The Binomial Probability Distribution

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PART 1: INTRODUCTION

The Binomial Distribution

- The binomial probability distribution is a ***discrete*** probability distribution, that is associated with a multiple step experiment that we call the ***binomial experiment***.

PART 2: THE BINOMIAL EXPERIMENT

The Binomial Distribution

A binomial experiment has the following properties.

<u>Property 1</u>

The binomial experiment consists of a sequence of **n** independent , identical trials.

Throwing a coin ten times is an example of a binomial experiment .

The Binomial Distribution

<u>Property 2</u>

- Two, and only two, outcomes are possible at each trial. We refer to one as a “**success”** and the other as a “**failure”**.

- Throwing a “**head**” could thought of as a success, whereas throwing a “**tail**” could be thought of as a failure.

The Binomial Distribution

<u>Property 2</u>

- In another example, the “success” could refer to a randomly selected component could be found to be broken during an inspection.
- Conversely, the “failure” would refer to the randomly selected component being good.

The Binomial Distribution

Property 3

- The probability of a success, denoted by **p**, does not change from trial to trial.

- Similarly the probability of a failure, denoted by **1-p**, also does not change from trial to trial.

The Binomial Distribution

Property 4

- The trials are independent.
- The outcome of one trial does not have any effect on the outcome of the next.
- The fact that we have thrown a head in the last step doest not increase the chances of throwing a head in the next step.

The Binomial Distribution

In a binomial experiment our interest is in the number of successes occurring in the ***n*** trials.

To find out the probability of a specified number of successes in **n** trials, given the probability of success **p,** we use the binomial probability distribution.

PART 3: THE BINOMIAL DISTRIBUTION FORMULA

The Binomial Distribution

The formula for the probability distribution is as follows:

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$$

We will now explain each term of this formula individually.

The Binomial Distribution

$$P(X = k)$$

- This term denotes the probability (***P***) that number of successes (***X***) will be "***k***".
- For example, the probability that the number of successes will be three is written as follows:

  P(X=3)

The Binomial Distribution

$$\binom{n}{k}$$

- This is the "***choose operator***".
- This is used to calculate the number of ways ***k*** successes can occur in ***n*** trials.
- It is also denoted $^{\mathbf{n}}\mathbf{C}_{\mathbf{k}}$.

The Binomial Distribution

$$\binom{n}{k} = \frac{n!}{(n-k)! \times k!}$$

The Binomial Distribution

$$p^k$$

This is the probability of a success ***p*** to the power of the number of successes ***k.***

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The Binomial Distribution

$$(1-p)^{n-k}$$

This is the probability of a failure ***1-p*** to the power of the number of failures ***n-k.***

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PART 4: AN SIMPLE EXAMPLE

The Binomial Distribution

If we throw a coin 4 times, what is the probability that we do not throw a "head" on any of those four times.

Firstly we know that the number of trials is four (***n = 4***) . Also throwing a "***head***" is what constitutes a "success". (Throwing a tail is a "failure".)

The Binomial Distribution

What is the probability that the number of success (***k***) is zero?

P(X=0)

If we have zero successes out of four trials, the number of failures **(n-k**) must be four.

We assume that the probability of a success ***p*** , and of a failure ***1-p*** is 0.5 (50%)

The Binomial Distribution

Here are our important values:

- ***k = 0***
- ***n = 4***
- ***n-k = 4-0 = 4***
- ***p = 0.5***
- ***1-p = 0.5***

The Binomial Distribution

So what is the probability of no successes in four trials?

Assigning our values to the relevant positions in the formula we write

$$P(X = 0) = \binom{4}{0}(0.5)^0(0.5)^4$$

The Binomial Distribution

$$\binom{4}{0} = \frac{4!}{(4-0)! \times 0!}$$

$$\binom{4}{0} = \frac{4!}{4! \times 1} = \frac{4!}{4!} = 1$$

$$N.B. \quad 0! = 1$$

Binomial Distribution

The value of the choose operator is 1.

$$\binom{4}{0} = 1$$

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Binomial Distribution

The any value to the power of zero is always one.

$$(0.5)^0 = 1$$

The last value is found using a calculator.

$$(0.5)^4 = 0.625$$

Binomial Distribution

$$P(X = 0) = \binom{4}{0}(0.5)^0(0.5)^4$$

$$P(X = 0) = 1 \times 1 \times 0.625$$

$$P(X = 0) = 0.625$$

Binomial Distribution

Therefore the probability of not throwing any heads in four throws of a coin is **6.25**%

PART 5: ANOTHER SIMPLE EXAMPLE

Binomial Distribution

What is the probability that the number of success (***k***) is 2?

P(X=2)

If we have two successes out of four trials, the number of failures **(n-k = 4-2**) must be ***2***.

We assume that the probability of a success ***p*** , and of a failure ***1-p*** is 0.5 (50%)

Binomial Distribution

Here are our important values:

- ***k* = 2**
- ***n* = 4**
- ***n-k* = 4-2 = 2**
- ***P* = 0.5**
- ***1-p* = 0.5**

Binomial Distribution

So what is the probability of two successes in four trials?

Assigning our values to the relevant positions in the formula we write:

$$P(X = 2) = \binom{4}{2}(0.5)^2(0.5)^2$$

The Binomial Distribution

$$\binom{4}{2} = \frac{4!}{(4-2)!}$$

$$\binom{4}{2} = \frac{4!}{2! \times 2!} = \frac{4 \times 3 \times 2!}{2! \times 2!}$$

$$\binom{4}{2} = \frac{4 \times 3}{2 \times 1} = \frac{12}{2} = 6$$

Binomial Distribution

The next two terms are identical, and both evaluate to 0.25:

$$(0.5)^2 = 0.25$$

Binomial Distribution

$$P(X=2)=\binom{4}{2}(0.5)^2(0.5)^2$$

$$P(X=2)=6\times 0.25\ \times 0.25$$

$$P(X=2)=0.375$$

Binomial Distribution

Therefore the probability of throwing two heads in four throws of a coin is 37.50%