Mathematics for Computing

Adjacency Matrix

kobriendublin.wordpress.com

Twitter: @StatsLabDublin

Adjacency Matrix

Important Graph Theory Terms:

- A **path** in a graph is a sequence of edges which connect a sequence of vertices.
- A **simple path** is a path in a graph which does not have repeating vertices.
- A **cycle**, also called a **closed walk**, consists of a sequence of vertices starting and ending at the same vertex.

Adjacency Matrix

The adjacency matrix of this graph ($\mathcal{A}$) is as follows

$$\mathcal{A} = \begin{pmatrix} & a & b & c & d & e & f \\ a & 0 & 1 & 0 & 1 & 0 & 0 \\ b & 1 & 0 & 0 & 0 & 1 & 1 \\ c & 0 & 0 & 0 & 1 & 1 & 1 \\ d & 1 & 0 & 1 & 0 & 0 & 1 \\ e & 0 & 1 & 1 & 0 & 0 & 0 \\ f & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Adjacency Matrix

How many paths of length 4 are there between vertices b and d?

$$\mathcal{A}^2 = \begin{pmatrix} & a & b & c & d & e & f \\ a & 2 & 0 & 1 & 0 & 1 & 2 \\ b & 0 & 3 & 2 & 2 & 0 & 0 \\ c & 1 & 2 & 3 & 1 & 0 & 1 \\ d & 0 & 2 & 1 & 3 & 1 & 1 \\ e & 1 & 0 & 0 & 1 & 2 & 2 \\ f & 2 & 0 & 1 & 1 & 2 & 3 \end{pmatrix}$$

Adjacency Matrix

How many cycles of length 3 are there for the vertices a and b?

$$\mathcal{A}^3 = \begin{pmatrix} & a & b & c & d & e & f \\ a & 0 & 5 & 3 & 5 & 1 & 1 \\ b & 5 & 0 & 2 & 2 & 5 & 7 \\ c & 3 & 2 & 2 & 5 & 5 & 6 \\ d & 5 & 2 & 5 & 2 & 3 & 6 \\ e & 1 & 5 & 5 & 3 & 0 & 1 \\ f & 1 & 7 & 6 & 6 & 1 & 2 \end{pmatrix}$$

Adjacency Matrix

How many paths of length 4 are there between vertices b and d?

$$\mathcal{A}^4 = \begin{pmatrix} & a & b & c & d & e & f \\ a & 10 & 2 & 7 & 4 & 8 & 13 \\ b & 2 & 17 & 14 & 14 & 2 & 4 \\ c & 7 & 14 & 16 & 11 & 4 & 9 \\ d & 4 & 14 & 11 & 16 & 7 & 9 \\ e & 8 & 2 & 4 & 7 & 10 & 13 \\ f & 13 & 4 & 9 & 9 & 13 & 19 \end{pmatrix}$$