Modeling Electromagnetic Wave Scattering Using the Finite Difference Time Domain Technique:

A Matrix Based Approach Using Matlab^{TM}

Attique Dawood
Department of Telecom Engineering, National University of Computer and Emerging Sciences, Islamabad, Pakistan.
attique.dawood@nu.edu.pk

Muhammad Adeel Raza
Department of Telecom Engineering, National University of Computer and Emerging Sciences,
Islamabad, Pakistan.
adeelshigri@yahoo.com

Abstract-**This paper analyzes the conventional loop based and matrix based implementations of the FDTD algorithm. Two canonical problems in this domain are solved using both approaches and time-space considerations are considered.**

Keyword-component: FDTD; loop-based; matrix-based; simulation order;

I. INTRODUCTION

Many numerical methods are used to solve Maxwell's equations. There are basically two approaches to formulate an electromagnetic scattering problem using Maxwell's equations; integral and differential. Examples of integral techniques are method of moments and finite elements. Finite Difference Time Domain (FDTD) is a differential technique used to numerically implement Maxwell's equations. The aim of this paper is to analyze the space-time considerations when using conventional loop-based and the newer matrix based implementations by solving "standard" electromagnetic scattering problems.

The FDTD method is conceptually simple and does not need formulation of integral equations. Conventionally FDTD is implemented using loops. While this requires little memory the computational time is excessive and has exponential order. With the evolution of computer systems, memory size has increased many-fold. Therefore, with more memory available matrix based approach can be used which have a linear order of execution. This however, requires the whole problem space to be stored in the memory during execution. There is still a limitation as larger and more complex structures cannot be modeled using matrices.

Two problems are solved involving plane wave scattering by a dielectric cylinder and a conducting plate. The results obtained are matched with published results to verify the correctness of the approach.

II. THE FDTD ALGORITHM

The FDTD method, first introduced by Yee in 1966 [1], is a direct solution of

Maxwell's time-dependent curl equations. The scheme treats the irradiation of the scattered as an initial value problem.

The starting point for the FDTD formulation is the descritization of curl equations, which are Ampere's Law and Faraday's Laws in differential form.

The derivatives in Ampere's and Faraday's laws are replaced with finite differences. The space and time are descritized so that the electric and magnetic fields are staggered in both space and time.

The resulting difference equations are solved for "future" fields in terms of "past" fields. Effectively these future fields become past fields when the time step is incremented [2].

The simulation is carried out in half time steps. Electric fields are solved in the first half using magnetic fields from the previous time step. The magnetic fields are solved in the next half time step using these electric fields.

The process is continued for desired number of time steps.

III. ELECTROMAGNETIC WAVE SCATTERING BY A SQUARE CONDUCTING PLATE

This problem is taken from Kane S. Yee [1]. A transverse magnetic (TM) wave is incident on a conducting plate as shown in figure 1. This is a two-dimensional problem. The TM wave travelling in –x direction encounters a perfectly conducting plate at i=49. The wave will only have Ez and Hy components . The width of the plate is taken as 4α units where α is a constant. The spatial increment δ is chosen as α /16 and the temporal increment Δt as δ/2 or α /16.

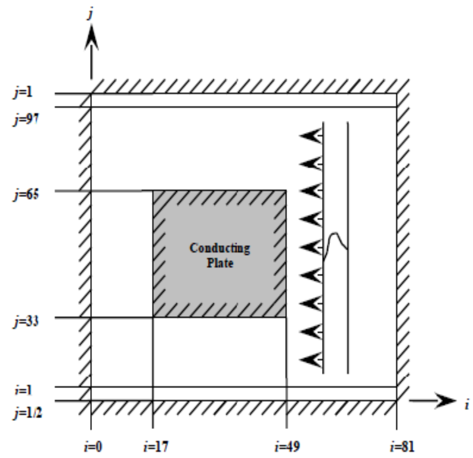

Figure 1. Geometry of Yee's Problem.

The results obtained using MatlabTM and Yee's results are compared in figures 2 and 3. The ordinate (y-axis) is in volts/meter and the abscissa (x-axis) is the number of horizontal increments, n is the number of time cycles.

The voltage pulse travels along –x direction until it encounters the conducting plate at i=49. After it hits the plate it reflects back after a phase change of 180 degrees. The results are taken at different time steps.

