HibColl Videos

1.A Converting from Decimal to Binary 1.B Converting from Decimal to Hexadecimal 1.C Converting from Binary to Decimal 1.D Converting from Hexadecimal to Decimal 1.E Binary Addition 1.F Binary Subtraction

2.A Membership Tables 2.B Venn Diagrams 2.C Set differences and symmetric difference 2.D 2.E

Chapter 1

Section 1

1.0.1 Significant Digits

There are three rules on determining how many significant figures are in a number: Non-zero digits are always significant. Any zeros between two significant digits are significant. A final zero or trailing zeros in the decimal portion ONLY are significant.

1.1 Floating Point Notation

In computing, floating point describes a method of representing an approximation of a real number in a way that can support a wide range of values.

The numbers are, in general, represented approximately to a fixed number of significant digits (the mantissa) and scaled using an exponent.

In essence, computers are integer machines and are capable of representing real numbers only by using complex codes. The most popular code for representing real numbers is called the IEEE Floating-Point Standard . The term floating point is derived from the fact that there is no fixed number of digits before and after the decimal point; that is, the decimal point can float. There are also representations in which the number of digits before and after the decimal point is set, called fixed-point representations. In general, floating-point representations are slower and less accurate than fixed-point representations, but they can handle a larger range of numbers.

Chapter 2

Set Theory

2.0.1 Cartesian Product

- Let X and Y be sets.
- The **cartesian product** $X \times Y$ is the set whose elements are **all** of the ordered pairs of elements (x, y) where $x \in X$ and $y \in Y$.
- Let $X = \{a, b, c\}$
- Let $Y = \{0, 1\}$
- The cartesian product $X \times Y$ is therefore:
- Importantly $X \times Y \neq Y \times X$
- Recall: Let $X = \{a, b, c\}$ and let $Y = \{0, 1\}$
- The cartesian product $Y \times X$ is therefore:

3.3.A
4.4.B Logarithms 4.4.C
5.2.A Graph Theory 5.2.B

2.0.2 The Cartesian Product

Exercises

Discrete Maths

A binary relation on a set A is the collection of ordered pairs of elements of A. In other words, it is the subset of the cartesian product A2= AA

Cartesian Product

This is a direct product pf 2 sets

XY = (x,y)— xXandyY

4 suits of cards and 13 Ranks, therefore 52 element cartesian prodcut.

N.B AB BA A=A =

Cartesian product is not associative

Chapter 3

graph theory

Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4.

3.1 Set Theory

1. The Universal Set $\mathcal{U}$
2. Union
3. Intersection
4. Set Difference
5. Relative Difference

Question 2B 2010 Zone A

- Let A and B be subsets of the a universal set U.
- Use membership tables to prove that $(A \cup B')' = A' \cap B$
- Shade the regions corresponding to this set on a Venn Diagram

A	B	B'	$A \cup B'$	$(A \cup B')'$
0	0	1	1	0
0	1	0	0	1
1	0	1	1	0
1	1	0	1	0

A	B	A'	$A' \cap B$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	0	0

Given the universal set U and subsets A and B, list the set $(A \cup B')'$

- $U = \{1, 2, \ldots, 8, 9\}$
- $A = \{2, 4, 6, 8\}$
- $B = \{4, 5, 6, 7\}$
- $B' = \{1, 2, 3, 8, 9\}$
- $A \cup B' = \{1, 2, 3, 4, 6, 8, 9\}$
- $(A \cup B')' = \{5, 7\}$

3.1.1 Number Sets

Blackboard Bold Typeface

- Conventionally the symbols for numbers sets are written in a special typeface, known as **blackboard bold**.
- Examples : $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$.

