Rules of Inclusion, Listing and Cardinality

For each of the following sets, a set is specified by the rules of inclusion method and listing method respectively. Also stated is the cardinality of that data set.

Worked example 1

- $\{x : x \text{ is an odd integer } 5 \leq x \leq 17\}$
- $x = \{5, 7, 9, 11, 13, 15, 17\}$
- The cardinality of set x is 7.

Worked example 2

- $\{y : y \text{ is an even integer } 6 \leq y < 18\}$
- $y = \{6, 8, 10, 12, 14, 16\}$
- The cardinality of set y is 6.

Worked example 3

A perfect square is a number that has a integer value as a square root. 4 and 9 are perfect squares ($\sqrt{4} = 2$, $\sqrt{9} = 3$).

- $\{z : z \text{ is an perfect square } 1 < z < 100\}$
- $z = \{4, 9, 16, 25, 36, 49, 64, 81\}$
- The cardinality of set z is 8.

Exercises

For each of the following sets, write out the set using the listing method. Also write down the cardinality of each set.

- $\{s : s \text{ is an negative integer } -10 \leq s \leq 0\}$
- $\{t : t \text{ is an even number } 1 \leq t \leq 20\}$
- $\{u : u \text{ is a prime number } 1 \leq u \leq 20\}$
- $\{v : v \text{ is a multiple of 3 } 1 \leq v \leq 20\}$

Power Sets

Worked Example

Consider the set Z:

$$Z = \{a, b, c\}$$

Q1 How many sets are in the power set of Z?

Q2 Write out the power set of Z.

Q3 How many elements are in each element set?

Solutions to Worked Example

Q1 There are 3 elements in Z. So there is $2^3 = 8$ element sets contained in the power set.

Q2 Write out the power set of Z.

$$\mathcal{P}(Z) = \{\{0\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$$

Q3

* One element set is the null set - i.e. containing no elements
* Three element sets have only elements
* Three element sets have two elements
* One element set contains all three elements
* 1+3+3+1=8

Exercise

For the set $Y = \{u, v, w, x\}$, answer the questions from the previous exercise

Complement of a Set

Consider the universal set U such that

$$U = \{2, 4, 6, 8, 10, 12, 15\}$$

For each of the sets A,B,C and D, specify the complement sets.

Set	Complement
$A = \{4, 6, 12, 15\}$	$A' = \{2, 8, 10\}$
$B = \{4, 8, 10, 15\}$	
$C = \{2, 6, 12, 15\}$	
$D = \{8, 10, 15\}$	

Set Operations

Consider the universal set U such that

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and the sets

$$A = \{2, 5, 7, 9\}$$

$$B = \{2, 4, 6, 8, 9\}$$

(a) $A - B$

(b) $A \otimes B$

(c) $A \cap B$

(d) $A \cup B$

(e) $A' \cap B'$

(f) $A' \cup B'$

Venn Diagrams

Draw a Venn Diagram to represent the universal set $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6\}$ with subsets $A = \{2, 4, 5\}$ $B = \{1, 4, 5, 6\}$

Find each of the following

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

(e) $A \otimes B$

Conversion to Binary Form

		Quotient	Remainder
507	*253.5*	253	1
253	*126.5*	126	1
126	*63*	63	0
63	*31.5*	31	1
31	*15.5*	15	1
15	*7.5*	7	1
7	*3.5*	3	1
3	*1.5*	1	1
1	*0.5*	0	1

Correct Answer : 111111011
2011 Question 1C
List all the set of positive integers with precisely 3 bits in binary notation

100	4
101	5
110	6
111	7

Let n be a positive integer. How many positive integers have precisely n bits in binary notation.

