Chapter 1

Session 1

Part 1 : Binary numbers

Question 1

Working in base 2 and showing all your workings, compute the following. $(10110)_2 \times (111)_2$

- Express the binary number $(1101.101)_2$ as a decimal, showing all your workings
- Express the decimal number $(3599)_{10}$ in base 2

(a) Express the following binary numbers as decimal numbers

 (i) 11011

 (ii) 100101

(b) Express the following decimal numbers as binary numbers

 (i) 6

 (ii) 15

 (iii) 37

(c) Perform the following binary additions

 (i) $1011 + 1111$

 (ii) $10101 + 10011$

 (iii) $1010 + 11010$

Part 1b : Hexadecimal numbers

(i) Calculate the decimal equivalent of the hexadecimal number $(A2F.D)_{16}$

(ii) Working in base 2, compute the following binary additions, showing all you workings

$$(1110)_2 + (11011)_2 + (1101)_2$$

(iv) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part E: Miscellaneous Questions

(i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation

(ii) From part (i), is x^8 a rational number?

Part A : Binary numbers

(a) Express the following binary numbers as decimal numbers

  (i) 11011
  (ii) 100101

(b) Express the following decimal numbers as binary numbers

  (i) 6
  (ii) 15
  (iii) 37

(c) Perform the following binary additions

  (i) $1011 + 1111$
  (ii) $10101 + 10011$
  (iii) $1010 + 11010$

Part A: Number Systems - Binary Numbers

1. Express the following decimal numbers as binary numbers.

   i) $(73)_{10}$ ii) $(15)_{10}$ iii) $(22)_{10}$

   All three answers are among the following options.

   a) $(10110)_2$ b) $(1111)_2$ c) $(1001001)_2$ d) $(1000010)_2$

2. Express the following binary numbers as decimal numbers.

   a) $(101010)_2$ b) $(10101)_2$ c) $(111010)_2$ d) $(11010)_2$

3. Express the following binary numbers as decimal numbers.

   a) $(110.10101)_2$ b) $(101.0111)_2$ c) $(111.01)_2$ d) $(110.1101)_2$

4. Express the following decimal numbers as binary numbers.

   a) $(27.4375)_{10}$ b) $(5.625)_{10}$ c) $(13.125)_{10}$ d) $(11.1875)_{10}$

Part B: Number Systems - Binary Arithmetic

1. Perform the following binary additions.

   a) $(110101)_2 + (1010111)_2$
   b) $(1010101)_2 + (101010)_2$
   c) $(11001010)_2 + (10110101)_2$
   d) $(1011001)_2 + (111010)_2$

2. Perform the following binary subtractions.

   a) $(110101)_2$ - $(1010111)_2$
   b) $(1010101)_2$ - $(101010)_2$
   c) $(11001010)_2$ - $(10110101)_2$
   d) $(1011001)_2$ - $(111010)_2$

3. Perform the following binary multiplications.

   a) $(1001)_2 \times (1000)_2$
   b) $(101)_2 \times (1101)_2$
   c) $(111)_2 \times (1111)_2$
   d) $(10000)_2 \times (11001)_2$

4. Perform the following binary multiplications.

   (a) Which of the following binary numbers is the result of this binary division: $(10)_2 \times (1101)_2$.

   a) $(11010)_2$
   b) $(11100)_2$
   c) $(10101)_2$
   d) $(11011)_2$

   (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \times (111)_2$.

   a) $(11000)_2$
   b) $(11001)_2$
   c) $(10101)_2$
   d) $(11011)_2$

   (c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \times (1101)_2$.

   a) $(11000)_2$
   b) $(11001)_2$
   c) $(10101)_2$
   d) $(11011)_2$

5. Perform the following binary divisions.

   (a) Which of the following binary numbers is the result of this binary division: $(111001)_2 \div (10011)_2$.

   a) $(10)_2$
   b) $(11)_2$
   c) $(100)_2$
   d) $(101)_2$

   (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \div (111)_2$.

a) $(11)_2$

b) $(100)_2$

c) $(101)_2$

d) $(110)_2$

(c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \div (1101)_2$.

a) $(100)_2$

b) $(110)_2$

c) $(111)_2$

d) $(1001)_2$

Part C: Number Bases - Hexadecimal

1. Answer the following questions about the hexadecimal number systems

   a) How many characters are used in the hexadecimal system?

   b) What is highest hexadecimal number that can be written with two characters?

   c) What is the equivalent number in decimal form?

   d) What is the next highest hexadecimal number?

2. Which of the following are not valid hexadecimal numbers?

   a) 73   b) $A5G$ c) 11011   d) EEF

3. Express the following decimal numbers as a hexadecimal number.

   a) $(73)_{10}$ b) $(15)_{10}$ c) $(22)_{10}$ d) $(121)_{10}$

4. Compute the following hexadecimal calculations.

   a) $5D2 + A30$ b) $702 + ABA$ c) $101 + 111$ d) $210 + 2A1$

Part D : Base 5 and Base 8 numbers

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

  (i) decimal number
  (ii) hexadecimal number
  (iii) binary number

(b) Perform the following binary additions

  (i) $1011 + 1111$
  (ii) $10101 + 10011$
  (iii) $1010 + 11010$

Part C : Base 5 and Base 8 numbers

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

  (i) decimal number
  (ii) hexadecimal number
  (iii) binary number

Part E: Natural, Rational and Real Numbers

1. State which of the following sets the following numbers belong to.

1) 18	3) π	5) 17/4	7) $\sqrt{\pi}$
2) 8.2347...	4) 1.33333...	6) 4.25	8) $\sqrt{25}$

The possible answers are

a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$
d) Real Number $\mathbb{R}$

- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$
- $\mathbb{Z}$: integers $\{-3, -2, -1, 0, 1, 2, 3, \ldots\}$
    - (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means "numbers" in German.)

- $\mathbb{Q}$: rational numbers
- $\mathbb{R}$: real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
  - (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

Exercises Real and Rational Numbers

(i) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part F : Scientific and Floating Point Notation

- Abscissa
- Exponent (power)

With floating point notation, the abscisa must be between 0 and 1. It is similar to scientific notation differing only by the fact that, with scientific notaiton, the abscissa is between 1 and 10.

- Floating Point Notation
- $pi \approx 0.31415 \times 10^1$
- $pi \approx 3.1415 \times 10^0$

Question 1

Part B : Hexadecimal numbers

(i) Calculate the decimal equivalent of the hexadecimal number $(A2F.D)_{16}$

(ii) Working in base 2, compute the following binary additions, showing all you workings

$$(1110)_2 + (11011)_2 + (1101)_2$$

(iv) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part D : Real and Rational Numbers

(i) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

(i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation.

(ii) From part (i), is x^8 a rational number?

