UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4402

SEMESTER: Autumn, 2003-2004

MODULE TITLE: Computing Maths

DURATION OF EXAMINATION: 2 1/2 hours

LECTURER: Dr. C. Nolan

PERCENTAGE OF TOTAL MARKS: 80%

EXTERNAL EXAMINER: Prof. J. Gibbon

INSTRUCTIONS TO CANDIDATES: **Full marks for 5 questions. Number your questions clearly.**

1 (a) Define what is meant by a function $f : A \to B$. 2%

(b) Which of the following are defines a function?

(i) $f : \mathbf{R} \to \mathbf{R}$

$$f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$

3%

(ii) $f : [-1, 1] \to [0, \pi],\ f(x) = \sin^{-1}(x)$ 3%

(iii) $f : [0, 1] \to [0, \pi/2],\ f(x) = \sin^{-1}(x)$ 3%

(c) Which of the following functions are bijective (1-1)?

(i) $f : \mathbf{R} \to \mathbf{R},\ f(x) = x^3 + 1$ 3%

(ii) $f : [0, 1] \to [0, 1],\ f(x) = x^2$ 3%

(iii) $f : [-1, 1] \to [0, 1],\ f(x) = x^2$ 3%

2 (a) Define what is meant by a sequence $\{a_n\}_{n=1}^{\infty}$ of real numbers. 3%

(b) Define what is meant by a convergent series $\displaystyle\sum_{n=1}^{\infty} a_n$ 3%

(c) Show that the series

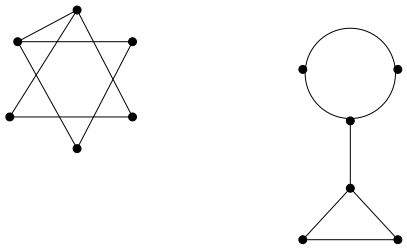
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-2}}{(2n-2)!}$$

defines a convergent series, for all $x \in \mathbf{R}$. Note that this series defines $\cos(x)$. 10%

(d) Use part (c) to estimate the value of $\cos(\pi/4)$. 4%

3 (a) What does it mean to say that two graphs are *isomorphic*? 5%

(b) Construct an isomorphism between the two following graphs



15%

4 (a) Recall that a simple, connected, planar graph with $n \geq 3$ nodes and a arcs satisfies the inequality

$$a \leq 3n - 6$$

Use this fact, to establish whether the graph K_6 is planar or not. 10%

(b) For what values of n is the graph K_n Eulerian?

10%

5 Using calculus (derivatives, etc), as well as the Newton-Raphson method for root-finding, sketch the graph of the function

$$f(x) = x^3 - 5x^2 + 8x - 3$$

You should give estimates for the root(s) of this function in order to aid in the drawing of the graph. 20%

6 (a) A line segment $A - B$ having end-point coordinates $(0, 1)$ and $(-1, 0)$ is first translated upward by one unit and then rotated about its centre point by and angle of θ. Calculate the coordinates of the end points after these two transformation. 15%

(b) Verify that the line segment is the same length after these transformations. 5%