UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics
Department *of* Mathematics and Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE : MA4402	SEMESTER: Autumn 2007
MODULE TITLE: Computer Mathematics 2	DURATION: $2\frac{1}{2}$ hours
LECTURER: Mr. Seán Lacey	GRADING SCHEME: Examination: 80%

INSTRUCTIONS TO CANDIDATES: Full marks for correct answers to any 5 questions. Calculators and logarithm tables may be used.

Q 1 (a) Define what is meant by a function $f : A \to B$ where A and B are given subsets of real numbers ($\mathbb{R}$). 3

(b) Let $A = \{1, 2, 3\}$ and let $B = \{a, b, c\}$. The function $f : A \to B$ is defined by $f(x)$, $\forall x \in A$. Give an example of an ordered pair of the function that is

1. bijective,
2. injective but not surjective,
3. surjective but not injective.

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(c) Consider the following functions

$$\begin{aligned} f &: \mathbb{R} \to \mathbb{R}, \quad f(x) = x^2 + 2. \\ g &: \mathbb{R} \to \mathbb{R}, \quad g(x) = x^3 - 1. \end{aligned}$$

Is f surjective? Explain your answer.

Is g injective? Explain your answer. 4

(d) Consider the function:

$$f : [0, 1] \to \mathbb{R}, \quad f(x) = 1 + x^2.$$

What could you replace the codomain of this function with in order to make it surjective? 4

(e) Is it possible for a function to be neither injective nor surjective? Illustrate your answer by way of an example. 3

Q 2 (a) Given the sequence $\{a_n\} = n^2 - 1$. Evaluate S_4. 4

(b) Show that the recursively defined sequence (which you may assume is convergent) defined by

$$a_1 = 1, \quad a_{n+1} = \frac{1}{2}\left(a_n + \frac{p}{a_n}\right)$$

converges to $\sqrt{p}$, for $p > 0$.

Use this recursive sequence to compute $\sqrt{5}$ to three decimal places. 8

(c) Show that the series

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

converges. 5

(d) Note that the series in Q2(c) can be used to estimate e^2. Estimate e^2 correct to three decimal places. 3

Q 3 (a) Give an outline of the Newton-Raphson algorithm for root finding and explain how it works. 6

(b) Given

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^3 - 2x - 2.$$

Evaluate $f(x)$ for $x = 1$ and $x = 2$. Hence use the Newton-Raphson algorithm to estimate the root of the function correct to 4 decimal places. 10

(c) Give two examples of instances when the Newton-Raphson algorithm fails. Illustrations can be used as part of your examples. 4

Q 4 (a) Define what is meant by the dot product of two vectors. 3

(b) Consider the two vectors

$$\mathbf{v} = \langle 4, 3 \rangle, \quad \mathbf{w} = \langle 5, 12 \rangle$$

1. Find $|\mathbf{v}|$ and $|\mathbf{w}|$
2. Find $\mathbf{v}.\mathbf{w}$
3. Find the acute angle between $\mathbf{v}$ and $\mathbf{w}$

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(c) Consider the line segment with endpoints $(2, 1)$ and $(3, 3)$. Using the rotation matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

rotate the above line segment about its endpoint $(2, 1)$ by $\frac{\pi}{3}$ radians. 8

Q 5 (a) Explain under what circumstances it is possible to (i) subtract and (ii) multiply the matrices A (order $m \times n$) and B (order $p \times q$). 4

(b) Let

$$A = \begin{bmatrix} 1 & 3 \\ -4 & 2 \\ 3 & -1 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 4 & -1 & 3 \\ 2 & 3 & 0 \end{bmatrix}$$

Calculate AB and BA, if possible. 7

(c) Show, using the matrices in Q5(b), that

1. $(A^T)^T = A$
2. $(AB)^T = B^T A^T$

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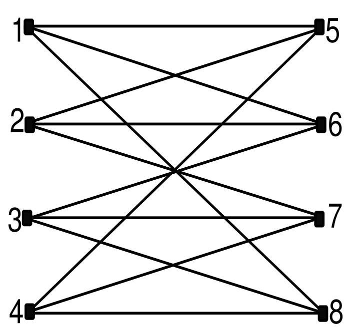
Q 6 (a) State the requirements necessary for a graph to be planar and show that the graph $K_{2,3}$ is planar. 4

(b) Write down the adjacency matrix for $K_{4,3}$. 4

(c) What is the expression tree for $(2 + x)(3 + x) - (1 - x)^2$? Is your result a full binary tree? 4

(d) The following two graphs are isomorphic.



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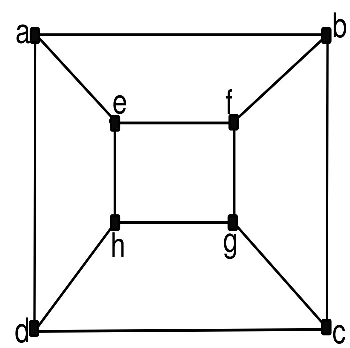
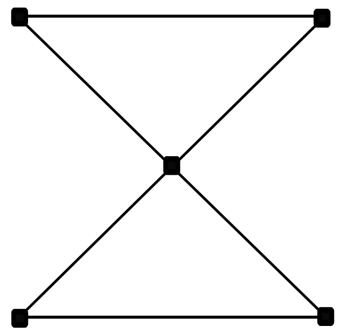


(b)

Find the isomorphic bijection for the two graphs. 4

(e) Are the following graphs (i) Hamiltonian, (ii) Eulerian? Clearly explain your answers.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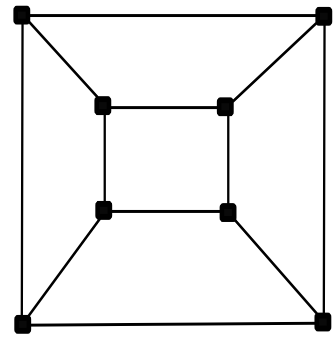
(b)

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