UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B. Sc. Examination 2005

COMPUTING

CIS102w Mathematics for Computing

Duration: 3 hours

Date and time:

There are TEN questions on this paper.

Full marks will be awarded for complete answers to TEN questions.

Electronic calculators may be used. The make and model should be specified on the script and the calculator must not be programmed prior to the examination.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1 (a) Express the binary number $(110101001.011)_2$ as

(i) a decimal number

(ii) a hexadecimal number [4]

(b) Showing all your working, express the repeating decimal

$$0.6363...$$

as a fraction in its simplest form. [3]

(c) Using the method of repeated division, or otherwise, convert the decimal number 4768 to base 8, showing all your working. [3]

Question 2 Let A and B and C be subsets of a universal set $\mathcal{U}$.

(a) Draw a labelled Venn diagram depicting A, B, C in such a way that they divide $\mathcal{U}$ into 8 disjoint regions. [1]

(b) The subset $X \subseteq \mathcal{U}$ is defined by the following membership table:

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Shade the region X on your diagram. Describe the region you have shaded in set notation as simply as you can. [3]

(c) The subset $Y \subseteq \mathcal{U}$ is defined as $Y = A \cup (C - B)$. Construct a membership table for Y. [3]

(d) For each of the following statements say whether it is true or false, **justifying your answer**, using the Venn diagram you drew earlier.

(i) $Y \subseteq X$

(ii) $Y' \subseteq X'$

(iii) $Y - X = A \cap B \cap C$. [3]

Question 3 (a) Given propositions p and q, construct the truth tables for:

$$p \wedge q; \quad q \rightarrow p; \quad \neg p \wedge (p \vee q).$$

[3]

(b) Given n is a positive integer and p and q the following statements about n :

$$p : nisodd; \quad q : n < 12.$$

(i) List the elements of the truth set for each of the compound statements: $p \wedge q; \quad \neg p \wedge (p \vee q)$.

(ii) Find one value of n which makes $q \rightarrow p$ **false**. [3]

(c) Draw a logic network that accepts independent inputs p and q and gives as output

$$\neg p \wedge (p \vee q)$$

Find a simpler expression that is logically equivalent to this final output. [4]

Question 4 (a) Let S be the set of all 4 bit binary strings. The function $f : S \rightarrow \mathbb{Z}$ is defined by the rule:

$$f(x) = thenumberofzerosinx, \ foreachbinarystringx \in S.$$

Find:

(i) the number of elements in the domain

(ii) $f(1000)$

(iii) the set of pre-images of 1

(iv) the range of f. [4]

(b) Decide whether the function f defined in part (a) has either the one to one or the onto property, justifying your answers. [2]

(c) State the condition to be satisfied by a function $f : X \rightarrow Y$ for it to have an inverse function $f^{-1} : Y \rightarrow X$. [1]

(d) Define the inverse functions for each of the following:

(i) $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(x) = 3x + 5$;

(ii) $h : A \rightarrow B$ where $A = \{1, 2, 3, 4, 5\}$, $B = \{u, v, w, x, y, z\}$ and h is defined by the following table:

x	1	2	3	4	5
$h(x)$	w	v	y	x	u

[3]

Question 5 Let the sequence u_n be defined by the recurrence relation

$$u_{n+1} = u_n + 2n, \quad for n = 1, 2, 3, ... and let u_1 = 1.$$

(a) Calculate u_2, u_3, u_4 and u_5, showing all your working. [2]

(b) Prove by mathematical induction that the nth term, where $n \geq 0$, is given by

$$u_n = n^2 - n + 1.$$

[5]

(c) Showing all your working, find the sum of the first 100 terms of this sequence. [3]

Question 6 (a) An American airline operates daily flights between 9 cities, c_1, c_2 , ..., c_9. Each city is connected to another by a direct flight and there is at most one flight a day between any two cities.

(i) Describe how such a network of flights can be modelled by a graph, saying what the vertices in the model represent and a rule for determining when two vertices are adjacent.

(ii) The following table gives the number of other cities to which each city is connected by a direct flight:

City	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
No. of connections	3	7	4	5	6	8	1	2	4

Calculate how many pairs of cities have a direct flight between them, giving a brief explanation of your method.