3.1.2 Number Sets

Natural Numbers ($\mathbb{N}$)

- The whole numbers from 1 upwards.
- The set of natural numbers is
$$\{1, 2, 3, 4, 5, 6, \ldots\}$$
- In some branches of mathematics, 0 might be counted as a natural number.
$$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

3.1.3 Number Sets

Integers ($\mathbb{Z}$)

- The integers are all the whole numbers, all the negative whole numbers and zero.
- The set of integers is
$$\{\ldots, -4, -3, -2, -1,\ 0,\ 1,\ 2,\ 3, \ldots\}$$
- The notation $\mathbb{Z}$ is from the German word for numbers: *Zahlen*.
- All natural numbers are integers.
$$\mathbb{Q} \subset \mathbb{Z}$$

3.1.4 Number Sets

Integers ($\mathbb{Z}$)

- Natural numbers may also be referred to as positive integers, denoted $\mathbb{Z}^+$.
(note the superscript)
- Negative integers are denoted $\mathbb{Z}^-$.
$$\{\ldots, -4, -3, -2, -1\}$$

3.1.5 Number Sets

Integers ($\mathbb{Z}$)

- 0 is neither positive nor negative. The following set of non-negative numbers

$$\{0, 1, 2, 3, 4, 5, 6, \dots\}$$

  might be denoted $0 \cup \mathbb{Z}^+$

- $\cup$ is the mathematical symbol for **union**.

3.1.6 Number Sets

Rational Numbers ($\mathbb{Q}$)

- Rational numbers, also known as quotients, are numbers you can make by dividing one integer by another (but not dividing by zero).
- If a number can be expressed as one integer divided by another, it is a rational number.

$$\mathbb{Q} = \left\{ \frac{p}{q} \middle| p \in \mathbb{Z},\ q \in \mathbb{Z},\ q \neq 0 \right\}$$

3.1.7 Number Sets

Rational Numbers ($\mathbb{Q}$)

- All integers are rational numbers

$$\mathbb{Z} \subset \mathbb{Q}$$

  (and by extension all natural numbers are rational numbers too)

- Examples of rational numbers

$$9500,\ 7,\ \frac{1}{2},\ \frac{3}{7},\ -2.6,\ 0.001$$

3.1.8 Number Sets

Irrational Numbers

- A number that can not be written as the ratio of two integers is known as an irrational number.
- Two famous examples of irrational numbers are π and $\sqrt{2}$.

$$\pi = 3.141592\dots$$

$$\sqrt{2} = 1.41421\dots$$

3.1.9 Number Sets

Real Numbers ($\mathbb{R}$)

- Irrational numbers are types of real numbers.
- Rational numbers are real numbers too.

$$\mathbb{Q} \subset \mathbb{R}$$

- A real number is simply any point anywhere on the number line.

3.1.10 Number Sets

Real Numbers ($\mathbb{R}$)

- There are numbers that are not real numbers, for example **imaginary numbers**, but we will not cover them in this presentation.

2010 Zone B Q 1
5n+1 Rules of Inclusion method
$A = \{5n + 1 : n \in Z\}$

Floating Point Notation

(Demonstration on white board)

2011 Zone A question 1d

Showing your workings, express the repeating decimal 0.012012012012... as a rational number in its simplest form.

- x = 0.012012012012...
- 10x = 0.12012012012... (not particularly useful)
- 100x = 1.2012012012... (not particularly useful either)
- 1000x= 12.012012012... (very useful)
- 999x = 12
- x= 12/999 = 4/333 (Answer!)

2008 Zone A question2a

$B = \{3n - 1 : n \in Z^+\}$ Describe the set B using the listing method

- Let $n = 1$. Consequently $3(1) - 1 = 2$
- Let $n = 2$. Likewise $3(2) - 1 = 5$

- Let $n = 3$. $3(3) - 1 = 8$
- The repeated differences are 3. The next few values are 11, 14 and 17
- So by the listing method $B = \{2, 5, 8, 11, 14, 17, \ldots\}$

$A = \{3, 5, 7, 9, ldots\}$ Describe the set A using the rules of inclusion method

- The repeated differences are 2.
- We can say the rule has the form $2n + k$
- For the first value n=1. Therefore $2 + k = 3$
- Checking this , for the second value , n=2. Therefore $4 + k = 5$
- Clearly k = 1.
- $A = \{2n + 1 : n \in Z^+\}$
- So by the listing method $B = \{2, 5, 8, 11, 14, 17, \ldots\}$

