APPENDIX FOR CSCW2014-239

Exhaustive Search of Puzzles in Operational Transformation

A. UE-CP1-complying Transformation Matrix and Transformation Functions

An UE-CP1-complying transformation matrix *TM(O_1,O_2)* and a collection of UE-CP1-complying transformation functions are given below. It can be verified that these transformation functions achieve the same transformation capabilities as specified in *TM(O_1,O_2)*.

O_1O_2	PR	TM(O_1,O_2)
I(p_1,c_1) I(p_2,c_2)	$p_1<p_2$	I(p_1,c_1)
	$p_1=p_2$	I(p_1,c_1) if $O_1>O_2$
		I(p_1+1,c_1) if $O_1<O_2$
	$p_1>p_2$	I(p_1+1,c_1)
I(p_1,c_1) D(p_2,c_2)	$p_1<p_2$	I(p_1,c_1)
	$p_1=p_2$	I(p_1,c_1)
	$p_1>p_2$	I(p_1-1,c_1)
D(p_1,c_1) I(p_2,c_2)	$p_1<p_2$	D(p_1,c_1)
	$p_1=p_2$	D(p_1+1,c_1)
	$p_1>p_2$	D(p_1+1,c_1)
D(p_1,c_1) D(p_2,c_2)	$p_1<p_2$	D(p_1,c_1)
	$p_1=p_2$	NULL
	$p_1>p_2$	D(p_1-1,c_1)

O_1O_2	Transformation Functions
I(p_1,c_1) I(p_2,c_2)	T(I(p_1,c_1),I(p_2,c_2)) { if $p_1<p_2$ or ($p_1=p_2$ and priority(O_1)>priority(O_2)) return I(p_1,c_1) ; else return I(p_1+1,c_1);}

I(p_1,c_1) D(p_2,c_2)	T(I(p_1,c_1),D(p_2,c_2)) {
	if $p_1\leq p_2$ return I(p_1,c_1);
	else return I(p_1-1,c_1);}

D(p_1,c_1) I(p_2,c_2)	T(D(p_1,c_1),I(p_2,c_2)) {
	if $p_1<p_2$ return D(p_1,c_1);
	else return D(p_1+1,c_1);}

D(p_1,c_1) D(p_2,c_2)	T(D(p_1,c_1),D(p_2,c_2)) {
	if $p_1<p_2$ return D(p_1,c_1);
	else if $p_1>p_2$ return D(p_1-1,c_1);
	else return NULL;}

B. Transformation Matrix Properties and Position Relation Derivation Matrix

Transformation Matrix *TM(O_1,O_2)* properties, that are related to the derivation of the Position Relation Derivation Matrix (PRDM) (shown below), are elaborated below. Notations:

1. *p* represents the original position; and *p'* represents the corresponding transformed position;
2. *PR* denotes Position Relationship between original positions p_1 and p_2; *and PR*' denotes Position Relationship between transformation positions p_1' and p_2'.

p_2'vsp_2 / p_1'vs p_1	$p_2'=p_2$	$p_2'=p_2+1$	$p_2'=p_2-1$
$p_1'=p_1$	PR'=PR	$p_1'<p_2'$	1.$p_1'\leq p_2'$ when $p_1=p_3$ 2.$p_1'<p_2'$ when $p_1<p_3$
$p_1'=p_1+1$	$p_2'<p_1'$	PR'=PR	X
$p_1'=p_1-1$	1.$p_2'\leq p_1'$ when $p_2=p_3$ 2.$p_2'<p_1'$ when $p_2<p_3$	X	PR'=PR

To derive position relationships between transformed operations, we use the following *TM(O_1,O_2)* properties, which are directly derivable from definitions and specifications related to *TM(O_1,O_2)*:

- **Property 1:** An *insert* operation O_2 may have *right-shifting* effect on another operation O_1 *only if* O_1 is positioned *at* or *on the right* of O_2 ($p_2 \leq p_1$); O_2 may not have *right*-shifting effect on O_1 *only if* O_1 is positioned *at* or *on the left* of O_2 ($p_2 \geq p_1$).
- **Property 2:** A *delete* operation O_2 may have *left-shifting* effect on another operation O_1 *if and only if* O_1 is positioned *on the right* of O_2 ($p_2 < p_1$).
- **Property 3:** An operation may not have both *left*- and *right*-shifting effects on two operations defined on the same state.
- **Property 4: (**Derived from Property 1)**:** Given O_1, O_2 and O_3 defined on the same state, after transforming O_1 and O_2 against O_3, respectively, if O_3 has a *right*-shifting effect on O_1 but not on O_2, then $p_2 \leq p_3 \leq p_1$; if O_3 has a *right*-shifting effect on O_2 but not on O_1, then $p_1 \leq p_3 \leq p_2$.
- **Property 5: (**Derived from Property 2). Given O_1, O_2 and O_3 defined on the same state. After transforming O_1 and O_2 against O_3, respectively, if O_3 has a *left*-shifting effect on O_1 but not on O_2, then $p_2 \leq p_3 < p_1$; if O_3 has a *left*-shifting effect on O_2 but not on O_1, then $p_1 \leq p_3 < p_2$.

Based on the transformation results in $TM(O_1,O_2)$, there exist only three (3) possible relations between the original position p and the corresponding transformed position p', i.e. $p' = p$, $p' = p + 1$, or $p' = p - 1$. For two operations O_1 and O_2, after transforming them against the same third operation O_3, there may exist $3 \times 3 = 9$ possible different combinations of p_1' vs p_1 and p_2' vs p_2 relations, corresponding to the *nine (9)* entries in the PRDM. These entries are elaborated below.

