

HibColl Videos

1.A Converting from Decimal to Binary 1.B Converting from Decimal to Hexadecimal 1.C Converting from Binary to Decimal 1.D Converting from Hexadecimal to Decimal 1.E Binary Addition 1.F Binary Subtraction  
2.A Membership Tables 2.B Venn Diagrams 2.C Set differences and symmetric difference 2.D 2.E

# Chapter 1

## Number Systems

### 1.0.1 Significant Digits

There are three rules on determining how many significant figures are in a number: Non-zero digits are always significant. Any zeros between two significant digits are significant. A final zero or trailing zeros in the decimal portion ONLY are significant.

### 1.1 Floating Point Notation

In computing, floating point describes a method of representing an approximation of a real number in a way that can support a wide range of values.

The numbers are, in general, represented approximately to a fixed number of significant digits (the mantissa) and scaled using an exponent.

In essence, computers are integer machines and are capable of representing real numbers only by using complex codes. The most popular code for representing real numbers is called the IEEE Floating-Point Standard. The term floating point is derived from the fact that there is no fixed number of digits before and after the decimal point; that is, the decimal point can float. There are also representations in which the number of digits before and after the decimal point is set, called fixed-point representations. In general, floating-point representations are slower and less accurate than fixed-point representations, but they can handle a larger range of numbers.

## Part A: Number Systems - Binary Numbers

1. Express the following decimal numbers as binary numbers.

i)  $(73)_{10}$

ii)  $(15)_{10}$

iii)  $(22)_{10}$

All three answers are among the following options.

- a)  $(10110)_2$                       b)  $(1111)_2$                       c)  $(1001001)_2$                       d)  $(1000010)_2$

2. Express the following binary numbers as decimal numbers.

- a)  $(101010)_2$                       b)  $(10101)_2$                       c)  $(111010)_2$                       d)  $(11010)_2$

3. Express the following binary numbers as decimal numbers.

- a)  $(110.10101)_2$                       b)  $(101.0111)_2$                       c)  $(111.01)_2$                       d)  $(110.1101)_2$

4. Express the following decimal numbers as binary numbers.

- a)  $(27.4375)_{10}$                       b)  $(5.625)_{10}$                       c)  $(13.125)_{10}$                       d)  $(11.1875)_{10}$

## Part B: Number Systems - Binary Arithmetic

1. Perform the following binary additions.

a)  $(110101)_2 + (1010111)_2$

c)  $(11001010)_2 + (10110101)_2$

b)  $(1010101)_2 + (101010)_2$

d)  $(1011001)_2 + (111010)_2$

2. Perform the following binary subtractions.

a)  $(110101)_2 - (1010111)_2$

c)  $(11001010)_2 - (10110101)_2$

b)  $(1010101)_2 - (101010)_2$

d)  $(1011001)_2 - (111010)_2$

3. Perform the following binary multiplications.

a)  $(1001)_2 \times (1000)_2$

c)  $(111)_2 \times (1111)_2$

b)  $(101)_2 \times (1101)_2$

d)  $(10000)_2 \times (11001)_2$

4. Perform the following binary multiplications.

5. Perform the following binary divisions.

(a) Which of the following binary numbers is the result of this binary division:  $(111001)_2 \div (10011)_2$ .

a)  $(10)_2$

c)  $(100)_2$

b)  $(11)_2$

d)  $(101)_2$

(b) Which of the following binary numbers is the result of this binary division:  $(101010)_2 \div (111)_2$ .

a)  $(11)_2$

c)  $(101)_2$

b)  $(100)_2$

d)  $(110)_2$

(c) Which of the following binary numbers is the result of this binary division:  $(1001110)_2 \div (1101)_2$ .

a)  $(100)_2$

c)  $(111)_2$

b)  $(110)_2$

d)  $(1001)_2$

## Part D: Natural, Rational and Real Numbers

- $\mathbb{N}$  : natural numbers (or positive integers)  $\{1, 2, 3, \dots\}$
- $\mathbb{Z}$  : integers  $\{-3, -2, -1, 0, 1, 2, 3, \dots\}$

\* (The letter  $\mathbb{Z}$  comes from the word *Zahlen* which means “numbers” in German.)

