



EST.MM

HIBERNIA COLLEGE DUBLIN

Computing



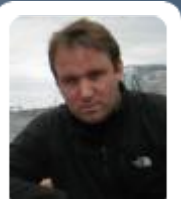
Tutor : Kevin O'Brien

Tutorial: Maths for Computing



Online Tutorial 4

Chapter 8 : Trees

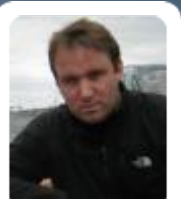


Trees

- A lot of concepts and definitions follows from Chapter 5 : Introduction to Graph Theory

Syllabus

- Properties of Trees
- Rooted Trees and Binary Trees
- Binary Search Trees



Trees : Properties of Trees

1) Characteristics of a Tree

A tree is a connected graph that contains no cycles. A tree has no loops and no multiple edges. All trees are simple graphs.

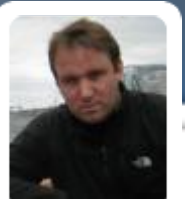
2) Path Graphs

A tree that contains only vertices of degree one or two is called a ***path graph***.

The length of a path graph is the number of edges in it.

3) Number of Edges

(Theorem 3.3) Let T be a tree with n vertices. Then T has $n - 1$ edges.

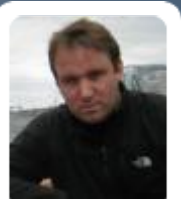


Trees : Properties of Trees

4) Spanning Subgraphs

The graph H is a **subgraph** of a graph G if H 's vertices are a subset of the G 's vertex, its edges are a subset of the edge set of G , and each edge of H has the same end-vertices in G .

H is called a **spanning subgraph** of G if the vertices of H are the **same** as the vertices of G .



Trees : Properties of Trees

5) Spanning Trees

If H is a spanning subgraph which is also a tree, then H is said to be a spanning tree of G . (G does not need to be a tree)



Trees : Properties of Trees 2008 Zone A Q9

Question 9

(a) A graph with 5 vertices: a, b, c, d, e has the following adjacency list:

$a : b, e$

$b : a, c, d$

$c : b, d$

$d : b, c, e$

$e : d, a.$

- (i) Draw this graph, G .
- (ii) Draw a spanning tree of G .
- (iii) Draw all the non-isomorphic spanning trees of G and call this set S .
- (iv) How many non-isomorphic trees can be created by adding a new vertex and edge to the trees in S . [6]



Trees : Properties of Trees 2008 Zone A Q9

$a : b, e$

$b : a, c, d$

$c : b, d$

$d : b, c, e$

$e : d, a.$



Trees : Properties of Trees 2008 Zone A Q9

- Part IV
- Examiner's Commentaries : Then it is a question of adding another vertex and edge to each of these in all possible places and finally eliminating the isomorphic ones to do part (iv).
- Simpler Exercise (2006 Q9)
 - (b) (i) Draw the 3 non-isomorphic trees on 5 vertices.
 - (ii) Draw, on a separate diagram, all the non-isomorphic trees on 6 vertices, by adding a vertex to copies of the trees you have drawn or otherwise. [6]



Trees : Properties of Trees 2006 Zone A Q9

- (b) (i) Draw the 3 non-isomorphic trees on 5 vertices.
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Trees : Properties of Trees

Question 7 (a) (i) What properties must a graph have in order for it to be a tree?

(ii) Say, with reason, whether or not it is possible to construct a tree with degree sequence $4, 3, 3, 1, 1$.

(iii) Say, with reason, whether it is possible to construct a tree with degree sequence $4, 3, 2, 2, 1$.

(iv) What properties must a graph have in order for it to be a binary tree?
[5]



Trees : Properties of Trees

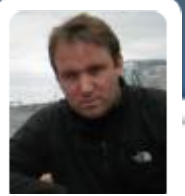


Trees : Properties of Trees (2005)

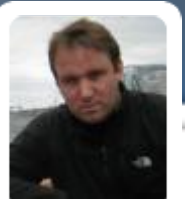
- (c) Let G be the simple graph with vertex set $V(G) = \{a, b, c, d, e\}$ and adjacency matrix

$$\mathbf{A} = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 1 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 0 & 1 & 0 & 1 & 0 \end{array}$$

- (i) What do the numbers on the leading diagonal of this matrix tell you about the graph?
- (ii) Say how the number of edges in G is related to the entries in the adjacency matrix \mathbf{A} and calculate this number.
- (iii) Draw G .
- (iv) Find a spanning tree T_1 for G and give its degree sequence.
- (v) Find a spanning tree T_2 for G which is **not** isomorphic to T_1 and give a reason why it is not isomorphic.



