# COMPUTATIONAL THINKING

Computational Thinking can be broad and include all matters related to computers, computing, and development of algorithms. In middle school and high school, though, we focus on understanding computing problems, breaking them down, and creating algorithms. Generally, you need to be able to

- Recognize and define computational problems
- Develop and use abstractions
- Create, test, and refine computational artifacts

There are many overlapping definitions of Computational Thinking, but they agree on the following four components:

- Decomposition—breaking a complex problem or system into parts that are easier to understand
- Pattern Recognition—finding patterns and regularities in a problem or in data.
- Generalization/Abstraction—removing unnecessary details and formulating general concepts or rules that apply to the common properties of specific instances of a problem or of data
- Algorithms—creating an unambiguous, well-defined sequence of instructions to solve a problem or accomplish a task.

Many of the materials in this section are puzzles. There isn't much difference between a math problem, a physics problem, and a puzzle.

Or, as Don Knuth, one of the greatest computer scientists, put it:

*y teachers who present them with well-chosen recreational probl*

“I’ve never been able to see any boundary between scientific research and game-playing. … I believe that the creation of a great puzzle or a great pattern is a scholarly achievement of great merit, an important contribution to world culture. [My view is] that students are best-served by teachers who present them with well-chosen recreational problems.”

## 1 - Decomposition

The best way to solve any relatively complex problem is to break it down into smaller parts that are easier to understand.

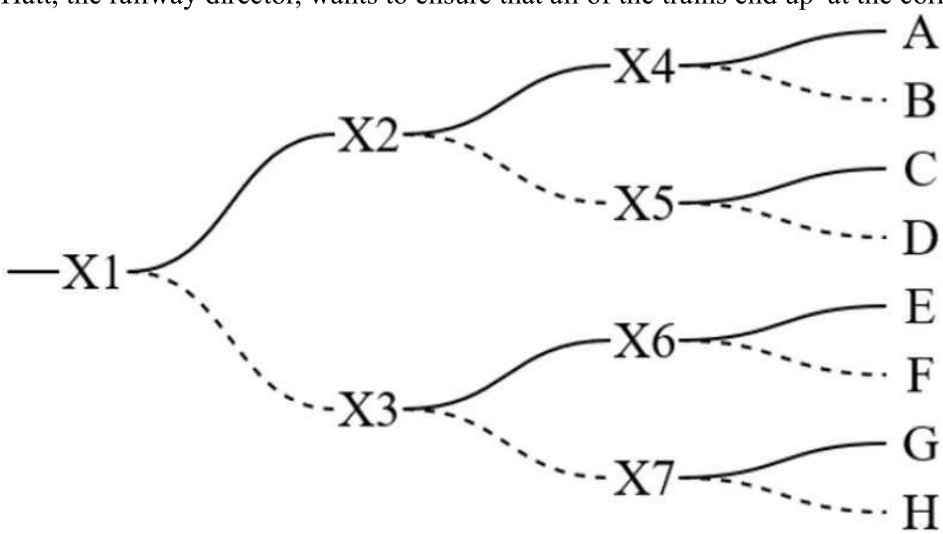
For example, let’s say you are preparing a grand buffet for your best friend. You are overwhelmed at first, but then you feel more certainty as you begin making a list of your tasks: to find a venue, to send out invitations, to prepare appetizers, to make entrees, to bake desserts, and to serve drinks.

Among the desserts you decide to prepare for the buffet, one of them is an apple pie. To make this apple pie, you have another set of instructions: to first make the pie crust, then make the filling, and then bake. You start by following your simple instructions to make the pie, and then once that’s finished, you move on to doing the same for each of the other dishes and tasks, one by one. And in the end, you succeed! Everything seemed to magically come together like pieces in a puzzle.

This is exactly what decomposition is—breaking larger tasks into several more manageable pieces, which you ultimately put together in the end. Now try applying this idea to solve some of the logic-based problems below.

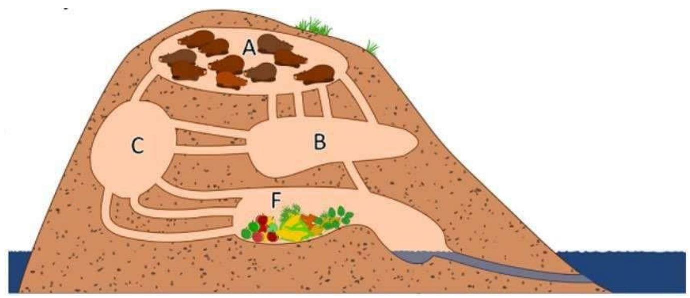
1.1 - In the diagram below, eight trains (named a, b, c, d, e, f, g, and h) enter the switch X1 from the left side of this page. Train a needs to go to station A, train b to station B, train c to station C, and so on. Each of the switches X1 through X7 are initially set to direct trains to the left. After a train has passed a switch, the switch changes to the other direction.

Sir Topham Hatt, the railway director, wants to ensure that all of the trains end up at the correct stations.




What is the correct order for the trains to pass through switch X1?

1.2 - Beaver I: Magic Potions




Boover Dam has four rooms (A, B, C, F) connected by a series of tunnels. The first three rooms (A, B, and C) are living rooms. The fourth room (F) is where the beavers store their food.

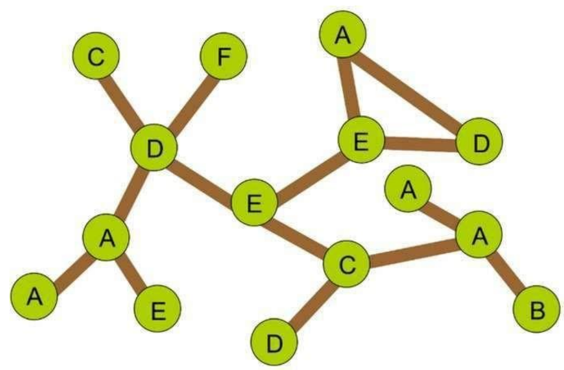
The 10 beavers in the figure were playing charades in room A. They just got hungry and now they want to go to room F to eat. They all are very hungry and want to get to the food storage room as soon as possible.

The rooms are connected by a series of tunnels: 4 tunnels connect A & B; 1 tunnel connects A & C; 2 tunnels connect B & C; 1 tunnel connects B & F; and 3 tunnels connect C & F.

It takes a beaver 1 minute to go through a tunnel. Only one beaver may go through a tunnel at the same time (a beaver may only enter a tunnel once the beaver before it is out the other side. All the rooms are big enough to hold all 10 beavers (there are no capacity limits).

Question: If the beavers plan their route optimally, how quickly can all 10 beavers get to the food storage room (F)?

1.3 - Evans Creek Park "Graph Theory"




To the right is a very abstract map of Evans Creek Park.

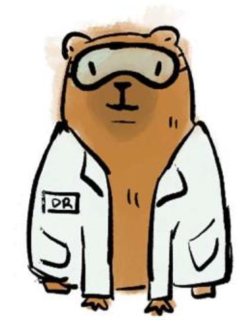
The circles with letters represent major landmarks and the lines are paths between them. Note that some letters are used to label more than one landmark. For example, walking from F to B can be described as F D E C A B.

Last Sunday, two families walked in the park. They started at the same time.

The Iyer family walk was B A A A C E D E E D A.

The Zhang family walk was F D C D A E A D E D A.

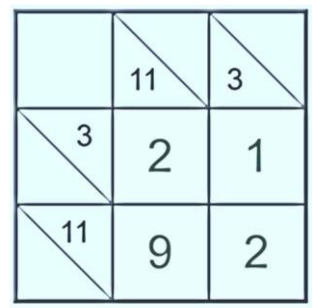
Walking from landmark to landmark, down one path, takes the same amount of time. How many times did the two families meet?



## 2 - Decomposition: Kakuro

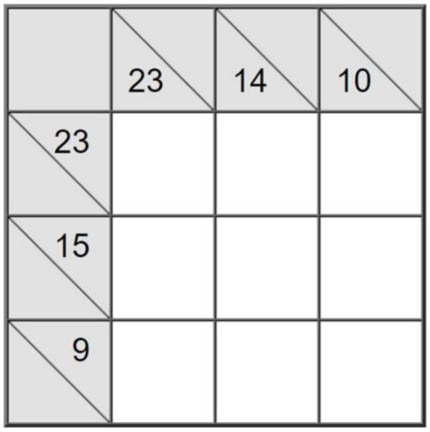
Kakuro were created in the U.S., became popular in Japan, then became popular in the U.S. and Europe.

Each puzzle is made up of horizontal and vertical sets of numbers, like a crossword. Each set may include up to 9 boxes. Each box contains one digit, from 1 to 9.



Solved Karuko puzzle

1. Each digit may only be used once in each set.
2. The numbers in each set add up to the sum in the triangular clue on both sides of the set. For example, if the clue is 4, then the set must include 1 and 3. It cannot be 2 and 2, because digits cannot repeat.
3. When you see big numbers and small numbers, there usually are only a few ways to make them—and that is how you solve them.
4. Here are a few easy puzzles that will help you decompose the next, harder ones: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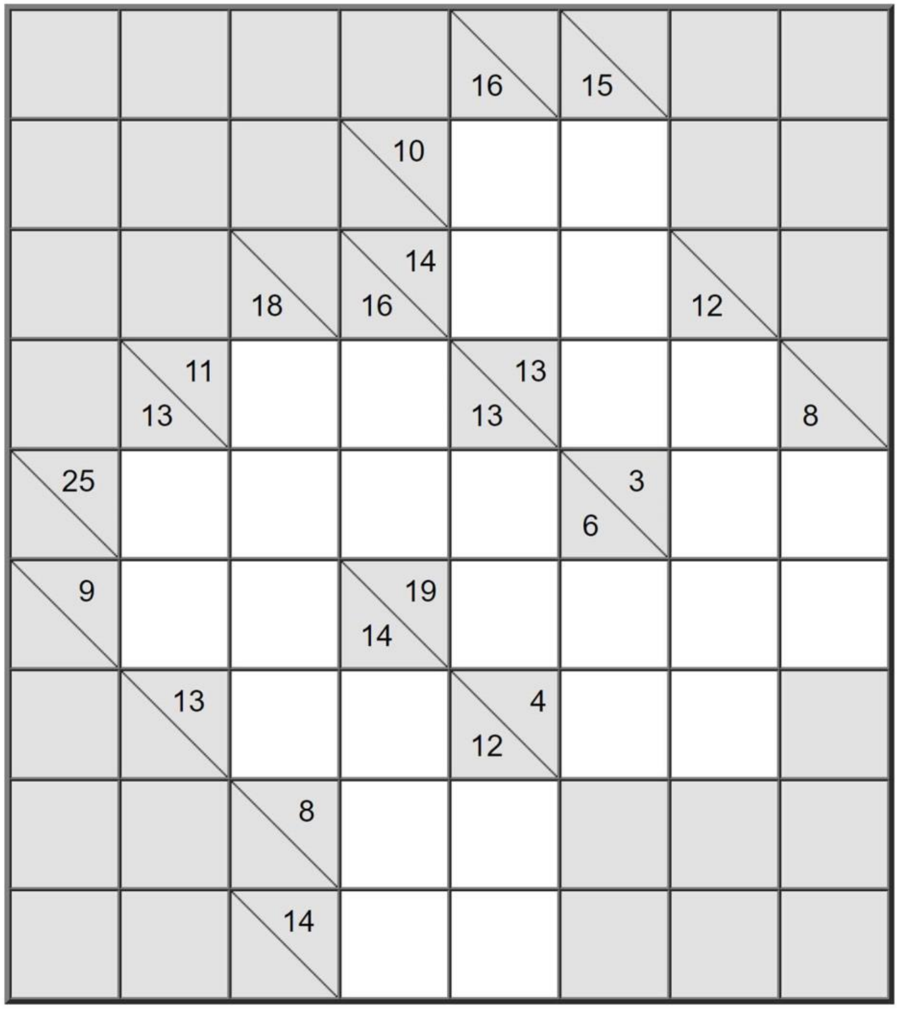
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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 29 | 3 | | | 6 | 3 | 17 | | | | |
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| | | 14 | | | | | 15 / 17 | | | | | 16 | | |
| | | 6 / 34 | | | 15 | | | | | 8 / 3 | | | | |
| | 20 / 3 | | | | 10 / 16 | | | | 12 | | | | | |
| 13 | | | | 9 | | | 29 | 10 | 7 | | | 24 | | |
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| | 8 | | | | | 8 | | | | | 9 | | | |
| | 11 | | | 16 | | 10 | | | 16 | 17 | 14 | | | 16 |
| | | 5 | | | 3 | 20 | | | | | | 10 / 29 | | |
| | | | 3 / 17 | | | | | 15 / 16 | | | 16 / 11 | | | |
| | | 15 | | | | | 8 / 7 | | | 14 | | | | |
| | | 11 | | | 16 | 13 / 17 | | | | 9 / 4 | | | | |
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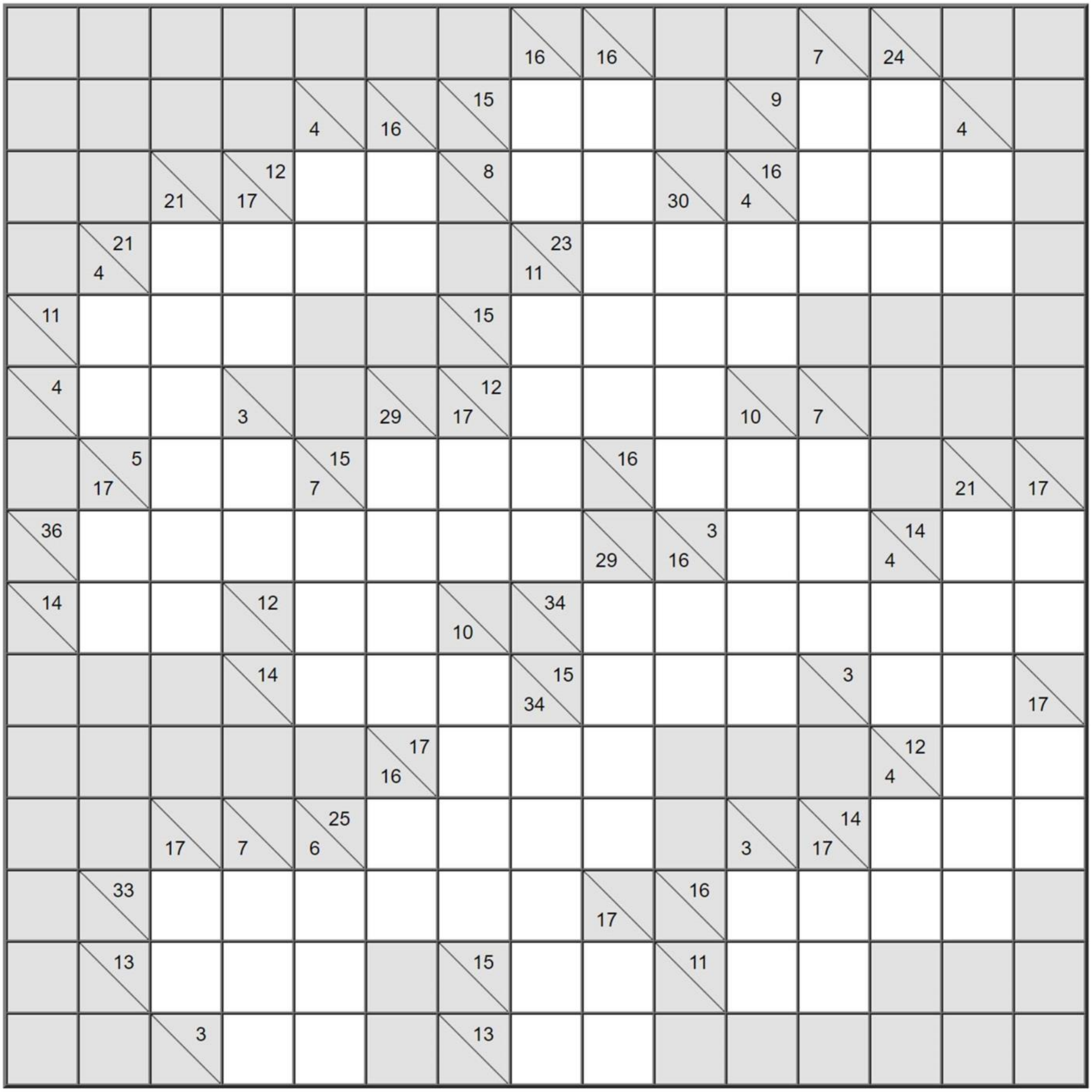