- $\mathbb{Q}$  : rational numbers
- $\mathbb{R}$  : real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

\* (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

1. State which of the following sets the following numbers belong to.

- |                  |                   |           |                 |
|------------------|-------------------|-----------|-----------------|
| 1) 18            | 3) $\pi$          | 5) $17/4$ | 7) $\sqrt{\pi}$ |
| 2) $8.2347\dots$ | 4) $1.33333\dots$ | 6) 4.25   | 8) $\sqrt{25}$  |

The possible answers are

- a) Natural number :  $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
- b) Integer :  $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
- c) Rational Number :  $\mathbb{Q} \subseteq \mathbb{R}$
- d) Real Number  $\mathbb{R}$

## Floating Point Notation

(Demonstration on white board)

## Part A: Number Systems - Binary Numbers

1. Express the following decimal numbers as binary numbers.

- |                |                 |                  |
|----------------|-----------------|------------------|
| i) $(73)_{10}$ | ii) $(15)_{10}$ | iii) $(22)_{10}$ |
|----------------|-----------------|------------------|

All three answers are among the following options.

- |                |               |                  |                  |
|----------------|---------------|------------------|------------------|
| a) $(10110)_2$ | b) $(1111)_2$ | c) $(1001001)_2$ | d) $(1000010)_2$ |
|----------------|---------------|------------------|------------------|

2. Express the following binary numbers as decimal numbers.

- a)  $(101010)_2$                       b)  $(10101)_2$                       c)  $(111010)_2$                       d)  $(11010)_2$

3. Express the following binary numbers as decimal numbers.

- a)  $(110.10101)_2$                       b)  $(101.0111)_2$                       c)  $(111.01)_2$                       d)  $(110.1101)_2$

4. Express the following decimal numbers as binary numbers.

- a)  $(27.4375)_{10}$                       b)  $(5.625)_{10}$                       c)  $(13.125)_{10}$                       d)  $(11.1875)_{10}$

## Part B: Number Systems - Binary Arithmetic

(See section 1.1.3 of the text)

1. Perform the following binary additions.

- a)  $(110101)_2 + (1010111)_2$                       c)  $(11001010)_2 + (10110101)_2$   
b)  $(1010101)_2 + (101010)_2$                       d)  $(1011001)_2 + (111010)_2$

2. Perform the following binary subtractions.

- a)  $(110101)_2 - (1010111)_2$                       c)  $(11001010)_2 - (10110101)_2$   
b)  $(1010101)_2 - (101010)_2$                       d)  $(1011001)_2 - (111010)_2$

3. Perform the following binary multiplications.

- a)  $(1001)_2 \times (1000)_2$                       c)  $(111)_2 \times (1111)_2$   
b)  $(101)_2 \times (1101)_2$                       d)  $(10000)_2 \times (11001)_2$

4. Perform the following binary multiplications.

(a) Which of the following binary numbers is the result of this binary division:  $(10)_2 \times (1101)_2$ .

a)  $(11010)_2$

c)  $(10101)_2$

b)  $(11100)_2$

d)  $(11011)_2$

(b) Which of the following binary numbers is the result of this binary division:  $(101010)_2 \times (111)_2$ .

a)  $(11000)_2$

c)  $(10101)_2$

b)  $(11001)_2$

d)  $(11011)_2$

(c) Which of the following binary numbers is the result of this binary division:  $(1001110)_2 \times (1101)_2$ .

a)  $(11000)_2$

c)  $(10101)_2$

b)  $(11001)_2$

d)  $(11011)_2$

5. Perform the following binary divisions.

(a) Which of the following binary numbers is the result of this binary division:  $(111001)_2 \div (10011)_2$ .

a)  $(10)_2$

c)  $(100)_2$

b)  $(11)_2$

d)  $(101)_2$

(b) Which of the following binary numbers is the result of this binary division:  $(101010)_2 \div (111)_2$ .

a)  $(11)_2$

c)  $(101)_2$

b)  $(100)_2$

d)  $(110)_2$

(c) Which of the following binary numbers is the result of this binary division:  $(1001110)_2 \div (1101)_2$ .

a)  $(100)_2$

c)  $(111)_2$

b)  $(110)_2$

d)  $(1001)_2$

## Part C: Number Bases - Hexadecimal

1. Answer the following questions about the hexadecimal number systems

a) How many characters are used in the hexadecimal system?

b) What is highest hexadecimal number that can be written with two characters?

c) What is the equivalent number in decimal form?

d) What is the next highest hexadecimal number?

