Chapter 1

Logic

Chapter 2

Session 3

Part B : Logical Operations

Let $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p and q be the following propostions concerning the integers n.

p n is event

q $n < 5$

Find the values of n for which each of the following compound statement is true,

(i) $\neg p$

(ii) $p \wedge q$

(iii) $\neg p \vee q$

(iv) $p \oplus q$

Question 3

Let $\mathcal{S} = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$ $n \in \mathcal{S}$ p : n is a multiple of two q : n is a multiple of five
Express the following statement using logic symbols n is not a multiple of either 2 or 5.
List the elements of S which are in the truth set for the statement in (ii).
Write the contrapositive of the following statement concerning an integer n.
If the last digit of n is 4, then n is divisible by 3

2.1 Conditional Connectives

Construct the truth table for the proposition $p \to q$.

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

2.2 Tautologies and Truth Tables

Truth Table for the Biconditional Connective.

P	Q	$P \leftrightarrow Q$
T	T	
T	F	
F	T	
F	F	T

P	Q	$P \vee Q$		
T	T			
T	F			
F	T			
F	F			

Question 3

Part A : Propositions

Let p, q be the following propositions:

- p : this apple is red,
- q : this apple is ripe.

Express the following statements in words as simply as you can:

(i) $p \to q$

(ii) $p \wedge \neg q$.

Express the following statements symbolically:

(iii) This apple is neither red nor ripe.

(iv) If this apple is not red it is not ripe.

Question 3

Question 6

2.2.1 Question 6

Say with reason whether or not $\mathcal{R}$ is

- reflexive
- symmetric
- transitive

In the cases where the given property does not hold provide a counter example to justify this.

Question 6 Part A : Digraphs

Suppose A =$\{1, 2, 3, 4\}$. Consider the following relation in A

$$\{(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)\}$$

Draw the direct graph of A. Based on the Digraph of A discuss whether or not a relation that could be depicted by the digraph could be described as the following, justifying your answer.

(i) Symmetric

(ii) Reflexive

(iii) Transitive

(iv) Antisymmetric

Part B : Relations

Determine which of the following relations xRy are reflexive, transitive, symmetric, or antisymmetric on the following - there may be more than one characteristic. if

(i) $x = y$

(ii) $x < y$

(iii) $x^2 = y^2$

(iv) $x \geq y$

Question 2

Let $A = \{0, 1, 2\}$ and $R = \{(0, 0), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$ and $S = \{(0, 0), (1, 1), (2, 2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Part C : Partial Orders

Let $A = \{0, 1, 2\}$ and $R = \{(0,0),(0,1),(0,2),(1,1),(1,2),(2,2)\}$ and $S = \{(0,0),(1,1),(2,2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Biconnective Operators

$$p \longleftrightarrow q$$

We could verbalize this as "p implies q and q implies p".
Maths for Computing Sprint

Part 0. - Numeracy

2.2.2 Factorials

Evaluate the following

- 6!
- 3!
- 1!
- 0!

Evaluate the following expressions

$$\frac{5!}{3!} \text{ and } \frac{6!}{2! \times 4!}$$

Laws of Logarithms

- Addition of Logarithms
- Subtraction of Logaritms
- Powers of Logarithms

Section 3. Logic

Proofs with Truth Tables

$$\neg(p \vee q) \wedge p \equiv q$$

p	q				

2.3 Section 3 Logic

2.3.1 Logical Operations

- $\neg p$ the negation of proposition p.
- $p \wedge q$ Both propositions p and q are simultaneously true (Logical State AND)
- $p \vee q$ One of the propositions is true, or both (Logical State : OR)
- $p \otimes q$ Only one of the propositions is true (Logical State : exclusive OR (i.e XOR)

p	q	$p \vee q$	$q \wedge p$	$p \otimes q$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

2.4 Logic Proposition

Let p, Q and r be the following propositions concerning integers n:

- p : n is a factor of 36 (2)
- q : n is a factor of 4 (2)
- r : n is a factor of 9 (3)

n	**p**	**q**	**r**
1	1	1	1
2	1	0	1
3	0	1	1
4	1	0	1
6	0	0	1
9	0	1	1
12	0	0	1
18	0	0	1
36	0	0	1

For each of the following compound statements, express it using the propositions P q and r, andng logical symbols, then given the truth table for it,

1) If n is a factor of 36, then n is a factor of 4 or n is a factor of 9
2) If n is a factor of 4 or n is a factor of 9 then n is a factor of 36

- $p \to q$ p implies q
- $p \lg q$

Part 1 : Logic

1.1 2010 Question 3

Let $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ and let p, q be the following propositions concerning the integer $n \in S$.

- p: n is a multiple of two. (i.,18e. $\{10, 12, 14, 16, 18\}$)
- q: n is a multiple of three. i.e. $\{12, 15, 18\}$

For each of the following compound statements find the sets of values n for which it is true.

