Discrete Maths : Logic
Logical Propostions

www.Stats-Lab.com

Twitter: @StatsLabDublin

Logical Propostions

Let p, q and r be the following propositions concerning integers n (where $n > 1$):

- p : n is a prime factor of 36
- q : n is a prime factor of 4
- r : n is a prime factor of 9

Logical Propostions

n	p	q	r
1	1	1	1
2	1	0	1
3	0	1	1

Logical Propostions

For each of the following compound statements, express it using the propositions p,q and r, and the appropriate logical symbols, then given the truth table for it,

1) If n is a prime factor of 36, then n is a prime factor of 4 or n is a prime factor of 9
2) If n is a prime factor of 4 or n is a prime factor of 9, then n is a prime factor of 36

Logical Propostions

Suppose $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$.

Let p, q be the following propositions concerning the integer $n \in S$.

p: n is a multiple of two.
(i.e. $\{10, 12, 14, 16, 18\}$)

q: n is a multiple of three.
(i.e. $\{12, 15, 18\}$)

Logical Propositions

For each of the following compound statements find the sets of values n for which it is true.

(i) $p \vee q$: (p *or* q) : $\{10, 12, 14, 15, 16, 18\}$

(ii) $p \wedge q$: (p *and* q): $\{12, 18\}$

(iii) $p \oplus q$: (p *or* q, *but not both*) : $\{10, 14, 15, 16\}$

Recall p $= \{10, 12, 14, 16, 18\}$ and q $= \{12, 15, 18\}$

Logical Propositions

For each of the following compound statements find the sets of values n for which it is true.

(iv) $\neg p$ (i.e. *not-p*)

(v) $\neg p \vee q$ (i.e. *not-p or q*)

Recall S= $\{10, 11, 12, \ldots, 18, 19\}$, p $= \{10, 12, 14, 16, 18\}$ and q $= \{12, 15, 18\}$

Logical Propositions

For each of the following compound statements find the sets of values n for which it is true.

(iv) $\neg p$ (i.e. *not-p*)

(v) $\neg p \vee q$ (i.e. *not-p or q*)

Recall S= $\{10, 11, 12, \ldots, 18, 19\}$, p $= \{10, 12, 14, 16, 18\}$ and q $= \{12, 15, 18\}$

Logical Propositions

For each of the following compound statements find the sets of values n for which it is true.

(iv) $\neg p$ (i.e. *not-p*) $= \{11, 13, 15, 17, 19\}$

(v) $\neg p \vee q$ (i.e. *not-p or q*) $=$
$\{11, 12, 13, 15, 17, 18, 19\}$

Recall S$= \{10, 11, 12, \ldots, 18, 19\}$, p $= \{10, 12, 14, 16, 18\}$ and q $= \{12, 15, 18\}$

Logical Propositions

For each of the following compound statements find the sets of values n for which it is true.

(vi) $\neg p \wedge q$ (i.e. *not-p and q*) $= \{15\}$

(vii) $\neg p \oplus q$ (i.e. *not-p or q but not both*)$=$ $\{11, 12, 13, 17, 18, 19\}$

Recall S$=$ $\{10,\ 11,\ 12,\ \ldots,18,\ 19\}$, $\neg$p $= \{11, 13, 15, 17, 19\}$ and q $= \{12, 15, 18\}$

Logical Propositions

For each of the following compound statements find the sets of values n for which it is true.

(vi) $\neg p \wedge q$ (i.e. *not-p and q*)

(vii) $\neg p \oplus q$ (i.e. *not-p or q but not both*)

Recall S= $\{10, 11, 12, \ldots, 18, 19\}$, $\neg$p $= \{11, 13, 15, 17, 19\}$ and q $= \{12, 15, 18\}$

Discrete Maths : Logic
Proof With Truth Tables

www.Stats-Lab.com

Twitter: @StatsLabDublin

Proof With Truth Tables

Let p and q be propositions. Use**Truth Tables** to prove that

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Proof With Truth Tables

Important

Remember to make a comment at the end to say why the table proves that the two statements are logically equivalent.

For example : *"Since the relevant columns are identical, then it can be said that both sides of the equation are equivalent"*.

