1 Section 1 Number Systems

2 Section 2 Set Theory

3 Section 3 Logic

3.1 Logical Operations

- $\neg p$ the negation of proposition p.
- $p \wedge q$ Both propositions p and q are simultaneously true (Logical State AND)
- $p \vee q$ One of the propositions is true, or both (Logical State : OR)
- $p \otimes q$ Only one of the propositions is true (Logical State : exclusive OR (i.e XOR)

p	q	$p \vee q$	$q \wedge p$	$p \otimes q$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

4 Conditional Connectives

Construct the truth table for the proposition $p \rightarrow q$.

p	q	$p \rightarrow q$	$q \rightarrow p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

5 Section 4 Functions

5.1 Invertible Functions

A function is invertible if it fulfils two criteria

- The function is ***onto***,
- The function is ***one-to-one***.

State the conditions to be satisfied by a function $f : X \leftarrow Y$ for it to have an inverse function $f^{-1} : Y \leftarrow X$.
$\lceil \frac{x^2+1}{4} \rceil$ where $f : A \rightarrow \mathbf{Z}$

(i) Find $f(4)$ and the ancestors of 3.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer

Given $f : \mathbf{R} \rightarrow \mathbf{R}$ where f(x) =3x-1,define fully the inverse of the function f ,i.e.f^{-1}. State the value of $f^{-1}(2)$

5.2 Precision Functions

- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
- Floor Function $\lfloor x \rfloor$

Question1.2: State the range and domain of the following function

$$F(x) = \lfloor x - 1 \rfloor$$

5.3 Powers

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

5.3.1 Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

5.4 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

5.5 Logarithmic Functions

5.5.1 Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$
3. $\log_b(X^Y) = Y\log_b(X)$

Question1.3 Compute the Logarithm of the following

- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$

6 graph theory

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

7 Digraphs and Relatiosn

Given a flock of chickens, between any two chickens one of them is dominant. A relation, R, is defined between chicken x and chicken y as xRy if x is dominant over y. This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

- Amy is dominant over Beth and Carol
- Beth is dominant over Eve and Carol
- Carol is dominant over Eve and Daisy
- Daisy is dominant over Eve, Amy and Beth
- Eve is dominant over Amy.

8 Counting

Given S is the set of all 5 digit binary strings, E is the set of a 5 digit binary strings beginning with a 1 and F is the set of all 5 digit binary strings ending with two zeroes.

(a) Find the cardinality of S, E and F.

(b) Draw a Venn diagram to show the relationship between the sets S, E and F. Show the relevant number of elements in each region of your diagram.