

Question 7 Given a flock of chickens, between any two chickens one of them is dominant. A relation, R , is defined between chicken x and chicken y as xRy if x is dominant over y . This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

Amy is dominant over Beth and Carol
Beth is dominant over Eve and Carol
Carol is dominant over Eve and Daisy
Daisy is dominant over Eve, Amy and Beth
Eve is dominant over Amy.

- (a) Draw a digraph to represent this pecking order, saying what the vertices represent and what it means when two vertices are connected by an edge. [2]
- (b) Say whether or not the pecking order R is
- (i) reflexive;
 - (ii) anti-symmetric;
 - (iii) transitive;
 - (iv) a partial order.
- Justify each answer in terms of a small proof or counter-example. [4]
- (c) Another relation, R_2 , is defined between the chickens on Home Farm. Let x and y be chickens on Home Farm, then

xR_2y if and only if x and y have the same mother.

The mothers of the chickens on Home Farm are either Flora or Harriet from a neighbouring farm. Harriet is the mother of Amy, Daisy and Eve. Flora is the mother of Beth and Carol.

Justifying your answer, say whether R_2 is an equivalence relation on the set of chickens at Home Farm. If this is an equivalence relation write down the equivalence classes. [4]