1. Two entries, corresponding to ($p_2' = p_2 + 1$, $p_1' = p_1 - 1$) and ($p_2' = p_2 - 1$, $p_1' = p_1 + 1$), are filled with "X", which means they are impossible to occur according to TM Property 3.
2. When O_3 has the same effect on both O_1 and O_2, corresponding to the three entries along the diagonal, the original position relationships between O_1 and O_2 must be preserved, represented as $PR' = PR$.
3. When O_3 has a *right*-shifting effect on one operation only, then: (1) $p_2' < p_1'$ if only O_1 is shifted, corresponding to the entry ($p_1' = p_1 + 1$, $p_2' = p_2$); and (2) $p_1' < p_2'$ if only O_2 is shifted, corresponding to the entry ($p_2' = p_2 + 1$, $p_1' = p_1$). This is because, if O_3 has a *right*-shifting effect on O_1 but not O_2, then $p_2 \leq p_3 \leq p_1$ according to TM Property 4, which leads to $p_2' < p_1'$ due to $p_2' = p_2$ and $p_1' = p_1 + 1$. The same reasoning follows if only O_2 is shifted.
4. When O_3 has a *left*-shifting effect on one operation only, then: (1) in case that only O_1 is shifted, corresponding to the entry ($p_1' = p_1 - 1$, $p_2' = p_2$), we have: $p_2' \leq p_1'$ if $p_2 = p_3$; or $p_2' < p_1'$ if $p_2 < p_3$; and (2) in case that only O_2 is shifted, corresponding to the entry ($p_2' = p_2 - 1$, $p_1' = p_1$), we have: $p_1' \leq p_2'$ if $p_2 = p_3$; or $p_1' < p_2'$ if $p_2 < p_3$. This is because, if O_3 has a left-shifting effect on O_1 but not on O_2 (i.e. $p_1' = p_1 - 1$, $p_2' = p_2$), then $p_2 \leq p_3 < p_1$ according to TM Property 5, which leads to either $p_2' \leq p_1'$ under the condition $p_2 = p_3$; or $p_2' < p_1'$ under the condition $p_2 < p_3$. The same reasoning follows if only O_2 is shifted.

C. *UE-CP1 Verification Framework and Results*

In this section, the UE-CP1 verification framework and verification results (drawn manually for clarity and convenience in describing the derivation process) are given in full detail. Columns in the UE-CP1 framework are explained below:

1. ***O_1* and *O_2*:** given operation combinations.
2. **PR:** given position relationships between O_1 and O_2 for each operation type combination.
3. **UE-CP1-Equation: $S_{12}=S_{21}=UE(O_1,O_2)$:** the UE-CP1 equation to be solved in this verification process.
   1) **Derive $S_{12}=S_1\circ O_2^{O1}=S\circ O_1\circ O_2^{O1}$:** steps in deriving S_{12}
   2) **Derive $S_{21}=S_2\circ O_1^{O2}=S\circ O_2\circ O_1^{O2}$:** steps in deriving S_{21}
   3) ***$UE(O_1, O_2)$***: given union effects for all verification cases
4. **VR:** verification results. A confirmation case is indicated as T (True); a violation case is indicated as F (False). In the UE-CP1 verification, there is no case indicated as F.

O_1 O_2	PR	UE-CP1-Equation: $S_{12}=S_{21}=UE(O_1,O_2)$			VR
		Derive $S_{12}=S_1\circ O_2^{O1}=S\circ O_1\circ O_2^{O1}$	Derive $S_{21}=S_2\circ O_1^{O2}=S\circ O_2\circ O_1^{O2}$	$UE(O_1,O_2)$	
$I(p_1,c_1)$; $I(p_2,c_2)$	$p_1<p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+"c_1"+S[p_1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_2-1]+"c_2"+S[p_2,\|S\|-1]$ $=S_2[0,p_2-1]+S_2[p_2]+S_2[p_2+1,\|S_2\|-1]$	$S[0,p_1-1]$ $+"c_1"$ $+S[p_1,$ $p_2-1]$ $+"c_2"$ $+S[p_2,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=I(p_2+1,c_2)$	$O_1^{O2}=T(O_1,O_2)=I(p_1,c_1)$		
		$S_{12}=S_1\circ O_2^{O1}=S_1[0,p_2]+"c_2"+S_1[p_2+1,\|S_1\|-1]$	$S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1-1]+"c_1"+S_2[p_1,\|S_2\|-1]$		
		$S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_2]+"c_2"+S_1[p_2+1,\|S_1\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+1,p_2]$ $+"c_2"+S_1[p_2+1,\|S_1\|-1]$ $=S[0,p_1-1]+"c_1"+S[p_1,p_2-1]+"c_2"+S[p_2,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1-1]+"c_1"+S_2[p_1,\|S_2\|-1]$ $=S_2[0,p_1-1]+"c_1"+S_2[p_1,p_2-1]$ $+S_2[p_2]+S_2[p_2+1,\|S_2\|-1]$ $=S[0,p_1-1]+"c_1"+S[p_1,p_2-1]+"c_2"+S[p_2,\|S\|-1]$		
	$p_1=p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+"c_1"+S[p_1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_1-1]+"c_2"+S[p_1,\|S\|-1]$ $=S_2[0,p_1-1]+S_2[p_1]+S_2[p_1+1,\|S_2\|-1]$	$S[0,p_1-1]$ $+"c_1c_2"$ $+S[p_1,$ $\|S\|-1]$ or $S[0,p_1-1]$ $+"c_2c_1"$ $+S[p_1,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=I(p_1+1,c_2)$ or $I(p_1,c_2)$	$O_1^{O2}=T(O_1,O_2)=I(p_1,c_1)$ or $I(p_1+1,c_1)$		
		$S_{12}=S_1\circ O_2^{O1}=S_1[0,p_1]+"c_2"+S_1[p_1+1,\|S_1\|-1]$ or $S_{12}=S_1\circ O_2^{O1}=S_1[0,p_1-1]+"c_2"+S_1[p_1,\|S_1\|-1]$	$S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1-1]+"c_1"+S_2[p_1,\|S_2\|-1]$ or $S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1]+"c_1"+S_2[p_1+1,\|S_2\|-1]$		
		$S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_1]+"c_2"+S_1[p_1+1,\|S_1\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+"c_2"+S_1[p_1+1,\|S_1\|-1]$ $=S[0,p_1-1]+"c_1c_2"+S[p_1,\|S\|-1]$ or $S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_1-1]+"c_2"+S_1[p_1,\|S_1\|-1]$ $=S_1[0,p_1-1]+"c_2"+S_1[p_1]+S_1[p_1+1,\|S_1\|-1]$ $=S[0,p_1-1]+"c_2c_1"+S[p_1,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1-1]+"c_1"+S_2[p_1,\|S_2\|-1]$ $=S_2[0,p_1-1]+"c_1"+S_2[p_1]+S_2[p_1+1,\|S_2\|-1]$ $=S[0,p_1-1]+"c_1c_2"+S[p_1,\|S\|-1]$ or $S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1]+"c_1"+S_2[p_1+1,\|S_2\|-1]$ $=S_2[0,p_1-1]+S_2[p_1]+"c_1"+S_2[p_1+1,\|S_2\|-1]$ $=S[0,p_1-1]+"c_2c_1"+S[p_1,\|S\|-1]$		
	$p_1>p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+"c_1"+S[p_1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_2-1]+"c_2"+S[p_2,\|S\|-1]$ $=S_2[0,p_2-1]+S_2[p_2]+S_2[p_2+1,\|S_2\|-1]$	$S[0,p_2-1]$ $+"c_2"$ $+S[p_2,$ $p_1-1]$ $+"c_1"$ $+S[p_1,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=I(p_2,c_2)$	$O_1^{O2}=T(O_1,O_2)=I(p_1+1,c_1)$		
		$S_{12}=S_1\circ O_2^{O1}=S_1[0,p_2-1]+"c_2"+S_1[p_2,\|S_1\|-1]$	$S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1]+"c_1"+S_2[p_1+1,\|S_2\|-1]$		
		$S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_2-1]+"c_2"+S_1[p_2,\|S_1\|-1]$ $=S_1[0,p_2-1]+"c_2"$ $+S_1[p_2,p_1-1]+S_1[p_1]+S_1[p_1+1,\|S_1\|-1]$ $=S[0,p_2-1]+"c_2"+S[p_2,p_1-1]+"c_1"+S[p_1,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1]+"c_1"+S_2[p_1+1,\|S_2\|-1]$ $=S_2[0,p_2-1]+S_2[p_2]+S_2[p_2+1,p_1]$ $+"c_1"+S_2[p_1+1,\|S_2\|-1]$ $=S[0,p_2-1]+"c_2"+S[p_2,p_1-1]+"c_1"+S[p_1,\|S\|-1]$		