# 2 - PATTERN RECOGNITION

Here are a few pattern recognition questions. Keep the front of your head that these are pattern recognition problems—look for the pattern, then solve the problem.

2.1 - There are an odd number of Tesla STEM students on a field. Everyone has a water gun and simultaneously shoots water at the student closest to them. There is at least one student who does not get wet. Why?

2.2 - A pattern that repeats every six symbols starts as shown below:

♥ ♣ ♥ ♦ ♥ ♠ ♥ ♣ ♥ ♦ ♥ ♠ …

What is the 1001$^{st}$ symbol? What is the 1,000,001$^{st}$ symbol?

Is there a rule you could use to find the symbol at any point in the sequence?

(Hint: Once you've figured out the length of the pattern, how can remainders help? If you really wanna get fancy, try using "modular arithmetic".)

2.3 - Samiksha the snail leaves on Monday to go and visit her Grandmother, 90m away. Except for rest days, Samiksha travels 1m each day (24 hour period) at a constant rate and without pause. However, Samiksha stops for a 24-hour rest every tenth day, that is, after nine days traveling. On which day does Samiksha arrive at Grandmother's?

2.4 - One year, there were exactly four Tuesdays and exactly four Fridays in October. On what day of the week did Hallowe'en (October 31$^{st}$) fall that year?

2.5 - When I glanced at my car odometer (which shows how many total miles the car has traveled), it showed 24942, a palindromic number. Two days later, I noticed that it showed the next palindromic number. How many miles did my car travel in those two days?

2.6 -

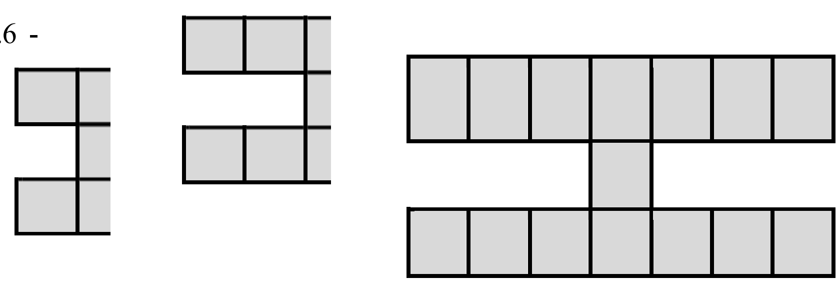


DIAGRAM 1 DIAGRAM 2 DIAGRAM 3

How many squares are there in Diagram 9? How many squares are there in Diagram 25? Write an algorithm: how many squares are there in Diagram n?

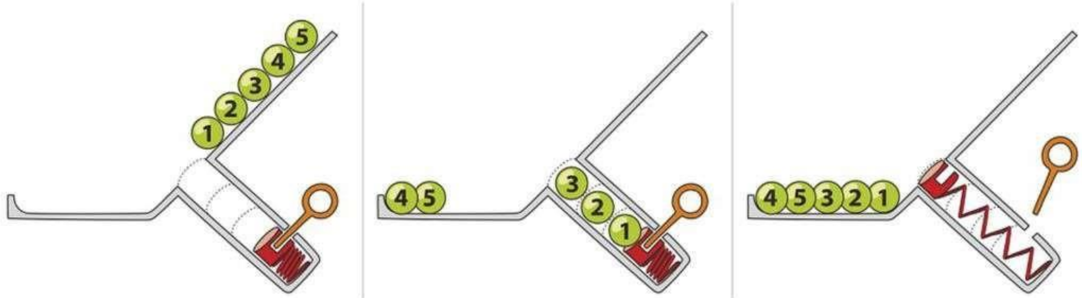
2.7 - Which of the following fractions is closest to 1? (This is in pattern recognition, not number sense, because you are comparing numbers—look for the pattern!)

A) 7/8 B) 8/7 C) 9/10 D) 10/11 E) 11/10

2.8 - Things falling in holes

This is a pretty common problem in computational thinking. It combines decomposition and pattern recognition, so we include it here:

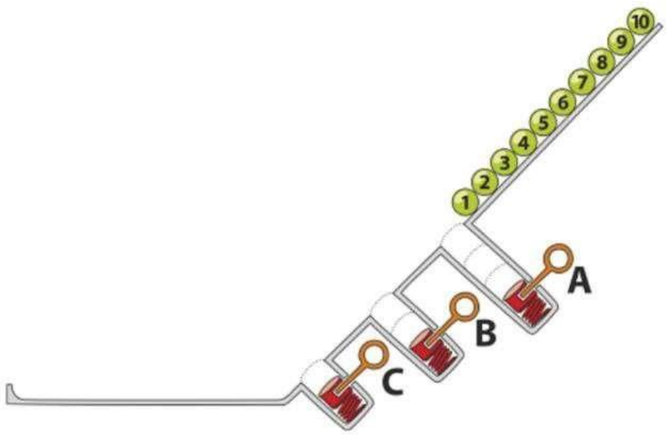
Let’s say we have numbered balls which are rolling down ramps. The order of the balls changes as they fall into holes. When a ball comes to a hole: if there is enough space, the ball falls in; otherwise, the ball rolls past the hole. A pin at the bottom of each hole can be pulled which ejects the balls. Here is an example with 5 balls:




1. Before balls rolling

2. After balls have stopped rolling

3. Final results after pulling the pin




Ten balls roll down the ramp pictured to the left.

Three holes labeled A, B, and C have space for 3, 2, and 1 balls as shown. The pins are pulled in the order A, B, C, but only after all the balls have stopped rolling. Which of the following will be the final result?





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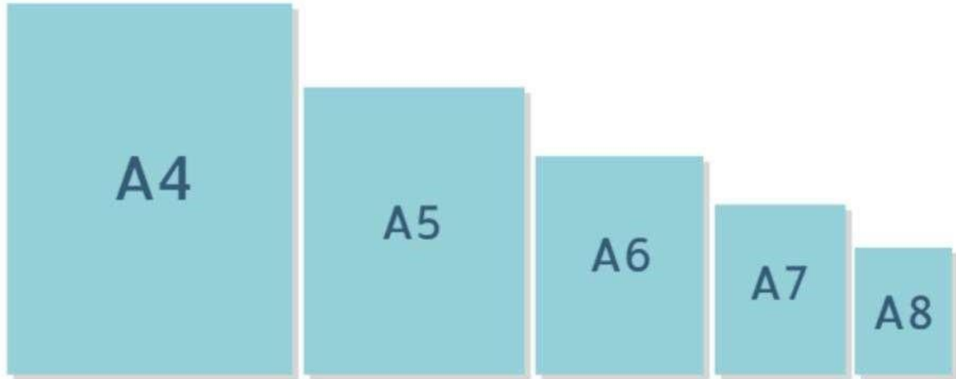
2.9 - Playing Cards

You want to make playing cards and you have 5 pieces of cardboard available for use. Each of the pieces of cardboard have different sizes: A4, A5, A6, A7, and A8.



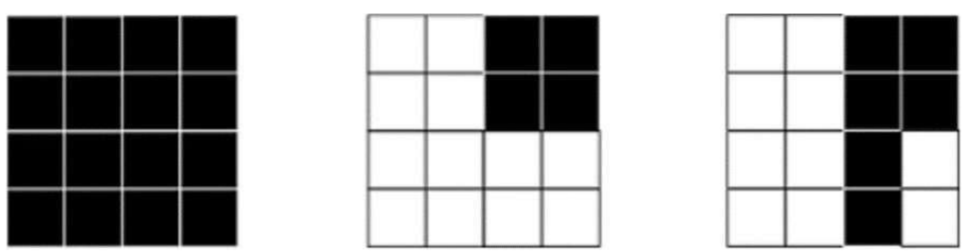

A4 is twice as big as A5, A5 is twice as big as A6, and so on.

You need exactly 12 playing cards of size A8 and you don't want to waste any paper. Which pieces of cardboard should you use?

1) A4 and A5
2) A5 and A6
3) A6 and A7
4) It's not possible!

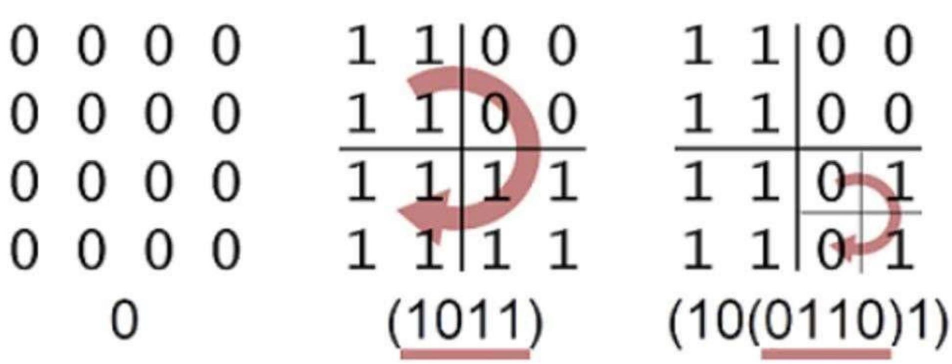
2.10 - Image Compression

Do you want to try some image compression?
Take a look at the following 4 x 4 black and white pixel images:



We can store these images using binary digits: "1" for white pixels and "0" for black pixels. For a 4 x 4 image, we would have to store 16 digits.

The following image compression method allows us to store images using less space, especially for simple patterns. These examples correspond with the 4 x 4 grids above.



The binary digits are arranged in a grid like the pixels in the images. Do you see the pattern? Here are some basic rules of this compression method, which you may have already noticed:

1. If all the digits in the grid are zeroes, the result is “0” (see left image). If all the digits in the grid are 1, the result is “1”.

2. Otherwise, the grid is divided into quarters. Step 1 is again applied, but this time, to each quarter sub-grid. If Step 1 is not applicable (the digits are not all the same), then the compression method records the digits starting from the top left and in clockwise order. All of the results are combined and surrounded by round brackets. Two different examples of this can be seen in the center and on the right above.

Think you’ve figured it out? Try this one.

Below is the binary digit grid for an 8 x 8 image. The above compression method is applied to this grid.

```
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 0 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
```

Keep in mind when solving this puzzle that since we are now working with a larger grid, each quarter sub-grid (4 x 4) can be further divided into quarters, if need be. So which string of digits will represent this image?

a. (1110)  b. (11(1011)1)  c. (111(1(1101)11))  d. (111(1(1011)11))

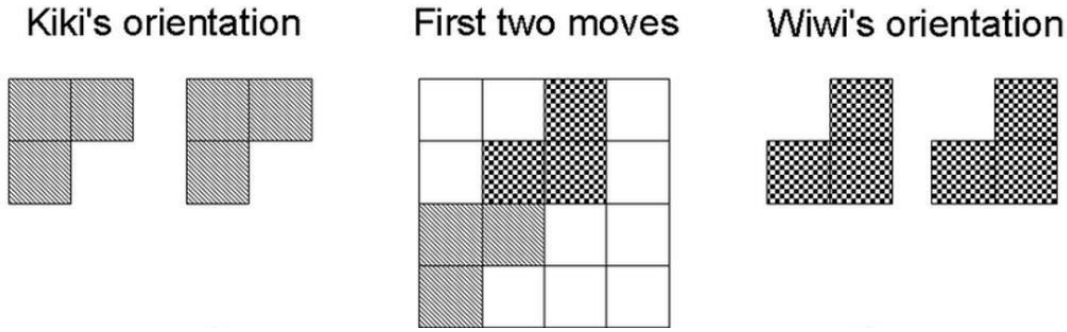
2.11 - Hurling is a fast-paced contact sport that has been played for 4000 years. After a long, sometimes violent match, players line up in a row and walk past the other team and, as they pass each other, they shake hands. At the beginning, only the first player on each team shakes hands. Next, the first two players shake hands. This continues until each player shakes hands with every player on the other team.

There are 15 players on each team. If each player takes one second to shake hands and move to the next player, how many seconds will it take for all of the hand-shaking?

