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MA4016 - Engineering Mathematics 6

Problem Sheet 5: Master theorem (March 05, 2010)

1. For each of the following recurrence relations, give an estimate for $T(n)$ if the Master theorem applies, or state why the theorem does not apply.

a) $T(n) = 2T(n/2) + n \log n$

b) $T(n) = 8T(n/4) + n$

c) $T(n) = 2T(n/4) + n^{0.6}$

d) $T(n) = \frac{1}{2}T(n/2) + n^2$

e) $T(n) = 3T(n/3) + n/2$

f) $T(n) = 4T(n/2) + \log n$

2. Find $f(n)$ when $n = 4^k$, where f satisfies the recurrence relation

$$f(n) = 5f(n/4) + 6n,$$

with $f(1) = 1$ and estimate f if f is an increasing function.

3. Suppose that the function f safisfies the recurrence relation

$$f(n) = 2f(\sqrt{n}) + \log_2 n$$

whenever n is a perfect square greater than 1 and $f(2) = 1$.

a) Find $f(16)$.

b) Find a big-$\mathcal{O}$ estimate for $f(n)$.
Hint: Make the substitution $m = \log_2 n$.

4. An efficient algorithm for evaluating polynomials is called Horner's method. It uses the representation

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = (((((a_n)x + a_{n-1})x + a_{n-2})x + \ldots)x + a_1)x + a_0.$$

Write an iterative pseudocode algorithm to compute the value of a polynomial given by a_0, a_1, ..., a_n at x. How many multiplications and additions are used to evaluate a polynomial of degree n at a position x?

5. What sequences of pseudorandom numbers is generated using the linear congruential generator

$$x_{n+1} = (4x_n + 1) \mod 7, \qquad \text{with seeds } x_0 = 1,\ 2 \text{ and } 3?$$

Explain this behaviour.