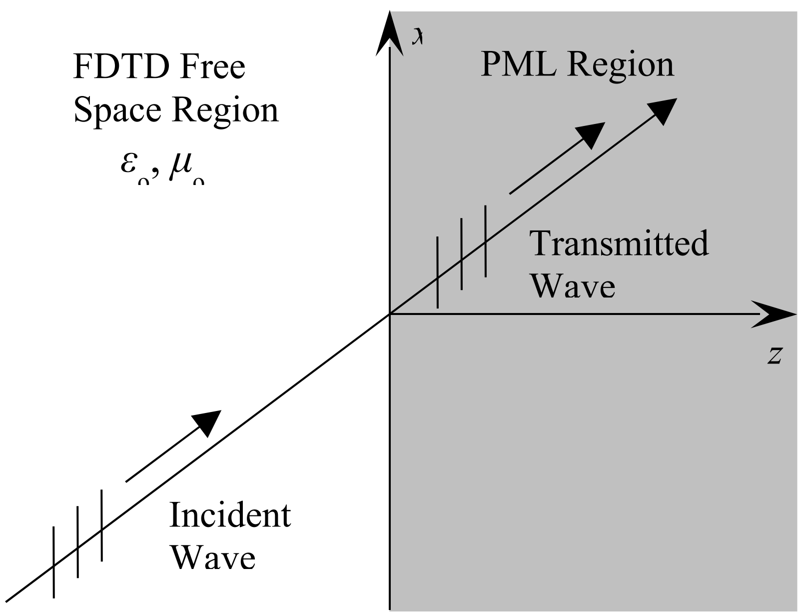

Figure 2.4: Reflectionless transmission of a plane at a PML interface

To illustrate PML technique, consider Maxwell's equation in two dimensions for transverse electric (TE) case with field components E_x, E_y and H_z and no variation with z. By setting $E_z = 0$ in Maxwell's equations we obtain,

$$\varepsilon_o \frac{\partial E_x}{\partial t} + \sigma E_x = \frac{\partial H_z}{\partial y} \qquad \text{… (2.26)}$$

$$\varepsilon_o \frac{\partial E_y}{\partial t} + \sigma E_y = -\frac{\partial H_z}{\partial x} \qquad \text{… (2.27)}$$

$$\mu_o \frac{\partial H_z}{\partial t} + \sigma^* H_z = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \qquad \text{… (2.28)}$$

The PML, as a lossy medium, is characterized by an electrical conductivity σ and a magnetic conductivity σ^*. The conductivities are related as,

$$\frac{\sigma}{\varepsilon_o} = \frac{\sigma^*}{\mu_o} \qquad \text{… (2.29)}$$

The relationship ensures a required level of attenuation and forces the wave impedance of the PML to be equal to that of the free space. Thus a reflectionless transmission of a plane wave propagation across the interface is permitted. For oblique incident angles, the conductivity of the PML must have a certain anisotropy characteristic to ensure reflectionless transmission. To achieve this, the H_z component must be split into two subcomponents, H_{zx} and H_{zy}, where $H_z = H_{zx} + H_{zy}$. This is the cornerstone of the PML technique. It leads to four components E_x, E_y, H_{zx} and H_{zy} and four coupled field equations. Notice that in obtaining equations 2.32 and 2.33, equation 2.28 has been split into two equations written in terms of H_{zx} and H_{zy} and σ^* is split into σ_x^* and σ_y^* to model and anisotropic medium.

$$\varepsilon_o \frac{\partial E_x}{\partial t} + \sigma_y E_x = \frac{\partial (H_{zx} + H_{zy})}{\partial y} \qquad \text{… (2.30)}$$

$$\varepsilon_o \frac{\partial E_y}{\partial t} + \sigma_x E_y = -\frac{\partial (H_{zx} + H_{zy})}{\partial x} \qquad \text{… (2.31)}$$

$$\mu_o \frac{\partial H_{zx}}{\partial t} + \sigma_x^* H_{zx} = -\frac{\partial E_y}{\partial x} \qquad \text{… (2.32)}$$

$$\mu_o \frac{\partial H_{zy}}{\partial t} + \sigma_y^* H_{zy} = \frac{\partial E_x}{\partial y} \qquad \text{… (2.33)}$$

These equations can be discretized to provide the FDTD time-stepping equations for the PML region.

Chapter 3

Electromagnetic Wave Scattering by a Dielectric Cylinder

3.1 Problem Description

This problem is taken from Sadiku [14]. It investigates the scattering of a TEM electromagnetic wave by a dielectric cylinder in free space. The cylinder has a radius of 6cm. The axis of the dielectric cylinder lies along the *z*-axis and the cylinder is infinitely long in z direction. The symmetry reduces it to a two-dimensional problem in *xy*-plane. The TEM wave propagates in +y direction and scatters when encounters the dielectric cylinder. The objective is to compute one of the components, such as E_z, at points within the cylinder. The dielectric is assumed lossless with,

$$\varepsilon_d = 4\varepsilon_o, \quad \mu_d = \mu_o, \quad \sigma_d = 0$$

A sine wave of frequency 2.5 GHz is applied as the exciting source. The simulation is run long enough so that steady-state is achieved.

3.2 Solution

The first step is to obtain the discretized Maxwell's equations using the finite difference scheme discussed in section 2.2. It is assumed that the electric field has only z-component and that z-component of magnetic field is zero. By setting partial derivative with respect to z zero in equations 2.3 to 2.8 we are left with following equations.

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\frac{\partial E_z}{\partial y} \qquad \ldots (3.1)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\frac{\partial E_z}{\partial x} \quad \ldots (3.2)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right) \quad \ldots (3.3)$$

Since conductivity throughout the problem space is zero we are left with,

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \quad \ldots (3.4)$$

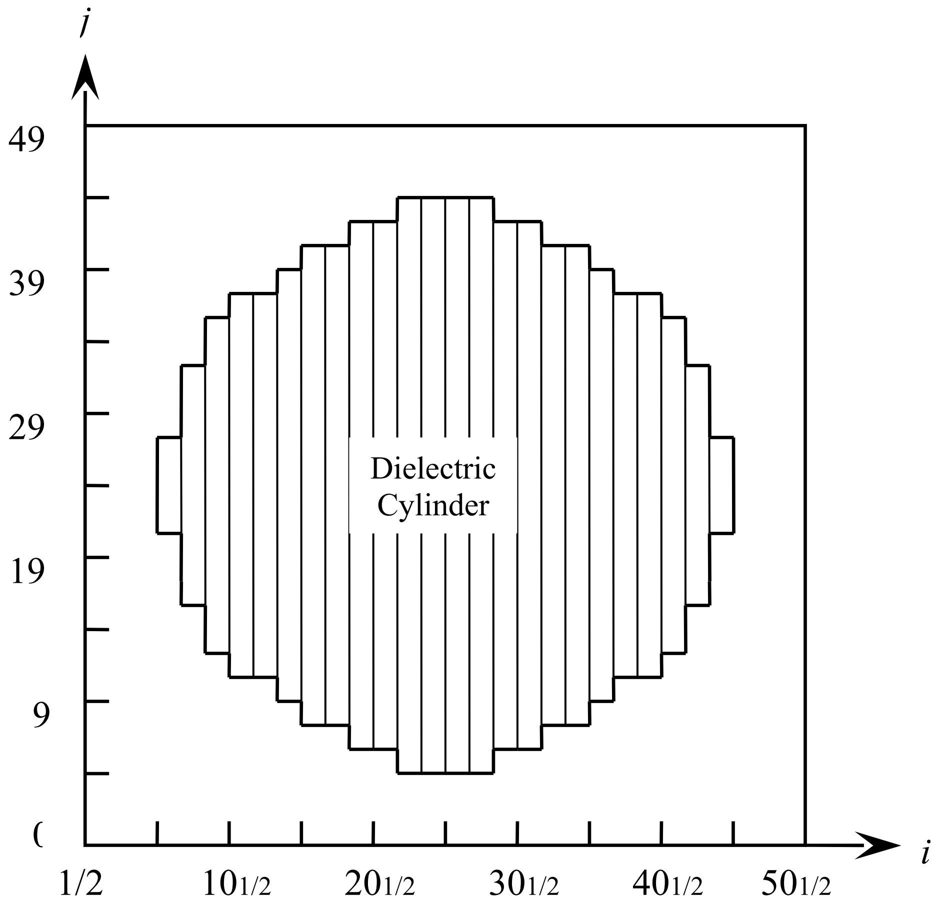

Figure 3.1: Problem geometry showing points internal and external to the cylinder