3.2 Set Theory

1. The Universal Set $\mathcal{U}$
2. Union
3. Intersection
4. Set Difference
5. Relative Difference

Set theory

3.2.1 Specifying Sets (2.1)

1. Listing Method
2. Rules of Inclusion

Subsets (2.2) Subsets and Proper Subsets Cardinality of a Set Power Set

3.2.2 Set operations (2.3)

complement union intersection set difference
venn diagrams 8 Disjoint Regions
DE Muorgan's Laws (Useful for Propositions)
membership tables
proof by truth tables AND OR NOT Set difference symmetric difference

Harmonic Mean

$$H_x = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

2,4,6,8 ¿ 1/mean(1/a) [1] 3.84 ¿ 1/a [1] 0.5000000 0.2500000 0.1666667 0.1250000 ¿ sum(1/a) [1] 1.041667 ¿ sum(1/a)*24 [1] 25 ¿ 96/25 [1] 3.84

3.2.3 Functions

Consider the floor function $f : R \to Z$ given the rule

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

1. evaluate $f(6)$ and $f(-6)$
2. Show that f(x) is not one-to-one
3.

3.2.4 Functions

Evaluate $f(6)$

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

$$f(6) = \lfloor \frac{\mathbf{6}+1}{2} \rfloor = \lfloor \frac{7}{2} \rfloor$$

$$\lfloor 3.5 \rfloor = 3$$

3.2.5 Functions

Evaluate $f(-6)$

$$f(x) = \lfloor \frac{x+1}{2} \rfloor$$

$$f(6) = \lfloor \frac{\mathbf{-6}+1}{2} \rfloor = \lfloor \frac{-5}{2} \rfloor$$

$$\lfloor -2.5 \rfloor = -3$$

3.2.6 Discrete Maths : Relations

- A relation R from a set A to a set B is a subset of the **cartesian product** A x B.
- Thus R is a set of **ordered pairs** where the first element comes from A and the second element comes from B i.e. (a, b)

3.2.7 Discrete Maths : Relations

- If $(a, b) \in R$ we say that a is related to b and write aRb.
- If $(a, b) \notin R$, we say that a is not related to b and write aRb. CHECK
- If R is a relation from a set A to itself then we say that "R is a relation on A".

3.2.8 Discrete Maths : Relations

Example

- Let $A = \{2, 3, 4, 6\}$ and $B = \{4, 6, 9\}$
- Let R be the relation from A to B defined by $\boldsymbol{xRy}$ if x divides y exactly.

3.2.9 Discrete Maths : Relations

Example

- Let $A = \{2, 3, 4, 6\}$ and $B = \{4, 6, 9\}$
- Let R be the relation from A to B defined by $\boldsymbol{xRy}$ if x divides y exactly.
- Then
$$R = (2, 4), (2, 6), (3, 6), (3, 9), (4, 4), (6, 6)$$

Chapter 4

Logic

4.1 Logic Proposition

Let p, Q and r be the following propositions concerning integers n:

- p : n is a factor of 36 (2)
- q : n is a factor of 4 (2)
- r : n is a factor of 9 (3)

n	p	q	r
1	1	1	1
2	1	0	1
3	0	1	1
4	1	0	1
6	0	0	1
9	0	1	1
12	0	0	1
18	0	0	1
36	0	0	1

For each of the following compound statements, express it using the propositions P q and r, andng logical symbols, then given the truth table for it,

1) If n is a factor of 36, then n is a factor of 4 or n is a factor of 9
2) If n is a factor of 4 or n is a factor of 9 then n is a factor of 36

Part 1 : Logic

1.1 2010 Question 3

Let $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ and let p, q be the following propositions concerning the integer $n \in S$.

- p: n is a multiple of two. (i.,18e. $\{10, 12, 14, 16, 18\}$)
- q: n is a multiple of three. i.e. $\{12, 15, 18\}$

For each of the following compound statements find the sets of values n for which it is true.

