

The Identity Matrix

The identity matrix or unit matrix of size n is the $n \times n$ square matrix with ones on the main diagonal and zeros elsewhere. It is denoted by I_n , or simply by I if the size is immaterial or can be trivially determined by the context.

$$I_1 = [1], \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \dots, \quad I_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

When A is a square matrix (i.e. $n \times n$), it is a property of matrix multiplication that

$$I_n A = A I_n = A.$$

When A is $m \times n$, it is a property of matrix multiplication that

$$I_m A = A I_n = A.$$

The Identity matrix itself is invertible, being its own inverse. The Identity matrix is symmetric, which is to say, it is its own transpose.