UNIVERSITY OF LONDON 0291 0102 ZA

BSc/Diploma Examination

for External Students

2010

Computing and Information Systems and Creative Computing

0291 0102 ZA Mathematics for Computing

Duration: 3 hours

Date and time: Monday 10 May 2010 : 10.00 – 1.00 pm

There are ten questions in this paper. Candidates should answer ***all ten*** *questions. All questions carry equal marks and full marks can be obtained for complete answers to* ***ten*** *questions.*

Questions involving a description or explanation should, wherever possible, be accompnaied by an appropriate example.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, texts or algebraic equations. The make and tupe of machine must be stated clearly on the front cover of the examination book.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

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Question 1

(a) Working in base 2 and showing all your working, compute the following:

i. $(10101)_2 + (11011)_2 - (101)_2$

ii. $(10101)_2 \times (101)_2$ [4]

(b) Express the binary number $(1011.011)_2$ as a decimal, showing all your working. [2]

(c) Express the decimal number $(347)_{10}$ in base 2. [2]

(d) Say to which of the sets $\mathbb{Z}$, $\mathbb{Q}$ or $\mathbb{R}$ the following numbers belong. If they belong to more than one of these sets give all the sets.

i. π

ii. $\frac{2}{3}$

[2]

Question 2

(a) i. Describe the following set by the rules of inclusion method:
the set of integers which have a remainder of 1 on division by 3.

ii. Describe the following set by the listing method:
the set of positive multiples of 5 which are less than 50.

[3]

(b) Let A and B be subsets of a universal set $\mathcal{U}$.

i. Use membership tables to prove that

$$(A \cup B')' = A' \cap B$$

ii. Shade the region corresponding to this set on a Venn diagram. [5]

(c) Given the universal set $\mathcal{U} = \{1,2,3,4,5,6,7,8,9\}$, and subsets $A = \{2,4,6,8\}$, $B = \{4,5,6,7\}$, list the set $(A \cup B')'$. [2]

Question 3

(a) Let $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ and let p, q be the following propositions concerning the integer $n \in S$.

$$p \;:\; n \text{ is a multiple of two}$$
$$q \;:\; n \text{ is a multiple of three.}$$

i. For each of the following compound statements find the set of values n for which it is true.

$$p \wedge q; \quad p \vee q; \quad \neg p \oplus q$$

ii. Express the following statement using logic symbols.

n is not a multiple of either two or three.

iii. List the elements of S which are in the truth set for the statement in (ii).

[6]

(b) (i) Let p and q be propositions. Use truth tables to prove that

$$p \rightarrow q \equiv \neg q \rightarrow \neg p.$$

(ii) Write the contrapositive of the following statement concerning an integer n.

If the last digit of n is 0, then n is divisible by 5.

[4]

Question 4

(a) Consider the two sets $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4, 5\}$. A function $f : X \rightarrow Y$ is given by $\{(c, 3), (a, 2), (b, 5), (d, 3)\}$, a subset of $X \times Y$.

i. Show f as an arrow diagram.

ii. State the domain, co-domain and range of f.

iii. Say why f does not have the one-to-one property and why f does not have the onto property, giving a specific counter example in each case.

[5]

(b) i. State the condition to be satisfied in order for a function to have an inverse.

ii. Given $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = 2x - 1$, define fully the inverse function f^{-1} and state the value of $f^{-1}(1)$.

iii. Given $g : \mathbb{R} \rightarrow \mathbb{R}^+$ where $g(x) = 3^x$, define fully the inverse function g^{-1} and state the value of $g^{-1}(1)$.

[5]

Question 5

(a) A sequence is defined by the formula

$u_n = 3n - 2$ for $n \geq 1$.

i. Calculate u_1, u_2 and u_3, showing your working.

ii. Prove by induction that

$$1 + 4 + 7 + 10 + \ldots + (3n - 2) = \frac{n(3n-1)}{2}$$

for all $n \geq 1$.

[6]

(b) The formula for the sum of the first n square numbers is

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

Write the following sums in $\sum$ notation and evaluate them.

i. $1^2 + 2^2 + 3^2 + \ldots + 50^2$

ii. $2 + 5 + 10 + \ldots + 2501$

iii. $2 + 8 + 18 + \ldots + 5000$.

