Examiners' reports 2010

2910102 Mathematics for computing – Zone A

General remarks

These scripts showed that most of the candidates had a thorough understanding of the syllabus and the ability to apply their knowledge and skills in the appropriate context. They were well prepared and demonstrated that they had a good grasp of the subject and had revised well.

When revising for the exam it is a good idea to work through the sample paper in the subject guide which has full solutions. You can then compare your answers with those given and if your approach is very different then you can consider why and perhaps modify your method. You can learn a lot from looking at the notation and wording used in the solution and the way any mathematical definitions or proofs are included. The way you present your solution may help you clarify the problem and develop the solution, as well as make it easier for the Examiner to follow your work. Please try to ensure your answers convey your meaning clearly and correctly and show your working in full, so that the Examiner can give you marks for correct method, even if you make an error which means your final solution is incorrect. It will help you greatly to work through other past papers as part of the revision process so that you are familiar with the type of questions which may arise on each topic, and the material and skills you need to answer them. It may also help to make a list of key points on each chapter as a revision guide, together with typical exam questions.

Comments on individual questions

Question 1

This question required students to transfer numbers from binary to decimal and back as well as to add, subtract and multiply numbers in binary. When the question specifies working in base 2 that is what students should do. Converting everything to another base in order to perform the arithmetic, such as base ten, will lose marks.

In part (d) the sets of integers, Z, Q, R, refer to the sets of integers, rational numbers and real numbers respectively. Pi is an irrational number and as such is neither an integer nor a rational number and thus belongs only to the set R. The fraction 2/3 is a rational number but not an integer and thus belongs to the sets Q and R.

Question 2

Part (a) asked students to describe the first set by the rule of inclusion method and the majority of students could do this and also list the elements in the second set.

Part (b) was well done by the majority of students. A membership table for two sets, A and B was required and this needed to have 4 rows in it with entries 0 0, 0 1, 1 0, 1 1.

Columns for both sides of the equation are required. The fact that the entries in these two columns are the same leads us to conclude that the expressions are equivalent. A sentence saying this is required for full marks. The entries of one in each of these columns correspond to shading that corresponding region in the Venn diagram showing two sets intersecting in the most general way. Doing this creates the necessary shading on the Venn diagram.

In part (c) the elements of the required set may be more easily found using the intersection of A complement and B, that is the intersection of {1,3,5,7,9} and {4,5,6,7}.

Question 3

In the first part of this question a set of values is required for each of the logical compound statements. This covered in the subject guide and students should ensure they can produce the relevant lists. In the next part of this question students were asked to translate from the English to the symbolic logical equivalent compound propositions. This skill (and translating back in the other direction) needs to be practised beforehand.

In part (b) a truth table was required with columns for p; q; p implies q; not q; not p and finally not q implies not p. The resulting column entries for both sides of the equation should be equal, leading us to conclude that 'since the columns are identical both sides of the equation are equivalent', or some such statement.

The contrapositive of the given statement is 'if n is not divisible by 5 then the last digit of n is not zero'. A number of students got this the wrong way round. Some students wrote down the contrapositive symbolically, usually in terms of p and q. However to obtain full marks for this they also needed to define both p and q in words.

Question 4

The first part of this question on functions involved drawing a simple arrow diagram showing the arrows connecting (c,3), (a,2), (b,5) and (d,3). The domain of this function is {a,b,c,d}, the co-domain {1,2,3,4,5} and the range {2,3,5}. Be careful not to confuse the co-domain and the range. The function is not one-to-one because 3 in the co-domain has more than one ancestor, namely c and d. The function is not onto because 1 and 4 in the co-domain have no ancestors. It was necessary to give these specific examples to gain full marks. Simply saying that the range is not equal to the co-domain is not specific enough. Whether or not the function is one to one and/or onto is a question which arises virtually every year and is well worth preparing for. A clear understanding of the concepts of domain, range and co-domain is necessary for this as well as the ability to find and interpret these according to the particular example.

Part (b) required candidates to demonstrate they know when a function has an inverse and to be able to find this inverse and define it fully. This requires giving the inverse function in algebraic terms and also its domain and co-domain. Many candidates missed this latter part of the definition and lost marks.

Question 5

The majority of students calculated the first three terms of the sequence correctly by substituting 1, 2 and 3 for n, in the formula 3n-2. They were then required to perform a proof by induction on the sum of the terms of this sequence. There is a standard procedure involved in such a proof which breaks down into several stages which students should know and have practised beforehand, so that although the individual proofs differ, their structure is the same.

