0.1 Special Mathematical Functions

0.1.1 Mathematical Operators

- The Square Root function
- The Floor and Ceiling functions
- The Absolute Value functions
- Root Functions
- Absolute Value Function
- Floor Function
- Ceiling Function

$$\lfloor 3.14 \rfloor = 3 \quad (1)$$
$$\lceil -4.5 \rceil = -5 \quad (2)$$
$$|-4| = 4 \quad (3)$$

For this course, only positive numbers have square roots. The square roots are positive numbers. (This statement is not strictly true. The square root of a negative number is called a complex number. However this is not part of the course).

Negative numbers can have cube roots

$$-27 = -3 \times -3 \times -3$$

$$\sqrt[3]{-27} = -3$$

Exponential and Logarithms

Rules

Expontials : Rules 4.18 Page 58
Logarithms : Rules 4.23 Page 61

$$log_a(a) = 1$$

$$log_a(b^c) = c \times log_a(b)$$

- $log_2(128) = 7$
- $log_2(1/4) = -2$

- $log_2(2) = 1$

$$log_a(b) = \frac{log_x(b)}{log_x(a)}$$

0.1.2 Logarithms

- Laws of Logarithms - Change of Base

$$\mathrm{Log}_b(x) = a$$
$$b^a = x$$
$$\mathrm{Log}_2(8) = 3$$
$$2^3 = 8$$

$$\mathrm{Log}_b(x) \times \mathrm{Log}_b(y) = \mathrm{Log}_b(x + y)$$
$$\mathrm{Log}_b(x^y) = y \times \mathrm{Log}_b(x)$$

$$\mathrm{Log}_y(x) = \frac{\mathrm{Log}_b(x)}{\mathrm{Log}_b(y)}$$

0.1.3 Exponents

- Rules of Exponents

$$(a^b)^c = a^{b\times c}$$

$$64^{2/3} = (4^3)^{2/3} = 4^{3\times 2/3} = 4^2 = 16$$

$$(a^b) \times (a^c) = a^{b+c}$$
$$(3^2) \times (3^3) = 3^{2+3} = 3^5 = 243$$

0.1.4 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

0.1.5 Logarithmic Functions

Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$
3. $\log_b(X^Y)$= $Y \log_b(X)$

Question1.3 Compute the Logarithm of the following

- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$
- $a^x = y \ log_a(y) = x$
- $e^x = y \ ln(y) = x$
- $log_a(x \times y) = log_a(x) + log_a(y)$
- $log_a(\frac{x}{y}) = log_a(x) - log_a(y)$
- $log_a(\frac{1}{x}) = -log_a(x)$
- $log_a(a) = 1$
- $log_a(1) = 0$
- $\lceil x \rceil$
- $\lfloor x \rfloor$

Sample value x	Floor $\lfloor x \rfloor$	Ceiling $\lceil x \rceil$	Fractional part $\{x\}$
$12/5 = 2.4$	2	3	$2/5 = 0.4$
2.7	2	3	0.7
-2.7	-3	-2	0.3
-2	-2	-2	0

0.1.6 Precision Functions

- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
- Floor Function $\lfloor x \rfloor$

Question1.2: State the range and domain of the following function

$$F(x) = \lfloor x - 1 \rfloor$$

0.1.7 Powers

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

0.1.8 Precision Functions

- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
- Floor Function $\lfloor x \rfloor$

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0.1.9 Powers

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0.1.10 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

0.2 Logarithmic Functions

Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$
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- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$

$$Log_b(x) = \frac{1}{Log_x(b)}$$

$$Log_b(x) = \frac{Log_a(b)}{Log_a(b)}$$

Example 1

$$log_3(x) + 3log_x(3) = 4$$

$$(log_3(x))^2 + 3 = 4log_3(x)$$

Example 1

$$log_3(x) + 3log_x(3) = 4$$

$$(log_3(x))^2 - 4log_3(x) + 3 = 0$$

0.2.1 Logarithmic Functions