1 Set Theory

1. The Universal Set $\mathcal{U}$
2. Union
3. Intersection
4. Set Difference
5. Relative Difference

(2.2.2) Cardinality The number of distinct elements in a finite set is called its cardinality.

(2.2.3) Power Set

(2.3) Operations on Sets

(2.3.1) Complement of a set (2.3.2) Binary Operations on Sets -Union -Intersection -Set Difference -Symmetric Difference

Dice Rolls

Consider rolls of a die. What is the universal set?

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$$

Worked Example

Suppose that the Universal Set $\mathcal{U}$ is the set of integers from 1 to 9.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and that the set $\mathcal{A}$ contains the prime numbers between 1 to 9 inclusive.

$$\mathcal{A} = \{1, 2, 3, 5, 7\},$$

and that the set $\mathcal{B}$ contains the even numbers between 1 to 9 inclusive.

$$\mathcal{B} = \{2, 4, 6, 8\}.$$

Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}'$ and $\mathcal{B}'$

$$\mathcal{A}' = \{4, 6, 8, 9\},$$

$$\mathcal{B}' = \{1, 3, 5, 7, 9\},$$

Intersection

- Intersection of two sets describes the elements that are members of both the specified Sets
- The intersection is denoted $\mathcal{A} \cap \mathcal{B}$

$$\mathcal{A} \cap \mathcal{B} = \{2\}$$

- only one element is a member of both A and B.

Set Difference

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

symbols

$\varnothing$, $\forall$, $\in$, $\notin$, $\cup$

Prepositional Logic

- $p \wedge q$
- $p \vee q$
- $p \rightarrow q$

2 Sequence and Series and Proof by Induction

$$\sum(n^2)$$

Relative Difference

- $A \otimes B$

Power Sets

- Consider the set A where $A = \{w, x, y, z\}$
- There are 4 elements in set A.
- The power set of A contains 16 element data sets.
- $$\mathcal{P}(A) = \{\{x\}, \{y\}\}$$
- (i.e. 1 null set, 4 single element sets, 6 two -elemnts sets, 4 three lement set and one 4- element set.)

- $p \to q$ p implies q
- $p \lg q$

Session 05 Graph Theory

- Eulerian Path
- Isomorphism
- Adjacency matrices

Adjacency Matrices

$$\begin{pmatrix} o & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Functions

- Domain of a Function
- Range of a function
- Inverse of a function
- one-one (surjective)
- onto (bijective)

Probability

Binomial Coefficients

- factorials

  $$n! = (n) \times (n-1) \times (n-2) \times \ldots \times 1$$

  - $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
  - $3! = 3 \times 2 \times 1$

- Zero factorial

  $$0! = 1$$

The complement rule in Probability
$P(C') = 1 - P(C)$
If the probability of C is 70% then the probability of C' is 30%

3 Matrices

What are the dimensions of the following matrix

$$\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}\begin{pmatrix} c_1 & d_1 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} (a_1 \times c_1) + (a_2 \times c_2) & (a_1 \times d_1) + (a_2 \times d_2) \\ (b_1 \times c_1) + (b_2 \times c_2) & (b_1 \times d_1) + (b_2 \times d_2) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} (1 \times 1) + (3 \times 4) & (1 \times 2) + (3 \times 1) \\ (0 \times 4) + (2 \times 4) & (0 \times 2) + (2 \times 1) \end{pmatrix} = \begin{pmatrix} 14 & 5 \\ 8 & 2 \end{pmatrix}$$

$$\left(\begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} \begin{matrix} 1 & 3 \\ 0 & 2 \end{matrix}\right) =?$$