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UNIVERSITY OF LIMERICK

MA4016 - Engineering Mathematics 6

Problem Sheet 6: Discrete Mathematics (March 12, 2010)

1. Are all elements of the sequence f_n, $n = 1, 2, \ldots$ with

$$f_n = n^2 - n + 41$$

primes?

2. If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple?

3. Show that whenever $n \geq 3$, $f_n > \alpha^{n-2}$, where f_n is the n-th Fibonacci number and $\alpha = (1 + \sqrt{5})/2$.

4. How many divisions are required to find $\gcd(34, 55)$ using the Euclidean algorithm? What is the bound from Lamé's theorem?

5. Apply the extended Euclidean algorithm to find the greatest common divisor and s, t in
a)
$$\gcd(1529, 14038) = 1529s + 14038t, \quad s, t \text{ integers,}$$

b)
$$\gcd(1529, 14039) = 1529s + 14039t, \quad s, t \text{ integers,}$$