2.12 - The L-Game: Kiki and Wiwi are playing L-game on a 4x4 board. They take turns placing L-shaped pieces according to the following rules:

- Every piece placed by Kiki is oriented as shown below.
- Every piece placed by Wiwi is oriented as shown below.
- Every piece is placed entirely on the board and pieces may not overlap.
- Pieces cannot be moved after they are placed.

A player loses the game when it is their turn, but it is not possible to place a piece according to the rules above. An example where Kiki goes first is shown below. In this example, Kiki can win the game by placing a piece in the bottom-right corner.

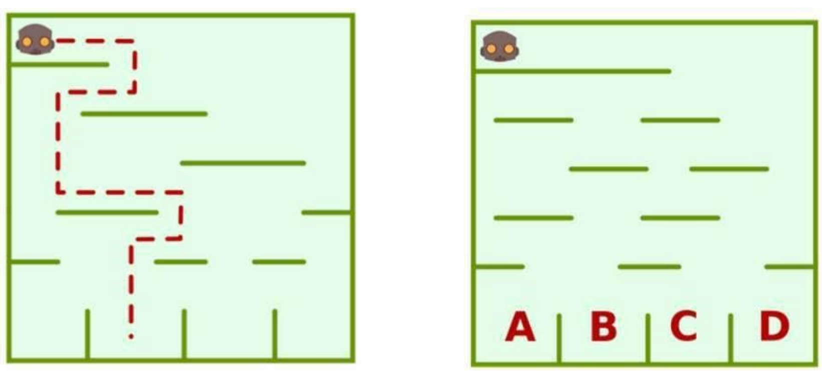



How many possible moves are there for each player?

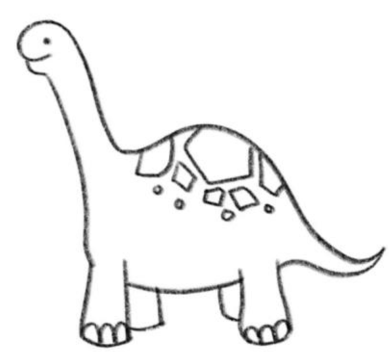
If Kiki goes first, for how many of those moves is she guaranteed to win no matter how pieces are placed in the following turns?

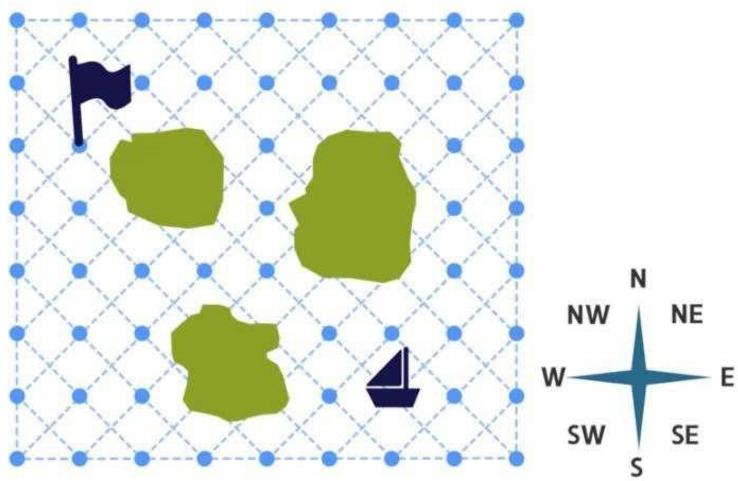
2.13 - Falling robot: A robot moves through a vertical maze. The maze consists of various platforms. The robot begins in the upper left corner and then moves to the right. When it reaches the end of a platform, it falls down onto the platform below. As soon as the robot lands it changes direction. Eventually the robot reaches one of the buckets at the bottom of the maze. The left image shows an example of how it might move.

Which bucket on the right image will the robot reach?




2.14 – Navigation: The Dread Pirate Roberts sails through an area with islands. Their goal is to go to a place marked on the map with a black flag. The computer that controls the course is constructed in such a way that the sailing vessel can sail from point to point. In addition, it keeps course following one of the eight directions of the compass rose. For example, "1N" means that the ship heads north to the next point. And "2SW 1S" means that the ship first sails twice to the next point to the southwest and then to the next point to the south.






Which program below will have the fewest waypoints to reach the target and ensure that the ship will not collide with an island?

A) 4NW 1W

B) 2NW 2W 2N 1W

C) 2NW 2N 1NW 1SW

D) 2NW 2W 1NW 1N

2.15 - Bebrocarina

The instructions for playing a bebrocarina are given with 3 symbols, an o, a +, and a -. They each mean:

The bebrocarina is a special musical instrument: it has only six different tones

And: after a tone is played, only the same tone or the tone directly above or below it can be played.

| | |
|---|---|
| O | Play the previous note |
| + | Play one note above the previous note |
| - | Play one note below the previous note |

Not all sequences of Os, +s, and -s can be played on a bebrocarina.

Which of the following sequences of instructions cannot be played?

A) + o o o + o o o + o o o + o o o +

B) - - - o + - o - - o o o +

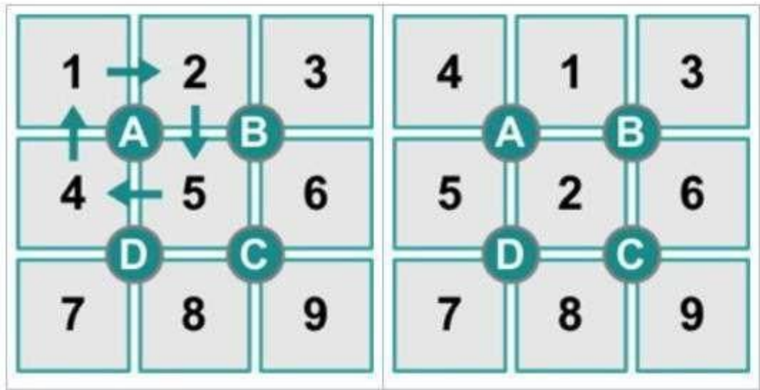
C) - - + - - + - - o - + - -

D) - - - - - o + + + + + o - - - - -

2.16 Turning Numbers

In the game of "Turning Numbers" you can scramble the numbers 1 to 9 at right. At the start of every game, the numbers are oriented as shown in the picture in the image on the left side.

Pushing one of the buttons (A, B, C, or D) rotates the four surrounding numbers clockwise, as shown in the image on the right side.

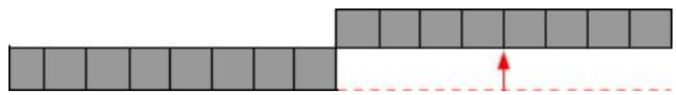



You start a new round and push the following buttons: D,C,B,B.
Where will the number 4 be at the end?

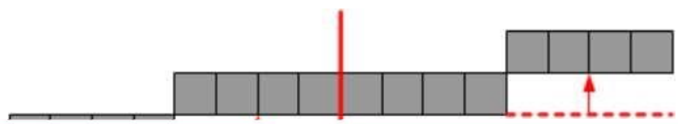
2.17 - Half Sliding

A paper strip is divided in 16 equal pieces:

Such a strip can be used for "half sliding". This is done by splitting the strip in half and sliding the right half up:

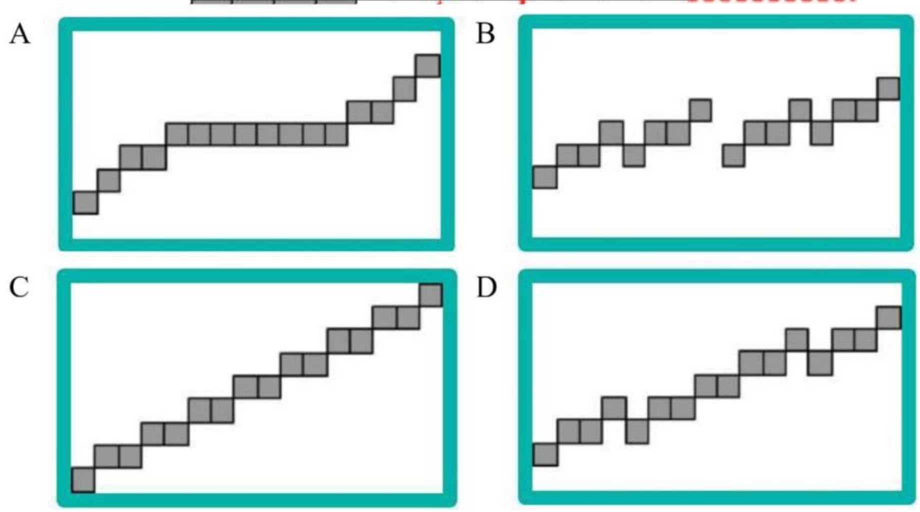


The two halves are also split in half and again, both right halves slide up. This would look like this:



We do this again with the four piece strips and, after that, with the two-piece strips.

What will the final result look like?



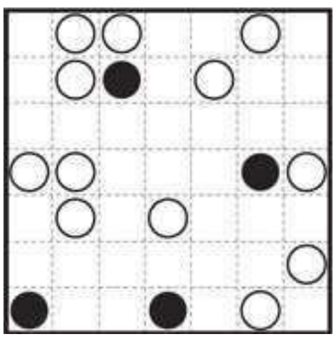
# 3 - PATTERN RECOGNITION: MASYU

(or black pearls and white pearls)

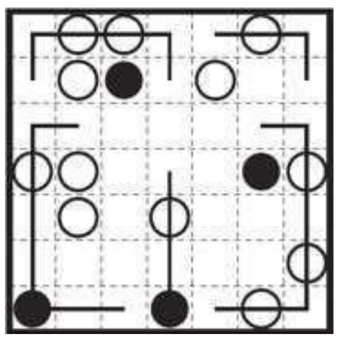
Masyu was created in Japan as a numberless puzzle that requires logic to solve and is aesthetically pleasing. The rules are simple:

- Each puzzle is solved by drawing a single non-intersecting loop.
- The loop enters and exits the center of one of the four sides of each cell that it passes through. All turns are 90 degrees.
- Black circles: the loop turns 90 degrees, but travels straight through the adjacent cells (the previous and the next cells) in its path.
- White circles: the loop travels straight through white circles, but must turn 90 degrees in one of the adjacent cells. Don't forget the turn!

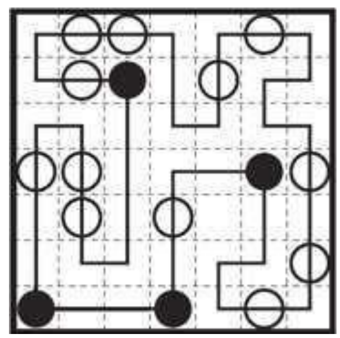
Here is an example from Nikoli (the publication that invented Masyu):