Following Sadiku [15] spatial step is choose as $\delta = \Delta x = \Delta y = 0.3$cm and $\delta t = \delta 2c = 5$ps. We use the 51x50 grid shown in figure 3.2.Sadiku uses symmetry condition to further decrease the size of the grid [16] but we will simulate the whole problem space as shown in figure 3.1. The cylinder axis is chosen to pass through the point (i,j) = (25.5, 24.5). Referring to figure 3.2 we see that the problem space is bounded by the z and y components of electric and magnetic fields respectively. So we need to apply boundary conditions on E_z at j = 0.5 and j = 49 and on H_y at i = 0 and i = 50.5. The FDTD code for the simulation is given in appendix B and the results obtained are shown in figures 3.3 and 3.4.

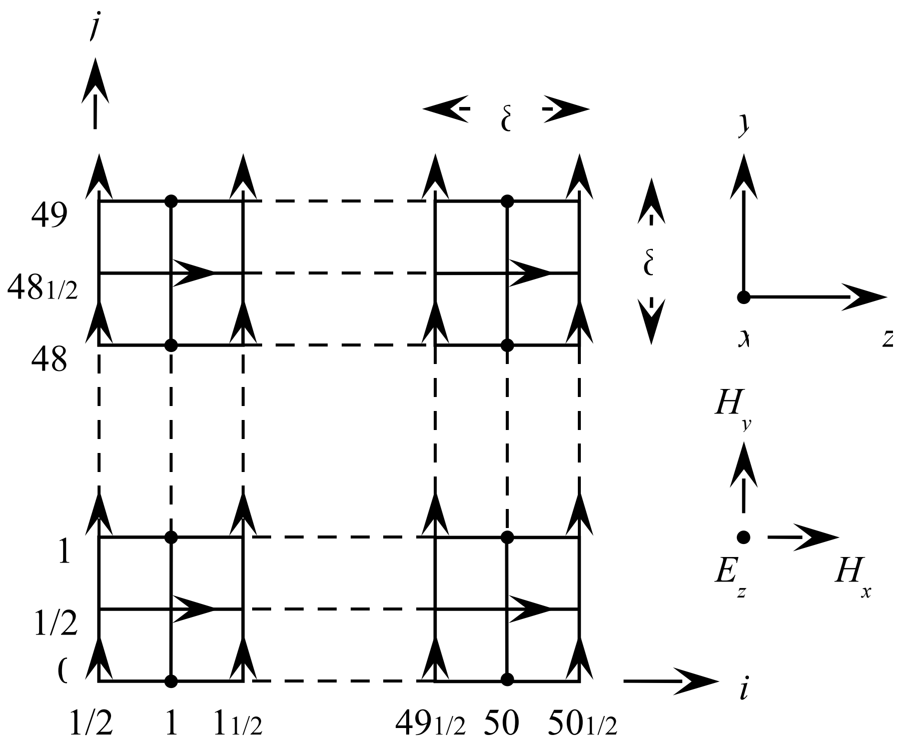

Figure 3.2: Two-dimensional lattice for the problem

3.3 Simulation Results

Figure 3.3 and 3.4 show results obtained after simulation in Matlab™. The results by Sadiku [17] are depicted in the small boxes. The graphs show the absolute or steady state value of E_z at i=15 and i=25. The jaginess of the graphs is due to the fact that a coarse grid was used. The results can be improved by using a finer grid. Nevertheless the results show good agreement with the published results.

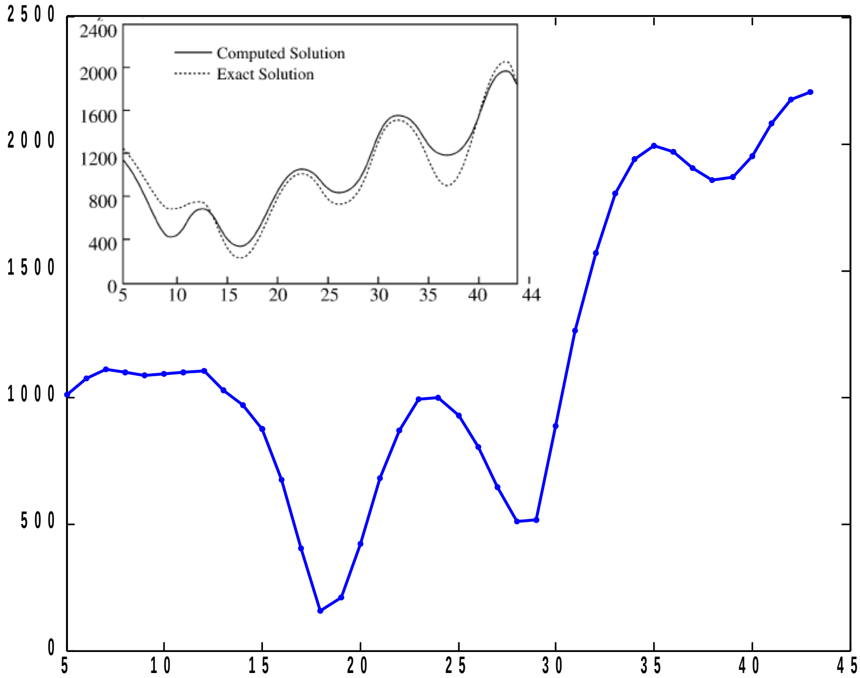

Figure 3.3: Magnitude of E_z at i = 15

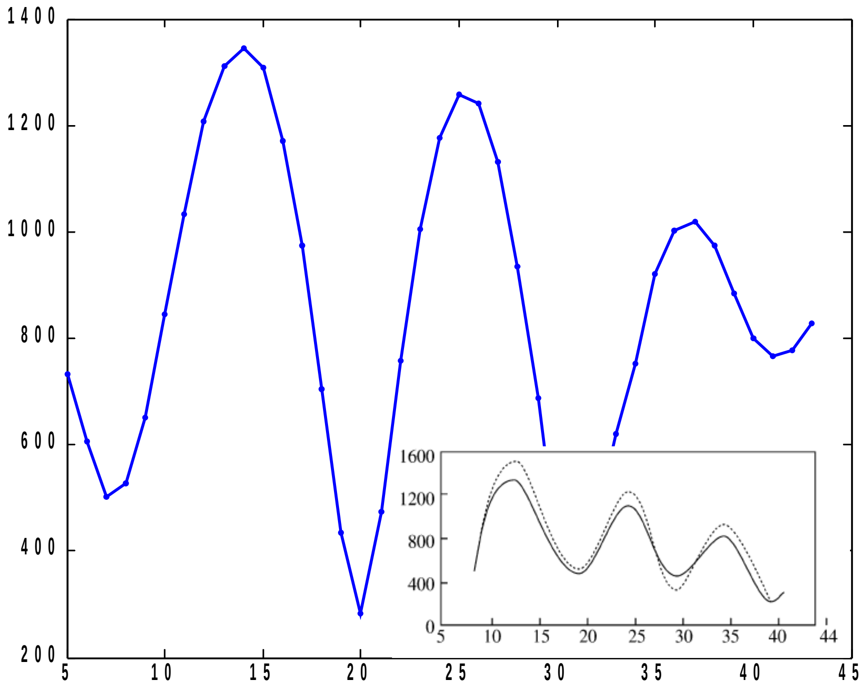

Figure 3.4: Magnitude of E_z at $i = 25$

Chapter 4

Electromagnetic Wave Scattering By a Conducting Square Plate

4.1 Introduction

FDTD was first introduced by Kane S. Yee, in his paper [18]. He showed that if the field points are chosen appropriately the boundary conditions can be incorporated in the set of finite difference equations. He demonstrated this by an example of scattering of an electromagnetic pulse by a conducting plate.

