Partitioning of Summations

For some integers m and n, with $m < n$.

$$\sum_{i=1}^{i=n} u_i = \sum_{i=1}^{i=m} u_i + \sum_{i=m+1}^{i=n} u_i$$

Suppose $n = 100$ and $m = 50$

$$\sum_{i=1}^{i=100} u_i = \sum_{i=1}^{i=50} u_i + \sum_{i=51}^{i=100} u_i$$

Partitioning of Summations

Example Evaluate the following expression:

$$\sum_{i=51}^{i=100} (i+1)$$

Step 1 Evaluate this expression using the identities (notice the lower bound)

$$\sum_{i=1}^{i=100} (i+1)$$

Step 2 From the outcome of step 1, subtract the following

$$\sum_{i=1}^{i=50} (i+1)$$

Partitioning of Summations

Step 1 Evaluate the following expression using the identities. In this step $n = 100$

$$\sum_{i=1}^{i=100}(i+1) = \sum_{i=1}^{i=100} i + \sum_{i=1}^{i=100} 1$$

(i) First term

$$\sum_{i=1}^{i=100} i = \frac{100 \times (100+1)}{2} = 5050$$

(ii) Second term

$$\sum_{i=1}^{i=100} i = 100$$

$$\sum_{i=1}^{i=100}(i+1) = 5050 + 100 = 5150$$

Partitioning of Summations

Step 2 Evaluate the following expression using the identities. In this step $n = 50$

$$\sum_{i=1}^{i=50}(i+1) = \sum_{i=1}^{i=50} i + \sum_{i=1}^{i=50} 1$$

(i) First term

$$\sum_{i=1}^{i=50} i = \frac{50 \times (50+1)}{2} = 1275$$

(ii) Second term

$$\sum_{i=1}^{i=50} i = 50$$

$$\sum_{i=1}^{i=50}(i+1) = 1275 + 50 = 1325$$

Partitioning of Summations

$$\sum_{i=51}^{i=100}(i+1) = \sum_{i=1}^{i=100}(i+1) - \sum_{i=1}^{i=50}(i+1)$$

$$\sum_{i=51}^{i=100}(i+1) = 5150 - 1325 = \mathbf{3825}$$