Example puzzle



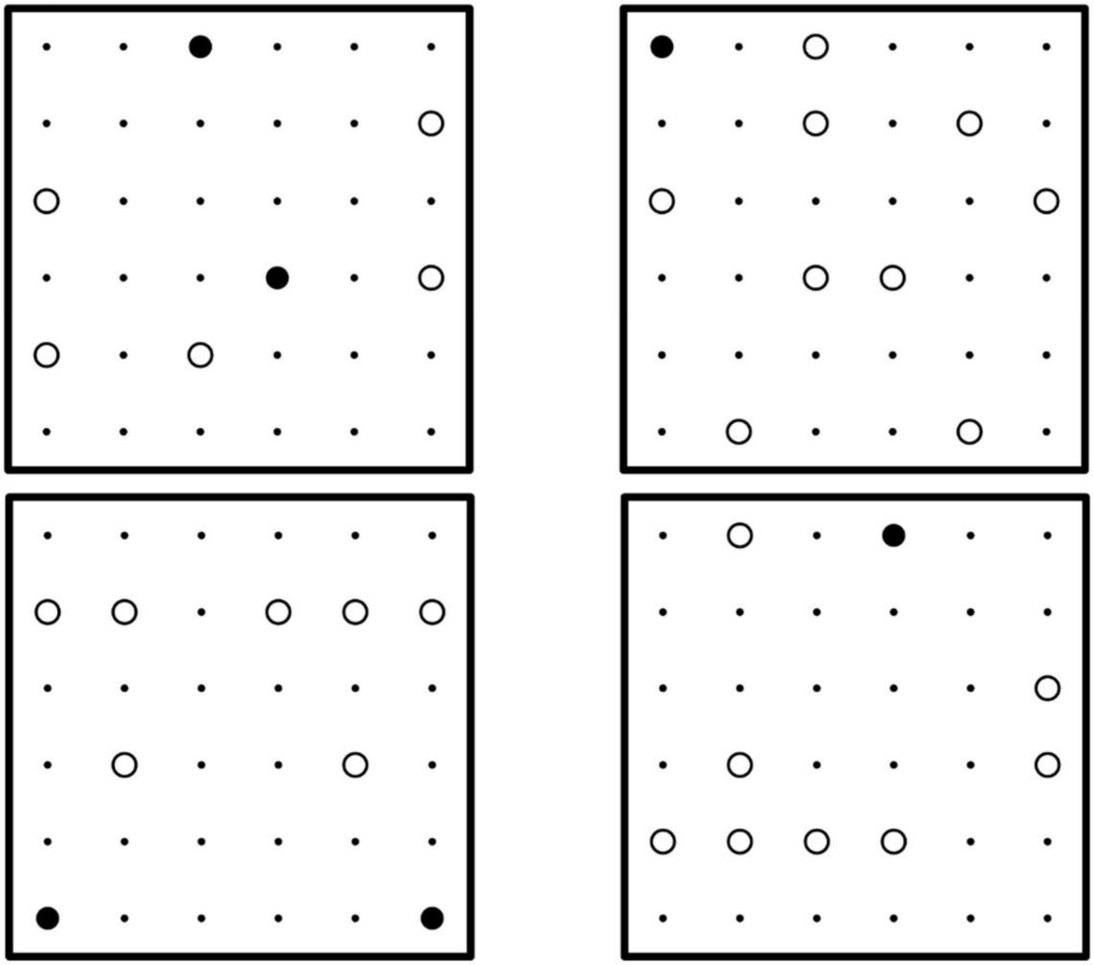
Solution in progress…



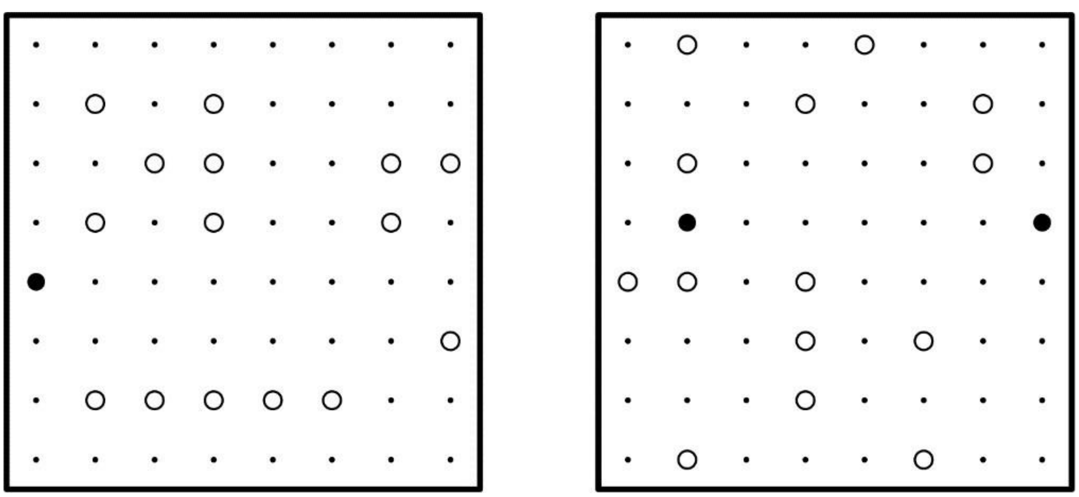
Example puzzle, solved

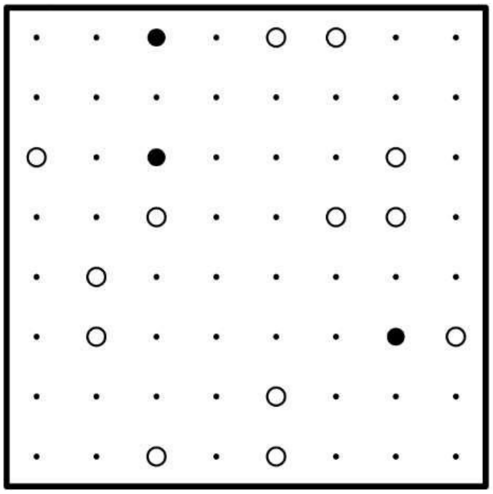
Each of the following puzzles has only one correct solution. You are trying to find it, of course, but you are also looking for patterns that will prove useful in solving later puzzles.

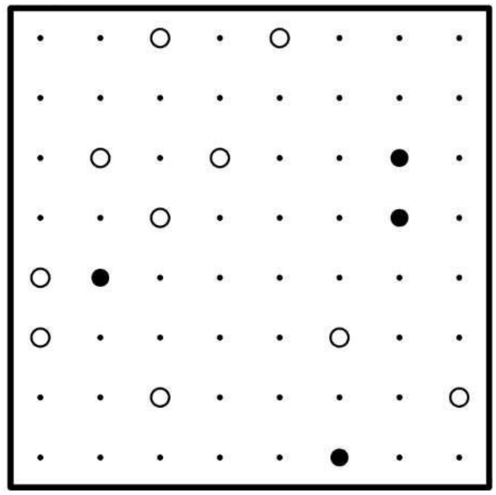
Hints: use a pencil… and don't forget, each solution is a single loop.

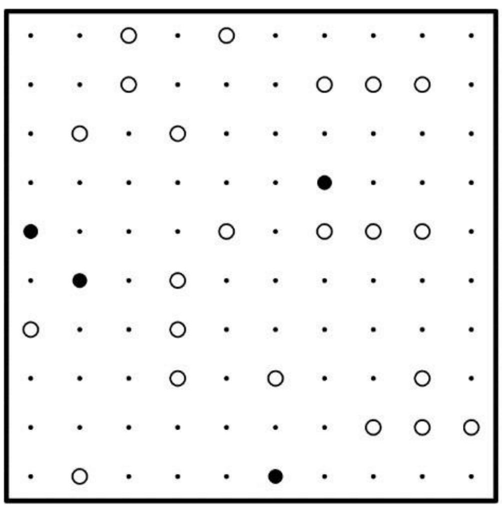


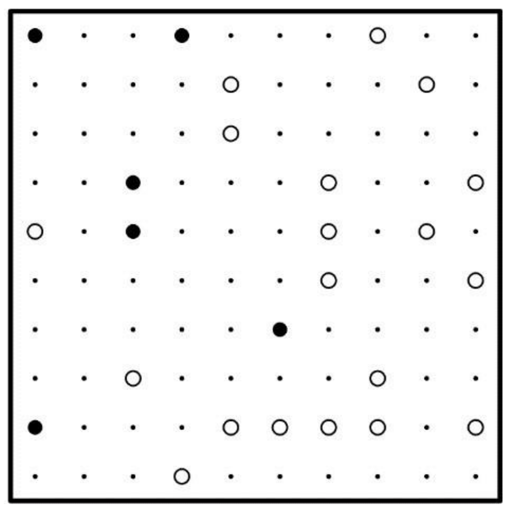
Are you spotting any patterns yet?

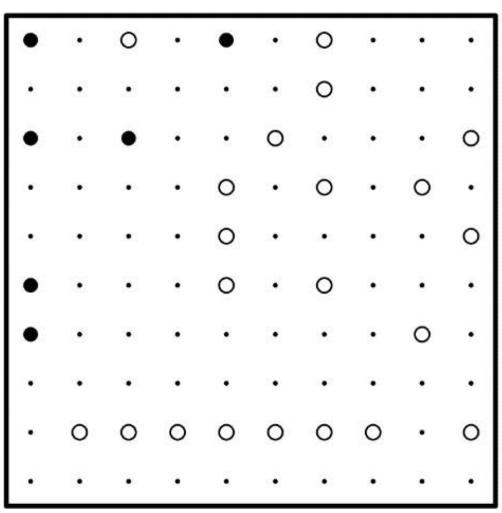


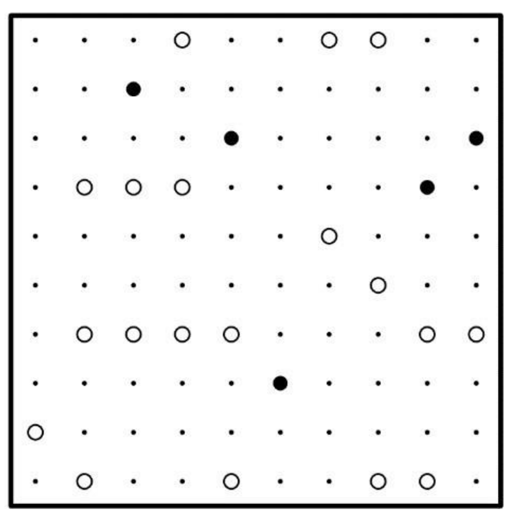












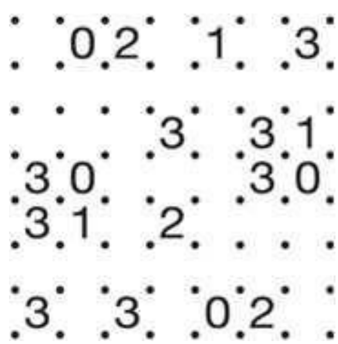
# 4 - PATTERN RECOGNITION: SLITHERLINK

Slitherlink is another aesthetic puzzle created in Japan by Nikoli, where it is called Sli-Lin. It is similar to Masyu in that you must solve each puzzle using a single loop. The rules are simple:

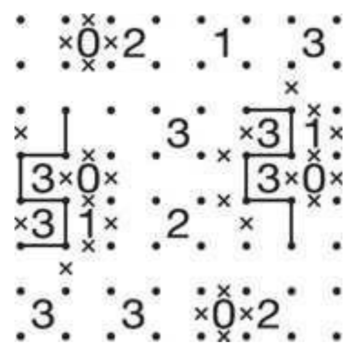
- Each puzzle is solved by drawing a single non-intersecting loop.
- The numbers indicate, for each cell, how many lines of the solution surround that cell.
- Empty cells may be surrounded by any number of lines.

In the square grid version, a 0 means the solution does not use any of the “lines surrounding the cell. A 1 means only one line of the square is used; a 2 means two lines are used and a 3 means 3 (of the four possible) lines are used.

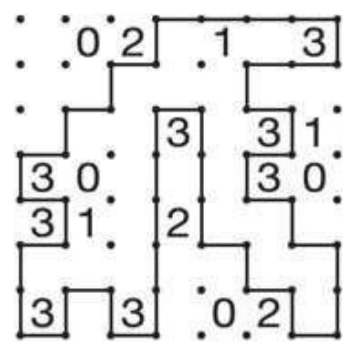
Here is an example of a solved puzzle from Nikoli (the publication that invented Masyu):



Example puzzle

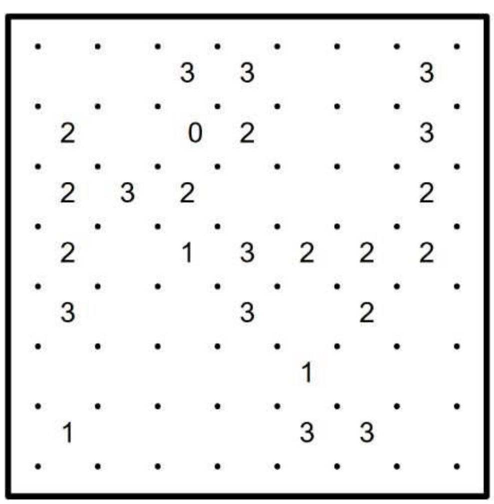


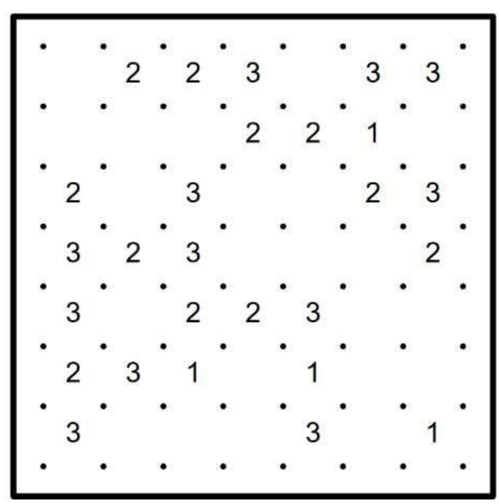
Solution in progress…

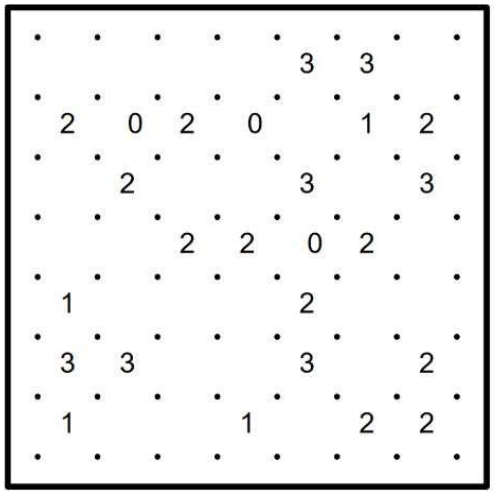


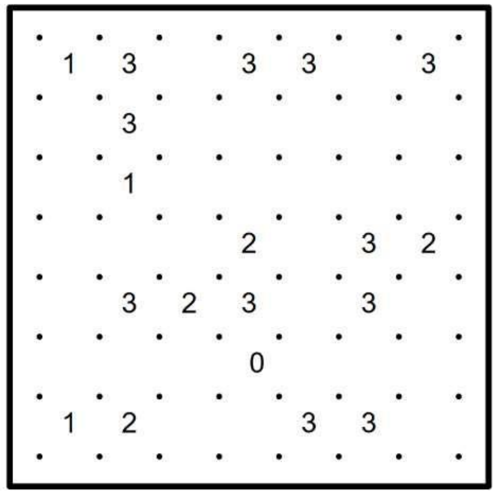
Example puzzle, solved

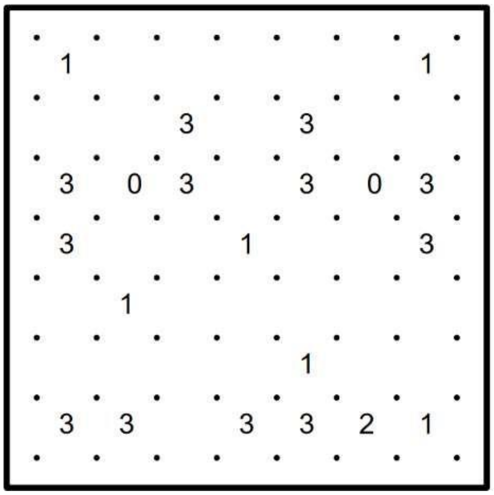
Like Masyu, you learn Slitherlinks by solving them—with a pencil. A few warmups:

