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I pledge my honor that I have abided by the Stevens Honor System.
Problem 1.
a) Claim: The language L={0^(i)1^(j): i<j} is not regular.
Proof:
Assume L is regular.
Therefore it is recognized by a DFA N with p states, for some number p.
By the pumping lemma, there is a string S in L that has length of at least p.
Let S=0^(p)1^(p+1).
By the pumping lemma S can be divided so S=xyz and satisfies the three conditions of the pumping lemma.
Let x=0^(k), y=0^(p-k), z=1^(p+1), and k<p.
By condition 3 of the pumping lemma y is a string of 0s.
By the pumping lemma xy^(i)z is an element of L for any value of i.
There are two cases where contradictions occur, when i<1 and when i>1
Case 1, i<1.
Let i=0, xy^(0)z = xz.
xz = 0^(k)1^(p+1).
This is accepted by N, and is within the language, but breaks the rule 2 of the pumping lemma, |y|>0, creating a contradiction.
Case 2, i>1.
Let i=2, xy^(2)z = xyyz.
xyyz = 0^(k)0^(p-k)0^(p-k)1^(p+1).
After pumping the string xyyz, while still accepted by the machine N, is not a member of L because the amount of 0s, which is p+p-k, is greater than the number of 1s, which is p+1, thus breaking the rule 1 of the pumping lemma.
This is a contradiction.
The strings xz, xyyz, xyyyz, … are accepted by N but breaks either rule 1 or 2 of the pumping lemma, resulting in contradictions.
Therefore the language L={0^(i)1^(j): i<j} is not regular.

b) Claim: The language L={0^(i)1^(j):i>j} is not regular
Proof:
Assume L is regular.
Let p be the pumping length for L (given by the pumping lemma).
Let S = 0^(p+1)1^p.
S can be split into xyz, satisfying the conditions of the pumping lemma.
By condition 3, y consists only of 0s.
By the pumping lemma xy^(i)z is an element of L for any value of i.
Let i=0, xy^(0)z = xz.
xz removes the y, decreasing the number of 0s in S, causing S to have more 1s than 0s.
Therefore xz cannot have more 0s than 1s, so it is not a member of L, causing a contradiction.
Therefore the language L={0^(i)1^(j):i>j} is not regular.

Problem 2.
Claim: The language B={0^i1^i:i≠j} is irregular.
Proof:
Assume that B is regular
Let the complement of B be represented by C.
Since regular languages are closed under complement, C is also regular.
Note that {0^i1^i:i>=0} = C ∩ 0*1*.
Note that 0*1* is regular.
Since regular languages are closed under intersection, and 0*1* is regular, {0^i1^i:i>=0} is also regular.
However, {0^i1^i:i>=0} is actually irregular, leading to a contradiction.
Therefore C is not regular.
Therefore the language B={0^(i)1^(j):i≠j} is irregular.

Problem 3.
1. 0001^*
The minimum pumping length is 4. The string 000 is in the language but cannot be pumped, so 3 is not a pumping length for this language. If s has length 4 or more, it contains 1s. By dividing s into xyz, where x is 000 and y is the first 1 and z is everything afterward, we satisfy the pumping lemma's three conditions.
2. 0^*1^*
The minimum pumping length is 1. The pumping length cannot be 0 because the string ε is in the language and it cannot be pumped.
3. $0^*1^*0^*1^* \cup 10^*1$
The minimum pumping length is 3. The pumping length cannot be 2 because the string 11 is in the language and it cannot be pumped.
4. $(01)^*$
The minimum pumping length is 2. The pumping length cannot be 1, as 1 is not in the language, but 0 is in the language and cannot be pumped by itself.
5. $1^*\ 01^*\ 01^*$
The minimum pumping length is 1. The pumping length cannot be 0 because the string ε is in the language and it cannot be pumped.
Problem 4.
L = {$a^ib^jc^k$: i,j, k ≥ 0 and i = 1 ⇒ j = k}
a. $\forall s \in A$, |s| ≥ p ⇒$\exists x, y, z$: $s = xyz$ where
1.$\forall i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy^i| \leq$ p
Let the pumping threshold be equal to two
String s = {ab^ic^i: i ≥ 0} ∈ L, where x = a, $y^i = b^ic^i$, and z = ε
Since |s| ≥ p and $|xy^i| \leq p$, ∃i: $xy^iz \in$ L

b. $\forall s \in A$, |s| ≥ p ⇒$\exists x, y, z$: $s = xyz$ where

1. $\forall i \geq 0$, $xy^iz \in A$,

2. $|y| > 0$, and

3. $|xy^i| \leq p$

Note that because of the closure of regular languages over union:

$aaa^*b^*c^* \cup b^*c^* \cup \{ab^ic^i: i \geq 0\} \in L$

For $s_2 = xy^iz: i \geq 0$

Let $s_2 = aaa^*b^*c^*$

If we say $x = aa$, $y = a^*b^*c^*$, and $z = \varepsilon$, $|xy^i| > p$

If we say $x = aaa^*$, $y = b^*$, $z = c^*$, we exit the language, as c is not within the window for p

Since $|xy^i| > p$, $\exists i: xy^iz \notin L$

c. Explain why parts (a) and (b) do not contradict the pumping lemma.

Parts a and b do not contradict the pumping lemma because the language is not regular. If a language is regular then it must satisfy all the conditions of the pumping lemma, but it doesn't mean that a non-regular language can't satisfy those conditions as well.