2910102 Mathematics for computing

Examiner's report: Zone A

General remarks

These scripts showed that most of the candidates had a thorough understanding of the syllabus and the ability to apply their knowledge and skills in the appropriate context. They were well prepared and demonstrated that they had a good grasp of the subject and had revised well.

When revising for the exam it is a good idea to work through the sample paper in the subject guide, which has full solutions. You can then compare your answers with those given and if your approach is very different then you can consider why and perhaps modify your method. You can learn a lot from looking at the notation and wording used in the solution and the way any mathematical definitions or proofs are included. The way you present your solution may help you clarify the problem and develop the solution, as well as make it easier for the examiner to follow your work. Please try to ensure your answers convey your meaning clearly and correctly and show your working in full so that the examiner may give you marks for correct method, even if you make an error that means your final solution is incorrect. It will help you greatly to work through other past papers as part of the revision process so that you are familiar with the type of questions that may arise on each topic, and the material and skills you need to answer them. It may also help to make a list of key points on each chapter as a revision guide, together with typical exam questions.

Comments on individual questions

Question 1

Part (a) of this question involved producing a standard table of the first 16 integers, together with their binary equivalents, and then using this to convert from hex to binary and vice versa. Some students failed to realise that 100101.01 can be converted by grouping of the digits in groups of 4, working from the decimal point outwards.

In part (b) the addition could be performed in two stages if preferred, or all in one. Most students did well at this.

In part (c) some students gave the correct definition of an irrational number but did not seem to understand what it meant, as they could not see that a repeating decimal is rational, by the very definition they just quoted. This is an example of rote learning without comprehension of its significance. This was compounded when some went on to express the given recurring decimal as a fraction, but still failed to realise it was therefore rational. The actual process of converting the decimal was done well on the whole.

Question 2

The sets A and B were described well. Some students included the number 0 in both, which was incorrect.

In part (b) most students constructed the membership table with 8 rows, but a few only had 4. Most produced columns for P, Q and R. Columns for notP, QnR and notPu(QnR) were also needed. It was then easy to construct the Venn diagram, shading the relevant regions which had a 1 in the corresponding column of the table. It was then easy to see whether the shaded region included QnR was contained in this shaded area or not.

Question 3

In the first part of this question most students could express the statements symbolically and produce the correct truth table for: if q then p. However some produced a table for: if p then q, which is different and was not requested. The contrapositive was not done as well. It may have helped if students had first translated the original statement into logical form and then attempted to produce the contrapositive. Some still fall into the trap of thinking that the contrapositive of if q then p (as was the case here) is: if not q then not p, rather that if not p then not q.

In part (b) the logic network was well done apart from some missing labels, and some having the gates in the wrong order. Comparisons with truth tables and working out in which order to calculate the necessary column headings may be helpful here. On the whole the truth table followed easily from a correct diagram and most found a simpler expression such as if q then p with ease.

Question 4

Students were able to evaluate the terms of the first series, u_1, u_2,… much more readily than the second, which involved the summation of terms 1 to n of the first. i.e. $s_3 = u_1 + u_2 + u_3$. The standard results given were then used to find the sum (5k+1) as k goes from 1 to n, by splitting into 5 times the sum of k plus the sum of 1, as k goes from 1 to n.

Part (b) involved a proof by induction, difficult for many students, who are advised to learn the format and layout of a general induction proof and complete as much of that as possible.

Question 5

The first part of this question on functions involved a function not yet met in this context, the mapping from a set of 2 by 2 matrices to the set of integers, which involved counting the number of zeros in the matrix. Some students interpreted each matrix as a four digit binary string, and counted the zeros in that, for which some marks were given if they followed this through consistently. Some students did not realise that the set of integers in the range could only be {0,1,2,3,4} as these are the only possible number of zeros in a 2 by 2 matrix. Whether or not the function is one to one and/or onto is a question which arises virtually every year and is well worth preparing for. A clear understanding of the concepts of domain, range and co-domain is necessary for this as well as the ability to find and interpret these according to the particular example.

Part (b) required further demonstration of candidates understanding of when a function has an inverse or if not why – which boils down to knowing what is meant by one to one and onto properties.

Question 6

This question proved very difficult for several reasons. Firstly many students were unable to draw the graph following the description given, despite the fact the complete and cycle graphs are found in the subject guide. Secondly, it was difficult for students to find the number edges in terms of n for a general graph K_n, C_n, W_n. Practice with specific graphs to see an underlying pattern may be helpful in this process. Thirdly, students did not always refer to the graphs drawn in part (a) in order to answer part (c)(i), which was very helpful. The last part of this question was completed successfully by a handful of students and required a good ability to see general patterns, and express them algebraically.

Question 7

The digraph for this question could be drawn by simply interpreting the given dominance relations between the 5 chickens (as the vertices) and an arc from one to another when the first has dominance over the second. It then follows as a standard procedure to decide whether or not the digraph is reflexive, transitive or anti-symmetric. Candidates should always be prepared for this in an examination on this subject. Justification of answers is at least as important as the answer itself in this type of question. Thus just saying yes it is reflexive will not earn full marks.

The last part of the question involved a new relation and introduced two new chickens, the mothers of the five previous chickens. If the digraph of this new relation is drawn it still has only five vertices. The two new chickens are not actually represented in the digraph, they are merely a reference point for deciding whether or not the original chickens are related to one another (i.e. are they siblings). Thus the digraph is disconnected into two groups, chickens in each group all have the same mother. It is an equivalence relation which partitions the chickens into two equivalence classes according to which mother they have.

Question 8

The cardinality of a set is the number of elements it contains, so it was not necessary to list all the elements of S, E and F to answer the first part of the question, but to understand and calculate how many of them there would be. It was not essential to enter all the elements into the Venn diagram either, just the number of elements in each region, although marks were not deducted for doing this.

In part (c) if the relevant diagram had been drawn with the correct numbers (including those in the overlapping regions) the probabilities followed directly. Students should prepare for this type of probability question as well as those using counting methods more directly.

Question 9

Once the graph had been constructed from the adjacency list students' knowledge of trees was tested. Since a tree with n vertices has n-1 edges, a tree with seven vertices will have six edges, which is the case with this example. A simple deduction tells us how many edges we must therefore remove to form a tree in this case.

Since the graph has two cycles, an edge must be removed from each in order to create a tree. For a cycle of four this may be done in four ways and for a cycle of three this may be done in three ways. Combining these there are $4 \times 3 = 12$ different ways of forming a tree, each of which has involved the removal of two edges. The list of these 12 pairs of edges was what was required in part (iii). Some of these 12 trees are obviously isomorphic, and grouping the 12 into separate subgroups of isomorphic trees is what was required in the last part of part (a).

Part (b) was a standard binary search tree with the record 25000 at its root. Students should ensure they can calculate the record numbers at each level and also calculate the height of a binary tree needed to store a certain number of records, and thus the maximum number of comparisons which would need to be made.

Question 10

The most straight forward way to see if the graphs are isomorphic is to draw them both. A^2 can be calculated by multiplying the matrix A by itself, not merely squaring each of its elements. Similarly A^4 can be found by multiplying A^2 by A^2. These matrices give information about walks of length 2 and 4 in the graphs.

Part (b) was a standard question involving Gaussian elimination. Marks were given for method. It is helpful if the row operations employed are clearly labelled and the order of transformations is shown, so any errors can be worked through and credit given.