

1. Does there exist a single connective from which all statements in propositional logic can be built ? It's well known that the answer is yes. We investigate some possibilities in the following.
 - (a) Define the connective "Nand" (not...and...) by $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$ for any statements p and q . Represent the following using only this connective:
 - (i) $\neg p$
 - (ii) $p \vee q$
 - (iii) $p \wedge q$
 - (iv) $p \rightarrow q$
 - (v) $p \leftrightarrow q$
 - (b) Define the connective "Nor" (not ... or...) by $(p \downarrow q) \Leftrightarrow \neg(p \vee q)$ for any statements p and q . Represent the statements of part (a) using only this connective.
2. (a) Prove that $(p \wedge q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are logically equivalent.
 (b) Consider the Pascal programme segments shown below (all variables are integers):

<pre> z := 4; For i := 1 to 10 do Begin x := z - i; y := z + 3 * i; If ((x > 0) and (y > 0)) then Writeln ('The value of the sum x + y is ', x + y) End; </pre>	<pre> z := 4; For i := 1 to 10 do Begin x := z - i; y := z + 3 * i; If x > 0 then If y > 0 then Writeln ('The value of the sum x + y is ', x + y) End; End; </pre>
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- (i) Can the segments be related to part (a) ?
 - (ii) Which segment is preferable and why ?
3. Write each of the following in symbolic form. Then either prove the validity of the argument, or disprove it by counterexample, as appropriate.
 - (a) If Clare win the "All Ireland" this year, then Des McInerney will be happy. Being home in Clarecastle is sufficient to make Des happy. Consequently, if Clare win the "All Ireland" this year, then Des's happiness is a necessary condition for his being home in Clarecastle.
 - (b) If Lisa is in college on Friday, then she will attend her two o'clock lecture if she can catch a later bus home. Lisa will be in college this Friday but she can't get a later bus. Therefore she won't attend her two o'clock lecture.
 - (c) If the Punt is strong, then exports decrease. Unemployment will rise unless a decrease in exports is halted. A drop in interest rates is necessary to weaken the Punt. Hence a drop in interest rates is sufficient to cause unemployment to fall.

4. Consider the following *open* statement

$$p(x, y) : \quad y - x = y + x^2$$

where the universe of discourse is the set of all integers. Determine whether the following are true or false:

- (a) $p(0, 0)$ (b) $p(1, 1)$ (c) $p(0, 1)$ (d) $p(0, 3)$ (e) $\forall y \, p(0, y)$
(f) $\exists y \, p(1, y)$ (g) $\forall x, y \, p(x, y)$ (h) $\forall x \, \exists y \, p(x, y)$
(i) $\exists y \, \forall x \, p(x, y)$ (j) $\forall y \, \exists x \, p(x, y)$

5. Express each of the following in symbolic form. (The universe is the set of all strictly positive real numbers.)

- (a) There is no smallest positive real number.
(b) There exists a unique positive real number that equals its square. (You'll need the symbol $\exists!$ " There exists a unique ...". Define it in terms of \exists and \forall .)
(c) Every positive real number has a unique multiplicative inverse.