I(p_1,c_1);D(p_2,c_2)	$p_1<p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+"c_1"+S[p_1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_2-1]+S[p_2+1,\|S\|-1]$ $=S_2[0,p_2-1]+S_2[p_2,\|S_2\|-1]$	$S[0,p_1-1]$ $+"c_1"$ $+S[p_1,$ $p_2-1]$ $+S[p_2+1,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=D(p_2+1,c_2)$	$O_1^{O2}=T(O_1,O_2)=I(p_1,c_1)$		
		$S_{12}=S_1\circ O_2^{O1}=S_1[0,p_2]+S_1[p_2+2,\|S_1\|-1]$	$S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1-1]+"c_1"+S_2[p_1,\|S_2\|-1]$		
		$S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_2]+S_1[p_2+2,\|S_1\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+1,p_2]$ $+S_1[p_2+2,\|S_1\|-1]$ $=S[0,p_1-1]+"c_1"+S[p_1,p_2-1]+S[p_2+1,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1-1]+"c_1"+S_2[p_1,\|S_2\|-1]$ $=S_2[0,p_1-1]+"c_1"+S_2[p_1,p_2-1]+S_2[p_2,\|S_2\|-1]$ $=S[0,p_1-1]+"c_1"+S[p_1,p_2-1]+S[p_2+1,\|S\|-1]$		
	$p_1=p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+"c_1"+S[p_1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_2-1]+S[p_2+1,\|S\|-1]$ $=S_2[0,p_2-1]+S_2[p_2,\|S_2\|-1]$	$S[0,p_1-1]$ $+"c_1"$ $+S[p_1+1,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=D(p_2+1,c_2)$	$O_1^{O2}=T(O_1,O_2)=I(p_1,c_1)$		
		$S_{12}=S_1\circ O_2^{O1}=S_1[0,p_1]+S_1[p_1+2,\|S_1\|-1]$	$S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1-1]+"c_1"+S_2[p_1,\|S_2\|-1]$		
		$S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_2]+S_1[p_2+2,\|S_1\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+2,\|S_1\|-1]$ $=S[0,p_1-1]+"c_1"+S[p_1+1,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1-1]+"c_1"+S_2[p_1,\|S_2\|-1]$ $=S[0,p_1-1]+"c_1"+S[p_1+1,\|S\|-1]$		
	$p_1>p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+"c_1"+S[p_1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1]+S_1[p_1+1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_2-1]+S[p_2+1,\|S\|-1]$ $=S_2[0,p_2-1]+S_2[p_2,\|S_2\|-1]$	$S[0,p_2-1]$ $+S[p_2+1,$ $p_1-1]$ $+"c_1"$ $+S[p_1,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=D(p_2,c_2)$	$O_1^{O2}=T(O_1,O_2)=I(p_1-1,c_1)$		
		$S_{12}=S_1\circ O_2^{O1}=S_1[0,p_2-1]+S_1[p_2+1,\|S_1\|-1]$	$S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1-2]+"c_1"+S_2[p_1-1,\|S_2\|-1]$		
		$S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_2-1]+S_1[p_2+1,\|S_1\|-1]$ $=S_1[0,p_2-1]+S_1[p_2+1,p_1-1]+S_1[p_1]$ $+S_1[p_1+1,\|S_1\|-1]$ $=S[0,p_2-1]+S[p_2+1,p_1-1]+"c_1"+S[p_1,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1-2]+"c_1"+S_2[p_1-1,\|S_2\|-1]$ $=S_2[0,p_2-1]+S_2[p_2,p_1-2]+"c_1"+S_2[p_1-1,\|S_2\|-1]$ $=S[0,p_2-1]+S[p_2+1,p_1-1]+"c_1"+S[p_1,\|S\|-1]$		
D(p_1,c_1);D(p_2,c_2)	$p_1<p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+S[p_1+1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_2-1]+S[p_2+1,\|S\|-1]$ $=S_2[0,p_2-1]+S_2[p_2,\|S_2\|-1]$	$S[0,p_1-1]$ $+S[p_1+1,$ $p_2-1]$ $+S[p_2+1,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=D(p_2-1,c_2)$	$O_1^{O2}=T(O_1,O_2)=D(p_1,c_1)$		
		$S_{12}=S_1\circ O_2^{O1}=S_1[0,p_2-2]+S_1[p_2,\|S_1\|-1]$	$S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1-1]+S_2[p_1+1,\|S_2\|-1]$		
		$S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_2-2]+S_1[p_2,\|S_1\|-1]$ $=S_1[0,p_1-1]+S_1[p_1,p_2-2]+S_1[p_2,\|S_1\|-1]$ $=S[0,p_1-1]+S[p_1+1,p_2-1]+S[p_2+1,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1-1]+S_2[p_1+1,\|S_2\|-1]$ $=S_2[0,p_1-1]+S_2[p_1+1,p_2-1]+S_2[p_2,\|S_2\|-1]$ $=S[0,p_1-1]+S[p_1+1,p_2-1]+S[p_2+1,\|S\|-1]$		
	$p_1=p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+S[p_1+1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_2-1]+S[p_2+1,\|S\|-1]$ $=S_2[0,p_2-1]+S_2[p_2,\|S_2\|-1]$	$S[0,p_1-1]$ $+S[p_1+1,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=NULL$	$O_1^{O2}=T(O_1,O_2)=NULL$		
		$S_{12}=S_1\circ O_2^{O1}=S_1$	$S_{21}=S_2\circ O_1^{O2}=S_2$		
		$S_{12}=S\circ O_1\circ O_2^{O1}=S_1$ $=S_1[0,p_1-1]+S_1[p_1,\|S_1\|-1]$ $=S[0,p_1-1]+S[p_1+1,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}=S_2$ $=S_2[0,p_1-1]+S_2[p_1,\|S_2\|-1]$ $=S[0,p_1-1]+S[p_1+1,\|S\|-1]$		
	$p_1>p_2$	$S_1=S\circ O_1$ $=S[0,p_1-1]+S[p_1+1,\|S\|-1]$ $=S_1[0,p_1-1]+S_1[p_1,\|S_1\|-1]$	$S_2=S\circ O_2$ $=S[0,p_2-1]+S[p_2+1,\|S\|-1]$ $=S_2[0,p_2-1]+S_2[p_2,\|S_2\|-1]$	$S[0,p_2-1]$ $+S[p_2+1,$ $p_1-1]$ $+S[p_1+1,$ $\|S\|-1]$	T
		$O_2^{O1}=T(O_2,O_1)=D(p_2,c_2)$	$O_1^{O2}=T(O_1,O_2)=D(p_1-1,c_1)$		
		$S_{12}=S_1\circ O_2^{O1}=S_1[0,p_2-1]+S_1[p_2+1,\|S_1\|-1]$	$S_{21}=S_2\circ O_1^{O2}=S_2[0,p_1-2]+S_2[p_1,\|S_2\|-1]$		
		$S_{12}=S\circ O_1\circ O_2^{O1}$ $=S_1[0,p_2-1]+S_1[p_2+1,\|S_1\|-1]$ $=S_1[0,p_2-1]+S_1[p_2+1,p_1-1]+S_1[p_1,\|S_1\|-1]$ $=S[0,p_2-1]+S[p_2+1,p_1-1]+S[p_1+1,\|S\|-1]$	$S_{21}=S\circ O_2\circ O_1^{O2}$ $=S_2[0,p_1-2]+S_2[p_1,\|S_2\|-1]$ $=S_2[0,p_2-1]+S_2[p_2,p_1-2]+S_2[p_1,\|S_2\|-1]$ $=S[0,p_2-1]+S[p_2+1,p_1-1]+S[p_1+1,\|S\|-1]$		

