Kinematics and Dynamics of a Rigid body

Discord Physics
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Chapter 1
Kinematics of a Rigid body

0.1 Definition: Rigid body .....

0.2 Chasles theorem: General motion of a rigid body has six degrees of freedom, three which represents translational motion of one given point of the rigid body, and three describing rotation around the same point.

Proof: Here will come the proof.

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0.3 Definition: Basis fixed in space .....

0.4 Definition: Co-rotating basis .....

1.1 Angular velocity

Let $\{\boldsymbol{e}_i\}$ be a basis fixed in space, $\{\boldsymbol{e}'_i(t)\}$ a basis co-rotating with a rigid body. As a co-rotating ortonormal basis is only rotated with respect to a fixed ortonormal basis, we can write $\boldsymbol{e}'_i(t) = A_{ik}(t)\boldsymbol{e}_k$. Consider now an arbitrary, time dependent vector $\boldsymbol{w} = \boldsymbol{w}(t)$ and express it in both bases, $\boldsymbol{w}(t) = w_i(t)\boldsymbol{e}_i = w'_i(t)\boldsymbol{e}'_i(t)$. Time derivative of $\boldsymbol{w}(t)$ with respect to the inertial fixed basis but expressed in the co-rotating bassis is

$$\frac{\mathrm{d}\boldsymbol{w}}{\mathrm{d}t} = \frac{\mathrm{d}w'_i}{\mathrm{d}t}\boldsymbol{e}'_i + w'_i\frac{\mathrm{d}\boldsymbol{e}'_i}{\mathrm{d}t} = \frac{\mathrm{d}w'_i}{\mathrm{d}t}\boldsymbol{e}'_i + w'_i\frac{\mathrm{d}A_{ik}}{\mathrm{d}t}\boldsymbol{e}_k = \frac{\mathrm{d}w'_i}{\mathrm{d}t}\boldsymbol{e}'_i + w'_i\frac{\mathrm{d}A_{ik}}{\mathrm{d}t}A_{jk}\boldsymbol{e}'_j. \tag{1}$$

Therefore, we can formulate a definition:

0.5 Definition: Let $\{\boldsymbol{e}_i\}$ be an ortonormal basis[1] fixed in space, $\{\boldsymbol{e}'_i(t)\}$ an ortonormal basis co-rotating with a rigid body and $A_{ij}(t) \in C^1(\mathbb{R})$ elements of orthogonal matrix[2] such that $\boldsymbol{e}'_i(t) = A_{ij}(t)\boldsymbol{e}_j$. Then the *tenzor of angular velocity* $\Omega : V \times V \longrightarrow \mathbb{R}$ is defined as

$$\Omega = \frac{\mathrm{d}A}{\mathrm{d}t}A^t, \tag{2}$$

with elements

$$\Omega = \frac{\mathrm{d}A_{ik}}{\mathrm{d}t}A_{jk}. \tag{3}$$

[1]Both bases are bases of V, what is defined as a vector space over $\mathbb{R}$, such that $\dim V = 3$.
[2]$A^t = A^{-1}$ and $A_{ik}A_{jk} = \delta_{ij}$