1. For each vertex, in-degree = out-degree.

2. maximum number = 2. The graph on the left can be drawn within the given restrictions.

3. For the graph on the right, labelling the vertices from top to bottom and from left to right gives a,c,d,e,f,g,h,i,j,k,b. Applying *Dijkstra*'s algorithm results in

a	b	c	d	e	f	g	h	i	j	k
$\boxed{0}$	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
$\boxed{0}$	∞	2	3	$\boxed{1}$	∞	∞	∞	∞	∞	∞
$\boxed{0}$	∞	$\boxed{2}$	3	$\boxed{1}$	∞	∞	9	∞	∞	6
$\boxed{0}$	∞	$\boxed{2}$	$\boxed{3}$	$\boxed{1}$	6	∞	9	5	∞	6
$\boxed{0}$	∞	$\boxed{2}$	$\boxed{3}$	$\boxed{1}$	6	$\boxed{4}$	9	5	∞	6
$\boxed{0}$	∞	$\boxed{2}$	$\boxed{3}$	$\boxed{1}$	6	$\boxed{4}$	8	$\boxed{5}$	11	6
$\boxed{0}$	10	$\boxed{2}$	$\boxed{3}$	$\boxed{1}$	$\boxed{6}$	$\boxed{4}$	8	$\boxed{5}$	11	6
$\boxed{0}$	10	$\boxed{2}$	$\boxed{3}$	$\boxed{1}$	$\boxed{6}$	$\boxed{4}$	8	$\boxed{5}$	11	$\boxed{6}$
$\boxed{0}$	$\boxed{8}$	$\boxed{2}$	$\boxed{3}$	$\boxed{1}$	$\boxed{6}$	$\boxed{4}$	8	$\boxed{5}$	8	$\boxed{6}$

The shortest path from a to b is of length 8. By backstepping, we find it to be:
a – e – k – b
Note: The permanent labels of h & j are 8 (why ?)

4. For the graph on the right, all vertices are even except for those labelled a & b in Question 3. By adding a virtual edge from a to b, we get an *Eulerian* graph. Thus the shortest route from any vertex which visits all other vertices and arrives back at the starting vertex will have length equal to the sum of the weights of the original graph plus the weight of the virtual edge. If the virtual edge is to be made up of existing edges we get as the minimum length route

$$\sum w_i + 8 = 79 + 8 = 87$$

5. There is a loop at vertex 3, and a double between vertices 3 & 4. There are 6 other single edges.

6. (In this matrix, the rows correspond to vertices, while the columns correspond to edges.) There is a loop at vertex 3, and a double edge between vertices 3 & 4. There are 5 other single edges. The only difference between this graph and that of Question 5 is that there is no edge between vertices 1 & 2 in this graph.

7.
   (a) Not isomorphic. The loop is one edge away from the double edge in the left hand side graph, while it is two edges away in the right hand side one.
   (b) Isomorphic.

8. The graph on the left (The *Petersen* graph) does not contain a Hamiltonian cycle. The graph on the right (dodecahedron) does (see Johnsonbaugh).