HibColl Videos

1.A Converting from Decimal to Binary 1.B Converting from Decimal to Hexadecimal 1.C Converting from Binary to Decimal 1.D Converting from Hexadecimal to Decimal 1.E Binary Addition 1.F Binary Subtraction

2.A Membership Tables 2.B Venn Diagrams 2.C Set differences and symmetric difference 2.D 2.E

Chapter 1

Number Systems

1.0.1 Significant Digits

There are three rules on determining how many significant figures are in a number: Non-zero digits are always significant. Any zeros between two significant digits are significant. A final zero or trailing zeros in the decimal portion ONLY are significant.

1.1 Floating Point Notation

In computing, floating point describes a method of representing an approximation of a real number in a way that can support a wide range of values.

The numbers are, in general, represented approximately to a fixed number of significant digits (the mantissa) and scaled using an exponent.

In essence, computers are integer machines and are capable of representing real numbers only by using complex codes. The most popular code for representing real numbers is called the IEEE Floating-Point Standard . The term floating point is derived from the fact that there is no fixed number of digits before and after the decimal point; that is, the decimal point can float. There are also representations in which the number of digits before and after the decimal point is set, called fixed-point representations. In general, floating-point representations are slower and less accurate than fixed-point representations, but they can handle a larger range of numbers.

Part A: Number Systems - Binary Numbers

1. Express the following decimal numbers as binary numbers.

   i) $(73)_{10}$ ii) $(15)_{10}$ iii) $(22)_{10}$

   All three answers are among the following options.

a) $(10110)_2$ b) $(1111)_2$ c) $(1001001)_2$ d) $(1000010)_2$

2. Express the following binary numbers as decimal numbers.

   a) $(101010)_2$ b) $(10101)_2$ c) $(111010)_2$ d) $(11010)_2$

3. Express the following binary numbers as decimal numbers.

   a) $(110.10101)_2$ b) $(101.0111)_2$ c) $(111.01)_2$ d) $(110.1101)_2$

4. Express the following decimal numbers as binary numbers.

   a) $(27.4375)_{10}$ b) $(5.625)_{10}$ c) $(13.125)_{10}$ d) $(11.1875)_{10}$

Part B: Number Systems - Binary Arithmetic

1. Perform the following binary additions.

   a) $(110101)_2 + (1010111)_2$
   b) $(1010101)_2 + (101010)_2$
   c) $(11001010)_2 + (10110101)_2$
   d) $(1011001)_2 + (111010)_2$

2. Perform the following binary subtractions.

   a) $(110101)_2$ - $(1010111)_2$
   b) $(1010101)_2$ - $(101010)_2$
   c) $(11001010)_2$ - $(10110101)_2$
   d) $(1011001)_2$ - $(111010)_2$

3. Perform the following binary multiplications.

   a) $(1001)_2 \times (1000)_2$
   b) $(101)_2 \times (1101)_2$
   c) $(111)_2 \times (1111)_2$
   d) $(10000)_2 \times (11001)_2$

4. Perform the following binary multiplications.

   (a) Which of the following binary numbers is the result of this binary division: $(10)_2 \times (1101)_2$.

   a) $(11010)_2$
   b) $(11100)_2$
   c) $(10101)_2$
   d) $(11011)_2$

   (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \times (111)_2$.

   a) $(11000)_2$
   b) $(11001)_2$
   c) $(10101)_2$
   d) $(11011)_2$

   (c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \times (1101)_2$.

   a) $(11000)_2$
   b) $(11001)_2$
   c) $(10101)_2$
   d) $(11011)_2$

5. Perform the following binary divisions.

   (a) Which of the following binary numbers is the result of this binary division: $(111001)_2 \div (10011)_2$.

   a) $(10)_2$
   b) $(11)_2$
   c) $(100)_2$
   d) $(101)_2$

   (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \div (111)_2$.

a) $(11)_2$

b) $(100)_2$

c) $(101)_2$

d) $(110)_2$

(c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \div (1101)_2$.

a) $(100)_2$

b) $(110)_2$

c) $(111)_2$

d) $(1001)_2$

Part C: Number Bases - Hexadecimal

1. Answer the following questions about the hexadecimal number systems
   a) How many characters are used in the hexadecimal system?
   b) What is highest hexadecimal number that can be written with two characters?
   c) What is the equivalent number in decimal form?
   d) What is the next highest hexadecimal number?

2. Which of the following are not valid hexadecimal numbers?

   a) 73   b) $A5G$ c) 11011   d) EEF

3. Express the following decimal numbers as a hexadecimal number.

   a) $(73)_{10}$ b) $(15)_{10}$ c) $(22)_{10}$ d) $(121)_{10}$

4. Compute the following hexadecimal calculations.

   a) $5D2 + A30$ b) $702 + ABA$ c) $101 + 111$ d) $210 + 2A1$

Part D: Natural, Rational and Real Numbers

- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$
- $\mathbb{Z}$: integers $\{-3, -2, -1, 0, 1, 2, 3, \ldots\}$
  - (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means "numbers" in German.)
- $\mathbb{Q}$: rational numbers
- $\mathbb{R}$: real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
  - (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

1. State which of the following sets the following numbers belong to.

1) 18	3) π	5) 17/4	7) $\sqrt{\pi}$
2) 8.2347...	4) 1.33333...	6) 4.25	8) $\sqrt{25}$

The possible answers are

a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$
d) Real Number $\mathbb{R}$

Floating Point Notation

(Demonstration on white board)

Part A: Number Systems - Binary Numbers

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Part B: Number Systems - Binary Arithmetic

(See section 1.1.3 of the text)

1. Perform the following binary additions.

   a) $(110101)_2 + (1010111)_2$
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      b) $(11100)_2$
      c) $(10101)_2$
      d) $(11011)_2$

   (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \times (111)_2$.

      a) $(11000)_2$
      b) $(11001)_2$
      c) $(10101)_2$
      d) $(11011)_2$

   (c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \times (1101)_2$.

      a) $(11000)_2$
      b) $(11001)_2$
      c) $(10101)_2$
      d) $(11011)_2$

5. Perform the following binary divisions.

   (a) Which of the following binary numbers is the result of this binary division: $(111001)_2 \div (10011)_2$.

      a) $(10)_2$
      b) $(11)_2$
      c) $(100)_2$
      d) $(101)_2$

   (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \div (111)_2$.

      a) $(11)_2$
      b) $(100)_2$
      c) $(101)_2$
      d) $(110)_2$

   (c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \div (1101)_2$.

      a) $(100)_2$
      b) $(110)_2$
      c) $(111)_2$
      d) $(1001)_2$

Part C: Number Bases - Hexadecimal

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   d) What is the next highest hexadecimal number?

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- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$
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  - (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means "numbers" in German.)
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- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
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1. State which of the following sets the following numbers belong to.

   1) 18
   2) $8.2347\ldots$
   3) π
   4) $1.33333\ldots$
   5) $17/4$
   6) 4.25
   7) $\sqrt{\pi}$
   8) $\sqrt{25}$

   The possible answers are

   a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
   b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
   c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$
   d) Real Number $\mathbb{R}$

1.1.1 Irrational Numbers

An irrational number cannot be expressed as a ratio between two numbers and it cannot be written as a simple fraction because there is not a finite number of numbers when written as a decimal. Instead, the numbers in the decimal would go on forever, without repeating.

In mathematics, the cardinality of a set is a measure of the "number of elements of the set". For example, the set A = 2, 4, 6 contains 3 elements, and therefore A has a cardinality of 3.

Chapter 2

Set Theory

2.1 Set Theory

1. The Universal Set $\mathcal{U}$
2. Union
3. Intersection
4. Set Difference
5. Relative Difference

NOTATIONS FOR A SET:

A set can be represented by two methods: 1.ROSTER METHOD 2.BUILDER METHOD ROSTER METHOD:

In this method the elements of a set are separated by commas and are enclosed within curly brackets . For example:

$A = 1, 2, 3, 4, 5, 6$ is a set of numbers.

B = Sunny, Joy, Kartik, Harish, Girish is a set of names.

C = a, e, i, o, u is a set of vowels.

D = apple, banana, guava, orange, pear is a set of fruits.

Listing the elements in this way is called Roster method. In this method, it is not necessary for the elements to be listed in a particular order. The elements of the set can be written just plainly, separated by commas and in any order.

BUILDER METHOD:

This method is also called Property method. In Builder method, a set is represented by stating all the properties which are satisfied by the elements of that particular set only.

For example:

If A is a set of elements less than 0, then in Builder method it will be written as

$A = x : x < 0$, this statement is read as ”the set of all x such that x is less than 0”

If A is a set of all real numbers less than 7, then in Builder method it is written as $A = xR : x < 7$

Similarly,

$A = 2i : iisaninteger$ is a set of all even integers.

$A = xR : x \neq 2$ is a set of all real numbers except 2.

$A = xR : x > 3andx < 7$ is a set of real numbers greater than 3 but less than 7.

$A = xZ : x > 6$ is a set of integers greater than 6.

$A = xZ : 2x + 1$ is a set of all odd integers.

2.1.1 Proper subset definition.

A proper subset of a set A is a subset of A that is not equal to A. In other words, if B is a proper subset of A, then all elements of B are in A but A contains at least one element that is not in B. For example, if A=1,3,5 then B=1,5 is a proper subset of A.

- Let A and B be subsets of the a universal set U.
- Use membership tables to prove that $(A \cup B')' = A' \cap B$
- Shade the regions corresponding to this set on a Venn Diagram

A	B	B'	$A \cup B'$	$(A \cup B')'$
0	0	1	1	0
0	1	0	0	1
1	0	1	1	0
1	1	0	1	0

A	B	A'	$A' \cap B$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	0	0

Given the universal set U and subsets A and B, list the set $(A \cup B')'$

- $U = \{1, 2, \ldots, 8, 9\}$
- $A = \{2, 4, 6, 8\}$
- $B = \{4, 5, 6, 7\}$
- $B' = \{1, 2, 3, 8, 9\}$
- $A \cup B' = \{1, 2, 3, 4, 6, 8, 9\}$
- $(A \cup B')' = \{5, 7\}$

2.1.2 Number Sets

Blackboard Bold Typeface

- Conventionally the symbols for numbers sets are written in a special typeface, known as **blackboard bold**.
- Examples : $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$.