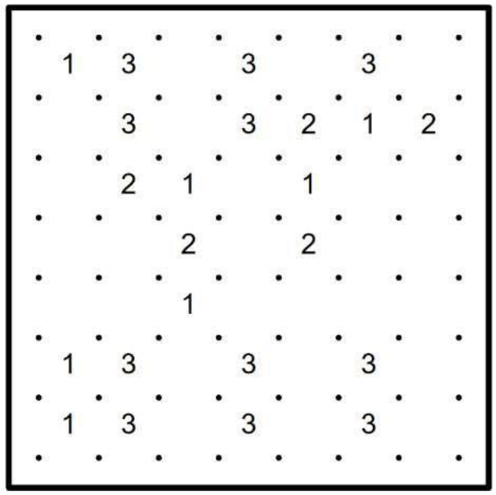




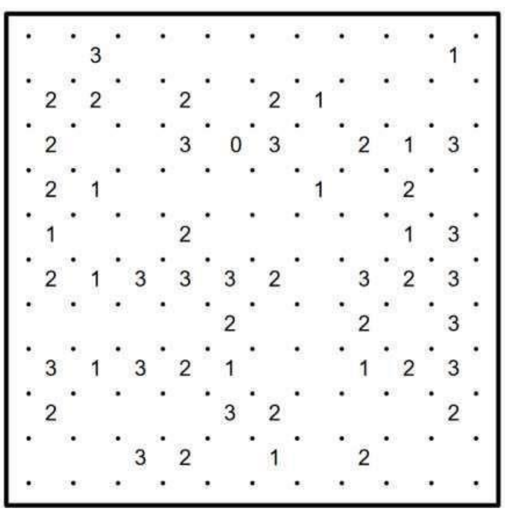


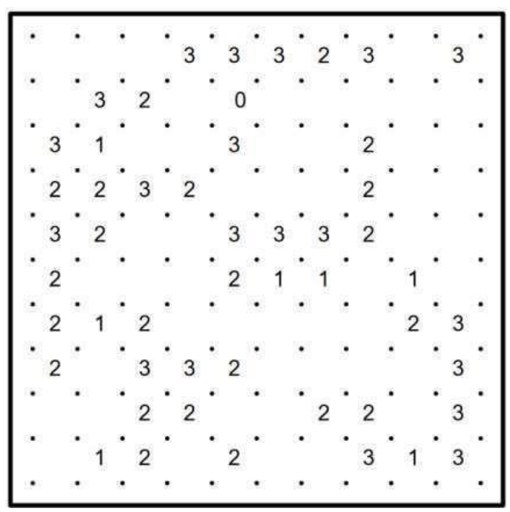


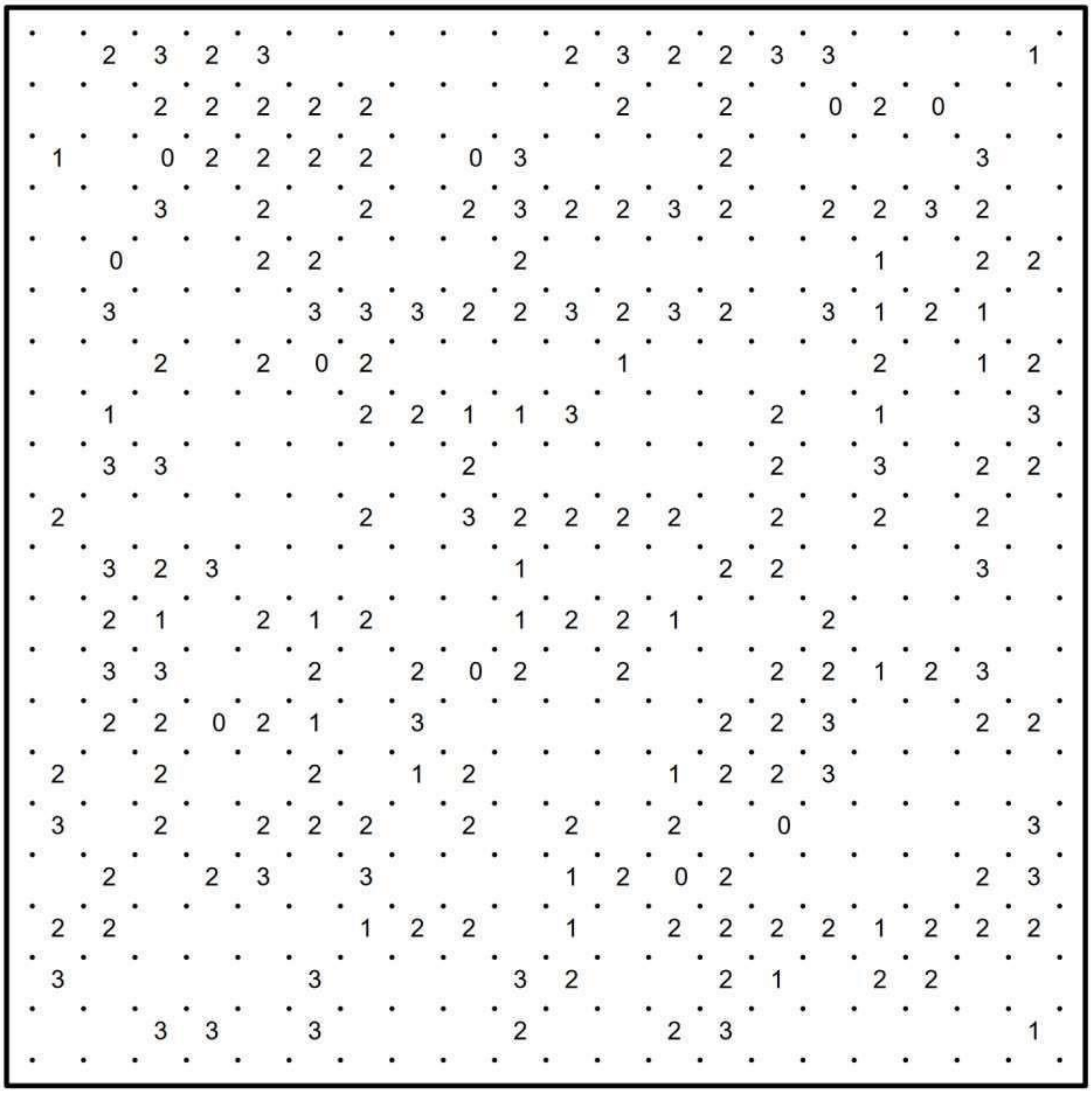




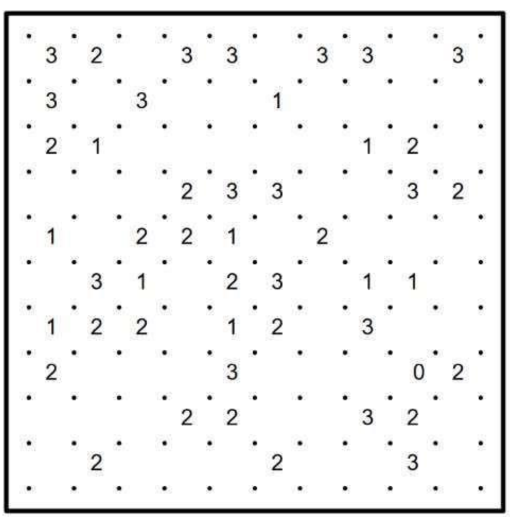
Try these, which are bigger, but not harder—just use the patterns you have seen:

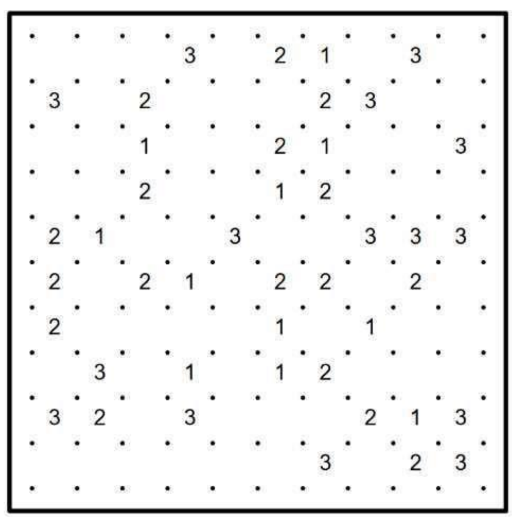




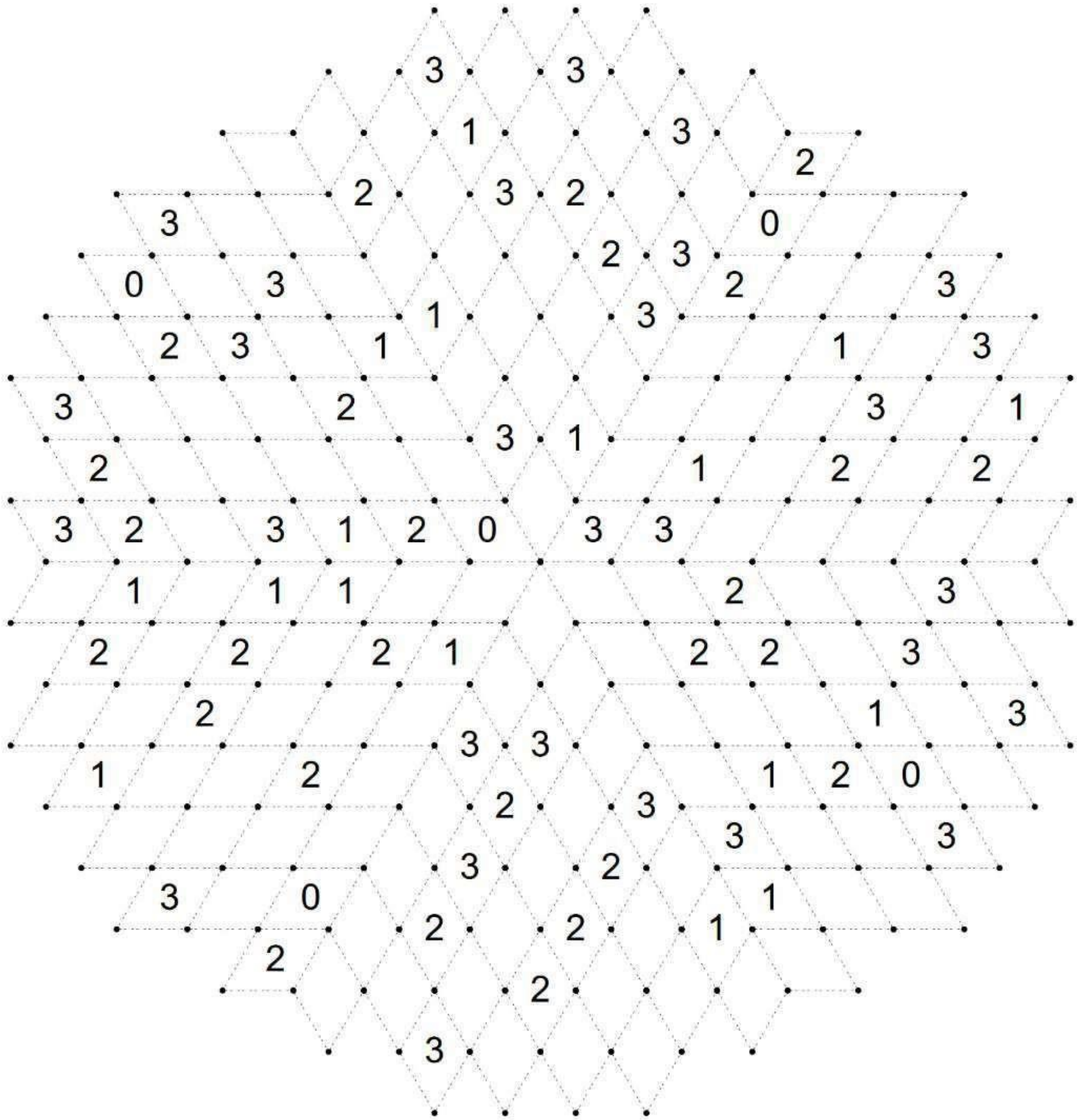


… and here are a couple of middle-difficulty ones:

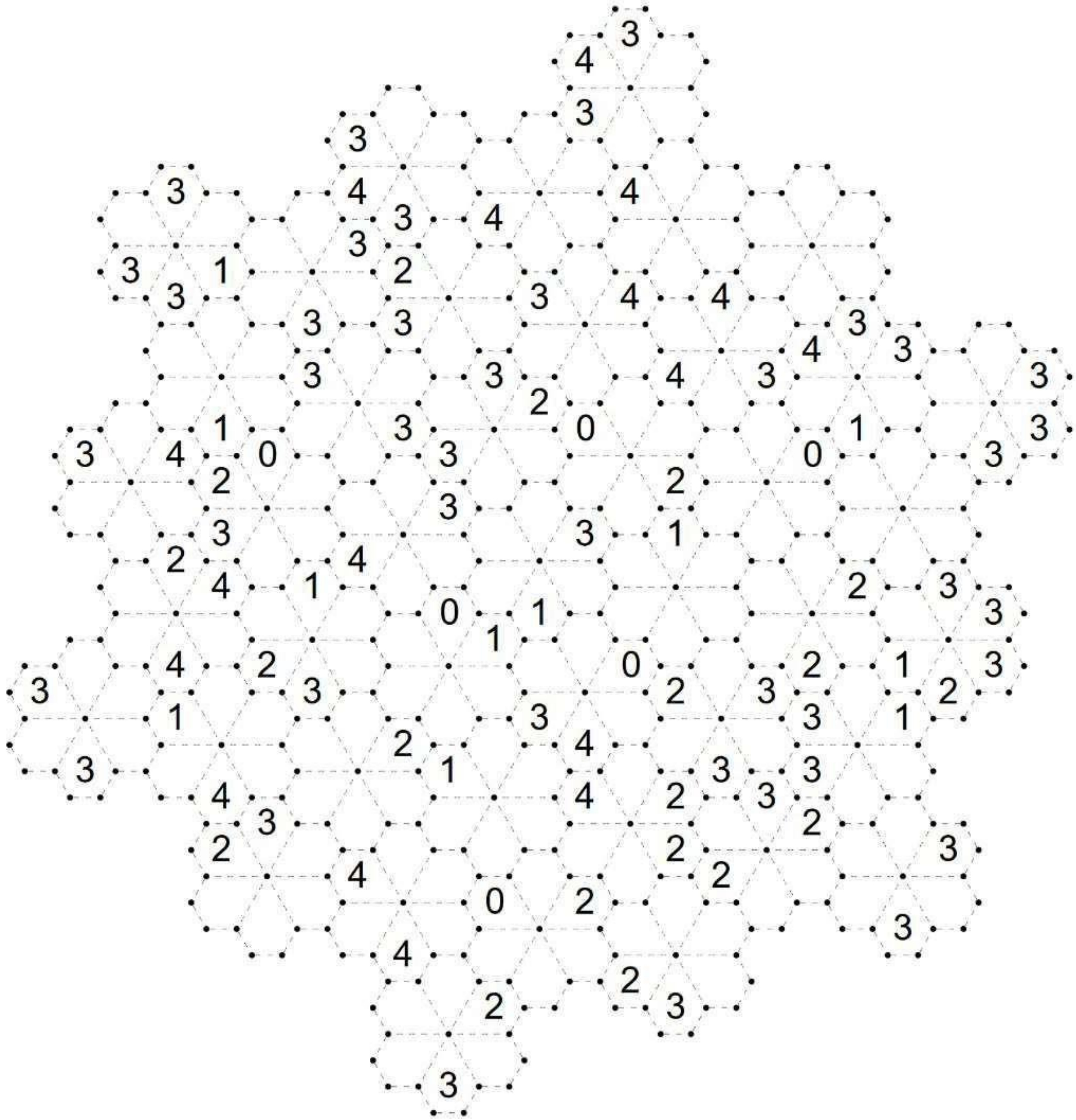




The thing about Slitherlinks is they don’t have to be square! Try these:



This one is tough… but you will see the same patterns you spotted earlier:



## 5 - PSYCHIC PATTERN RECOGNITION

You've made it this far in the book already! You aren't sure whether to continue, so you visit your local psychic to read your fortune and advise you. We knew you would visit. (We're psychic.)