D. CP2 Verification Framework and Results

In this section, the CP2 verification framework and verification results (generated by OTX) are given in full detail. Columns in the CP2 framework are explained below:

1. ***O_1, O_2, O_3*:** given operation type permutations.
2. **PR:** given position relationships among O_1, O_2 and O_3 for each operation type permutation.
3. **CP2-Equation $O_1^{O2,O3}=O_1^{O3,O2}$:** the CP2 equation to be solved in this verification process.
   1) **Derive $O_1^{O2,O3}=T(O_1^{O2}, O_3^{O2})$:** steps in deriving $O_1^{O2,O3}$
      (1) **$O_1^{O2}=T(O_1, O_2)$:** deriving O_1^{O2} by consulting the TM according to operation types and position relationships among O_1 and O_2.
      (2) **$O_3^{O2}=T(O_3, O_2)$:** deriving O_3^{O2} by consulting the TM according to operation types and position relationships among O_3 and O_2.
      (3) **Derived PR(O_1^{O2}, O_3^{O2}):** deriving position relationships between the transformed operations O_1^{O2} and O_3^{O2}, according to the Position Relation Derivation Matrix (PRDM) shown in APPENDIX B.
      (4) **$O_1^{O2,O3}=T(O_1^{O2}, O_3^{O2})$:** deriving $O_1^{O2,O3}$ according to the operation types of O_1 and O_3 and the derived position relation by consulting the TM.
   2) **Derive $O_1^{O3,O2}=T(O_1^{O3}, O_2^{O3})$:** steps in deriving $O_1^{O3,O2}$ (similar steps in deriving $O_1^{O2,O3}$)
      (1) **$O_1^{O3}=T(O_1, O_3)$**
      (2) **$O_2^{O3}=T(O_2, O_3)$**
      (3) **Derived PR(O_1^{O3}, O_2^{O3})**
      (4) **$O_1^{O3,O2}=T(O_1^{O3}, O_2^{O3})$**
4. **VR:** verification results. A confirmation case is indicated as T (True); a violation case is indicated as F (False). In the CP2 verification, only one case (Case *20*) is indicated as F.
5. **#:** verification case number.

From the verification results, we can see that there is only one CP2-violation case (Case 20) among a total of 58 verification cases. Case 20 has the following operation types and position relationships: $O_1 = I(p_1,c_1)$, $O_2 = D(p_2,c_2)$ and $O_3 = I(p_3,c_3)$, with $(p_1 - 1) = p_2 = p_3$. This case gives a general description of the scenario-based FT puzzle in Figure 2, where the document state $S =$"abc", $O_1 = I(2,x)$, $O_2 = D(1,b)$ and $O_3 = I(1,y)$, respectively, and $((p_1=2) - 1) = (p_2=1) = (p_3=1)$. Case 20 can be converted into a CP2-confirmation case by changing the priority to $priority(O_1) < priority(O_2) < priority(O_3)$, but Case 10, which is a CP2-confirmation case under $priority(O_1) > priority(O_2) > priority(O_3)$, will be turned to a CP2-violation case. In other words, no priority among the three operations is able to make both Case 20 and Case 10 comply with CP2.

