

Rules of Inclusion, Listing and Cardinality

For each of the following sets, a set is specified by the rules of inclusion method and listing method respectively. Also stated is the cardinality of that data set.

Worked example 1

- $\{x : x \text{ is an odd integer } 5 \leq x \leq 17\}$
- $x = \{5, 7, 9, 11, 13, 15, 17\}$
- The cardinality of set x is 7.

Worked example 2

- $\{y : y \text{ is an even integer } 6 \leq y < 18\}$
- $y = \{6, 8, 10, 12, 14, 16\}$
- The cardinality of set y is 6.

Worked example 3

A perfect square is a number that has a integer value as a square root. 4 and 9 are perfect squares ($\sqrt{4} = 2$, $\sqrt{9} = 3$).

- $\{z : z \text{ is an perfect square } 1 < z < 100\}$
- $z = \{4, 9, 16, 25, 36, 49, 64, 81\}$
- The cardinality of set z is 8.

Exercises

For each of the following sets, write out the set using the listing method. Also write down the cardinality of each set.

- $\{s : s \text{ is an negative integer } -10 \leq s \leq 0\}$
- $\{t : t \text{ is an even number } 1 \leq t \leq 20\}$
- $\{u : u \text{ is a prime number } 1 \leq u \leq 20\}$
- $\{v : v \text{ is a multiple of 3 } 1 \leq v \leq 20\}$

Power Sets

Worked Example

Consider the set Z :

$$Z = \{a, b, c\}$$

Q1 How many sets are in the power set of Z ?

Q2 Write out the power set of Z .

Q3 How many elements are in each element set?

Solutions to Worked Example

Q1 There are 3 elements in Z . So there is $2^3 = 8$ element sets contained in the power set.

Q2 Write out the power set of Z .

$$\mathcal{P}(Z) = \{\{0\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

Q3 * One element set is the null set - i.e. containing no elements

- Three element sets have only elements
- Three element sets have two elements
- One element set contains all three elements
- $1+3+3+1=8$

Exercise

For the set $Y = \{u, v, w, x\}$, answer the questions from the previous exercise

Complement of a Set

Consider the universal set U such that

$$U = \{2, 4, 6, 8, 10, 12, 15\}$$

For each of the sets A, B, C and D , specify the complement sets.

Set	Complement
$A = \{4, 6, 12, 15\}$	$A' = \{2, 8, 10\}$
$B = \{4, 8, 10, 15\}$	
$C = \{2, 6, 12, 15\}$	
$D = \{8, 10, 15\}$	

Set Operations

Consider the universal set U such that

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and the sets

$$A = \{2, 5, 7, 9\}$$

$$B = \{2, 4, 6, 8, 9\}$$

(a) $A - B$

(b) $A \otimes B$

(c) $A \cap B$

(d) $A \cup B$

(e) $A' \cap B'$

(f) $A' \cup B'$

Venn Diagrams

Draw a Venn Diagram to represent the universal set $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6\}$ with subsets:

$$A = \{2, 4, 5\}$$

$$B = \{1, 4, 5, 6\}$$

Find each of the following

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

(e) $A \otimes B$