Let your spirit guide you to pick a four-card hand from the 16 Minor Arcana cards below. You can choose whichever cards you want, but you must not repeat any suit (Wands, Cups, Swords, Pentacles) and you must not repeat any rank (Page, Knight, Queen, King).

Once you have made your hand, take the sum of the positions of the four cards. Find the entry in the table on the following page and that will tell you what to do.

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

| SUM OF YOUR CARDS | YOUR FORTUNE… |
|---|---|
| 29 | TURN BACK |
| 30 | BE AFRAID |
| 31 | GO PLAY VIDEO GAMES |
| 32 | DON'T DO IT |
| 33 | HIDE |
| 34 | GO FORTH AND FINISH! GREATNESS AWAITS! |

| | |
|---|---|
| 35 | BURN THIS BOOK (BUT CHECK YOUR MATH FIRST) |
| 36 | AVOID FURTHER PUZZLES |
| 37 | RUN AWAY |
| 38 | ASK YOUR PARENTS |
| 39 | TRY KNITTING INSTEAD |

Seeing the puzzle psychic is a very serious step in your journey.

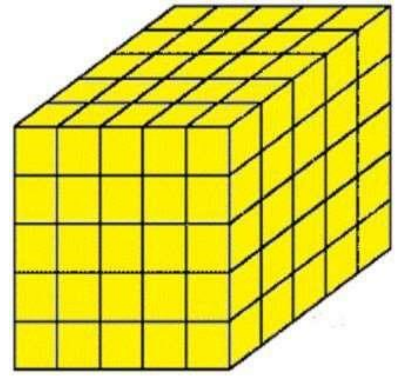
You should think very hard about not following this advice.

## 6 - ABSTRACTION

To abstract means to focus on the larger picture and ignore the smaller details that are rather irrelevant. This section has been abstracted away for your pleasure—refer to the computational thinking section for abstractious thinking.

### 6.1 - Painted Cubicles

A 5x5x5 cube consisting of 125 smaller cubes (let's call them cubicles) is painted yellow so that each of its faces is completely covered.
How many cubicles have exactly 4 faces painted yellow? How many cubicles have exactly 3 faces painted yellow?
How many cubicles have exactly 2 faces painted yellow? How many cubicles have exactly 1 face painted yellow?
How many cubicles have exactly 0 faces painted yellow?
What is the answer for an n x n x n cube?
(That is, can you "abstract away" the specific dimension of the cube to determine a way to solve this for any cube?)



What digit does each square end in? Anything ending in 1, squared, will end in 1.
Anything ending in 2, squared, will end in 4.
We can extrapolate this out for all digits:

| Last digit of n | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Last digit of $n^2$ | 0 | 1 | 4 | 9 | 6 | 5 | 6 | 9 | 4 | 1 |

How many squares have 8 as their units digit?

What is the most popular units digit for a square? Do you see a pattern?

This can be abstracted even further! All last digits, when further multiplied, cycle:

| Last digit of n | Last digit of $n^2$ | Last digit of $n^3$ | Last digit of $n^4$ | Last digit of $n^5$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 6 | 2 |
| 3 | 9 | 7 | 1 | 3 |
| 4 | 6 | 4 | 6 | 4 |
| 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 |
| 7 | 9 | 3 | 1 | 7 |
| 8 | 4 | 2 | 6 | 8 |
| 9 | 1 | 9 | 1 | 9 |

The remarkable thing is that no matter the digit, it cycles completely in, at most, four steps—and, in the case of 0, 1, 5, and 6, the last digit never changes!
How is this useful? In solving problems, you can quickly check whether the multiplication is wrong (but not whether it is right!) by looking at its last digit.

6.2 Which of these statements is true? (Hint: Don't calculate 56.)

a) $15{,}614 = 1 + 5^6 - 1 \times 4$

b) $15{,}615 = 1 + 5^6 - 1 \times 5$

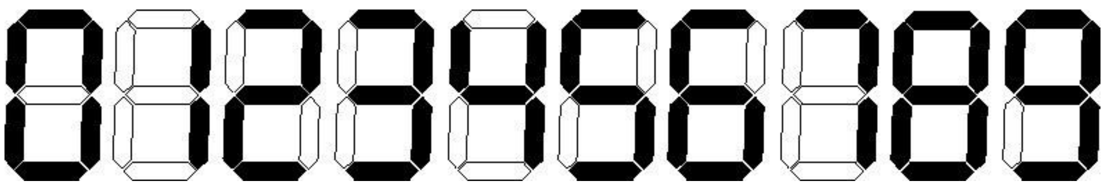
c) $15{,}616 = 1 + 5^6 - 1 \times 6$

d) $15{,}617 = 1 + 5^6 - 1 \times 7$

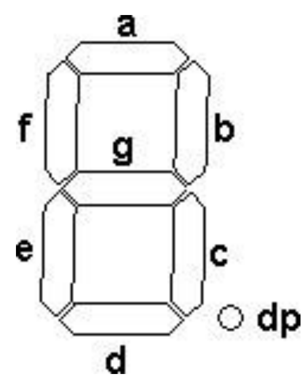
e) $15{,}618 = 1 + 5^6 - 1 \times 8$

6.3 - Seven Segments

Systems that recognize or display digits often use a "7-segment" display like this:



Each digit is made up of two (1) to all seven (8) segments.
Not all segments are necessary to recognize a digit. It is possible to understand a digit if only some of the segments are visible.




Which of the following segments are absolutely necessary in order to identify all of the ten digits (0 through 9, as pictured above) unambiguously?

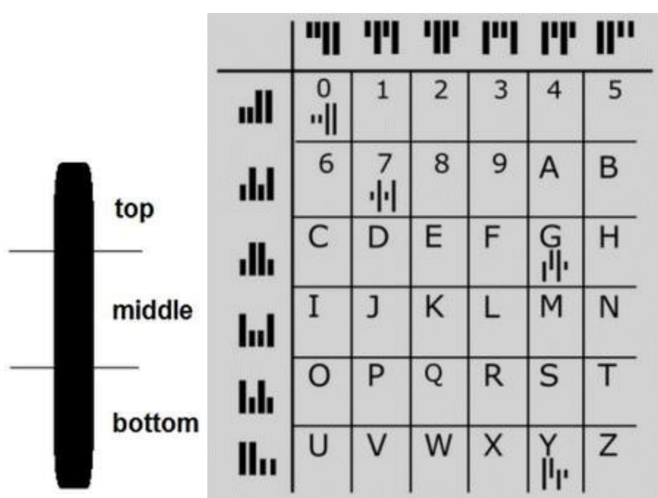
Hint: Each segment can only be on or off… so to display ten digits, what is the minimum number of segments required? (Once you know that, you can work out the required ones & "abstract away" the rest. It might be more than the minimum.)

6.4 STEM Code

The STEM building post office uses address codes that contain four characters. To make the codes readable by machines, they convert the postal codes into STEM codes.

In a STEM code, each character is represented by 4 vertical bars.

A code has 2 sections: upper and lower. The upper section contains only the middle and top bars; the lower section contains only the middle and bottom bars. The table at right shows the codes for several characters:




For example, the STEM code for "G7Y0" is

What is the postal code for this STEM code?

What is the STEM code for N3V9?

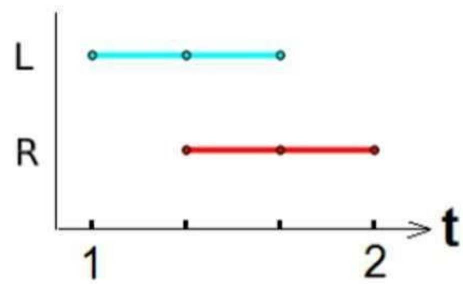
What is the STEM code for K5T2?

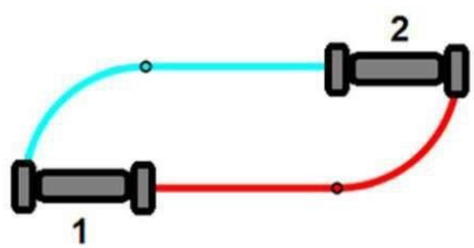
6.5 - Segway Buttons

Esther has a vehicle that looks like a Segway. She moves it by pressing two buttons: a button on the left handle and a button on the right handle.

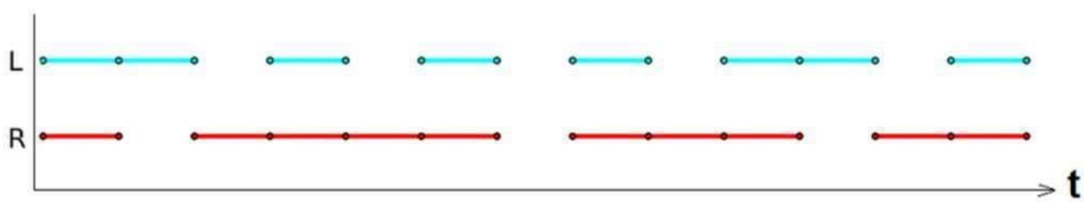
When she presses a button, the wheel on that side of the vehicle rotates: if both buttons are pressed at the same time, both wheels rotate and the vehicle moves forward. If she pushes a single button, only one wheel rotates and the vehicle turns.

For example, the following diagrams show which buttons were pressed and how the vehicle moved:




The vehicle started facing north. Here is a record of the button presses from her journey:



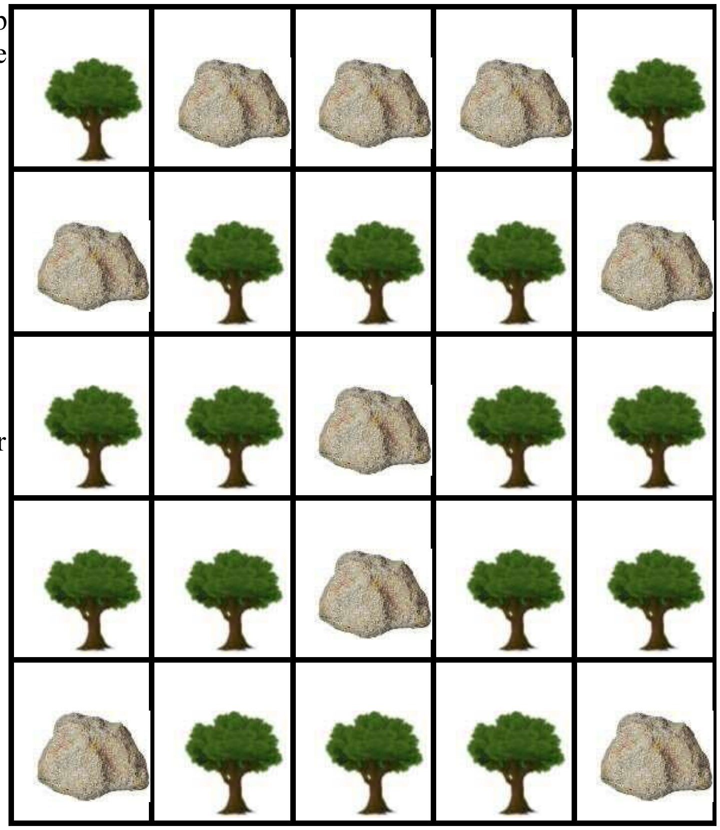

In which direction was the vehicle facing in the end? How far did it travel?

6.6 - The 5 Imps

There are 5 imps living in a forest. Each imp wants to bury its treasure somewhere in the forest.
Forest imps only bury their treasure under trees. Each imp has a map that divides the forest into squares, each of which has a tree or a rock.

Forest imps can see in all directions—including diagonally. They cannot see through rocks, however.

In how many ways can the 5 imps bury their treasure in this forest without seeing each other?



6.7 - Turning Glasses

There are five drinking glasses on a table. One of them, as you can see, is turned upside
down

The goal of this game is to get all of the glasses standing upright. For every turn, you will get the chance to turn exactly three glasses (if the glass is facing up, turn it down, and if the glass is facing down, turn it up).

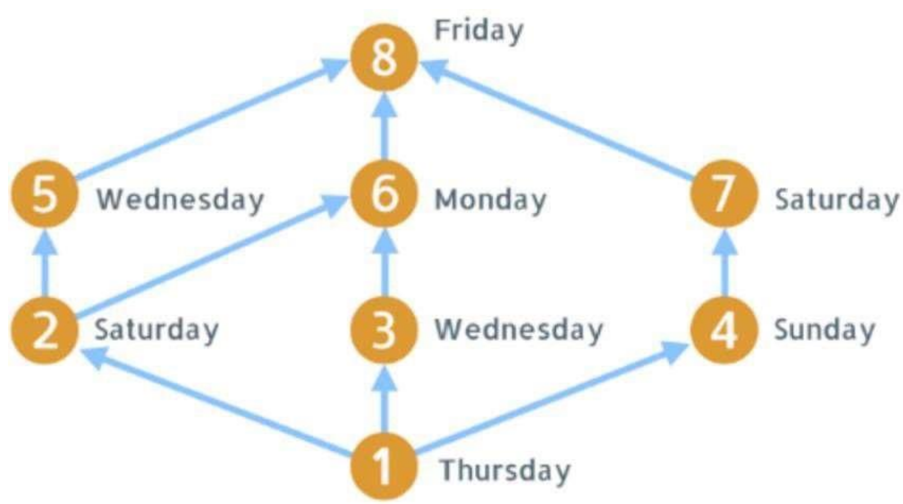
At the minimum, how many turns do you need to get all the glasses standing upright?

A) 2 turns
B) 3 turns
C) 5 turns
D) It is impossible!

6.8 - Stage Coaches

In the Wild West, where the deer and the antelope play, the Fells Cargo Stage Coach Company built a network of stage coach routes between eight settlements.