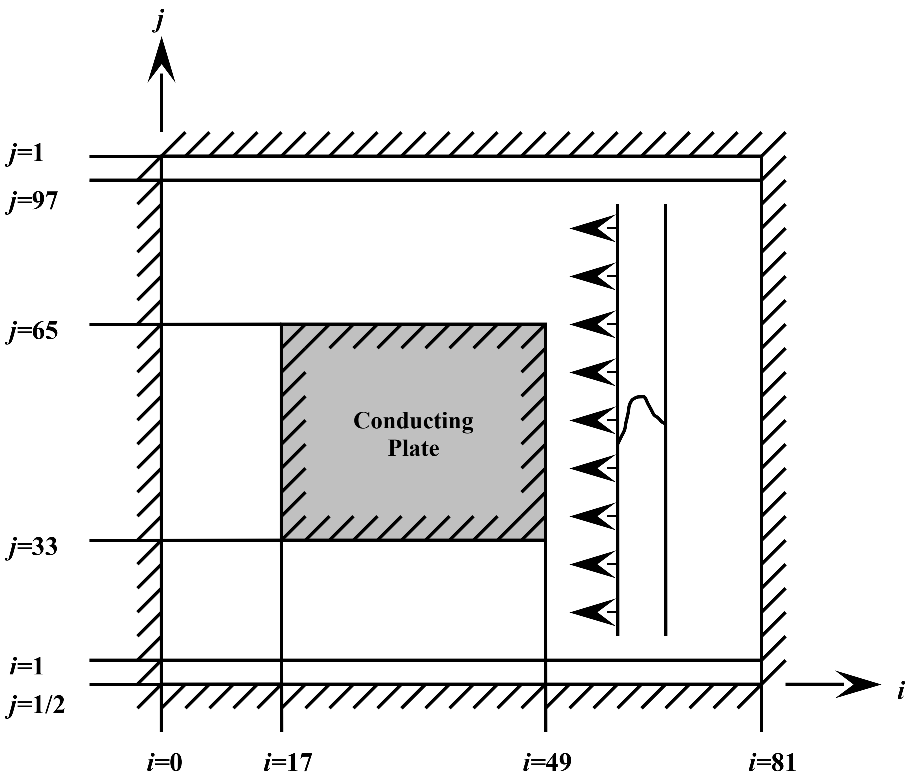

Figure 4.1: Geometry of Yee's problem

4.2 Problem Description

A transverse magnetic (TM) wave is incident on a conducting plate as shown in figure 4.1. This is also a two-dimensional problem. The TM wave travelling in $-x$ direction encounters a perfectly conducting plate at i=49. The wave will only have E_z and H_y components [19]. The width of the plate is taken as 4α units where α is a constant. The spatial increment δ is chosen as α /16 and the temporal increment Δt as δ/2 or α /16. The Matlab™ code for the simulation is given in appendix C. The results obtained using Matlab™ and Yee's results are compared in figures 4.2-4.

4.3 Comparison of Results

Figure 4.2 shows results in the absence of any obstacle where the voltage pulse travels along $-x$ direction in free space unhindered.

Yee's Results **Matlab™ Simulation**

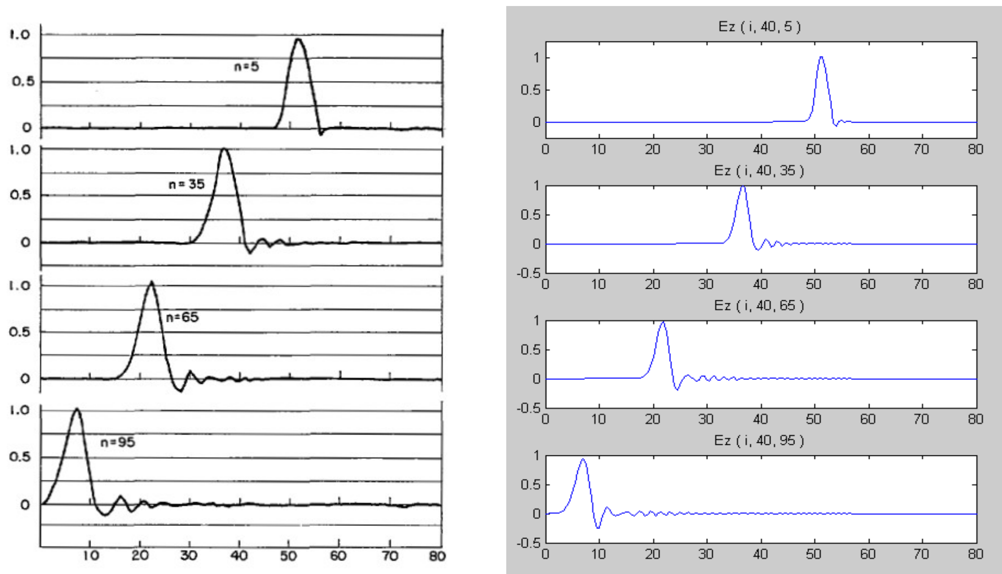

Figure 4.2: Results in the absence of any obstacle. The ordinate (y-axis) is in volts/meter and the abscissa (x-axis) is the number of horizontal Increments. n is the number of time cycles.

In figure 4.3 the voltage voltage pulse travels along $-x$ direction until it encounters the conducting plate at i=49. After it hits the plate it reflects back after a phase change of 180 degrees. The results are taken at different time instants at j=50.

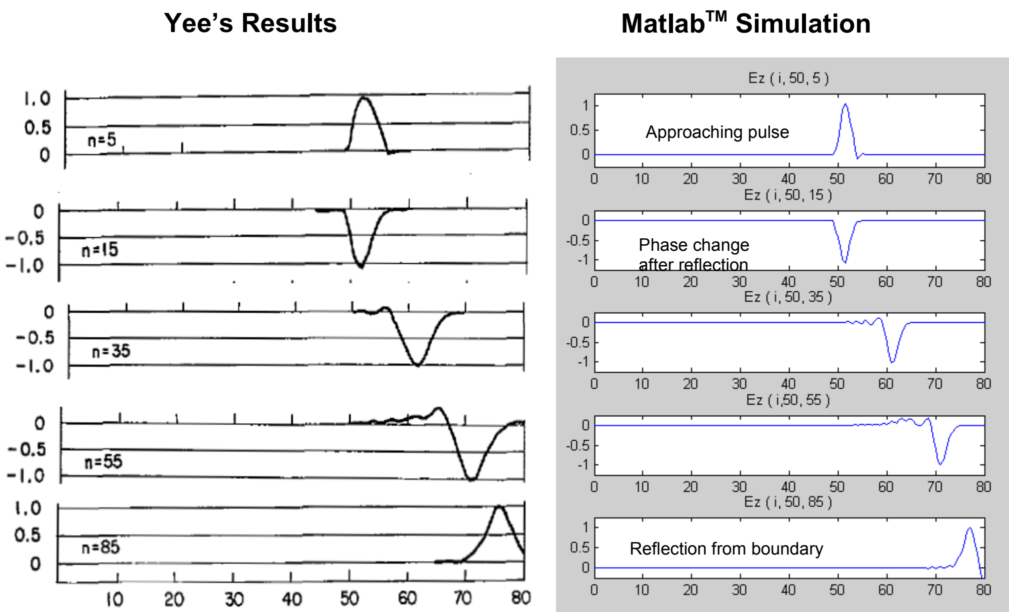

Figure 4.3: Results in the presence of obstacle. The ordinate (y-axis) is in volts/meter and the abscissa (x-axis) is the number of horizontal Increments. n is the number of time cycles.

