1. The number of vertices is 6. Therefore the minimal spanning tree has 5 edges. The edges in non-decreasing order are:
10(a-d), 13(e-f), 21(b-c), 35(a-c), 36(c-d), 51(a-e), 51(d-e), 56(a-b), 57(b-d), 60(a-f), 60(d-f), 68(c-e), 68(c-f), 70(b-f), 76(b-e).

   Kruskal's Algorithm: Choose edges in order
a-d, e-f, b-c, a-c, a-e. Total weight = 130

   One version of *Prim*'s Algorithm: choose edges from above list in order
a-d, a-c, b-c, a-e, e-f. Total weight = 130.
Another version: Start with any vertex, say a, and add the smallest weight incident edge to the existing tree (without creating a cycle). Repeat until $n-1$ edges are found. In this example, the gives the same list of edges as above.

2. To sort the edges in non-decreasing order requires of the order of
$m - 1 + (m - 1)\log_2 m)$ comparisons.
To check whether the i-th candidate edge makes a cycle with the previously chosen $i-1$ edges requires at least $1+\log_2(i-1)$ and at most $2+2\log_2(i-1)$ comparisons. Thus in the case where no chosen edge is rejected, we require at least

$$\sum_{i=2}^{n-1} 1 + \log_2(i-1) = n - 2 + \log_2(n-2)!$$

   of these comparisons, and at most $2(n-2) + 2\log_2(n-2)!$ comparisons.

3. Assuming the set (of cardinality p) of pre-ordained edges does not contain a cycle, select these p edges at first, then choose each of the remaining $n-1-p$ edges using the normal procedure.

6 Hint: Start with the vertex labelled $(8, 0, 0)$ and create the digraph with each vertex labelled of the form (a, b, c) where vertex v_i has an edge going to vertex v_j if there is a single pouring which converts the label of v_i into the label of v_j. For example, the two out edges from $(8, 0, 0)$ lead to $(5, 0, 3)$ and $(3, 5, 0)$.
The solution is a path from $(8, 0, 0)$ to $(4, 4, 0)$. The minimal path is of length 7.

For Questions 4,5 & 7 see *Johnsonbaugh*, Discrete Mathematics.