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MA4016 - Engineering Mathematics 6

Problem Sheet 2: Algorithms (February 12, 2010)

1. Determine whether each of these functions is $\mathcal{O}(x^2)$, $\Omega(x^2)$ or $\Theta(x^2)$.

a) $f(x) = 10$, **b)** $f(x) = 17x + 7$, **c)** $f(x) = x^2 + x + 1$
d) $f(x) = x \log x$, **e)** $f(x) = x^4/2$, **f)** $f(x) = 2^x$
g) $f(x) = \lfloor x \rfloor \lceil x \rceil$, **h)** $f(x) = x \log(x^2)$

2. Give as good a big-$\mathcal{O}$ estimate as possible for each of these functions

a) $f(n) = (n^3 + n^2 \log n)(\log n + 1) + (17 \log n + 19)(n^3 + 2)$,
b) $f(n) = (2^n + n^2)(n^3 + 3^n)$,
c) $f(n) = (n^n + n2^n + 5^n)(n! + 5^n)$

3. Explain what it means for a function to be $\mathcal{O}(1)$, $\Omega(1)$ or $\Theta(1)$.

4. The *ternary search algorithm* locates an element in a list of increasing integers by successively splitting the list into three sublists of (almost) equal size, and restricting the search to the appropriate piece. Specify the steps of this algorithm and describe its complexity in terms of comparisons. Is the algorithm better (in terms of complexity) than binary search?

5. Use the bubble sort to sort $6, 2, 3, 1, 5, 4$, showing the lists obtained in each step.

6. Adapt the bubble sort algorithm so that it stops when no interchanges required. Express this modified algorithm in pseudocode. What is the best/worst/average-case complexity of this new algorithm?

7. Prove that the Strassen algorithm and the standard algorithm for matrix multiplication give the same result. Hint: Use general matrices $A, B \in \mathbb{R}^{2\times 2}$ for this.