Binary and Hex

1A.1 Coverting from Binomial to Decimal

1A.2 Converting to Decimal

1A.3 Priority of Operation

1A.4

Numbers

1B.1 Real Numbers

1B.2 Rational Numbers

1B.3 Floating Point Aritmetic

1B.4

Part 1. Number Systems

Section 1a. Binary Numbers

1. $1101001_{(2)}$
2. $1101001_{(2)}$
3. $1101001_{(2)}$

1.1 Inequality Operators

Given $x = \sqrt{2}$ determine whether the following statements are true or false:

(i) $x \leq 2$

(ii) $1.42 > x > 1.41$

(iii) x is a rational number

(iv) $\sqrt{2} = 2$

1.2 Revision Questions

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Mathematics for Computing

Session 2 : Set Theory

Mathematics for Computing

Session 3 : Logic

Mathematics for Computing
Digraphs and Relations

Question 1 : Binary numbers

(a) Express the following binary numbers as decimal numbers

(i) 101
(ii) 1101
(iii) 11011
(iv) 100101

(b) Express the following decimal numbers as binary numbers

(i) 6
(ii) 15
(iii) 37
(iv) 77

Question 2

A number is expressed in base 5 as $(234)_5$. What is it as decimal number? Suppose you multiply $(234)_5$ by 5. what would be the answer in base 5.

Question 3

Perform the following binary additions

(i) $1011 + 1111$
(ii) $10101 + 10011$
(iii) $1010 + 11010$
(iv) $101010 + 10101 + 101$

Question 4

Perform the binary additions

- $(10111)_2 + (111010)_2$
- $(1101)_2 + (1011)_2 + (1111)_2$

Question 5

Perform the binary subtractions using both the bit-borrowing method and the two's complement method.

- $(1001)_2 - (111)_2$
- $(110000)_2 - (10111)_2$

Question 6

Perform the binary multiplications

- $(1101)_2 \times (101)_2$
- $(1101)_2 \times (1101)_2$

Question 7

(a) What is highest Hexadecimal number that can be written with two characters, and what is it's equivalent in decimal form? What is the next highest hexadecimal number?

(b) Which of the following are not valid hexadecimal numbers?

(i) A5G
(ii) 73
(iii) EEF
(iv) 101

Question 8 : Binary Substraction

Exercises:

(i) 110 - 10
(ii) 101 - 11
(iii) 1001 - 11
(iv) 10001 - 100
(v) 101001 - 1101
(vi) 11010101-1101

Question 9

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

  (i) decimal number
  (ii) hexadecimal number
  (iii) binary number

(b) Perform the following binary additions

  (i) $1011 + 1111$
  (ii) $10101 + 10011$
  (iii) $1010 + 11010$

Question 10

Calculate working in hexadecimal

(i) $(BBB)_{16} + (A56)_{16}$

(ii) $(BBB)_{16} - (A56)_{16}$

Question 11

Write the hex number $(EC4)_{16}$ in binary. Write the binary number $(11110110101|)_2$ in hex.

Question 12

Express the decimal number 753 in binary , base 5 and hexadecimal.

Question 13

Express 42900 as a product of its prime factors, using index notation for repeated factors.

Question 14

Expresse the recuring decimals

(i) $0.727272\ldots$

(ii) 0.126126126....

(iii) 0.7545454545...

as rational numbers in its simplest form.

Question 15

Given that π is an irrational number, can you say whether $\frac{\pi}{2}$ is rational or irrational. or is it impossible to tell?

Question 16

(i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation

(ii) From part (i), is x^8 a rational number?

Question 17

(i) 5/7 lies between 0.714 and 0.715.

(ii) $\sqrt(2)$ is at least 1.41.

(iii) $\sqrt(3)$ 9s at lrast 1.732 and at most 1.7322.

Question 18

(i) Write down the numbers 0.0000526 in floating point form.

(ii) How is the number 1 expressed in floating point form.

Question 19

- Deduce that every composite integer n has a prime factor such that $p \leq \sqrt{n}$.
- Decide whether 899 is a prime.

Question 20

- What would be the maximum numbber of digits that a decimal fraction with denominator 13 could have in a recurring block in theory?
- Can you predict which other fractions with denominator 13 will have the same digits as 1/13 in their recurring block?

1.3 Video 6

Convert the following statements into symbols:

- $\sqrt{2}$ is less than 1.5 and greater than 1.4
- $\sqrt{2}$ is greater than or equal to 5

Chapter 2

Session 2

The Universal Set and the Empty Set

- The first is the ***universal set***, typically denoted U. This set is all of the elements that we may choose from. This set may be different from one setting to the next.
- For example one universal set may be the set of all real numbers, denoted $\mathbb{R}$, whereas for another problem the universal set may be the whole numbers $\{0, 1, 2, \ldots\}$.
- The other set that requires consideration is called the ***empty set***. The empty set is the unique set is the set with no elements. We write this as $\{\}$ and denote this set by $\emptyset$.

Number Sets

The font that the following symbols are written in (i.e. $\mathbb{N}$, $\mathbb{R}$) is known as ***blackboard font***.

- $\mathbb{N}$ Natural Numbers $(1, 2, 3, \ldots)$
- $\mathbb{Z}$ Integers $(-3, -2, -1, 0, 1, 2, 3, \ldots)$
  - $\mathbb{Z}^+$ Positive Integers

  - $\mathbb{Z}^-$ Negative Integers
  - 0 is not considered as either positive or negative.
- $\mathbb{Q}$ Rational Numbers
- $\mathbb{R}$ Real Numbers
- $\mathbb{C}$ Complex Numbers

Rules of Inclusion, Listing and Cardinality

For each of the following sets, a set is specified by the rules of inclusion method and listing method respectively. Also stated is the cardinality of that data set.

Worked example 1

- $\{x : x$ is an odd integer $5 \leq x \leq 17\}$
- $x = \{5, 7, 9, 11, 13, 15, 17\}$
- The cardinality of set x is 7.

Worked example 2

- $\{y : y$ is an even integer $6 \leq y < 18\}$
- $y = \{6, 8, 10, 12, 14, 16\}$
- The cardinality of set y is 6.

Worked example 3

A perfect square is a number that has a integer value as a square root. 4 and 9 are perfect squares ($\sqrt{4} = 2$, $\sqrt{9} = 3$).

- $\{z : z$ is an perfect square $1 < z < 100\}$
- $z = \{4, 9, 16, 25, 36, 49, 64, 81\}$
- The cardinality of set z is 8.

Exercises

For each of the following sets, write out the set using the listing method. Also write down the cardinality of each set.

- $\{s : s \text{ is an negative integer } -10 \leq s \leq 0\}$
- $\{t : t \text{ is an even number } 1 \leq t \leq 20\}$
- $\{u : u \text{ is a prime number } 1 \leq u \leq 20\}$
- $\{v : v \text{ is a multiple of 3 } 1 \leq v \leq 20\}$

Power Sets

Worked Example

Consider the set Z:

$$Z = \{a, b, c\}$$

(i) How many sets are in the power set of Z?

