Recurrence Relations : Proof by Induction

- A sequence is determined by the recurrence relation $u_{n+1} = 3u_n + 2$ and initial term $u_1 = 2$.

- Prove by induction the following statement, for all $n \in \mathbb{Z}^+$.

$$u_n = 3^n - 1$$

Step 1 : Base Step

- Demonstrate the expression is valid for $n = 1$ (i.e. $u_1 = 2$).

$$u_n = 3^n - 1$$

$$u_1 = 3^1 - 1$$

$$u_1 = 3 - 1 = 2$$

Step 2 : Induction Hypothesis

- Demonstrate the expression is valid for $n = k$.

$$u_k = 3^k - 1$$

Step 3 : Induction Step

▸ Demonstrate the expression is valid for
$n = k + 1$.

$$u_{k+1} = 3^{k+1} - 1$$

$$u_{k+1} = 3.3^k - 1$$

$$u_{k+1} = (1 + 2).3^k - 1$$

$$u_{k+1} = 3^k + 2.3^k - 1$$