Question 5

(a) There are 16 different 2 by 2 matrices whose entries may consist only of zeroes and ones, for example

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} and \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} are two such matrices.$$

Let S be the set all such matrices. We define a function f on S by the rule

$$f(\mathbf{X}) = the number of zeroes in \mathbf{X} where f : S \to \mathbb{Z} and \mathbf{X} \in S.$$

(i) Find a numerical value for both $f(\mathbf{A})$ and $f(\mathbf{B})$.

(ii) Write down the set of pre-images or ancestors of 1.

(iii) Write down the range of f.

(iv) Say whether or not this function is one to one, justifying your answer.

(v) Say whether or not this function is onto, justifying your answer. [6]

(b) Say whether or not each of the following functions has an inverse, justifying your answer. In the cases where there is an inverse define it.

(i) $f : S \to \mathbb{Z}$ defined in part (a).

(ii) $g : \mathbb{R} \to \mathbb{Z}$ defined by $g(x) = \lfloor x \rfloor$.

(iii) $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = 2x + 5$. [4]

Question 6 Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, ..., v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}, ...\{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex, v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4. [$2\frac{1}{2}$]

(b) Giving your answer in terms of n, write down an expression for the number of edges in K_n, C_n, and W_n. [$2\frac{1}{2}$]

(c) (i) Find the number of different paths of length two in each of the graphs in part (a), where a path does not contain the same edge more than once, and a path from v_x to v_y is different from a path from v_y to v_x.

(ii) Giving your answer in terms of n, write down an expression for the number of different paths of length two there are in K_n. [5]