# Chapter 1

# Set theory

## Set Theory

### Union and intersection of sets

- The **union** of two sets A and B is a set containing all the elements in either A or B (or both) i.e.

$$A \cup B = x/x \in A \text{ or } x \in B.$$

- The **intersection** of two sets A and B is a set containing all the elements that are both in A and B i.e.

$$A \cap B = x/x \in A \text{ and } x \in B$$

  .

- If sets A and B have no elements in common, i.e. $A \cap B = \emptyset$,then A and B are termed **disjoint sets**.

### Subsets

- Proper Subsets

### The Power Set

## 1.1 Video 2 : Set Theory

Given the following sets

| | |
|---|---|
| $\mathcal{U}$ | $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ |
| $\mathcal{A}$ | $\{1, 2, 5, 6, 8\}$ |
| $\mathcal{B}$ | $\{3, 5, 7, 8\}$ |
| $\mathcal{C}$ | $\{5, 6, 7, 8, 9\}$ |

List the elements of the following $A' \cap B$
$A' \cap C$

## Venn Diagrams

Subsets of the universal set $\mathcal{U}$, intersecting in the most general way (Essentially this means - the venn diagram allows for all possible combinations of overlap.)

| | |
|---|---|
| | |

## Question 2

HibCollWorkSheet2

$\in \subset$

univeral Set $\mathcal{U}$ Laws for Binary Operations Membership Tables

De Morgan's Law

$$A' \cup B' = A \cap B$$

### Part A : Builder Method

The following sets have been defined using the **Building Method** of notation. Re-write them by listing **some** of the elements.

1. $\{p|p$ is a capital city, p is in Europe$\}$
2. $\{x|x = 2n - 5$, x and n are natural numbers$\}$
3. $\{y|2y^2 = 50$, y is an integer$\}$
4. $\{z|z = n^3$, z and n are natural numbers$\}$

### Part B : Sets

U = natural numbers; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. State whether each of the following is true or false:

(i) $A \subset U$

(ii) $B \subseteq A$

(iii) $\emptyset \subset U$

## Question 2

### Part A : Builder Method

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(ii) $B \subseteq A$

(iii) $\emptyset \subset U$

**Relative Difference**

- $A \otimes B$

**Power Sets**

- Consider the set A where $A = \{w, x, y, z\}$
- There are 4 elements in set A.
- The power set of A contains 16 element data sets.
- $$\mathcal{P}(A) = \{\{x\}, \{y\}\}$$
- (i.e. 1 null set, 4 single element sets, 6 two -elemnts sets, 4 three lement set and one 4- element set.)

- $p \to q$ p implies q
- $p \lg q$

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## Dice Rolls

Consider rolls of a die. What is the universal set?

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$$

## Worked Example

Suppose that the Universal Set $\mathcal{U}$ is the set of integers from 1 to 9.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and that the set $\mathcal{A}$ contains the prime numbers between 1 to 9 inclusive.

$$\mathcal{A} = \{1, 2, 3, 5, 7\},$$

and that the set $\mathcal{B}$ contains the even numbers between 1 to 9 inclusive.

$$\mathcal{B} = \{2, 4, 6, 8\}.$$

### Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}'$ and $\mathcal{B}'$

$$\mathcal{A}' = \{4, 6, 8, 9\},$$
$$\mathcal{B}' = \{1, 3, 5, 7, 9\},$$

### Intersection

- Intersection of two sets describes the elements that are members of both the specified Sets
- The intersection is denoted $\mathcal{A} \cap \mathcal{B}$

$$\mathcal{A} \cap \mathcal{B} = \{2\}$$

- only one element is a member of both A and B.

### Set Difference

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

### symbols

$\varnothing, \forall, \in, \notin, \cup$

### 2008 Zone A question2a

$B = \{3n - 1 : n \in Z^+\}$ Describe the set B using the listing method

- Let $n = 1$. Consequently $3(1) - 1 = 2$
- Let $n = 2$. Likewise $3(2) - 1 = 5$
- Let $n = 3$. $3(3) - 1 = 8$
- The repeated differences are 3. The next few values are 11, 14 and 17
- So by the listing method $B = \{2, 5, 8, 11, 14, 17, \ldots\}$

$A = \{3, 5, 7, 9, ldots\}$ Describe the set A using the rules of inclusion method

- The repeated differences are 2.
- We can say the rule has the form $2n + k$
- For the first value n=1. Therefore $2 + k = 3$
- Checking this , for the second value , n=2. Therefore $4 + k = 5$
- Clearly k = 1.
- $A = \{2n + 1 : n \in Z^+\}$
- So by the listing method $B = \{2, 5, 8, 11, 14, 17, \ldots\}$

### 1.1.1 Cartesian Product

- Let $X$ and $Y$ be sets.
- The **cartesian product** $X \times Y$ is the set whose elements are **all** of the ordered pairs of elements $(x, y)$ where $x \in X$ and $y \in Y$.
- Let $X = \{a, b, c\}$
- Let $Y = \{0, 1\}$
- The cartesian product $X \times Y$ is therefore:
- Importantly $X \times Y \neq Y \times X$
- Recall: Let $X = \{a, b, c\}$ and let $Y = \{0, 1\}$
- The cartesian product $Y \times X$ is therefore:

### 1.1.2 The Cartesian Product

Exercises

Discrete Maths

A binary relation on a set A is the collection of ordered pairs of elements of A. In other words, it is the subset of the cartesian product A2= AA

Cartesian Product

This is a direct product pf 2 sets

XY = (x,y)— xXandyY

4 suits of cards and 13 Ranks, therefore 52 element cartesian prodcut.

N.B AB BA A=A =

Cartesian product is not associative

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**Set Difference**

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$