Figure 4.4 shows the results obtained at j = 30 which is near the lower edge of the conducting plate. It shows that some portion of the pulse travels forward while some of it reflected back showing a partial reflection. Again good match is obtained between the code and published results. This verifies the correctness of the code.

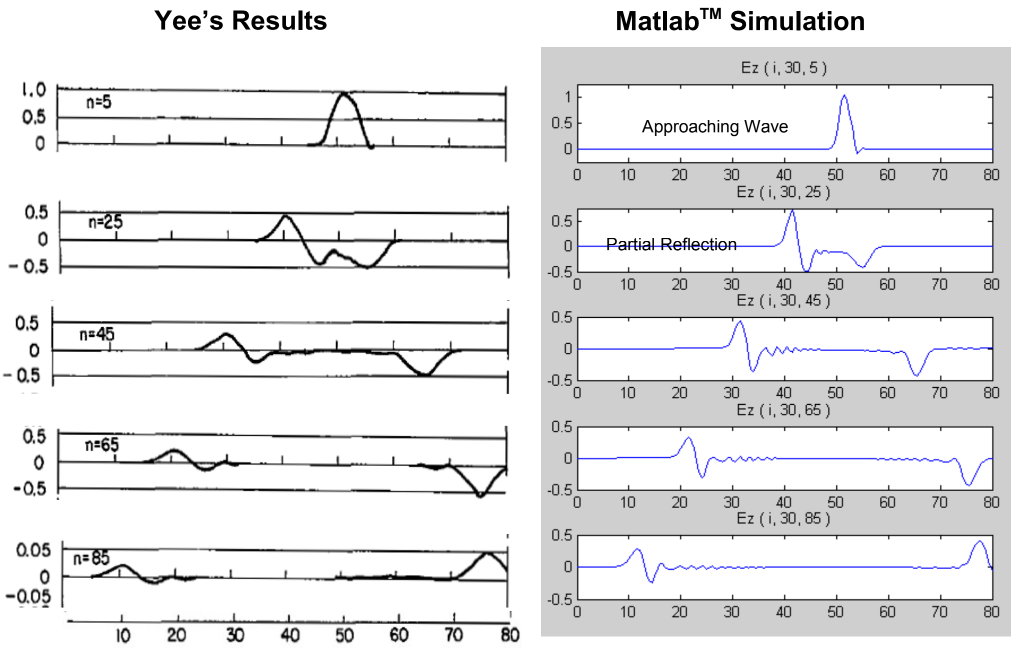

Figure 4.4: Results in the presence of obstacle near the boundary. The ordinate (y-axis) is in volts/meter and the abscissa (x-axis) is the number of horizontal Increments. n is the number of time cycles.

Chapter 5

Simulation of a Monopole Using XFDTD™

5.1 Introduction

This problem is from Maloney et. al [20]. In this paper two antennas, a cylindrical monopole and a conical monopole, are analyzed using the FDTD method. The computed results are matched with experimental results to show the accuracy of the FDTD technique. Here a cylindrical FDTD scheme is used which conforms to the geometry of the monopole and reduces the problem to a two-dimensional problem in cylindrical coordinates.

After writing the FDTD code for the two problems discussed in previous chapters a commercial software XFDTD™ was used to model the cylindrical monopole from the abovementioned paper as we have already verified the validity of the written code. This will enable us to solve practical electromagnetic problems using commercially available softwares.

5.2 Problem Description

The problem domain consists of a coaxial cable whose inner conductor is extended out of the ground plane forming a monopole. A voltage pulse is applied at one end of the coaxial cable. As the pulse travels on the monopole some of it reflects at $z = 0$, which is occupied by the ground plane, due to discontinuity at the interface between the two media. The remaining pulse travels on the monopole and some of it reflects back from the end of the monopole, again due to mismatch in media, while some part of the pulse is radiated as electromagnetic wave in space. The travelling pulse again encounters the ground plane where again some of it reflects back with a phase change while the rest passes into the coaxial cable. The problem geometry constructed in XFDTD™ is shown in figure 5.1. Appendix D gives a description of XFDTD™ usage.

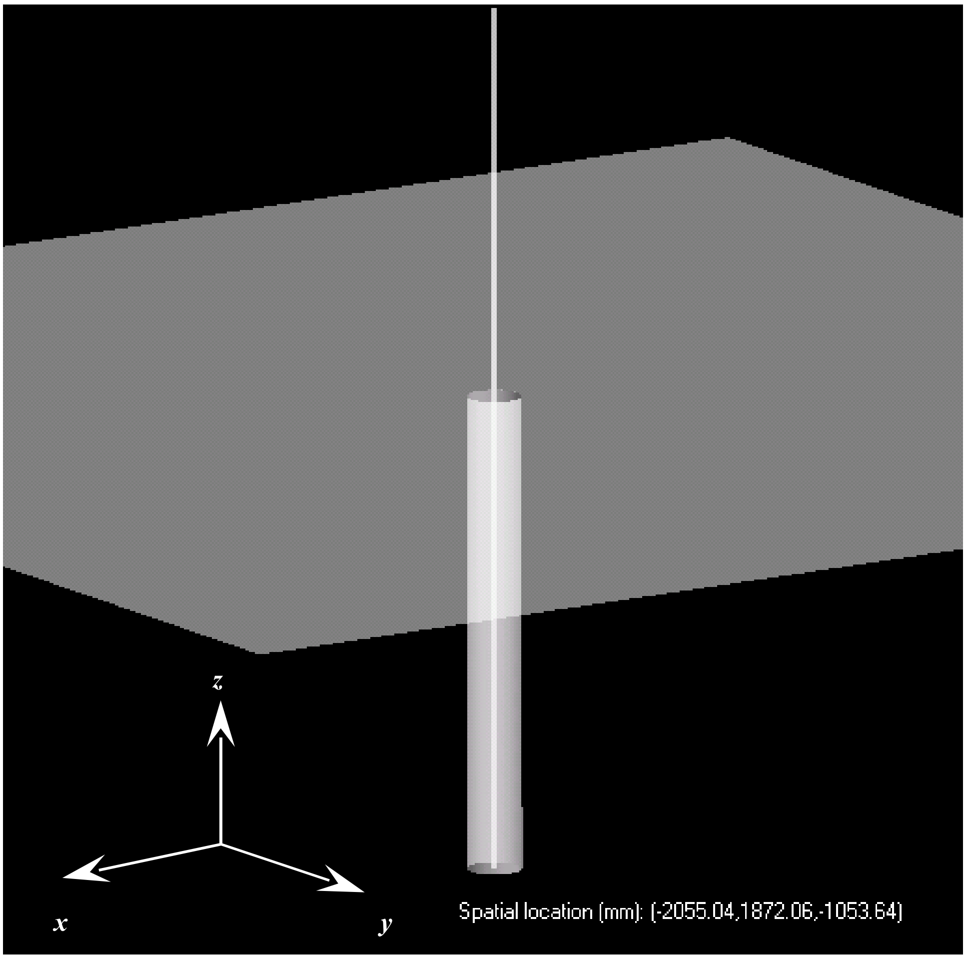

Figure 5.1: Monopole geometry in XFDTD™

5.3 Simulation Results

Figure 5.2 shows the measured voltage on the monopole just below the ground plane on the coaxial cable. This figure is taken from the paper [x] for comparison with simulation results. Figure 5.3 shows results obtained using XFDTD™. The first peak shows the initial voltage pulse. The second peak shows the voltage pulse reflected back at the end of the antenna ($0 < \Gamma < 1$). It has a smaller magnitude owing to some loss due to radiation from the antenna. The third peak is much smaller than the first two and shows the voltage pulse reflected at $z = 0$ ($-1 < \Gamma < 0$) and being 180 degrees out of phase.