(ii) Write out the power set of Z.

(iii) How many elements are in each element set?

Solutions to Worked Example

(i) There are 3 elements in Z. So there is $2^3 = 8$ element sets contained in the power set.

(ii) Write out the power set of Z.

$$\mathcal{P}(Z) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

(iii)
- One element set is the null set - i.e. containing no elements
- Three element sets have only elements
- Three element sets have two elements
- One element set contains all three elements
- 1+3+3+1=8

Exercise

For the set $Y = \{u, v, w, x\}$, answer the questions from the previous exercise

Complement of a Set

Consider the universal set U such that

$$U = \{2, 4, 6, 8, 10, 12, 15\}$$

For each of the sets A,B,C and D, specify the complement sets.

Set	Complement
$A = \{4, 6, 12, 15\}$	$A' = \{2, 8, 10\}$
$B = \{4, 8, 10, 15\}$	
$C = \{2, 6, 12, 15\}$	
$D = \{8, 10, 15\}$	

Set Operations

- Union ($\cup$) - also known as the **OR** operator. A union signifies a bringing together. The union of the sets A and B consists of the elements that are in either A or B.
- Intersection ($\cap$) - also known as the **AND** operator. An intersection is where two things meet. The intersection of the sets A and B consists of the elements that in both A and B.
- Complement (A' or A^c) - The complement of the set A consists of all of the elements in the universal set that are not elements of A.

Exercise

Consider the universal set U such that

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and the sets

$$A = \{2, 5, 7, 9\}$$
$$B = \{2, 4, 6, 8, 9\}$$

(a) $A - B$

(b) $A \otimes B$

(c) $A \cap B$

(d) $A \cup B$

(e) $A' \cap B'$

(f) $A' \cup B'$

Venn Diagrams

Draw a Venn Diagram to represent the universal set $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6\}$ with subsets:
$A = \{2, 4, 5\}$
$B = \{1, 4, 5, 6\}$

Find each of the following

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

(e) $A \otimes B$

(i) Describe the following set by the listing method

$$\{2r + 1 : r \in Z^{+} and r \leq 5\}$$

(ii) Let A,B be subsets of the universal set U.

Question 1

- $\{s :$ s is an odd integer and $2 \leq s \leq 10\}$
- $\{2m : m \in Z$ and $5 \leq m \leq 10\}$
- $\{2^{t} : t \in Z$ and $0 \leq t \leq 5\}$

Question 2

- $\{12, 13, 14, 15, 16, 17\}$
- $\{0, 5, -5, 10, -10, 15, -15, ...\}$
- $\{6, 8, 10, 12, 14, 16, 18\}$

Question 7 : Membership Tables

Using membership tables

A	B	C	x	y	z
0	0	0	1	1	1
0	0	1	0	0	1
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	1
1	1	1	1	0	1

(i) Draw a venn diagram to show three subsets A,B and C of a universal set U intersecting in the most general way?

(ii) How are sets X and Z related?

(iii) Can you describe each of the subsets X,Y and Z in terms of the sets A,B,C using the operations union intersection and set complement.

2.0.1 Ellipsis

When using Ellipsis, it should be clear what the pattern is

Three Sets

Propositional Logic A statement is a declarative sentence that is either true or false.

- $\tilde{q}$ not q
- $p \vee q$
- $p \wedge \tilde{q}$

Question 5
Let A, B be subsets of the universal set $\mathcal{U}$.
Use membership tables to prove De Morgan's Laws.

a. (1 mark) Write out the sample space for the outcomes for both players A and B.

b. (1 mark) Write out the sample space for the outcomes of C, where C is the difference of the two scores (i.e. B-A)

c. (1 mark) Are the sample points for the sample space of C equally probable? Provide a brief justification for your answer.

Section B: Set Operations

B.1 complement of A A'

B.2 Union $A \cup B$

B.3 Intersection $A \cap B$

B.4 Relative Difference $A \otimes B$

A.5

A.6

A.7

A.8

- Specifying Sets
- Listing Method
- Rules of Inclusion method
- Subsets Notation of a subset
- Cardinality of a set
- Power of a set

Operation on Sets

- The complement of Set
- Binary Operations
  - Union
  - Intersection
- Membership tables
- Laws for Combining Sets

Associative Laws

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distributive Laws

$$(A \cup B) \cap C = (A \cup B) \cap (A \cup C)$$
$$(A \cap B) \cup C = (A \cap B) \cup (A \cap C)$$

$$(A \cup B) \cap B'$$

Section C: Real and Rational Numbers

Formulae

- Bayes' Theorem:

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)}.$$

- Binomial probability distribution:

$$P(X = k) = {}^{n}C_k \times p^k \times (1-p)^{n-k} \qquad \left(\text{where} \qquad {}^{n}C_k = \frac{n!}{k!\,(n-k)!}.\right)$$

Part 2: $\{\frac{1}{n} : 1 < n < 4, n \in \mathbb{Z}\}$

- Poisson probability distribution:

$$P(X = k) = \frac{m^k e^{-m}}{k!}.$$

2.2 Video 2 : Set Theory

Given the following sets

2.1 Video 1 : Set Theory Listing Method

Part 1: $\{10^m : -2 \le m \le 4, m \in \mathbb{Z}\}$

$\mathcal{U}$	$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$\mathcal{A}$	$\{1, 2, 5, 6, 8\}$
$\mathcal{B}$	$\{3, 5, 7, 8\}$
$\mathcal{C}$	$\{5, 6, 7, 8, 9\}$

Describe the following sets using the Listing Method

(i) $\{10^m : -2 \le m \le 4, m \in \mathbb{Z}\}$

(ii) $\{\frac{1}{n} : 1 < n < 4, n \in \mathbb{Z}\}$

List the elements of the following $A' \cap B$

$A' \cap C$

Venn Diagrams

Subsets of the universal set $\mathcal{U}$, intersecting in the most general way (Essentially this means - the venn diagram allows for all possible combinations of overlap.)

Question 2

HibCollWorkSheet2

$\in \subset$

univeral Set $\mathcal{U}$ Laws for Binary Operations Membership Tables

De Morgan's Law

$$A' \cup B' = A \cap B$$

Part A : Builder Method

The following sets have been defined using the **Building Method** of notation. Re-write them by listing **some** of the elements.

1. $\{p|p$ is a capital city, p is in Europe$\}$
2. $\{x|x = 2n - 5$, x and n are natural numbers$\}$
3. $\{y|2y^2 = 50$, y is an integer$\}$
4. $\{z|z = n^3$, z and n are natural numbers$\}$

Part B : Sets

U = natural numbers; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. State whether each of the following is true or false:

(i) $A \subset U$

(ii) $B \subseteq A$

(iii) $\emptyset \subset U$

Question 2

Part A : Builder Method

The following sets have been defined using the **Building Method** of notation. Re-write them by listing **some** of the elements.

