Question 1

Part A : Binary numbers

(a) Express the following binary numbers as decimal numbers

(i) 11011

(ii) 100101

(b) Express the following decimal numbers as binary numbers

(i) 6

(ii) 15

(iii) 37

(c) Perform the following binary additions

(i) $1011 + 1111$

(ii) $10101 + 10011$

(iii) $1010 + 11010$

Part B : Hexadecimal numbers

(i) Calculate the decimal equivalent of the hexadecimal number $(A2F.D)_{16}$

(ii) Working in base 2, compute the following binary additions, showing all you workings

$$(1110)_2 + (11011)_2 + (1101)_2$$

(iv) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part C : Base 5 and Base 8 numbers

(a) Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:

(i) decimal number

(ii) hexadecimal number

(iii) binary number

Part D : Real and Rational Numbers

(i) Express the recurring decimal $0.727272\ldots$ as a rational number in its simplest form.

Part E : Real and Rational Numbers

(i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation.

(ii) From part (i), is x^8 a rational number?

Question 2

Part A : Builder Method

The following sets have been defined using the **Building Method** of notation. Re-write them by listing **some** of the elements.

1. $\{p|p$ is a capital city, p is in Europe$\}$
2. $\{x|x = 2n - 5$, x and n are natural numbers$\}$
3. $\{y|2y^2 = 50$, y is an integer$\}$
4. $\{z|z = n^3$, z and n are natural numbers$\}$

Part B : Sets

U = natural numbers; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. State whether each of the following is true or false:

(i) $A \subset U$

(ii) $B \subseteq A$

(iii) $\emptyset \subset U$

Question 3

Part A : Propositions

Let p, q be the following propositions:

- p : this apple is red,
- q : this apple is ripe.

Express the following statements in words as simply as you can:

(i) $p \rightarrow q$

(ii) $p \wedge \neg q$.

Express the following statements symbolically:

(iii) This apple is neither red nor ripe.

(iv) If this apple is not red it is not ripe.

Part B : Logical Operations

Let $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p and q be the following propostions concerning the integers n.

p n is event

q $n < 5$

Find the values of n for which each of the following compound statement is true,

(i) $\neg p$

(ii) $p \wedge q$

(iii) $\neg p \vee q$

(iv) $p \oplus q$

Question 4

Part A : Functions

Given a real number x, say how the floor of x $\lfloor x \rfloor$ is defined.

(i) Find the values of $\lfloor 2.97 \rfloor$ and $\lfloor -2.97 \rfloor$.

(ii) Find an example of a real number x such that $\lfloor 2x \rfloor \neq 2\lfloor x \rfloor$, justifying your answer.

Part B : Logarithms

Evaluate the following expression.

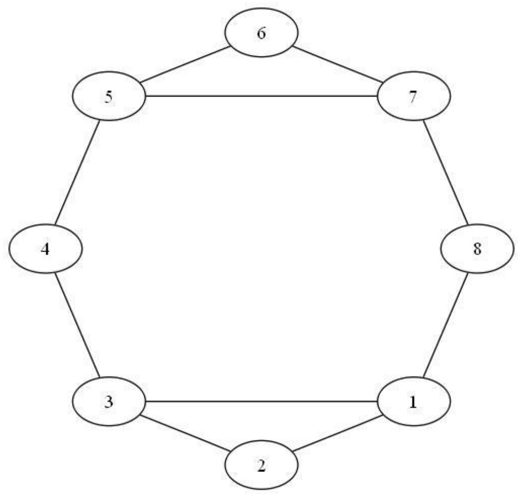
$$\mathrm{Log}_4 64 + \mathrm{Log}_5 625 + \mathrm{Log}_9 3$$

Question 5

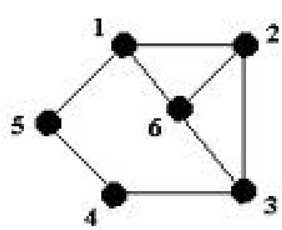
1. Draw two non-isomorphic graphs with the following degree sequence.

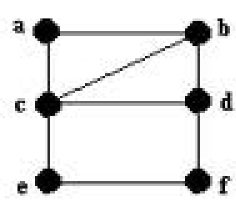
$$4, 3, 3, 2, 2, 2, 2, 1, 1$$

2. Write out the degree sequence of the following graph.



3. State the vertices that comprise a cycle of length 5 in both of the following graphs.





Question 6

Part A : Digraphs

Suppose $A = \{1, 2, 3, 4\}$. Consider the following relation in A

$$\{(1,1),(2,2),(2,3),(3,2),(4,2),(4,4)\}$$

Draw the direct graph of A. Based on the Digraph of A discuss whether or not a relation that could be depicted by the digraph could be described as the following, justifying your answer.

(i) Symmetric

(ii) Reflexive

(iii) Transitive

(iv) Antisymmetric

Part B : Relations

Determine which of the following relations xRy are reflexive, transitive, symmetric, or antisymmetric on the following - there may be more than one characteristic. if

(i) $x = y$

(ii) $x < y$

(iii) $x^2 = y^2$

(iv) $x \geq y$

Part C : Partial Orders

Let $A = \{0, 1, 2\}$ and $R = \{(0,0),(0,1),(0,2),(1,1),(1,2),(2,2)\}$ and $S = \{(0,0),(1,1),(2,2)\}$ be 2 relations on A. Show that

(i) R is a partial order relation.

(ii) S is an equivalence relation.

Question 7

Part A : Recurrence Relations

A sequence is defined by the recurrence relations

$$x_{n+2} = 3x_{n+1} - 2x_n$$

with initial terms $x_1 = 1$ and $x_2 = 3$.

(i) Calculate x_3, x_4 and x_5, showing your workings.

(ii) Prove by induction that $x_n = 2^n - 1$ for all $n \geq 1$

Part B : Summations

Compute the following summation

$$\sum_{i=25}^{i=100} i^2 + 3i - 5)$$

Question 8

Part A : Spanning Trees

1. How many edges are in the spanning tree T ?
2. What is the sum of the degree sequence of T?
3. Write down all the possible degree sequences for the spanning tree T.

Part B : Binary Search Trees

Suppose a database, comprised of 30,000 internal nodes, is structured as a Binary Search Tree.

1. What is the key (number) of the Root node?
2. What are the keys of the nodes at level 1?
3. For the nodes at level1, how many subtrees are there?
4. State which nodes are in the substrees of the level 1 nodes?
5. How many nodes are the between the root (level 0) and level 7.] (Hint: use a summation theorem mentioned in session 7
6. What is the maximum number of searchs in this database?