

CIS102 Tutorial 8 Answers

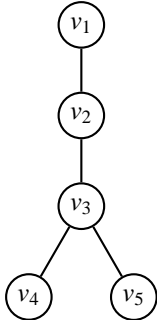
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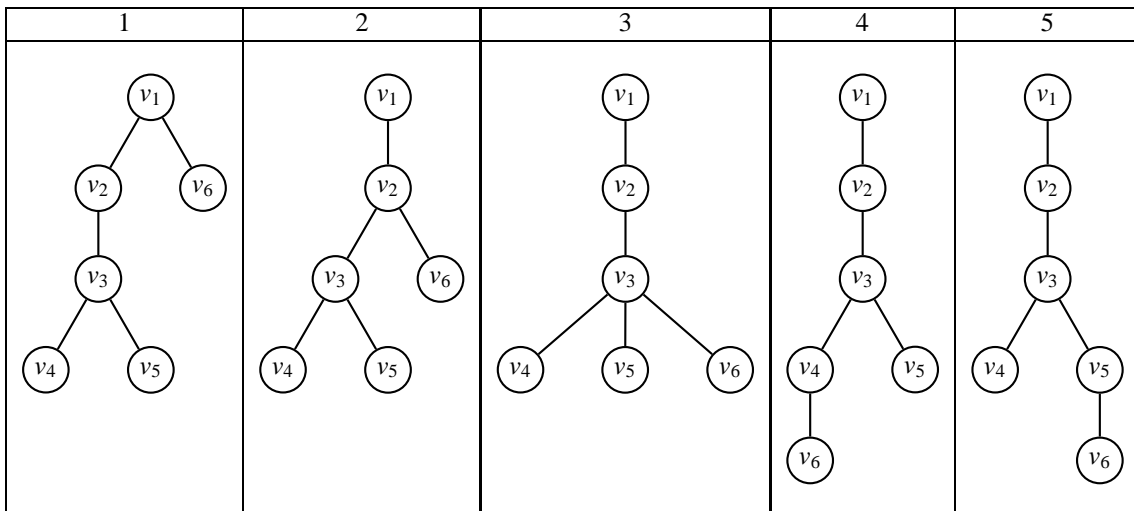
1. In order for G to be a tree it needs to be:

- connected;
- acyclic.

2. (a)



(b)

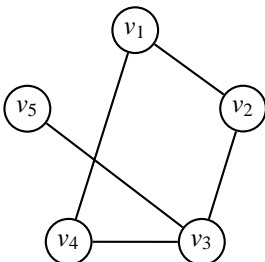


(c) The non-isomorphic trees in 2b are 1, 2, 3 and either 4 or 5.

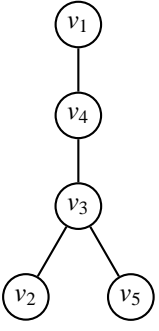
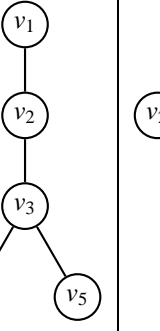
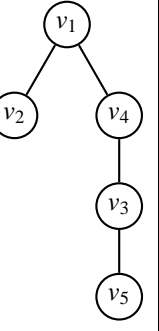
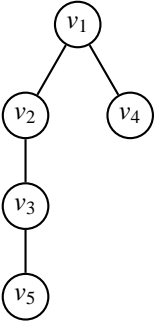
3. If G is a graph with 50 vertices and T is a spanning tree of G then:

- T has 50 vertices;
- T has 49 edges.

4.

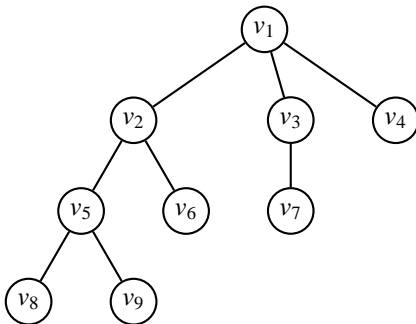


(a)

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 3, 2, 1, 1, 1 | 3, 2, 1, 1, 1 | 2, 2, 2, 1, 1 | 2, 2, 2, 1, 1 |
|  |  |  |  |

