

Part IA — Groups

Based on lectures by Prof H. Winton

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Michaelmas 2023

Examples of groups

Axioms for groups. Examples from geometry: symmetry groups of regular polygons, cube, tetrahedron. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The Möbius group; cross-ratios, preservation of circles, the point at infinity. Conjugation. Fixed points of Möbius maps and iteration. [4]

Lagrange's theorem

Cosets. Lagrange's theorem. Groups of small order (up to order 8). Quaternions. Fermat-Euler theorem from the group-theoretic point of view. [5]

Group actions

Group actions; orbits and stabilizers. Orbit-stabilizer theorem. Cayley's theorem (every group is isomorphic to a subgroup of a permutation group). Conjugacy classes. Cauchy's theorem. [4]

Quotient groups

Normal subgroups, quotient groups and the isomorphism theorem. [4]

Matrix groups

The general and special linear groups; relation with the Möbius group. The orthogonal and special orthogonal groups. Proof (in \mathbb{R}^3) that every element of the orthogonal group is the product of reflections and every rotation in \mathbb{R}^3 has an axis. Basis change as an example of conjugation. [3]

Permutations

Permutations, cycles and transpositions. The sign of a permutation. Conjugacy in S_n and in A_n . Simple groups; simplicity of A_5 . [4]

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