for all $x > 1$. Hence $f(x)$ is $O(x^2)$, where in the formal definition, we have $M = 16$, $g(x) = x^2$ and $x_0 = 1$. □

Example 4.42 We show that $f(x) = 2x^3 - 5x^2 + 27$ is $O(x^3)$.

Suppose that $x > 1$. This polynomial contains a negative coefficient. In this case we can say that the absolute value of $f(x)$, calculated at any given value of x, is at most the value we would get by *adding* all the terms, instead of adding some and subtracting others. Thus we have

$$|f(x)| \leq |2x^3| + |5x^2| + |27|.$$

Now each of the terms $2x^3$, $5x^2$, 27 is positive when $x > 1$. Hence

$$|2x^3| + |5x^2| + |27| = 2x^3 + 5x^2 + 27,$$

and using Result 4.27 again, we have

$$2x^3 + 5x^2 + 27 < 2x^3 + 5x^3 + 27x^3 = 34x^3,$$

for all $x > 1$. Thus $|f(x)| < 34|x^3|$, for all $x > 1$, and hence $f(x)$ is $O(x^3)$. □

Following the method of the previous two examples, we can prove the following result.

Result 4.29 *Suppose that m is the highest exponent of x present in a polynomial function $f(x)$, then $f(x)$ is $O(x^m)$.* □

4.4.4 Comparing the exponential and logarithmic functions with the power functions

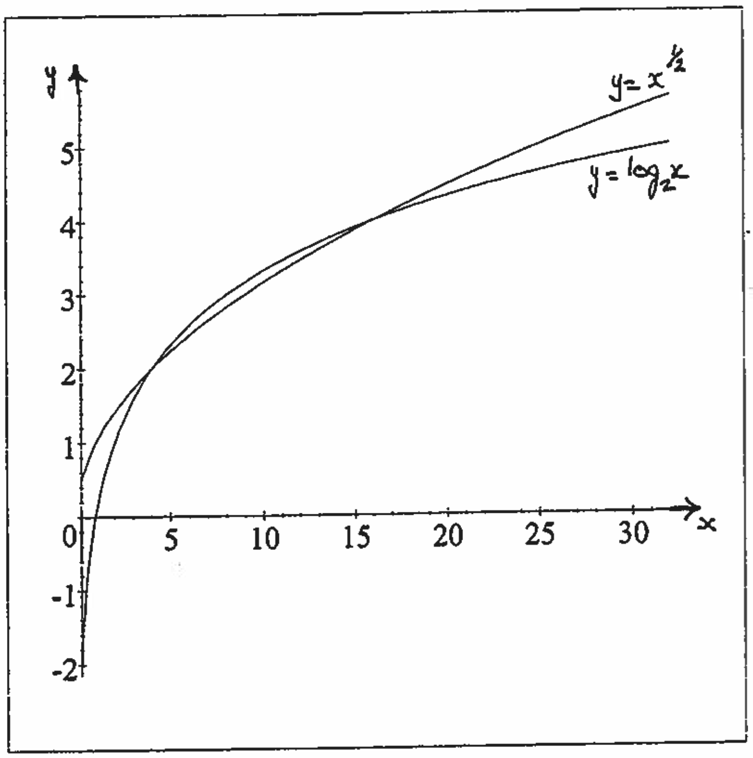

Figure 4.4.

It remains to compare the sizes of the exponential functions and the logarithmic functions with the power functions for large values of x. The following result can be established by calculus.