University of Limerick
Department of Mathematics and Statistics
Dr. S. Franz

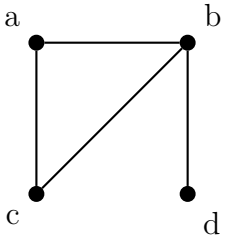
OLLSCOIL LUIMNIGH
UNIVERSITY OF LIMERICK

MA4016 - Engineering Mathematics 6

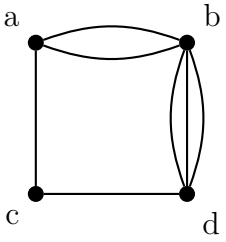
Problem Sheet 8: Graph Theory (March 26, 2010)

1. Determine the type of graph for the shown ones and find for each undirected graph that is not simple a set of edges to remove to make it simple.

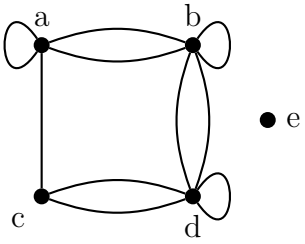
a)



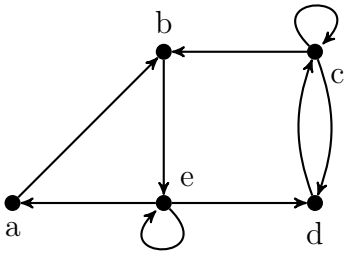
b)



c)



d)

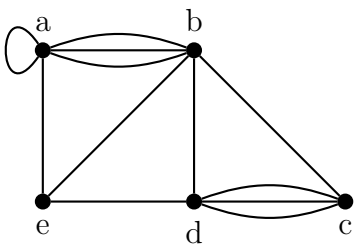


2. Draw a precedence graph (without inherited dependencies) for the following program:

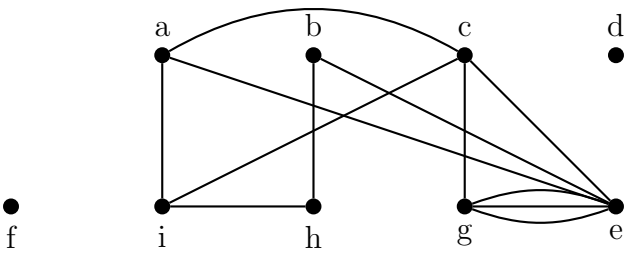
```
S1: x:=0
S2: x:=x+1
S3: y:=2
S4: z:=y
S5: x:=x+2
S6: y:=x+z
S7: z:=4
```

3. Find the number of vertices and edges, and the degree of each vertex in the given graphs. Identify all isolated and pendant vertices.

a)



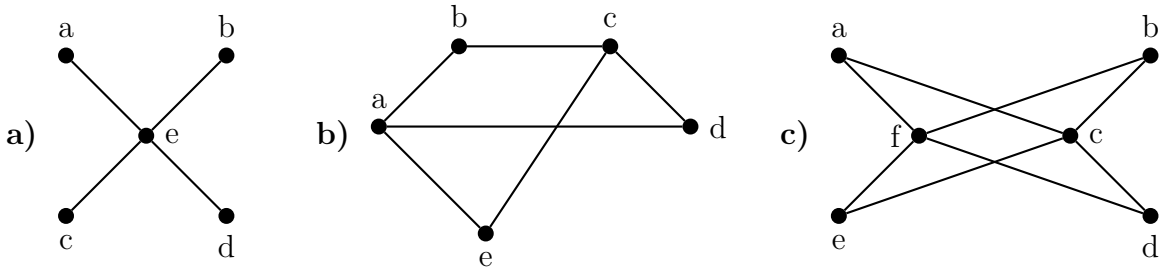
b)



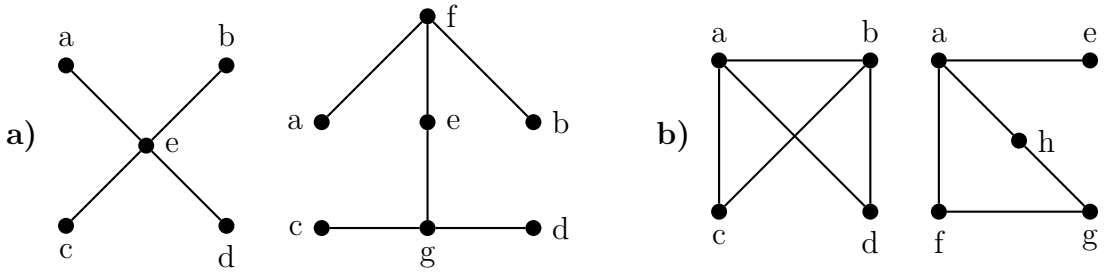
4. Can a simple graph exist with 15 vertices each of degree five?

5. Draw these graphs.
a) K_5 **b)** $K_{1,8}$ **c)** $K_{4,4}$ **d)** C_7 **e)** W_7

6. Are these graphs bipartite?



7. Find the union of the given graphs. Assume edges with the same endpoints are the same.



8. V. Bain invented an algorithm to draw the n-cube in the plane. In the algorithm, all vertices are on the unit-circle in the xy-plane. The angle of a point is the angle from the positive x-axis counterclockwise to the ray from the origin to the point. The input is n.

1. If $n = 0$, put an unlabelled vertex at $(-1, 0)$ and stop.
2. Recursively invoke this algorithm with input $n - 1$.
3. Move each vertex so that its new angle is half the current angle, maintaining edge connections.
4. Reflect each vertex and edge in the x-axis.
5. Connect each vertex above the x-axis to its mirror below the x-axis.
6. Prefix 0 to the label of each vertex above the x-axis, and similarly with 1 below.

Use this algorithm to draw a 4-cube.