1 Fundamental of Mathematics

1.1 Powers

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

1.1.1 Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Mathematics for Computing

Session 2 : Set Theory

Mathematics for Computing

Set Theory

2 Section 2 Set Theory

- A Set is a collection of distinct and defined elements.
- Sets are represented by using French braces {} with commas to separate the elements in a Set.

(a)

(b)

(c)

2.1 Important Operations in Set Theory

- Union ($\cup$) - also known as the OR operator. A union signifies a bringing together. The union of the sets A and B consists of the elements that are in either A or B.
- Intersection ($\cap$) - also known as the AND operator. An intersection is where two things meet. The intersection of the sets A and B consists of the elements that in both A and B.
- Complement (^{c}) - The complement of the set A consists of all of the elements in the universal set that are not elements of A.

2.a Describe the following set by the listing method

$$\{2r + 1 : r \in Z^{+} and r \leq 5\}$$

2.b Let A,B be subsets of the universal set U.

3.a Let n be an element of the set $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$, and p and q be the propositions: p : n is even, $q : n > 15$. Draw up truth tables for the following statements and find the values of n for which they are true: (i) $p \vee \neg q$ (ii) $\neg p \wedge q$

2.2 Universal Set and the Empty Set

The first is the ***universal set***, typically denoted U. This set is all of the elements that we may choose from. This set may be different from one setting to the next.

For example one universal set may be the set of real numbers whereas for another problem the universal set may be the whole numbers $\{0, 1, 2, \ldots\}$.

The other set that requires consideration is called the ***empty set***. The empty set is the unique set is the set with no elements. We write this as $\{\}$ and denote this set by $\emptyset$.

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%--------------------------------%
\section{Number Sets}
 The font that the symbols are written in (i.e. $\mathbb{N}$, $\mathbb{R}$) is kn
\begin{itemize}
 \item $\mathbb{N}$ Natural Numbers ($0,1,2,3$)
(Not used in the CIS102 Syllabus)
\item $\mathbb{Z}$ Integers ($-3,-2,-1,0,1,2,3, \ldots$)
\begin{itemize}
\item[$\ast$] $\mathbb{Z}^{+}$ Positive Integers
\item[$\ast$] $\mathbb{Z}^{-}$ Negative Integers
\end{itemize}
\item $\mathbb{Q}$ Rational Numbers
\item $\mathbb{R}$ Real Numbers
\end{itemize}
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(a)

(b)

(c)

Mathematics for Computing

Session 3 : Logic

3 Logic Proposition

Let p, Q and r be the following propositions concerning integers n:

- p : n is a factor of 36 (2)
- q : n is a factor of 4 (2)
- r : n is a factor of 9 (3)

n	**p**	**q**	**r**
1	1	1	1
2	1	0	1
3	0	1	1
4	1	0	1
6	0	0	1
9	0	1	1
12	0	0	1
18	0	0	1
36	0	0	1

For each of the following compound statements, express it using the propositions P q and r, andng logical symbols, then given the truth table for it,

1) If n is a factor of 36, then n is a factor of 4 or n is a factor of 9

2) If n is a factor of 4 or n is a factor of 9 then n is a factor of 36

Part 1 : Logic

1.1 2010 Question 3

Let $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ and let p, q be the following propositions concerning the integer $n \in S$.

- p: n is a multiple of two. (i.,18e. $\{10, 12, 14, 16, 18\}$)
- q: n is a multiple of three. i.e. $\{12, 15, 18\}$

For each of the following compound statements find the sets of values n for which it is true.

- $p \vee q$: (p or q : 10 12 14 15 16 18)
- $p \wedge q$: (p and q: 12 18)
- $\neg p \oplus q$: (not-p or q, but not both)
  - $\neg p$ not-p $= \{1113151719\}$
  - $\neg p \vee q$ not-p or q $\{11121315171819\}$
  - $\neg p \wedge q$ not-p and q $\{15\}$
  - $\neg p \oplus q = \{11, 12, 13, 17, 18, 19\}$

1.2 2010 Question 3

Let p and q be propositions. Use Truth Tables to prove that

$$p \to q \equiv \neg q \to \neg$$

Important Remember to make a comment at the end to say why the table proves that the two statements are logically equivalent. e.g. *since the columns are identical both sides of the equation are equivalent.*

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

(Key "difference" is first and last rows)

1.3 Membership Tables for Laws

Page 44 (Volume 1) Q8. Also see Section 3.3 Laws of Logic.

Construct a truth table for each of the following compound statement and hence find simpler propositions to which it is equivalent.

- $p \vee F$
- $p \wedge T$

Solutions

p	T	$p \vee T$	$p \wedge T$
0	1	1	0
1	1	1	1

- Logical OR: $p \vee T = T$
- Logical AND: $p \wedge T = p$

p	F	$p \vee F$	$p \wedge F$
0	0	0	0
1	0	1	0

- Logical OR: $p \vee F = p$
- Logical AND: $p \wedge F = F$

1.4 Propositions

Page 67 Question 9 Write the contrapositive of each of the following statements:

- If n= 12, then n is divisible by 3.
- If n=5, then n is positive.
- If the quadrilateral is square, then four sides are equal.

Solutions

- If n is not divisible by 3, then n is not equal to 12.
- If n is not positive, then n is not equal to 5.
- If the four sides are not equal, then the quadrilateral is not a square.

1.5 Truth Sets

2009

Let $n = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p, q be the following propositions concerning the integer n.

- p: n is even,
- q: $n \geq 5$.