At right, you can see at which days of the week the stage coaches will depart from the different settlements. For example, the stage coach will leave settlement 2 on Saturday.




A stage coach departs early in the morning and arrives at the next settlement in the evening of the same day.

What is the fastest way to get a package from settlement 1 to settlement 8?

A) 1-2-5-8 C) 1-4-7-8

B) 1-2-6-8 D) 1-3-6-8

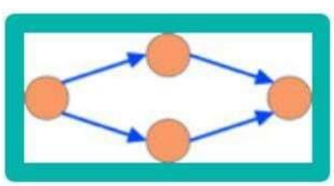
6.9 - Group Assignment

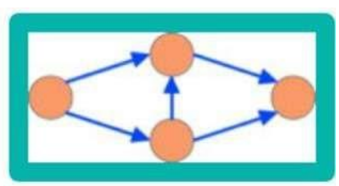
A class is split up into four groups for a group assignment. Each group divides the different tasks between the group members. Three groups manage to finish the complete task, but one group fails to do so. What happened?

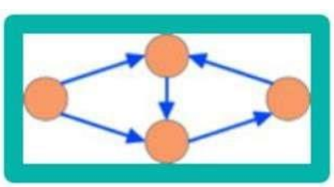
The smartest students, Ada and Charles, have analyzed the four groups. They found out that most group members have to wait for other group members before they can start with their own tasks. Ada and Charles have made a diagram for each group to show which group members every student has to wait for before he/she can start.

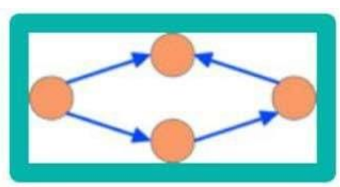
Looking at the diagrams below, (1) a circle represents a student, and (2) An arrow from student 1 to student 2 means that person 2 has to wait for person 1 to finish his/her tasks.:

Which diagram represents the group that did not finish the assignment in time?

A) 

B) 

C) 

D) 

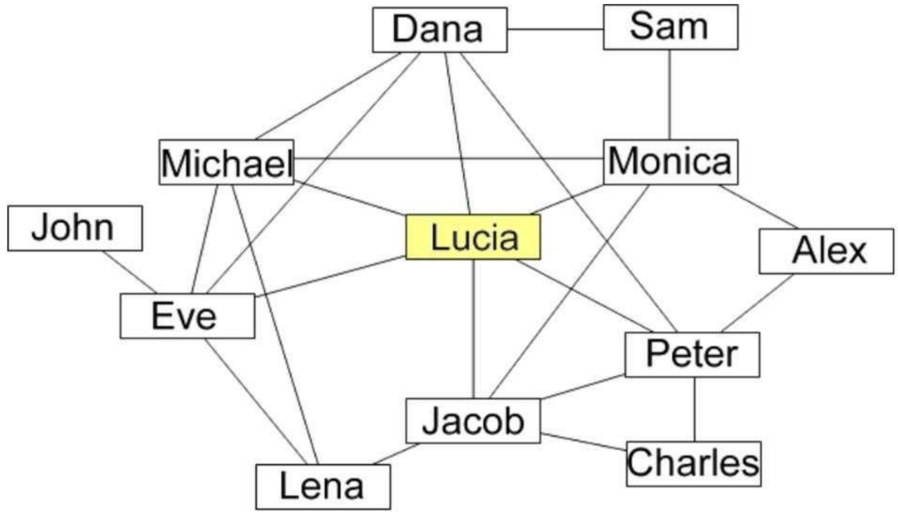
## 7 - ALGORITHM CONSTRUCTION

An algorithm is a precise and concise sequence of instructions that can be used to solve a problem or accomplish a task. (You'll hear that definition again. And again.) Think of it like a numbered, step-by-step procedure from your lab science classes!

7.1 - Social Networks

Lucia and her friends are part of a social network. The diagram below shows some of the friendships in that network. Each line indicates a friendship.

Lucia sends a photo of Jacob's birthday present to some of her friends. Each of those friends sends a photo to all of their friends. Write an algorithm to determine which friends Lucia could send her photo to so that Jacob is not sent the photo.




7.2 - Party planning!

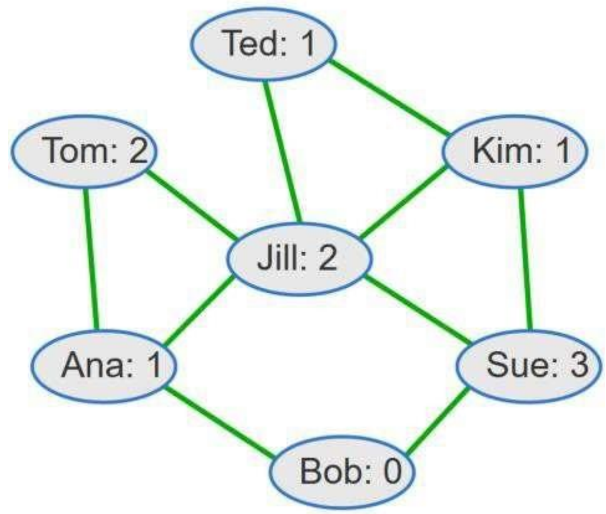
Some friends are planning a party. The diagram shows the relationships—two people are friends with each other if a line connects their names.

For each pair of friends, only one of them is to buy and bring a gift for the other.

The numbers in the diagram show how many gifts each person can buy. No person may buy more than this number of gifts.

Draw an arrowhead on each line to indicate who gives and receives gifts.

Can you think of an algorithm to solve questions like this?

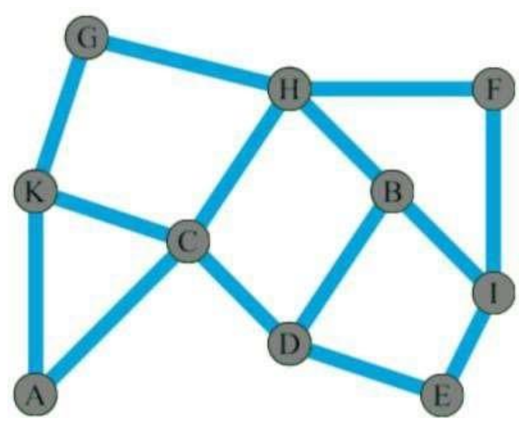



7.3 - Regional Hospitals

Dr. Sutton wants to build three hospitals to service 10 small towns. Each hospital must be located in a town (indicated by a letter in the map). No person in the region should have to travel through more than one town to get to a hospital.

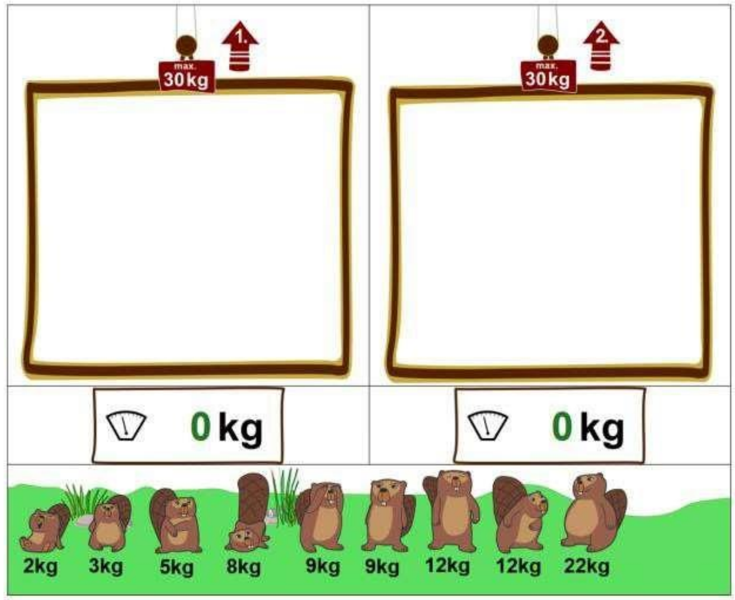
Write an algorithm to determine where the three hospitals should be built.

For the map at right, what is the solution?



7.4 - Beavers in an Elevator

A lot of beavers are using two elevators. Each elevator can hold a maximum of 30 kg. Write an algorithm to maximize the number of beavers on the two elevators.



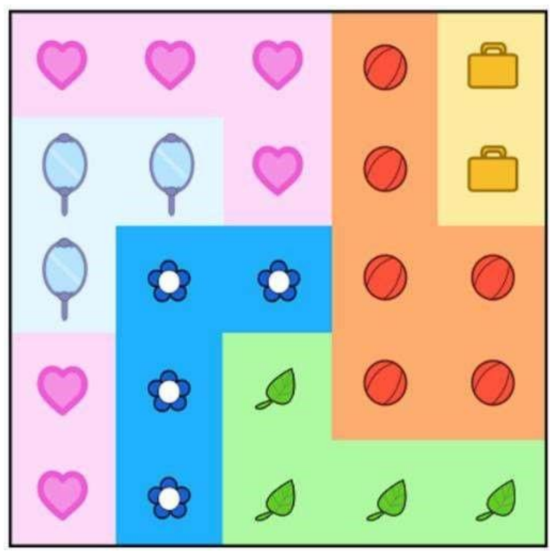

Write an algorithm to maximize the mass of beavers on the two elevators.

7.5 – Wallpapering

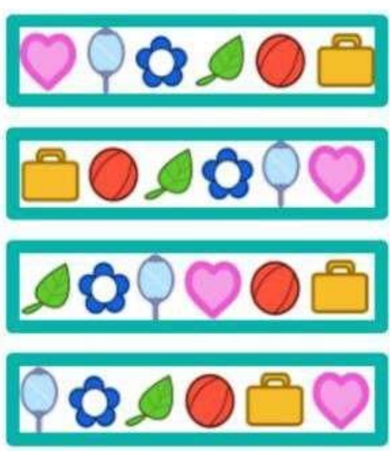
Neelum is wallpapering. She uses one piece of each type of wallpaper on a wall, each piece being different sizes but all rectangular (e.g.. 4x3 squares of a specific pattern and color).

Each wallpaper piece has one color and one pattern on it.

Neelum never puts wallpaper beyond the edge of the wall, but sometimes she covers part of one piece of wallpaper with a piece of a different color. At left is her latest wall:



In which of the orders displayed at right did Neelum put the wallpaper on the wall?



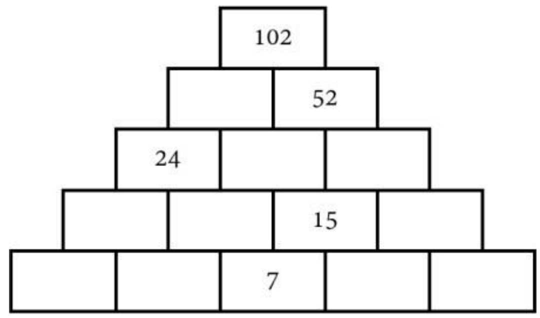
7.6 – Counterfeit

You have three coins, which appear identical, except one of them is counterfeit and does not have the same weight as the other two. You have a set of balance scales, so you can compare coins against each other.

Write an algorithm to find the fake coin in the fewest number of steps.
Given a pile of coins, which are identical except one counterfeit that has a different weight, write an algorithm to find the fake coin in the fewest number of steps.

7.7 - Pyramid

In the partially-completed pyramid at right, each rectangle must be filled in with the sum of the two numbers in the rectangles immediately below it.




Write an algorithm to completely fill in the pyramid.
If you are given one number on each level of the pyramid, can you always solve it? Try to write an algorithm that does so.
Will this work even for larger pyramids, e.g., a 12-level pyramid?

7.8 - Pardon the Interruption

One of the three symbols +, -, x, is inserted somewhere between the digits of 2016 to give a new number. For example, 20-16 gives 4.

How many of the following four numbers can be obtained this way?

36 195 207 320

How many total numbers can be obtained this way?

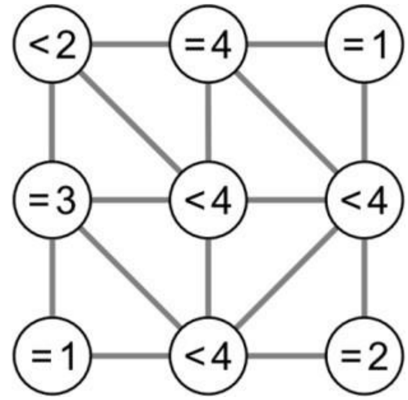
How would you write an algorithm to solve this problem and similar problems?

7.9 - I recently discovered a silly mistake I made in my 700-page novel about prime numbers: I reversed the 1s and the 11s!
Every 1 should be replaced with an 11 and every 11 should be replaced with a 1.
Luckily, I use a Microsoft Word, which empowers me to replace every sequence of characters with a different sequence.

Which of the following should I do to fix my text?

A) Replace every 1 with $, then replace every $ with 11, then replace every 11 with 1.
B) Replace every 11 with 1, then replace every 1 with 11.
C) Replace every 11 with $, then replace every 1 with 11, then replace every $ with 1.
D) Replace every 1 with 11, then replace every 11 with 1.

E) Replace every 11 with $, then replace every $ with 1, then replace every 1 with 11.




7.10 - Coloring

Your job is to color in some of the circles in the picture.
The circles have connections to some of their neighbors.
The number and symbol inside each circle indicate the number of neighbor circles that need to be colored in. For example, a circle with “=3” must have exactly 3 of its connected neighbors colored in.
Similarly, a circle with “<3” must have fewer than 3 of its neighbors colored in.
Write an algorithm for coloring in the required circles.

When you don’t have a calculator and are dealing with large numbers, how do you break problems into calculatable bits? Can you find an efficient way to answer some of the questions below?

7.11 - Wood Streams

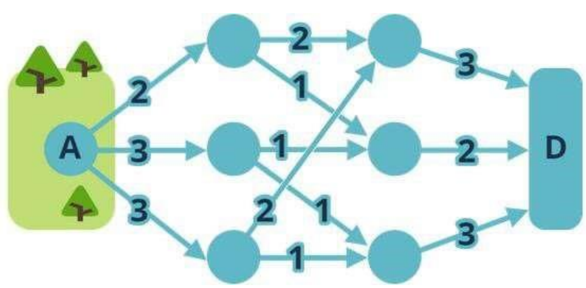
In forest (A) is an area where the beavers fell trees for their dams. They transport the tree trunks to their new project – the biggest dam of all time (D) – through an infrastructure of channels.

