UNIVERSITY OF LONDON 291 0102 ZA

EXTERNAL PROGRAM
BSc/Diploma Examinations
for External Students
COMPUTING AND INFORMATION SYSTEMS AND CREATIVE COMPUTING
COMPUTING

CIS102w Mathematics for Computing

Duration: 3 hours

Date and time: Tuesday 6 May 2008 : 10.00 – 1.00 pm

There are TEN questions on this paper.
Full marks will be awarded for complete answers to TEN questions.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

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Question 1

(a) The first 16 hexadecimal integers ≥ 0 can be represented by 4 bit binary strings as follows:

$0000 : 0$	$0100 : 4$	$1000 : 8$	$1100 : C$
$0001 : 1$	$0101 : 5$	$1001 : 9$	$1101 : D$
$0010 : 2$	$0110 : 6$	$1010 : A$	$1110 : E$
$0011 : 3$	$0111 : 7$	$1011 : B$	$1111 : F$

(i) Find the hexadecimal equivalent of the binary numeral 1101101.01

(ii) Find the binary equivalent of the hexadecimal numeral $A9.B$

(iii) Working in the hexadecimal system compute the following sum, showing all your working:

$$5C2 + A3E$$

(iv) Working in the binary system compute the following sum, showing all your working:

$$11001001 - 1100111$$

[6]

(b) (i) Define what is meant by a rational number. Say whether or not the repeating decimal 0.151515..... is a rational or irrational number, justifying your answer.

(ii) Showing all your working, express the repeating decimal 0.272727..... as a fraction in its simplest terms. [4]

Question 2

(a) Let $A = \{3, 5, 7, 9, ...\}$ and $B = \{3n - 1 : n \in \mathbb{Z}^+\}$ be two sets of numbers.

(i) Describe the set A by the rules of inclusion method.

(ii) Describe the set B by the listing method.

(iii) Describe the two sets $A \cap B$ and $A - B$, by the listing method. [4]

(b) Let P, Q and R be subsets of a universal set $\mathcal{U}$.

(i) Construct a membership table for the set $X = P \cap (Q' \cup R)$.

(ii) Draw a labelled Venn diagram showing P, Q, and R intersecting in the most general way.

(iii) Shade the region X on your diagram. [6]

Question 3 (a) Let p and q be the following propositions about an object:

p : "this object is a circle"
q : "this object is yellow".

(i) Express each of the three following compound propositions concerning positive integers symbolically by using p, q and appropriate logical symbols.

"this object is a yellow circle"
"if this object is yellow then it is a circle"
"this object is not yellow but it is a circle".

(ii) Construct the truth table for the statement $q \rightarrow p$.

(iii) Write in words the contrapositive of the statement given symbolically by "$q \rightarrow p$". [6]

(b) Construct a logic network that accepts as inputs p and q, which may independently have the value 0 or 1, and gives as final output $\neg(p \wedge \neg q)$. Show that $\neg(p \wedge \neg q)$ is equivalent to $p \rightarrow q$. [4]

Question 4

(a) The terms of a sequence are defined by the formula $u_k = 4k - 1$.

(i) Calculate $u_1, u_2,$ u_3 and u_4.

(ii) What value of k gives the term which equals 2999?

(iii) Use the standard formula $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ to find an expression for the sum of the series given by $\sum_{k=1}^{n} (4k - 1)$, in terms of n. Use this to find the sum of this series when $n = 100$. [6]

(b) Prove by induction that the series with recurrence relation $u_{n+1} = u_n + n$ has general term given by

$$u_n = \frac{n(n-1)}{2}, \quad \text{for all positive integers } n.$$

[4]

Question 5

(a) Let S be the set of names of students on a particular course in a college and let f be the function that counts the number of letters in a student's name. So if X is the name of an individual student then

$$f(X) = \text{ the number of letters in } X \text{ where } f : X \to \mathbb{Z}^+ \text{ and } X \in S.$$

For example if Y is the name "Tom Patey" then $f(Y) = 8$.