- $p \vee q$: (p or q : 10 12 14 15 16 18)
- $p \wedge q$: (p and q: 12 18)
- $\neg p \oplus q$: (not-p or q, but not both)
  - $\neg p$ not-p = $\{1113151719\}$
  - $\neg p \vee q$ not-p or q $\{11121315171819\}$
  - $\neg p \wedge q$ not-p and q $\{15\}$
  - $\neg p \oplus q = \{11, 12, 13, 17, 18, 19\}$

1.2 2010 Question 3

Let p and q be propositions. Use Truth Tables to prove that

$$p \rightarrow q \equiv \neg q \rightarrow \neg$$

Important Remember to make a comment at the end to say why the table proves that the two statements are logically equivalent. e.g. *since the columns are identical both sides of the equation are equivalent.*

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

p	q	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

(Key "difference" is first and last rows)

1.3 Membership Tables for Laws

Page 44 (Volume 1) Q8. Also see Section 3.3 Laws of Logic.

Construct a truth table for each of the following compound statement and hence find simpler propositions to which it is equivalent.

- $p \vee F$
- $p \wedge T$

Solutions

p	T	$p \vee T$	$p \wedge T$
0	1	1	0
1	1	1	1

- Logical OR: $p \vee T = T$
- Logical AND: $p \wedge T = p$

p	F	$p \vee F$	$p \wedge F$
0	0	0	0
1	0	1	0

- Logical OR: $p \vee F = p$
- Logical AND: $p \wedge F = F$

1.4 Propositions

Page 67 Question 9 Write the contrapositive of each of the following statements:

- If n= 12, then n is divisible by 3.
- If n=5, then n is positive.
- If the quadrilateral is square, then four sides are equal.

Solutions

- If n is not divisible by 3, then n is not equal to 12.
- If n is not positive, then n is not equal to 5.
- If the four sides are not equal, then the quadrilateral is not a square.

1.5 Truth Sets

2009

Let $n = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p, q be the following propositions concerning the integer n.

- p: n is even,
- q: $n \geq 5$.

By drawing up the appropriate truth table nd the truth set for each of the propositions $p \vee \neg q$ and $\neg q \rightarrow p$

n	p	q	$\neg q$	$p \vee \neg q$
1	0	0	1	1
2	1	0	1	0
3	0	0	1	1
4	1	0	1	0
5	0	1	0	1
6	1	1	0	1
7	0	1	0	1
8	1	1	0	1
9	0	1	0	1

$$\text{Truth Set} = \{1, 3, 5, 6, 7, 8, 9\}$$

n	p	q	$q \to p$	$q \to p$
1	0	0	1	0
2	1	0	1	0
3	0	0	1	0
4	1	0	1	0
5	0	1	0	1
6	1	1	1	0
7	0	1	0	1
8	1	1	1	0
9	0	1	0	1

$$\text{Truth Set} = \{5, 7, 9\}$$

1.6 Biconditional

See Section 3.2.1.

Use truth tables to prove that $\neg p \leftrightarrow \neg q$ is equivalent to $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

1.7 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $\neg(p \wedge \not q)$ (i.e. $\equiv p \to q$).

Logic Gates

- AND
- OR
- NOT

Examiner's Comments:*Many diagrams were carefully and clearly drawn and well labelled, gaining full marks. The logic table was also well done by most, but there were a few marks lost in the final part by failing to deduce that since the columns of the table are identical the expressions are equivalent.*

1.8 2008 Q3b Logic Networks

Construct a logic network that accepts as input p and q, which may independently have the value 0 or 1, and gives as final input $(p \wedge q) \vee \neg q$ (i.e. $\equiv p \to q$).

Important Label each of the gates appropriately and label the diagram with a symblic expression for the output after each gate.

Prepositional Logic

4.1.1 five basic connectives

Reflexive, Symmetric and Transitive

- Reflexive
- Symmetric
- Transitive

4.1.2 Logarithms

Here we assume x and y are positive real numbers. 1. $loga(xy) = loga(x) + loga(y)$ 2. $loga(x/y) = loga(x) - loga(y)$ 3. $loga(x^r) = rloga(x)$ for any real number r. Invertible Functions

4.1.3 Proof by Induction

Another sequence is defined by the recurrence relation un = un-1+2n-1 and u1 = 1. (i) Calculate u2, u3, u4 and u5. 1,4,9,16,25