1. $\{p|p$ is a capital city, p is in Europe$\}$
2. $\{x|x = 2n - 5$, x and n are natural numbers$\}$
3. $\{y|2y^2 = 50$, y is an integer$\}$
4. $\{z|z = n^3$, z and n are natural numbers$\}$

Part B : Sets

U = natural numbers; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. State whether each of the following is true or false:

(i) $A \subset U$

(ii) $B \subseteq A$

(iii) $\emptyset \subset U$

Question 2

Describe the following set by the rules of inclusion method.

Describe the following set by the listing method the set of positive multiples of 3 which are less than 20.

Let A and B be subsets of univerisal set U

Use the membership rule to prove that

$$(A' \cap B)' = A \cup B'$$

shade the region corresponding to this set on a Venn Diagram

Given the universal set $\mathcal{U} = 1, 2, 3, 4, 5, 6, 7, 8, 9$ and the subsets $A = \{1, 3, 5, 7\}$ $B = \{6, 7, 8, 9\}$ list the set $A' \cap B)'$

(i) $\{5, 8\}$

(ii) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

2.3 Set Theory

A $\{2n : n \in \mathbb{Z}^+\}$

B $\{3, 6, 9, 12, 15, 18, \ldots\}$

Questions

(i) **A** is described by the rules of inclusion. Describe **A** with the listing method.

(ii) **B** is described by the listing method. Describe **B** with the rules of inclusion.

Question 4

$$\text{Log}_4 64 + \text{Log}_5 625 + \text{Log}_9 3$$

Question 5

1. Draw two non-isomorphic graphs with the following degree sequence.

$$4, 3, 3, 2, 2, 2, 2, 1, 1$$

2.

Question 7b

Compute the following summation

$$\sum_{i=25}^{i=100} i^2 + 3i - 5)$$

Section 2. Set Theory

1.
2.
3.

Membership Tables

For the following venn diagrams, complete the membership.

The events W, X,Y and Z correspond to the shaded areas in each of the venn diagrams.

Chapter 3

Session 3

Question 3

Part A : Propositions

Let p, q be the following propositions:

- p : this apple is red,
- q : this apple is ripe.

Express the following statements in words as simply as you can:

(i) $p \to q$

(ii) $p \wedge \neg q$.

Express the following statements symbolically:

(iii) This apple is neither red nor ripe.

(iv) If this apple is not red it is not ripe.

Part B : Logical Operations

Let $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p and q be the following propostions concerning the integers n.

p n is event

q $n < 5$

Find the values of n for which each of the following compound statement is true,

(i) $\neg p$

(ii) $p \wedge q$

(iii) $\neg p \vee q$

(iv) $p \oplus q$

Question 3

Let $\mathcal{S} = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$ $n \in \mathcal{S}$ p : n is a multiple of two q : n is a multiple of five

Express the following statement using logic symbols n is not a multiple of either 2 or 5.

List the elements of S which are in the truth set for the statement in (ii).

Write the contrapositive of the following statement concerning an integer n.

If the last digit of n is 4, then n is divisible by 3

3.1 Conditional Connectives

Construct the truth table for the proposition $p \to q$.

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

3.2 Tautologies and Truth Tables

Truth Table for the Biconditional Connective.

P	Q	$P \leftrightarrow Q$
T	T	
T	F	
F	T	
F	F	T

P	Q	$P \vee Q$		
T	T			
T	F			
F	T			
F	F			

Question 3

Part A : Propositions

Let p, q be the following propositions:

- p : this apple is red,
- q : this apple is ripe.

Express the following statements in words as simply as you can:

(i) $p \to q$

(ii) $p \wedge \neg q$.

Express the following statements symbolically:

(iii) This apple is neither red nor ripe.

(iv) If this apple is not red it is not ripe.

Part B : Logical Operations

Let $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p and q be the following propostions concerning the integers n.

p n is event

q $n < 5$

Find the values of n for which each of the following compound statement is true,

(i) $\neg p$

(ii) $p \wedge q$

(iii) $\neg p \vee q$

(iv) $p \oplus q$

Question 3

Question 6

3.2.1 Question 6

Say with reason whether or not $\mathcal{R}$ is

- reflexive
- symmetric
- transitive

In the cases where the given property does not hold provide a counter example to justify this.

Question 6 Part A : Digraphs

Suppose $A = \{1, 2, 3, 4\}$. Consider the following relation in A

$$\{(1,1),(2,2),(2,3),(3,2),(4,2),(4,4)\}$$

Draw the direct graph of A. Based on the Digraph of A discuss whether or not a relation that could be depicted by the digraph could be described as the following, justifying your answer.

(i) Symmetric

(ii) Reflexive

(iii) Transitive

(iv) Antisymmetric

Part B : Relations

Determine which of the following relations xRy are reflexive, transitive, symmetric, or antisymmetric on the following - there may be more than one characteristic. if

(i) $x = y$

(ii) $x < y$

(iii) $x^2 = y^2$

(iv) $x \geq y$

Question 2

Let $A = \{0, 1, 2\}$ and $R = \{(0, 0), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$ and $S = \{(0, 0), (1, 1), (2, 2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Part C : Partial Orders

Let $A = \{0, 1, 2\}$ and $R = \{(0, 0), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$ and $S = \{(0, 0), (1, 1), (2, 2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Biconnective Operators

$$p \longleftrightarrow q$$

We could verbalize this as "p implies q and q implies p".

Maths for Computing Sprint

Part 0. - Numeracy

3.2.2 Factorials

Evaluate the following

- 6!
- 3!
- 1!
- 0!

Evaluate the following expressions

$$\frac{5!}{3!} \text{ and } \frac{6!}{2! \times 4!}$$

Laws of Logarithms

- Addition of Logarithms
- Subtraction of Logaritms
- Powers of Logarithms

Section 3. Logic

Proofs with Truth Tables

$$\neg(p \vee q) \wedge p \equiv q$$

p	q				

3.3 Section 3 Logic

3.3.1 Logical Operations

- $\neg p$ the negation of proposition p.
- $p \wedge q$ Both propositions p and q are simultaneously true (Logical State AND)
- $p \vee q$ One of the propositions is true, or both (Logical State : OR)
- $p \otimes q$ Only one of the propositions is true (Logical State : exclusive OR (i.e XOR)

p	q	$p \vee q$	$q \wedge p$	$p \otimes q$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Chapter 4

Session 4

Invertible Functions

Necessary Condtions for Invertibility of a Function

- The function must be one-to-one
- The fucntion must be onto.

