CIS 102 2001

**Question 1**

(a) (i) Calculate, showing your working, the decimal equivalent of the hexadecimal numeral $(A2F.D)_{16}$. [2]

(ii) Make a table representing each of the hexits $(0)_{16}, (1)_{16}, \ldots, (F)_{16}$ as a 4-bit binary string. [2]

(iii) Explain how to use your table to express the binary number $(1011100.101)_2$ in hexadecimal. [3]

(b) Working in base 2, compute the following binary addition, *showing all your working*:

$$(1110)_2 + (11011)_2 + (1101)_2.$$

[3]

**Question 2**

(a) Describe the following sets by the listing method.

(i) $\{2r + 1 : r \in \mathbf{Z}^+ \text{ and } r \leq 5\}$

(ii) $\{10^t : r \in \mathbf{Z} \text{ and } -2 \leq t \leq 2\}$ [3]

(b) Let $A, B$ be subsets of a universal set $\mathcal{U}$.

(i) Use membership tables to prove that $(A' \cup B)' = A \cap B'$. [5]

(ii) Suppose that $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 7\}$ and $B = \{3, 4, 5, 6\}$. Find the set $(A' \cup B)'$. [2]

## Question 3

(a) (i) Draw a logic network that accepts independent inputs $p$ and $q$ and gives as output $\neg p \wedge (p \vee q)$. Label your diagram to show the symbolic output after each gate. [4]

(ii) Make a table to show the truth value of the output from the network corresponding to each combination of truth values of $p$ and $q$. [2]

(b) (i) Construct the truth table for the proposition $p \to q$. [2]

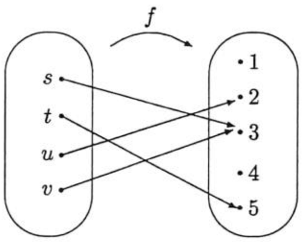
(ii) Let $n$ be an element of the set $\{1, 2, 3, 4, 5, 6, 7\}$. Let $p, q$ be the propositions

$$p\text{: } n \textit{ is even}; \quad q\text{: } n > 4\ .$$

Find the values of $n$ for which $p \to q$ is true. [2]

## Question 4

(a) A function $f$ is represented by the arrow diagram shown below.




(i) Give the *domain*, *co-domain* and *range* of $f$. [3]

(ii) Say why $f$ does not have the *one-to-one* property and why it does not have the *onto* property, giving a specific counter-example in each case. [2]

(b) (i) State the conditions to be satisfied by a function $f : X \to Y$ for it to have an inverse function $f^{-1} : Y \to X$. [2]

(ii) Define $f^{-1}$ when $X = \{1, 2, 3, 4\}$, $Y = \{a, b, c, d\}$, and $f$ is given by the table below.

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | $b$ | $c$ | $d$ | $a$ |

[3]

## Question 5

(a) For each of the following equations, give *two* different examples of a real number $x$ which satisfies the equation:

$$\text{(i) } \lfloor x \rfloor = 3; \quad \text{(ii) } \lceil x \rceil = -1; \quad \text{(iii) } | x - 5 | = 12.$$

[3]

(b) Let $b$ be a positive real number and let $p, q$ be integers with $q > 0$. Express $b^{1/q}$ and $b^{p/q}$ as roots of $b$.

Illustrate your definitions by showing how to evaluate $9^{0.5}$ and $32^{3/5}$ without using a calculator. [4]

(c) Given a positive real number $x$, say what is meant by the *logarithm of $x$ to the base* 2. Use your definition to evaluate (i) $\log_2 8$ and (ii) $\log_2 1/16$ without using a calculator. [3]

## Question 6

(a) Let $G$ be a graph and let $v$ be a vertex of $G$. Say what is meant by the *degree* of $v$. [1]

(b) A graph is called $k$-regular if each of its vertices has degree $k$. Construct an example of:

(i) a 2-regular graph with 5 vertices; [2]

(ii) a 3-regular graph with 6 vertices. [2]

(c) (i) State, without proving, a result connecting the degrees of the vertices of a graph $G$ with the number of its edges. [1]

(ii) Use this result to find the number of edges of a 3-regular graph with 10 vertices. [2]

(iii) Explain why it is not possible to construct a 3-regular graph with 9 vertices. [2]

### Question 7

Let $G$ be a graph with vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and adjacency lists as follows.

$$\begin{array}{ll} v_1: & v_2, v_5 \\ v_2: & v_1, v_3, v_4, v_5 \\ v_3: & v_2, v_4 \\ v_4: & v_2, v_3, v_5 \\ v_5: & v_1, v_2, v_4. \end{array}$$

(a) Find

(i) $\deg(v_1)$; [1]

(ii) two distinct paths of length 2, starting at $v_4$ and ending at $v_1$; [2]

(iii) a 5-cycle in $G$. [2]

(b) Construct the adjacency matrix $\mathbf{A(G)}$. [2]

(c) Say how the number of edges in a graph is related to the sum of the entries in its adjacency matrix.

