HIBERNIA COLLEGE DUBLIN

EST.MM

Computing

Tutor : Kevin O’Brien

Tutorial: Maths for Computing

Trees

- A lot of concepts and definitions follows from Section 5 : ***Introduction to Graph Theory***

Syllabus

- Properties of Trees
- Rooted Trees and Binary Trees
- Binary Search Trees

Trees : Properties of Trees

1) Characteristics of a Tree

A tree is a connected graph that contains no cycles. A tree has no loops and no multiple edges. All trees are simple graphs.

2) Path Graphs

A tree that contains only vertices of degree one or two is called a ***path graph***. The length of a path graph is the number of edges in it.

3) Number of Edges

(***Theorem***) Let T be a tree with n vertices. Then T has $n - 1$ edges.

Therefore, the sum of the degree sequence is necessarily ***2n-2***

Trees : Properties of Trees

4) Spanning Subgraphs

The graph H is a **subgraph** of a graph G if H's vertices are a subset of the G's vertex, its edges are a subset of the edge set of G, and each edge of H has the same end-vertices in G and H.

H is called a **spanning subgraph** of G if the vertices of H are the same as the vertices of G.

5) Spanning Trees

If H is a spanning subgraph which is also a tree, then H is said to be a spanning tree of G. (G does not need to be a tree)

Trees : Properties of Trees 2008 Zone A Q9

Question 9

(a) A graph with 5 vertices: a, b, c, d, e has the following adjacency list:

$a : b,\ e$

$b : a,\ c,\ d$

$c : b,\ d$

$d : b,\ c,\ e$

$e : d,\ a.$

(i) Draw this graph, G.

(ii) Draw a spanning tree of G.

(iii) Draw all the non-isomorphic spanning trees of G and call this set S.

(iv) How many non-isomorphic trees can be created by adding a new vertex and edge to the trees in S. [6]

Trees : Properties of Trees 2008 Zone A Q9

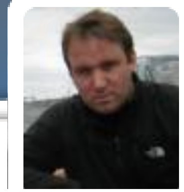
$a : b,\ e$

$b : a,\ c,\ d$

$c : b,\ d$

$d : b,\ c,\ e$

$e : d,\ a.$



2001

Question 10

(a) Draw the tree T with vertex set $V(T) = \{v_1, v_2, v_3, v_4\}$ and edge set $E(T) = \{v_1v_2, v_2v_3, v_3v_4\}$.

(i) Construct all the *non-isomorphic* trees with five vertices which can be obtained by attaching a new vertex of degree one to a vertex of T. [2]

(ii) Explain briefly why the trees you obtain in (i) are not isomorphic to each other. [2]

(iii) Constuct a tree with five vertices which is not isomorphic to any tree you constructed in (i). [2]

2001

Trees : Properties of Trees 2008 Zone A Q9

- Part IV
- Examiner's Commentaries : Then it is a question of adding another vertex and edge to each of these in all possible places and finally eliminating the isomorphic ones to do part (iv).

- Simpler Exercise (2006 Q9)

(b) (i) Draw the 3 non-isomorphic trees on 5 vertices.

(ii)Draw, on a separate diagram, all the non-isomorphic trees on 6 vertices, by adding a vertex to copies of the trees you have drawn or otherwise. [6]

2001

(b) A binary search tree is designed for an ordered list of 3185 records.

(i) Find which record is stored at the root (at level 0) of the tree and at each of the nodes at level 1. [2]

(ii) What is the maximum number of comparisons that would need to be made to match a target with any existing record? [2]

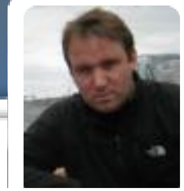
2002

Question 9 (a) What properties must a graph satisfy in order for it to be a *tree*? [2]

(b) (i) Design a balanced binary search tree for an ordered list of 11 records. Label the records $1, 2, \ldots, 11$ in your tree. [4]

(ii) What is the height of the tree that you have constructed? [1]

(iii) What is the maximum number of comparisons that would have to be made to match any existing record? This number should include the final comparison that determines the match. [1]



2002

2002

(c) Calculate the least height of a binary search tree that has 1000 records stored at its internal nodes. [2]

2003

Question 8 (a) In a tennis match two players, A and B, play up to 3 sets and the winner of the match is the first player to win a total of 2 sets. Each set is either won or lost, it cannot be drawn.