2. Which of the following are not valid hexadecimal numbers?

a) 73

b)  $A5G$

c) 11011

d)  $EEF$

3. Express the following decimal numbers as a hexadecimal number.

a)  $(73)_{10}$

b)  $(15)_{10}$

c)  $(22)_{10}$

d)  $(121)_{10}$

4. Compute the following hexadecimal calculations.

a)  $5D2 + A30$

b)  $702 + ABA$

c)  $101 + 111$

d)  $210 + 2A1$

## Part D: Natural, Rational and Real Numbers

- $\mathbb{N}$  : natural numbers (or positive integers)  $\{1, 2, 3, \dots\}$
- $\mathbb{Z}$  : integers  $\{-3, -2, -1, 0, 1, 2, 3, \dots\}$ 
  - (The letter  $\mathbb{Z}$  comes from the word *Zahlen* which means “numbers” in German.)
- $\mathbb{Q}$  : rational numbers
- $\mathbb{R}$  : real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$ 
  - (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

### 1.1.1 Irrational Numbers

An irrational number cannot be expressed as a ratio between two numbers and it cannot be written as a simple fraction because there is not a finite number of numbers when written as a decimal. Instead, the numbers in the decimal would go on forever, without repeating.

In mathematics, the cardinality of a set is a measure of the “number of elements of the set”. For example, the set  $A = \{2, 4, 6\}$  contains 3 elements, and therefore  $A$  has a cardinality of 3.



# Chapter 2

## graph theory

Given the following definitions for simple, connected graphs:

- $K_n$  is a graph on  $n$  vertices where each pair of vertices is connected by an edge;
- $C_n$  is the graph with vertices  $v_1, v_2, v_3, \dots, v_n$  and edges  $\{v_1, v_2\}, \{v_2, v_3\}, \dots, \{v_n, v_1\}$ ;
- $W_n$  is the graph obtained from  $C_n$  by adding an extra vertex,  $v_{n+1}$ , and edges from this to each of the original vertices in  $C_n$ .

(a) Draw  $K_4$ ,  $C_4$ , and  $W_4$ .

### 2.1 Set Theory

1. The Universal Set  $\mathcal{U}$
2. Union
3. Intersection
4. Set Difference
5. Relative Difference

DE Muorgan's Laws (Useful for Propositions)

membership tables

proof by truth tables AND OR NOT Set difference symmetric difference

Harmonic Mean

$$H_x = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$$

2,4,6,8  $\downarrow$  1/mean(1/a) [1] 3.84  $\downarrow$  1/a [1] 0.5000000 0.2500000 0.1666667 0.1250000  $\downarrow$  sum(1/a) [1] 1.041667  
 $\downarrow$  sum(1/a)\*24 [1] 25  $\downarrow$  96/25 [1] 3.84

### 2.1.1 Functions

Consider the floor function  $f : R \rightarrow Z$  given the rule

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

1. evaluate  $f(6)$  and  $f(-6)$
2. Show that  $f(x)$  is not one-to-one
- 3.

### 2.1.2 Functions

Evaluate  $f(6)$

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

$$f(6) = \lfloor \frac{6+1}{2} \rfloor = \lfloor \frac{7}{2} \rfloor$$

$$\lfloor 3.5 \rfloor = 3$$

### 2.1.3 Functions

Evaluate  $f(-6)$

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

$$f(6) = \lfloor \frac{-6+1}{2} \rfloor = \lfloor \frac{-5}{2} \rfloor$$

$$\lfloor -2.5 \rfloor = -3$$

### 2.1.4 Discrete Maths : Relations

- A relation  $R$  from a set  $A$  to a set  $B$  is a subset of the **cartesian product**  $A \times B$ .
- Thus  $R$  is a set of **ordered pairs** where the first element comes from  $A$  and the second element comes from  $B$  i.e.  $(a, b)$

### 2.1.5 Discrete Maths : Relations

- If  $(a, b) \in R$  we say that  $a$  is related to  $b$  and write  $aRb$ .
- If  $(a, b) \notin R$ , we say that  $a$  is not related to  $b$  and write  $aRb$ . CHECK
- If  $R$  is a relation from a set  $A$  to itself then we say that “ $R$  is a relation on  $A$ ”.

