1 Set Theory

1. The Universal Set $\mathcal{U}$
2. Union
3. Intersection
4. Set Difference
5. Relative Difference

Part 1 : Logic

1.1 2010 Question 3

Let $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ and let p, q be the following propositions concerning the integer $n \in S$.

- p: n is a multiple of two. (i.,18e. $\{10, 12, 14, 16, 18\}$)
- q: n is a multiple of three. i.e. $\{12, 15, 18\}$

For each of the following compound statements find the sets of values n for which it is true.

- $p \vee q$: (p or q : 10 12 14 15 16 18)
- $p \wedge q$: (p and q: 12 18)
- $\neg p \oplus q$: (not-p or q, but not both)
  - $\neg p$ not-p = $\{1113151719\}$
  - $\neg p \vee q$ not-p or q $\{11121315171819\}$
  - $\neg p \wedge q$ not-p and q $\{15\}$
  - $\neg p \oplus q$ = $\{11, 12, 13, 17, 18, 19\}$

1.2 2010 Question 3

Let p and q be propositions. Use Truth Tables to prove that

$$p \to q \equiv \neg q \to \neg$$

Important Remember to make a comment at the end to say why the table proves that the two statements are logically equivalent. e.g. *since the columns are identical both sides of the equation are equivalent.*

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

(Key “difference” is first and last rows)

1.3 Membership Tables for Laws

Page 44 (Volume 1) Q8. Also see Section 3.3 Laws of Logic.

Construct a truth table for each of the following compound statement and hence find simpler propositions to which it is equivalent.

- $p \vee F$
- $p \wedge T$

Solutions

p	T	$p \vee T$	$p \wedge T$
0	1	1	0
1	1	1	1

- Logical OR: $p \vee T = T$
- Logical AND: $p \wedge T = p$

p	F	$p \vee F$	$p \wedge F$
0	0	0	0
1	0	1	0

- Logical OR: $p \vee F = p$
- Logical AND: $p \wedge F = F$

1.4 Propositions

Page 67 Question 9 Write the contrapositive of each of the following statements:

- If n= 12, then n is divisible by 3.
- If n=5, then n is positive.
- If the quadrilateral is square, then four sides are equal.

Solutions

- If n is not divisible by 3, then n is not equal to 12.
- If n is not positive, then n is not equal to 5.
- If the four sides are not equal, then the quadrilateral is not a square.

1.5 Truth Sets

2009

Let $n = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p, q be the following propositions concerning the integer n.

- p: n is even,
- q: $n \geq 5$.

By drawing up the appropriate truth table nd the truth set for each of the propositions $p \vee \neg q$ and $\neg q \to p$

n	p	q	$\neg q$	$p \vee \neg q$
1	0	0	1	1
2	1	0	1	0
3	0	0	1	1
4	1	0	1	0
5	0	1	0	1
6	1	1	0	1
7	0	1	0	1
8	1	1	0	1
9	0	1	0	1

$$\text{Truth Set} = \{1, 3, 5, 6, 7, 8, 9\}$$

n	p	q	$q \to p$	$q \to p$
1	0	0	1	0
2	1	0	1	0
3	0	0	1	0
4	1	0	1	0
5	0	1	0	1
6	1	1	1	0
7	0	1	0	1
8	1	1	1	0
9	0	1	0	1

$$\text{Truth Set} = \{5, 7, 9\}$$

1.6 Biconditional

See Section 3.2.1.
Use truth tables to prove that $\neg p \leftrightarrow \neg q$ is equivalent to $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

1.7 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $\neg(p\wedge \not q)$ (i.e. $\equiv p \rightarrow q$).

Logic Gates

- AND
- OR
- NOT

Examiner's Comments:*Many diagrams were carefully and clearly drawn and well labelled, gaining full marks. The logic table was also well done by most, but there were a few marks lost in the final part by failing to deduce that since the columns of the table are identical the expressions are equivalent.*

1.8 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $(p \wedge q) \vee \neg q$ (i.e. $\equiv p \rightarrow q$).

Important Label each of the gates appropriately and label the diagram with a symblic expression for the output after each gate.

Dice Rolls

Consider rolls of a die. What is the universal set?

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$$

Worked Example

Suppose that the Universal Set $\mathcal{U}$ is the set of integers from 1 to 9.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and that the set $\mathcal{A}$ contains the prime numbers between 1 to 9 inclusive.

$$\mathcal{A} = \{1, 2, 3, 5, 7\},$$

and that the set $\mathcal{B}$ contains the even numbers between 1 to 9 inclusive.

$$\mathcal{B} = \{2, 4, 6, 8\}.$$

Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}'$ and $\mathcal{B}'$

$$\mathcal{A}' = \{4, 6, 8, 9\},$$

$$\mathcal{B}' = \{1, 3, 5, 7, 9\},$$

Intersection

- Intersection of two sets describes the elements that are members of both the specified Sets
- The intersection is denoted $\mathcal{A} \cap \mathcal{B}$

$$\mathcal{A} \cap \mathcal{B} = \{2\}$$

- only one element is a member of both A and B.

Set Difference

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

symbols

$\varnothing$, $\forall$, $\in$, $\notin$, $\cup$

Prepositional Logic

1.1 five basic connectives