Equivalence Relations

-
-
-

4.1 Logarithmic Functions

Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$

3. $\log_b(X^Y)= Y\log_b(X)$

Question1.3 Compute the Logarithm of the following

- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125)$ + $\log_3(729)$
- $\log_2(64/4)$

$$Log_b(x) = \frac{1}{Log_x(b)}$$

$$Log_b(x) = \frac{Log_a(b)}{Log_a(b)}$$

Example 1

$$log_3(x) + 3log_x(3) = 4$$

$$(log_3(x))^2 + 3 = 4log_3(x)$$

Example 1

$$log_3(x) + 3log_x(3) = 4$$

$$(log_3(x))^2 - 4log_3(x) + 3 = 0$$

4.2 Digraphs and Relatiosn

Given a flock of chickens, between any two chickens one of them is dominant. A relation, R, is defined between chicken x and chicken y as xRy if x is dominant over y. This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

- Amy is dominant over Beth and Carol
- Beth is dominant over Eve and Carol
- Carol is dominant over Eve and Daisy
- Daisy is dominant over Eve, Amy and Beth
- Eve is dominant over Amy.

4.3 Section 4 Functions

4.3.1 Invertible Functions

A function is invertible if it fulfils two criteria

- The function is ***onto***,
- The function is ***one-to-one***.

State the conditions to be satisfied by a function $f : X \leftarrow Y$ for it to have an inverse function $f^{-1} : Y \leftarrow X$.
$\lceil \frac{x^2+1}{4} \rceil$ where $f : A \rightarrow \mathbf{Z}$

(i) Find $f(4)$ and the ancestors of 3.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer

Given $f : \mathbf{R} \rightarrow \mathbf{R}$ where f(x) =3x-1,define fully the inverse of the function f ,i.e.f^{-1}. State the value of $f^{-1}(2)$

4.3.2 Precision Functions

- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
- Floor Function $\lfloor x \rfloor$

Question1.2: State the range and domain of the following function

$$F(x) = \lfloor x - 1 \rfloor$$

4.3.3 Powers

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

4.3.4 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

4.3.5 Logarithmic Functions

Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$
3. $\log_b(X^Y)$= $Y\log_b(X)$

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- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$

Question 4

Session 04:Functions

- Definitions

Domain

Co-domain

Image

Ancestor

Range

Part A : Functions

Given a real number x, say how the floor of x $\lfloor x \rfloor$ is defined.

(i) Find the values of $\lfloor 2.97 \rfloor$ and $\lfloor -2.97 \rfloor$.

(ii) Find an example of a real number x such that $\lfloor 2x \rfloor \neq 2\lfloor x \rfloor$, justifying your answer.

Part B : Logarithms

Evaluate the following expression.

$$\mathrm{Log}_4 64 + \mathrm{Log}_5 625 + \mathrm{Log}_9 3$$

$$\mathrm{Log}_4 64 + \mathrm{Log}_5 625 + \mathrm{Log}_9 3$$

Absolute Value Function (4.1.3)

- The absolute value of some real number x is denoted $|x|$.
- If the number is positive, the asbolute value is the same number.
- If the number is negative, the asbolute value is the number without the minus sign.
- $|2| = 2$
- $|-2| = 2$

Floor and Ceiling Function (4.1.4)

Polynomial Functions (4.1.5)

Constants (P_0)

near Functions (P_1)

ratic Functions (P_2)

ubic Functions (P_3)

Equality of Functions (4.1.6)

$$f(x) = g(x)$$

Encoding and Decoding Functions (4.2)

Onto Functions (4.2.2)

One-to-One Functions (4.2.3)

$f(x)$, must be *One-to-One* and *Onto*

Exponential and Logarithmic Functions (4.3)

The Laws of Logarithms

-
- $log_b(x^y) = y \times log_b(x)$
-
-

Big O-notation

Comparing the size of Functions (4.4)
Using O-notations

Power Notation (4.4.2)

Question 4

A function f: X-¿ Y , where $X = \{p, q, r, s\}$ and $Y = \{1, 2, 3, 4, 5\}$ is given by the subset of $X \times Y$

- Show f as an arrow diagram
- state the domain, the co-domain, and the range of f
- Say why f does not have the one-to-one property and why f does not have the "onto" property, giving a specific counter example in each case.

4.4 Section 4 Functions

4.4.1 Invertible Functions

A function is invertible if it fulfils two criteria

- The function is ***onto***,

- The function is ***one-to-one***.

State the conditions to be satisfied by a function $f : X \leftarrow Y$ for it to have an inverse function $f^{-1} : Y \leftarrow X$.
$\lceil \frac{x^2+1}{4} \rceil$ where $f : A \rightarrow \mathbf{Z}$

(i) Find $f(4)$ and the ancestors of 3.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer

Given $f : \mathbf{R} \rightarrow \mathbf{R}$ where f(x) =3x-1,define fully the inverse of the function f ,i.e.f^{-1}. State the value of $f^{-1}(2)$

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- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
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$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

4.4.4 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

Functions

- Domain of a Function
- Range of a function
- Inverse of a function
- one-one (surjective)
- onto (bijective)

Functions

- Domain of a Function
- Range of a function
- Inverse of a function
- one-one (surjective)
- onto (bijective)

Chapter 5

Session 5

5.1 Video 4 : Graph Theory

Draw the graph **G**, which has the vertices v_1,v_2,v_3,. . .,v_7, and the adjacency list:

v_1 : v_2,v_4

v_2 : v_1,v_3

v_3 : v_2,v_4

v_4 : v_1,v_3,v_5

v_5 : v_4,v_6

v_6 : v_5,v_7

v_7 : v_5,v_6

5.2 Video 6 : Graph Theory - Isomorphic Graphs

[fragile]

- If the graphs are not simple, we need more sophisticated methods to check for when two graphs are isomorphic.

- However, it is often straightforward to show that two graphs are not isomorphic.
- You can do this by showing any of the following seven conditions are true.

[fragile]

1. The two graphs have different numbers of vertices.
2. The two graphs have different numbers of edges.
3. One graph has parallel edges and the other does not.
4. One graph has a loop and the other does not.
5. One graph has a vertice of degree k (for example) and the other does not.
6. One graph is connected and the other is not.
7. One graph has a cycle and the other has not.