Part (b) involved knowledge and understanding of Sigma notation. The first sum required substitution into the given formula of n=50 to obtain the solution 42925. The next sum has general term k^2 +1 and so is the previous sum + 50. The final sum has general term 2k^2 and so is twice 42925.

Question 6

There are a few basic definitions in each chapter of the subject guide which are key to understanding the concepts in that section. These should be noted and learnt as part of the revision process. These include knowing what is meant by the degree of a vertex of a graph and also what is meant by saying two vertices are adjacent, which was part (a) of the question.

Part (b) again was standard bookwork from the subject guide.

In part (c) this result could be used to find the number of edges in a 5-regular graph with 8 vertices, and also to show that the number of edges in a 5-regular graph on 11 vertices would be 55/2. This shows the graph is not possible since we cannot have half an edge. Some such statement was needed to gain full marks, rather than just showing the arithmetic. In the last part of the question students were asked to generalise their understanding to a 5-regular graph on 2n vertices and show it has 5n edges. Students at this level should not leave their solution in the form 10n/2.

Question 7

In part (a) a relation was defined on S and the digraph requested. Most students were able to produce this. It is not necessary in a digraph to draw separate edges when there is a symmetric relation between two elements, such as between 1 and 2. They should just draw one edge and show arrows on it in both directions. In this case there were only loops on 3 and 6. The formal definitions for reflexivity, symmetry and transitivity should be revised before the exam and students should be able to reproduce them, with appropriate counter examples where required in the exam.

In part (b) the relation produces a digraph in three unconnected parts. The elements 1 and 4 form one part; the elements 2 and 5 another; the elements 3 and 6 the third. This relation is an equivalence relation as it is reflexive, symmetric and transitive. It is not a partial order since it is not anti-symmetric. The equivalence classes are {1,4},{2,5},{3,6}. Notation for equivalence classes is specific and the appropriate notation should be used.

Question 8

Most students were able to draw the tree T correctly. There are four non-isomorphic trees which can be constructed by adding a new vertex and edge to T. Some students missed one or more of these. Two of them have different degree sequences and the others are not isomorphic since their adjacency properties differ. Students needed to explain clearly why all four trees are non-isomorphic to obtain full marks. There is more than one tree with six vertices which are not isomorphic to any of these four, such as a path graph.

In part (b) a ternary tree was defined. The nodes on level 4 were 81 in number and those on level ten 3^10. The total number of internal nodes was a Sigma expression and the correct limits for the expression they gave were not always found by students. The smallest height of such a tree with 1000 internal nodes is 7, but as the question defined the tree's height to be greater than or equal to ten the answer 10 was also accepted. Some students took logs to base two instead of base three when performing this calculation.

Question 9

Part (a) required students to draw a tree diagram to model the process of throwing a coin three times. This is covered in the subject guide. The resulting tree in this case is of height 3 and there are 8 different nodes on level three, which show the eight equally likely outcomes of the process, such as (HHH), (HHT). The event A contains the outcomes {HHH, HTH, THH, TTH}. The event B contains the outcomes {HTT, THT,TTH}. Since each outcome has probability 1/8 we find P(A)=4/8 and P(B)=3/8. The probabilities of the union and intersection of A and B can similarly be found, by considering the number of outcomes in each set. A and B are not independent since P(A intersection B) is not equal to P(A).P(B).

Part (b) was about permutations and combinations and some students mixed these up.

Question 10

The first part of this question required students to know that the first line of the matrix shows the number of edges between vertex one and all the other vertices, as well as itself. The first entry in the matrix of 1indicates a loop at vertex one. To calculate the number of edges in g students should realise that they cannot simply add up all the entries and divide by 2. The loops only count as one in this calculation so they should be removed from it and added in later, or an appropriate adjustment should be made for them. The square of the matrix gives the number of walks of length two between pairs of vertices in the graph. It is calculated by multiplying G by itself to produce another 3 by 3 matrix.

Part (b) was a standard question involving Gaussian elimination. Marks were given for method. Many students lost a mark as they did not fully reduce the matrix to one with ones on the leading diagonal, but left other numbers there. It is helpful if the row operations employed are clearly labelled and the order of transformations is shown, so any errors can be worked through and credit given.