MA4016 Problem Sheet 10 (Machines)

1. Let $M = \{S, \mathcal{I}, \mathcal{O}, \nu, \omega\}$ be a finite state machine with $\mathcal{I} = \{a, b, c\}$ and $\mathcal{O} = \{0, 1\}$ defined by the following table:

	ν			ω		
	a	b	c	a	b	c
s_0	s_0	s_3	s_2	0	1	1
s_1	s_1	s_1	s_3	0	0	1
s_2	s_1	s_1	s_3	1	1	0
s_3	s_2	s_3	s_0	1	0	1

(a) Starting at s_0, what is the output for the input string *abbccc* ?

(b) Draw the state diagram (digraph) for this machine.

2. Let $M = \{S, \mathcal{I}, \mathcal{O}, \nu, \omega\}$ be a finite state machine with $\mathcal{I} = \mathcal{O} = \{0, 1\}$ defined by the following table:

	ν		ω	
	0	1	0	1
s_0	s_4	s_1	0	0
s_1	s_3	s_2	0	0
s_2	s_3	s_2	0	1
s_3	s_3	s_3	0	0
s_4	s_5	s_3	0	0
s_5	s_5	s_3	1	0

(a) Draw the state diagram for this machine.

(b) Let $i \in \mathcal{I}^*$ with $\|i\| = 4$. If 1 is a suffix of $\omega(s_0, i)$, what are the possibilities for the string i?

(c) Let $A \subset \{0, 1\}^*$ be the language where $\omega(s_0, i)$ has 1 as a suffix for all $i \in A$. Determine A.

(d) Find the language $A \subset \{0, 1\}^*$ where $\omega(s_0, i)$ has 111 as a suffix for all $i \in A$.

3. Let $M = \{S, \mathcal{I}, \mathcal{O}, \nu, \omega\}$ be a finite state machine with $\mathcal{I} = \mathcal{O} = \{0, 1\}$ defined by the following table:

	ν		ω	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_0	s_1	1	1

(a) Draw the state diagram for this machine.

(b) Determine the output for the following input sequences starting at s_0 in each case:

$(a)\quad i = 111 \qquad (b)\quad i = 1010 \qquad (c)\quad i = 00011$

(c) Describe in words what M does.

4. Construct $M = \{S, \mathcal{I}, \mathcal{O}, \nu, \omega\}$ with $\mathcal{I} = \mathcal{O} = \{0, 1\}$ that recognises each occurrence of (a) 0110 and (b) 1010.

5. Apply the state minimisation process to each of the machines below:

(a)

	ν		ω	
	0	1	0	1
s_0	s_5	s_2	0	0
s_1	s_4	s_3	0	1
s_2	s_5	s_1	1	1
s_3	s_3	s_2	1	0
s_4	s_1	s_3	0	1
s_5	s_3	s_5	0	0

(b)

	ν		ω	
	0	1	0	1
s_1	s_6	s_3	0	0
s_2	s_3	s_1	0	0
s_3	s_2	s_4	0	0
s_4	s_7	s_4	0	0
s_5	s_6	s_7	0	0
s_6	s_5	s_2	1	0
s_7	s_4	s_1	0	0

6. Construct *Turing* Machines which perform the following computations on the natural numbers:

   (a) $T(m) = 3m$

   (b) $T(m) = \begin{cases} 0, & \text{if } m \text{ is even} \\ 1, & \text{if } m \text{ is odd} \end{cases}$

   (c) determine if m is divisible by 4.

   (d) $T(m) = m^2$

7. Construct *Turing* Machines which recognise the following languages:

   (a) $\{0^n 1^n ; n = 0, 1, 2, \ldots\}$

   (b) $\{(01)^n ; n = 0, 1, 2, \ldots\}$