

Part IA — Vector and Matrices

Based on lectures by Prof N. Peake

Notes taken by Marcus Ng

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Complex numbers

Review of complex numbers, including complex conjugate, inverse, modulus, argument and Argand diagram. Informal treatment of complex logarithm, n -th roots and complex powers. de Moivre's theorem. [2]

Vectors

Review of elementary algebra of vectors in \mathbb{R}^3 , including scalar product. Brief discussion of vectors in \mathbb{R}^n and \mathbb{C}^n ; scalar product and the Cauchy-Schwarz inequality. Concepts of linear span, linear independence, subspaces, basis and dimension.

Suffix notation: including summation convention, δ_{ij} and ε_{ijk} . Vector product and triple product: definition and geometrical interpretation. Solution of linear vector equations. Applications of vectors to geometry, including equations of lines, planes and spheres. [5]

Matrices

Elementary algebra of 3×3 matrices, including determinants. Extension to $n \times n$ complex matrices. Trace, determinant, non-singular matrices and inverses. Matrices as linear transformations; examples of geometrical actions including rotations, reflections, dilations, shears; kernel and image. [4]

Simultaneous linear equations: matrix formulation; existence and uniqueness of solutions, geometric interpretation; Gaussian elimination. [3]

Symmetric, anti-symmetric, orthogonal, hermitian and unitary matrices. Decomposition of a general matrix into isotropic, symmetric trace-free and antisymmetric parts. [1]

Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors; geometric significance. [2]

Proof that eigenvalues of hermitian matrix are real, and that distinct eigenvalues give an orthogonal basis of eigenvectors. The effect of a general change of basis (similarity transformations). Diagonalization of general matrices: sufficient conditions; examples of matrices that cannot be diagonalized. Canonical forms for 2×2 matrices. [5]

Discussion of quadratic forms, including change of basis. Classification of conics, cartesian and polar forms. [1]

Rotation matrices and Lorentz transformations as transformation groups. [1]

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