5.3 Video 7 : Numbers

mantissa

abscissa

radix point

- Number Systems
- Set Theory
- Function
-
- Graph Theory

- Digraphs
- Set Theory
- Function
- Probability
- MAtrices
- Continuously divide the decimal number by 2.
- Keep record of the remainder, either 0 or 1.
- The sequence of remainders is the binary number required.
- Hex Characters $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$
-
- $\neq$ Not Equal
- ¡ Less than
- ¿ greater than
- $\geq$ greater than or equal to
- $\leq$ Not Equal to
- Natural Numbers $\{1, 2, 3, 4, \ldots\}$
- Integers $\{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$
- Rational Numbers e.g 4/7 , 11/25
- Real Numbers Any number e.g. 3.1415

‘ Floating Point Noation
Section 1 Section 1.2 Section 1.3 Section 14

- Decmal Number Systesm

  - Base 10

- Binary Number Systems
  - Base 2
  - allowable characters are 0.1 only
- Base 16 Hexadecimal
  - Use all of the decimal digits, in addition to 6 more A,B ,D,D,E,F

  (where might you see this - specifying colours RGB Numbers

For example FF in hexadecimal is is 255 in decimal
Rational Numbers
Natural Numbers indies Integers Z
Computing a binary number
Useful

$2^0 = 1$	$2^4 = 16$
$2^1 = 2$	$2^5 = 32$
$2^2 = 4$	$2^6 = 64$
$2^3 = 8$	$2^7 = 128$

Firslly determine the highest power
Suppose the number we wish to convert is 58
Hwhat is the highest power of two that deivides

5.4 graph theory

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

Session 05:Graphs

5A.1 What is a Graph?

5A.2 Paths Cycles and Connectivity

5A.3 Isomorphisms of a graph

5A.4 Adjacency Matrices and Adjacency Lists

Isomorphism

- They have a different number of connected components
- They have a different number of vertices
- They have different degrees sequences
- They have a different number of paths of any given length
- They have a different number of cycles of any length.

Adjacency Lists

u : $\{v\}$

v : $\{w, x\}$

w : $\{v, x\}$

z : $\{v, w\}$

- Spanning Subgraphs of G.
- a vertex is said to be an **emph isolated vertex** if it has a degree of zero.
- a vertex is said to be an **emph end-vertex** if it has a degree of one.
- a vertex is said to be an **emph even vertex** if it has a degree of an even number.

- a vertex is said to be an **emph odd vertex** if it has a degree of an odd number.
- A graph is said to be **emphk-regular** if the degree of each vertex is k.
- Every Graph has an even number of odd vertices.
- A cubic graph is a graph where every vertex has degree three.

Section 5. Graph Theory

Adjacency Lists

1.
2.
3.
4.

Question 5

1. Draw two non-isomorphic graphs with the following degree sequence.

$$4, 3, 3, 2, 2, 2, 2, 1, 1$$

2. Write out the degree sequence of the following graph.
3. State the vertices that comprise a cycle of length 5 in both of the following graphs.

Session 05 Graph Theory

- Eulerian Path
- Isomorphism

- Adjacency matrices

Adjacency Matrices

$$\begin{pmatrix} o & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Session 05 Graph Theory

- Eulerian Path
- Isomorphism
- Adjacency matrices

Adjacency Matrices

$$\begin{pmatrix} o & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

5.5 graph theory

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

Condtions for Isomorphism

-
-
-

Question 6

Chapter 6

Session 6

Session 06: Digraphs and Relations

6A.1 In-degree and out-degree

6A.2

6A.3

Relations

6B.1 Equivalence RElations (6.2.2)

6B.2

6B.3 Relations and Cartesian Products (6.3)

- **Reflexive**:
- **Symmetric**:
- **Transitive**:
- **Anti-symmetric**:
- **Equivalence Relation**:
- **Partial Order**:
- **Order**:

Question 6

Part A : Digraphs

Suppose $A = \{1, 2, 3, 4\}$. Consider the following relation in A

$$\{(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)\}$$

Draw the direct graph of A. Based on the Digraph of A discuss whether or not a relation that could be depicted by the digraph could be described as the following, justifying your answer.

(i) Symmetric

(ii) Reflexive

(iii) Transitive

(iv) Antisymmetric

Part B : Relations

Determine which of the following relations xRy are reflexive, transitive, symmetric, or antisymmetric on the following - there may be more than one characteristic. if

(i) $x = y$

(ii) $x < y$

(iii) $x^2 = y^2$

(iv) $x \geq y$

Part C : Partial Orders

Let $A = \{0, 1, 2\}$ and $R = \{(0, 0), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$ and $S = \{(0, 0), (1, 1), (2, 2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

6.1 Digraphs and Relatiosn

Given a flock of chickens, between any two chickens one of them is dominant. A relation, R, is defined between chicken x and chicken y as xRy if x is dominant over y. This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

- Amy is dominant over Beth and Carol
- Beth is dominant over Eve and Carol
- Carol is dominant over Eve and Daisy
- Daisy is dominant over Eve, Amy and Beth
- Eve is dominant over Amy.

Chapter 7

Session 7

Session 07:Sequences and Series

7A.1 Sequences

7A.2 Induction

7A.3 Series and the Sigma Notation

Recurrence Relations(7.1.1)

$u_1 = 2$ $u_2 = u_1 + 3 = 2 + 3 = 5$ $u_3 = u_2 + 3 = 5 + 3 = 8$ Airthmetic Progression

Proof by Induction(7.2.2)

Step 1 Base case

Step 2 Induction hypothesis

Step 3 Induction step

Series and Sigma Notation(7.2.3)

Question 7

Part A : Recurrence Relations

A sequence is defined by the recurrence relations

$$x_{n+2} = 3x_{n+1} - 2x_n$$

with initial terms $x_1 = 1$ and $x_2 = 3$.

(i) Calculate x_3, x_4 and x_5, showing your workings.

(ii) Prove by induction that $x_n = 2^n - 1$ for all $n \geq 1$

Part B : Summations

Compute the following summation

$$\sum_{i=25}^{i=100} i^2 + 3i - 5)$$

Say which of the set the following numbers belong to.
If they belong to more than one of these sets, give all the sets.
$\sqrt{2}$ $\frac{3}{7}$

Section 8 Exercises

- $8^{\frac{1}{3}}$ Recall $a^{\frac{b}{c}} = a^{\frac{b}{c}}$
-
-

Question 7b

Compute the following summation

$$\sum_{i=25}^{i=100} i^2 + 3i - 5)$$

Question 7

7.0.1 Question 7a

A sequence is defined by the formula $u_n = 5n - 3$ for $n \geq 1$

$$\frac{n(n+1)(2n+1)}{6}$$

Write the following sums in the Σ notation and evaluate them

- $1^2 + 2^2 + 3^2 + \ldots + 40^2 = ?$
- $2 + 5 + 10 + \ldots + 1601 = ?$
- $2 + 8 + 18 + \ldots + 3200 = ?$

Part A : Recurrence Relations

A sequence is defined by the recurrence relations

$$x_{n+2} = 3x_{n+1} - 2x_n$$

with initial terms $x_1 = 1$ and $x_2 = 3$.

(i) Calculate x_3, x_4 and x_5, showing your workings.

