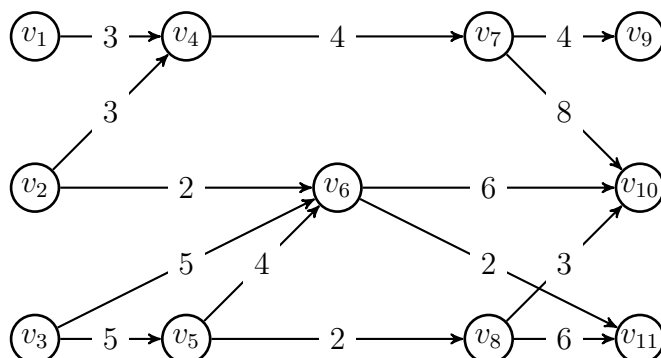


MA4016 - Engineering Mathematics 6

Problem Sheet 11: Maximal Flow and Turing machines

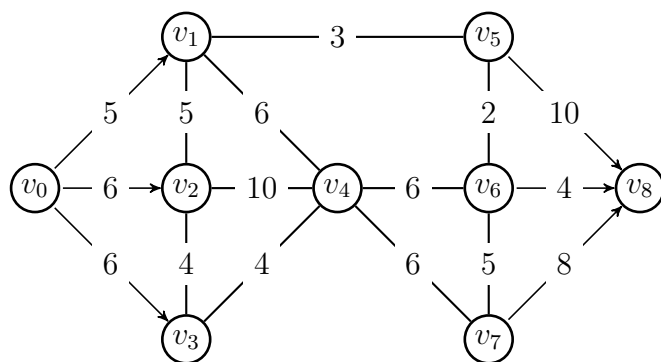
(April 23, 2010)

- Find a maximal flow and a minimal cut in the following pumping network.



Find a maximal flow after changing the following labels: $(v_6, v_{11}) = 4$, $(v_4, v_7) = 8$.

- We want to maximise the flow from v_0 to v_8 . The flow between two vertices, neither of which is v_0 or v_8 , can be in either direction. Model this as system as a network and find a maximal flow and a minimal cut.



- Five students, V , W , X , Y , and Z , are members of four committees, C_1 , C_2 , C_3 , and C_4 . The members of C_1 are V , X , and Y ; the members of C_2 are X and Z ; the members of C_3 are V , Y , and Z ; and the members of C_4 are V , W , X , and Z . Each committee is to send a representative to the administration. No student can represent more than one committee.
 Model this situation in a matching network and find a maximal matching. Is this a complete matching?
- Construct a Turing machine with tape symbols 0, 1, and B , when given a bit string as input, adds a 1 to the end of the bit string and does not change any of the other symbols on the tape.

5. Construct a Turing machine with tape symbols 0, 1, and B , when given a bit string as input, replaces the first 0 with a 1 and does not change any of the other symbols on the tape.
6. Construct a Turing machine that recognises the set of all bit strings that end with a 0.
7. Construct a Turing machine that computes the function $f(n) = n + 2$ for all nonnegative integers n .
8. Construct a Modulo-4 Machine, i.e. a *Turing* Machine which takes as tape input a string of symbols representing an integer and produces as tape output the string of symbols representing the remainder after division by 4 of the integer.
Illustrate the operation of the Modulo-4 Machine on the input string representing the number *seven*.