Chapter 1

Session 1

Tutorial Sheet for Session 1

Part A: Number Systems - Binary Numbers

1. Express the following decimal numbers as binary numbers.

   i) $(73)_{10}$ ii) $(15)_{10}$ iii) $(22)_{10}$

   All three answers are among the following options.

   a) $(10110)_2$ b) $(1111)_2$ c) $(1001001)_2$ d) $(1000010)_2$

2. Express the following binary numbers as decimal numbers.

   a) $(101010)_2$ b) $(10101)_2$ c) $(111010)_2$ d) $(11010)_2$

3. Express the following binary numbers as decimal numbers.

   a) $(110.10101)_2$ b) $(101.0111)_2$ c) $(111.01)_2$ d) $(110.1101)_2$

4. Express the following decimal numbers as binary numbers.

   a) $(27.4375)_{10}$ b) $(5.625)_{10}$ c) $(13.125)_{10}$ d) $(11.1875)_{10}$

Part B: Number Systems - Binary Arithmetic

(See section 1.1.3 of the text)

1. Perform the following binary additions.

a) $(110101)_2 + (1010111)_2$
b) $(1010101)_2 + (101010)_2$
c) $(11001010)_2 + (10110101)_2$
d) $(1011001)_2 + (111010)_2$

2. Perform the following binary subtractions.

a) $(110101)_2$ - $(1010111)_2$
b) $(1010101)_2$ - $(101010)_2$
c) $(11001010)_2$ - $(10110101)_2$
d) $(1011001)_2$ - $(111010)_2$

3. Perform the following binary multiplications.

a) $(1001)_2 \times (1000)_2$
b) $(101)_2 \times (1101)_2$
c) $(111)_2 \times (1111)_2$
d) $(10000)_2 \times (11001)_2$

4. Perform the following binary multiplications.

5. Perform the following binary divisions.

Part 1 : Binary numbers

Question 1

Working in base 2 and showing all your workings, compute the following. $(10110)_2 \times (111)_2$

- Express the binary number $(1101.101)_2$ as a decimal, showing all your workings
- Express the decimal number $(3599)_{10}$ in base

(a) Express the following binary numbers as decimal numbers
  (i) 11011
  (ii) 100101

(b) Express the following decimal numbers as binary numbers
  (i) 6
  (ii) 15
  (iii) 37

(c) Perform the following binary additions
  (i) $1011 + 1111$
  (ii) $10101 + 10011$
  (iii) $1010 + 11010$

Part 1b : Hexadecimal numbers

(i) Calculate the decimal equivalent of the hexadecimal number $(A2F.D)_{16}$

(ii) Working in base 2, compute the following binary additions, showing all you workings

$$(1110)_2 + (11011)_2 + (1101)_2$$

(iv) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part E: Miscellaneous Questions

(i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation

(ii) From part (i), is x^8 a rational number?

Part A : Binary numbers

(a) Express the following binary numbers as decimal numbers

(i) 11011

(ii) 100101

(b) Express the following decimal numbers as binary numbers

(i) 6

(ii) 15

(iii) 37

(c) Perform the following binary additions

(i) $1011 + 1111$

(ii) $10101 + 10011$

(iii) $1010 + 11010$

Part A: Number Systems - Binary Numbers

(a) Express the following decimal numbers as binary numbers.

i) $(73)_{10}$ ii) $(15)_{10}$ iii) $(22)_{10}$

All three answers are among the following options.

a) $(10110)_2$ b) $(1111)_2$ c) $(1001001)_2$ d) $(1000010)_2$

(b) Express the following binary numbers as decimal numbers.

a) $(101010)_2$ b) $(10101)_2$ c) $(111010)_2$ d) $(11010)_2$

(c) Express the following binary numbers as decimal numbers.

a) $(110.10101)_2$ b) $(101.0111)_2$ c) $(111.01)_2$ d) $(110.1101)_2$

(d) Express the following decimal numbers as binary numbers.

a) $(27.4375)_{10}$ b) $(5.625)_{10}$ c) $(13.125)_{10}$ d) $(11.1875)_{10}$

Part B: Number Systems - Binary Arithmetic

(a) Perform the following binary additions.

a) $(110101)_2 + (1010111)_2$
b) $(1010101)_2 + (101010)_2$
c) $(11001010)_2 + (10110101)_2$
d) $(1011001)_2 + (111010)_2$

(b) Perform the following binary subtractions.

a) $(110101)_2$ - $(1010111)_2$
b) $(1010101)_2$ - $(101010)_2$
c) $(11001010)_2$ - $(10110101)_2$
d) $(1011001)_2$ - $(111010)_2$

(c) Perform the following binary multiplications.

a) $(1001)_2 \times (1000)_2$
b) $(101)_2 \times (1101)_2$
c) $(111)_2 \times (1111)_2$
d) $(10000)_2 \times (11001)_2$

(d) Perform the following binary multiplications.