OTX CP2 Verification Framework (Priority Setting:O1>O2>O3)

O_1 O_2 O_3	Original PR(O_1,O_2,O_3)	CP2-Equation: $O_1^{O2,O3}=O_1^{O3,O2}$								VR	#
		Derive $O_1^{O2,O3}=T(O_1^{O3},O_2^{O3})$				Derive $O_1^{O3,O2}=T(O_1^{O2},O_3^{O2})$					
		$O_1^{O2}=T(O_1,O_2)$	$O_3^{O2}=T(O_3,O_2)$	Derived PR(O_1^{O2},O_3^{O2})	$O_1^{O2,O3}=T(O_1^{O2},O_3^{O2})$	$O_1^{O3}=T(O_1,O_3)$	$O_2^{O3}=T(O_2,O_3)$	Derived PR(O_1^{O3},O_2^{O3})	$O_1^{O3,O2}=T(O_1^{O3},O_2^{O3})$		
I(p1,c1) I(p2,c2) I(p3,c3)	p1<p2<p3	I(p1,c1)	I(p3+1,c3)	p1<p3+1	I(p1,c1)	I(p1,c1)	I(p2,c2)	p1<p2	I(p1,c1)	T	1
	p1=p2<p3	I(p1,c1)	I(p3+1,c3)	p1<p3+1	I(p1,c1)	I(p1,c1)	I(p2,c2)	p1=p2	I(p1,c1)	T	2
	p1=p2=p3	I(p1,c1)	I(p3+1,c3)	p1<p3+1	I(p1,c1)	I(p1,c1)	I(p2,c2)	p1=p2	I(p1,c1)	T	3
	p1<p2=p3	I(p1,c1)	I(p3+1,c3)	p1<p3+1	I(p1,c1)	I(p1,c1)	I(p2,c2)	p1<p2	I(p1,c1)	T	4
	p2<p1<p3	I(p1+1,c1)	I(p3+1,c3)	p1+1<p3+1	I(p1+1,c1)	I(p1,c1)	I(p2,c2)	p1>p2	I(p1+1,c1)	T	5
	p2<p1=p3	I(p1+1,c1)	I(p3+1,c3)	p1+1=p3+1	I(p1+1,c1)	I(p1,c1)	I(p2,c2)	p1>p2	I(p1+1,c1)	T	6
	p2<p3<p1	I(p1+1,c1)	I(p3+1,c3)	p1+1>p3+1	I(p1+2,c1)	I(p1+1,c1)	I(p2,c2)	p1+1>p2	I(p1+2,c1)	T	7
	p2=p3<p1	I(p1+1,c1)	I(p3+1,c3)	p1+1>p3+1	I(p1+2,c1)	I(p1+1,c1)	I(p2,c2)	p1+1>p2	I(p1+2,c1)	T	8
I(p1,c1) D(p2,c2) I(p3,c3)	p1<p2<p3	I(p1,c1)	I(p3-1,c3)	p1<p3-1	I(p1,c1)	I(p1,c1)	D(p2,c2)	p1<p2	I(p1,c1)	T	9
	p1=p2<p3	I(p1,c1)	I(p3-1,c3)	p1<p3-1	I(p1,c1)	I(p1,c1)	D(p2,c2)	p1=p2	I(p1,c1)	T	10
				p1=p3-1	I(p1,c1)					T	10
	p1=p2=p3	I(p1,c1)	I(p3,c3)	p1=p3	I(p1,c1)	I(p1,c1)	D(p2+1,c2)	p1<p2+1	I(p1,c1)	T	11
	p1<p2=p3	I(p1,c1)	I(p3,c3)	p1<p3	I(p1,c1)	I(p1,c1)	D(p2+1,c2)	p1<p2+1	I(p1,c1)	T	12
	p1<p3<p2	I(p1,c1)	I(p3,c3)	p1<p3	I(p1,c1)	I(p1,c1)	D(p2+1,c2)	p1<p2+1	I(p1,c1)	T	13
	p1=p3<p2	I(p1,c1)	I(p3,c3)	p1=p3	I(p1,c1)	I(p1,c1)	D(p2+1,c2)	p1<p2+1	I(p1,c1)	T	14
	p2<p1<p3	I(p1-1,c1)	I(p3-1,c3)	p1-1<p3-1	I(p1-1,c1)	I(p1,c1)	D(p2,c2)	p1>p2	I(p1-1,c1)	T	15
	p2<p1=p3	I(p1-1,c1)	I(p3-1,c3)	p1-1=p3-1	I(p1-1,c1)	I(p1,c1)	D(p2,c2)	p1>p2	I(p1-1,c1)	T	16
	p3<p1<p2	I(p1,c1)	I(p3,c3)	p1>p3	I(p1+1,c1)	I(p1+1,c1)	D(p2+1,c2)	p1+1<p2+1	I(p1+1,c1)	T	17
	p3<p1=p2	I(p1,c1)	I(p3,c3)	p1>p3	I(p1+1,c1)	I(p1+1,c1)	D(p2+1,c2)	p1+1=p2+1	I(p1+1,c1)	T	18
	p2<p3<p1	I(p1-1,c1)	I(p3-1,c3)	p1-1>p3-1	I(p1,c1)	I(p1+1,c1)	D(p2,c2)	p1+1>p2	I(p1,c1)	T	19
	p2=p3<p1	I(p1-1,c1)	I(p3,c3)	p1-1>p3	I(p1,c1)	I(p1+1,c1)	D(p2+1,c2)	p1+1>p2+1	I(p1,c1)	T	20
				p1-1=p3	I(p1-1,c1)					F	20
	p3<p2<p1	I(p1-1,c1)	I(p3,c3)	p1-1>p3	I(p1,c1)	I(p1+1,c1)	D(p2+1,c2)	p1+1>p2+1	I(p1,c1)	T	21
I(p1,c1) D(p2,c2) D(p3,c3)	p1<p2<p3	I(p1,c1)	D(p3-1,c3)	p1<p3-1	I(p1,c1)	I(p1,c1)	D(p2,c2)	p1<p2	I(p1,c1)	T	22
	p1=p2<p3	I(p1,c1)	D(p3-1,c3)	p1<p3-1	I(p1,c1)	I(p1,c1)	D(p2,c2)	p1=p2	I(p1,c1)	T	23
				p1=p3-1	I(p1,c1)					T	23
	p1=p2=p3	I(p1,c1)	NULL	ANY	I(p1,c1)	I(p1,c1)	NULL	ANY	I(p1,c1)	T	24
	p1<p2=p3	I(p1,c1)	NULL	ANY	I(p1,c1)	I(p1,c1)	NULL	ANY	I(p1,c1)	T	25
	p2<p1<p3	I(p1-1,c1)	D(p3-1,c3)	p1-1<p3-1	I(p1-1,c1)	I(p1,c1)	D(p2,c2)	p1>p2	I(p1-1,c1)	T	26