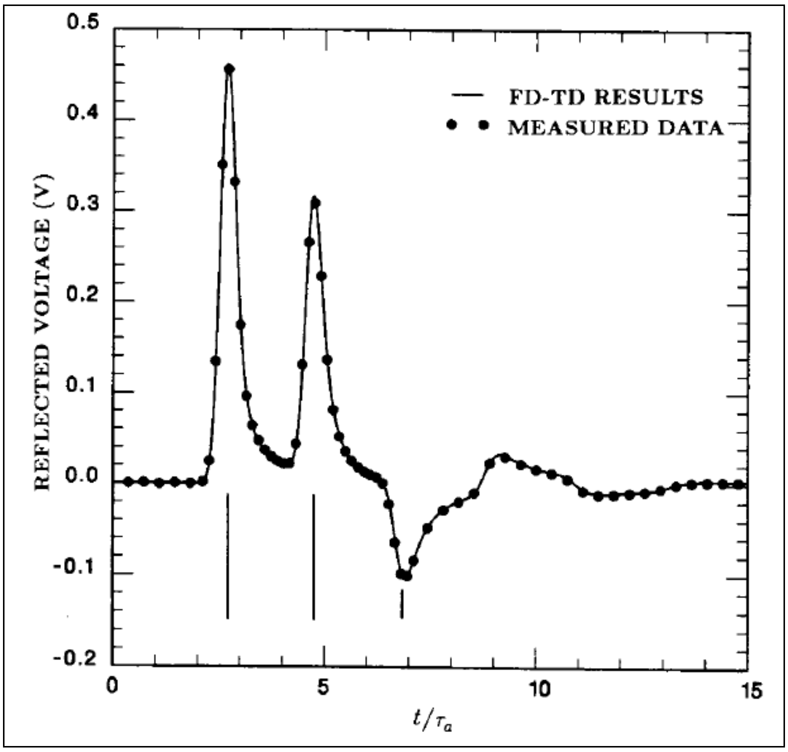

Figure 5.2: Results from the paper (shows measured reflected voltage pulse on the monopole)

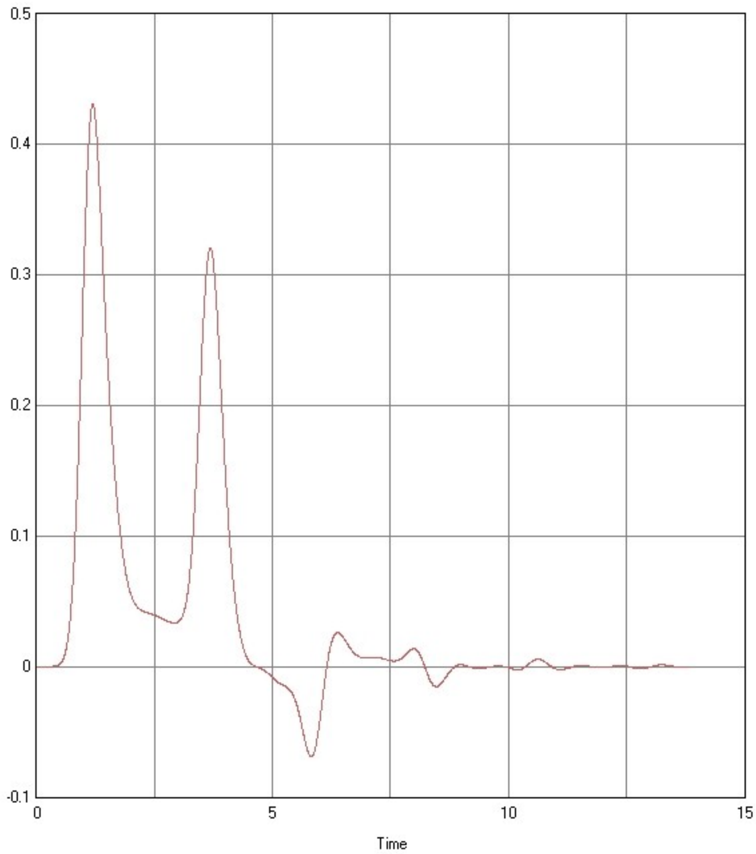

Figure 5.3: Results obtained using XFDTD™

Conclusion

The aim of the project was to get acquainted with computational electromagnetics. Basically three problems were modelled in the project. The first was modelling electromagnetic scattering by a dielectric cylinder. The second problem was that of Yee's original scattering problem which provided proof of evidence. The third problem was a more practical problem involving simulation of a monopole antenna using commercially available software.

Solving the first two problems has enabled us to understand the basic working of the FDTD algorithm as the code was written by us. The third problem helped us get an idea of how practical electromagnetic problems can be solved using commercially available softwares.

References

[1] A. Taflove, Computational Electrodynamics: The Finite-Difference Time-Domain Method. Boston, MA: Artech House, 1995.

[2] M.N. O. Sadiku, “Numerical Techniques in Electromagnetics” CRC Press, second ed., 2001, pp. 173–197.

[3] Numerical Solution of Initial Boundary Value Problems Involving Maxwell’s Equations in Isotropic Media, Kane S. Yee, Vol. AP-14, No. 8 IEEE Transactions on Antennas and Propagation.

[4] Accurate Computation of the Radiation From Simple Antennas Using the FDTD Technique, Maloney et al. Vol 38, No. 7, IEEE Transactions on Antennas and Propagation.

Appendix A

Matlab™ FDTD Code for Simulation of a TEM Wave Propagation in One Dimension

```
% Simulation parameters.

SIZE = 200;             % No. of spatial steps
MaxTime = 200;          % No. of time steps
imp0 = 377.0;           % Impedence of free space

% Initialization.

Ez = zeros ( SIZE, MaxTime );    % z-component of E-field
Hy = zeros ( SIZE, MaxTime );    % y-component of H-field

PLOT1(1) = 0;           % Data for plotting.

% Outer loop for time-stepping.
for q = 2:MaxTime

    % Calculation of Hy using update difference equation for
Hy. This is time step q.
    for m = 1:SIZE-1

        Hy(m,q) = Hy(m,q-1) + ( ( Ez(m+1,q-1) - Ez(m,q-1) ) /
imp0 );

    end

    % Calculation of Ez using updated difference equation for
Ez. This is time step q+1/2.
    for m = 2:SIZE

        Ez(m,q) = Ez(m,q-1) + ( imp0 * ( Hy(m,q) -
Hy(m-1,q) ) );

    end

    % Activating a plane-wave source.
    Ez(1,q) = exp ( - 1 * ( (q-31)^2) / 100 );

end

% Plotting Electric field at differenct times.
figure
plot ( Ez(:,10) )

figure
plot ( Ez(:,30) )
```

```
figure
plot ( Ez(:,50) )

figure
plot ( Ez(:,70) )

figure
plot ( Ez(:,90) )

figure
plot ( Ez(:,110) )

figure
plot ( Ez(:,120) )

figure
plot ( Ez(:,130) )

figure
plot ( Ez(:,150) )
```