[4]

Question 6

Let G be a graph and let u and v be vertices of G.

(a) i. Say what is meant by the degree of v.

ii. Say what is meant by saying u and v are adjacent. [2]

(b) State, without proof, a result connecting the number of edges of G with the degree of its vertices. [1]

(c) A graph is called k-regular if each of its vertices has degree k.

i. Use the result from (b) to find the number of edges in a 5-regular graph with 8 vertices.

ii. Explain why it is not possible to construct a 5-regular graph on 11 vertices.

iii. Construct an example of a 3-regular graph on 4 vertices.

iv. Construct an example of a 2-regular graph on 5 vertices.

v. How many edges are there in a 5-regular graph on $2n$ vertices where $n \geq 3$?

[7]

Question 7

Given S is the set of integers $\{1, 2, 3, 4, 5, 6\}$.

(a) Let R be a relation defined on S such that, for all $x, y \in S$, x is related to y if $(x + y)$ is a multiple of 3.

i. Draw the digraph of R.

ii. Say with reason whether or not R is

- reflexive;
- symmetric;
- transitive.

In the cases where the given property does not hold provide a counter-example to justify this.

[5]

(b) Another relation is defined on S such that, for all $x, y \in S$, x is related to y if $(x - y)$ is a multiple of 3.

i. Draw the digraph of this relation on S.

ii. Explain why this relation is an equivalence relation but not a partial order.

iii. Write down the equivalence classes for this relation. [5]

Question 8

(a) A tree T has vertex set $V(T)$ and edge set $E(T)$ where

$$\begin{aligned} V(T) &= \{v_1, v_2, v_3, v_4, v_5\} \text{ and} \\ E(T) &= \{v_1v_2, v_2v_3, v_3v_4, v_2v_5\}. \end{aligned}$$

i. Draw this tree.

ii. Construct all the non-isomorphic trees with 6 vertices which can be obtained by attaching a new vertex of degree one to a vertex of T.

iii. Explain briefly why the trees obtained in (ii) are not isomorphic to each other.

iv. Construct a tree with 6 vertices which is not isomorphic to any tree you constructed in (ii). [6]

(b) A ternary tree is a rooted tree in which each internal node has exactly 3 children. Let Q be a ternary tree of height $h \geq 10$ in which all the external nodes lie on level h.

i. Determine the number of nodes on level 4 and level 10.

ii. Find an expression in terms of $\sum$ and k for the number of internal nodes in such a tree.

iii. What is the smallest possible height of a tree if it has 1000 internal nodes? [4]

Question 9

(a) In an experiment a coin is tossed three times and each time it is noted whether the coin comes up heads (H) or tails (T). The final result is recorded as an ordered triple, such as (H,H,T). Let A be the event that the last toss comes up as a head and B be the event that there is only one head in the triple.

i. Draw a rooted tree to model this process.

ii. Calculate the probabilities of the events A, B, $A \cap B$ and $A \cup B$.

iii. Are A and B independent events? Justify your answer.

[7]

(b) In a class of 50 students in how many different ways can

i. a group of 3 students be chosen?

ii. a first, second and third prize be awarded in a class competition if each student can receive at most one prize? [3]

Question 10

(a) The matrix $\mathbf{A}(G)$ given below is the adjacency matrix of a graph, G, with vertex set $V(G) = \{v_1, v_2, v_3\}$.

$$\mathbf{A}(G) = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 0 & 2 \\ 3 & 2 & 0 \end{pmatrix}.$$

i. Say what information the first row of this matrix contains.

ii. Find the number of edges in G.

iii. Calculate $(\mathbf{A}(G))^2$, and say what information it gives us about the graph G.

[5]

(b) i. Write down the augmented matrix for the following system of equations.

$$\begin{aligned} x + y + 2z &= 2 \\ 2x + y + 3z &= 5 \\ x - 2y + 5z &= 11 \end{aligned}$$

ii. Use Gaussian elimination to solve the system.

[5]