1. $\forall n \in \mathbf{N}_0$, prove that
   (a) 5 divides $n^5 - n$.
   (b) 3 divides $2^{2n+1} + 1$.
   (c) if $0 < a < 1$, then $(1-a)^n \geq 1 - na$.

2. $\forall n \in \mathbf{N}$, prove that
   (a)
   $$1 \cdot 2 \ + \ 2 \cdot 3 \ + \ 3 \cdot 4 \ + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$
   (b)
   $$\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$
   (c) if $n \geq 3$, then $2^n \geq 2n + 1$.

3. Guess a formula for the given sum; then use induction to prove it.
   (a)
   $$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{n(n+1)}$$
   (b)
   $$4(\frac{1}{2 \times 3}) + 8(\frac{2}{3 \times 4}) + \cdots + 2^{n+1}(\frac{n}{(n+1)(n+2)})$$

4. Use induction to show that n straight lines divide the plane into $(n^2 + n + 2)/2$ regions. Assume that no two lines are parallel, and that no three lines have a common point.

5. What's wrong with the following "proof" ?
   Theorem:
   $$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{(n-1) \times n} = \frac{3}{2} - \frac{1}{n}$$
   Proof: Using induction on n. For $n = 1$, $3/2 - 1/n = 1/(1 \times 2)$ and assuming the result is true for n,
   $$\begin{aligned} \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{(n-1) \times n} + \frac{1}{n \times (n+1)} &= \frac{3}{2} - \frac{1}{n} + \frac{1}{n(n+1)} \\ &= \frac{3}{2} - \frac{1}{n} + (\frac{1}{n} - \frac{1}{n+1}) \\ &= \frac{3}{2} - \frac{1}{n+1}. \end{aligned}$$