Question 8 Given S is the set of all 5 digit binary strings, E is the set of a 5 digit binary strings beginning with a 1 and F is the set of all 5 digit binary strings ending with two zeroes.

(a) Find the cardinality of S, E and F. [3]

(b) Draw a Venn diagram to show the relationship between the sets S, E and F. Show the relevant number of elements in each region of your diagram. [3]

(c) What is the probability that a 5 digit binary string chosen at random :

(i) begins with a 1;

(ii) ends with two zeroes;

(iii) both begins with a 1 and ends with two zeroes;

(iv) either begins with a 1 or ends with two zeroes or both? [3]

(d) Say whether or not E and F are independent events, justifying your answer. [1]

Question 9

(a) Given the graph G with vertices v_1, v_2, ... v_7 and adjacency list

$v_1 : v_2, v_4$

$v_2 : v_1, v_3$

$v_3 : v_2, v_4$

$v_4 : v_1, v_3, v_5$

$v_5 : v_4, v_6$

$v_6 : v_5, v_7$

$v_7 : v_5, v_6$.

(i) Draw this graph.

(ii) Say how many edges there are in a tree with n vertices. Hence explain how many edges must be removed from G to create a spanning tree.

(iii) The graph G has precisely 12 different spanning trees, list the twelve distinct pairs of edges which, when removed, give the 12 spanning trees, T_1, T_2, ... T_{12}.

(iv) By partitioning the set $\{T_1, T_2, ... T_{12}\}$ into subsets where the trees of a subset are all isomorphic to one another, while the two trees from different subsets are non-isomorphic, or otherwise, draw the four non-isomorphic spanning trees of G. [7]