Appendix B

Matlab™ FDTD Code for TEM Wave Scattering By a Dielectric Cylinder

```
% FDTD example 3.7 from Numerical techniques in
Electromagnetics by Sadiku.
% Main m file for the FDTD simulation.

clc
clear all

% Simulation related parameters.

IHx = 50;
JHx = 49;
IHy = 51;
JHy = 50;
IEz = 50;
JEz = 50;

NMax = 2; NNMax = 500;     % Maximum time.
NHW = 40;                  % One half wave cycle.
Med = 2;                   % No of different media.
Js = 2;                    % J-position of the plane wave front.

% Different Constants.
delta = 3e-3;
Cl = 3e8;
f = 2.5e9;
pi = 3.141592654;
e0 = (1e-9) / (36*pi);
u0 = (1e-7) * 4 * pi;

DT = delta / ( 2 * Cl );
TwoPIFDeltaT = 2 * pi * f * DT;

% Data arrays.

CHx = zeros ( IHx, JHx );        % Conductance
CHy = zeros ( IHy, JHy );        % Conductance
REz = zeros ( IEz, JEz );        % Impedance

RaEz = zeros ( IEz, JEz );       % Scaling parameter dependent
on material conductivity.

Hx = zeros ( IHx, JHx, NNMax );
Hy = zeros ( IHy, JHy, NNMax );
Ez = zeros ( IEz, JEz, NNMax );
AbsEz = zeros ( IEz, JEz );     % Absolute value of Ez for
steady state solution
```

```
% ############ Initialization #############

fprintf ( 1, 'Initializing' );

fprintf ( 1, '.' );

% Initializing the eta and 1/eta arrays.

for i=1:IHx
    for j=1:JHx
        CHx ( i, j ) = DT / ( u0 * ur ( i, j-0.5 ) * delta );
    end
end

fprintf ( 1, '.' );

for i=1:IHy
    for j=1:JHy
        CHy ( i, j ) = DT / ( u0 * ur ( i-0.5, j-1 ) * delta );
    end
end

fprintf ( 1, '.' );

for i=1:IEz
    for j=1:JEz
        REz ( i, j ) = DT / ( e0 * er ( i, j-0.5 ) * delta );
        RaEz ( i, j ) = ( 1 - ( s(i, j-0.5) * DT )/( e0 * er
( i, j-0.5 ) ) );
    end
end

fprintf ( 1, 'done.\n' );

% ############ Initialization Complete ##############

% ########### 2. Now running the Simulation #############

fprintf ( 1, 'Simulation started... \n', 0 );

for n=0:498

    fprintf ( 1, '%g %% \n', (n*100)/500 );

    % *** Calculation of Magnetic Field Components ***

    % * Calculation of Hx.
    Hx ( :, :, n+2 ) = Hx ( :, :, n+1 ) +  ( CHx .* ( Ez ( :,
1:JHx, n+1 ) - Ez ( :, 2:JHx+1, n+1 ) ));

    %  * Calculation of Hy.
    Hy ( 2:IHy-1, :, n+2 ) = Hy ( 2:IHy-1, :, n+1 ) +  ( CHy
( 2:IHy-1, : ) .* ( Ez ( 2:IHy-1, :, n+1 ) - Ez ( 1:IHy-2, :,
n+1 ) ));

    % Boundary conditions on Hy. Soft grid truncation.
```

```
    Hy ( 1, 2:JHy-1, n+2 ) = (1/3) * ( Hy ( 2, 1:JHy-2, n+1 ) +
Hy ( 2, 2:JHy-1, n+1 ) + Hy ( 2, 3:JHy, n+1 ) );
    Hy ( IHy, 2:JHy-1, n+2 ) = (1/3) * ( Hy ( IHy-1, 1:JHy-2,
n+1 ) + Hy ( IHy-1, 2:JHy-1, n+1 ) + Hy ( IHy-1, 3:JHy,
n+1 ) );
    Hy ( 1, 1, n+2 ) = (1/2) * ( Hy ( 2, 1, n+1 ) + Hy ( 2, 2,
n+1 ) );
    Hy ( 1, JHy, n+2 ) = (1/2) * ( Hy ( 2, JHy, n+1 ) + Hy ( 2,
JHy-1, n+1 ) );
    Hy ( IHy, 1, n+2 ) = (1/2) * ( Hy ( IHy-1, 1, n+1 ) + Hy
( IHy-1, 2, n+1 ) );
    Hy ( IHy, JHy, n+2 ) = (1/2) * ( Hy ( IHy-1, JHy, n+1 ) +
Hy ( IHy-1, JHy-1, n+1 ) );

    % ************************************************

    % *** Calculation of Electric Field Components ***

    % * Calculation of Ez.
    Ez ( :, 2:JEz-1, n+2 ) = ( RaEz ( :, 2:JEz-1 ) .* Ez ( :,
2:JEz-1, n+1 ) ) +  ( REz ( :, 2:JEz-1 ) .* ( Hy ( 2:JEz+1,
2:JEz-1, n+2 ) - Hy ( 1:JEz, 2:JEz-1, n+2 ) + Hx ( :, 1:JEz-2,
n+2 ) - Hx ( :, 2:JEz-1, n+2 ) ));

    % Boundary conditions on Ez. Soft grid truncation.

    Ez ( 2:IEz-1, 1, n+2 ) = (1/3) * ( Ez ( 1:IEz-2, 2, n+1 ) +
Ez ( 2:IEz-1, 2, n+1 ) + Ez ( 3:IEz, 2, n+1 ) );
    Ez ( 2:IEz-1, JEz, n+2 ) = (1/3) * ( Ez ( 1:IEz-2, JEz-1,
n+1 ) + Ez ( 2:IEz-1, JEz-1, n+1 ) + Ez ( 3:IEz, JEz-1,
n+1 ) );
    Ez ( 1, 1, n+2 ) = (1/2) * ( Ez ( 1, 2, n+1 ) + Ez ( 2, 2,
n+1 ) );
    Ez ( IEz, 1, n+2 ) = (1/2) * ( Ez ( IEz, 2, n+1 ) + Ez
( IEz-1, 2, n+1 ) );
    Ez ( 1, JEz, n+2 ) = (1/2) * ( Ez ( 1, JEz-1, n+1 ) + Ez
( 2, JEz-1, n+1 ) );
    Ez ( IEz, JEz, n+2 ) = (1/2) * ( Ez ( IEz, JEz-1, n+1 ) +
Ez ( IEz-1, JEz-1, n+1 ) );

    % ************************************************

    % Applying a plane wave source at Js.

    Ez ( :, Js, n+2 ) = 1000 * sin ( TwoPIFDeltaT * (n+2) );

    % 4. Retaining absolute values during the last half cycle.
    if ( n > 460 )

        A = abs ( Ez(:,:,n+2) ) >= AbsEz;
        B = abs ( Ez(:,:,n+2) ) < AbsEz;
        C = A .* abs ( Ez(:,:,n+2) );
        D = B .* AbsEz;
        AbsEz =  C + D;

    end

end
```

```
fprintf ( 1, '100 %% \n' );
fprintf ( 1, 'Simulation complete! \n', 0 );

% Plotting Ez.

subplot ( 211 )
plot ( 5:43, AbsEz(25,5:43) )
title ( 'Ez ( 25, j )' )

subplot ( 212 )
plot ( 5:43, AbsEz(15,5:43) )
title ( 'Ez ( 15, j )'
```

