UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

MID-TERM ASSESSMENT PAPER

MODULE CODE: MA4402 SEMESTER: Autumn, 2004

MODULE TITLE: Computing Maths DURATION OF EXAMINATION: 2hrs 30 mins

LECTURER: Dr. C. Nolan PERCENTAGE OF TOTAL MARKS: 20%

INSTRUCTIONS TO CANDIDATES: **Answer 5 questions only. Do not attempt more than 5 questions, as only your first 5 questions will be evaluated.**

1 Let $f : [0, 1] \to \mathbf{R}$, $f(x) = 1 + 2x^2$

(a) Define what the graph of the above function is (it is not necessary to provide any sketch to answer this). 4%

(b) Explain why the function is not bijective 5%

(c) What could you replace the range of this function by in order to make it bijective? 5%

(d) Suppose $g : A \to B$ and $h : C \to D$ are given functions. What restrictions (if any) are needed to define a function $F : A \to D, F(x) = h(g(x))$, for any element x in the domain A? 6%

2 (a) Define what is meant by the magnitude $|V|$of a vector V. 4%

(b) Define what is meant by the dot product $V \cdot W$ of two vectors V and W (of the same dimension). 4%

(c) Let V and W be two vectors of the same dimension, inclined at an angle θ to each other. Show that they satisfy

$$V \cdot W = |V|\ |W|\ \cos(\theta)$$

8%

(d) Let

$$V = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ W = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Compute the (acute) angle between these vectors. 4%

3 (a) Give an outline of what the Newton-Raphson algorithm for root finding, and how it works. 6%

(b) Use this algorithm to estimate the roots of the function $f : \mathbf{R} \to \mathbf{R}, f(x) = x^3 - 3x - 2$. 9%

(c) Sketch the graph of this function, incorporating roots (from the last part of the question) as well as critical points. 5%

4 (a) Define what is meant by an Eulerian graph. 4%

(b) Recall the Konigsberg problem (depicted in the figure below) By analyzing the associated graph, show that it is not possible visit all of the bridges exactly once. 8%

(c) Is the graph in part (b) Hamiltonian? What does this answer mean practically?

(d) Show that by removing a particular bridge (you find one), it is possible visit all of the bridges exactly once, so that the associate graph is Eulerian. Show how this fits in with Eulerian graph theory. 5%

5 (a) Explain how $f : \mathbf{N} \to \mathbf{R}$ defines a sequence $\{a_n\}_{n=1}^{\infty}$. (Note: $\mathbf{N}$ denotes the natural (counting) numbers) 5%

(b) Let $a_n = (-1)^{n+1}x^{2n}/(2n!)$. Show that the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges. 9%

(c) Note that the series in part (b) converges to $\cos(x)$ (where x is measured in radians). Use this fact to estimate $\cos(\pi/3)$ to two decimal places. 6%

6 (a) Under what conditions is it possible to multiply two matrices $A \in \mathbf{R}^{m\times n}$, $B \in \mathbf{R}^{p\times q}$ to form the product AB? 3%

(b) Calculate

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 5 \\ 2 & 2 \\ 4 & 1 \end{bmatrix}$$

5%

(c) Use matrix-vector theory to calculate the end-points of the line segment joining $(1, 2)$ to $(-4, 5)$, which is rotating about its mid-point at an angular speed of $\pi/4$ radians per second. 8%

(d) What would the end-points be if its mid-point were also being translated at an angle of $\pi/3$ (with the x-axis) with speed 3? 4%