	p2<p1=p3	I(p1-1,c1)	D(p3-1,c3)	p1-1=p3-1	I(p1-1,c1)	I(p1,c1)	D(p2,c2)	p1>p2	I(p1-1,c1)	T	27
	p2<p3<p1	I(p1-1,c1)	D(p3-1,c3)	p1-1>p3-1	I(p1-2,c1)	I(p1-1,c1)	D(p2,c2)	p1-1>p2	I(p1-2,c1)	T	28
	p2=p3<p1	I(p1-1,c1)	NULL	ANY	I(p1-1,c1)	I(p1-1,c1)	NULL	ANY	I(p1-1,c1)	T	29
D(p1,c1) I(p2,c2) I(p3,c3)	p1<p2<p3	D(p1,c1)	I(p3+1,c3)	p1<p3+1	D(p1,c1)	D(p1,c1)	I(p2,c2)	p1<p2	D(p1,c1)	T	30
	p1=p2<p3	D(p1+1,c1)	I(p3+1,c3)	p1+1<p3+1	D(p1+1,c1)	D(p1,c1)	I(p2,c2)	p1=p2	D(p1+1,c1)	T	31
	p1=p2=p3	D(p1+1,c1)	I(p3+1,c3)	p1+1=p3+1	D(p1+2,c1)	D(p1+1,c1)	I(p2,c2)	p1+1>p2	D(p1+2,c1)	T	32
	p1<p2=p3	D(p1,c1)	I(p3+1,c3)	p1<p3+1	D(p1,c1)	D(p1,c1)	I(p2,c2)	p1<p2	D(p1,c1)	T	33
	p2<p1<p3	D(p1+1,c1)	I(p3+1,c3)	p1+1<p3+1	D(p1+1,c1)	D(p1,c1)	I(p2,c2)	p1>p2	D(p1+1,c1)	T	34
	p2<p1=p3	D(p1+1,c1)	I(p3+1,c3)	p1+1=p3+1	D(p1+2,c1)	D(p1+1,c1)	I(p2,c2)	p1+1>p2	D(p1+2,c1)	T	35
	p2<p3<p1	D(p1+1,c1)	I(p3+1,c3)	p1+1>p3+1	D(p1+2,c1)	D(p1+1,c1)	I(p2,c2)	p1+1>p2	D(p1+2,c1)	T	36
	p2=p3<p1	D(p1+1,c1)	I(p3+1,c3)	p1+1>p3+1	D(p1+2,c1)	D(p1+1,c1)	I(p2,c2)	p1+1>p2	D(p1+2,c1)	T	37
D(p1,c1) D(p2,c2) I(p3,c3)	p1<p2<p3	D(p1,c1)	I(p3-1,c3)	p1<p3-1	D(p1,c1)	D(p1,c1)	D(p2,c2)	p1<p2	D(p1,c1)	T	38
	p1=p2<p3	NULL	I(p3-1,c3)	ANY	NULL	D(p1,c1)	D(p2,c2)	p1=p2	NULL	T	39
	p1=p2=p3	NULL	I(p3,c3)	ANY	NULL	D(p1+1,c1)	D(p2+1,c2)	p1+1=p2+1	NULL	T	40
	p1<p2=p3	D(p1,c1)	I(p3,c3)	p1<p3	D(p1,c1)	D(p1,c1)	D(p2+1,c2)	p1<p2+1	D(p1,c1)	T	41
	p1<p3<p2	D(p1,c1)	I(p3,c3)	p1<p3	D(p1,c1)	D(p1,c1)	D(p2+1,c2)	p1<p2+1	D(p1,c1)	T	42
	p1=p3<p2	D(p1,c1)	I(p3,c3)	p1=p3	D(p1+1,c1)	D(p1+1,c1)	D(p2+1,c2)	p1+1<p2+1	D(p1+1,c1)	T	43
	p2<p1<p3	D(p1-1,c1)	I(p3-1,c3)	p1-1<p3-1	D(p1-1,c1)	D(p1,c1)	D(p2,c2)	p1>p2	D(p1-1,c1)	T	44
	p2<p1=p3	D(p1-1,c1)	I(p3-1,c3)	p1-1=p3-1	D(p1,c1)	D(p1+1,c1)	D(p2,c2)	p1+1>p2	D(p1,c1)	T	45
	p3<p1<p2	D(p1,c1)	I(p3,c3)	p1>p3	D(p1+1,c1)	D(p1+1,c1)	D(p2+1,c2)	p1+1<p2+1	D(p1+1,c1)	T	46
	p3<p1=p2	NULL	I(p3,c3)	ANY	NULL	D(p1+1,c1)	D(p2+1,c2)	p1+1=p2+1	NULL	T	47
	p2<p3<p1	D(p1-1,c1)	I(p3-1,c3)	p1-1>p3-1	D(p1,c1)	D(p1+1,c1)	D(p2,c2)	p1+1>p2	D(p1,c1)	T	48
	p2=p3<p1	D(p1-1,c1)	I(p3,c3)	p1-1>p3	D(p1,c1)	D(p1+1,c1)	D(p2+1,c2)	p1+1>p2+1	D(p1,c1)	T	49
				p1-1=p3	D(p1,c1)					T	49
	p3<p2<p1	D(p1-1,c1)	I(p3,c3)	p1-1>p3	D(p1,c1)	D(p1+1,c1)	D(p2+1,c2)	p1+1>p2+1	D(p1,c1)	T	50
D(p1,c1) D(p2,c2) D(p3,c3)	p1<p2<p3	D(p1,c1)	D(p3-1,c3)	p1<p3-1	D(p1,c1)	D(p1,c1)	D(p2,c2)	p1<p2	D(p1,c1)	T	51
	p1=p2<p3	NULL	D(p3-1,c3)	ANY	NULL	D(p1,c1)	D(p2,c2)	p1=p2	NULL	T	52
	p1=p2=p3	NULL	NULL	ANY	NULL	NULL	NULL	ANY	NULL	T	53
	p1<p2=p3	D(p1,c1)	NULL	ANY	D(p1,c1)	D(p1,c1)	NULL	ANY	D(p1,c1)	T	54
	p2<p1<p3	D(p1-1,c1)	D(p3-1,c3)	p1-1<p3-1	D(p1-1,c1)	D(p1,c1)	D(p2,c2)	p1>p2	D(p1-1,c1)	T	55
	p2<p1=p3	D(p1-1,c1)	D(p3-1,c3)	p1-1=p3-1	NULL	NULL	D(p2,c2)	ANY	NULL	T	56
	p2<p3<p1	D(p1-1,c1)	D(p3-1,c3)	p1-1>p3-1	D(p1-2,c1)	D(p1-1,c1)	D(p2,c2)	p1-1>p2	D(p1-2,c1)	T	57
	p2=p3<p1	D(p1-1,c1)	NULL	ANY	D(p1-1,c1)	D(p1-1,c1)	NULL	ANY	D(p1-1,c1)	T	58

