UNIVERSITY OF LONDON

EXTERNAL PROGRAMME

B. Sc. Examination 2004

COMPUTING

CIS102w Mathematics for Computing

Duration: 3 hours

Date and time:

There are TEN questions on this paper.
Full marks will be awarded for complete answers to TEN questions.
Electronic calculators may be used. The make and model should be specified on the script and the calculator must not be programmed prior to the examination.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1 (a) Perform the following subtraction in hexadecimal, showing all your working.

$$(A4D)_{16} - (9EF)_{16}.$$

[2]

(b) Convert the binary number $(110111001.01)_2$ to hexadecimal. [2]

(c) Convert the hexadecimal number $(A2B.8)_{16}$ to decimal. [2]

(d) Given $x = \sqrt{2}$ determine whether the following statements are true or false:

(i) $x \leq 2$
(ii) $1.42 > x > 1.41$
(iii) x is a rational number
(iv) $\sqrt{2}x = 2$ [2]

(e) Convert the following statements into symbols:

$$\sqrt{2} is less than 1.5 and greater than 1.4$$
$$2\sqrt{2} is greater than or equal to \frac{5}{2}.$$

[2]

Question 2 Let p and q be the following propositions about the positive integer n:

$$p : n < 20 \qquad q : n is prime.$$

(a) List the truth sets for: (i) p (ii) $p \wedge q$. [2]

(b) Express each of the following compound propositions symbollically using p, q :

(i) $n < 20$ and n is not prime
(ii) $n < 20$ if n is prime
(iii) $n < 20$ or n is prime.

[3]

(c) For each of the compound expressions in (b) give ONE example of n for which the proposition is FALSE. [3]

(d) Write the contrapositive of the folowing proposition:

$$"if n = 14 then n is divisible by 7."$$

[2]

Question 3 Given any number $x \in \mathbb{R}$ the floor value is denoted by $\lfloor x \rfloor$ and the absolute value is denoted by $|x|$.

(a) Find $\lfloor \sqrt{2} \rfloor$ and $|-2|$. [2]

(b) Find the set of values of a such that $\lfloor a \rfloor = 1$ and the set of values of b such that $|b| = 1$. [2]

(c) Consider the functions $f : \mathbb{R} \to \mathbb{Z}$ and $g : \mathbb{R} \to \mathbb{R}$ given by:

$$f(x) = \lfloor x - 1 \rfloor \, and g(x) = |x - 1|.$$

(i) Write down the domain, co-domain and range of f and g. [2]

(ii) For each function, say whether or not it is one to one, justifying your answer. [2]

(iii) For each function, say whether or not it is onto, justifying your answer. [2]

Question 4 (a) List the following sets:

$$\begin{array}{lcl} \{2^r & : & r \in \mathbb{Z} \; and \; 0 \leq r \leq 5\} \\ \{r^2 & : & r \in \mathbb{Z} and 1 \leq r \leq \; 6\}. \end{array}$$

[2]

(b) Let A, B and C be subsets of a universal set $\mathcal{U}$.

(i) Draw a labelled Venn diagram to illustrate the relationship between A, B and C such that they divide $\mathcal{U}$ into 8 separate regions. [1]

(ii) The subset $X \subseteq \mathcal{U}$ is defined by the following membership table.

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Shade the area X on your Venn diagram. [2]

(iii) Use set operations to express the set X as a combination of the subsets A, B and C. [1]

(iv) The subset $Y \subseteq \mathcal{U}$ is defined as $Y = A \cap (B \cup C')$. Construct a membership table for Y. [2]

(v) For each of the following statements state whether it is true or false.
$X \subset Y$; $Y \subset X$, $Y = (A \cap B) \cap C'$. [2]

Question 5 (a) What properties should a graph have in order for it to be:

(i) a simple graph;

(ii) a connected graph. [2]

(b) Let K_n be the simple graph with vertices $v_1, v_2, v_3, ..., v_n$ in which each vertex is joined to every other vertex by an edge.