- $p \vee q$: (p or q : 10 12 14 15 16 18)
- $p \wedge q$: (p and q: 12 18)
- $\neg p \oplus q$: (not-p or q, but not both)
  - $\neg p$ not-p = $\{1113151719\}$
  - $\neg p \vee q$ not-p or q $\{11121315171819\}$
  - $\neg p \wedge q$ not-p and q $\{15\}$
  - $\neg p \oplus q = \{11, 12, 13, 17, 18, 19\}$

1.2 2010 Question 3

Let p and q be propositions. Use Truth Tables to prove that

$$p \to q \equiv \neg q \to \neg$$

Important Remember to make a comment at the end to say why the table proves that the two statements are logically equivalent. e.g. *since the columns are identical both sides of the equation are equivalent.*

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

(Key “difference” is first and last rows)

1.3 Membership Tables for Laws

Page 44 (Volume 1) Q8. Also see Section 3.3 Laws of Logic.

Construct a truth table for each of the following compound statement and hence find simpler propositions to which it is equivalent.

- $p \vee F$
- $p \wedge T$

Solutions

p	T	$p \vee T$	$p \wedge T$
0	1	1	0
1	1	1	1

- Logical OR: $p \vee T = T$
- Logical AND: $p \wedge T = p$

p	F	$p \vee F$	$p \wedge F$
0	0	0	0
1	0	1	0

- Logical OR: $p \vee F = p$
- Logical AND: $p \wedge F = F$

1.4 Propositions

Page 67 Question 9 Write the contrapositive of each of the following statements:

- If n= 12, then n is divisible by 3.
- If n=5, then n is positive.
- If the quadrilateral is square, then four sides are equal.

Solutions

- If n is not divisible by 3, then n is not equal to 12.
- If n is not positive, then n is not equal to 5.
- If the four sides are not equal, then the quadrilateral is not a square.

Prepositional Logic

2.4.1 five basic connectives

Reflexive, Symmetric and Transitive

- Reflexive
- Symmetric
- Transitive

2.4.2 Logarithms

Here we assume x and y are positive real numbers. 1. $loga(xy) = loga(x) + loga(y)$ 2. $loga(x/y) = loga(x) - loga(y)$ 3. $loga(x^r) = rloga(x)$ for any real number r. Invertible Functions

2.4.3 Proof by Induction

Another sequence is defined by the recurrence relation un = un-1+2n-1 and u1 = 1. (i) Calculate u2, u3, u4 and u5. 1,4,9,16,25

(ii) Prove by induction that $u_n = n^2$ for all $n \geq 1$

Exponentials

Prepositional Logic

- $p \wedge q$
- $p \vee q$
- $p \rightarrow q$

1.5 Truth Sets

2009

Let $n = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p, q be the following propositions concerning the integer n.

- p: n is even,
- q: $n \geq 5$.

By drawing up the appropriate truth table nd the truth set for each of the propositions $p \vee \neg q$ and $\neg q \rightarrow p$

n	p	q	$\neg q$	$p \vee \neg q$
1	0	0	1	1
2	1	0	1	0
3	0	0	1	1
4	1	0	1	0
5	0	1	0	1
6	1	1	0	1
7	0	1	0	1
8	1	1	0	1
9	0	1	0	1

Truth Set $= \{1, 3, 5, 6, 7, 8, 9\}$

n	p	q	$q \rightarrow p$	$q \rightarrow p$
1	0	0	1	0
2	1	0	1	0
3	0	0	1	0
4	1	0	1	0
5	0	1	0	1
6	1	1	1	0
7	0	1	0	1
8	1	1	1	0
9	0	1	0	1

Truth Set $= \{5, 7, 9\}$

1.6 Biconditional

See Section 3.2.1.

Use truth tables to prove that $\neg p \leftrightarrow \neg q$ is equivalent to $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

1.7 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $\neg(p \wedge \not{q})$ (i.e. $\equiv p \rightarrow q$).

Logic Gates

- AND
- OR
- NOT

Examiner's Comments:*Many diagrams were carefully and clearly drawn and well labelled, gaining full marks. The logic table was also well done by most, but there were a few marks lost in the final part by failing to deduce that since the columns of the table are identical the expressions are equivalent.*

2.4.4 logical implication

Logical implication is a type of relationship between two statements or sentences. The relation translates verbally into "logically implies" or "if/then" and is symbolized by a double-lined arrow pointing toward the right ($\rightarrow$). If A and B represent statements, then $A \rightarrow B$ means "A implies B" or "If A, then B." The word "implies" is used in the strongest possible sense.

As an example of logical implication, suppose the sentences A and B are assigned as follows:

A = The sky is overcast. B = The sun is not visible.

In this instance, A B is a true statement (assuming we are at the surface of the earth, below the cloud layer.) However, the statement B A is not necessarily true; it might be a clear night. Logical implication does not work both ways. However, the sense of logical implication is reversed if both statements are negated. That is,

$$(A \rightarrow B) \equiv (-B \rightarrow -A)$$

Using the above sentences as examples, we can say that if the sun is visible, then the sky is not overcast. This is always true. In fact, the two statements A B and -B -A are logically equivalent.

1.8 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $(p \wedge q) \vee \neg q$ (i.e. $\equiv p \to q$).

Important Label each of the gates appropriately and label the diagram with a symblic expression for the output after each gate.