Proof With Truth Tables

Left hand side of expression : p *implies* q.

$$p \rightarrow q$$

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Proof With Truth Tables

Right hand side of expression : *not-q implies not-p*

$$\neg q \rightarrow \neg p$$

p	q	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

Proof With Truth Tables

Side by Side

$$p \to q \equiv \neg q \to \neg p$$

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

(only “difference” is first and last rows)

Discrete Maths : Logic

Laws of Logic

www.Stats-Lab.com

Twitter: @StatsLabDublin

Laws of Logic

Construct a truth table for each of the following compound statement and hence find simpler propositions to which it is equivalent.

(i) $p \vee F$

(ii) $p \wedge T$

Laws of Logic

Solutions

p	T	$p \vee T$	$p \wedge T$
0	1		
1	1		

Solutions

p	T	$p \vee T$	$p \wedge T$
0	1	1	0
1	1	1	1

(i) $p \vee F \equiv T$

(ii) $p \wedge T \equiv p$

Laws of Logic

Construct a truth table for each of the following compound statement and hence find simpler propositions to which it is equivalent.

(iii) $p \vee F$

(iv) $p \wedge F$

Solutions

p	F	$p \vee F$	$p \wedge F$
0	0		
1	0		

Laws of Logic

Solutions

p	F	$p \vee F$	$p \wedge F$
0	0	0	0
1	0	1	0

(iii) $p \vee F = p$

(iv) $p \wedge F = F$

Discrete Maths : Logic

Contra-positives

www.Stats-Lab.com

Twitter: @StatsLabDublin

Contra-positive

Write the contra-positive of each of the following statements:

- If n= 12, then n is divisible by 3.
- If n=5, then n is positive.
- If the quadrilateral is square, then four sides are equal.

Contra-positives

Solutions

- If n is not divisible by 3, then n is not equal to 12.
- If n is not positive, then n is not equal to 5.
- If the four sides are not equal, then the quadrilateral is not a square.

Discrete Maths : Logic

Truth Sets

www.Stats-Lab.com

Twitter: @StatsLabDublin

Truth Sets

2009

Let $n = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p and q be the following propositions concerning the integer n.

- p: n is even,
- q: $n \geq 5$.

By drawing up the appropriate truth table nd the truth set for each of the propositions $p \vee \neg q$ and $\neg q \rightarrow p$

Truth Sets

n	p	q	$\neg q$	$p \vee \neg q$
1	0	0	1	
2	1	0	1	
3	0	0	1	
4	1	0	1	
5	0	1	0	
6	1	1	0	
7	0	1	0	
8	1	1	0	
9	0	1	0	

Truth Sets

n	p	q	$\neg q$	$p \vee \neg q$
1	0	0	1	1
2	1	0	1	0
3	0	0	1	1
4	1	0	1	0
5	0	1	0	1
6	1	1	0	1
7	0	1	0	1
8	1	1	0	1
9	0	1	0	1

$$\text{Truth Set} = \{1, 3, 5, 6, 7, 8, 9\}$$

Truth Sets

n	p	q	$p \to q$	$q \to p$
1	0	0	1	0
2	1	0	1	0
3	0	0	1	0
4	1	0	1	0
5	0	1	0	1
6	1	1	1	0
7	0	1	0	1
8	1	1	1	0
9	0	1	0	1

$$\text{Truth Set} = \{5, 7, 9\}$$

Biconditional

See Section 3.2.1.

Use truth tables to prove that $\neg p \leftrightarrow \neg q$ is equivalent to $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Biconditional

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

Discrete Maths : Logic

Logic Networks

www.Stats-Lab.com

Twitter: @StatsLabDublin

Logic Networks

(2008 Q3b)

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $\neg(p \wedge \not q)$ (i.e. $\equiv p \rightarrow q$).

Logic Networks

Logic Gates

- AND
- OR
- NOT

Examiner's Comments: *Many diagrams were carefully and clearly drawn and well labelled, gaining full marks. The logic table was also well done by most, but there were a few marks lost in the final part by failing to deduce that since the columns of the table are identical the expressions are equivalent.*

1.8 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $(p \wedge q) \vee \neg q$ (i.e. $\equiv p \to q$).

Important Label each of the gates appropriately and label the diagram with a symblic expression for the output after each gate.