Express the binary number (1011.011) as a decimal, showing all of your working

Digit	Power	Weight	
1	3	$2^3 = 8$	8
0	2	$2^2 = 4$	0
1	1	$2^1 = 2$	2
1	0	$2^0 = 1$	1
.	.	.	.
0	-1	$2^{-1} = 0.5$	0
1	-2	$2^{-1} = 0.25$	0.25
1	-3	$2^{-3} = 0.125$	0.125
			11.375

2011 Question 1b

Working in Binary, and showing all carries, compute the following:
$(11010)_2 + (111)_2$ (i.e. $(26)_10 + 7_{10}$)
(Demonstration on whiteboard)
Answer: 11010 + 111 = 100001

- Remark 0 is not a positive integer.
- How many have 1 bit : Answer only one 1.
- How many have 2 bits : Answer only two : 10 and 11 (i.e. 2 and 3 in decimal)
- From above: 4 have 3 bits.
- What is the first 4 bit numbers? : 1000 (i.e 8 in decimal)
- What is the last 4 bit number? 1111 (i.e 15 in decimal)
- How many numbers have 4 bits? Answer :8
- Answer to question: 2^{n-1}
- Return to this after "Proof by Induction".

Hexadecimal

- Hexadecimal basic concepts:
- Hexadecimal is base 16.
- There are 16 digits in counting (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)
- When you reach 16, you carry a 1 over to the next column
- The number after F (decimal 15) is 10 in hex (or 16 in decimal)

Hex	A	B	C	D	E	F
Dec	10	11	12	13	14	15

Conversion to Decimal

$$\mathbf{5} \times 16^2 + \mathbf{A} \times 16^1 + \mathbf{9} \times 16^0$$

$$= 5 \times 16^2 + 10 \times 16^1 + 9 \times 16^0$$

$$= 1280 + 160 + 9$$

Hexadecimal Addition

- Add one column at a time.
- Convert to decimal and add the numbers.
- If the result of step two is 16 or larger subtract the result from 16 and carry 1 to the next column.
- If the result of step two is less than 16, convert the number to hexadecimal.

$$\begin{array}{cccc} & 5 & A & 9 \\ - & 6 & 9 & 4 \\ \hline \end{array}$$

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$$\begin{array}{cccc} & 5 & A & 9 \\ + & 6 & 9 & 4 \\ \hline \end{array}$$

1. Add one column at a time
2. Convert to decimal and add (9 + 4 = 13)
3. Decimal 13 is hexadecimal D
4. Next column :Convert to decimal and add (10 + 9 = 19)
5. (19 larger than 16) 14 in Hexadecimal
6. Leave 4 carry to one
7. Next Colulm add 1 + 5 + 6 = 12)
8. Decimal 12 is hexadecimal C

$$\begin{array}{cccc} & 5 & A & 9 \\ + & 6 & 9 & 4 \\ \hline & C & 4 & D \end{array}$$

Hexadecimal Subtraction

$$\begin{array}{cccc} & B & B & B \\ - & A & 5 & D \\ \hline \end{array}$$

$$\begin{array}{ccc} B & \boldsymbol{A} & \boldsymbol{1B} \\ A & 5 & D \\ \hline \end{array}$$

```
If U = {2, 3, 4, 7, 9, 10, 11, 13} and A = {3, 4, 9, 10}

Then

Compliment of A = A  = U - A
= {2, 3, 4, 7, 9, 10, 11, 13}  {3, 4, 9, 10}
= {2, 7, 11, 13}

\begin{verbatim}
UNIVERSAL SET:

A universal set is a set of all elements under consideration. It is denoted by U.
A Universal set is always a non-empty set.

For example: The set of real numbers R is a universal set
for the operations related to real numbers.

Example:

Given that U = {5, 6, 7, 8, 9, 10, 11, 12}, list the elements of the following sets.
A = {x : x is a prime number}
B = {x : x is a factor of 60}
Solution:

The elements of sets A and B can only be selected
from the given universal set U
A = {5, 7, 11}
B = {5, 6, 10, 12}
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MUTUALLY EXCLUSIVE SETS OR DISJOINT SETS:

Two sets are called disjoint if they don't have any common element.

For example, A = {2, 3, 4} and B = {5, 6, 7} are disjoint sets.