Hence find the number of edges of the graph $G$. [3]

### Question 8

Let $S = \{a, b, c, d\}$ and suppose that a relation $\mathcal{R}$ is defined on $S$ in precisely the following cases:

$$a\mathcal{R}a,\ a\mathcal{R}b,\ a\mathcal{R}c,\ b\mathcal{R}b,\ b\mathcal{R}c,\ c\mathcal{R}d.$$

(a) Draw the relational digraph for $\mathcal{R}$ on $S$. [2]

(b) The relation $\mathcal{R}$ is not reflexive. Which minimal set of pairs should be added to $\mathcal{R}$ to make it reflexive? [2]

(c) The relation $\mathcal{R}$ is not symmetric. Which minimal set of pairs should be added to $\mathcal{R}$ to make it symmetric? [2]

(d) The relation $\mathcal{R}$ is not transitive. Which minimal set of pairs should be added to $\mathcal{R}$ to make it transitive? [2]

(e) Is the relation $R$ anti-symmetric? Justify your answer. [2]

## Question 9

(a) Calculate the terms $u_3$ and $u_4$ of the sequence defined for $n \geq 2$ by the recurrence relation

$$u_{n+1} = u_n + 3u_{n-1},$$

when $u_1 = 1$ and $u_2 = 4$. [2]

(b) For each of the following sequences, find a recurrence relation that gives $u_{n+1}$ in terms of $u_n$.

(i) 12, 1.2, 0.12, 0.012, 0.0012, ...;

(ii) 3, 7, 11, 15, 19, .... [2]

(c) Use the formula $\sum_{r=1}^{n} r = n(n+1)/2$ to evaluate the following sums.

(i) $1 + 2 + 3 + \cdots + 100$; [2]

(ii) $21 + 22 + 23 + \cdots + 100$; [2]

(iii) $5 + 10 + 15 + 20 + 25 + \cdots + 100$. [2]

## Question 10

(a) Draw the tree $T$ with vertex set $V(T) = \{v_1, v_2, v_3, v_4\}$ and edge set $E(T) = \{v_1v_2, v_2v_3, v_3v_4\}$.

(i) Construct all the *non-isomorphic* trees with five vertices which can be obtained by attaching a new vertex of degree one to a vertex of $T$. [2]

(ii) Explain briefly why the trees you obtain in (i) are not isomorphic to each other. [2]

(iii) Constuct a tree with five vertices which is not isomorphic to any tree you constructed in (i). [2]

(b) A binary search tree is designed for an ordered list of 3185 records.

(i) Find which record is stored at the root (at level 0) of the tree and at each of the nodes at level 1. [2]

(ii) What is the maximum number of comparisons that would need to be made to match a target with any existing record? [2]

**Question 11**

(a) The code to open a combination lock is an ordered sequence of four digits chosen from the set $\{1, 2, 3, 4, 5, 6\}$. How many different codes are possible

(i) if repetition is allowed?

(ii) if repetition is not allowed? [2]

(b) Twelve balls numbered $1, 2, 3, \ldots, 12$, are placed in a container and three balls are drawn at random without replacement. How many different selections of three balls are possible, if the order of selection is not important? [2]

(c) In the experiment described in part (b), let $A$ be the event that the number on each ball drawn is at most 5. Let $B$ be the event that the number on each ball drawn is odd. Calculate the probability of each of the events $A$, $B$ and $A \cap B$. [6]

**Question 12**

(a) Let $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 1 & 1 \end{pmatrix}$.

(i) Calculate the matrix sum $\mathbf{A} + \mathbf{B}$.

(ii) Calculate the matrix product $\mathbf{CB}$.

(iii) Explain briefly why the matrix product $\mathbf{AC}$ is not defined. [3]

(b) Write the system of equations

$$\begin{array}{rcrcrcl} x_1 & + & x_2 & + & x_3 & = & 4 \\ 2x_1 & & & - & x_3 & = & 1 \\ & & x_2 & + & x_3 & = & 3 \end{array}$$

in the matrix form $\mathbf{Ax} = \mathbf{b}$, identifying clearly the matrices $\mathbf{A}$, $\mathbf{x}$ and $\mathbf{b}$. [2]

(c) Solve the system in part (b) by Gaussian elimination. [5]