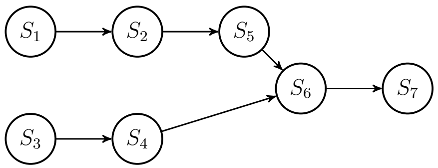
MA4016 - Engineering Mathematics 6

Solution Sheet 8: Graph Theory (March 26, 2010)

1. Determine the type of graph for the shown ones and find for each undirected graph that is not simple a set of edges to remove to make it simple.
a) simple (undirected) graph, **b)** (undirected) multigraph, **c)** (undirected) pseudeograph, **d)** directed multigraph.
Remove in **b)** one of the $\{a,b\}$ edges and two of the $\{b,d\}$ edges to get C_4.
Remove in **c)** all loops and one of all the doubled edges to get the union of C_4 and an isolated vertex e.

2. Draw a precedence graph (without inherited dependencies) for the following program:



3. Find the number of vertices and edges, and the degree of each vertex in the given graphs. Identify all isolated and pendant vertices.
a) 5 vertices, 12 edges, no isolated or pendant vertices. $\deg(a) = \deg(b) = 6$, $\deg(c) = 4$, $\deg(d) = 5$, $\deg(e) = 3$, $\sum \deg(v) = 24 = 2{\cdot}12$.
b) 9 vertices, 12 edges, 2 isolated vertices d and f, no pendant vertex. $\deg(a) = \deg(i) = 3$, $\deg(b) = \deg(h) = 2$, $\deg(c) = \deg(g) = 4$, $\deg(d) = \deg(f) = 0$, $\deg(e) = 6$, $\sum \deg(v) = 24 = 2 \cdot 12$.

4. Can a simple graph exist with 15 vertices each of degree five?
No, because the sum of the degrees of the vertices cannot be odd.

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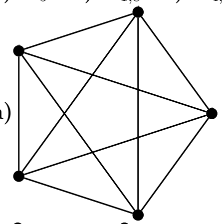
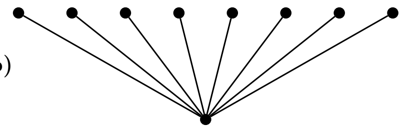
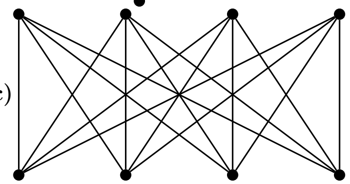
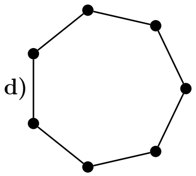
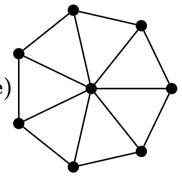
5. Draw these graphs.
a) K_5 **b)** $K_{1,8}$ **c)** $K_{4,4}$ **d)** C_7 **e)** W_7

a) 

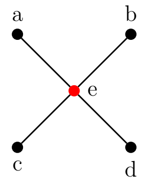
b) 

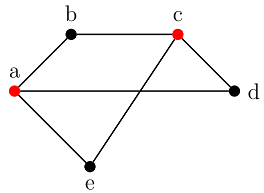
c) 

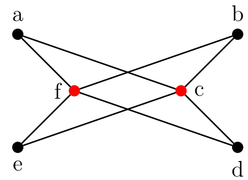
d) 

e) 

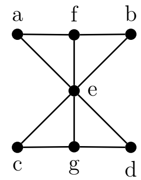
6. Are these graphs bipartite? Yes to all. **a)** is $K_{1,4}$, **b)** is $K_{2,3}$ and **c)** is $K_{2,4}$.

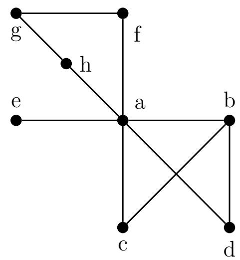
a) 

b) 

c) 

7. Find the union of the given graphs. Assume edges with the same endpoints are the same?

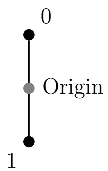
a) 

b) 

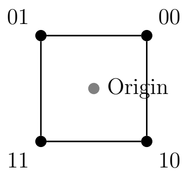
8. 1. If $n = 0$, put an unlabelled vertex at $(-1, 0)$ and stop.
2. Recursively invoke this algorithm with input $n - 1$.
3. Move each vertex so that its new angle is half the current angle, maintaining edge connections.
4. Reflect each vertex and edge in the x-axis.
5. Connect each vertex above the x-axis to its mirror below the x-axis.
6. Prefix 0 to the label of each vertex above the x-axis, and similarly with 1 below.

Use this algorithm to draw a 4-cube.

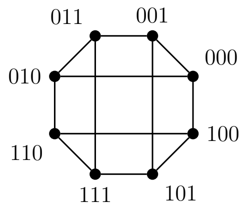
1-cube:



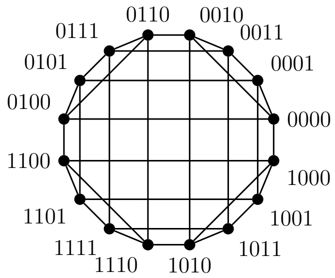
2-cube:



3-cube:



4-cube:



5-cube: