

UNIVERSITY OF LONDON

0291 0102 ZA

BSc/Diploma Examination

for External Students

2011

Computing and Information Systems and Creative
Computing

0291 0102 ZA Mathematics for Computing

Duration: 3 hours

Date and time: Thursday 5 May 2011 : 2.30 – 5.30 pm

*There are ten questions in this paper. Candidates should answer **all ten** questions. All questions carry equal marks and full marks can be obtained for complete answers to **ten** questions.*

Questions involving a description or explanation should, wherever possible, be accompanied by an appropriate example.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, texts or algebraic equations. The make and type of machine must be stated clearly on the front cover of the examination book.

**THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM**

Question 1

- (a) Convert the decimal integer $(507)_{10}$ to binary notation. [2]
- (b) Working in binary and showing all carries, compute $(11010)_2 + (111)_2$. [2]
- (c) i. List the set of positive integers with precisely 3 bits in binary notation.
ii. Let n be a positive integer. How many positive integers have precisely n bits in binary notation? [3]
- (d) Showing your working, express the repeating decimal

$0.012012012012\dots$

as a rational number in its simplest form. [3]

Question 2

- (a) Let the two sets A and B be given by

$$A = \{3, \frac{1}{3}, \pi\} \quad \text{and} \quad B = \{x \in \mathbb{Q} : x \notin \mathbb{Z}\}.$$

Give each of the following sets by using the listing method.

- i. $A \cap \mathbb{Q}$;
ii. $A \cap \mathbb{R}$;
iii. $A \cap B$;
iv. $A - B$. [3]
- (b) Describe by using the rules of inclusion method the set of non-negative integers which have a remainder of 0 on division by 10. [2]
- (c) Define a relation on the set of all pairs of integers $\mathbb{Z} \times \mathbb{Z}$ by

$$(a, b) R (x, y) \text{ if and only if } a \leq x \text{ and } b \leq y.$$

Justifying your answers, say whether the relation R on the set $\mathbb{Z} \times \mathbb{Z}$ is

- i. reflexive;
ii. anti-symmetric;
iii. transitive;
iv. a partial order;
v. an order. [5]

Question 3

Let p , q and r be the following propositions concerning integers n .

p : n is a multiple of two

q : $n < 20$

r : $n \leq 20$.

(a) List the truth set of the compound proposition $\neg q \wedge p$. [2]

(b) Express each of the following statements using the propositions p , q and r and logical symbols.

i. n is an integer less than 20 which is even;

ii. n is an integer larger than 20 which is odd;

iii. $n = 20$. [3]

(c) i. Use truth tables to prove that

$$p \rightarrow (q \vee r) \equiv (\neg q \wedge \neg r) \rightarrow \neg p.$$

ii. Write in plain English the contrapositive of the statement

“If n is a positive integer and $n < 2$ then $n = 1$ ”.

[5]

Question 4

(a) Given a real number x , say how $\lfloor x \rfloor$, the *floor* of x , is defined. [1]

(b) The function $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by the rule

$$f(x) = \lfloor x/2 \rfloor.$$

i. Find $f(-3)$ and $f(3)$.

ii. Justifying your answer, say whether f is one-to-one.

iii. Justifying your answer, say whether f is onto. [6]

(c) Explain what it means for a function to be $O(x)$. [1]

(d) Justifying your answer, say whether the function $f(x)$ from part (b) is $O(x)$. [2]

Question 5

A sequence is given by the recurrence relation

$$u_n = u_{n-1} + 2n \quad \text{for } n \geq 2$$

and the initial term $u_1 = 1$.

- (a) Showing your working, calculate u_2, u_3, u_4 and u_5 . [2]
- (b) Prove by induction that $u_n = n^2 + n - 1$ for all $n \geq 1$. [6]
- (c) Showing all your working, compute

$$\sum_{n=1}^{100} (u_n - (n-1)^2).$$

[2]

Question 6

- (a) Let G be a simple graph. How is the sum of the degrees of the vertices of G related to the number of edges of G ? [2]
- (b) Justifying your answer, say why it is not possible to construct a simple graph G with degree sequence

$$3, 1, 1, 1, 1.$$

[2]

- (c) Justifying your answer, say why it is not possible to construct a simple graph G with degree sequence

$$4, 3, 2, 1, 0.$$

[2]

- (d) Justifying your answer, say whether there exists a positive integer $n > 5$ for which it is possible to construct a simple graph G with degree sequence

$$n-1, n-2, \dots, 3, 2, 1, 0.$$

[2]

- (e) Justifying your answer, say whether it is possible to construct a simple graph with precisely 50 vertices and 1500 edges. [2]

Question 7

- (a) i. Construct a binary search tree to store the following ordered list of 12 integers at its internal nodes.

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23.

- ii. What is the maximum number of comparisons needed in order to find an existing integer in the tree? [4]

- (b) A binary search tree is designed to store an ordered list of 300 records at its internal nodes.

- i. Which record is stored at the root (at level 0) of the tree?
ii. Which records are stored at level 1 of the tree?
iii. Determine the number of records stored at level 8 of the tree. [6]

Question 8

- (a) Let G be the simple graph on the vertex set $V(G) = \{1, 2, 3, 4, 5\}$ with adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- i. Draw G .
ii. For each pair of distinct vertices of G , find the number of paths of length 2 between them.
iii. Compute the matrix A^3 and explain what the entries say about the graph G .
iv. For each pair of distinct vertices of G , find the number of paths of length 3 between them. [6]

- (b) Let H be the graph with vertex set $V(H) = \{a, b, c, d, e\}$ and edge set $E(H) = \{ba, bc, bd, be\}$.

- i. What are the conditions for a function $f : V(H) \rightarrow V(G)$ to be an isomorphism between the graphs H and G ?
ii. Justifying your answer, find an isomorphism between the graphs H and G . [4]

Question 9

The college refectory has 4 different starters, 3 main courses and 5 desserts on the lunch menu and offers a small, a medium and a large set lunch. A small lunch consists of just a main course, a medium lunch consists of either a starter and a main course or a main course together with a dessert. A large lunch consists of a starter, a main course and a dessert. One of the starters and two of the desserts contain nuts.

- (a) Describe a sample space to model all possible set lunches. [2]
- (b) Give the total number of possible set lunches. [2]
- (c) You ask the dinner lady to serve you a random set lunch. What is the probability that
 - i. you get a large lunch?
 - ii. you do not get a dessert?
 - iii. your lunch does not contain any nuts? [6]

Question 10

- (a) Use Gaussian elimination to solve the following system of equations

$$\begin{aligned}3y + 2z &= 2 \\2x + y + 3z &= 4 \\2x + 3y + z &= 2.\end{aligned}$$

[4]

- (b) Given the matrices

$$A = \begin{pmatrix} 0 & 3 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix},$$

- i. Compute AB .
- ii. Find a matrix X such $AX = B$.
- iii. Find a matrix Y such that $AY = A$. [6]