UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics
Department *of* Mathematics and Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE : MA4402	SEMESTER: Autumn 2005/2006
MODULE TITLE: Computing Maths	DURATION: $2\frac{1}{2}$ hours
LECTURER: Ms. Maria Pickett	PERCENTAGE OF TOTAL MARKS: 80%

INSTRUCTIONS TO CANDIDATES: Full marks for correct answers to 5 questions

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Q 1 (a) Explain what it means for a function to be surjective? 3%

(b) Explain what it means for a function to be injective? 3%

(c) Consider the following functions:

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^3 + 3,$$
$$g : \mathbb{R} \to \mathbb{R}, \quad g(x) = 2x^2 - 3.$$

(i) Is f surjective? Explain your answer. 2%

(iii) Is f injective? Explain your answer. 2%

(ii) Is g surjective? Explain your answer. 2%

(iv) Is g injective? Explain your answer. 2%

Note: $\mathbb{R}$ is the set of real numbers.

(d) For each of the functions in part (c) that are not bijective (both surjective and injective), change the domain and/or range of the function so it becomes bijective. 6%

Q 2 (a) Define what is meant by a sequence $\{a_n\}_{n=1}^{\infty}$ of real numbers. 3%

(b) Consider the sequence defined by $\left\{\frac{n}{1+2n}\right\}_{n=1}^{\infty}$. Is this sequence convergent? If so what is its limit? 5%

(c) Show that the series

$$\sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!}(-1)^{n-1}$$

defines a convergent series for all $x \in \mathbb{R}$. Note this series defines $\sin(x)$ 8%

(d) Use the series in part (c) to estimate the value of $\sin(\pi/2)$. 4%

Q 3 (a) Consider the following function

$$f \ : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^3 - 6x^2 + 9x + 1.$$

Using derivative information, sketch the graph of $f(x)$ (find critical points, etc.). 9%

(b) Use Newton's method to approximate the root(s) of this function. Note, you can use the graph from part (a) to determine appropriate value(s) for our initial guess(es) for x_0. 7%

(c) Refine the graph in part (a), incorporating the root(s) obtained in part (b). 4%

Q 4 (a) Consider the line segment with endpoints

$$(1,1), \qquad (2,3).$$

Rotate this line segment anti-clockwise about its endpoint $(1,1)$ by $\pi/2$ radians. (Hint: first translate the segment to put the endpoint $(1,1)$ at the origin.) 10%

(b) Consider the two following vectors,

$$v = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \qquad w = \begin{pmatrix} 5 \\ 12 \end{pmatrix}.$$

(i) Find $|v|$ and $|w|$. 4%

(ii) Find $v.w$ (dot product of v and w). 3%

(iii) Find the angle between v and w. 3%

Q 5 (a) Explain under what conditions two matrices, A and B, can be: (i) added together and (ii) multiplied together. 5%

(b) Let

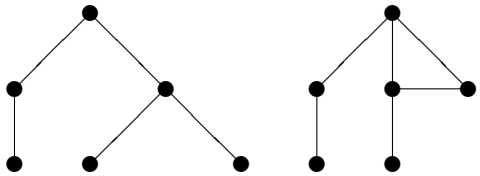
$$A = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & -3 \\ 3 & -2 & 1 \end{pmatrix}.$$

(i) Find AB and BA (if possible). 7%

(ii) Show that $(AB)^T = B^T A^T$. 8%

Q 6 (a) (i) What are the main characteristics of a tree. 3%

(ii) Which of the following graphs represent trees? 3%



graph 1 *graph 2*

(iii) Draw a tree to represent the following algebraic expression: 4%

$$(2+x)^2 + (1-x).$$

(b) Consider the complete graph K_4:

(i) Draw K_4 and construct its Adjacency Matrix. 6%

(ii) Is K_4 Eulerian? Explain your answer. 2%

(iii) Is K_4 Hamiltonian? Explain your answer. 2%