Trees : Properties of Trees



Trees : Properties of Trees



Trees : Rooted Trees and Binary Trees

Terminology (Page 37)

- Root
- Nodes
- Key
- Children and Parents
- Ancestors and Descendants
- Height



Binary Search Tree

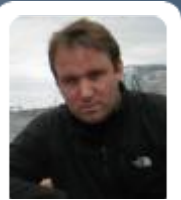
A Binary Search Tree is a binary tree in symmetric order

Symmetric order means that:

- every node has a key (or number)
- every node's key is
 - larger than all keys in its left subtree
 - smaller than all keys in its right subtree

The root r is the record

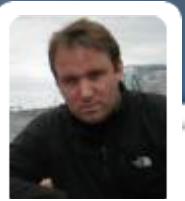
$$\# \lfloor (1 + N) / 2 \rfloor.$$



Binary Search Tree

If the first record in the **subtree** is **#a** and the last record is **#b**, then the **root** of the subtree is

$$\# \lfloor (a + b) / 2 \rfloor .$$



Trees : Binary Search Trees

The height h of a binary search tree with N records stored at internal nodes is

$$h = \lceil \log_2 (N + 1) \rceil .$$



Trees : Binary Search Trees (2011 Zone B)

- (a) i. Construct a binary search tree to store the following ordered list of 12 integers at its internal nodes.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

- ii. What is the maximum number of comparisons needed in order to find an existing integer in the tree?

[4]



Trees : Binary Search Trees



Trees : Binary Search Trees (2004)

- (b) A binary search tree is designed to store an ordered list of 3000 records at its internal nodes.
- (i) Find which record is stored at the root (level 0) of the tree and at each of the nodes at level 1.
 - (ii) What is the height of the tree?
 - (iii) What is the maximum number of comparisons needed in order to find an existing record in the tree? [5]



Trees : Binary Search Trees



Trees : Binary Search Trees

Question 9

- (a) A binary search tree is designed to store an ordered list of 10000 records numbered 1,2,3,...10000 at its internal nodes.
- (i) Draw levels 0, 1 and 2 of this tree showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.
 - (ii) What is the maximum number of comparisons that would have to be made in order to locate an existing record from the list of 10000? [4]



Trees : Binary Search Trees (2006)



Trees : Binary Search Trees

Proof. Since each vertex at level i has exactly two children at level $i + 1$ for all levels $i = 0 \leq i \leq h - 2$, it follows that the number of vertices at level $i + 1$ is exactly twice the number of vertices at level i .

The only vertex at level 0 is the root, so the number of vertices at level 0 is $1 = 2^0$, hence we have exactly 2^i vertices at level i for all i where $0 \leq i \leq h - 1$.



Trees : Binary Search Trees

Theorem 2.1 *Let n be a positive integer. Then*

$$(a) \sum_{r=1}^n 1 = n.$$

$$(b) \sum_{r=1}^n r = n(n+1)/2.$$

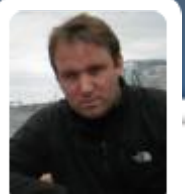
$$(c) \sum_{r=1}^n r^2 = n(n+1)(2n+1)/6.$$

$$(d) \sum_{r=0}^n x^r = \frac{x^{n+1}-1}{x-1}, \text{ for any } x \in \mathbb{R} \text{ with } x \neq 1.$$



Trees : Binary Search Trees

- (b) A binary search tree is designed to store an ordered list of 600 records at its internal nodes.
- i. Which record is stored at the root (at level 0) of the tree?
 - ii. Which records are stored at level 1 of the tree?
 - iii. Determine the number of records stored at level 9 of the tree.



Trees : Binary Search Trees