(ii) Prove by induction that $x_n = 2^n - 1$ for all $n \geq 1$

Proof By Induction

- **Base Step**
- **Induction Step**
- **Some Step**

7.0.2 Sequence and Series and Proof by Induction

$$\sum_{i=1}^{n}(n^2)$$

Chapter 8

Session 8

Question 8

Part A : Spanning Trees

1. How many edges are in the spanning tree T ?
2. What is the sum of the degree sequence of T?
3. Write down all the possible degree sequences for the spanning tree T.

Part B : Binary Search Trees

Suppose a database, comprised of 30,000 internal nodes, is structured as a Binary Search Tree.

1. What is the key (number) of the Root node?
2. What are the keys of the nodes at level 1?
3. For the nodes at level1, how many subtrees are there?
4. State which nodes are in the substrees of the level 1 nodes?
5. How many nodes are the between the root (level 0) and level 7.] (Hint: use a summation theorem mentioned in session 7
6. What is the maximum number of searchs in this database?

8.1 Question 8B

Suppose a database, comprised of 30,000 internal nodes, is structured as a Binary Search Tree.

1. What is the Key of the Root node?
2. What are the keys of the nodes at level 1?
3. For the nodes at level1, how many subtrees are there?
4. State which nodes are in the substrees of the level 1 nodes?
5. How many nodes are the between the root (level 0) and level 7.] (Hint: use a summation theorem mentioned in session 7
6. What is the maximum number of searchs in this database?

Binary Search Trees

What is a Binary Search Tree

$$\lfloor \frac{\log_2}{T} \rfloor$$

Session 08:Trees

8A.1 Trees

8A.2 Spanning Trees

8A.3 Rooted trees

8A.4 Binary Search Trees

A tree is a directed graph that contains no cycles.

8.2 Question 8

1) Draw this tree 2) Construct all the isomorphic tress with 6 vertices which can be obtainbed by attaching a new vertex of degreee one to a vertex of T. 3) Explain briefly why the tree obtained in (ii) is not isomorphic to each other. 4) Construct a tree with 6 vertices which is not isomorphic to any tree you constructed in (ii)

Part b Determine the number of nodes on level 5 and level 10 Find an expression in terms of Σ and h for the number of internal nodes in such a tree. What is the smallest possible height of such a tree if there are at least 900 internal nodes.

8.3 Question 8A

1. How many edges are in the spanning tree T ?
2. What is the sum of the degree sequence of T?
3. Write down all the possible degree sequences for the spanning tree T.

Chapter 9

Session 9

9.1 Probability and Counting

Given S is the set of all 5 digit binary strings, E is the set of a 5 digit binary strings beginning with a 1 and F is the set of all 5 digit binary strings ending with two zeroes.

(a) Find the cardinality of S, E and F.

(b) Draw a Venn diagram to show the relationship between the sets S, E and F.

(c) Show the relevant number of elements in each region of your diagram.

9.1.1 Axioms of Probability

The Axioms of Probability

- The probability of a certain event is 1.
- The probability of an impossible event is 0.
-

Session 09: Probability

9A.1 Counting Methods

9A.2 Counting using Sets

9A.3 Probability

9A.4 Independent Events

9B.1 Permutation

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$\binom{6}{3} = \frac{6!}{(6-3)!3!} = \frac{6!}{3! \times 3!}$$

$$\frac{6!}{3! \times 3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = \frac{120}{6} = 120$$

- $\binom{6}{2} = 15$
- $\binom{5}{2} = 10$
- $\binom{4}{0} = 1$
- $\binom{4}{3} = 4$

- pairwise disjoint sets
- The addition principle

Theorem

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Probability

9B.2 The sample space of an experiment (S)

9B.3 The size of a sample space

9B.4 Indepedent Evcents (9.3.1)

Session 9 Probability

Binomial Coefficients

- factorials
$$n! = (n) \times (n-1) \times (n-2) \times \ldots \times 1$$
  - $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
  - $3! = 3 \times 2 \times 1$
- Zero factorial
$$0! = 1$$

The complement rule in Probability
$P(C') = 1 - P(C)$
If the probability of C is 70% then the probability of C' is 30%

Probability

Binomial Coefficients

- factorials
$$n! = (n) \times (n-1) \times (n-2) \times \ldots \times 1$$
  - $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
  - $3! = 3 \times 2 \times 1$
- Zero factorial
$$0! = 1$$

The complement rule in Probability
$P(C') = 1 - P(C)$
If the probability of C is 70% then the probability of C' is 30%

9.2 Counting

Given S is the set of all 5 digit binary strings, E is the set of a 5 digit binary strings beginning with a 1 and F is the set of all 5 digit binary strings ending with two zeroes.

(a) Find the cardinality of S, E and F.

(b) Draw a Venn diagram to show the relationship between the sets S, E and F.

(c) Show the relevant number of elements in each region of your diagram.

Probability: Binomial Coefficients

- factorials
$$n! = (n) \times (n-1) \times (n-2) \times \ldots \times 1$$

- $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
- $3! = 3 \times 2 \times 1$

- Zero factorial

$$0! = 1$$

The complement rule in Probability
$P(C') = 1 - P(C)$
If the probability of C is 70% then the probability of C' is 30%

Chapter 10

Session 10

Systems of Linear Equation

10.1 Matrices

What are the dimensions of the following matrix

$$\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} c_1 & d_1 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} (a_1 \times c_1) + (a_2 \times c_2) & (a_1 \times d_1) + (a_2 \times d_2) \\ (b_1 \times c_1) + (b_2 \times c_2) & (b_1 \times d_1) + (b_2 \times d_2) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} (1 \times 1) + (3 \times 4) & (1 \times 2) + (3 \times 1) \\ (0 \times 4) + (2 \times 4) & (0 \times 2) + (2 \times 1) \end{pmatrix} = \begin{pmatrix} 14 & 5 \\ 8 & 2 \end{pmatrix}$$

$$\left(\begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} \begin{matrix} 1 & 3 \\ 0 & 2 \end{matrix} \right) =?$$

Reduced Echelon Form

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-
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Session 10: Matrices and Systems of Equations

10A.1 Dimensions of a Matrix

10A.2 Matrix Multiplication

10A.3 Matrix Calculations

10A.4

10B.1 Systems of Equations

10B.2 Expression Systems of Equations as Matrices

10B.3 Augmented Matrices

10B.4 Guassian Elimination

Question 10

Session 10: Matrices and Systems of Equations

10A.1 Dimensions of a Matrix

10A.2 Matrix Multiplication

10A.3 Matrix Calculations

10A.4

10B.1 Systems of Equations

10B.2 Expression Systems of Equations as Matrices

10B.3 Augmented Matrices

10B.4 Guassian Elimination

Question 10A

Say what information the first row of the matrix contains. Find the number of edges of G.

Write doen the augmented matrix for the following system of equations. $x+y+2z=7$ $2x+y+3z=11$ $x-27+5z=4$

Use Gaussian elimination to solve the system.

