HIBERNIA COLLEGE DUBLIN

EST.MM

Computing

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Tutorial: Maths for Computing

Online Tutorial 4

Chapter 8 : Trees

Trees

- A lot of concepts and definitions follows from Chapter 5 : Introduction to Graph Theory

<u>Syllabus</u>

- Properties of Trees
- Rooted Trees and Binary Trees
- Binary Search Trees

Trees : Properties of Trees

1) Characteristics of a Tree

A tree is a connected graph that contains no cycles. A tree has no loops and no multiple edges. All trees are simple graphs.

2) Path Graphs

A tree that contains only vertices of degree one or two is called a ***path graph***. The length of a path graph is the number of edges in it.

3) Number of Edges

(Theorem 3.3) Let $\boldsymbol{T}$ be a tree with $\boldsymbol{n}$ vertices. Then $\boldsymbol{T}$ has $\boldsymbol{n - 1}$ edges.

Trees : Properties of Trees

4) Spanning Subgraphs

The graph H is a **subgraph** of a graph G if H’s vertices are a subset of the G’s vertex, its edges are a subset of the edge set of G, and each edge of H has the same end-vertices in G.

H is called a **spanning subgraph** of G if the vertices of H are the **same** as the vertices of G.

Trees : Properties of Trees

5) Spanning Trees

If H is a spanning subgraph which is also a tree, then H is said to be a spanning tree of G. (G does not need to be a tree)

Trees : Properties of Trees 2008 Zone A Q9

Question 9

(a) A graph with 5 vertices: a, b, c, d, e has the following adjacency list:

$a : b,\ e$

$b : a,\ c,\ d$

$c : b,\ d$

$d : b,\ c,\ e$

$e : d,\ a.$

(i) Draw this graph, G.

(ii) Draw a spanning tree of G.

(iii) Draw all the non-isomorphic spanning trees of G and call this set S.

(iv) How many non-isomorphic trees can be created by adding a new vertex and edge to the trees in S.

[6]

Trees : Properties of Trees 2008 Zone A Q9

$a : b,\ e$

$b : a,\ c,\ d$

$c : b,\ d$

$d : b,\ c,\ e$

$e : d,\ a.$

Trees : Properties of Trees 2008 Zone A Q9

- Part IV
- Examiner's Commentaries : Then it is a question of adding another vertex and edge to each of these in all possible places and finally eliminating the isomorphic ones to do part (iv).

- Simpler Exercise (2006 Q9)

(b) (i) Draw the 3 non-isomorphic trees on 5 vertices.

(ii)Draw, on a separate diagram, all the non-isomorphic trees on 6 vertices, by adding a vertex to copies of the trees you have drawn or otherwise. [6]

Trees : Properties of Trees 2006 Zone A Q9

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Trees : Properties of Trees

Question 7 (a) (i) What properties must a graph have in order for it to be a tree?

(ii) Say, with reason, whether or not it is possible to construct a tree with degree sequence $4, 3, 3, 1, 1$.

(iii) Say, with reason, whether it is possible to construct a tree with degree sequence $4, 3, 2, 2, 1$.

(iv) What properties must a graph have in order for it to be a binary tree?

[5]

Trees : Properties of Trees

Trees : Properties of Trees (2005)

(c) Let G be the simple graph with vertex set $V(G) = \{a, b, c, d, e\}$ and adjacency matrix

$$\mathbf{A} = \boxed{\begin{array}{cccccc} & a & b & c & d & e \\ a & 0 & 1 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 1 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 0 & 1 & 0 & 1 & 0 \end{array}}.$$

(i) What do the numbers on the leading diagonal of this matrix tell you about the graph?

(ii) Say how the number of edges in G is related to the entries in the adjacency matrix $\mathbf{A}$ and calculate this number.

(iii) Draw G.

(iv) Find a spanning tree T_1 for G and give its degree sequence.

(v) Find a spanning tree T_2 for G which is **not** isomorphic to T_1 and give a reason why it is not isomorphic. [7]

Trees : Properties of Trees

Trees : Properties of Trees

Trees : Rooted Trees and Binary Trees

Terminology (Page 37)

- Root
- Nodes
- Key
- Children and Parents
- Ancestors and Descendants
- Height

Binary Search Tree

A Binary Search Tree is a binary tree in symmetric order

Symmetric order means that:

- every node has a key (or number)
- every node's key is
  - larger than all keys in its left subtree
  - smaller than all keys in its right subtree

The root ***r*** is the record

$$\# \lfloor (1+N)/2 \rfloor .$$

Binary Search Tree

If the first record in the ***subtree*** is ***#a*** and the last record is ***#b***, then the **root of the subtree is**

$$\# \lfloor (a+b)/2 \rfloor .$$

Trees : Binary Search Trees

The height h of a binary search tree with N records stored at internal nodes is

$$h = \lceil \log_2 (N + 1) \rceil .$$

Trees : Binary Search Trees (2011 Zone B)

(a) i. Construct a binary search tree to store the following ordered list of 12 integers at its internal nodes.

$$2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.$$

ii. What is the maximum number of comparisons needed in order to find an existing integer in the tree? [4]

Trees : Binary Search Trees

Trees : Binary Search Trees (2004)

(b) A binary search tree is designed to store an ordered list of 3000 records at its internal nodes.

(i) Find which record is stored at the root (level 0) of the tree and at each of the nodes at level 1.

(ii) What is the height of the tree?

(iii) What is the maximum number of comparisons needed in order to find an existing record in the tree? [5]

Trees : Binary Search Trees

Trees : Binary Search Trees

Question 9

(a) A binary search tree is designed to store an ordered list of 10000 records numbered 1,2,3,...10000 at its internal nodes.

(i) Draw levels 0, 1 and 2 of this tree showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.

(ii) What is the maximum number of comparisons that would have to be made in order to locate an existing record from the list of 10000? [4]

Trees : Binary Search Trees (2006)

Trees : Binary Search Trees

Proof. Since each vertex at level i has exactly two children at level $i+1$ for all levels $i = 0 \leq i \leq h-2$, it follows that the number of vertices at level $i+1$ is exactly twice the number of vertices at level i.

The only vertex at level 0 is the root, so the number of vertices at level 0 is $1 = 2^0$, hence we have exactly 2^i vertices at level i for all i where $0 \leq i \leq h-1$.

Trees : Binary Search Trees

Theorem 2.1 *Let n be a positive integer. Then*

(a) $\sum_{r=1}^{n} 1 = n.$

(b) $\sum_{r=1}^{n} r = n(n+1)/2.$

(c) $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6.$

(d) $\sum_{r=0}^{n} x^r = \frac{x^{n+1}-1}{x-1}$, *for any* $x \in \mathbb{R}$ *with* $x \neq 1$.

Trees : Binary Search Trees

(b) A binary search tree is designed to store an ordered list of 600 records at its internal nodes.

i. Which record is stored at the root (at level 0) of the tree?

ii. Which records are stored at level 1 of the tree?

iii. Determine the number of records stored at level 9 of the tree.

Trees : Binary Search Trees