Natural Numbers ($\mathbb{N}$)

- The whole numbers from 1 upwards.
- The set of natural numbers is
$$\{1, 2, 3, 4, 5, 6, \ldots\}$$
- In some branches of mathematics, 0 might be counted as a natural number.
$$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

2.1.3 Number Sets

Integers ($\mathbb{Z}$)

- The integers are all the whole numbers, all the negative whole numbers and zero.
- The set of integers is
$$\{\ldots, -4, -3, -2, -1,\ 0,\ 1,\ 2,\ 3, \ldots\}$$
- The notation $\mathbb{Z}$ is from the German word for numbers: *Zahlen*.
- All natural numbers are integers.
$$\mathbb{Q} \subset \mathbb{Z}$$

2.1.4 Number Sets

Integers ($\mathbb{Z}$)

- Natural numbers may also be referred to as positive integers, denoted $\mathbb{Z}^+$.
  (note the superscript)
- Negative integers are denoted $\mathbb{Z}^-$.
  $$\{\ldots, -4, -3, -2, -1\}$$

2.1.5 Number Sets

Integers ($\mathbb{Z}$)

- 0 is neither positive nor negative. The following set of non-negative numbers
  $$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$$
  might be denoted $0 \cup \mathbb{Z}^+$
- $\cup$ is the mathematical symbol for **union**.

2.1.6 Number Sets

Rational Numbers ($\mathbb{Q}$)

- Rational numbers, also known as quotients, are numbers you can make by dividing one integer by another (but not dividing by zero).
- If a number can be expressed as one integer divided by another, it is a rational number.
  $$\mathbb{Q} = \left\{ \frac{p}{q} \middle| p \in \mathbb{Z},\ q \in \mathbb{Z},\ q \neq 0 \right\}$$

2.1.7 Number Sets

Rational Numbers ($\mathbb{Q}$)

- All integers are rational numbers
  $$\mathbb{Z} \subset \mathbb{Q}$$
  (and by extension all natural numbers are rational numbers too)
- Examples of rational numbers
  $$9500,\ 7,\ \frac{1}{2},\ \frac{3}{7},\ -2.6,\ 0.001$$

2.1.8 Number Sets

Irrational Numbers

- A number that can not be written as the ratio of two integers is known as an irrational number.
- Two famous examples of irrational numbers are π and $\sqrt{2}$.

$$\pi = 3.141592\ldots$$

$$\sqrt{2} = 1.41421\ldots$$

Real Numbers ($\mathbb{R}$)

- Irrational numbers are types of real numbers.
- Rational numbers are real numbers too.

$$\mathbb{Q} \subset \mathbb{R}$$

- A real number is simply any point anywhere on the number line.

Real Numbers ($\mathbb{R}$)

- There are numbers that are not real numbers, for example **imaginary numbers**, but we will not cover them in this presentation.

2010 Zone B Q 1
5n+1 Rules of Inclusion method
$A = \{5n + 1 : n \in Z\}$

2011 Zone A question 1d

Showing your workings, express the repeating decimal 0.012012012012... as a rational number in its simplest form.

- x = 0.012012012012...
- 10x = 0.12012012012... (not particularly useful)
- 100x = 1.2012012012... (not particularly useful either)
- 1000x= 12.012012012... (very useful)
- 999x = 12
- x= 12/999 = 4/333 (Answer!)

2008 Zone A question2a

$B = \{3n - 1 : n \in Z^+\}$ Describe the set B using the listing method

- Let $n = 1$. Consequently $3(1) - 1 = 2$
- Let $n = 2$. Likewise $3(2) - 1 = 5$
- Let $n = 3$. $3(3) - 1 = 8$
- The repeated differences are 3. The next few values are 11, 14 and 17
- So by the listing method $B = \{2, 5, 8, 11, 14, 17, \ldots\}$

$A = \{3, 5, 7, 9, ldots\}$ Describe the set A using the rules of inclusion method

- The repeated differences are 2.
- We can say the rule has the form $2n + k$
- For the first value n=1. Therefore $2 + k = 3$
- Checking this , for the second value , n=2. Therefore $4 + k = 5$
- Clearly k = 1.
- $A = \{2n + 1 : n \in Z^+\}$
- So by the listing method $B = \{2, 5, 8, 11, 14, 17, \ldots\}$

2.1.9 Cartesian Product

- Let X and Y be sets.
- The **cartesian product** $X \times Y$ is the set whose elements are **all** of the ordered pairs of elements (x, y) where $x \in X$ and $y \in Y$.
- Let $X = \{a, b, c\}$
- Let $Y = \{0, 1\}$
- The cartesian product $X \times Y$ is therefore:
- Importantly $X \times Y \neq Y \times X$
- Recall: Let $X = \{a, b, c\}$ and let $Y = \{0, 1\}$
- The cartesian product $Y \times X$ is therefore:

2.1.10 The Cartesian Product

Exercises

Discrete Maths

A binary relation on a set A is the collection of ordered pairs of elements of A. In other words, it is the subset of the cartesian product A2= AA

Cartesian Product

This is a direct product pf 2 sets

XY = (x,y)— xXandyY

4 suits of cards and 13 Ranks, therefore 52 element cartesian prodcut.

N.B AB BA A=A =

Cartesian product is not associative

Worked Example

Suppose that the Universal Set $\mathcal{U}$ is the set of integers from 1 to 9.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and that the set $\mathcal{A}$ contains the prime numbers between 1 to 9 inclusive.

$$\mathcal{A} = \{1, 2, 3, 5, 7\},$$

and that the set $\mathcal{B}$ contains the even numbers between 1 to 9 inclusive.

$$\mathcal{B} = \{2, 4, 6, 8\}.$$

Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}'$ and $\mathcal{B}'$

$$\mathcal{A}' = \{4, 6, 8, 9\},$$

$$\mathcal{B}' = \{1, 3, 5, 7, 9\},$$

Intersection

- Intersection of two sets describes the elements that are members of both the specified Sets
- The intersection is denoted $\mathcal{A} \cap \mathcal{B}$

$$\mathcal{A} \cap \mathcal{B} = \{2\}$$

- only one element is a member of both A and B.

Set Difference

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

Chapter 3

graph theory

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

3.1 Set Theory

1. The Universal Set $\mathcal{U}$
2. Union
3. Intersection
4. Set Difference
5. Relative Difference

Set theory

3.1.1 Specifying Sets (2.1)

1. Listing Method
2. Rules of Inclusion

Subsets (2.2) Subsets and Proper Subsets Cardinality of a Set Power Set

3.1.2 Set operations (2.3)

complement union intersection set difference

venn diagrams 8 Disjoint Regions

DE Muorgan's Laws (Useful for Propositions)

membership tables

proof by truth tables AND OR NOT Set difference symmetric difference

Harmonic Mean

$$H_x = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

2,4,6,8 ¿ 1/mean(1/a) [1] 3.84 ¿ 1/a [1] 0.5000000 0.2500000 0.1666667 0.1250000 ¿ sum(1/a) [1] 1.041667 ¿ sum(1/a)*24 [1] 25 ¿ 96/25 [1] 3.84

3.1.3 Functions

Consider the floor function $f : R \to Z$ given the rule

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

1. evaluate $f(6)$ and $f(-6)$
2. Show that f(x) is not one-to-one
3.

3.1.4 Functions

Evaluate $f(6)$

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

$$f(6) = \lfloor \frac{\mathbf{6}+1}{2} \rfloor = \lfloor \frac{7}{2} \rfloor$$

$$\lfloor 3.5 \rfloor = 3$$

3.1.5 Functions

Evaluate $f(-6)$

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

$$f(6) = \lfloor \frac{\mathbf{-6}+1}{2} \rfloor = \lfloor \frac{-5}{2} \rfloor$$

$$\lfloor -2.5 \rfloor = -3$$

3.1.6 Discrete Maths : Relations

- A relation R from a set A to a set B is a subset of the **cartesian product** A x B.
- Thus R is a set of **ordered pairs** where the first element comes from A and the second element comes from B i.e. (a, b)

3.1.7 Discrete Maths : Relations

- If $(a, b) \in R$ we say that a is related to b and write aRb.
- If $(a, b) \notin R$, we say that a is not related to b and write aRb. CHECK
- If R is a relation from a set A to itself then we say that “R is a relation on A”.

3.1.8 Discrete Maths : Relations

Example

- Let $A = \{2, 3, 4, 6\}$ and $B = \{4, 6, 9\}$
- Let R be the relation from A to B defined by $\boldsymbol{xRy}$ if x divides y exactly.

3.1.9 Discrete Maths : Relations

Example

- Let $A = \{2, 3, 4, 6\}$ and $B = \{4, 6, 9\}$
- Let R be the relation from A to B defined by $\boldsymbol{xRy}$ if x divides y exactly.
- Then

$$R = (2, 4), (2, 6), (3, 6), (3, 9), (4, 4), (6, 6)$$

3.2 Sets of Numbers

- $\mathbb{Z}$ Set of all integers
- $\mathbb{Q}$ Set of all rational numbers
- $\mathbb{R}$ Set of all real numbers
- $\mathbb{Z}^+$ Set of all positive integers
- $\mathbb{Z}^-$ Set of all negative integers
- $\mathbb{R}^+$ Set of all positive real numbers
- $\mathbb{R}^-$ Set of all negative real numbers

3.3 Arrow Diagrams

- Domain
- Co-Domain
- Range

$$f(x) : \text{Domain} \rightarrow \text{Co-Domain}$$
$$f(x) : \mathbb{R} \rightarrow \mathbb{R}$$

Polynomial Functions (4.1.5)

Constants (P_0)

Linear Functions (P_1)

Quadratic Functions (P_2)

Cubic Functions (P_3)

Equality of Functions (4.1.6)

$$f(x) = g(x)$$

3.4 Special Mathematical Functions

3.4.1 Mathematical Operators

- The Square Root function
- The Floor and Ceiling functions
- The Absolute Value functions
- Root Functions
- Absolute Value Function
- Floor Function
- Ceiling Function

$$\lfloor 3.14 \rfloor = 3 \tag{3.1}$$
$$\lceil -4.5 \rceil = -5 \tag{3.2}$$
$$|-4| = 4 \tag{3.3}$$

For this course, only positive numbers have square roots. The square roots are positive numbers. (This statement is not strictly true. The square root of a negative number is called a complex number. However this is not part of the course).

