Question 3 (a) Let n be a positive integer and p and q be the following propositions:

$$p \ : \ n \leq 12$$
$$q \ : \ n is odd.$$

(i) Express each of the three following compound propositions concerning positive integers symbolically by using p, q and appropriate logical symbols.

$$n \leq 12 and n is even.$$
$$if n \leq 12 then n is even$$
$$n > 12 and n is odd.$$

(ii) Construct the truth table for the statement $q \rightarrow p$. Hence find a value of n that makes this statement false.

(iii) Write in logical symbols the contrapositive of the statement:

$$if n is odd then n \leq 12.$$

[6]

(b) Construct a logic network that accepts as inputs p and q, which may independently have the value 0 or 1, and gives as final output

$$\neg(\neg p \wedge q).$$

Show the truth table for this output and hence give a simple expression (without using negation) that is equivalent to $\neg(\neg p \wedge q)$. [4]

Question 4

(a) Given $u_k = 5k + 1$ and $s_n = \sum_{k=1}^{n} (5k + 1)$ for all positive integers n.

(i) Calculate u_1, u_2, u_3 and u_4.

(ii) Calculate s_1 s_2 and s_3.

(iii) Use the formula $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ to find a formula for $s_n = \sum_{k=1}^{n} (5k + 1)$ in terms of n. Use this formula to find this sum when $n = 10$. [6]

(b) Prove by induction that

$$3 + 7 + 11 + 15 + ... + (4n - 1) = n(2n + 1) for all positive integers n.$$

[4]