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MA4016 - Engineering Mathematics 6

Problem Sheet 4: Recurrence Relations (February 26, 2010)

1. Solve the homogeneous recurrence relations for the given initial conditions.

a) $a_n = 2a_{n-1} + 8a_{n-2},\ n \geq 2,\ a_0 = 4,\ a_1 = 10$

b) The Lucas sequence: $L_n = L_{n-1} + L_{n-2},\ n \geq 2,\ L_0 = 2,\ L_1 = 1$

c) $a_n = a_{n-1} + a_{n-2} - a_{n-3},\ n \geq 3,\ a_0 = 0,\ a_1 = 3,\ a_2 = 2$

d) $a_n = 5a_{n-2} - 4a_{n-4},\ n \geq 4,\ a_0 = 3,\ a_1 = 2,\ a_2 = 6,\ a_3 = 8$

2. What is the general form of the solution of a homogeneous linear recurrence relation if its characteristic equation has the roots

a) $1, 1, 1, 1, -2, -2, -2, 3, 3, -4$?

b) $-1, -1, -1, 2, 2, 5, 5, 7$?

3. What is the ansatz function for the general solution of the nonhomogeneous linear recurrence relation

$$a_n = 8a_{n-2} - 16a_{n-4} + F(n) \text{ if}$$

a) $F(n) = n^3$?
b) $F(n) = (-2)^n$?
c) $F(n) = n2^n$?
d) $F(n) = n^2 4^n$?
e) $F(n) = (n^2 - 2)(-2)^n$?
f) $F(n) = n^4 2^n$?
g) $F(n) = 2$?
h) $F(n) = n + 2^n$?
i) $F(n) = 2^n + (-2)^2$?

4. Use a nonhomogeneous recurrence relation to find a formula for $\sum_{k=1}^{n} k^2$.

5. Solve the simultaneous recurrence relations

$$\begin{aligned} a_n &= 3a_{n-1} + 2b_{n-1}, & a_0 &= 1 \\ b_n &= a_{n-1} + 2b_{n-1}, & b_0 &= 2 \end{aligned}$$

a) by elimination.

b) with the discrete Putzer algorithm.