Negative numbers can have cube roots

$$-27 = -3 \times -3 \times -3$$

$$\sqrt[3]{-27} = -3$$

3.5 Exponential and Logarithms

Laws of Logarithms

- Law 1 : Multiplcation of Logarithms
$$Log(a) \times Log(b) = Log(a + b)$$
- Law 2 : Division of Logarithms
$$\frac{Log(a)}{Log(b)} = Log(a - b)$$
- Law 3 : Powers of Logarithms
$$Log(a^b) = b \times Log(a)$$

3.5.1 Exercise

$h(x) : \mathbb{R} \to \mathbb{R}$ $g(x) : \mathbb{R} \to \mathbb{R}$

$$f(x) = sqrt(x)$$
$$g(x) = \sqrt{3}x + 2$$
$$h(x) = 2^x$$

- Is the function $h(x)$ an *onto* function?
- determine the inverse function of $h(x)$ and $g(x)$
- Simplify the following function.
$$j(x) = \log_4(h(6x))$$

3.5.2 Onto Functions

Definition: If every element in the co-domain of the function has an ancestor, the function is said to be "onto". An onto function has the property that the domain is equal to the co-domain.

Example 4.26 Page 53

Exponential and Logarithms

Rules

Expontials : Rules 4.18 Page 58
Logarithms : Rules 4.23 Page 61

$$log_a(a) = 1$$
$$log_a(b^c) = c \times log_a(b)$$

- $log_2(128) = 7$

- $log_2(1/4) = -2$
- $log_2(2) = 1$

$$log_a(b) = \frac{log_x(b)}{log_x(a)}$$

3.5.3 Logarithms

- Laws of Logarithms - Change of Base

$$\text{Log}_b(x) = a$$
$$b^a = x$$
$$\text{Log}_2(8) = 3$$
$$2^3 = 8$$

$$\text{Log}_b(x) \times \text{Log}_b(y) = \text{Log}_b(x + y)$$
$$\text{Log}_b(x^y) = y \times \text{Log}_b(x)$$

$$\text{Log}_y(x) = \frac{\text{Log}_b(x)}{\text{Log}_b(y)}$$

3.5.4 Exponents

- Rules of Exponents

$$(a^b)^c = a^{b\times c}$$

$$64^{2/3} = (4^3)^{2/3} = 4^{3\times 2/3} = 4^2 = 16$$

$$(a^b) \times (a^c) = a^{b+c}$$
$$(3^2) \times (3^3) = 3^{2+3} = 3^5 = 243$$

Exercises

(a) Complete the following table for the functions

i) $g(x) = \log_3 x$,
ii) $h(x) = \sqrt[3]{x}$.

x	1				81	
$g(x)$		1	2			5
$h(x)$				3.00		

Express your answers to 2 decimal places only.

3.6 *One-to-One* Functions and *Onto* Functions

3.6.1 Invertible Functions

- One-to-One Function
- Onto Function

Onto Functions : Range and Co-Domain are equivalent

3.6.2 Inverting a Function

- You are given $f(x)$ in terms of x
- Re-arrange the equation so that x is given in terms of $f(x)$
- Replace x with $f^{-1}(x)$ and $f(x)$ with x

Example

- Determine the inverse function of $f(x)$. Re-arrange the equation so that x is given in terms of $f(x)$

$$f(x) : \mathbb{R} \to \mathbb{R} \ f(x) = \sqrt{x+1}$$

- Square both sides of the equation.

$$[f(x)]^2 = x + 1$$

- Subtract 1 from both sides of the equation. We have the equation written in terms of x.

$$f(x)^2 - 1 = x$$

- Replace x with $f^{-1}(x)$ and $f(x)$ with x

$$x^2 - 1 = f^{-1}(x)$$

- Re-arrange equation and specify domain and co-domain.

$$f(x) : \mathbb{R} \to \mathbb{R} \ f^{-1}(x) = x^2 - 1$$

3.7 Big O-Notation

(b) Let S be the set of all 4 bit binary strings.

The function $f : S \to \mathbb{Z}$ is defined by the rule:

$$f(x) = \text{the number of zeros in x}$$

for each binary string $x \in S$.
Find:

1. the number of elements in the domain
2. $f(1000)$
3. the set of pre-images of 1
4. the range of f.

(c)

4.a $\lfloor x - y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$

4.b

4.c

3.8 Section 4 Functions

3.8.1 Invertible Functions

A function is invertible if it fulfils two criteria

- The function is ***onto***,
- The function is ***one-to-one***.

State the conditions to be satisfied by a function $f : X \leftarrow Y$ for it to have an inverse function $f^{-1} : Y \leftarrow X$. $\lceil \frac{x^2+1}{4} \rceil$ where $f : A \rightarrow \mathbf{Z}$

(i) Find $f(4)$ and the ancestors of 3.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer

Given $f : \mathbf{R} \rightarrow \mathbf{R}$ where f(x) =3x-1,define fully the inverse of the function f ,i.e.f^{-1}. State the value of $f^{-1}(2)$

3.8.2 Precision Functions

- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
- Floor Function $\lfloor x \rfloor$

Question1.2: State the range and domain of the following function

$$F(x) = \lfloor x - 1 \rfloor$$

3.8.3 Powers

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

3.8.4 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

3.8.5 Logarithmic Functions

Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$
3. $\log_b(X^Y)$= $Y \log_b(X)$

Question1.3 Compute the Logarithm of the following

- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$
- $a^x = y \ log_a(y) = x$
- $e^x = y \ ln(y) = x$
- $log_a(x \times y) = log_a(x) + log_a(y)$
- $log_a(\frac{x}{y}) = log_a(x) - log_a(y)$
- $log_a(\frac{1}{x}) = -log_a(x)$
- $log_a(a) = 1$
- $log_a(1) = 0$
- $\lceil x \rceil$
- $\lfloor x \rfloor$

Sample value x	Floor $\lfloor x \rfloor$	Ceiling $\lceil x \rceil$	Fractional part $\{x\}$
12/5 = 2.4	2	3	2/5 = 0.4
2.7	2	3	0.7
-2.7	-3	-2	0.3
-2	-2	-2	0

Set Theory

1.1 Introduction

1.2 Sets

1.3 Sub-sets

1.4 The order of sets: finite and infinite sets .

1.5 Union and intersection of sets

1.6 Differences and complements

1.7 Venn diagrams

1.8 Logic analysis

Union and intersection of sets

- The **union** of two sets A and B is a set containing all the elements in either A or B (or both) i.e.

$$A \cup B = x/x \in A \text{ or } x \in B.$$

- The **intersection** of two sets A and B is a set containing all the elements that are both in A and B i.e.

$$A \cap B = x/x \in A \text{ and } x \in B$$

.

- If sets A and B have no elements in common, i.e. $A \cap B = \emptyset$,then A and B are termed **disjoint sets**.

Subsets

- Proper Subsets

The Power Set

Venn Diagrams

IMAGE

Chapter 4

Logic

4.1 Logic Proposition

Let p, Q and r be the following propositions concerning integers n:

- p : n is a factor of 36 (2)
- q : n is a factor of 4 (2)
- r : n is a factor of 9 (3)

n	**p**	**q**	**r**
1	1	1	1
2	1	0	1
3	0	1	1
4	1	0	1
6	0	0	1
9	0	1	1
12	0	0	1
18	0	0	1
36	0	0	1

For each of the following compound statements, express it using the propositions P q and r, andng logical symbols, then given the truth table for it,

1) If n is a factor of 36, then n is a factor of 4 or n is a factor of 9
2) If n is a factor of 4 or n is a factor of 9 then n is a factor of 36

Part 1 : Logic

1.1 2010 Question 3

Let $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ and let p, q be the following propositions concerning the integer $n \in S$.

- p: n is a multiple of two. (i.,18e. $\{10, 12, 14, 16, 18\}$)
- q: n is a multiple of three. i.e. $\{12, 15, 18\}$

For each of the following compound statements find the sets of values n for which it is true.

- $p \vee q$: (p or q : 10 12 14 15 16 18)
- $p \wedge q$: (p and q: 12 18)
- $\neg p \oplus q$: (not-p or q, but not both)
  - $\neg p$ not-p = $\{1113151719\}$
  - $\neg p \vee q$ not-p or q $\{11121315171819\}$
  - $\neg p \wedge q$ not-p and q $\{15\}$
  - $\neg p \oplus q = \{11, 12, 13, 17, 18, 19\}$

1.2 2010 Question 3

Let p and q be propositions. Use Truth Tables to prove that

$$p \to q \equiv \neg q \to \neg$$

Important Remember to make a comment at the end to say why the table proves that the two statements are logically equivalent. e.g. *since the columns are identical both sides of the equation are equivalent.*

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

(Key “difference” is first and last rows)

1.3 Membership Tables for Laws

Page 44 (Volume 1) Q8. Also see Section 3.3 Laws of Logic.

Construct a truth table for each of the following compound statement and hence find simpler propositions to which it is equivalent.

- $p \vee F$
- $p \wedge T$

Solutions

p	T	$p \vee T$	$p \wedge T$
0	1	1	0
1	1	1	1

- Logical OR: $p \vee T = T$
- Logical AND: $p \wedge T = p$

p	F	$p \vee F$	$p \wedge F$
0	0	0	0
1	0	1	0

- Logical OR: $p \vee F = p$
- Logical AND: $p \wedge F = F$

1.4 Propositions

Page 67 Question 9 Write the contrapositive of each of the following statements:

- If n= 12, then n is divisible by 3.
- If n=5, then n is positive.
- If the quadrilateral is square, then four sides are equal.

Solutions

- If n is not divisible by 3, then n is not equal to 12.
- If n is not positive, then n is not equal to 5.
- If the four sides are not equal, then the quadrilateral is not a square.

1.5 Truth Sets

2009

Let $n = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p, q be the following propositions concerning the integer n.

- p: n is even,
- q: $n \geq 5$.

