# Chapter 1

# Session 5

## 1.1 Video 4 : Graph Theory

Draw the graph **G**, which has the vertices $v_1$,$v_2$,$v_3$,...,$v_7$, and the adjacency list:

$v_1$ : $v_2$,$v_4$

$v_2$ : $v_1$,$v_3$

$v_3$ : $v_2$,$v_4$

$v_4$ : $v_1$,$v_3$,$v_5$

$v_5$ : $v_4$,$v_6$

$v_6$ : $v_5$,$v_7$

$v_7$ : $v_5$,$v_6$

## 1.2 graph theory

Given the following definitions for simple, connected graphs:

- $K_n$ is a graph on $n$ vertices where each pair of vertices is connected by an edge;
- $C_n$ is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- $W_n$ is the graph obtained from $C_n$ by adding an extra vertex,$v_{n+1}$, and edges from this to each of the original vertices in $C_n$.

(a) Draw $K_4$, $C_4$, and $W_4$.

# Session 05:Graphs

5A.1 What is a Graph?

5A.2 Paths Cycles and Connectivity

5A.3 Isomorphisms of a graph

5A.4 Adjacency Matrices and Adjacency Lists

## Isomorphism

- They have a different number of connected components
- They have a different number of vertices
- They have different degrees sequences
- They have a different number of paths of any given length
- They have a different number of cycles of any length.

## Adjacency Lists

u : $\{v\}$

v : $\{w, x\}$

w : $\{v, x\}$

z : $\{v, w\}$

- Spanning Subgraphs of G.
- a vertex is said to be an **emph isolated vertex** if it has a degree of zero.
- a vertex is said to be an **emph end-vertex** if it has a degree of one.
- a vertex is said to be an **emph even vertex** if it has a degree of an even number.
- a vertex is said to be an **emph odd vertex** if it has a degree of an odd number.
- A graph is said to be **emphk-regular** if the degree of each vertex is $k$.
- Every Graph has an even number of odd vertices.
- A cubic graph is a graph where every vertex has degree three.

## 1.3 Graph Theory - Isomorphic Graphs

- If the graphs are not simple, we need more sophisticated methods to check for when two graphs are isomorphic.
- However, it is often straightforward to show that two graphs are not isomorphic.
- You can do this by showing any of the following seven conditions are true.

## 1.4 Isomorphic Graphs

1. The two graphs have different numbers of vertices.
2. The two graphs have different numbers of edges.
3. One graph has parallel edges and the other does not.
4. One graph has a loop and the other does not.
5. One graph has a vertice of degree k (for example) and the other does not.
6. One graph is connected and the other is not.
7. One graph has a cycle and the other has not.

# Section 5. Graph Theory

### Adjacency Lists

1.
2.
3.
4.

# Question 5

1. Draw two non-isomorphic graphs with the following degree sequence.

$$4, 3, 3, 2, 2, 2, 2, 1, 1$$

2. Write out the degree sequence of the following graph.
3. State the vertices that comprise a cycle of length 5 in both of the following graphs.

# Session 05 Graph Theory

- Eulerian Path
- Isomorphism
- Adjacency matrices

Adjacency Matrices

$$\begin{pmatrix} o & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

# Session 05 Graph Theory

- Eulerian Path
- Isomorphism
- Adjacency matrices

Adjacency Matrices

$$\begin{pmatrix} o & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

## 1.5 graph theory

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(a) Draw $K_4$, $C_4$, and $W_4$.

## Condtions for Isomorphism

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## Question 5

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(a) Draw $K_4$, $C_4$, and $W_4$.

a) (i) A simple, connected graph has 7 vertices, all having the same degree d. State the possible values of d and for each value also give the number of edges in the corresponding graph. (ii) Another simple, connected graph has 6 vertices, all having the same degree, n. Draw such a graph when n = 3 and state the other possible values of n. [4]