## Dr. S. Franz

## MA4016 - Engineering Mathematics 6

## Solution Sheet 1 (February 04, 2010)

1. Proof by contraposition: Assume not (m is even or n is even).

8gl jug | 5gl jug | 3gl jug

- 2. Proof by contradiction: Assume that at most 2 days of the 25 have the same month. We have 12 different month and therefore only 24 days fulfil this assumption.
- **3.** Existence proof: 3 = 1 + 2Uniqueness proof:  $\sum_{i=1}^n i = \frac{n(n+1)}{2} \stackrel{!}{=} n+1$  gives as only positive solution n=2 and therefore the number n+1=3.

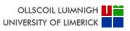
4.	Existence proof:	8	0	0
		3	5	0
		3	2	3
		6	2	0
		6	0	2
		1	5	2
		1	4	3

**5.** Proof by mathematical induction: IS:

$$\sum_{i=1}^{k+1} i^3 = \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 = (k+1)^2 \left(\frac{k^2 + 4k + 4}{4}\right) = \left(\frac{(k+1)(k+2)}{2}\right)^2.$$

- **6.** The sequence of numbers  $c_n = \sum_{i=1}^n 1/2^i$  leads to the conjecture  $\sum_{i=1}^n 1/2^i = 1-2^{-n}$ . Proof with mathematical induction. Alternatively the formula for the geometric sum can be applied.
- 7. Proof by mathematical induction: IS:

$$(1+x)^{k+1} = (1+x)(1+x)^k \ge (1+x)(1+kx) = 1 + (k+1)x + \underbrace{kx^2}_{\ge 0} \ge 1 + (k+1)x$$



8. We check n = 1, 2, 3, 4, 5 and see n = 1 works and n > 4 too. The later can be proved with mathematical induction.

$$(k+1)! = (k+1)k! \ge (k+1)k^2 = (k+1)^2 \frac{k^2}{k+1} > (k+1)^2$$

where we used

IS:

$$\frac{k^2}{k+1} = \frac{k^2 - 1}{k+1} + \frac{1}{k+1} = \underbrace{k-1}_{\geq 3} + \underbrace{\frac{1}{k+1}}_{> 0} \geq 3 > 1.$$

Thus the inequality is true for n = 1 and n > 4.

Alternatively an direct proof instead of the induction can be used, starting with

$$0 < n^2 - 4n + 2 = (n-1)(n-2) - n$$
 for  $n > 4$ .

9. Mathematical induction: IS:

$$(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 3k + 1 + 2k + 2 = k^3 + 2k + 3(k^2 + k + 1) = 3 \cdot (m + k^2 + k + 1).$$

Direct proof: Use the three distinct cases n = 3m, n = 3m + 1 and n = 3m + 2 with m positive integer and prove separately.

Alternatively use the equivalence

"3 divides 
$$n^3 + 2n$$
"  $\equiv$  "3 divides  $n^3 - n = (n-1)n(n+1)$ .