THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON **291 0102 ZB**

BSc/Diploma Examination
for External Students

COMPUTING AND INFORMATION SYSTEMS AND CREATIVE COMPUTING

Mathematics for Computing

Dateline: Monday 11 May 2009 : 10.00 – 1.00 pm

Duration: 3 hours

There are ten questions in this paper. Candidates should answer **TEN** questions. Full marks will be awarded for complete answers to **TEN** questions.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

Question 1

(a) What hexadecimal number has to be added to $(A97)_{16}$ to obtain $(D03)_{16}$? [2]

(b) Write the decimal number 249 in

(i) base 2 (ii) base 16. [3]

(c) (i) Express the fraction $\frac{9}{11}$ as a recurring decimal.

(ii) Express the recurring decimal 0.1717.... as a fraction in its lowest terms.

[4]

(d) Give an example of (i) a rational number and (ii) an irrational number. [1]

Question 2

(a) (i) Describe the set A by the listing method giving the first three elements and last element of A where $A = \{4r - 1 : r \in \mathbb{Z}^+ \text{ and } -1 \leq r \leq 50\}$.

(ii) Describe the set B by the rule of inclusion method where $B = \{4, 16, 64, \ldots, 4096\}$.

[4]

(b) (i) Write out and complete the following table:

A	B	C	$B \cap C$	$(B \cap C)'$	$(B \cap C)'$-A	X
0	0	0				0
0	0	1				0
0	1	0				0
0	1	1				1
1	0	0				1
1	0	1				1
1	1	0				1
1	1	1				1

(ii) Draw a labelled Venn diagram showing A, B and C intersecting in the most general way and shade the region X on it.

(iii) Find an expression which defines the set X in terms of A, B and C and set operations. [6]

Question 3 (a) Let $n \in \{1,2,3,4,5,6,7,8,9\}$ and let p, q be the following propositions concerning the integer n.

$$p: \ n \text{ is odd} \qquad q: n < 5.$$

By drawing up the appropriate truth table find the truth set for each of the propositions $p \vee \neg q$; $\neg(q \rightarrow p)$. [4]

(b) (i) Construct and draw a logic network that accepts as inputs p and q, which may independently have the value 0 or 1, and gives as final output $(p \wedge q) \vee \neg q$. Label all the gates appropriately and also give labels to show the output from each gate.

(ii) Construct a logic table to show the value of the output corresponding to each combination of values (0 or 1) for the inputs p and q.

(iii) Show that $(p \wedge q) \vee \neg q$ is equivalent to $q \rightarrow p$. [6]

Question 4 (a) What properties should a graph possess if it is

(i) simple (ii) connected? [2]

(b) Given a graph G with degree sequence

$$4, 3, 3, 3, 2, 1.$$

(i) How many vertices are there in G?

(ii) Find the number of edges in G, explaining how you obtain your answer.

(iii) Draw an example of a simple graph G with the degree sequence

$$4, 3, 3, 3, 2, 1.$$

[6]

(c) (i) Say why it is not possible to construct a simple graph with the degree sequence

$$4, 2, 2, 2.$$

(ii) Show it is possible to construct a graph with this degree sequence if we do not require it to be simple. [2]

Question 5

(a) Given the ceiling function: $\lceil x \rceil = n$ where $n - 1 < x \leq n$, $n \in \mathbb{Z}$.

Let $A = \{1, 2, 3, 4, 5\}$ and $f(x) = \left\lceil \frac{x^2-1}{4} \right\rceil$ where $f : A \to \mathbb{Z}$.

(i) Find $f(2)$ and the ancestor of 0.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer. [3]

(b) Let $g(n) = \lceil \frac{n-1}{4} \rceil$ where $g : \mathbb{Z} \to \mathbb{Z}$.

(i) Find $g(4)$ and the set of ancestors of 0.

(ii) Find the range of g.

