MA4016 Problem Sheet 9 (Languages)

1. For $\mathbf{\Sigma} = \{0,\ 1\}$ determine whether the string 00010 is in each of the following languages ($\subset \mathbf{\Sigma}^*$).

$$(a)\ \ \{0,\ 1\}^* \qquad (b)\ \ \{000,\ 101\}\{10,\ 11\} \qquad (c)\ \ \{00\}\{0\}^*\{10\}$$
$$(d)\ \ \{000\}^*\{1\}^*\{0\} \qquad (e)\ \ \{00\}^*\{10\}^* \qquad (f)\ \ \{0\}^*\{1\}^*\{0\}^*$$

2. For $\mathbf{\Sigma} = \{x,\ y,\ z\}$, let $A, B \subset \mathbf{\Sigma}^*$ be given by $A = \{xy\}$ and $B = \{\lambda,\ x\}$. Determine $AB,\ BA,\ B^3,\ B^+,\ A^*$.

3. If $A(\neq \emptyset)$ is a language and $A^2 = A$, prove that $A = A^*$.

4. Let $\mathbf{\Sigma} = \{0,\ 1\}$. $A \subset \mathbf{\Sigma}^*$ is recursively defined by

   (i) $0,\ 1 \in A$.

   (ii) $\forall x \in A, \qquad 0x1 \in A$.

   Reword this definition in terms of a phrase-structure grammar $G = (\mathbf{\Sigma}, \mathrm{T}, \mathrm{S}, \mathrm{P})$.
   Is $0001111 \in A$? Is 00001111 ? Why ?

5. Provide a recursive definition for each of the following languages $A \subset \mathbf{\Sigma}^*$ over $\mathbf{\Sigma} = \{0,\ 1\}$.

   (a) $x \in A$ if and only if the number of 0's in x is even.

   (b) $x \in A$ if and only if there is only one 0 in x.

   (c) $x \in A$ if and only if all of the 0's in x precede all of the 1's.

   Reword these definitions in terms of grammars G.

6. Let $G = (\mathbf{\Sigma}, \mathrm{T}, \mathrm{S}, \mathrm{P})$ where $\mathbf{\Sigma} = \{\mathrm{S}, \mathrm{A}, \mathrm{B}, \mathrm{a}, \mathrm{b}\}$ and $\mathrm{T} = \{\mathrm{a}, \mathrm{b}\}$. Determine the type classification(s) of G when P is the set of productions given by:

   (a) S $\rightarrow$ aAB, A $\rightarrow$ Bb, B $\rightarrow$ λ.

   (b) S $\rightarrow$ aA, A $\rightarrow$ a, A $\rightarrow$ b.

   (c) S $\rightarrow$ ABa, AB $\rightarrow$ a.

   (d) S $\rightarrow$ ABA, A $\rightarrow$ aB, B $\rightarrow$ ab.

   (e) S $\rightarrow$ bA, A $\rightarrow$ B, B $\rightarrow$ a.

   (f) S $\rightarrow$ aA, aA $\rightarrow$ B, B $\rightarrow$ aA, A $\rightarrow$ b.

   (g) S $\rightarrow$ bA, A $\rightarrow$ b, S $\rightarrow$ λ.

   (h) S $\rightarrow$ AB, B $\rightarrow$ aAb, aAb $\rightarrow$ b.

   (i) S $\rightarrow$ aA, A $\rightarrow$ bB, B $\rightarrow$ b, B $\rightarrow$ λ.

   (j) S $\rightarrow$ A, A $\rightarrow$ B, B $\rightarrow$ λ.