Mathematics for Computing
Graph Theory

DRAFT

Graph theory is the study of points and lines. In particular, it involves the ways in which sets of points, called ***vertices***, can be connected by lines or arcs, called ***edges***.

Graphs are classified according to their complexity, the number of edges allowed between any two vertices, and whether or not directions (for example, up or down) are assigned to edges.

Adjacency

(a) An ***adjacency list*** representation of a graph is a collection of unordered lists, one for each vertex in the graph. Each list describes the set of neighbors of its vertex.

(b) An ***adjacency matrix*** is a means of representing which vertices (or nodes) of a graph are adjacent to which other vertices. Another matrix representation for a graph is the incidence matrix.

(c)

Graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A "graph" in this context is made up of "vertices" or "nodes" and lines called edges that connect them. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another

1 Simple Connected Graphs

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

Let G be a simple graph with vertex set $V(G) = \{v1, v2, v3, v4, v5\}$ and adjacency lists as follows:

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v1 : v2 v3 v4
v2 : v1 v3 v4 v5
v3 : v1 v2 v4
v4 : v1 v2 v3.
v5 : v2
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5.a List the degree sequence of G. Draw the graph of G.

5.b Find two distinct paths of length 3, starting at v3 and ending at v4. Find a 4 cycle in G.

5.c Let G be a graph and let v be a vertex of G. Say what is meant by the degree of v. State, without proving, a result connecting the degrees of the vertices of a graph G with the number of its edges.

5.d Degree sequence 4,3,2,2,2 Degree sequence 4,3,3,2,2

5.e K8 has degree sequence 7,7,7,7,7,7,7,7 so every vertex has degree 7.