



EST.MM

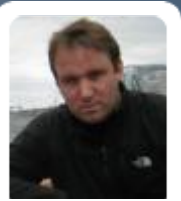
HIBERNIA COLLEGE DUBLIN

Computing



Tutor : Kevin O'Brien

Tutorial: Maths for Computing



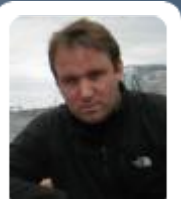
Online Tutorial 4

Chapter 7 : Sequence, Series and Proof by Induction

Covered heavily in the last onsite tutorial. Leave until end of this class.

Chapter 8 : Trees

Emphasis of Today's Class

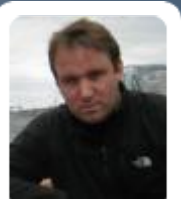


Trees

- A lot of concepts and definitions follows from Chapter 5 : Introduction to Graph Theory

Syllabus

- Properties of Trees
- Rooted Trees and Binary Trees
- Binary Search Trees



Trees : Properties of Trees

1) Characteristics of a Tree

A tree is a connected graph that contains no cycles. A tree has no loops and no multiple edges. All trees are simple graphs.

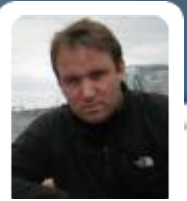
2) Path Graphs

A tree that contains only vertices of degree one or two is called a ***path graph***.

The length of a path graph is the number of edges in it.

3) Number of Edges

(Theorem 3.3) Let T be a tree with n vertices. Then T has $n - 1$ edges.



Trees : Properties of Trees

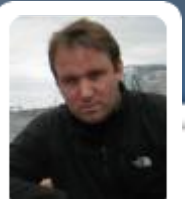
4) Spanning Subgraphs

The graph H is a **subgraph** of a graph G if H 's vertices are a subset of the G 's vertex, its edges are a subset of the edge set of G , and each edge of H has the same end-vertices in G and H .

H is called a **spanning subgraph** of G if the vertices of H are the same as the vertices of G .

5) Spanning Trees

If H is a spanning subgraph which is also a tree, then H is said to be a spanning tree of G . (G does not need to be a tree)



Trees : Properties of Trees

- Spanning Trees (Figure)



Trees : Properties of Trees 2008 Zone A Q9

Question 9

(a) A graph with 5 vertices: a, b, c, d, e has the following adjacency list:

$a : b, e$

$b : a, c, d$

$c : b, d$

$d : b, c, e$

$e : d, a.$

- (i) Draw this graph, G .
- (ii) Draw a spanning tree of G .
- (iii) Draw all the non-isomorphic spanning trees of G and call this set S .
- (iv) How many non-isomorphic trees can be created by adding a new vertex and edge to the trees in S . [6]



Trees : Properties of Trees 2008 Zone A Q9

$a : b, e$

$b : a, c, d$

$c : b, d$

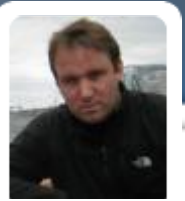
$d : b, c, e$

$e : d, a.$



Trees : Properties of Trees 2008 Zone A Q9

- Part IV
- Examiner's Commentaries : Then it is a question of adding another vertex and edge to each of these in all possible places and finally eliminating the isomorphic ones to do part (iv).
- Simpler Exercise (2006 Q9)
 - (b) (i) Draw the 3 non-isomorphic trees on 5 vertices.
 - (ii) Draw, on a separate diagram, all the non-isomorphic trees on 6 vertices, by adding a vertex to copies of the trees you have drawn or otherwise. [6]



Trees : Properties of Trees 2006 Zone A Q9



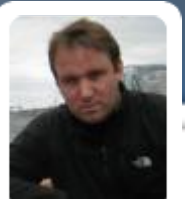
Trees : Properties of Trees

Question 7 (a) (i) What properties must a graph have in order for it to be a tree?

(ii) Say, with reason, whether or not it is possible to construct a tree with degree sequence $4, 3, 3, 1, 1$.

