# Graph Theory

**Isomorphic Graph:** Two graphs $(V_1, E_1, g_1)$ and $(V_2, E_2, g_2)$ are *isomorphic* if there are bijections $f_1 : V_1 \to V_2$ and $f_2 : E_1 \to E_2$ such that for each edge $a \in E_1$, $g_1(a) = x - y$ if and only if $g_2[f_2(a)] = f_1(x) - f_1(y)$.

It is not always easy to establish if 2 graphs are isomorphic or not. An exception is the case where the graphs are simple. In this case, we just need to check if there is a bijection $f : V_1 \to V_2$ which preserves adjacent vertices (i.e. if $v_1$, $v_2$ are adjacent in graph 1, then $f(v_1)$, $f(v_2)$ must be adjacent in graph 2).

If the graphs are not simple, we need more sophisticated methods to check for when two graphs are isomorphic. However, it is often straightforward to show that two graphs are *not* isomorphic. You can do this by showing *any* of the following seven conditions are true.

1. The two graphs have different numbers of vertices.
2. The two graphs have different numbers of edges.
3. One graph has parallel edges and the other does not.
4. One graph has a loop and the other does not.
5. One graph has a vertice of degree $k$ (for example) and the other does not.
6. One graph is connected and the other is not.
7. One graph has a cycle and the other has not.