UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics
Department *of* Mathematics and Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE : MA4402	SEMESTER: Autumn 2006/2007
MODULE TITLE: Computer Mathematics 2	DURATION: $2\frac{1}{2}$ hours
LECTURER: Dr. Patrick Johnson	GRADING SCHEME: Examination: 70%

INSTRUCTIONS TO CANDIDATES: Full marks for correct answers to any 5 questions. Calculators and logarithm tables may be used.

Q 1 (a) Define what is meant by a function $f : A \to B$ where A and B are given subsets of real numbers ($\mathbb{R}$). 3

(b) Explain what is meant by saying that a functions is

1. Injective,
2. Surjective,
3. Bijective.

Give an example in each case. 6

(c) Consider the following functions

$$\begin{aligned} f &: \mathbb{R} \to \mathbb{R}, \quad f(x) = x^3 + 3. \\ g &: \mathbb{R} \to \mathbb{R}, \quad g(x) = 2x^2 + 1. \end{aligned}$$

Is f surjective? Explain your answer.

Is g injective? Explain your answer. 4

(d) Consider the function:

$$f : [0, 1] \to \mathbb{R}, \quad f(x) = 2 + 2x^2.$$

What could you replace the codomain of this function with in order to make it surjective? 4

(e) Is it possible for a function to be neither injective nor surjective? Illustrate your answer by way of an example. 3

Q 2 (a) Explain how $f : \mathbb{N} \to \mathbb{R}$ defines a sequence $\{a_n\}_{n=1}^{\infty}$. (**Note**: $\mathbb{N}$ denotes the set of natural (counting) numbers.) 4

(b) Show that the recursively defined sequence (which you may assume is convergent) defined by

$$a_1 = 1, \quad a_{n+1} = \frac{1}{2}\left(a_n + \frac{3}{a_n^2}\right)$$

converges to $\sqrt[3]{3}$. Use this to compute $\sqrt[3]{3}$ to two decimal places. 8

(c) Show that the series defined by

$$\left\{ \frac{x^n}{n!} \right\}_{n=0}^{\infty}$$

is convergent. Note that this series defines e^x. 5

(d) Use the series in Q2(c) to estimate e^4 correct to three decimal places. 3

Q 3 (a) Give an outline of the Newton-Raphson algorithm for root finding and explain how it works. 6

(b) Use the Newton-Raphson algorithm to estimate the **roots** of the function

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^3 - 3x - 2.$$

correct to 3 decimal places. 10

(c) Give two examples of instances when the Newton-Raphson algorithm fails. Illustrations can be used as part of your examples. 4

Q 4 (a) Define what is meant by the magnitude of a vector. 3

(b) Consider the two vectors

$$\mathbf{v} = \langle 3, 1 \rangle, \quad \mathbf{w} = \langle 5, 7 \rangle$$

1. Find $|\mathbf{v}|$ and $|\mathbf{w}|$
2. Find $\mathbf{v}.\mathbf{w}$ (dot product of $\mathbf{v}$ and $\mathbf{w}$)
3. Find the acute angle between $\mathbf{v}$ and $\mathbf{w}$

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(c) Consider the line segment with endpoints $(1, 2)$ and $(3, 3)$. Using the rotation matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

rotate the above line segment about its endpoint $(1, 2)$ by $\frac{\pi}{4}$ radians. 8

Q 5 (a) Explain under what circumstances it is possible to (i) add and (ii) multiply the matrices A (order $m \times n$) and B (order $p \times q$). 4

(b) Let

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 3 & -1 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 4 & 1 & 2 \\ 0 & -3 & 2 \end{bmatrix}$$

Calculate AB and BA, if possible. 7

(c) Show, using the matrices in Q5(b), that

1. $(A^T)^T = A$
2. $(AB)^T = B^T A^T$

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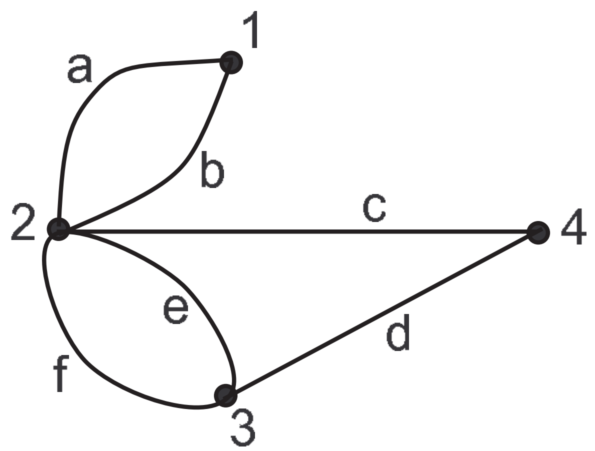
Q 6 (a) State the requirements necessary for a graph to be planar and show that the graph K_4 is planar. 4

(b) Write down the adjacency matrix for $K_{3,2}$. 4

(c) Given any simple undirected graph, list the features of its adjacency matrix. 4

(d) Construct a graph that is isomorphic to the following graph and list the bijections necessary so that the two graphs are isomorphic. $\{1, 2, 3, 4\}$ are the vertices and {a,b,c,d,e,f} are the edges.



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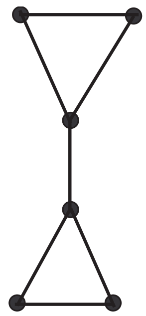
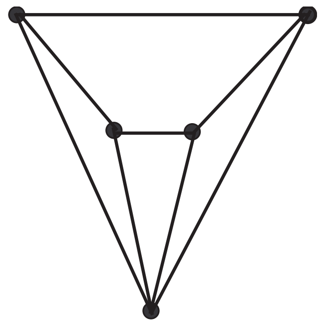
(e) Are the following graphs (i) Eulerian, (ii) Hamiltonian? Clearly explain your answers.



(a)



(b)

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