(ii) Prove by induction that $u_n = n^2$ for all $n \geq 1$

Exponentials

4.2 Laws of Exponents

Here are the Laws (explanations follow):

Law Example x1 = x 61 = 6 x0 = 1 70 = 1 x-1 = 1/x 4-1 = 1/4 xmxn = xm+n x2x3 = x2+3 = x5 xm/xn = xm-n x6/x2 = x6-2 = x4 (xm)n = xmn (x2)3 = x23 = x6 (xy)n = xnyn (xy)3 = x3y3 (x/y)n = xn/yn (x/y)2 = x2 / y2 x-n = 1/xn x-3 = 1/x3 Z Score

$$Z = \frac{X - \mu}{\sigma}$$

4.2.1 Exercises

Showing your workings, use the rules of indices and logarithms to give the following two expression in their simplest form.

- **Exercise 1**

$$4 \cdot 2^x - 2^{x+1}$$

- **Exercise 2**

$$\frac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$$

4.2.2 Exercise 1

$$4 \cdot 2^x - 2^{x+1}$$

Remarks:
(looking at the second term)

1 Using the following rule

$$a^b \cdot a^c = a^{(b+c)}$$

2 Using this rule in reverse we can say

$$2^{x+1} = 2^x \cdot 2^1 = 2 \cdot (2^x)$$

$$4 \cdot 2^x - 2^{x+1} \;=\; (4 \cdot 2^x) - (2 \cdot 2^x)$$

4.2.3 Exercise 1

Remarks:

3 This expression is in the form

$$(a \cdot b) - (c \cdot b)$$

which can be re-expressed as follows

$$(a - c\sqrt{b}) \cdot b$$

$$(4 \cdot 2^x) - (2 \cdot 2^x) = (4 - 2) \cdot 2^x$$
$$= 2 \cdot 2^x = 2^{x+1}$$

4.2.4 Exercise 2

$$\frac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$$

Useful Rule of Logarithms

$$\ln(a^b) = b \cdot \ln(a)$$

$$\frac{\ln(2) + 2 \cdot \ln(2) + 3 \cdot \ln(2) + 4 \cdot \ln(2) + 5 \cdot \ln(2)}{\ln(4)}$$

4.2.5 Exercise 2

Adding up all the terms in the numerator

$$\frac{1 \cdot \ln(2) + 2 \cdot \ln(2) + 3 \cdot \ln(2) + 4 \cdot \ln(2) + 5 \cdot \ln(2)}{\ln(4)}$$

$$= \frac{15 \cdot \ln(2)}{\ln(4)}$$

4.2.6 Exercise 2

Our expression has now simplified to

$$\frac{15 \cdot \ln(2)}{\ln(4)}$$

We can simplify the denominator too

$$\ln(4) = \ln(2^2) = 2 \cdot \ln(2)$$

4.2.7 Exercise 2

Our expression has now simplified to

$$\frac{15 \cdot \ln(2)}{\ln(4)} = \frac{15 \cdot \ln(2)}{2 \cdot \ln(2)}$$

We can divide above and below by $\ln(2)$ to get our final answer

$$\frac{15 \cdot \ln(2)}{2 \cdot \ln(2)} = \frac{15}{2} = 7.5$$

4.3 Tree Definition

What properties must a graph have in order for it to be a tree? (ii) Say, with reason, whether or not it is possible to construct a tree with degree sequence 4, 3, 3, 1, 1.

Dice Rolls

Consider rolls of a die. What is the universal set?

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$$

Worked Example

Suppose that the Universal Set $\mathcal{U}$ is the set of integers from 1 to 9.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and that the set $\mathcal{A}$ contains the prime numbers between 1 to 9 inclusive.

$$\mathcal{A} = \{1, 2, 3, 5, 7\},$$

and that the set $\mathcal{B}$ contains the even numbers between 1 to 9 inclusive.

$$\mathcal{B} = \{2, 4, 6, 8\}.$$

Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}'$ and $\mathcal{B}'$

$$\mathcal{A}' = \{4, 6, 8, 9\},$$
$$\mathcal{B}' = \{1, 3, 5, 7, 9\},$$

Intersection

- Intersection of two sets describes the elements that are members of both the specified Sets
- The intersection is denoted $\mathcal{A} \cap \mathcal{B}$

$$\mathcal{A} \cap \mathcal{B} = \{2\}$$

- only one element is a member of both A and B.

