UNIVERSITY OF LONDON 0291 0102 ZB

BSc/Diploma Examination

for External Students

2011

Computing and Information Systems and Creative Computing

0291 0102 ZB Mathematics for Computing

Duration: 3 hours

Date and time: Thursday 5 May 2011 : 2.30 – 5.30 pm

There are ten questions in this paper. Candidates should answer ***all ten*** *questions. All questions carry equal marks and full marks can be obtained for complete answers to* ***ten*** *questions.*

Questions involving a description or explanation should, wherever possible, be accompanied by an appropriate example.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, texts or algebraic equations. The make and type of machine must be stated clearly on the front cover of the examination book.

THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM

Question 1

(a) Convert the decimal integer $(503)_{10}$ to binary notation. [2]

(b) Working in binary and showing all carries, compute $(110101)_2 + (1110)_2$. [2]

(c) i. List the set of positive integers with precisely 3 bits in binary notation.

ii. Let n be a positive integer. How many positive integers have precisely n bits in binary notation? [3]

(d) Showing your working, express the repeating decimal

$$0.021021021021\ldots$$

as a rational number in its simplest form. [3]

Question 2

(a) Let the two sets A and B be given by

$$A = \{2, \tfrac{1}{2}, \pi\} \qquad \text{and} \qquad B = \{x \in \mathbb{Q} : x \notin \mathbb{Z}\}.$$

Give each of the following sets by using the listing method.

i. $A \cap \mathbb{Q}$;

ii. $A \cap \mathbb{R}$;

iii. $A \cap B$;

iv. $A - B$. [3]

(b) Describe by using the rules of inclusion method the set of non-negative integers which have a remainder of 0 on division by 100. [2]

(c) Define a relation on the set of all pairs of integers $\mathbb{Z} \times \mathbb{Z}$ by

$$(a, b)\, R\, (x, y) \text{ if and only if } a \geq x \text{ and } b \geq y.$$

Justifying your answers, say whether the relation R on the set $\mathbb{Z} \times \mathbb{Z}$ is

i. reflexive;

ii. anti-symmetric;

iii. transitive;

iv. a partial order;

v. an order. [5]

Question 3

Let p, q and r be the following propositions concerning integers n.

$$\begin{array}{lcl} p & : & n \text{ is a multiple of two} \\ q & : & n < 10 \\ r & : & n \leq 10. \end{array}$$

(a) List the truth set of the compound proposition $\neg q \wedge p$. [2]

(b) Express each of the following statements using the propositions p, q and r and logical symbols.

i. n is an integer less than 10 which is even;

ii. n is an integer larger than 10 which is odd;

iii. $n = 10$. [3]

(c) i. Use truth tables to prove that

$$p \rightarrow (q \vee r) \equiv (\neg q \wedge \neg r) \rightarrow \neg p.$$

ii. Write in plain English the contrapositive of the statement

"If n is a positive integer and $n < 2$ then $n = 1$".

[5]

Question 4

(a) Given a real number x, say how $\lfloor x \rfloor$, the *floor* of x, is defined. [1]

(b) The function $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by the rule

$$f(x) = \lfloor x/10 \rfloor.$$

i. Find $f(-3)$ and $f(3)$.

ii. Justifying your answer, say whether f is one-to-one.

iii. Justifying your answer, say whether f is onto. [6]

(c) Explain what it means for a function to be $O(x)$. [1]

(d) Justifying your answer, say whether the function $f(x)$ from part (b) is $O(x)$. [2]

Question 5

A sequence is given by the recurrence relation

$$u_n = u_{n-1} + 2n \qquad \text{for } n \geq 2$$

and the initial term $u_1 = 1$.

(a) Showing your working, calculate u_2, u_3, u_4 and u_5. [2]

(b) Prove by induction that $u_n = n^2 + n - 1$ for all $n \geq 1$. [6]

(c) Showing all your working, compute

$$\sum_{n=1}^{100}(u_n - (n-1)^2).$$

[2]

Question 6

(a) Let G be a simple graph. How is the sum of the degrees of the vertices of G related to the number of edges of G? [2]

(b) Justifying your answer, say why it is not possible to construct a simple graph G with degree sequence

$$5, 1, 1, 1, 1.$$

[2]

(c) Justifying your answer, say why it is not possible to construct a simple graph G with degree sequence

$$4, 3, 2, 1, 0.$$

[2]

(d) Justifying your answer, say whether there exists a positive integer $n > 5$ for which it is possible to construct a simple graph G with degree sequence

$$n-1, n-2, \ldots, 3, 2, 1, 0.$$

[2]

(e) Justifying your answer, say whether it is possible to construct a simple graph with precisely 40 vertices and 800 edges. [2]

Question 7

(a) i. Construct a binary search tree to store the following ordered list of 12 integers at its internal nodes.

$$2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.$$

ii. What is the maximum number of comparisons needed in order to find an existing integer in the tree? [4]

(b) A binary search tree is designed to store an ordered list of 600 records at its internal nodes.

i. Which record is stored at the root (at level 0) of the tree?

ii. Which records are stored at level 1 of the tree?

iii. Determine the number of records stored at level 9 of the tree. [6]

Question 8

(a) Let G be the simple graph on the vertex set $V(G) = \{1, 2, 3, 4, 5\}$ with adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

i. Draw G.

ii. For each pair of distinct vertices of G, find the number of paths of length 2 between them.

iii. Compute the matrix A^3 and explain what the entries say about the graph G.

iv. For each pair of distinct vertices of G, find the number of paths of length 3 between them. [6]

(b) Let H be the graph with vertex set $V(H) = \{a, b, c, d, e\}$ and edge set $E(H) = \{ca, cb, cd, ce\}$.

i. What are the conditions for a function $f : V(H) \rightarrow V(G)$ to be an isomorphism between the graphs H and G?

ii. Justifying your answer, find an isomorphism between the graphs H and G. [4]

Question 9

The college refectory has 3 different starters, 4 main courses and 5 desserts on the lunch menu and offers a small, a medium and a large set lunch. A small lunch consists of just a main course, a medium lunch consists of either a starter and a main course or a main course together with a dessert. A large lunch consists of a starter, a main course and a dessert. One of the starters and two of the desserts contain nuts.

(a) Describe a sample space to model all possible set lunches. [2]

(b) Give the total number of possible set lunches. [2]

(c) You ask the dinner lady to serve you a random set lunch. What is the probability that

i. you get a large lunch?

ii. you do not get a dessert?

iii. your lunch does not contain any nuts? [6]

Question 10

(a) Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} 2y + 3z &= 2 \\ 2x + 3y + z &= 4 \\ 2x + y + 3z &= 2. \end{aligned}$$

[4]

(b) Given the matrices

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix},$$

i. Compute AB.

ii. Find a matrix X such $AX = B$.

iii. Find a matrix Y such that $AY = A$. [6]