(i) Draw K_6.

(ii) Determine the number of edges of K_6.

(iii) Determine the number of paths from v_1 to v_2 of length two.

(iv) Find an expression in terms of n for the number of paths from v_1 to v_2 of length two in k_n. [5]

(c) Draw two different (that is non-isomorphic) connected graphs each having the degree sequence $3, 3, 2, 1, 1, 1, 1$. Give one reason why the graphs you have drawn are not isomorphic. [3]

Question 6 (a) Consider the sequence given by $1, 4, 7, 10, 13, ...$
State a recurrence relation which expresses the nth term, u_n, in terms of the $(n-1)th$ term, u_{n-1}. [2]

(b) Another sequence is defined by the recurrence relation $u_n = u_{n-1} + 2n - 1$ and $u_1 = 1$.

(i) Calculate u_2, u_3, u_4 and u_5.

(ii) Prove by induction that $u_n = n^2$ for all $n \geq 1$.

(iii) Find the sum of the first 50 terms of this sequence.
You may assume the formula for $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, [8]

Question 7 (a) (i) What properties must a graph have in order for it to be a tree?

(ii) Say, with reason, whether or not it is possible to construct a tree with degree sequence $4, 3, 3, 1, 1$.

(iii) Say, with reason, whether it is possible to construct a tree with degree sequence $4, 3, 2, 2, 1$.

(iv) What properties must a graph have in order for it to be a binary tree? [5]

(b) A binary search tree is designed to store an ordered list of 3000 records at its internal nodes.

(i) Find which record is stored at the root (level 0) of the tree and at each of the nodes at level 1.

(ii) What is the height of the tree?

(iii) What is the maximum number of comparisons needed in order to find an existing record in the tree? [5]

Question 8 An ordered sequence of four digits is formed by choosing digits without repetition from the set $\{1, 2, 3, 4, 5, 6, 7\}$.

(a) Determine:

(i) the total number of such sequences;

(ii) the number of sequences which begin with an odd number;

(iii) the number of sequences which end with an odd number;

(iv) the number of sequences which begin and end with an odd number;

(v) the number of sequneces which begin with an odd number or end with an odd number or both;

(vi) the number of sequences which begin with an odd number or end with an odd number but not both. [6]

(b) By finding the number of such sequences or otherwise find the probability that the sequence:

(i) ends with an even number;

(ii) begins and ends with an even number. [4]

Question 9 Let S be the set $\{a, b, c, d\}$.

(a) (i) Describe briefly how each subset of S can be represented by a unique 4-bit binary string.

(ii) Write down the string corresponding to the subset $\{a, c, d\}$ and the subset corresponding to the string 0110.

(iii) What is the total number of subsets of S? [4]

(b) R is a relation defined on S in precisely the following cases:

$$_bR_b; \;\; _bR_c; \;\; _cR_b; \;\; _cR_c; \;\; _cR_d; \;\; _dR_a.$$

(i) Draw the relationship digraph for R on S.

(ii) The relation R is not reflexive. Which minimal set of pairs should be added to R to make it relexive?

(iii) The relation R is not symmetric. Which minimal set of pairs should be added to R to make it symmetric?

(iv) The relation R is not transitive. Which minimal set of pairs should be added to R to make it transitive?

(v) Is the relation R anti-symmetric? Justify your answer. [6]

Question 10 Consider the three matrices

$$A = \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & 4 \\ 2 & 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 0 \end{pmatrix}.$$

(a) Find the following matrices:

(i) $BC + A$.

(ii) $A^2 + (AB)C$. [5]

(b) Let D be a 2×4 matrix and E be a 4×3 matrix. Given R is the relation on two matrices X and Y where X is related to Y if XY is a valid product of the two matrices:

(i) draw the digraph of the relation on the matrices A, B, C, D, E;

(ii) write down the adjacency matrix of this digraph. [5]

END OF EXAMINATION