THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALLS

UNIVERSITY OF LONDON **CO1102 ZA**

(291 0102 ZA)

BSc/Diploma Examination

COMPUTING AND INFORMATION SYSTEMS AND CREATIVE COMPUTING

Mathematics for Computing

Friday 4 May 2012 : 2.30 – 5.30 pm

Duration: 3 hours

There are 10 questions in this paper. Candidates should answer all **10** questions. All questions carry equal marks and full marks can be obtained for complete answers to **10** questions. Each question carries equal marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Calculators may be used. Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

Question 1

(a) Convert the decimal integer $(247)_{10}$ to binary notation. [2]

(b) Working in binary and showing all carries, compute $(11110)_2 + (1101)_2$. [2]

(c) Let the two sets A and B be given by

$$A = \{\sqrt{2}, \tfrac{3}{2}, 2\} \qquad \text{and} \qquad B = \{x \in \mathbb{R} : x \notin \mathbb{Q}\}.$$

Give each of the following sets.

i. $A \cap \mathbb{Q}$;
ii. $A \cap B$;
iii. $B \cup \mathbb{Q}$. [3]

(d) Showing your working, express the repeating decimal

$$1.247247247247\ldots$$

as a rational number in its simplest form. [3]

Question 2

Let p, q and r be the following propositions concerning integers n.

$$\begin{aligned} p &: \ n \text{ is a factor of } 36 \\ q &: \ n \text{ is a factor of } 4 \\ r &: \ n \text{ is a factor of } 9. \end{aligned}$$

(a) List the truth set for the proposition p. [1]

(b) For each of the following compound statements, express it using the propositions p, q and r and logical symbols and give the truth set for it.

i. If n is a factor of 36 then n is a factor of 4 or n is a factor of 9;
ii. If n is a factor of 4 and n is a factor of 9 then n is a factor of 36. [4]

(c) Use truth tables to prove that

$$(p \wedge q) \vee (\neg p \wedge \neg q) \equiv p \leftrightarrow q.$$

[3]

(d) Draw a logic network that accepts independent inputs p and q and gives as output

$$p \leftrightarrow q.$$

[2]

Question 3

(a) Let A,B and C be subsets of a universal set $\mathcal{U}$, and let $X = (B \cap C)' - A$.

i. Draw a labelled Venn diagram depicting the sets A, B and C in such a way that they divide $\mathcal{U}$ into 8 disjoint regions and shade the region corresponding to the set X on this diagram.

ii. Give a membership table for X.

iii. Give the membership table for $Y = (A \cap B \cap C)'$.

iv. Justifying your answer say whether $Y \subseteq X$. [6]

(b) Give the set $\{-2^n : n \in \mathbb{Z}, -2 \leq n \leq 3\}$ by the listing method. [1]

(c) Express the set $\{\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \ldots, \frac{99}{100}\}$ by using rules of inclusion. [2]

(d) Let $S = \{0, 2, 4\}$. Give the power set $\mathcal{P}(S)$ by the listing method. [1]

Question 4

(a) Showing your working, use the rules of indices and logarithms to give the following two expressions in their simplest possible form.

i. $4 \cdot 2^x - 2^{x+1}$; ii. $\dfrac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$.

[2]

(b) i. Given a positive real number x, say how $\log_2(x)$ and $\log_4(x)$ are defined.

ii. Compute $\log_2(16)$ and $\log_2(\frac{1}{16})$.

iii. Explain what it means for a function to be $O(\log_2(x))$.

iv. Justifying your answer, say whether $\log_4(x)$ is $O(\log_2(x))$. [5]

(c) Consider the function $f : \mathbb{A} \to \mathbb{A}$ where $\mathbb{A} = \{1, 2, 3, 4, 5, 6\}$ and f is defined by the table

x	1	2	3	4	5	6
$f(x)$	3	1	2	5	6	4

Let $g : \mathbb{A} \to \mathbb{A}$ be the function defined by $g(x) = f(f(x))$.

