

Sequences

- ▶ A sequence is any succession of numbers.
- ▶ A general sequence is denoted by

$$u_1, u_2, \dots, u_n, \dots$$

in which u_1 is the first term, u_2 is the second term and u_n is the n -th term in the sequence.

- ▶ If the sequence goes on forever it is called an **infinite sequence**, otherwise it is called a **finite sequence**.
- ▶ A sequence usually has a rule, which is a way to find the value of each term.

Sequences

Examples of Sequences

- ▶ $\{1, 2, 3, 4, \dots\}$ is a very simple sequence (and it is an infinite sequence)
- ▶ $\{20, 25, 30, 35, \dots\}$ is also an infinite sequence
- ▶ $\{1, 3, 5, 7\}$ is the sequence of the first 4 odd numbers (and is a finite sequence)

Sequences: Recursive Formulas

- ▶ Often the rule for evaluating the current term in the sequence depends on the values of one or more previous terms.
- ▶ In such cases, these rules are called **recursive formulas**.
- ▶ Recursive Rules also have initial values that allow the terms to be evaluated.
- ▶ The rule defining the *Fibonacci* sequence is a recursive formula.

Fibonacci Sequence

$$u_n = u_{n-1} + u_{n-2} \quad \text{for } n \geq 3, \quad u_1 = 0, \quad u_2 = 1$$

The first few terms of the Fibonacci Sequence looks like this:

1, 1, 2, 3, 5, 8, ...

Series

- ▶ A series is the sum of the terms of a sequence.

$$S_n = u_1 + u_2 + u_3 + \dots + u_n$$

- ▶ A series is usually expressed in terms of **sigma notation**.
- ▶ It is useful to remember the following, particularly in the context of proof by induction.

$$S_1 = u_1$$

$$S_{n+1} = S_n + u_{n+1}$$