(iii) Say, with reason, whether it is possible to construct a tree with degree sequence $4, 3, 2, 2, 1$.

(iv) What properties must a graph have in order for it to be a binary tree?
[5]



Trees : Properties of Trees

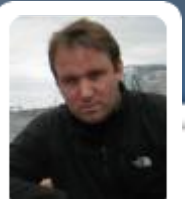


Trees : Properties of Trees (2005)

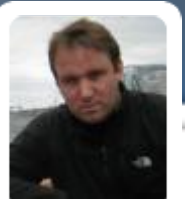
- (c) Let G be the simple graph with vertex set $V(G) = \{a, b, c, d, e\}$ and adjacency matrix

$$\mathbf{A} = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 1 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 0 & 1 & 0 & 1 & 0 \end{array}$$

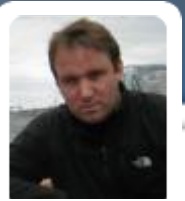
- (i) What do the numbers on the leading diagonal of this matrix tell you about the graph?
- (ii) Say how the number of edges in G is related to the entries in the adjacency matrix \mathbf{A} and calculate this number.
- (iii) Draw G .
- (iv) Find a spanning tree T_1 for G and give its degree sequence.
- (v) Find a spanning tree T_2 for G which is **not** isomorphic to T_1 and give a reason why it is not isomorphic.



Trees : Properties of Trees



Trees : Properties of Trees



Trees : Rooted Trees and Binary Trees

Terminology (Page 37)

- Root
- Nodes
- Key
- Children and Parents
- Ancestors and Descendants
- Height



Binary Search Tree

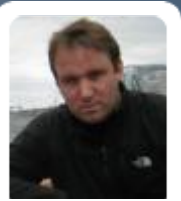
A Binary Search Tree is a binary tree in symmetric order

Symmetric order means that:

- every node has a key (or number)
- every node's key is
 - larger than all keys in its left subtree
 - smaller than all keys in its right subtree

The root r is the record

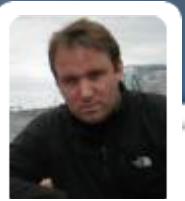
$$\# \lfloor (1 + N) / 2 \rfloor.$$



Binary Search Tree

If the first record in the subtree is #a and the last record is #b, then the **root of the subtree** is

$$\# \lfloor (a + b) / 2 \rfloor .$$



Trees : Binary Search Trees

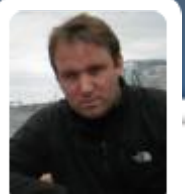
The height h of a binary search tree with N records stored at internal nodes is

$$h = \lceil \log_2 (N + 1) \rceil .$$



Trees : Binary Search Trees (2004)

- (b) A binary search tree is designed to store an ordered list of 3000 records at its internal nodes.
 - (i) Find which record is stored at the root (level 0) of the tree and at each of the nodes at level 1.
 - (ii) What is the height of the tree?
 - (iii) What is the maximum number of comparisons needed in order to find an existing record in the tree? [5]



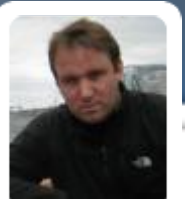
Trees : Binary Search Trees



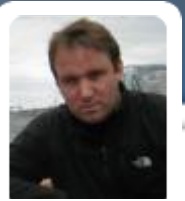
Trees : Binary Search Trees (2006)

Question 9

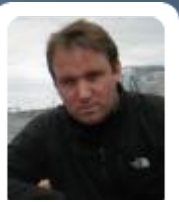
- (a) A binary search tree is designed to store an ordered list of 10000 records numbered 1,2,3,...10000 at its internal nodes.
- (i) Draw levels 0, 1 and 2 of this tree showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.
 - (ii) What is the maximum number of comparisons that would have to be made in order to locate an existing record from the list of 10000? [4]

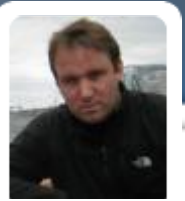


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