1. An iterative procedure uses a test at the end of each iteration to determine which of 3 possible sub-processes to do next: A, B or C (finish). The transition probabilities are independent of the number of iterations so far and are as shown in the table below

↓	A	B	C
A	0.5	0.4	0.0
B	0.5	0.4	0.0
C	0.0	0.2	1.0

Let $p_A(k), p_B(k)$ and $p_C(k)$ represent the probabilities that sub- processes A, B, C are performed on the k-th iteration respectively. Suppose the procedure starts off by performing A.

(a) Represent the procedure as a weighted digraph with sub-processes as vertices and transition probabilities as edges.

(b) Find a system of three recurrence equations which show how the probabilities change from one iteration to the next.

(c) Solve this recurrence for the initial conditions given.

(d) What is the expected number of iterations before the procedure ends?

2. An iterative procedure uses a test at the end of each iteration to determine whether it should finish (F) or which of 2 possible sub-processes (A or B) to do next. The transition probabilities are independent of the number of iterations so far and are as shown in the table below:

↓	A	B	F
A	0.4	0.8	0.0
B	0.4	0.0	0.0
F	0.2	0.2	1.0

Let $p_A(k)$ and $p_B(k)$ represent the probabilities that sub-processes A and B are performed on the k-th iteration respectively, and $p_F(k)$ the probability that the procedure finishes on or before the k-th iteration. Find a system of three recurrence equations which shows how the probabilities change from one iteration to the next. Solve this recurrence (using diagonalisation to compute the k-th power of the appropriate system matrix) if the system initially performs sub-process A. Find an expression for Prob$\{T = k\}$, the probability that the procedure ends on the k-th iteration. What is the expected value of T?

3. An iterative procedure uses a test at the end of each iteration to determine whether it should finish (F) or which of 2 possible sub-processes (A or B) to do next. The transition probabilities are independent of the number of iterations so far and are as shown in the table below:

↓	A	B	F
A	0.6	0.675	0.0
B	0.4	0.0	0.0
F	0.0	0.325	1.0

Let $p_A(k)$ and $p_B(k)$ represent the probabilities that sub-processes A and B are performed on the k-th iteration respectively, and $p_F(k)$ the probability that the procedure finishes on or before the k-th iteration.

(a) Find a system of three recurrence equations which shows how the probabilities change from one iteration to the next.

(b) Solve this recurrence (using Discrete *Putzer* Algorithm to compute the k-th power of the appropriate system matrix) if the system initially performs sub-process A.

(c) Find an expression for Prob$\{T = k\}$, the probability that the procedure ends on the k-th iteration.

(d) What is the expected number of iterations before the procedure ends?