(iii) Define a simple graph. Is the graph model of this network a simple graph or not? Justify your answer. [6]

(b) Say what is meant by an n-regular graph.
Can you draw the following graphs? If so do so. If not explain why not.

(i) A 3 regular graph with six vertices.

(ii) A 3 regular graph with seven vertices. [4]

Question 7 (a) In a tournament with four players $A,\ B,\ C,\ D$ every player plays every other player exactly once. The results are as follows:

$$AbeatsBandC$$
$$BbeatsC$$
$$DbeatsAandBandC.$$

(i) Draw a digraph to model this information. Explain what the vertices of the digraph represent and what an arc from one vertex to another represents.

(ii) $\mathcal{R}$ is the relation represented by this digraph. Determine whether or not $\mathcal{R}$ is reflexive, symmetric or transitive. Give an example to support your answer in each case where the property does **not** hold and justify your answer in each case where the property **does** hold. [6]

(b) Let $S = \{a, b, c\}$. For each of the following statements concerning relations, say whether or not it is true for **any** relation $\mathcal{R}$ on S. If a statement is false give a reason for your answer.

(i) $\mathcal{R}$ is reflexive if ${}_a\mathcal{R}_a$ and ${}_b\mathcal{R}_b$.

(ii) If $\mathcal{R}$ is symmetric then ${}_a\mathcal{R}_b$ and ${}_b\mathcal{R}_a$.

(iii) If $\mathcal{R}$ is anti-symmetric then it is not symmetric.

(iv) If $\mathcal{R}$ is not an equivalence relation then it is a partial order.

[4]

Question 8 A college teaches courses in mathematics, computing and statistics, Students choose a range of courses from these three subject areas. Currently 600 students are enrolled of whom 300 study mathematics courses, 120 study statistics and 380 study computing courses. 40 students study courses from all three subject areas. 200 mathematics students study computing as well. 60 computing students also study statistics and 70 statistics students also study mathematics.

(a) Let the subject areas be represented by the letters M for mathematics, C for computing and S for statistics. Draw a labelled Venn diagram showing the areas M, C, and S in such a way as to represent the students studying at the college. On your diagram show the number of students studying in each region of the Venn diagram. [3]

(i) How many students study none of these courses at all?

(ii) How many students study mathematics but not computing or statistics?

(iii) How many students study courses from precisely two of these subject areas? [3]

(b) What is the probability that a student at this college will study precisely

(i) one course?

(ii) two courses?

(iii) three courses? [3]

(c) What is the probability that a student from the college who is studying mathematics is also studying computing? [1]

Question 9 (a) What two properties must a graph, G, satisfy in order for it to be a tree? [1]

(b) Let H be a subgraph of a graph G. Explain what it means for H to be a spanning tree of G. [2]

(c) Let G be the simple graph with vertex set $V(G) = \{a, b, c, d, e\}$ and adjacency matrix

$$\mathbf{A} = \begin{array}{|cccccc|} \hline & a & b & c & d & e \\ a & 0 & 1 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 1 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 0 & 1 & 0 & 1 & 0 \\ \hline \end{array}.$$

(i) What do the numbers on the leading diagonal of this matrix tell you about the graph?

(ii) Say how the number of edges in G is related to the entries in the adjacency matrix **A** and calculate this number.

(iii) Draw G.

(iv) Find a spanning tree T_1 for G and give its degree sequence.

(v) Find a spanning tree T_2 for G which is **not** isomorphic to T_1 and give a reason why it is not isomorphic. [7]

Question 10 (a) Consider the following matrices

$$\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \; and \; \mathbf{C} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}.$$

(i) Compute the matrix products $\mathbf{AB}$ and $\mathbf{BA}$. Is matrix multiplication commutative in this case?

(ii) Calculate $\mathbf{A}(\mathbf{B}+\mathbf{C})$ and $\mathbf{AB}+\mathbf{AC}$. Does this show that matrix multiplication is distributive over matrix addition? [6]

(b) A digraph with the following adjacency matrix:

$$\mathbf{M} = \begin{array}{cccc} & v_1 & v_2 & v_3 \\ v_1 & 1 & 1 & 0 \\ v_2 & 1 & 0 & 0 \\ v_3 & 0 & 1 & 0 \end{array}$$

Calculate $\mathbf{M}^2$ and, by drawing its digraph or otherwise, say what this matrix represents. [4]

END OF EXAMINATION