Appendix C

Matlab™ FDTD Code for TEM Wave Scattering By a Conducting Plate

```
% Simulation related parameters.

IHy = 163;
JHy = 195;
IEz = 164;
JEz = 195;

NMax = 2; NNMax = 200;          % Maximum time.
NHW = 40;                       % One half wave cycle.
Med = 2;                        % No of different media.
Js = 2;                         % J-position of the plane wave
front.

% Constants.
alpha = 16;
delta = alpha/8;
Cl = 3e8;
f = 2.5e9;
pi = 3.141592654;
e0 = (1e-9) / (36*pi);
u0 = (1e-7) * 4 * pi;

DT = delta / ( 2 * Cl );
TwoPIFDeltaT = 2 * pi * f * DT;

% Data arrays.

CHy = zeros ( IHy, JHy );
REz = zeros ( IEz, JEz );
RaEz = zeros ( IEz, JEz );

Hy = zeros ( IHy, JHy, NNMax );
Ez = zeros ( IEz, JEz, NNMax );
AbsEz = zeros ( IEz, JEz );

% ############ Initialization ##############

fprintf ( 1, 'Initializing' );

fprintf ( 1, '.' );

% Initializing the eta and 1/eta arrays.

fprintf ( 1, '.' );

for i=1:IHy
    for j=1:JHy
```

```
        CHy ( i, j ) = DT / ( u0 * ur ( i-0.5, j ) * delta );
    end
end

fprintf ( 1, '.' );

for i=1:IEz
    for j=1:JEz
        REz ( i, j ) = DT / ( e0 * er ( i-1, j ) * delta );
        RaEz ( i, j ) = ( 1 - ( s(i-1, j) * DT ))/( 1 +
( s(i-1, j) * DT ));
    end
end

% Applying a plane wave.
for i=-15:16
    Ez ( 140+i, :, 1 ) = exp ( -(1/16) * ( i )^2 );
    Hy ( 140+i, :, 1 ) = Ez ( 140+i, :, 1 )/377;
end
fprintf ( 1, 'done.\n' );

% ########## Initialization Complete ############

% ##### 2. Now running the Simulation ######

fprintf ( 1, 'Simulation started... \n', 0 );

for n=0:NNMax-2

    fprintf ( 1, '%g %% \n', (n*100)/NNMax );

    % *** Calculation of Magnetic Field Components ***

    %  * Calculation of Hy.
    Hy ( :, :, n+2 ) = Hy ( :, :, n+1 ) +  ( CHy .* ( Ez
( 2:IHy+1, :, n+1 ) - Ez ( 1:IHy, :, n+1 ) ));

    % ***********************************************

    % *** Calculation of Electric Field Components ***

    % * Calculation of Ez.
    Ez ( 2:IEz-1, :, n+2 ) = ( RaEz ( 2:IEz-1, : ) .* Ez
( 2:IEz-1, :, n+1 ) ) +  ( REz ( 2:IEz-1, : ) .* ( Hy
( 2:IEz-1, :, n+2 ) - Hy ( 1:IEz-2, :, n+2 ) ));


end

fprintf ( 1, '100 %% \n' );
fprintf ( 1, 'Simulation complete! \n', 0 );
```

Appendix D

Monopole Construction in XFDTD^{TM}

When XFDTD^{TM} is started it shows the main screen. From here we can move to the geometry construction and viewing menu by clicking on the geometry tab. The geometry construction screen is shown in figure D1.

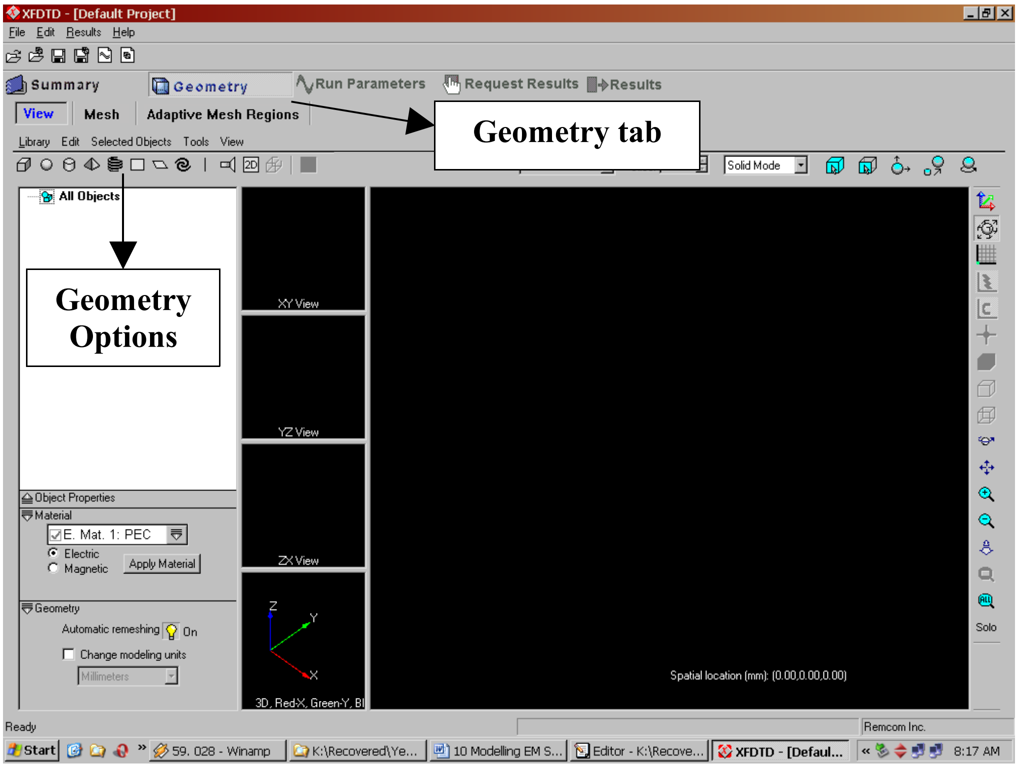

Figure D1: The Geometry Viewing/Construction Screen

From here we add a conducting plate at $z = 0$ for the ground plane. It must be kept in mind that all measurements are in millimeters. We select the plate option as shown in figure D2. This opens a where we can input the dimensions of the plate and select the material for the plate. We choose the plate to occupy $z = 0$ plane and made of perfectly conducting material (PEC) as shown in figure D3. We want our monopole to located at (0, 0, z) along z-axis. We model the monopole as a wire and the coaxial cable as a conducting cylinder surrounding the wire.

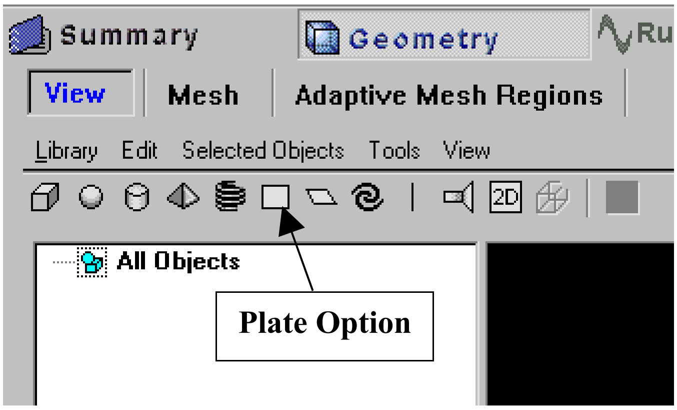

Figure D2: Adding a plate.

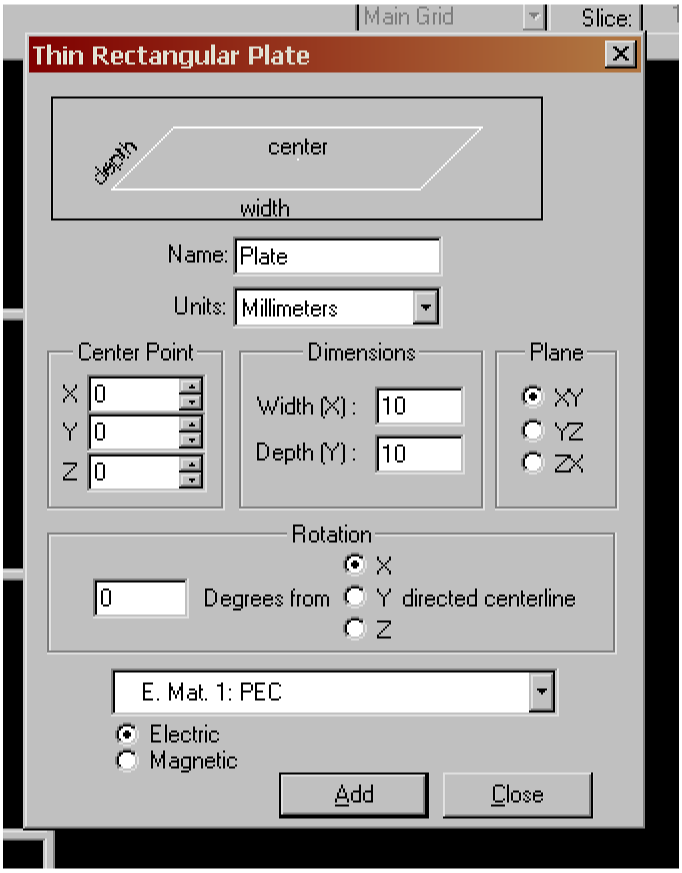

Figure D3: Selecting parameters of the plate.

