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MA4016 - Engineering Mathematics 6

Problem Sheet 1: Proofs (February 04, 2010)

1. Prove that if m and n are integers and mn is even, then m is even or n is even.

2. Show that at least 3 days of any 25 days of a year chosen must fall in the same month of the year.

3. Prove that there is a positive integer that equals the sum of the positive integers not exceeding it.

4. Prove or disprove that if you have an 8-gallon jug of water and two empty jugs with capacities of 5 gallons and 3 gallons, respectively, then you can measure 4 gallons by successively pouring some or all of the water in a jug into another jug.

5. Prove for every positive integer n

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

6. Find a formula for

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

and prove the formula you conjectured.

7. Prove for every positive integer n and real number $x \geq -1$

$$(1+x)^n \geq 1 + nx.$$

8. For which non-negative integers n is $n^2 \leq n!$? Prove your answer.

9. Prove that 3 divides $n^3 + 2n$ whenever n is a positive integer with a direct proof and with mathematical induction.