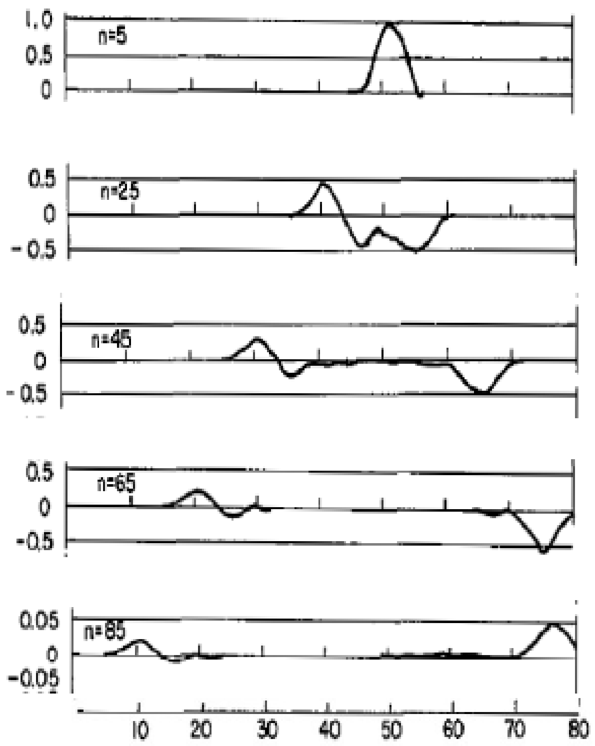

Figure 2. Yee's results in the presence of obstacle

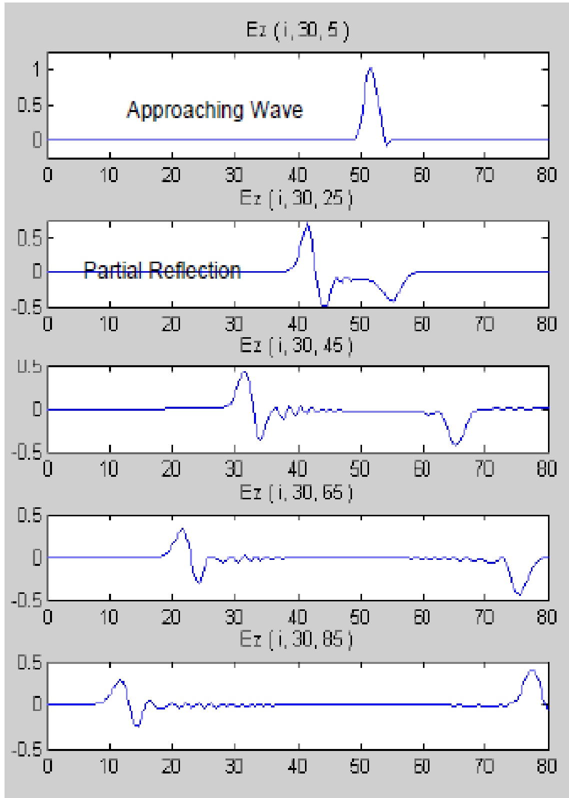

Figure 3. Matlab™ results in the presence of obstacle.

IV. Electromagnetic Wave Scattering by a Dielectric Cylinder

This problem is taken from Sadiku [3]. Again a 2D problem, it investigates the scattering of a TEM electromagnetic wave by a dielectric cylinder in free space. The cylinder has a radius of 6cm. The axis of the dielectric cylinder lies along the z-axis and the cylinder is infinitely long in z direction. The symmetry reduces it to a two dimensional problem in xy-plane. The TEM wave propagates in +y direction and scatters when encounters the dielectric cylinder. The objective is to compute one of the components, such as Ez, at points within the cylinder. The dielectric is assumed lossless with, $\varepsilon d = 4\varepsilon o$, $\mu d = \mu o$ and $\zeta = 0$. A sine wave of frequency 2.5 GHz is applied as the exciting source. The simulation is run long enough so that steady-state is achieved.

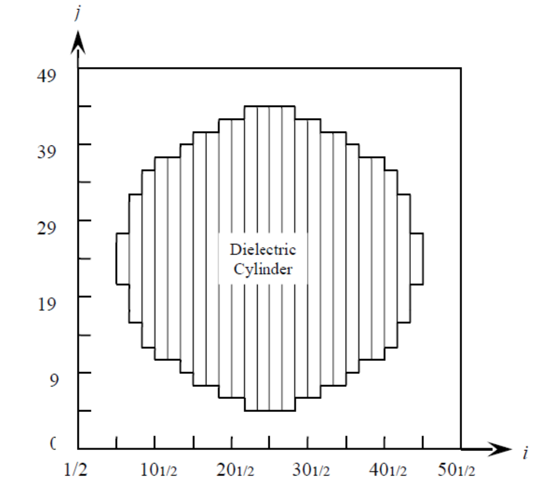

Figure 4. Problem geometry showing points internal and external to the cylinder.

Following Sadiku spatial step is choose as δ = Δx=Δy = 0.3cm and δt = δ2c =5ps. We have used the 51x50 grid shown in figure 4. Sadiku uses symmetry condition to further decrease the size of the grid but we have simulated the whole problem space as shown in figure 4. The cylinder axis is chosen to pass through the point (i,j) = (25.5, 24.5). Referring to figure 5 we see that the problem space is bounded by the z and y components of electric and magnetic fields respectively. So we need to apply boundary conditions on Ez at j = 0.5 and j = 49 and on Hy at i = 0 and i = 50.5. The FDTD code for the simulation is given in appendix B and the results obtained are shown in figures 6 and 7.

