Introduction

One of the first people to experiment with graph theory was a man by the name of Euler (pronounced "oiler")(1707-1783). He attempted to solve the problem of crossing seven bridges onto an island without using any of them more than once.

From that point on, the study of graphs has been applied to a large number of real world problems. Today, graphs are all around us. They are used in many diverse industries, from urban planning, to shipping lanes, to computer networks such as the Internet.

Terminology

A *graph* consists of a nonempty set of points or *vertices*(or *nodes*), and a set of *edges*(or *arcs*) that link together the vertices. A simple real world example of a graph would be an airport flight network. The airports are the vertices and the flight paths between them are the edges connecting any two vertices.

The relationship between the vertices and the edges in a graph can be thought of as a function. This function associated with each edge e an unordered pair $(a - b)$ of vertices called the endpoints of e - the edge e joins the vertices a and b.

The vertices u and v are said to be *adjacent* if there is an edge e connecting the two of them. In such a case, u and v are called the *endpoints* of e, and e is said to *connect* u and v.

A *loop* in a graph is an edge connecting a node to itself. Two edges are *parallel* if they have common endpoints.

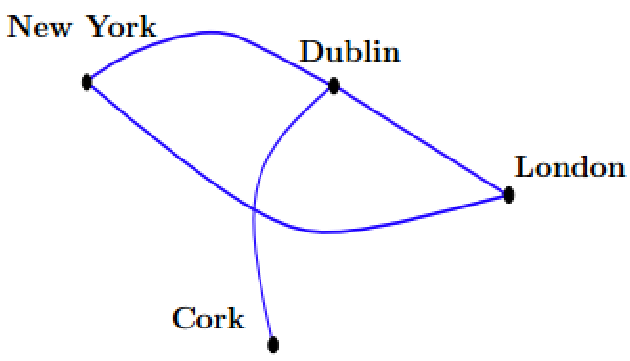

Figure 1: Graph of flight network between airports

An *isolated node* is a node which is not the end point of any edge. A graph is called *simple* if it has no loops or parallel edges. Otherwise it is called a *multigraph*. The *degree* of a vertex v, written $\deg(v)$, is the number of edge ends at that vertex.

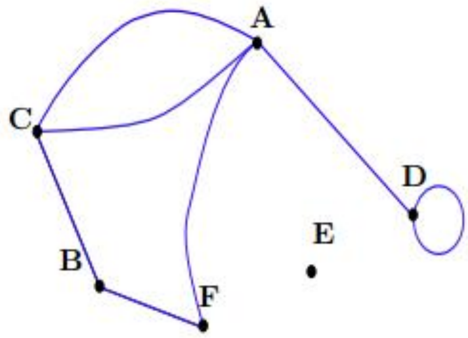

Figure 2: An example of a multigraph

A *path* from vertex v_0 to vertex v_k is a sequence

$$v_0, e_0, v_1, e_1, v_2, e_2, ..., v_{k-1}, e_{k-1}, v_k$$

of vertices and edges where for each i, the endpoints of edge e_i are $v_i - v_{i+1}$. The *length* of a path is the number of edges it contains; if an edge is used more than once, it is counted each time it is used. A graph is *connected* if there is a path from any vertex to any other vertex. A *cycle* in a graph is a path from some vertex v_0 back to v_0, where no edge appears more than once in the path sequence, v_0 is the only vertex appearing more than once, and v_0 occurs only at the ends. A graph with no cycles is called *acyclic*.

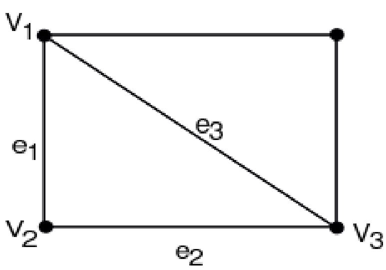

Figure 3: An example of a cycle

$V_1, e_1, V_2, e_2, V_3, e_3, V_1$ is an example of a cycle.

A graph can take on many forms: directed or undirected. A *directed graph* is one in which the direction of any given edge is defined. Conversely, in an *undirected graph* you can move in both directions between vertices. The edges can also be *weighted* or *unweighted*. Finally, a *complete graph* is a graph in which every pair of vertices is adjacent e.g. a triangle.