Set Difference

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

symbols

$\varnothing$, $\forall$, $\in$, $\notin$, $\cup$

Prepositional Logic

- $p \wedge q$
- $p \vee q$
- $p \rightarrow q$

4.4 Sequence and Series and Proof by Induction

$$\sum(n^2)$$

Relative Difference

- $A \otimes B$

Power Sets

- Consider the set A where $A = \{w, x, y, z\}$
- There are 4 elements in set A.
- The power set of A contains 16 element data sets.
- $$\mathcal{P}(A) = \{\{x\}, \{y\}\}$$
- (i.e. 1 null set, 4 single element sets, 6 two -elemnts sets, 4 three lement set and one 4- element set.)

- $p \to q$ p implies q
- $p \lg q$

Relative Difference

- $A \otimes B$

Power Sets

- Consider the set A where $A = \{w, x, y, z\}$
- There are 4 elements in set A.
- The power set of A contains 16 element data sets.
- $$\mathcal{P}(A) = \{\{x\}, \{y\}\}$$
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Worked Example

Suppose that the Universal Set $\mathcal{U}$ is the set of integers from 1 to 9.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and that the set $\mathcal{A}$ contains the prime numbers between 1 to 9 inclusive.

$$\mathcal{A} = \{1, 2, 3, 5, 7\},$$

and that the set $\mathcal{B}$ contains the even numbers between 1 to 9 inclusive.

$$\mathcal{B} = \{2, 4, 6, 8\}.$$

Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}'$ and $\mathcal{B}'$

$$\mathcal{A}' = \{4, 6, 8, 9\},$$
$$\mathcal{B}' = \{1, 3, 5, 7, 9\},$$

Intersection

- Intersection of two sets describes the elements that are members of both the specified Sets
- The intersection is denoted $\mathcal{A} \cap \mathcal{B}$

$$\mathcal{A} \cap \mathcal{B} = \{2\}$$

- only one element is a member of both A and B.

Set Difference

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

symbols

$\varnothing$, $\forall$, $\in$, $\notin$, $\cup$

Prepositional Logic

4.5 Section 3 Logic

4.5.1 Logical Operations

- $\neg p$ the negation of proposition p.
- $p \wedge q$ Both propositions p and q are simultaneously true (Logical State AND)
- $p \vee q$ One of the propositions is true, or both (Logical State : OR)
- $p \otimes q$ Only one of the propositions is true (Logical State : exclusive OR (i.e XOR)

p	q	$p \vee q$	$q \wedge p$	$p \otimes q$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

4.6 Conditional Connectives

Construct the truth table for the proposition $p \to q$.

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

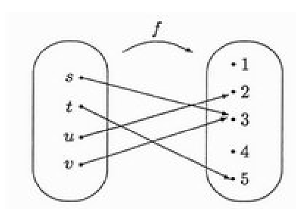
Question 1

(b) Express the following hexadecimal number as a decimal number: (A32.8)16. [3]

(c) Convert the following decimal number into base 2, showing all your working: $(253)_{10}$. [2]

(d) Express the recurring decimal 0.4242424 . . . as a rational number in its simplest form. [2]

Question 4



Question 6

Let $\mathcal{S}$ be a set and let $\mathcal{R}$ be a relation on $\mathcal{S}$ Explain what it means to say that $\mathcal{R}$ is

(i) reflexive

(ii) symmetrix

(iii) anti-symmetric

(iv) Transitive

Question 10

(a) Given the following adjacency matrices A and B where

(i) Say whether or not the graphs they represent are isomorphic. (ii) Calculate A2 and A4 and say what information each gives about the graph corresponding to A. [6] (b) (i) Write down the augmented matrix for the following system of equations.

$$2x + y - z = 2$$

$$x - y + z = 4$$

$$x + 2y + 2z = 10$$

(ii) Use Gaussian elimination to solve the system. [4]