Mathematics for Computing

Binary Strings and Functions

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Binary Strings and Functions

Let S be the set of all 4 bit binary strings.
The function $f : S \to \mathbb{Z}$ is defined by the rule:

$$f(x) = \text{the number of zeros in x}$$

for each binary string $x \in S$.
Find:

1. the number of elements in the domain
2. $f(1000)$
3. the set of pre-images of 1
4. the range of f.

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Part 1: Find the number of all elements in the domain.

- In this question, the domain is S
- S is the set of all 4 bit binary strings.

$$1010 \quad 1101 \quad 0101 \quad \ldots$$
$$0010 \quad 1001 \quad 1110 \quad \ldots$$

- There are 2^4 4-bit binary strings.

$$2^4 = \mathbf{16}$$

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Part 2: Find $f(1000)$

- Recall

$$f(x) = \text{the number of zeros in x}$$

- How many zeros in the binary string 1000?

$$f(1000) = 3$$

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Part 3: Find the set of pre-images of 1.

- Find all the 4-bit binary strings with only one zero.

$$\{1110,\ 1101,\ 1011,\ 0111\ \}$$

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Part 4: Find the range of f.

- What is minimum and maximum numbers of 0s in a 4-bit binary string
- There can be no 0s or as many as 4.
- $f(1111) = 0$ and $f(0000) = 4$
- The range of f is therefore

$$\{0, 1, 2, 3, 4\}$$