EST.MM

Hibernia College Dublin

Computing

Tutor : Kevin O'Brien

Tutorial: Maths for Computing

Review Tutorial 1

Session 10

- MATRICES
- SYSTEMS OF LINEAR EQUATIONS

Session 9

- PROBABILITY
- COUNTING

2011 Q 10 Zone B Part A

Question 10

(a) Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} 2y + 3z &= 2 \\ 2x + 3y + z &= 4 \\ 2x + y + 3z &= 2. \end{aligned}$$

2011 Q 10 Zone B Part A

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2011 Q 10 Zone B Part A

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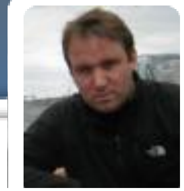

2011 Q 10 Zone B Part b

(b) Given the matrices

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix},$$

i. Compute AB.

ii. Find a matrix X such $AX = B$.

iii. Find a matrix Y such that $AY = A$.

2011 Q 10 Zone B Part b

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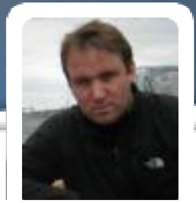
2009 Zone B Question 10 Part A

$$A = \begin{pmatrix} -4 & 3 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 4 & -2 \\ 1 & 3 \end{pmatrix}, C = \begin{pmatrix} -4 & 0 & 5 \\ 1 & -3 & 2 \end{pmatrix}$$

(i) Calculate 2BC.

(ii) Calculate (A + B)C.



2011 Q 10 Zone B Part b

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2009 Zone B Question 10 Part A

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(a) Given the following matrices A and B and C where

$$\mathbf{A} = \begin{pmatrix} -4 & 3 \\ 2 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & -2 \\ 1 & 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 & 0 & 5 \\ 1 & -3 & 2 \end{pmatrix}$$

(i) Calculate $2\mathbf{BC}$.

(ii) Calculate $(\mathbf{A}+\mathbf{B})\mathbf{C}$. [4]

2009 Zone B Question 10 Part A

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(i) Calculate $2\mathbf{BC}$.

(ii) Calculate $(\mathbf{A}+\mathbf{B})\mathbf{C}$.

2009 Zone B Question 10 Part B

(b) (i) Write down the augmented matrix for the following system of equations.

$$\begin{aligned} 2x + y - z &= 1 \\ x - y + z &= 2 \\ x + 2y + 2z &= 11 \end{aligned}$$

(ii) Use Gaussian elimination to solve the system. [6]

2007 Zone B Question 10 Part A

Question 10

(a) Given the following adjacency matrices **A** and **B** where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

(i) Say whether or not the graphs they represent are isomorphic.

(ii) Calculate $\mathbf{A}^2$ and $\mathbf{A}^4$ and say what information each gives about the graph corresponding to **A**. [6]

2007 Zone B Question 10 Part A

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2004 Q 10 Zone B Part A

Question 10 Consider the three matrices

$$A = \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & 4 \\ 2 & 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 0 \end{pmatrix}.$$

(a) Find the following matrices:

(i) $BC + A$.

(ii) $A^2 + (AB)C$. [5]

2004 Q 10 Zone B Part A

(b) Let D be a 2×4 matrix and E be a 4×3 matrix. Given R is the relation on two matrices X and Y where X is related to Y if XY is a valid product of the two matrices:

(i) draw the digraph of the relation on the matrices A, B, C, D, E;

(ii) write down the adjacency matrix of this digraph. [5]

2006 Zone B Q7

Question 7 A 4 letter code is made from the letters {a,b,c,d,e}, where repetitions are allowed and the order of the letters in the code is significant - for example "a,a,e,c" is a different code to "a,c,e,a".

Let U be the set of all such codes.

Let V be the set of all such codes beginning with a vowel.

Let P be the set of all such codes which are palindromic.

(A palindromic code is a string of letters which read the same backwards as forwards, for example "a,e,c,e,a" is a 5 letter palindromic code.)