i. Complete the following table so it defines the function $g(x)$:

x	1	2	3	4	5	6
$g(x)$						

ii. Show that the function g is the inverse of f. [3]

Question 5

(a) Consider the floor function $f : \mathbb{R} \to \mathbb{Z}$ given by the rule

$$f(x) = \lfloor \tfrac{x+1}{2} \rfloor.$$

i. Compute $f(-6)$ and $f(6)$.

ii. Show that f is not one-to-one.

iii. Justifying your answer, say whether f is onto. [6]

(b) A binary search tree T_n is designed to store an ordered list of n records at its internal nodes with the record $f(n) = \lfloor \frac{n+1}{2} \rfloor$ at its root.

i. Which record is stored at the root of the tree T_{200}?

ii. Which records are stored at level 1 of the tree T_{200}?

iii. Given that the binary search tree T_n has record number r at its root, explain why it is not possible to determine the value of n from this information. [4]

Question 6

(a) Suppose that we have a group consisting of 5 women and 10 men. A committee consisting of 6 people is chosen from this group. Find the number of different committees possible and compute the probability that the committee chosen has

i. all 5 women in it;

ii. at least one woman in it. [6]

(b) Consider the set $S = \{1, 2, 3, \ldots, 600\}$. Justifying your answer, find the number of integers in S which

i. are divisible by 5;

ii. are divisible by 4, 5 or 6. [4]

Question 7

(a) Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} 4x + y + z &= 5 \\ 3x + 4y + z &= 1 \\ 4x + 7y + 2z &= 1. \end{aligned}$$

[4]

(b) For each of the following statements concerning matrices, state whether it is true or false, justifying your answer in each case.

i. For all 2×2 matrices A, B and C we have $(A + B)C = BC + AC$;

ii. For all 2×2 matrices A there exists a matrix B such that $AB = BA = A$;

iii. For all 2×2 matrices A and B we have $(A + B)^2 = A^2 + 2AB + B^2$. [4]

(c) Write the contrapositive of the following statement.

"The number of columns in A and the number of rows in B are the same if the product AB can be computed."

[2]

Question 8

(a) Suppose that it is given that a graph G has degree sequence $4, 3, 3, 3, 2, 1$.

i. Explain why this information is not sufficient to enable us to draw G.

ii. Justifying your answer, find the number of vertices in G.

iii. Justifying your answer, find the number of edges in G. [5]

(b) Let G be the simple graph on the vertex set $V = \{1, 2, 3, 4, 5, 6\}$ with adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

i. Draw G.

ii. Let H be a subgraph of a graph G. Explain what it means for H to be a spanning tree of G.

iii. Find two non-isomorphic spanning trees H_1 and H_2 for G with degree sequence $3, 2, 2, 1, 1, 1$. Explain why your two trees are not isomorphic. [5]

Question 9

Let G be the simple graph on the vertex set $V = \{1, 2, 3, 4, 5, 6\}$ with adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

(a) Give an example of a walk in G which is

i. a path of length 3;

ii. a 3-cycle;

iii. a walk of length 3 which is neither a path nor a cycle. [3]

(b) Let W denote the set of all walks in G, and define a relation R on W by

$w_1 \, R \, w_2$ if and only if the walks w_1 and w_2 have the same length.

i. Explain briefly why the relation R is an equivalence relation on W.

ii. Describe the equivalence class containing the walk 123.

iii. Compute the matrix A^2 and use it to find the cardinality of the equivalence class containing the walk 123. [7]

Question 10

Let n be a non-negative integer and consider the sum

$$s_n = \sum_{i=0}^{n} 2^{2i}.$$

(a) Showing your working, calculate s_0, s_1, s_2, s_3 and s_4. [2]

(b) For $n \geq 0$ find a recurrence relation giving s_{n+1} as a function of s_n. [2]

(c) Prove by induction that $3s_n = 2^{2n+2} - 1$ for all $n \geq 0$. [6]