PR: Position Relationship VR: Verification Result #: Case Number

E. IP2 Verification Framework and Results

In this section, the IP2 verification framework and verification results (generated by OTX) are given in full detail. Columns in the IP2 framework are explained below:

1. ***O_1, O_2, $!O_2$*:** given O_1 and O_2 operation type permutations. $!O_2$ is the inverse of O_2.
2. **PR:** given position relationships among O_1 and O_2 for each operation type permutation.
3. **IP2-Equation $O_1^{O2,!O2} = O_1$:** solving the IP2 equation.
   1) **$O_1^{O2} = T(O_1,O_2)$:** deriving O_1^{O2} by consulting the TM according to operation types and position relationships among O_1 and O_2.
   2) **Derived PR(O_1^{O2},$!O_2$):** deriving position relationships between the transformed operations O_1^{O2} and inverse operation $!O_2$, according to the Position Relation Derivation Matrix (PRDM) shown in APPENDIX B.
   3) **$O_1^{O2,!O2} = T(O_1^{O2},!O_2)$:** deriving $O_1^{O2,!O2}$ according to the operation types of O_1 and $!O_2$ and the derived position relationships by consulting the TM.
4. **VR:** verification results. A confirmation case is indicated as T (True); a violation case is indicated as F (False). In the IP2 verification, Case 6 and Case 11 are indicated as F.
5. **#:** verification case number.

From the verification results, we can see that there are two IP2-violation cases (Case 6 and Case 11) among a total of 12 verification cases. Case 6 has the following operation types and position relationships: $O_1 = I(p_1,c_1)$ and $O_2 = D(p_2,c_2)$, where $(p_1 - 1) = p_2$. This case gives a general description of the scenario-based undo puzzles in Figure 3(a), where the document state $S =$"axb", $O_1 = I(2,y)$ and $O_2 = D(1,x)$, and $((p_1 = 2) - 1) = (p_2 = 1)$. Case 6 can be converted into an IP2-confirmation case by changing the priority to $priority(O_1) < priority(O_2)$, but Case 5, which is an IP2-confirmation case under $priority(O_1) > priority(O_2)$, will be turned to an IP2-violation case. In other words, no priority between these two operations is able to make both Case 5 and Case 6 comply with IP2.

Case 11 has the following operation types and position relationships: $O_1 = D(p_1,c_1)$ and $O_2 = D(p_2,c_2)$, where $p_1 = p_2$. This case gives a general description of the scenario-based undo puzzle in Figure 3(b), where the document state $S =$"axb", $O_1 = D(1,x)$ and $O_2 = D(1,x)$, and $(p_1 = 1) = (p_2 = 1)$. Case 11 is always an IP2-violation case no matter which priority we are using.

OTX IP2 Verification Framework (Priority Setting: O1>O2)

O_1 O_2 $!O_2$	Original PR(O_1,O_2)	IP2-Equation: $O_1^{O2,!O2} = O_1$			VR	#
		O_1^{O2}=T(O_1,O_2)	Derived PR(O_1^{O2},$!O_2$)	$O_1^{O2,!O2}$=T(O_1^{O2},$!O_2$)		
I(p1,c1) I(p2,c2) D(p2,c2)	p1<p2	I(p1,c1)	p1<p2	I(p1,c1)	T	1
	p1=p2	I(p1,c1)	p1=p2	I(p1,c1)	T	2
	p1>p2	I(p1+1,c1)	p1+1>p2	I(p1,c1)	T	3
I(p1,c1) D(p2,c2) I(p2,c2)	p1<p2	I(p1,c1)	p1<p2	I(p1,c1)	T	4
	p1=p2	I(p1,c1)	p1=p2	I(p1,c1)	T	5
	p1>p2	I(p1-1,c1)	p1-1>p2	I(p1,c1)	T	6
			p1-1=p2	I(p1-1,c1)	F	6
D(p1,c1) I(p2,c2) D(p2,c2)	p1<p2	D(p1,c1)	p1<p2	D(p1,c1)	T	7
	p1=p2	D(p1+1,c1)	p1+1>p2	D(p1,c1)	T	8
	p1>p2	D(p1+1,c1)	p1+1>p2	D(p1,c1)	T	9
D(p1,c1) D(p2,c2) I(p2,c2)	p1<p2	D(p1,c1)	p1<p2	D(p1,c1)	T	10
	p1=p2	NULL	ANY	NULL	F	11
	p1>p2	D(p1-1,c1)	p1-1>p2	D(p1,c1)	T	12
			p1-1=p2	D(p1,c1)	T	12

