are two general methods for tackling this type of problem; they are illustrated in the following two examples.

Example 3.15 We use truth tables to simplify $w = (p \wedge \neg q) \vee (p \wedge q)$.

p	q	$\neg q$	$p \wedge \neg q$	$p \wedge q$	$(p \wedge \neg q) \vee (p \wedge q)$
0	0	1	0	0	0
0	1	0	0	0	0
1	0	1	1	0	1
1	1	0	0	1	1

From the table, we see that the column for the output $w = (p \wedge \neg q) \vee (p \wedge q)$ is identical to the column for p. Hence this network can be replaced by the input p, and no input q or gates are necessary. □

Example 3.16 We use the laws of logic to simplify the expression for the final output $w = (p \wedge \neg q) \vee (p \wedge q)$.
First note that we have "$p\wedge$(something)" in both brackets. By the distributive law, we can take "$p\wedge$" out of these brackets as "a common factor" and write:

$$(p \wedge \neg q) \vee (p \wedge q) = p \wedge (\neg q \vee q).$$

Now $(\neg q \vee q) = (q \vee \neg q) = T$, by the commutative law and the complement law. Thus

$$w = p \wedge T.$$

However, $p \wedge T = p$, by the identity law. Hence we obtain $w = p$, as in the previous example. □

3.5 Exercises 3

1. The following propositions relate to a 3-bit binary string s.

   p: *Only one bit of s is 0.*
   q: *The first two bits of s are the same.*

   Find the truth set for each of the following statements:

   $$p; \quad q; \quad p \wedge q; \quad p \vee q.$$

2. Let p, q denote the following propositions concerning an integer n.

   $$p\text{: } n \leq 50; \qquad q\text{: } n \geq 10.$$

   Express in words, as simply as you can, the following statements as conditions on n.

   $$\neg p; \quad p \wedge q; \quad \neg(\neg q); \quad \neg p \vee \neg q.$$

3. Let p, q be the following propositions.

   p: *This book is on Databases.*
   q: *This book is on Programming.*

   Express each of the following compound statements symbolically in TWO different ways:

   (a) This book is not on Databases or Programming.

   (b) This book is not on Databases and Programming.

4. Use truth tables to prove that $(p \wedge q) \vee (\neg p \wedge q) = q$. (Hint: you will need to construct columns for p, q and $p \wedge q, \neg p, \neg p \wedge q, (p \wedge q) \vee (\neg p \wedge q)$. Remember to make a comment at the end to say why the table proves that the two statements are logically equivalent.)