Proof By Induction

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Proof By Induction

Prove by Induction the following expression

$$\sum_{r=1}^{r=n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

for all $n \in N_0$.

Proof By Induction

- **Step 1** : Is the proposition true for $n = 1$?
- **Step 2** : Show that if the proposition is true for $n = k$, then it is also true for $n = k + 1$.
- **Step 3** : Conclude that the proposition is true for all natural numbers greater than or equal to one.

Proof By Induction

Prove by induction the following expression

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for all $n \in N_0$.

- **Step 1** : Is the proposition true for $n = 1$?

Left-hand side

$$\sum_{r=1}^{r=1} r^2 = 1^2 = 1$$

Proof By Induction

Left-hand side

$$\sum_{r=1}^{r=1} r^2 = 1^2 = \mathbf{1}$$

Right-hand side

$$\frac{n(n+1)(2n+1)}{6} = \frac{1(1+1)(2\cdot 1+1)}{6}$$
$$= \frac{1\times 2\times 3}{6} = \frac{6}{6} = \mathbf{1}$$

Proof By Induction

- **Step 1** : Is the proposition true for $n = 1$?

Yes: The left-hand side and right-hand side of the equation yield the same value.

Proof By Induction

- **Step 2** : Show that if the proposition is true for $n = k$, then it is also true for $n = k + 1$.

Given:

$$\sum_{r=1}^{r=k} r^2 = \frac{k(k+1)(2k+1)}{6}$$

To Prove:

$$\sum_{r=1}^{r=k+1} r^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

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$$\sum_{r=1}^{r=k+1} r^2 = 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2$$

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$$\sum_{r=1}^{r=k+1} r^2 = \sum_{\mathbf{r=1}}^{\mathbf{r=k}} \mathbf{r^2} + (k+1)^2$$

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$$\sum_{r=1}^{r=k+1} r^2 = \sum_{\mathbf{r=1}}^{\mathbf{r=k}} \mathbf{r^2} + (k+1)^2$$

$$\sum_{r=1}^{r=k+1} r^2 = \frac{k(k+1(2k+1))}{6} + (k+1)^2$$

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