By drawing up the appropriate truth table nd the truth set for each of the propositions $p \vee \neg q$ and $\neg q \to p$

n	p	q	$\neg q$	$p \vee \neg q$
1	0	0	1	1
2	1	0	1	0
3	0	0	1	1
4	1	0	1	0
5	0	1	0	1
6	1	1	0	1
7	0	1	0	1
8	1	1	0	1
9	0	1	0	1

Truth Set $= \{1, 3, 5, 6, 7, 8, 9\}$

n	p	q	$q \to p$	$q \to p$
1	0	0	1	0
2	1	0	1	0
3	0	0	1	0
4	1	0	1	0
5	0	1	0	1
6	1	1	1	0
7	0	1	0	1
8	1	1	1	0
9	0	1	0	1

Truth Set $= \{5, 7, 9\}$

1.6 Biconditional

See Section 3.2.1.

Use truth tables to prove that $\neg p \leftrightarrow \neg q$ is equivalent to $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

1.7 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $\neg(p\wedge \not q)$ (i.e. $\equiv p \rightarrow q$).

Logic Gates

- AND
- OR
- NOT

Examiner's Comments:*Many diagrams were carefully and clearly drawn and well labelled, gaining full marks. The logic table was also well done by most, but there were a few marks lost in the final part by failing to deduce that since the columns of the table are identical the expressions are equivalent.*

1.8 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $(p \wedge q) \vee \neg q$ (i.e. $\equiv p \rightarrow q$).

Important Label each of the gates appropriately and label the diagram with a symblic expression for the output after each gate.

Prepositional Logic

4.1.1 five basic connectives

Reflexive, Symmetric and Transitive

- Reflexive
- Symmetric
- Transitive

4.1.2 Logarithms

Here we assume x and y are positive real numbers. 1. $loga(xy) = loga(x) + loga(y)$ 2. $loga(x/y) = loga(x) - loga(y)$ 3. $loga(x^r) = rloga(x)$ for any real number r. Invertible Functions

4.1.3 Proof by Induction

Another sequence is defined by the recurrence relation un = un-1+2n-1 and u1 = 1. (i) Calculate u2, u3, u4 and u5. 1,4,9,16,25

(ii) Prove by induction that $u_n = n^2$ for all $n \geq 1$

Exponentials

4.2 Laws of Exponents

Here are the Laws (explanations follow):

LawExample x1 = x61 = 6 x0 = 170 = 1 x-1 = 1/x4-1 = 1/4 xmxn = xm+nx2x3 = x2+3 = x5 xm/xn = xm-nx6/x2 = x6-2 = x4 (xm)n = xmn(x2)3 = x23 = x6 (xy)n = xnyn(xy)3 = x3y3 (x/y)n = xn/yn(x/y)2 = x2 / y2 x-n = 1/xnx-3 = 1/x3 Z Score

$$Z = \frac{X - \mu}{\sigma}$$

4.2.1 Exercises

Showing your workings, use the rules of indices and logarithms to give the following two expression in their simplest form.

- **Exercise 1**

$$4 \cdot 2^x - 2^{x+1}$$

- **Exercise 2**

$$\frac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$$

4.2.2 Exercise 1

$$4 \cdot 2^x - 2^{x+1}$$

Remarks:
(looking at the second term)

1 Using the following rule

$$a^b \cdot a^c = a^{(b+c)}$$

2 Using this rule in reverse we can say

$$2^{x+1} = 2^x \cdot 2^1 = 2 \cdot (2^x)$$

$$4 \cdot 2^x - 2^{x+1} \;=\; (4 \cdot 2^x) - (2 \cdot 2^x)$$

4.2.3 Exercise 1

Remarks:

3 This expression is in the form

$$(a \cdot b) - (c \cdot b)$$

which can be re-expressed as follows

$$(a - c\sqrt{b}) \cdot b$$

$$\begin{aligned}(4 \cdot 2^x) - (2 \cdot 2^x) &= (4 - 2) \cdot 2^x \\ &= 2 \cdot 2^x = 2^{x+1}\end{aligned}$$

4.2.4 Exercise 2

$$\frac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$$

Useful Rule of Logarithms

$$\ln(a^b) = b \cdot \ln(a)$$

$$\frac{\ln(2) + 2 \cdot \ln(2) + 3 \cdot \ln(2) + 4 \cdot \ln(2) + 5 \cdot \ln(2)}{\ln(4)}$$

4.2.5 Exercise 2

Adding up all the terms in the numerator

$$\frac{1 \cdot \ln(2) + 2 \cdot \ln(2) + 3 \cdot \ln(2) + 4 \cdot \ln(2) + 5 \cdot \ln(2)}{\ln(4)}$$

$$= \frac{15 \cdot \ln(2)}{\ln(4)}$$

4.2.6 Exercise 2

Our expression has now simplified to

$$\frac{15 \cdot \ln(2)}{\ln(4)}$$

We can simplify the denominator too

$$\ln(4) = \ln(2^2) = 2 \cdot \ln(2)$$

4.2.7 Exercise 2

Our expression has now simplified to

$$\frac{15 \cdot \ln(2)}{\ln(4)} = \frac{15 \cdot \ln(2)}{2 \cdot \ln(2)}$$

We can divide above and below by $\ln(2)$ to get our final answer

$$\frac{15 \cdot \ln(2)}{2 \cdot \ln(2)} = \frac{15}{2} = 7.5$$

Prepositional Logic

4.3 Section 3 Logic

4.3.1 Logical Operations

- $\neg p$ the negation of proposition p.
- $p \wedge q$ Both propositions p and q are simultaneously true (Logical State AND)
- $p \vee q$ One of the propositions is true, or both (Logical State : OR)
- $p \otimes q$ Only one of the propositions is true (Logical State : exclusive OR (i.e XOR)

p	q	$p \vee q$	$q \wedge p$	$p \otimes q$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

4.4 Conditional Connectives

Construct the truth table for the proposition $p \to q$.

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

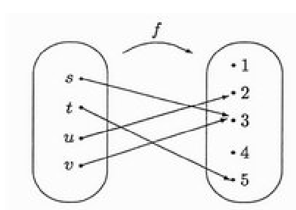
Question 1

(b) Express the following hexadecimal number as a decimal number: (A32.8)16. [3]

(c) Convert the following decimal number into base 2, showing all your working: $(253)_{10}$. [2]

(d) Express the recurring decimal $0.4242424\ldots$ as a rational number in its simplest form. [2]

Question 4



Question 6

Let $\mathcal{S}$ be a set and let $\mathcal{R}$ be a relation on $\mathcal{S}$ Explain what it means to say that $\mathcal{R}$ is

(i) reflexive

(ii) symmetrix

(iii) anti-symmetric

(iv) Transitive

Question 10

(a) Given the following adjacency matrices A and B where

(i) Say whether or not the graphs they represent are isomorphic. (ii) Calculate A2 and A4 and say what information each gives about the graph corresponding to A. [6] (b) (i) Write down the augmented matrix for the following system of equations.

$$2x + y - z = 2$$

$$x - y + z = 4$$

$$x + 2y + 2z = 10$$

(ii) Use Gaussian elimination to solve the system. [4]

Dice Rolls

Consider rolls of a die. What is the universal set?

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$$

symbols

$\varnothing$, $\forall$, $\in$, $\notin$, $\cup$

Prepositional Logic

- $p \wedge q$
- $p \vee q$
- $p \rightarrow q$

Chapter 5

Sequences and Series, and Proof by Induction

5.1 Sequence and Series and Proof by Induction

$$\sum (n^2)$$

Relative Difference

- $A \otimes B$

Power Sets

- Consider the set A where $A = \{w, x, y, z\}$
- There are 4 elements in set A.
- The power set of A contains 16 element data sets.
-

$$\mathcal{P}(A) = \{\{x\}, \{y\}\}$$

- (i.e. 1 null set, 4 single element sets, 6 two -elemnts sets, 4 three lement set and one 4- element set.)

- $p \rightarrow q$ p implies q
- $p \lg q$

Relative Difference

- $A \otimes B$

Power Sets

- Consider the set A where $A = \{w, x, y, z\}$
- There are 4 elements in set A.
- The power set of A contains 16 element data sets.
- $$\mathcal{P}(A) = \{\{x\}, \{y\}\}$$
- (i.e. 1 null set, 4 single element sets, 6 two -elemnts sets, 4 three lement set and one 4- element set.)

- $p \to q$ p implies q
- $p \lg q$

Dice Rolls

Consider rolls of a die. What is the universal set?

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$$

Worked Example

Suppose that the Universal Set $\mathcal{U}$ is the set of integers from 1 to 9.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and that the set $\mathcal{A}$ contains the prime numbers between 1 to 9 inclusive.

$$\mathcal{A} = \{1, 2, 3, 5, 7\},$$

and that the set $\mathcal{B}$ contains the even numbers between 1 to 9 inclusive.

$$\mathcal{B} = \{2, 4, 6, 8\}.$$

Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}'$ and $\mathcal{B}'$

$$\mathcal{A}' = \{4, 6, 8, 9\},$$
$$\mathcal{B}' = \{1, 3, 5, 7, 9\},$$

Intersection

- Intersection of two sets describes the elements that are members of both the specified Sets
- The intersection is denoted $\mathcal{A} \cap \mathcal{B}$

$$\mathcal{A} \cap \mathcal{B} = \{2\}$$

- only one element is a member of both A and B.

Set Difference

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

symbols

$\varnothing$, $\forall$, $\in$, $\notin$, $\cup$

Chapter 6

Session 1

Part 1 : Binary numbers

Question 1

Working in base 2 and showing all your workings, compute the following. $(10110)_2 \times (111)_2$

- Express the binary number $(1101.101)_2$ as a decimal, showing all your workings
- Express the decimal number $(3599)_{10}$ in base 2

(a) Express the following binary numbers as decimal numbers

  (i) 11011
  (ii) 100101

(b) Express the following decimal numbers as binary numbers

  (i) 6
  (ii) 15
  (iii) 37

(c) Perform the following binary additions

  (i) $1011 + 1111$
  (ii) $10101 + 10011$
  (iii) $1010 + 11010$

Part 1b : Hexadecimal numbers

(i) Calculate the decimal equivalent of the hexadecimal number $(A2F.D)_{16}$

(ii) Working in base 2, compute the following binary additions, showing all you workings

$$(1110)_2 + (11011)_2 + (1101)_2$$

(iv) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part E: Miscellaneous Questions

(i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation

(ii) From part (i), is x^8 a rational number?