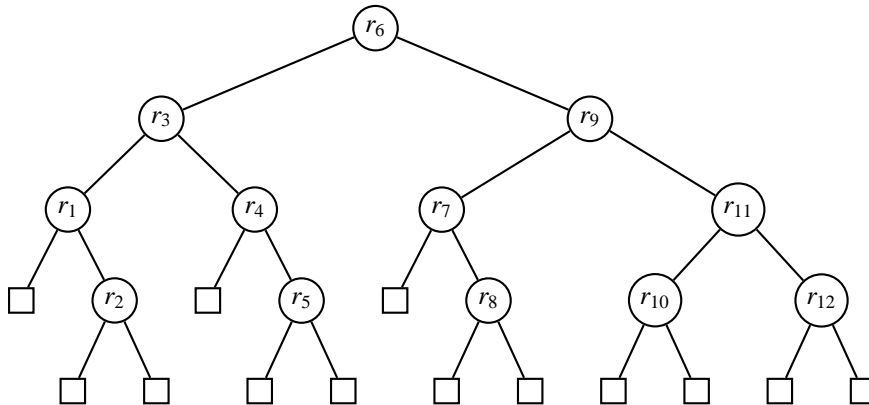
(b) The non-isomorphic graphs are (1 or 2) and (3 or 4).

5.



- (a) Node a is an internal node of T if node a has one or more descendants. *e.g.* in the tree above v_1 , v_2 , v_3 and v_5 are internal nodes.
- (b) Node a is an external node of T if node a has no descendants. *e.g.* in the tree above v_4 , v_6 , v_7 , v_8 and v_9 are external nodes.
- (c) The height of T , h , is the length of the longest path in T starting at the root. *e.g.* in the tree above the height is 3.
- (d) If a is an ancestor of b then the unique path from the root to b passes through a .

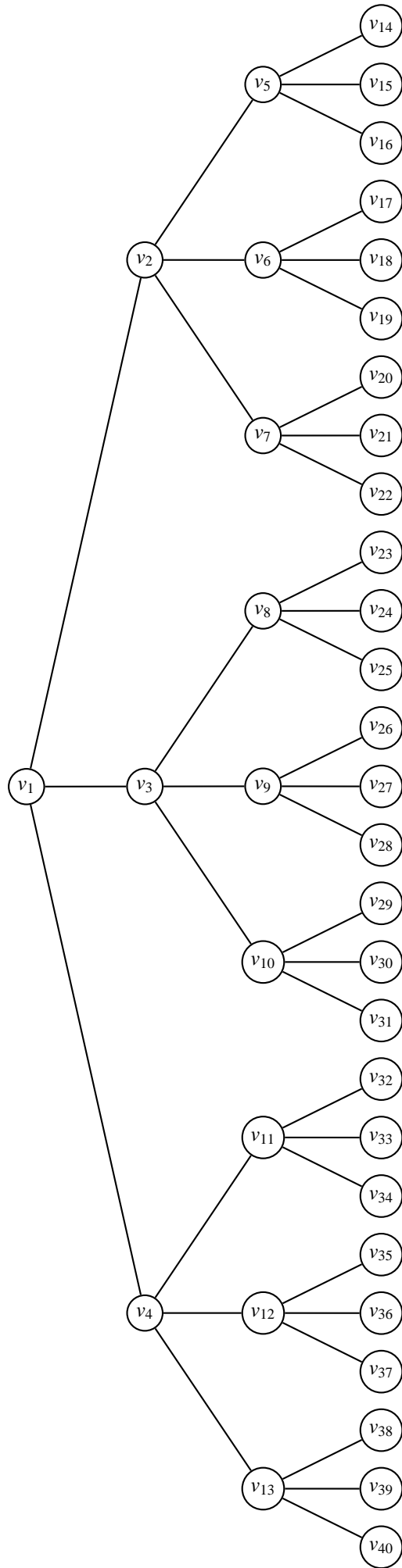
6. (a)



- (b) The ancestors of r_5 are r_6 , r_3 and r_4 .
- (c) The nodes at level 2 are r_1 , r_4 , r_7 and r_{11}
- (d) The maximum number of comparisons that the computer would need to make to match any existing record in a binary tree is $\lceil \log_2 n \rceil$, where n is the number of nodes in the tree. In this case it would be $\lceil \log_2 12 \rceil = 4$
- (e) The external nodes (*leaf* nodes) indicate that there are no further records on that particular path. *i.e.* if you are at an external node whose value is not the same as the record you are looking up, then there is no matching record in the tree. They also allow for the insertion of new records at those points.

7. $\lceil \log_2 2534 \rceil = 12$

8. (a)



- (b) For a n -ary tree of depth h where all the external nodes lie on level h , the number of nodes at level k , where $0 \leq k \leq h$, is n^k .
 $3^6 = 729$
- (c) The height, h of an n -ary tree with m nodes is $\lceil \log_n m \rceil \leq h \leq m$
 $\lceil \log_3 150 \rceil \leq h \leq 150$
 $5 \leq h \leq 150$