### 2.1.6 Discrete Maths : Relations

Example

- Let  $A = \{2, 3, 4, 6\}$  and  $B = \{4, 6, 9\}$
- Let  $R$  be the relation from  $A$  to  $B$  defined by  $\mathbf{xRy}$  if  $x$  divides  $y$  exactly.

### 2.1.7 Discrete Maths : Relations

Example

- Let  $A = \{2, 3, 4, 6\}$  and  $B = \{4, 6, 9\}$
- Let  $R$  be the relation from  $A$  to  $B$  defined by  $\mathbf{xRy}$  if  $x$  divides  $y$  exactly.
- Then

$$R = (2, 4), (2, 6), (3, 6), (3, 9), (4, 4), (6, 6)$$

## 2.2 Arrow Diagrams

- Domain
- Co-Domain
- Range

$$f(x) : \text{Domain} \rightarrow \text{Co-Domain}$$

$$f(x) : \mathbb{R} \rightarrow \mathbb{R}$$

## Polynomial Functions (4.1.5)

Constants ( $P_0$ )

Linear Functions ( $P_1$ )

Quadratic Functions ( $P_2$ )

Cubic Functions ( $P_3$ )

## Equality of Functions (4.1.6)

$$f(x) = g(x)$$

### 2.2.1 Exercise

$$h(x) : \mathbb{R} \rightarrow \mathbb{R} \quad g(x) : \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = \text{sqrt}(x)$$

$$g(x) = \sqrt{3}x + 2$$

$$h(x) = 2^x$$

- Is the function  $h(x)$  an *onto* function?
- determine the inverse function of  $h(x)$  and  $g(x)$
- Simplify the following function.

$$j(x) = \log_4(h(6x))$$

### 2.2.2 Onto Functions

Definition: If every element in the co-domain of the function has an ancestor, the function is said to be "onto". An onto function has the property that the domain is equal to the co-domain.

**Example 4.26 Page 53**

## 2.3 *One-to-One* Functions and *Onto* Functions

### 2.3.1 Invertible Functions

- One-to-One Function
- Onto Function

Onto Functions : Range and Co-Domain are equivalent

### 2.3.2 Inverting a Function

- You are given  $f(x)$  in terms of  $x$
- Re-arrange the equation so that  $x$  is given in terms of  $f(x)$
- Replace  $x$  with  $f^{-1}(x)$  and  $f(x)$  with  $x$

#### Example

- Determine the inverse function of  $f(x)$ . Re-arrange the equation so that  $x$  is given in terms of  $f(x)$

$$f(x) : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \sqrt{x+1}$$

- Square both sides of the equation.

$$[f(x)]^2 = x + 1$$

- Subtract 1 from both sides of the equation. We have the equation written in terms of  $x$ .

$$f(x)^2 - 1 = x$$

- Replace  $x$  with  $f^{-1}(x)$  and  $f(x)$  with  $x$

$$x^2 - 1 = f^{-1}(x)$$

- Re-arrange equation and specify domain and co-domain.

$$f(x) : \mathbb{R} \rightarrow \mathbb{R} \quad f^{-1}(x) = x^2 - 1$$

## 2.4 Big O-Notation

(b) Let  $S$  be the set of all 4 bit binary strings.

The function  $f : S \rightarrow \mathbb{Z}$  is defined by the rule:

$$f(x) = \text{the number of zeros in } x$$

for each binary string  $x \in S$ .

Find:

1. the number of elements in the domain
2.  $f(1000)$
3. the set of pre-images of 1
4. the range of  $f$ .

