MA4016 Problem Sheet 3: Analysis of Linear Algebra Algorithms

1. Find the number of multiplications and number of additions needed to multiply a $n \times m$ matrix by a $m \times p$ matrix using the standard algorithm. In particular what are the results when $m = n$?

2. Given A a $n \times n$ matrix of real entries, $\mathbf{b}$ a $n \times 1$ vector of real entries and m a positive integer, consider the following algorithms for computing $A^m\mathbf{b}$.

   Algorithm 1: compute A^m recursively using $A^{k+1} = AA^k$, and then compute $A^m\mathbf{b}$.
   Algorithm 2: let $\mathbf{v}_k = A^k\mathbf{b}$; compute $\mathbf{v}_m$ recursively using $\mathbf{v}_{k+1} = A\mathbf{v}_k,\ \mathbf{v}_0 = \mathbf{b}$.

   Determine the number of multiplications and additions used in each algorithm. Which algorithm is more efficient?

3. Write down recurrence relations and initial conditions for the number of additions/subtractions required in the (i) elimination and (ii) substitution phases of the *Gauss* Elimination algorithm for solving a system of n equations in n unknowns. Solve the equations.

4. Describe a polynomial time algorithm for computing the determinant of a $n \times n$ matrix and determine the number of arithmetic operations it requires.

5. Compare and contrast *Gauss* Elimination with *Cramer*'s Rule for solving a system of n equations in n unknowns from the point of view of the number of arithmetic operations required by each method.