Powers and Logarithms

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Youtube: StatsLabDublin

Exercises

Showing your workings, use the rules of indices and logarithms to give the following two expression in their simplest form.

- **Exercise 1**

$$4 \cdot 2^x - 2^{x+1}$$

- **Exercise 2**

$$\frac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$$

Exercise 1

$$4 \cdot 2^x - 2^{x+1}$$

Remarks:

(looking at the second term)

1. Using the following rule

$$a^b \cdot a^c = a^{(b+c)}$$

2. Using this rule in reverse we can say

$$2^{x+1} = 2^x \cdot 2^1 = 2 \cdot (2^x)$$

$$4 \cdot 2^x - 2^{x+1} \; = \; (4 \cdot 2^x) - (2 \cdot 2^x)$$

Exercise 1

Remarks:

3 This expression is in the form

$$(a \cdot b) - (c \cdot b)$$

which can be re-expressed as follows

$$(a - c\sqrt{b}) \cdot b$$

$$(4 \cdot 2^x) - (2 \cdot 2^x) = (4 - 2) \cdot 2^x$$
$$= 2 \cdot 2^x = 2^{x+1}$$

Exercise 2

$$\frac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$$

Useful Rule of Logarithms

$$\ln(a^b) = b \cdot \ln(a)$$

$$\frac{\ln(2) + 2 \cdot \ln(2) + 3 \cdot \ln(2) + 4 \cdot \ln(2) + 5 \cdot \ln(2)}{\ln(4)}$$

Exercise 2

Adding up all the terms in the numerator

$$\frac{1 \cdot \ln(2) + 2 \cdot \ln(2) + 3 \cdot \ln(2) + 4 \cdot \ln(2) + 5 \cdot \ln(2)}{\ln(4)}$$

$$= \frac{15 \cdot \ln(2)}{\ln(4)}$$

Exercise 2

Our expression has now simplified to

$$\frac{15 \cdot \ln(2)}{\ln(4)}$$

We can simplify the denominator too

$$\ln(4) = \ln(2^2) = 2 \cdot \ln(2)$$

Exercise 2

Our expression has now simplified to

$$\frac{15 \cdot \ln(2)}{\ln(4)} = \frac{15 \cdot \ln(2)}{2 \cdot \ln(2)}$$

We can divide above and below by $\ln(2)$ to get our final answer

$$\frac{15 \cdot \ln(2)}{2 \cdot \ln(2)} = \frac{15}{2} = 7.5$$

The End