

1. Consider the following table of towns and their distances apart. The EU is funding the construction of a motorway network which will link the towns. Assuming that the cost of building a motorway is proportional to its length, find the cheapest network that will enable travel between any two towns without leaving the network.

	A	B	C	D	E	F
A	-	56	35	10	51	60
B	56	-	21	57	76	70
C	35	21	-	36	68	68
D	10	57	36	-	51	60
E	51	76	68	51	-	13
F	60	70	68	60	13	-

Detail the steps when using (i) *Kruskal's* algorithm, (ii) *Prim's* algorithm.

2. Analyse *Kruskal's* algorithm in terms of the number of comparisons needed to find the minimal spanning tree of a graph with  $n$  vertices and  $m$  edges.
3. Modify *Kruskal's* algorithm to find the minimal spanning tree of a connected graph which includes a set of pre-ordained edges.
4. Construct a binary decision tree for sorting the set of numbers  $\{b_1, b_2, b_3, b_4\}$  into increasing order.
5. Show the details of the (i) merge sort and (ii) tournament sort of the list:  
8, 21, 35, 16, 5, 98, 15, 51

### Miscellaneous

6. Bill and Ben have an 8 litre jug full of wine, and two empty jugs of capacities 5 and 3 litres. How can they divide the wine equally, using only the jugs.
7. You have  $n$  coins, which are identical in appearance, but one of which has a different weight to the others, which all weigh the same. Using only a balance, what is the minimum number of weighings necessary to (i) identify the odd coin and (ii) tell whether it is lighter or heavier.  
To get you started, consider the cases  $n = 5, 4, 8, 12$ , and 13.