Sigma Notation

The Greek letter $\sum$ (pronounced *sigma*), which means *the sum of*, is generally used to express a series in a concise way.

$$2+4+8+16+32 = 2+2^2+2^3+2^4+2^5 = \sum_{r=1}^{r=5} 2^r$$

Notice 2^r is the r^{th} term in the sequence. Since we are summing the terms from 1 to 5 inclusive, the least value of r is placed below the sigma sign and the greatest value of r is placed above.

A finite series always ends with the last term even if several middle terms are omitted.

$$\sum_{r=1}^{r=20} \frac{1}{2r+2} = \frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\ldots+\frac{1}{42}$$

An infinite series may also be written in sigma notation, where ∞ is used to indicate that there is no upper limit for r.

$$2+4+6+8+\ldots\ldots = \sum_{r=1}^{\infty} 2r$$

Example

Write the following series using $\sum$ notation:

1. $1+4+9+16+\ldots+81$
2. $1-x+x^2-x^3+x^4-\ldots$

$$1+4+9+16+\ldots+81 = 1+2^2+3^2+\ldots+9^2 = \sum_{r=1}^{r=9} r^2$$

In a series where the sign alternates from positive to negative, $(-1)^r$ may be used for the sign. If for example the counter r started at $r=1$ then $(-1)^r$ would result in even terms being positive and odd terms being negative whereas $(-1)^{r+1}$ would result in even terms being negative and odd terms being positive.

$$1-x+x^2-x^3+x^4-\ldots = \sum_{r=0}^{\infty}(-1)^r x^r$$