Part A : Binary numbers

(a) Express the following binary numbers as decimal numbers

(i) 11011

(ii) 100101

(b) Express the following decimal numbers as binary numbers

(i) 6

(ii) 15

(iii) 37

(c) Perform the following binary additions

(i) $1011 + 1111$

(ii) $10101 + 10011$

(iii) $1010 + 11010$

Part A: Number Systems - Binary Numbers

1. Express the following decimal numbers as binary numbers.

   i) $(73)_{10}$ ii) $(15)_{10}$ iii) $(22)_{10}$

   All three answers are among the following options.

   a) $(10110)_2$ b) $(1111)_2$ c) $(1001001)_2$ d) $(1000010)_2$

2. Express the following binary numbers as decimal numbers.

   a) $(101010)_2$ b) $(10101)_2$ c) $(111010)_2$ d) $(11010)_2$

3. Express the following binary numbers as decimal numbers.

   a) $(110.10101)_2$ b) $(101.0111)_2$ c) $(111.01)_2$ d) $(110.1101)_2$

4. Express the following decimal numbers as binary numbers.

   a) $(27.4375)_{10}$ b) $(5.625)_{10}$ c) $(13.125)_{10}$ d) $(11.1875)_{10}$

Part B: Number Systems - Binary Arithmetic

1. Perform the following binary additions.

    a) $(110101)_2 + (1010111)_2$
    b) $(1010101)_2 + (101010)_2$
    c) $(11001010)_2 + (10110101)_2$
    d) $(1011001)_2 + (111010)_2$

2. Perform the following binary subtractions.

    a) $(110101)_2$ - $(1010111)_2$
    b) $(1010101)_2$ - $(101010)_2$
    c) $(11001010)_2$ - $(10110101)_2$
    d) $(1011001)_2$ - $(111010)_2$

3. Perform the following binary multiplications.

    a) $(1001)_2 \times (1000)_2$
    b) $(101)_2 \times (1101)_2$
    c) $(111)_2 \times (1111)_2$
    d) $(10000)_2 \times (11001)_2$

4. Perform the following binary multiplications.

    (a) Which of the following binary numbers is the result of this binary division: $(10)_2 \times (1101)_2$.

    a) $(11010)_2$
    b) $(11100)_2$
    c) $(10101)_2$
    d) $(11011)_2$

    (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \times (111)_2$.

    a) $(11000)_2$
    b) $(11001)_2$
    c) $(10101)_2$
    d) $(11011)_2$

    (c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \times (1101)_2$.

    a) $(11000)_2$
    b) $(11001)_2$
    c) $(10101)_2$
    d) $(11011)_2$

5. Perform the following binary divisions.

    (a) Which of the following binary numbers is the result of this binary division: $(111001)_2 \div (10011)_2$.

    a) $(10)_2$
    b) $(11)_2$
    c) $(100)_2$
    d) $(101)_2$

    (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \div (111)_2$.

a) $(11)_2$

b) $(100)_2$

c) $(101)_2$

d) $(110)_2$

(c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \div (1101)_2$.

a) $(100)_2$

b) $(110)_2$

c) $(111)_2$

d) $(1001)_2$

Part C: Number Bases - Hexadecimal

1. Answer the following questions about the hexadecimal number systems

   a) How many characters are used in the hexadecimal system?

   b) What is highest hexadecimal number that can be written with two characters?

   c) What is the equivalent number in decimal form?

   d) What is the next highest hexadecimal number?

2. Which of the following are not valid hexadecimal numbers?

   a) 73 b) $A5G$ c) 11011 d) EEF

3. Express the following decimal numbers as a hexadecimal number.

   a) $(73)_{10}$ b) $(15)_{10}$ c) $(22)_{10}$ d) $(121)_{10}$

4. Compute the following hexadecimal calculations.

   a) $5D2 + A30$ b) $702 + ABA$ c) $101 + 111$ d) $210 + 2A1$

Part D : Base 5 and Base 8 numbers

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

- (i) decimal number
- (ii) hexadecimal number
- (iii) binary number

(b) Perform the following binary additions

- (i) $1011 + 1111$
- (ii) $10101 + 10011$
- (iii) $1010 + 11010$

Part C : Base 5 and Base 8 numbers

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

- (i) decimal number
- (ii) hexadecimal number
- (iii) binary number

Part E: Natural, Rational and Real Numbers

1. State which of the following sets the following numbers belong to.

1) 18	3) π	5) 17/4	7) $\sqrt{\pi}$
2) 8.2347...	4) 1.33333...	6) 4.25	8) $\sqrt{25}$

The possible answers are

- a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
- b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
- c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$
- d) Real Number $\mathbb{R}$

- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$
- $\mathbb{Z}$: integers $\{-3, -2, -1, 0, 1, 2, 3, \ldots\}$
  - ∗ (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means “numbers” in German.)

- $\mathbb{Q}$: rational numbers
- $\mathbb{R}$: real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
  - (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

Exercises Real and Rational Numbers

(i) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part F : Scientific and Floating Point Notation

- Abscissa
- Exponent (power)

With floating point notation, the abscisa must be between 0 and 1. It is similar to scientific notation differing only by the fact that, with scientific notaiton, the abscissa is between 1 and 10.

- Floating Point Notation
- $pi \approx 0.31415 \times 10^1$
- $pi \approx 3.1415 \times 10^0$

Question 1

Part B : Hexadecimal numbers

(i) Calculate the decimal equivalent of the hexadecimal number $(A2F.D)_{16}$

(ii) Working in base 2, compute the following binary additions, showing all you workings

$$(1110)_2 + (11011)_2 + (1101)_2$$

(iv) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part D : Real and Rational Numbers

(i) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

(i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation.

(ii) From part (i), is x^8 a rational number?

Binary and Hex

1A.1 Coverting from Binomial to Decimal

1A.2 Converting to Decimal

1A.3 Priority of Operation

1A.4

Numbers

1B.1 Real Numbers

1B.2 Rational Numbers

1B.3 Floating Point Aritmetic

1B.4

Part 1. Number Systems

Section 1a. Binary Numbers

1. $1101001_{(2)}$
2. $1101001_{(2)}$
3. $1101001_{(2)}$

6.1 Inequality Operators

Given $x = \sqrt{2}$ determine whether the following statements are true or false:

(i) $x \leq 2$

(ii) $1.42 > x > 1.41$

(iii) x is a rational number

(iv) $\sqrt{2} = 2$

6.2 Revision Questions

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Mathematics for Computing

Session 2 : Set Theory

Mathematics for Computing

Session 3 : Logic

Mathematics for Computing

Digraphs and Relations

Question 1 : Binary numbers

(a) Express the following binary numbers as decimal numbers

(i) 101

(ii) 1101

(iii) 11011

(iv) 100101

(b) Express the following decimal numbers as binary numbers

(i) 6

(ii) 15

(iii) 37

(iv) 77

Question 2

A number is expressed in base 5 as $(234)_5$. What is it as decimal number? Suppose you multiply $(234)_5$ by 5. what would be the answer in base 5.

Question 3

Perform the following binary additions

(i) $1011 + 1111$

(ii) $10101 + 10011$

(iii) $1010 + 11010$

(iv) $101010 + 10101 + 101$

Question 4

Perform the binary additions

- $(10111)_2 + (111010)_2$
- $(1101)_2 + (1011)_2 + (1111)_2$

Question 5

Perform the binary subtractions using both the bit-borrowing method and the two's complement method.

- $(1001)_2 - (111)_2$
- $(110000)_2 - (10111)_2$

Question 6

Perform the binary multiplications

- $(1101)_2 \times (101)_2$
- $(1101)_2 \times (1101)_2$

Question 7

(a) What is highest Hexadecimal number that can be written with two characters, and what is it's equivalent in decimal form? What is the next highest hexadecimal number?

(b) Which of the following are not valid hexadecimal numbers?

(i) A5G
(ii) 73
(iii) EEF
(iv) 101

Question 8 : Binary Substraction

Exercises:

(i) 110 - 10
(ii) 101 - 11
(iii) 1001 - 11
(iv) 10001 - 100
(v) 101001 - 1101
(vi) 11010101-1101

Question 9

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

(i) decimal number
(ii) hexadecimal number
(iii) binary number

(b) Perform the following binary additions

(i) $1011 + 1111$
(ii) $10101 + 10011$
(iii) $1010 + 11010$

Question 10

Calculate working in hexadecimal

(i) $(BBB)_{16} + (A56)_{16}$

(ii) $(BBB)_{16} - (A56)_{16}$

Question 11

Write the hex number $(EC4)_{16}$ in binary. Write the binary number $(11110110101|)_2$ in hex.

Question 12

Express the decimal number 753 in binary , base 5 and hexadecimal.

Question 13

Express 42900 as a product of its prime factors, using index notation for repeated factors.

Question 14

Expresse the recuring decimals

(i) $0.727272\ldots$

(ii) 0.126126126....

(iii) 0.7545454545...

as rational numbers in its simplest form.

Question 15

Given that π is an irrational number, can you say whether $\frac{\pi}{2}$ is rational or irrational. or is it impossible to tell?

Question 16

(i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation

(ii) From part (i), is x^8 a rational number?

Question 17

(i) 5/7 lies between 0.714 and 0.715.

(ii) $\sqrt{(}2)$ is at least 1.41.

(iii) $\sqrt{(}3)$ 9s at lrast 1.732 and at most 1.7322.

Question 18

(i) Write down the numbers 0.0000526 in floating point form.

(ii) How is the number 1 expressed in floating point form.

Question 19

- Deduce that every composite integer n has a prime factor such that $p \leq \sqrt{n}$.
- Decide whether 899 is a prime.

Question 20

- What would be the maximum numbber of digits that a decimal fraction with denominator 13 could have in a recurring block in theory?
- Can you predict which other fractions with denominator 13 will have the same digits as 1/13 in their recurring block?

6.3 Video 6

Convert the following statements into symbols:

- $\sqrt{2}$ is less than 1.5 and greater than 1.4
- $\sqrt{2}$ is greater than or equal to 5

Chapter 7

Session 2

The Universal Set and the Empty Set

- The first is the ***universal set***, typically denoted U. This set is all of the elements that we may choose from. This set may be different from one setting to the next.
- For example one universal set may be the set of all real numbers, denoted $\mathbb{R}$, whereas for another problem the universal set may be the whole numbers $\{0, 1, 2, \ldots\}$.
- The other set that requires consideration is called the ***empty set***. The empty set is the unique set is the set with no elements. We write this as $\{\}$ and denote this set by $\emptyset$.