(i) Use a binary tree to model the possible outcomes of the match. [2]

(ii) The probability of A winning any given set is $\frac{3}{5}$ and the probability of B winning any given set is $\frac{2}{5}$. Find the probablities that: the match is won by A in 3 sets; the match lasts for 2 sets. Show your calculations clearly. [4]

2003

(b) Construct a balanced binary search tree for an ordered list of 15 records, labelling them $1, 2, 3, \ldots 15$ in your tree. What is the maximum number of comparisons a computer would have to make to match any existing record? [4]

2003

2004

Question 7 (a) (i) What properties must a graph have in order for it to be a tree?

(ii) Say, with reason, whether or not it is possible to construct a tree with degree sequence $4, 3, 3, 1, 1$.

(iii) Say, with reason, whether it is possible to construct a tree with degree sequence $4, 3, 2, 2, 1$.

(iv) What properties must a graph have in order for it to be a binary tree?

[5]

2004

(b) A binary search tree is designed to store an ordered list of 3000 records at its internal nodes.

(i) Find which record is stored at the root (level 0) of the tree and at each of the nodes at level 1.

(ii) What is the height of the tree?

(iii) What is the maximum number of comparisons needed in order to find an existing record in the tree? [5]

2004

2005

Question 9 (a) What two properties must a graph, G, satisfy in order for it to be a tree? [1]

(b) Let H be a subgraph of a graph G. Explain what it means for H to be a spanning tree of G. [2]

2005

(c) Let G be the simple graph with vertex set $V(G) = \{a, b, c, d, e\}$ and adjacency matrix

$$\mathbf{A} = \boxed{\begin{array}{cccccc} & a & b & c & d & e \\ a & 0 & 1 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 1 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 0 & 1 & 0 & 1 & 0 \end{array}}.$$

(i) What do the numbers on the leading diagonal of this matrix tell you about the graph?

(ii) Say how the number of edges in G is related to the entries in the adjacency matrix $\mathbf{A}$ and calculate this number.

2005

(iii) Draw G.

(iv) Find a spanning tree T_1 for G and give its degree sequence.

(v) Find a spanning tree T_2 for G which is **not** isomorphic to T_1 and give a reason why it is not isomorphic. [7]

2006

Question 9

(a) A binary search tree is designed to store an ordered list of 10000 records numbered 1,2,3,...10000 at its internal nodes.

(i) Draw levels 0, 1 and 2 of this tree showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.

(ii) What is the maximum number of comparisons that would have to be made in order to locate an existing record from the list of 10000? [4]

2006

(b) (i) Draw the 3 non-isomorphic trees on 5 vertices.

(ii)Draw, on a separate diagram, all the non-isomorphic trees on 6 vertices, by adding a vertex to copies of the trees you have drawn or otherwise. [6]

2007

Question 9

(a) Given the graph G with vertices v_1, $v_2, \ldots v_7$ and adjacency list

$v_1 : v_2, v_4$

$v_2 : v_1, v_3$

$v_3 : v_2, v_4$

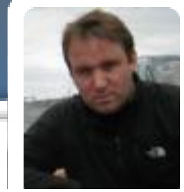
$v_4 : v_1, v_3, v_5$

$v_5 : v_4, v_6$

$v_6 : v_5, v_7$

$v_7 : v_5, v_6.$

(i) Draw this graph.



2007

(ii) Say how many edges there are in a tree with n vertices. Hence explain how many edges must be removed from G to create a spanning tree.

(iii) The graph G has precisely 12 different spanning trees, list the twelve distinct pairs of edges which, when removed, give the 12 spanning trees, $T_1,\ T_2, \ldots.\ T_{12}$.

(iv) By partitioning the set $\{T_1,\ T_2, \ldots.\ T_{12}\}$ into subsets where the trees of a subset are all isomorphic to one another, while the two trees from different subsets are non-isomorphic, or otherwise, draw the four non-isomorphic spanning trees of G. [7]

2007

(b) A binary search tree is designed to store an ordered list of 50000 records, numbered 1,2,3....50000 at its internal nodes.

(i) Draw levels 0, 1 and 2 of this tree, showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.

(ii) What is the maximum number of comparisons that would have to be made in order to locate an existing record from the list of 50000? [3]

2007