i. Which of the following binary numbers is the result of this binary division: $(10)_2 \times (1101)_2$.

a) $(11010)_2$
b) $(11100)_2$
c) $(10101)_2$
d) $(11011)_2$

ii. Which of the following binary numbers is the result of this binary division: $(101010)_2 \times (111)_2$.

a) $(11000)_2$
b) $(11001)_2$
c) $(10101)_2$
d) $(11011)_2$

iii. Which of the following binary numbers is the result of this binary division: $(1001110)_2 \times (1101)_2$.

a) $(11000)_2$
b) $(11001)_2$
c) $(10101)_2$
d) $(11011)_2$

(e) Perform the following binary divisions.

i. Which of the following binary numbers is the result of this binary division: $(111001)_2 \div (10011)_2$.

a) $(10)_2$
b) $(11)_2$
c) $(100)_2$
d) $(101)_2$

ii. Which of the following binary numbers is the result of this binary division: $(101010)_2 \div (111)_2$.

a) $(11)_2$
b) $(100)_2$
c) $(101)_2$
d) $(110)_2$

iii. Which of the following binary numbers is the result of this binary division: $(1001110)_2 \div (1101)_2$.

a) $(100)_2$
b) $(110)_2$
c) $(111)_2$
d) $(1001)_2$

Part C: Number Bases - Hexadecimal

(a) Answer the following questions about the hexadecimal number systems

a) How many characters are used in the hexadecimal system?
b) What is highest hexadecimal number that can be written with two characters?
c) What is the equivalent number in decimal form?
d) What is the next highest hexadecimal number?

(b) Which of the following are not valid hexadecimal numbers?

a) 73 b) $A5G$ c) 11011 d) EEF

(c) Express the following decimal numbers as a hexadecimal number.

a) $(73)_{10}$ b) $(15)_{10}$ c) $(22)_{10}$ d) $(121)_{10}$

(d) Compute the following hexadecimal calculations.

a) $5D2 + A30$ b) $702 + ABA$ c) $101 + 111$ d) $210 + 2A1$

Part D : Base 5 and Base 8 numbers

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

(i) decimal number
(ii) hexadecimal number
(iii) binary number

(b) Perform the following binary additions

(i) $1011 + 1111$
(ii) $10101 + 10011$
(iii) $1010 + 11010$

Part C : Base 5 and Base 8 numbers

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

(i) decimal number
(ii) hexadecimal number
(iii) binary number

Part E: Natural, Rational and Real Numbers

(a) State which of the following sets the following numbers belong to.

1) 18	3) π	5) 17/4	7) $\sqrt{\pi}$
2) 8.2347...	4) 1.33333...	6) 4.25	8) $\sqrt{25}$

The possible answers are

a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$
d) Real Number $\mathbb{R}$

- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$
- $\mathbb{Z}$: integers $\{-3, -2, -1, 0, 1, 2, 3, \ldots\}$
  - (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means "numbers" in German.)
- $\mathbb{Q}$: rational numbers
- $\mathbb{R}$: real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
  - (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

Exercises Real and Rational Numbers

(i) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part F : Scientific and Floating Point Notation

- Abscissa
- Exponent (power)

With floating point notation, the abscisa must be between 0 and 1. It is similar to scientific notation differing only by the fact that, with scientific notaiton, the abscissa is between 1 and 10.

- Floating Point Notation
- $pi \approx 0.31415 \times 10^1$
- $pi \approx 3.1415 \times 10^0$

(a) Which of the following binary numbers is the result of this binary division: $(111001)_2 \div (10011)_2$.

a) $(10)_2$
b) $(11)_2$
c) $(100)_2$
d) $(101)_2$

(b) Which of the following binary numbers is the result of this binary division: $(101010)_2 \div (111)_2$.

a) $(11)_2$
b) $(100)_2$
c) $(101)_2$
d) $(110)_2$

(c) Which of the following binary numbers is the result of this binary division: $(1001110)_2 \div (1101)_2$.

a) $(100)_2$
b) $(110)_2$
c) $(111)_2$
d) $(1001)_2$

Part D: Natural, Rational and Real Numbers

- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$
- $\mathbb{Z}$: integers $\{-3, -2, -1, 0, 1, 2, 3, \ldots\}$
  - (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means "numbers" in German.)
- $\mathbb{Q}$: rational numbers
- $\mathbb{R}$: real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
  - (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

Question 1

Part B : Hexadecimal numbers

(i) Calculate the decimal equivalent of the hexadecimal number $(A2F.D)_{16}$

(ii) Working in base 2, compute the following binary additions, showing all you workings

$$(1110)_2 + (11011)_2 + (1101)_2$$

(iv) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

1.1 Questions

Question 1

(b) Express the following hexadecimal number as a decimal number: (A32.8)16. [3]

(c) Convert the following decimal number into base 2, showing all your working: $(253)_{10}$. [2]

(d) Express the recurring decimal $0.4242424\ldots$ as a rational number in its simplest form. [2]