The arrows represent the channels, the dots are points where the water splits up or comes together.

Every channel has a restricted capacity.

The numbers next to the channels show how many tree trunks can be transported through the channels in one minute.

How many tree trunks can be transported from A to D at most in one minute?



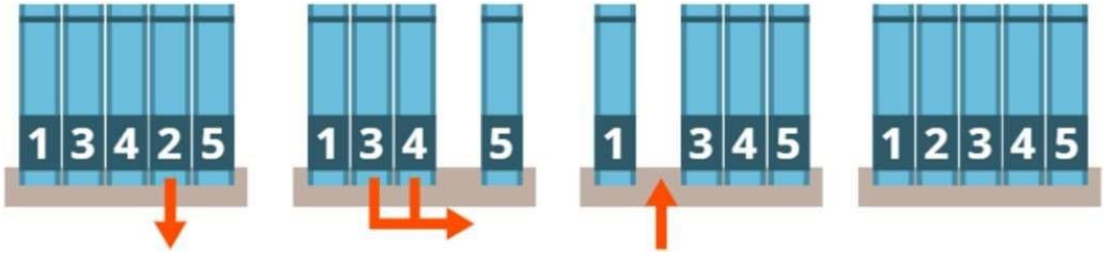

7.12 Book Dance

Abhinav has a grand collection of encyclopedias. But his encyclopedias are disorganized, and he would like to put the different parts of the encyclopedia in the right order.

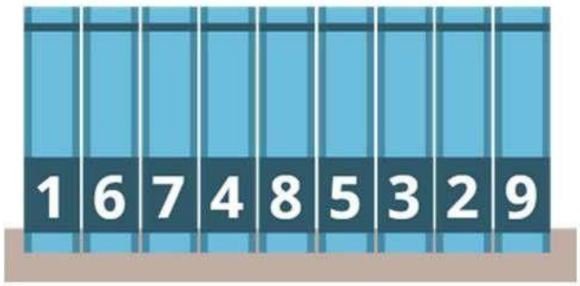
Abhinav repeats the following step to accomplish his goal:

- He takes a book off the shelf.
- He moves one or more books to the left or the right.
- He puts the book he took off the shelf back in the open spot.

Below is an example of what he did to place five sections of Encyclopedia Collection A using the process above:




On another day, Abhinav decides to organize Encyclopedia Collection B, which has nine parts to it.




What is the minimum number of steps Abhinav must take to get the books in Encyclopedia Collection B in the right order?

# CRYPTOLOGY

Cryptograms involve having a “Plain text” message encrypted in some way to become the “Cipher text”, which is text that cannot be easily understood until it is decrypted.This is used to send messages where people other than the sender or receiver cannot understand the contents of the super-secret message.

There are over 200 common methods of encrypting, ranging from using math/modulo (RSA, Vigenère, Affine), morse code (Pollux, Morbit), matrices (Hill), or just simply swapping letters (Cryptograms). Ciphers such as RSA ciphers are the algorithms used for transmitting secure information across the internet.

These might be helpful:
Most used letters in order: E, T, A, O, I, N Most
used words: the, be, to, of, you

## 1 - Caesar Cipher

This is the most basic form of a cipher, where you essentially “shift” the letters in a set direction. For example, with a shift of 5, A would be replaced with F because A is the 1st letter, 1 + 5 = 6, and the 6th letter is F.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

↓

shifted 5

V W X Y Z A B C D E F G H I J K L M N O P Q R S T U

For example:

| | | |
|---|---|---|
| Plain Text | A | ALLENDER TEACHES PSYCH AND FORENSICS |
| Cipher Text | F | FQQJSIJW YJFHMJX UXDHM FSI KTWJSXNHX |

Try encoding the following with a shift of 9:

THE CLUBS AT STEM INCLUDE SCIENCE OLYMPIAD, FBLA, HOSA AND TSA.

Decode (go in the other direction) the following message that has a shift of 5:

DTZ XMTZQI ANXNY JAFS'X HWJJP XTRJ YNRJ!

Challenge: Decode the following message with an unknown shift:
Hint: There are only 25 possible shifts as there are only 26 letters in the alphabet. All other numbers for shifts just become multiples of 26.

UGFKAVWJAFY QGM OWJW YGGV WFGMYZ LG VWUJQHL LZAK EWKKSYW,

QGM KZGMDV UGFKAVWJ BGAFAFY KUAWFUW GDQEHASV!

## 2 - Vigenère Cipher:

The Vigenère Cipher uses a keyword to shift letters variably (unlike Caesar's fixed shift). Each letter of the keyword determines the shift for the corresponding plaintext letter (A=0, B=1, etc.). The keyword repeats to match the message length.

**Example:**

Plaintext: ATTACK
Keyword: LEMON (L=11, E=4, M=12, O=14, N=13)
Ciphertext: LXFOPV

**Basic Encryption:**

Plaintext: HELLO

Keyword: KEY

**Easy Challenges:**

1. Ciphertext: VXUKG LWRRG KZAXQ FLQ
   Keyword: BEAKER

2. Ciphertext: QSGVI ZMGIQ BAMMZ XG
   Keyword: NEWTON

3. Ciphertext: YFHPJ QTLWJ RJSET HZQ
   Keyword: CELL

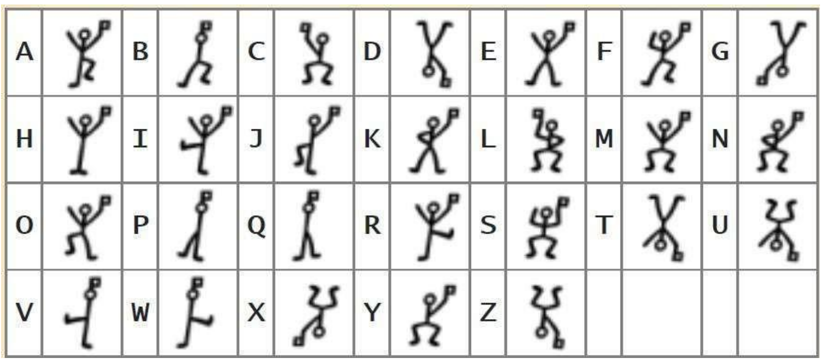
**Hard Challenges:**

1. Ciphertext: WVPZM ZYBTK XGWQL IIQVX UOVVD RZSE
   Keyword: BLACKHOLE

2. Ciphertext: HRVTX MTRQV XUVRV STXOV KGZXG XTQXS QVJXU VRVST XOVKG ZXGXT QXS
   Keyword: CIPHER

## 3 - Dancing Man

The dancing man cipher is relatively simple: each letter is correlated to a cute drawing of a stick figure, the “dancing man”.



(Key from dcode.fr)

D A N C I N G M E N

Decode the following using the Dancing Man cipher:

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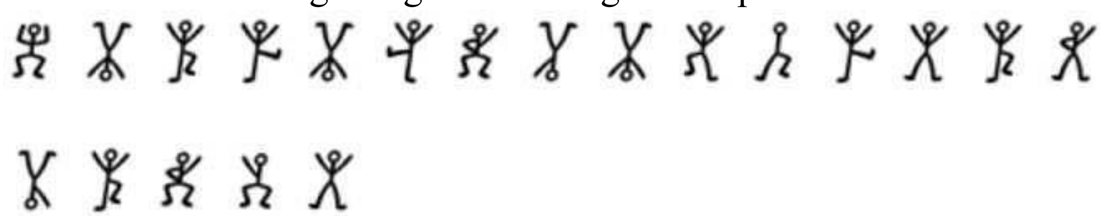

Encode the following message using the Dancing Man cipher

Y O U ' V E  G O T T A  D A N C E  L I K E

# THERE'S NOBODY WATCHING

-William W. Purkey

## 4 - Traditional Cryptogram

DKN JAUNTXY RWO BZMN UT LWPPNCN ZAN JAUNTXY RWO'PP KZFN

JWA PUJN, NFNT UJ RWO XWT'D DZPM JWA RNZAY ZD Z DUBN.

A few letters have been given as a hint…

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 6 | 2 | 1 | 5 | | 2 | | | | 6 | 2 | 1 | 2 | 13 | 3 | 6 | | 4 | | 5 | 6 | | 7 | 3 | 3 | 7 |
| Replacement | R | | | | | | | | | | | C | | E | | | | | | | | | | | | A |

NRD KMVBI OX UMWOJE XM SQXN NRDXD IQLX NRQN NRD UQJ KRM

XQLX ON GQJ'N TD IMJD OX EDJDVQBBL OJNDVVCFNDI TL XMUDMJD
IMOJE ON.

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | | 3 | 1 | 12 | 3 | 1 | 1 | | 5 | 8 | 2 | 4 | 8 | 11 | 7 | | 7 | 5 | 1 | 2 | 3 | 4 | 1 | 9 | | |
| Replacement | | | | | | | | | D | | | | | | I | | | | | | | | | | | |

RWS RWBUNV ZQO CQU'R JUQP QA OUCSAVRFUC FAS FV BGXQARFUR
FV ZQOA CSVBAS RQ JUQP RWSG. RWBV BV RWS ASTFRBQUVWBX QI
GFU RQ GZVRSAZ.

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 8 | 7 | 4 | | | 7 | 4 | | 1 | 2 | | | | 1 | 3 | 2 | 11 | 13 | 9 | 1 | 9 | 9 | 6 | 2 | | 4 |
| Replacement | | | | | | | | | | | | | | | | | | | | | | | | | | |

IKNCT MHEG KAF MFOCFPI THIKFB SR MX USDFNRFIIFI, IEASSOI, HRB
IKHKFI, XSJ PCRB CRBFOCMOF KNJKAI HK SRF'I ESNF.

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 3 | 4 | 5 | 1 | 3 | 12 | 1 | 5 | 12 | 2 | 8 | | 4 | 4 | 4 | 2 | | 6 | 7 | 2 | 1 | | | 2 | | 4 |
| Replacement | | | | | | | | | | | | | | | | | | | | | | | | | | |

OL ITMDQ NMPO CWZOLC OLWC W UGWST RLGSE QLNTDGFOLQ RH

GDDLEWD RMDDQTALCF ZOWS W NTAWCZ PTSPLCZT

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 2 | | 6 | 7 | 2 | 2 | 4 | 1 | 1 | | | 9 | 3 | 3 | 6 | 3 | 4 | 3 | 4 | 7 | 1 | | 8 | | | 4 |
| Replacement | | | | | | | | | | | | | | | | | | | | | | | | | | |

## 5 – Additional Ciphers

As mentioned earlier, there are seemingly infinite possibilities of ciphers and over 200 common cipher types. Here are some more fun ciphers you can solve. We recommend searching them up to see if you are interested in learning how to decrypt them!

5.1 - Decode the following message encoded with a Morbit Cipher. Use the following table:

| ●● | ●– | ●× | –● | –– | –× | ×● | ×– | ×× |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

4 3 1 8 1 9 4 5 8 5 7 2 9 1 6 1 3 3 7 6 8 1

7 8 2 3 5 6 4 3 3 2 3 8 8 5 9 1 3 5 6 2 1 7

1 6 3 8 7 1 3 1 7 1

5.2 - Decode the following message encoded using the Hill Cipher and using the matrix:

$$\begin{pmatrix} N & U \\ M & B \end{pmatrix}$$

P P O Y M L K S X W G H

5.3 - Decode the following message encoded with a Baconian Cipher

ABAAA ABABA ABABA AAAAB BAABB ABAAA ABABA

AAABB AAAAA ABBAA AAABB AABAA ABABB ABBBA

ABAAA BAAAA AABAA

5.4 - Decode the following message encoded with a Porta Cipher with the key WRECK X

Q W G L P M I Z J T D D X M E N D B R F A I Y Z P I O Y D O N C P

5.5 - Decode the following Patristocrat (Cryptogram without Spaces)

BPQBS PRUQW CASPR UASPJ UPHEF PESQO PMWPV JQKPI ESSPV AOWEO

PMOJA SRPAO NPOJP VWCEF JBUQO PWEOV WQIAE SCPVA E

- Mokokoma Mokhonoana

5.6 - Decode the following message encoded with a Beaufort Cipher and with the key FEEL

HQK TTWTH UWUH HQKUB XWIXRY TRSASYWRF

5.7 - Decode the following message encoded with an Affine cipher knowing a=11 and b=8

JHSY YAEJSGV SY I HUWA IP LGN YESAVEA GZMKRSIP IVP

EGPATUYJANY

Ciphers in this section were made with toebes.com/codebusters and dcode.fr

## 5 - Cryptarithms

Cryptarithms involve swapping each letter with a number from 0 to 9, so that the equation makes sense. Each digit (0-9) can only be used once, and pay attention to the operation (addition, subtraction, etc.) See the example on the right.

SUN 1U6 136
\+ FUN + F3N + 936
JUNE

Maybe a little less of a cipher, but fun nonetheless.

Try some for yourself!

PHYSIC
\- SLEEP
CHEM

SEND
\+ MORE
MONEY

Hint: O is 0

NOON
MOON
\+ SOON

SWIM S0IM 1072

CROSS
+ ROADS
DANGER

Hint: S is 3

BOB x
BOB
MARLEY

SCIENCE
+ CHEM
STUDENT

LETS
+ WAVE
LATER

BONOMO
+ LESLIE
ERIKSEN

OSTLIE
+ LESLIE
NGUYEN

Hint: E

# SERPENT CIPHER:

So this is a serpent cipher which is basically when you decrypt encrypted text that is in the shape of a pyramid. For example:

THE WORD:
THE SECRET MEETING
GETS ENCRYPTED TO:

```
  T   S   E   E   G
 H  E R T E T N G
    E C     M   I
```

The Serpent Trail Cipher is a way of hiding messages using a zigzag writing pattern. To encode a message, first remove spaces and punctuation, then decide how many rows to use (three is common). Write the message in a serpent pattern that moves down and up the rows like this: row 1 -> row 2 -> row 3 -> row 2 -> row 1, and repeat. Fill one letter per column following this motion. Once the message is fully written out in the pattern, read it vertically column by column (top to bottom) to get the final cipher text. To decode it, you need to know the number of rows used and the original zigzag path. Reconstruct the letter placement in the pattern and read it by tracing the same serpent path to reveal the original message. This cipher can be made harder by adding decoy letters or hiding the number of rows.