CIS102 2006 Solutions

1.

(a) (i)

0000	0	1000	8
0001	1	1001	9
0010	2	1010	A
0011	3	1011	B
0100	4	1100	C
0101	5	1101	D
0110	6	1110	E
0111	7	1111	F

.

(ii) 0011/0111.0100=37.4 and C2.6=11000010.011

(b) (i) 10101×101 = 1101001 and 1101001-101=1100100.

(ii) 37+9B=D2 in base 16.

(c) (i) $\left(\sqrt{2}\right)^8 = 2^4 = 10000$ in base 2

(ii) Yes it is rational as it equals 16.

(iii) $\left(\frac{1}{\sqrt{2}}\right)^3 = 2^{-\frac{3}{2}}$.

2.

(a) (i) {1/100, 1/10, 1, 10, 100, 1000} (ii) {1/2, 1/3, 1/4, 1/5}.

(b) (i) See page 2 of solutions

(ii)

A	B	C	A'	$A'\cap B$	$A'\cap C$	$(A'\cap B)\cup(A'\cap C)$	$B\cup C$	$A'\cap(B\cup C)$
0	0	0	1	0	0	0	0	0
0	0	1	1	0	1	1	1	1
0	1	0	1	1	0	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	1	0
1	1	0	0	0	0	0	1	0
1	1	1	0	0	0	0	1	0

Since the columns are equal we have $A'\cap(B\cup C) = (A'\cap B)\cup(A'\cap C)$. This illustrates the distributive law that set intersection is distributive over set union.

(c) (i) A´={3,4,7,9} B∪C={3,5,6,7,8,9} A´∩B={3,7} A´∩C={7,9}

(ii) A$\cap B\cap C = \{5,8\}$ A$\cup B\cup C = \{1,2,3,5,6,7,8,9\}$.

3.

(a) (i) $\sum_{k=1}^{33}(3k-1) = 2+5+8+...+92+95+98$

(ii) $u_{n+1} = u_n + 3$ and $u_1 = 2$.

b. (i) 2+5+8=...200= $\sum_{k=1}^{67}(3k-1)$ (ii) 101+104+...299= $\sum_{k=34}^{100}(3k-1)$

$\sum_{k=1}^{67}(3k-1) = 3\sum k - \sum 1 = \frac{3\times 67\times 68}{2} - 67 = 6767.$

c. 3+7+11+...+399=2×$100^2 + 100$ since there are 100 terms= 20100

403+407+...+999=2×$250^2 + 250 - 20100 = 105150.$

4.

(a) (i) arrow diagram - see examples in subject guide and lecture notes.

(ii) domain ={a,b,c,d} co-domain={2,4,6,8,10} range={6,8,10}

(iii) f is not onto since range is not equal to the co-domain or 2 and 4 have no ancestors.

(iv) remove (c,6) or (a,6)

add (c,4) or (c,2).

(b) (i) g(5)=2 g(10)=1

(ii) Ancestors of 0 are {3,6,9,...}

(iii) Range of g is {0,1,2} so g is not onto as the co-domain={0,1,-1,2,-1,...} so the range is not equal to the co-domain.

(iv) g is not one to one since for example g(5)=g(7)=2.

5.

(a) (ii)

p	q	$\neg q$	$p \vee q$	$\neg q \wedge (p \vee q)$
0	0	1	0	0
0	1	0	1	0
1	0	1	1	1
1	1	0	1	0

(iii) p$\wedge \neg q$.

(b) (i) p$\rightarrow q$: if this animal is a cat then it has a tail

(ii) if this animal does not have a tail then it is not a cat $\neg q \rightarrow \neg p$

(iii)p$\wedge \neg q$, $\neg p \wedge \neg q$.

6.

(a) (i) 7 vertices, degree of each d gives 7×d edges. So d must be even: d=2,4,6. d=2 gives 7 edges, d=4 fives 14 edges, d=6 gives 21 edges.

(ii) 6 vertices each of degree 3 can be drawn as a hexagon with opposite vertices conected.

n can also be 2,4,5.

(b) (i) Italy Belgium France all share a border with Germany.

(ii) The matrix is symmetric since if Austria shares a border with Italy then Italy must also share a border with Austria.

(iii) (v) See page 2 of solutions.

(iv) the number of edges is half the sum of the entries in the matrix which is 16/2=8.

7.

(a) $|U| = 5 \times 5 \times 5 \times 5 = 625$
$|V| = 2 \times 5 \times 5 \times 5 = 250$
$|P| = 5 \times 5 \times 1 \times 1 = 25$

(b) See page 2 of solutions

c. (i) 250/625=2/5 (ii) 25/625=1/25 (iii)10/625=2/125.

8.

(a) (i) See page 2 of solutions

(b) iii) R_2 is an equivalence relation with equivalence classes {0,2,4} and {1,3,5}. The partition is {[0],[1]}.

$$\begin{array}{llcccc} & & reflexive & symmetric & anti-symetric & transitive \\ (ii) & R_1 & \times & \times & \checkmark & \times \\ & R_2 & \checkmark & \checkmark & \times & \checkmark \end{array}.$$

(c) R is not relexive since no-one is a brother of themselves

R is not symmetric since x may be a brother of y but y is not always a brother of x.

R is transitive since, if x is a brother of y and y is a brother of z then x is also a brother of z for all x,y and z in the population.

9.

(a) (i)

$$\begin{array}{lccccccc} level\ 0 & & & & 5000 & & & \\ level\ 1 & & 2500 & & & & 7500 & \\ level\ 2 & 1250 & & 3750 & & 6250 & & 8750 \end{array}$$

(ii)$\lceil \log_2 10001 \rceil = 14$ or $2^{13} < 10001 + 1 < 2^{14}$. So answer is 14.

b. See page 2 of solutions.

10.

(a) (i) AB=$\begin{pmatrix} 4 & -3 \\ 13 & 0 \end{pmatrix}$ $C^2 + \begin{pmatrix} 4 & -1 \\ 0 & 9 \end{pmatrix}$ $X = \begin{pmatrix} 6 & -2 \\ 13 & -3 \end{pmatrix}$ $Z = \begin{pmatrix} -2 & 4 \\ -13 & -3 \end{pmatrix}$.

(b) (i)

$$\begin{aligned} x - y - z &= 0 \\ 2x + y - z &= 8 \\ x + 2y + 2z &= 6 \end{aligned}$$

(ii)

$$\begin{array}{lrrrr} r1 & 1 & -1 & -1 & 0 \\ r2 & 2 & 1 & -1 & 8 \\ r3 & 1 & 2 & 2 & 6 \end{array}$$

$$\begin{array}{crrrr} r1 & 1 & -1 & -1 & 0 \\ r1-r2 & 0 & -3 & -3 & -6 \\ 2r1-r2 & 0 & -3 & -1 & -8 \end{array}$$

$$\begin{array}{crrrr} r1 & 1 & -1 & -1 & 0 \\ r2 \div -3 & 0 & 1 & 1 & 2 \\ r2-r3 & 0 & 0 & -2 & 2 \end{array}$$

$$\begin{array}{crrrr} r1 & 1 & -1 & -1 & 0 \\ r2 & 0 & 1 & 1 & 2 \\ r2 \div -2 & 0 & 0 & 1 & -1 \end{array}$$

So using back substitution we have the three equations: $z = -1,\;\; y = z = 2,\; x - y - z = 0$ and we get $x = 2, y = 3, z = -1$.