(c)

4.a  $\lfloor x - y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$

4.b

4.c

## 2.5 Section 4 Functions

### 2.5.1 Invertible Functions

A function is invertible if it fulfils two criteria

- The function is *onto*,
- The function is *one-to-one*.

State the conditions to be satisfied by a function  $f : X \leftarrow Y$  for it to have an inverse function  $f^{-1} : Y \leftarrow X$ .  
 $\lceil \frac{x^2+1}{4} \rceil$  where  $f : A \rightarrow \mathbf{Z}$

- Find  $f(4)$  and the ancestors of 3.
- Find the range of  $f$ .
- Is  $f$  invertible? Justify your answer

Given  $f : \mathbf{R} \rightarrow \mathbf{R}$  where  $f(x) = 3x-1$ , define fully the inverse of the function  $f$ , i.e.  $f^{-1}$ . State the value of  $f^{-1}(2)$

## 2.6 Laws of Exponents

Here are the Laws (explanations follow):

LawExample  $x^1 = x^6 \cdot 1 = 6 \cdot x^0 = 170 = 1 \cdot x^{-1} = 1/x^4 \cdot 1 = 1/4$   $x^m x^n = x^{m+n} x^2 x^3 = x^{2+3} = x^5$   $x^m/x^n = x^{m-n} x^6/x^2 = x^{6-2} = x^4$   $(x^m)^n = x^{mn} (x^2)^3 = x^{23} = x^6$   $(xy)^n = x^n y^n (xy)^3 = x^3 y^3$   $(x/y)^n = x^n/y^n (x/y)^2 = x^2/y^2$   $x^{-n} = 1/x^n x^{-3} = 1/x^3$  Z Score

$$Z = \frac{X - \mu}{\sigma}$$

### 2.6.1 Exercises

Showing your workings, use the rules of indices and logarithms to give the following two expression in their simplest form.

- **Exercise 1**

$$4 \cdot 2^x - 2^{x+1}$$

- **Exercise 2**

$$\frac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$$



### 2.6.2 Exercise 1

$$4 \cdot 2^x - 2^{x+1}$$

**Remarks:**

(looking at the second term)

1 Using the following rule

$$a^b \cdot a^c = a^{(b+c)}$$

2 Using this rule in reverse we can say

$$2^{x+1} = 2^x \cdot 2^1 = 2 \cdot (2^x)$$

$$4 \cdot 2^x - 2^{x+1} = (4 \cdot 2^x) - (2 \cdot 2^x)$$

### 2.6.3 Exercise 1

**Remarks:**

3 This expression is in the form

$$(a \cdot b) - (c \cdot b)$$

which can be re-expressed as follows

$$(a - c\sqrt{b}) \cdot b$$

$$\begin{aligned}(4 \cdot 2^x) - (2 \cdot 2^x) &= (4 - 2) \cdot 2^x \\ &= 2 \cdot 2^x = 2^{x+1}\end{aligned}$$

### 2.6.4 Exercise 2

$$\frac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$$

Useful Rule of Logarithms

$$\ln(a^b) = b \cdot \ln(a)$$

$$\frac{\ln(2) + 2 \cdot \ln(2) + 3 \cdot \ln(2) + 4 \cdot \ln(2) + 5 \cdot \ln(2)}{\ln(4)}$$

### 2.6.5 Exercise 2

Adding up all the terms in the numerator

$$\begin{aligned}\frac{1 \cdot \ln(2) + 2 \cdot \ln(2) + 3 \cdot \ln(2) + 4 \cdot \ln(2) + 5 \cdot \ln(2)}{\ln(4)} \\ = \frac{15 \cdot \ln(2)}{\ln(4)}\end{aligned}$$