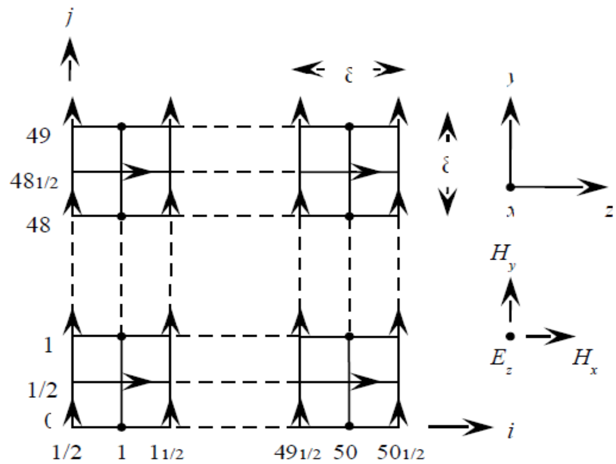

Figure 5. Two-dimensional lattice for the problem

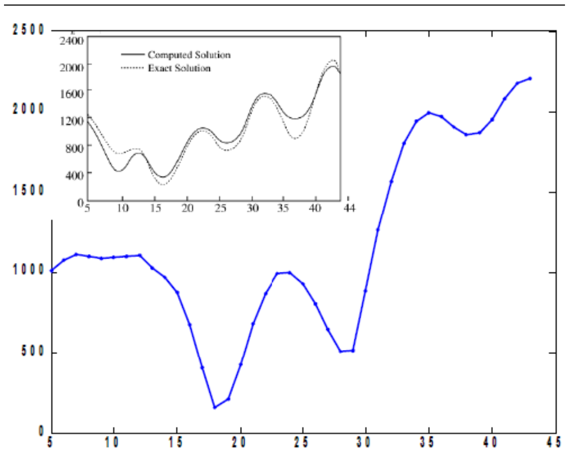

Figure 6. Magnitude of Ez at i=15

The results by Sadiku are depicted in the small boxes. The graphs show the absolute or steady state value of Ez at i=15 and i=25. The jaginess of the graphs is due to the fact that a coarse grid was used. The results can be improved by using a finer grid. Nevertheless the results show good agreement with the published results.

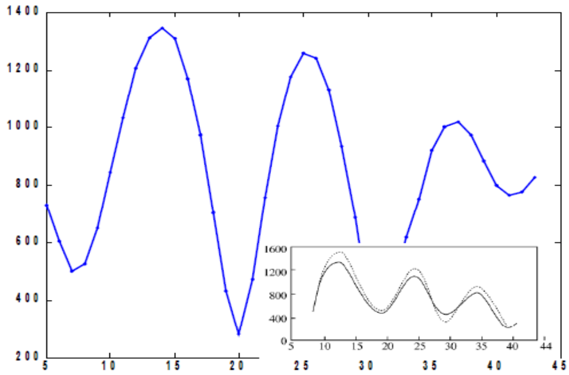

Figure 7. Magnitude of Ez at i=25

V. Time-Space Analysis

Simulation with a 50x50 grid for 500 time steps using matrices is almost instantaneous while time required for loop based approach increases exponential as the grid size increase. A comparison of grid size and simulation time is presented in figure 8.

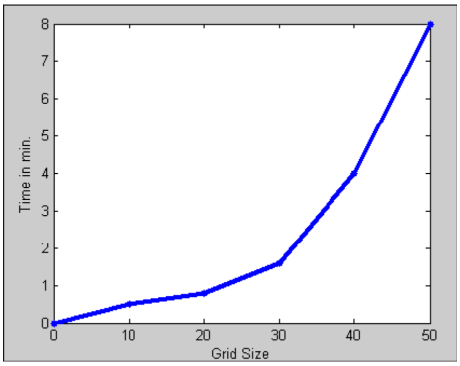

Figure 8. Exponential time requirement for loop-based approach on an Intel™ based Core 2 Duo machine.

Using the loop-based approach memory requirement depends on the number points at which we want to evaluate fields. For Matrix-based implementation all the lattice points in problem space must be stored in memory for all six components of fields.

For a 50x50 grid the memory required to store the field components is around 60KB. For loop based simulation we only require about 1.2KB of memory.

VI. Conclusion

While matrix based approach is much efficient as far as time requirements are concerned, the memory requirement increases exponentially for complex problems involving three-dimensional geometry and large grid sizes. In such situation it is better to either use integral techniques or switch to loop-based approach if the solution can be obtained in a reasonable amount of time. For sizeable problems, such as the ones analyzed in this paper, matrix-based implementation is better.

References

[1] *Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media*, Kane S. Yee, Vol. AP-14, No. 8 IEEE Transactions on Antennas and Propagation.

[2] *Accurate Computation of the Radiation From Simple Antennas Using the FDTD Technique*, Maloney et al. Vol 38, No. 7, IEEE Transactions on Antennas and Propagation.

[3] *Numerical Techniques in Electromagnetics*, M.N. O. Sadiku, CRC Press, second ed., 2001, pp. 173–197.