CIS102 Tutorial 5 Answers

Goldsmiths College, University of London

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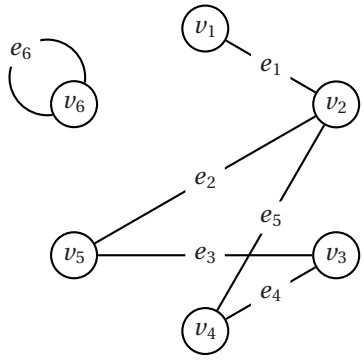
1.



(a) $\{v_1, v_4, v_5\}$

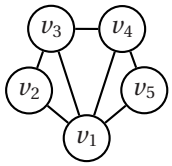
(b) $\{e_3, e_4\}$

(c) $v_2, e_5, v_4, e_4, v_3, e_3, v_5$

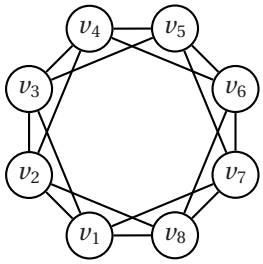
(d) $v_2, e_5, v_4, e_4, v_3, e_3, v_5, e_2, v_2$

2. (a) The degree sequence 4,3,2,2 has a sum of 11. The sum should be equal to *twice* the number of edges, which would lead to 5.5 edges, which is clearly impossible.

(b) Degree sequence 4,3,3,2,2



(c) 4 regular graph with 8 vertices



3. (a) The *vertices* represent the sites, the *edges* connections. Two vertices are joined by an edge when the corresponding sites have a connection.

(b) It has 7 vertices.

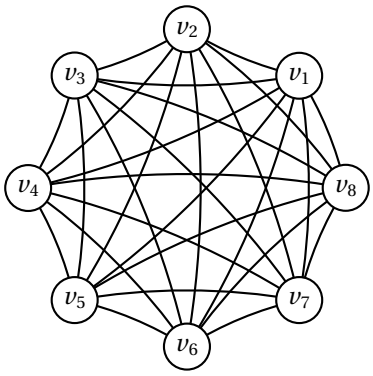
(c) The *sum* of the degree sequence is *twice* the number of edges in the graph. $7+4+3+3+2+2+1=22$, so there are 11 edges or connections between pairs of sites.

(d) There are 7 *vertices* or sites, but one of them has 7 *incident edges*. There are only 6 other sites for these edges to connect to, so one edge must be a parallel edge or a loop. By definition, a simple graph has no parallel edges or loops, so this degree sequence cannot be of a simple graph.

(e) A 9 vertex graph where each vertex is connected to exactly 5 of the other sites would have degree sequence 5,5,5,5,5,5,5,5,5 which has an odd sum. The sum is twice the number of edges, so such a graph could not be constructed.

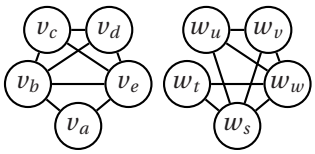
4. (a) A *complete* graph is one where every pair of vertices is joined by exactly one edge

   (b) K_8 has degree sequence 7,7,7,7,7,7,7,7 so every vertex has degree 7.
   K_8 has $\frac{8\times 7}{2} = 28$ edges.



   (c) K_n has each vertex of degree $n-1$. The total number of edges $= \frac{n(n-1)}{2}$

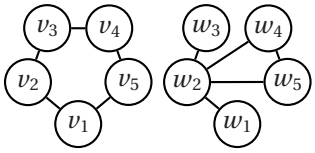
5.



Yes, the graphs are isomorphic.

v	v_a	v_b	v_c	v_d	v_e
$f(v)$	w_t	w_s	w_u	w_v	w_w

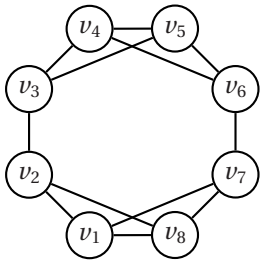
6.



7. (a)

d	no. of edges
0	0
2	7
4	14
6	21

   (b) Possible values of d when there are 8 vertices are 0,2,3,4,5,6,7.



8.