Mathematics for Computing
Mock Exam 2015

Question 1

(i) (1 Mark) When is the positive integer p said to be a prime number?

(ii) (2 Marks) Express the following hexadecimal number as a decimal number, and as a binary number:

$$(A32.8)_{16}$$

(iii) (2 Marks) Convert the following decimal number into base 2, showing all your working: $(253)_{10}$.

(iv) (2 Marks) Covert the decimal integer $(407)_{10}$ to binary notation.

(v) (2 Marks) Showing your working, express the following number

$$1.024024024024\ldots$$

as a ration number in its simplest form.

(vi) (1 Mark) Compute the following $101101_2 + 1101_2$

Question 2

Let A and B and C be subsets of a universal set U.

(a) (1 Mark) Draw a labelled Venn diagram depicting A,B,C in such a way that they divide U into 8 disjoint regions. [1]

(b) (3 Marks) The subset $X \subseteq U$ is defined by the following membership table below. Shade the region X on your diagram. Describe the region you have shaded in set notation as simply as you can.

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

(c) (3 Marks) The subset $Y \subseteq U$ is defined as $Y = A \cup (CB)$. Construct the membership table for Y.

(d) (3 Marks) For each of the following statements say whether it is true or false, justifying your answer, using the Venn diagram you drew earlier.

(i) $Y \subseteq X$

(ii) $Y' \subseteq X'$

(iii) $YX = A \cap B \cap C$.

Question 3

Let n be an element of the set $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$, and p and q be the propositions:

$$p : n \text{ is odd}, q : n < 15$$

.

(a) Draw up truth tables for the following statements and find the values of n for which they are true:

(i) $p \vee \neg q$

(ii) $\neg p \wedge q$

(b) Use truth tables to find a statement that is logically equivalent to $\neg p \rightarrow q$.

(i) Let p and q be propositions. Use Truth Tables to prove that

$$p \rightarrow q \equiv \neg q \rightarrow \neg$$

Question 4

Let S be the set of all 4 bit binary strings. The function $f : S \rightarrow Z$ is defined by the rule:

$$f(x) = \text{ the number of zeros in x}$$

for each binary string $x \in S$. Find:

(a) (4 Marks) Answer the following questions

(i) the number of elements in the domain

(ii) f(1010)

(iii) the set of pre-images of 1

(iv) the range of f.

(b) (2 Marks) Decide whether the function f, as defined above, has either the one to one or the onto property, justifying your answers.

(c) (2 Marks) State the condition to be satisfied by a function $f : X \rightarrow Y$ for it to have an inverse function $f^1 : Y \rightarrow X$.

(d) (2 Marks) Define the inverse functions for each of the following:

Question 5

Question 5

Given the following definitions for simple, connected graphs:

(a) Let G be a simple graph. Explain why the sum of the degrees of the vertices of G is twice the number of its edges. [2]

(b) Justifying your answer, say why it is not possible to construct a simple graph G with degree sequence

$$6, 5, 3, 2, 2, 1, 1, 1.$$

[2]

(c) Justifying your answer, say whether it is possible to construct a simple graph with degree sequence $3, 3, 3, 3, 3, 3$. [2]

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

a) (i) A simple, connected graph has 7 vertices, all having the same degree d. State the possible values of d and for each value also give the number of edges in the corresponding graph. (ii) Another simple, connected graph has 6 vertices, all having the same degree, n. Draw such a graph when n = 3 and state the other possible values of n. [4]

Question 6

Given a flock of chickens, between any two chickens one of them is dominant. A relation, R, is defined between chicken x and chicken y as xRy if x is dominant over y. This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

- Amy is dominant over Beth and Carol
- Beth is dominant over Eve and Carol
- Carol is dominant over Eve and Daisy
- Daisy is dominant over Eve, Amy and Beth
- Eve is dominant over Amy.

Question 6

Let $A = \{0, 1, 2\}$ and $R = \{(0,0), (0,1), (0,2), (1,1), (1,2), (2,2)\}$ and $S = \{(0,0), (1,1), (2,2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Let S be a set and let R be a relation on S Explain what it means to say thet $\mathcal{R}$ is

(i) reflexive

(ii) symmetrix

(iii) anti-symmetric

(iv) Transitive

Question 7

Let the sequence un be defined by the recurrence relation

$$u_{n+1} = u_n + 2n, \text{ for n = 1, 2, 3, ...}$$

and let $u_1 = 1$.

(a) Calculate u_2, u_3, u_4 and u_5, showing all your working. [2]

(b) Prove by mathematical induction that the nth term, where $n \geq 0$, is given by

$$u_n = n^2 - n + 1.$$

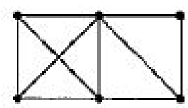
[5]

(c) Showing all your working, find the sum of the first 100 terms of this sequence. [3]

Question 8

(Part A : Spanning Trees - 5 Marks)

Find three non-isomorphic spanning trees for the following simple graph. Explain why your three trees are not isomorphic.



(Part B :Binary Search Trees - 5 Marks)
Suppose a database, comprised of 30,000 internal nodes, is structured as a Binary Search Tree.

(i) What is the Key of the Root node?

(ii) What are the keys of the nodes at level 1?

(iii) For the nodes at level 1, how many subtrees are there?

(iv) How many nodes are the between the root (level 0) and level 4.]

(v) What is the maximum number of searchs in this database?

Question 9

Given S is the set of all 5 digit binary strings, E is the set of a 5 digit binary strings beginning with a 1 and F is the set of all 5 digit binary strings ending with two zeroes.

(a) Find the cardinality of S, E and F.

(b) Draw a Venn diagram to show the relationship between the sets S, E and F. Show the relevant number of elements in each region of your diagram.

- A college teaches courses in the following subjects areas: mathematics, computing and statistics.
- Students in the college may choose their courses from these three subject areas.
- Students are not obliged to take courses from these three subject areas, and may instead take courses in other subject areas.
- Let the subject areas be represented by the letters **M** for mathematics, **C** for computing and **S** for statistics.
- Draw a labelled Venn diagram showing the areas **M**, **C**, and **S** in such a way as to represent the students studying at the college.
- On your diagram show the number of students studying in each region of the Venn diagram.
  - Currently 600 students are enrolled in the college.
  - 300 students are taking mathematics courses.
  - 120 student are taking statistics courses.
  - 380 students are taking computing courses.
  - 40 students study courses from all three subject areas.
  - 200 mathematics students are taking computing courses as well.
  - 60 computing students are also takings statistics courses.
  - 70 statistics students are also taking mathematics course.

(i) How many students study none of these courses at all?

(ii) How many students are taking mathematics courses but not computing or statistics courses.

(iii) How many students study courses from precisely two of these subject areas?

Question 10

Part A : Matrix Operations - 4 Marks

i. Compute AB.

ii. Compute C^2.

iii. Find a matrix X such that $2X + C^2 = AB$.

iv. Find a matrix Y such that $YC = C$.

Part B : Gaussian Elimination - 5 Marks

(i) Say whether or not the graphs they represent are isomorphic.

(ii) Calculate A^2 and A^4 and say what information each gives about the graph corresponding to A. [6]

(i) Write down the augmented matrix for the following system of equations.

$$2x + y - z = 2$$
$$x - y + z = 4$$
$$x + 2y + 2z = 10$$

(ii) Use Gaussian elimination to solve the system.