Number Sets

The font that the following symbols are written in (i.e. $\mathbb{N}$, $\mathbb{R}$) is known as ***blackboard font***.

- $\mathbb{N}$ Natural Numbers $(1, 2, 3, \ldots)$
- $\mathbb{Z}$ Integers $(-3, -2, -1, 0, 1, 2, 3, \ldots)$
  - $\mathbb{Z}^+$ Positive Integers
  - $\mathbb{Z}^-$ Negative Integers
  - 0 is not considered as either positive or negative.
- $\mathbb{Q}$ Rational Numbers
- $\mathbb{R}$ Real Numbers
- $\mathbb{C}$ Complex Numbers

Rules of Inclusion, Listing and Cardinality

For each of the following sets, a set is specified by the rules of inclusion method and listing method respectively. Also stated is the cardinality of that data set.

Worked example 1

- $\{x : x \text{ is an odd integer } 5 \leq x \leq 17\}$
- $x = \{5, 7, 9, 11, 13, 15, 17\}$
- The cardinality of set x is 7.

Worked example 2

- $\{y : y \text{ is an even integer } 6 \leq y < 18\}$
- $y = \{6, 8, 10, 12, 14, 16\}$
- The cardinality of set y is 6.

Worked example 3

A perfect square is a number that has a integer value as a square root. 4 and 9 are perfect squares ($\sqrt{4} = 2$, $\sqrt{9} = 3$).

- $\{z : z \text{ is an perfect square } 1 < z < 100\}$
- $z = \{4, 9, 16, 25, 36, 49, 64, 81\}$
- The cardinality of set z is 8.

Exercises

For each of the following sets, write out the set using the listing method. Also write down the cardinality of each set.

- $\{s : s \text{ is an negative integer } -10 \leq s \leq 0\}$
- $\{t : t \text{ is an even number } 1 \leq t \leq 20\}$
- $\{u : u \text{ is a prime number } 1 \leq u \leq 20\}$
- $\{v : v \text{ is a multiple of 3 } 1 \leq v \leq 20\}$

Power Sets

Worked Example

Consider the set Z:

$$Z = \{a, b, c\}$$

(i) How many sets are in the power set of Z?

(ii) Write out the power set of Z.

(iii) How many elements are in each element set?

Solutions to Worked Example

(i) There are 3 elements in Z. So there is $2^3 = 8$ element sets contained in the power set.

(ii) Write out the power set of Z.

$$\mathcal{P}(Z) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

(iii)
- One element set is the null set - i.e. containing no elements
- Three element sets have only elements
- Three element sets have two elements
- One element set contains all three elements
- 1+3+3+1=8

Exercise

For the set $Y = \{u, v, w, x\}$, answer the questions from the previous exercise

Complement of a Set

Consider the universal set U such that

$$U = \{2, 4, 6, 8, 10, 12, 15\}$$

For each of the sets A,B,C and D, specify the complement sets.

Set	Complement
$A = \{4, 6, 12, 15\}$	$A' = \{2, 8, 10\}$
$B = \{4, 8, 10, 15\}$	
$C = \{2, 6, 12, 15\}$	
$D = \{8, 10, 15\}$	

Set Operations

- Union ($\cup$) - also known as the **OR** operator. A union signifies a bringing together. The union of the sets A and B consists of the elements that are in either A or B.
- Intersection ($\cap$) - also known as the **AND** operator. An intersection is where two things meet. The intersection of the sets A and B consists of the elements that in both A and B.
- Complement (A' or A^c) - The complement of the set A consists of all of the elements in the universal set that are not elements of A.

Exercise

Consider the universal set U such that

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and the sets

$$A = \{2, 5, 7, 9\}$$

$$B = \{2, 4, 6, 8, 9\}$$

(a) $A - B$

(b) $A \otimes B$

(c) $A \cap B$

(d) $A \cup B$

(e) $A' \cap B'$

(f) $A' \cup B'$

Venn Diagrams

Draw a Venn Diagram to represent the universal set $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6\}$ with subsets:
$A = \{2, 4, 5\}$
$B = \{1, 4, 5, 6\}$

Find each of the following

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

(e) $A \otimes B$

(i) Describe the following set by the listing method

$$\{2r + 1 : r \in Z^{+} and r \leq 5\}$$

(ii) Let A,B be subsets of the universal set U.

Question 1

- $\{s :$ s is an odd integer and $2 \leq s \leq 10\}$
- $\{2m : m \in Z$ and $5 \leq m \leq 10\}$
- $\{2^t : t \in Z$ and $0 \leq t \leq 5\}$

Question 2

- $\{12, 13, 14, 15, 16, 17\}$
- $\{0, 5, -5, 10, -10, 15, -15, ...\}$
- $\{6, 8, 10, 12, 14, 16, 18\}$

Question 7 : Membership Tables

Using membership tables

A	B	C	x	y	z
0	0	0	1	1	1
0	0	1	0	0	1
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	1
1	1	1	1	0	1

(i) Draw a venn diagram to show three subsets A,B and C of a universal set U intersecting in the most general way?

(ii) How are sets X and Z related?

(iii) Can you describe each of the subsets X,Y and Z in terms of the sets A,B,C using the operations union intersection and set complement.

7.0.1 Ellipsis

When using Ellipsis, it should be clear what the pattern is

Three Sets

Propositional Logic A statement is a declarative sentence that is either true or false.

- $\tilde{q}$ not q
- $p \vee q$
- $p \wedge \tilde{q}$

Question 5
Let A, B be subsets of the universal set $\mathcal{U}$.
Use membership tables to prove De Morgan's Laws.

a. (1 mark) Write out the sample space for the outcomes for both players A and B.

b. (1 mark) Write out the sample space for the outcomes of C, where C is the difference of the two scores (i.e. B-A)

c. (1 mark) Are the sample points for the sample space of C equally probable? Provide a brief justification for your answer.

Section B: Set Operations

B.1 complement of A A'

B.2 Union $A \cup B$

B.3 Intersection $A \cap B$

B.4 Relative Difference $A \otimes B$

A.5

A.6

A.7

A.8

- Specifying Sets
- Listing Method
- Rules of Inclusion method
- Subsets Notation of a subset
- Cardinality of a set
- Power of a set

Operation on Sets

- The complement of Set
- Binary Operations
  - Union
  - Intersection
- Membership tables
- Laws for Combining Sets

Associative Laws

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distributive Laws

$$(A \cup B) \cap C = (A \cup B) \cap (A \cup C)$$
$$(A \cap B) \cup C = (A \cap B) \cup (A \cap C)$$

$$(A \cup B) \cap B'$$

Section C: Real and Rational Numbers

Formulae

- Bayes' Theorem:

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)}.$$

- Binomial probability distribution:

$$P(X = k) = {}^n C_k \times p^k \times (1-p)^{n-k} \qquad \left(\text{where} \qquad {}^nC_k = \frac{n!}{k!\,(n-k)!}.\right)$$

- Poisson probability distribution:

$$P(X = k) = \frac{m^k \mathrm{e}^{-m}}{k!}.$$

Part 2: $\{\frac{1}{n} : 1 < n < 4, n \in \mathbb{Z}\}$

7.2 Video 2 : Set Theory

Given the following sets

$\mathcal{U}$	$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$\mathcal{A}$	$\{1, 2, 5, 6, 8\}$
$\mathcal{B}$	$\{3, 5, 7, 8\}$
$\mathcal{C}$	$\{5, 6, 7, 8, 9\}$

Part 1: $\{10^m : -2 \leq m \leq 4, m \in \mathbb{Z}\}$

7.1 Video 1 : Set Theory Listing Method

Describe the following sets using the Listing Method

(i) $\{10^m : -2 \leq m \leq 4, m \in \mathbb{Z}\}$

(ii) $\{\frac{1}{n} : 1 < n < 4, n \in \mathbb{Z}\}$

List the elements of the following $A' \cap B$
$A' \cap C$

Venn Diagrams

Subsets of the universal set $\mathcal{U}$, intersecting in the most general way (Essentially this means - the venn diagram allows for all possible combinations of overlap.)

Question 2

HibCollWorkSheet2

$\in$ $\subset$

univeral Set $\mathcal{U}$ Laws for Binary Operations Membership Tables

De Morgan's Law

$$A' \cup B' = A \cap B$$

Part A : Builder Method

The following sets have been defined using the **Building Method** of notation. Re-write them by listing **some** of the elements.

1. $\{p|p$ is a capital city, p is in Europe$\}$
2. $\{x|x = 2n - 5$, x and n are natural numbers$\}$
3. $\{y|2y^2 = 50$, y is an integer$\}$
4. $\{z|z = n^3$, z and n are natural numbers$\}$

Part B : Sets

U = natural numbers; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. State whether each of the following is true or false:

(i) $A \subset U$

(ii) $B \subseteq A$

(iii) $\emptyset \subset U$

Question 2

Part A : Builder Method

The following sets have been defined using the **Building Method** of notation. Re-write them by listing **some** of the elements.

1. $\{p|p$ is a capital city, p is in Europe$\}$
2. $\{x|x = 2n - 5$, x and n are natural numbers$\}$

3. $\{y|2y^2 = 50$, y is an integer$\}$

4. $\{z|z = n^3$, z and n are natural numbers$\}$

Part B : Sets

U = natural numbers; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. State whether each of the following is true or false:

(i) $A \subset U$

(ii) $B \subseteq A$

(iii) $\emptyset \subset U$

Question 2

Describe the following set by the rules of inclusion method.

Describe the following set by the listing method the set of positive multiples of 3 which are less than 20.

Let A and B be subsets of univerisal set U

Use the membership rule to prove that

$$(A' \cap B)' = A \cup B'$$

shade the region corresponding to this set on a Venn Diagram

Given the universal set $\mathcal{U} = 1, 2, 3, 4, 5, 6, 7, 8, 9$ and the subsets $A = \{1, 3, 5, 7\}$ $B = \{6, 7, 8, 9\}$ list the set $A' \cap B)'$

(i) $\{5, 8\}$

(ii) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

7.3 Set Theory

A $\{2n : n \in \mathbb{Z}^+\}$

B $\{3, 6, 9, 12, 15, 18, \ldots\}$

Questions

(i) **A** is described by the rules of inclusion. Describe **A** with the listing method.