Part B : Summations

Compute the following summation

$$\sum_{i=25}^{i=100} i^2 + 3i - 5)$$

10.2 Matrices

What are the dimensions of the following matrix

$$\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} c_1 & d_1 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} (a_1 \times c_1) + (a_2 \times c_2) & (a_1 \times d_1) + (a_2 \times d_2) \\ (b_1 \times c_1) + (b_2 \times c_2) & (b_1 \times d_1) + (b_2 \times d_2) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} (1 \times 1) + (3 \times 4) & (1 \times 2) + (3 \times 1) \\ (0 \times 4) + (2 \times 4) & (0 \times 2) + (2 \times 1) \end{pmatrix} = \begin{pmatrix} 14 & 5 \\ 8 & 2 \end{pmatrix}$$

$$\left(\begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} \begin{matrix} 1 & 3 \\ 0 & 2 \end{matrix} \right) =?$$

Question 1

(i) (1 Mark) When is the positive integer p said to be a prime number?

(ii) (2 Marks) Express the following hexadecimal number as a decimal number, and as a binary number:

$$(A32.8)_{16}$$

(iii) (2 Marks) Convert the following decimal number into base 2, showing all your working: $(253)_{10}$.

(iv) (2 Marks) Covert the decimal integer $(407)_{10}$ to binary notation.

(v) (2 Marks) Showing your working, express the following number

$$1.024024024024\ldots$$

as a ration number in its simplest form.

(vi) (1 Mark) Compute the following $101101_2 + 1101_2$

Question 2

Let A and B and C be subsets of a universal set U.

(a) (1 Mark) Draw a labelled Venn diagram depicting A,B,C in such a way that they divide U into 8 disjoint regions. [1]

(b) (3 Marks) The subset $X \subseteq U$ is defined by the following membership table below. Shade the region X on your diagram. Describe the region you have shaded in set notation as simply as you can.

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

(c) (3 Marks) The subset $Y \subseteq U$ is defined as $Y = A \cup (C - B)$. Construct the membership table for Y.

(d) (3 Marks) For each of the following statements say whether it is true or false, justifying your answer, using the Venn diagram you drew earlier.

(i) $Y \subseteq X$

(ii) $Y' \subseteq X'$

(iii) $Y - X = A \cap B \cap C$.

Question 3

(a) Let n be an element of the set $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$, and p and q be the propositions:

$$p : n \text{ is odd}, q : n < 15$$

. Draw up truth tables for the following statements and find the values of n for which they are true:

(i) $p \vee \neg q$

(ii) $\neg p \wedge q$

(b) Use truth tables to find a statement that is logically equivalent to $\neg p \to q$.

(i)

Question 4

Let S be the set of all 4 bit binary strings. The function $f : S \to Z$ is defined by the rule:

$$f(x) = \text{ the number of zeros in x}$$

for each binary string $x \in S$. Find:

(a) (4 Marks) Answer the following questions

(i) the number of elements in the domain

(ii) f(1010)

(iii) the set of pre-images of 1

(iv) the range of f.

(b) (2 Marks) Decide whether the function f, as defined above, has either the one to one or the onto property, justifying your answers.

(c) (2 Marks) State the condition to be satisfied by a function $f : X \to Y$ for it to have an inverse function $f^{-1} : Y \to X$.

(d) (2 Marks) Define the inverse functions for each of the following:

Question 5

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

a) (i) A simple, connected graph has 7 vertices, all having the same degree d. State the possible values of d and for each value also give the number of edges in the corresponding graph. (ii) Another simple, connected graph has 6 vertices, all having the same degree, n. Draw such a graph when n = 3 and state the other possible values of n. [4]

Question 6

Given a flock of chickens, between any two chickens one of them is dominant. A relation, R, is defined between chicken x and chicken y as xRy if x is dominant over y. This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

- Amy is dominant over Beth and Carol
- Beth is dominant over Eve and Carol
- Carol is dominant over Eve and Daisy
- Daisy is dominant over Eve, Amy and Beth
- Eve is dominant over Amy.

Question 6

Let $A = \{0, 1, 2\}$ and $R = \{(0, 0), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$ and $S = \{(0, 0), (1, 1), (2, 2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Let S be a set and let R be a relation on S Explain what it means to say thet $\mathcal{R}$ is

(i) reflexive

(ii) symmetrix

(iii) anti-symmetric

(iv) Transitive

Question 7

Let the sequence un be defined by the recurrence relation

$$u_{n+1} = u_n + 2n, \text{ for n = 1, 2, 3, ...}$$

and let $u_1 = 1$.

Question 8

(Part B :Binary Search Trees - 5 Marks)
Suppose a database, comprised of 30,000 internal nodes, is structured as a Binary Search Tree.

(i) What is the Key of the Root node?

(ii) What are the keys of the nodes at level 1?

(iii) For the nodes at level 1, how many subtrees are there?

(iv) How many nodes are the between the root (level 0) and level 4.]

(v) What is the maximum number of searchs in this database?

Question 9

Given S is the set of all 5 digit binary strings, E is the set of a 5 digit binary strings beginning with a 1 and F is the set of all 5 digit binary strings ending with two zeroes.

(a) Find the cardinality of S, E and F.

(b) Draw a Venn diagram to show the relationship between the sets S, E and F. Show the relevant number of elements in each region of your diagram.

- A college teaches courses in the following subjects areas: mathematics, computing and statistics.
- Students in the college may choose their courses from these three subject areas.
- Students are not obliged to take courses from these three subject areas, and may instead take courses in other subject areas.
- Let the subject areas be represented by the letters **M** for mathematics, **C** for computing and **S** for statistics.
- Draw a labelled Venn diagram showing the areas **M**, **C**, and **S** in such a way as to represent the students studying at the college.
- On your diagram show the number of students studying in each region of the Venn diagram.
  - Currently 600 students are enrolled in the college.
  - 300 students are taking mathematics courses.

- 120 student are taking statistics courses.
- 380 students are taking computing courses.
- 40 students study courses from all three subject areas.
- 200 mathematics students are taking computing courses as well.
- 60 computing students are also takings statistics courses.
- 70 statistics students are also taking mathematics course.

(i) How many students study none of these courses at all?

(ii) How many students are taking mathematics courses but not computing or statistics courses.

(iii) How many students study courses from precisely two of these subject areas?

Question 10

Part B : Gaussian Elimination - 5 Marks

(i) Say whether or not the graphs they represent are isomorphic.

(ii) Calculate A^2 and A^4 and say what information each gives about the graph corresponding to A. [6]

(i) Write down the augmented matrix for the following system of equations.

$$2x + y - z = 2$$

$$x - y + z = 4$$

$$x + 2y + 2z = 10$$

(ii) Use Gaussian elimination to solve the system.