

# Recurrence Relations

*CIS102 2004 Question 7 2 Marks*

- ▶ Consider the sequence given by

$$1, 4, 7, 10, 13, \dots$$

- ▶ State a recurrence relation which expresses the  $n$ th term,  $u_n$ , in terms of the  $(n - 1)$ th term,  $u_{n-1}$ ,
- ▶ State a recurrence relation which expresses the  $n$ th term,  $u_n$ , in terms of the first term  $u_1$ .

# Recurrence Relations

- ▶  $u_1 = 1$  ,  $u_2 = 4$ ,  $u_3 = 7$  etc
- ▶ Difference in successive terms is 3.
- ▶ Therefore we can say

$$u_n = u_{n-1} + 3$$

# Recurrence Relations

- ▶ Difference between  $u_2$  and  $u_1$  is 3 (i.e.  $1 \times 3$ ).
- ▶ Difference between  $u_3$  and  $u_1$  is 6 (i.e.  $2 \times 3$ )
- ▶ Difference between  $u_4$  and  $u_1$  is 9 (i.e.  $3 \times 3$ )
- ▶ In general the difference between  $u_n$  and  $u_1$  is  $(n - 1) \times 3$ .

$$u_n = u_1 + 3 \times (n - 1)$$

$$u_n = 1 + (3n - 3) = 3n - 2$$

- ▶ Equivalently

$$u_{n+1} = u_1 + 3n = 3n + 1$$

# Recurrence Relations

- ▶ Another sequence is defined by the recurrence relation

$$u_n = u_{n-1} + 2n - 1$$

and  $u_1 = 1$ .

- ▶ Calculate  $u_2$  ,  $u_3$  ,  $u_4$  and  $u_5$  .
- ▶ (Answers 1,4,9,16,25)