Question 1

(a) Convert the hexadecimal integer $(B07)_{16}$ to binary notation.

(b) Working in binary and showing all carries, compute $(101110)_2 + (110$

(c) Give an example of an element of the set

$$A = \{x \in \mathbb{R} : 1 < x \leq 2\}.$$

which is

i. an integer;

ii. rational, but not an integer;

iii. irrational.

(d) Showing your working, express the repeating decimal

$$3.14314314314\ldots$$

as a rational number in its simplest form.

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(a) The function $f:\ \mathbb{Z} \to \mathbb{Z}^{+}$ is given by the rule

$$f(x) = |5 - x^2|.$$

i. Find $f(2)$ and $f(-3)$.

ii. Find all the pre-images (ancestors) of 4 under f.

iii. Justifying your answer, say whether f is one-to-one.

iv. Justifying your answer, say whether f is onto.

(b) Let $\mathbb{A} = \{1, 2, 3, 4, 5, 6\}$ and consider the function $g : \mathbb{A} \to \mathbb{A}$ defined by the table

x	1	2	3	4	5	6
$g(x)$	4	5	2	1	3	6

i. Find $g(3)$ and $g(g(3))$.

ii. Explain why the function g is invertible and give the function table of g^{-1}.

Question 8

(a) What properties must a graph satisfy in order for it to be a *tree*?

(b) Justifying your answer, say whether it is possible to construct a tree with degree sequence $4, 3, 3, 2, 2, 1, 1$.

(c) Justifying your answer, say whether it is possible to construct a tree with degree sequence $2, 2, 2, 2, 2, 1, 1$.

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degree sequence $4, 3, 3, 2, 2, 1, 1$.

(c) Justifying your answer, say whether it is possible to construct a tree w
degree sequence $2, 2, 2, 2, 2, 1, 1$.

(c) Draw two non-isomorphic simple graphs with degree sequence $3, 2, 2,$
Explain why your two graphs are not isomorphic.

(d) Explain why it is not possible to draw a simple graph with degree seque

$$n, 3, 3, 3, 2, 1$$

if $n \neq 4$.

Question 10

(a) Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} 3y + 2z &= 2 \\ 2x + y + 3z &= 4 \\ 2x + 3y + z &= 2. \end{aligned}$$

Figure 1:

(b) Given the matrices

$$A = \begin{pmatrix} 0 & 3 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix},$$

i. Compute AB.

ii. Find a matrix X such $AX = B$.

iii. Find a matrix Y such that $AY = A$.

Question 2

(a) Let the two sets A and B be given by

$$A = \{3,\ \tfrac{1}{3},\ \pi\} \qquad \text{and} \qquad B = \{x \in \mathbb{Q} : x \notin \mathbb{Z}\}.$$

Give each of the following sets by using the listing method.

i. $A \cap \mathbb{Q}$;

ii. $A \cap \mathbb{R}$;

iii. $A \cap B$;

iv. $A - B$.

(b) A binary search tree is designed to store an ordered list of 300 records at its internal nodes.

i. Which record is stored at the root (at level 0) of the tree?

ii. Which records are stored at level 1 of the tree?

iii. Determine the number of records stored at level 8 of the tree.

Question 10

Let n be a non-negative integer and consider the sum

$$s_n = \sum_{i=0}^{n} 2^{2i}.$$

(a) Showing your working, calculate s_0, s_1, s_2, s_3 and s_4.

(b) For $n \geq 0$ find a recurrence relation giving s_{n+1} as a function of s_n.

(c) Prove by induction that $3s_n = 2^{2n+2} - 1$ for all $n \geq 0$.

Question 1

(a) Convert the decimal integer $(247)_{10}$ to binary notation.

(b) Working in binary and showing all carries, compute $(11110)_2 + (11$

(c) Let the two sets A and B be given by

$$A = \{\sqrt{2}, \tfrac{3}{2}, 2\} \qquad \text{and} \qquad B = \{x \in \mathbb{R} : x \notin \mathbb{Q}\}.$$

Give each of the following sets.

i. $A \cap \mathbb{Q}$;

ii. $A \cap B$;

iii. $B \cup \mathbb{Q}$.

Question 5

(a) Consider the floor function $f : \mathbb{R} \to \mathbb{Z}$ given by the rule

$$f(x) = \lfloor \tfrac{x+1}{2} \rfloor.$$

i. Compute $f(-6)$ and $f(6)$.

ii. Show that f is not one-to-one.

iii. Justifying your answer, say whether f is onto.

(b) A binary search tree T_n is designed to store an ordered list of n records a
internal nodes with the record $f(n) = \lfloor \frac{n+1}{2} \rfloor$ at its root.

i. Which record is stored at the root of the tree T_{200}?

ii. Which records are stored at level 1 of the tree T_{200}?

iii. Given that the binary search tree T_n has record number r at its root, ex
why it is not possible to determine the value of n from this information.

Question 6

(a) Suppose that we have a group consisting of 5 women and 10 men. A comm
consisting of 6 people is chosen from this group. Find the number of diff
committees possible and compute the probability that the committee chosen

i. all 5 women in it;

ii. at least one woman in it.

(b) Consider the set $S = \{1, 2, 3, \ldots, 600\}$. Justifying your answer, find the number of integers in S which

i. are divisible by 5;

ii. are divisible by 4, 5 or 6.

Question 8

(a) Suppose that it is given that a graph G has degree sequence $4, 3, 3, 3, 2, 1$.

i. Explain why this information is not sufficient to enable us to draw G.

ii. Justifying your answer, find the number of vertices in G.

iii. Justifying your answer, find the number of edges in G.