(i) Given A is the name Don Whillans and B is the name Jo Brown find $f(A)$ and $f(B)$.

(ii) Give two different examples of a possible pre-image or ancestor of 6.

(iii) Say under what circumstances you think this function is one to one, justifying your answer.

(iv) Say whether or not this function is onto, justifying your answer. [6]

(b) Say whether or not each of the following functions has an inverse, justifying your answer. In the cases where there is an inverse define it fully.

(i) $f : S \to \mathbb{Z}^+$ defined in part (a).

(ii) $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2$.

(iii) $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = 3x - 1$. [4]

Question 6

(a) What properties should a graph possess if it is

(i) simple (ii) connected? [2]

(b) Given K_n is the simple graph with vertices $v_1,\ v_2,\ v_3, ..., v_n$ in which each vertex is connected to every other vertex by a single edge

(i) Draw K_5.

(ii) State the number of edges in K_5.

(iii) Find the number of different paths of length 2 from v_1 to v_2 in K_5 (not counting cycles of length 2).

(iv) Find an expression for the number of edges in K_n.

(v) Find an expression for the number of paths of length 2 from v_1 to v_2 in K_n. [8]

Question 7

(a) Given the set $S = \{m,\ o,\ u,\ s,\ e\}$.

(i) Describe how each subset of S can be represented by a unique 5 digit binary string.

(ii) Write down the string corresponding to the subset $\{m,\ s,\ e\}$ and the subset corresponding to the string 01010.

(iii) What is the total number of subsets of S? [5]

(b) R is a relation defined on S as follows:

$$xRy \text{ if } x \text{ and } y \text{ are vowels.}$$

Draw the relationship digraph for R on S and say, with reason, whether this relation is

(i) reflexive

(ii) symmetric

(iii) transitive. [5]

Question 8 (a) Consider all possible arrangements of the letters in the word "exams", where each letter may be used once only.

(i) How many of these arrangements are possible?

(ii) How many of these arrangements begin with a vowel? Call this set B.

(iii) How many of these arangements end with a vowel? Call this set E

(iv) How many of these arrangements both begin and end with a vowel? Call this set B∩E.

(v) Show the sets B and E on a Venn diagram. Show the relevant number of elements in each region of your diagram. [7]

(b) What is the probability that an arrangement of the letters in the word "exams" chosen at random

(i) begins and ends with a vowel;

(ii) either begins or ends with a vowel or both;

(iii) has two vowels next to one another. [3]

Question 9

(a) A graph with 5 vertices: a, b, c, d, e has the following adjacency list:

$a : b,\ e$

$b : a,\ c,\ d$

$c : b,\ d$

$d : b,\ c,\ e$

$e : d,\ a.$

(i) Draw this graph, G.

(ii) Draw a spanning tree of G.

(iii) Draw all the non-isomorphic spanning trees of G and call this set S.

(iv) How many non-isomorphic trees can be created by adding a new vertex and edge to the trees in S. [6]

(b) A binary search tree is designed to store an ordered list of 4000 records, numbered 1,2,3,...,4000 at its internal nodes.

(i) Draw levels 0, 1 and 2 of this tree, showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.

(ii) What is the height of this tree? [4]

Question 10

(a) Given the following adjacency matrices **A** and **B** and **C** where

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & -3 & 7 \\ 1 & 0 & 2 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & -2 \\ 4 & 3 \\ 0 & 5 \end{pmatrix}$$

(i) Calculate A+BC.

(ii) Calculate CB. [4]

(b) (i) Write down the augmented matrix for the following system of equations.

$$\begin{aligned} 2x + y - z &= 6 \\ x - y + z &= 0 \\ x + 2y + 2z &= 2 \end{aligned}$$

(ii) Use Gaussian elimination to solve the system. [6]

END OF EXAMINATION