UNIVERSITY OF LONDON

291 0102E

B. Sc. Examination 2006

for External Students

COMPUTING AND INFORMATION SYSTEMS

COMPUTING

CIS102e Mathematics for Computing

Duration: 3 hours

Date and time: Tuesday 16 May 2006: 2.30 - 5.30pm

There are TEN questions on this paper.

Full marks will be awarded for complete answers to TEN questions.

Electronic calculators may be used. The make and model should be specified on the script and the calculator must not be programmed prior to the examination.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

(a) Consider a set $S = \{1, 2, 3, 4, 5, 6\}$. R_1 is the relation such that xR_1y if $2x - y = 3$ and R_2 is the relation such that xR_2y if $x \bmod 3 = y \bmod 3$ (the remainder when x is divided by 3 equals the remainder when y is divided by 3), for all x and y in S.

(i) Illustrate the relations R_1 and R_2 , using a separate digraph for each.

(ii) Complete the following table:

	Reflexive	Symmetric	Anti-symmetric	Transitive
R_1	$\times$			
R_2		$\checkmark$		

(iii) One of these relations is an equivalence relation. Say which relation this is and give the partition on S created by this relation. [6]

(b) Another relation is defined on a population of people such that x is related to y if x is a sister of y, for all x and y in the population. Say whether or not this relation is reflexive, symmetric or transitive, explaining briefly what this means in terms of the relation in each case.(Note: in this instance "sister of" means x and y have the same parents and are both female.) [4]

Question 2

(a) A binary search tree is designed to store an ordered list of 5000 records numered 1,2,...5000 at its internal nodes.

(i) Draw levels 0, 1 and 2 of this tree showing which record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.

(ii) What is the maximum number of comparisons that would have to be made in order to locate an existing record from this list of 5000? [4]

(b) (i) Draw the 3 non-isomorphic trees on 5 vertices.

(ii) Draw, on a separate diagram, all the non-isomorphic trees on 6 vertices, by adding a vertex to copies of the trees you have drawn or otherwise. [6]

Question 3

(a) (i) A simple, connected graph has 5 vertices, all having the same degree d. State the possible values of d and for each value also give the number of edges in the corresponding graph.

(ii) Another simple, connected graph has 8 vertices, all having the same degree, n. Draw such a graph when $n = 3$ and state the other possible values of n.[4]

(b) The following adjacency matrix shows several African countries and an entry of 1 indicates the countries concerned share a common border, whereas a zero entry indicates they do not.

$$\begin{array}{c} \\ \text{Algeria} \\ \text{Libya} \\ \text{Mali} \\ \text{Niger} \\ \text{Chad} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Algeria} & \text{Libya} & \text{Mali} & \text{Niger} & \text{Chad} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(i) Write down the countries which share a border with Chad.

(ii) Is this matrix symmetric or not? Give an example to show what this means.

(iii) Draw the graph, G, associated with this matrix.

(iv) Explain how the number of edges of the graph can be calculated from the entries in the matrix and find this number.

(v) Draw another graph, H, which has 5 vertices and the same degree sequence as G but is not isomorphic to it. Give a reason why G and H are not isomorphic. [6]

Question 4

(a) A logic network accepts inputs p and q, which may each independently have the value 0 or 1, and gives as the final output

$$(p \wedge q) \vee \neg p.$$

(i) Draw this network. Label each of the gates appropriately and also label the diagram with a symbolic expression for the output after each gate.

(ii) Construct a truth table to show the value of the output corresponding to each combination of values (0 or 1) for the inputs p and q.

(iii) Hence, or otherwise, find a simpler expression that is logically equivalent to the final output. [5]

(question continues on next page)

(b) Let p be the proposition "this animal is an elephant" and q be the proposition "this animal has a trunk".

(i) Expain in words the meaning of the logical statement $p \rightarrow q$.

(ii) Write the contrapositive of this statement in logical symbols and explain its meaning in an English sentence.

(iii) Write each of the following as a logical statement involving p and q :

"This animal is an elephant and it does not have a trunk";
"This animal neither is an elephant nor has a trunk". [5]

Question 5

(a) (i) Write down the first three and last three terms of the series given by $\sum_{k=1}^{50}(4k-1)$.

(ii) Write the terms of this series as a recurrence relation that gives u_{n+1} in terms of u_n and give the value of the initial term. [3]

(b) Write the following series in $\sum$ notation:

(i) $3+7+9+11+...+399$

(ii) $199+203+207+...+299$.

Use the formula $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ to evaluate the first of these two sums. [4]

(c) It can be proved by induction that the series $2+5+8+...$ has the sum to r terms given by S_r, where

$$S_r = \frac{r+3r^2}{2}$$

Use this result to evaluate the following sums:

(i) $2+5+8+...+200$

(ii) $203+206+209...+500$. [3]

Question 6

(a) Given $A = \{1, 2, 3, 4\}$ and $B = \{a, e, i, o, u\}$, a function f is defined as a subset of $A \times B$, where f consists of the ordered pairs:

$$(1, i), (2, o), (3, o), (4, u).$$

(i) Illustrate this function using an arrow diagram.

