

## De Morgan's Laws

The De Morgan's Laws allow the expression of conjunctions and disjunctions purely in terms of each other via negation.

For two propositions A and B, the laws can be verbalized as:

- The negation of a conjunction is the disjunction of the negations.
- The negation of a disjunction is the conjunction of the negations.

Using Pseudo-Notation

- (i) "not (A and B)" is the same as "(not A) or (not B)"
- (ii) "**not (A or B)**" is the same as "**(not A) and (not B)**"

### 0.1 Exercise

Use Truth Tables to prove De Morgan's Laws.

$$\neg(p \vee q) = \neg p \wedge \neg q$$

p	q	$p \vee q$ (1)	$p \wedge q$ (2)	$\neg(p \vee q)$ (3)	$\neg(p \wedge q)$ (4)
0	0	0	0	1	1
0	1	1	0	0	1
1	0	1	0	0	1
1	1	1	1	0	0

Looking at the lefthand side of equation

$$\neg(p \vee q) = \neg p \wedge \neg q$$

p	q	$p \vee q$ (1)	$p \wedge q$ (2)	$\neg(p \vee q)$ (3)	$\neg(p \wedge q)$ (4)
0	0	0	0	1	1
0	1	1	0	0	1
1	0	1	0	0	1
1	1	1	1	0	0

Looking at the righthand side of equation

$$\neg(p \vee q) = \neg p \wedge \neg q$$

p	q	$\neg p$ (5)	$\neg q$ (6)	$\neg p \wedge \neg q$ (7)	$\neg p \vee \neg q$ (8)
0	0	1	1	1	1
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	0

## 1 De Morgan's Laws