PR: Position Relationship VR: Verification Result #: Case Number

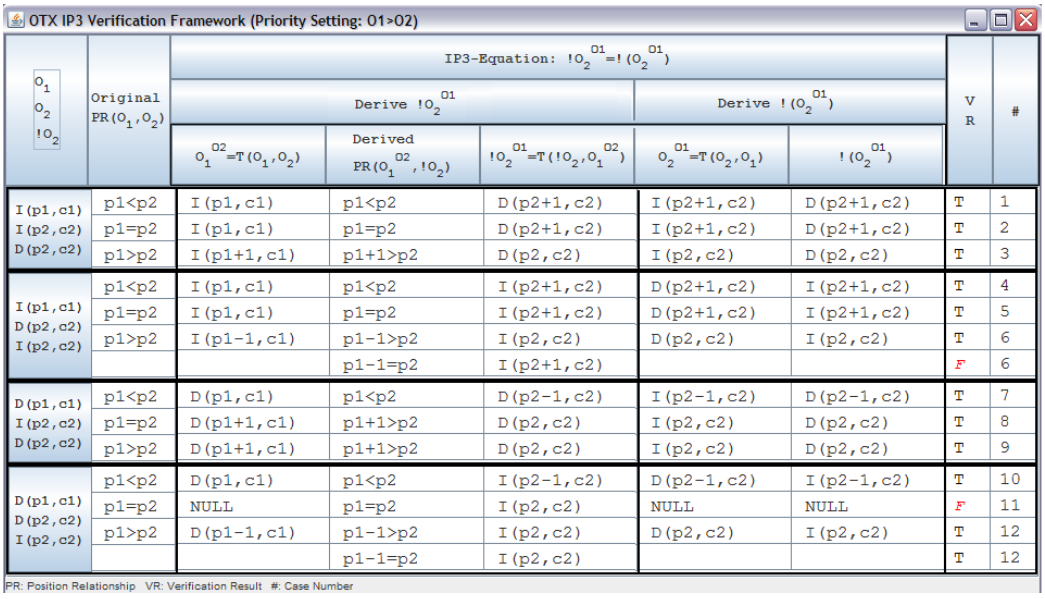
F. IP3 Verification Framework and Results

In this section, the IP3 verification framework and verification results (generated by OTX) are given in full detail. Columns in the IP3 framework are explained below:

1. ***O_1, O_2, $!O_2$*:** given O_1 and O_2 operation type permutation. $!O_2$ is the inverse of O_2.
2. **PR:** given position relationships among O_1 and O_2 for each operation type permutation.
3. **IP3-Equation $!O_2^{O1} = !(O_2^{O1})$:** solving the IP3 equation.
   1) **Derive $!O_2^{O1}$:** steps in deriving $!O_2^{O1}$
      (1) **$O_1^{O2} = T(O_1,O_2)$:** deriving O_1^{O2} by consulting the TM according to operation types and position relationships between O_1 and O_2.
      (2) **Derived PR(O_1^{O2},$!O_2$):** deriving position relationships between the transformed operations O_1^{O2} and $!O_2$, according to the Position Relation Derivation Matrix (PRDM) shown in APPENDIX B.
      (3) **$!O_2^{O1} = T(!O_2, O_2^{O1})$:** deriving $!O_2^{O1}$ according to the operation types of O_1 and $!O_2$, and the derived position relationships by consulting the TM.
   2) **Derive $!(O_2^{O1})$:** steps in deriving $!(O_2^{O1})$
4. **VR:** verification results. A confirmation case is indicated as T (True); a violation case is indicated as F (False). In the IP3 verification, Case 6 and Case 11 are indicated as F.
5. **#:** verification case number.

Case 6 has the following operation types and position relationships: $O_1 = I(p_1,c_1)$ and $O_2 = D(p_2,c_2)$, where $p_1 > p_2$. This case gives a general description of the scenario-based undo puzzles in Figure 3(a), where the document state $S =$"axb", $O_1 = I(2,y)$ and $O_2 = D(1,x)$, and $((p_1 = 2) - 1) = (p_2 = 1)$. Case 6 can be converted into an IP3-confirmation case by changing the priority to $priority(O_1) < priority(O_2)$, but Case 5, which is an IP2-confirmation case under $priority(O_1) > priority(O_2)$, will be turned to an IP3-violation case. In other words, no priority between these two operations is able to make both Case 5 and Case 6 comply with IP3.

Case 11 has the following operation types and position relationships: $O_1 = D(p_1,c_1)$ and $O_2 = D(p_2,c_2)$, where $p_1 = p_2$. This case gives a general description of the scenario-based undo puzzle in Figure 3(b), where the document state $S =$"axb", $O_1 = D(1,x)$ and $O_2 = D(1,x)$, and $(p_1 = 1) = (p_2 = 1)$. Case 11 is always an IP3-violation case no matter which priority is used.



OTX IP3 Verification Framework (Priority Setting: O1>O2)

O_1 O_2 $!O_2$	Original PR(O_1,O_2)	IP3-Equation: $!O_2^{O1}=!(O_2^{O1})$					VR	#
		Derive $!O_2^{O1}$			Derive $!(O_2^{O1})$			
		$O_1^{O2}=T(O_1,O_2)$	Derived PR(O_1^{O2},$!O_2$)	$!O_2^{O1}=T(!O_2,O_1^{O2})$	$O_2^{O1}=T(O_2,O_1)$	$!(O_2^{O1})$		
I(p1,c1) I(p2,c2) D(p2,c2)	p1<p2	I(p1,c1)	p1<p2	D(p2+1,c2)	I(p2+1,c2)	D(p2+1,c2)	T	1
	p1=p2	I(p1,c1)	p1=p2	D(p2+1,c2)	I(p2+1,c2)	D(p2+1,c2)	T	2
	p1>p2	I(p1+1,c1)	p1+1>p2	D(p2,c2)	I(p2,c2)	D(p2,c2)	T	3
I(p1,c1) D(p2,c2) I(p2,c2)	p1<p2	I(p1,c1)	p1<p2	I(p2+1,c2)	D(p2+1,c2)	I(p2+1,c2)	T	4
	p1=p2	I(p1,c1)	p1=p2	I(p2+1,c2)	D(p2+1,c2)	I(p2+1,c2)	T	5
	p1>p2	I(p1-1,c1)	p1-1>p2	I(p2,c2)	D(p2,c2)	I(p2,c2)	T	6
			p1-1=p2	I(p2+1,c2)			F	6
D(p1,c1) I(p2,c2) D(p2,c2)	p1<p2	D(p1,c1)	p1<p2	D(p2-1,c2)	I(p2-1,c2)	D(p2-1,c2)	T	7
	p1=p2	D(p1+1,c1)	p1+1>p2	D(p2,c2)	I(p2,c2)	D(p2,c2)	T	8
	p1>p2	D(p1+1,c1)	p1+1>p2	D(p2,c2)	I(p2,c2)	D(p2,c2)	T	9
D(p1,c1) D(p2,c2) I(p2,c2)	p1<p2	D(p1,c1)	p1<p2	I(p2-1,c2)	D(p2-1,c2)	I(p2-1,c2)	T	10
	p1=p2	NULL	p1=p2	I(p2,c2)	NULL	NULL	F	11
	p1>p2	D(p1-1,c1)	p1-1>p2	I(p2,c2)	D(p2,c2)	I(p2,c2)	T	12
			p1-1=p2	I(p2,c2)			T	12

PR: Position Relationship VR: Verification Result #: Case Number