(ii) List the domain, co-domain and range of this function,

(iii) Say whether or not the function f has the onto property, justifying your answer.

(iv) Which pair or pairs should be altered in order to make the function have the one-to-one property?

[5]

(b) Another function g is given by

$$g(n) = n \bmod 5 \text{ where } g : \mathbb{Z}^+ \to \mathbb{Z}$$

i.e. $g(n)$ is the remainder when n is divided by 5, so that $g(10) = 0$ and $g(6) = 1$.

(i) Find $g(12)$ and $g(16)$.

(ii) List the ancestors of 0.

(iii) List the range of g, and say whether or not g is onto, justifying your answer.

(iv) Say whether or not g is a one-to-one function, giving a reason for your answer. [5]

Question 7

(a) Consider the following matrices

$$\mathbf{A} = \begin{pmatrix} -1 & 1 & 0 \\ 2 & 0 & -3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 4 \\ -2 & 0 \\ 1 & 3 \end{pmatrix} \quad \text{and} \quad \mathbf{C} = \begin{pmatrix} -2 & 0 \\ 1 & 3 \end{pmatrix}.$$

(i) Compute the matrix products $\mathbf{AB}$ and $\mathbf{C}^2$.

(ii) Find a matrix $\mathbf{X}$ such that $\mathbf{X} = \mathbf{AB} + \mathbf{C}$

(iii) Find a matrix $\mathbf{Z}$ such that $\mathbf{AB} + \mathbf{Z} = \mathbf{C}$ [4]

(question continues on next page)

(b) Consider the system of linear equations given by the matrix equation:

$$\begin{pmatrix} 1 & -1 & -1 \\ 2 & 1 & -1 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}.$$

(i) Write down the 3 linear equations which correspond to the above equation.

(ii) Use Gaussian elimination to solve the system. [6]

Question 8

(a) (i) Make a table to show how each of the hexadecimal numbers

$$(0)_{16}, (1)_{16}, (2)_{16}, ...(\mathrm{F})_{16}$$

can be represented as a 4 bit binary string.

(ii) Hence express the binary number 110111.01 in hexadecimal and the hex number 3F.7 in binary. [4]

(b) Compute the following, showing all you working:

(i) $(11010 \times 11)_2 - (1011)_2$;

(ii) $(79)_{16} + (\mathrm{CB})_{16}$. [3]

(c) Given x is the irrational positive number $\sqrt{2}$:

(i) express x^6 in binary notation;

(ii) is x^6 a rational number?

(iii) write $\left(\frac{1}{x}\right)^4$ in the form 2^y where $y \in \mathbb{Q}$. [3]

Question 9

(a) Describe the following sets using the listing method:

(i) $\{5^n : -1 \leq n \leq 4, n \in \mathbb{Z}\}$ (ii) $\{\frac{m}{2} : 2 \leq m \leq 7, m \in \mathbb{Z}\}$ [2]

(b) (i) Given 3 sets, A, B and C, subsets of a universal set $\mathcal{U}$, draw a labelled Venn diagram and shade the region corresponding to $(A' \cap B) \cup C$.

(ii) Show, using membership tables or Venn diagrams, that this region is equivalent to $(A' \cup C) \cap (B \cup C)$.

(iii) What law does this illustrate? [5]

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(c) Given the sets

$$\begin{aligned} \mathcal{U} &= \{a, b, c, d, e, f, g, h, i\} \\ A &= \{a, b, e, f, i\} \\ B &= \{c, e, g, i\} \\ C &= \{e, f, g, h, i\}. \end{aligned}$$

(i) List separately the elements of $A' \cup C$ and $B \cup C$.

(ii) Describe, as simply as you can in terms of set operations on A, B and C, the sets $\{e, h\}$ and $\{a, b, c, e, f, g, h, i\}$. [3]

Question 10 A 4 letter code is made from the letters {l,m,n,o,p}, where repetitions are allowed and the order of the letters in the code is significant - for example "l,l,p,n" is a different code to "l,n,p,l".

Let $\mathcal{U}$ be the set of all such codes.

Let $\mathcal{V}$ be the set of all such codes beginning with a vowel.

Let $\mathcal{P}$ be the set of all such codes which are palindromic.

(A palindromic code is a string of letters which read the same backwards as forwards, for example "m,p,l,p,m" is a 5 letter palindromic code.)

(a) How many elements are there in the sets $\mathcal{U}, \mathcal{V}$ and $\mathcal{P}$? [3]

(b) Draw a Venn diagram to show the relationship between the sets $\mathcal{U}$, $\mathcal{V}$ and $\mathcal{P}$. Show the relevant number of elements in each region of your diagram. [4]

(c) What is the probability that a code chosen in this way:

(i) begins with a vowel;

(ii) is palindromic;

(iii) both begins with a vowel and is palindromic? [3]

END OF EXAMINATION