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MA4016 - Engineering Mathematics 6

Solution Sheet 3: Recursion (February 19, 2010)

1. **a)** $f(n) = (-1)^n$

b) $f(n) = \begin{cases} 2^{n-1} & n = 3k \text{ for an integer } k \\ 0 & n = 3k+1 \text{ for an integer } k\ 5 \\ 2^n & n = 3k+2 \text{ for an integer } k \end{cases}$

c) not valid

d) not valid

e) $f(n) = \begin{cases} 2 & \text{for odd } n \\ 2^{n/2+1} & \text{for even} n \end{cases}$

3. **a)**

$$f_{n+2}^2 - f_{n+1}^2 = (f_{n+2} - f_{n+1})(f_{n+2} + f_{n+1}) = f_n f_{n+3}$$

b) Inductive step

$$\begin{aligned} f_k f_{k+2} + (-1)^{k+2} &= f_k(f_k + f_{k+1}) + (-1)^{k+2} = f_k f_{k+1} + f_k^2 + (-1)^{k+2} \\ &= f_k f_{k+1} + f_{k-1} f_{k+1} + (-1)^{k+1} + (-1)^{k+2} \\ &= (f_k + f_{k-1}) f_{k+1} = f_{k+1}^2 \end{aligned}$$

c)

$$\begin{aligned} f_{n-2} f_{n+2} + (-1)^n = f_{n-2}(f_n + f_{n+1}) &= \underbrace{f_n f_{n-2}}_{3b)} + \underbrace{f_{n-2} f_{n+1}}_{3a)} \\ &= f_{n-1}^2 + (-1)^{n+1} + f_n^2 - f_{n-1}^2 = (-1)^{n+1} + f_n^2 \end{aligned}$$

d) Inductive step

$$\sum_{k=1}^{m+1} f_k^2 = f_{m+1}^2 + \sum_{k=1}^{m} f_k^2 = f_{m+1}^2 + f_m f_{m+1} = (f_{m+1} + f_m) f_{m+1} = f_{m+1} f_{m+2}$$

4. You can tile a 2×1 board in exactly one way and a 2×2 board in two ways. So $T(1) = 1$, $T(2) = 2$. Tiling a $n \times 2$ board with $n \geq 3$ can be done with a vertical tile at the end or two horizontal ones. Thus $T(n) = T(n-1) + T(n-2)$. Comparing to the recurrence relation of the Fibonacci numbers gives the solution.