CIS102 Tutorial 4 Answers

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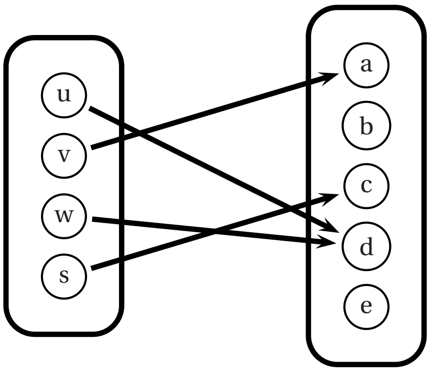
1. (a)



   (b) domain of $f = \{u, v, w, s\}$
   co-domain of $f = \{a, b, c, d, e\}$
   range of $f = \{a, c, d\}$

   (c) $f(v) = a$
   the set of pre-images of $d = \{a, w\}$

   (d) f is not one-to-one, since $f(u) = f(w)$
   f is not onto, since b and e have no pre-image.

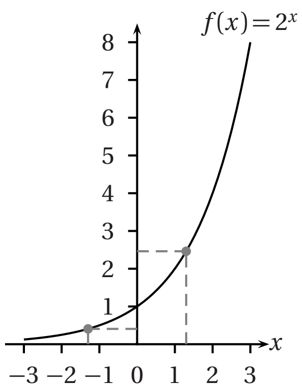
2. $\lfloor x \rfloor = n$ where $n \leq x < n+1$, $n \in \mathbb{Z}$

   (a) $\lfloor 5.67 \rfloor = 5$
   $\lfloor -2.97 \rfloor = -3$
   $\lfloor 17 \rfloor = 17$

   (b) $\lfloor x \rfloor = n$
   $\lfloor x - 3 \rfloor = \lfloor x \rfloor - 3$
   $\lfloor x - 3 \rfloor = n - 3$

   (c) $\lfloor 2.1 - 0.9 \rfloor = \lfloor 1.2 \rfloor = 1$
   $\lfloor 2.1 \rfloor - \lfloor 0.9 \rfloor = 2 - 0 = 2$
   $\therefore \lfloor x - y \rfloor \neq \lfloor x \rfloor - \lfloor y \rfloor$

3. (a)



   (b) range of *exp* is $y : y \in \mathbb{R}, y > 0$

   (c) $\lfloor exp(1.3) \rfloor = 2$
   $\lfloor exp(-1.3) \rfloor = 0$

4. A function has an inverse if and only if it is one-to-one and onto.

   (a) domain of $f^{-1} = \{1,2,3,4\}$
   co-domain of $f^{-1} = \{a,b,c,d\}$

x	1	2	3	4
$f^{-1}(x)$	c	b	a	d

   (b) $\lfloor x \rfloor$ is *not* invertible, since it is not one-to-one

5. $g : \mathbb{R} \to \mathbb{R}$, $g(x) = 3x - 5$

   (a) g(6)=13

   (b) $y = 3x - 5$
   $\frac{y+5}{3} = x$
   For any y we can find an x
   $\therefore g$ is onto

   (c) $g(a) = g(b)$
   $3a - 5 = 3b - 5$
   $3a = 3b$
   $a = b$
   $\therefore g$ is one-to-one

   (d) $g^{-1}(x) = \frac{x+5}{3}$

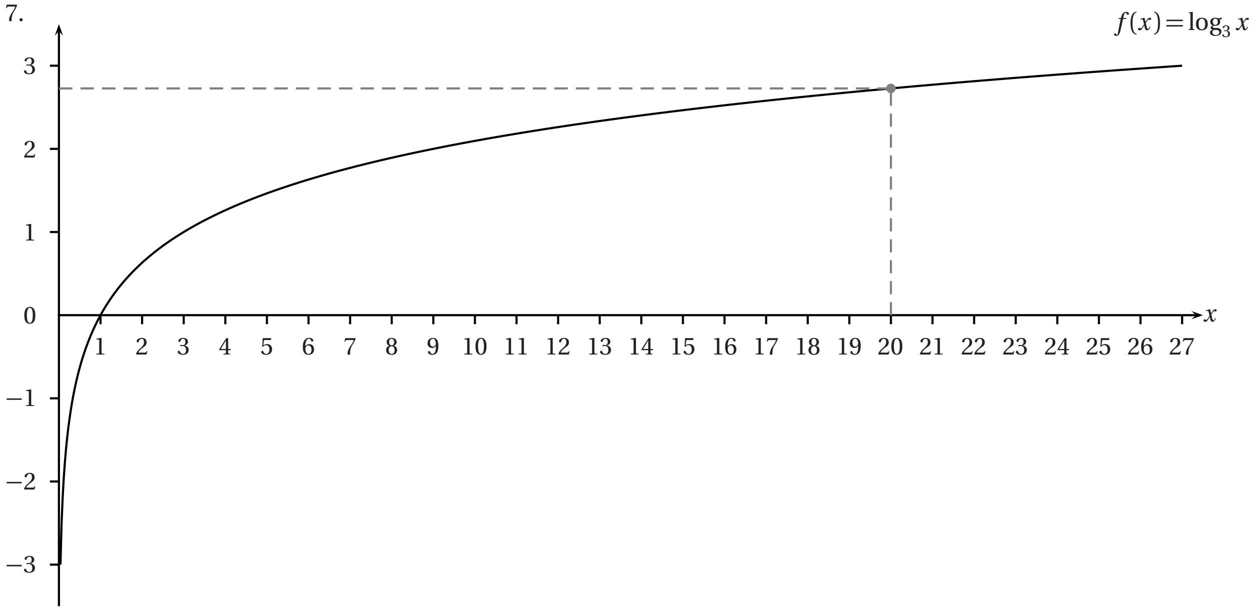
6. (a) $81 = 3^4$
   $\therefore \log_3 81 = 4$

   (b) $9\sqrt{3} = 3^{2.5}$
   $\therefore \log_3 9\sqrt{3} = 2.5$

   (c) $\frac{1}{\sqrt{3}} = 3^{-0.5}$
   $\therefore \log_3 \frac{1}{\sqrt{3}} = -0.5$

   (d) $\frac{1}{9\sqrt{3}} = 3^{-2.5}$
   $\therefore \log_3 \frac{1}{9\sqrt{3}} = -2.5$

7.



$\lfloor \log_3 20 \rfloor = 2$

8. (a) $f(x) = x^3$
   $f^{-1}(x) = \sqrt[3]{x}$
   $f^{-1} : X \to X$

   (b) $g(x) = x^{\frac{1}{2}}$
   $g^{-1}(x) = x^2$
   $g^{-1} : X \to X$