### 2.6.6 Exercise 2

Our expression has now simplified to

$$\frac{15 \cdot \ln(2)}{\ln(4)}$$

We can simplify the denominator too

$$\ln(4) = \ln(2^2) = 2 \cdot \ln(2)$$

### 2.6.7 Exercise 2

Our expression has now simplified to

$$\frac{15 \cdot \ln(2)}{\ln(4)} = \frac{15 \cdot \ln(2)}{2 \cdot \ln(2)}$$

We can divide above and below by  $\ln(2)$  to get our final answer

$$\frac{15 \cdot \ln(2)}{2 \cdot \ln(2)} = \frac{15}{2} = 7.5$$

## Propositional Logic

## 2.7 Section 3 Logic

### 2.7.1 Logical Operations

- $\neg p$  the negation of proposition  $p$ .
- $p \wedge q$  Both propositions  $p$  and  $q$  are simultaneously true (Logical State AND)
- $p \vee q$  One of the propositions is true, or both (Logical State : OR)
- $p \otimes q$  Only one of the propositions is true (Logical State : exclusive OR (i.e XOR))

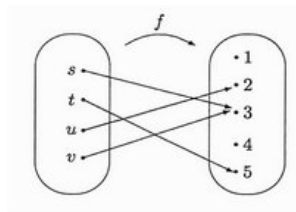
p	q	$p \vee q$	$q \wedge p$	$p \otimes q$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

## 2.8 Conditional Connectives

Construct the truth table for the proposition  $p \rightarrow q$ .

p	q	$p \rightarrow q$	$q \rightarrow p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

### Question 4



### Question 6

Let  $\mathcal{S}$  be a set and let  $\mathcal{R}$  be a relation on  $\mathcal{S}$ . Explain what it means to say that  $\mathcal{R}$  is

- reflexive
- symmetric
- anti-symmetric
- Transitive

### Question 10

(a) Given the following adjacency matrices A and B where

(i) Say whether or not the graphs they represent are isomorphic. (ii) Calculate  $A^2$  and  $A^4$  and say what information each gives about the graph corresponding to A. [6] (b) (i) Write down the augmented matrix for the following system of equations.

$$2x + y - z = 2$$

$$x - y + z = 4$$

$$x + 2y + 2z = 10$$

(ii) Use Gaussian elimination to solve the system. [4]

### Dice Rolls

Consider rolls of a die. What is the universal set?

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$$

### symbols

$\emptyset, \forall, \in, \notin, \cup$

## Chapter 3

# Sequences and Series, and Proof by Induction

### 3.1 Sequence and Series and Proof by Induction

$$\sum (n^2)$$

#### Part D : Real and Rational Numbers

- (i) Express the recurring decimal  $0.727272\dots$  as a rational number in its simplest form.
- (i) Given  $x$  is the irrational positive number  $\sqrt{2}$ , express  $x^8$  in binary notation.
- (ii) From part (i), is  $x^8$  a rational number?

#### Binary and Hex

1A.1 Converting from Binomial to Decimal

1A.2 Converting to Decimal

1A.3 Priority of Operation

1A.4

#### Numbers

1B.1 Real Numbers

1B.2 Rational Numbers

1B.3 Floating Point Arithmetic

1B.4

## Part 1. Number Systems

### Section 1a. Binary Numbers

1.  $1101001_{(2)}$
2.  $1101001_{(2)}$
3.  $1101001_{(2)}$

### 3.2 Inequality Operators

Given  $x = \sqrt{2}$  determine whether the following statements are true or false:

- (i)  $x \leq 2$
- (ii)  $1.42 > x > 1.41$
- (iii)  $x$  is a rational number
- (iv)  $\sqrt{2} = 2$

### 3.3 Revision Questions

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

#### Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of } X$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

# Mathematics for Computing

## Session 2 : Set Theory

# Mathematics for Computing

## Session 3 : Logic

### 3.4 Video 6

Convert the following statements into symbols:

- $\sqrt{2}$  is less than 1.5 and greater than 1.4
- $\sqrt{2}$  is greater than or equal to 5



# Chapter 4

## Session 4

### Invertible Functions

Necessary Conditions for Invertibility of a Function

- The function must be one-to-one
- The function must be onto.

### Equivalence Relations

#### 4.1 Digraphs and Relations

Given a flock of chickens, between any two chickens one of them is dominant. A relation,  $R$ , is defined between chicken  $x$  and chicken  $y$  as  $xRy$  if  $x$  is dominant over  $y$ . This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

- Amy is dominant over Beth and Carol
- Beth is dominant over Eve and Carol
- Carol is dominant over Eve and Daisy
- Daisy is dominant over Eve, Amy and Beth
- Eve is dominant over Amy.