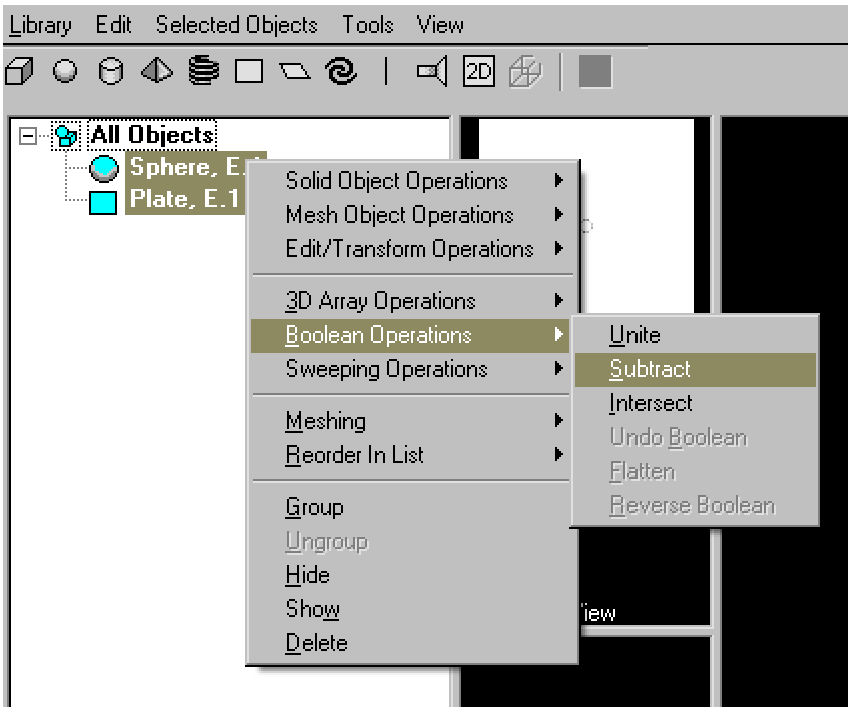

Figure D4: Performing a Boolean operation of subtraction between sphere and plate.

We add a hole in the conducting plate by first adding a sphere of radius 0.5mm at location (0, 0, 0) and then subtracting the sphere from plate as shown in figure D4. Next we add a wire, from the wire option, at (0, 0, z) along z-direction starting at z = -10 to z = 10 (figure D5). Lastly we add a hollow cylinder from z = -10 to z = 0 to simulate a coaxial cable (figure D6).

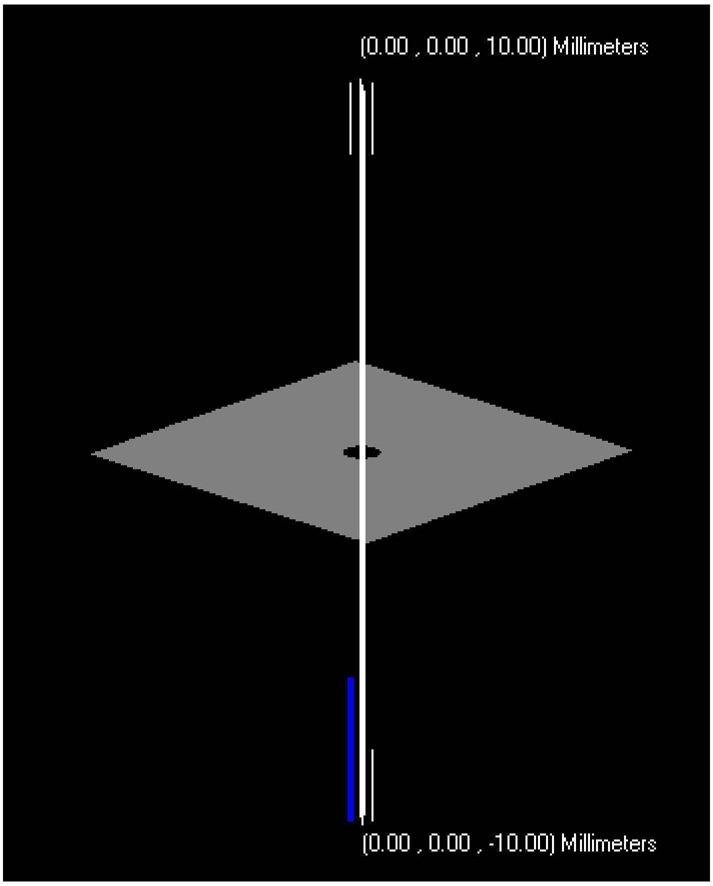

Figure D5: Geometry after adding wire.

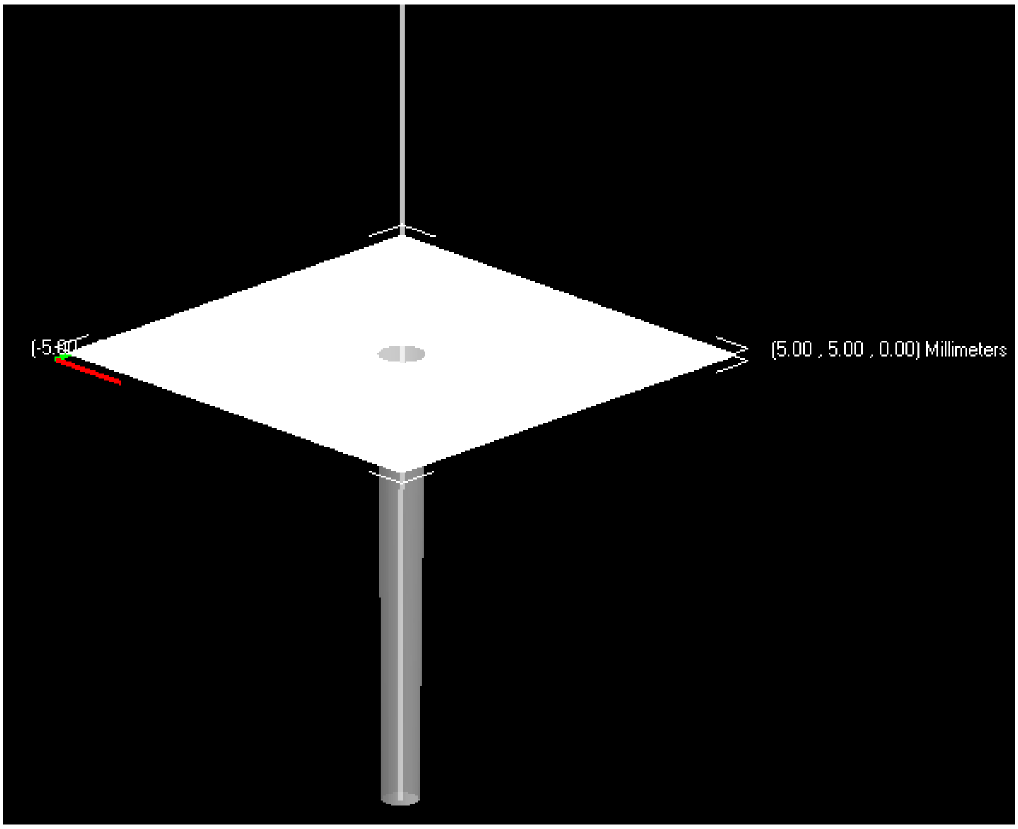

Figure D6: Final Geometry.