Number Systems - Tutorial Sheet 2

Part A: Number Systems - Binary Numbers

1. Express the following decimal numbers as binary numbers.

   i) $(73)_{10}$ ii) $(15)_{10}$ iii) $(22)_{10}$

   All three answers are among the following options.

   a) $(10110)_2$ b) $(1111)_2$ c) $(1001001)_2$ d) $(1000010)_2$

2. Express the following binary numbers as decimal numbers.

   a) $(101010)_2$ b) $(10101)_2$ c) $(111010)_2$ d) $(11010)_2$

3. Express the following binary numbers as decimal numbers.

   a) $(110.10101)_2$ b) $(101.0111)_2$ c) $(111.01)_2$ d) $(110.1101)_2$

4. Express the following decimal numbers as binary numbers.

   a) $(27.4375)_{10}$ b) $(5.625)_{10}$ c) $(13.125)_{10}$ d) $(11.1875)_{10}$

Part B: Number Systems - Binary Arithmetic

1. Perform the following binary additions.

   a) $(110101)_2 + (1010111)_2$
   b) $(1010101)_2 + (101010)_2$
   c) $(11001010)_2 + (10110101)_2$
   d) $(1011001)_2 + (111010)_2$

2. Perform the following binary subtractions.

   a) $(110101)_2$ - $(1010111)_2$
   b) $(1010101)_2$ - $(101010)_2$
   c) $(11001010)_2$ - $(10110101)_2$
   d) $(1011001)_2$ - $(111010)_2$

3. Perform the following binary multiplications.

   a) $(1001)_2 \times (1000)_2$
   b) $(101)_2 \times (1101)_2$
   c) $(111)_2 \times (1111)_2$
   d) $(10000)_2 \times (11001)_2$

4. Perform the following binary multiplications.

   i) Which of the following binary numbers is the result of this binary division: $(10)_2 \times (1101)_2$.

      a) $(11010)_2$
      b) $(11100)_2$
      c) $(10101)_2$
      d) $(11011)_2$

   ii) Which of the following binary numbers is the result of this binary division: $(101010)_2 \times (111)_2$.

      a) $(11000)_2$
      b) $(11001)_2$
      c) $(10101)_2$
      d) $(11011)_2$

   iii) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \times (1101)_2$.

a) $(11000)_2$ c) $(10101)_2$

b) $(11001)_2$ d) $(11011)_2$

5. Perform the following binary divisions.

   i) Which of the following binary numbers is the result of this binary division: $(111001)_2 \div (10011)_2$.

   a) $(10)_2$ c) $(100)_2$

   b) $(11)_2$ d) $(101)_2$

   ii) Which of the following binary numbers is the result of this binary division: $(101010)_2 \div (111)_2$.

   a) $(11)_2$ c) $(101)_2$

   b) $(100)_2$ d) $(110)_2$

   iii) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \div (1101)_2$.

   a) $(100)_2$ c) $(111)_2$

   b) $(110)_2$ d) $(1001)_2$

Part C: Number Bases - Hexadecimal

1. Answer the following questions about the hexadecimal number systems

   a) How many characters are used in the hexadecimal system?

   b) What is highest hexadecimal number that can be written with two characters?

   c) What is the equivalent number in decimal form?

   d) What is the next highest hexadecimal number?

2. Which of the following are not valid hexadecimal numbers?

   a) 73 b) $A5G$ c) 11011 d) EEF

3. Express the following decimal numbers as a hexadecimal number.

   a) $(73)_{10}$ b) $(15)_{10}$ c) $(22)_{10}$ d) $(121)_{10}$

4. Compute the following hexadecimal calculations.

   a) $5D2 + A30$ b) $702 + ABA$ c) $101 + 111$ d) $210 + 2A1$

Part D: Natural, Rational and Real Numbers

- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$
- $\mathbb{Z}$: integers $\{-3, -2, -1, 0, 1, 2, 3, \ldots\}$
  - ∗ (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means "numbers" in German.)
- $\mathbb{Q}$: rational numbers
- $\mathbb{R}$: real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
  - ∗ (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

1. State which of the following sets the following numbers belong to.

1) 18	3) π	5) 17/4	7) $\sqrt{\pi}$
2) 8.2347...	4) 1.33333...	6) 4.25	8) $\sqrt{25}$

The possible answers are

a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$

d) Real Number $\mathbb{R}$