## Section 4. Functions

### 4.2 Section 4 Functions

#### Question 4

##### Session 04: Functions

- Definitions

Domain

Co-domain

Image

Ancestor

Range

#### Part A : Functions

Given a real number  $x$ , say how the floor of  $x$   $\lfloor x \rfloor$  is defined.

- Find the values of  $\lfloor 2.97 \rfloor$  and  $\lfloor -2.97 \rfloor$ .
- Find an example of a real number  $x$  such that  $\lfloor 2x \rfloor \neq 2\lfloor x \rfloor$ , justifying your answer.

#### Absolute Value Function (4.1.3)

- The absolute value of some real number  $x$  is denoted  $|x|$ .
- If the number is positive, the absolute value is the same number.
- If the number is negative, the absolute value is the number without the minus sign.
- $|2| = 2$
- $|-2| = 2$

#### Floor and Ceiling Function (4.1.4)

#### Polynomial Functions (4.1.5)

Constants ( $P_0$ )

Linear Functions ( $P_1$ )

Quadratic Functions ( $P_2$ )

Cubic Functions ( $P_3$ )

## Equality of Functions (4.1.6)

$$f(x) = g(x)$$

## Encoding and Decoding Functions (4.2)

### Onto Functions (4.2.2)

### One-to-One Functions (4.2.3)

$f(x)$ , must be *One-to-One* and *Onto*

## Exponential and Logarithmic Functions (4.3)

The Laws of Logarithms

- 
- $\log_b(x^y) = y \times \log_b(x)$
- 
- 

## Big O-notation

Comparing the size of Functions (4.4)

Using O-notations

## Power Notation (4.4.2)

## Question 4

A function  $f: X \rightarrow Y$ , where  $X = \{p, q, r, s\}$  and  $Y = \{1, 2, 3, 4, 5\}$  is given by the subset of  $X \times Y$

- Show  $f$  as an arrow diagram
- state the domain, the co-domain, and the range of  $f$
- Say why  $f$  does not have the one-to-one property and why  $f$  does not have the "onto" property, giving a specific counter example in each case.

## 4.3 Section 4 Functions

### Functions

- Domain of a Function
- Range of a function

- Inverse of a function
- one-one (surjective)
- onto (bijective)

## 4.4 Video 7 : Numbers

**mantissa**

**abscissa**

**radix point**

- Number Systems
- Set Theory
- Function
- 
- Graph Theory
- Digraphs
- Set Theory
- Function
- Probability
- MATrices
- Continuously divide the decimal number by 2.
- Keep record of the remainder, either 0 or 1.
- The sequence of remainders is the binary number required.
- Hex Characters  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$
- 
- $\neq$  Not Equal
- $\downarrow$  Less than
- $\downarrow$  greater than
- $\geq$  greater than or equal to
- $\leq$  Not Equal to

- Natural Numbers  $\{1, 2, 3, 4, \dots\}$
- Integers  $\{\dots, -2, -1, 0, 1, 2, 3, \dots\}$
- Rational Numbers e.g  $4/7$  ,  $11/25$
- Real Numbers Any number e.g. 3.1415

‘ Floating Point Noation

Section 1 Section 1.2 Section 1.3 Section 14

- Decmal Number Systesm
  - Base 10
- Binary Number Systems
  - Base 2
  - allowable characters are 0.1 only
- Base 16 Hexadecimal
  - Use all of the decimal digits, in addition to 6 more A,B ,D,D,E,F

(where might you see this - specifying colours RGB Numbers

For example FF in hexadecimal is is 255 in decimal

Rational Numbers

Natural Numbers indies Integers Z

Computing a binary number

Useful

$2^0 = 1$	$2^4 = 16$
$2^1 = 2$	$2^5 = 32$
$2^2 = 4$	$2^6 = 64$
$2^3 = 8$	$2^7 = 128$

Firsly determine the highest power  
Suppose the number we wish to convert is 58  
Hwhat is the highest power of two that deivides