2006 Zone B Q7

(a) How many elements are there in the sets U, V and P? [3]

2006 Zone B Q7

(b) Draw a Venn diagram to show the relationship between the sets $\mathcal{U}$, $\mathcal{V}$ and $\mathcal{P}$. Show the relevant number of elements in each region of your diagram. [4]

2006 Zone B Q7

(c) What is the probability that a code chosen in this way:

(i) begins with a vowel;

(ii) is palindromic;

(iii) both begins with a vowel and is palindromic? [3]

2004 Question 8

Question 8 An ordered sequence of four digits is formed by choosing digits without repetition from the set $\{1, 2, 3, 4, 5, 6, 7\}$.

(a) Determine:

(i) the total number of such sequences;

(ii) the number of sequences which begin with an odd number;

(iii) the number of sequences which end with an odd number;

2004 Question 8

(iv) the number of sequences which begin and end with an odd number;

(v) the number of sequneces which begin with an odd number or end with an odd number or both;

(vi) the number of sequences which begin with an odd number or end with an odd number but not both. [6]

2011 Question 9

Question 9

The college refectory has 3 different starters, 4 main courses and 5 desserts on the lunch menu and offers a small, a medium and a large set lunch. A small lunch consists of just a main course, a medium lunch consists of either a starter and a main course or a main course together with a dessert. A large lunch consists of a starter, a main course and a dessert. One of the starters and two of the desserts contain nuts.

(a) Describe a sample space to model all possible set lunches. [2]

2011 Question 9

(b) Give the total number of possible set lunches. [2]

2011 Question 9

(c) You ask the dinner lady to serve you a random set lunch. What is the probability that

i. you get a large lunch?

ii. you do not get a dessert?

iii. your lunch does not contain any nuts? [6]

2010

Question 9

(a) In an experiment a coin is tossed three times and each time it is noted whether the coin comes up heads (H) or tails (T). The final result is recorded as an ordered triple, such as (H,H,T). Let A be the event that the last toss comes up as a tail and B be the event that there is only one tail in the triple.

2010 Zone B

i. Draw a rooted tree to model this process.

ii. Calculate the probabilities of the events A, B, $A \cap B$ and $A \cup B$.

iii. Are A and B independent events? Justify your answer.

[7]

2010 Zone B

(b) In a class of 60 students in how many different ways can

i. a group of 3 students be chosen?

ii. a first, second and third prize be awarded in a class competition if each student can receive at most one prize? [3]

2009 Zone B

Question 8 (a) Determine the number of different 3-digit strings using only digits from the set $\{1,2,3,4,5,6,7\}$ where repetitions are allowed. How many of these strings will have all their digits distinct? [2]

2009 Zone B

(b) A deck of cards contains six cards numbered $1, 2, 3, 4, 5, 6$ and 7. An experiment is carried out in which three cards are chosen from this deck **without** replacement and the result is recorded as an ordered triple, such as (1,2,4), where this result is different from the result (2,4,1).

2009 Zone B

(i) Let A be the event that the first card is odd and B the event that the last card is a 7. Calculate the number of elements in each of the sets A, B, $A \cap B$ and $A \cup B$.

(ii) Hence calculate the probabilities of A, B, $A \cap B$ and $A \cup B$. [8]

2008 Zone A

Question 8 (a) Consider all possible arrangements of the letters in the word "exams", where each letter may be used once only.

(i) How many of these arrangements are possible?

(ii) How many of these arrangements begin with a vowel? Call this set B.

(iii) How many of these arangements end with a vowel? Call this set E

2008 Zone A

(iv) How many of these arrangements both begin and end with a vowel? Call this set B∩E.

(v) Show the sets B and E on a Venn diagram. Show the relevant number of elements in each region of your diagram. [7]

2008 Zone A

(b) What is the probability that an arrangement of the letters in the word “exams” chosen at random

(i) begins and ends with a vowel;

(ii) either begins or ends with a vowel or both;

(iii) has two vowels next to one another. [3]