

Part IA — Numbers and Sets

Based on lectures by Prof J. Wolf

Notes taken by Marcus Ng

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Introduction to number systems and logic

Overview of the natural numbers, integers, real numbers, rational and irrational numbers, algebraic and transcendental numbers. Brief discussion of complex numbers; statement of the Fundamental Theorem of Algebra.

Ideas of axiomatic systems and proof within mathematics; the need for proof; the role of counter-examples in mathematics. Elementary logic; implication and negation; examples of negation of compound statements. Proof by contradiction. [2]

Sets, relations and functions

Union, intersection and equality of sets. Indicator (characteristic) functions; their use in establishing set identities. Functions; injections, surjections and bijections. Relations, and equivalence relations. Counting the combinations or permutations of a set. The Inclusion-Exclusion Principle. [4]

The integers

The natural numbers: mathematical induction and the well-ordering principle. Examples, including the Binomial Theorem. [2]

Elementary number theory

Prime numbers: existence and uniqueness of prime factorisation into primes; highest common factors and least common multiples. Euclid's proof of the infinity of primes. Euclid's algorithm. Solution in integers of $ax + by = c$.

Modular arithmetic (congruences). Units modulo n . Chinese Remainder Theorem. Wilson's Theorem; the Fermat-Euler Theorem. Public key cryptography and the RSA algorithm. [8]

The real numbers

Least upper bounds; simple examples. Least upper bound axiom. Sequences and series; convergence of bounded monotonic sequences. Irrationality of $\sqrt{2}$ and e . Decimal expansions. Construction of a transcendental number. [4]

Countability and uncountability

Definitions of finite, infinite, countable and uncountable sets. A countable union of countable sets is countable. Uncountability of \mathbb{R} . Non-existence of a bijection from a set to its power set. Indirect proof of existence of transcendental numbers. [4]

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