(ii) **B** is described by the listing method. Describe **B** with the rules of inclusion.

Question 4

$$\text{Log}_4 64 + \text{Log}_5 625 + \text{Log}_9 3$$

Question 5

1. Draw two non-isomorphic graphs with the following degree sequence.

$$4, 3, 3, 2, 2, 2, 2, 1, 1$$

2.

Question 7b

Compute the following summation

$$\sum_{i=25}^{i=100} i^2 + 3i - 5)$$

Section 2. Set Theory

1.
2.
3.

Membership Tables

For the following venn diagrams, complete the membership.

The events W, X,Y and Z correspond to the shaded areas in each of the venn diagrams.

7.3.1 logical implication

Logical implication is a type of relationship between two statements or sentences. The relation translates verbally into "logically implies" or "if/then" and is symbolized by a double-lined arrow pointing toward the right ($\rightarrow$). If A and B represent statements, then $A \rightarrow B$ means "A implies B" or "If A, then B." The word "implies" is used in the strongest possible sense.

As an example of logical implication, suppose the sentences A and B are assigned as follows:

A = The sky is overcast. B = The sun is not visible.

In this instance, A B is a true statement (assuming we are at the surface of the earth, below the cloud layer.) However, the statement B A is not necessarily true; it might be a clear night. Logical implication does not work both ways. However, the sense of logical implication is reversed if both statements are negated. That is,

$$(A \rightarrow B) \equiv (-B \rightarrow -A)$$

Using the above sentences as examples, we can say that if the sun is visible, then the sky is not overcast. This is always true. In fact, the two statements

A B and -B -A are logically equivalent.

Chapter 8

Session 3

Question 3

Part A : Propositions

Let p, q be the following propositions:

- p : this apple is red,
- q : this apple is ripe.

Express the following statements in words as simply as you can:

(i) $p \to q$

(ii) $p \wedge \neg q$.

Express the following statements symbolically:

(iii) This apple is neither red nor ripe.

(iv) If this apple is not red it is not ripe.

Part B : Logical Operations

Let $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p and q be the following propostions concerning the integers n.

p n is event

q $n < 5$

Find the values of n for which each of the following compound statement is true,

(i) $\neg p$

(ii) $p \wedge q$

(iii) $\neg p \vee q$

(iv) $p \oplus q$

Question 3

Let $\mathcal{S} = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$ $n \in \mathcal{S}$ p : n is a multiple of two q : n is a multiple of five

Express the following statement using logic symbols n is not a multiple of either 2 or 5.

List the elements of S which are in the truth set for the statement in (ii).

Write the contrapositive of the following statement concerning an integer n.

If the last digit of n is 4, then n is divisible by 3

8.1 Conditional Connectives

Construct the truth table for the proposition $p \to q$.

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

8.2 Tautologies and Truth Tables

Truth Table for the Biconditional Connective.

P	Q	$P \leftrightarrow Q$
T	T	
T	F	
F	T	
F	F	T

P	Q	$P \vee Q$		
T	T			
T	F			
F	T			
F	F			

Question 3

Part A : Propositions

Let p, q be the following propositions:

- p : this apple is red,
- q : this apple is ripe.

Express the following statements in words as simply as you can:

(i) $p \to q$

(ii) $p \wedge \neg q$.

Express the following statements symbolically:

(iii) This apple is neither red nor ripe.

(iv) If this apple is not red it is not ripe.

Part B : Logical Operations

Let $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p and q be the following propostions concerning the integers n.

p n is event

q $n < 5$

Find the values of n for which each of the following compound statement is true,

(i) $\neg p$

(ii) $p \wedge q$

(iii) $\neg p \vee q$

(iv) $p \oplus q$

Question 3

Question 6

8.2.1 Question 6

Say with reason whether or not $\mathcal{R}$ is

- reflexive
- symmetric
- transitive

In the cases where the given property does not hold provide a counter example to justify this.

Question 6 Part A : Digraphs

Suppose $A = \{1, 2, 3, 4\}$. Consider the following relation in A

$$\{(1,1), (2,2), (2,3), (3,2), (4,2), (4,4)\}$$

Draw the direct graph of A. Based on the Digraph of A discuss whether or not a relation that could be depicted by the digraph could be described as the following, justifying your answer.

(i) Symmetric

(ii) Reflexive

(iii) Transitive

(iv) Antisymmetric

Part B : Relations

Determine which of the following relations xRy are reflexive, transitive, symmetric, or antisymmetric on the following - there may be more than one characteristic. if

(i) $x = y$

(ii) $x < y$

(iii) $x^2 = y^2$

(iv) $x \geq y$

Question 2

Let $A = \{0, 1, 2\}$ and $R = \{(0, 0), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$ and $S = \{(0, 0), (1, 1), (2, 2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Part C : Partial Orders

Let $A = \{0, 1, 2\}$ and $R = \{(0, 0), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$ and $S = \{(0, 0), (1, 1), (2, 2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Biconnective Operators

$$p \longleftrightarrow q$$

We could verbalize this as "p implies q and q implies p".

Maths for Computing Sprint

Part 0. - Numeracy

8.2.2 Factorials

Evaluate the following

- 6!
- 3!
- 1!
- 0!

Evaluate the following expressions

$$\frac{5!}{3!} \text{ and } \frac{6!}{2! \times 4!}$$

Laws of Logarithms

- Addition of Logarithms
- Subtraction of Logaritms
- Powers of Logarithms

Section 3. Logic

Proofs with Truth Tables

$$\neg(p \vee q) \wedge p \equiv q$$

p	q				

8.3 Section 3 Logic

8.3.1 Logical Operations

- $\neg p$ the negation of proposition p.
- $p \wedge q$ Both propositions p and q are simultaneously true (Logical State AND)
- $p \vee q$ One of the propositions is true, or both (Logical State : OR)
- $p \otimes q$ Only one of the propositions is true (Logical State : exclusive OR (i.e XOR)

p	q	$p \vee q$	$q \wedge p$	$p \otimes q$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Chapter 9

Session 4

Invertible Functions

Necessary Condtions for Invertibility of a Function

- The function must be one-to-one
- The fucntion must be onto.

Equivalence Relations

-
-
-

9.1 Logarithmic Functions

Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$

3. $\log_b(X^Y) = Y \log_b(X)$

Question1.3 Compute the Logarithm of the following

- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$

$$Log_b(x) = \frac{1}{Log_x(b)}$$

$$Log_b(x) = \frac{Log_a(b)}{Log_a(b)}$$

Example 1

$$log_3(x) + 3log_x(3) = 4$$

$$(log_3(x))^2 + 3 = 4log_3(x)$$

Example 1

$$log_3(x) + 3log_x(3) = 4$$

$$(log_3(x))^2 - 4log_3(x) + 3 = 0$$

9.2 Digraphs and Relatiosn

Given a flock of chickens, between any two chickens one of them is dominant. A relation, R, is defined between chicken x and chicken y as xRy if x is dominant over y. This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

- Amy is dominant over Beth and Carol
- Beth is dominant over Eve and Carol
- Carol is dominant over Eve and Daisy
- Daisy is dominant over Eve, Amy and Beth
- Eve is dominant over Amy.

Section 4. Functions

9.3 Section 4 Functions

9.3.1 Invertible Functions

A function is invertible if it fulfils two criteria

- The function is ***onto***,
- The function is ***one-to-one***.

State the conditions to be satisfied by a function $f : X \leftarrow Y$ for it to have an inverse function $f^{-1} : Y \leftarrow X$.

$\lceil \frac{x^2+1}{4} \rceil$ where $f : A \rightarrow \mathbf{Z}$

(i) Find $f(4)$ and the ancestors of 3.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer

Given $f : \mathbf{R} \rightarrow \mathbf{R}$ where f(x) =3x-1,define fully the inverse of the function f ,i.e.f^{-1}. State the value of $f^{-1}(2)$

9.3.2 Precision Functions

- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
- Floor Function $\lfloor x \rfloor$

Question1.2: State the range and domain of the following function

$$F(x) = \lfloor x - 1 \rfloor$$

9.3.3 Powers

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

9.3.4 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

9.3.5 Logarithmic Functions

Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$
3. $\log_b(X^Y)$= $Y\log_b(X)$

Question1.3 Compute the Logarithm of the following

- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$

Question 4

Session 04:Functions

- Definitions

Domain

Co-domain

Image

Ancestor

Range

Part A : Functions

Given a real number x, say how the floor of x $\lfloor x \rfloor$ is defined.

(i) Find the values of $\lfloor 2.97 \rfloor$ and $\lfloor -2.97 \rfloor$.

(ii) Find an example of a real number x such that $\lfloor 2x \rfloor \neq 2\lfloor x \rfloor$, justifying your answer.

Part B : Logarithms

Evaluate the following expression.

$$\mathrm{Log}_4 64 + \mathrm{Log}_5 625 + \mathrm{Log}_9 3$$

$$\mathrm{Log}_4 64 + \mathrm{Log}_5 625 + \mathrm{Log}_9 3$$

Absolute Value Function (4.1.3)

- The absolute value of some real number x is denoted $|x|$.
- If the number is positive, the asbolute value is the same number.
- If the number is negative, the asbolute value is the number without the minus sign.
- $|2| = 2$
- $|-2| = 2$

Floor and Ceiling Function (4.1.4)

Polynomial Functions (4.1.5)

Constants (P_0)

near Functions (P_1)

ratic Functions (P_2)

ubic Functions (P_3)

Equality of Functions (4.1.6)

$$f(x) = g(x)$$

Encoding and Decoding Functions (4.2)

Onto Functions (4.2.2)

One-to-One Functions (4.2.3)

$f(x)$, must be *One-to-One* and *Onto*

Exponential and Logarithmic Functions (4.3)

The Laws of Logarithms

-
- $log_b(x^y) = y \times log_b(x)$
-
-

Big O-notation

Comparing the size of Functions (4.4)
Using O-notations

Power Notation (4.4.2)

Question 4

A function f: X-¿ Y , where $X = \{p, q, r, s\}$ and $Y = \{1, 2, 3, 4, 5\}$ is given by the subset of $X \times Y$

- Show f as an arrow diagram
- state the domain, the co-domain, and the range of f
- Say why f does not have the one-to-one property and why f does not have the "onto" property, giving a specific counter example in each case.