(iii) Is g invertible? Justify your answer. [4]

(c) Let $L = \{a, b, c\}$, $M = \{1, 2, 3, 4\}$ and $N = \{x, y\}$. Draw arrow diagrams for the following functions:

(i) a function from $L \to M$ that is one to one but not onto;

(ii) a function from $L \to N$ that is onto but not one to one;

(iii) a function from $L \to L$ that is both one to one and onto. [3]

Question 6 Let n be a positive integer.

(a) Given $s_n = 1 + 2 + 3 + 4 + \ldots + n$

(i) Calculate s_1, s_2 and s_3.

(ii) Give a recurrence relation which expresses s_{n+1} in terms of s_n for all $n \geq 1$.

(iii) It can be proved by induction that $s_n = \frac{n(n+1)}{2}$. USe this result to find the sum of the first 300 positive integers. [4]

(b) Write the following two expressions in $\sum$ notation with appropriate limits and use (a)(iii) to calculate each sum:

(i) $2 + 4 + 6 + 8 + \ldots + 1000$

(ii) $4 + 7 + \ldots + ((3 \times 1000) + 2)$. [6]

Question 7

(a) Given the set $S = \{a, b, c, d\}$.

(i) Describe briefly how each subset of S can be represented by a unique 4 digit binary string.

(ii) Write down the string corresponding to the subset $\{a, c, d\}$ and the subset corresponding to the string 1011.

(iii) What is the total number of subsets of S? [4]

(b) T is the set $\{\{a, b\}, \{a\}, \{b\}\{a, b, d\}\}$. R is a relation defined on T as follows:

$${}_X R_Y \text{ if } X \subseteq Y \text{ where } X \text{ and } Y \text{ are elements of } T.$$

Draw the relationship digraph for R on T and say, with reason, whether this relation is

(i) reflexive

(ii) symmetric

(iii) transitive

(iv) a partial order. [6]

Question 8 (a) Determine the number of different 3-digit strings using only digits from the set $\{1, 2, 3, 4, 5, 6, 7\}$ where repetitions are allowed. How many of these strings will have all their digits distinct? [2]

(b) A deck of cards contains six cards numbered $1, 2, 3, 4, 5, 6$ and 7. An experiment is carried out in which three cards are chosen from this deck **without** replacement and the result is recorded as an ordered triple, such as (1,2,4), where this result is different from the result (2,4,1).

(i) Let A be the event that the first card is odd and B the event that the last card is a 7. Calculate the number of elements in each of the sets A, B, $A \cap B$ and $A \cup B$.

(ii) Hence calculate the probabilities of A, B, $A \cap B$ and $A \cup B$. [8]

Question 9

(a) What two properties must a graph satisfy in order to be a tree? [2]

(b) Let H be a subgraph of a graph G Explain what it means to say that H is a spanning tree of G. [1]

(c) Let G be the graph with the following adjacency list

$a : b,$

$b : a, c, e$

$c : b, d$

$d : c, e$

$e : b, d, f$

$f : e.$

(i) Draw this graph, G.

(ii) Find and draw all spanning trees of G.

(iii) How many non-isomorphic spanning trees does G have? [4]

(d) A binary search tree is designed to store an ordered list of 6000 records, numbered 1,2,3,...,6000 at its internal nodes.

(i) Draw levels 0, 1 and 2 of this tree, showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.

(ii) What is the height of this tree? [3]

Question 10

(a) Given the following matrices **A** and **B** and **C** where

$$\mathbf{A} = \begin{pmatrix} -4 & 3 \\ 2 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & -2 \\ 1 & 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 & 0 & 5 \\ 1 & -3 & 2 \end{pmatrix}$$

(i) Calculate $2\mathbf{BC}$.

(ii) Calculate $(\mathbf{A} + \mathbf{B})\mathbf{C}$. [4]

(b) (i) Write down the augmented matrix for the following system of equations.

$$\begin{aligned} 2x + y - z &= 1 \\ x - y + z &= 2 \\ x + 2y + 2z &= 11 \end{aligned}$$

(ii) Use Gaussian elimination to solve the system. [6]

END OF EXAMINATION

CIS102e IS51002A 2009