Onsite Tutorial

Hibernia College

Saturday 9th February 2013

Tutorial

Today’s Class

Part 1 Graph Theory

Part 2 Digraphs and Relations

Part 3 Overview of previous material

Tutorial

Part 1 : Graph Theory

Important Terminology

Vertex / Vertices

Edges

Incidence

Adjacency

Simple Graphs

Connected Graphs

Degree

Parallel Edges

Loops

Isolated Vertex

Tutorial

Part 1 : Graph Theory

Degree Sequence of a graph

Computing the number of edges of a graph

Tutorial

Part 1 : Graph Theory

Special graphs

- *n-regular graphs*
- *Complete graphs (K-graphs)*

Tutorial

More terminology

- Cycles (also known as trails and tours)
- Paths

(Remark: Important definitions for more advanced algorithms, such as Travelling Salesman Problem and Chinese Postman Problem)

2003 Question 6

Question 6 Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, ..., v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}, ...\{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex, v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4. $[2\frac{1}{2}]$

(b) Giving your answer in terms of n, write down an expression for the number of edges in K_n, C_n, and W_n. $[2\frac{1}{2}]$

2006 Question 6

Question 6

(a) (i) A simple, connected graph has 7 vertices, all having the same degree d. State the possible values of d and for each value also give the number of edges in the corresponding graph.

(ii) Another simple, connected graph has 6 vertices, all having the same degree, n. Draw such a graph when $n = 3$ and state the other possible values of n.

[4]

Tutorial

Isomorphism

Graphs that appear different are isomorphic if, in fact, they have same mathematical structure.

Mathematical structure of a graph can be considered as

1) Adjacency Lists
2) Adjacency Matrices

2003 Question 5

Question 5 (a) Let G be a simple graph with vertex set $V(G) = \{v_1,\ v_2,\ v_3,\ v_4,\ v_5\}$ and adjacency lists as follows:

$$\begin{array}{lcl} v_1 & : & v_2\ v_3\ v_4 \\ v_2 & : & v_1\ v_3\ v_4\ v_5 \\ v_3 & : & v_1\ v_2\ v_4 \\ v_4 & : & v_1\ v_2\ v_3. \\ v_5 & : & v_2 \end{array}$$

(i) List the degree sequence of G.

(ii) Draw the graph of G.

(iii) Find two distinct paths of length 3, starting at v_3 and ending at v_4.

(iv) Find a 4 cycle in G. [6]

Digraphs

- Directed Graphs

- Adjacency Matrix and Adjacency Lists
- Indegree and Outdegree of a vertex

Relations

Relations

- Reflexive xRx?
- Symmetric if xRy then yRx?
- Transitive if xRy and yRz then yRz?

Relations

- Consider the children of Emer and Finbar

Ann, Barry, Ciara and Dermot

Suppose the relation we are interested is “is the brother of”

dRc : Dermot is the brother of Ciara

bRd : Barry is the brother of Dermot

Relations

(Ciara, Dermot, Emer and Finbar)

Is this relation

- Reflexive
- Symmetric
- Transitive

Relations

Suppose the relation is defined as

1) “ is the sibling of “
2) “ has the same parents as”

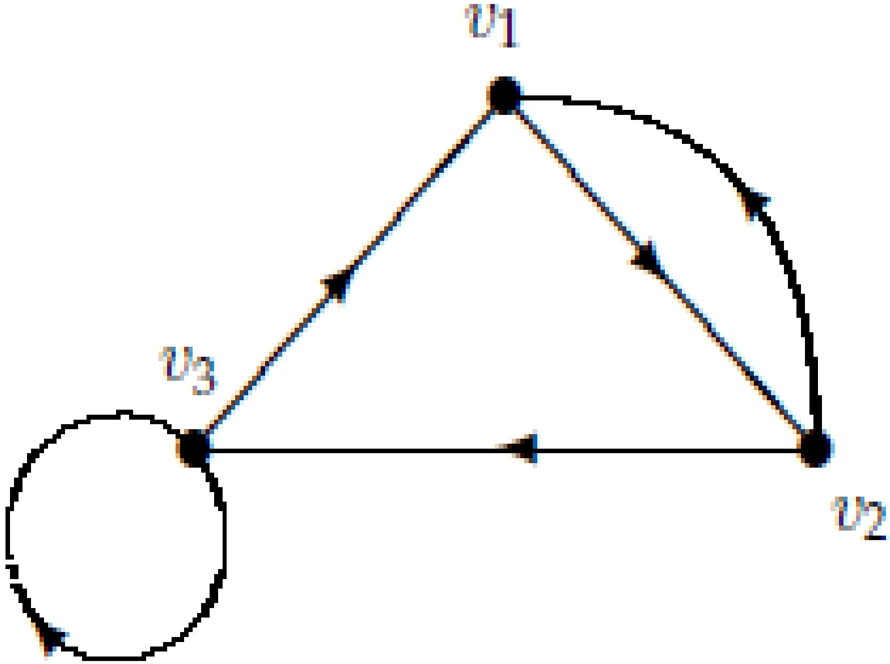
Relations

Transitive

- A, B and C live in a row of three houses
- A and B are next door neighbours (symm)
- B and C are next door neighbours (symm)

- Not transitive. A and C don't live beside each other.

Digraphs and Relations



Relations

- Equivalence Relations: A relation that is **reflexive**, **symmetric** and **transitive.**

- **Equivalence Classes**

Relations

Question 8

(a) Consider a set $S = \{0, 1, 2, 3, 4, 5\}$. R_1 is the relation such that xR_1y, if $x - y = 2$ and R_2 is the relation such that xR_2y if $x - y$ is even, for all x and $y \in S$.

(i) Illustrate the relations R_1 and R_2,using a separate digraph for each.

(ii) Complete the following table:

	Reflexive	Symmetric	Anti-symmetric	Transitive
R_1	$\times$			
R_2		$\checkmark$		

(iii) One of these relations is an equivalence relation. Say which relation this is and give the partition on S created by this relation. [6]

Cartesian Relations

- Every possible ordered pairing of elements of two sets