9.4 Section 4 Functions

9.4.1 Invertible Functions

A function is invertible if it fulfils two criteria

- The function is ***onto***,

- The function is ***one-to-one***.

State the conditions to be satisfied by a function $f : X \leftarrow Y$ for it to have an inverse function $f^{-1} : Y \leftarrow X$.

$\lceil \frac{x^2+1}{4} \rceil$ where $f : A \rightarrow \mathbf{Z}$

(i) Find $f(4)$ and the ancestors of 3.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer

Given $f : \mathbf{R} \rightarrow \mathbf{R}$ where f(x) =3x-1,define fully the inverse of the function f ,i.e.f^{-1}. State the value of $f^{-1}(2)$

9.4.2 Precision Functions

- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
- Floor Function $\lfloor x \rfloor$

Question1.2: State the range and domain of the following function

$$F(x) = \lfloor x - 1 \rfloor$$

9.4.3 Powers

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

Special Cases

Anything to the power of zero is always 1

$$X^0 = 1 \text{ for all values of X}$$

Sometimes the power is a negative number.

$$X^{-Y} = \frac{1}{X^Y}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

9.4.4 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

Functions

- Domain of a Function
- Range of a function
- Inverse of a function
- one-one (surjective)
- onto (bijective)

Functions

- Domain of a Function
- Range of a function
- Inverse of a function
- one-one (surjective)
- onto (bijective)

Chapter 10

Session 5

10.1 Video 4 : Graph Theory

Draw the graph **G**, which has the vertices v_1,v_2,v_3,...,v_7, and the adjacency list:

v_1 : v_2,v_4

v_2 : v_1,v_3

v_3 : v_2,v_4

v_4 : v_1,v_3,v_5

v_5 : v_4,v_6

v_6 : v_5,v_7

v_7 : v_5,v_6

10.2 Graph Theory - Isomorphic Graphs

- If the graphs are not simple, we need more sophisticated methods to check for when two graphs are isomorphic.
- However, it is often straightforward to show that two graphs are not isomorphic.

- You can do this by showing any of the following seven conditions are true.

[fragile]

1. The two graphs have different numbers of vertices.
2. The two graphs have different numbers of edges.
3. One graph has parallel edges and the other does not.
4. One graph has a loop and the other does not.
5. One graph has a vertice of degree k (for example) and the other does not.
6. One graph is connected and the other is not.
7. One graph has a cycle and the other has not.

10.3 Video 7 : Numbers

mantissa

abscissa

radix point

- Number Systems
- Set Theory
- Function
-
- Graph Theory
- Digraphs
- Set Theory
- Function

- Probability
- MAtrices
- Continuously divide the decimal number by 2.
- Keep record of the remainder, either 0 or 1.
- The sequence of remainders is the binary number required.
- Hex Characters $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$
-
- $\neq$ Not Equal
- ¡ Less than
- ¿ greater than
- $\geq$ greater than or equal to
- $\leq$ Not Equal to
- Natural Numbers $\{1, 2, 3, 4, \ldots\}$
- Integers $\{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$
- Rational Numbers e.g 4/7 , 11/25
- Real Numbers Any number e.g. 3.1415

‘ Floating Point Noation
Section 1 Section 1.2 Section 1.3 Section 14

- Decmal Number Systesm
  - Base 10
- Binary Number Systems
  - Base 2

– allowable characters are 0.1 only

- Base 16 Hexadecimal

  – Use all of the decimal digits, in addition to 6 more A,B ,D,D,E,F

  (where might you see this - specifying colours RGB Numbers

For example FF in hexadecimal is is 255 in decimal
Rational Numbers
Natural Numbers indies Integers Z
Computing a binary number
Useful

$2^0 = 1$	$2^4 = 16$
$2^1 = 2$	$2^5 = 32$
$2^2 = 4$	$2^6 = 64$
$2^3 = 8$	$2^7 = 128$

Firslly determine the highest power
Suppose the number we wish to convert is 58
Hwhat is the highest power of two that deivides

10.4 graph theory

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

Session 05:Graphs

5A.1 What is a Graph?

5A.2 Paths Cycles and Connectivity

5A.3 Isomorphisms of a graph

5A.4 Adjacency Matrices and Adjacency Lists

Isomorphism

- They have a different number of connected components
- They have a different number of vertices
- They have different degrees sequences
- They have a different number of paths of any given length
- They have a different number of cycles of any length.

Adjacency Lists

u : $\{v\}$

v : $\{w, x\}$

w : $\{v, x\}$

z : $\{v, w\}$

- Spanning Subgraphs of G.
- a vertex is said to be an **emph isolated vertex** if it has a degree of zero.
- a vertex is said to be an **emph end-vertex** if it has a degree of one.
- a vertex is said to be an **emph even vertex** if it has a degree of an even number.

- a vertex is said to be an **emph odd vertex** if it has a degree of an odd number.
- A graph is said to be **emphk-regular** if the degree of each vertex is k.
- Every Graph has an even number of odd vertices.
- A cubic graph is a graph where every vertex has degree three.

Section 5. Graph Theory

Adjacency Lists

1.
2.
3.
4.

Question 5

1. Draw two non-isomorphic graphs with the following degree sequence.

$$4, 3, 3, 2, 2, 2, 2, 1, 1$$

2. Write out the degree sequence of the following graph.
3. State the vertices that comprise a cycle of length 5 in both of the following graphs.

Session 05 Graph Theory

- Eulerian Path
- Isomorphism

- Adjacency matrices

Adjacency Matrices

$$\begin{pmatrix} o & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Session 05 Graph Theory

- Eulerian Path
- Isomorphism
- Adjacency matrices

Adjacency Matrices

$$\begin{pmatrix} o & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

10.5 graph theory

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

Condtions for Isomorphism

-
-
-

Question 5

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

a) (i) A simple, connected graph has 7 vertices, all having the same degree d. State the possible values of d and for each value also give the number of edges in the corresponding graph. (ii) Another simple, connected graph has 6 vertices, all having the same degree, n. Draw such a graph when n = 3 and state the other possible values of n. [4]

Tutorial Sheet for Session 1

Part A: Number Systems - Binary Numbers

1. Express the following decimal numbers as binary numbers.

   i) $(73)_{10}$ ii) $(15)_{10}$ iii) $(22)_{10}$

   All three answers are among the following options.

   a) $(10110)_2$ b) $(1111)_2$ c) $(1001001)_2$ d) $(1000010)_2$

2. Express the following binary numbers as decimal numbers.

   a) $(101010)_2$ b) $(10101)_2$ c) $(111010)_2$ d) $(11010)_2$

3. Express the following binary numbers as decimal numbers.

   a) $(110.10101)_2$ b) $(101.0111)_2$ c) $(111.01)_2$ d) $(110.1101)_2$

4. Express the following decimal numbers as binary numbers.

   a) $(27.4375)_{10}$ b) $(5.625)_{10}$ c) $(13.125)_{10}$ d) $(11.1875)_{10}$

Part B: Number Systems - Binary Arithmetic

(See section 1.1.3 of the text)

1. Perform the following binary additions.

   a) $(110101)_2 + (1010111)_2$ c) $(11001010)_2 + (10110101)_2$
   b) $(1010101)_2 + (101010)_2$ d) $(1011001)_2 + (111010)_2$

2. Perform the following binary subtractions.

a) $(110101)_2$ - $(1010111)_2$

b) $(1010101)_2$ - $(101010)_2$

c) $(11001010)_2$ - $(10110101)_2$

d) $(1011001)_2$ - $(111010)_2$

3. Perform the following binary multiplications.

a) $(1001)_2 \times (1000)_2$

b) $(101)_2 \times (1101)_2$

c) $(111)_2 \times (1111)_2$

d) $(10000)_2 \times (11001)_2$

4. Perform the following binary multiplications.

   (a) Which of the following binary numbers is the result of this binary division: $(10)_2 \times (1101)_2$.

      a) $(11010)_2$
      b) $(11100)_2$
      c) $(10101)_2$
      d) $(11011)_2$

   (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \times (111)_2$.

      a) $(11000)_2$
      b) $(11001)_2$
      c) $(10101)_2$
      d) $(11011)_2$

   (c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \times (1101)_2$.

      a) $(11000)_2$
      b) $(11001)_2$
      c) $(10101)_2$
      d) $(11011)_2$

5. Perform the following binary divisions.

   (a) Which of the following binary numbers is the result of this binary division: $(111001)_2 \div (10011)_2$.

      a) $(10)_2$
      b) $(11)_2$
      c) $(100)_2$
      d) $(101)_2$

   (b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \div (111)_2$.

      a) $(11)_2$
      b) $(100)_2$
      c) $(101)_2$
      d) $(110)_2$

(c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \div (1101)_2$.

a) $(100)_2$
b) $(110)_2$
c) $(111)_2$
d) $(1001)_2$

Part C: Number Bases - Hexadecimal

1. Answer the following questions about the hexadecimal number systems
   a) How many characters are used in the hexadecimal system?
   b) What is highest hexadecimal number that can be written with two characters?
   c) What is the equivalent number in decimal form?
   d) What is the next highest hexadecimal number?
2. Which of the following are not valid hexadecimal numbers?

   a) 73 b) $A5G$ c) 11011 d) EEF
3. Express the following decimal numbers as a hexadecimal number.

   a) $(73)_{10}$ b) $(15)_{10}$ c) $(22)_{10}$ d) $(121)_{10}$
4. Compute the following hexadecimal calculations.

   a) $5D2 + A30$ b) $702 + ABA$ c) $101 + 111$ d) $210 + 2A1$

Part D: Natural, Rational and Real Numbers

- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$
- $\mathbb{Z}$: integers $\{-3, -2, -1, 0, 1, 2, 3, \ldots\}$

  - (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means "numbers" in German.)

- $\mathbb{Q}$: rational numbers
- $\mathbb{R}$: real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
  - (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

1. State which of the following sets the following numbers belong to.

1) 18	3) π	5) $17/4$	7) $\sqrt{\pi}$
2) $8.2347\ldots$	4) $1.33333\ldots$	6) 4.25	8) $\sqrt{25}$

The possible answers are

a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$

d) Real Number $\mathbb{R}$