# Mathematics for Computing Irrational Numbers

kobriendublin.wordpress.com

Twitter: @StatsLabDublin

Given x is the irrational positive number  $\sqrt{2}$ :

- (i) Express  $x^8$  in binary notation.
- (ii) Is  $x^8$  a rational number?
- (iii) Write  $\left(\frac{1}{x}\right)^3$  in the form  $2^y$  where  $y \in \mathbb{Q}$ .

# **Part (i)** Express $x^8$ in binary notation

- $(\sqrt{2})^8 = 2^4 = 16$
- ► The binary equivalent of 16 is 1000.

**Part (ii)** Is  $x^8$  a rational number?

$$x^8 = 16$$

Yes 16 is an integer and therefore also a rational number.

$$x^8 \in \mathbb{Q}$$

**Part (iii)** Write  $\left(\frac{1}{x}\right)^3$  in the form  $2^y$  where  $y \in \mathbb{Q}$ .

$$\left(\frac{1}{x}\right)^3 = \left(\frac{1}{\sqrt{2}}\right)^3$$
$$\left(\frac{1}{\sqrt{2}}\right)^3 = \left(2^{-1/2}\right)^3$$
$$\left(2^{-1/2}\right)^3 = 2^{-3/2}$$
$$y = \frac{-3}{2}$$