By drawing up the appropriate truth table nd the truth set for each of the propositions $p \vee \neg q$ and $\neg q \rightarrow p$

n	p	q	$\neg q$	$p \vee \neg q$
1	0	0	1	1
2	1	0	1	0
3	0	0	1	1
4	1	0	1	0
5	0	1	0	1
6	1	1	0	1
7	0	1	0	1
8	1	1	0	1
9	0	1	0	1

$$\text{Truth Set} = \{1, 3, 5, 6, 7, 8, 9\}$$

n	p	q	$q \rightarrow p$	$q \rightarrow p$
1	0	0	1	0
2	1	0	1	0
3	0	0	1	0
4	1	0	1	0
5	0	1	0	1
6	1	1	1	0
7	0	1	0	1
8	1	1	1	0
9	0	1	0	1

$$\text{Truth Set} = \{5, 7, 9\}$$

1.6 Biconditional

See Section 3.2.1.

Use truth tables to prove that $\neg p \leftrightarrow \neg q$ is equivalent to $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

1.7 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $\neg(p \wedge \not q)$ (i.e. $\equiv p \rightarrow q$).

Logic Gates

- AND
- OR
- NOT

Examiner's Comments:*Many diagrams were carefully and clearly drawn and well labelled, gaining full marks. The logic table was also well done by most, but there were a few marks lost in the final part by failing to deduce that since the columns of the table are identical the expressions are equivalent.*

1.8 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $(p \wedge q) \vee \neg q$ (i.e. $\equiv p \rightarrow q$).

Important Label each of the gates appropriately and label the diagram with a symblic expression for the output after each gate.

Prepositional Logic

3.1 five basic connectives

3.2 Conditional Connectives

- $p \rightarrow q$ is logically equivalent to $\neg(p \wedge \neg q)$.

4 De Morgan's Laws

The De Morgan's Laws allow the expression of conjunctions and disjunctions purely in terms of each other via negation.

The laws can be verbalized as:

- The negation of a conjunction is the disjunction of the negations.
- The negation of a disjunction is the conjunction of the negations.

The Rules can be verbalized as

(i) "not (A and B)" is the same as "(not A) or (not B)"

(ii) "*not (A or B)*" is the same as "*(not A) and (not B)*"

Use Truth Tables to prove De Morgan's Laws (see page 40).

$$\neg(p \vee q) = \neg p \wedge \neg q$$

p	q	$p \vee q$ (1)	$p \wedge q$ (2)	$\neg(p \vee q)$ (3)	$\neg(p \wedge q)$ (4)
0	0	0	0	1	1
0	1	1	0	0	1
1	0	1	0	0	1
1	1	1	1	0	0

p	q	¬p (5)	¬q (6)	$\neg p \wedge \neg q$ (7)	$\neg p \vee \neg q$ (8)
0	0	1	1	1	1
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	0

4.1 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

4.2 Logarithmic Functions

4.2.1 Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$
3. $\log_b(X^Y)$= $Y \log_b(X)$

Question1.3 Compute the Logarithm of the following

- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$

Mathematics for Computing
Digraphs and Relations

5 Relations

Let R be an equivalence relation defined on a set S and let x 2 S. Then the equivalence class of x is the subset of S containing all elements of S which are related to x.

We denote this by [x]. Thus

$$[x] = y2S : yRx$$

.

Theorem 1.3

Let R be an equivalence relation on a set S. Then:

- The set of distinct equivalence classes of R on S is a partition of S.
- Two elements of S are related if and only if they belong to the same equivalence class.

Definition 1.6

We say that a relation R on a set S is anti-symmetric if for all x, y 2 S such that xRy and yRx, we have x = y. Thus R is **anti-symmetric** if and only if the relationship digraph of R has no directed cycles of length two.

6 Digraphs and Relations

Given a flock of chickens, between any two chickens one of them is dominant. A relation, R, is defined between chicken x and chicken y as xRy if x is dominant over y. This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

- Amy is dominant over Beth and Carol
- Beth is dominant over Eve and Carol
- Carol is dominant over Eve and Daisy
- Daisy is dominant over Eve, Amy and Beth
- Eve is dominant over Amy.

Types of Relations

- Antisymmetric
- Symmetric
- Reflexive
- Transitive

Antisymmetric Relations

- A binary relation R on a set X is **antisymmetric** if there is no pair of distinct elements of X each of which is related by R to the other.
- More formally, R is antisymmetric precisely if for all a and b in X :
  if $R(a, b)$ and $R(b, a)$, $then a = b$,
- Intuitively, an antisymmetric relation has no symmetric pairs. Consider the relation:

  $$R = (0, 0),(0, 1),(1, 0),$$

  this relation is symmetric.

  In the example stated, the pair (1,0) and (0,1) are symmetric, so this violates the antisymmetric condition. An example of a relation that is antisymmetric is $R = \{(0, 0),(0, 1)\}$.

The Cartesian Product

The Cartesian product (or cross product) of sets A and B, denoted by $A \times B$, is the set defined as

$$A \times B = \{(a, b)|a \in Aandb \in B\}.$$

Importantly the elements (a, b) are an *ordered pair* from A and B respectively.

Example

Given two sets A and B

- $A = \{2, 34\}$
- $B = \{4, 5\}$

Compute the Cartesian Producs $A \times B$ and $B \times A$.
Solutions:

$$A \times B = \{(2, 4),(2, 5),(3, 4),(3, 5),(4, 4),(4, 5)\}$$

$$B \times A = \{(4, 2),(4, 3),(4, 4),(5, 2),(5, 3),(5, 4)\}$$